Relational Calculus

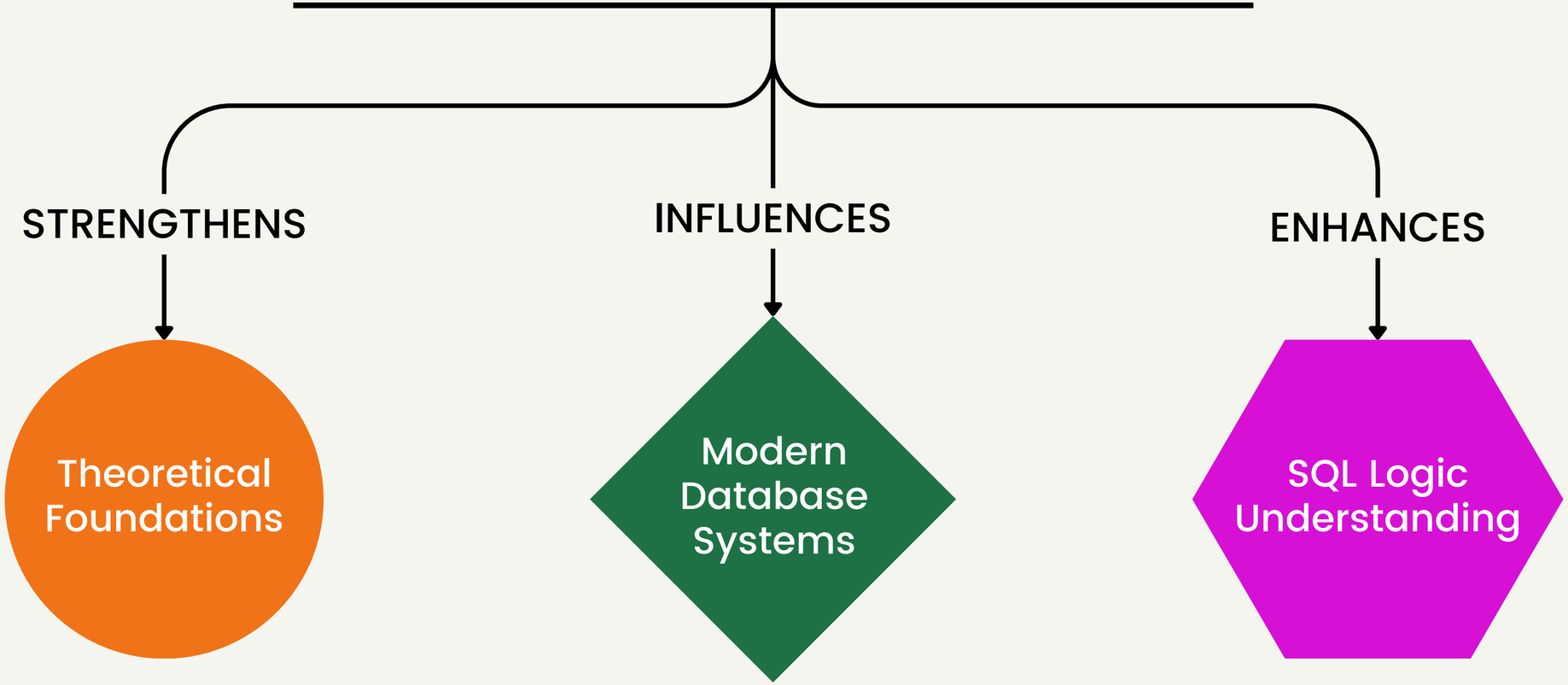
in Database Management Systems

Presented by
Om Gupta, Vikash Kumar, Aditya Raj,
Parkhi Jain, Adarsh Sharma & Gurnam Singh

BOOK REFERRED:

Elmasri, R. (2008).
Fundamentals of Database Systems.
Pearson Education India.
6th Edition

Why we need Relational Calculus ?

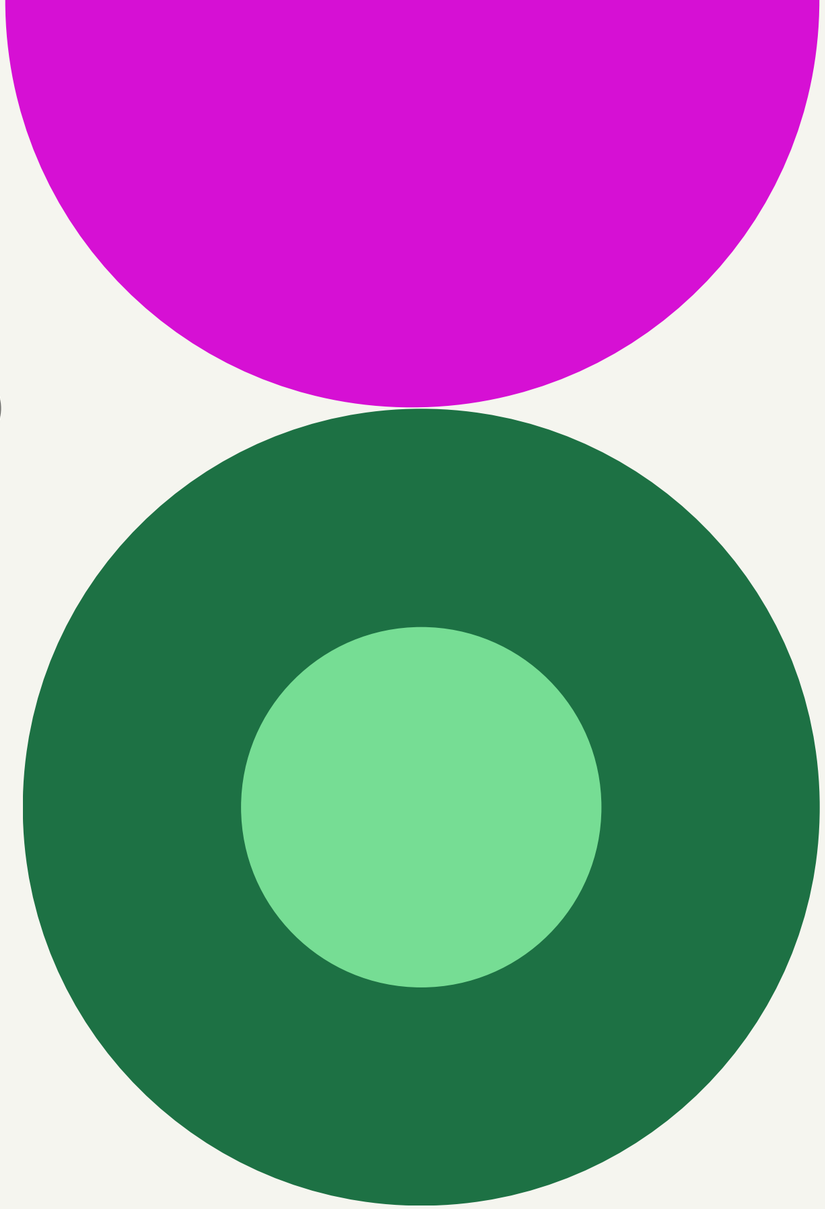


What is Relational Calculus?

It is a **Non-Procedural Query Language.**

We specify **what** we want rather than **how** to retrieve it. Hence, also called **declarative language**.

We indirectly use it to query a database, that is, **retrieve** data from a database.

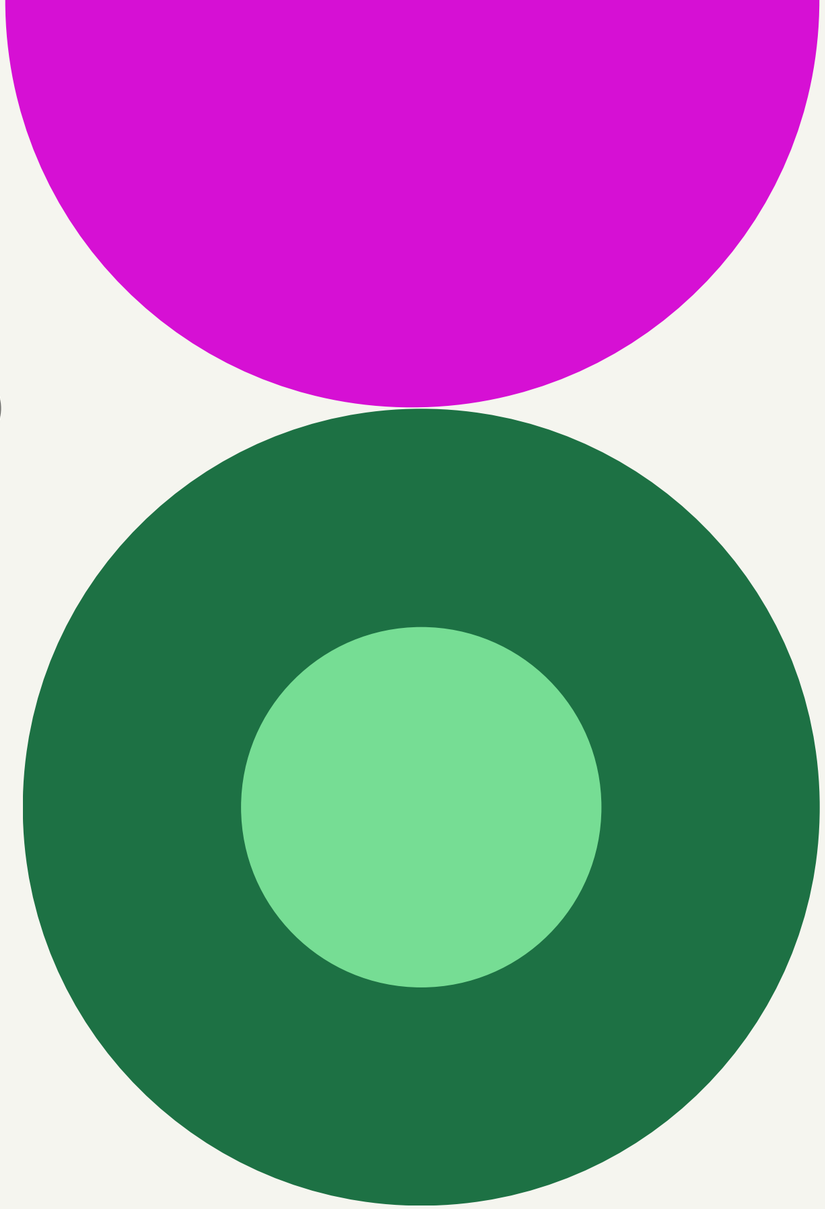


What is Relational Calculus?

It is the "**Grammar**" of databases.

Just like natural languages have different ways to form sentences, databases have two forms of Relational Calculus:

- **Tuple Relational Calculus (TRC):** Describes what tuples (rows) satisfy a given condition.
- **Domain Relational Calculus (DRC):** Works with domain values rather than entire tuples.



What is Relational Calculus?

It is based on **Predicate Calculus**.

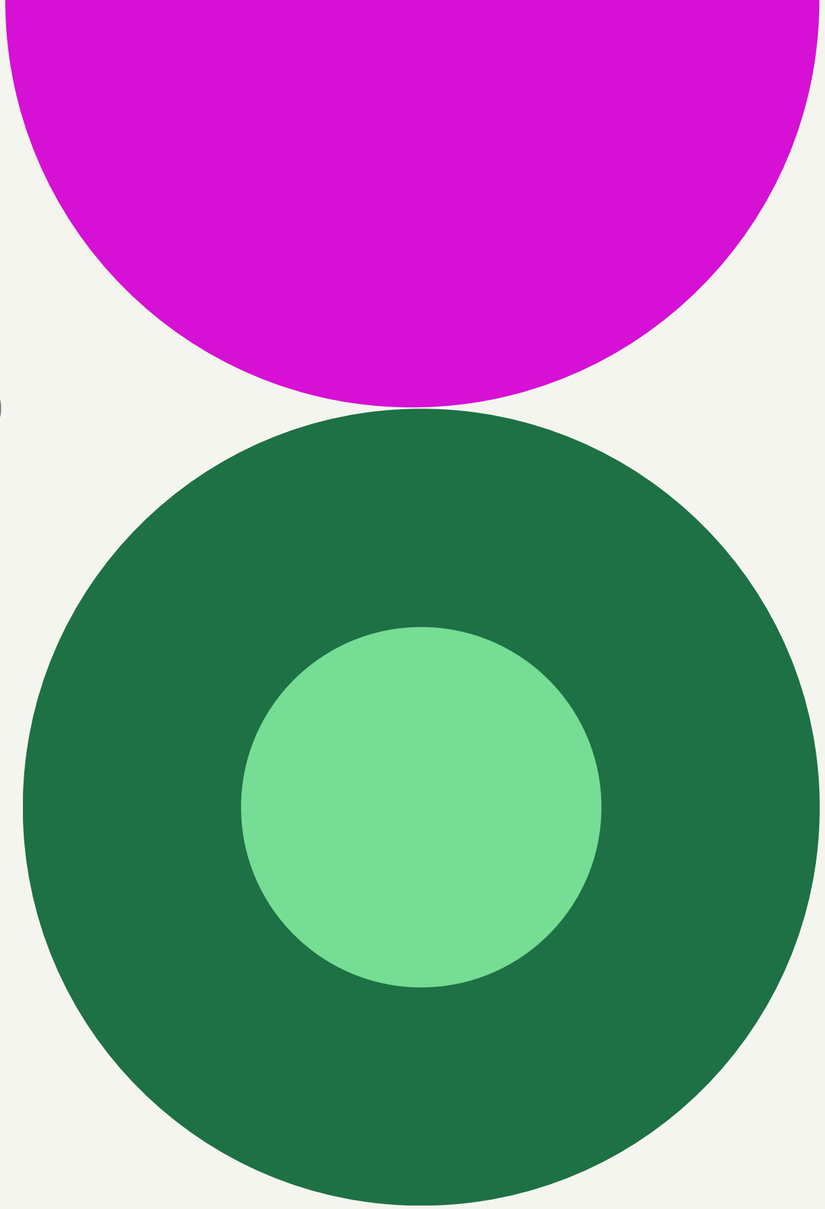
- Relational calculus uses **predicates** and **quantifiers** to express conditions on data.
- Queries are written as logical statements, specifying what data to retrieve rather than how to retrieve it.

Example query:

$$\{ t \mid t \in Student \land t.Age > 20 \}$$

Result
(Tuple variable 't')

Predicate
(Condition used to fetch tuple 't')



Relational...

Calculus V/S Algebra

- **Non-Procedural** Query Language
- Specifies ***what*** data to retrieve (without specifying steps)
- Uses **logical predicates** in order to define conditions
- Query defined by **logical formulas** (predicate calculus)
- More **abstract**, closer to first-order logic
- SQL query **formulation** (internally) is closer to relational calculus

- **Procedural** Query Language
- Specifies ***how*** to retrieve data (step-by-step operations)
- Uses **operations** like Selection, Projection, Join, Union, Difference, etc
- Query defined by a **sequence of operations** on relations
- More **intuitive**, closer to set operations.
- SQL **execution** (internally) is closer to relational algebra

FIND NAMES OF ALL STUDENTS ENROLLED IN COURSE C1

$$\{ s.\text{name} \mid s \in \text{Student} \wedge \exists e \, (e \in \text{Enrolled} \wedge s.\text{sid} = e.\text{sid} \wedge e.\text{cid} =' C1') \}$$

$$\pi_{\text{name}} \left(\sigma_{\text{Enrolled.cid}='C1'} (\text{Student} \bowtie \text{Enrolled}) \right)$$

Basics of Predicate Calculus

used in the following presentation

COMPARISON OPERATORS

$<, \leq, =, \neq, >, \geq$

CONNECTIVES

and ($\wedge$), or ($\vee$), not ($\neg$)

IMPLICATION

$P1 \Rightarrow P2$

QUANTIFIERS

Universal: $\forall\ t \in r\ (Q(t))$

Existential: $\exists\ t \in r\ (Q(t))$

Tuple Relational Calculus (TRC)

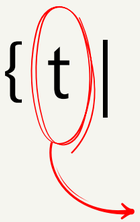
Selecting **tuples** in a relation that satisfy a given condition (or predicate).

- The **result** of the query can consist of one or more tuples
- Basic **syntax**:

- **Result:** Set of tuples '**t**' such that predicate '**P**' is true for '**t**'.
- **Notations:**
  - `t:` Tuple Variable
  - `t.A:` Value of `t` on attribute `A`
  - `t ∈ r:` Denotes that tuple `t` is in relation `r`
  - `P:` predicate i.e, the condition on tuples

NOTE

Variables with quantifiers are **bound** variables, all others are free.



DATABASE

used in the following presentation

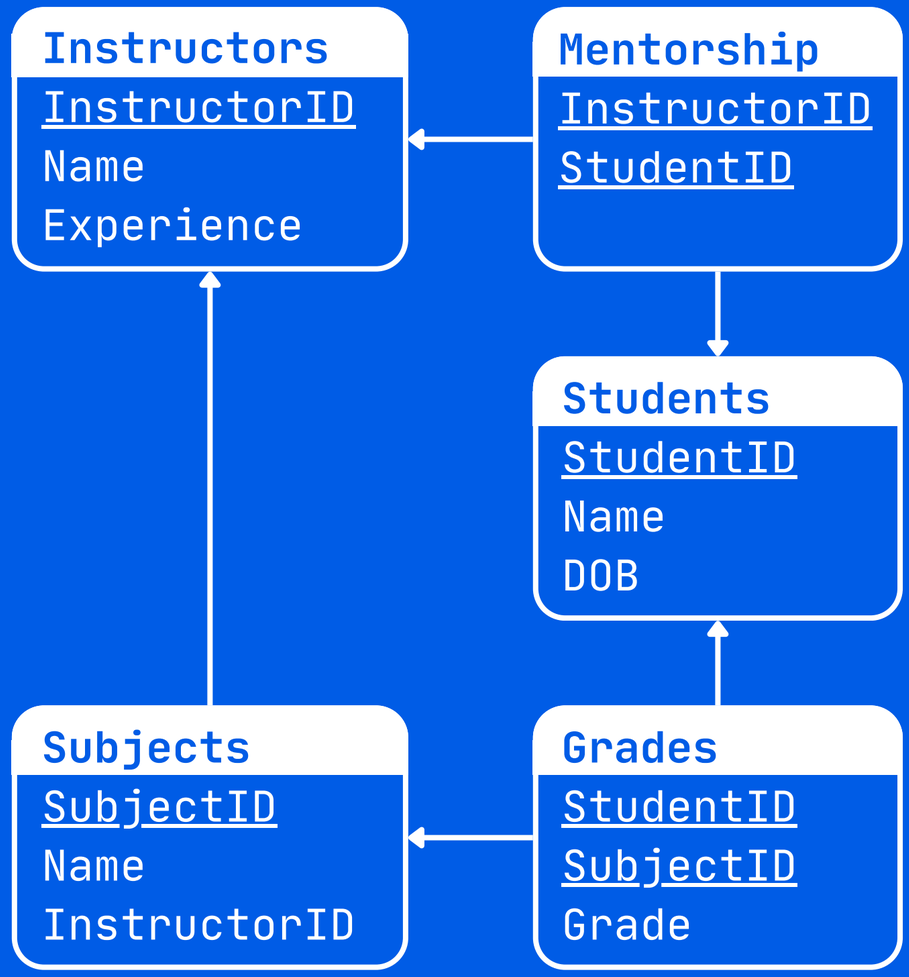
Students: (StudentID, Name, DOB)

Instructors: (InstructorID, Name, Experience)

Subjects: (SubjectID, Name, InstructorID)

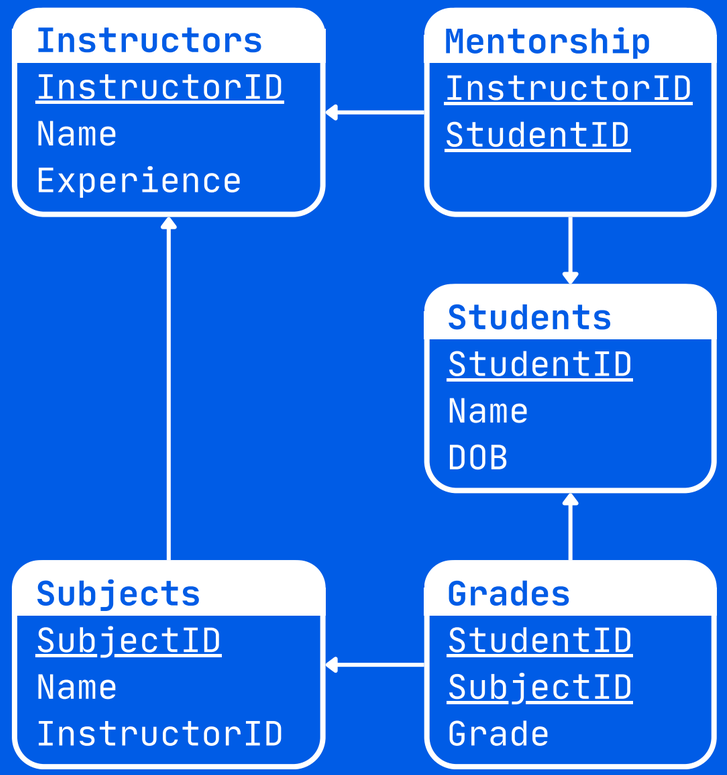
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)



TRC Queries

1 Retrieve details of all the students from the Students table.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

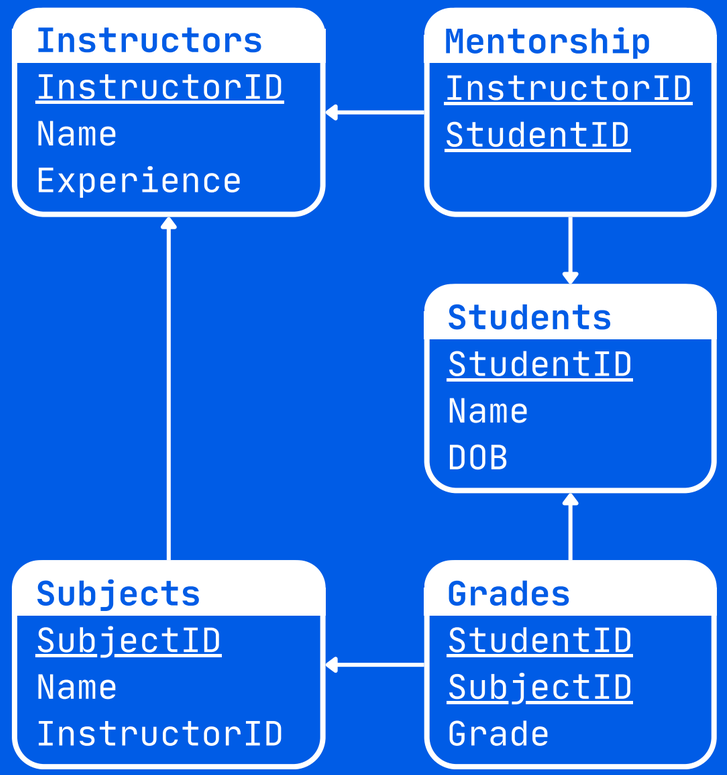
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

1 Retrieve details of all the students from the Students table.

$$\{t \mid t \in Students\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

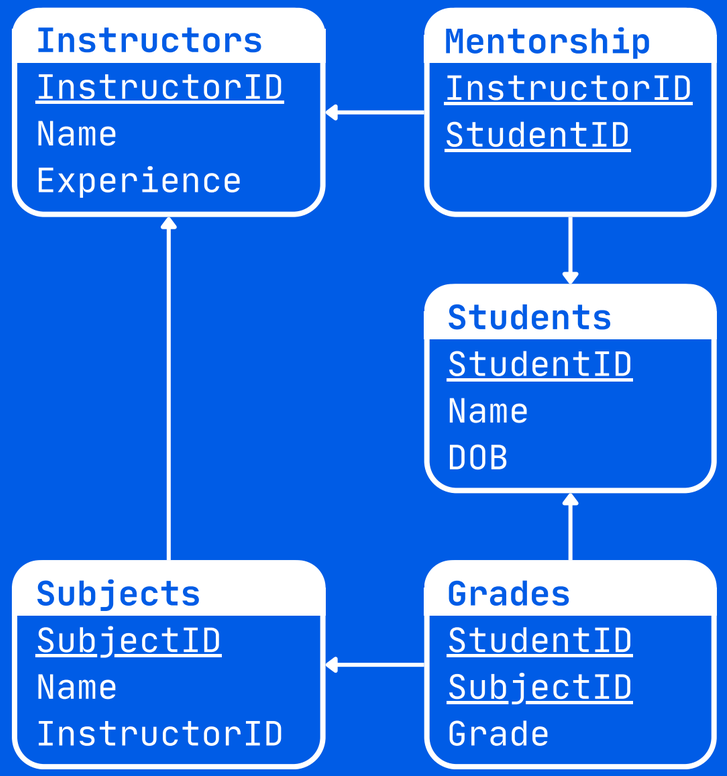
Mentorship: (InstructorID, StudentID)

TRC Queries

1 Retrieve details of all the students from the Students table.

$$\{t \mid t \in Students\}$$

2 Retrieve the names and DOB of all the students.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

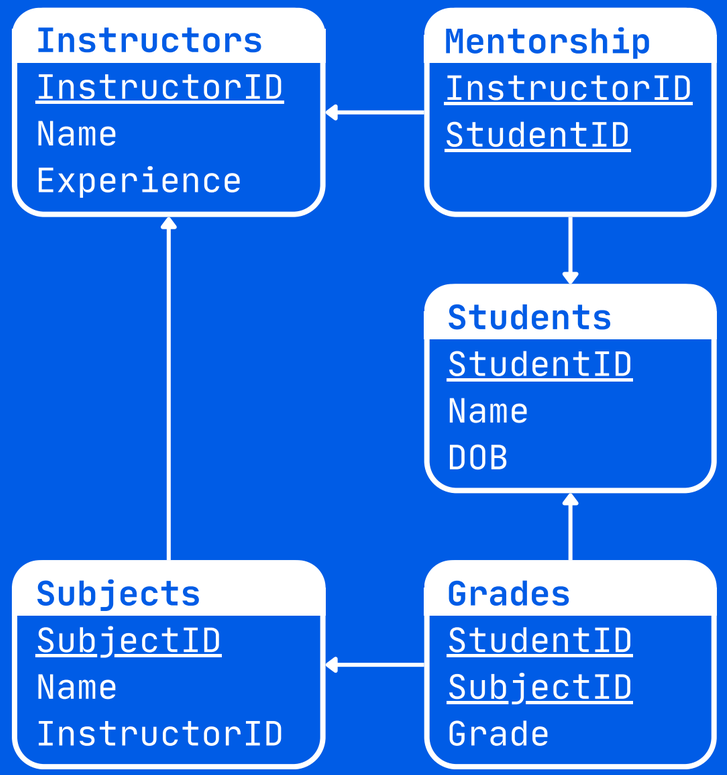
TRC Queries

1 Retrieve details of all the students from the Students table.

$$\{t \mid t \in Students\}$$

2 Retrieve the names and DOB of all the students.

$$\{t.Name, t.DOB \mid t \in Students\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

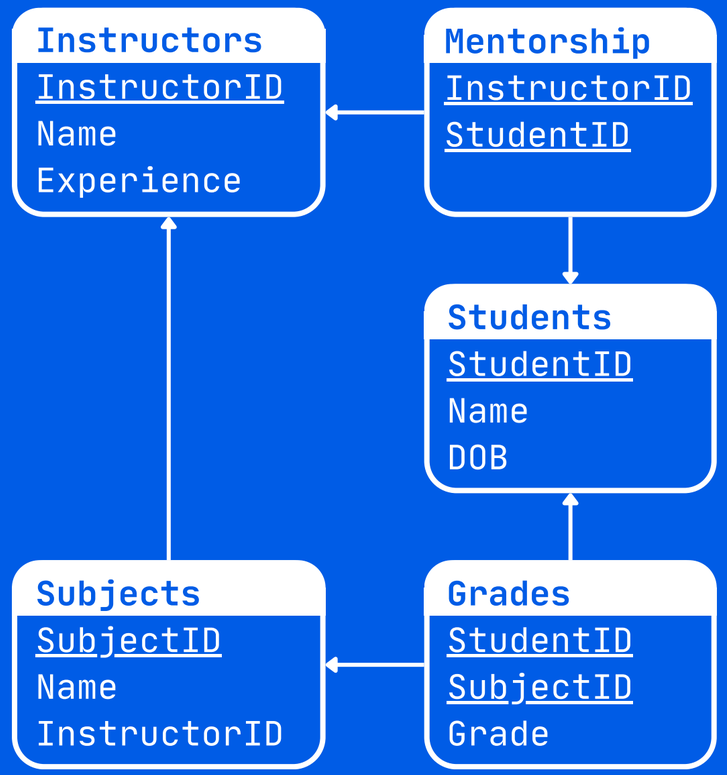
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

3 Find StudentID of students who scored grade “A”.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

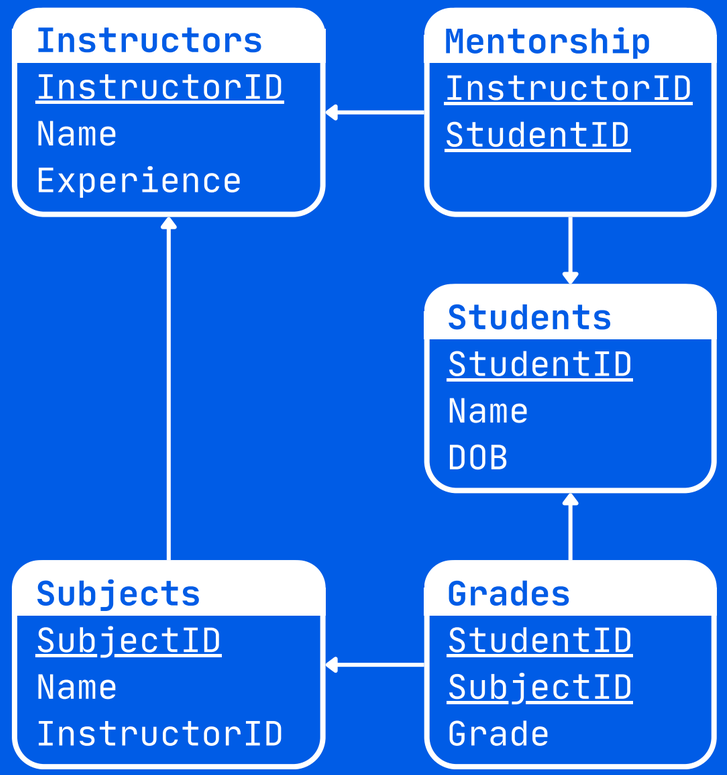
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

3 Find StudentID of students who scored grade “A”.

$$\{t.StudentID \mid t \in Grades \land t.Grade = \text{“A”}\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

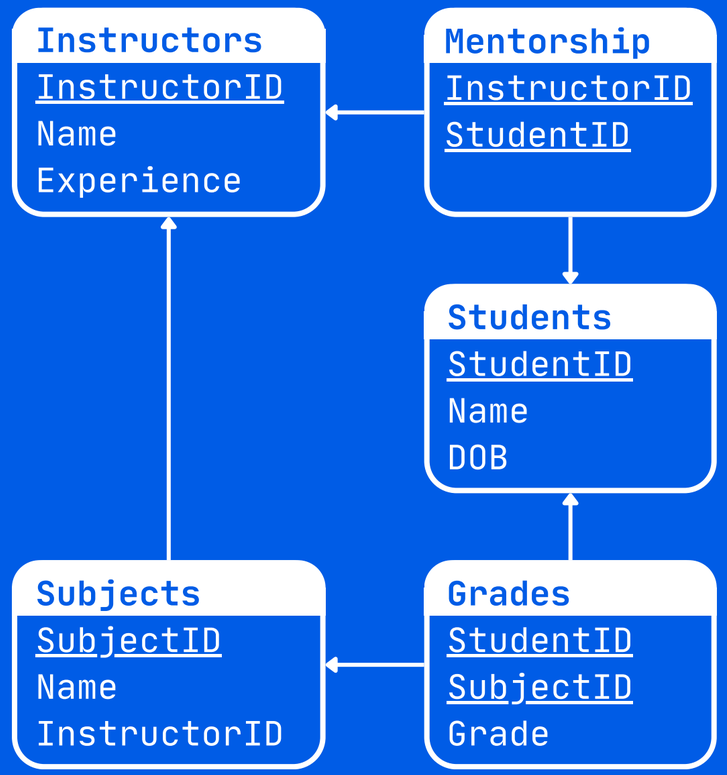
Mentorship: (InstructorID, StudentID)

TRC Queries

3 Find StudentID of students who scored grade “A”.

$$\{t.StudentID \mid t \in Grades \land t.Grade = \text{“A”} \}$$

4 Retrieve the instructor ID and the Name from the Instructor table who have more than 5 year of experience.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

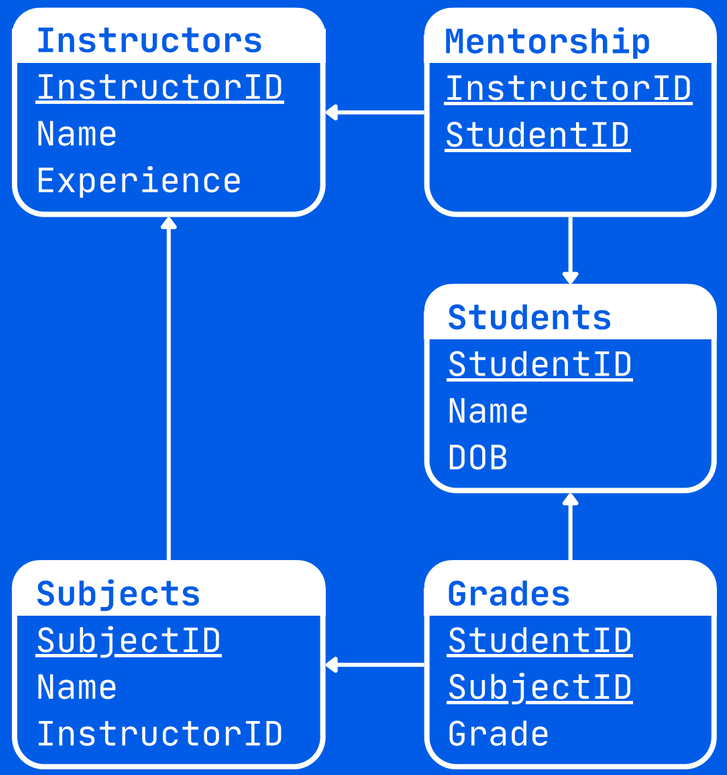
TRC Queries

3 Find StudentID of students who scored grade “A”.

$$\{t.StudentID \mid t \in Grades \land t.Grade = \text{“A”}\}$$

4 Retrieve the instructor ID and the Name from the Instructor table who have more than 5 year of experience.

$$\{t.InstructorID, t.Name \mid t \in Instructors \land t.Experience > 5\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

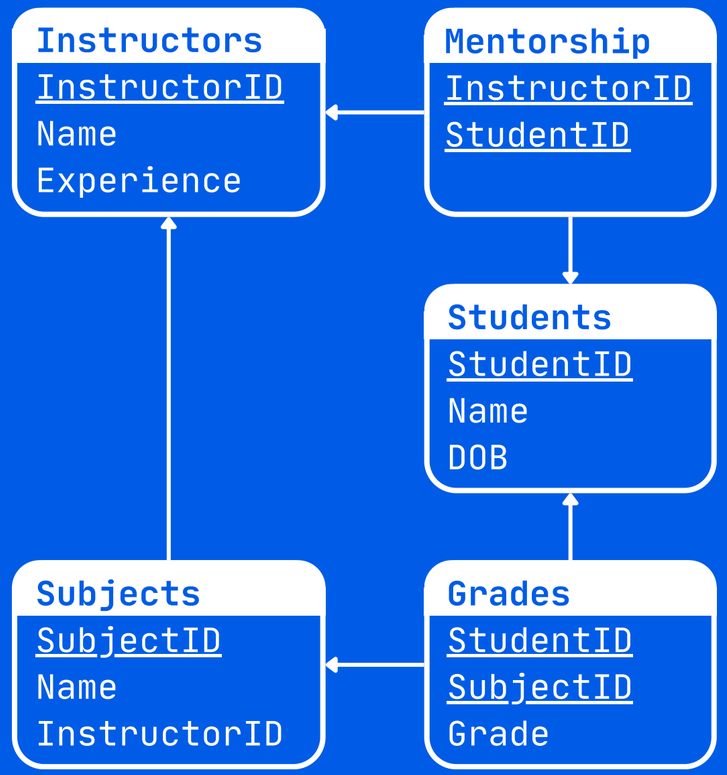
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

5 Find StudentID and their names of students who were born in or after year 2000.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

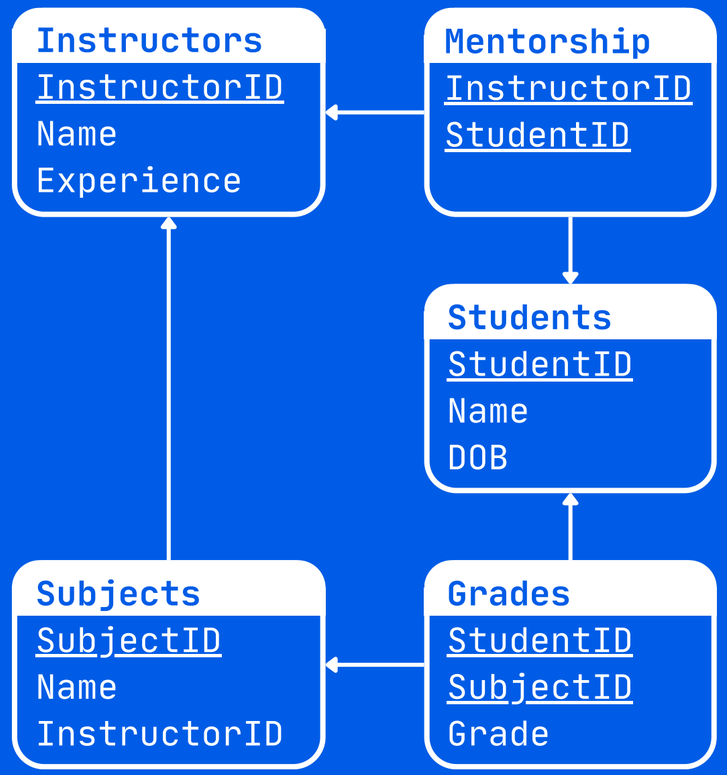
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

5 Find StudentID and their names of students who were born in or after year 2000.

$$\{t.StudentID, t.Name \mid t \in Students \land t.DOB >= \text{‘2000-01-01’}\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

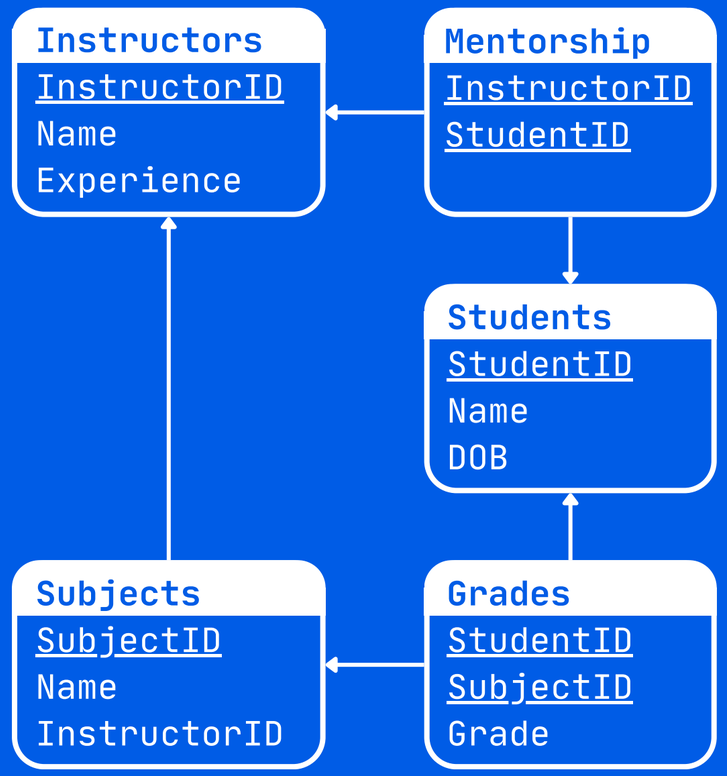
Mentorship: (InstructorID, StudentID)

TRC Queries

5 Find StudentID and their names of students who were born in or after year 2000.

$$\{t.StudentID, t.Name \mid t \in Students \land t.DOB >= \text{‘2000-01-01’}\}$$

6 Retrieve InstructorIDs along with the SubjectIDs of the subjects taught by those instructors who have more than 5 years of experience.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

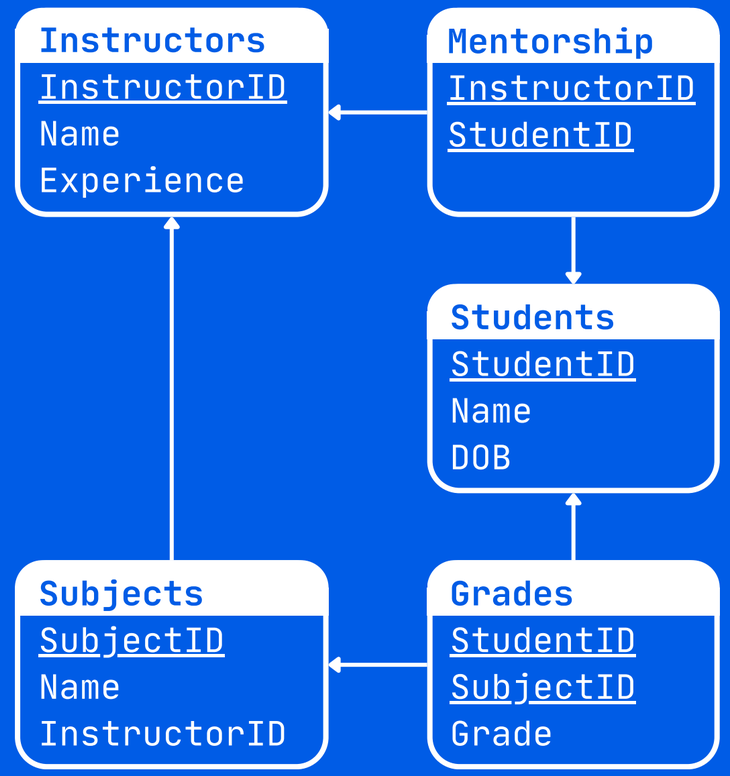
TRC Queries

5 Find StudentID and their names of students who were born in or after year 2000.

$$\{t.StudentID, t.Name \mid t \in Students \land t.DOB >= \text{‘2000-01-01’}\}$$

6 Retrieve InstructorIDs along with the SubjectIDs of the subjects taught by those instructors who have more than 5 years of experience.

$$\{t.InstructorID\ ,\ t.SubjectID \mid t \in Subjects \land \exists\ s\ (\ s \in Instructors \land s.InstructorID = t.InstructorID \land s.Experience > 5)\ \}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

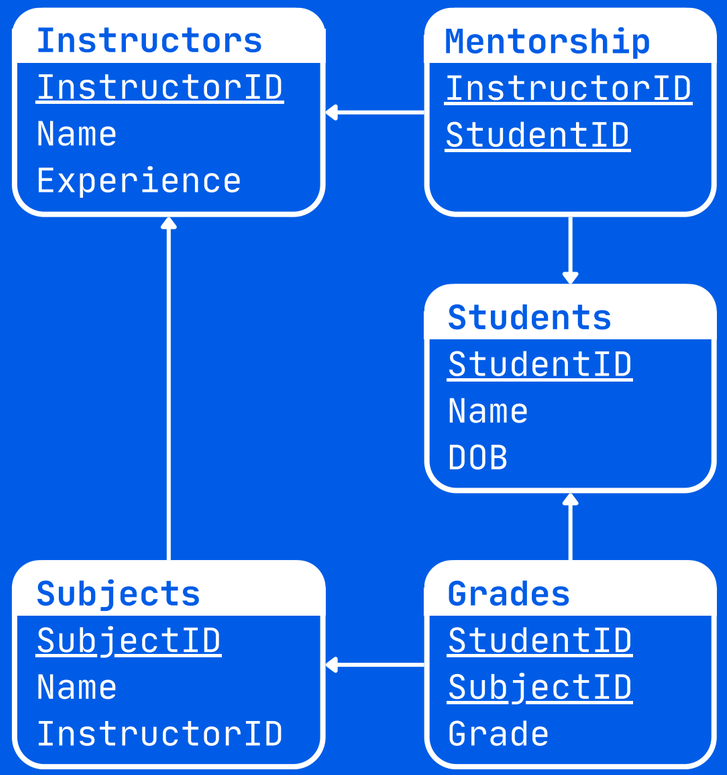
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

7 Retrieve InstructorID and name of instructors who mentor at least one student.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

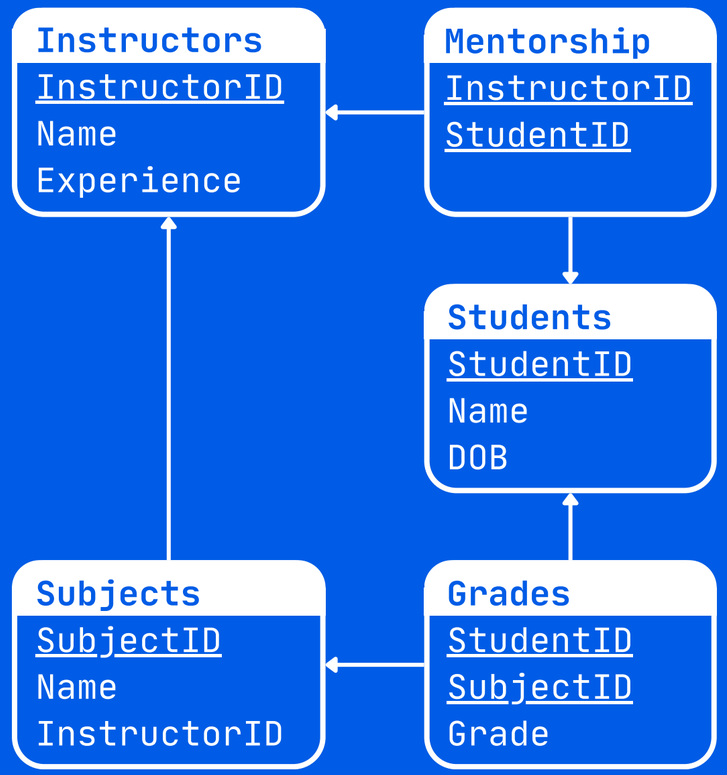
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

7 Retrieve InstructorID and name of instructors who mentor at least one student.

$$\{ t.InstructorID , t.Name \mid t \in Instructor \wedge \exists s (s \in Mentorship \wedge s.InstructorID = t.InstructorID) \}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

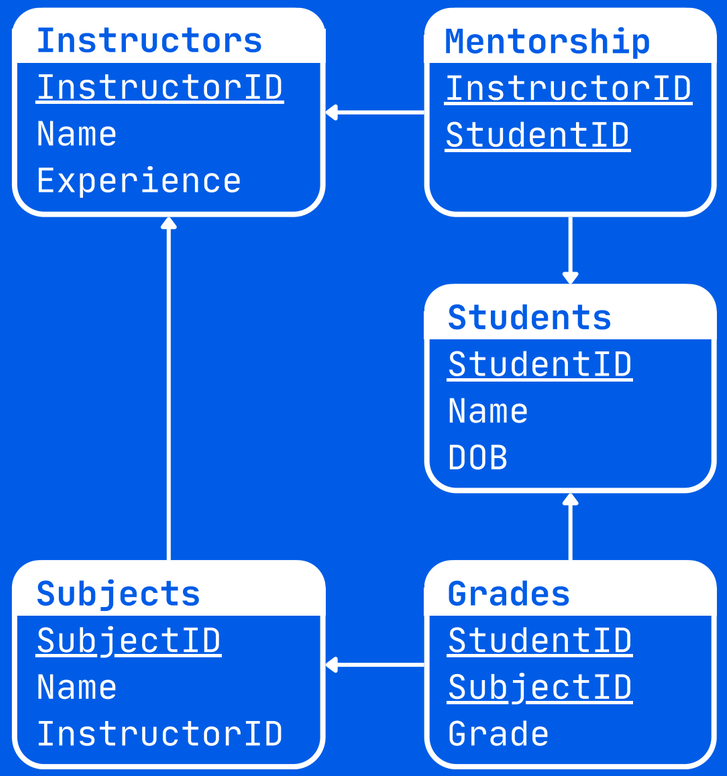
Mentorship: (InstructorID, StudentID)

TRC Queries

7 Retrieve InstructorID and name of instructors who mentor at least one student.

$$\{ t.InstructorID , t.Name \mid t \in Instructor \wedge \exists s (s \in Mentorship \wedge s.InstructorID = t.InstructorID) \}$$

8 Retrieve StudentID and SubjectID for the students who have received grade “A” and are also mentored by an instructor.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

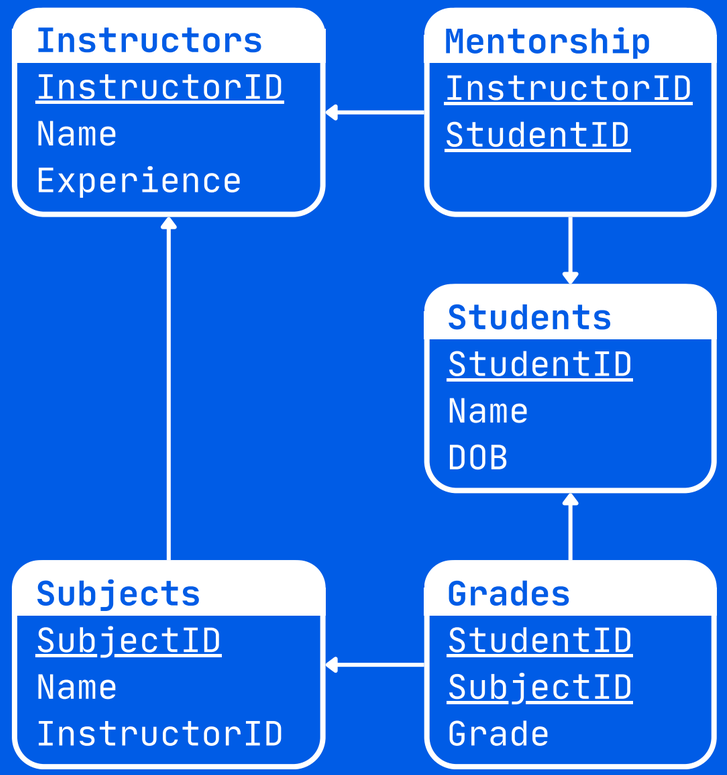
TRC Queries

7 Retrieve InstructorID and name of instructors who mentor at least one student.

$$\{ t.InstructorID , t.Name \mid t \in Instructor \wedge \exists s (s \in Mentorship \wedge s.InstructorID = t.InstructorID) \}$$

8 Retrieve StudentID and SubjectID for the students who have received grade “A” and are also mentored by an instructor.

$$\{t.StudentID,t.SubjectID \mid t \in Grades \wedge t.Grades = \text{“A”} \wedge \exists s (s \in Mentorship \wedge t.StudentID = s.StudentID) \}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

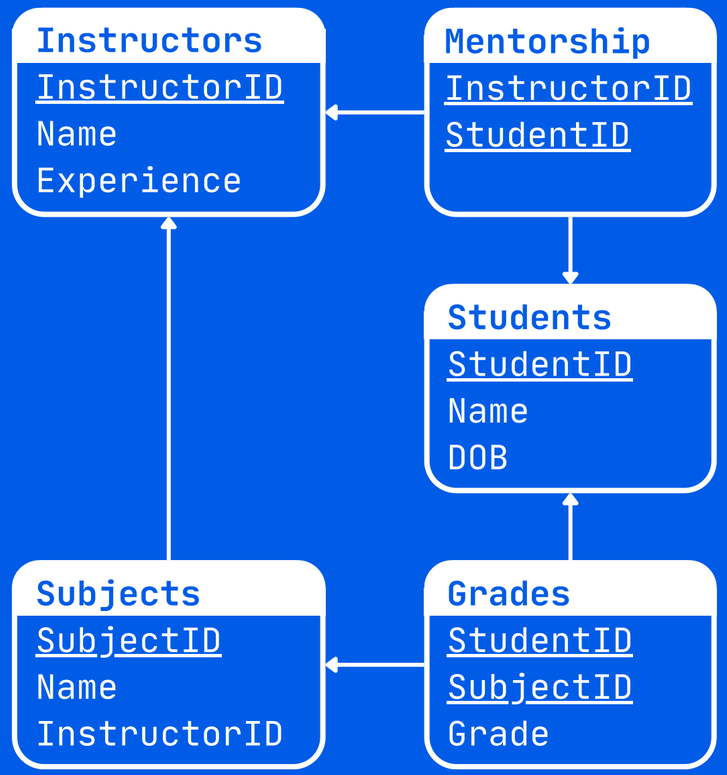
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

9 List the details of the students whose DOB is of before year 2000 or got grade “A”.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

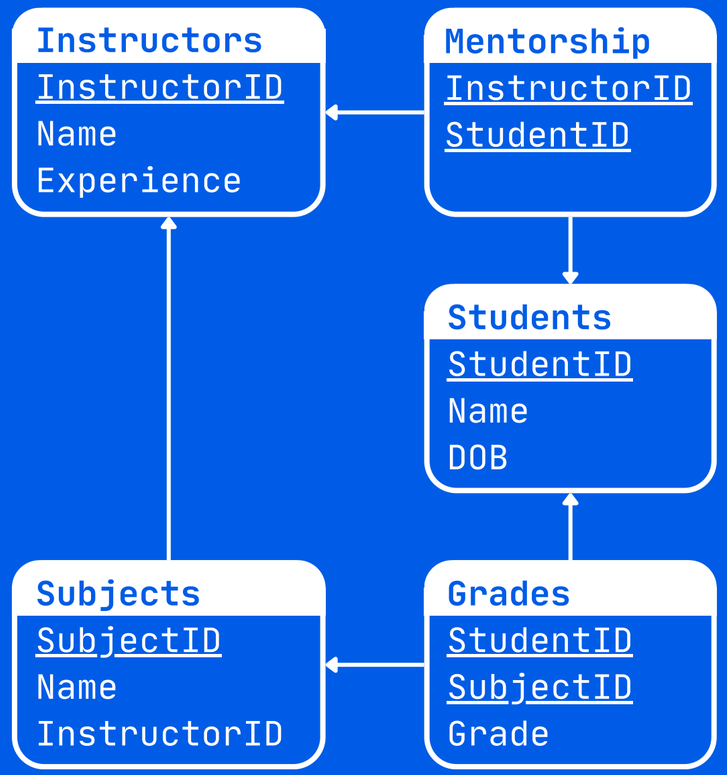
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

9 List the details of the students whose DOB is of before year 2000 or got grade “A”.

$$\{t \mid t \in \textit{Students} \land t.DOB < \text{‘2000-01-01’} \lor \exists s(\, s \in \textit{Grades} \land s.Grade = \text{“A”} \land s.StudentID = t.StudentID)\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

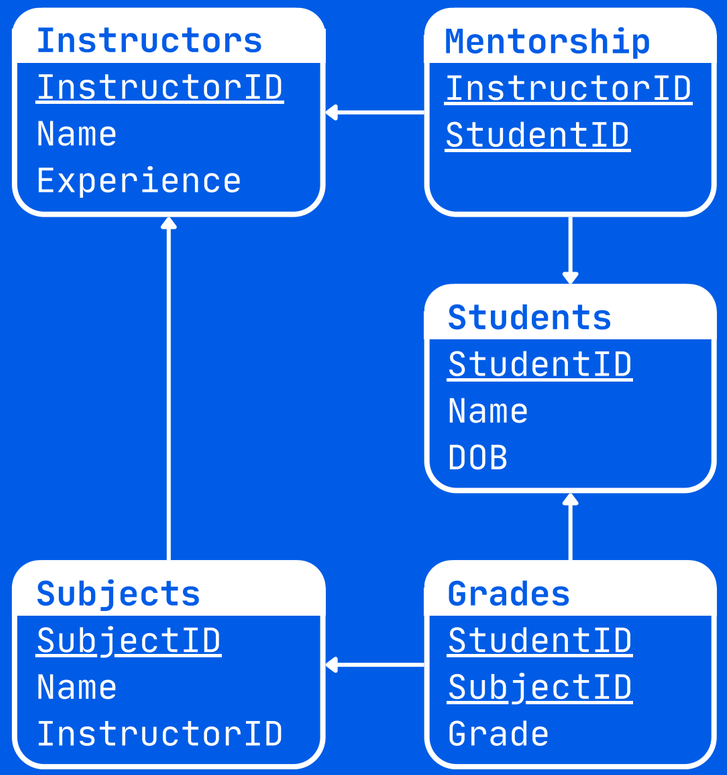
Mentorship: (InstructorID, StudentID)

TRC Queries

9 List the details of the students whose DOB is of before year 2000 or got grade “A”.

$$\{t \mid t \in \textit{Students} \land t.DOB < \text{‘2000-01-01’} \lor \exists s(\, s \in \textit{Grades} \land s.Grade = \text{“A”} \land s.StudentID = t.StudentID)\}$$

10 Find the names of students who have taken a subject taught by 'Korth’.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

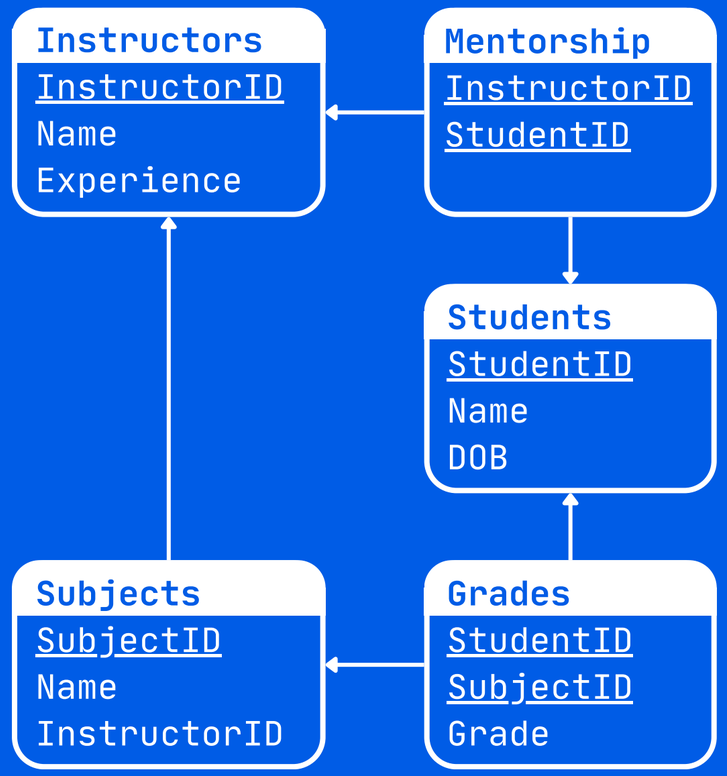
TRC Queries

9 List the details of the students whose DOB is of before year 2000 or got grade “A”.

$$\{t \mid t \in Students \land t.DOB < \text{‘2000-01-01’} \lor \exists s(\ s \in Grades \land s.Grade = \text{“A”} \land s.StudentID = t.StudentID)\}$$

10 Find the names of students who have taken a subject taught by 'Korth'.

$$\{t.name \mid t \in Students \land \exists g(\ g \in Grades \land t.StudentID=g.StudentID \land \exists s(\ s \in Subjects \land g.SubjectID=s.SubjectID \land \exists i(\ i \in Instructors \land i.InstructorID=s.InstructorID \land i.Name = \text{“Korth”})))\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

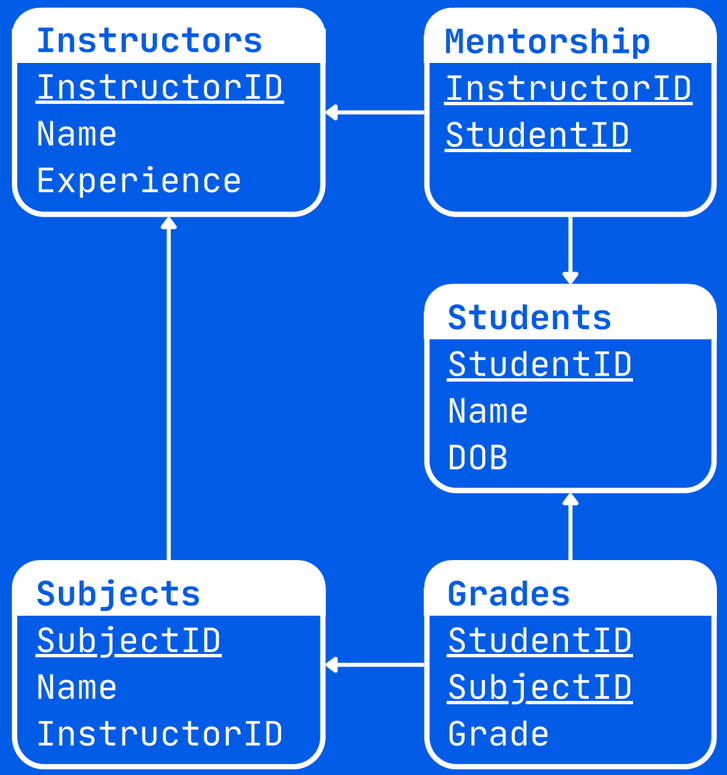
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

11 Retrieve instructor name and student name for each pair of instructor and student from mentorship table.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

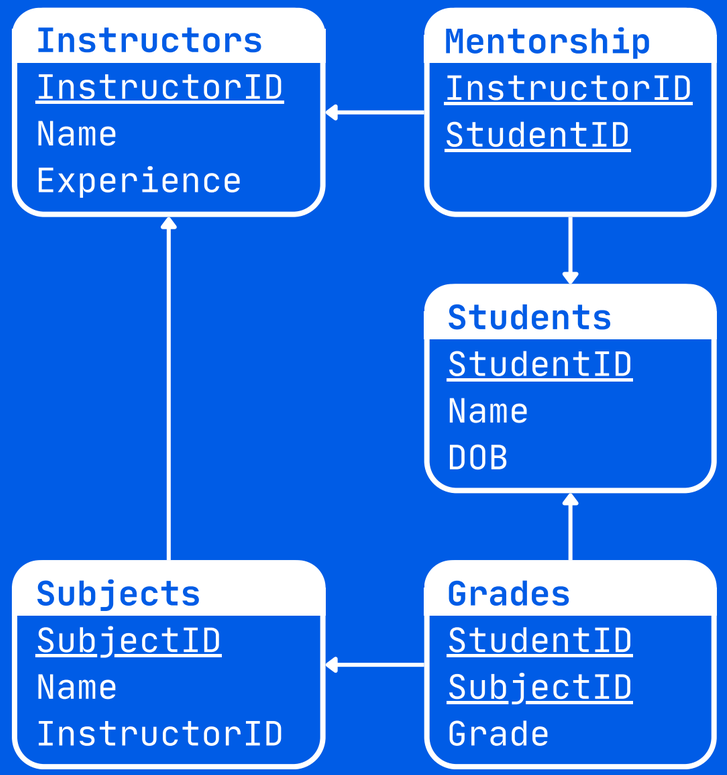
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC Queries

11 Retrieve instructor name and student name for each pair of instructor and student from mentorship table.

$$\{ t \mid \exists i(i \in Instructors \wedge \exists s(s \in Students \wedge \exists m(m \in Mentorship \wedge i.InstructorID = m.InstructorID \wedge s.StudentID = m.StudentID \wedge t.InstructorName = i.Name \wedge t.StudentName = s.Name)))\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC V/S SQL

TRC	SQL
• **Theoretical** query language used to describe what data to fetch	• **Practical** and industry-standard query language
• Based on mathematical logic (**predicate logic**)	• Based on **relational calculus**, but **extended** with many features
• Primarily used in **academic** and **theoretical contexts**	• Widely used in **real-world applications** and **DBMSs**
• Tuple relational calculus focuses only on **data retrieval logic**	• Supports **data retrieval**, **manipulation**, and **control**
• No user interface or tools, exists as **conceptual model**.	• Supported by a vast ecosystem of **tools**, **extensions**, and **GUIs**
• `Eg: {t | t ∈ Students}`	• `SELECT * FROM Students;`

Domain Relational Calculus (DRC)

Selecting **attributes** in a relation that satisfy a given condition (or predicate).

- Working same as that of TRC, except we are selecting **columns**.
- Basic **syntax**:

$$\{ \langle a_1, a_2, \ldots a_n \rangle \mid P(a_1, a_2, \ldots a_n) \}$$

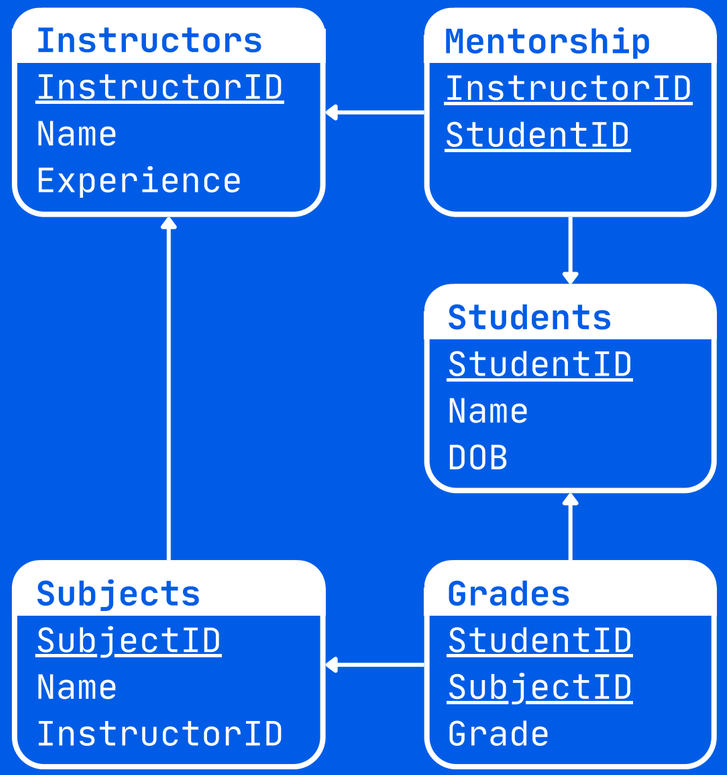
Result
(Domain variables 'a_i')

Predicate
(Condition used to fetch attributes 'a_i')

- **Result:** Set of attributes $\mathbf{a_1}$, $\mathbf{a_2}$,...$\mathbf{a_n}$ such that predicate '**P**' is true for attributes $\mathbf{a_1}$, $\mathbf{a_2}$,...$\mathbf{a_n}$.
- **Notations:**
  - $\mathbf{<a_1, a_2, \ldots a_n> \in r}$**:** Where **r** is a relation on n attributes and $a_1, a_2, \ldots a_n$ are domain variables/constants.
  - **P:** predicate i.e, the condition on tuples

DRC Queries

1 List the names of all students in Students relation.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

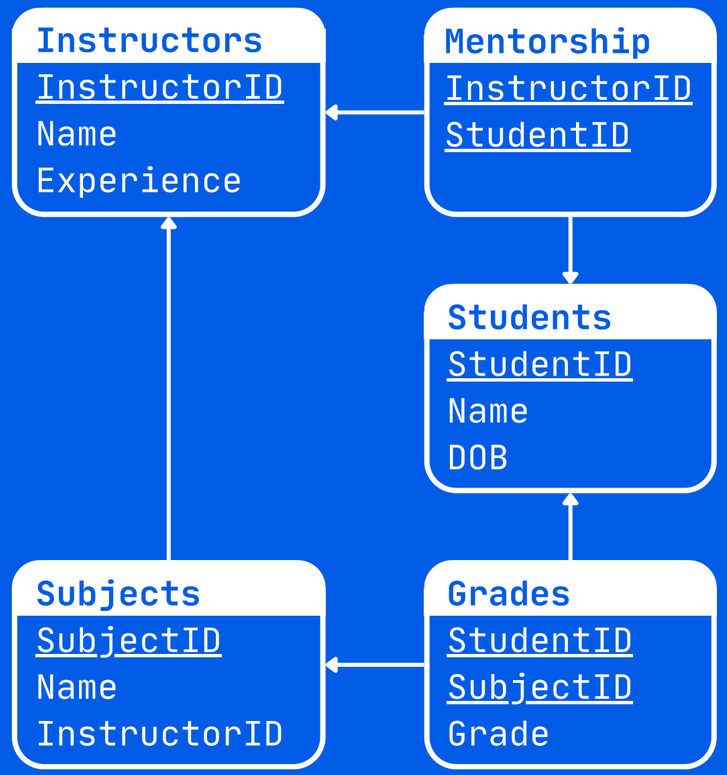
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

DRC Queries

1 List the names of all students in Students relation.

$$\{ n \mid (\exists r)(\exists d)(\mathit{Students}(r, n, d))\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

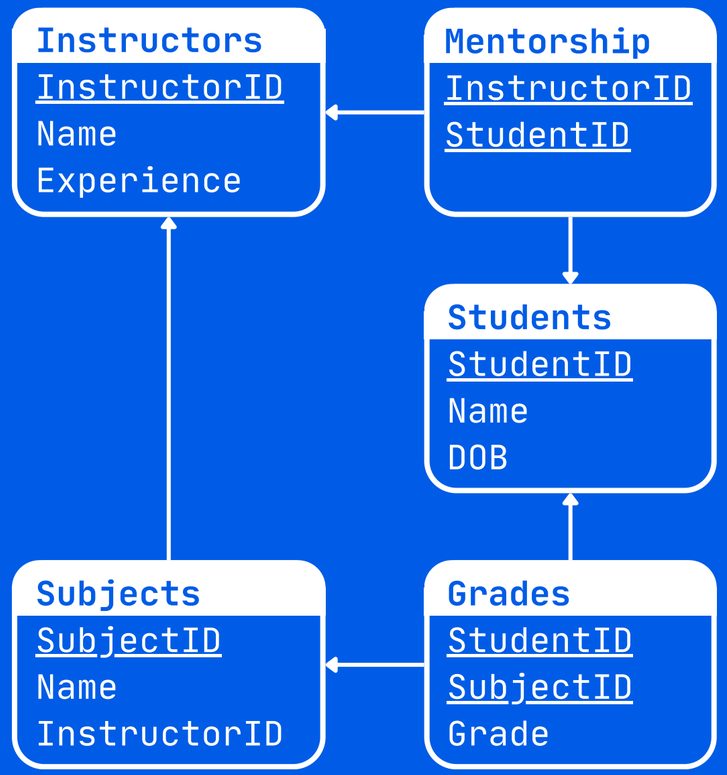
Mentorship: (InstructorID, StudentID)

DRC Queries

1 List the names of all students in Students relation.

$$\{ n \mid (\exists r)(\exists d)(\mathit{Students}(r, n, d))\}$$

2 Find names of instructors with more than 10 years of experience.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

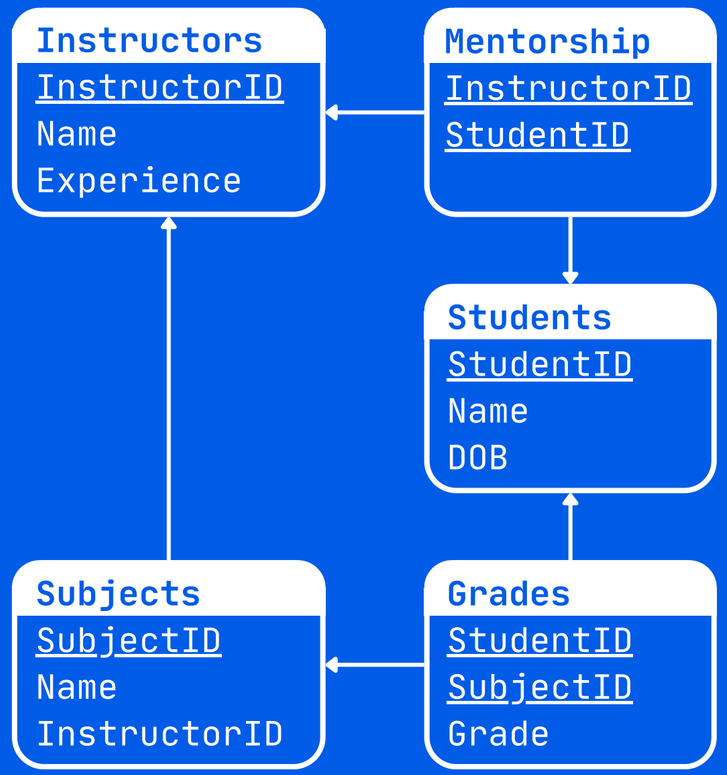
DRC Queries

1 List the names of all students in Students relation.

$$\{ n \mid (\exists r)(\exists d)(Students(r, n, d))\}$$

2 Find names of instructors with more than 10 years of experience.

$$\{ x \mid (\exists i)(\exists e)(Instructors(i, x, e) \land e > 10)\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

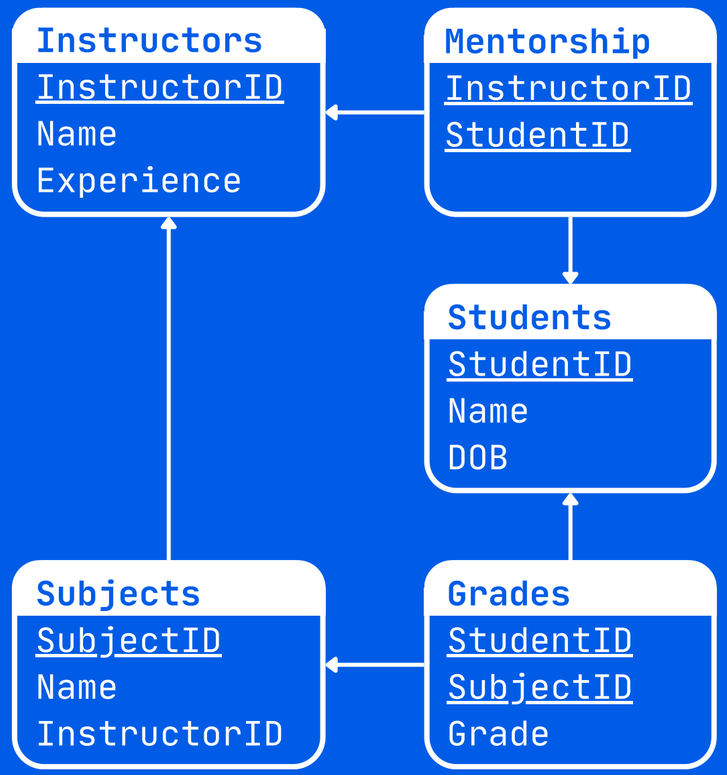
Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

DRC Queries

3 List names of students who got an A grade in any subject.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

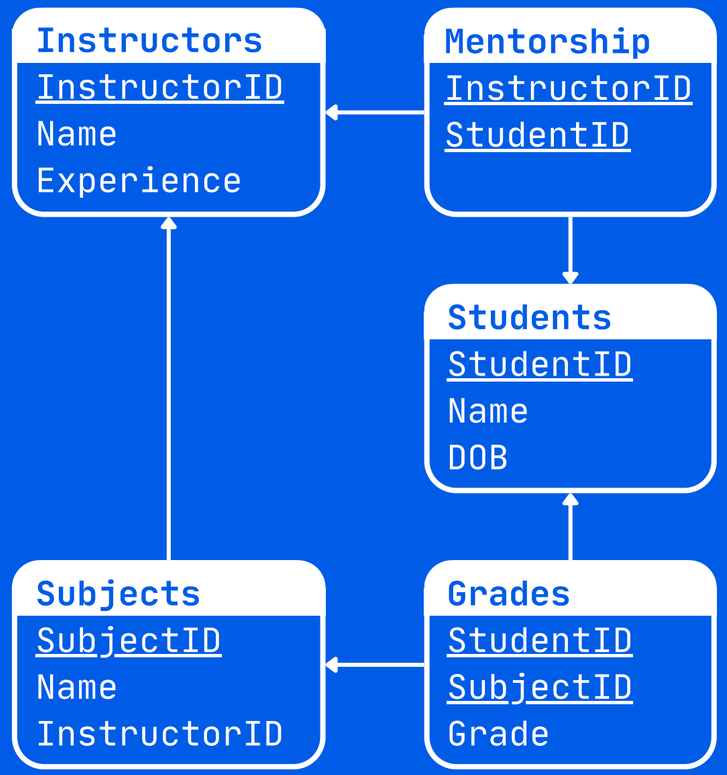
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

DRC Queries

3 List names of students who got an A grade in any subject.

$$\{ n \mid (\exists r)(\exists d)(\exists s)(\exists g)(\mathit{Students}(r, n, d) \wedge \mathit{Grades}(r, s, g) \wedge g='A') \}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

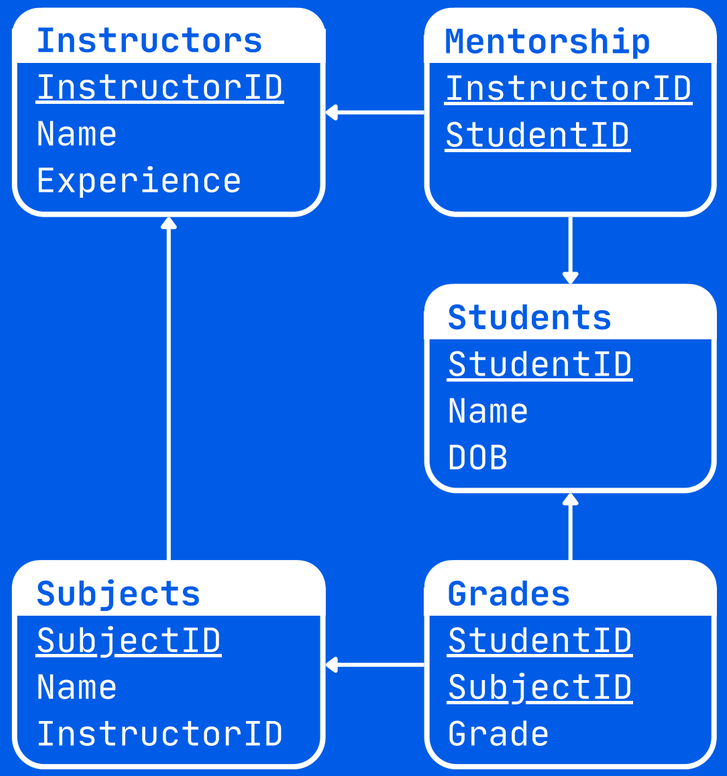
Mentorship: (InstructorID, StudentID)

DRC Queries

3 List names of students who got an A grade in any subject.

$$\{ n \mid (\exists r)(\exists d)(\exists s)(\exists g)(\mathit{Students}(r, n, d) \wedge \mathit{Grades}(r, s, g) \wedge g='A') \}$$

4 List student names mentored by instructors with over 15 years of experience.



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

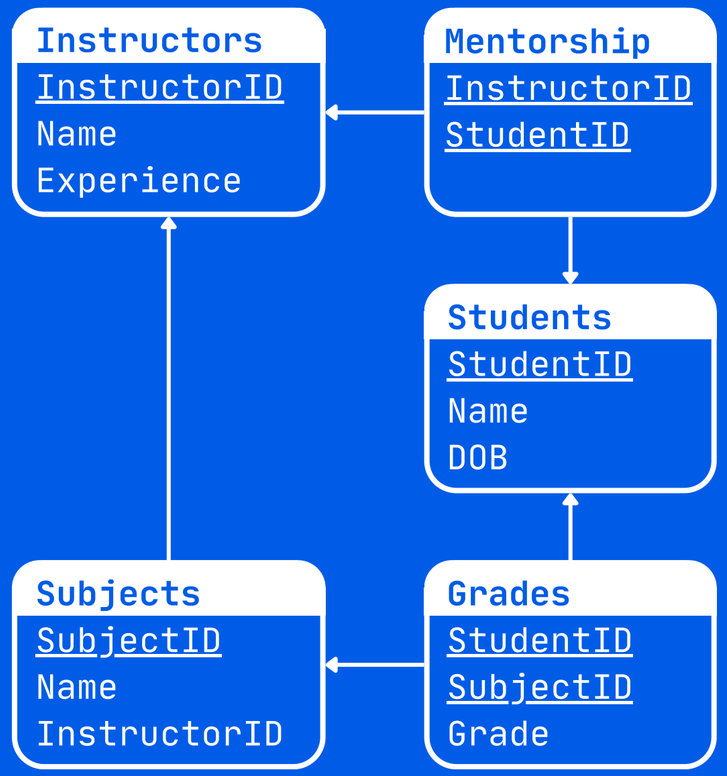
DRC Queries

3 List names of students who got an A grade in any subject.

$$\{ n \mid (\exists r)(\exists d)(\exists s)(\exists g)(Students(r, n, d) \wedge Grades(r, s, g) \wedge g='A')\}$$

4 List student names mentored by instructors with over 15 years of experience.

$$\{ n \mid (\exists r)(\exists d)(\exists i)(\exists x)(\exists e)(Students(r, n, d) \wedge Mentorship(i,r) \wedge Instructors(i, x, e) \wedge e>15)\}$$



Students: (StudentID, Name, DOB)

Instructors: (InstructorID,Name,Experience)

Subjects: (SubjectID, Name, InstructorID)

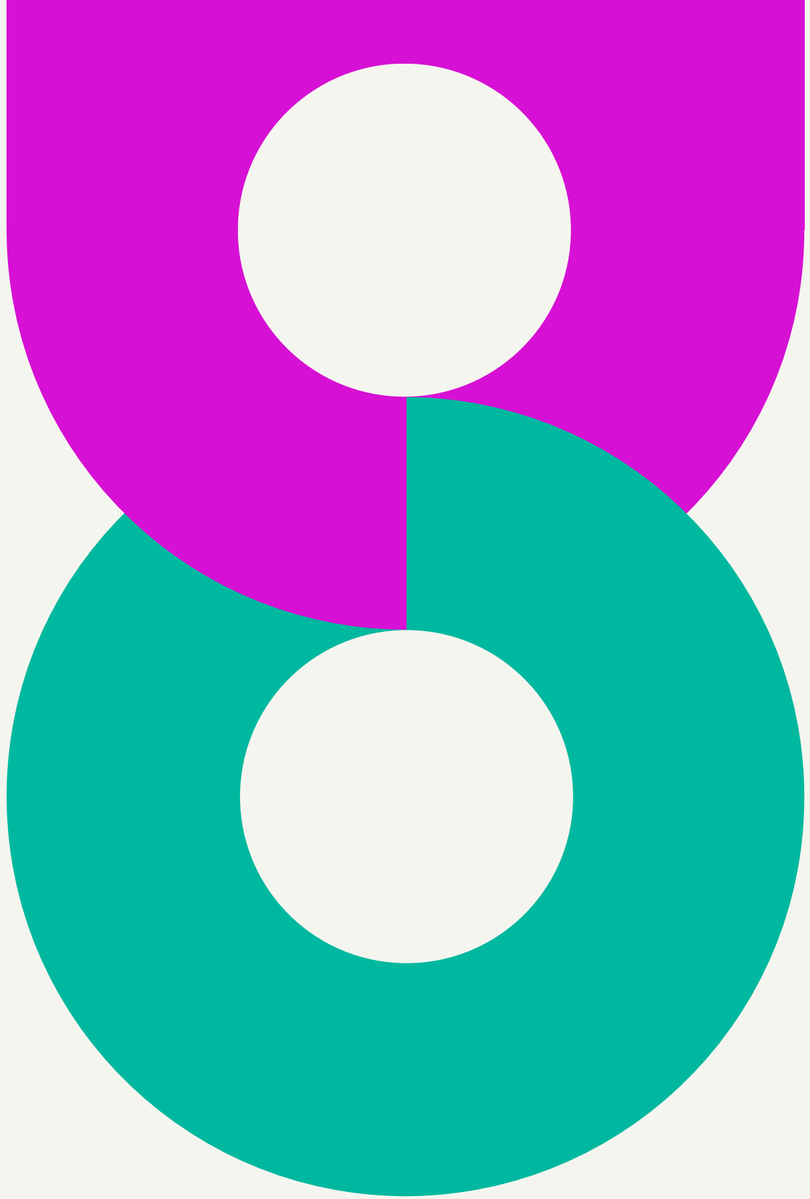
Grades: (StudentID, SubjectID, Grade)

Mentorship: (InstructorID, StudentID)

TRC	DRC
• Based on **tuples as variables**, each representing an entire row	• Based on **attribute-level variables**, each representing a value
• Queries are expressed in terms of **relations** and **tuples**	• Queries are expressed using **individual domain values**
• Queries tend to be **shorter** and **cleaner** for multi-attribute relations	• Queries can become **verbose** as each field is explicitly named
• Better for understanding **row-based query formulation**	• Better for understanding **column-level constraints**
• Often preferred for **practical query formulation**	• Often used in **academic** and **logic-based proofs**
• Eg: { t.Name,t.DOB|t∈*Students*}	• {<n, d> | ∃i(Students(n,d,i))}

What's wrong with the below query?

$$\{t \mid \neg(t \in student)\}$$



What's **wrong** with the below query?

$$\{t \mid \neg(t \in \text{student})\}$$

∵ There are infinitely many tuples that are not in *student.*

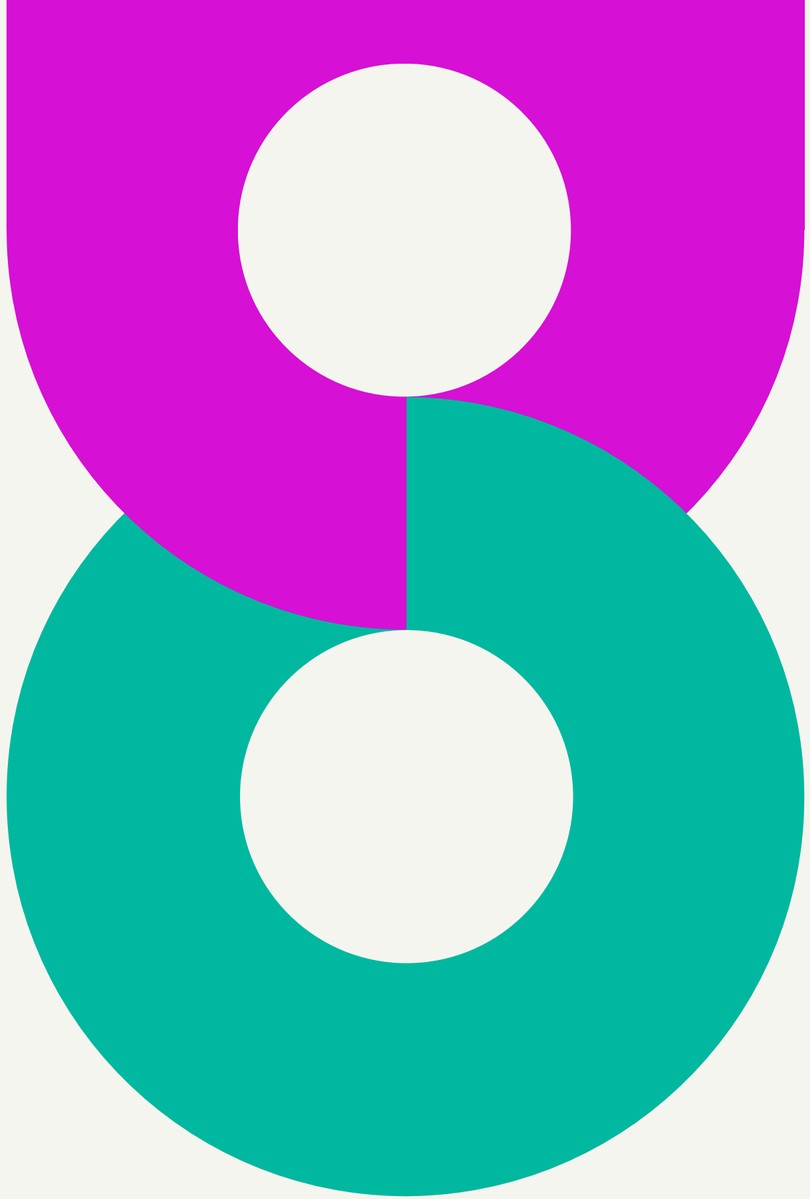
∴ This leads to a result with **infinite** tuples.

Such queries are known as **Unsafe Queries**

Safe Expressions in Relational Calculus

A query is safe if it produces a **finite** result set.

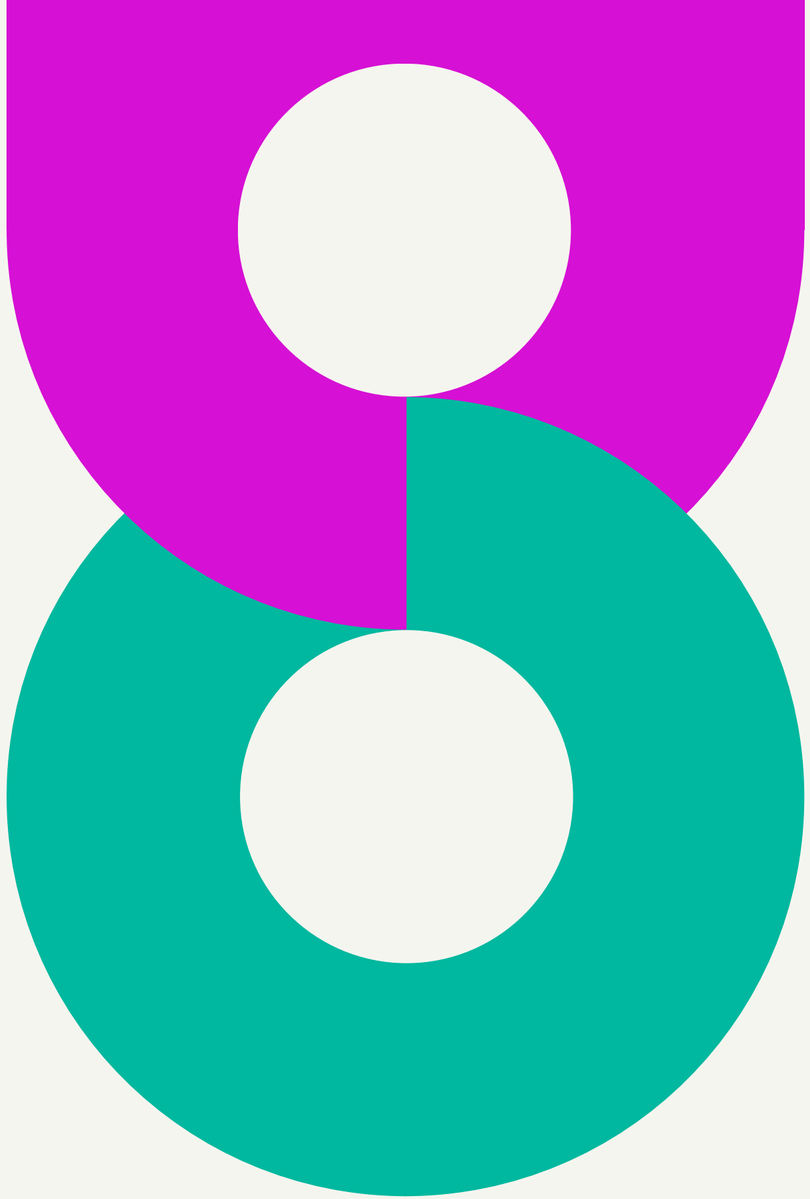
- A safe expression in Relational Calculus ensures that the result of a query is **finite** and **computable**.
- It avoids returning an infinite number of tuples, which is essential for **practical database execution**.
- Guarantees that results come from **actual**, **existing data** in the database and not hypothetical or undefined values.



Safe Expressions in Relational Calculus

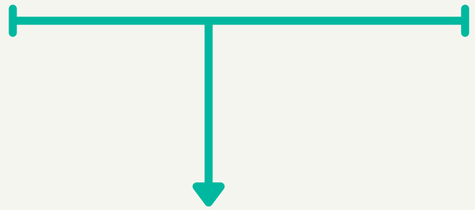
How to ensure **Safety of Expressions**?

1. **Restrict variables to finite domains:** Every variable must range over a finite relation (i.e., a known table), not the entire universal domain.
2. **Use only values from the database or constants:** Ensure all values in the query come from existing data or explicit constants, not hypothetical or infinite values.
3. **Apply negation and quantifiers within restricted domains:** Use ¬ (NOT) and ∀ (FOR ALL) only when the scope is limited to a defined relation, avoiding unbounded results.



Safe version of the previous query!

$$\{t \mid t \in Person \wedge \neg(t \in Student)\}$$



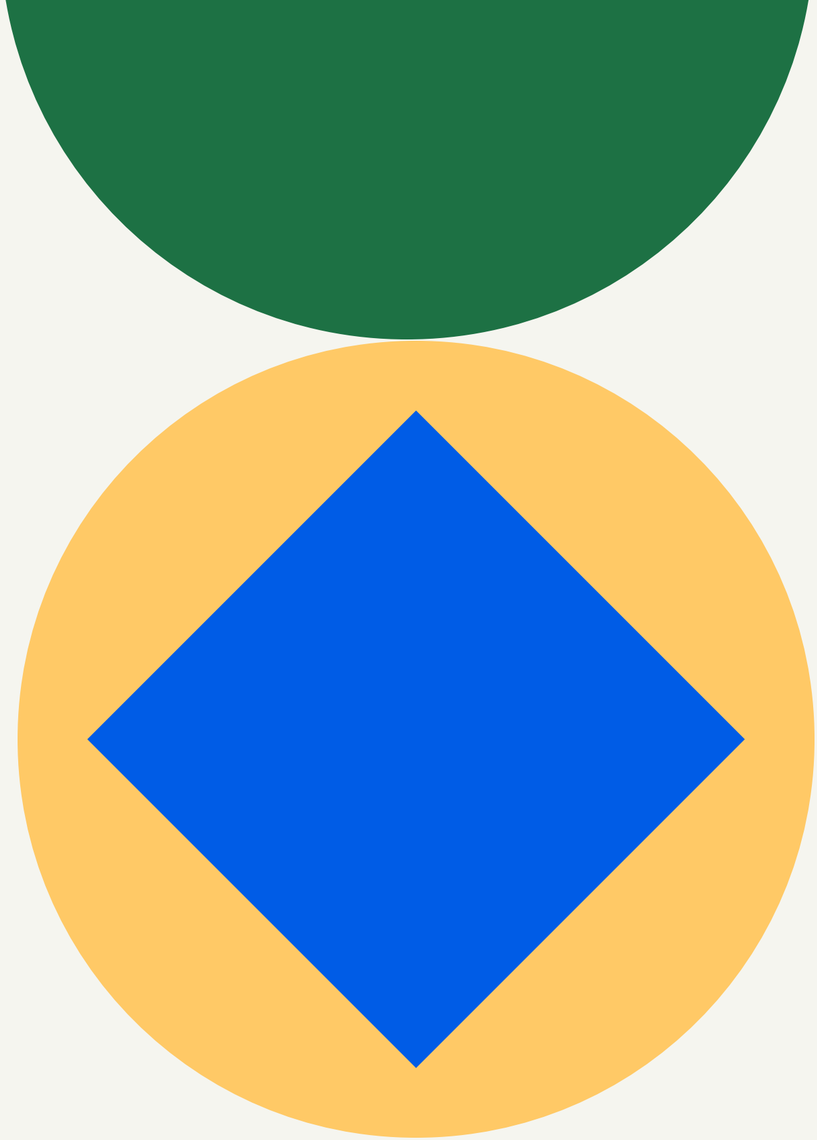
∵ It restricts the domain to known values from the *Person* relation.

∴ This leads to a result with **finite** tuples.

Leading to a **Safe Query**

Summary

- **Relational Calculus:** Non-procedural query language that focuses on what to retrieve, not how to retrieve it.
- Based on **Predicate Calculus**, using logical expressions, variables, and quantifiers.
- **Two types:** Tuple Relational Calculus (**TRC**) and Domain Relational Calculus (**DRC**).
- Differs from Relational Algebra by **being declarative**, not operational.
- **Safe expressions** are essential to ensure queries return finite, computable results.



Presented by
Om Gupta, Vikash Kumar, Aditya Raj,
Parkhi Jain, Adarsh Sharma & Gurnam Singh

THE END