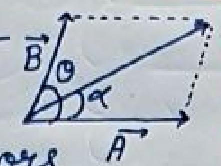
Motion in a plane MINDMAP

• Vectors:→

→ Vector in plane– $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$



* Addition of Vectors

* Parallelogram Method→

$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

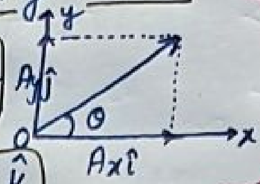
$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + \cos\theta}}$$

$$\boxed{R_{max} = A + B \qquad R_{min} = |A - B|}$$

Resolution of Vectors→

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}} \quad \boxed{A_x = A\cos\theta} \quad \boxed{A_y = A\sin\theta}$$

$$\boxed{A = \sqrt{A_x^2 + A_y^2}} \quad \boxed{\tan\theta = \frac{A_y}{A_x}}$$



When, $\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Dot Product of Vectors→

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ $\quad \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\boxed{\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2}$$

$$\boxed{\hat{i}.\hat{i} = 1,\ \hat{j}.\hat{j} = 1,\ \hat{k}.\hat{k} = 1} \quad \boxed{\hat{i}.\hat{j} = 0,\ \hat{j}.\hat{k} = 0,\ \hat{k}.\hat{i} = 0}$$

$$\boxed{\vec{a}.\vec{b} = ab\cos\theta} \quad \boxed{\cos\theta = \frac{\vec{a}.\vec{b}}{ab}}$$

→ projection of a vector on another vector.

$$\boxed{A\cos\theta = A\left[\frac{\vec{A}.\vec{B}}{AB}\right]}$$

• Cross Product.→

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k},\ \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$= (b_1c_2 - b_2c_1)\hat{i} - (a_1c_2 - a_2c_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

$\vec{a} \times \vec{b} = ab\sin\theta\,\hat{n}$

→ Direction of $\hat{n}$ can be found using right hand rule.

→ Circle Rule→

$\hat{i} \times \hat{j} = \hat{k}$ $\quad \hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad \hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad \hat{k} \times \hat{k} = 0$

Kinematics→

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

$$\boxed{\vec{a} = \frac{dv}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}}$$

→ equations of Motion

1. $\vec{v} = \vec{u} + \vec{a}t$
2. $\Delta\vec{x} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
3. $|\vec{v}|^2 = |\vec{u}|^2 + 2\vec{a}.\Delta\vec{x}$

$$\boxed{\begin{aligned} \vec{a} &= a_x\hat{i} + a_y\hat{j} \\ \Delta\vec{x} &= x\hat{i} + y\hat{j} \\ \vec{v} &= u_x\hat{i} + u_y\hat{i} \end{aligned}}$$

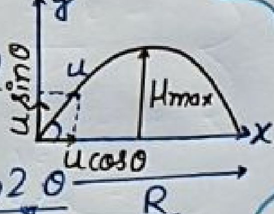
$$\boxed{\begin{aligned} V_x &= u_x + a_xt \\ V_y &= u_y + a_yt \\ \Delta x &= u_xt + \frac{1}{2}a_xt^2 \\ \Delta y &= u_yt + \frac{1}{2}a_yt^2 \end{aligned}}$$

Projectile Motion→

$T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g}$

$H = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2\theta}{2g}$

$R = \frac{2u_xu_y}{a_y} = \frac{u^2\sin 2\theta}{g}$



→ Eq^n of Trajectory.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

If, a) $u = 0$ → linear path ($a \neq 0$)

b) $u \neq 0$ and $u \parallel a$ → linear path

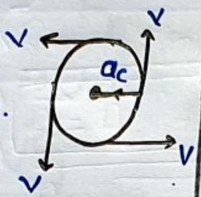
c) $u \neq 0$ and u not $\parallel a$ → parabolic

Circular Motion →

- **Uniform Circular Motion →**

→ Speed = Constant,
Velocity ≠ Constant.

→ Acceleration ≠ Constant.

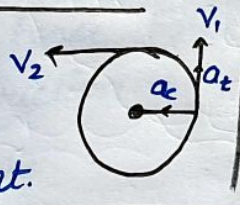


$\boxed{|\vec{a_c}| = \frac{v^2}{r}}$ $\boxed{T = \frac{2\pi r}{v}}$

- **Non-Uniform Circular Motion →**

→ Speed ≠ Constant,
Velocity ≠ Constant.

→ Acceleration ≠ Constant.



$\boxed{|\vec{a_c}| = \frac{|\vec{v}|^2}{r}}$ $\boxed{|\vec{a_t}| = \frac{d|\vec{v}|}{dt}}$ $\boxed{a_{net} = \vec{a_c} + \vec{a_t}}$

- **Relative Motion →**

$\vec{V_{AB}} = \vec{V_A} - \vec{V_B}$

→ **River-Boat**

* For minimum time →

$\boxed{t = \frac{d}{V_m}}$

* For minimum distance →

$\boxed{t = \frac{d}{\sqrt{V_m^2 - V_r^2}}}$

V_m = Velocity of man w.r.t. river
V_r = Velocity of river w.r.t. ground.

NEET
SLAYER