微分方程数值解法

第十三周作业

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1 P207 DuFort-Frankel 格式截断误差

证明.

$$R_i^n = \frac{u_i^{n+1} - u^{n+1}}{2\tau} - a\frac{u_{i+1}^n - (u_i^{n+1} + u_i^{n-1}) + u_{i-1}^n}{h^2} - (u_t\big|_i^n - au_{xx}\big|_i^n)$$

分别定义 $R_t\big|_i^n, R_x\big|_i^n$

$$\begin{aligned}
R_t\big|_i^n &= \frac{u_i^{n+1} - u^{n+1}}{2\tau} - u_t\big|_i^n \\
&= \frac{\tau^2}{6}u_{tt}\big|_i^n + O(\tau^3) \\
&= O(\tau^2) \\
R_x\big|_i^n &= -a\frac{u_{i+1}^n - (u_i^{n+1} + u_i^{n-1}) + u_{i-1}^n}{h^2} + au_{xx}\big|_i^n \\
&= -\frac{a}{h^2}\left((u_{i+1}^n + u_{i-1}^n) - (u_i^{n+1} + u_i^{n-1})\right) + au_{xx}\big|_i^n \\
&= -\frac{a}{h^2}\left((2u_i^n + h^2u_{xx}\big|_i^n + O(h^4)) - (2u_i^n + \tau^2u_{tt}\big|_i^n + O(\tau^4))\right) + au_{xx}\big|_i^n \\
&= -\frac{a}{h^2}\left(O(h^4) - \tau^2u_{tt}\big|_i^n - O(\tau^4)\right) \\
&= O(h^2) + O(\tau^2h^{-2})
\end{aligned}$$

所以，$R_i^n = O(\tau^2 + h^2) + O(\tau^2h^{-2})$

□