

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = v_{x0}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = t(v_{x0} + \frac{1}{2}a_x t)$$

$$x - x_0 = t(\frac{1}{2}(v_{x0} + a_x t) + \frac{1}{2}v_{x0})$$

$$\text{Since } v_x = v_{x0} + a_x t, \frac{1}{2}v_x = \frac{1}{2}v_{x0} + \frac{1}{2}a_x t, \text{ so } x - x_0 = t(\frac{1}{2}v_x + \frac{1}{2}v_{x0})$$

$$x - x_0 = \frac{1}{2}t(v_x + v_{x0})$$

$$\frac{2(x-x_0)}{t} = v_x + v_{x0}$$

$$v_x = \frac{2(x-x_0)}{t} - v_{x0}$$

$$\text{Since } a_x = \frac{\Delta v_x}{t} = \frac{v_x - v_{x0}}{t}, t = \frac{v_x - v_{x0}}{a}, \text{ so } v_x = \frac{2a(x-x_0)}{v_x - v_{x0}} - v_{x0}$$

$$v_x = \frac{2a(x-x_0) - v_x v_{x0} + v_{x0}^2}{v_x - v_{x0}}$$

$$v_x^2 - v_x v_{x0} = 2a(x - x_0) - v_x v_{x0} + v_{x0}^2$$

$$v_x^2 = v_{x0}^2 + 2a(x - x_0) \text{ Voila, the third equation appears.}$$

Brought to you by the lead developer of the jonkler67 website!