ECBS 6060: International Trade
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Lecture 8: The Ricardian model

A simple example

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- Germany and Greece produce olive oils and steam turbines.
- Both are only produced using labor.
- Labor requirements:

	Germany	Greece
Olive oil	10 minutes	30 minutes
Steam turbines	10 hours	100 hours

A simple example

1. Calculate the autarky prices.
2. What is the pattern of trade?
3. What can you say about the equilibrium prices?
4. Why doesn't Germany produce both goods?
5. What can you say about the equilibrium relative wage?

General setup

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- There is one factor of production, say, labor L.
- I goods, each produced with CRS technology:

$$c_i^j(w^j, x_i^j) = w^j a_i^j x_i^j.$$

- Unit labor requirement a_i^j differs across sectors *and countries*.

Autarky equilibrium

- Because we will want to evoke the Law of Comparative Advantage, we study the autarky equilibrium first.
- Relative prices equal the marginal rates of transformation.
- With linear technology, MRT equals relative productivities.
- (I drop country superscript j, but *everything* is country specific.)

Equilibrium conditions

- Profit maximization:
$$w^a a_i \geq p_i^a$$
with equality if $x_i^a > 0$.
- Labor market clearing:
$$\sum_{i \in I} a_i x_i^a = L$$
$$a\mathbf{x}^a = L$$
- Utility maximization and goods market clearing:
$$x_i^a = e_i(p^a, u).$$

The supply side

- In autarky, all goods are produced.
- Hence
$$p_i^a = w^a a_i$$
for all i.
- Relative prices are pinned down by the supply side alone:
$$\frac{p_i^a}{p_j^a} = \frac{a_i}{a_j}.$$
- (This is no longer true in the open economy. Why?)

Pattern of trade

The law of comparative advantage

- The law of comparative advantage says that the autarky value of the net import vector is positive:
$$\sum_i p_i^a m_i > 0.$$
- Substituting in, and dividing by autarky wages:
$$\sum_i a_i m_i > 0.$$
- This looks very much like *absolute advantage*. Why is still about CA?

The law of comparative advantage

- Divide by a_1:
$$\sum_i \frac{a_i}{a_1} m_i > 0.$$

Integrated equilibrium

- In the integrated equilibrium, not all products will be produced.
- In fact, all products are only produced in one country. Why?
- Let us consider a small open economy first, facing world prices p^w.

Balanced trade

- Balanced trade implies
$$\sum_i p_i^w m_i = 0.$$
- Clearly, some goods are exported (in net terms), some are imported.
- Divide by p_1^w:
$$\sum_i \frac{p_i^w}{p_1^w} m_i = 0.$$

Comparative advantage

- Subtract the balanced trade equation from the LOCA:

$$\sum_i \left(\frac{a_i}{a_1} - \frac{p_i^w}{p_1^w} \right) m_i > 0.$$

- If balanced trade holds together with the LOCA, then *on average* a_i / p_i^w tends to be high for imported goods, and low for exported goods.
- In this sense it is *relative* productivity that matters.

Pattern of production

Specialization in the small open economy

- Suppose the wage rate is w. What products do we produce at this wage rate?
- Profit maximization:

$$p_i^w \leq wa_i$$

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- This gives I inequalities.
- There is only one unknown, w.
- Only one good will be produced.

Specialization in the small open economy

- Sort goods by p_i^w/a_i:

$$\frac{p_1^w}{a_1} > \frac{p_2^w}{a_2} > ... > \frac{p_I^w}{a_I}$$

- These are all $\leq w$:

$$w \geq \frac{p_1^w}{a_1} > \frac{p_2^w}{a_2} > ... > \frac{p_I^w}{a_I}$$

- Clearly, only good 1 (the one with the lowest autarky relative price) will be produced and

$$w = \frac{p_1^w}{a_1}.$$

Specialization in the 2-country case

- Good i has unit labor requirement a_i^1 in country 1, and a_i^2 in country 2.
- Profit maximization in both countries requires

$$p_i^w \leq w^1 a_i^1,$$
$$p_i^w \leq w^2 a_i^2,$$

  with equality if there is production.
- If country 1 produces good i,

$$p_i^w = w^1 a_i^1.$$

- Then country 2 does not produce the good iff

$$w^1 a_i^1 < w^2 a_i^2,$$

  or

$$\frac{a_i^2}{a_i^1} > \frac{w^1}{w^2}.$$

Specialization in the 2-country case

- Country 1 produces all the goods in which it has a *comparative advantage* relative to wages.

$$\frac{a_i^2}{a_i^1} > \frac{w^1}{w^2}.$$

- Country 2 produces all the goods for which the opposite holds

$$\frac{a_i^2}{a_i^1} < \frac{w^1}{w^2}.$$

The 2-good case

The 2-good case

- If there are only two goods, the case is simpler.
- In general,

$$\frac{p_1^w}{p_2^w} \neq \frac{a_1}{a_2},$$

  so the conditions cannot hold both with equality.
- Only one of the two goods will be produced. Let it be good 1.

Integrated equilibrium

- Profit maximization:

$$p_1^w = wa_1$$
$$p_2^w < wa_2$$

- The first equation pins down the wage rate.
- The second inequality holds if

$$\frac{a_1}{a_2} < \frac{p_1^w}{p_2^w}.$$

- The country produces the good in which it has a *comparative advantage*.

Continuum of goods

- The Ricardian model can be generalized to a continuum of goods (Dornbusch, Fischer and Samuelson, 1977, AER).
- There are two countries, so world prices will also be endogenized.

Tastes

- There is a continuum of goods, indexed by $z \in [0, 1]$.
- The consumption basket is a function $c(z) : [0, 1] \to \mathbb{R}$.
- Consumers have Cobb-Douglas utility over them:

$$u[c(\cdot)] = \exp[\int_0^1 \ln c(z) dz].$$

- What is the corresponding expenditure function?

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$$\min_{c(\cdot)} \int_0^1 p(z)c(z) \text{ s.t. } \exp[\int_0^1 \ln c(z) dz] \geq u.$$

Consumer demand

- Each good receives a constant fraction of expenditure:

$$p(z)c(z) = \lambda.$$

- So if aggregate expenditure is E,

$$E = \int_0^1 p(z)c(z)dz = \int_0^1 \lambda dz,$$

so is good-by-good expenditure:

$$p(z)c(z) = E.$$

Technology

- Good z is produced with unit labor requirements $a^1(z)$ and $a^2(z)$.
- Let us order goods such that

$$A(z) = \frac{a^2(z)}{a^1(z)}$$

  is *decreasing*.
  - A higher z means higher *comparative advantage* of that good in country 1.

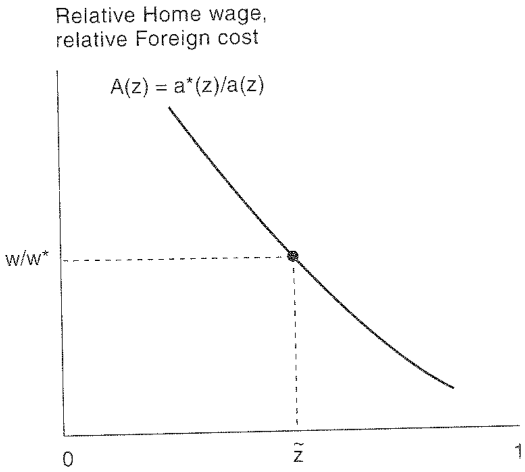
Patterns of specialization with given wages

- Suppose wages are w^1 and w^2. What is the pattern of specialization?
- The law of one price prevails for each good, $p^1(z) = p^2(z) = p(z)$.
- Clearly, $w^1 a^1(z) = p(z) = w^2 a^2(z)$ cannot hold for both countries, except for one particular good.
  - This is where continuum comes handy.
- Goods with

$$A(z) = \frac{a^2(z)}{a^1(z)} > \frac{w^1}{w^2}$$

  have lower autarky prices in country 1, and are produced there.
- These goods are in $[0, \tilde{z}]$. The rest, $(\tilde{z}, 1]$ is produced in country 2.

Determining the pattern of specialization



Trading equilibrium

- Of course, w^1/w^2 is endogenous.
- We close the model with the demand side.
- Aggregate expenditure:

$$\int_0^1 p(z)[c^1(z) + c^2(z)]dz = \int_0^1 p(z)x^j(z)dz = w^1L^1 + w^2L^2.$$

- Each good receives the same expenditure,

$$p(z)c(z) = \lambda.$$

Market clearing

- If the first $\tilde{z}$ goods are produced in country 1, then country 1's revenue is

$$\int_0^{\tilde{z}} p(z)x^1(z)dz = \int_0^{\tilde{z}} p(z)c(z) = \tilde{z}(w^1L^1 + w^2L^2).$$

- Because of zero profits, revenue is the same as wage costs,

$$\tilde{z}(w^1L^1 + w^2L^2) = w^1L^1.$$

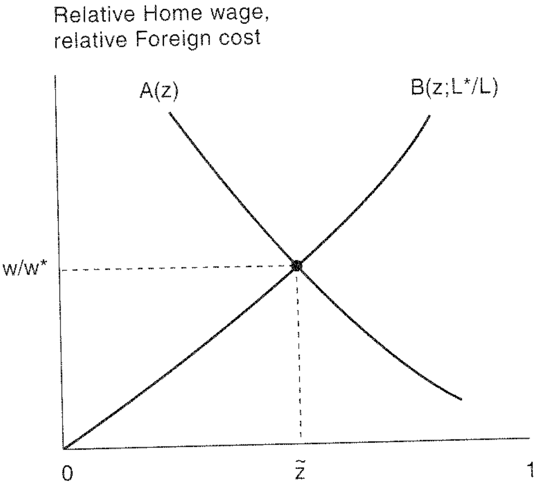
- We can express the relative wage as

$$\frac{w^1}{w^2} = B(\tilde{z}) \equiv \frac{\tilde{z}}{1-\tilde{z}}\frac{L^2}{L^1}$$

.

- To find the equilibrium we solve for $\tilde{z}$ and w^1/w^2 *jointly*.
  - Why not w^1 and w^2?
  - Can you verify that this is an equilibrium?

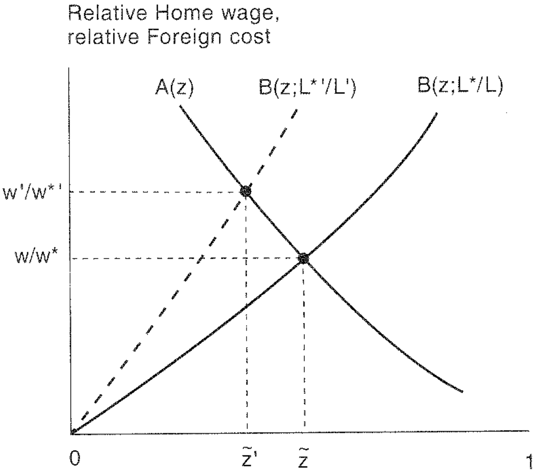
Joint determination of wages and specialization



Comparative statics

- We can conduct simple comparative statics.
- Bigger and more productive countries produce a wider range of goods.

A rise in country 2's labor supply



Trade costs

- Suppose there are trade costs involved in crossing the border.
- You have to send $\tau > 1$ goods so that 1 unit arrives.
  - Samuelson's (1954) iceberg formulation.
- How does this change the pattern of specialization?
  - Some goods are produced in both countries and are not traded.
  - More on this later in the course.

Trade costs and specialization

- Good z is only shipped from country 1 to 2 if

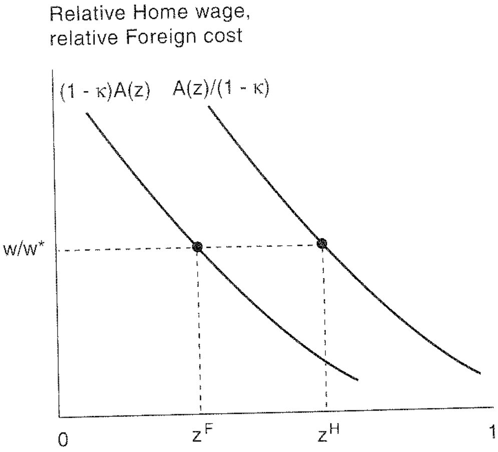
$$A(z) > \frac{\tau w^1}{w^2}.$$

- Good z is only shipped from country 2 to 2 if

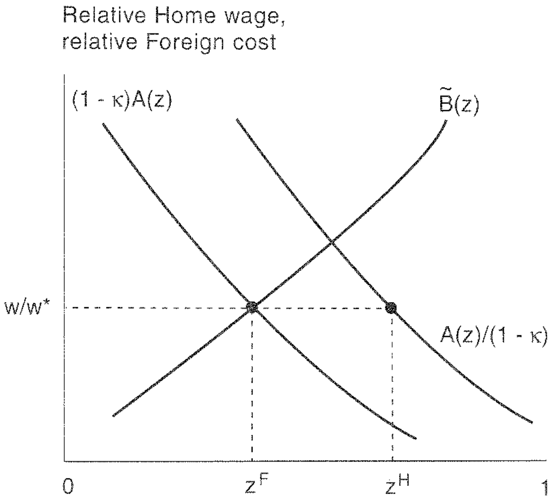
$$A(z) < \frac{w^1}{\tau w^2}.$$

- For $\tau > 1$, there is a range of zs for which neither inequality holds.
- The flatter the $A(z)$, the wider this range.

Specialization with transport costs



Some goods are not traded



Appendix

Aside: Cardinality of the continuum

- The *cardinality* of a set is a measure of the "number of elements" in it.
  - $|A| = |B|$ if there is a bijection between the elements of A and B.
  - $|A| > |B|$ if $|a| = |B|$ for a proper subset $a \subset A$, but not $|A| = |B|$.
- *Continuum* is the cardinality of the real line.
  - $|\mathbb{R}| = |[0, 1]| = |\mathbb{R}^2|$
  - $|\mathbb{R}| > |\mathbb{N}|$
- The cardinality of the continuum is *infinite*. What does this mean?

Why continuum?

- The two-good model implies sever restrictions.
  - Many results change if we add more goods.
- It is often easier to work with a continuum of goods than with 3 or N goods.
  - Especially when working with inequalities.
  - Or when things are random. Why?
- In many cases, we can simply think of the continuum case as $N \to \infty$.
- But often our intuition breaks down, and special techniques are needed.
  - differentiation
  - double continuums