Interstellar Medium 2020

Lecture 11: Molecular clouds and their properties

Paul van der Werf

Course Contents

Today’s lecture

Molecular clouds and their properties

- Measuring molecular cloud masses
- Photon trapping
- Molecular cloud masses from CO(1-0)
- Real molecular clouds

Corresponding textbook material: Draine Ch. 19 & 32

Possible ways to measure molecular gas masses

- ^{12}CO (1–0) line emission
- 1-0 line emission of other CO isotopologues (^{13}CO, $C^{18}O$ etc.)
- Dynamical masses
- Optically thin dust emission

Column density from optically thin emission

Recapitulation from Lecture 2

In the optically thin case, the Equation of Transfer reduces to

$$I_\nu = \tau_\nu B_\nu(T_{\mathrm{ex}})$$

The optical depth is given by

$$\tau_\nu = N_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) \varphi_\nu = \frac{N_l}{\sigma_v} \frac{g_u}{g_l} \frac{c^3}{8\pi\sqrt{2\pi}\nu_{ul}^3} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) e^{-\frac{v^2}{2\sigma_v^2}}$$

for a Gaussian line with velocity dispersion σ_v

Conclusion: if the line is optically thin, it measures column density (note: for CO typically $T_{ex} \approx$ 8K but you can also get it by measuring multiple lines, e.g., not only 1-0 but also 2-1)

Optical depth of ^{12}CO(1-0)

Line parameters: CO J = 1-0

ν = 115 GHz (λ =2.6 mm)

$A_{10} = 6.78 \cdot 10^{-8}\ \mathrm{s}^{-1}$

For a typical molecular cloud: $n_{\mathrm{H}} = 10^{3}\ \mathrm{cm}^{-3}$

$R = 10^{19}$ cm (≈ 3 pc)

$n(\mathrm{CO})\ /\ n(\mathrm{H_2}) = 7 \cdot 10^{-5}$ (25% of C in CO)

T_{ex} = 8 K

Then we find a peak optical depth (optical depth at line centre)

$$\tau_0 = \frac{N_l}{\sigma_v}\frac{g_u}{g_l}\frac{c^3}{8\pi\sqrt{2\pi}\nu_{ul}^3}A_{ul}\left(1-e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) = 46\,\frac{n_{\mathrm{H}}}{1000\ \mathrm{cm}^{-3}}\ \frac{R}{10^{19}\mathrm{cm}}\ \frac{n_{\mathrm{CO}}/n_{\mathrm{H}}}{7\cdot 10^{-5}}\ \frac{1.4\ \mathrm{km\ s}^{-1}}{\sigma_v}$$

So the CO(1-0) line will be quite optically thick!

Peak optical depth

Note that $\tau_0 \propto \frac{N(\mathrm{H_2})}{\sigma_v}[\mathrm{CO}]/[\mathrm{H_2}]$ where [CO] / [H_2] is the abundance of CO with respect to H_2.

Note the dependence on line width (will become important later).

Rarer CO isotopologues (^{13}CO, $C^{18}O$, $C^{17}O$, $^{13}C^{18}O$ etc,) will be optically thin, so they measure H_2 column density and mass if you know their abundance.

What if the line is optically thick?

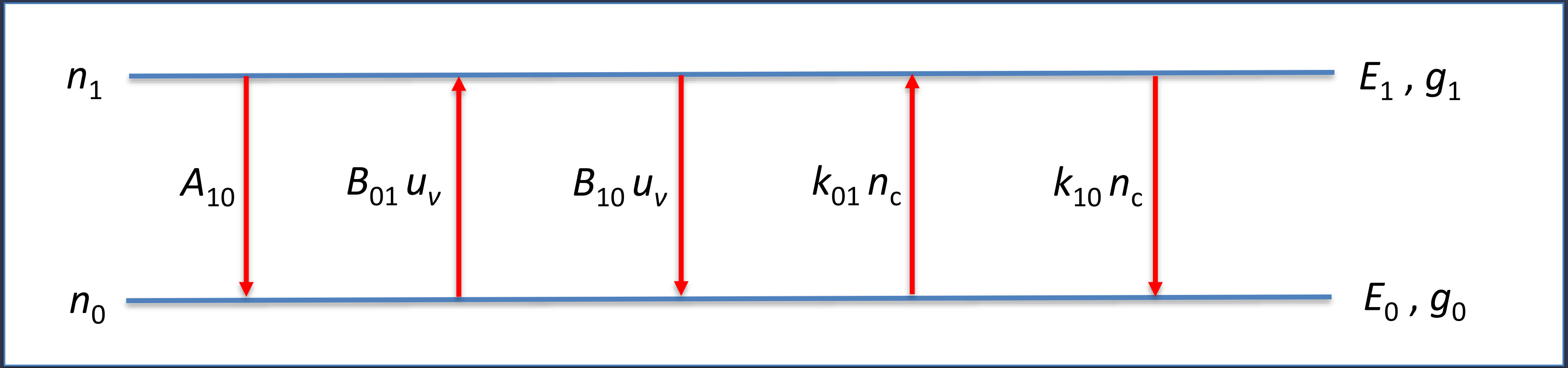
To answer this, look in detail at excitation and radiative transfer (For simplicity, take a 2-level system with levels 1 and 0).

Today's lecture

Molecular clouds and their properties

- Measuring molecular cloud masses
- Photon trapping
- Molecular cloud masses from CO(1-0)
- Real molecular clouds

Optically thick line in a 2-level system



So:

$$\frac{dn_1}{dt} = n_0\left(k_{01}n_c + \bar{n}_\gamma \frac{g_1}{g_0} A_{10}\right) - n_1\left[k_{10}n_c + (1+\bar{n}_\gamma)A_{10}\right]$$

collisional excitation

absorption

collisional deexcitation

stimulated emission

spontaneous emission

This can be rearranged to:

$$\frac{dn_1}{dt} = n_c k_{01} n_0 - n_c k_{10} n_1 - A_{10} n_1 + n_0 \frac{g_1}{g_0} A_{10} \bar{n}_\gamma \left(1 - \frac{g_0 n_1}{g_1 n_0}\right)$$

Excitation and radiative transfer

$$\frac{dn_1}{dt} = n_c k_{01} n_0 - n_c k_{10} n_1 - A_{10} n_1 + n_0 \frac{g_1}{g_0} A_{10} \bar{n}_\gamma \left(1 - \frac{g_0 n_1}{g_1 n_0}\right)$$

Excitation depends on n_γ (photon occupation number), but this follows from the Equation of Transfer. But the optical depth (which goes into the Equation of Transfer) will depend on the excitation.

So: for optically thick lines, excitation and radiative transfer are coupled in a complicated and non-local way.
Extremely nasty to solve (requires Monte Carlo codes)

NB: if optically thin, no need to solve Equation of Transfer, and solving excitation is then simple.

Solution: escape probability formalism

Define β_ν as the probability that a photon of frequency ν escapes from the cloud, then averaging over direction:

$$\bar{\beta}_\nu(\vec{r}) = \frac{1}{4\pi}\int e^{-\tau_\nu(\vec{r},\hat{n})}d\Omega$$

Now also average over the line profile: this gives the probability that a photon emitted in a particular spectral line will escape:

$$\langle\bar{\beta}(\vec{r})\rangle = \frac{\int \bar{\beta}_\nu(\vec{r})\varphi_\nu d\nu}{\int \varphi_\nu d\nu} = \int \bar{\beta}_\nu(\vec{r})\varphi_\nu d\nu$$

Two approximations

1. Excitation is uniform → single T_{ex}

2. If a photon does not escape, it is absorbed on the spot

These two approximations turn the non-local problem into a local problem.

Solving the local problem (1)

Using Approximation 1, the Equation of Transfer becomes

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{\rm ex})(1 - e^{-\tau_\nu})$$

where $I_\nu(0)$ is the incident radiation field.

Using $n_\gamma(\nu) = \frac{c^2}{2h\nu^3} I_\nu$ and $\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{h\nu_{10}}{kT_{ex}}}$ this becomes

$$n_\gamma(\nu) = n_\gamma^{(0)} e^{-\tau_\nu} + \frac{1 - e^{-\tau_\nu}}{\frac{g_1 n_0}{g_0 n_1} - 1}$$

where

$$n_\gamma^{(0)} = \frac{c^2}{2h\nu^3} I_\nu(0)$$

Solving the local problem (2)

Because of Approximation 2, replace $e^{-\tau_\nu}$ by the direction-averaged escape probability.

This yields
$$\bar{n}_\gamma(\nu) = n_\gamma^{(0)} \bar{\beta}_\nu + \frac{1 - \bar{\beta}_\nu}{\frac{g_1 n_0}{g_0 n_1} - 1}$$

Now also average over the line profile:
$$\bar{n}_\gamma = n_\gamma^{(0)} \langle \bar{\beta} \rangle + \frac{1 - \langle \bar{\beta} \rangle}{\frac{g_1 n_0}{g_0 n_1} - 1}$$

This can now be substituted into the expression we had for dn_1/dt.

Photon trapping

We had the expression:

$$\frac{dn_1}{dt} = n_c k_{01} n_0 - n_c k_{10} n_1 - A_{10} n_1 + n_0 \frac{g_1}{g_0} A_{10} \bar{n}_\gamma \left(1 - \frac{g_0 n_1}{g_1 n_0}\right)$$

The new expression becomes:

$$\frac{dn_1}{dt} = n_c k_{01} n_0 - n_c k_{10} n_1 - \langle \bar{\beta} \rangle A_{10} n_1 + n_0 \frac{g_1}{g_0} \langle \bar{\beta} \rangle A_{10} n_\gamma^{(0)} \left(1 - \frac{g_0 n_1}{g_1 n_0}\right)$$

Photon trapping made simple

So the net effect is:

- all Einstein coefficients reduced by factor $\langle\bar{\beta}\rangle$
  (only this fraction of the photons escapes, the rest is “trapped” in excited states of the particles: photon trapping)

- now $n_\gamma^{(0)}$ in stead of n_γ : no internally generated radiation field
  (internally generated radiation absorbed on-the-spot)

- transparent to external radiation field

Still this includes all effects of internally generated photons and stimulated emission.

What is the escape probability?

To actually use this formalism, we need an expression for the escape probability.

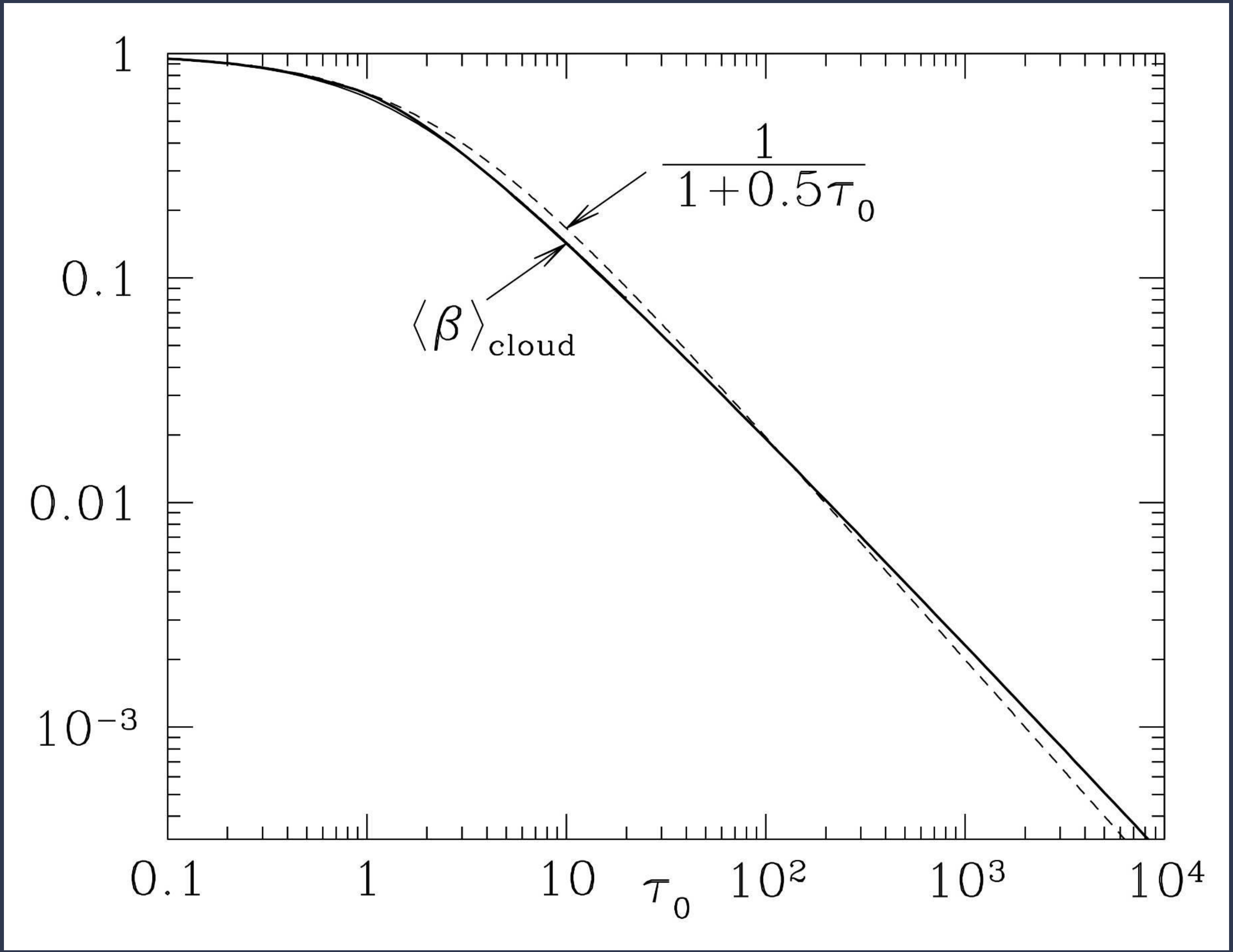
Simple example: static spherical cloud

Escape probability can now be calculated numerically and a very good fit is given by

$$\langle \bar{\beta} \rangle = \frac{1}{1 + 0.5\tau_0}$$

Since we found for CO(1-0) in a typical molecular cloud $\tau_0 \approx 50$, the escape probability will be only ≈0.04, so optical depth and photon trapping has a strong effect on fluxes, excitation etc.

Escape probability for a spherical cloud



Draine, Fig. 19.1

Escape probability models

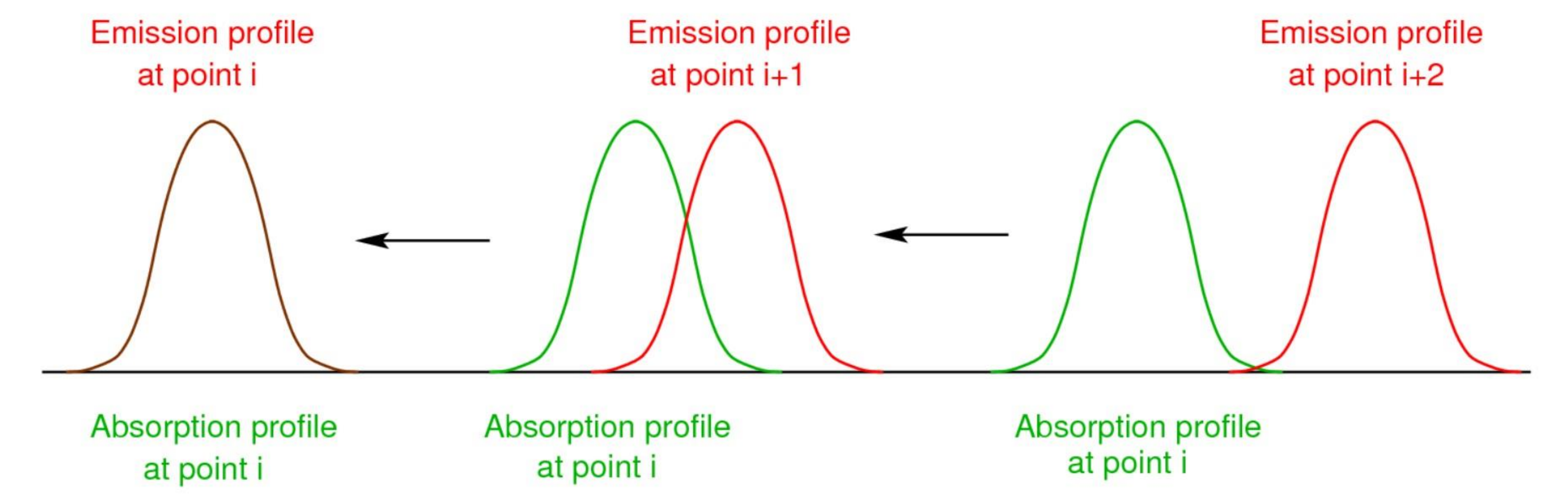
- Static spherical cloud (see before)

- Sobolev approximation: uniformly expanding cloud with $v = AR$
  → every point in the cloud maps to a point in velocity-space
  → every point in the line profile maps to a point in the cloud
  (no coverage of layers of the same velocity)

- Large Velocity Gradient (LVG) approximation (most important):
  now the velocities are in random order in the cloud
  → represents a turbulent cloud (coverage of layers of the same velocity is possible but unlikely)
  → every point in the line profile maps to a point in the cloud

In all cases, very similar expressions for the escape probabilities.

The LVG trick



Distant points are not radiatively coupled

Photon trapping and excitation

Recall critical density: $$n_{\rm crit} = \frac{A_{10}}{k_{10}}$$

This now becomes: $$n_{\rm crit} = \frac{\langle \bar{\beta} \rangle A_{10}}{k_{10}}$$

So if a transition becomes optically thick, its critical density goes down, due to photon trapping.

Example: CO(1-0): $$k_{10} \approx 6 \cdot 10^{-11} \left(\frac{T}{100\ \rm K} \right)^{0.2} \rm cm^3 s^{-1}$$

$$A_{10} \approx 6.78 \cdot 10^{-8} \rm s^{-1}$$

So if optically thin: $$n_{\rm crit} \approx 1100 \left(\frac{T}{100\ \rm K} \right)^{-0.2} \rm cm^{-3}$$

But if optically thick with β ≈ 0.04: $$n_{\rm crit} \approx 50 \left(\frac{T}{100\ \rm K} \right)^{-0.2} \rm cm^{-3}$$

Photon trapping: summary

- Escape probability methods can be used to analyse optically thick lines
- In practice this is an iterative process: take starting populations, calculate β, recalculate populations, recalculate β, etc.
- Always multi-level
- Computer codes (e.g., RADEX)
- Line ratios depend on n , T , N , σ_v

Today's lecture

Molecular clouds and their properties

- Measuring molecular cloud masses
- Photon trapping
- Molecular cloud masses from CO(1-0)
- Real molecular clouds

Tracing molecular cloud mass with $^{12}CO(1\text{-}0)$

Key assumption: cloud is self-gravitating, in virial equilibrium (this is normally valid)

Use escape probability formalism derived before.

Step 1: Virial equilibrium

In virial equilibrium, there is balance between gravity and internal random motions, such that for a spherical cloud with radius *R* and mass *M*.

$$\sigma_v = \sqrt{\frac{GM}{5R}}$$

The next step is connecting this to the line peak optical depth.

Step 2: Connect virial linewidth to optical depth

Use the previously derived expression for the peak optical depth of a Gaussian line with velocity dispersion σ_v, and substitute the expression we just got for σ_v in:

$$\tau_0 = \frac{N_l}{\sigma_v}\frac{g_u}{g_l}\frac{c^3}{8\pi\sqrt{2\pi}\nu_{ul}^3}A_{ul}\left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) = \frac{g_u}{g_l}\frac{A_{ul}\lambda_{ul}^3}{8\pi}\left(\frac{5}{2\pi G}\right)^{\frac{1}{2}}\frac{n_l R^{\frac{3}{2}}}{M^{\frac{1}{2}}}\left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right)$$

Step 3: Use the escape probability

Recall for an optically thin line $$I_\nu = \frac{h\nu}{4\pi} A_{ul} N_u \varphi_\nu$$

But now we have $$I_\nu = \frac{h\nu}{4\pi} \langle \bar{\beta} \rangle A_{ul} N_u \varphi_\nu$$

With $$\langle \bar{\beta} \rangle = \frac{1}{1 + 0.5\tau_0} \approx \frac{2}{\tau_0}$$

This gives $$I_\nu = \frac{h\nu}{4\pi} \frac{2}{\tau_0} A_{ul} N_u \varphi_\nu$$

Step 3: Go to brightness temperature

An observer will measure the velocity-integrated line brightness temperature, where $I_\nu = \frac{2k\nu^2}{c^2} T_b$

Combining everything, we obtain $\int T_b dv = \sqrt{\frac{2}{15}} 4\pi \frac{hc}{\lambda_{ul}} \left(\frac{G\mu m_{\mathrm{H}}}{n_{\mathrm{H}_2}}\right)^{\frac{1}{2}} \frac{N(\mathrm{H}_2)}{e^{\frac{h\nu_{ul}}{kT_{ex}}} - 1}$

where μ is the average mass of a molecule in the cloud in units of m_{H} (in a typical molecular cloud, $\mu \approx 2.8$).

The *X*-factor

$$\int T_b dv = \sqrt{\frac{2}{15}4\pi\frac{hc}{\lambda_{ul}}}\left(\frac{G\mu m_{\mathrm{H}}}{n_{\mathrm{H}_2}}\right)^{\frac{1}{2}}\frac{N(\mathrm{H}_2)}{e^{\frac{h\nu_{ul}}{kT_{ex}}}-1}$$

This shows that we can measure H_2 column density and mass from optically thick CO emission.

So we can define the conversion factor X_{CO} :

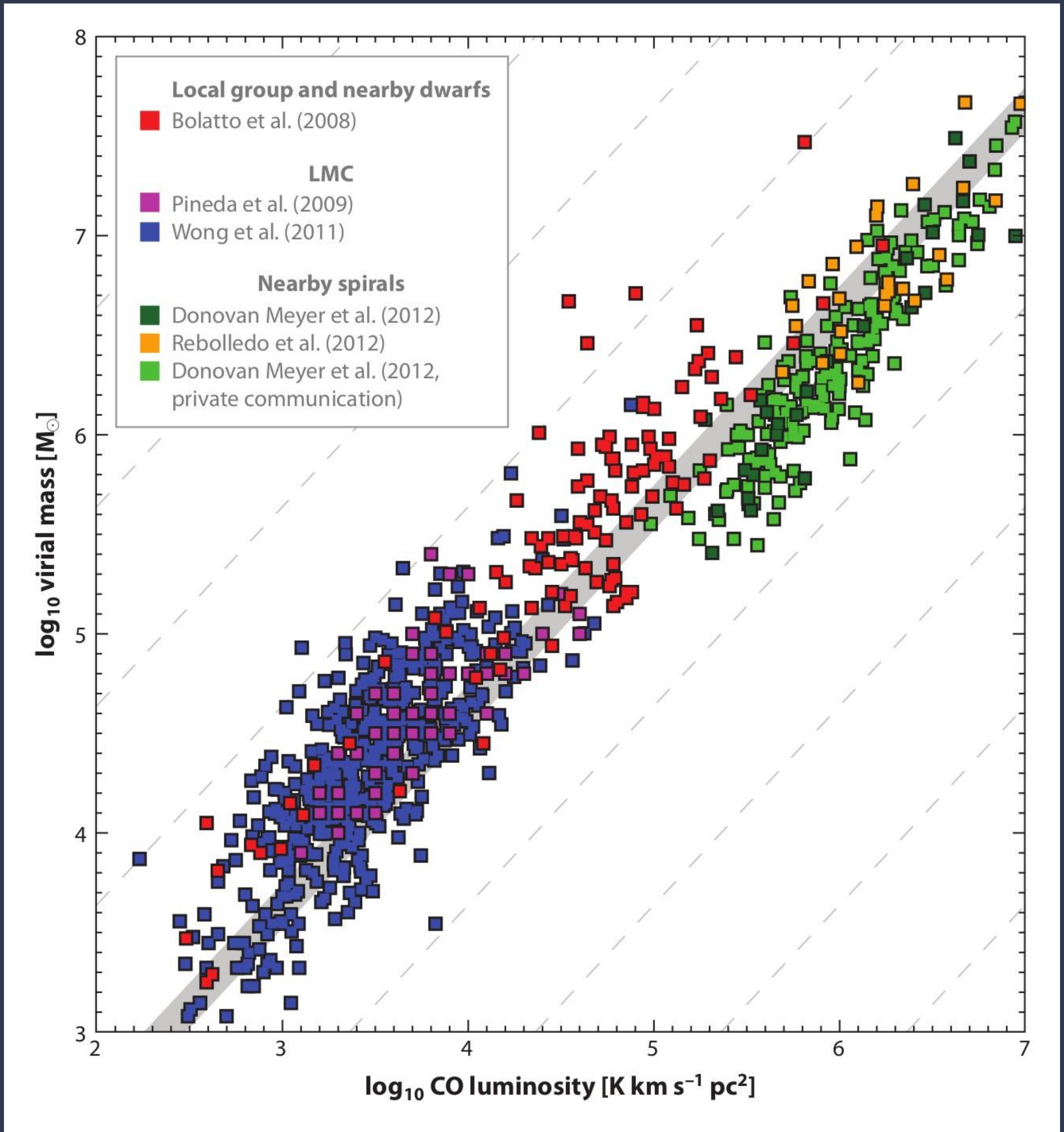
$$\int T_b dv = X_{\mathrm{CO}} N(\mathrm{H}_2)$$

Note that X_{CO} does not depend on CO abundance (!) but does depend on n and T_{ex}

In numbers:

$$X_{CO} = \frac{N(H_2)}{\int T_b dv} = 1.6 \cdot 10^{20}\left(\frac{n_{\mathrm{H}_2}}{1000\ \mathrm{cm}^{-3}}\right)^{\frac{1}{2}}\left(e^{\frac{5.5\ \mathrm{K}}{T_{ex}}}-1\right)^{-1}\frac{\mathrm{cm}^{-2}}{\mathrm{K\ km\ s}^{-1}}$$

Virial mass – L_{CO} relation



(Bolatto *et al*, 2012)

CO/H_2 conversion factor: summary

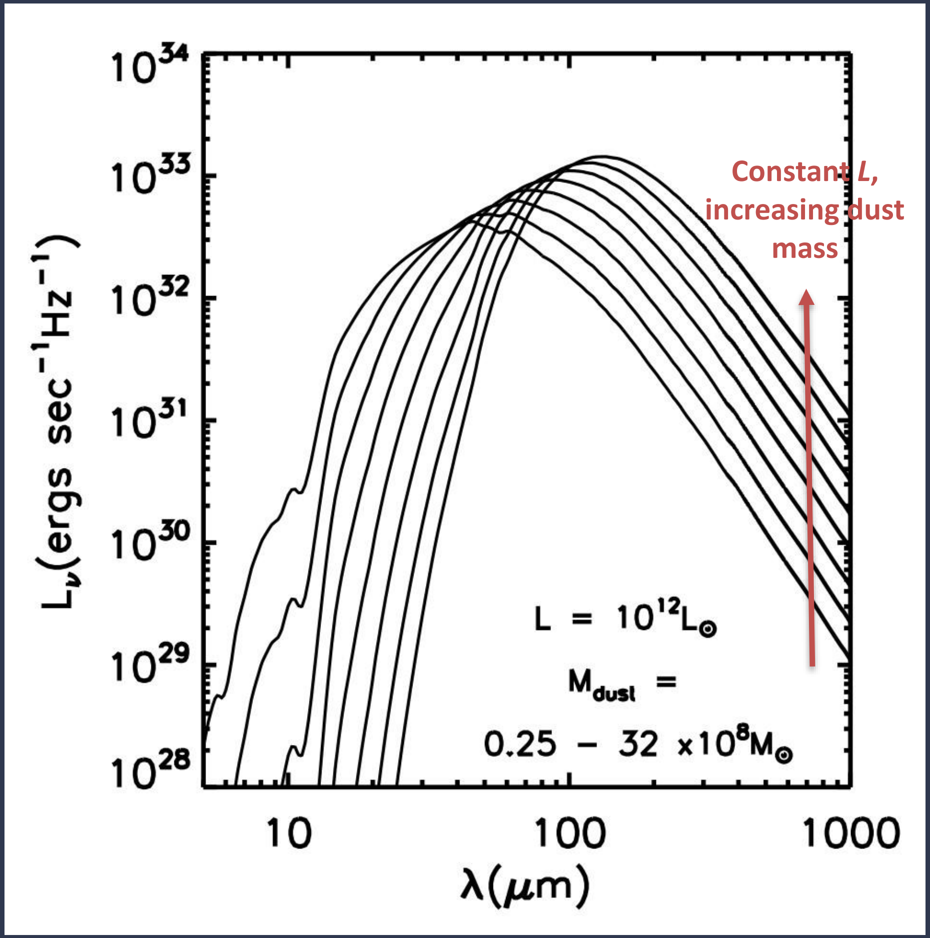
- Various methods (CO(1-0), rare CO isotopologues, virial masses) agree remarkably well
- CO(1-0) method relies on clouds being in virial equilibrium
- Conversion factor for CO(1-0) does not depend on CO abundance but does depend on T and n; in spite of this, remarkably constant throughout Milky Way disk
- Probably also valid for disks of galaxies similar to the Milky Way
- Conversion factor derived for Milky Way disk is not valid for galactic nuclei (including Galactic Center region) or metal-poor systems; possibly also not for ultraluminous galaxies

Dust emission as a gas mass probe

Rayleigh-Jeans tail traces dust column density ($\lambda > 200\mu m$).

Conversion factor proportional to $T_d\ \kappa_d\ (M_g\ /\ M_d)$

Recall: dust mass dominated by largest grains; these have lowest T hence radiate at longest λ



(Scoville, 2012)

Calculating molecular gas mass: summary

- CO(1-0) and long-wavelength dust emission both seem to work
- Surprising since conversion factors depend on local conditions (T and, for CO, also n)
- A possible reason may be that regions of enhanced T are only cloud edges which do not represent much mass

- Caution is needed (with both methods):
  - if *bulk* material has different temperatures: strongly star-forming galaxies, AGNs.
  - in low-metallicity galaxies (at any redshift)

Today's lecture

Molecular clouds and their properties

- Measuring molecular cloud masses
- Photon trapping
- Molecular cloud masses from CO(1-0)
- Real molecular clouds

Real molecular clouds

- Formation and destruction
- Molecular cloud structure: clumps and filaments
- Larson’s laws
- Stability and star formation

Molecular cloud formation and lifetimes

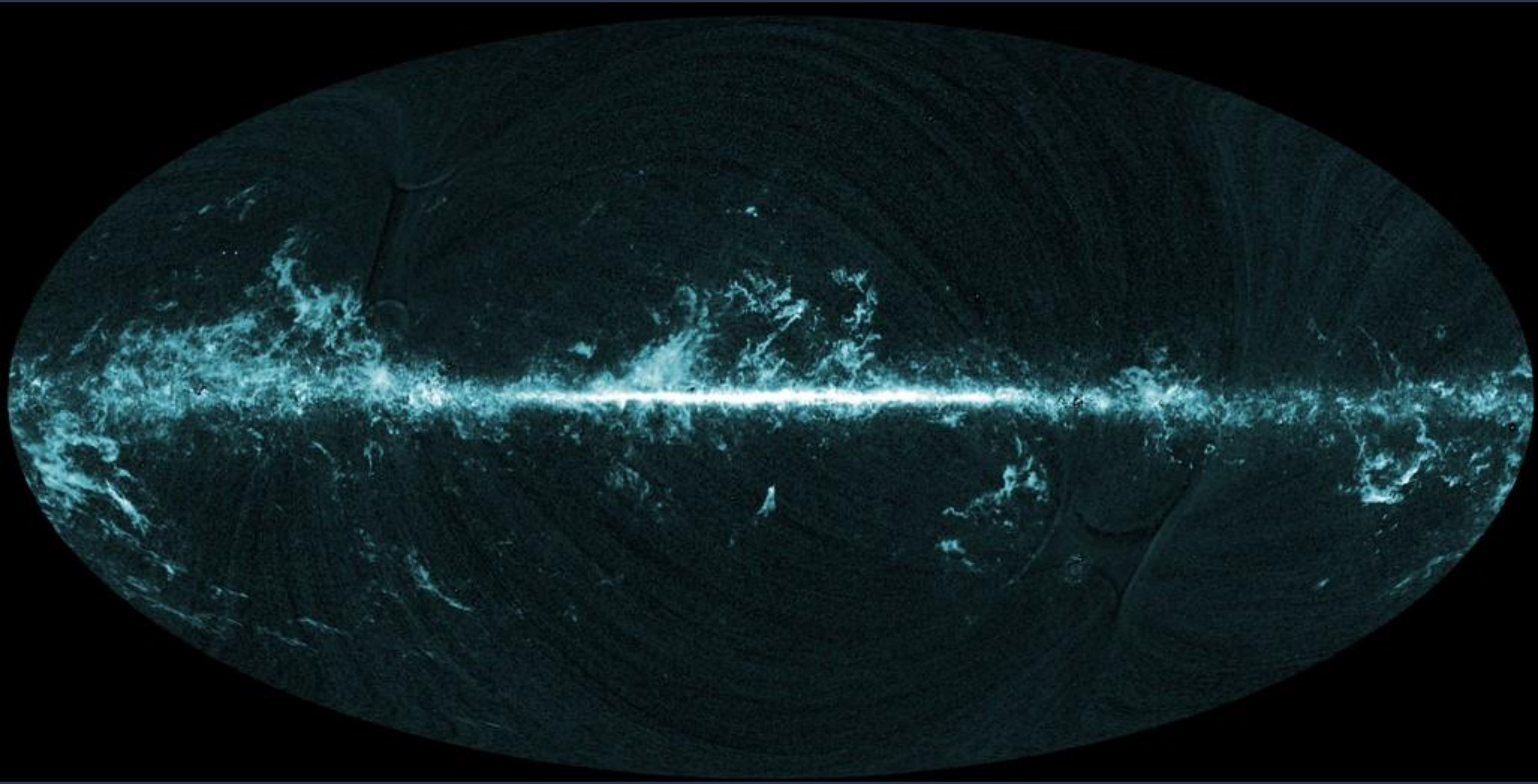
GMC Lifetimes: controversial

- Long estimates: $>10^8$ yr based on *z*-distribution and presence GMCs in interarm regions
- Short estimates: $\sim 2 \cdot 10^7$ yr because OB stars destroy GMC rapidly and GMCs mostly confined to spiral arms

Formation of GMCs: controversial

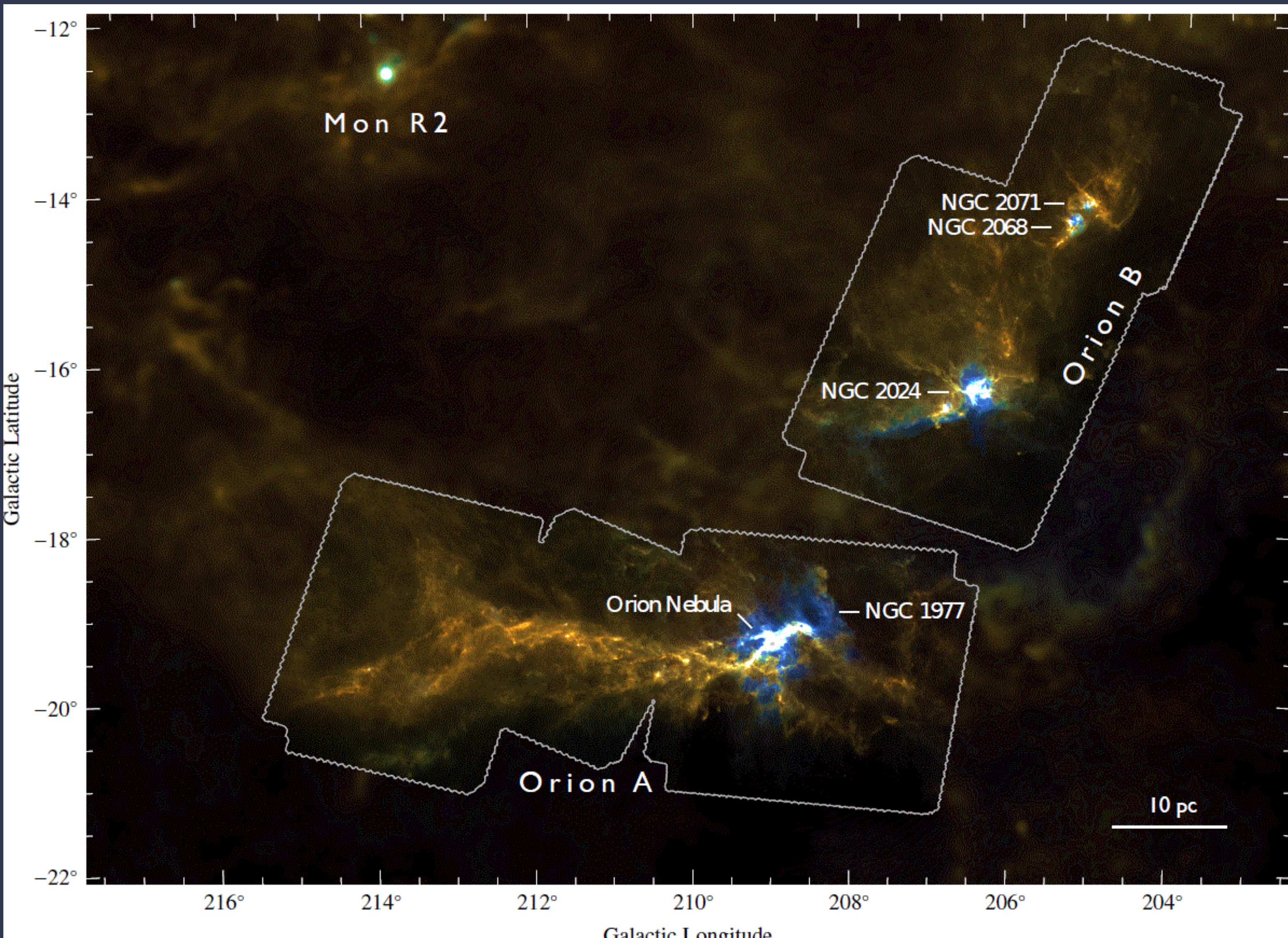
- random collisions of smaller clouds?
- spiral density wave?
- gravitational instability in atomic medium?

Molecular cloud structure



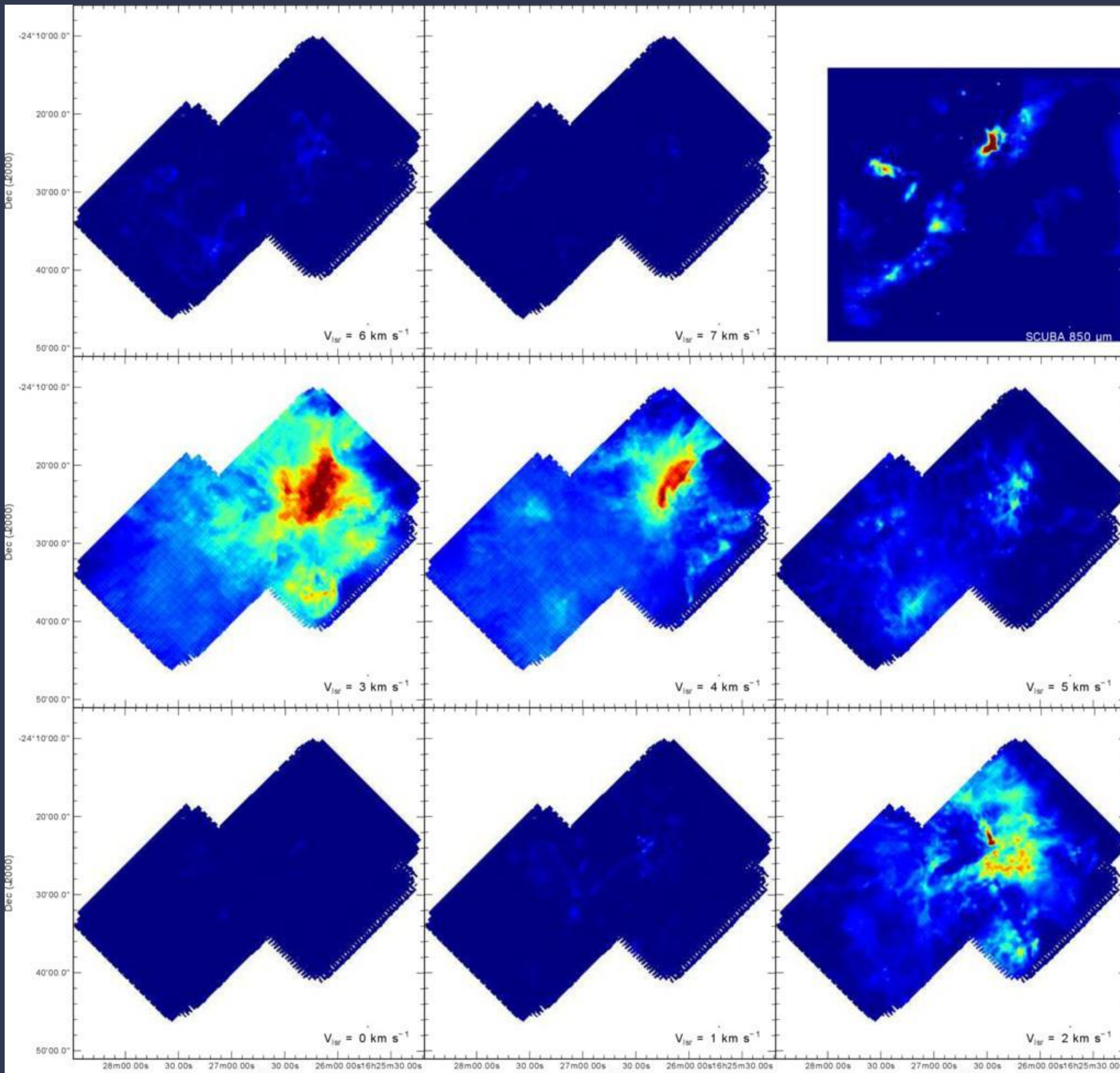
CO (as proxy for H_2)
Planck Satelite

Orion-Monoceros Giant Molecular Clouds



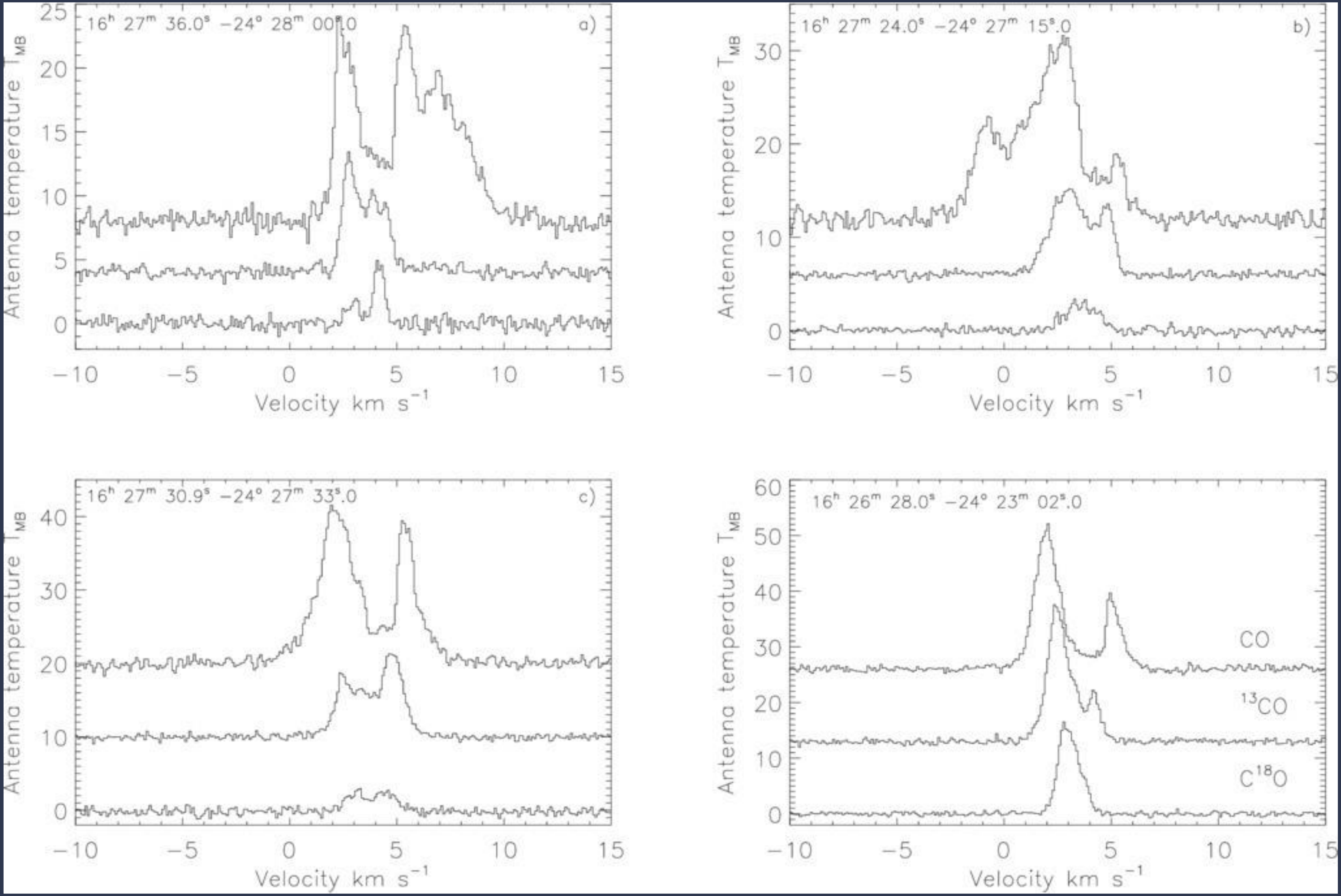
Herschel data (Lombardi et al., 2014)

^{12}CO in the Ophiuchus Molecular Cloud



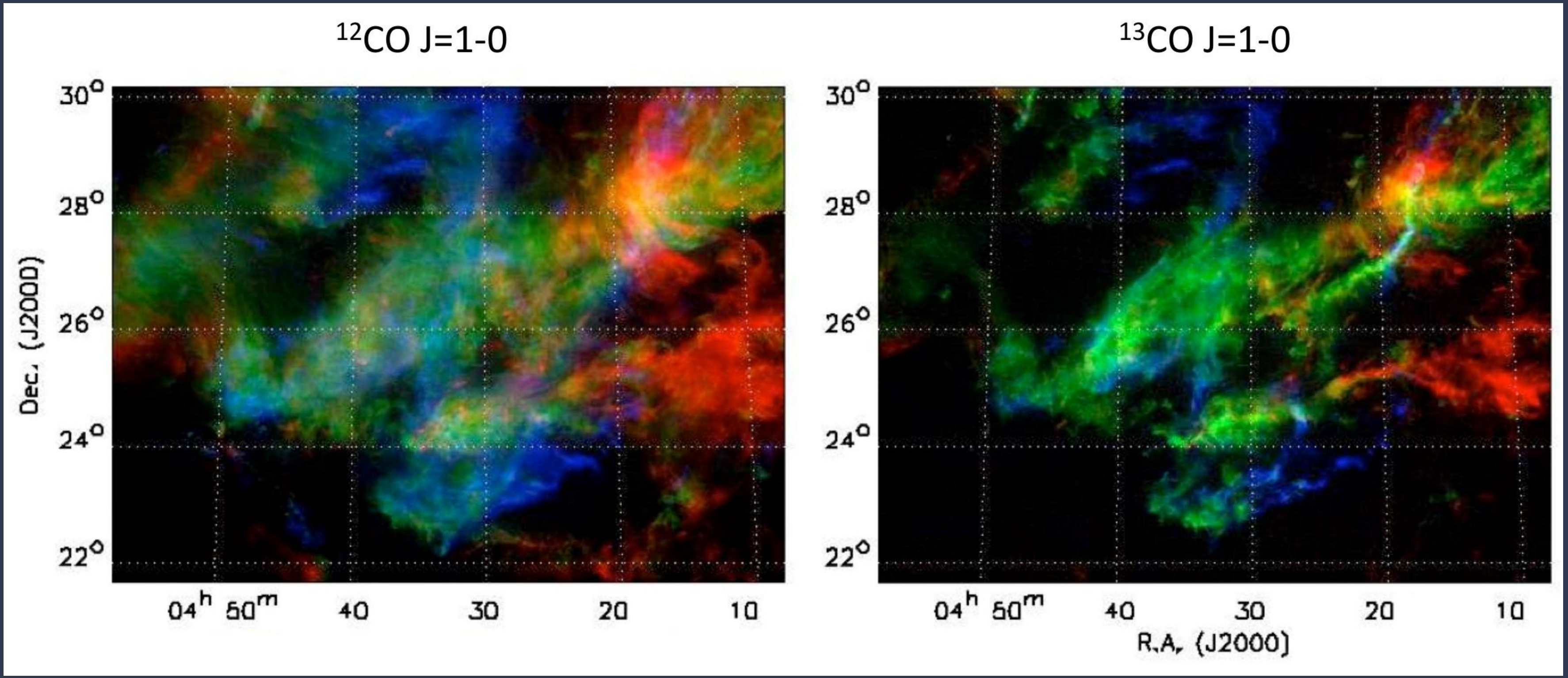
White *et al.*, 2014

CO lines in the Ophiuchus molecular cloud



White *et al.*, 2014

CO isotopogues in the Taurus molecular cloud



(Goldsmith *et al.*, 2008)

Larson's "Laws"

- Molecular clouds are virialized

- Column density of molecular clouds is approximately constant ($A_V = 5$): $\Sigma_{\mathrm{H_2}} \approx 100\ M_\odot\ \mathrm{pc}^{-2}$

- Size-linewidth relation: $\sigma_v \approx 1.10 \left(\frac{L}{\mathrm{pc}}\right)^{\gamma} \mathrm{km\ s^{-1}}$

Note: only 2 of the 3 laws are independent

(Larson, 1981)

Larson's "Laws"

- Molecular clouds are virialized

  Low mass clouds are not… And what about star formation?

- Column density of molecular clouds is approximately constant ($A_V = 5$): $\boxed{\Sigma_{H_2} \approx 100\ M_\odot\ \mathrm{pc}^{-2}}$

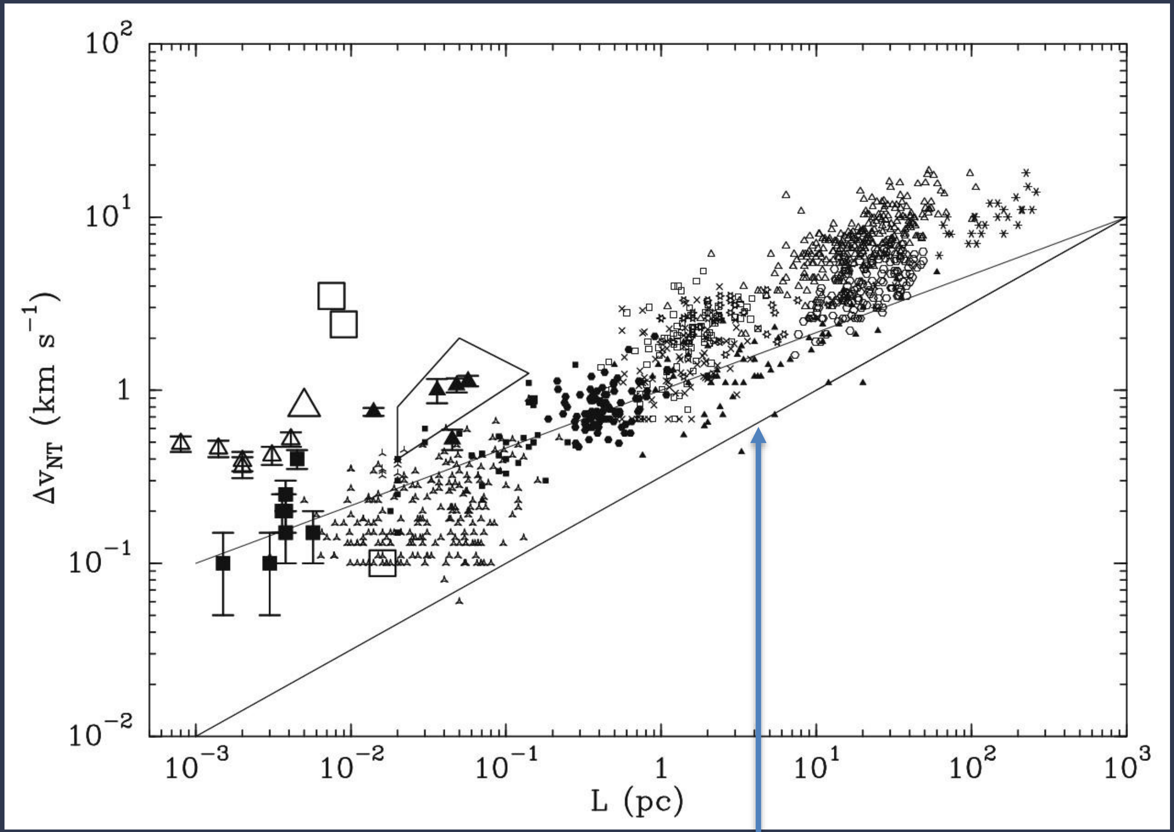
  Universal? Selection effect? Valid at what scale?

- Size-linewidth relation: $\boxed{\sigma_v \approx 1.10 \left(\frac{L}{\mathrm{pc}}\right)^{\gamma} \mathrm{km\ s^{-1}}}$

  Value of exponent?

After 50 years, we still have no good understanding of this…

Size-linewidth relation



(Falgarone *et al*, 2009)

$$\sigma_v \approx 1.10 \left(\frac{L}{\mathrm{pc}}\right)^{\gamma} \mathrm{km\ s^{-1}}$$

with $\gamma \approx$ 0.5

Note for L > 0.02 pc: supersonic turbulence

Stability and star formation

Molecular clouds pressures:

Thermal pressure $P/k = nT \approx 3{\cdot}10^5$ K cm^{-3}
Mean ISM pressure: $P/k \approx 3000$ K cm^{-3}

no pressure balance, confined by self-gravity (like stars)

Note: turbulent pressure and magnetic pressure may exceed thermal pressure!

Star formation

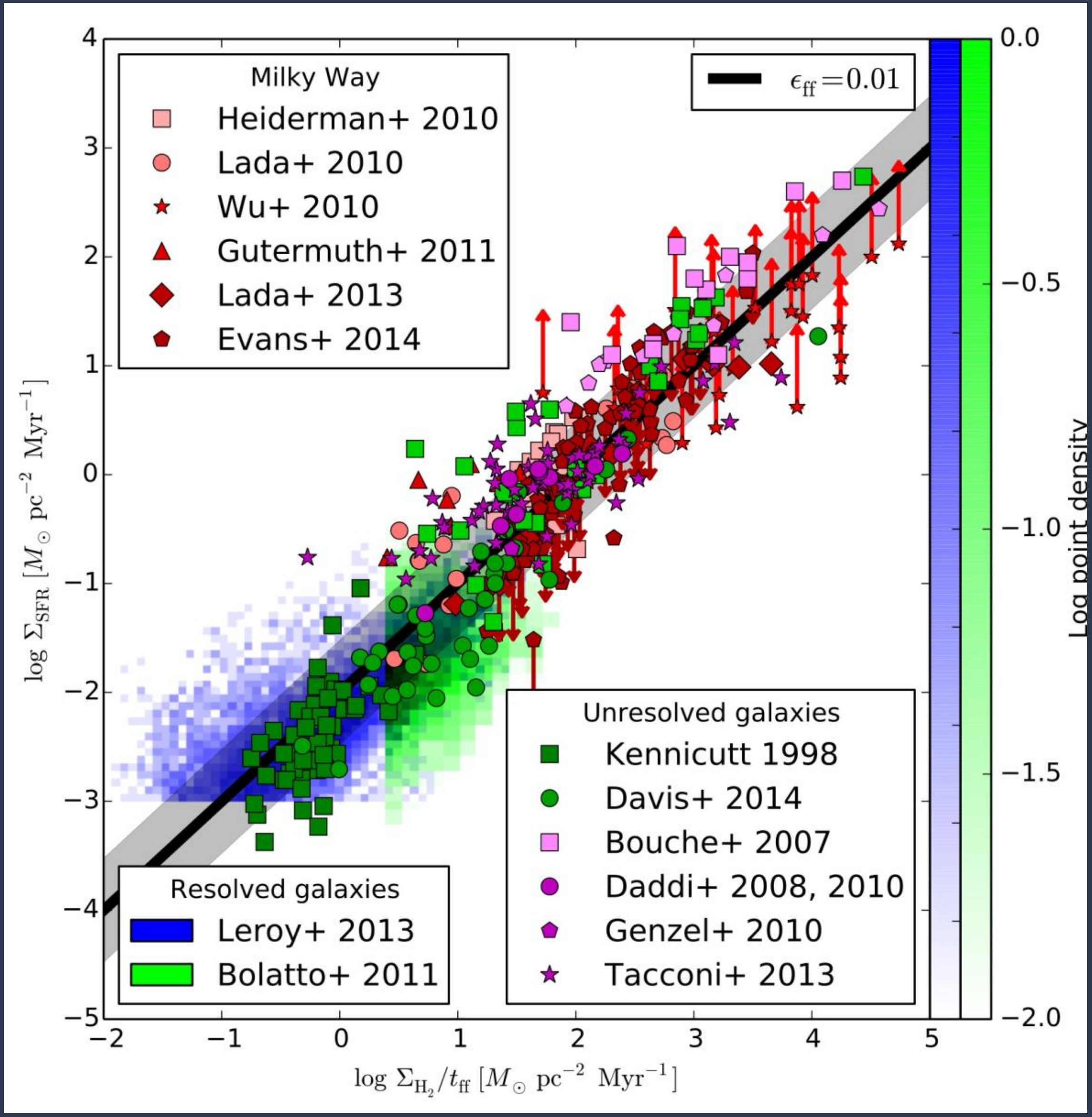
- Free fall time $$t_{ff} = \left(\frac{3\pi}{32}\frac{1}{G\rho}\right)^{1/2} = 1.4 \times 10^5 \left(\frac{2n(\mathrm{H_2})}{10^5\ \mathrm{cm^{-3}}}\right)^{-1/2} \mathrm{yr} \approx 3 \times 10^6\ \mathrm{yr}$$

- Milky Way H_2 mass ~$4\times10^9\ M_\odot$
- With no support, SFR = M_{H_2} / t_{ff} ~ 1300 $M_\odot$/yr
- Observed star formation rate across the MW only ~ 3 $M_\odot$/yr
  cf. starburst galaxy: tens to hundreds $M_\odot$/yr (e.g., in ULIRGs)

Implication:

Clouds are supported against free collapse, possibly by a self-regulating mechanism resulting from feedback.
Many unknowns and uncertainties.

Kennicutt-Schmidt Law



A molecular cloud converts about 1% of its mass into stars in one free-fall time.

(credit Mark Krumholz)

Next week's lecture

- Shock waves: principles & examples
- Supernova remnants
- 3-phase Interstellar Medium