ECBS 6060: International Trade Winter 2020

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Lecture 8: The Ricardian model



A simple example

- ▶ Germany and Greece produce olive oils and steam turbines.
- ▶ Both are only produced using labor.
- Labor requirements:

	Germany	Greece
Olive oil	10 minutes	30 minutes
Steam turbines	10 hours	100 hours

A simple example

- 1. Calculate the autarky prices.
- 2. What is the pattern of trade?
- 3. What can you say about the equilibrium prices?
- 4. Why doesn't Germany produce both goods?
- 5. What can you say about the equilibrium relative wage?

General setup

General setup

- ightharpoonup There is one factor of production, say, labor L.
- ▶ *I* goods, each produced with CRS technology:

$$c_i^j(w^j, x_i^j) = w^j a_i^j x_i^j.$$

▶ Unit labor requirement a_i^j differs across sectors and countries.

Autarky equilibrium

- ▶ Because we will want to evoke the Law of Comparative Advantage, we study the autarky equilibrium first.
- ▶ Relative prices equal the marginal rates of transformation.
- ▶ With linear technology, MRT equals relative productivities.
- ightharpoonup (I drop country superscript j, but everything is country specific.)

Equilibrium conditions

Profit maximization:

$$w^a a_i \ge p_i^a$$

with equality if $x_i^a > 0$.

► Labor market clearing:

$$\sum_{i \in I} a_i x_i^a = L$$
$$a\mathbf{x}^a = L$$

Utility maximization and goods market clearing:

$$x_i^a = e_i(p^a, u).$$

The supply side

- In autarky, all goods are produced.
- ► Hence

$$p_i^a = w^a a_i$$

for all i.

▶ Relative prices are pinned down by the supply side alone:

$$\frac{p_i^a}{p_j^a} = \frac{a_i}{a_j}.$$

► (This is no longer true in the open economy. Why?)

Pattern of trade

The law of comparative advantage

► The law of comparative advantage says that the autarky value of the net import vector is positive:

$$\sum_{i} p_i^a m_i > 0.$$

Substituting in, and dividing by autarky wages:

$$\sum_{i} a_i m_i > 0.$$

► This looks very much like absolute advantage. Why is still about CA?

The law of comparative advantage

ightharpoonup Divide by a_1 :

$$\sum_{i} \frac{a_i}{a_1} m_i > 0.$$

Integrated equilibrium

- In the integrated equilibrium, not all products will be produced.
- In fact, all products are only produced in one country. Why?
- lacktriangle Let us consider a small open economy first, facing world prices p^w .

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Balanced trade

Balanced trade implies

$$\sum_{i} p_i^w m_i = 0.$$

- Clearly, some goods are exported (in net terms), some are imported.
- ightharpoonup Divide by p_1^w :

$$\sum_{i} \frac{p_i^w}{p_1^w} m_i = 0.$$

Comparative advantage

Subtract the balanced trade equation from the LOCA:

$$\sum_{i} \left(\frac{a_i}{a_1} - \frac{p_i^w}{p_1^w} \right) m_i > 0.$$

- ▶ If balanced trade holds together with the LOCA, then *on average* a_i/p_i^w tends to be high for imported goods, and low for exported goods.
- In this sense it is *relative* productivity that matters.

Pattern of production

- ightharpoonup Suppose the wage rate is w. What products do we produce at this wage rate?
- ► Profit maximization:

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- ► This gives *I* inequalities.
- ightharpoonup There is only one unknown, w.
- Only one good will be produced.

▶ Sort goods by p_i^w/a_i :

$$\frac{p_1^w}{a_1} > \frac{p_2^w}{a_2} > \dots > \frac{p_I^w}{a_I}$$

ightharpoonup These are all $\leq w$:

$$w \ge \frac{p_1^w}{a_1} > \frac{p_2^w}{a_2} > \dots > \frac{p_I^w}{a_I}$$

► Clearly, only good 1 (the one with the lowest autarky relative price) will be produced and

$$w = \frac{p_1^w}{a_1}.$$

Specialization in the 2-country case

- ▶ Good i has unit labor requirement a_i^1 in country 1, and a_i^2 in country 2.
- ▶ Profit maximization in both countries requires

$$p_i^w \le w^1 a_i^1,$$

$$p_i^w \le w^2 a_i^2,$$

with equality if there is production.

► If country 1 produces good *i*,

$$p_i^w = w^1 a_i^1.$$

▶ Then country 2 does not produce the good iff

$$w^1 a_i^1 < w^2 a_i^2,$$

or

$$\frac{a_i^2}{a_i^1} > \frac{w^1}{w^2}.$$

Specialization in the 2-country case

Country 1 produces all the goods in which it has a comparative advantage relative to wages.

$$\frac{a_i^2}{a_i^1} > \frac{w^1}{w^2}.$$

► Country 2 produces all the goods for which the opposite holds

$$\frac{a_i^2}{a_i^1} < \frac{w^1}{w^2}.$$

The 2-good case

The 2-good case

- ▶ If there are only two goods, the case is simpler.
- ► In general,

$$\frac{p_1^w}{p_2^w} \neq \frac{a_1}{a_2},$$

so the conditions cannot hold both with equality.

▶ Only one of the two goods will be produced. Let it be good 1.

Integrated equilibrium

Profit maximization:

$$p_1^w = wa_1$$
$$p_2^w < wa_2$$

- ▶ The first equation pins down the wage rate.
- ► The second inequality holds if

$$\frac{a_1}{a_2} < \frac{p_1^w}{p_2^w}.$$

▶ The country produces the good in which it has a *comparative advantage*.

Continuum of goods

Continuum of goods

- ► The Ricardian model can be generalized to a continuum of goods (Dornbusch, Fischer and Samuelson, 1977, AER).
- ▶ There are two countries, so world prices will also be endogenized.

Tastes

- ▶ There is a continuum of goods, indexed by $z \in [0, 1]$.
- ▶ The consumption basket is a function $c(z):[0,1] \to \mathbb{R}$.
- Consumers have Cobb-Douglas utility over them:

$$u[c(\cdot)] = \exp\left[\int_0^1 \ln c(z)dz\right].$$

What is the corresponding expenditure function?

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What is the corresponding expenditure function?

$$\min_{c(\cdot)} \int_0^1 p(z)c(z) \text{ s.t. } \exp[\int_0^1 \ln c(z)dz] \ge u.$$

Consumer demand

► Each good receives a constant fraction of expenditure:

$$p(z)c(z) = \lambda.$$

 \triangleright So if aggregate expenditure is E,

$$E = \int_0^1 p(z)c(z)dz = \int_0^1 \lambda dz,$$

so is good-by-good expenditure:

$$p(z)c(z) = E.$$

Technology

- ▶ Good z is produced with unit labor requirements $a^1(z)$ and $a^2(z)$.
- ► Let us order goods such that

$$A(z) = \frac{a^2(z)}{a^1(z)}$$

is decreasing.

lacktriangle A higher z means higher comparative advantage of that good in country 1.

Patterns of specialization with given wages

- ▶ Suppose wages are w^1 and w^2 . What is the pattern of specialization?
- ▶ The law of one price prevails for each good, $p^1(z) = p^2(z) = p(z)$.
- ▶ Clearly, $w^1a^1(z) = p(z) = w^2a^2(z)$ cannot hold for both countries, except for one particular good.
 - This is where continuum comes handy.
- Goods with

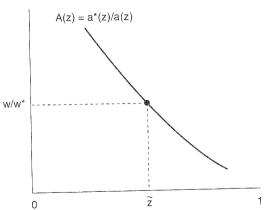
$$A(z) = \frac{a^2(z)}{a^1(z)} > \frac{w^1}{w^2}$$

have lower autarky prices in country 1, and are produced there.

▶ These goods are in $[0, \tilde{z}]$. The rest, $(\tilde{z}, 1]$ is produced in country 2.

Determining the pattern of specialization

Relative Home wage, relative Foreign cost



Trading equilibrium

- ▶ Of course, w^1/w^2 is endogenous.
- ▶ We close the model with the demand side.
- Aggregate expenditure:

$$\int_0^1 p(z)[c^1(z) + c^2(z)]dz = \int_0^1 p(z)x^j(z)dz = w^1L^1 + w^2L^2.$$

► Each good receives the same expenditure,

$$p(z)c(z) = \lambda.$$

Market clearing

If the first \tilde{z} goods are produced in country 1, then country 1's revenue is

$$\int_0^{\tilde{z}} p(z)x^1(z)dz = \int_0^{\tilde{z}} p(z)c(z) = \tilde{z}(w^1L^1 + w^2L^2).$$

Because of zero profits, revenue is the same as wage costs,

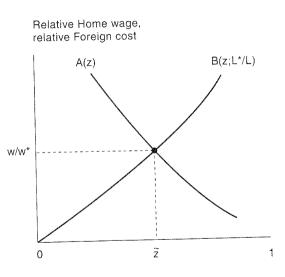
$$\tilde{z}(w^1L^1 + w^2L^2) = w^1L^1.$$

▶ We can express the relative wage as

$$\frac{w^1}{w^2} = B(\tilde{z}) \equiv \frac{\tilde{z}}{1 - \tilde{z}} \frac{L^2}{L^1}$$

- ▶ To find the equilibrium we solve for \tilde{z} and w^1/w^2 jointly.
 - ightharpoonup Why not w^1 and w^2 ?
 - Can you verify that this is an equilibrium?

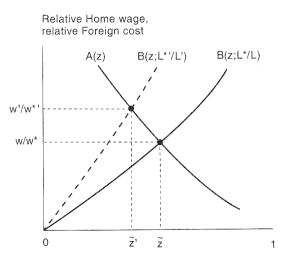
Joint determination of wages and specialization



Comparative statics

- ▶ We can conduct simple comparative statics.
- ▶ Bigger and more productive countries produce a wider range of goods.

A rise in country 2's labor supply



Trade costs

- ▶ Suppose there are trade costs involved in crossing the border.
- ▶ You have to send $\tau > 1$ goods so that 1 unit arrives.
 - ► Samuelson's (1954) iceberg formulation.
- ▶ How does this change the pattern of specialization?
 - Some goods are produced in both countries and are not traded.
 - More on this later in the course.

Trade costs and specialization

▶ Good z is only shipped from country 1 to 2 if

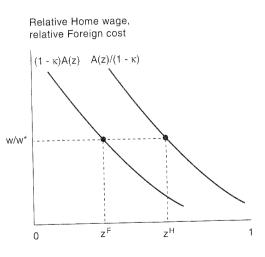
$$A(z) > \frac{\tau w^1}{w^2}.$$

Good z is only shipped from country 2 to 2 if

$$A(z) < \frac{w^1}{\tau w^2}.$$

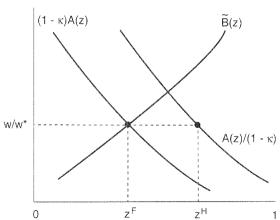
- ▶ For $\tau > 1$, there is a range of zs for which neither inequality holds.
- ▶ The flatter the A(z), the wider this range.

Specialization with transport costs



Some goods are not traded





Appendix

Aside: Cardinality of the continuum

- ▶ The cardinality of a set is a measure of the "number of elements" in it.
 - ightharpoonup |A| = |B| if there is a bijection between the elements of A and B.
 - lacksquare |A| > |B| if |a| = |B| for a proper subset $a \subset A$, but not |A| = |B|.
- Continuum is the cardinality of the real line.
 - $|\mathbb{R}| = |[0,1]| = |\mathbb{R}^2|$
 - ightharpoonup $|\mathbb{R}| > |\mathbb{N}|$
- ▶ The cardinality of the continuum is *infinite*. What does this mean?

Why continuum?

- ▶ The two-good model implies sever restrictions.
 - Many results change if we add more goods.
- ightharpoonup It is often easier to work with a continuum of goods than with 3 or N goods.
 - **E**specially when working with inequalities.
 - Or when things are random. Why?
- ▶ In many cases, we can simply think of the continuum case as $N \to \infty$.
- But often our intuition breaks down, and special techniques are needed.
 - differentiation
 - double continuums