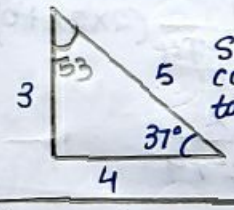
BASIC MATHEMATICS

Trigonometry

θ	Sin θ	Cos θ	tan θ
0°	0	1	0
30°	1/2	√3/2	1/√3
45°	1/√2	1/√2	1
60°	√3/2	1/2	√3
90°	1	0	Not defined

$\sin = \frac{1}{\text{cosec}}$, $\cos = \frac{1}{\text{sec}}$, $\tan = \frac{1}{\text{cot}}$, $\frac{\sin}{\cos} = \tan$



Sin 37° = 3/5
cos 37° = 4/5
tan 37° = 3/4

Sin 53° = 4/5
cos 53° = 3/5
tan 58° = 4/3

Angle in terms of Radian →

$\theta = \frac{l}{r}$ ↓ in radians.

Radians	Degree
2π	360°
π	180°
π/2	90°
π/3	60°
π/4	45°
π/6	30°

Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta + \tan^2\theta = 1$

$\text{cosec}^2\theta + \cot^2\theta = 1$

Double Angle Formulae

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta, 2\cos^2\theta - 1, 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Addition Formulae

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Four Quadrants and Sign Conventions

II	I
sin, cosec (+ve)	all (+ve)
III	IV
tan, cot (+ve)	cos, sec (+ve)

Add Sugar to Coffee.

(90 − θ) → 1st Quad	(270 − θ) → III rd Quad
$\sin(90-\theta) = \cos\theta$	$\sin(270-\theta) = -\cos\theta$
$\cos(90-\theta) = \sin\theta$	$\cos(270-\theta) = -\sin\theta$
$\tan(90-\theta) = \cot\theta$	$\tan(270-\theta) = +\cot\theta$
(90 + θ) → II nd Quad	(270 + θ) → IV th Quad.
$\sin(90+\theta) = +\cos\theta$	$\sin(270+\theta) = -\cos\theta$
$\cos(90+\theta) = -\sin\theta$	$\cos(270+\theta) = +\sin\theta$
$\tan(90+\theta) = -\cot\theta$	$\tan(270+\theta) = -\cot\theta$
(180 − θ) → II nd Quad.	(360 − θ) → IV th Quad.
$\sin(180-\theta) = \sin\theta$	$\sin(360-\theta) = -\sin\theta$
$\cos(180-\theta) = -\cos\theta$	$\cos(360-\theta) = \cos\theta$
$\tan(180-\theta) = -\tan\theta$	$\tan(360-\theta) = -\tan\theta$
(180 + θ) → III rd Quad.	(360 + θ) → I st Quad
$\sin(180+\theta) = -\sin\theta$	$\sin(360+\theta) = \sin\theta$
$\cos(180+\theta) = -\cos\theta$	$\cos(360+\theta) = \cos\theta$
$\tan(180+\theta) = \tan\theta$	$\tan(360+\theta) = \tan\theta$

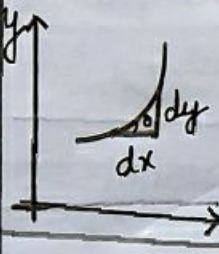
Trigonometric function of angle (2nπ + θ) where, n = 1, 2, 3, 4... will remain same

$\sin(2\pi+\theta) = \sin\theta$	$\cos(10\pi+\theta) = \cos\theta$
$\cos(2\pi+\theta) = \cos\theta$	$\sin(100\pi+\theta) = \sin\theta$
$\tan(2\pi+\theta) = \tan\theta$	

If θ is very small →

$\sin\theta = \theta$, $\cos\theta = 1$, $\tan\theta = \theta$

Differentiation



$\frac{dy}{dx}$ = Instantaneous rate of change of y w.r.t. x.
= Slope of y v/s x
= tanθ

Rules for Differentiation →

① $y = C \Rightarrow \frac{dy}{dx} = 0$

② $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$

Examples →

① $y = 10 = \frac{dy}{dx} = 0$

② $y = x = \frac{dy}{dx} = 1x^{1-1} = 1$

③ $y = x^5 = \frac{dy}{dx} = 5x^4$

④ $y = \frac{1}{x} = \frac{dy}{dx} = -\frac{1}{x^2}$

⑤ $y = \frac{1}{x^9} = \frac{dy}{dx} = -\frac{9}{x^{10}}$

⑥ $y = \sqrt{x} = \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

@sashaistic_

Derivative of Trigonometric Functions

① $y = \sin x = \frac{dy}{dx} = \cos x$

② $y = \cos x = \frac{dy}{dx} = -\sin x$

③ $y = \tan x = \frac{dy}{dx} = \sec^2 x$

④ $y = \sec x = \frac{dy}{dx} = \sec x \tan x$

⑤ $y = \cot x = \frac{dy}{dx} = -\operatorname{cosec}^2 x$

⑥ $y = \operatorname{cosec} x = \frac{dy}{dx} = -\operatorname{cosec} x \cot x$

#NEET SLAYER

Derivative of Log and Exponentials →

① $y = \log_e x = \frac{dy}{dx} = \frac{1}{x}$

② $y = e^x = \frac{dy}{dx} = e^x$

③ $y = a^x = \frac{dy}{dx} = a^x \log_e a$.

example →
$y = 10^x = \frac{dy}{dx} = 10^x \log_e 10$

Addition Rule

$y = x + x^4 + \frac{1}{x} + 2\sqrt{x} + \cos x - 3e^x$

$\frac{dy}{dx} = 1 + 4x^3 - \frac{1}{x^2} + 2 \times \frac{1}{2\sqrt{x}} - \sin x - 3e^x$

UV Method

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Example →
$y = x^u \log_e^v x$

$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log_e x (1)$

$= 1 + \log_e x$

$\frac{u}{v}$ method

$y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Example →
$y = \frac{x}{\sin x}^{u}_{v}$

$\frac{dy}{dx} = \frac{\sin x(1) - x(\cos x)}{\sin^2 x}$

Chain Rule (Assumption, Maan lo)

① $y = \sin(x^2)$

$\frac{dy}{dx} = \cos(x^2) 2x$

$= 2x \cos(x^2)$

② $y = \log_e(2x^2+4)$

$\frac{dy}{dx} = \frac{1}{2x^2+4}(2x2x+0)$

$= \frac{4x}{2x^2+4}$

Double Differentiate

$y = \sin x$

$\frac{dy}{dx} = y' = \cos x$

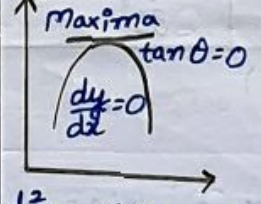
$\frac{d^2y}{dx^2} = y'' = -\sin x$

$y = x^3$

$\frac{dy}{dx} = y' = 3x^2$

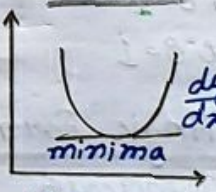
$\frac{d^2y}{dx^2} = y'' = 3 \times 2x = 6x$

Maxima



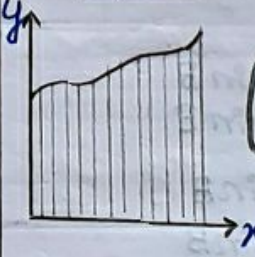
$\frac{d^2y}{dx^2} < 0$ (-ve)

Minima



$\frac{d^2y}{dx^2} > 0$ (+ve)

Basic Integration →



$\int y\,dx$ = Area Under Curve

Basic Formulae

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (not valid for → $n = -1$)

Examples →

① $\int dx = \int x^0 dx = \frac{x^{0+1}}{0+1} + C = \frac{x}{1} + C = x + C$

② $\int x\,dx = \frac{x^2}{2} + C$

③ $\int x^2 dx = \frac{x^3}{3} + C$

④ $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C$

For $n = -1$

$= \frac{-1}{x} + C$

$\int \frac{1}{x} dx = \log_e x + C$

$\int 5\,dx = 5\int dx = 5x + C$

$\int 10x\,dx = 10\int x\,dx = 10\frac{x^2}{2} + C = 5x^2 + C$

$\int e^x dx = e^x + C$

$\int \log_e x\,dx = x\log_e x - x$

Trigonometric Functions →

① $\int \sin x\,dx = -\cos x + C$

② $\int \cos x\,dx = \sin x + C$

③ $\int \sec^2 x\,dx = \tan x + C$

④ $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

ax + b types (x Ki jagah, ax+b)

① $\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + C$

② $\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + C$

③ $\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + C$

④ $\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{\cot(ax+b)}{a} + C$

⑤ $\int \frac{dx}{ax+b} = \frac{\log_e(ax+b)}{a} + C$ $\left[\int \frac{1}{x} dx = \log_e x\right]$

⑥ $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$ $\left[\int e^x dx = e^x + C\right]$

Definite Integral →

① $\int_2^4 x\,dx = \left[\frac{x^2}{2}\right]_2^4 = \left[\frac{4^2}{2} - \frac{2^2}{2}\right] = 6$

② $\int_1^2 x^3 dx = \left[\frac{x^4}{4}\right]_1^2 = \left[\frac{2^4}{4} - \frac{1^4}{4}\right] = \frac{15}{4}$

③ $\int_0^{\pi} \sin\theta = [-\cos\theta]_0^{\pi} = -[\cos\theta]_0^{\pi} = -[\cos\pi - \cos 0]$

$e^0 = 1,\ e^{-\infty} = 0,\ e^{\infty} = \infty$ ★

VECTORS

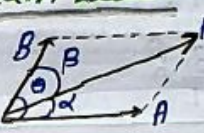
Vectors → $\vec{A} = |A|\hat{A}$ → Direction (Unit Vector)

↓ magnitude

Addition of Vector → (PYQ)

→ Parallelogram Law:—

$\vec{A} + \vec{B} = \vec{R}$



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$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ★

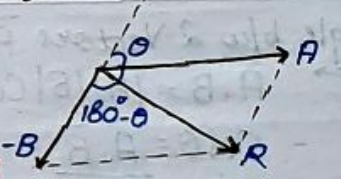
$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$, $\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$

Subtraction Of Vector →

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$\vec{R} = \vec{A} + (-\vec{B})$

$|R| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$ ★



$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}| = 90°$ ★

$\theta = 120° \rightarrow \vec{A} + \vec{B} = \vec{R}$ (all are unit Vectors)

$\vec{A} - \vec{B}$ = Difference = $\sqrt{3}$

Multiplication of Vectors

Dot product → Scalar →

$\vec{A}.\vec{B} = |A||B|\cos\theta$

↓ smaller angle

$A \perp B \rightarrow A.B = 0$

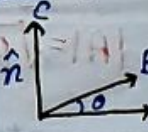
$\vec{A}.\vec{B} = \vec{B}.\vec{A}$ ✓

$A.A = A^2 = |A|^2$

Cross Product → Vector

$\vec{A} \times \vec{B} = \vec{C}$ (screw rule)

$\vec{C} \perp$ to plane containing both $\vec{A}$ and $\vec{B}$.



$\vec{C} = A \times B = |A||B|\sin\theta\,\hat{n}$ ★

magnitude dir

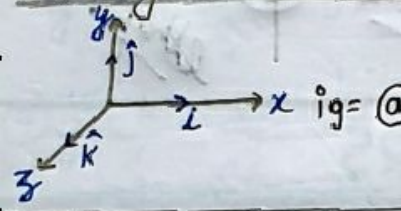
$A \parallel B \rightarrow \vec{A} \times \vec{B} = 0$

$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ ✗

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{A} \times \vec{A} = 0$

Orthogonal Unit Vector ($\hat{i}, \hat{j}, \hat{k}$)



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Que) Find Unit Vector, in direction of $\vec{A} = 6\hat{i}, 2\hat{j}, 3\hat{k}$

Sol → $\hat{A} = \frac{\vec{A}}{|A|} = \frac{6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{6^2+2^2+3^2}} = \frac{6\hat{i}+2\hat{j}+3\hat{k}}{7}$

$\hat{A} = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$

Que) $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$
$\vec{B} = 2\hat{i} + \hat{j} - 3\hat{k}$] dot me simple mul.]

Sol → $\vec{A} + \vec{B} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{A} - \vec{B} = \hat{i} + \hat{j} + 4\hat{k}$

$\vec{A} \cdot \vec{B} = 6\hat{i} + 2\hat{j} - 3\hat{k} = 5$

→ angle b/w 2 Vectors $\vec{A}$ and $\vec{B}$.

Sol → $\vec{A} \cdot \vec{B} = |A||B|\cos\theta$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$

Cross Product ✗

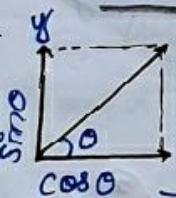
$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 2 & 1 & -3 \end{vmatrix}$ Jisko le rahe ho, usko Chupa do

$= \hat{i} = (2 \times -3) - (1 \times 1) = -7\hat{i}$

$+ \hat{j} = (3 \times -3) - 1 \times 2 = -11\hat{j}$ $= -7\hat{i} - 11\hat{j} + 1\hat{k}$

$+ \hat{k} = 2 \times 2 - 3 \times 1 = 1\hat{k}$

Resolution Of Vectors →



$|A| = \sqrt{Ax^2 + Ay^2}$

→ Components

UNIT AND DIMENSION

Fundamental Quantities →

1. Length → Meter → m
2. Mass → Kilogram → Kg
3. Time → Second → s
4. Temperature → Kelvin → K
5. Current → Ampere → A
6. Luminous Intensity → Candela → cd
7. Amount of Substance → Mole → mol

Sakshi

plane angle (2D) → □ △ ○

$\theta = \frac{\text{arc}}{\text{radius}}$ [Radian]

Solid angle (3D) → □ ◁

Solid angle = $\frac{\text{Area}}{(\text{Radius})^2}$ [steredian]

Principle Of Homogenity →

Only physical quantities having same dimensions can be added or subtracted.

Conversion Of Units → like♥

nu → Constant

n = numerical Value $n \propto \frac{1}{u}$

u = unit

Derived Quantities → Derived from fundamental quantities.

e.g. - Work, power, force

Dimensionless Quantity →

Angles, Trigonometric functions (sin, cos), Exponents ($e^x = M^0 L^0 T^0$), Logarithm function, Solid angle, plane angle

Applications of Dimensional Analysis →

→ Checking correctness of formulae.
→ Conversion of Units.
→ Derivation of formula

Dimension → Power to which, fundamental quantity is raised to express a quantity

Some Important Dimensions →

→ Coefficient of Viscosity = $ML^{-1}T^{-1}$
→ Electric field = $MLT^{-3}I^{-1}$
→ Resistance = $ML^2T^{-3}I^{-2}$
→ Impulse = MLT^{-1} (PYQ)
→ Permeability = $MLT^{-2}I^{-2}$
→ Permitivity = $M^{-1}L^{-3}T^4I^2$
→ Stefan's Constant = $ML^0T^{-3}\theta^{-4}$

NEET SLAYER..
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Errors (PYQ)

Errors → Systematic Error, Random Error, Least Count Error, Personal Error

① Absolute Error (Δa) → True Value - Measured Value.

② Mean absolute Error →

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 ---- \Delta a_n}{n}$$

③ Relative Error →

$$\delta a = \frac{\Delta a}{a} \quad a \rightarrow \text{true Value}$$

④ Percentage Error →

$$\delta a_x = \frac{\Delta a}{a} \times 100$$

→ Error always gets added.

★ For (+ve) (-ve) →

$6 \pm 0.1 + 3 \pm 0.1 = 9 \pm 0.2$
$6 \pm 0.1 - 3 \pm 0.1 = 3 \pm 0.2$

★ For (×) (÷) →

$x = A \cdot B$

$$\left(\frac{\Delta x}{x}\right) = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Relative error

Significant Figures → (PYQ)

→ All non zero → Significant.
→ 0 between two non zero → Significant.
e.g. → 401 = S.F = 3.
→ All 0, left to non zero digits, in decimal no. → Non Significant
e.g → 0.002 = S.F. = 1
→ Tailing 0 in decimal no. → Significant.
e.g. → 0.00200 = SF = 3
→ Tailing 0, without decimal → Non-Significant
e.g. → 2300 = SF = 2
→ Exponents = Non Significant.

Significant Digits →

(+ve) (-ve) → Minimum decimal place wala answer.

(×) (÷) → Minimum significant figure wala answer.

Rounding Off → (★)

>5 → ahge wala digit
<5 → utna hi rakho, baaki no. gayab.
5 → 51, 52, 53... 59 dekho, hai toh ahge wala digit.
50 → odd → ahge wala digit.
→ even → chordh do vese hi.

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