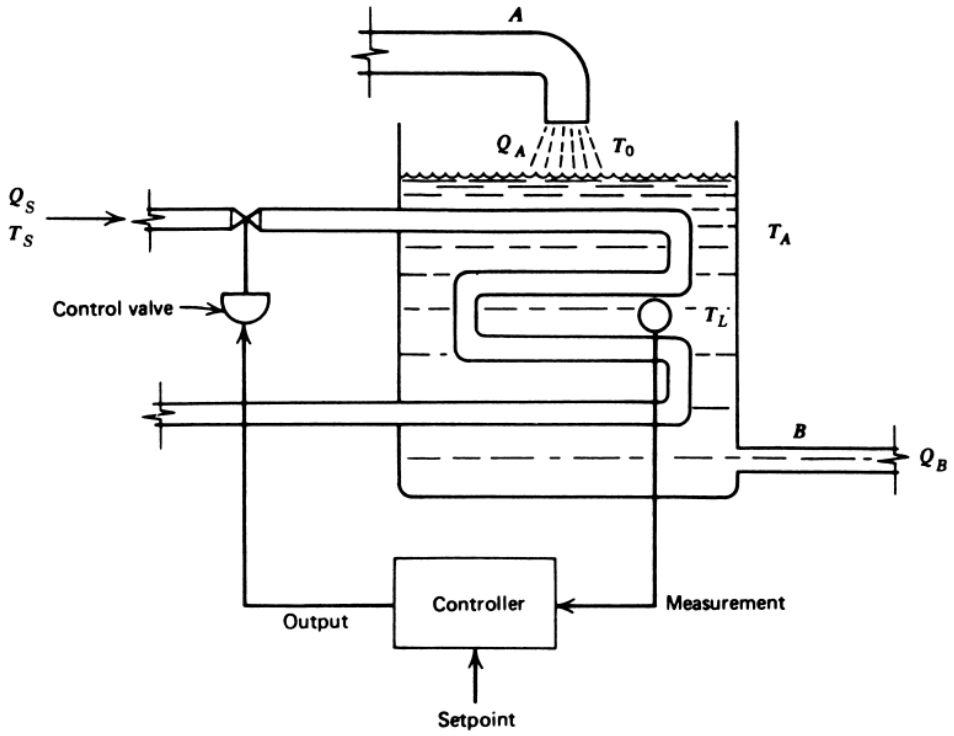
CPS Implementation

Electronics Controller Design

Controller Design Techniques

Control of Liquid Temperature in a Tank



Controlled Variable is Liquid Temperature T_L
Controlling Parameter is Steam Flow Rate Q_S

$$T_L = F(Q_A, Q_B, Q_S, T_A, T_S, T_0)$$

where
Q_A, Q_B = flow rates in pipes A and B
Q_S = steam flow rate
T_A = ambient temperature
T_0 = inlet fluid temperature
T_S = steam temperature

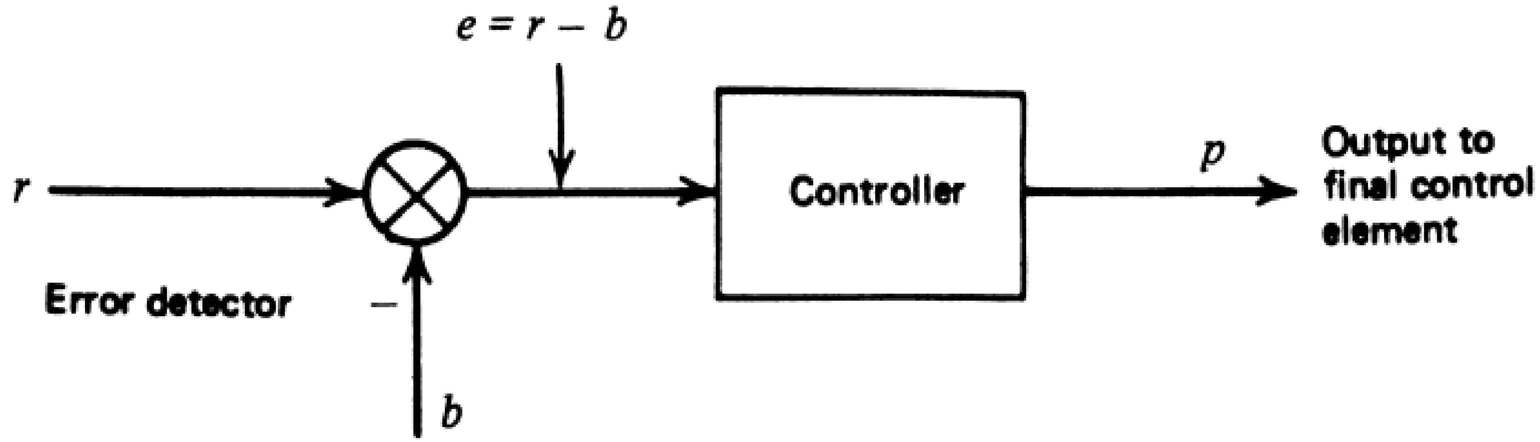
Process Equation

- From the process equation, or knowledge of and experience with the process, it is possible to identify a set of values for the process parameters that results in the controlled variable having the setpoint value. This set of parameters is called the nominal set. The term process load refers to this set of all parameters, excluding the controlled variable
- A temporary variation of one of the load parameters. After the excursion, the parameter returns to its nominal value. This variation is called a transient.

Process Lag

- Process-control operations are essentially a time-variation problem. At some point in time, a process-load change or transient causes a change in the controlled variable.
- The process-control loop responds to ensure that, some finite time later, the variable returns to the setpoint value. Part of this time is consumed by the process itself and is called the process lag.
- A significant characteristic of some processes is the tendency to adopt a specific value of the controlled variable for nominal load with no control operations. The control operations may be significantly affected by such self-regulation.

Control System Parameters



Error Detector and Controller

The deviation or error of the controlled variable from the setpoint is given by

$$e = r - b$$

where e = error
b = measured indication of variable
r = setpoint of variable (reference)

Controller Equations

- The measured value of a variable can be expressed as percent of span over a range of measurement by the equation

$$c_p = \frac{c - c_{\min}}{c_{\max} - c_{\min}} \times 100$$

where c_p = measured value as percent of measurement range
c = actual measured value
$c_{\max}$ = maximum of measured value
$c_{\min}$ = minimum of measured value

To express error as percent of span, it is necessary only to write both the setpoint and measurement in terms of percent of span and take the difference

$$e_p = \frac{r - b}{b_{max} - b_{min}} \times 100$$

e_p = error expressed as percent of span

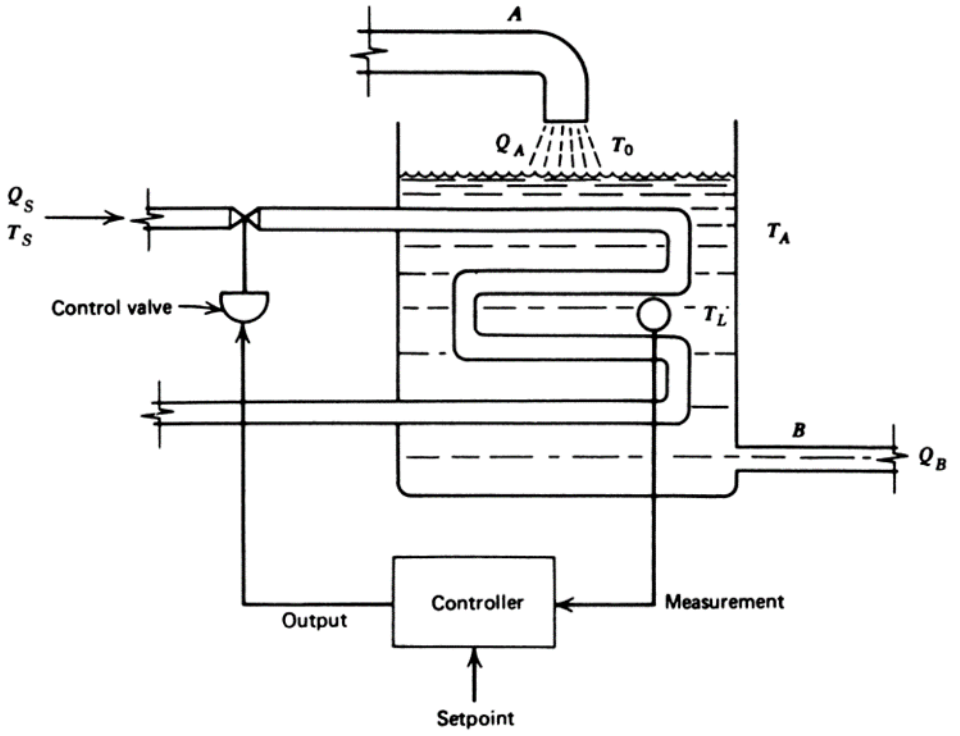
You can see the convenience of using a standard measured indication range like 4 to 20 mA, because the span is always 16 mA. Suppose we have a setpoint of 10.5 mA and a measurement of 13.7 mA. Then, without even knowing what is being measured, we know the error is

$$e_p = \frac{10.5\text{ mA} - 13.7\text{ mA}}{20\text{ mA} - 4\text{ mA}} \times 100$$

$$e_p = -20\%$$

A positive error indicates a measurement below the setpoint, and a negative error indicates a measurement above the setpoint.

Example



The temperature in Figure has a range of 300 to 440 K and a setpoint of 384 K. Find the percent of span error when the temperature is 379 K.

Solution

The percent error is

$$e_p = \frac{r - b}{b_{\text{max}} - b_{\text{min}}} \times 100$$

$$e_p = \frac{384 - 379}{440 - 300} \times 100$$

$$e_p = \mathbf{3.6\%}$$

Example

- A velocity control system has a range of 220 to 460 mm/s. If the setpoint is 327 mm/s and the measured value is 294 mm/s, calculate the error as percentage of span.

Controller Parameter Range

- Often, the output is expressed as a percentage where 0% is the minimum controller output and 100% the maximum.
- The controller output as a percent of full scale when the output varies between specified limits is given by

$$p = \frac{u - u_{\min}}{u_{\max} - u_{\min}} \times 100$$

where
p = controller output as percent of full scale
u = value of the output
$u_{\max}$ = maximum value of controlling parameter
$u_{\min}$ = minimum value of controlling parameter

Example

A controller outputs a 4- to 20-mA signal to control motor speed from 140 to 600 rpm with a linear dependence. Calculate **(a)** current corresponding to 310 rpm, and **(b)** the value of (a) expressed as the percent of control output.

We find the slope m and intersect of the linear relation between current I and speed S, where

$$S_p = mI + S_0$$

Solution

Knowing S_p and I at the two given positions, we write two equations:

$$140 = 4m + S_0$$
$$600 = 20m + S_0$$

Solving these simultaneous equations, we get $m = 28.75$ rpm/mA and $S_0 = 25$ rpm. Thus, at 310 rpm we have $310 = 28.75I + 25$, which gives $I = 9.91$ mA.

b. Expressed as a percentage of the 4- to 20-mA range, this controller output is

$$p = \frac{u - u_{\min}}{u_{\max} - u_{\min}} \times 100$$

$$p = \left[\frac{9.91 - 4}{20 - 4}\right] \times 100$$

$$p = \mathbf{36.9\%}$$

Example

- A controlling variable is a motor speed that varies from 800 to 1750 rpm. If the speed is controlled by a 25- to 50-V dc signal, calculate (a) the speed produced by an input of 38 V, and (b) the speed calculated as a percent of span.

Other Parameters

- Control Lag: Control lag refers to the time for the process-control loop to make necessary adjustments to the final control element.

- Dead Time: This is the elapsed time between the instant a deviation (error) occurs and when the corrective action first occurs.

- Cycling: One of the most important modes is an oscillation of the error about zero. This means the variable is cycling above and below the setpoint value. Such cycling may continue indefinitely, in which case we have steady-state cycling.

Controller Modes

- Discontinuous controller Mode
- Continuous controller Mode

Discontinuous Controller Mode

- Two Position Mode
- Multiposition Mode
- Floating control Mode

Two Position (ON/OFF) Mode

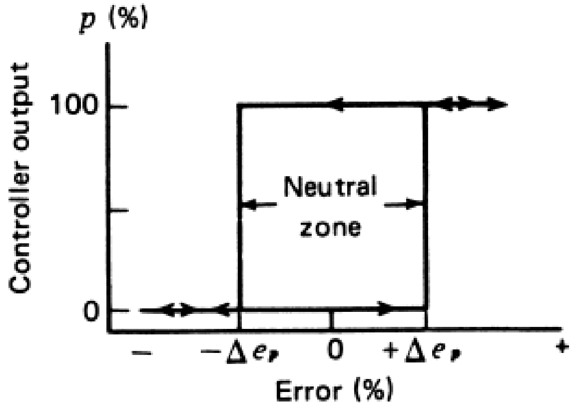
- The most elementary controller mode is the ON/OFF, or two-position, mode. This is an example of a discontinuous mode. It is the simplest and the cheapest, and often suffices when its disadvantages are tolerable.

$$p = \begin{cases} 0\% & e_p < 0 \\ 100\% & e_p > 0 \end{cases}$$

This relation shows that when the measured value is less than the setpoint, full controller output results. When it is more than the setpoint, the controller output is zero.

Neutral Zone

- In virtually any practical implementation of the two-position controller, there is an overlap as increases through zero or decreases through zero. In this span, no change in controller output occurs.
- We see that until an increasing error changes by above zero, the controller output will not change state. In decreasing, it must fall below zero before the controller changes to the 0% rating. The range , which is referred to as the neutral zone or differential gap, is often purposely designed above a certain minimum quantity to prevent excessive cycling. The existence of such a neutral zone is an example of desirable hysteresis in a system.



Example

A liquid-level control system linearly converts a displacement of 2 to 3 m into a 4- to 20-mA control signal. A relay serves as the two-position controller to open or close an inlet valve. The relay closes at 12 mA and opens at 10 mA. Find **(a)** the relation between displacement level and current, and **(b)** the neutral zone or displacement gap in meters.

a. The relation between level and a current is a linear equation such as

$$H = KI + H_0$$

We find K and H_0 by writing two equations:

$$2 \text{ m} = K(4 \text{ mA}) + H_0$$
$$3 \text{ m} = K(20 \text{ mA}) + H_0$$

Solving these simultaneous equations yields $K = 0.0625$ m/mA and $H_0 = 1.75$ m, at the intersection of the linear relations.

b. The relay closes at 12 mA, which is a high level, H_H, of

$$H_H = (0.0625 \text{ m/mA})(12 \text{ mA}) + 1.75 \text{ m}$$
$$H_H = 2.5 \text{ m}$$

The low level, H_L, occurs at 10 mA, which is

$$H_L = (0.0625 \text{ m/mA})(10 \text{ mA}) + 1.75 \text{ m}$$
$$H_L = 2.375 \text{ m}$$

Thus, the neutral zone is $H_H - H_L = (2.5 - 2.375)$ m, or **0.125 m.**

Applications

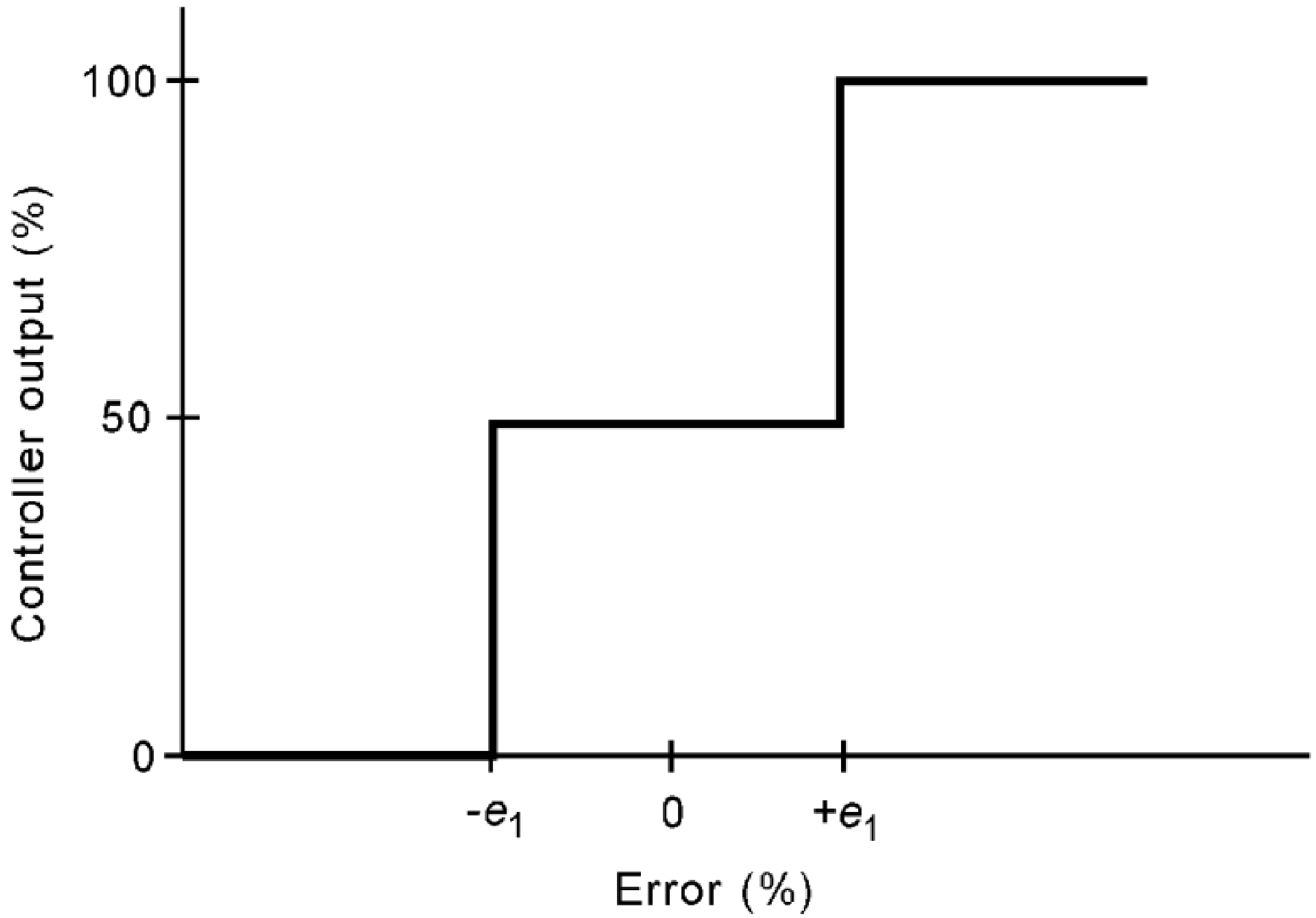
- Generally, the two-position control mode is best adapted to large-scale systems with relatively slow process rates. Thus, in the example of either a room heating or air-conditioning system, the capacity of the system is very large in terms of air volume, and the overall effect of the heater or cooler is relatively slow.
- Sudden, large-scale changes are not common to such systems. Other examples of two position control applications are liquid bath-temperature control and level control in large-volume tanks.
- The process under two-position control must allow continued oscillation in the controlled variable because, by its very nature, this mode of control always produces such oscillation.

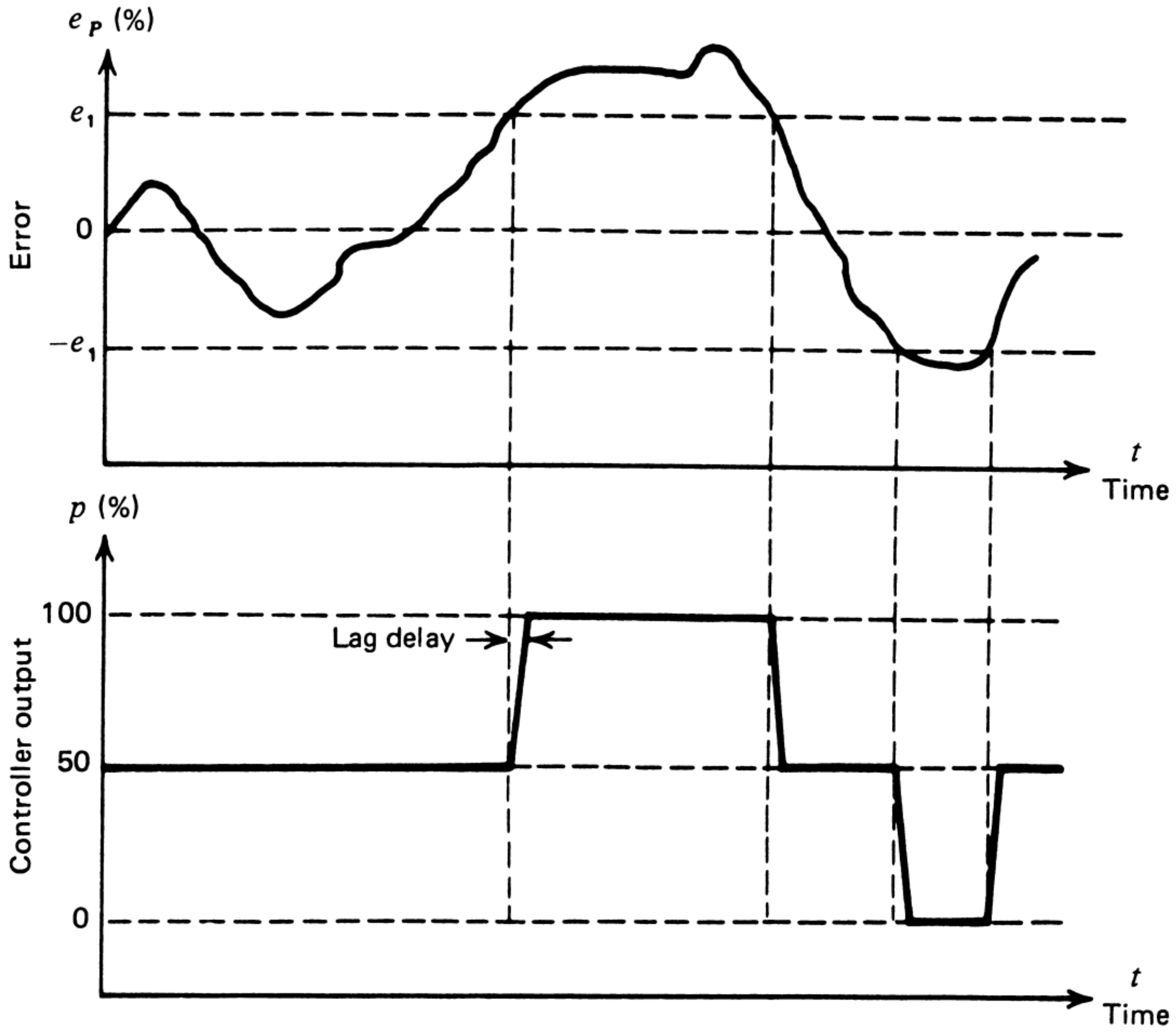
Multi position Mode

- This discontinuous control mode is used in an attempt to reduce the cycling behavior and overshoot and undershoot inherent in the two-position mode.
- As the error exceeds certain set limits +/- e_i, the controller output is adjusted to preset values p_i.
- Three position controller

$$p = \begin{cases} 100 & e_p > e_2 \\ 50 & -e_1 < e_p < e_2 \\ 0 & e_p < -e_1 \end{cases}$$

Three Position Controller Design





e_P (%)
e_1
0
-e_1
Error
t
Time
p (%)
100
50
0
Lag delay
Controller output
t
Time

Floating Control Mode

- In the previous mode, if the error exceeded some preset limit, the output was changed to a new setting as quickly as possible.
- In floating control, the specific output of the controller is not uniquely determined by the error.
- If the error is zero, the output does not change but remains (floats) at whatever setting it was .
- When the error moves off zero, the controller output again begins to change.

Single Speed Floating Mode

- In the single-speed floating-control mode, the output of the control element changes at a fixed rate when the error exceeds the neutral zone.

$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p$$

where
$$\frac{dp}{dt} = \text{rate of change of controller output with time}$$
$$K_F = \text{rate constant (\%/s)}$$
$$\Delta e_p = \text{half the neutral zone}$$

$$p = \pm K_F t + p(0) \qquad |e_p| > \Delta e_p$$

where $p(0)$ = controller output at $t = 0$

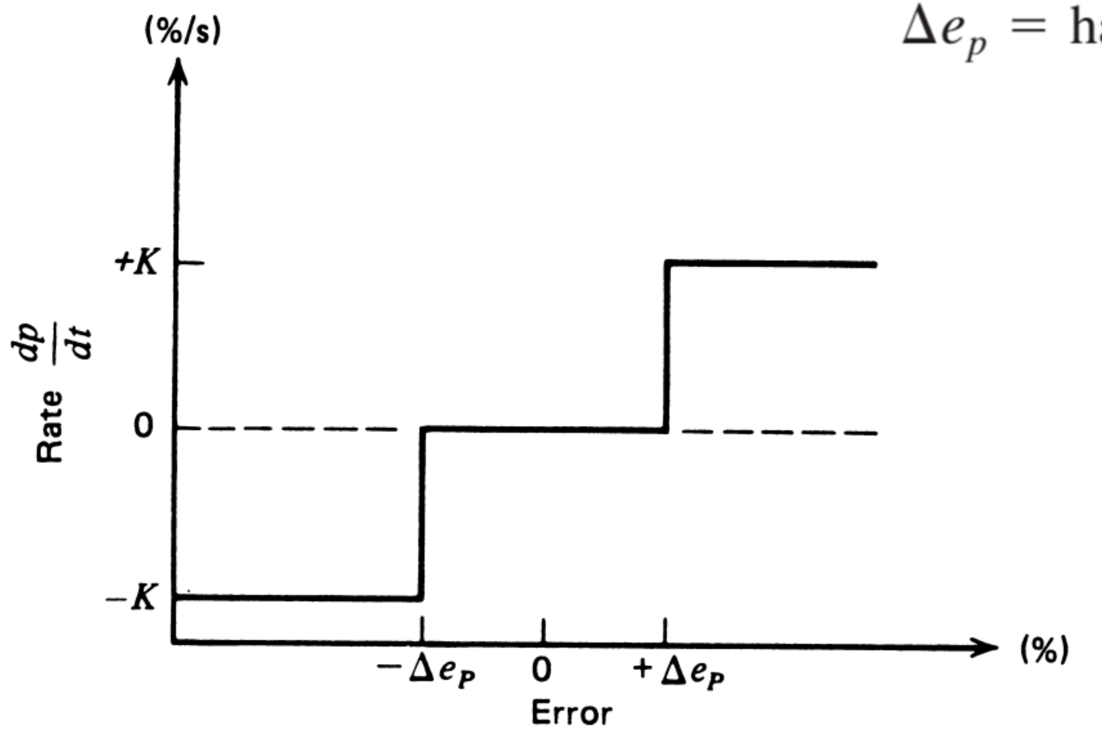
- Here, present output depends on the time history of errors that have previously occurred.
- If the deviation persists, then the controller saturates at either 100% or 0% and remains there until an error drives it toward the opposite extreme.

$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p$$

where $\dfrac{dp}{dt}$ = rate of change of controller output with time

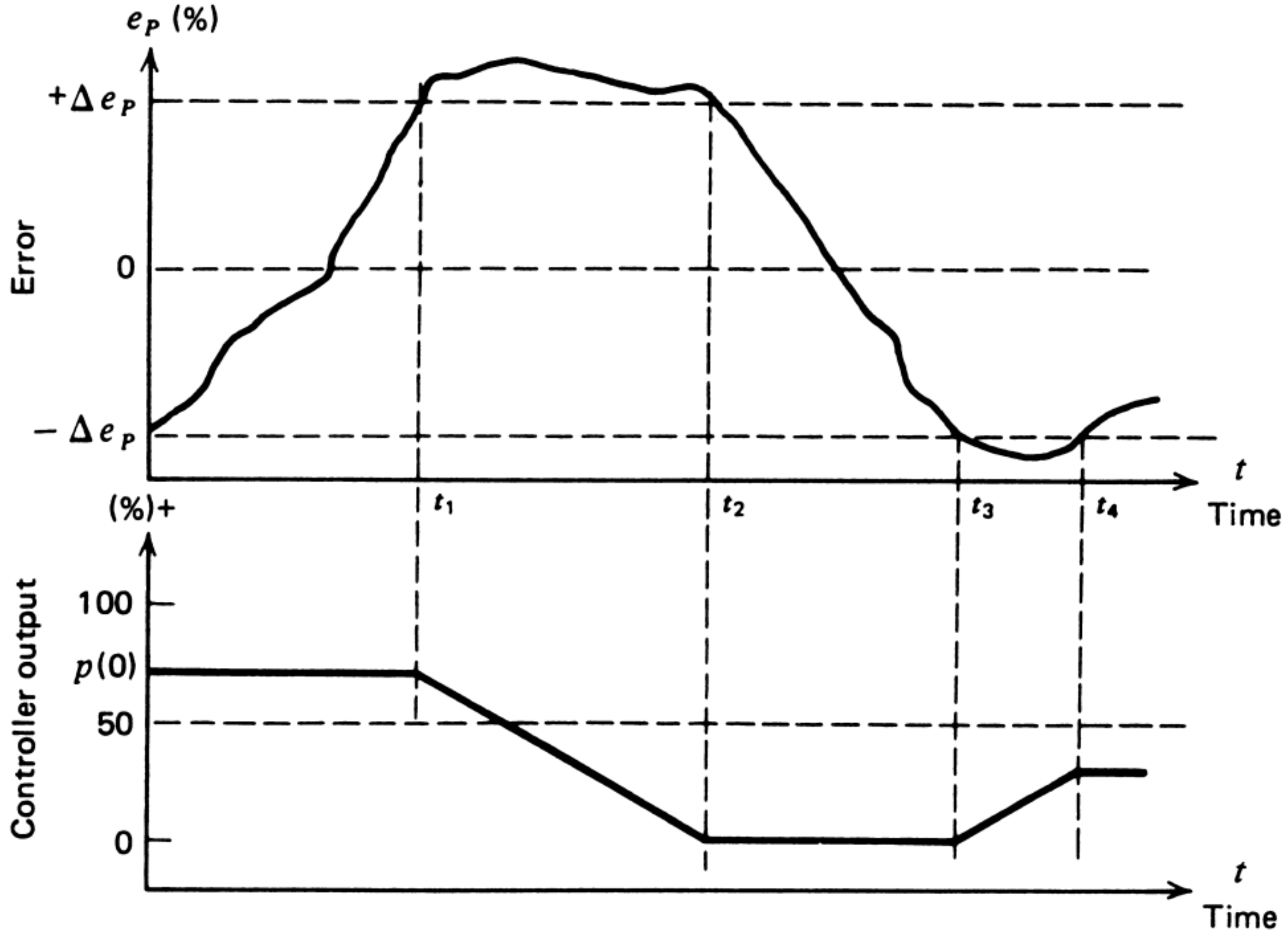
K_F = rate constant (%/s)

Δe_p = half the neutral zone



(a)

Error v/s Controller Response



(b)

Example

Suppose a process error lies within the neutral zone with $p = 25\%$. At $t = 0$, the error falls below the neutral zone. If $K = +2\%$ per second, find the time when the output saturates.

$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p$$

where $\frac{dp}{dt}$ = rate of change of controller output with time

K_F = rate constant (%/s)

Δe_p = half the neutral zone

$$p = \pm K_F t + p(0) \qquad |e_p| > \Delta e_p$$

where $p(0)$ = controller output at $t = 0$

Solution

The relation between controller output and time is

$$p = K_F t + p(0)$$

When $p = 100$

$$100\% = (2\%/\mathrm{s})(t) + 25\%$$

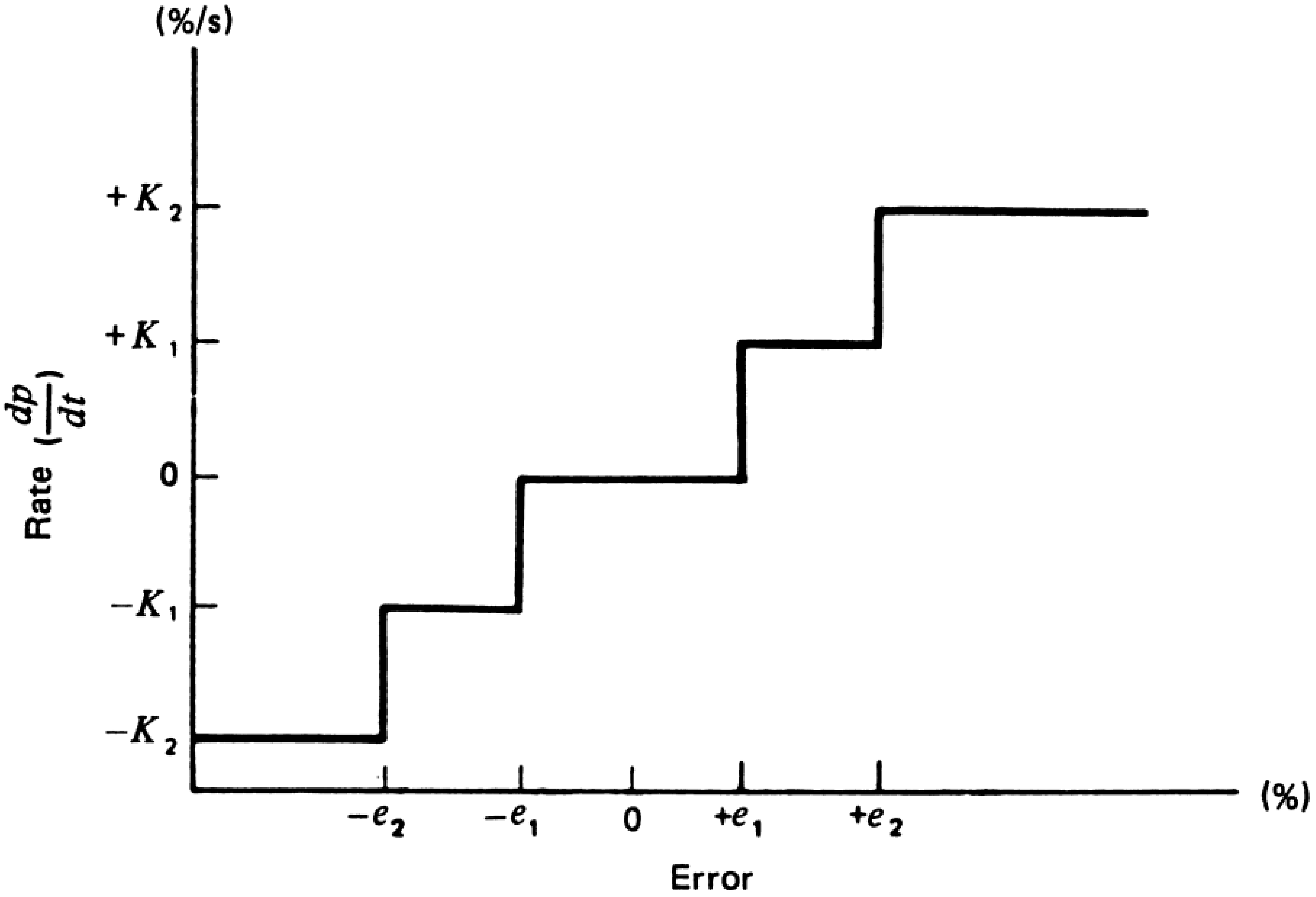
that, when solved for t, yields

$$t = \mathbf{37.5\ s}$$

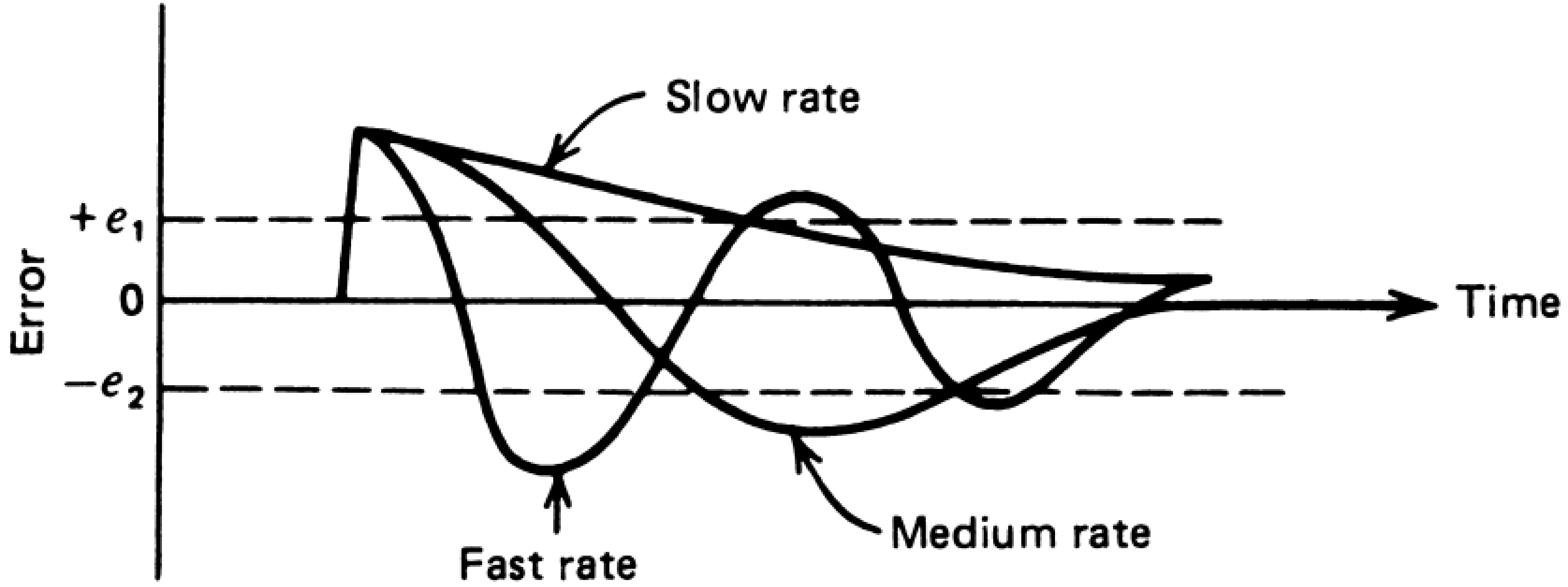
Multiple Speed In the floating multiple-speed control mode, not one but several possible speeds (rates) are changed by controller output. Usually, the rate increases as the deviation exceeds certain limits. Thus, if we have certain speed change points, e_{pi}, depending on the error, then each has its corresponding output rate change, K_i. We can then say

$$\frac{dp}{dt} = \pm K_{Fi} \qquad |e_p| > e_{pi}$$

If the error exceeds e_{pi}, then the speed is K_{Fi}. If the error rises to exceed e_{p2}, the speed is increased to K_{F2}, and so on. Actually, this mode is a discontinuous attempt to realize an in-


(%/s)
$+K_2$
$+K_1$
0
$-K_1$
$-K_2$
Rate $(\frac{dp}{dt})$
$-e_2$
$-e_1$
0
$+e_1$
$+e_2$
(%)
Error

Rate of controller output in Floating Controller



Continuous Controller Modes

- The most common controller action used in process control is one or a combination of continuous controller modes. In these modes, the output of the controller changes smoothly in response to the error or rate of change of error.
- Proportional Control Mode
- Integral Control Mode
- Derivative Control Mode

Proportional Control Mode

- The two-position mode had the controller output of either 100% or 0%, depending on the error being greater or less than the neutral zone.
- In multiple-step modes, more divisions of controller outputs versus error are developed.
- The natural extension of this concept is the proportional mode, where a smooth, linear relationship exists between the controller output and the error.
- Thus, over some range of errors about the setpoint, each value of error has a unique value of controller output in one-to-one correspondence.
- The range of error to cover the 0% to 100% controller output is called the proportional band, because the one-to-one correspondence exists only for errors in this range.

$$p = K_P e_p + p_0$$

Error can be positive or negative

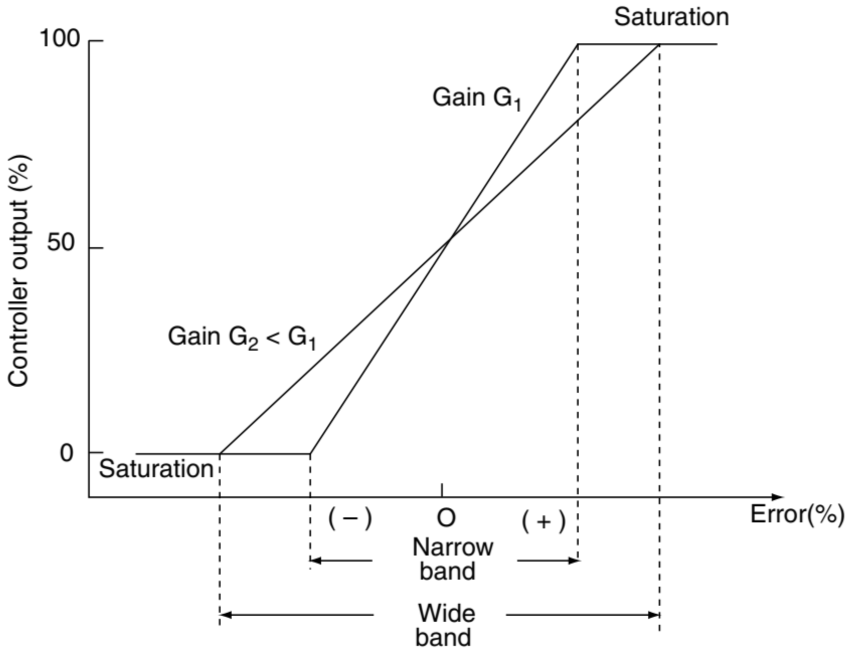
K_P = proportional gain between error and controller output (% per %)
p_0 = controller output with no error (%)

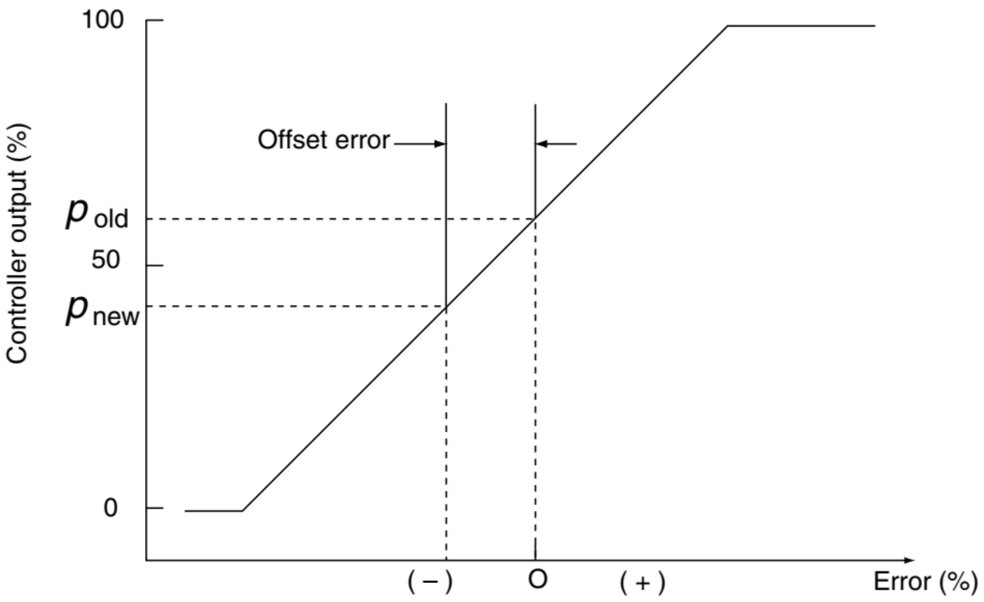
Proportional Band is defined as

$$PB = \frac{100}{K_P}$$

Characteristics of Proportional Mode

1. If the error is zero, the output is a constant equal to p_0.
2. If there is error, for every 1% of error, a correction of K_P percent is added to or subtracted from p_0, depending on the sign of the error.
3. There is a band of error about zero of magnitude PB within which the output is not saturated at 0% or 100%.

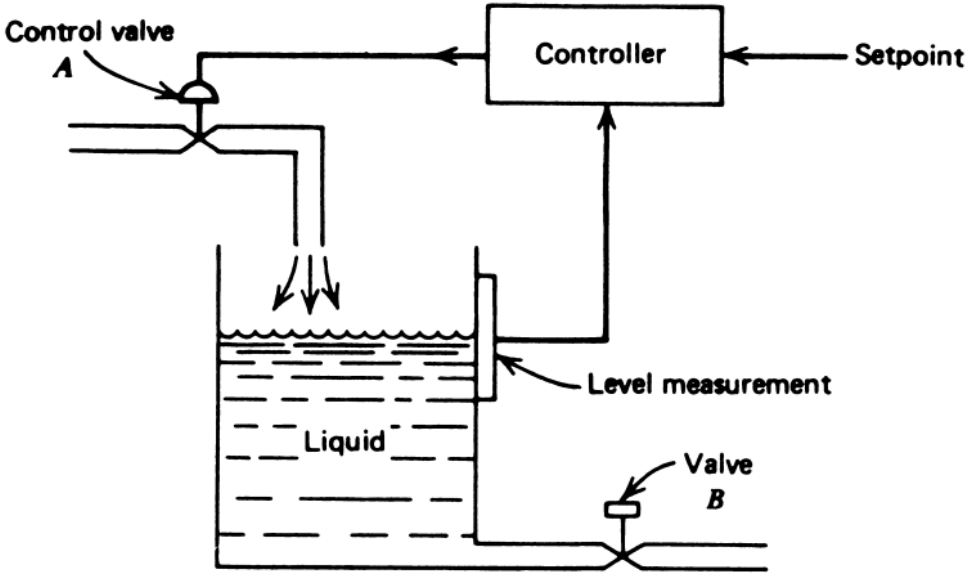




In between load change. Offset Error occurs

Example

Consider the proportional-mode level-control system of Figure Value A is linear, with a flow scale factor of 10 m^3/h per percent controller output. The controller output is nominally 50% with a constant of $K_P = 10\%$ per %. A load change occurs when flow through valve B changes from 500 m^3/h to 600 m^3/h. Calculate the new controller output and offset error.



Certainly, valve A must move to a new position of 600 m^3/h flow or the tank will empty. This can be accomplished by a 60% new controller output because

$$Q_A = \left(\frac{10 \text{ m}^3/\text{h}}{\%}\right)(60\%) = 600 \text{ m}^3/\text{h}$$

as required. Because this is a proportional controller, we have

$$p = K_P e_p + p_0$$

with the nominal condition $p_0 = 50\%$. Thus

$$e_p = \frac{p - p_0}{K_P} = \frac{60 - 50}{10}\%$$

$$e_p = 1\%$$

so a 1% offset error occurred because of the load change.

Applications

- Proportional control generally is used in processes where large load changes are unlikely or with moderate to small process lag times.
- process lag time is small, the proportional band can be made very small (large), which reduces offset error.

Integral Control Mode

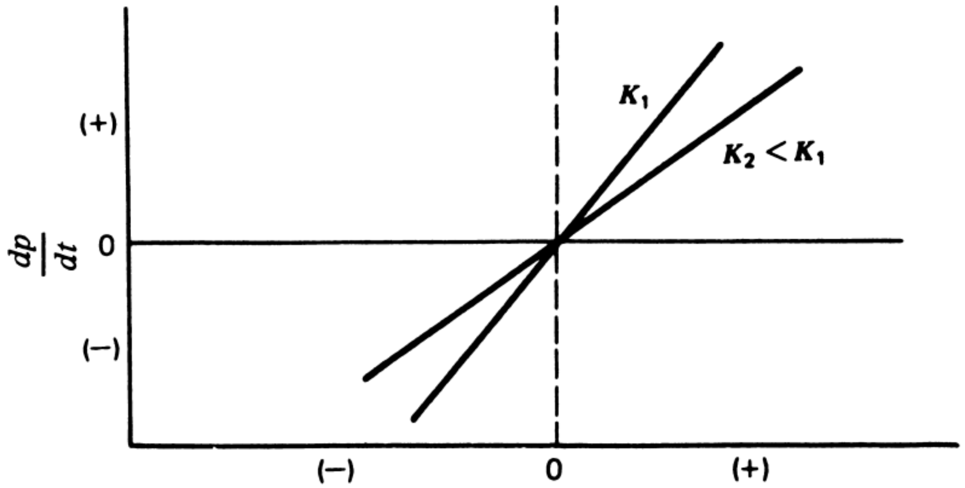
- The offset error of the proportional mode occurs because the controller cannot adapt to changing external conditions—that is, changing loads.
- The error may reduce, but it does not go to zero; in fact, it may become constant. Therefore Integral action is needed.
- Integral action is provided by summing the error overtime, multiplying that sum by a gain, and adding the result to the present controller output.
- if the error becomes positive or negative for an extended period of time, the integral action will begin to accumulate and make changes to the controller output.

$$p(t) = K_I \int_0^t e_P \, dt + p(0)$$

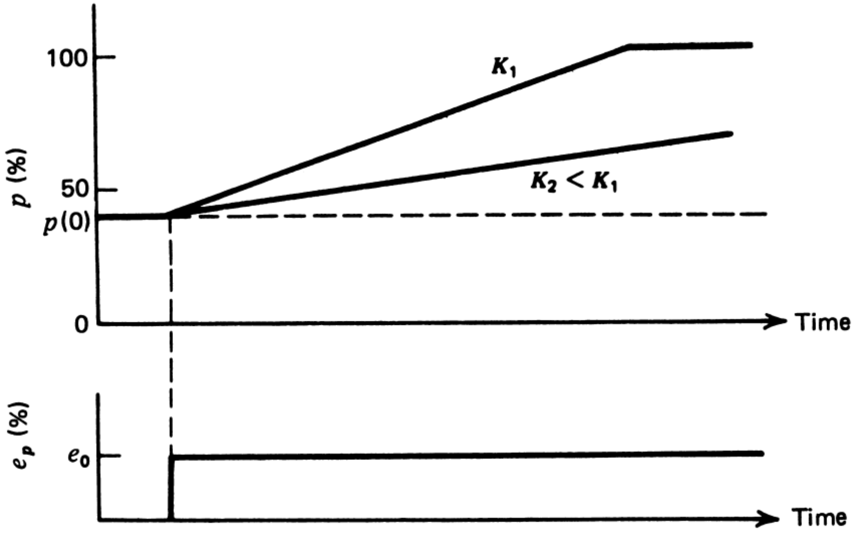
where p(0) is the controller output when the integral action starts. The gain expresses how much controller output in percent is needed for every percent-time accumulation of error. Taking the derivative of Equation

$$\frac{dp}{dt} = K_I e_p$$

This equation shows that when an error occurs, the controller begins to increase (or decrease) its output at a rate that depends upon the size of the error and the gain.
If the error is zero, the controller output is not changed.
If there is positive error, the controller output begins to ramp up at a rate determined by equation.



(a)



(b)

1. If the error is zero, the output stays fixed at a value equal to what it was when the error went to zero.
2. If the error is not zero, the output will begin to increase or decrease at a rate of K_I percent/second for every 1% of error.

Example

An integral controller is used for speed control with a setpoint of 12 rpm within a range of 10 to 15 rpm. The controller output is 22% initially. The constant $K_I = -0.15\%$ controller output per second per percentage error. If the speed jumps to 13.5 rpm, calculate the controller output after 2 s for a constant e_p.

Steps: First find out error from the set point and span equation
Put the value of error in rate of change of controller output equation
Find the controller output with integral equation

Steps: First find out error from the set point and span equation
Put the value of error in rate of change of controller output equation
Find the controller output with integral equation

$$e_p = \frac{r - b}{b_{max} - b_{min}} \times 100$$

$$\frac{dp}{dt} = K_I e_p$$

$$p = K_I \int_0^t e_p \, dt + p(0)$$

but because e_p is constant,

$$p = K_I e_p t + p(0)$$

Solution

$$e_p = \frac{r - b}{b_{\max} - b_{\min}} \times 100$$

$$e_p = \frac{12 - 13.5}{15 - 10} \times 100$$

$$e_p = -30\%$$

$$\frac{dp}{dt} = K_I e_p = (-0.15\ \mathrm{s}^{-1})(-30\%)$$

$$\frac{dp}{dt} = 4.5\%/\mathrm{s}$$

The rate of controller output change

$$p = K_I \int_0^t e_p\,dt + p(0)$$

but because e_p is constant,

$$p = K_I e_p t + p(0)$$

After 2 s, we have

$$p = (0.15)(30\%)(2) + 22$$

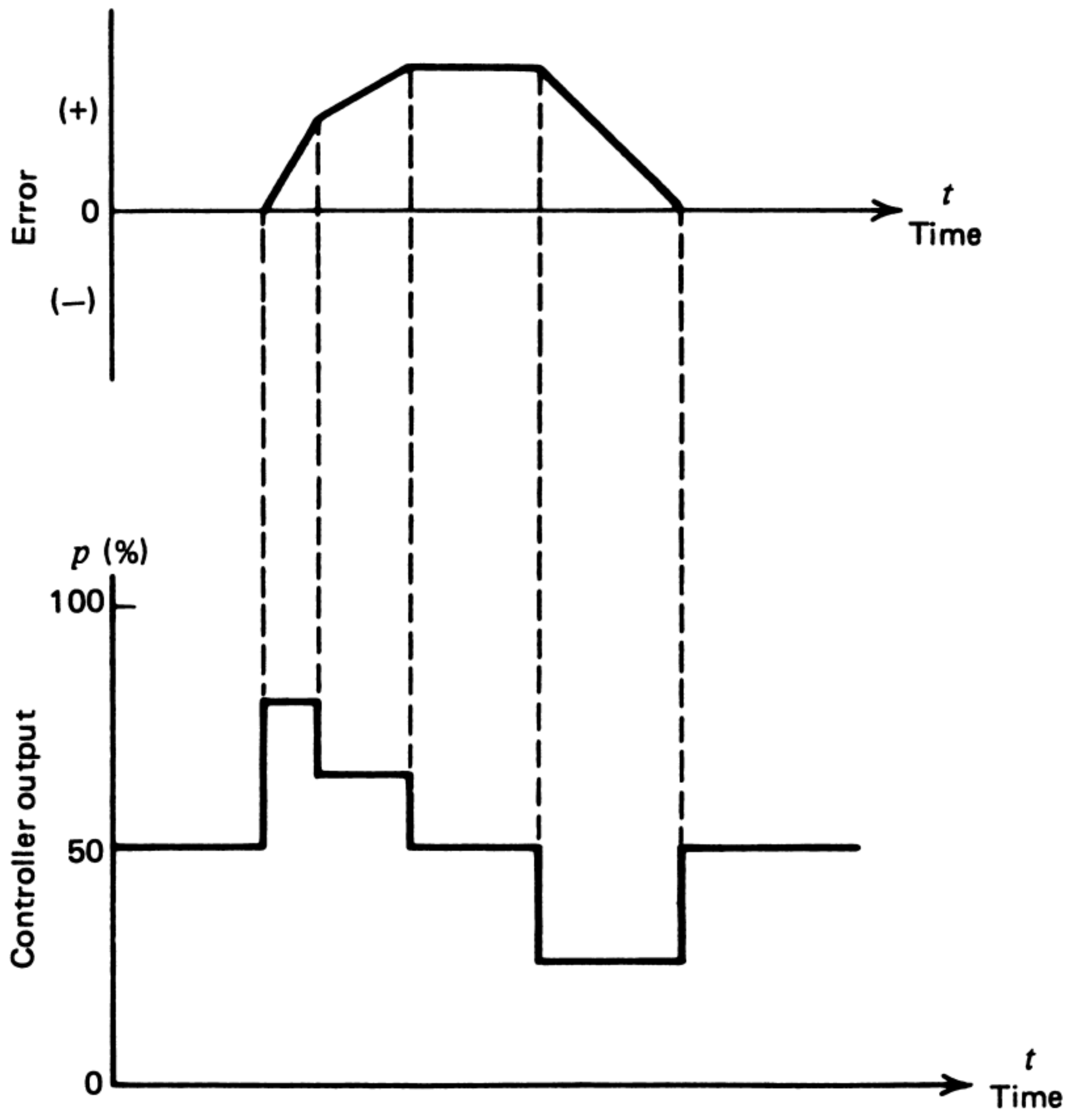
$$p = \mathbf{31\%}$$

Derivative Control Mode

- Derivation controller action responds to the rate at which the error is changing—that is, the derivative of the error.

$$p(t) = K_D \frac{de_p}{dt}$$

where the gain, , tells us by how much percent to change the controller output for every percent-per-second rate of change of error.
Derivative action is not used alone because it provides no output when the error is constant.
Derivative controller action is also called rate action and anticipatory control.



For this example, it is assumed that the controller output with no error or rate of change of error is 50%.

When the error changes very rapidly with a positive slope, the output jumps to a large value, and when the error is not changing, the output returns to 50%.

Finally, when the error is decreasing—that is, has a negative slope—the output discontinuously changes to a lower value.

Characteristics of Derivative Mode

- The derivative mode must be used with great care and usually with a small gain, because a rapid rate of change of error can cause very large, sudden changes of controller output. Such an event can lead to instability.

1. If the error is zero, the mode provides no output.
2. If the error is constant in time, the mode provides no output.
3. If the error is changing in time, the mode contributes an output of K_D percent for every 1%-per-second rate of change of error.
4. For direct action, a positive rate of change of error produces a positive derivative mode output.

Composite Control Modes

- It is common in the complex of industrial processes to find control requirements that do not fit the application norms of any of the discussed controller modes.
- It is both possible and expedient to combine several basic modes, thereby gaining the advantages of each mode.
- In some cases, an added advantage is that the modes tend to eliminate some limitations they individually possess.

Proportional Integral Control

- This is a control mode that results from a combination of the proportional mode and the integral mode.

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

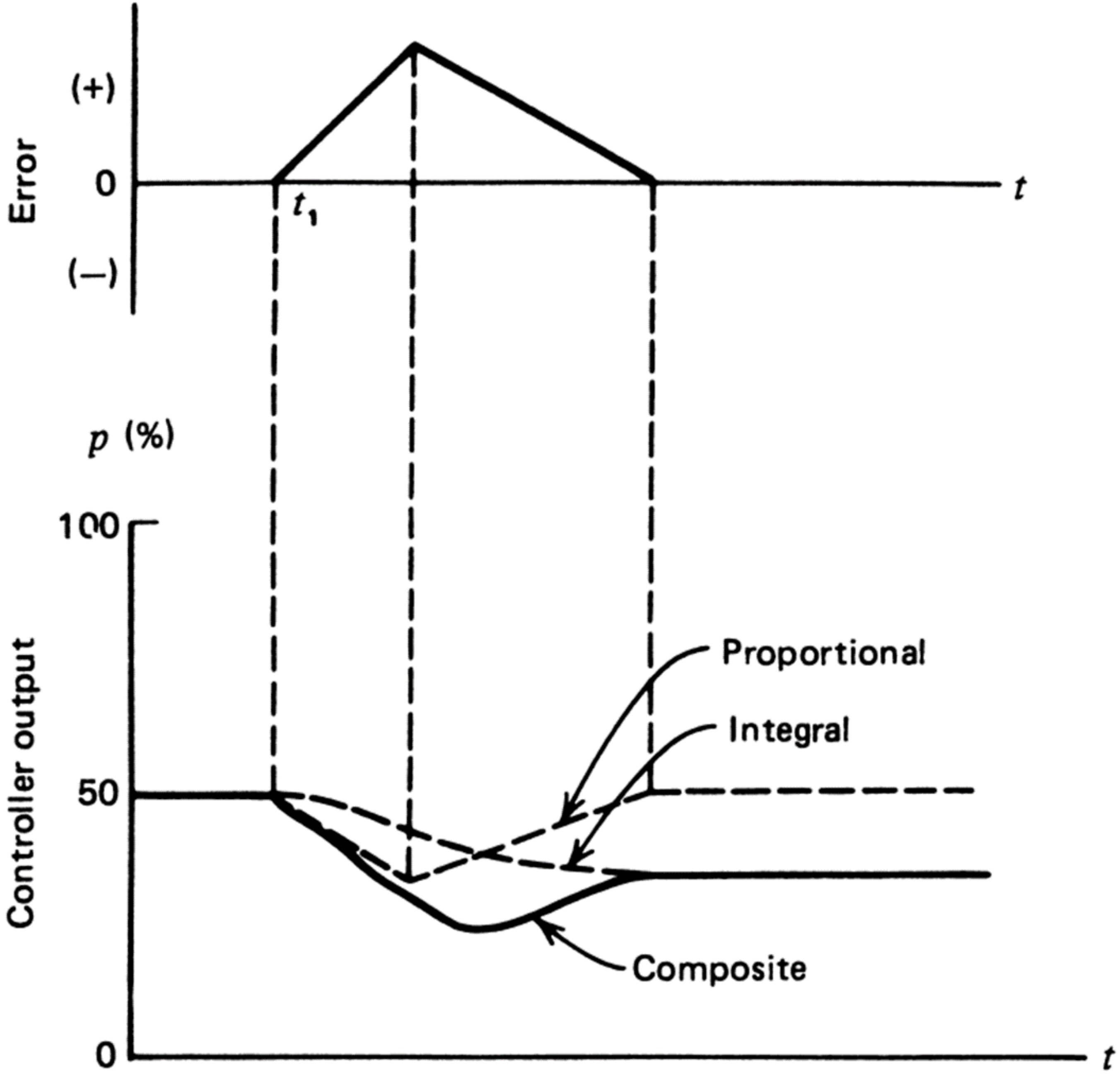
$$p_I(0) = \text{integral term value at } t = 0 \, (\text{initial value})$$

The main advantage of this composite control mode is that the one-to-one correspondence of the proportional mode is available and the integral mode eliminates the inherent offset.

Characteristics of PI Mode

1. When the error is zero, the controller output is fixed at the value that the integral term had when the error went to zero. This output is given by $p_I(0)$ in Equation simply because we chose to define the time at which observation starts as $t = 0$.
2. If the error is not zero, the proportional term contributes a correction, and the integral term begins to increase or decrease the accumulated value [initially, $p_I(0)$], depending on the sign of the error and the direct or reverse action.

The integral term cannot become negative. Thus, it will saturate at zero if the error and action try to drive the area to a net negative value.

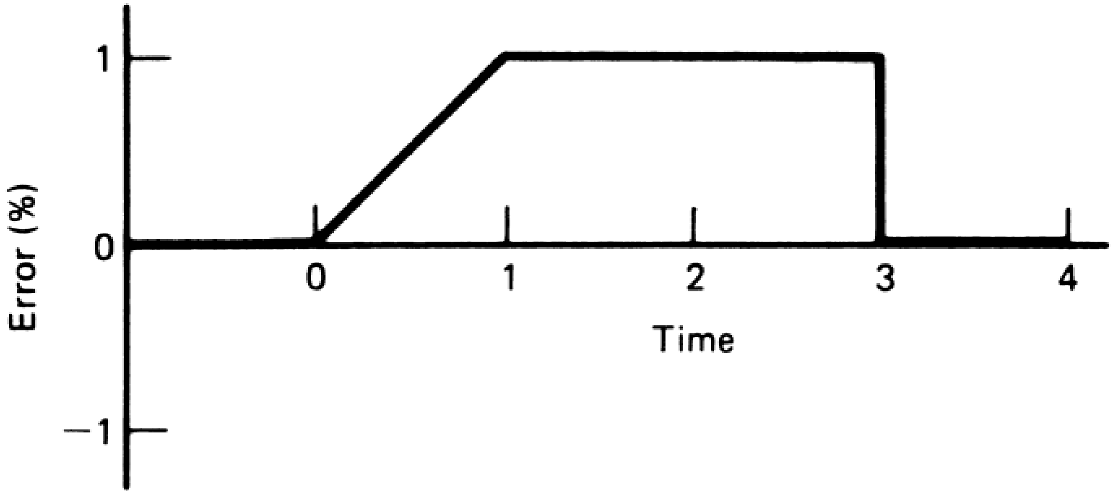


(+)
Error
0
(–)
t_1
t
p (%)
100
50
0
Controller output
Proportional
Integral
Composite
t

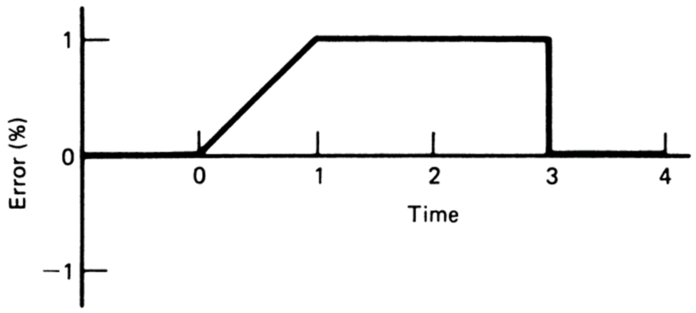
Example

Given the error of Figure, plot a graph of a proportional-integral controller output as a function of time.

$$K_P = 5,\ K_I = 1.0\ \mathrm{s}^{-1},\ \text{and } p_I(0) = 20\%$$



$$p = K_P e_p + K_P K_I \int_0^t e_p\, dt + p_I(0)$$



$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

$$K_P = 5,\ K_I = 1.0\ \mathrm{s}^{-1},\ \text{and}\ p_I(0) = 20\%$$

The error can be expressed in three time regions.

$$0 \le t \le 1 \qquad (t \text{ between 0 and 1 s})$$

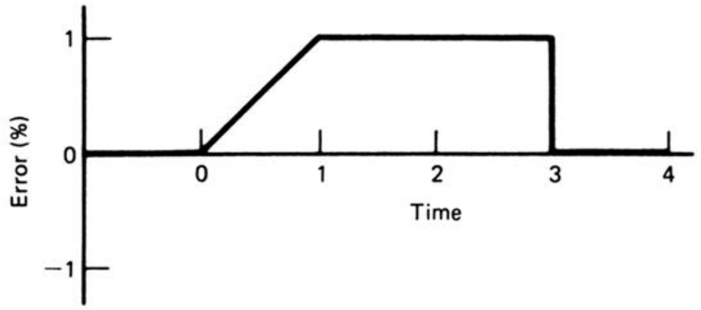
The error rises from 0% to 1% in 1 s. Thus, it is given by $e_p = t$.

$$1 \le t \le 3$$

For this time span, the error is constant and equal to 1%; therefore, it is given by $e_p = 1$.

$$t \ge 3$$

For this time, the error is zero, $e_p = 0$.



$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

$$K_P = 5,\ K_I = 1.0\ \mathrm{s}^{-1},\ \text{and}\ p_I(0) = 20\%$$

$$0 \le t \le 1 \qquad e_p = t$$

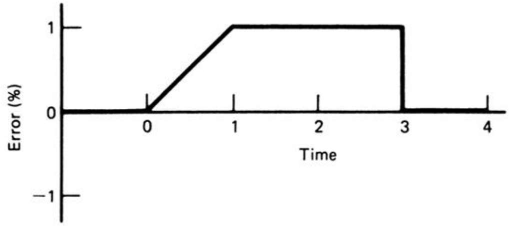
$$p_1 = 5t + 5\int_0^t t\,dt + 20$$

$$p_1 = 5t + 5\left[\frac{t^2}{2}\right]\int_0^t + 20$$

$$p_1 = 5t + 2.5t^2 + 20$$

Put values 0 and 1 and plot the graph.

The curvature will come because of the squared term.



$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0) \qquad K_P = 5,\ K_I = 1.0\ \mathrm{s}^{-1}, \text{ and } p_I(0) = 20\%$$

Remember that only the integral term accumulates values, so in finding the output at 1 s, the contribution of the proportional term, 5t, is not included. Therefore, the starting value for the next time span is given by $p_1(1) = 2.5t^2 + 20 = 22.5\%.$

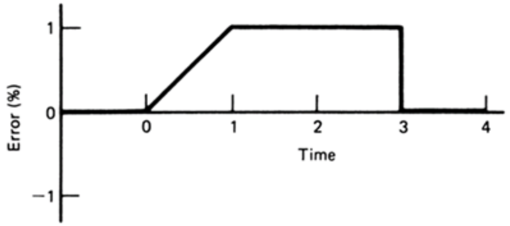
$$1 \leq t \leq 3 \qquad e_p = 1$$

$$p_2 = 5 + 5 \int_1^t 1 \ dt + 22.5$$

The integral term accumulation from 0 to 1 s forms the initial condition for this new equation.

$$p_2 = 5 + 5[t]_1^t + 22.5$$
$$p_2 = 5 + 5(t - 1) + 22.5$$

This function is plotted in Figure $\boxed{}$ from 1 to 3 s. At the end of this period, the integral term has accumulated a value of $p_2(3) = 32.5\%$.



$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

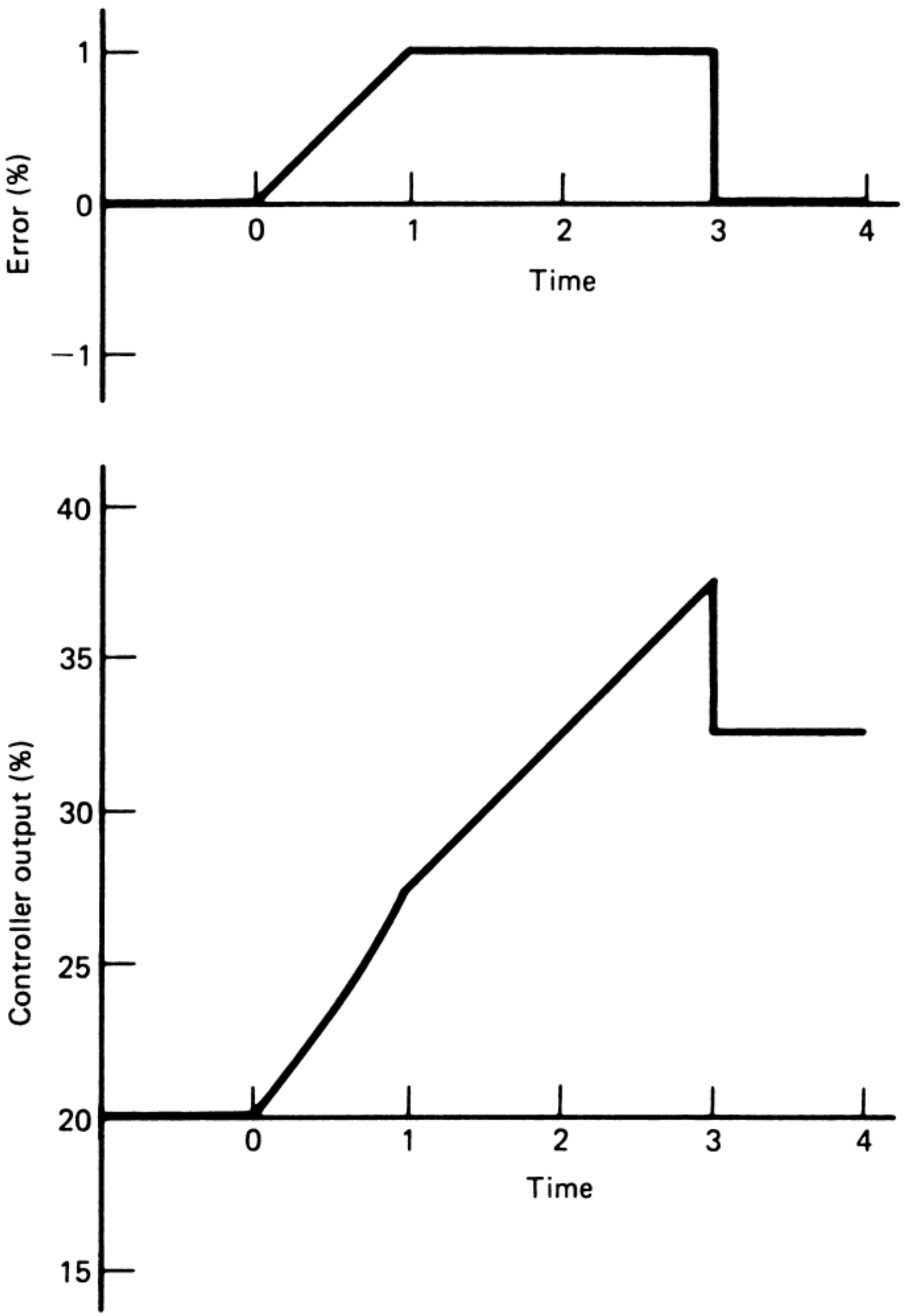
$$K_P = 5, K_I = 1.0 \text{ s}^{-1}, \text{and } p_I(0) = 20\%$$

$$t \geq 3 \qquad e_p = 0$$

$$p_3 = 5[0] + 5 \int_3^t 0 \; dt + 32.5$$

$$p_3 = 32.5$$

The output will stay constant at 32.5% from 3 s.
The sudden drop of 5% is due to the sudden change of error from 1% to 0% at
t = 3s.



T1 =27.5%

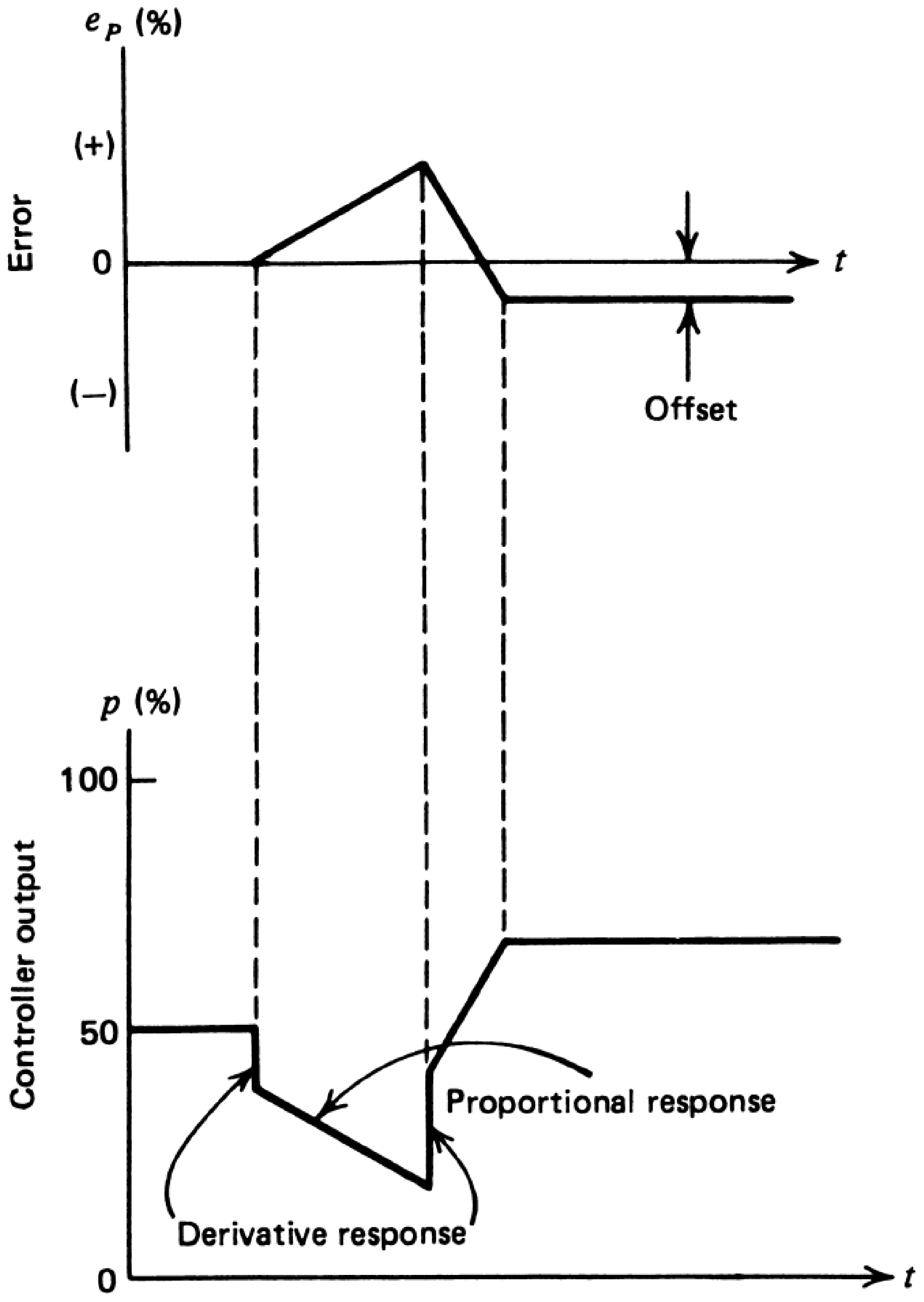
T2 =37.5%

T3 =32.5%

Proportional Derivative Control Mode (PD)

- Serial (cascaded) use of the proportional and derivative modes.
- This system cannot eliminate the offset of proportional controllers.
- It can, however, handle fast process load changes as long as the load change offset error is acceptable.

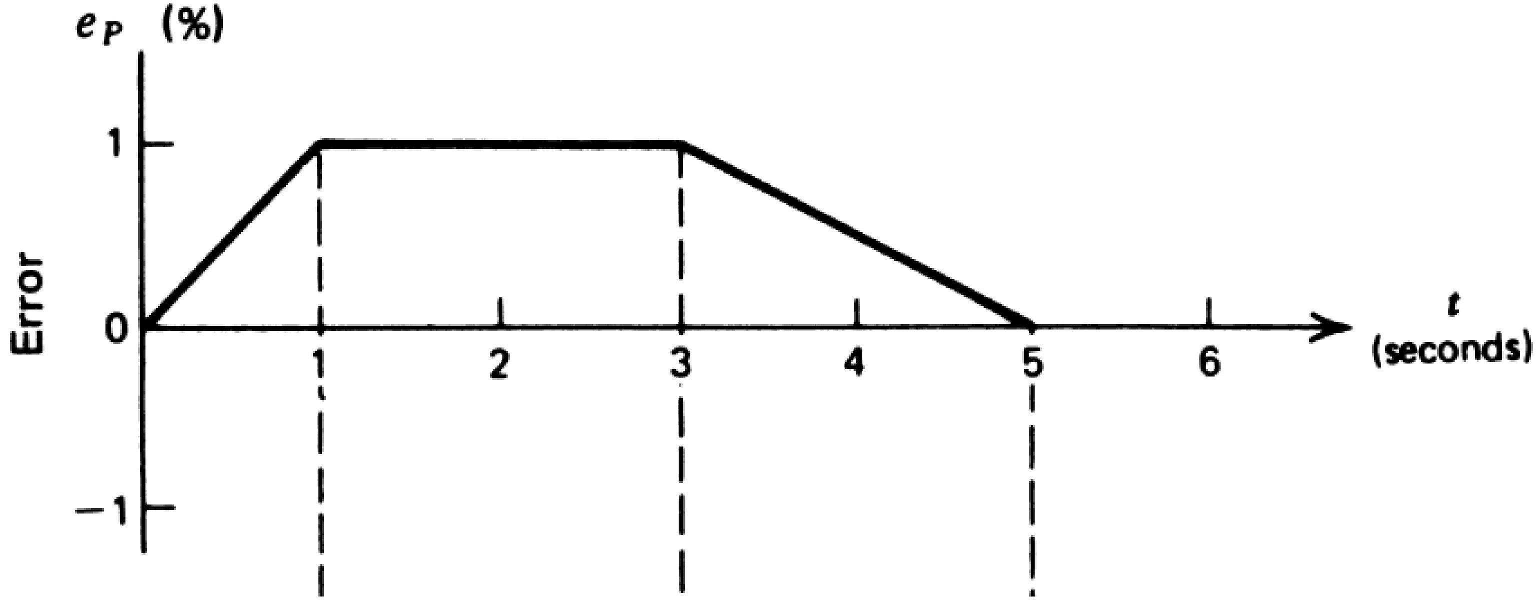
$$p = K_P e_p + K_P K_D \frac{de_p}{dt} + p_0$$



e_P (%)
Error
(+)
0
(−)
t
Offset
p (%)
Controller output
100
50
0
t
Proportional response
Derivative response

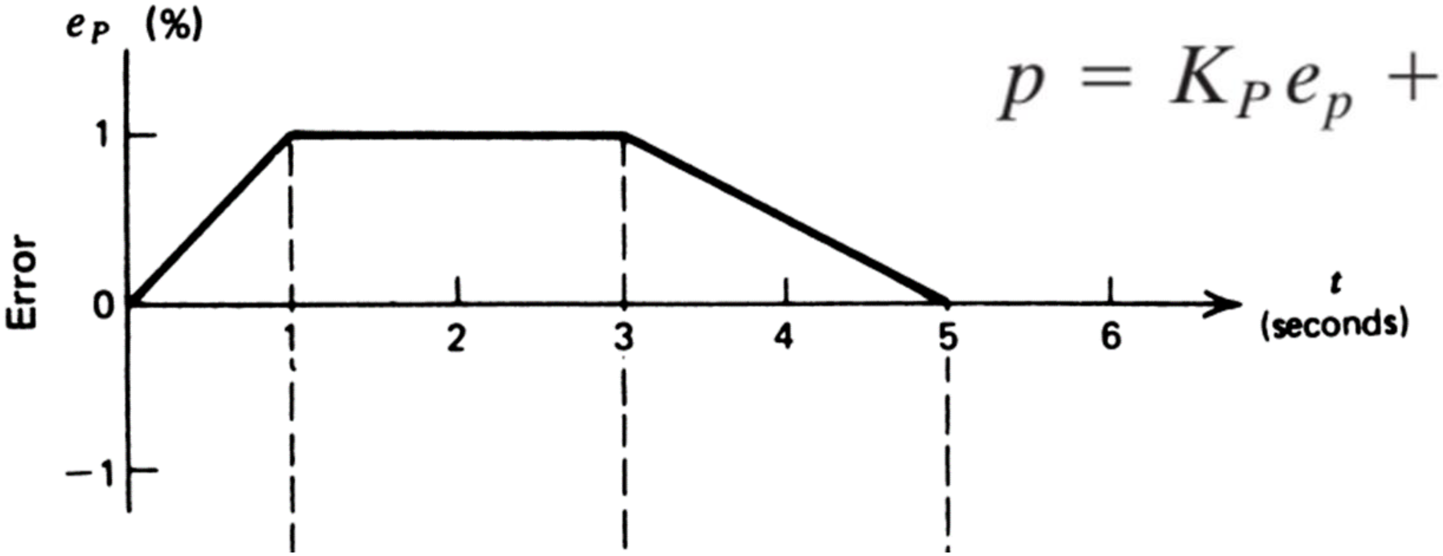
Example

Suppose the error, as shown in Figure, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $P_0 = 20\%$. Draw a graph of the resulting controller output.



Solution

Suppose the error, as shown in Figure, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $P_0 = 20\%$. Draw a graph of the resulting controller output.

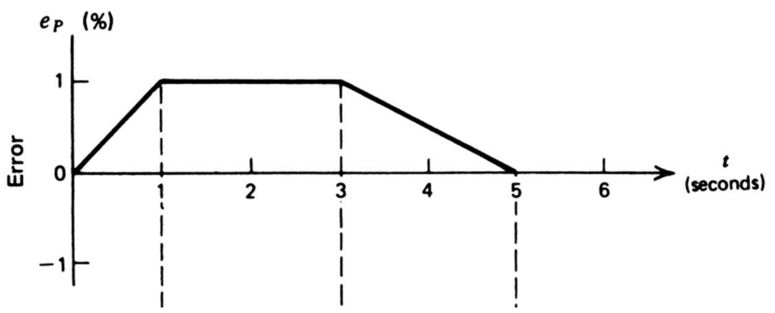


$$p = K_P e_p + K_P K_D \frac{de_p}{dt} + p_0$$

0–1 s $\quad e_p = t\%$

1–3 s $\quad e_p = 1\%$

3–5 s $\quad e_p = -\frac{1}{2}t + 2.5\%$



$$p = K_P\, e_p + K_D K_P \frac{de_P}{dt} + p_0$$

over the two spans of the error. In the time of 0 to 1 s where $e_p = at,$ we have

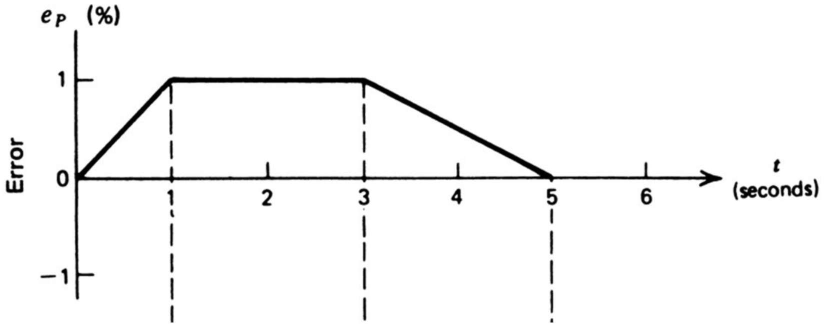
$$p_1 = K_P at + K_D K_P a + p_0$$

or, because $a = 1\%/\text{s}$,

$$p_1 = 5t + 2.5 + 20$$

Note the instantaneous change of 2.5% produced by this error. In the span from 1 to 3 s, we have

$$p_2 = 5 + 20 = 25$$



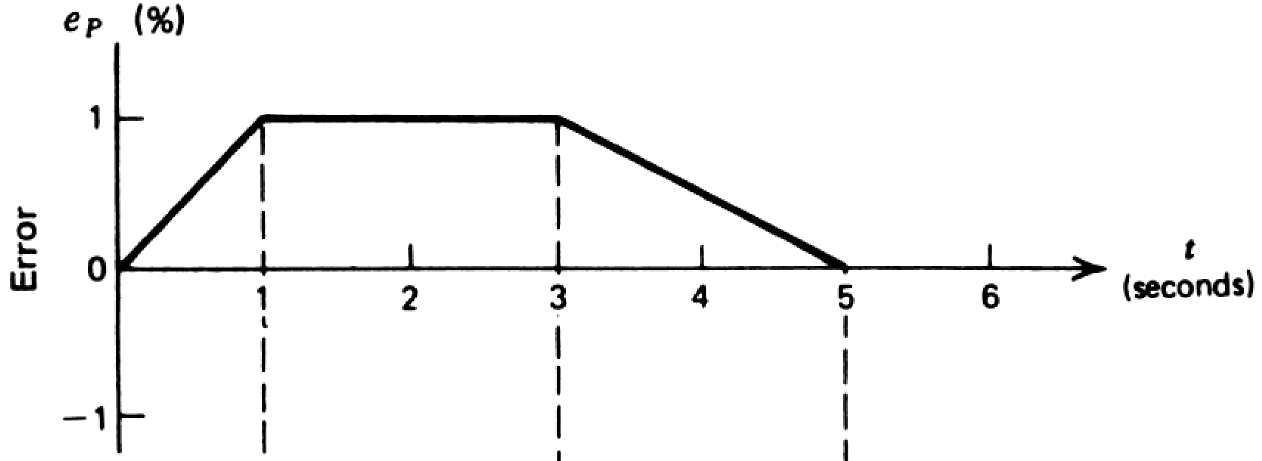
Suppose the error, as shown in Figure, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $P_0 = 20\%$. Draw a graph of the resulting controller output.

The span from 3 to 5 s has an error of $e_p = -0.5t + 2.5$, so that we get for 3 to 5 s

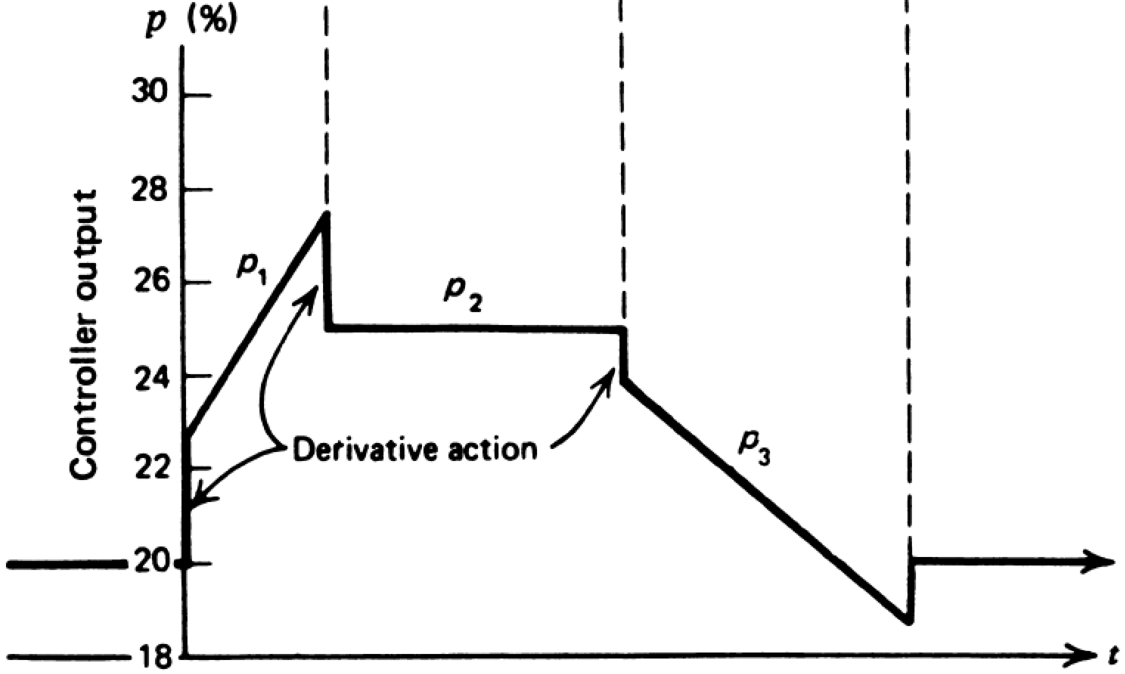
$$p_3 = -2.5t + 12.5 - 12.5 + 20$$

or

$$p_3 = -2.5t + 31.25$$



a) Error input



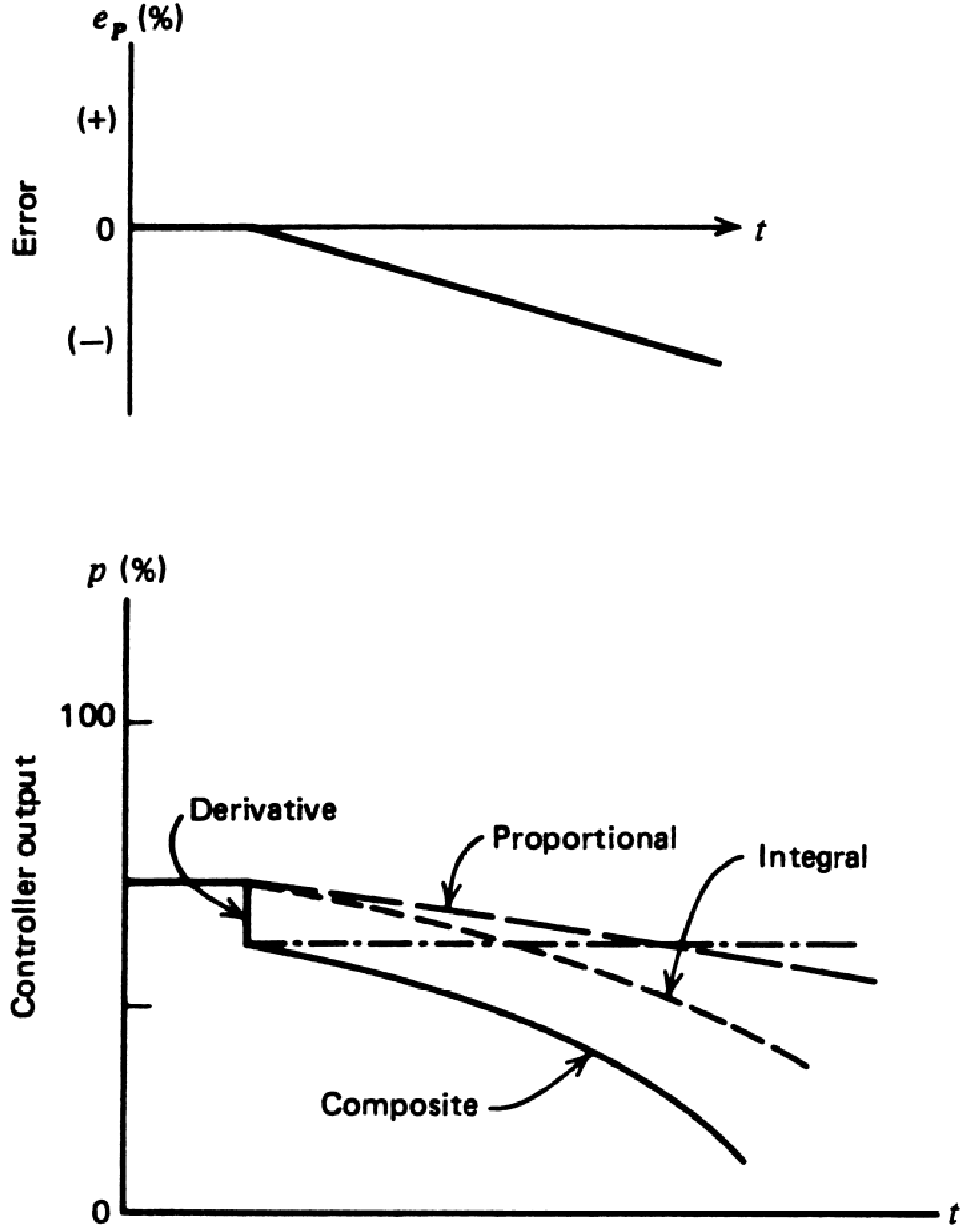
b) Controller output

Three Mode Controller (PID)

- One of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes.

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

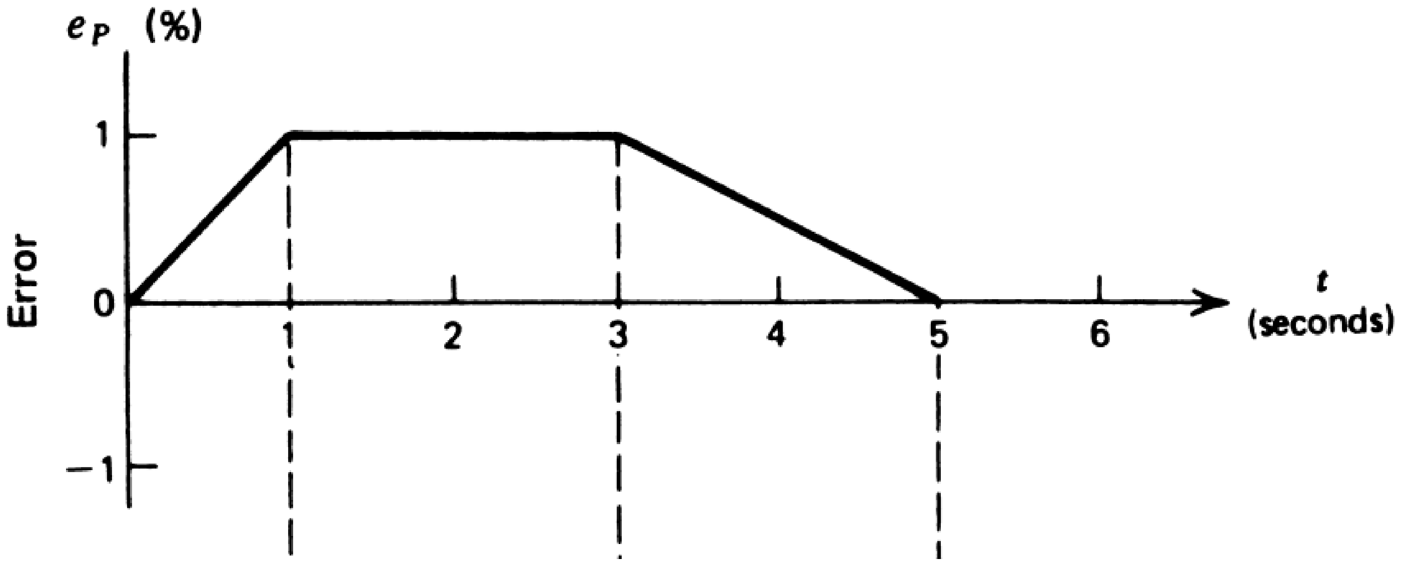
This mode eliminates the offset of the proportional mode and still provides fast response.



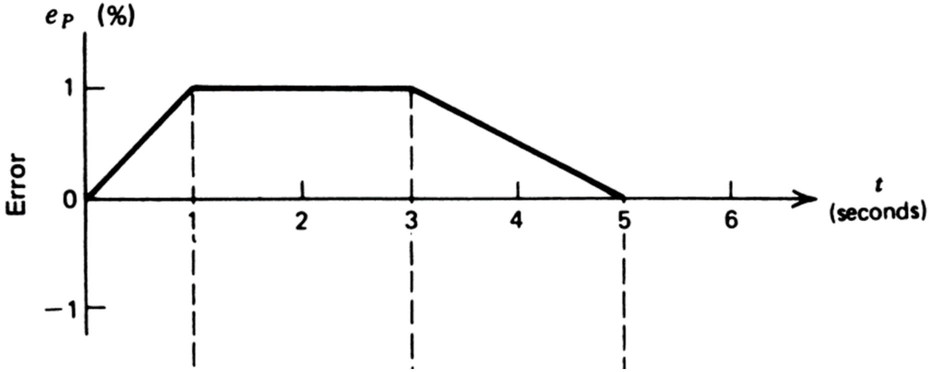
e_p (%)
(+)
Error
0
(−)
t
p (%)
100
Controller output
Derivative
Proportional
Integral
Composite
0
t

Example

Let us combine everything and see how the error of Figure produces an output in the three-mode controller with $K_P = 5$, $K_I = 0.7\ \mathrm{s}^{-1}$, $K_D = 0.5$ s, and $p_I(0) = 20\%$. Draw a plot of the controller output.



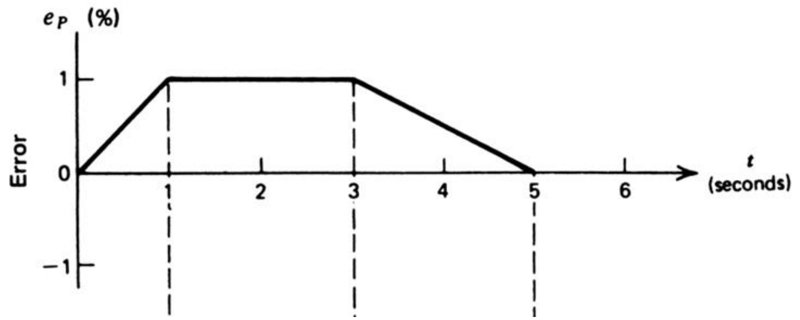
Solution



$$
\begin{array}{ll}
0\text{–}1 \text{ s} & e_p = t\% \\
1\text{–}3 \text{ s} & e_p = 1\% \\
3\text{–}5 \text{ s} & e_p = -\dfrac{1}{2} t + 2.5\%
\end{array}
$$

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

Let us combine everything and see how the error of Figure 22a produces an output in the three-mode controller with $K_P = 5$, $K_I = 0.7\ \text{s}^{-1}$, $K_D = 0.5$ s, and $p_I(0) = 20\%$. Draw a plot of the controller output.

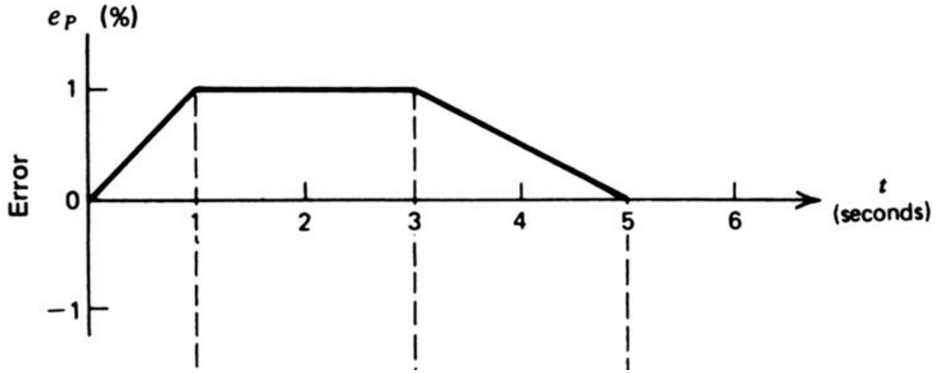


We must apply each of these spans to the three-mode equation for controller output:

$$p = K_P e_p + K_P K_I \int_0^t e_p\, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

or

$$p = 5e_p + 3.5 \int_0^t e_p\, dt + 2.5 \frac{de_p}{dt} + 20$$



$$
\begin{array}{ll}
0\text{–}1\ \text{s} & e_p = t\% \\
1\text{–}3\ \text{s} & e_p = 1\% \\
3\text{–}5\ \text{s} & e_p = -\dfrac{1}{2}t + 2.5\%
\end{array}
$$

From 0 to 1 s, we have

$$p_1 = 5t + 3.5\int_0^t t\ dt + 2.5 + 20$$

or

$$p_1 = 5t + 1.75t^2 + 22.5$$

Plot this by putting values 0 and 1. At the end of 1 s, the integral term has accumulated to PI(1) = 21.75%.

from 1 to 3 s, we have

$$p_2 = 5 + 3.5 \int_1^t (1)\, dt + 21.75$$

$$p_2 = 3.5(t - 1) + 26.75$$

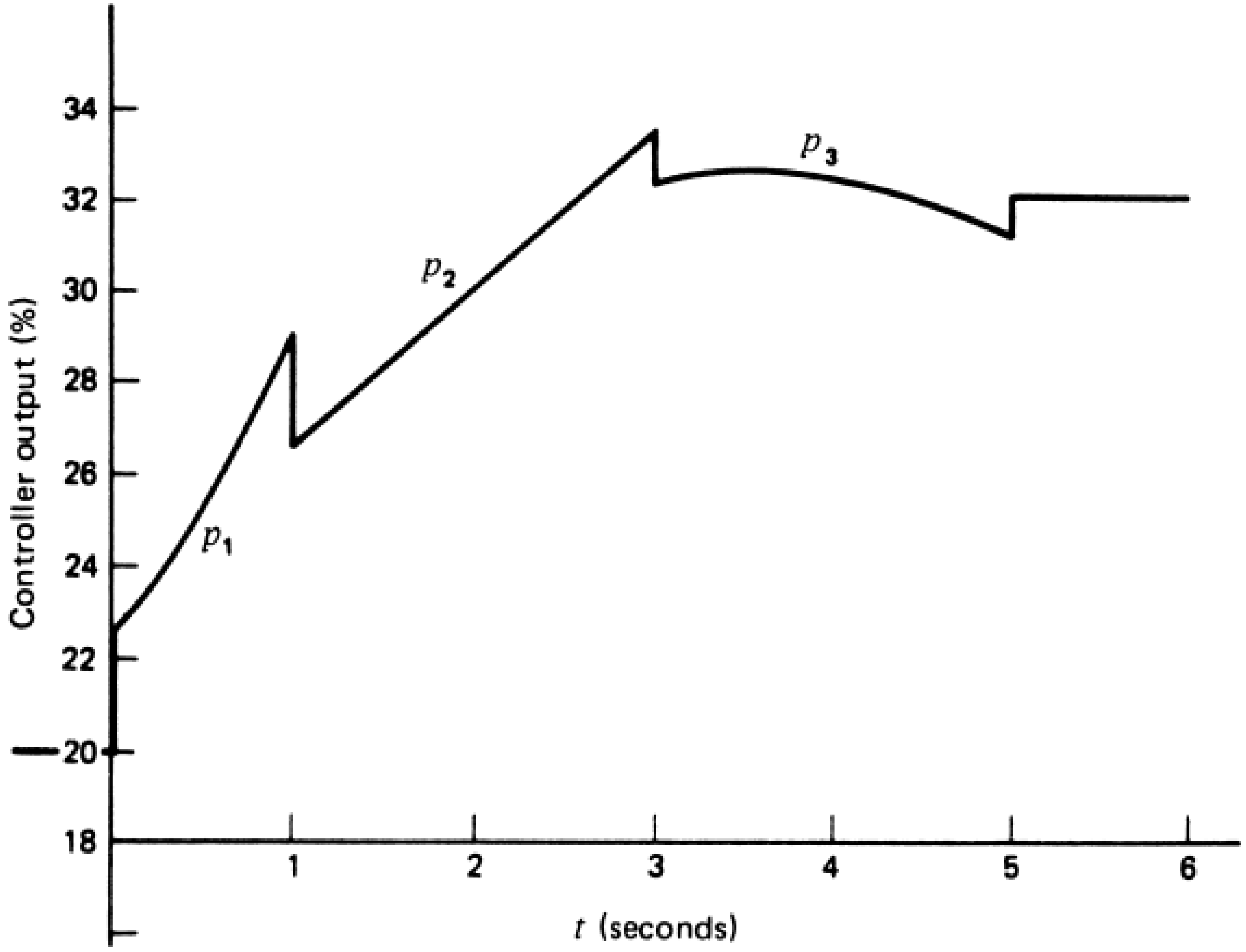
At the end of 3 s, the integral term has accumulated to a value of PI(3) = 28.75%

from 3 to 5 s, we have

$$p_3 = 5\left(-\frac{1}{2}t + 2.5\right) + 3.5\int_3^t \left(-\frac{1}{2}t + 2.5\right)dt - \frac{2.5}{2} + 28.75$$

$$p_3 = -0.875t^2 + 6.25t + 21.625$$

After 5 s, the error is zero. Therefore, the output will simply be the accumulated integral response providing a constant output of PI = 32.25%



Controller output (%)
34
32
30
28
26
24
22
20
18
1
2
3
4
5
6
t (seconds)
p_1
p_2
p_3