Interstellar Medium 2020
Problem Set 2

Paul van der Werf

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1 Problem 1: Radii of Strömgren spheres

Consider a spherical H II region ionized by an O9V star. Relevant parameters for a star of this type are given in Draine, Table 15.1. Assume that the He/H abundance ratio is 10% by number. What will be the ratio $R_{\rm HeII}/R_{\rm HII}$, where $R_{\rm HII}$ is the radius out to which hydrogen is ionized and $R_{\rm HeII}$ is the radius out to which helium is singly ionized? An answer to $\sim 10\%$ accuracy is OK. You will also need Draine Table 14.7. Don't worry over details but do state your assumptions.

Solution

Recombination rates for H and He are found in Draine Table 14.7: $\alpha_{\rm B} = 2.59 \cdot 10^{-13}$ cm^3 s^{-1} for H and $\alpha_{\rm B} = 2.72 \cdot 10^{-13}$ cm^3 s^{-1} for He, where we have assumed $T_{\rm e} = 10^4$ K. If $n_{\rm H}$ is constant (and dust absorption is neglegible), then n_e is larger by 10% in the zone where helium is also ionized compared to where only hydrogen is ionized. The radii of the ionized zones are now given by the ionization balance equations:

$$Q_0 = n_{\rm H} \cdot n_{\rm H}\, \alpha_{\rm B}({\rm H}) \frac{4\pi}{3} R_{\rm HII}^3 \tag{1}$$

$$Q_1 = 0.1\, n_{\rm H} \cdot 1.1\, n_{\rm H}\, \alpha_{\rm B}({\rm He}) \frac{4\pi}{3} R_{\rm HeII}^3, \tag{2}$$

where Q_0 and Q_1 are the ionizing photon production rates above the hydrogen ionization edge (Q_0) and the helium ionization edge (Q_1), which are found from Draine Table 15.1. Hence

$$\frac{R_{\rm HeII}^3}{R_{\rm HII}^3} = \frac{Q_1}{Q_0} \frac{1 \cdot 1}{0.1 \cdot 1.1} \frac{\alpha_{\rm B}({\rm H})}{\alpha({\rm He})} \tag{3}$$

$$= 0.0145\, \frac{1}{0.11} \frac{2.59 \cdot 10^{-13}}{2.72 \cdot 10^{-13}} = 0.101. \tag{4}$$

So

$$\frac{R_{\rm HeII}}{R_{\rm HII}} \approx 0.466. \tag{5}$$

2 Problem 2: Radio recombination lines

In class we briefly discussed radio recombination lines. The interpretation of these lines is complicated by stimulated emission. However, if you take line ratios such as for instance He166α/H166α (which we use in this problem), the interpretation becomes very simple. This is because in this line ratio, all complicating factors cancel out. The only difference between the hydrogen line and the corresponding helium line is then a small frequency difference, resulting from the more massive nucleus of the He atom. We will now use such a line ratio to estimate the temperature of the central star of an H II region. To be specific, we assume that we use a radio telescope to observe the H166α and He166α recombination lines from an H II region. The line ratio (integrated over the H II region) is found to be He166α/H166α=0.032.

(a) Use Draine Tables 14.7 and 15.1 to estimate the temperature of the exciting star of the H II region, assuming it to be of luminosity class V. Assume that all photons above the ionization edge of helium are absorbed by helium. Assume case B recombination at a temperature $T_\mathrm{e} = 10^4$ K. Since in the line ratio all complicating factors (resulting from stimulated emission, etc.) cancel out, you can simply use the expressions for recombination line fluxes/intensities derived in class. Note also that the line emissivity coefficients for these high-n lines are to good accuracy the same for hydrogen and helium.

(b) The observed emission lines are measured to have Gaussian line profiles with a full width at half maximum (FWHM) of 23.5 and 15.3 km s^{-1} for H and He respectively. The spectrometer with which the lines are detected has a instrumental line width (FWHM) of 5.0 km s^{-1}. Assume that the only motions are thermal motions due to the temperature of the gas, and turbulence (constant throughout the H II region) with an unknown velocity dispersion. Assume that instrumental line profile of the spectrograph and the thermal and turbulent velocity distributions are all Gaussians. What is the one-dimensional turbulent velocity dispersion in the nebula? What is the kinetic temperature T_kin in the nebula?

Solution

(a) We have measured

$$0.032 = \frac{F(\mathrm{He}166\alpha)}{F(\mathrm{H}166\alpha)}, \tag{6}$$

where F denotes the flux density integrated over the line. Given that complicating factors due to stimulated emission and atomic parameters cancel out in the ratio, we can write this (see Draine Sect. 14.2.3) as

$$\begin{aligned} 0.032 &= \frac{\int d\Omega\, I(\mathrm{He}166\alpha)}{\int d\Omega\, I(\mathrm{H}166\alpha)} && (7)\\ &= \frac{\int dA\, I(\mathrm{He}166\alpha)}{\int dA\, I(\mathrm{H}166\alpha)} && (8)\\ &= \frac{\int dA \int ds\, n_e n(\mathrm{He}^+)}{\int dA \int ds\, n_e n(\mathrm{H}^+)} && (9)\\ &= \frac{\int dV\, n_e n(\mathrm{He}^+)}{\int dV\, n_e n(\mathrm{H}^+)}, && (10) \end{aligned}$$

where I is the specific intensity integrated over the line, and we have used the fact that the emissivity coefficients of the H and He line are equal. We need to relate this ratio to properties of the central star, using ionization-recombination balance. This is done as follows:

$$
\begin{aligned}
0.032 &= \frac{\int dV\, n_e n(\mathrm{He}^+)}{\int dV\, n_e n(\mathrm{H}^+)} && (11)\\
&= \frac{\alpha_{\mathrm{B}}(\mathrm{H})}{\alpha_{\mathrm{B}}(\mathrm{He})} \frac{\int dV\, \alpha_{\mathrm{B}}(\mathrm{He}) n_e n(\mathrm{He}^+)}{\int dV\, \alpha_{\mathrm{B}}(\mathrm{H}) n_e n(\mathrm{H}^+)} && (12)\\
&= \frac{\alpha_{\mathrm{B}}(\mathrm{H})}{\alpha_{\mathrm{B}}(\mathrm{He})} \frac{Q_1}{Q_0}, && (13)
\end{aligned}
$$

where Q_0 and Q_1 are again the ionizing photons production rates for hydrogen and helium ionization, respectively. So

$$
\begin{aligned}
\frac{Q_1}{Q_0} &= \frac{\alpha_{\mathrm{B}}(\mathrm{He})}{\alpha_{\mathrm{B}}(\mathrm{H})} \cdot 0.032 && (14)\\
&= \frac{2.72}{2.54} \cdot 0.032 = 0.034. && (15)
\end{aligned}
$$

From Draine Table 15.1, we see that an O8.5 star with $T_{\mathrm{eff}} = 33900$ K has $Q_1/Q_0 = 0.0347$. Hence the conclusion is that the exciting star must be of spectral type approximately O8.5V with $T_{\mathrm{eff}} \approx 33900$ K.

(b) For a Gaussian line profile we can easily derive

$$\mathrm{FWHM} = \sqrt{8 \ln 2}\, \sigma. \tag{16}$$

A Maxwellian velocity distribution has a Gaussian 1-dimensional velocity dispersion $\sigma_{\mathrm{therm}} = \sqrt{kT/m}$ where T is the kinetic temperature and m the mass of the particle. Thus we get

$$\sigma^2 = \frac{kT}{m} + \sigma_{\mathrm{turb}}^2 + \sigma_{\mathrm{instr}}^2. \tag{17}$$

This gives

$$\frac{kT}{m} + \sigma_{\mathrm{turb}}^2 = \frac{(\mathrm{FWHM})^2 - (\mathrm{FWHM}_{\mathrm{instr}})^2}{8 \ln 2}, \tag{18}$$

which gives for the hydrogen line

$$\frac{kT}{m_{\mathrm{H}}} + \sigma_{\mathrm{turb}}^2 = \frac{(23.5 \text{ km s}^{-1})^2 - (5 \text{ km s}^{-1})^2}{8 \ln 2} \tag{19}$$

and for the helium line

$$\frac{kT}{4m_{\mathrm{H}}} + \sigma_{\mathrm{turb}}^2 = \frac{(15.3 \text{ km s}^{-1})^2 - (5 \text{ km s}^{-1})^2}{8 \ln 2}. \tag{20}$$

Subtracting, we get

$$\frac{kT}{m_{\mathrm{H}}} - \frac{kT}{4m_{\mathrm{H}}} = \frac{(23.5 \text{ km s}^{-1})^2 - (15.3 \text{ km s}^{-1})^2}{8 \ln 2}, \tag{21}$$

which gives $T = 9260$ K. Inserting this into one of the earlier expressions involving σ_{turb}, we find $\sigma_{\mathrm{turb}} = 4.3 \text{ km s}^{-1}$.

3 Problem 3: An obscured H II region

We observe Hα (λ = 6563 Å) and Hβ (λ = 4861 Å) emission from an H II region with $T_e = 10^4$ K at a distance of D = 450 pc. The observed flux densities integrated over the lines are: $F(\text{H}\alpha) = 1.0 \cdot 10^{-7}$ erg s^{-1} cm^{-2} and $F(\text{H}\beta) = 2.0 \cdot 10^{-8}$ erg s^{-1} cm^{-2}. The H II region is observed through a foreground H I cloud, which may be dusty in which case it obscures Hβ more strongly than Hα, with $\tau_{\text{H}\beta}/\tau_{\text{H}\alpha} = 1.60$.

(a) What is the spectral type of the ionizing star (assumed to be of luminosity class V) of this H II region?

(b) Now you also carry out a measurement in the Brγ line and you find $F(\text{Br}\gamma) = 4.3 \cdot 10^{-9}$ erg s^{-1} cm^{-2}. How can you reconcile this with the Hα and Hβ fluxes given before, given that the extinction curve has the property $\tau_{\text{H}\alpha}/\tau_{\text{Br}\gamma} = 6.80$?

Solution

(a) The observed flux density ratio is $F(\text{H}\alpha)/F(\text{H}\beta) = 5.0$, which is higher than the case B ratio at $T_e = 10^4$ K, which is 2.86 (Draine, Table 14.2). There must therefore be dust in the foreground H I cloud, which suppresses the Hβ more than the Hα. We must first correct for this and derive the intrinsic line flux densities, which we call F_0. In order to do this, we write down the equation of transfer for the lines through the absorbing foreground cloud (which, since it is neutral, of course does not itself emit Hα or Hβ):

$$F(\text{H}\alpha) = F_0(\text{H}\alpha)\, e^{-\tau_{\text{H}\alpha}} \tag{22}$$

$$F(\text{H}\beta) = F_0(\text{H}\beta)\, e^{-\tau_{\text{H}\beta}}. \tag{23}$$

Taking the ratio, we get

$$\frac{F(\text{H}\alpha)}{F(\text{H}\beta)} = \frac{F_0(\text{H}\alpha)}{F_0(\text{H}\beta)}\, e^{\tau_{\text{H}\beta}-\tau_{\text{H}\alpha}} \tag{24}$$

$$5.0 = 2.86\, e^{1.60\tau_{\text{H}\alpha}-\tau_{\text{H}\alpha}} = 2.86\, e^{0.60\tau_{\text{H}\alpha}}, \tag{25}$$

which yields $\tau_{\text{H}\alpha} = 0.93$ so that $F_0(\text{H}\alpha) = 2.5 \cdot 10^{-7}$ erg s^{-1} cm^{-2}. The line luminosity is then

$$L(\text{H}\alpha) = 4\pi D^2 F_0(\text{H}\alpha) = 6.0 \cdot 10^{36} \text{ erg s}^{-1}. \tag{26}$$

In class we have derived the equation relating this to Q_0, which was

$$L(\text{H}\alpha) = \frac{\alpha_{\text{H}\alpha}}{\alpha_\text{B}}\, h\nu\, Q_0, \tag{27}$$

where α_B is the total case B recombination rate, found in Draine Table 14.2. The factor $\alpha_{\text{H}\alpha}$ is the rate of emission of Hα photons, and this number is also found in Draine Table 14.2, where it is indicated as $\alpha_{\text{eff}}(\text{H}\alpha)$. Using these numbers, we find $Q_0 = 4.4 \cdot 10^{48}$ s^{-1} and the spectral type can now be found from Draine Table 15.1 and is O7.5V.

(b) Since $\tau_{\text{H}\alpha}/\tau_{\text{Br}\gamma} = 6.80$ and $\tau_{\text{H}\alpha} = 0.93$, we find $\tau_{\text{Br}\gamma} = 0.14$, so the intrinsic Br$\gamma$ flux becomes $F_0(\text{Br}\gamma) = 5.0 \cdot 10^{-9}$ erg s^{-1} cm^{-2}. From Table 14.2 in Draine, we find that the intrinsic Hα/Brγ case B ratio at 10^4 K is $F_0(\text{H}\alpha)/F_0(\text{Br}\gamma) = 2.86/0.159/0.174 \approx 103$, so based on the extinction-corrected Hα flux we would expect $F_0(\text{Br}\gamma) = 2.4 \cdot 10^{-9}$ erg s^{-1} cm^{-2}. But the value we measure is about a factor of 2 larger! Assuming a different temperature for the H II region is not making this discrepancy go away. Evidently something else is going on.

What is happening here is that the dust is not located in a foreground layer but mixed with the ionized gas. To see this, begin by calculating the extinction from the longest wavelength observations (which are least affected by extinction), which are Hα and Brγ. Repeating the above analysis we get

$$\frac{F(\mathrm{H}\alpha)}{F(\mathrm{Br}\gamma)} = \frac{F_0(\mathrm{H}\alpha)}{F_0(\mathrm{Br}\gamma)}\, e^{\tau_{\mathrm{Br}\gamma} - \tau_{\mathrm{H}\alpha}} \tag{28}$$

$$23.3 = 103\, e^{\tau_{\mathrm{Br}\gamma} - 6.80\tau_{\mathrm{Br}\gamma}} = 103\, e^{-5.80\tau_{\mathrm{Br}\gamma}}, \tag{29}$$

which yields $\tau_{\mathrm{Br}\gamma} = 0.26$ and $\tau_{\mathrm{H}\alpha} = 1.74$, so almost a factor two more extinction than before.

But why is this higher extinction not reflected in the observed Hα/Hβ ratio? The reason is that in the situation where the dust and gas are mixed (and extinction is significant), for every spectral line we detect the line only to a certain depth into the cloud, and the rest is so extincted by dust that it does not produce a significant signal. This depth is typically the depth at which the optical depth at the observing wavelength reaches about a value of 1. As a result, at every observing wavelength you will measure an optical depth of the order of 1. If the optical depth is in reality much larger than 1, you will only detect the outer "skin" of cloud, where the skin is the layer over which optical depth reaches unity. The best way to handle this is to make observations of recombination lines over a range of wavelengths and adopt an equation of transfer that takes into account that the dust if mixed with the gas. If this is not possible, the best approach is to use the longest wavelength observations, which are least affected by extinction.