2ECDE62 Computer Vision
Numerical Problems

1 Consider a camera with a focal length of 20 where all the rays pass through its camera center. The principal point is located at (0,0) in the coordinate system of the image plane. The image plane is parallel to the XY plane of the camera-centric world coordinate system and its coordinate axes are parallel to X and Y axes, respectively. Compute the image coordinates for the world coordinate (60, 80, 40) using the perspective projective mapping.
If the principal point is located at (1, 2) in the coordinate system of the image plane, what would be the image coordinates of the world point (6, 7, 9) in a camera-centric coordinate convention, using a pinhole camera with focal length $f = 3$?

2 Derive the 2D rotational matrices transformation and show that rotation matrices are characterized by the following properties: (a) the inverse of a rotational matrix is its transpose and (b) its determinant is 1.

3 What is the concatenated transformation matrix for translation by vector [1 1] followed by rotation by angle 45 degrees in 2D.

4 The following Figure 1 is first scaled by a factor 2 and then translated by a vector [2 2]. What is the transformed figure? Determine the figure if the translation is applied first followed by scaling on Figure 1.

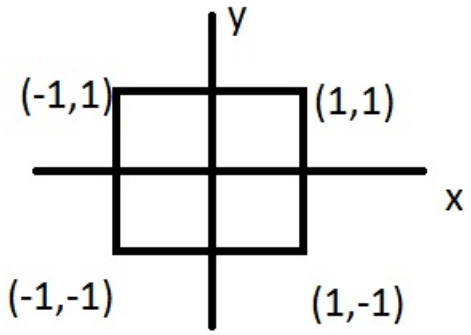

Figure 1

5 A camera lens has a focal length of 5. Find out the image point corresponding to a world point at location (50, 70, 100). Assume the image coordinate system and the world coordinate system to be perfectly aligned.

6 A camera with a focal length of 0.04 m is placed at a height of 1.0 m and is looking vertically downwards to take images of the XY plane. If the size of the image sensor plate is 4 mm x 3 mm, find the area on the XY plane that can be imaged.

7 Given a Projection matrix

$$P = \begin{bmatrix} 1 & 2 & 5 & -1 \\ -5 & 2 & 1 & 2 \\ 2 & 1 & -1 & 0 \end{bmatrix}$$

(a) Computer the image of the point (2, 4, 3)
(b) Compute the vanishing point of the line with direction cosines 3 : 4 : 2.

(c) Find the camera center.

8 Suppose P and P' are the projection matrix for left and right cameras of Stereo Geometry. Images of a scene point are formed at (0, 3.5), and (2/3, -1/3), respectively. Find the 3D coordinate of the scene point.

$$P = \begin{bmatrix} 7 & 4 & -6 & 3 \\ 8 & -1 & 2 & -5 \\ 9 & -10 & 4 & 1 \end{bmatrix} \quad \text{and } P' = \begin{bmatrix} 6 & 4 & -6 & 10 \\ 8 & -5 & 2 & -7 \\ 9 & -10 & 6 & 2 \end{bmatrix}$$

9 Consider two images of the scene formed using two cameras P and P', with the same camera center, as I and I', respectively. Camera projection matrices are given as $P = \begin{bmatrix} -8 & 5 & -4 & 2 \\ -1 & 8 & -9 & 3 \\ 2 & 1 & -1 & 4 \end{bmatrix}$ and $P' = \begin{bmatrix} -4 & 3 & -2 & 2 \\ -1 & 4 & -3 & 3 \\ -1 & 1 & -1 & 4 \end{bmatrix}$

Compute the homography H from I to I'.

10 Given a Homography

$$H = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

Compute the transformed point of intersection of the straight lines given by 2x + 3y + 2 = 0 and 3x + 3y + 5 = 0 in R^2

11 Consider a camera with a focal length of 20 where all the rays pass through its camera center. The principal point is located at (0,0) in the coordinate system of the image plane. The image plane is parallel to the XY plane of the camera-centric world coordinate system and its coordinate axes are parallel to X and Y axes, respectively. Compute the image coordinates for the world coordinate (60, 80, 40) using the perspective projective mapping.
If the principal point is located at (1, 2) in the coordinate system of the image plane, what would be the image coordinates of the world point (6, 7, 9) in a camera-centric coordinate convention, using a pinhole camera with focal length f = 3?

12 Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

13 Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

14 Consider the following Homography H between two projective spaces.

$$H = \begin{bmatrix} 3 & 4 & -6 \\ 1 & 3 & -8 \\ 0 & 5 & 1 \end{bmatrix}$$

Compute the transformation of the line formed by two points (2,4,2) and (6,9,3) in P^2.

15 What is the concatenated transformation matrix for translation by vector [1 1] followed by rotation by angle 45 degrees in 2D.

16 A scene point at coordinates (400,600,1200) is perspectively projected into an image at coordinates (24,36), where both coordinates are given in millimeters in the camera coordinate frame and the camera's principal point is at coordinates (0,0,f) (i.e., $u_0 = 0$ and $v_0 = 0$). Assuming the aspect ratio of the pixels in the camera is 1, what is the focal length of the camera? (Note: the aspect ratio is defined as the ratio between the width and the height of a pixel.)