System Of Particles and Rotational Motion MINDMAP

Centre Of Mass :→

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$\vec{V}_{COM} = \frac{m_1\vec{V_1} + m_2\vec{V_2} + m_3\vec{V_3}}{m_1 + m_2 + m_3}$$

$$\vec{a}_{COM} = \frac{m_1\vec{a_1} + m_2\vec{a_2} + m_3\vec{a_3}}{m_1 + m_2 + m_3}$$

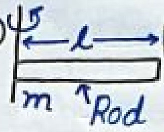
→ $\vec{F}_{ext} = M\vec{a}_{COM}$

→ When there's no external force

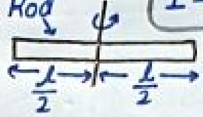
↳ $\vec{a}_{COM} = 0$

If, $\vec{V}_{COM} = 0 \rightarrow \Delta \vec{x}_{COM} = 0, \Delta \vec{y}_{COM} = 0$

Moment Of Inertia of Different Bodies :→

(a)


$I = \frac{Ml^2}{3}$

(b)


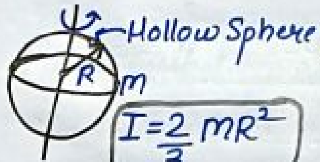
$I = \frac{Ml^2}{12}$

(C)


$I = MR^2$

(d)

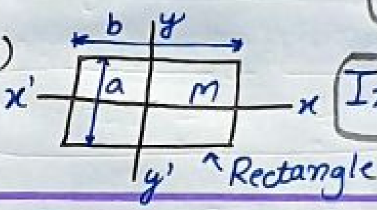

$I = \frac{MR^2}{2}$

(e)


$I = \frac{2}{3}MR^2$

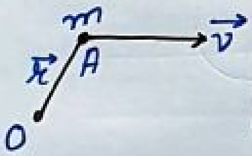
(f)


$I = \frac{2}{5}MR^2$

(g)


$I_{xx'} = \frac{Ma^2}{12} + \frac{Mb^2}{12}$

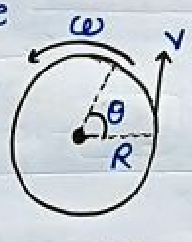
Angular Momentum :→



$\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = \vec{r} \times m\vec{v}$

* Angular Variable



$\theta = \frac{arc}{R} \Rightarrow arc = R\theta$

$|\vec{V}| = R\omega$

$\omega = \frac{d\theta}{dt}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad \omega = \frac{2\pi n}{60}$

n = revolution per minute.

→ If α = Constant :→

i) $\omega = \omega_0 + \alpha t$

ii) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

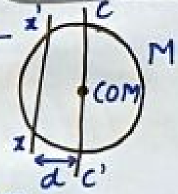
iii) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

→ Relation b/w $\vec{\tau}$ and $\vec{L}$:→

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

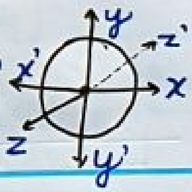
Moment Of Inertia :→ $I = \int r^2 dm$

→ Parallel Axis Theorem :—



$I_{xx'} = I_{cc'} + Md^2$

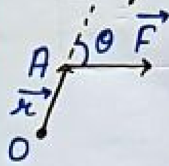
→ Perpendicular Axis Theorem →



$I_{zz'} = I_{xx'} + I_{yy'}$

Only for planar bodies.

Torque :→



$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp$

* Equilibrium →

Translational Equilibrium = $\sum \vec{F} = 0$

Rotational Equilibrium = $\sum \vec{\tau} = 0$

If, $F_{net} = 0$ on a rigid body, then $\vec{\tau}_{net}$ is same about every point.

Rolling Motion :→

→ $\boxed{\sum \vec{F}_{ext} = M\vec{a}_{com}}$ → $\boxed{\sum \vec{\tau}_{ext} = I\vec{\alpha}}$

→ $\boxed{v = R\omega}$ $\boxed{a = R\alpha}$ → Only for contact point.

$$\boxed{\text{Total K.E.} = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} m v_{com}^2}$$

$$\boxed{\vec{L}_{system} = I_{com}\vec{\omega} + m\vec{r}_{com} \times \vec{v}_{com}}$$

* Work done by Torque

$W = \int \tau \, d\theta$ Power, $P = \vec{\tau} \cdot \vec{\omega}$

* Conservation of Angular Momentum –

When,

$\vec{\tau}_{ext} = 0 \Rightarrow \vec{L} = \text{constant}$