# 概率论与数理统计*
## 作业 LᴬTEX

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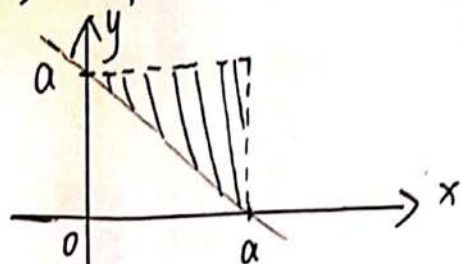*Department of Computer Science and Technology*

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13. 解: 不妨设 3段长度为 $x, y, 2a-x-y$

构成三角形条件为:
$$\begin{cases} x+y > 2a-x-y \\ x+2a-x-y > y \\ y+2a-x-y > x \end{cases} \Rightarrow \begin{cases} x+y > a \\ 0 < y < a \\ 0 < x < a \end{cases}$$

几何概率



总的域空间为
$$\begin{cases} x > 0 \\ y > 0 \\ x+y < 2a \end{cases}$$

∴ 能构成三角形的概率为 $\dfrac{a \times a \div 2}{2a \times 2a \div 2} = \dfrac{1}{4}$

14. 解: $P(B|A) = \dfrac{P(AB)}{P(A)} = \dfrac{P(B) - P(B-A)}{P(A \cup B) - P(B-A)} = \dfrac{\frac{2}{5} - \frac{1}{3}}{\frac{3}{4} - \frac{1}{3}} = \dfrac{4}{25}$

15. 解: $P(AB) = P(A)\,P(B|A) = \dfrac{1}{12}$

$P(B) = \dfrac{P(AB)}{P(A|B)} = \dfrac{\frac{1}{12}}{\frac{1}{2}} = \dfrac{1}{6}$

$P(\bar{A}\bar{B}) = P(\overline{A \cup B}) = 1 - \left( P(A) + P(B) - P(AB) \right)$

$\qquad = 1 - \left( \dfrac{1}{4} + \dfrac{1}{6} - \dfrac{1}{12} \right) = \dfrac{2}{3}$

16. 解: $A_1$: 至少有一次品 ; $A_2$: 两件都是次品

$P(A_2|A_1) = \dfrac{P(A_1 A_2)}{P(A_1)} = \dfrac{P(A_2)}{P(A_1)}$

$\qquad = \dfrac{C_4^2}{C_6^1 C_4^1 + C_4^2} = \dfrac{1}{5}$

17.解: (1) $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{1 - P(\bar{A} \cup \bar{B})}{P(B)}$

$= \dfrac{1 - \left( P(\bar{A}) + P(\bar{B}) - P(\bar{A}\bar{B}) \right)}{P(B)}$

$= \dfrac{1 - \left( 1 - P(A) + 1 - P(B) - P(\bar{A}\bar{B}) \right)}{P(B)}$

$= \dfrac{a + b - 1 + P(\bar{A}\bar{B})}{b}$

又 $P(\bar{A}\bar{B}) \geqslant 0$ $\therefore P(A|B) \geqslant \dfrac{a+b-1}{b}$

(2) $\dfrac{P(AB)}{P(B)} + \dfrac{P(\bar{A}\bar{B})}{P(\bar{B})} = 1$

$\Rightarrow \dfrac{P(AB)(1-P(B)) + P(B)\left\{ 1 - [P(A) + P(B) - P(AB)] \right\}}{P(B)(1-P(B))} = 1$

不妨令 $P(A) = a$, $P(B) = b$, $P(AB) = p$

上式 $\Rightarrow p(1-b) + b(1-a-b+p) = b(1-b)$

$\Rightarrow p - bp + b - ab - b^2 + bp = b - b^2$

$\Rightarrow p = ab$

即 $P(AB) = P(A)P(B)$    得证

18.解: $A_1, A_2, A_3$ 分别表示甲、乙、丙车间生产

B 表示次品

(1) $P(B) = \sum\limits_{i=1}^{3} P(B|A_i) P(A_i)$

$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02$

$= 0.0345$

(2) $P(A_1|B) = \dfrac{P(B|A_1) P(A_1)}{\sum\limits_{i=1}^{3} P(B|A_i) P(A_i)}$

$= 0.3623$