GRAVITATION MINDMAP

Kepler's Laws and Newton's Law of Gravitation :→

*** Kepler's 1st Law →**

All planets revolve around the sun in elliptical orbits and the sun lies at one of the focii of the orbit.

*** Kepler's 2nd Law →**

Planets cover equal area in equal time intervals.

Areal velocity → $\boxed{\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m_p}}$

*** Kepler's 3rd Law →**

$\boxed{T^2 \propto r^3}$

*** Newton's Law of Gravitation →**

$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ $G = 6.67 \times 10^{-11} Nm^2/Kg^2$

Acceleration Due to Gravity :→

* Acc.ⁿ due to Gravity = $\boxed{g_p = \frac{GM_p}{R_p^2}}$

For Earth, $g_e = 9.8\, m/s^2$

*** Effects on acceler.ⁿ due to Gravity :→**

(a) Effect of Altitude :-

$$\boxed{g' = \frac{GM}{(R+h)^2} = \frac{gR^2}{(R+h)^2}}$$

For small heights :-

$$\boxed{g' = g\left[1 - \frac{2h}{R}\right]}$$

(b) Effect of Depth :-

$$\boxed{g'' = \frac{GM(R-d)}{R^3}} = g\left[1 - \frac{d}{R}\right]$$

(c) Effect of Rotation : $\boxed{g_{eff} = g - \omega^2 R\cos^2\lambda}$
(λ = Latitude)

(At poles – $g' = g$)

(At equator – $g' = g - \omega^2 R$)

Gravitational Potential :→

• Due to point mass : $\boxed{V = -\frac{GM}{r}}$

• Due to ring : $V_{centre} = -\frac{GM}{R}$, $V_{axis} = \frac{-GM}{\sqrt{R^2 + x^2}}$

• Due to shell : $V_{in} = \frac{-GM}{R}$, $V_{out} = \frac{-GM}{r}$

• Due to solid sphere –

$V_{in} = \frac{-GM}{2R^3}(3R^2 - r^2)$, $V_{out} = \frac{-GM}{r}$, $V_{surface} = \frac{-GM}{R}$

• Relation b/w V and $\vec{I}$: $V(\vec{r}) = -\int \vec{I} \cdot d\vec{r}$

$\rightarrow \vec{I} = -\frac{\delta V}{\delta x}\hat{i} - \frac{\delta V}{\delta y}\hat{j} - \frac{\delta V}{\delta z}\hat{k}$

Gravitational Potential Energy :→

• Two point mass-system :

$$\boxed{U = -\frac{GM_1M_2}{r}}$$

• Self potential energy of shell :

$$U_{shell} = -\frac{GM^2}{2R}$$

• Self potential energy of solid sphere :

$$U_{sphere} = -\frac{3GM^2}{5R}$$

• Gravitational potential energy of Earth Body System :

For large height

$$\Delta U = -Gm_1m_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

When $h << R$

$$\Delta U = mgh$$

Escape Velocity and Satellites →

- Escape Velocity → $\boxed{v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}}$
- Geostationery Satellite – appear to be Stationary bcoz it moves with same angular velocity as that of earth with time period of 24 hours.
- Polar Satellite – Orbital plane coincides with axis of rotation of Earth.
- Orbital Velocity – $\boxed{v_0 = \frac{v_e}{\sqrt{2}} = \sqrt{gR}}$
- Time period – $\boxed{T = 2\pi\sqrt{\frac{R}{g}} \text{ If, } h << R}$
- Energy → $K.E. = \frac{GMm}{2r}$

  $P.E. = -\frac{GMm}{r}$

Gravitational Field (I) →

- Due to a point mass –

  $\boxed{|\vec{I_g}| = \frac{\vec{F}}{m_0} = \frac{GM}{r^2}}$
- Due to a ring –

  $\boxed{I_{centre} = 0 \quad I_{axis} = \frac{Gmx}{(a^2+x^2)^{3/2}}}$
- Due to a solid sphere –

  $\boxed{I_{centre} = 0 \quad I_{in.} = \frac{GMr}{R^3} \quad I_{out} = \frac{GM}{r^2}}$
- Due to a spherical shell –

  $\boxed{I_{in.} = 0 \quad I_{out} = \frac{GM}{r^2}}$
- Due to a disc –

  $\boxed{I_{axis} = \frac{GM}{R^2}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]}$