Interstellar Medium 2020

Lecture 6: Collisional excitation and nebular diagnostics

Sterrewacht Leiden

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Course Contents

1. Introduction and ecology of the interstellar medium
2. Physical conditions and radiative processes
3. The atomic interstellar medium
4. Ionization and recombination
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6. Collisional excitation and nebular diagnostics
7. Molecules, molecular excitation and molecular clouds
8. Interstellar dust
9. Thermal balance
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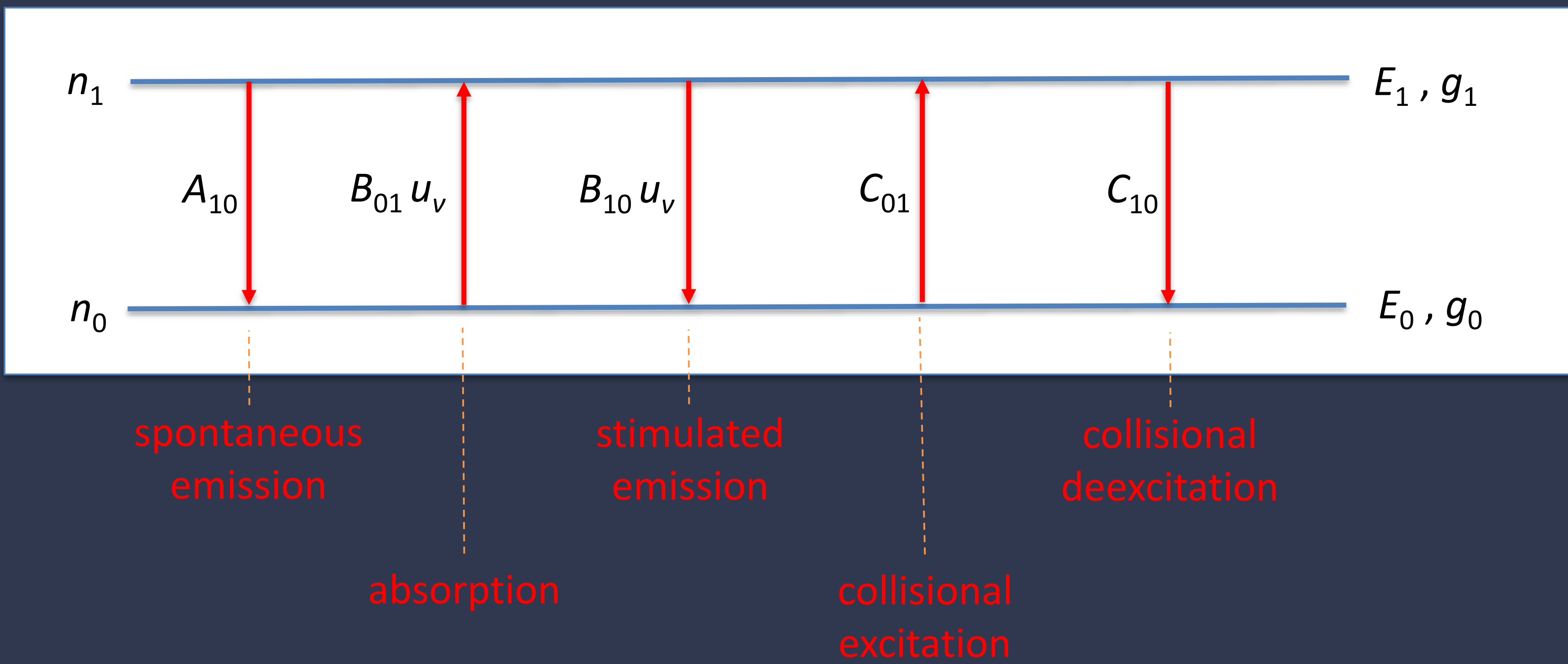
Today's lecture

1. Excitation by collisions
2. Critical density
3. Nebular diagnostics of temperature and density

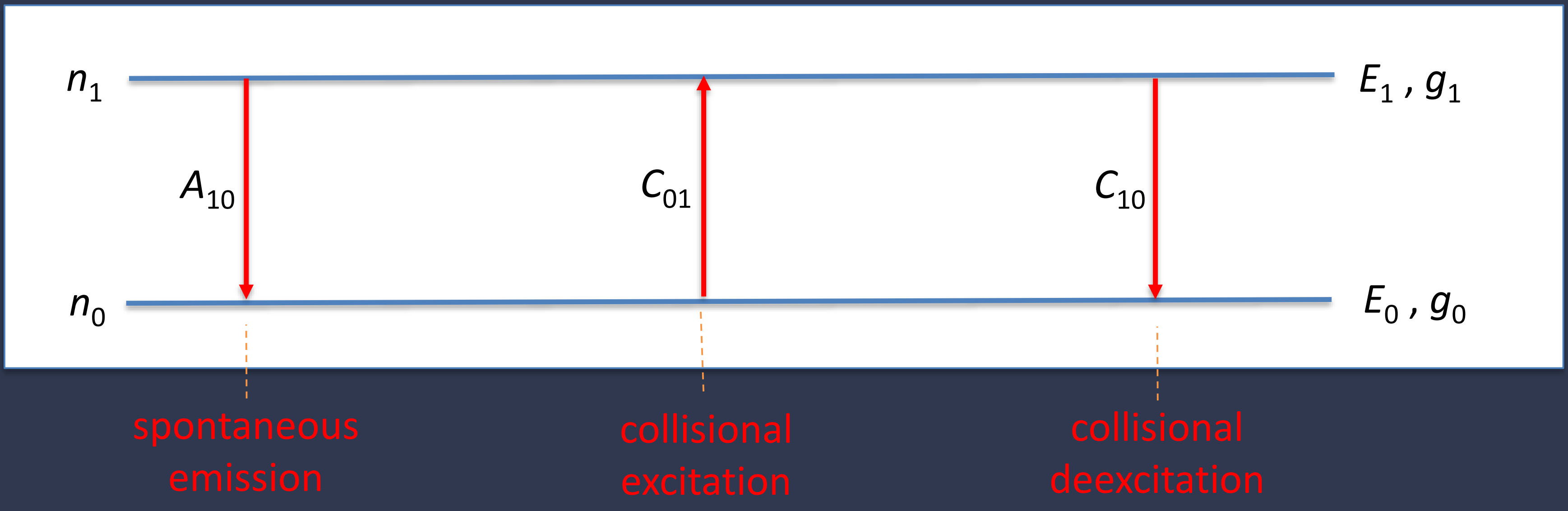
Corresponding textbook material: Draine Ch 17 & 18

Excitation in a 2-level system



Excitation without induced radiative transitions

(when is this valid?)



Collisional (de)excitation rates must be proportional to density n_c of the collision partner (typically H_2 , H or e^- , depending on the environment).

So we write: $C_{01} = k_{01}\, n_c$ and $C_{10} = k_{10}\, n_c$

Collisional (de)excitation coefficients

Collisional (de)excitation coefficients k_{01} & k_{10} [$\mathrm{cm}^3\ \mathrm{s}^{-1}$] depend on:

- nature of the collision partner (and, in principle, its quantum state): H_2, H or e^-
- kinetic temperature T_{kin} (coefficient involves collision cross section integrated over a Maxwell distribution); we will usually not write this explicitly, to keep the equations simple.

There is a relation between excitation and deexcitation coefficients:

$$k_{01} = \frac{g_1}{g_0} k_{10} e^{-\frac{E_{10}}{kT_{\mathrm{kin}}}}$$

Collision strengths

For collisional (de)excitation of ions by electrons, we often use the dimensionless collision strength Ω_{10} (Draine uses this extensively):

$$k_{10} = \frac{h^2}{(2\pi m_e)^{3/2}} \frac{1}{\sqrt{kT_{\mathrm{kin}}}} \frac{\Omega_{10}(T)}{g_1}$$

Inserting numbers: $k_{10} = \frac{8.629 \cdot 10^{-8}}{\sqrt{T_4}} \frac{\Omega_{10}}{g_1}$ [cm^3 s^{-1}] with $T_4 = T/10^4$ K

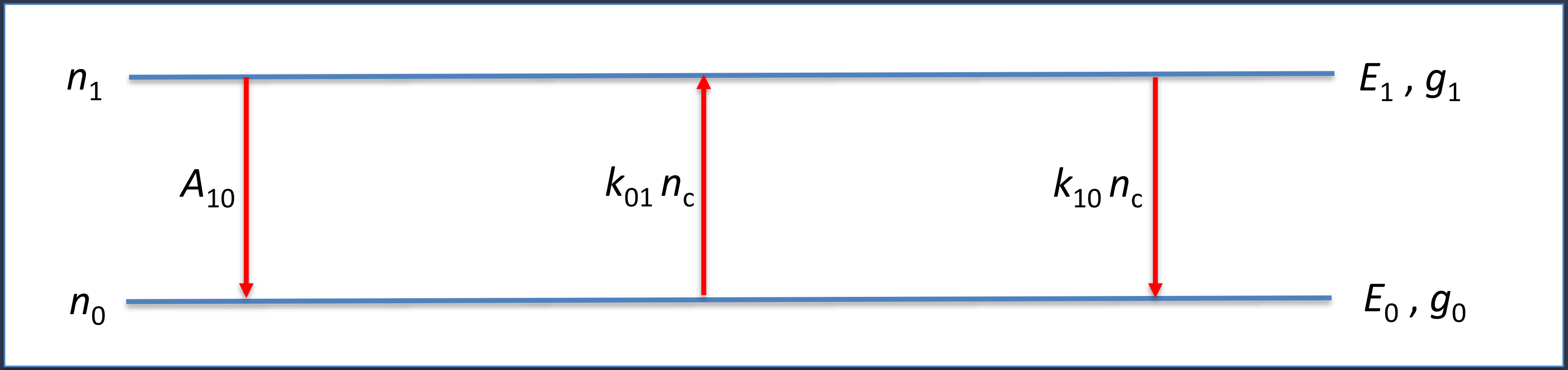
Advantages:

- for T up to 10^4 K, Ω_{10} approximately independent of T
- Ω_{10} typically between 1 and 10

Today's lecture

1. Excitation by collisions
2. Critical density
3. Nebular diagnostics of temperature and density

Excitation without induced radiative transitions



Considering level 1, we can write: $$\frac{dn_1}{dt} = -A_{10}n_1 + k_{01}n_c n_0 - k_{10}n_c n_1$$

In statistical equilibrium, $dn_1 / dt = 0$, so $$\frac{n_1}{n_0} = \frac{k_{01}n_c}{k_{10}n_c + A_{10}}$$

Using the relation between k_{01} and k_{10} , this becomes

$$\frac{n_1}{n_0} = \frac{1}{1 + \frac{A_{10}}{k_{10}n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\mathrm{kin}}}}$$

Critical density

Now define the critical density: $\boxed{n_{\text{crit}} = \frac{A_{10}}{k_{10}}}$

(recall: A_{10} [s^{-1}] and k_{10} [$cm^3\ s^{-1}$])

So we can finally write: $\boxed{\frac{n_1}{n_0} = \frac{1}{1 + \frac{n_{\text{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}}}$

How large is n_{crit} ?

- small for forbidden transitions
- large for permitted transitions

High density limit

$$\frac{n_1}{n_0} = \frac{1}{1 + \frac{n_{\text{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}}$$

If $n_c >> n_{crit}$:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}}$$

so $T_{ex} = T_{kin}$: thermalized levels

Population ratio has its LTE value (independent of density)

Time between collisions much shorter than radiative lifetime (note Einstein *A* coefficient has disappeared).

Low density limit

$$\frac{n_1}{n_0} = \frac{1}{1 + \frac{n_{\text{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}}$$

If $n_c << n_{crit}$:

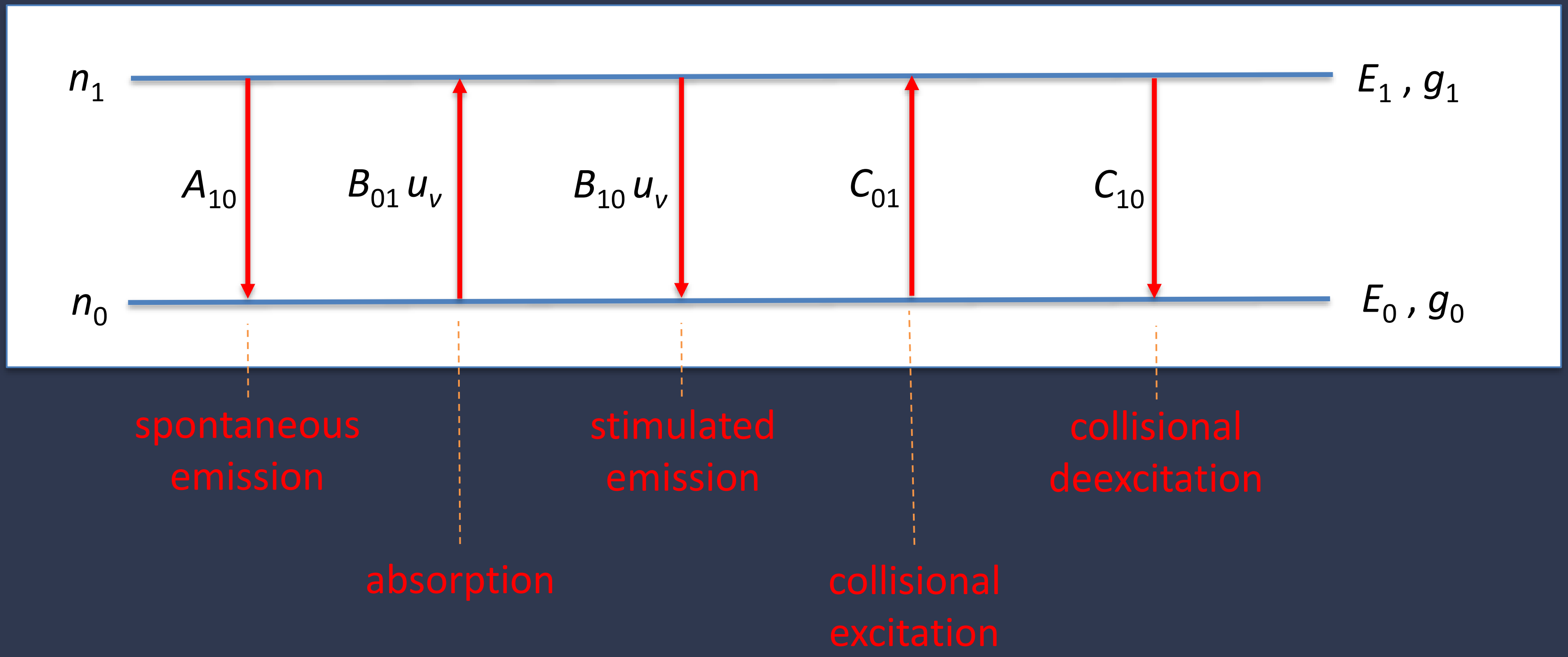
$$\frac{n_1}{n_0} = \frac{n_c}{n_{\text{crit}}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}}$$

so $T_{ex} < T_{kin}$: subthermal excitation

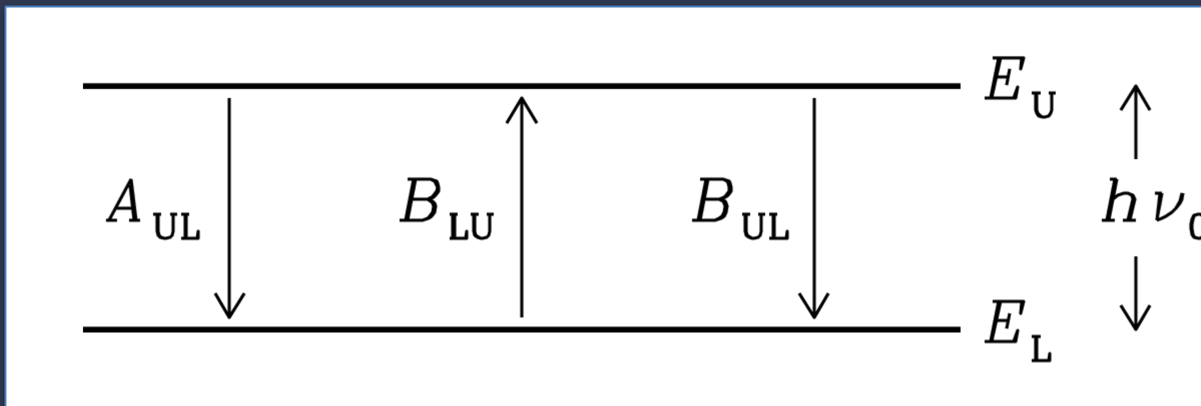
Population ratio has its LTE value × n_c / n_{crit} (density-dependent)

Time between collisions comparable to or longer than radiative lifetime.

Excitation of a 2-level system



Recall: excitation with only radiative transitions



Recall $\boxed{n_\gamma = \dfrac{c^2}{8h\nu^3} I_\nu}$ so $\boxed{\bar{n}_\gamma = \dfrac{c^2}{2h\nu^3} \bar{I}_\nu = \dfrac{c^3}{8\pi h\nu^3} u_\nu}$.

In the blackbody case: $\boxed{\bar{n}_\gamma = \dfrac{1}{e^{h\nu/kT_{\mathrm{rad}}} - 1}}$

but we can also write this generally, as a definition of T_{rad}.

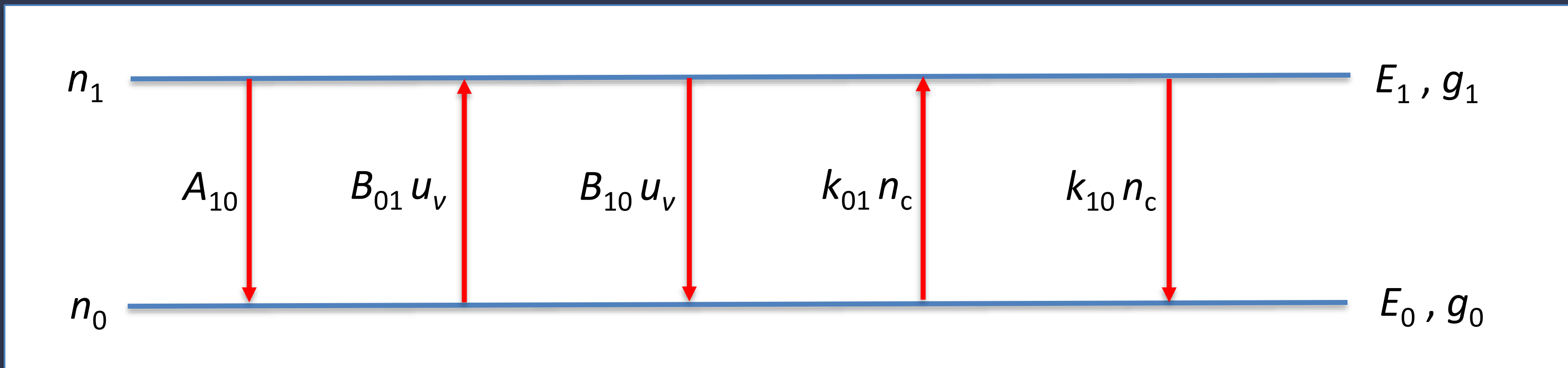
stimulated emission

This gave: $$\frac{dn_1}{dt} = \left[n_0 \bar{n}_\gamma \frac{g_1}{g_0} - n_1 (1 + \bar{n}_\gamma) \right] A_{10}$$

absorption

spontaneous emission

Full treatment of 2-level excitation



Now we get: $$\frac{dn_1}{dt} = n_0\left(k_{01}n_c + \bar{n}_\gamma \frac{g_1}{g_0} A_{10}\right) - n_1\left[k_{10}n_c + \left(1 + \bar{n}_\gamma\right)A_{10}\right]$$

collisional excitation

absorption

collisional deexcitation

stimulated emission

spontaneous emission

so .

Statistical equilibrium: $dn_1 / dt = 0$, so:

$$\frac{n_1}{n_0} = \frac{k_{01}n_c + \bar{n}_\gamma \frac{g_1}{g_0} A_{10}}{k_{10}n_c + \left(1 + \bar{n}_\gamma\right)A_{10}}$$

Full treatment of 2-level excitation

$$\frac{n_1}{n_0} = \frac{k_{01} n_c + \bar{n}_\gamma \frac{g_1}{g_0} A_{10}}{k_{10} n_c + (1 + \bar{n}_\gamma) A_{10}}$$

Now define a more general critical density:
(why does this make sense?)

$$n_{\text{crit}} = \frac{(1 + \bar{n}_\gamma) A_{10}}{k_{10}}$$

This gives

$$\frac{n_1}{n_0} = \frac{1}{1 + \frac{n_{\text{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}} + \frac{1}{1 + \frac{n_c}{n_{\text{crit}}}} \frac{g_1}{g_0} \frac{\bar{n}_\gamma}{1 + \bar{n}_\gamma}$$

only important if $n_\gamma >> 1$
→ radio

Now using $\bar{n}_\gamma = \frac{1}{e^{h\nu/kT_{\text{rad}}} - 1}$ we finally get

$$\frac{n_1}{n_0} = \frac{1}{1 + \frac{n_{\text{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{kin}}}} + \frac{1}{1 + \frac{n_c}{n_{\text{crit}}}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\text{rad}}}}$$

Limiting cases

$$\frac{n_1}{n_0} = \frac{1}{1+\frac{n_{\mathrm{crit}}}{n_c}}\frac{g_1}{g_0}e^{-\frac{E_{10}}{kT_{\mathrm{kin}}}} + \frac{1}{1+\frac{n_c}{n_{\mathrm{crit}}}}\frac{g_1}{g_0}e^{-\frac{E_{10}}{kT_{\mathrm{rad}}}}$$

Behaviour in limiting cases now depends on both n_c and n_γ .

- $n_\gamma \ll 1$: back to what we discussed before

- $n_\gamma \gg 1$:
  - if $n_c \ll n_{\mathrm{crit}}$: $T_{ex} = T_{rad}$
  - if $n_c \gg n_{\mathrm{crit}}$: both collisions and radiation important: more complicated

Application 1: the HI 21cm line

$$\frac{n_1}{n_0} = \frac{1}{1+\frac{n_{\mathrm{crit}}}{n_c}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\mathrm{kin}}}} + \frac{1}{1+\frac{n_c}{n_{\mathrm{crit}}}} \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT_{\mathrm{rad}}}}$$

$E_{10} / k = 0.0682$ K
$A_{10} = 2.88 \cdot 10^{-15}$ s^{-1}
$k_{10} \approx 1.2 \cdot 10^{-10}$ cm^3 s^{-1} (at 100K, for collisions with H)

Now calculate $$n_{\mathrm{crit}} = \frac{(1+\bar{n}_\gamma)A_{10}}{k_{10}}$$

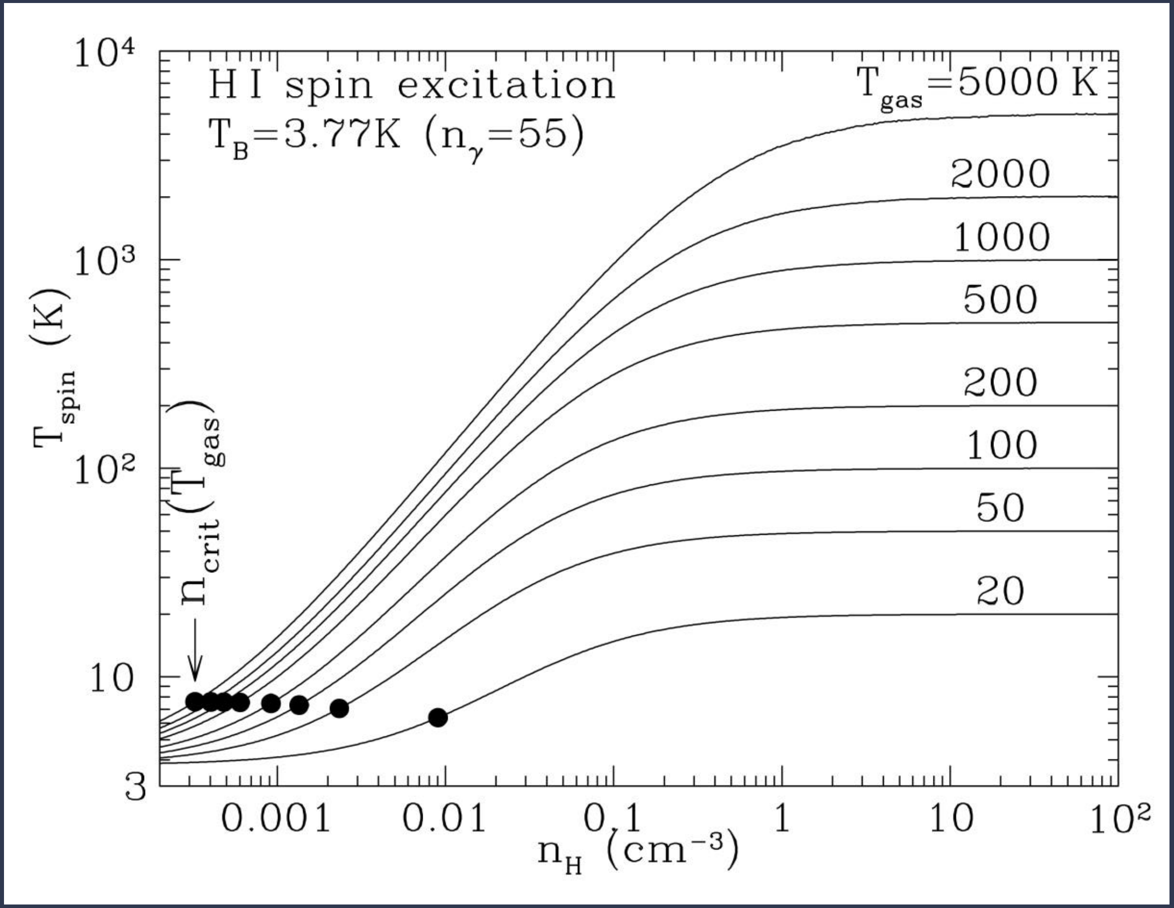
What is n_γ ?

$T_{\mathrm{rad}} = \underbrace{2.73\text{ K}}_{\text{CMB}} + \underbrace{1\text{ K}}_{\text{diffuse Galactic 21cm emission}} = 3.73$ K

→ $n_\gamma \approx 55$

Insert numbers: $n_{\mathrm{crit}} \approx 1.7 \cdot 10^{-3}$ cm^{-3}.

HI 21cm line spin temperature

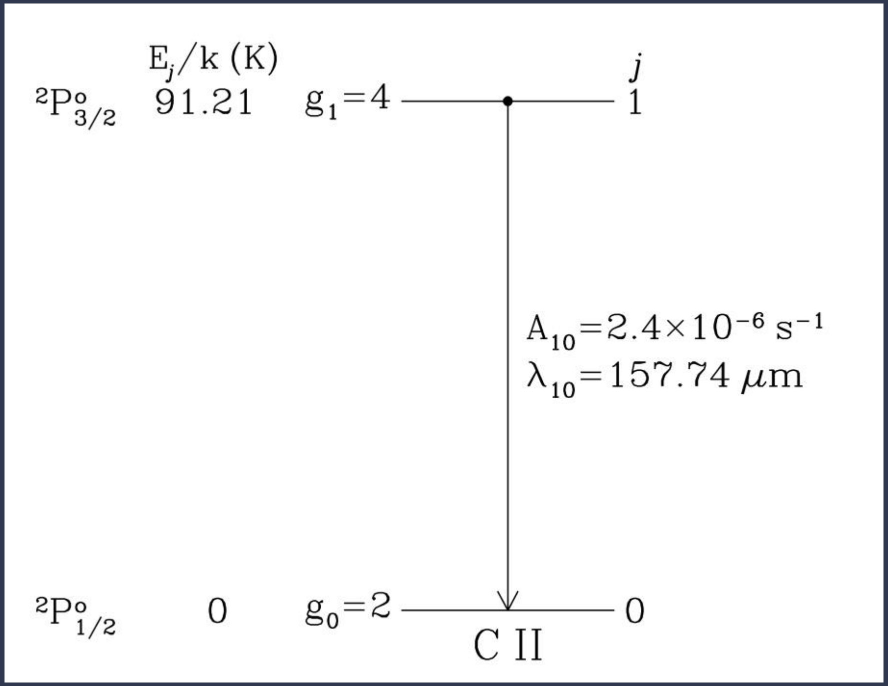


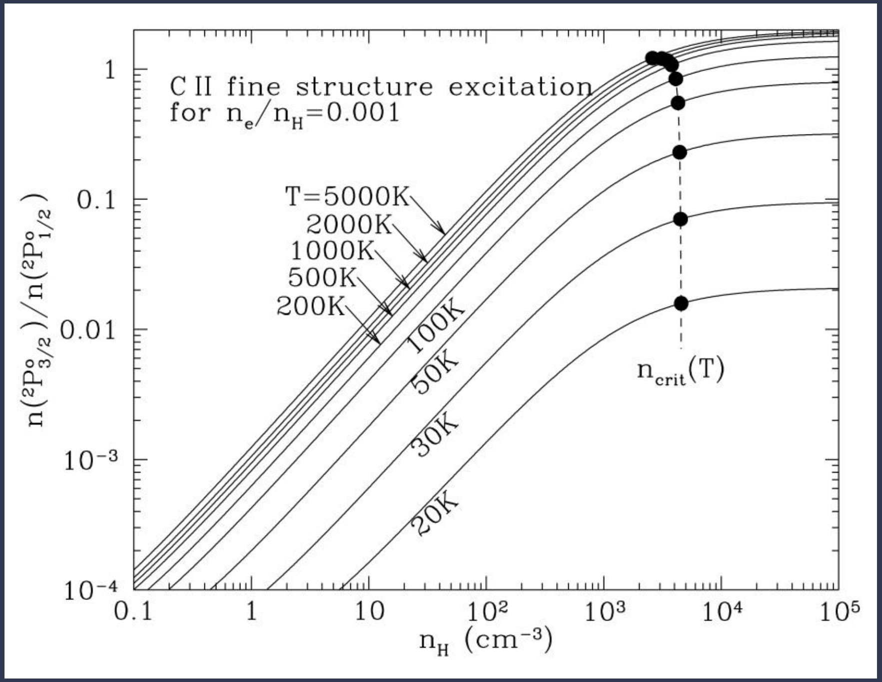
Draine, Fig. 17.2

$T_s = T_{kin}$ for Milky Way Conditions (both CNM & WNM).

So the differences in T_s between the WNM and CNM are actually differences in T_{kin}.

Application 2: the [CII] 158 μm line



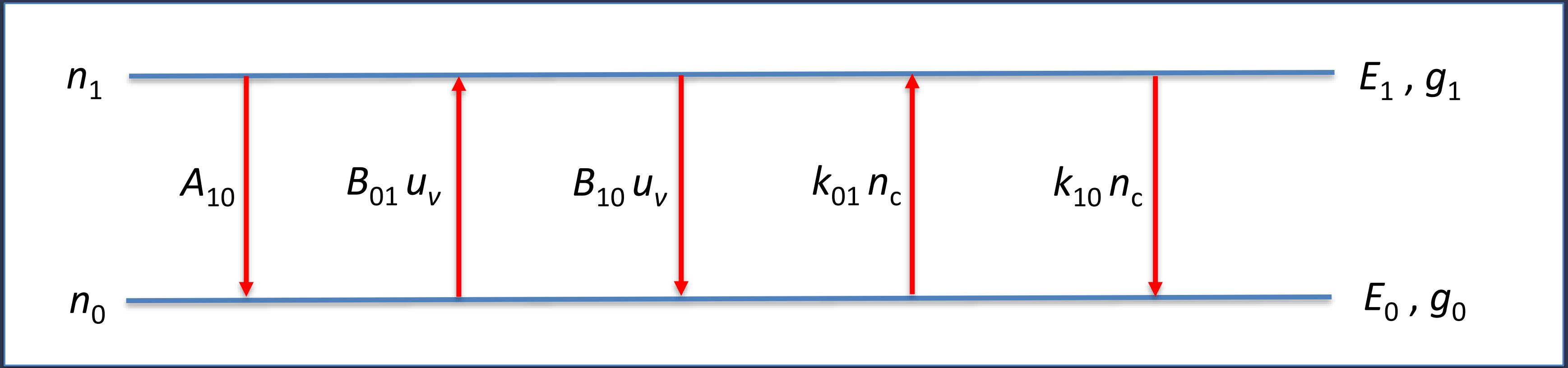


Draine, Figs. 17.3 & 4

$$\frac{n_1}{n_0} = \frac{1}{1+\frac{n_{\rm crit}}{n_c}}\frac{g_1}{g_0}e^{-\frac{E_{10}}{kT_{\rm kin}}}$$

Exercise: explain the behaviour of the curves on the right-hand side.

Generalization to N-level system



Requires a modified expression for the critical density of the upper level u:

$$n_{\mathrm{crit,u}} = \frac{\sum_{l<u} \left(1 + \bar{n}_{\gamma,ul}\right) A_{ul}}{\sum_{l<u} k_{ul}}$$

Statistical equilibrium then gives N linear equations with N unknowns: the N level populations – can be solved by standard linear algebra methods.

Today's lecture

1. Excitation by collisions
2. Critical density
3. Nebular diagnostics of temperature and density

Line ratios as nebular diagnostics

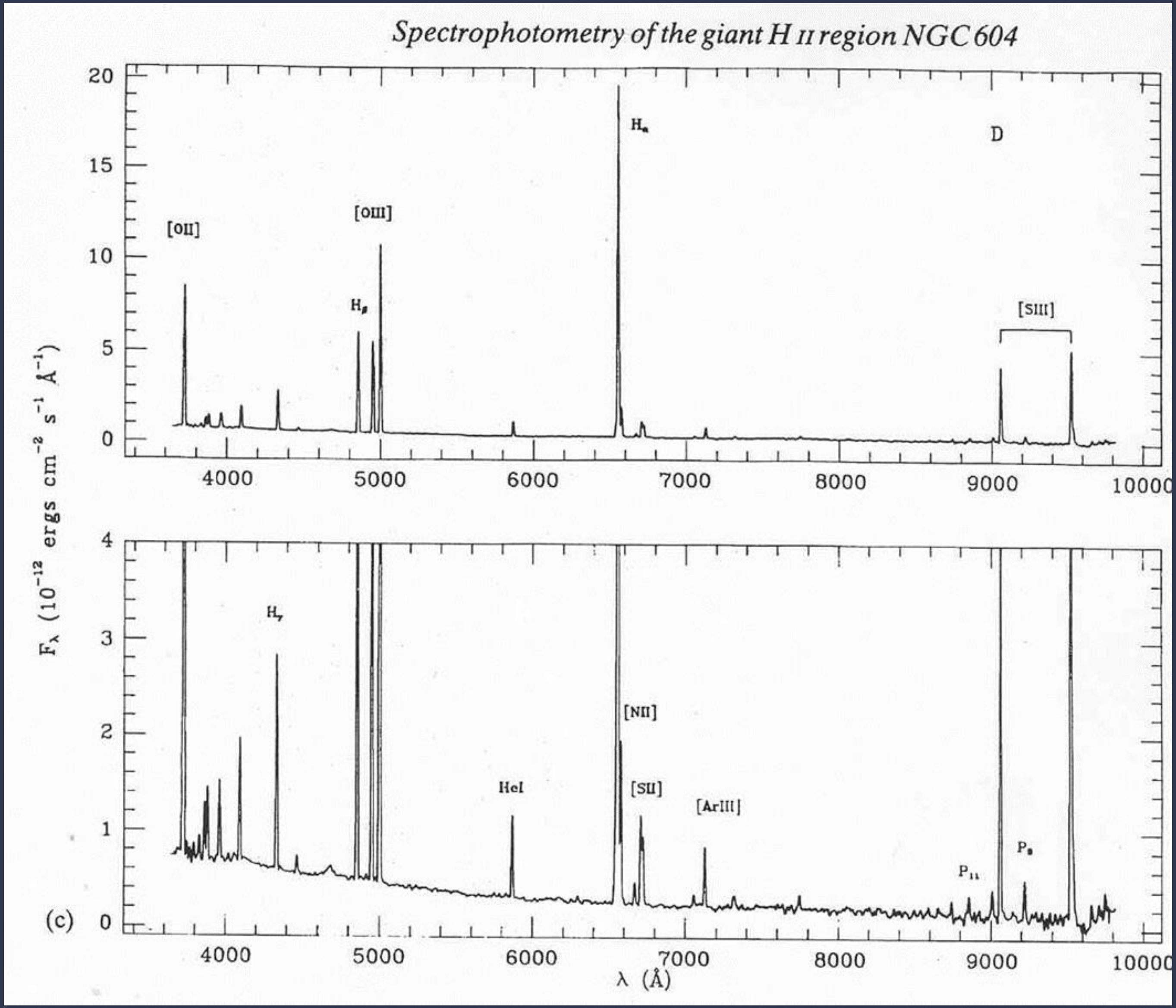
1. Temperature probes
2. Density probes
3. Abundance probes

Relevant parameters:

- Upper level temperatures (with respect to expected temperatures)
- Critical densities (with respect to expected densities)

Here we focus on HII regions as a case study, but it works in exactly the same way for neutral and molecular clouds.

HII Region spectra



Line ratios as temperature probes

Use 2 lines that:

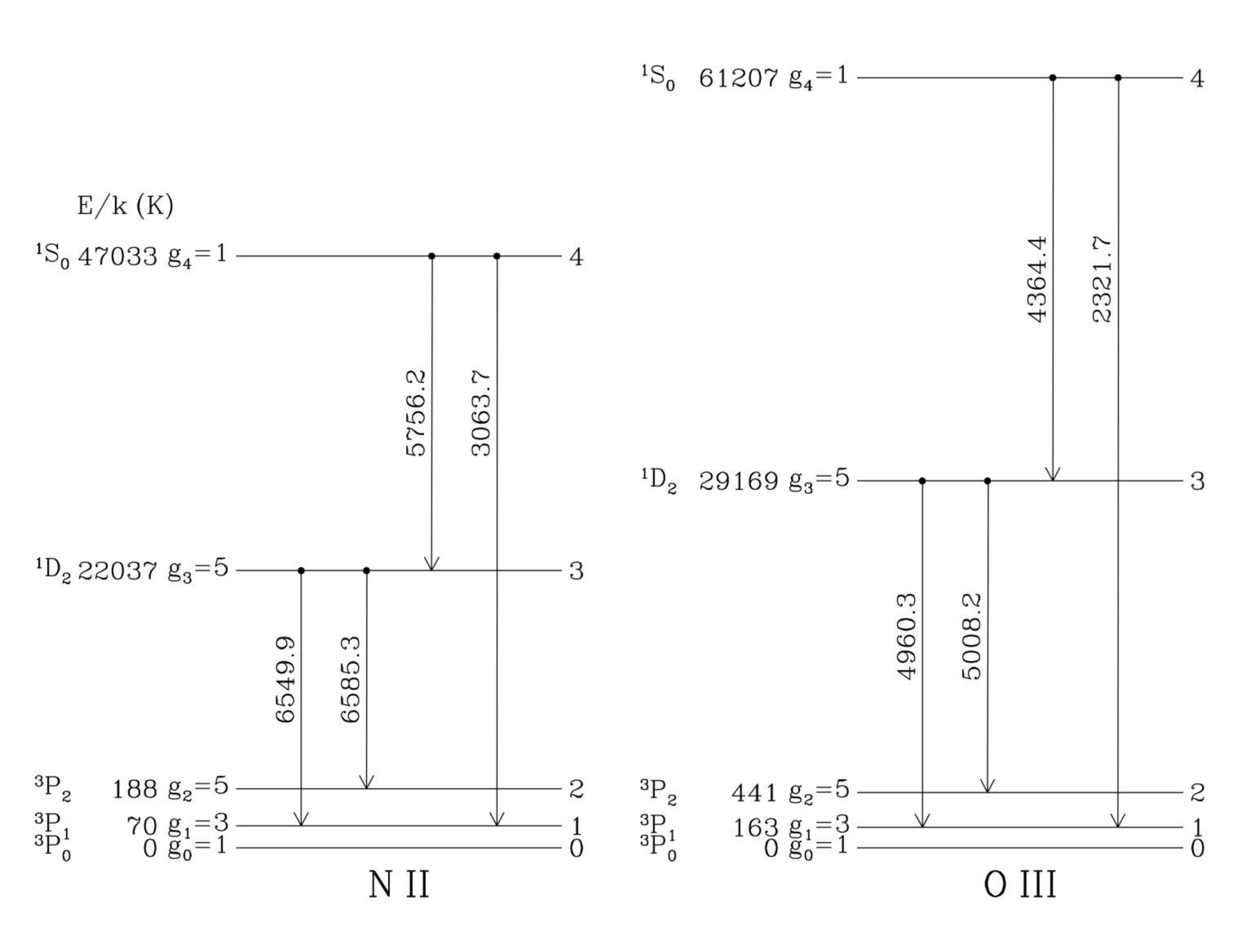
- have different upper level temperatures (in the relevant range)
- come from the same atom/ion

Note that both lines have a (different) critical density:

- $n_e \gg n_{\mathrm{crit}}$ for both lines (rare): ratio independent of n_e
- $n_e \ll n_{\mathrm{crit}}$ for both lines (more often): ratio independent of n_e

So: need to avoid the region where the n_e is between the two critical densities of the two lines.

Temperature probes



Draine, Fig. 18.1

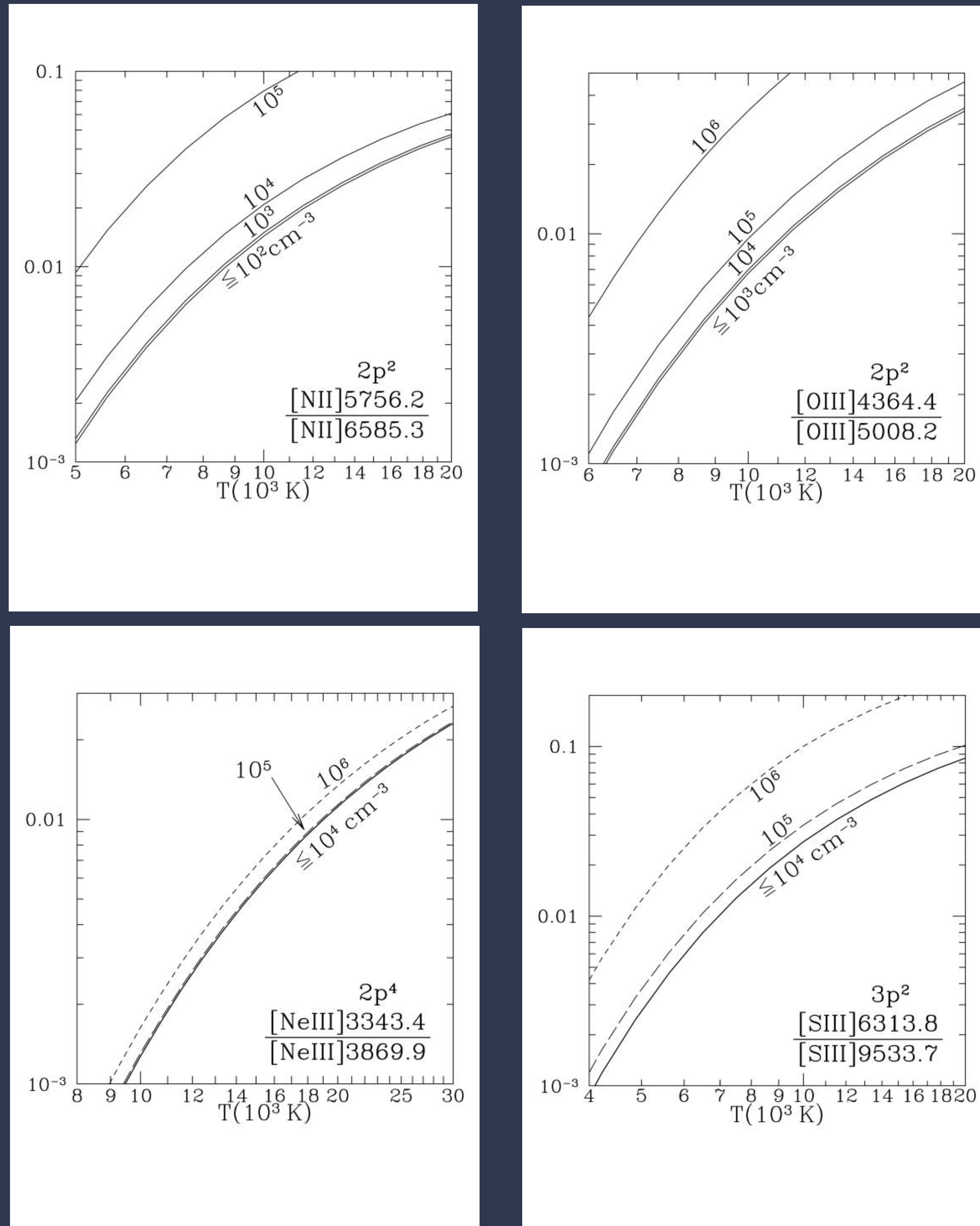
Critical densities for electron collisions at 10^4 K

Table 18.1 Critical Electron Density $n_{\rm crit}(e^-)$ ($\rm cm^{-3}$) for Selected np^2 and np^4 Ions

Configuration	Ion	$n_{\rm crit}(e)$ at $T = 10^4$ K: $^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^1S_0$
$1s^22s^22p^2$	C I	–	7.37×10^0	1.21×10^1		
	N II	–	1.67×10^2	2.96×10^2	7.68×10^4	1.23×10^7
	O III	–	1.74×10^3	3.79×10^3	6.40×10^5	2.78×10^7
	Ne V	–	3.19×10^5	3.48×10^5	1.44×10^8	9.58×10^8
$1s^22s^22p^4$	O I	3.11×10^3	2.87×10^4	–	1.62×10^6	4.04×10^8
	Ne III	3.02×10^4	2.76×10^6	–	9.47×10^6	1.37×10^8
	Mg V	4.36×10^6	4.75×10^7	–	1.07×10^9	8.07×10^9
$1s^22s^22p^63s^23p^2$	Si I	–	7.72×10^2	1.92×10^3		
	S III	–	4.22×10^3	1.31×10^4	7.33×10^5	1.52×10^7
	Ar V	–	1.09×10^7	1.16×10^7	3.65×10^8	2.49×10^8
$1s^22s^22p^23s^23p^4$	S I	1.04×10^5	1.55×10^5	–	4.12×10^7	1.38×10^9
	Ar III	2.49×10^5	2.67×10^6	–	1.26×10^7	4.54×10^8

Draine, Table 18.1

Line ratios as temperature probes



Draine, Fig. 18.2

Temperatures of HII regions

	[N II]			[O III]	
Nebula	$\frac{I(\lambda 6548)+I(\lambda 6583)}{I(\lambda 5755)}$	$T(^\circ\ K)$	$N_e/T^{1/2}$	$\frac{I(\lambda 4959)+I(\lambda 5007)}{I(\lambda 4363)}$	$T(^\circ\ K)$
NGC 1976 2b	81	10,000	51	338	8,700
NGC 1976 1a	102	9,100	68	371	8,500
NGC 1976 5b	111	8,900	21	310	8,900
NGC 1976 5a	189	7,500	12	263	9,300
M 8 I	162	7,900	(10)	445	8,100
M 17 I	257	6,900	(10)	330	8,700

Temperatures of Planetary Nebulae

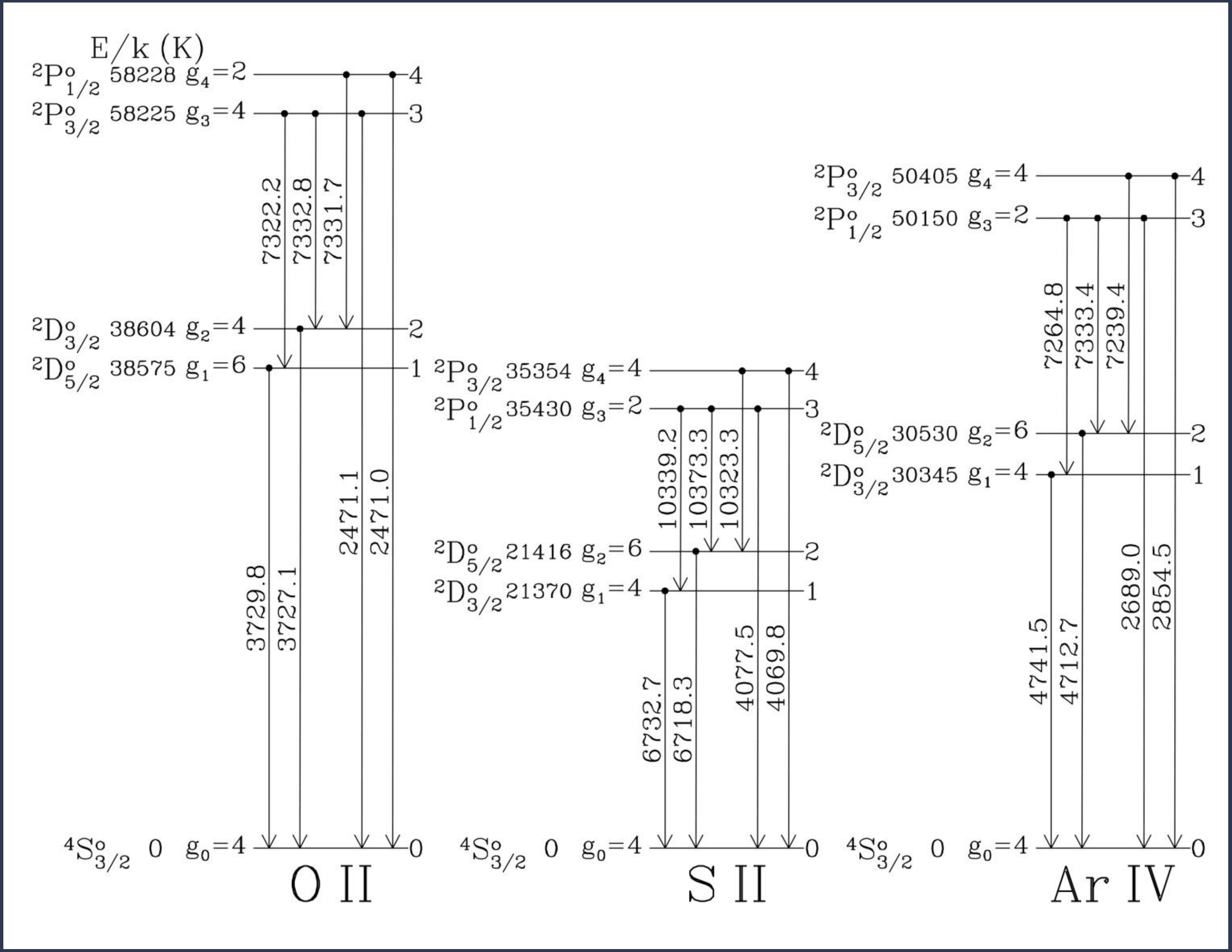
Nebula	T[N II] (° K)	T[O III] (° K)
NGC 650	9,500	10,700
NGC 4342	10,100	11,300
NGC 6210	10,700	9,700
NGC 6543	9,000	8,100
NGC 6572	—	10,300
NGC 6720	10,600	11,100
NGC 6853	10,000	11,000
NGC 7027	—	12,400
NGC 7293	9,300	11,000
NGC 7662	10,600	12,800
IC 418	—	9,700
IC 5217	—	11,600
BB 1	10,500	12,900
Haro 4-1	—	12,000
K 648	—	13,100

Line ratios as density probes

Use 2 lines that:

- have different critical densities (in the relevant range)
- come from the same atom/ion
- have upper level temperatures close together OR much lower than the expected T_{kin} (e.g., IR lines from an HII region)

Density Probes



Draine
Fig. 18.3

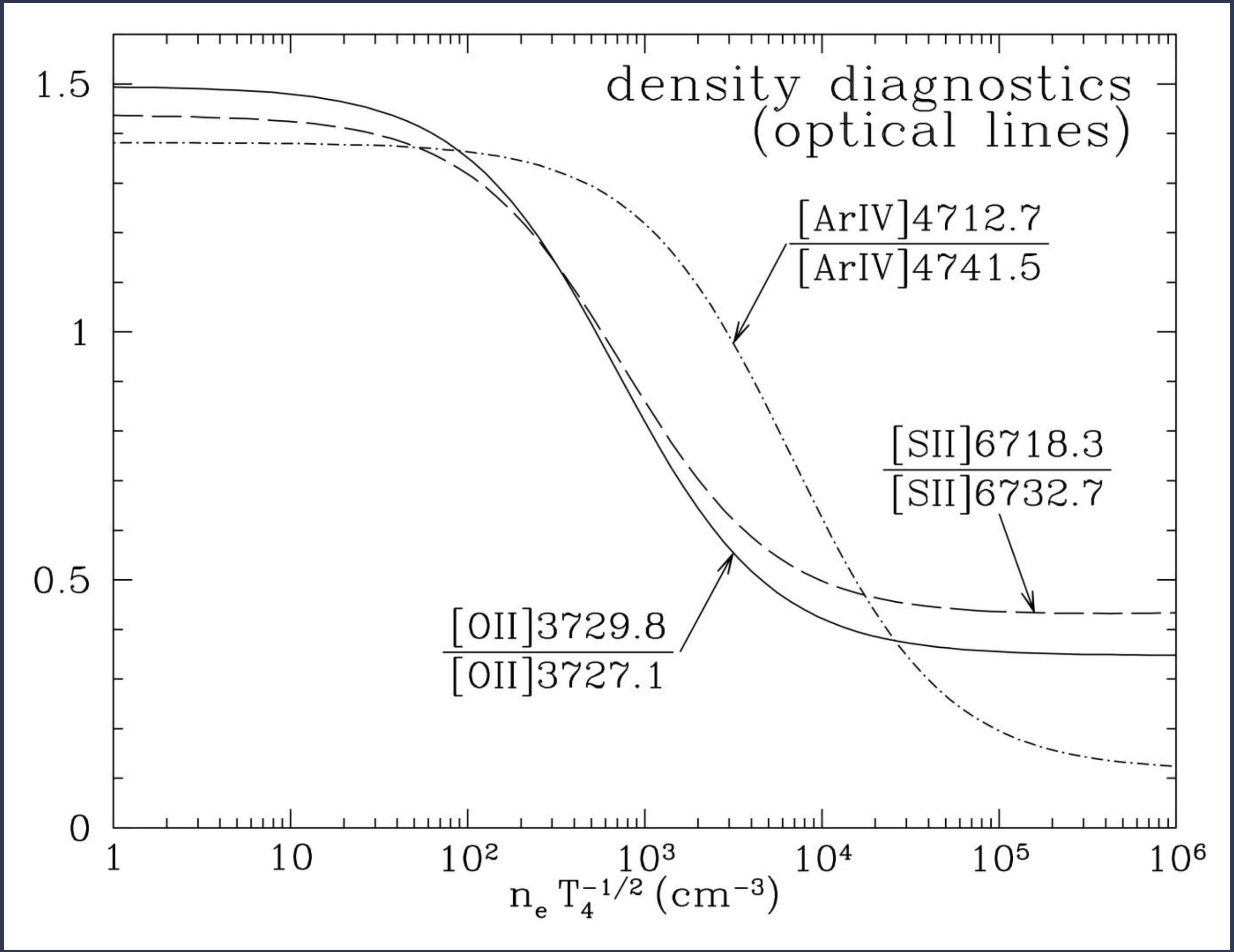
Critical densities for density probes

Table 18.2 Critical Electron Density $n_{crit}(e^-)$ (cm^{-3}) for Selected np^3 Ions, for $T = 10^4$ K

		$n_{crit}(e)$ at $T = 10^4$ K			
Configuration	Ion	$^2D^o_{3/2}$	$^2D^o_{5/2}$	$^2P^o_{1/2}$	$^2P^o_{3/2}$
$1s^2 2s^2 2p^3$	N I	2.18×10^4	1.19×10^4	7.11×10^7	3.15×10^7
	O II	4.49×10^3	3.31×10^3	5.30×10^6	1.03×10^7
	Ne IV	1.40×10^6	4.66×10^5	4.17×10^8	2.79×10^8
$1s^2 2s^2 2p^6 3s^2 3p^3$	S II	1.49×10^4	1.57×10^3	1.49×10^6	1.91×10^6
	Ar IV	1.35×10^6	1.55×10^4	1.06×10^7	1.81×10^7

Draine, Table 18.2

Line ratios as function of n_e



Draine
Fig. 18.4

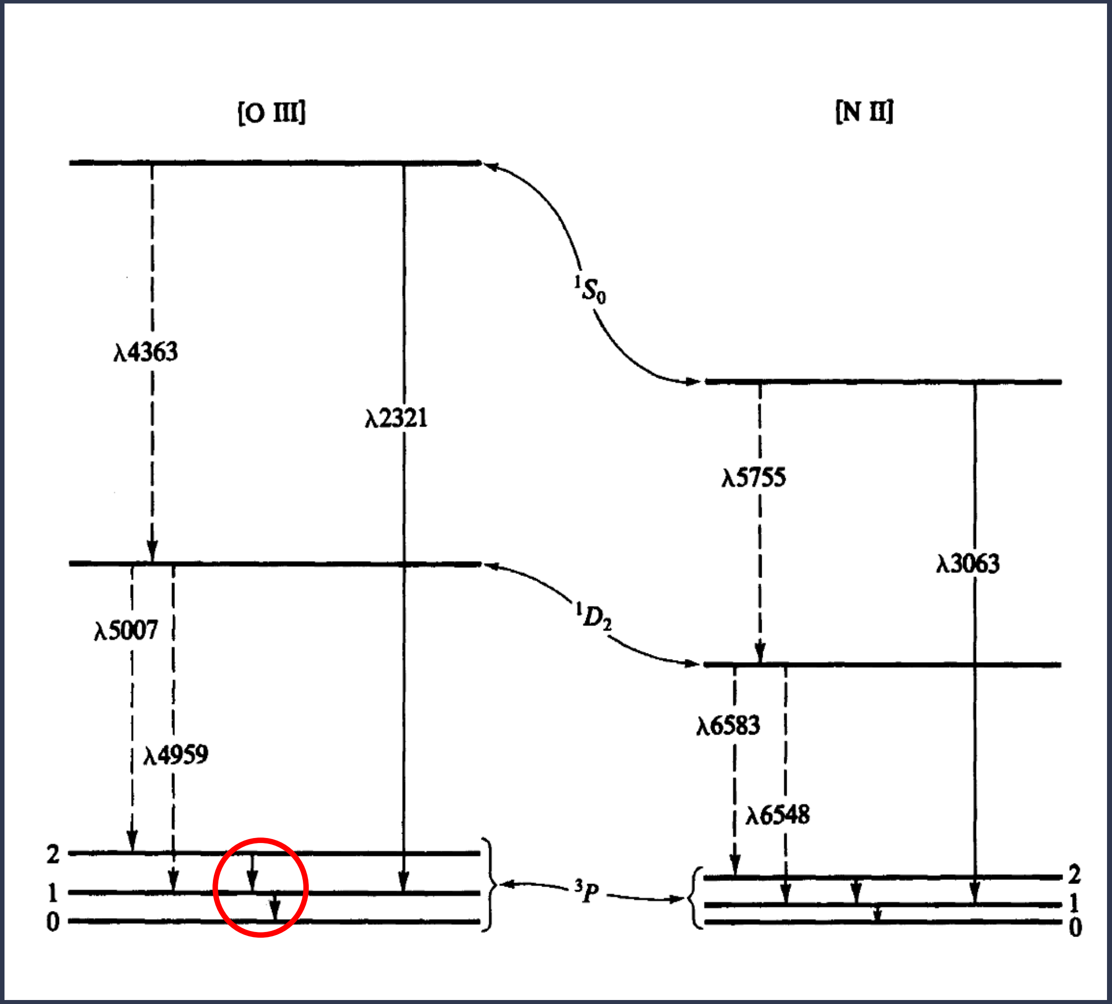
Exercise: explain the behaviour of these curves

Densities from [OII] and [SII] line ratios

Nebula	[O II] $\frac{\lambda 3729}{\lambda 3726}$	[O II] N_e [a] (cm^{-3})	[S II] $\frac{\lambda 6716}{\lambda 6731}$	[S II] N_e [a] (cm^{-3})
NGC 40	0.78	1.1×10^3	0.69	2.1×10^3
NGC 650/1	1.23	2.1×10^2	1.08	4.0×10^2
NGC 2392	0.78	1.1×10^3	0.88	9.1×10^2
NGC 2440	0.64	1.9×10^3	0.62	3.2×10^3
NGC 3242	0.62	2.2×10^3	0.64	2.8×10^3
NGC 3587	1.30	1.4×10^2	1.25	1.8×10^2
NGC 6210	0.47	5.8×10^3	0.66	2.5×10^3
NGC 6543	0.44	7.9×10^3	0.54	5.9×10^3
NGC 6572	0.38	2.1×10^4	0.51	8.9×10^3
NGC 6720	1.04	4.7×10^2	1.14	3.2×10^2
NGC 6803	0.57	2.8×10^3	–	–
NGC 6853	1.16	2.9×10^2	–	–
NGC 7009	0.50	4.6×10^3	0.61	3.3×10^3
NGC 7027	0.48	5.2×10^3	0.59	4.0×10^3
NGC 7293	1.32	1.3×10^2	1.28	1.6×10^2
NGC 7662	0.56	3.0×10^3	0.64	2.8×10^3
IC 418	0.37	3.2×10^5	0.49	9.5×10^3
IC 2149	0.56	3.0×10^3	0.57	4.6×10^3
IC 4593	0.63	2.0×10^3	–	–
IC 4997	0.34	1.0×10^6	0.45	1.0×10^5

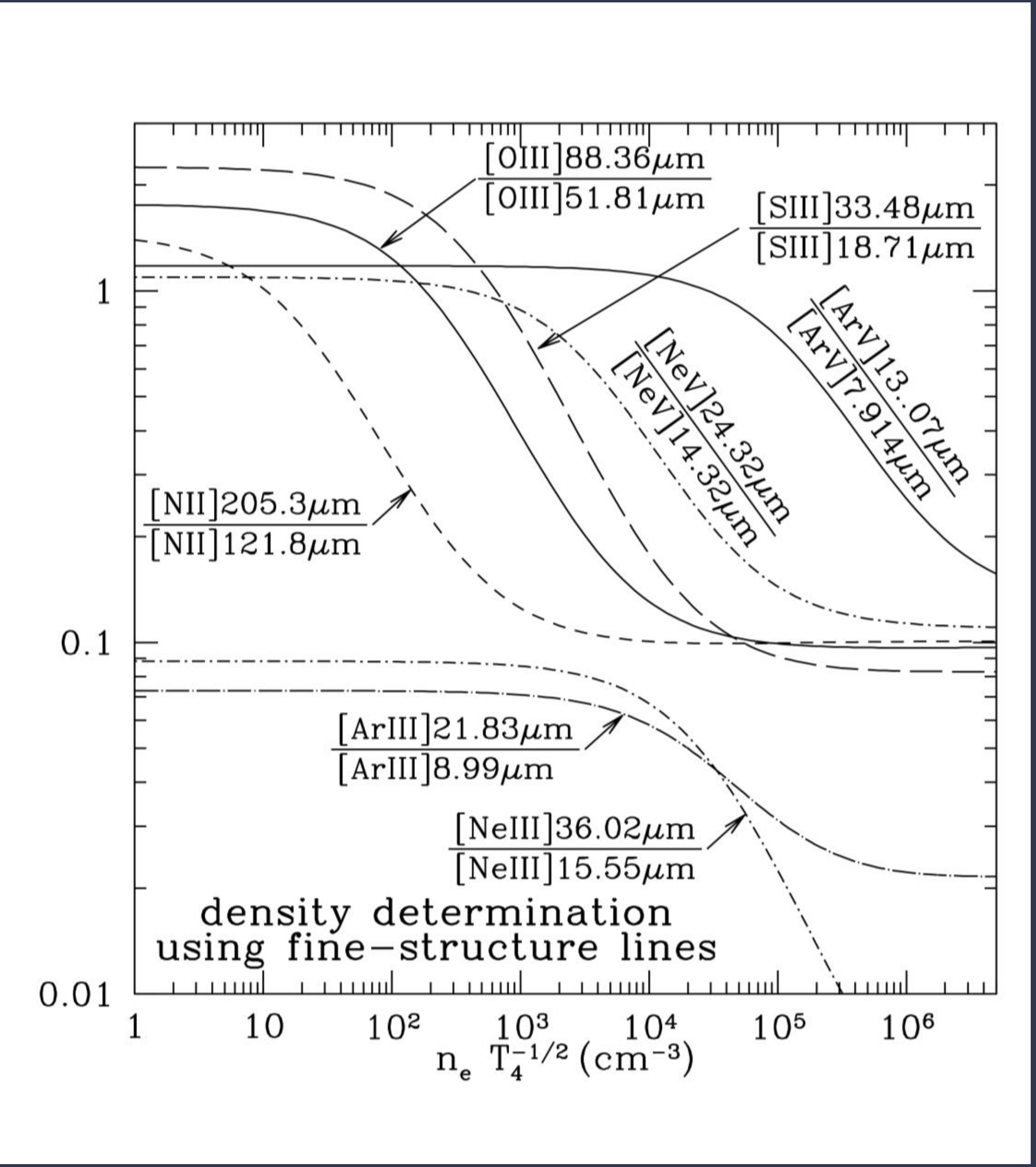
[a] N_e given for assumed $T = 10^4$ °K; for any other T divide listed value by $(T/10^4)^{1/2}$.

Far-infrared lines



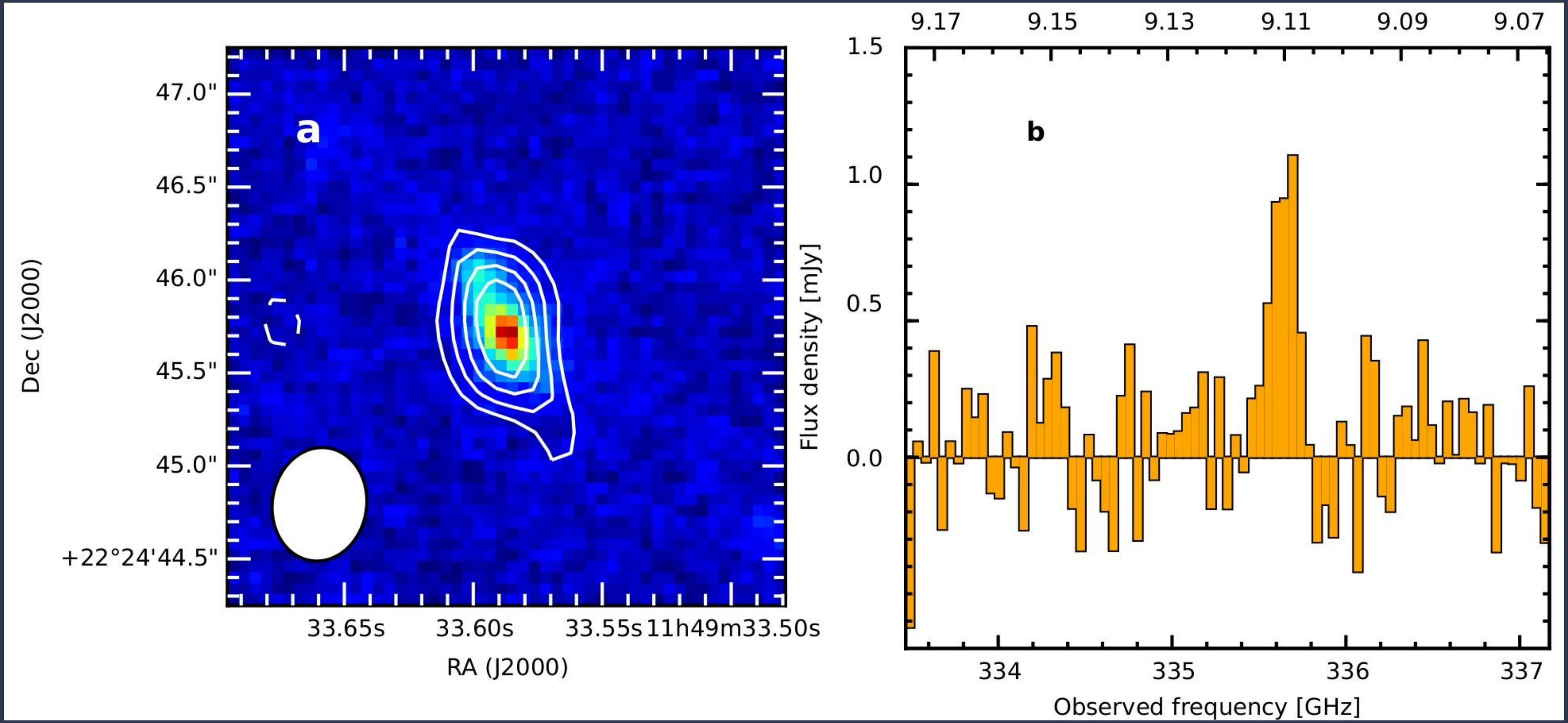
- Far-IR fine structure lines can be detected from space (e.g., Herschel) or (with large redshifts) in the submm regime with ALMA
- Example: [O III] $^3P_0 - {}^3P_1$ 88 µm, $^3P_1 - {}^3P_2$ 52 µm

Far-IR line ratios as function of n_e



Draine, Fig. 18.5

ISM at $z = 9.11$: redshifted [OIII] 88 μm



Hashimoto *et al.*, 2018

Next lecture

Molecules, molecular excitation and molecular clouds

1. Molecular structure and molecular spectra
2. Critical densities
3. Molecular hydrogen and molecular clouds