

Interstellar Medium 2020

# Lecture 2: Physical conditions and radiative processes

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# Course Contents

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# Previous Lecture

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## Introduction and Ecology of the Interstellar Medium

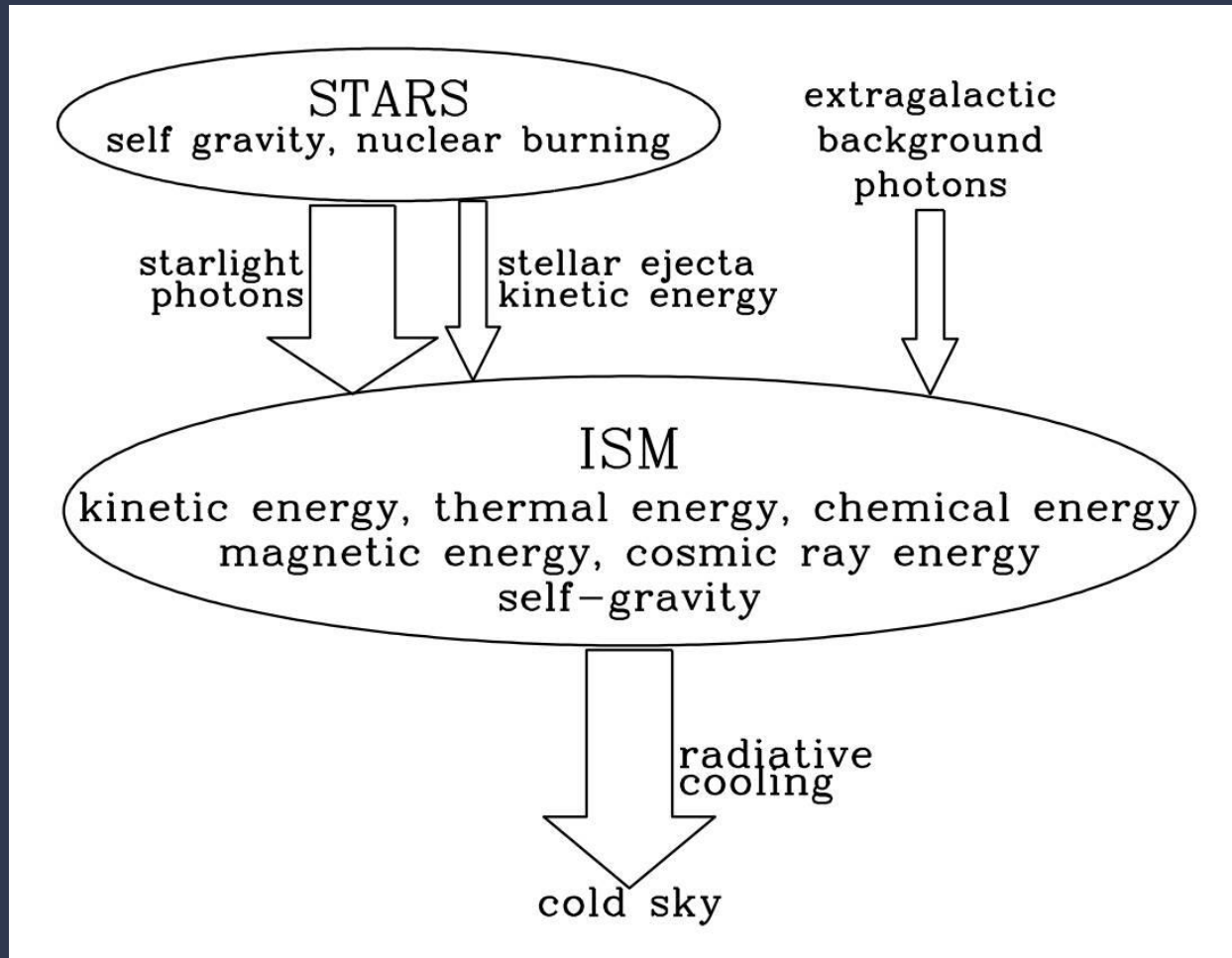
1. Discovery of the ISM and brief history
2. Constituents of the ISM
  - Objects
  - Phases
3. Energy densities

# Today's Lecture

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1. Basic physical conditions
2. Radiation quantities
3. Radiative transitions
4. Radiative transfer

# The ISM is not in thermodynamic equilibrium



(Draine,  
Fig. 1.3)

# Detailed balance

Thermodynamic Equilibrium implies:

detailed balance

every process is balanced exactly by its counterprocess

This is manifestly untrue in the ISM

It is also untrue locally:

hence no Local Thermodynamic Equilibrium (LTE)  
(in contrast to, e.g., stellar interiors)

# Are these distributions valid in the ISM?

Maxwell distribution of particle kinetic energies?

Planck distribution of radiation energies?

Boltzmann distribution of level population?

# Maxwell distribution

- Elastic collisions are sufficiently frequent to thermalize velocity distribution (see e.g., Spitzer 1978, Physical Processes in the Interstellar Medium)
- Maxwell distribution is valid in the ISM
- Usually  $T_{\text{kinetic}} \equiv T_{\text{electrons}} = T_{\text{ions}} = T_{\text{neutrals}}$   
(NB: exceptions exist)



# Planck distribution

- The interstellar radiation field deviates strongly from a Planck function
- Planck distribution of radiation energies is not generally valid in the ISM

# Boltzmann distribution

$$n_i = \frac{g_i e^{-\frac{E_i}{kT}}}{Z(T)}$$

$$Z(T) = \sum_i g_i e^{-\frac{E_i}{kT}}$$

$g_i$  : level degeneracies (statistical weights)

$Z(T)$  : partition function

# Boltzmann distribution

- Boltzmann distribution is not generally valid in the ISM
- Define *excitation temperature*  $T_{\text{ex}}$  by

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT_{\text{ex}}}$$

- The excitation temperature is defined per transition
- In general  $T_{\text{ex}} \neq T_{\text{kin}}$ ; if  $T_{\text{ex}} = T_{\text{kin}}$  the level populations are called *thermalized*; if  $T_{\text{ex}} < T_{\text{kin}}$  we speak of *subthermal excitation*

# Statistical equilibrium

- In the absence of thermodynamic equilibrium (and hence detailed balance) in the ISM the weaker condition of **statistical equilibrium** is valid:

sum of rates of all processes populating level  $i$  =  
sum of rates of all processes depopulating level  $i$

$$\frac{dn_i}{dt} = \sum_j (-R_{ij}n_i + R_{ji}n_j) = 0 \quad \forall i$$

- $R_{ij}$  is the transition rate (in  $\text{s}^{-1}$ ) for a particular process causing a transition from level  $i$  to level  $j$

# Today's Lecture

1. Basic physical conditions
2. Radiation quantities
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# Radiation

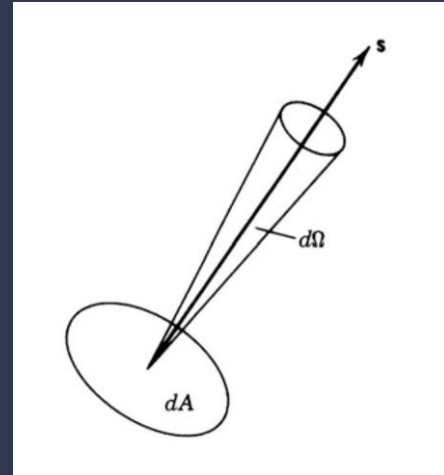
Energy in radiation: let  $dE_\nu$  be the energy in radiation at frequency  $\nu$ , in frequency interval  $d\nu$ , from solid angle  $d\Omega$ , flowing through an area  $dA$ , in a time interval  $dt$  :

$$dE_\nu = I_\nu d\nu d\Omega dA dt$$

$$I_\nu(\nu, \hat{n}, \vec{r}, t)$$

is called the **specific intensity** (sometimes: surface brightness)

Units:  $\text{erg s}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$   
(or something equivalent)



NB:

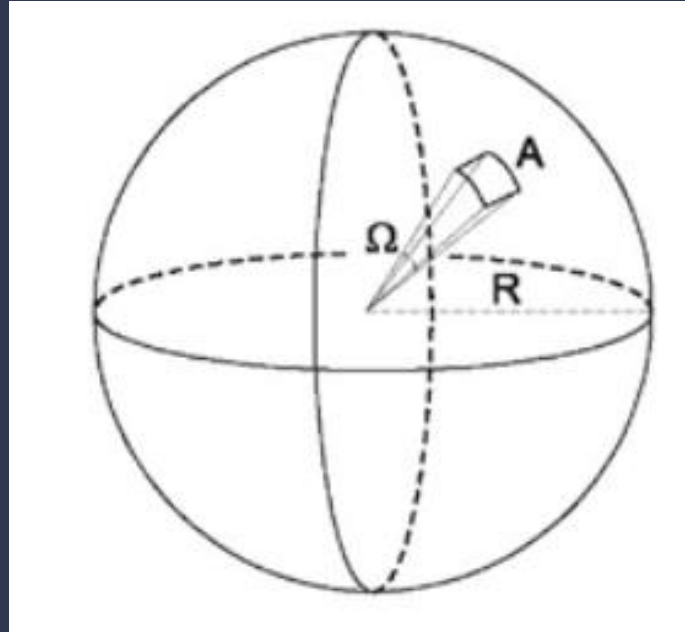
1. we can also use  $I_\lambda$  (with corresponding units)
2. In LTE:

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

3. Specific intensity is distance-independent

# Reminder: solid angle

$$d\Omega = \frac{dA}{R^2}$$



Entire sphere has solid angle  $4\pi$  sr (steradian).

Useful:  $1 \text{ arcsec}^2 = 2.35 \cdot 10^{-11} \text{ sr}$

# Photon occupation number

An alternative, equivalent (and often simpler) way to express specific intensity is by the **photon occupation number**:

$$n_\gamma(\nu, \hat{n}, \vec{r}, t) = \frac{c^2}{2h\nu^3} I_\nu(\nu, \hat{n}, \vec{r}, t)$$

Note that in LTE we have:

$$n_\gamma(\nu, \hat{n}, \vec{r}, t) = \frac{1}{e^{h\nu/kT} - 1}$$

with the limiting cases:

$$h\nu \gg kT \Rightarrow n_\gamma \ll 1$$

$$h\nu \ll kT \Rightarrow n_\gamma \gg 1$$



# Brightness temperature

Another equivalent quantity is the (Rayleigh-Jeans) brightness temperature:

$$T_b(\nu) = \frac{c^2}{2k\nu^2} I_\nu$$

In the radio regime (where  $h\nu \ll kT$ ) this is actually a temperature of a Planck curve:

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$h\nu \ll kT \Rightarrow I_\nu \approx \frac{2h\nu^3}{c^2} \frac{1}{\frac{h\nu}{kT}} = \frac{2k\nu^2}{c^2} T$$

NB: Draine calls this  $T_a$  (antenna temperature)

# Flux density

The quantities discussed so far are suitable for extended objects. But very often we observe an object as a point source (e.g., stars, distant galaxies). What then?

Only specific intensity integrated over solid angle is now available, so:

$$F_\nu = \int I_\nu d\Omega$$

This is called **flux density**, with units  $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ . A more practical unit is a **Jansky**, where  $1 \text{ Jy} = 10^{-23} \text{ erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ .

Note that  $F_\nu$  is not an intrinsic property of the source but proportional to  $D^{-2}$  (why?).

# Luminosity

We can now calculate the total energy radiated by an object, which we call **specific luminosity** (if per spectral interval) or **luminosity** (integrated over the spectrum, or over part of it):

$$L_\nu = 4\pi D_L^2 F_\nu$$

$$L = \int L_\nu d\nu$$

The units of luminosity are  $\text{erg s}^{-1}$ , or, more practical, solar luminosities, where  $L_\odot = 3.8 \cdot 10^{33} \text{ erg s}^{-1}$ .

# Specific energy density

We can first write down the **direction-averaged specific intensity**

$$\bar{I}_\nu = \frac{1}{4\pi} \int I_\nu d\Omega = \frac{1}{4\pi} F_\nu$$

from which we can easily derive the **specific energy density**, with units  $\text{erg cm}^{-3} \text{ Hz}^{-1}$ :

$$u_\nu = \frac{1}{c} \int I_\nu d\Omega = \frac{4\pi}{c} \bar{I}_\nu = \frac{1}{c} F_\nu$$

(think about why the factor  $1/c$  appears)

# What when

	continuum	integrated spectral line
point source (or extended object spatially integrated)	$F_\nu$ in Jy	e.g., $F_\nu \Delta\nu$ in Jy km s <sup>-1</sup>
extended object	$I_\nu$ or $I_\lambda$ with units such as erg s <sup>-1</sup> cm <sup>-2</sup> Hz <sup>-1</sup> arcsec <sup>-2</sup> , erg s <sup>-1</sup> cm <sup>-2</sup> μm <sup>-1</sup> arcsec <sup>-2</sup> , Jy sr <sup>-1</sup> in radio: $T_b$ (in K) or $I_\nu$ in e.g., mJy beam <sup>-1</sup>	$I$ with units such as erg s <sup>-1</sup> arcsec <sup>-2</sup> or Jy km s <sup>-1</sup> sr <sup>-1</sup> in radio: $T_b \Delta\nu$ (in K km s <sup>-1</sup> ) or $I$ in e.g., mJy km s <sup>-1</sup> beam <sup>-1</sup>

# Extended object, continuum

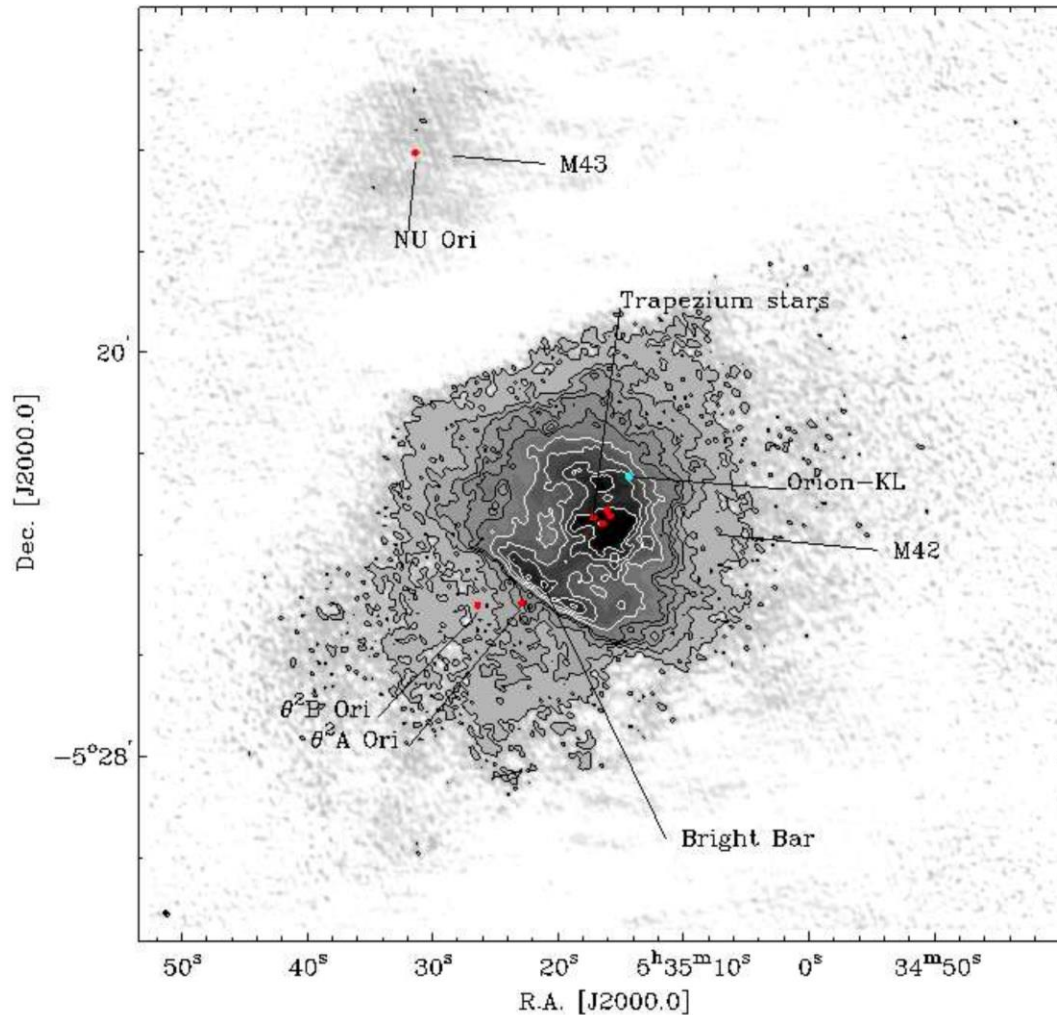
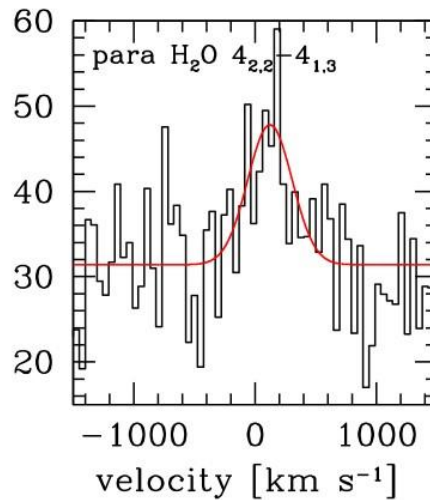
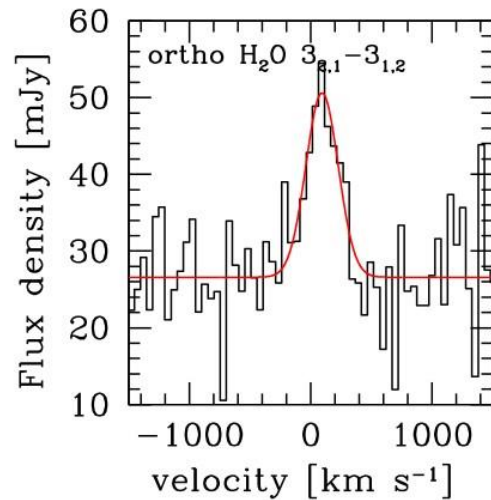
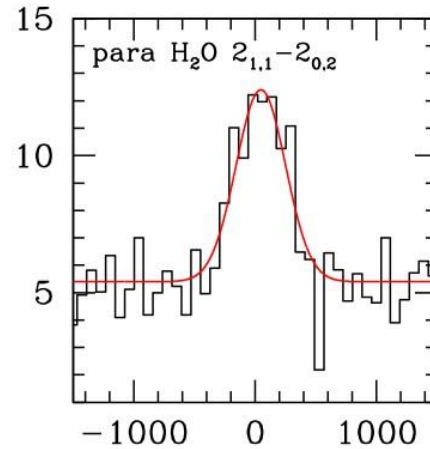
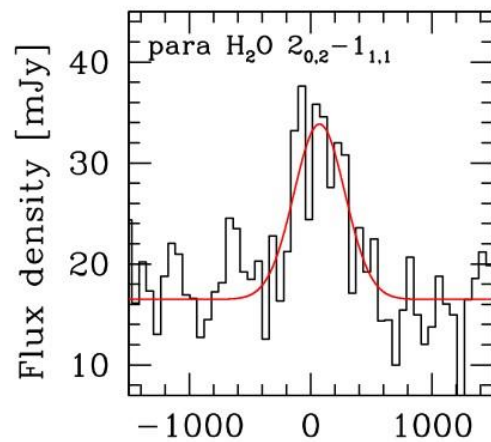


FIG. 1.— Continuum emission of the Orion Nebula at 1420.4 MHz, constructed using a multi-scale deconvolution (see Sect. 2.2). Contours indicate surface brightness levels of 20, 40, 60, 80 and 100 mJy beam<sup>-1</sup> (black contours) and 150, 200, 250, 300 and 350 mJy beam<sup>-1</sup> (white contours). The small ellipse in the lower lefthand corner indicates the FWHM size and the orientation of the synthesized beam (7''2 × 5''7 at a position angle of 29.7°). The image has been corrected for primary beam attenuation. The principal massive young stars are indicated by red dots. A cyan dot indicates the position of the Orion-KL region.

Van der Werf *et al.*,  
2013 (VLA data)

# Spectral line



Van der Werf *et al.*, 2011

# Today's Lecture

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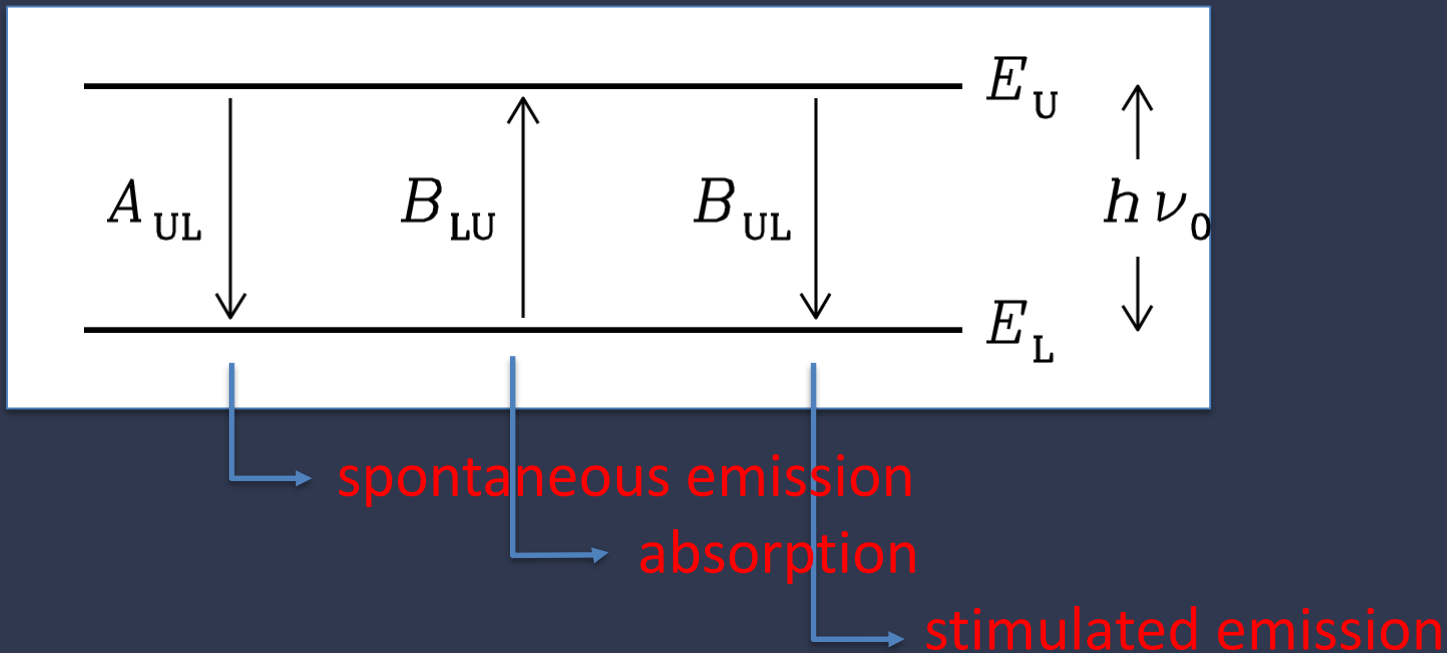
# Einstein coefficients

**Caution:** note different definitions of Einstein  $B$  coefficients between Draine (which we follow) and Rybicki & Lightman.

The definitions are related by  $B_D = \frac{4\pi}{c} B_{R\&L}$  .

There is no difference for the  $A$  coefficients.

# Radiative transitions and Einstein coefficients



In all cases, a photon of frequency  $\nu_{ul} = \frac{E_u - E_l}{h}$  is emitted or absorbed.

In reality this is not monochromatic but has a line profile function  $\varphi_\nu$  centred on  $\nu_{ul}$  and normalized such that  $\int \varphi_\nu d\nu = 1$ .

# 1) Spontaneous emission

$$X_u \rightarrow X_l + h\nu_{ul}$$

The number of spontaneous transitions per unit of time and volume is  $-\frac{dn_u}{dt}$  (units:  $\text{cm}^{-3} \text{s}^{-1}$ ).

This rate must be proportional to the density of particles in the upper state. Hence for spontaneous emission:

$$-\frac{dn_u}{dt} = \frac{dn_l}{dt} = A_{ul}n_u$$

where  $A_{ul}$  (in  $\text{s}^{-1}$ ) is the Einstein coefficient for spontaneous emission.

NB:  $1/A_{ul}$  is then the radiative lifetime of the upper level.

## 2) Absorption



The rate for this process must be proportional to both the density of particles in the lower state and the radiation energy density at frequency  $\nu_{ul}$  (taking into account the line profile). Hence for absorption:

$$\frac{dn_u}{dt} = -\frac{dn_l}{dt} = B_{lu}n_l \int u_\nu \phi_\nu d\nu$$

where  $B_{lu}$  is the Einstein coefficient for absorption.

### 3) Stimulated emission

$$X_u + h\nu_{ul} \rightarrow X_l + 2h\nu_{ul}$$

The rate for this process must be proportional to both the density of particles in the upper state and the radiation energy density at frequency  $\nu_{ul}$  (taking into account the line profile). Hence for stimulated emission:

$$\frac{dn_l}{dt} = -\frac{dn_u}{dt} = B_{ul}n_u \int u_\nu \phi_\nu d\nu$$

where  $B_{ul}$  is the Einstein coefficient for stimulated emission.

NB: the newly generated photon has same frequency, direction, phase and polarization as the original photon. The laser is based on this.

# Relations between Einstein coefficients

It is easy to derive the following relations between the Einstein coefficients:

$$B_{lu} = \frac{g_u}{g_l} B_{ul}$$
$$B_{ul} = \frac{c^3}{8\pi h \nu^3} A_{ul}$$

## Footnote: oscillator strength

In stead of  $A_{ul}$ , a quantity that is often used (and tabulated) in the literature is the oscillator strength  $f_{lu}$ .

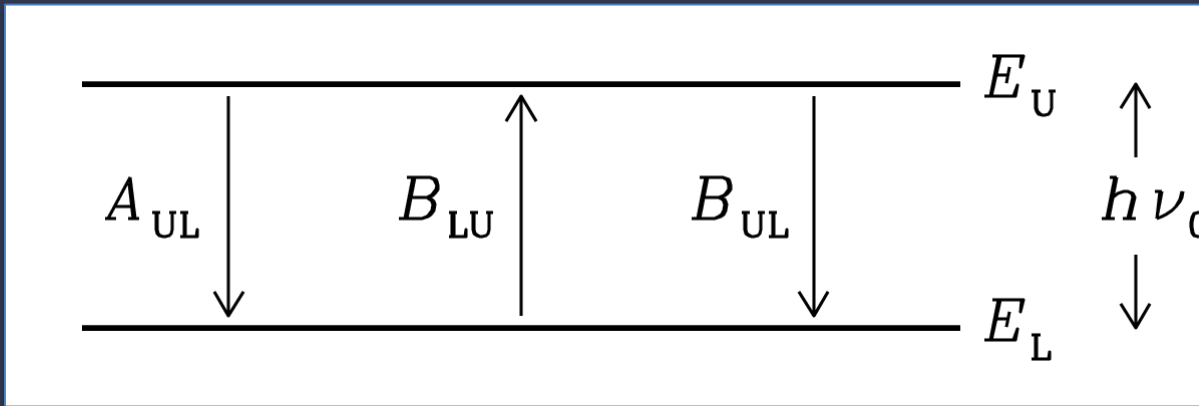
This is related to  $A_{ul}$  by

$$A_{ul} = \frac{8\pi^2 e^2 \nu_{lu}^2}{m_e c^3} \frac{g_l}{g_u} f_{lu}$$

where  $m_e$  is the mass of the electron.

See Draine Sect 6.3 for further information.

# Radiative transition rate equations with $n_\gamma$



Recall  $n_\gamma = \frac{c^2}{8h\nu^3} I_\nu$  so  $\bar{n}_\gamma = \frac{c^2}{2h\nu^3} \bar{I}_\nu = \frac{c^3}{8\pi h\nu^3} u_\nu$ .

Now the rate equations can be written (assume  $\varphi_\nu$  narrow)

$$\left(\frac{dn_l}{dt}\right)_{u \rightarrow l} = n_u A_{ul} (1 + \bar{n}_\gamma)$$

spontaneous emission

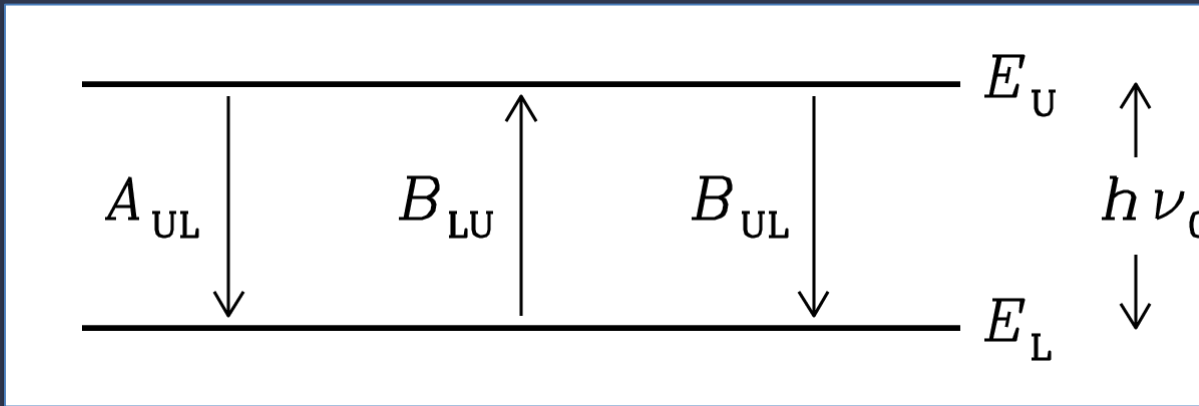
stimulated emission

$$\left(\frac{dn_u}{dt}\right)_{l \rightarrow u} = n_l \frac{g_u}{g_l} A_{ul} \bar{n}_\gamma$$

absorption



# When is stimulated emission important?



spontaneous emission

$$\left(\frac{dn_l}{dt}\right)_{u \rightarrow l} = n_u A_{ul} (1 + \bar{n}_\gamma)$$

stimulated emission

$$LTE: n_\gamma = \frac{1}{e^{h\nu/kT} - 1}$$

$$h\nu \gg kT \Rightarrow n_\gamma \ll 1$$

$$h\nu \ll kT \Rightarrow n_\gamma \gg 1$$

Importance of stimulated emission depends on  $n_\gamma$

optical, UV, X-ray,...

radio

# Line profile

The line profile is a combination of

1. natural linewidth (quantum uncertainty)

This gives a **Lorentz profile** with wings falling off as  $v^{-2}$

2. Doppler motions (thermal + bulk motion)

Thermal motions give a **Gaussian profile** with wings falling off as  $\exp(-v^2)$

Their convolution is a **Voigt profile**.

The natural linewidth is always much smaller than the thermal linewidth, so in practice the line profile is Gaussian (with one exception – see later).

# Gaussian line profile

Maxwell distribution:

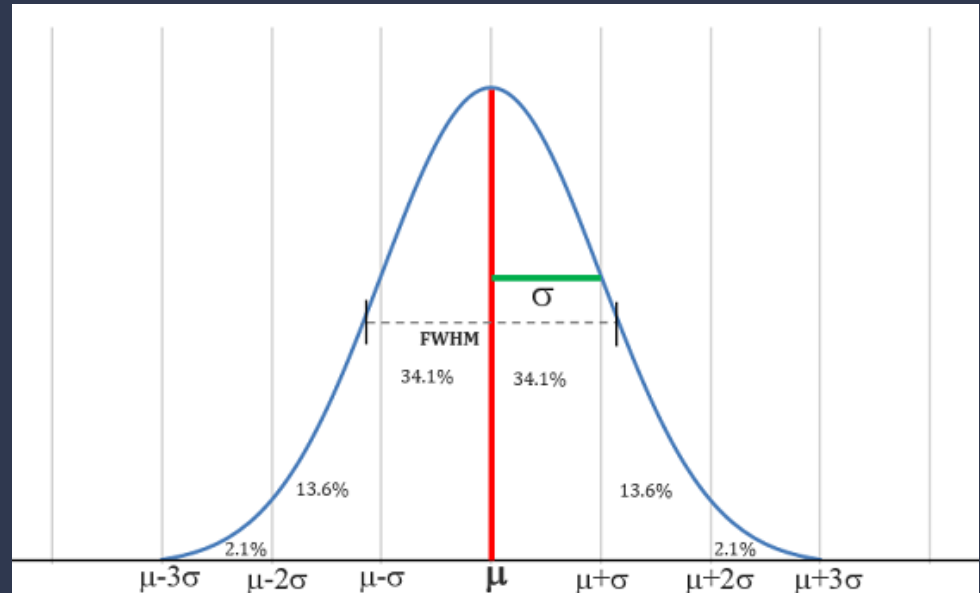
fraction  $f_v dv$  of particles with velocity between  $v$  and  $v+dv$  is

$$f_v dv = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{(v-v_0)^2}{2\sigma_v^2}} dv$$

$$\sigma_v = \sqrt{\frac{kT_k}{m}}$$

$$\Delta v_{\text{FWHM}} = \sqrt{8 \ln 2} \sigma_v = 2.35 \sigma_v$$

1-dimensional velocity  
dispersion



In frequency:

$$\varphi_v = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{(v-v_0)^2}{2\sigma_v^2}}$$

$$\sigma_v = \sigma_v \frac{v_0}{c}$$

For hydrogen:

phase	$T_k$ [K]	$\sigma_v$ [km s <sup>-1</sup> ]
CNM	100	1
WNM, WIM	10 <sup>4</sup>	10
HIM	10 <sup>6</sup>	100

## Core vs. wings

Sufficiently far from the line centre, the Lorentz wings (“damping wings”) will dominate (see  $\nu$ -dependence given earlier).

This happens for  $|v - v_0| \gg 4.5 \sigma_v$  .

This will always be irrelevant except for extremely strong lines (in practice only in specific HI Lyman  $\alpha$  absorption lines, so-called damped Lyman  $\alpha$  absorbers).

# Today's Lecture

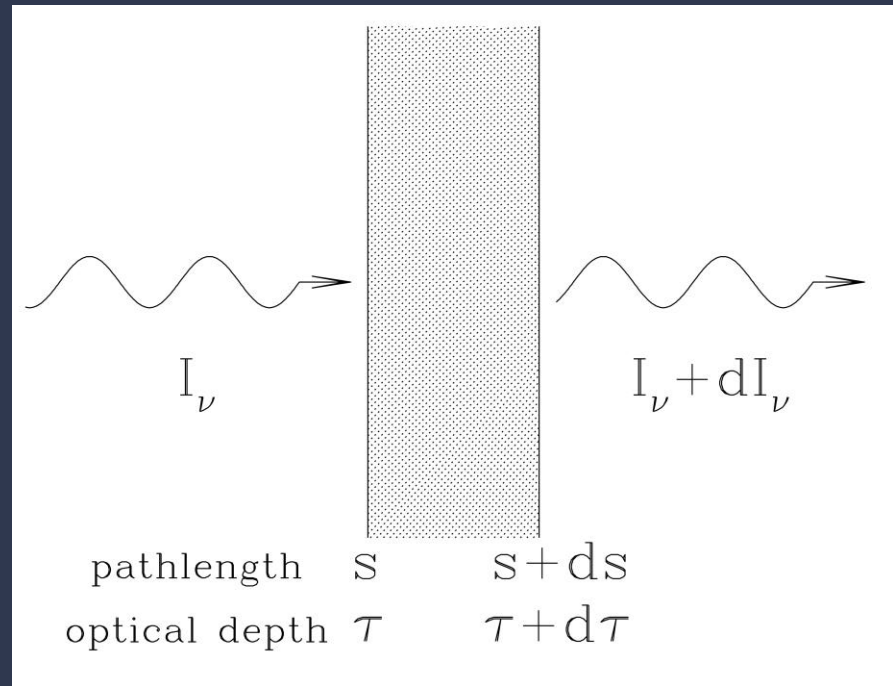
1. Basic physical conditions
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# Radiative transfer

Incident radiation:  $I_\nu$   
At the far side:  $I_\nu + dI_\nu$

What is  $dI_\nu$ ?

NB: scattering ignored  
(only radiative transitions)



$$dI_\nu = -I_\nu \kappa_\nu ds + j_\nu ds$$

spontaneous emission

net absorption (=absorption  
corrected for stimulated emission)

$j_\nu$  : emissivity [ $\text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$ ]

$\kappa_\nu$  : absorption coefficient [ $\text{cm}^{-1}$ ]

normally positive but not always (when not?)

# Emissivity

- Spontaneous emission: number of radiative decays  $u \rightarrow l$  per unit of time and volume:  $n_u A_{ul}$
- $\rightarrow$  power radiated in this transition per unit volume:  $h\nu_{ul} n_u A_{ul}$
- and per steradian:  $(h\nu_{ul} / 4\pi) n_u A_{ul}$
- and with a line profile  $\varphi_\nu$ :

$$j_\nu = \frac{h\nu_{ul}}{4\pi} A_{ul} n_u \varphi_\nu$$

all information on line  
shape and central  
frequency are encoded  
in  $\varphi_\nu$

# Absorption coefficient

Easiest approach is by considering cross sections:

$$\kappa_\nu = n_l \sigma_{lu}(\nu) - n_u \sigma_{ul}(\nu)$$

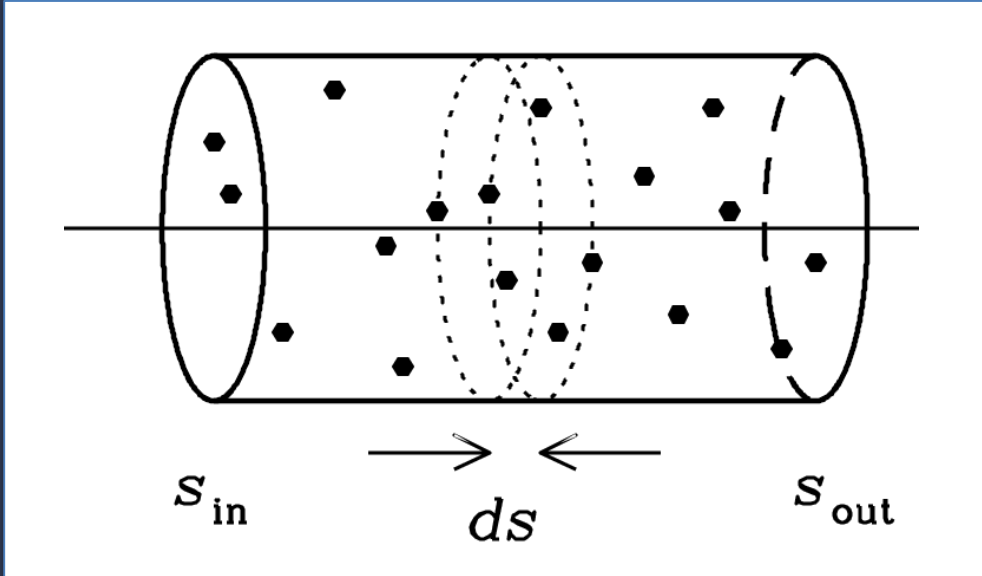
absorption

stimulated emission

So: how do we connect the cross sections to the Einstein coefficients?



# Cross section and Einstein coefficients



Cylinder length:  $c \, dt$

Absorption cross section for a particle:  
 $\sigma_{lu}(\nu)$

$$dn_u = n_l c \frac{u_\nu}{h\nu} \sigma_{lu}(\nu) d\nu dt$$

$$\left( \frac{dn_u}{dt} \right)_{l \rightarrow u} = n_l \int \sigma_{lu}(\nu) c \frac{u_\nu}{h\nu} d\nu \approx n_l c \frac{u_\nu}{h\nu} \int \sigma_{lu}(\nu) d\nu$$

# Absorption cross section and line profile

$$dn_u = n_l c \frac{u_\nu}{h\nu} \sigma_{lu}(\nu) d\nu dt$$

$$\left(\frac{dn_u}{dt}\right)_{l \rightarrow u} = n_l c \frac{u_\nu}{h\nu} \int \sigma_{lu}(\nu) d\nu = B_{lu} n_l \int u_\nu \varphi_\nu d\nu$$

Combining with the relations between the Einstein coefficients, we find

$$\sigma_{lu}(\nu) = \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \varphi_\nu$$

with

$$\int \varphi_\nu d\nu = 1$$

# Absorption coefficient

$$\kappa_\nu = n_l \sigma_{lu}(\nu) - n_u \sigma_{ul}(\nu)$$

Use relation  $\sigma_{ul} \leftrightarrow B_{ul}$  & relation  $B_{ul} \leftrightarrow B_{lu}$   
 $\rightarrow$  relation  $\sigma_{ul} \leftrightarrow \sigma_{lu}$ :

$$\kappa_\nu = n_l \sigma_{lu}(\nu) \left( 1 - \frac{n_u/n_l}{g_u/g_l} \right) = n_l \sigma_{lu}(\nu) \left( 1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}} \right)$$

Use definition of  $T_{ex}$

Now use relation  $\sigma_{lu} \leftrightarrow B_{lu}$  & relation  $A_{ul} \leftrightarrow B_{lu}$ :

# Absorption coefficient

$$\kappa_\nu = n_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left( 1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}} \right) \varphi_\nu$$

absorption

stimulated emission

frequency dependence is in  $\varphi_\nu$

# Equation of transfer

Define **optical depth**  $\tau_v$ :

$$d\tau_v = \kappa_v ds$$

so 
$$dI_v = -I_v \kappa_v ds + j_v ds = -I_v d\tau_v + \frac{j_v}{\kappa_v} d\tau_v$$

Also define the **source function**  $S_v$ : 
$$S_v = \frac{j_v}{\kappa_v}$$

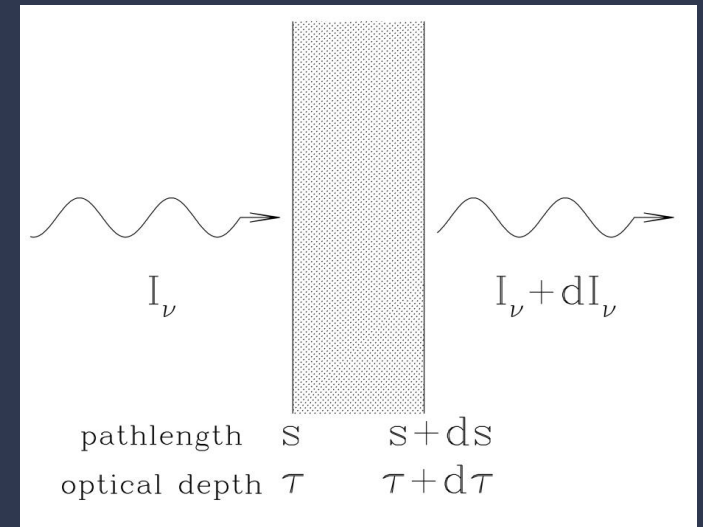
so 
$$dI_v = (S_v - I_v) d\tau_v$$

with the formal solution (see Draine Sect. 7.4 for derivation):

$$I_v(\tau_v) = I_v(0)e^{-\tau_v} + \int_0^{\tau_v} e^{-(\tau_v - \tau'_v)} S_v d\tau'_v$$

radiation emitted by the slab  
but also attenuated by it

incoming radiation  
attenuated by the slab



# Kirchhoff's Law for spectral line radiation

Evaluate  $S_\nu$  using the derived expressions for  $j_\nu$  &  $\kappa_\nu$ :

$$S_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\frac{h\nu}{e^{kT_{ex}} - 1}} = B_\nu(T_{ex})$$

This is the spectral line version of **Kirchhoff's Law** (which can be shown to be valid for all thermal radiation).

So 
$$I_\nu = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} B_\nu(T_{ex}(\tau'_\nu)) d\tau'_\nu$$

Simplify by taking  $T_{ex}$  constant:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{ex})(1 - e^{-\tau_\nu})$$

radiation thermalizes more and more as it passes through the slab

incoming radiation  
attenuated by the slab

# Equation of transfer

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{ex})(1 - e^{-\tau_\nu})$$

or

$$T_b = T_c e^{-\tau_\nu} + T_{ex}(1 - e^{-\tau_\nu})$$

background continuum

Limiting cases:

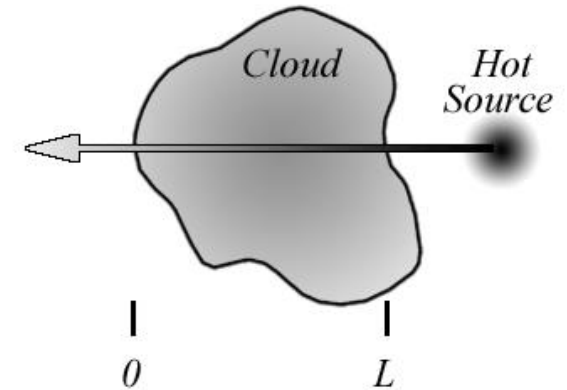
1. **Optically thick:**  $\tau_\nu \gg 1$ :

$$I_\nu = B_\nu(T_{ex})$$

(where has the spectral line gone?)

# Limiting cases

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{ex})(1 - e^{-\tau_\nu})$$

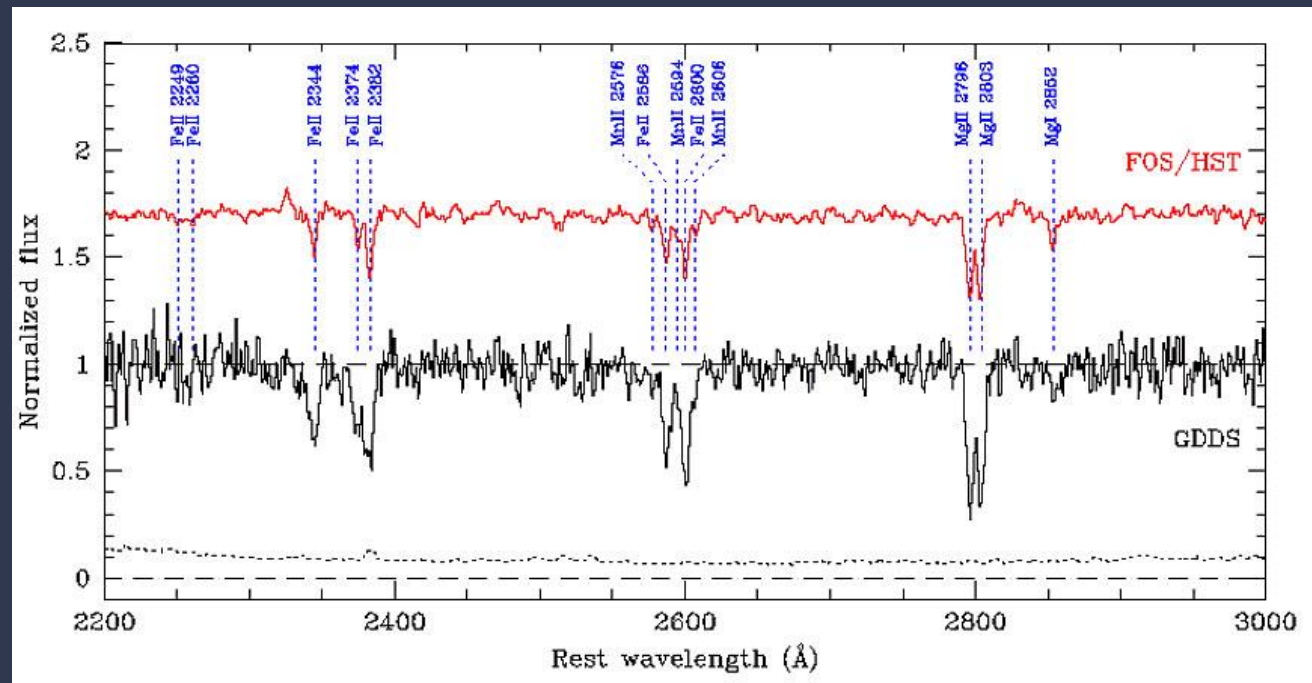


## 2. Bright background object

optical/UV regime: upper level almost unpopulated

→  $T_{ex}$  very low, so

$$I_\nu = I_\nu(0)e^{-\tau_\nu}$$





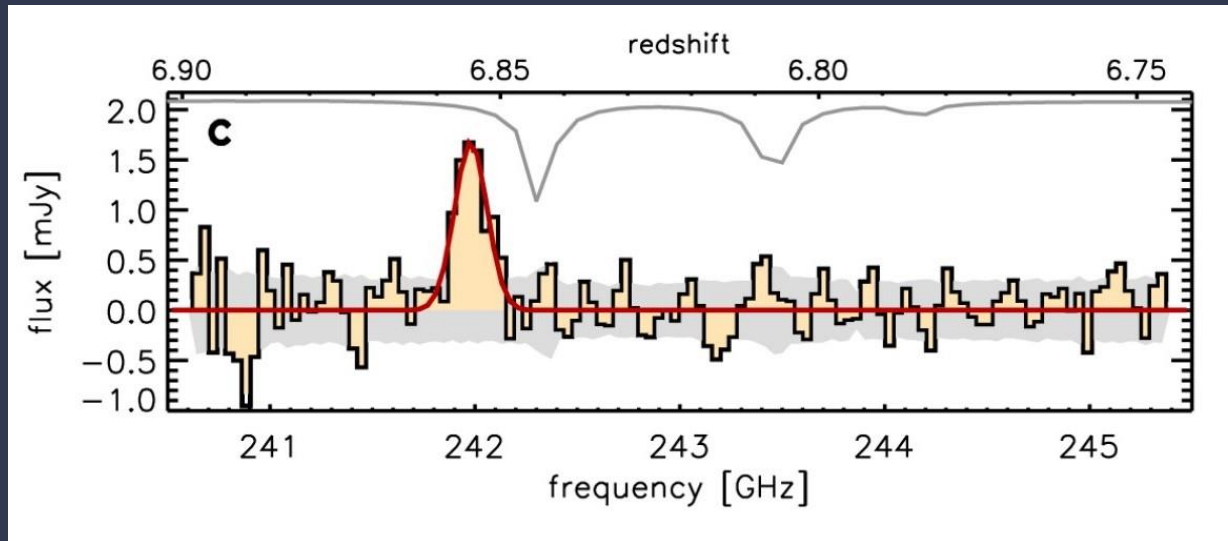
# Limiting cases

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{ex})(1 - e^{-\tau_\nu})$$

3. **Optically thin:**  $\tau_\nu \ll 1$  (for simplicity also assume no background source):

$$I_\nu = B_\nu(T_{ex})(1 - e^{-\tau_\nu}) \approx B_\nu(T_{ex})(1 - 1 + \tau_\nu) = \tau_\nu B_\nu(T_{ex})$$

line shape and central frequency are encoded in  $\tau_\nu$



Smit *et al.*, 2018

# Optical depth and column density

$$\kappa_\nu = n_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) \varphi_\nu \quad \text{so}$$

$$\tau_\nu = n_l ds \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) \varphi_\nu = N_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) \varphi_\nu \quad \text{so}$$

$$\int \tau_\nu d\nu = N_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) = N_u \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(e^{\frac{h\nu_{ul}}{kT_{ex}}} - 1\right)$$

Now if optically thin:

$$\int I_\nu d\nu \approx \int \tau_\nu B_\nu(T_{ex}) d\nu \approx B_\nu(T_{ex}) \int \tau_\nu d\nu = \frac{h\nu}{4\pi} A_{ul} N_u$$

Here  $N_i$  is the **column density** [ $\text{cm}^{-2}$ ] in level  $i$ .

# Maser emission

The excitation process may give rise to **inversion**:  $\frac{n_u}{g_u} > \frac{n_l}{g_l}$

Using  $\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-\frac{h\nu_{ul}}{kT_{ex}}}$  this gives  $T_{ex} < 0$

Now with  $\tau_\nu = N_l \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{ul}^2} A_{ul} \left(1 - e^{-\frac{h\nu_{ul}}{kT_{ex}}}\right) \varphi_\nu$  also  $\tau_\nu < 0$

and since  $I_\nu = I_\nu(0)e^{-\tau_\nu} + \dots$  incoming radiation is now **amplified**:

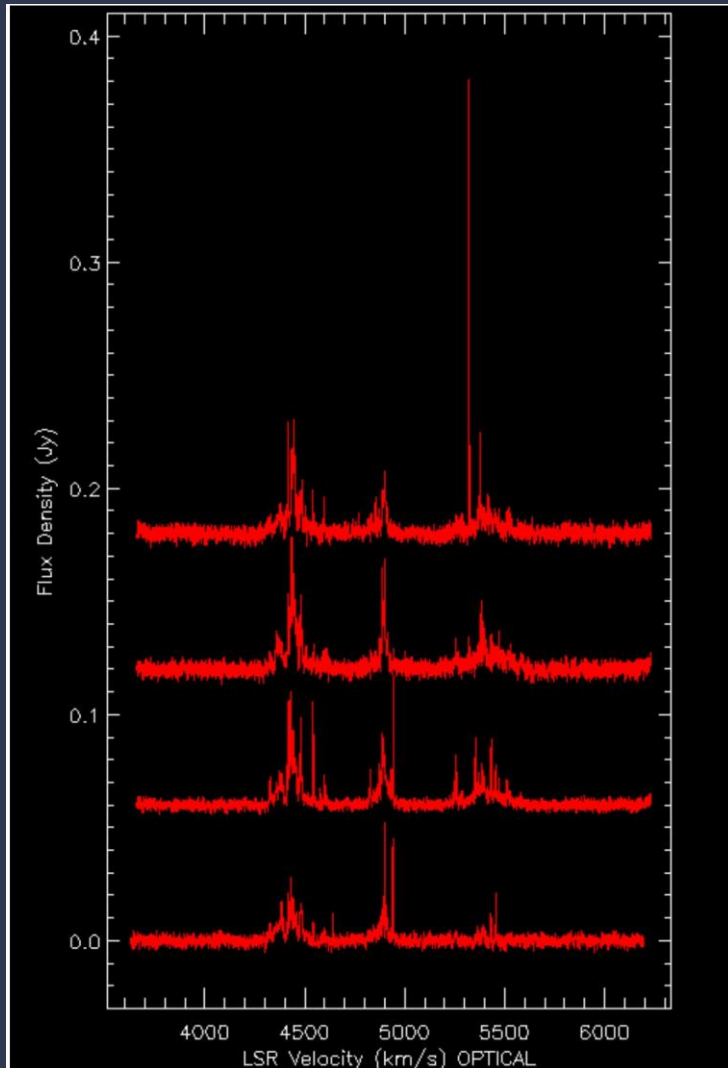
**maser emission**

# Interstellar masers

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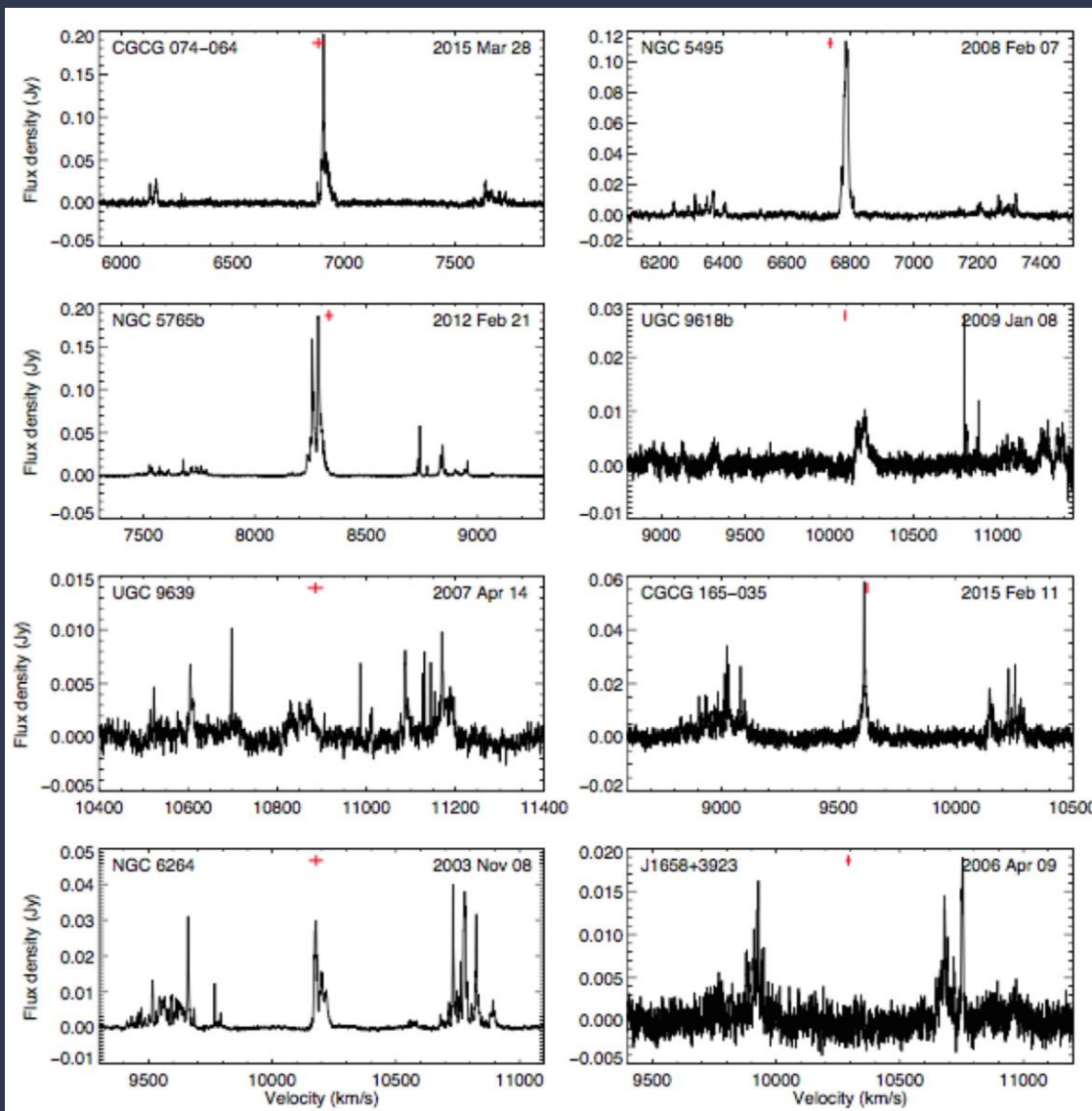
1. Maser lines are very bright
2. Lines are narrow and peaked
3. Pronounced variability
4. Maser emission comes from small “maser spots”
5. Molecules with observed maser transitions: OH, H<sub>2</sub>O, NH<sub>3</sub>, H<sub>2</sub>CO, CH<sub>3</sub>OH, SiO, HCN, etc.
6. Needs strong (IR) radiation field (and very high densities)
7. In IR-luminous galactic nuclei: OH, H<sub>2</sub>O megamasers

# Maser variability

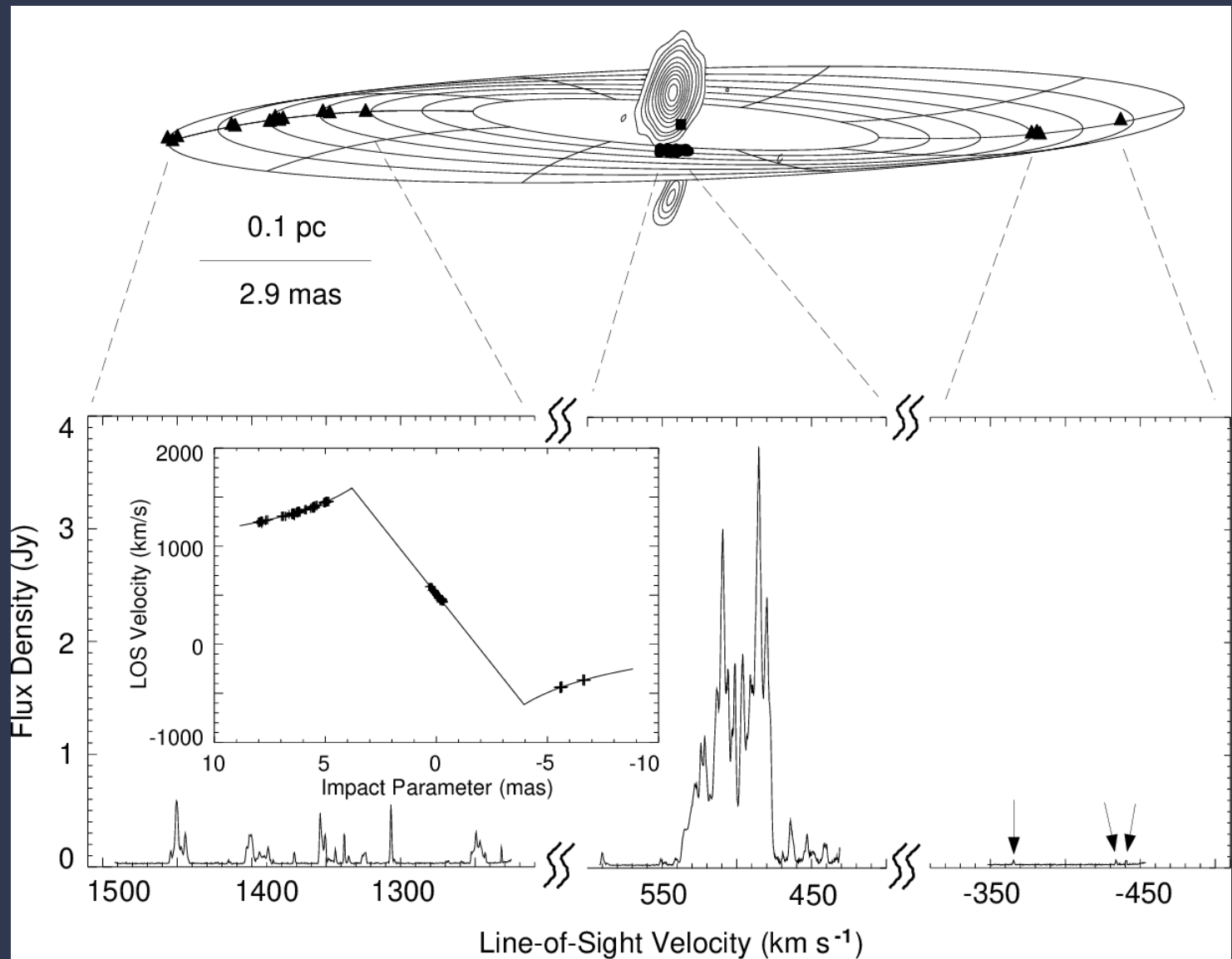


variable H<sub>2</sub>O maser emission in IR-luminous  
galaxy Mrk1419

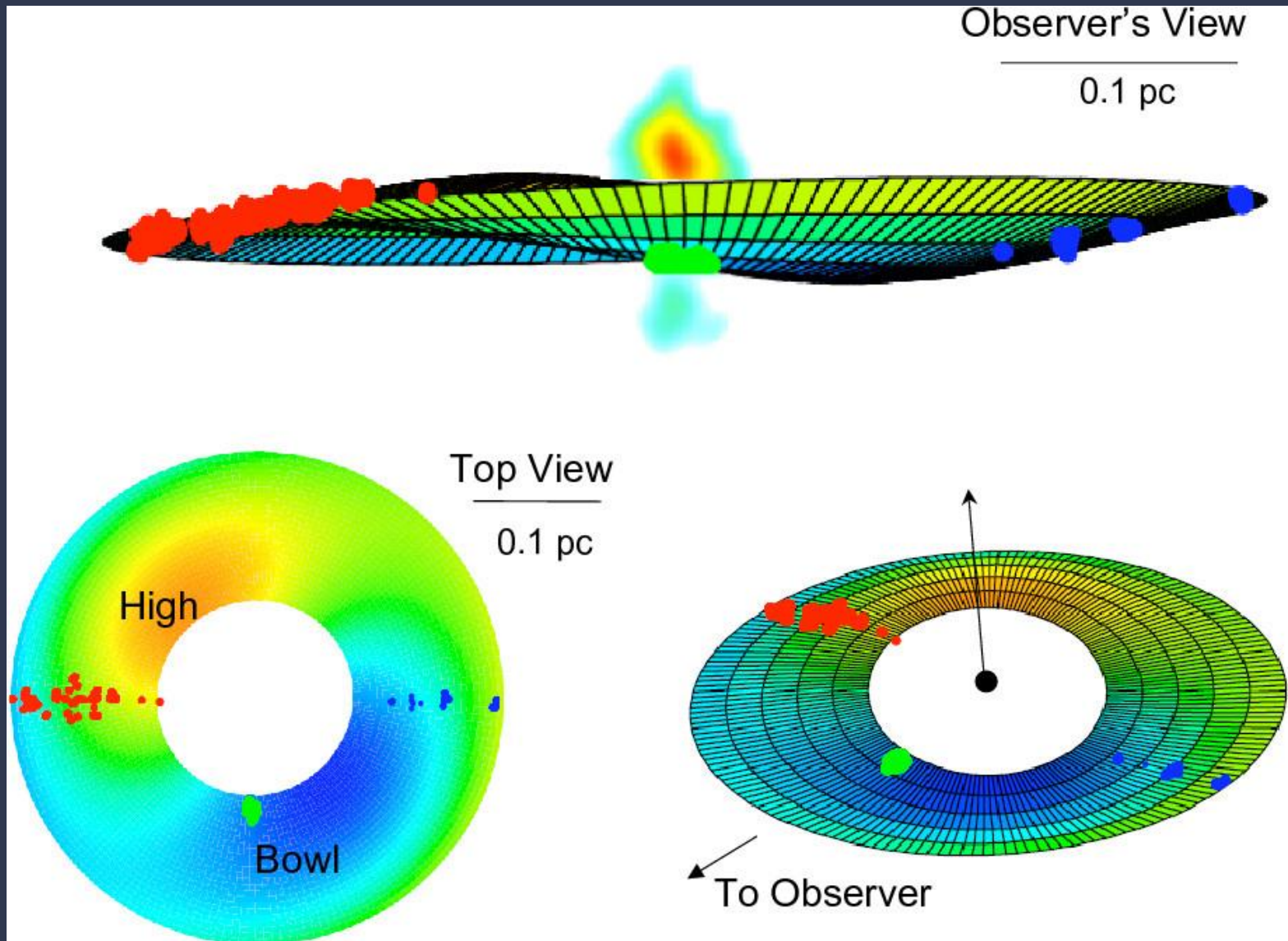
# Extragalactic H<sub>2</sub>O maser spectra



# Circumnuclear H<sub>2</sub>O masers in NGC4258



# Circumnuclear H<sub>2</sub>O masers in NGC4258





# Next lecture

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The HI 21cm line and the 2-phase ISM

1. Application of radiative process to the HI 21cm line
2. Line emission and absorption
3. Discovery and nature of the 2-phase ISM