msg_code_challenge

July 19, 2020

1 "Code & Win Challenge" from msg and get-in-it

This jupyter notebook contains my contribution to the "Code & Win Challenge", that was organized by msg and get-in-it in June and July 2020. The notebook is structured into 4 major parts:

1. The Challenge

2. The Distances from Google Maps API

3. The TSP-solving using Branch-and-Bound

4. The Result I executed the code entirely, so all cells will show output. You have no need to run the code on your machine. However, if you want to for example check the execution speed, it is quite simple to execute the notebook. You can just click on "Kernel" > "Restart & Run All".

However, there are two sections of the code, that I per default plugged out: 1. The code calls the Google Maps Disstance API several times. Doing that too often would lead to a credit card charge. Hence, I did this once, saved the result in a csv-file and load the results again later in the file. Nevertheless, if you want to check for yourself that my Google API calls really worked, you can do so. Set the following variable to True and provide a Google Maps API Key. (As mentioned, the key is linked to my personal credit card, so I can not make it public.)

[1]:
```
call_google_maps_api = False

"""
The key to the google directions API is stored locally on my harddrive.
Because it is linked to an account with my credit card information, I will not␣
↪make it public in this notebook.
If you want to run the entire code yourself, you would need to insert your own␣
↪google API key here.
This tutorial explains how to get a key:
https://developers.google.com/maps/premium/apikey/distance-matrix-apikey
If you do not want to get your own key, you can just use the distance data that␣
↪I gathered in distances.csv
"""
```

```python
if call_google_maps_api:
    # INSERT YOUR OWN KEY HERE, IF YOU WANT TO RUN THE GOOGLE API CALLS YOURSELF
    api_key = open("google_api_key.txt", "r").read(39)
```

2. Later on I visualize the algorithm that I use to solve the challenge as a graph. You need four more imports for a need visualization, which are all unnecessary for the actual algorithm. Hence, I made it optional to run this section. I included a picture of the graph. If you want to check the displaying on your own machine and install the packages via pip, you can set the following variable to True.

[2]:
```python
draw_branch_and_bound_graph = False
```

A last preliminary for running the code on your machines are the used python libaries. All imports are listed in the cell below. If you are missing one of those in your environment, you can easily install them via `pip`. You can even install it within the jupyter-notebook. Just uncomment the following lines for the packages you are missing. Or run `pip install name-of-missing-package` in your terminal. `pip` itself should be already installed together with a rather up-to-date version of python. If not, you can get pip here: https://pip.pypa.io/en/stable/installing/

[3]:
```python
#!pip install pandas
#!pip install numpy
#!pip install requests
#!pip install networkx
#!pip install queue
#!pip install time
```

[4]:
```python
# Staples of python projects
import pandas as pd
import numpy as np

# Used for calling the Google Distance Matrix API
import requests

# Used for modelling the state space graph during the Branch-and-Bound algorithm
import networkx as nx

# Used for sorting which node should be processed next in the approach of␣
↪finding
from queue import PriorityQueue

# Used for tracking the execution time of the algorithm
import time

# Two packages only needed for a nice plot, but not essential for the algorithm
if draw_branch_and_bound_graph:
    import matplotlib.pyplot as plt
```

```
from matplotlib.lines import Line2D
import pydot
from networkx.drawing.nx_pydot import graphviz_layout
```

Now that those preliminaries are out of the way, let's dive in.

1.1 1. The Challenge

The challenge is to find the shortest path to all 21 locations of the company msg in Germany. This tour should start and end in msg's headquarter in Ismaning. Each other location should be visited only once. A csv-file with the addresses and coordinates of the company locations is provided. It is left open how I determine the distances between the location.

So, this challenge is obviously an instance of the wellknown **Travelling Salesperson Problem**. Even though the problem is known, it is not trivial to solve it. For 21 locations, which are all connected to each other with symmetric distances, the number of possible paths can be calculated like this:

$$\frac{(n-1)!}{2} = \frac{20!}{2} = 1,216,451,004,088,320,000$$

[5]:
```
# Calculating the number of possible paths
np.math.factorial(20) / 2
```

[5]:
```
1.21645100408832e+18
```

Even if we could calculate the length of 1 million paths in just 1 milisecond, it would take 38 years to check all paths. So clearly, a brute force approach does not work. In fact, this problem is known to be NP-hard, so the time for the solution is non-polynomial in regards of the input size.

To come to a solution in reasonable time, I chose the **Branch-and-Bound Algorithm**. More on that in section 3 of this notebook.

One important aspect is that I am free in the determination of the distances. Hence, I decided to look for the distances related to my favourite mode of transportation: **the bike!**

[6]:
```python
# Load the list of locations, that could be downloaded on the website of the␣
↪challenge
cities = pd.read_csv("msg_standorte_deutschland.csv")
cities
```

[6]:
```
   Nummer                 msg Standort                 Straße  \
0       1  Ismaning/München (Hauptsitz)    Robert-Bürkle-Straße
1       2                        Berlin           Wittestraße
2       3                  Braunschweig             Mittelweg
3       4                      Bretten           Edisonstraße
```

4	5	Chemnitz	Zwickauer Straße
5	6	Düsseldorf	Gladbecker Straße
6	7	Essen	Am Thyssenhaus
7	8	Frankfurt	Mergenthalerallee
8	9	Görlitz	Melanchthonstraße
9	10	Hamburg	Dammtorwall
10	11	Hannover	Hildesheimerstraße
11	12	Ingolstadt	Pascalstraße
12	13	Köln/Hürth	Max-Planck-Straße
13	14	Lingen (Ems)	Kaiserstraße
14	15	Münster	Schulstraße
15	16	Nürnberg	Südwestpark
16	17	Passau	Dr. Hans-Kapfinger-Straße
17	18	Schortens/Wilhelmshaven	Beethovenstraße
18	19	St. Georgen	Leopoldstraße
19	20	Stuttgart	Humboldtstraße
20	21	Walldorf	Altrottstraße

	Hausnummer	PLZ	Ort	Breitengrad	Längengrad
0	1	85737	Ismaning	48.229035	11.686153
1	30	13509	Berlin	52.580911	13.293884
2	7	38106	Braunschweig	52.278748	10.524797
3	2	75015	Bretten	49.032767	8.698372
4	16a	9122	Chemnitz	50.829383	12.914737
5	3	40472	Düsseldorf	51.274774	6.794912
6	1.3	45128	Essen	51.450577	7.008871
7	73-75	65760	Eschborn	50.136479	8.570963
8	19	2826	Görlitz	51.145511	14.970028
9	7a	20354	Hamburg	53.557577	9.986065
10	265-267	30519	Hannover	52.337987	9.769706
11	4	85057	Ingolstadt	48.784417	11.399106
12	40	50354	Hürth	50.886726	6.913119
13	10b	49809	Lingen	52.519154	7.322185
14	22	48149	Münster	51.969304	7.614280
15	60	90449	Nürnberg	49.429596	11.017404
16	30	94032	Passau	48.571989	13.453256
17	46	26419	Schortens	53.537779	7.936809
18	1	78112	St. Georgen	48.126258	8.325873
19	35	70771	Leinfelden-Echterdingen	48.694648	9.161239
20	31	69190	Walldorf	49.295011	8.649036

1.2 2. The Distances from Google Maps API

For measuring the distances I use the real road distances, provided by the Google Maps API.

[7]:
```python
# Setting up a dataframe for the distance matrix
all_cities = cities["msg Standort"].values
number_of_cities = len(all_cities)
zero_matrix = np.zeros((number_of_cities,number_of_cities), dtype = int)
distance_matrix = pd.DataFrame(data = zero_matrix, index = all_cities, columns␣
↪= all_cities)
duration_matrix = distance_matrix.copy()
distance_matrix
```

[7]:
```
                             Ismaning/München (Hauptsitz)  Berlin  \
Ismaning/München (Hauptsitz)                            0       0
Berlin                                                  0       0
Braunschweig                                            0       0
Bretten                                                 0       0
Chemnitz                                                0       0
Düsseldorf                                              0       0
Essen                                                   0       0
Frankfurt                                               0       0
Görlitz                                                 0       0
Hamburg                                                 0       0
Hannover                                                0       0
Ingolstadt                                              0       0
Köln/Hürth                                              0       0
Lingen (Ems)                                            0       0
Münster                                                 0       0
Nürnberg                                                0       0
Passau                                                  0       0
Schortens/Wilhelmshaven                                 0       0
St. Georgen                                             0       0
Stuttgart                                               0       0
Walldorf                                                0       0

                             Braunschweig  Bretten  Chemnitz  Düsseldorf  \
Ismaning/München (Hauptsitz)            0        0         0           0
Berlin                                  0        0         0           0
Braunschweig                            0        0         0           0
Bretten                                 0        0         0           0
Chemnitz                                0        0         0           0
Düsseldorf                              0        0         0           0
Essen                                   0        0         0           0
Frankfurt                               0        0         0           0
Görlitz                                 0        0         0           0
Hamburg                                 0        0         0           0
Hannover                                0        0         0           0
Ingolstadt                              0        0         0           0
Köln/Hürth                              0        0         0           0
Lingen (Ems)                            0        0         0           0
```

Münster	0	0	0	0
Nürnberg	0	0	0	0
Passau	0	0	0	0
Schortens/Wilhelmshaven	0	0	0	0
St. Georgen	0	0	0	0
Stuttgart	0	0	0	0
Walldorf	0	0	0	0

	Essen	Frankfurt	Görlitz	Hamburg	…	\
Ismaning/München (Hauptsitz)	0	0	0	0	…	
Berlin	0	0	0	0	…	
Braunschweig	0	0	0	0	…	
Bretten	0	0	0	0	…	
Chemnitz	0	0	0	0	…	
Düsseldorf	0	0	0	0	…	
Essen	0	0	0	0	…	
Frankfurt	0	0	0	0	…	
Görlitz	0	0	0	0	…	
Hamburg	0	0	0	0	…	
Hannover	0	0	0	0	…	
Ingolstadt	0	0	0	0	…	
Köln/Hürth	0	0	0	0	…	
Lingen (Ems)	0	0	0	0	…	
Münster	0	0	0	0	…	
Nürnberg	0	0	0	0	…	
Passau	0	0	0	0	…	
Schortens/Wilhelmshaven	0	0	0	0	…	
St. Georgen	0	0	0	0	…	
Stuttgart	0	0	0	0	…	
Walldorf	0	0	0	0	…	

	Ingolstadt	Köln/Hürth	Lingen (Ems)	Münster	\
Ismaning/München (Hauptsitz)	0	0	0	0	
Berlin	0	0	0	0	
Braunschweig	0	0	0	0	
Bretten	0	0	0	0	
Chemnitz	0	0	0	0	
Düsseldorf	0	0	0	0	
Essen	0	0	0	0	
Frankfurt	0	0	0	0	
Görlitz	0	0	0	0	
Hamburg	0	0	0	0	
Hannover	0	0	0	0	
Ingolstadt	0	0	0	0	
Köln/Hürth	0	0	0	0	
Lingen (Ems)	0	0	0	0	
Münster	0	0	0	0	

Nürnberg	0	0	0	0
Passau	0	0	0	0
Schortens/Wilhelmshaven	0	0	0	0
St. Georgen	0	0	0	0
Stuttgart	0	0	0	0
Walldorf	0	0	0	0

	Nürnberg	Passau	Schortens/Wilhelmshaven \
Ismaning/München (Hauptsitz)	0	0	0
Berlin	0	0	0
Braunschweig	0	0	0
Bretten	0	0	0
Chemnitz	0	0	0
Düsseldorf	0	0	0
Essen	0	0	0
Frankfurt	0	0	0
Görlitz	0	0	0
Hamburg	0	0	0
Hannover	0	0	0
Ingolstadt	0	0	0
Köln/Hürth	0	0	0
Lingen (Ems)	0	0	0
Münster	0	0	0
Nürnberg	0	0	0
Passau	0	0	0
Schortens/Wilhelmshaven	0	0	0
St. Georgen	0	0	0
Stuttgart	0	0	0
Walldorf	0	0	0

	St. Georgen	Stuttgart	Walldorf
Ismaning/München (Hauptsitz)	0	0	0
Berlin	0	0	0
Braunschweig	0	0	0
Bretten	0	0	0
Chemnitz	0	0	0
Düsseldorf	0	0	0
Essen	0	0	0
Frankfurt	0	0	0
Görlitz	0	0	0
Hamburg	0	0	0
Hannover	0	0	0
Ingolstadt	0	0	0
Köln/Hürth	0	0	0
Lingen (Ems)	0	0	0
Münster	0	0	0
Nürnberg	0	0	0

```
Passau                                  0          0          0
Schortens/Wilhelmshaven                 0          0          0
St. Georgen                             0          0          0
Stuttgart                               0          0          0
Walldorf                                0          0          0

[21 rows x 21 columns]
```

[8]:
```
# Build a function that gets a name of a msg company location and returns the␣
↪coordinates as a string in the needed format
def get_city_coordinates(cityname):
    # Get the line from the imported csv file
    row = cities[cities["msg Standort"] == cityname]

    # The degrees need to be rounded to 6 decimal digits.
    coordinates = str(np.round(row["Breitengrad"].values[0], 6)) + "," + str(np.
↪round(row["Längengrad"].values[0], 6))
    return coordinates

# Quick check if the output looks alright
get_city_coordinates("Ismaning/München (Hauptsitz)")
```

[8]:
```
'48.229035,11.686153'
```

[9]:
```
# Set up a dict with all msg location and there coordinates in the specified␣
↪format
coordinate_dict = dict()
for location in all_cities:
    coordinate_dict[location] = get_city_coordinates(location)
coordinate_dict
```

[9]:
```
{'Ismaning/München (Hauptsitz)': '48.229035,11.686153',
 'Berlin': '52.580911,13.293884',
 'Braunschweig': '52.278748,10.524797',
 'Bretten': '49.032767,8.698372',
 'Chemnitz': '50.829383,12.914737',
 'Düsseldorf': '51.274774,6.794912',
 'Essen': '51.450577,7.008871',
 'Frankfurt': '50.136479,8.570963',
 'Görlitz': '51.145511,14.970028',
 'Hamburg': '53.557577,9.986065',
 'Hannover': '52.337987,9.769706',
 'Ingolstadt': '48.784417,11.399106',
 'Köln/Hürth': '50.886726,6.913119',
 'Lingen (Ems)': '52.519154,7.322185',
 'Münster': '51.969304,7.61428',
 'Nürnberg': '49.429596,11.017404',
```

```
 'Passau': '48.571989,13.453256',
 'Schortens/Wilhelmshaven': '53.537779,7.936809',
 'St. Georgen': '48.126258,8.325873',
 'Stuttgart': '48.694648,9.161239',
 'Walldorf': '49.295011,8.649036'}
```

[10]:
```
# Check if the Google API should be called. If not, api_key would not be defined
if call_google_maps_api:

    # Parameters to specify the distance measuring from Google Maps.
    # Most important here is to set the mode to bicycling, otherwise Google␣
 ↪will check for highways as well.
    parameters = {
                    "mode": "bicycling",
                    "traffic_model": "best_guess",
                    "departure_time": "now",
                    "key": api_key
                }
```

[11]:
```
# Defining a function that for one origin and a list of destinations retrieves␣
 ↪the distances using the given parameters
def get_bike_distance(origin, destination_list, parameters):
    # The base url are the fixed part of the API call.
    # The documentation can be found here:
    # https://developers.google.com/maps/documentation/distance-matrix/intro
    base_url = ["https://maps.googleapis.com/maps/api/distancematrix/json?
 ↪origins=",
                "&destinations=",
                "&mode=",
                "&traffic_model=",
                "&departure_time=",
                "&key="]

    # Getting the needed coordinates out of the dict
    origin_coordinates = coordinate_dict[origin]
    destination_coordinates_list = [coordinate_dict[destination] for␣
 ↪destination in destination_list]
    destination_coordinates = "|".join(destination_coordinates_list)

    # Composing the fixed and the variable parts of the url
    composed_url = base_url[0] + origin_coordinates + base_url[1] +␣
 ↪destination_coordinates + base_url[2] + parameters["mode"] + base_url[3] +␣
 ↪parameters["traffic_model"] + base_url[4] + parameters["departure_time"] +␣
 ↪base_url[5] + parameters["key"]

    # Making the actual request and parsing the result as a json
    gmaps_response = requests.get(composed_url).json()
```

```
    return gmaps_response
```

So, lets make a test call. From Munich to Berlin or to Hamburg.

[12]:
```
if call_google_maps_api:
    api_response = get_bike_distance("Ismaning/München (Hauptsitz)", ["Berlin",␣
↪"Hamburg"], parameters)
    print(api_response)
```

{'destination_addresses': ['Wittestraße 30, 13509 Berlin, Germany', 'Dammtorwall 7A, 20354 Hamburg, Germany'], 'origin_addresses': ['Robert-Bürkle-Straße 1, 85737 Ismaning, Germany'], 'rows': [{'elements': [{'distance': {'text': '623 km', 'value': 622638}, 'duration': {'text': '1 day 9 hours', 'value': 119656}, 'status': 'OK'}, {'distance': {'text': '742 km', 'value': 742394}, 'duration': {'text': '1 day 15 hours', 'value': 140069}, 'status': 'OK'}]}], 'status': 'OK'}

The result of the Google Distance API call is a json.

[13]:
```
if call_google_maps_api:
    # Navigating within the nested json to get to the distance in km and the␣
↪duration in seconds
    km_between_cities = np.
↪round((api_response['rows'][0]['elements'][0]['distance']['value']/1000),␣
↪decimals=1)
    seconds_between_cities =␣
↪api_response['rows'][0]['elements'][0]['duration']['value']

    print(f"The distance between the locations in Munich and Berlin is␣
↪{km_between_cities} km.\n\
    It takes {seconds_between_cities} seconds to cycle so far.")
```

```
The distance between the locations in Munich and Berlin is 622.6 km.
    It takes 119656 seconds to cycle so far.
```

[14]:
```
# In this cell the actual api calls to get all the distances are performed
if call_google_maps_api:

    # Get the list of all msg locations and initialize a list of cities, which␣
↪are already called by the API as origin
    all_cities = cities["msg Standort"].values
    called_cities = []

    for current_city in all_cities:
        # Mark the current city as called by the API
        called_cities.append(current_city)
```

```
        # Build the list of all cities, for which the distance needs to be␣
↪retrieved from the API
        list_of_open_cities = [city for city in all_cities if city not in␣
↪called_cities]

        if len(list_of_open_cities) > 0:
            # Make the api call to get the distance from the current city to␣
↪all cities, which are uncalled so far
            api_response = get_bike_distance(current_city, list_of_open_cities,␣
↪parameters)

            # Parse through the json result to write the result in the matrices
            for i in range(len(list_of_open_cities)):
                # Get the destination city
                destination_city = list_of_open_cities[i]

                # Grab the distance (rounded to km) and the duration (in␣
↪seconds) from the json, that was returned from the API
                km_between_cities = np.
↪round((api_response['rows'][0]['elements'][i]['distance']['value']/1000),␣
↪decimals=1)
                seconds_between_cities =␣
↪api_response['rows'][0]['elements'][i]['duration']['value']

                # Write the values in the matrices
                distance_matrix.loc[current_city, destination_city] =␣
↪km_between_cities
                duration_matrix.loc[current_city, destination_city] =␣
↪seconds_between_cities

    # Show the resulting distance matrix
    display(distance_matrix)
```

```
                              Ismaning/München (Hauptsitz)  Berlin  \
Ismaning/München (Hauptsitz)                             0   622.6
Berlin                                                   0     0.0
Braunschweig                                             0     0.0
Bretten                                                  0     0.0
Chemnitz                                                 0     0.0
Düsseldorf                                               0     0.0
Essen                                                    0     0.0
Frankfurt                                                0     0.0
Görlitz                                                  0     0.0
Hamburg                                                  0     0.0
Hannover                                                 0     0.0
Ingolstadt                                               0     0.0
Köln/Hürth                                               0     0.0
```

Lingen (Ems)	0	0.0
Münster	0	0.0
Nürnberg	0	0.0
Passau	0	0.0
Schortens/Wilhelmshaven	0	0.0
St. Georgen	0	0.0
Stuttgart	0	0.0
Walldorf	0	0.0

	Braunschweig	Bretten	Chemnitz	Düsseldorf	\
Ismaning/München (Hauptsitz)	580.7	287.6	403.0	641.6	
Berlin	238.8	638.2	256.4	579.4	
Braunschweig	0.0	492.8	297.2	341.8	
Bretten	0.0	0.0	457.8	361.0	
Chemnitz	0.0	0.0	0.0	562.2	
Düsseldorf	0.0	0.0	0.0	0.0	
Essen	0.0	0.0	0.0	0.0	
Frankfurt	0.0	0.0	0.0	0.0	
Görlitz	0.0	0.0	0.0	0.0	
Hamburg	0.0	0.0	0.0	0.0	
Hannover	0.0	0.0	0.0	0.0	
Ingolstadt	0.0	0.0	0.0	0.0	
Köln/Hürth	0.0	0.0	0.0	0.0	
Lingen (Ems)	0.0	0.0	0.0	0.0	
Münster	0.0	0.0	0.0	0.0	
Nürnberg	0.0	0.0	0.0	0.0	
Passau	0.0	0.0	0.0	0.0	
Schortens/Wilhelmshaven	0.0	0.0	0.0	0.0	
St. Georgen	0.0	0.0	0.0	0.0	
Stuttgart	0.0	0.0	0.0	0.0	
Walldorf	0.0	0.0	0.0	0.0	

	Essen	Frankfurt	Görlitz	Hamburg	...	\
Ismaning/München (Hauptsitz)	673.2	388.0	585.8	742.4	...	
Berlin	547.7	542.0	237.8	292.8	...	
Braunschweig	310.1	355.1	394.2	172.8	...	
Bretten	392.6	168.3	637.6	644.0	...	
Chemnitz	539.5	411.9	178.5	440.9	...	
Düsseldorf	35.5	271.3	739.7	390.6	...	
Essen	0.0	303.3	705.2	356.2	...	
Frankfurt	0.0	0.0	623.2	499.3	...	
Görlitz	0.0	0.0	0.0	515.5	...	
Hamburg	0.0	0.0	0.0	0.0	...	
Hannover	0.0	0.0	0.0	0.0	...	
Ingolstadt	0.0	0.0	0.0	0.0	...	
Köln/Hürth	0.0	0.0	0.0	0.0	...	
Lingen (Ems)	0.0	0.0	0.0	0.0	...	
Münster	0.0	0.0	0.0	0.0	...	

Nürnberg	0.0	0.0	0.0	0.0	...
Passau	0.0	0.0	0.0	0.0	...
Schortens/Wilhelmshaven	0.0	0.0	0.0	0.0	...
St. Georgen	0.0	0.0	0.0	0.0	...
Stuttgart	0.0	0.0	0.0	0.0	...
Walldorf	0.0	0.0	0.0	0.0	...

	Ingolstadt	Köln/Hürth	Lingen (Ems)	Münster	\
Ismaning/München (Hauptsitz)	77.1	591.4	710.2	651.8	
Berlin	592.3	606.5	472.0	466.6	
Braunschweig	502.4	368.8	263.4	234.4	
Bretten	244.3	310.9	530.0	466.2	
Chemnitz	336.2	544.9	517.6	485.2	
Düsseldorf	579.6	53.0	171.3	118.1	
Essen	611.6	85.0	147.8	83.7	
Frankfurt	325.3	220.4	351.7	284.9	
Görlitz	515.2	766.5	656.1	632.1	
Hamburg	668.7	442.6	245.6	273.3	
Hannover	505.8	324.7	201.7	181.6	
Ingolstadt	0.0	525.6	632.3	573.8	
Köln/Hürth	0.0	0.0	222.3	158.5	
Lingen (Ems)	0.0	0.0	0.0	72.3	
Münster	0.0	0.0	0.0	0.0	
Nürnberg	0.0	0.0	0.0	0.0	
Passau	0.0	0.0	0.0	0.0	
Schortens/Wilhelmshaven	0.0	0.0	0.0	0.0	
St. Georgen	0.0	0.0	0.0	0.0	
Stuttgart	0.0	0.0	0.0	0.0	
Walldorf	0.0	0.0	0.0	0.0	

	Nürnberg	Passau	Schortens/Wilhelmshaven	\
Ismaning/München (Hauptsitz)	170.2	164.9	794.2	
Berlin	502.8	605.9	456.1	
Braunschweig	412.8	656.4	264.7	
Bretten	215.0	420.4	667.0	
Chemnitz	271.5	391.4	561.8	
Düsseldorf	491.8	736.7	308.3	
Essen	523.8	768.6	282.6	
Frankfurt	242.5	487.3	467.5	
Görlitz	450.5	570.4	654.5	
Hamburg	579.2	822.8	175.9	
Hannover	416.3	659.9	213.0	
Ingolstadt	92.3	181.4	716.3	
Köln/Hürth	441.7	686.6	359.3	
Lingen (Ems)	541.0	784.6	141.9	
Münster	481.8	725.4	205.4	
Nürnberg	0.0	245.5	625.8	
Passau	0.0	0.0	871.0	

```
Schortens/Wilhelmshaven            0.0     0.0                         0.0
St. Georgen                        0.0     0.0                         0.0
Stuttgart                          0.0     0.0                         0.0
Walldorf                           0.0     0.0                         0.0

                              St. Georgen  Stuttgart  Walldorf
Ismaning/München (Hauptsitz)        310.5      231.0     309.0
Berlin                              766.4      664.5     631.7
Braunschweig                        624.9      530.7     452.2
Bretten                             140.3       65.1      39.4
Chemnitz                            578.2      463.1     438.7
Düsseldorf                          485.0      418.8     322.8
Essen                               517.0      450.8     354.8
Frankfurt                           283.6      211.6     111.0
Görlitz                             757.2      642.1     617.7
Hamburg                             759.6      685.4     586.8
Hannover                            615.7      527.0     443.0
Ingolstadt                          294.3      204.6     261.2
Köln/Hürth                          435.0      368.7     272.7
Lingen (Ems)                        655.2      589.0     493.0
Münster                             588.2      521.9     425.9
Nürnberg                            310.0      204.5     215.6
Passau                              462.9      377.1     437.5
Schortens/Wilhelmshaven             795.4      729.1     633.1
St. Georgen                           0.0      109.6     175.7
Stuttgart                             0.0        0.0      99.0
Walldorf                              0.0        0.0       0.0

[21 rows x 21 columns]
```

The resulting matrix for the distance has 0-values on the main diagonal, because the distance from a location to itself is 0. The retrieved distances in km from the API are written to the upper triangel. For simplicity, I assume that the distance are symmetric. Hence, I will sum the matrix with its own transpose.

[15]:
```
# This cell can only be executed, if there are results from the API call, in
↪particular the distance matrix
if call_google_maps_api:

    # Get the underlying array from the pandas data frame
    distance_array_upper_triangular = np.array(distance_matrix.values)

    # Transpose the array to make get the symmetric lower triangular matrix
    distance_array_lower_triangular = distance_array_upper_triangular.T

    # Sum the lower and the upper triangular and set this array as the data in
↪the data frame
```

```
distance_array = distance_array_upper_triangular +␣
↪distance_array_lower_triangular
distance_matrix = pd.DataFrame(data = distance_array, index = all_cities,␣
↪columns = all_cities)
display(distance_matrix)
```

```
                              Ismaning/München (Hauptsitz)  Berlin  \
Ismaning/München (Hauptsitz)                           0.0   622.6
Berlin                                               622.6     0.0
Braunschweig                                         580.7   238.8
Bretten                                              287.6   638.2
Chemnitz                                             403.0   256.4
Düsseldorf                                           641.6   579.4
Essen                                                673.2   547.7
Frankfurt                                            388.0   542.0
Görlitz                                              585.8   237.8
Hamburg                                              742.4   292.8
Hannover                                             584.3   281.1
Ingolstadt                                            77.1   592.3
Köln/Hürth                                           591.4   606.5
Lingen (Ems)                                         710.2   472.0
Münster                                              651.8   466.6
Nürnberg                                             170.2   502.8
Passau                                               164.9   605.9
Schortens/Wilhelmshaven                              794.2   456.1
St. Georgen                                          310.5   766.4
Stuttgart                                            231.0   664.5
Walldorf                                             309.0   631.7

                              Braunschweig  Bretten  Chemnitz  Düsseldorf  \
Ismaning/München (Hauptsitz)         580.7    287.6     403.0       641.6
Berlin                               238.8    638.2     256.4       579.4
Braunschweig                           0.0    492.8     297.2       341.8
Bretten                              492.8      0.0     457.8       361.0
Chemnitz                             297.2    457.8       0.0       562.2
Düsseldorf                           341.8    361.0     562.2         0.0
Essen                                310.1    392.6     539.5        35.5
Frankfurt                            355.1    168.3     411.9       271.3
Görlitz                              394.2    637.6     178.5       739.7
Hamburg                              172.8    644.0     440.9       390.6
Hannover                              62.3    480.0     354.9       289.0
Ingolstadt                           502.4    244.3     336.2       579.6
Köln/Hürth                           368.8    310.9     544.9        53.0
Lingen (Ems)                         263.4    530.0     517.6       171.3
Münster                              234.4    466.2     485.2       118.1
Nürnberg                             412.8    215.0     271.5       491.8
Passau                               656.4    420.4     391.4       736.7
```

Schortens/Wilhelmshaven	264.7	667.0	561.8	308.3
St. Georgen	624.9	140.3	578.2	485.0
Stuttgart	530.7	65.1	463.1	418.8
Walldorf	452.2	39.4	438.7	322.8

	Essen	Frankfurt	Görlitz	Hamburg	... \
Ismaning/München (Hauptsitz)	673.2	388.0	585.8	742.4	...
Berlin	547.7	542.0	237.8	292.8	...
Braunschweig	310.1	355.1	394.2	172.8	...
Bretten	392.6	168.3	637.6	644.0	...
Chemnitz	539.5	411.9	178.5	440.9	...
Düsseldorf	35.5	271.3	739.7	390.6	...
Essen	0.0	303.3	705.2	356.2	...
Frankfurt	303.3	0.0	623.2	499.3	...
Görlitz	705.2	623.2	0.0	515.5	...
Hamburg	356.2	499.3	515.5	0.0	...
Hannover	254.6	335.3	455.0	158.6	...
Ingolstadt	611.6	325.3	515.2	668.7	...
Köln/Hürth	85.0	220.4	766.5	442.6	...
Lingen (Ems)	147.8	351.7	656.1	245.6	...
Münster	83.7	284.9	632.1	273.3	...
Nürnberg	523.8	242.5	450.5	579.2	...
Passau	768.6	487.3	570.4	822.8	...
Schortens/Wilhelmshaven	282.6	467.5	654.5	175.9	...
St. Georgen	517.0	283.6	757.2	759.6	...
Stuttgart	450.8	211.6	642.1	685.4	...
Walldorf	354.8	111.0	617.7	586.8	...

	Ingolstadt	Köln/Hürth	Lingen (Ems)	Münster \
Ismaning/München (Hauptsitz)	77.1	591.4	710.2	651.8
Berlin	592.3	606.5	472.0	466.6
Braunschweig	502.4	368.8	263.4	234.4
Bretten	244.3	310.9	530.0	466.2
Chemnitz	336.2	544.9	517.6	485.2
Düsseldorf	579.6	53.0	171.3	118.1
Essen	611.6	85.0	147.8	83.7
Frankfurt	325.3	220.4	351.7	284.9
Görlitz	515.2	766.5	656.1	632.1
Hamburg	668.7	442.6	245.6	273.3
Hannover	505.8	324.7	201.7	181.6
Ingolstadt	0.0	525.6	632.3	573.8
Köln/Hürth	525.6	0.0	222.3	158.5
Lingen (Ems)	632.3	222.3	0.0	72.3
Münster	573.8	158.5	72.3	0.0
Nürnberg	92.3	441.7	541.0	481.8
Passau	181.4	686.6	784.6	725.4
Schortens/Wilhelmshaven	716.3	359.3	141.9	205.4
St. Georgen	294.3	435.0	655.2	588.2

Stuttgart	204.6	368.7	589.0	521.9
Walldorf	261.2	272.7	493.0	425.9

	Nürnberg	Passau	Schortens/Wilhelmshaven \
Ismaning/München (Hauptsitz)	170.2	164.9	794.2
Berlin	502.8	605.9	456.1
Braunschweig	412.8	656.4	264.7
Bretten	215.0	420.4	667.0
Chemnitz	271.5	391.4	561.8
Düsseldorf	491.8	736.7	308.3
Essen	523.8	768.6	282.6
Frankfurt	242.5	487.3	467.5
Görlitz	450.5	570.4	654.5
Hamburg	579.2	822.8	175.9
Hannover	416.3	659.9	213.0
Ingolstadt	92.3	181.4	716.3
Köln/Hürth	441.7	686.6	359.3
Lingen (Ems)	541.0	784.6	141.9
Münster	481.8	725.4	205.4
Nürnberg	0.0	245.5	625.8
Passau	245.5	0.0	871.0
Schortens/Wilhelmshaven	625.8	871.0	0.0
St. Georgen	310.0	462.9	795.4
Stuttgart	204.5	377.1	729.1
Walldorf	215.6	437.5	633.1

	St. Georgen	Stuttgart	Walldorf
Ismaning/München (Hauptsitz)	310.5	231.0	309.0
Berlin	766.4	664.5	631.7
Braunschweig	624.9	530.7	452.2
Bretten	140.3	65.1	39.4
Chemnitz	578.2	463.1	438.7
Düsseldorf	485.0	418.8	322.8
Essen	517.0	450.8	354.8
Frankfurt	283.6	211.6	111.0
Görlitz	757.2	642.1	617.7
Hamburg	759.6	685.4	586.8
Hannover	615.7	527.0	443.0
Ingolstadt	294.3	204.6	261.2
Köln/Hürth	435.0	368.7	272.7
Lingen (Ems)	655.2	589.0	493.0
Münster	588.2	521.9	425.9
Nürnberg	310.0	204.5	215.6
Passau	462.9	377.1	437.5
Schortens/Wilhelmshaven	795.4	729.1	633.1
St. Georgen	0.0	109.6	175.7
Stuttgart	109.6	0.0	99.0
Walldorf	175.7	99.0	0.0

[21 rows x 21 columns]

[16]:
```
# This cell can only be executed, if there are results from the API call, in␣
↪particular the duration matrix.
if call_google_maps_api:

    # The exact same needs to be done for the duration matrix.
    duration_array_upper_triangular = np.array(duration_matrix.values)
    duration_array_lower_triangular = duration_array_upper_triangular.T
    duration_array = duration_array_upper_triangular +␣
↪duration_array_lower_triangular
    duration_matrix = pd.DataFrame(data = duration_array, index = all_cities,␣
↪columns = all_cities)
    display(duration_matrix)
```

	Ismaning/München (Hauptsitz)	Berlin	\
Ismaning/München (Hauptsitz)	0	119656	
Berlin	119656	0	
Braunschweig	112281	44283	
Bretten	55932	124878	
Chemnitz	81474	48845	
Düsseldorf	120927	108156	
Essen	127151	102701	
Frankfurt	74406	105265	
Görlitz	118719	45350	
Hamburg	140069	53633	
Hannover	111378	51765	
Ingolstadt	15060	114594	
Köln/Hürth	111445	114300	
Lingen (Ems)	137765	85619	
Münster	127242	86866	
Nürnberg	32746	97472	
Passau	30550	121436	
Schortens/Wilhelmshaven	149982	82767	
St. Georgen	62390	152534	
Stuttgart	45606	130647	
Walldorf	60419	122313	

	Braunschweig	Bretten	Chemnitz	Düsseldorf	\
Ismaning/München (Hauptsitz)	112281	55932	81474	120927	
Berlin	44283	124878	48845	108156	
Braunschweig	0	94396	57038	64108	
Bretten	94396	0	92341	67737	
Chemnitz	57038	92341	0	108679	
Düsseldorf	64108	67737	108679	0	
Essen	58653	73961	104250	7048	

Frankfurt	68223	31723	83866	51098
Görlitz	73975	129278	36885	138465
Hamburg	31019	121514	81158	72496
Hannover	11706	92132	66795	54394
Ingolstadt	98228	47714	70003	110521
Köln/Hürth	70252	58255	108489	10310
Lingen (Ems)	48626	99327	97280	32064
Münster	43311	88084	91697	22227
Nürnberg	81106	43987	56896	95712
Passau	127234	80773	81528	142042
Schortens/Wilhelmshaven	48013	124310	104401	57047
St. Georgen	123414	31458	118266	96264
Stuttgart	104642	14424	94514	82021
Walldorf	86589	7376	89011	61472

	Essen	Frankfurt	Görlitz	Hamburg	...	\
Ismaning/München (Hauptsitz)	127151	74406	118719	140069	...	
Berlin	102701	105265	45350	53633	...	
Braunschweig	58653	68223	73975	31019	...	
Bretten	73961	31723	129278	121514	...	
Chemnitz	104250	83866	36885	81158	...	
Düsseldorf	7048	51098	138465	72496	...	
Essen	0	57018	132326	66358	...	
Frankfurt	57018	0	120542	94460	...	
Görlitz	132326	120542	0	94455	...	
Hamburg	66358	94460	94455	0	...	
Hannover	48256	65078	84400	29452	...	
Ingolstadt	116441	62922	106869	129089	...	
Köln/Hürth	16230	40797	143479	83664	...	
Lingen (Ems)	28242	68409	121320	45184	...	
Münster	16089	55820	116538	51175	...	
Nürnberg	101631	48854	93762	111967	...	
Passau	147961	95184	118394	158095	...	
Schortens/Wilhelmshaven	51911	89906	120073	32747	...	
St. Georgen	102184	57825	155132	149920	...	
Stuttgart	87940	41864	131380	134051	...	
Walldorf	67392	20667	125876	113096	...	

	Ingolstadt	Köln/Hürth	Lingen (Ems)	Münster	\
Ismaning/München (Hauptsitz)	15060	111445	137765	127242	
Berlin	114594	114300	85619	86866	
Braunschweig	98228	70252	48626	43311	
Bretten	47714	58255	99327	88084	
Chemnitz	70003	108489	97280	91697	
Düsseldorf	110521	10310	32064	22227	
Essen	116441	16230	28242	16089	
Frankfurt	62922	40797	68409	55820	
Görlitz	106869	143479	121320	116538	

Hamburg	129089	83664	45184	51175
Hannover	96941	62157	37035	34192
Ingolstadt	0	98186	123002	112479
Köln/Hürth	98186	0	41817	30675
Lingen (Ems)	123002	41817	0	13640
Münster	112479	30675	13640	0
Nürnberg	17983	85918	106475	95322
Passau	34199	132247	152603	141450
Schortens/Wilhelmshaven	135219	66800	26085	37786
St. Georgen	59001	86470	127809	116181
Stuttgart	40440	72226	113565	101937
Walldorf	49934	51678	93017	81389

	Nürnberg	Passau	Schortens/Wilhelmshaven	\
Ismaning/München (Hauptsitz)	32746	30550	149982	
Berlin	97472	121436	82767	
Braunschweig	81106	127234	48013	
Bretten	43987	80773	124310	
Chemnitz	56896	81528	104401	
Düsseldorf	95712	142042	57047	
Essen	101631	147961	51911	
Frankfurt	48854	95184	89906	
Görlitz	93762	118394	120073	
Hamburg	111967	158095	32747	
Hannover	79819	125947	39408	
Ingolstadt	17983	34199	135219	
Köln/Hürth	85918	132247	66800	
Lingen (Ems)	106475	152603	26085	
Münster	95322	141450	37786	
Nürnberg	0	46464	117627	
Passau	46464	0	163708	
Schortens/Wilhelmshaven	117627	163708	0	
St. Georgen	64349	92103	153796	
Stuttgart	41666	72175	139552	
Walldorf	43098	83552	119004	

	St. Georgen	Stuttgart	Walldorf
Ismaning/München (Hauptsitz)	62390	45606	60419
Berlin	152534	130647	122313
Braunschweig	123414	104642	86589
Bretten	31458	14424	7376
Chemnitz	118266	94514	89011
Düsseldorf	96264	82021	61472
Essen	102184	87940	67392
Frankfurt	57825	41864	20667
Görlitz	155132	131380	125876
Hamburg	149920	134051	113096
Hannover	121631	104364	84806

```
Ingolstadt                         59001     40440     49934
Köln/Hürth                         86470     72226     51678
Lingen (Ems)                      127809    113565     93017
Münster                           116181    101937     81389
Nürnberg                           64349     41666     43098
Passau                             92103     72175     83552
Schortens/Wilhelmshaven           153796    139552    119004
St. Georgen                            0     22621     33316
Stuttgart                          22621         0     19267
Walldorf                           33316     19267         0

[21 rows x 21 columns]
```

[17]:
```
# This cell can only be executed, if there are results from the API call.
if call_google_maps_api:
    # Save the results of the API calls in csv files
    distance_matrix.to_csv('distance_matrix.csv')
    duration_matrix.to_csv('duration_matrix.csv')
```

1.3 3. The TSP-solving using Branch-and-Bound

In this part I implement the actual algorithm needed to solve the TSP for a given adjacency matrix. I chose a Branch-and_Bound algorithm Wikipedia, because it can find an exact solution to the problem. For instances of TSP in this size, the execution time is reasonable.

I will use the adjacency matrix of the 21 msg locations and also a smaller 5x5 asymmetric metric for testing purposes. The test matrix was used in other resources so I can compare the result of my implementation with the published result.

[18]:
```
# Import the dataframe with the distance between the cities.
adjacency_df = pd.read_csv('distance_matrix.csv', index_col = 0)
adjacency_df
```

[18]:
```
                              Ismaning/München (Hauptsitz)  Berlin  \
Ismaning/München (Hauptsitz)                           0.0   622.6
Berlin                                               622.6     0.0
Braunschweig                                         580.7   238.8
Bretten                                              287.6   638.2
Chemnitz                                             403.0   256.4
Düsseldorf                                           641.6   579.4
Essen                                                673.2   547.7
Frankfurt                                            388.0   542.0
Görlitz                                              585.8   237.8
Hamburg                                              742.4   292.8
Hannover                                             584.3   281.1
```

Ingolstadt	77.1	592.3
Köln/Hürth	591.4	606.5
Lingen (Ems)	710.2	472.0
Münster	651.8	466.6
Nürnberg	170.2	502.8
Passau	164.9	605.9
Schortens/Wilhelmshaven	794.2	456.1
St. Georgen	310.5	766.4
Stuttgart	231.0	664.5
Walldorf	309.0	631.7

	Braunschweig	Bretten	Chemnitz	Düsseldorf	\
Ismaning/München (Hauptsitz)	580.7	287.6	403.0	641.6	
Berlin	238.8	638.2	256.4	579.4	
Braunschweig	0.0	492.8	297.2	341.8	
Bretten	492.8	0.0	457.8	361.0	
Chemnitz	297.2	457.8	0.0	562.2	
Düsseldorf	341.8	361.0	562.2	0.0	
Essen	310.1	392.6	539.5	35.5	
Frankfurt	355.1	168.3	411.9	271.3	
Görlitz	394.2	637.6	178.5	739.7	
Hamburg	172.8	644.0	440.9	390.6	
Hannover	62.3	480.0	354.9	289.0	
Ingolstadt	502.4	244.3	336.2	579.6	
Köln/Hürth	368.8	310.9	544.9	53.0	
Lingen (Ems)	263.4	530.0	517.6	171.3	
Münster	234.4	466.2	485.2	118.1	
Nürnberg	412.8	215.0	271.5	491.8	
Passau	656.4	420.4	391.4	736.7	
Schortens/Wilhelmshaven	264.7	667.0	561.8	308.3	
St. Georgen	624.9	140.3	578.2	485.0	
Stuttgart	530.7	65.1	463.1	418.8	
Walldorf	452.2	39.4	438.7	322.8	

	Essen	Frankfurt	Görlitz	Hamburg	…	\
Ismaning/München (Hauptsitz)	673.2	388.0	585.8	742.4	…	
Berlin	547.7	542.0	237.8	292.8	…	
Braunschweig	310.1	355.1	394.2	172.8	…	
Bretten	392.6	168.3	637.6	644.0	…	
Chemnitz	539.5	411.9	178.5	440.9	…	
Düsseldorf	35.5	271.3	739.7	390.6	…	
Essen	0.0	303.3	705.2	356.2	…	
Frankfurt	303.3	0.0	623.2	499.3	…	
Görlitz	705.2	623.2	0.0	515.5	…	
Hamburg	356.2	499.3	515.5	0.0	…	
Hannover	254.6	335.3	455.0	158.6	…	
Ingolstadt	611.6	325.3	515.2	668.7	…	

Köln/Hürth	85.0	220.4	766.5	442.6	…
Lingen (Ems)	147.8	351.7	656.1	245.6	…
Münster	83.7	284.9	632.1	273.3	…
Nürnberg	523.8	242.5	450.5	579.2	…
Passau	768.6	487.3	570.4	822.8	…
Schortens/Wilhelmshaven	282.6	467.5	654.5	175.9	…
St. Georgen	517.0	283.6	757.2	759.6	…
Stuttgart	450.8	211.6	642.1	685.4	…
Walldorf	354.8	111.0	617.7	586.8	…

	Ingolstadt	Köln/Hürth	Lingen (Ems)	Münster	\
Ismaning/München (Hauptsitz)	77.1	591.4	710.2	651.8	
Berlin	592.3	606.5	472.0	466.6	
Braunschweig	502.4	368.8	263.4	234.4	
Bretten	244.3	310.9	530.0	466.2	
Chemnitz	336.2	544.9	517.6	485.2	
Düsseldorf	579.6	53.0	171.3	118.1	
Essen	611.6	85.0	147.8	83.7	
Frankfurt	325.3	220.4	351.7	284.9	
Görlitz	515.2	766.5	656.1	632.1	
Hamburg	668.7	442.6	245.6	273.3	
Hannover	505.8	324.7	201.7	181.6	
Ingolstadt	0.0	525.6	632.3	573.8	
Köln/Hürth	525.6	0.0	222.3	158.5	
Lingen (Ems)	632.3	222.3	0.0	72.3	
Münster	573.8	158.5	72.3	0.0	
Nürnberg	92.3	441.7	541.0	481.8	
Passau	181.4	686.6	784.6	725.4	
Schortens/Wilhelmshaven	716.3	359.3	141.9	205.4	
St. Georgen	294.3	435.0	655.2	588.2	
Stuttgart	204.6	368.7	589.0	521.9	
Walldorf	261.2	272.7	493.0	425.9	

	Nürnberg	Passau	Schortens/Wilhelmshaven	\
Ismaning/München (Hauptsitz)	170.2	164.9	794.2	
Berlin	502.8	605.9	456.1	
Braunschweig	412.8	656.4	264.7	
Bretten	215.0	420.4	667.0	
Chemnitz	271.5	391.4	561.8	
Düsseldorf	491.8	736.7	308.3	
Essen	523.8	768.6	282.6	
Frankfurt	242.5	487.3	467.5	
Görlitz	450.5	570.4	654.5	
Hamburg	579.2	822.8	175.9	
Hannover	416.3	659.9	213.0	
Ingolstadt	92.3	181.4	716.3	
Köln/Hürth	441.7	686.6	359.3	

```
Lingen (Ems)                    541.0   784.6                        141.9
Münster                         481.8   725.4                        205.4
Nürnberg                          0.0   245.5                        625.8
Passau                          245.5     0.0                        871.0
Schortens/Wilhelmshaven         625.8   871.0                          0.0
St. Georgen                     310.0   462.9                        795.4
Stuttgart                       204.5   377.1                        729.1
Walldorf                        215.6   437.5                        633.1

                             St. Georgen  Stuttgart  Walldorf
Ismaning/München (Hauptsitz)       310.5      231.0     309.0
Berlin                             766.4      664.5     631.7
Braunschweig                       624.9      530.7     452.2
Bretten                            140.3       65.1      39.4
Chemnitz                           578.2      463.1     438.7
Düsseldorf                         485.0      418.8     322.8
Essen                              517.0      450.8     354.8
Frankfurt                          283.6      211.6     111.0
Görlitz                            757.2      642.1     617.7
Hamburg                            759.6      685.4     586.8
Hannover                           615.7      527.0     443.0
Ingolstadt                         294.3      204.6     261.2
Köln/Hürth                         435.0      368.7     272.7
Lingen (Ems)                       655.2      589.0     493.0
Münster                            588.2      521.9     425.9
Nürnberg                           310.0      204.5     215.6
Passau                             462.9      377.1     437.5
Schortens/Wilhelmshaven            795.4      729.1     633.1
St. Georgen                          0.0      109.6     175.7
Stuttgart                          109.6        0.0      99.0
Walldorf                           175.7       99.0       0.0

[21 rows x 21 columns]
```

[19]:
```python
# Setting the main diagonal from 0 to infinity
for i in range(adjacency_df.shape[0]):
    adjacency_df.iloc[i,i] = np.inf

# Matrix gets displayed as floating point, because inf is a floating point in␣
 ↪IEEE 754
adjacency_matrix = adjacency_df.values
adjacency_matrix
```

[19]:
```
array([[  inf, 622.6, 580.7, 287.6, 403. , 641.6, 673.2, 388. , 585.8,
        742.4, 584.3,  77.1, 591.4, 710.2, 651.8, 170.2, 164.9, 794.2,
        310.5, 231. , 309. ],
       [622.6,   inf, 238.8, 638.2, 256.4, 579.4, 547.7, 542. , 237.8,
```

```
 292.8, 281.1, 592.3, 606.5, 472. , 466.6, 502.8, 605.9, 456.1,
 766.4, 664.5, 631.7],
[580.7, 238.8,   inf, 492.8, 297.2, 341.8, 310.1, 355.1, 394.2,
 172.8,  62.3, 502.4, 368.8, 263.4, 234.4, 412.8, 656.4, 264.7,
 624.9, 530.7, 452.2],
[287.6, 638.2, 492.8,   inf, 457.8, 361. , 392.6, 168.3, 637.6,
 644. , 480. , 244.3, 310.9, 530. , 466.2, 215. , 420.4, 667. ,
 140.3,  65.1,  39.4],
[403. , 256.4, 297.2, 457.8,   inf, 562.2, 539.5, 411.9, 178.5,
 440.9, 354.9, 336.2, 544.9, 517.6, 485.2, 271.5, 391.4, 561.8,
 578.2, 463.1, 438.7],
[641.6, 579.4, 341.8, 361. , 562.2,   inf,  35.5, 271.3, 739.7,
 390.6, 289. , 579.6,  53. , 171.3, 118.1, 491.8, 736.7, 308.3,
 485. , 418.8, 322.8],
[673.2, 547.7, 310.1, 392.6, 539.5,  35.5,   inf, 303.3, 705.2,
 356.2, 254.6, 611.6,  85. , 147.8,  83.7, 523.8, 768.6, 282.6,
 517. , 450.8, 354.8],
[388. , 542. , 355.1, 168.3, 411.9, 271.3, 303.3,   inf, 623.2,
 499.3, 335.3, 325.3, 220.4, 351.7, 284.9, 242.5, 487.3, 467.5,
 283.6, 211.6, 111. ],
[585.8, 237.8, 394.2, 637.6, 178.5, 739.7, 705.2, 623.2,   inf,
 515.5, 455. , 515.2, 766.5, 656.1, 632.1, 450.5, 570.4, 654.5,
 757.2, 642.1, 617.7],
[742.4, 292.8, 172.8, 644. , 440.9, 390.6, 356.2, 499.3, 515.5,
   inf, 158.6, 668.7, 442.6, 245.6, 273.3, 579.2, 822.8, 175.9,
 759.6, 685.4, 586.8],
[584.3, 281.1,  62.3, 480. , 354.9, 289. , 254.6, 335.3, 455. ,
 158.6,   inf, 505.8, 324.7, 201.7, 181.6, 416.3, 659.9, 213. ,
 615.7, 527. , 443. ],
[ 77.1, 592.3, 502.4, 244.3, 336.2, 579.6, 611.6, 325.3, 515.2,
 668.7, 505.8,   inf, 525.6, 632.3, 573.8,  92.3, 181.4, 716.3,
 294.3, 204.6, 261.2],
[591.4, 606.5, 368.8, 310.9, 544.9,  53. ,  85. , 220.4, 766.5,
 442.6, 324.7, 525.6,   inf, 222.3, 158.5, 441.7, 686.6, 359.3,
 435. , 368.7, 272.7],
[710.2, 472. , 263.4, 530. , 517.6, 171.3, 147.8, 351.7, 656.1,
 245.6, 201.7, 632.3, 222.3,   inf,  72.3, 541. , 784.6, 141.9,
 655.2, 589. , 493. ],
[651.8, 466.6, 234.4, 466.2, 485.2, 118.1,  83.7, 284.9, 632.1,
 273.3, 181.6, 573.8, 158.5,  72.3,   inf, 481.8, 725.4, 205.4,
 588.2, 521.9, 425.9],
[170.2, 502.8, 412.8, 215. , 271.5, 491.8, 523.8, 242.5, 450.5,
 579.2, 416.3,  92.3, 441.7, 541. , 481.8,   inf, 245.5, 625.8,
 310. , 204.5, 215.6],
[164.9, 605.9, 656.4, 420.4, 391.4, 736.7, 768.6, 487.3, 570.4,
 822.8, 659.9, 181.4, 686.6, 784.6, 725.4, 245.5,   inf, 871. ,
 462.9, 377.1, 437.5],
```

```
[794.2, 456.1, 264.7, 667. , 561.8, 308.3, 282.6, 467.5, 654.5,
 175.9, 213. , 716.3, 359.3, 141.9, 205.4, 625.8, 871. ,   inf,
 795.4, 729.1, 633.1],
[310.5, 766.4, 624.9, 140.3, 578.2, 485. , 517. , 283.6, 757.2,
 759.6, 615.7, 294.3, 435. , 655.2, 588.2, 310. , 462.9, 795.4,
   inf, 109.6, 175.7],
[231. , 664.5, 530.7,  65.1, 463.1, 418.8, 450.8, 211.6, 642.1,
 685.4, 527. , 204.6, 368.7, 589. , 521.9, 204.5, 377.1, 729.1,
 109.6,   inf,  99. ],
[309. , 631.7, 452.2,  39.4, 438.7, 322.8, 354.8, 111. , 617.7,
 586.8, 443. , 261.2, 272.7, 493. , 425.9, 215.6, 437.5, 633.1,
 175.7,  99. ,   inf]])
```

[20]:

```
# Setup a test matrix to compare my function with the theoretical example
test_matrix = np.array([[np.inf, 20, 30, 10, 11],
                        [15, np.inf, 16, 4, 2],
                        [3, 5, np.inf, 2, 4],
                        [19, 6, 18, np.inf, 3],
                        [16, 4, 7, 16, np.inf]
                       ])
```

Now that we have the two adjacency matrices, let's start with the actual algorithm. We need a function to calculate the minimal cost in a matrix. Therefore, we find the minimum in each row and subtract this value from each entry in the row. This basically relates to the fact, that we need to leave each city at least once at the minimum cost. We do the same per column. The reduction per column means, that we at least once need to arrive in each city. We sum up the minima for the rows and the columns. This sum is the minimal cost to at least arrive and leave each city once. This is a lower bound for the cost of a branch.

[21]:

```
# Define a function to reduce a matrix by subtracting the minimum of each row␣
↪and column individually.
def reduce_matrix(input_matrix):

    # Necessary to use the function without changing the input matrix
    matrix = np.copy(input_matrix)

    # Find the minimum in each row (axis 1).
    min_in_rows = np.amin(matrix, axis=1)

    # If a row consists only of infinity entries, we change this min to 0.
    # Thereby, no further changes will be applied to this row and it will not␣
↪be accounted into the costs.
    min_in_rows[min_in_rows == np.inf] = 0

    # The minimum for each row will be subtracted from each value in the row.
    # Thereby, at least one value in the row will become 0.
    for i in range(len(min_in_rows)):
```

```
        matrix[i] = matrix[i] - min_in_rows[i]

    # Find the minimum in each column (axis 0).
    # If the minimum is 0 already, the subtraction will have no effect.
    min_in_cols = np.amin(matrix, axis=0)

    # Similar to the min_in_rows, also for the columns the inf will be replaced␣
↪by 0.
    min_in_cols[min_in_cols == np.inf] = 0

    # Again, similar procedure for the columns as for the rows.
    for i in range(len(min_in_cols)):
        matrix[:,i] = matrix[:,i] - min_in_cols[i]

    # Calculate the minimum cost for the path as the sum of the subtracted␣
↪minima per row and column.
    minimum_cost = np.sum(min_in_rows) + np.sum(min_in_cols)

    return matrix, minimum_cost
```

[22]:
```
# Test the reduce_matrix function with our test matrix
reduce_matrix(test_matrix)
```

[22]:
```
(array([[inf, 10., 17.,  0.,  1.],
        [12., inf, 11.,  2.,  0.],
        [ 0.,  3., inf,  0.,  2.],
        [15.,  3., 12., inf,  0.],
        [11.,  0.,  0., 12., inf]]), 25.0)
```

The function worked on the `test_matrix`, each row and each column contain at least one 0. The minimal cost of 25 also fits to the example.

The entire algorithm works on the basis of a graph. The next important function `make_edge_to_child` is to add a new child node in the graph. This function basically transfers to travelling from one city (parent node) to the other city (child node). In this function, several values of a node need to be updated, e.g. the current tour of cities represented by that node. Most importantly, the adjacency matrix will be adapted: - First, the column of the child node gets set to infinity, because we do not visit the city again. - Second, the row of the parent node gets set to infinity, because we will have no other outgoing journeys from the city we just left. - Third, the value for the way from the child node city to the parent node city gets set to ininify as well, because we do not want to travel back directly to the city we just came from.

Overall, by travelling from one city to another, we narrow down the possibilities for the rest of the tour. On this updated matrix, the `reduce_cost` function gets again applied to estimate the minimum cost of the remaining options.

[23]:
```
# Function for adding a new node from one city to another.
```

```python
def make_edge_to_child(parent_node, child_city_j, graph, global_node_counter, nodes_queue):

    # Define the new tour by taking the tour until the parent and adding the new child as the next step
    tour_to_child = parent_node['tour'] + [child_city_j]

    # Increment the level by 1 from parent to child
    child_level = parent_node['level'] + 1

    # Before transforming the remaining adjacency matrix, first safe the info about the cost for the edge
    current_city_i = parent_node['current_city']
    cost_parent_to_j = parent_node['reduced_matrix'][current_city_i][child_city_j]

    child_reduced_matrix = np.copy(parent_node['reduced_matrix'])

    # Set the jth column to infinity, because we do not visit city j afterwards again
    child_reduced_matrix[:,child_city_j] = np.inf

    # Set the ith row to infinity, because we do not want to have another outgoing arc from city i
    child_reduced_matrix[current_city_i] = np.inf

    # Set the entry in jth row and ith column to infinity, because we do not want to go back from j to i
    child_reduced_matrix[child_city_j, current_city_i] = np.inf

    # Reduce the matrix in terms of 0s in rows and columns and get the associated costs
    child_reduced_matrix, reduction_cost = reduce_matrix(child_reduced_matrix)

    # Calculate the lower bound for the cost for all path from this node j forwards.
    # It is composed out of three parts:
    # (1) The cost until node i
    # (2) The cost for the edge between node i and node j
    # (3) The cost for the reduction of the adjacency matrix of j
    lower_bound_node_j = parent_node["cost"] + cost_parent_to_j + reduction_cost

    # Add the node with all the defined parameters to the graph.
    child = graph.add_node(global_node_counter,
                           node_id = global_node_counter,
                           current_city = child_city_j,
```

```
                        tour = tour_to_child,
                        level = child_level,
                        reduced_matrix = child_reduced_matrix,
                        cost = lower_bound_node_j)

    # Add an edge between parent and child note in the graph
    graph.add_edge(parent_node["node_id"], global_node_counter)

    # Add to priority queue
    nodes_queue.put((lower_bound_node_j, global_node_counter))

    return
```

The next function needed is `initialize_for_given_adjaceny_matrix`. It basically takes the adjacency matrix and transforms the problem into two data structures: A graph and a priority queue.

The graph at this point only contains one node, the root. This basically transfers to the fact, that we know, that the tour will start in one particular place, e.g. in Ismaning. This root node also contains the adjacency_matrix after the first reduction.

The priority queue is a waiting line for the nodes to be processed. The nodes are ordered by the minimum cost, that we can calculate with the `reduce_matrix` function. Thereby, the most promising nodes with the lowest cost so far are taken into account first. After the initialization, the queue contains only the root node. But each explored child node will be a part of the queue, so it grows rather fast.

[24]:
```
def initialize_for_given_adjaceny_matrix(adjacency_matrix):
    # Priority queue to efficiently store the nodes
    nodes_queue = PriorityQueue()

    # Initalising the directed graph to store the nodes for later printing
    G = nx.DiGraph()

    """
    The root node consists of the following variables:

    node_id, which is 0 due to the initialisation of the global node counter
    currenty_city, which is 0 as Munich is top of the list
    tour, which is [0], so basically starting in Munich
    level, which is 0 because so far no other city was visited
    reduced matrix, which is the adacency matrix after the reduction with the␣
↪row and column minima
    cost, which are the inital cost that arised from the reduction of the␣
↪adjacency matrix
    """
    reduced_inital_matrix, initial_cost = reduce_matrix(adjacency_matrix)
```

```
    # The number of cities is also the size of the matrix
    N = reduced_inital_matrix.shape[0]

    G.add_node(0,
               node_id = 0,
               current_city = 0,
               tour = [0],
               level = 0,
               reduced_matrix = reduced_inital_matrix,
               cost = initial_cost)

    # The root node gets added to the priority queue
    nodes_queue.put((initial_cost, 0))

    return nodes_queue, G, N
```

The last function for the implementation of the algorithm is `branch_and_bound`. The function takes as arguments the output of the `initialize_for_given_adjaceny_matrix` function and initiates the parameters, which are independet of the actual adjacency matrix. The basic idea is, that the function takes the first node out of the priority queue and explores all possible paths to child nodes. This translates into being in city A and calculating the paths for all cities, which are still unvisited. This child nodes than get added to the priority queue as well.

When the first path comes to a complete loop, so all cities have been visited, the `minimum_solution_cost` will be updated. Then, it gets checked, if there is still a node in the priority queue with lower estimated cost, so a path, which could terminate even cheaper. If no such path is found, the search is complete. The result is the solution node in the graph, which contains all the information, most imprtantly the optimal tour and the cost of this tour.

For statistic purposes, I also measure the number of nodes which can be cut with Branch-and-Bound and also the time needed for the execution of the algorithm.

[25]:
```
# Setting up the function for the actual algorithm
def branch_and_bound(nodes_queue, graph, size_of_matrix):
    # Initalize the parameters, which are independent of the adjacency matrix
    # Global node counter for numbering the nodes uniquely, initalized with 0
    global_node_counter = 0

    # Minimal found solution so far (needed to evaluate if it still makes sense␣
↪to follow a path further)
    minimum_solution_cost = np.inf

    # Declare the node number that corresponds to the end of the path of the␣
↪minimal solution
    minimum_solution_node = -1

    # Just for statistic the number of nodes from which the children are␣
↪explored is also counted
```

```python
    number_of_parent_nodes = 0

    # Start the timer
    start = time.perf_counter()

    # Start the actual while-loop, that checks for the optimal solution
    while(not nodes_queue.empty()):

        # Get the node with the lowest cost from prio queue
        next_node = nodes_queue.get()

        # Increase for statistical purposes
        number_of_parent_nodes += 1

        # Check if this nodes cost are higher than the cost of the best found␣
↪complete solution so far.
        # This is the termination criteria. All other nodes in the queue can␣
↪not lead to a better solution.
        if next_node[0] > minimum_solution_cost:
            print("The search has ended!")
            break

        # If the search is not terminated, declare the node to a parent node to␣
↪find the children
        parent_node_i = graph.nodes[next_node[1]]
        current_city_i = parent_node_i['current_city']

        # Check, if the parent node is already the end of a path, so there␣
↪would not be any children
        if parent_node_i["level"] == (N - 1):
            # Add the way back from final city to starting city
            parent_node_i["tour"] = parent_node_i["tour"] + [0]

            # The cost for the last trip back to the starting point are 0 by␣
↪now due to the reduction
            # Update the minimal solution costs
            if(parent_node_i["cost"] < minimum_solution_cost):
                minimum_solution_cost = parent_node_i["cost"]
                minimum_solution_node = parent_node_i["node_id"]
                print(f"A minimal solution was found with a length of␣
↪{minimum_solution_cost :.1f} km.")

        # If the parent note is not the end of a tour yet
        else:
            # Explore the possible cities to visit
            for child_city_j in range(N):
```

```
                # Check if the adjacency is already set to infinity
                if␣
↪parent_node_i['reduced_matrix'][current_city_i][child_city_j] != np.inf:

                    # Increase global node counter
                    global_node_counter += 1
                    # This arc can still be explored, so a child node will be␣
↪created
                    make_edge_to_child(parent_node_i, child_city_j, graph,␣
↪global_node_counter, nodes_queue)

    # End the timer
    end = time.perf_counter()

    # Give out some statisitcs
    print(f"In total {number_of_parent_nodes} nodes got explored during the␣
↪execution.")
    print(f"At the end there were still {nodes_queue.qsize()} nodes in the␣
↪queue, that got effectively cut by Branch-and-Bound.")
    print("Execution time : {0:.2f} seconds".format(end - start))

    return minimum_solution_node, graph
```

Now that everything is set up, let's test the algorithm. First, on the small 5x5 `test_matrix`.

[26]:
```
# Run the algorithm on the small test array
nodes_queue, H, N = initialize_for_given_adjaceny_matrix(test_matrix)
minimum_solution_node_test, H = branch_and_bound(nodes_queue, H, N)
H.nodes[minimum_solution_node_test]
```

```
A minimal solution was found with a length of 28.0 km.
The search has ended!
In total 6 nodes got explored during the execution.
At the end there were still 7 nodes in the queue, that got effectively cut by
Branch-and-Bound.
Execution time : 0.00 seconds
```

[26]:
```
{'node_id': 12,
 'current_city': 2,
 'tour': [0, 3, 1, 4, 2, 0],
 'level': 4,
 'reduced_matrix': array([[inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf],
        [ 0., inf, inf, inf, inf],
        [inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf]]),
```

'cost': 28.0}

[27]:
```
nodes_queue_test, H, N = initialize_for_given_adjaceny_matrix(test_matrix)
minimum_solution_node_test, H = branch_and_bound(nodes_queue_test, H, N)
```

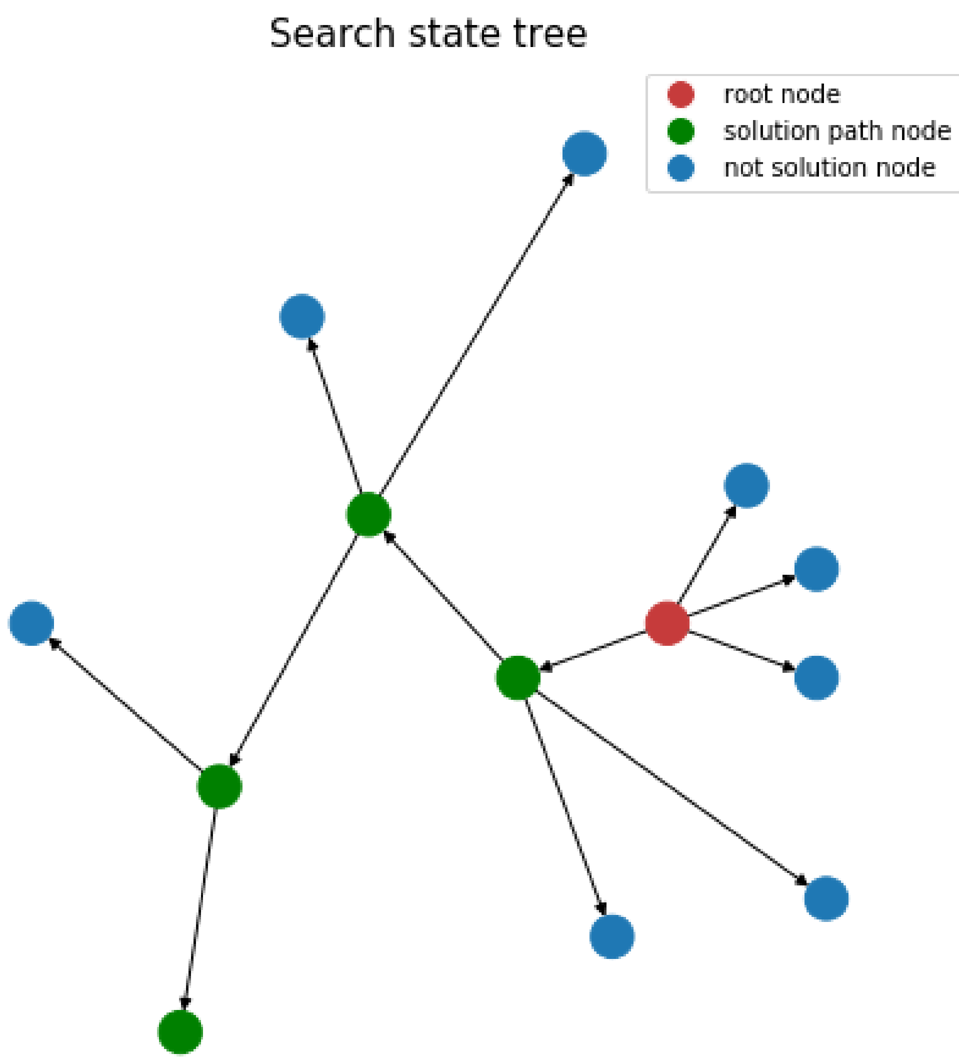
```
A minimal solution was found with a length of 28.0 km.
The search has ended!
In total 6 nodes got explored during the execution.
At the end there were still 7 nodes in the queue, that got effectively cut by
Branch-and-Bound.
Execution time : 0.00 seconds
```

The result fits to the example! The search state tree can also be visualized. Therefore, we need a function to find the path from the solution node to the root of the graph.

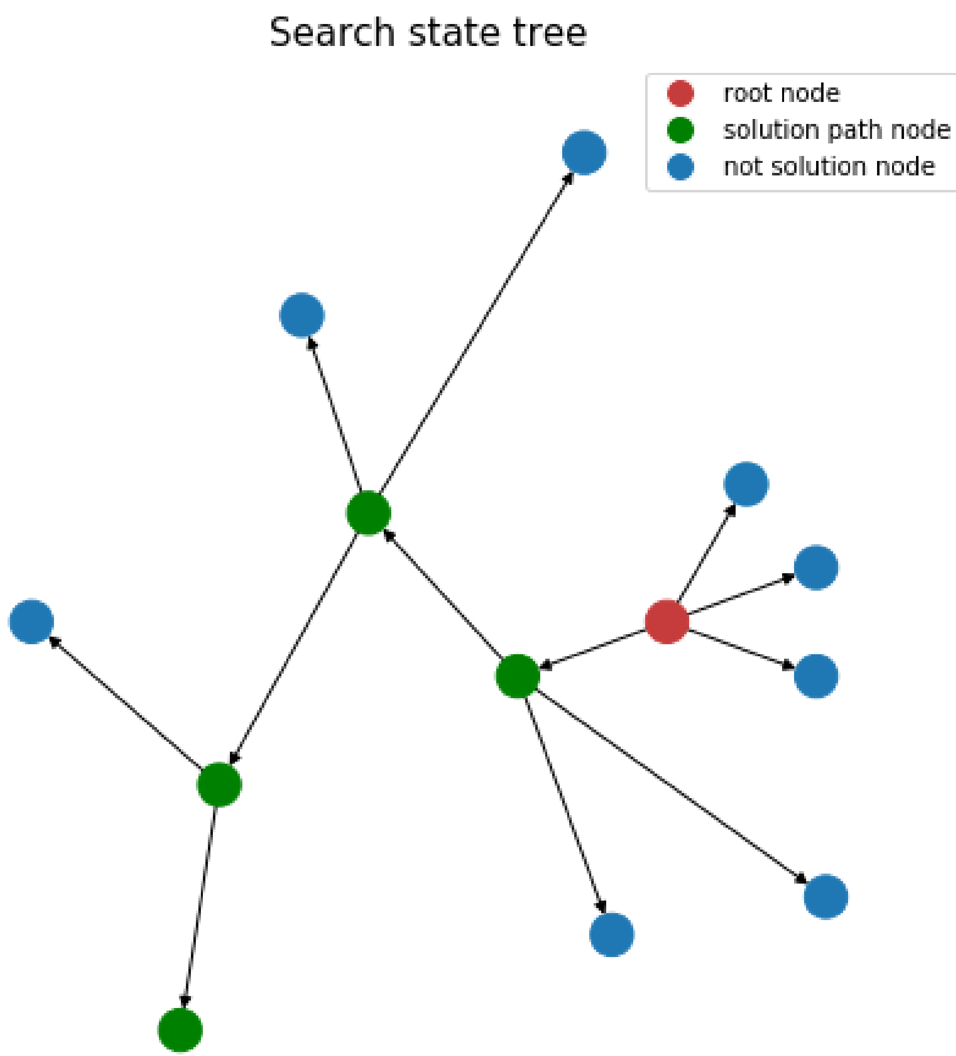
[28]:
```
# Define a function to find all nodes in the search graph from solution to root
# This is needed to color them during printing
def get_root_path(node, graph):
    path = [node]
    cur_node = node
    while (next(graph.predecessors(cur_node)) != 0):
        cur_node = next(graph.predecessors(cur_node))
        path.append(cur_node)

    path.append(0)

    return path

# List comprehension to have a list of the node colors
solution_path = get_root_path(12,H)
color_list = ['#1f78b4' if x not in solution_path  else '#008000' for x in␣
 ↪range(len(H.nodes))]
color_list[0] = '#c63b3b'
```

[42]:
```
# Draw a graph of the search space for the small example
if draw_branch_and_bound_graph:

    # Plot the graph directly in the jupyter notebook
    %matplotlib inline

    # This line finds the positions of the nodes in the graph
    pos = graphviz_layout(H, prog='twopi')
    fig = plt.figure(figsize=(8, 8))

    plt.title("Search state tree", fontsize=15)

    # Put together the legend.
    colors = ['#c63b3b', '#008000', '#1f78b4']
```

```
    dots = [Line2D([0], [0], marker='o', color='w', label='Scatter',␣
↪markerfacecolor=c, markersize=12) for c in colors]
    labels = ['root node', 'solution path node', 'not solution node']
    plt.legend(dots, labels)

    # Draw the actual graph.
    # Equal axis is necessary for the circular apperance.
    nx.draw(H, pos, node_size=300, node_color=color_list, with_labels=False)
    plt.axis('equal')
    plt.show()
```



If you do not run the graph drawing yourself, you can see the result here anyway. It should look like below. One can see, that for this small 5x5 matrix, there are 4 possible destinations after the starting city. This options are represented by the 4 nodes closest to the red root node. However, the algorithm only needs to explore more deeply one of these four alternatives, because after the reduction on the first level the estimated cost for the three other nodes are already higher than the found solution. The solution can be seen as the green path.



[30]:
```
# Run the algorithm on the real msg data
nodes_queue_real, G, N = initialize_for_given_adjaceny_matrix(adjacency_matrix)
minimum_solution_node, G = branch_and_bound(nodes_queue_real, G, N)
```

```
A minimal solution was found with a length of 2989.2 km.
The search has ended!
In total 12447 nodes got explored during the execution.
At the end there were still 134379 nodes in the queue, that got effectively cut
by Branch-and-Bound.
Execution time : 18.34 seconds
```

So, the algorithm came to a result. It was really efficient in terms of execution. On my laptop (Intel i7, 16 GB RAM) it takes far less than a minute to find the optimal solution.

Let's see how the content of the solution node looks like:

[31]:
```
G.nodes[minimum_solution_node]
```

[31]:
```
{'node_id': 146783,
 'current_city': 11,
 'tour': [0,
  16,
  4,
  8,
  1,
  2,
  10,
  9,
  17,
  13,
  14,
  6,
  5,
  12,
  7,
  20,
  3,
  18,
  19,
  15,
  11,
  0],
 'level': 20,
 'reduced_matrix': array([[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
         inf, inf, inf, inf, inf, inf, inf, inf],
        [inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
```

```
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[ 0., inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf],
[inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf, inf,
 inf, inf, inf, inf, inf, inf, inf, inf]]),
'cost': 2989.2000000000003}
```

The matrix is completly processed, all entries are infinity except for the last trip back to the starting location. The following tour is the order of the cities.

[32]:
```
solution_tour = G.nodes[minimum_solution_node]['tour']
solution_tour
```

[32]:
```
[0, 16, 4, 8, 1, 2, 10, 9, 17, 13, 14, 6, 5, 12, 7, 20, 3, 18, 19, 15, 11, 0]
```

1.4 4. The Result

So, the solution path is found, let's have a look on what it means in this last section. The function `summarize_the_result` pretty prints the results by interpreting the numbers as cities as well as setting up a summarizing data frame. The function `sec_to_pretty_time` helps with printing the durations.

[33]:
```
# Import the dataframe with the durations between the cities.
duration_df = pd.read_csv('duration_matrix.csv', index_col = 0)
duration_matrix = duration_df.values
```

[34]:
```
# The durations are stored in seconds.
# This function helps to pretty print it
def sec_to_pretty_time(seconds):
    days = int(np.floor(seconds / (60 * 60 * 24)))
    seconds_after_days = seconds % (60 * 60 * 24)

    hours = int(np.floor(seconds_after_days / (60 * 60)))
    seconds_after_hours = seconds_after_days % (60 * 60)

    minutes = int(np.floor(seconds_after_hours / 60))
    seconds_after_minutes = int(seconds_after_hours % 60)
    if days > 0:
        string = f"{days:02} d, {hours:02} h, {minutes:02} m,␣
↪{seconds_after_minutes:02} s"
    else:
        string = f"{hours:02} h, {minutes:02} m, {seconds_after_minutes:02} s"

    return string
```

[35]:
```
# City list is the ordered solution, the matrices are needed to assign values␣
↪to the trips
def summarize_the_result(city_list, adjacency_matrix, duration_matrix):

    # Initalize strings and a new data frame
    simple_string = ""
    pretty_string = ""
    result_df = pd.DataFrame(columns=['start', 'destination', 'distance in km',␣
↪'duration in sec', 'duration'])

    # Parsing through the solution list and appending the pretty string as well␣
↪as the data frame
    for i in range(len(city_list) - 1):
        start = cities.loc[city_list[i]]["msg Standort"]
        destination = cities.loc[city_list[i + 1]]["msg Standort"]
        distance = adjacency_matrix[city_list[i]][city_list[i+1]]
        duration_in_sec = duration_matrix[city_list[i]][city_list[i+1]]
        duration = sec_to_pretty_time(duration_in_sec)

        simple_string = simple_string + start +', '



        pretty_string += start
        pretty_string += "    {0} km       ".format(distance)
```

```
        # Appending a row to the data frame
        result_df = result_df.append({'start': start,
                                      'destination': destination,
                                      'distance in km': distance,
                                      'duration in sec': duration_in_sec,
                                      'duration': duration,
                                     }, ignore_index=True)

    simple_string += cities.loc[0]["msg Standort"]
    pretty_string += cities.loc[0]["msg Standort"]

    return result_df, pretty_string, simple_string
```

[36]:
```
result_df, pretty_string, simple_string = summarize_the_result(solution_tour,␣
 ↪adjacency_matrix, duration_matrix)
print(pretty_string)
```

```
Ismaning/München (Hauptsitz)    164.9 km      Passau    391.4 km
Chemnitz    178.5 km      Görlitz    237.8 km      Berlin    238.8 km
     Braunschweig    62.3 km      Hannover    158.6 km
Hamburg    175.9 km      Schortens/Wilhelmshaven    141.9 km
Lingen (Ems)    72.3 km      Münster    83.7 km      Essen    35.5 km
     Düsseldorf    53.0 km      Köln/Hürth    220.4 km
Frankfurt    111.0 km      Walldorf    39.4 km      Bretten    140.3
km      St. Georgen    109.6 km      Stuttgart    204.5 km
Nürnberg    92.3 km      Ingolstadt    77.1 km      Ismaning/München
(Hauptsitz)
```

[37]:
```
simple_string
```

[37]:
```
'Ismaning/München (Hauptsitz), Passau, Chemnitz, Görlitz, Berlin, Braunschweig,
Hannover, Hamburg, Schortens/Wilhelmshaven, Lingen (Ems), Münster, Essen,
Düsseldorf, Köln/Hürth, Frankfurt, Walldorf, Bretten, St. Georgen, Stuttgart,
Nürnberg, Ingolstadt, Ismaning/München (Hauptsitz)'
```

So, above is the order of the journey! Important side note: Because I took the assumption of symmetric distances, the route could also be done in the exact opposite order, so from Ismaning to Ingolstadt, to Nürnberg, and so on until Passau and then Ismaning.

And the table below summarizes the steps.

[38]:
```
result_df
```

[38]:
```
                          start                  destination  \
0  Ismaning/München (Hauptsitz)                       Passau
1                        Passau                     Chemnitz
2                      Chemnitz                      Görlitz
```

```
3                      Görlitz                        Berlin
4                       Berlin                   Braunschweig
5                 Braunschweig                       Hannover
6                     Hannover                        Hamburg
7                      Hamburg       Schortens/Wilhelmshaven
8      Schortens/Wilhelmshaven                  Lingen (Ems)
9                 Lingen (Ems)                       Münster
10                     Münster                         Essen
11                       Essen                    Düsseldorf
12                  Düsseldorf                    Köln/Hürth
13                  Köln/Hürth                     Frankfurt
14                   Frankfurt                      Walldorf
15                    Walldorf                       Bretten
16                     Bretten                   St. Georgen
17                 St. Georgen                     Stuttgart
18                   Stuttgart                      Nürnberg
19                    Nürnberg                    Ingolstadt
20                  Ingolstadt  Ismaning/München (Hauptsitz)

    distance in km duration in sec          duration
0            164.9           30550  08 h, 29 m, 10 s
1            391.4           81528  22 h, 38 m, 48 s
2            178.5           36885  10 h, 14 m, 45 s
3            237.8           45350  12 h, 35 m, 50 s
4            238.8           44283  12 h, 18 m, 03 s
5             62.3           11706  03 h, 15 m, 06 s
6            158.6           29452  08 h, 10 m, 52 s
7            175.9           32747  09 h, 05 m, 47 s
8            141.9           26085  07 h, 14 m, 45 s
9             72.3           13640  03 h, 47 m, 20 s
10            83.7           16089  04 h, 28 m, 09 s
11            35.5            7048  01 h, 57 m, 28 s
12            53.0           10310  02 h, 51 m, 50 s
13           220.4           40797  11 h, 19 m, 57 s
14           111.0           20667  05 h, 44 m, 27 s
15            39.4            7376  02 h, 02 m, 56 s
16           140.3           31458  08 h, 44 m, 18 s
17           109.6           22621  06 h, 17 m, 01 s
18           204.5           41666  11 h, 34 m, 26 s
19            92.3           17983  04 h, 59 m, 43 s
20            77.1           15060  04 h, 11 m, 00 s
```

Let's have a look at some details.

[39]:
```
max_row = result_df[result_df["distance in km"] == np.max(result_df["distance␣
↪in km"])]
```

```python
print(f"The longest part is the trip from {max_row.start.values[0]} to {max_row.destination.values[0]} and takes {max_row.duration.values[0]} for {max_row['distance in km'].values[0]} km.")

min_row = result_df[result_df["distance in km"] == np.min(result_df["distance in km"])]
print(f"The shortest part is the trip from {min_row.start.values[0]} to {min_row.destination.values[0]} and takes {min_row.duration.values[0]} for {min_row['distance in km'].values[0]} km.")
```

```
The longest part is the trip from Passau to Chemnitz and takes 22 h, 38 m, 48 s
for 391.4 km.
The shortest part is the trip from Essen to Düsseldorf and takes 01 h, 57 m, 28
s for 35.5 km.
```

And of course the overall sum of distance and duration:

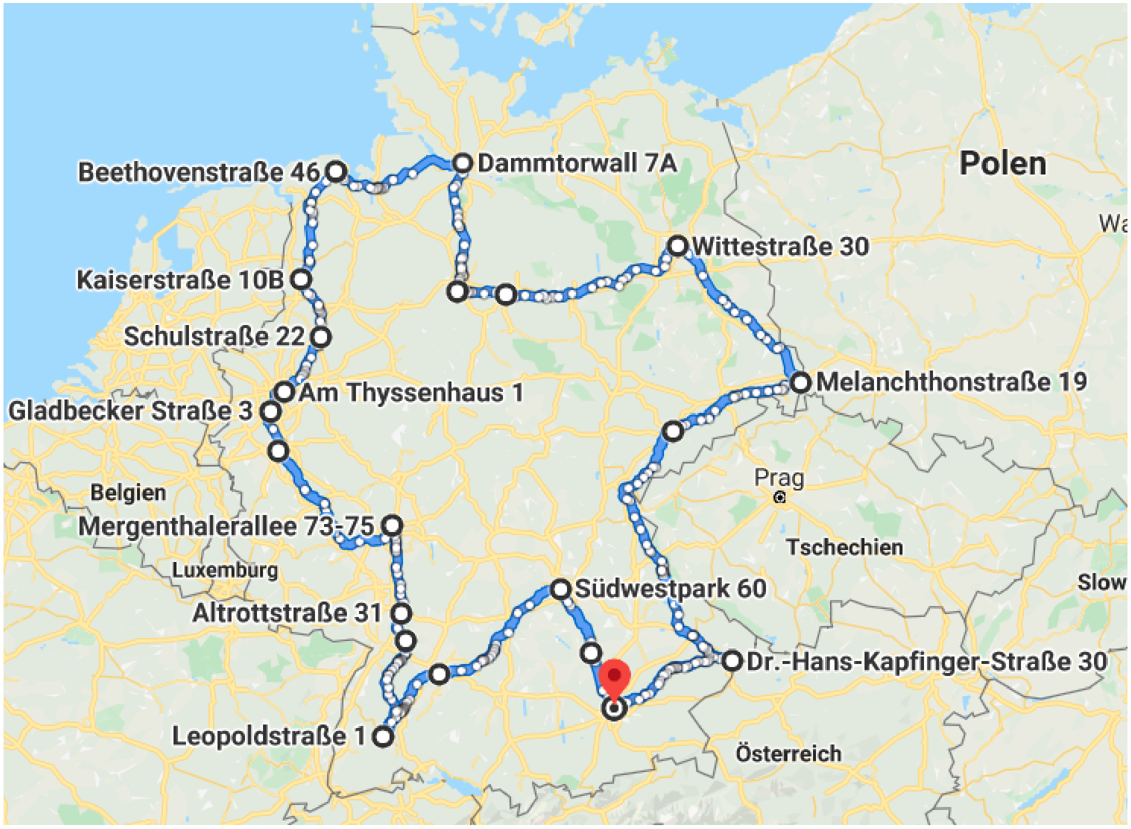
[40]:
```python
sum_of_km = np.round(np.sum(result_df['distance in km']), decimals=1)
sum_of_seconds = np.sum(result_df['duration in sec'])

print(f"The entire trip has a length of {sum_of_km} km.")
print(f"The entire trip will take {sum_of_seconds} seconds on a bike according to Google Maps.")
print(f"To read it more nicely, the seconds are equal to a duration of {sec_to_pretty_time(sum_of_seconds)}.")
```

```
The entire trip has a length of 2989.2 km.
The entire trip will take 583301 seconds on a bike according to Google Maps.
To read it more nicely, the seconds are equal to a duration of 06 d, 18 h, 01 m,
41 s.
```

So, after almost **3.000 km and almost a week of pure cycling time**, my shortest bike trip to all 21 msg locations would come to an end.

I noticed one fact regarding the exact length: Because I extract the distances from Google Maps, the shortest distance can vary a bit due to time of day, traffic, etc. The difference is only minor. The order of cities stays the same.

With the webtool https://www.morethan10.com/ I was able to concatenate the 21 trips in Google Maps for bikes. The result on a map looks as shown below. The solution appears to be optimal and the concatenated distance is equal to the distance that the algorithm calculated.



If you arrived at this place: Thank you for reading so far!

I had a lot of fun putting this together. I hope you had a little bit of entertainment along the way, too.