Exoplanets: Interiors & Atmospheres 2020
Practical Assignment

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1 Introduction

For the practical assignment of the course Exoplanets: Interiors & Atmospheres we will simulate the evolution of the interiors of 10 low mass planets using the MESA software (Paxton et al., 2013). After installing the software, we will simulate the 10 planets according to the assignment instruction, this will be briefly discussed in the methods section, section 2. From here we will be able to generate 4 figures that will show:

1. The evolution of the radius of each planet with time.
2. The final mass vs. the final radius of all the 10 planets, including some real exoplanets obtained from www.exoplanets.org or exoplanet.eu.
3. The final radiative gradient and the adiabatic gradient vs. the radius for each planet.
4. The final temperature as a function of pressure for all planets.

These 4 plots will be shown and discussed in the results, section 3. The report will be concluded with section 4.

2 Methods

The simulation of the 10 planets can be divided in 5 parts:

1. **Creating the planet**. The planets are initialised without a core and with a mass of $30M_{\odot}$, and a radius of $3M_{\mathrm{Jup}}$.

2. **Adding a core to the planet**. Assuming the core has the same composition as the Earth, 5 different planets are created in this step, each with a different core mass, see table 1. Using the relation (Valencia et al., 2006):

$$\frac{R}{R_{\oplus}} = \left(\frac{M}{M_{\oplus}}\right)^{0.27} \tag{1}$$

the density can be obtained, which is also shown in table 1.

Table 1: The computed core densities for the 5 different core masses.

Core Mass ($M_{\oplus}$)	Density (g/cm^3)
3	6.801
5	7.494
7	7.989
10	8.549
12	8.851

3. **Reducing the mass of the planets**. Because we want to end with a planet with lower mass, the mass will be reduced. The reduction will not be in the core, but in the envelope. For all the 5 planets currently simulated, two mass fraction values are explored:

$$f_{\mathrm{env}} = \frac{M_{\mathrm{env}}}{M_{\mathrm{p}}} = 0.01 \text{ and } 0.1 \tag{2}$$

This will result in the 10 planets mentioned in section 1. Using equation 2 and $M_p = M_{core} + M_{env}$, the different final masses of the planets can be calculated, see table 2. The obtained formula equals:

$$M_p = \left(1 + \frac{f_{env}}{1 - f_{env}}\right) M_{core} \tag{3}$$

Table 2: The computed final planet masses for the 5 different core masses and 2 different mass envelope fractions.

Core Mass ($M_\oplus$)	M_p for $f_{env} = 0.01(M_\oplus)$	M_p for $f_{env} = 0.1(M_\oplus)$
3	3.030	3.333
5	5.051	5.556
7	7.071	7.778
10	10.101	11.111
12	12.121	13.333

4. **Setting the initial entropy at the base of the envelope**. The initial conditions at the base of the envelope are changed. An entropy $S = 9$ kB/baryon at $t = 0$ is ued.

5. **Evolving the planet**. In the final step of the simulation the planets will evolve over a period of 5 Gyr. Using a core energy generation rate $\epsilon_{core} = 5 \cdot 10^{-8}$ erg/g/s, the luminosity of the core can be calculated. Since the units of the luminosity are an amount of energy per unit of time, the luminosity can simply be calculated by multiplying ϵ_{core} with M_{core}. The results are shown in table 3.

Table 3: The computed core densities for the 5 different core masses.

Core Mass ($M_\oplus$)	L_{core} (erg/s)
3	$8.9464 \cdot 10^{20}$
5	$1.4941 \cdot 10^{21}$
7	$2.0917 \cdot 10^{21}$
10	$2.9882 \cdot 10^{21}$
12	$3.5858 \cdot 10^{21}$

Now that all planets are evolved, the simulated data will be visualized in the next section.

3 Results

3.1 Evolution of the radius over time

Figure 1 show the evolution of the radius of each planet over time. On the horizontal axis the time is displayed, which stops at approximately 5 Gyr. On the vertical axis the radius is displayed in units of Earth radii. As expected, the planets with $f_{env} = 0.1$ have a larger radius. Something that may seem a bit odd at first sight, is that the planets with the smallest mass have the largest radii at first and after evolving they shrink to the smallest. This can be explained by the fact the all planets are initialized with the same amount of energy. In the beginning planets are a hot ideal gas, and being initialized with the same amount energy means that the planets with the lowest mass will have the largest radii.

3.2 The final mass vs. the radius

Figure 2 shows the final mass as function of the final radius of each planet. So, this data comes from the final step in the evolution part of the simulations. The mass is shown on the horizontal axis in units of Earth mass and the radius on the vertical axis in units of Jupiter radii. Six other planets were added from the site www.exoplanets.org. These planets have similar masses and radii, so that the figure could maintain its original size. What becomes clear from this figure is that, the more massive the planet, the larger the radii. Also, planets with the higher envelope mass fraction have higher radii.
As was explained in lecture 3, there is mass-radius relation for planets, depending on the polytropic index, $P = K\rho^{1+\frac{1}{n}}$. The polytropic index $n = 0$ corresponds to an ideal gas, and $n = 1.5$ to a degenerate gas. Choosing $n = 0$ results in the mass scaling positively ($R \propto M^{\frac{1}{5}}$), and $n = 1.5$ gives $R \propto M^{\frac{-1}{3}}$. This means that if the mass becomes larger, the radius becomes smaller. Planets with low mass or are very young the radius increases with mass. Planets that are evolved or have a very high mass, have a degenerate gas, so the the radius

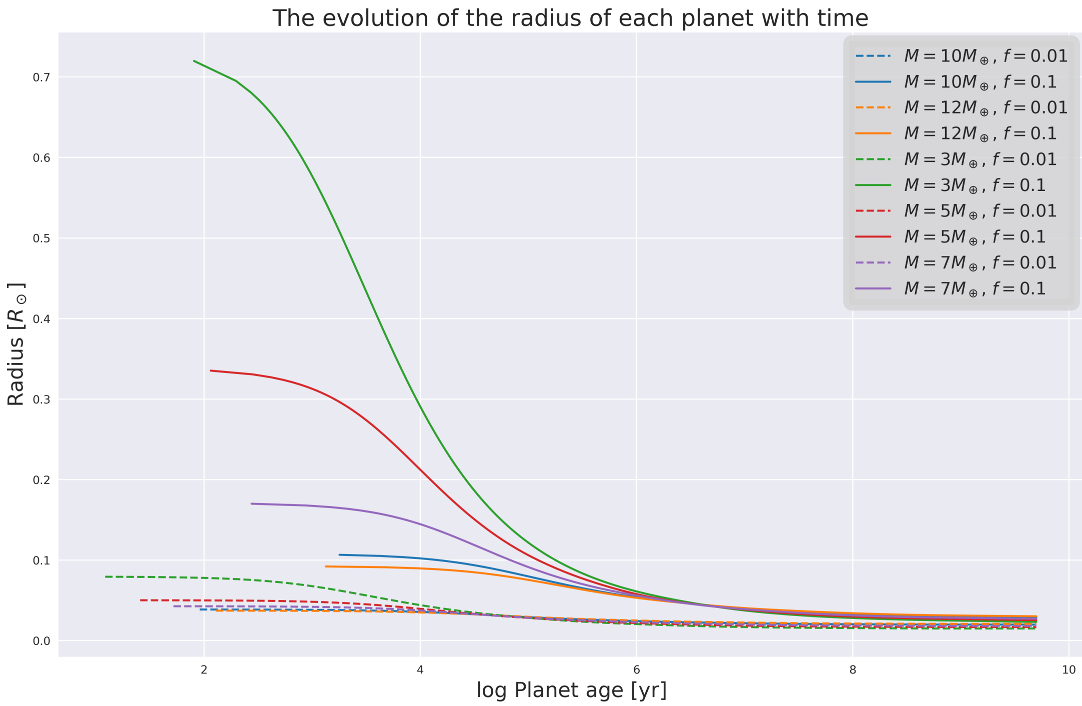

Figure 1: Evolution of the radius over time, **x-axis**: time, **y-axis**: radius.

decreases with mass. From figure 2 we see that the radius does increase with mass, so the planets are best approximated by an ideal gas and not a degenerate gas. So they do not reach degeneracy pressure during their evolution.

3.3 The final gradients vs. the radius

Figure 3 shows the final radiative and adiabatic gradients as function of the radius for each of the 10 planets. On the horizontal axis the radius is shown in Earth radii and on the vertical axis the gradients. Since this figure contains 20 data points, the legend is reduced so that the figure becomes more clear. There are only 5 different colours, each corresponding to one of the masses, and 4 different linestyles, 2 for the different mass envelope fractions and 2 for the different gradients.
For all the 10 different planets, $\nabla_{\rm rad} > \nabla_{\rm ad}$ for almost 100 percent in the interior of the planet. Only very close to the surface, $\nabla_{\rm rad} < \nabla_{\rm ad}$. A layer is stable against convection of $\nabla_{\rm rad} < \nabla_{\rm ad}$ (Pols, 2011), which means convection is the primary energy transport mechanism in the interior of the planets. Only very close to the surface where $\nabla_{\rm rad}$ decreases rapidly and $\nabla_{\rm ad}$ increases steadily, this is not the case.

3.4 The final temperature vs. the pressure

Figure 4 shows the final temperature as function of pressure for all the 10 planets. On the horizontal axis the pressure is shown which functions as the depth parameter, which means the r-coordinate in the spherical coordinate system. On the vertical axis the temperature is shown in Kelvin. Since some of the lines are very close together, the ending points have been marked with a large dot.
At the surface of the planet where the pressure is zero, the temperature is slightly above 0 Kelvin. For all the planets, the temperature increases rapidly when going deeper into the planet's interior. As expected the the most massive planets with the highest envelope mass fraction reaches the highest pressure and the highest temperature.

4 Conclusion

References

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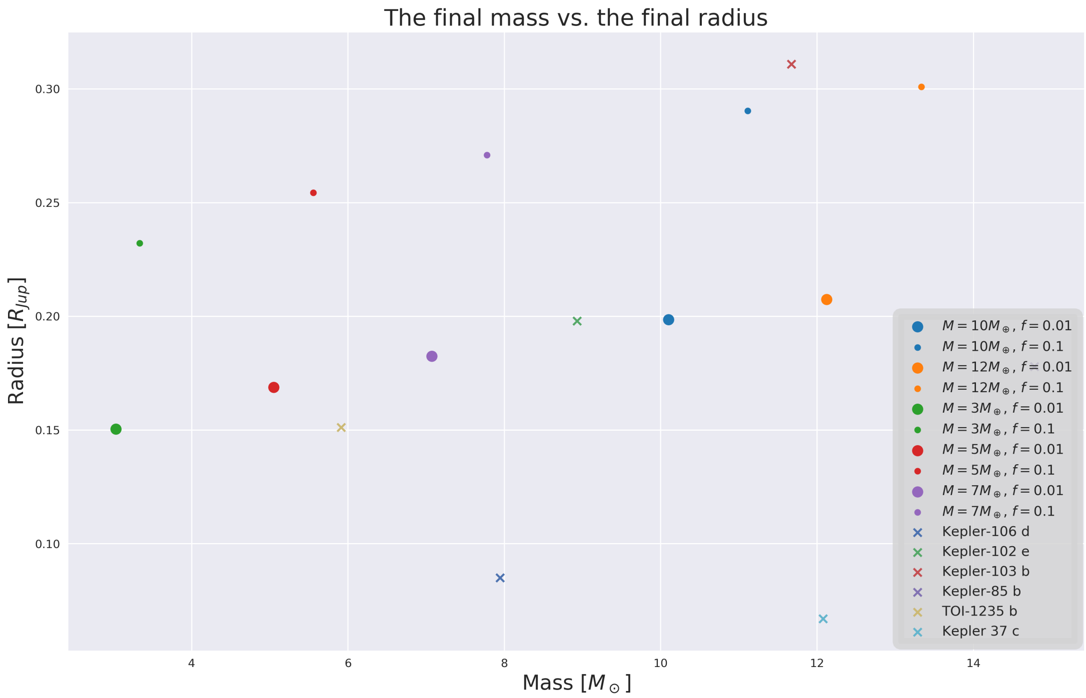

Figure 2: The final mass as function of radius, **x-axis**: mass, **y-axis**: radius.

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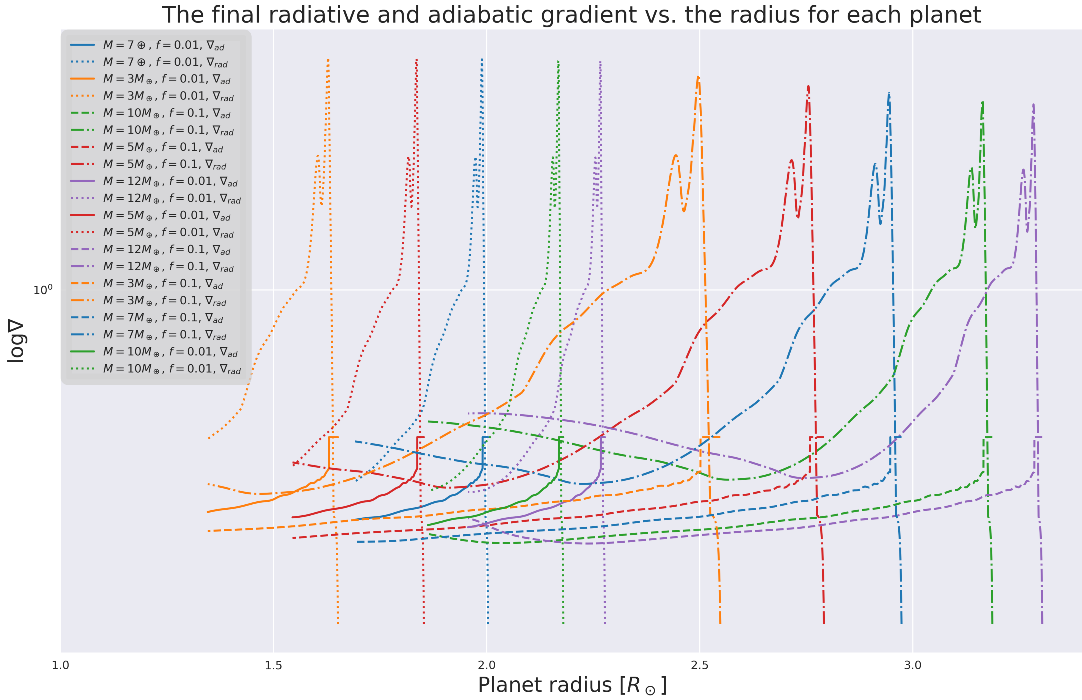

Figure 3: The final gradients as function of radius, **x-axis**: radius, **y-axis**: gradients.

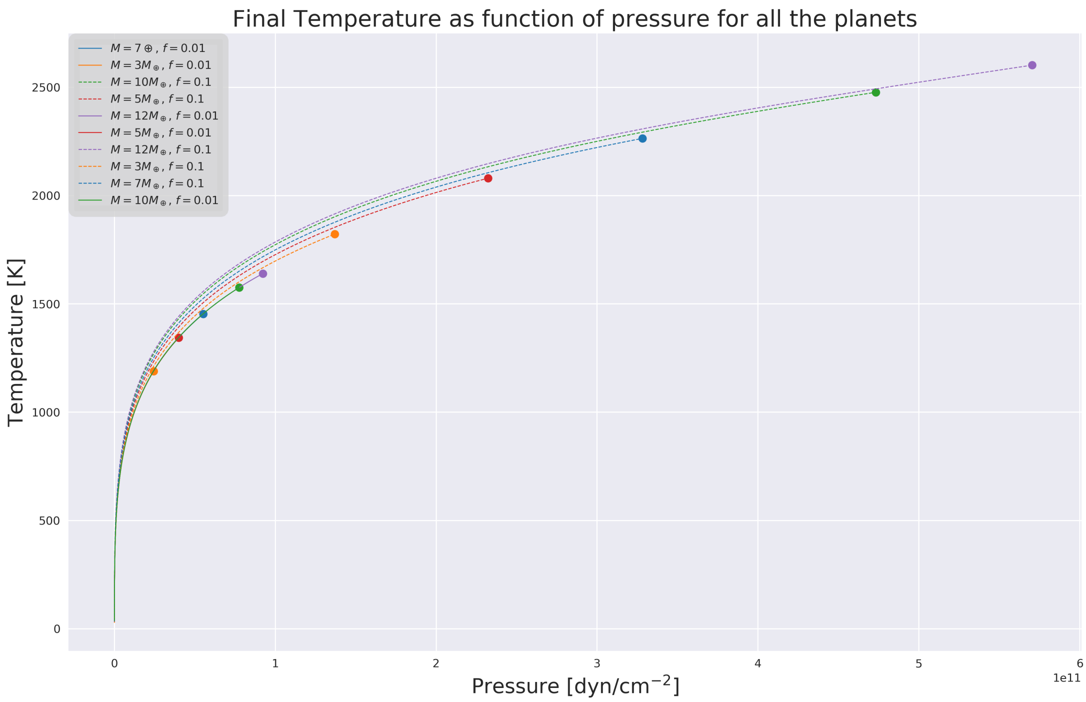

Figure 4: The final temperature as function of pressure, **x-axis**: pressure, **y-axis**: temperature.