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AN ALGORITHM FOR POSITION DETERMINATION
USING LORAN-C TRIPLETS WITH A
BASIC PROGRAM FOR THE
COMMODORE 2001 MICROCOMPUTER

by

R. H. Shudde

March 1980

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Rear Admiral J. J. Ekelund Superintendent Jack R. Borsting Provost

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ABSTRACT

An algorithm for position determination using Loran-C triplets and an implementation of the algorithm in the BASIC language of the Commodore 2001 microcomputer are presented. Modifications of the geodesic direct solution and reverse solution algorithms of P. D. Thomas are also presented; these modifications eliminate all of the special positioning and quadrant determination requirements of the original algorithms.

A. Introduction

Loran-C position determination programs for the Hewlett-Packard HP-67 programmable calculator for use aboard U. S. Navy patrol aircraft (P-3) have been prepared and are reported elsewhere [Ref. 1]. During the development and testing of the HP-67 programs the need for a high accuracy program for cross checking purposes arose. The program presented here was developed as a refinement of the HP-67 algorithm.

The basic methodology of this program is contained in the formulas of Paul D. Thomas [Ref. 2] for the direct and reverse problems of oblate spheroidal geodesy. The direct problem is to determine the latitude and longitude of a point $\mathbf{P_2}$ when the latitude and longitude of a point $\mathbf{P_1}$ as well as the azimuth and distance of P_2 from P_1 are known. The reverse (or inverse) problem is to determine the distance, forward azimuth, and backward azimuth between two points, P_1 and P_2 , when the latitude and longitude of both P_1 and P_2 are known. In both problems P₁ and P₂ are on the surface of a spheroid. Unfortunately, Thomas' algorithms contain several restrictions and numerous subcases that are too awkward to be programmed on the HP-67. For this reason portions of these algorithms were rewritten so as to eliminate all of the original restrictions and subcases. These modified algorithms for the direct and reverse problems are presented here with notations about the modifications made.

B. Loran-C Fixing Algorithms

The principles of Loran lines of position (LOP's) and fixing are adequately covered in Reference 3 and will not be repeated here.

The basic Loran-C equation [Ref. 4] can be written as

$$T = [T_S + p(T_S)] - [T_M + p(T_M)] + [T_B + p(T_B)] + \delta$$
 (1)

where

T is the "indicated time difference" in microseconds,

 T_{M} is the distance, in microseconds, from the master to the receiver,

 T_S is the distance, in microseconds, from the slave to the receiver,

 $\mathbf{T}_{\mathbf{B}}$ is the distance, in microseconds, between the master and the slave,

 δ is the assigned coding delay, in microseconds, and $p(\mathbf{T})$ is the secondary phase correction, in microseconds, for a surface water path of length T. The quantity

$$\Delta t = [T_B + p(T_B)] + \delta$$

is a constant for each master/slave pair. The following World Geodetic System 1972 (WGS 72) values have been adopted for Loran-C navigation [Ref. 4]:

 v_0 = 299792458 meters/second is the velocity of light in free space,

η = 1.000338 is the index of refraction of the surface of the earth for standard atmosphere and 100 kHz electromagnetic waves, and f = 1/298.26 is the flattening factor $(1-b/a_e)$, where b is the polar radius) of the earth.

With these parameters the secondary phase correction for an all seawater path has been taken to be of the form

$$p(T) = a_0/T + a_1 + a_2T$$
 (2)

where T is in microseconds and,

1. For T>537 µsec:

 $a_0 = 129.04398$,

 $a_1 = -0.40758$,

and $a_2 = 0.00064576438$.

2. For T<537 μsec:

 $a_0 = 2.7412979$,

 $a_1 = -0.011402$,

and $a_2 = 0.00032774624$.

If one uses a spherical approximation to the earth's surface (f = 0), then a spherical hyperbola can be represented by the equation [Ref. 3, page 175]

$$tan r = 2 \frac{\cos 2a - \cos 2c}{\sin 2c \cos \omega + \zeta \sin 2a}$$
 (3)

where the origin of the coordinate system is at the prime focus of the spherical hyperbola, 2c is the spherical arc joining the foci, 2a is a constant for any one hyperbola, and r and ω are the spherical coordinates of a point on the

hyperbola. If the base line of the coordinate system is the arc joining the foci then ω is the spherical polar angle from the baseline to a point P on the spherical hyperbola and r is spherical polar distance (or arc) from the prime focus to P. Using the Loran system we take $\zeta = +1$ if the prime focus is at a master station and we take $\zeta = -1$ if the prime focus is at a slave station.

If we take $v = v_0/\eta$ to be the velocity of 100kHz electromagnetic radiation at the earth's surface then, for a spherical earth, we can relate the parameters in Equations 1 and 3 as follows:

$$2c = vT_B/a_e,$$
and
$$2a = v(T_S - T_M)/a_e.$$

Using the spherical approximation for now, we see that 2c is known for any Loran pair. The "indicated time delay" T is measured by the receiver at point P, and to determine a hyperbolic line of position we must determine 2a, but $T_S - T_M$ cannot be computed from Equations 1 and 2. If a_0 were zero in Equation 2, then it would be possible to determine $T_S - T_M$ uniquely. As a first approximation we use the following parameters in Equation 2:

$$a_0 = 0,$$
 $a_1 = -0.321,$
 $a_2 = 0.000635.$

and

These values have been obtained by setting $a_0 = 0$ and determining

 a_1 and a_2 by linear regression of the T > 537 values over the interval of 1000 < T < 8000. This approximation is quite good (within 0.03 μs) for distances up to 10,000 microseconds where small changes in the LOP's can cause large position errors. At short distances the error increases from 0.05 μs at 1000 μs to 0.58 μs at 10 μs ; although these errors are large for small distances, the LOP's are not as sensitive to these changes as they would be at large distances. These errors are well within the 4 n.mi (24 μs) accuracy that was requested by COMPATWINGSPAC. When this approximation is substituted into Equation 1, we obtain

$$[T_S + a_1 + a_2T_S] - [T_M + a_1 + a_2T_M] = T - \Delta t$$

or

$$T_S - T_M = (T - \Delta t)/(1 + a_2)$$
 (4)

and hence $2a = v(T_S - T_M)/a_e$ is determined for use in the spherical approximation.

Consider a Loran-C triplet with the master stations colocated. Let ξ_1 and ξ_2 denote the azimuth angles of slave 1 (S₁) and slave 2 (S₂), respectively, measured from North toward the East from the master stations (M) (see Figure 1). Further, let α and r be the azimuth and spherical polar arc (distance) of the receiver (R) from M. For this geometry, Equation 3 can be written in the form

$$tan r_{i} = \frac{B_{i}}{C_{i} \cos (\alpha - \xi_{i}) + A_{i}}, \qquad (5)$$

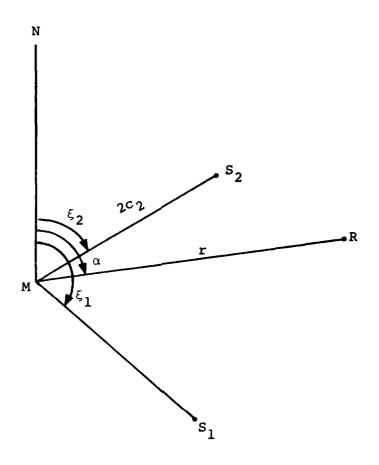


Figure 1. Geometry of a Loran Triplet and a Receiver.

where

$$A_i = \zeta_i \sin 2a_i$$

$$B_i = \cos 2a_i - \cos 2c_i$$

and

$$c_i = \sin 2c_i$$

for the i^{th} Loran pair, i = 1,2. Since $r_1 = r_2 = r$, we can eliminate tan r between the two equations. The resulting equation can be rewritten as

$$C \cos \alpha + S \sin \alpha = K,$$
 (6)

where

$$C = B_1 C_2 \cos \xi_2 - B_2 C_1 \cos \xi_1$$

$$s = B_1 c_2 \sin \xi_2 - B_2 c_1 \sin \xi_1$$

and

$$K = B_2A_1 - B_1A_2$$
.

If we define $\rho > 0$ and γ by the equations

$$\rho \cos \gamma = C, \tag{7}$$

and

$$\rho \sin \gamma = S$$
,

then

$$_{o}=\sqrt{c^{2}+s^{2}}.$$

and

$$\gamma = qatn (s,C)$$
.

Here the function qatn(y,x) is the arctangent of y/x adjusted for the proper quadrant according to the signs of x and y.

A compact form of this function is

$$qatn(y,x) = tan^{-1} \frac{y}{x + 10^{-9} t(x = 0?)} + \pi t(x < 0?)$$

where t(z) = 1 when z is true

and t(z) = 0 when z is false.

When convenient we will use the notation qatn(y/x) interchangeably with qatn(y,x). Now we can substitute Eq.(7) into Eq.(6) and solve for

$$\alpha = \gamma \pm \cos^{-1}(K/\rho) \tag{8}$$

to obtain the azimuth angle α of the two points of intersection of the spherical hyperbolic LOP's. Finally we can obtain a value for r by substituting each α into either Eq. (5). We find that

r = qatn
$$\left[\frac{B_i}{C_i \cos (\alpha - \xi_i) + A_i}\right]$$
 for $i = 1$ or 2.

To obtain the first solution, r and α are entered into the "direct" solution algorithm; latitude ϕ and longitude λ are the outputs.

The solution is improved using a two-variable Newton-Raphson search for the zeroes of the functions

$$f_{i}(\phi,\lambda) = f_{i} = (d_{Si} - d_{M})/a_{e} - 2a_{i}$$
 for $i = 1,2$,

where d_M , d_{S1} , and d_{S2} are the distances from R to M, S_1 , and S_2 , respectively, and are computed using the "inverse" solution algorithm. The flow of the improvement algorithm follows:

- 1. Input the latest approximation to the receiver latitude ϕ and longitude λ_{\star}
- 2. Compute $\mathbf{d_{M}},~\mathbf{d_{S1}}$ and $\mathbf{d_{S2}}$ using the "inverse" solution algorithm.
- 3. Compute $T_M = d_M/v$ and $T_{Si} = d_{Si}/v$ for i = 1,2.
- 4. Compute $T_{Si} T_{M} = (T \Delta t)_{i} p(T_{Si}) + p(T_{M})$ and $2a_{i} = v(T_{Si} T_{M})/a_{e}$ for i = 1, 2.
- 5. Compute $f_i(\phi,\lambda) = (d_{Si}-d_M)/a_e 2a_i$ for i = 1,2.
- 6. Compute

$$\frac{\partial \mathbf{f_i}}{\partial \phi} = \frac{\mathbf{f_i} (\phi + \Delta \phi, \lambda) - \mathbf{f_i} (\phi, \lambda)}{\Delta \phi}$$

and

$$\frac{\partial \mathbf{f_i}}{\partial \lambda} = \frac{\mathbf{f_i}(\phi, \lambda + \Delta \lambda) - \mathbf{f_i}(\phi, \lambda)}{\Delta \lambda} \quad \text{for i = 1,2.}$$

($\Delta\lambda = \Delta\phi = 10^{-4}$ radians is used in the BASIC program.)

7. Replace ϕ by

$$\phi - \left[\mathbf{f_1} \frac{\partial \mathbf{f_2}}{\partial \lambda} - \mathbf{f_2} \frac{\partial \mathbf{f_1}}{\partial \lambda} \right] / \mathbf{J}$$

and replace λ by

$$\lambda - \left[\mathbf{f_1} \, \frac{\partial \mathbf{f_2}}{\partial \phi} - \mathbf{f_2} \, \frac{\partial \mathbf{f_1}}{\partial \phi} \right] / \mathbf{J}$$

where

$$\mathbf{J} = \frac{\partial \mathbf{f}_1}{\partial \phi} \frac{\partial \mathbf{f}_2}{\partial \lambda} - \frac{\partial \mathbf{f}_1}{\partial \lambda} \frac{\partial \mathbf{f}_2}{\partial \phi}$$

8. Repeat from Step 1 until $_{\Phi}$ and $_{\lambda}$ are stationary.

C. The Direct Solution Algorithm

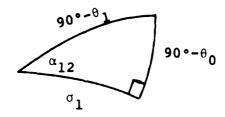
This direct solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas [Ref. 2, pp. 7-8]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qath function is defined in the previous section. West longitudes and South latitudes are negative. We are given the point $P_1(\phi_1,\lambda_1)$ on the spheroid, where ϕ_1 , λ_1 are the geodetic latitude and longitude (geographic coordinates); the forward azimuth α_{12} and distance S to a second point $P_2(\phi_2,\lambda_2)$; and from these we are to find the geographic coordinates ϕ_2,λ_2 and the back azimuth α_{21} . The given quantities are ϕ_1 , λ_1 , α_{12} and S. No assumptions about the relative location of P_1 and P_2 are required. The modified direct solution algorithm is:

$$\begin{array}{l} \theta_1 = \tan^{-1}[(1-f) \ \tan \ \phi_1], \quad M = \cos \ \theta_1 \ \sin \ \alpha_{12}, \\ N = \cos \ \theta_1 \ \cos \ \alpha_{12}, \quad c_1 = fM, \quad c_2 = f(1-M^2)/4, \\ D = (1-c_2)(1-c_2-c_1M), \quad P = c_2[1+(1/2)c_1M]/D, \\ \sigma_1 = \text{qatn}(N, \sin \ \theta_1), \quad d = S/(a_eD), \\ u = 2(\sigma_1-d), \quad W = 1-2P \cos u, \quad V = \cos \ (u+d), \\ X = c_2^2 \sin \ d \cos \ d \ (2V^2-1), \quad Y = 2PVW \sin \ d, \\ \Delta\sigma = d + X - Y, \quad \Sigma\sigma = 2\sigma_1 - \Delta\sigma, \\ \alpha_{21} = \text{qatn}[-M, -(N \cos \Delta\sigma - \sin \ \theta_1 \sin \Delta\sigma)], \end{array}$$

 $K = (1-f) [M^{2} + (N \cos \Delta \sigma - \sin \theta_{1} \sin \Delta \sigma)^{2}]^{1/2},$ $\phi_{2} = \tan^{-1} [(\sin \theta_{1} \cos \Delta \sigma + N \sin \Delta \sigma)/K],$ $\Delta \eta = \operatorname{qatn}(\sin \Delta \sigma \sin \alpha_{12}, \cos \theta_{1} \cos \Delta \sigma - \sin \theta_{1} \sin \Delta \sigma \cos \alpha_{12}),$ $H = c_{1}(1-c_{2})\Delta \sigma - c_{1}c_{2} \sin \Delta \sigma \cos \Sigma \sigma,$ $\Delta \lambda = \Delta \eta - H, \quad \lambda_{2} = \lambda_{1} + \Delta \lambda$

In addition to the introduction of the gath function for proper quadrant determination the following changes have been made to the original algorithm:

- 1. θ_0 is no longer computed from the equation M = $\cos\theta_0$. Since P_1 is no longer required to be westerly of P_2 , α_{12} is no longer required to lie in the interval $[0, 180^{\circ}]$. Consequently M, which was originally required to be positive, can be negative. With this change θ_0 was no longer properly determined from M = $\cos\theta_0$.
- 2. With the elimination of θ_0 the determination of σ_1 from the equation $\cos \sigma_1 = \cos \theta_1/\sin \theta_0$ became impossible. The following figure shows the spherical triangle involving σ_1 and θ_0 .



Using the "4-parts formula" [Ref. 5, pg. 12] we can determine σ_1 directly from the equation

 $\sigma_1 = qatn(cos \theta_1 cos \alpha_{12}, sin \theta_1)$

3. The original equation for ϕ_2 ,

becomes indeterminate when $\alpha_{12}=0^{\circ}$ or 180° since both M and $\sin^{\alpha}\alpha_{21}$ become zero simultaneously. From the equation for α_{21} one can determine that $\sin^{\alpha}\alpha_{21}=-M/[M^2+(N\cos^{\alpha}\alpha-\sin^{\theta}\theta_1\sin^{\alpha}\alpha_2)^2]^{1/2}.$ Using this equation to eliminate $\sin^{\alpha}\alpha_{21}$ in the equation for $\tan^{\phi}\theta_2$ allows the M in the numerator and denominator to cancel thus eliminating the indeterminancy in the equation for Φ_2 .

 $\tan \phi_2 = -(\sin \theta_1 \cos \Delta \sigma + N \sin \Delta \sigma) \sin \alpha_{21}/(1-f)M$

With these changes the restrictions that P_1 be West of P_2 , and that if P_1 and P_2 both have negative latitude the symmetric positive latitude problem be solved, have been eliminated. Also eliminated are rules for the determination of the quadrants of α_{21} and Δn .

D. The Reverse (Inverse) Solution Algorithm

This reverse solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas [Ref. 2, pp. 8-10]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qath function is defined in Section B. West longitudes (λ) and

South latitudes (ϕ) are negative. We are given the points $P_1(\phi_1,\lambda_1)$, $P_2(\phi_2,\lambda_2)$ on the spheroid and are to find the distance S between the points and the forward and back azimuths, α_{12} and α_{21} . Given quantities are ϕ_1 , λ_1 , ϕ_2 and λ_2 . No assumptions about the relative location of P_1 and P_2 are required. The modified reverse solution algorithm is:

$$\begin{array}{l} \theta_{\bf i} = \tan^{-1}\{(1-f) \ \tan \ \phi_{\bf i}\}, \quad {\bf i} = 1,2, \\ \\ \theta_{\bf m} = (\theta_1 + \theta_2)/2, \quad \Delta \theta_{\bf m} = (\theta_2 - \theta_1)/2, \quad \Delta \lambda = \lambda_2 - \lambda_1, \\ \\ \Delta \lambda_{\bf m} = \Delta \lambda/2, \quad {\bf H} = \cos^2 \Delta \theta_{\bf m} - \sin^2 \theta_{\bf m} = \cos^2 \theta_{\bf m} - \sin^2 \Delta \theta_{\bf m} = \cos \theta_1 \cos \theta_2 \\ \\ {\bf L} = \sin^2 \Delta \theta_{\bf m} + {\bf H} \sin^2 \Delta \lambda_{\bf m} = \sin^2 (d/2), \\ \\ {\bf l} - {\bf L} = \cos^2 (d/2), \quad {\bf d} = \cos^{-1} (1-2{\bf L}), \\ \\ {\bf U} = 2 \sin^2 \theta_{\bf m} \cos^2 \Delta \theta_{\bf m}/(1-{\bf L}), \quad {\bf V} = 2 \sin^2 \Delta \theta_{\bf m} \cos^2 \theta_{\bf m}/{\bf L}, \\ \\ {\bf X} = {\bf U} + {\bf V}, \quad {\bf Y} = {\bf U} - {\bf V}, \quad {\bf T} = {\bf d}/\sin \ {\bf d}, \quad {\bf D} = 4{\bf T}^2, \\ \\ {\bf E} = 2 \cos \ {\bf d}, \quad {\bf A} = {\bf DE}, \quad {\bf B} = 2{\bf D}, \quad {\bf C} = {\bf T} - ({\bf A} - {\bf E})/2 \\ \\ {\bf n}_1 = {\bf X}({\bf A} + {\bf C}{\bf X}), \quad {\bf n}_2 = {\bf Y}({\bf B} + {\bf E}{\bf Y}), \quad {\bf n}_3 = {\bf D}{\bf X}{\bf Y}, \\ \\ \delta_1 {\bf d} = {\bf f}({\bf T}{\bf X} - {\bf Y})/4, \quad \delta_2 {\bf d} = {\bf f}^2({\bf n}_1 - {\bf n}_2 + {\bf n}_3)/64, \\ \\ {\bf S} = {\bf a}_{\bf e}({\bf T} - \delta_1 {\bf d} + \delta_2 {\bf d}) \ \sin \ {\bf d}, \quad {\bf F} = 2{\bf Y} - {\bf E}(4 - {\bf x}) \\ \\ {\bf M} = 32{\bf T} - (20{\bf T} - {\bf A}){\bf X} - ({\bf B} + {\bf 4}){\bf Y}, \\ \\ {\bf G} = {\bf f}{\bf T}/2 + {\bf f}^2 {\bf M}/64, \quad {\bf Q} = -({\bf F}{\bf G} \ \tan \Delta \lambda)/4, \\ \end{array}$$

 $\Delta \lambda_{m}' = (\Delta \lambda + Q)/2,$ $t_{1} = \operatorname{qatn}(-\sin \Delta \theta_{m} \cos \Delta \lambda_{m}', \cos \theta_{m} \sin \Delta \lambda_{m}'),$ $t_{2} = \operatorname{qatn}(\cos \Delta \theta_{m} \cos \Delta \lambda_{m}', \sin \theta_{m} \sin \Delta \lambda_{m}'),$ $\alpha_{12} = t_{1} + t_{2}, \quad \alpha_{21} = t_{1} - t_{2}$

The only changes made to the original algorithm are the computation of t_1 and t_2 and the determination of α_{12} and α_{21} as the sum and difference of t_1 and t_2 . Minor though these changes may seem, they have eliminated the requirement that P_1 is West of P_2 and they have eliminated four cases for the quadrant determination of α_{12} and α_{21} . Quadrant determination is still required, but it is implemented with the qath function; the qath function is available on the better handheld calculators in the form of the rectangular-to-polar function.

E. Program Accuracy

Several programs were written for the Commodore 2001 computer. One program implemented only the direct and reverse solution algorithms presented in the previous two sections. Initially these algorithms were validated against the long line computations with the Clarke 1866 spheroid model presented in Reference 2. Later the reverse solution algorithm was compared with the WGS 72 data for 40 Loran-C pairs contained in Reference 4 (data for the 9930 stations were not included). In these comparisons the algorithm error for the baseline distances was 0.16 meters high for the 7980W pair and 0.15 meters low for the 7980Y pair. For the remaining stations all of the baseline errors were high, but the largest error

was only 0.08 meters. It should be mentioned that the Microsoft BASIC for the Commodore 2001 carries 10 digits of floating point internally, but will display only a maximum of nine digits; the baselines reported in Reference 4 are printed to the nearest 0.0001 meters. The computed one way baseline time plus secondary phase correction was also computed and compared to the 40 Loran-C pairs data. In three cases the algorithm was high by 0.01 μ s, in three cases the algorithm was low by 0.01 μ s, and there was no error in the remaining cases.

Another program was used to generate the indicated time delay between Loran-C pairs and test positions for a receiver. Combining the time delay for Loran-C pairs into allowable triplets enabled the position fixing program in the Appendix to be evaluated. Typical results are presented in Table I. In all cases the Commodore 2001, running at BASIC interpreter speed required 8 seconds from entry of the time delay input to the display of the first pair of solutions. The improvement algorithm will improve only one user selected solution. Each iteration requires 14.4 seconds, and each improvement is displayed so that the user can monitor the improvement process. Usually the solution has stabilized after two iterations, but occasionally a third iteration is generated. In Table I the worst fix took six iterations for the triplet comprised of pairs 9930W and 9930X where the radial position error is 0.91 n.mi. In fairness to the algorithm it should be noted that the three stations are 776 n.mi, 2130 n.mi, and 2514 n.mi from the receiver and are all within 6° of azimuth as viewed from the receiver.

Table I

Receiver Location: 520N, 350W

Station Pair	Computed Delay	Station Pair	Computed Delay
20.20	20245 22	50000	14472 77
7970W 7970X	32845.12 18889.03	7930W 7930X	16673.77 30899.81
9930W	16067.09	7930x 7930z	49369.30
9930X	28012.61		

Triad	First Solution	Improved Solution	No. Iter.
7970W/7970X	52 ⁰ 00'14"N, 34 ⁰ 59'34"W	52 ⁰ 00'00".14N, 34 ⁰ 59'59".66W	2
9930W/9930X	51 ⁰ 56'13"N, 35 ⁰ 21'23"W	51 ⁰ 59'45".25N, 35 ⁰ 01'25".37W	6
7930W/7930X	52 ⁰ 00'01"N, 35 ⁰ 00'01"W	51 ⁰ 59'59".69N, 35 ⁰ 00'00".02W	2
7930W/7930Z	52 ⁰ 00'01"N, 35 ⁰ 00'01"W	52 ⁰ 00'00".03N, 35 ⁰ 00'00".07W	2
7930X/7930Z	52 ⁰ 00'01"N, 35 ⁰ 00'01"W	52 ⁰ 00'00".00N, 34 ⁰ 59'59".96W	2
7930X/7970W	52 ⁰ 00'04"N, 34 ⁰ 59'57"W	52 ⁰ 00'00".01N, 34 ⁰ 59'59".96W	2
7930z/9930X	51 ⁰ 59'52"N, 34 ⁰ 59'26"W	51 ⁰ 59'59".60N 34 ⁰ 59'58".27W	2

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APPENDIX. BASIC Program Listing

This appendix contains the listing of the BASIC position fixing program for the Commodore 2001. A number of the cursor control and formatting symbols on the listing require explanation. These symbols are given below by line number at first occurrence.

- 130 Reverse field 'heart' means 'clear the screen'
- 130 Reverse field 'Q' means 'move cursor down'
- 250 Reverse field 'R' means 'turn on field reverse'
- 250 Reverse field ' ' means 'turn off field reverse'
- Use of the field reverse symbols highlights the 'Y' in 'YES' and the 'N' in 'NO' to prompt the user to type a 'Y' or 'N' on the keyboard. Similar highlighting is used on lines 390, 790, 800 and 810
- 740 CHR\$(34) generates the double quote symbol to designate 'seconds of arc'
- 740 The symbol between the double quotes which are between N\$(1) and N\$(2) is used to designate ' O ', the 'degrees of arc' symbol
- 910 The first symbol following the double quote means 'move cursor up', and the next symbols each mean 'move cursor right'

The DATA statements on lines 50001 through 50044 contain the pertinent parameters for each Loran-C pair. The fields contain:

- 1. The Loran-C station pair designator.
- 2. Δt , the sum of the coding delay plus one way baseline time, including the secondary phase correction for an all seawater path, in microseconds.
- 3. The master station latitude.
- 4. The master station longitude.
- 5. The slave station latitude.
- 6. The slave station longitude.

Negative longitudes are West longitudes. The latitudes and longitudes appear to be in decimal form, but the actual format is DDD.MMSSFF where

DDD designates degrees,

MM designates minutes,

ss designates seconds,

and FF designates hundredths of seconds.

```
50001 DATA 4990X,15972.23,16.444395,-169.303120,20.144916,-155.530970
50002 DATA 4990Y,34253.18,16.444395,-169.303120,28.234177,-178.173020
50003 DATA 5930X,13131.88,46.482720,-067.553771,41.151193,-069.583909
50004 DATA 5930Y,28755.02,46.482720,-067.553771,46.463218,-053.102816
           5990X,13343.60,51.575878,-122.220224,55.262085,-131.151965
50005 DATA
50006 DATA 5990Y,28927.36,51.575878,-122.220224,47.034799,-119.443953
50007 DATA
           5990Z,42266.63,51.575878,-122.220224,50.362972,-127.212935
50008 DATA 7930W.15068.02,59.591727,-045.102747,64.542658,-023.552175
50009 DATA 7930X,27803.77,59.591727,+045.102747,62.175968,-007.042671
50010 DATA 7930Z,48212.20,59.591727,-045.102747,46.463218,-053.102816
50011 DATA 7960X,13804.45,63.194281,-142.483190,57.262021,-152.221122
50012 DATA 7960Y,29651.14,63.194281,-142.483190,55.262085,-131.151965
50013 DATA 7970W,30065.64,62.175968,-007.042671,54.482980,+008.173633
50014 DATA 7970X,15048.10,62.175968,-007.042671,68.380615,+014.274700
50015 DATA 7970Y,48944.53,62.175968,-007.042671,64.542658,-023.552175
50016 DATA 7970Z,63216.30,62.175968,-007.042671,70.545261,-008.435869
50017 DATA 7980W,12809.54,30.593874,-085.100930,30.433302,-090.494360
50018 DATA 7980X,27443.38,30.593874,-085.100930,26.315501,-097.500009
50019 DATA 7980Y.45201.88,30.593874,-085.100930,27.015849,-080.065352
50020 DATA 7980Z,61542.72,30.593874,-085.100930,34.034604,-077.544676
50021 DATA 7990X,12755.97,38.522061, 016.43<mark>05</mark>96,35.312088, 012.312996
50022 DATA 7990Y,32273.30,38.522061, 016.430596,40.582095, 027.520152
50023 DATA 7990Z,50999.69,38.522061, 016.430596,42.033649, 003.121590
50024 DATA 8970W,14355.11,39.510754,-087.291214,30.593874,-085.100930
50025 DATA 8970%,31162.06,39.510754,-087.291214,42.425060,-076.493386
50026 DATA 8970Y,47753.74,39.518754,-087.291214,48.364984,-094.331847
50027 DATA 9930W,13695.51,34.034604,-077.544676,27.015849,-080.065352
50028 DATA 9930X,36389.66,34.034604,-077.544676,46.463218,-053.102816
50029 DATA 9930Y.52541.31,34.034604,-077.544676,41.151193,-069.583909
50930 DATA 9930Z,68560.72,34.034604,-077.544676,39.510754,-087.291214
50031 DATA
          9940W,13796.90,39.330662,-118.495637,47.034799,-119.443953
50032 DATA 9940X,28094.50,39.330662,-118.495637,38.465699,-122.294453
50033 DATA 9940Y,41967.30,39.330662,-118.495637,35.191818,-114.481743
58834 DAT9 9960W.13797.28.42.425060.-076.493386.46.482720.-067.553771
50035 DATA 9960X,26969.93,42.425060,-076.493386,41.151193,-069.583909
50036 DATA 9960Y,42221.65.42.425060,-076.493386,34.034604,-077.544676
30037 DATA 9960Z,57162.06,42.425060,-076.493386,39.510754,-087.291214
50038 DATA 9970W,15283.94,24.48041 , 141.19290 ,24.17077 , 153.58515
50039 DATA 9970X,36685.12,24.48041 / 141.19290 /42.443700/ 143.430906
50040 DATA
           9970Y,59463.18,24.48041 , 141.19290 ,26.362499, 128.085621
           9970Z,80746.79,24.48041 , 141.19290 ,09.324566, 138.095523
59041 DATA
50042 DATA 9990X,14875.32,57.090988,-170.145981,52.494505,+173.105231
50043 DATA 9990Y,32069.09,57.090988,-170.145981,65.144012,-166.531447
50044 DATA 9990Z,46590.10,57.090988,-170.145981,57.262021,-152.221122
```

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DIM ID$(NP),DI(NP),PS(2,2,NP):REM 1ST ARG=LAT,LONG. 2ND ARG=(M),(S). 3RD=1D
                                                                                                                                                                                                                                                                                                                                                                 P=n+n:RD=n/180:DEFFNS(X)=SIN(X*RD):DEFFNC(X)=COS(X*RD):DEFFNT(X)=TAN(X*RD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    IMPUT"Ø 15T ID",M$(1) FORI1=17ONP IFID$(11)=M$(1)6070180
NEXTIT PRINT"Ø THAT ID IS NOT IN THE CATALOG. PLEASE REENTER.":6070160
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                INPUT" # 2ND ID"; M*(2): FORIZ=1TONP: IFID*(12)=M*(2)GOTO200
MEXTIZ: PRINT" # THAT ID IS NOT IN THE CATALOG. PLEASE REENTER.": GOTO180
PRINT": INDM"; TAB(6); "YOUR STATION ID'S ARE: MA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            FORI=1TOMP:READID$(1),DT(1):FORK=1TO2:FORJ=1TO2:READPS(J,K,I):NEXTJ,K,I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 REM DETERMINE CO-LOCATED STATIONS
IFPS(1,1,11)=PS(1,1,12)ANDPS(2,1,11)=PS(2,1,12)THENL1=1:L2=1:GOTO430
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     CC IN M/SEC
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DEFFNM(X)=X-360*INT(X/360):DEFFNR(X)=INT(100*X+.5)/100
FL=1/298.26:RE=6378135:CC=299792.458E3:REM RE IN METERS,
RE=RE/CC*1E6:REM RE IS IN MICROSECONDS
IR=1.000338:REM INDEX OF REFRACTION OF RIR
GM=.000634:REM SECONDARY PHASE RPPROXIMATION
               REM LORAN NAVIGATION: 01-13-80, REV 03-18-80, R. SHUDDE NP=44:REM NUMBER OF LORAN PAIRS ABOARD.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              OK... 6010388
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DEFFNQ(X)=2.7412979/X-.011402+3.2774624E-4#X
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     DEFFNP(X)=129.04398/X-.40758+6.4576438E-4*X
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        PRINT MOMOINPUT THE ID'S OF THE LORAN PRIRS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         FORI=1102 PRINTIAB(13);M#(I):NEXTI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       PRINTIAB(8), "MARKY CORRECTIONS?"
PRINTIAB(11), "MARKES OR AMED.
REM LORAN NAVIGATION: 01-13-80.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              PRINT"IN THE TRIPLET.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         GETG# : IFG#=""GOT025@
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   IDX(1)=11:IDX(2)=12
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       (FG###1711] [ FG###171] [ FG###TT] [ FG##TT] [ FG##TT]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 RX=1+0M: BY=REXIR
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\begin{array}{cccc}
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u'_3 & u'_4 & u'_4 & u'_4 & u'_4 \\
u'_4 & u'_4 & u'_4 & u'_4 & u'_4 \\
u'_5 & u'_5 & u'_5 & u'_5 & u'_4 \\
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HX=0:K1=0:FORKZ=1TOZ:GOSUB1150:C2(K2)=S2/AE:AZ(K2)=A(1):1X=IDX(K2)
LB(K2)=100*INT((DT(1X)-S2)/100+.5):UB(K2)=LB(K2)+2*(DT(1X)-LB(K2)):NEXTK2
REM COMPUTE TWO SOLUTIONS
                                                                                                                                                                                                                                                                                                                            L=PS(1,1,1);60SUB1298:PH(X)=L:L=PS(2,1,1);60SUB1298:LM(X)=L:X=X+1:NEXTI
IFPS(1,2,11)=PS(1,2,I2)ANDPS(2,2,I1)=PS(2,2,I2)THENL1=-1:L2=-1:GOT0438 IFPS(1,1,I1)=PS(1,2,I2)THENL1=-1:L2=-1:GOT0438 IFPS(1,1,I1)=PS(1,2,I2)ANDPS(2,1,I1)=PS(2,2,I2)THENL1=1:L2=-1:GOT0438 PRINT "JUNNANOVOR LORAN STATIONSMA"
                                                                                         FORI=1702:PRINTTAB(13):M$(1):NEXTI:PRINT"MMDO NOT FORM A LORAN TRIPLET
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Q2(1)=L1*(IT(1)-DT(I1))/RY:Q2(2)=L2*(IT(2)-DT(I2))/RY
R2(1)=Q2(1)/RX:R2(2)=Q2(2)/RX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     PRINT"INPUT TIME FOR ";ID$(12);:INPUTIT(2)
IFLB(2)<*IT(2)ANDIT(2)<=UB(2)GOTO590
PRINTIT(2);"IS NOT VALID, PLEASE RE-ENTER.W":GOTO560
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 PRINT"INPUT TIME FOR ";ID$(11);:INPUTIT(1)
IFLB(1)<≈IT(1)ANDIT(1)<=UB(1)GOTO560
PRINTIT(1);"IS NOT VALID, PLEASE RE-ENTER.W":GOTO530
                                                                                                                                       新 70 CB.
                                                                                                                                                                                                                                                          REM SET UP TRIPLET AND COMPUTE BASELINES
                                                                                                               PRINT"XXXE CANNOT CONTINUE.
PRINT"XXPRESS XC™ TO CONTINUE OR
GETG$:IFG$=""GOTO380
                                                                                                                                                                                                                                                                                                                                                                          L=PS(1,K1,I2):GOSUB1298:PH(2)=L
L=PS(2,K1,I2):GOSUB1298:LM(2)=L
                                                                                                                                                                                                                                                                               K1=1:K2=2:IFL1<0THENK1=2:K2=1
                                                                                                                                                                                                                                                                                                   X=0:FORI=K1TOK2STEPK2-K1
                                                                                                                                                                                                                                                                                                                                                K1=2: IFL2<6THENK1=1
                                                                                                                                                                                   |FG$="C"60T0138
|FG$<>"E"60T0388
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FORI=1102:B(I)=COS(A2(I)>-COS(C2(I)>:C(I)=SIN(C2(I)>:A(I)=SIN(A2(I))
                                                                                                                                                                                                              REM TN=SP/SQR(1-SP*SP):TL(1)=ATN(TN):LT=ATN(TN/(1-FL))/RD
REM Y=SIN(R)*SIN(NU):X=COS(TH(0))*COS(R)-SIN(TH(0))*COS(NU)*SIN(R)
REM GOSUBI0400:LN=AT/RD+LM(0)
                                                                                                                                                                                                                                                                                                   SL(1,I)=P2:A=P2:GOSUB1480:NG$=" N":IFNS$="-"THENNQ$=" S"
PRINT" LATITUDE = ";N$(1);""";N$(2);"'";N$(3);CHR$(34);NQ$
SL(2,I)=Z2:A=Z2:GOSUB1480:NQ$=" W":IFNS$=" "THENNQ$=" E"
                                                                                                                                                                                                                                                                                                                                                                  LONGITUDE = ").N#(1);""";N#(2);""";N#(3);CHR#(34);NQ#
                                                                                                     Y=SQR(LR*LA-BK*BK):X=BK:GOSUB1460:KL=AT
SN=-1:FORI=1TO2:SN=-SN:NU=RL+SN*KL
Y=B(1):X=C(1)*COS(NU-AZ(1))+A(1):GOSUB1460:R=AT:R(1)=R
                                                                                                                                                                   TH=TH(0):LM=LM(0):AZ=NU/RD:S=R*AE:GOSUB1330
REM SP=SIN(TH(0))*COS(R)+COS(TH(0))*SIN(R)*COS(NU)
                                                                                 LA=SQR(BC*BC+BS*BS):Y=BS:X=BC:GOSUB1460:AL=AT
                BC=B(1)*C(2)*COS(AZ(2))-B(2)*C(1)*COS(AZ(1))
BS=B(1)*C(2)*SIN(AZ(2))-B(2)*C(1)*SIN(AZ(1))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  REM COMPUTE IMPROVED SOLUTION BY ITERATION PRINT MIMPROVE SOLUTION ALE OR ACET
                                                                                                                                                                                                                                                                                                                                                                                                                               PRINT MANIEW PROVE SOLUTION, PRINT MANIEW STATIONS OR PRINT MANIEWE STATIONS?
                                                            BK=B(2)*E(1)-B(1)*E(2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               GETC#: IFC#=""GOT0840
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              6ETC#: IFC#=""GOT0988
                                                                                                                                                                                                                                                                                                                                                                                                             REM DISPLAY OPTIONS
                                                                                                                                                                                                                                                                               PRINT"XSOLUTION";I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             FC#(\"I"G0T084Ø
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    FC#="S"6010528
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        FC#="N"60T0138
                                                                                                                                                                                                                                                                                                                                                                  PRINT"
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TM=(TH(K1)+TH(K2))/2:DT=(TH(K2)-TH(K1))/2:DL=(LM(K2)-LM(K1))*RD:DM=DL/2 H=COS(DT)12-SIN(TM)12:L=SIN(DT)12+H#SIN(DM)12:SD=2#RTN(SGR(L/(1-L))) U=2*((SIN(TM)*COS(DT))12:(1-L):V=2*((SIN(DT)*COS(TM))12:/L X=U+V:Y=U-V:T=SD/SIN(SD):D=4#T#T:E=2*COS(SD):R=D#E:B=D+D FORK1=1T02:60SUB1150:P(K1,2)=(P(K1,2)+S2/AE-A2(K1)-F(K1))/DR:NEXTK1 FORK1 = 1702 : GOSUB1150 : P(K1, 1) = (P(K1, 1) + S2/9E - P2(K1) - F(K1)) / DR : NEXTK1FORK1=1T02:60SUB1150:R2(K1)=02(K1)-(P2-P0)/RY:F(K1)=S2/RE-R-R2(K1) IFJK=0THENPRINT"AJACOBIAN = 0. CANNOT IMPROVE":60T0810 DP=(F(1)*P(2,2)-F(2)*P(1,2))/JK'DL=(F(1)*P(2,1)-F(2)*P(1,1))/JK PRINT" LATITUDE = ".N\$(1);""";N\$(2);"'";N\$(3);CHR\$(34);N0\$ A=LM(3):GOSUB1480:NQ\$=" W":IFNS\$=" "THENNQ\$=" E" LONGITUDE = ";N\$(1);""";N\$(2);"";N\$(3);CHR\$(34);NQ\$ FORI=KITOK2STEPK2-K1:TH(I)=ATN((1-FL)*FNT(PH(I))):NEXTI A=PH(3):60SUB1480:N0\$=" N":IFNS\$="-"THENNO\$=" S" C=T-(B-E)/2:N1=X*(B+C*X):N2=9*(B+E*V):N3=D*X*V LM=LM(0):PH(3)=SL(1,C%):LM(3)=SL(2,C%):DR=1E-4 K1=0:K2=3:GOSUB1150:R=S2/AE:PQ=P2 PH(3)=PH(3)-DR:LM(3)=LM(3)+DR K1=0:60SUB1150:P(1,2)=-52/AE:P(2,2)=-52/AE IFABS(DP)>DRORABS(LD)>DRTHENPRINT G0T0950 K1=0:COSUB1150:P(1,1)=-52/RE:P(2,1)=-52/REJK=P(1,1)*P(2,2)-P(1,2)*P(2,1) PH(3)=PH(3)-DP:LM(3)=LM(3)+DL REM REVERSE SOLUTION FC\$<>"2"G0T0988 LM(3)=LM(3)-DR PH(3)=PH(3)+DR GOT0818 PRINT" 0 0 0 0 0 0 0 0 0 010 838 848 166 116 999 020 888 898 986 928 965 978

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RI=RIK(Y/(X-1E-9*(X=0)))-**(X/0):RETURN:REM QIRN(Y,X) FUNCTION
                                                                                                                                                                                                                                                                                                                                                                                                    K=COS(TH)*FNC(6Z):C1=FL*M:C2=FL*(1-M*M)/4:D=(1-C2)*(1-C2-C1*M)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DOMESTIC TOTAL CONTRACT TO SECURITY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              P2=ATN((SIN(TH)*COS(DS)+N*SIN(DS))/SQR(X*X+Y*Y)/(1-FL))
Y=SIN(DS)*FNS(AZ):X=COS(TH)*COS(DS)-SIN(TH)*SIN(DS)*FNC(AZ)
                                                                                                                                                                                                                                                                                                                                                                                                                            P=C2*(1+C1*M/2)/D:Y=N:X=SIN(TH):GOSUB1460:SG=AT
SD=S/(AE*D):SU=2*(SG-SD):W=1-2*P*COS(SU):V=COS(SU+SD)
X=C2*C2*SIN(SD)*COS(SD)*(2*V*V-1)
                                                                                                                                                        V=-SIN(DI)*COS(DP):X=COS(TM)*SIN(DP):GOSUB1460:T1=RT
                                                                                                                                                                                  7#COS(D1)*COS(D2):X=SIN(TM)*SIN(D2):GOSUB1460:T2=81
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Y=2*P*V*L*SIN(SD):DS=SD+X-Y:SS=SG+SG-DS
Y=-A:X=-(N*COS(DS)-SIN(TH)*SIN(DS)):GOSUB1460:R2=RT
                                                                                                                               G=FL#T/2+FL#FL#M/64:Q=-/F#G#TRN(DL))/4:DP=(DL+Q)/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           H=C1*(1-C2)*DS-C1*C2*SIN(DS)*COS(SS)
DL=DN-H:22=LM+DL/RD:P2=P2/RD:1F22>180THEN22=22-360
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           4 % NE
                                                                                                                                                                                                                                                               S=SGN(L):L=ABS(L)+1/36E6:M1=INT(L):M2=100*(L-M1)
                        SN=RE#SIN(SD): S2=SN#(T-D1+D2): JFHN=060101230
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           N(2)=INT(B):N(3)=INT(60000*(B-N(2)))/1E3
                                                                                                     4*(++0)-X*(0-1*00)-1*00=5:(X-+)*0-4+6=6
D1=FL*(1*X-Y)/4:D2=FL*FL*(N1-N2+N3)/64
                                                PZ=FNP(S2): IFS2<537THENPZ=FNQ(S2)
                                                                                                                                                                                                              A(1)=(T1+T2):A(2)=(T1-T2):RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     N$(1)=RIGHT$(STR$(1000+N(1)),3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               N$(2)=RIGHT$(STR$(1000+N(2)),2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           N$(3)=MID$(STR$(1000+N(3)),4.6)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        L=S*((M4/60+M3)/60+M1) RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  N(1)=INI(B):B=60*(B-N(1))
                                                                                                                                                                                                                                                                                             M3=INT(M2): M4=100*(M2-M3)
                                                                                                                                                                                                                                                                                                                                              REM DIRECT SOLUTION
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              IFE<-180THENE=8+360
                                                                                                                                                                                                                                                                                                                                                                        M=COS(TH)*FNS(RZ)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    G0SUB1460: DN=RT
                                                                                                                                                                                                                                        REM DMS TO DEG
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   RETURN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       510
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