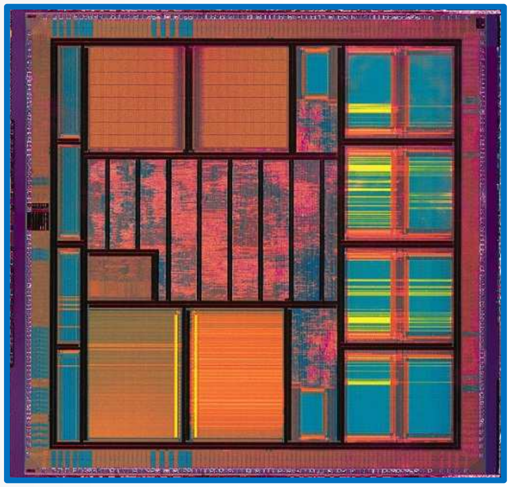
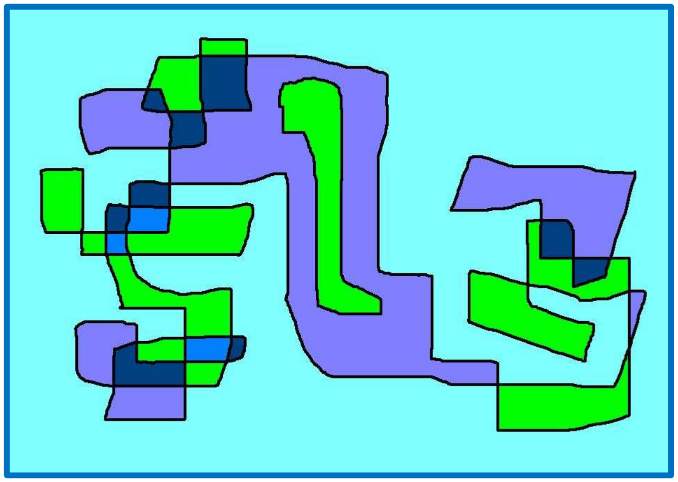
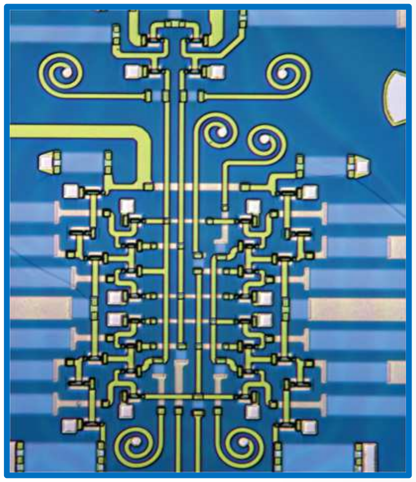






EIE 211 : Electronic Devices and Circuit Design II

Lecture 4x: Frequency Response

Introduction

- s-Domain analysis: poles, zeros and bode plots
- The amplifier transfer function
- Low-frequency response of the common-source and common-emitter amplifier
- High-frequency response of the CS and CE amplifiers
- The CB, CG and cascode configurations

Introduction

Why shall we study the frequency response?

Actual transistors exhibit charge storage phenomena that limit the speed and frequency of their operation.

- Aims: the emphasis in this lecture is on analysis, focusing attention on the mechanisms that limit frequency response and on methods for extending amplifier bandwidth.

Introduction

There are 3 main parts:

- s-Domain analysis and the amplifier transfer function
- High frequency model of BJT and MOS; Low-frequency and High-frequency response of the common-source and common-emitter amplifier
- Frequency response of cascode, Emitter and source followers and differential amplifier

Part 1

- s-Domain analysis
- Zeros and poles
- Bode plots
- The amplifier transfer function

s-Domain Analysis-
Frequency Response

- Transfer function: poles, zeros
- Examples: high pass and low pass
- Bode plots: Determining the 3-dB frequency

Transfer Function: poles, zeros

- Most of our work in this lecture will be concerned with finding amplifier voltage gain as a transfer function of the complex frequency s.
- A capacitance C: is equivalent an impedance 1/sC
- An inductance L: is equivalent an impedance sL
- Voltage transfer function: by replacing s by jω, we can obtain its magnitude response and phase response

$$T(s) = V_o(s)/V_i(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_0}$$

Transfer Function: poles, zeros

$$T(s) = V_o(s)/V_i(s) = a_m \frac{(s - Z_1)(s - Z_2)......(s - Z_m)}{(s - P_1)(s - P_2)......(s - P_n)}$$

- Z_1, Z_2, … Z_m are called the transfer-function zeros or transmission zeros.
- P_1, P_2, … P_m are called the transfer-function poles or natural modes.
- The poles and zeros can be either real or complex numbers, the complex poles(zeros) must occur in conjugate pairs.

First-order Functions

- All the transfer functions encountered in this chapter have real poles and zeros and can be written as the product of first-order transfer functions.

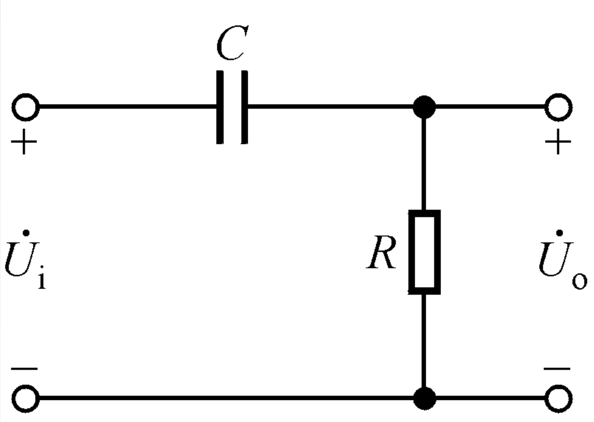
$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

- ω_0, called the pole frequency, is equal to the inverse of the time constant of circuit network(STC).

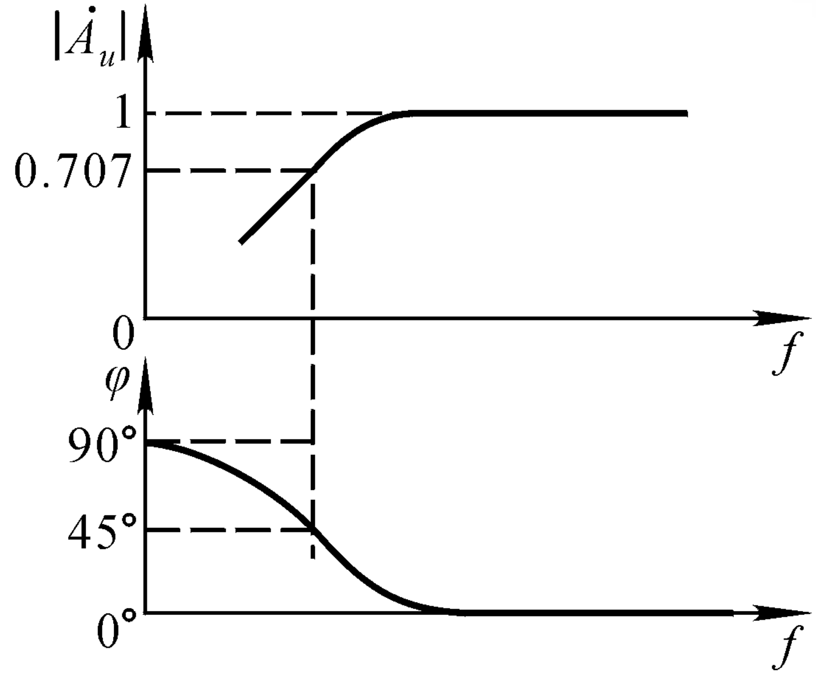
$$\text{Low pass}: T(s) = \frac{a_0}{s + \omega_0}$$

$$\text{High pass}: T(s) = \frac{a_1 s}{s + \omega_0}$$

Example 1: High-pass circuit



(a)



(b)

$$|\dot{A}_u| = \frac{\frac{f}{f_L}}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}$$

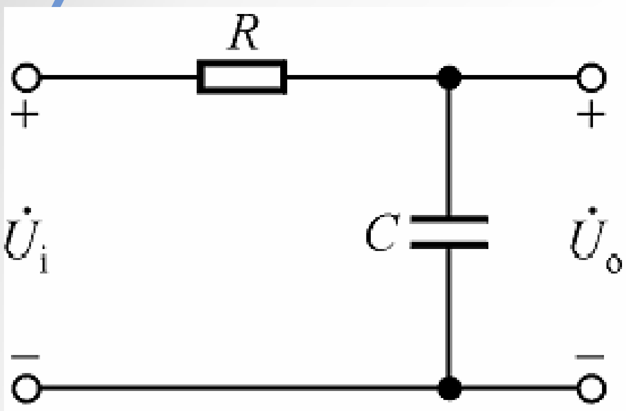
$$\varphi = 90^\circ - \arctan\frac{f}{f_L}$$

$$\dot{A}_u = \frac{\dot{U}_o}{\dot{U}_i} = \frac{R}{\frac{1}{j\omega C}+R} = \frac{1}{1+\frac{1}{j\omega RC}}$$

$$\boxed{T(s) = \frac{j\omega RC}{j\omega RC+1} = \frac{RCs}{RCs+1} = \frac{s}{s+1/RC}}$$

RC is the time constant; ωL=1/RC $f_L = \frac{\omega_L}{2\pi} = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$

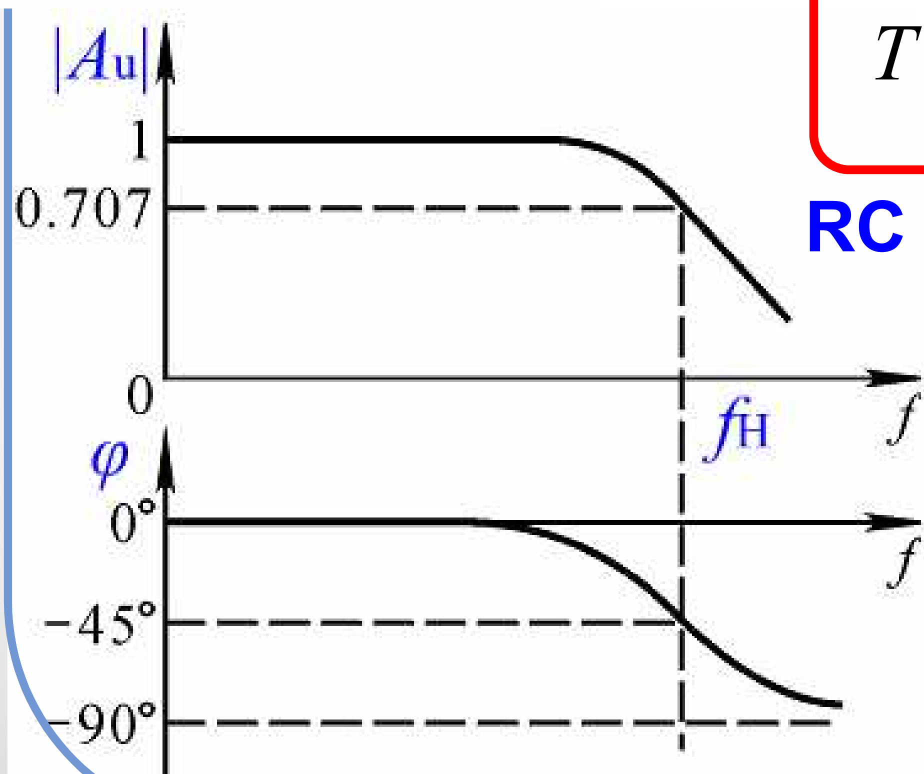
Example 2: Low-pass circuit



$$\dot{A}_u = \frac{\dot{U}_o}{\dot{U}_i} = \frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

$$\boxed{T(s) = \frac{1/RC}{s + 1/RC}}$$

RC is the time constant; $\omega_H = 1/RC$



$$\left|\dot{A}_u\right| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_H}\right)^2}}$$

$$\varphi = -\arctan\frac{f}{f_H}$$

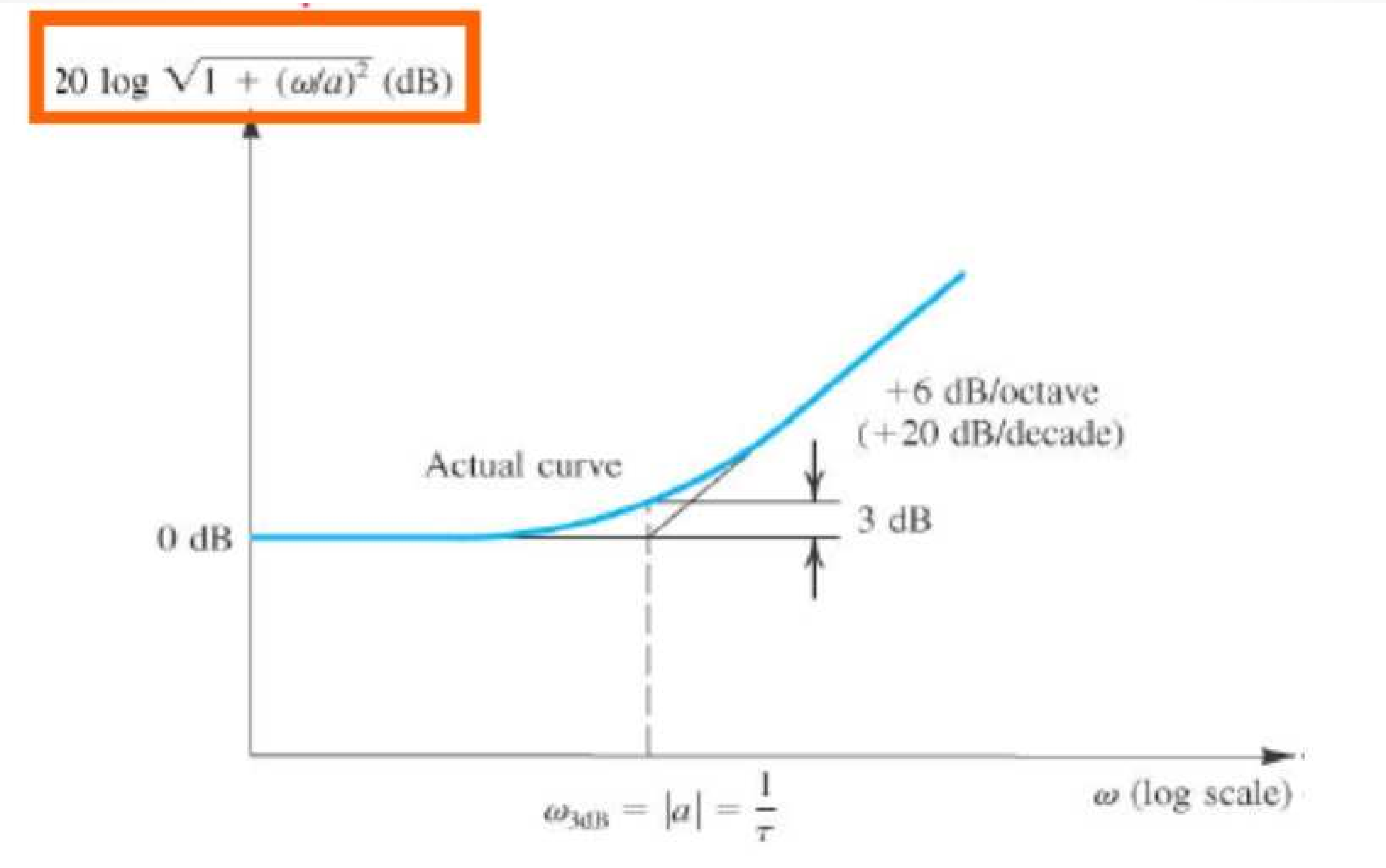
Bode Plots

- A simple technique exists for obtaining an approximate plot of the magnitude and phase of a transfer function given its poles and zeros. The resulting diagram is called Bode plots
- A transfer function consists of A product of factors of the form s+a

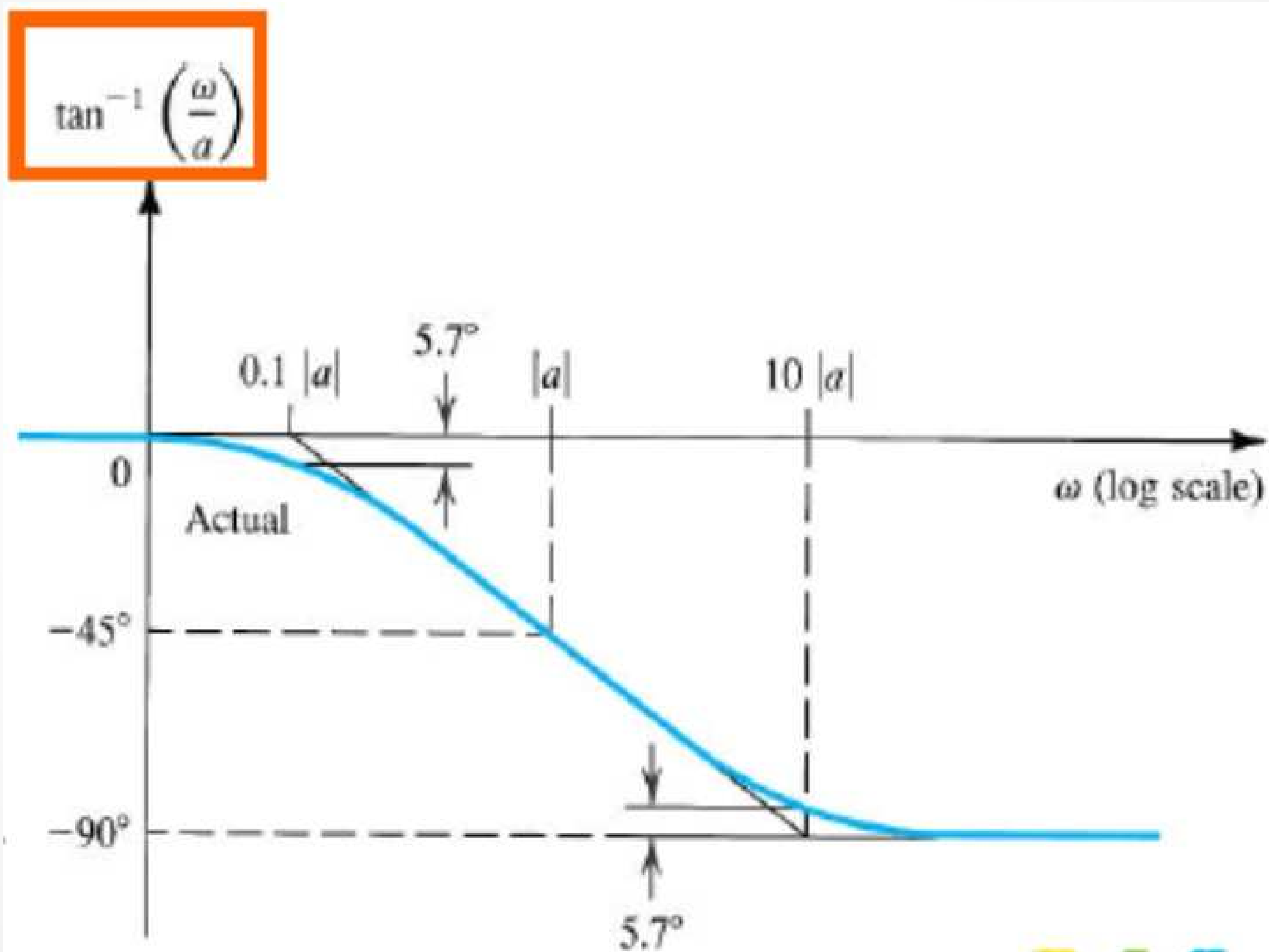
$$20\log_{10}\sqrt{a^2+\omega^2} \Rightarrow 20\log_{10}\sqrt{1+(\omega/a)^2}$$

Bode Plots

- For the case of a zero, Bode plot for the typical magnitude and phase term are shown below



Bode Plots



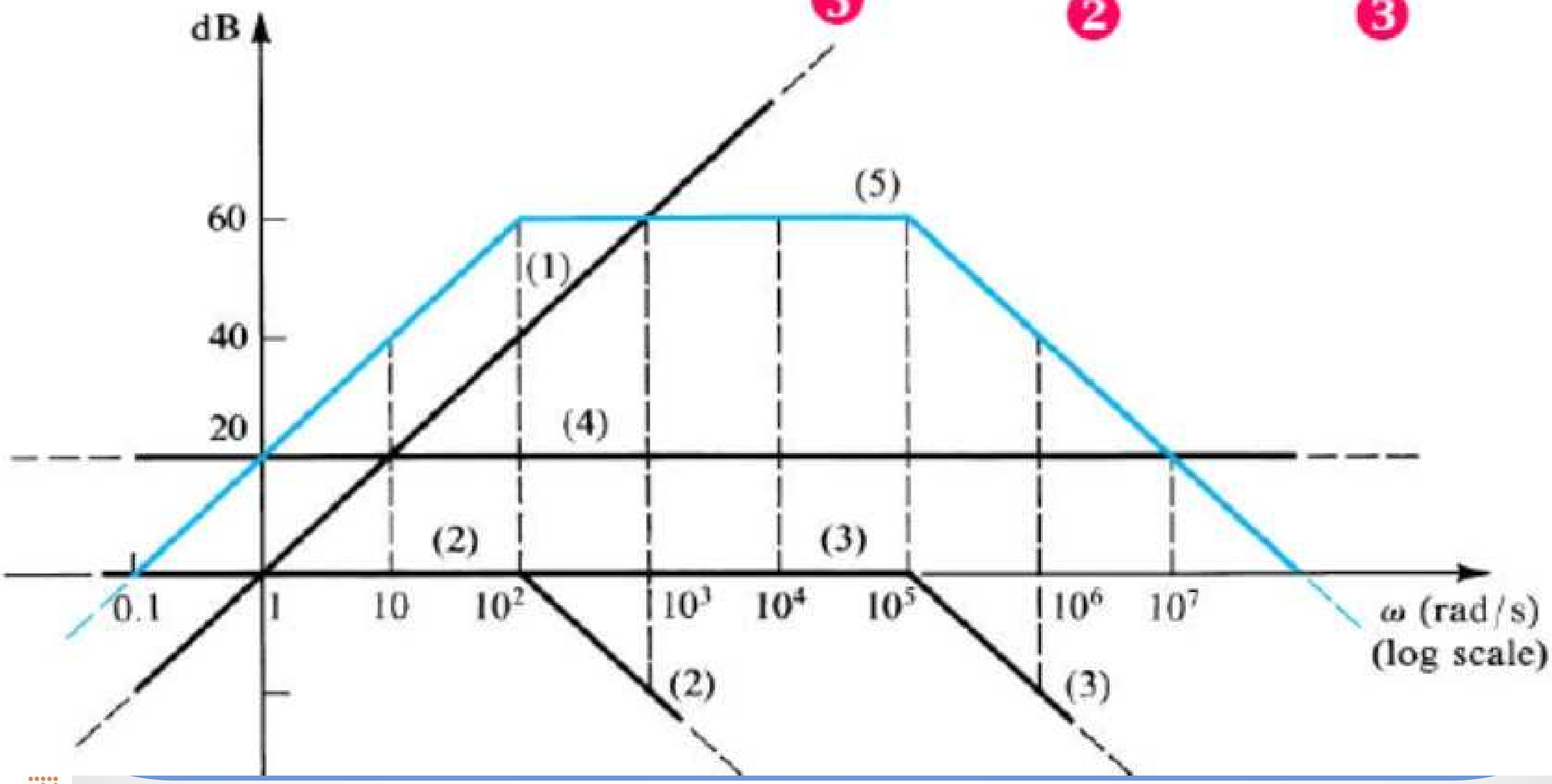
$\tan^{-1}\left(\frac{\omega}{a}\right)$
0.1 |a|
5.7°
|a|
10 |a|
0
Actual
ω (log scale)
−45°
−90°
5.7°

Example

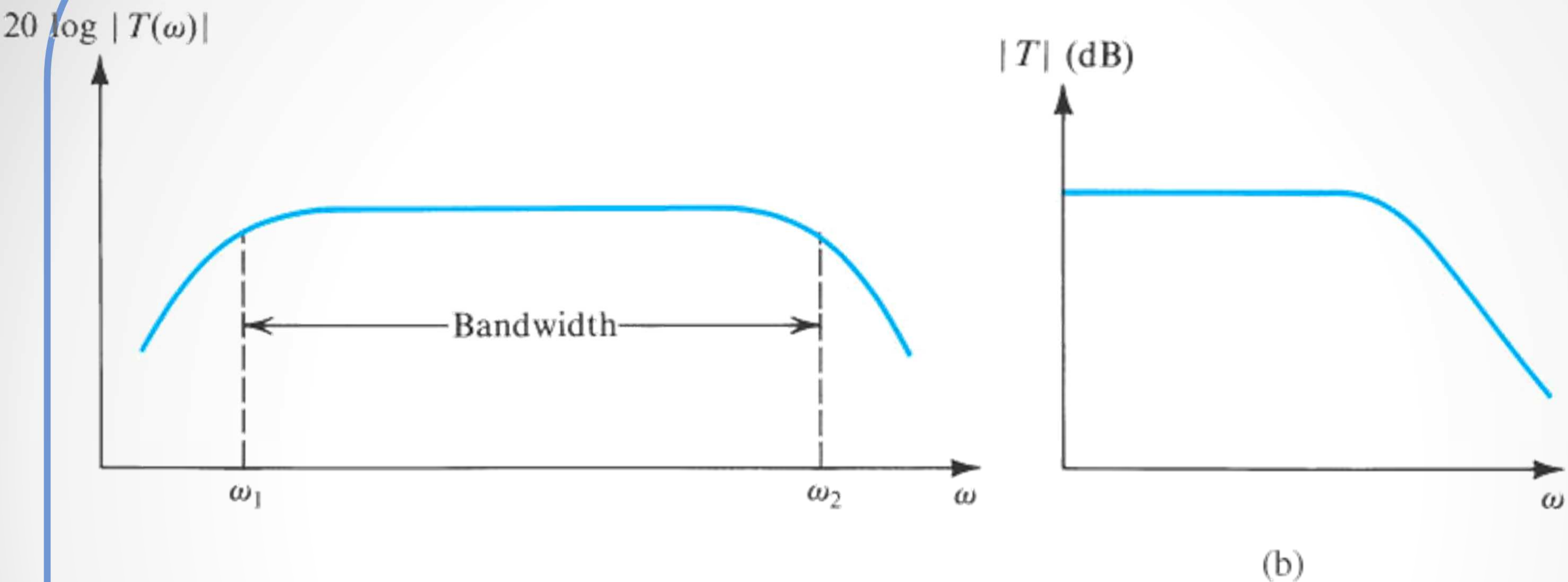
➢ An amplifier has the voltage transfer function, the bode plot is shown below.

$$T(s) = \frac{\overset{4}{10}\,\overset{1}{s}}{\underset{2}{(1+s/10^2)}\underset{3}{(1+s/10^5)}}$$

5



Amplifier Transfer Function



(a) a capacitively coupled amplifier **(b)** a direct-coupled amplifier

$$\text{Bandwidth} : BW = \omega_H - \omega_L$$

$$\text{Gain - bandwidth product} : GB = A_M BW \approx A_M \omega_H$$

The Gain Function

- Gain function $$\boxed{A(s) = A_M F_L(s) F_H(s)}$$
- Midband: No capacitors in effect

$$A(s) \approx A_M$$

- Low-frequency band: coupling and bypass capacitors in effect

$$A(s) \approx A_M F_L(s)$$

- High-frequency band: transistor internal capacitors in effect

$$A(s) \approx A_M F_H(s)$$

The Low Frequency Gain Function

- Gain function

$$A(s) = A_M F_L(s)$$

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})\ldots\ldots(s+\omega_{ZnL})}{(s+\omega_{p1})(s+\omega_{p2})\ldots\ldots(s+\omega_{pnL})}$$

- ω_{P1} , ω_{P2} , ….ω_{Pn} are positive numbers representing the frequencies of the *n* real poles.
- ω_{Z1} , ω_{Z2} , ….ω_{Zn} are positive, negative, or zero numbers representing the frequencies of the *n* real transmission zeros.

Determining the 3-dB Frequency

- Definition

$$\boxed{A(\omega_L) = A_M - 3dB}$$ or $$\boxed{A(\omega_L) = A_M / \sqrt{2}}$$

- Assume $\omega_{P1} > \omega_{P2} > \ldots > \omega_{Pn}$ and $\omega_{Z1} > \omega_{Z2} > \ldots > \omega_{Zn}$

$$\boxed{\omega_L \cong \sqrt{\omega_{P1}{}^2 + \omega_{P2}{}^2 + \ldots - 2(\omega_{Z1}{}^2 + \omega_{Z2}{}^2 + \ldots)}}$$

Determining the 3-dB Frequency

- Dominant pole

  If the highest-frequency pole is at least two octaves (a factor of 4) away from the nearest pole or zero, it is called dominant pole. Thus the 3-dB frequency is determined by the dominant pole.

- Single pole system,

$$A(s) = \frac{A_M s}{s + \omega_{P1}}$$

$$\omega_L \cong \omega_{P1}$$

The High Frequency Gain Function

- Gain function

$$A(s) = A_M F_H(s)$$

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2}).....(1+s/\omega_{Zn})}{(1+s/\omega_{P1})(1+s/\omega_{P2}).....(1+s/\omega_{Pn})}$$

- ω_{P1} , ω_{P2} , ….ω_{Pn} are positive numbers representing the frequencies of the *n* real poles.
- ω_{Z1} , ω_{Z2} , ….ω_{Zn} are positive, negative, or infinite numbers representing the frequencies of the *n* real transmission zeros.

Determining the 3-dB Frequency

- Definition

$$\boxed{A(\omega_H) = A_M - 3dB} \text{ or } \boxed{A(\omega_H) = A_M / \sqrt{2}}$$

- Assume $\omega_{P1} < \omega_{P2} < \ldots . < \omega_{Pn}$ and $\omega_{Z1} < \omega_{Z2} < \ldots . < \omega_{Zn}$

$$\boxed{\omega_H \cong 1 \Big/ \sqrt{\left(\frac{1}{\omega_{P1}{}^2} + \frac{1}{\omega_{P2}{}^2} + \ldots .\right) - 2\left(\frac{1}{\omega_{Z1}{}^2} + \frac{1}{\omega_{Z2}{}^2} + \ldots .\right)}}$$

Determining the 3-dB Frequency

- Dominant pole

  If the lowest-frequency pole is at least two octaves (a factor of 4) away from the nearest pole or zero, it is called dominant pole. Thus the 3-dB frequency is determined by the dominant pole.

- Single pole system,

$$A(s) = \frac{A_M}{1 + s / \omega_{P1}}$$

$$\omega_H \cong \omega_{P1}$$

Approx. Determination of Corner Frequency

- Using open-circuit time constants for computing high-frequency 3-dB Frequency: reduce all other C to zero; reduce the input source to zero.

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \dots + a_{nH} s^{nH}}{1 + b_1 s + b_2 s^2 + \dots + b_{nH} s^{nH}}$$

$$b_1 = \frac{1}{\omega p_1} + \frac{1}{\omega p_2} + \dots + \frac{1}{\omega p_{nH}}$$

$$b_1 = \sum_{i=1}^{nH} C_i R_{io} \cong \frac{1}{\omega p_1}$$

$$\omega_H \approx \frac{1}{\sum_{i=1}^{nH} C_i R_{io}}$$

Approx. Determination of Corner Frequency

- Using short-circuit time constants for computing low-frequency 3-dB Frequency: replace all other C with short circuit; reduce the input source to zero.

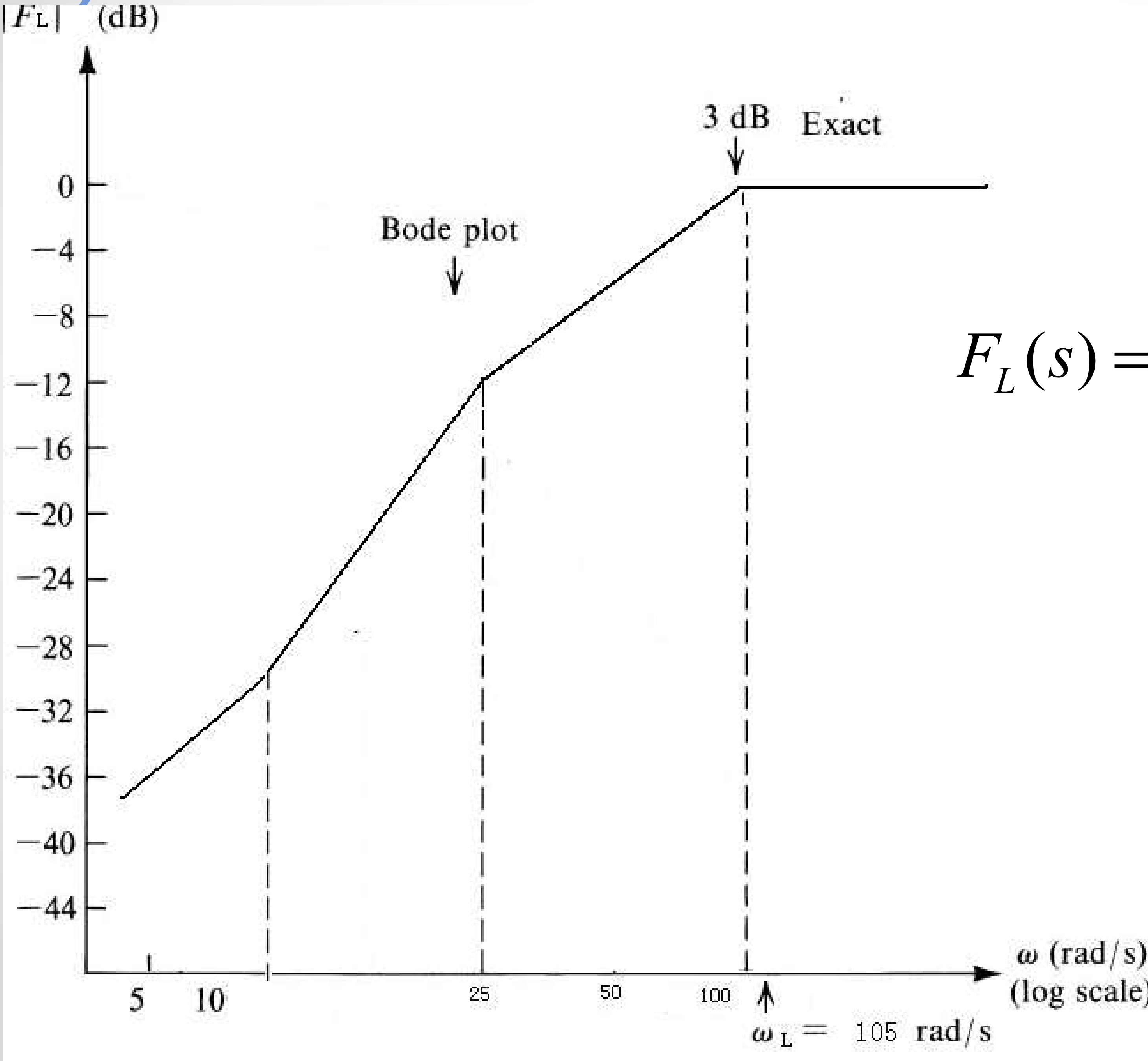
$$F_L(s) = \frac{s^{nL} + d_1 s^{nL-1} + d_2 s^2 + \ldots}{s^{nL} + e_1 s^{nL-1} + e_2 s^2 + \ldots}$$

$$e_1 = \omega p_1 + \omega p_2 + \ldots + \omega p_{nL}$$

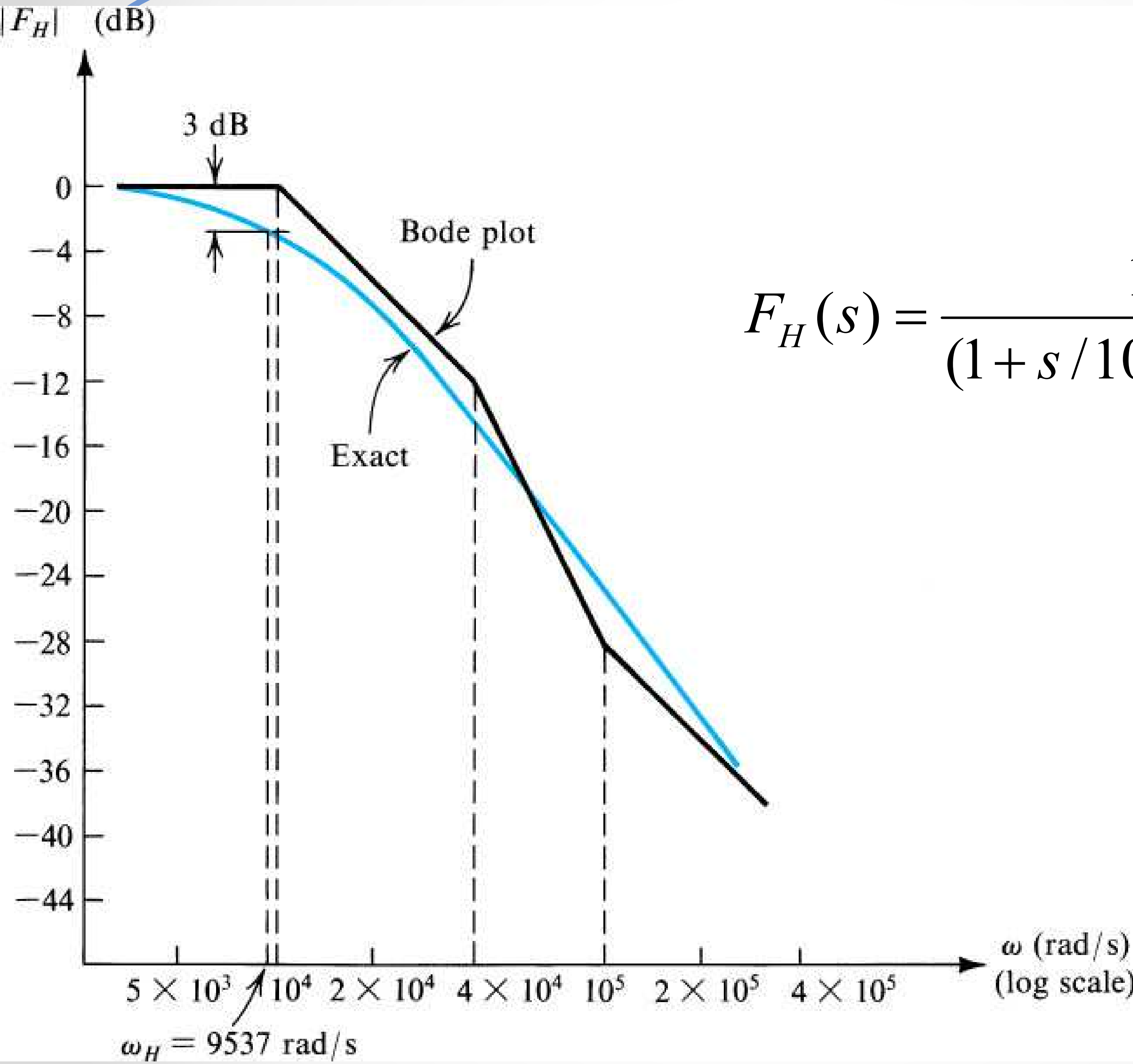
$$e_1 = \sum_{i=1}^{nL} \frac{1}{C_i R_{io}} \cong \omega_{p1} \text{ if dominant pole exists}$$

$$\omega_L \approx \sum_{i=1}^{nL} \frac{1}{C_i R_{io}}$$

Example



$$F_L(s) = \frac{s(s+10)}{(s+100)(s+25)}$$



$$F_H(s) = \frac{1 - s/10^5}{(1 + s/10^4)(1 + s/4\times10^4)}$$

Summary

(A) Poles and zeros are known or can be easily determined

Low-frequency

$$A(s) = A_M F_L(s)$$

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2}).....(s+\omega_{ZnL})}{(s+\omega_{p1})(s+\omega_{p2}).....(s+\omega_{pnL})}$$

$$A(s) = \frac{A_M s}{s+\omega_{P1}}$$

$$\omega_L \cong \omega_{P1}$$

$$\omega_L \cong \sqrt{\omega_{P1}{}^2 + \omega_{P2}{}^2 + \ldots - 2(\omega_{Z1}{}^2 + \omega_{Z2}{}^2 + \ldots)}$$

High-frequency

$$A(s) = A_M F_H(s)$$

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2}).....(1+s/\omega_{Zn})}{(1+s/\omega_{P1})(1+s/\omega_{P2}).....(1+s/\omega_{Pn})}$$

$$A(s) = \frac{A_M}{1+s/\omega_{P1}}$$

$$\omega_H \cong \omega_{P1}$$

$$\omega_H \cong 1 \Big/ \sqrt{\left(\frac{1}{\omega_{P1}{}^2} + \frac{1}{\omega_{P2}{}^2} + ...\right) - 2\left(\frac{1}{\omega_{Z1}{}^2} + \frac{1}{\omega_{Z2}{}^2} + ...\right)}$$

(B) Poles and zeros can not be easily determined

Low-frequency

$$F_L(s) = \frac{s^{nL} + d_1 s^{nL-1} + d_2 s^2 + \ldots}{s^{nL} + e_1 s^{nL-1} + e_2 s^2 + \ldots}$$

$$e_1 = \omega p_1 + \omega p_2 + \ldots + \omega p_{nL}$$

$$e_1 = \sum_{i=1}^{nL} \frac{1}{C_i R_{io}} \cong \omega_{p1} \text{ if dominant pole exists}$$

$$\omega_L \approx \sum_{i=1}^{nL} \frac{1}{C_i R_{io}}$$

High-frequency

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_{nH} s^{nH}}{1 + b_1 s + b_2 s^2 + \ldots + b_{nH} s^{nH}}$$

$$b_1 = \frac{1}{\omega p_1} + \frac{1}{\omega p_2} + \ldots + \frac{1}{\omega p_{nH}}$$

$$b_1 = \sum_{i=1}^{nH} C_i R_{io} \cong \frac{1}{\omega p_1} \text{ if dominant pole exists}$$

$$\omega_H \approx \frac{1}{\sum_{i=1}^{nH} C_i R_{io}}$$

Part II

- Internal Capacitances of the BJT
- BJT High Frequency Model
- Internal Capacitances of the MOS
- MOS High Frequency Model
- Low-frequency of CS and CE amplifiers

Internal Capacitances of the BJT and High Frequency Model

- Internal capacitance
  - The base-charging or diffusion capacitance
  - Junction capacitances
    - The base-emitter junction capacitance
    - The collector-base junction capacitance
- High frequency small signal model
- Cutoff frequency and unity-gain frequency

The Base-Charging or Diffusion Capacitance

- Diffusion capacitance almost entirely exists in forward-biased *pn* junction
- Expression of the small-signal diffusion capacitance

$$C_{de} = \tau_F g_m = \tau_F \frac{I_C}{V_T}$$

$$\tau_F \text{ : forward base - transit time, represents the average time a charge carrier spends in crossing the base 10ps - 100ps}$$

- Proportional to the biased current

Junction Capacitances

- The Base-Emitter Junction Capacitance

$$C_{je} = \frac{C_{je0}}{(1 - \frac{V_{BE}}{V_{oe}})^m} \approx 2C_{je0}$$

C_{je0} : is the value of C_{je} at zero voltage

V_{oe} : is the build in voltage (tipically , 0.9v)

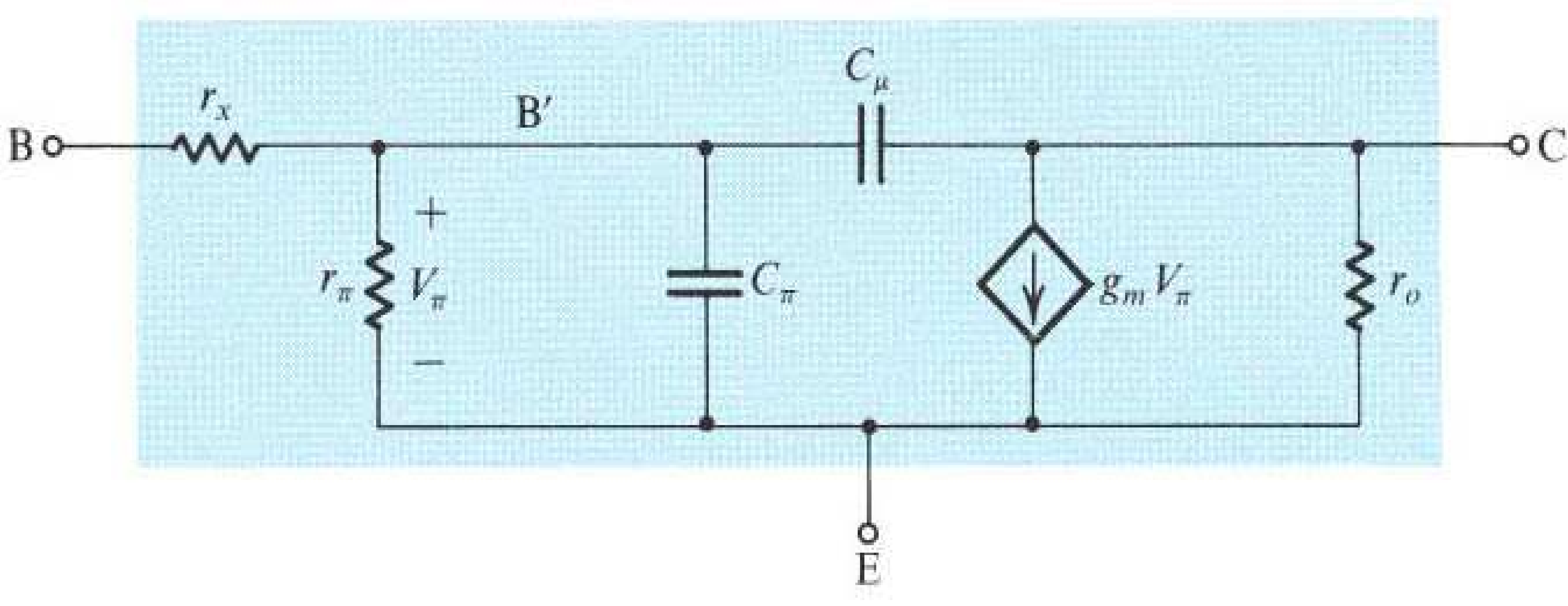
- The collector-base junction capacitance

$$C_{\mu} = \frac{C_{\mu 0}}{(1 + \frac{V_{CB}}{V_{oc}})^m}$$

$C_{\mu 0}$:is the value of C_{μ} at zero voltage

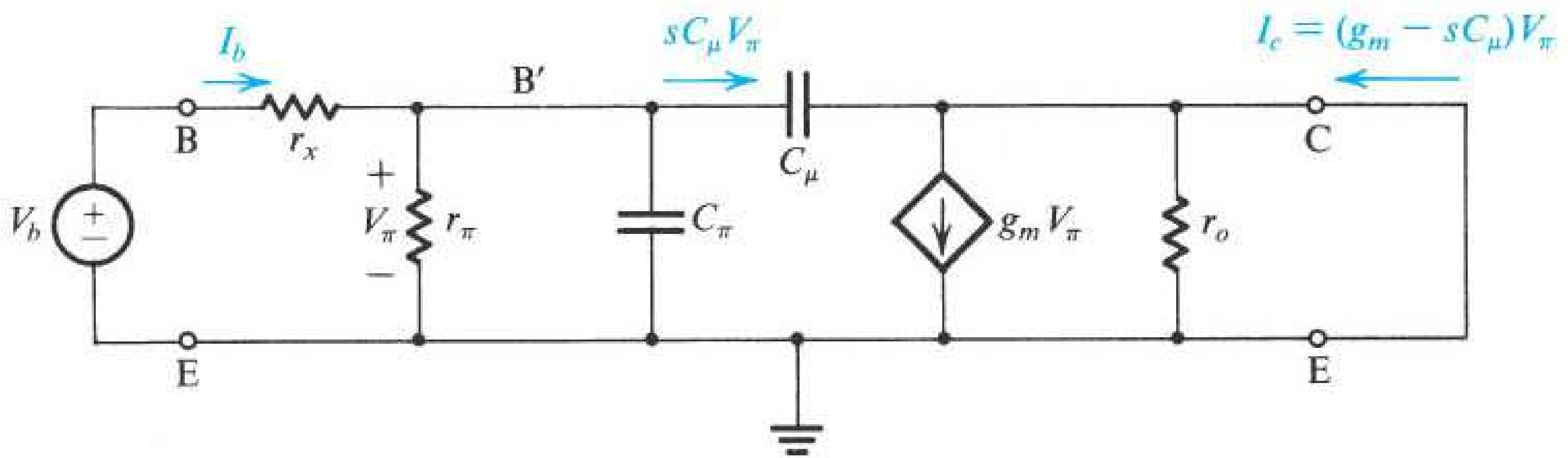
V_{oc} : is the CBJ bulid in voltage (typically , 0.75v)

The High Frequency Hybrid-π Model



- Two capacitances C_π and C_μ, where $\boxed{C_\pi = C_{de} + C_{je}}$
- One resistance r_x. Accurate value is obtained from high frequency measurement. $r_x << r_\pi$

The Cutoff and Unity Gain Frequencies



- Circuit for deriving an expression for $h_{fe}(s) \equiv \left.\frac{I_C}{I_B}\right|_{v_{CE}=0}$
- According to the definition, output port is short circuit

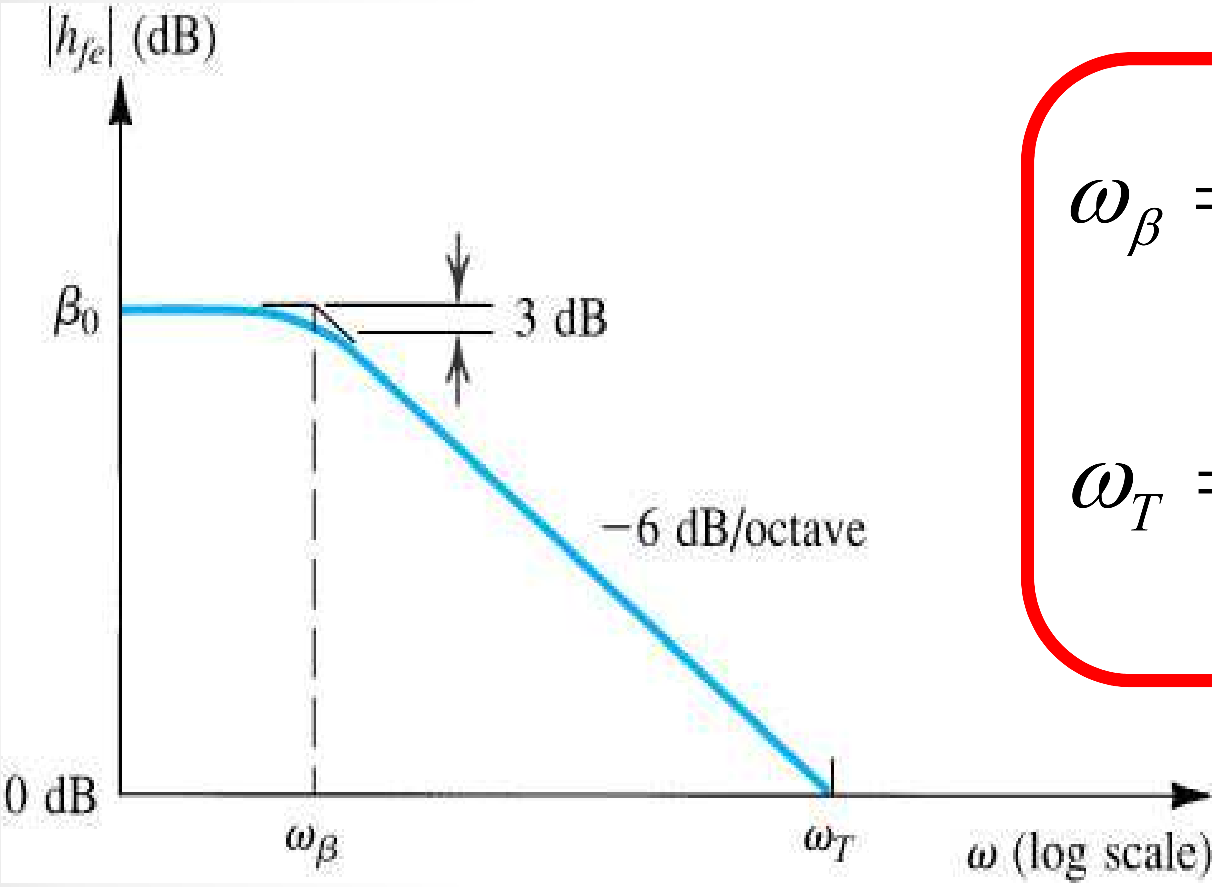
The Cutoff and Unity Gain Frequencies

- Expression of the short-circuit current transfer function

$$\boxed{h_{fe}(s) = \frac{\beta_0}{1 + s(C_\pi + C_\mu)r_\pi}}$$

- Characteristic is similar to the one of first-order low-pass filter

The Cutoff and Unity Gain Frequencies



$$\omega_\beta = \frac{1}{(C_\pi + C_\mu) r_\pi}$$

$$\omega_T = \beta_0 \omega_\beta = \frac{g_m}{(C_\pi + C_\mu)}$$

The MOSFET Internal Capacitances and High Frequency Model

- Internal capacitances
  - The gate capacitive effect
    - Triode region
    - Saturation region
    - Cutoff region
    - Overlap capacitance
  - The junction capacitances
    - Source-body depletion-layer capacitance
    - drain-body depletion-layer capacitance
- High-frequency model

The Gate Capacitive Effect

- MOSFET operates at triode region

$$C_{gs} = C_{gd} = \tfrac{1}{2} WLC_{ox}$$

- MOSFET operates at saturation region

$$\begin{cases} C_{gs} = \tfrac{2}{3} WLC_{ox} \\ C_{gd} = 0 \end{cases}$$

- MOSFET operates at cutoff region

$$\begin{cases} C_{gs} = C_{gd} = 0 \\ C_{gb} = WLC_{ox} \end{cases}$$

Overlap Capacitance

- Overlap capacitance results from the fact that the source and drain diffusions extend slightly under the gate oxide.
- The expression for overlap capacitance

$$C_{ov} = WL_{ov}C_{ox}$$

- Typical value

$$L_{ov} = 0.05 - 0.1L$$

- This additional component should be added to C_{gs} and C_{gd} in all preceding formulas.

Junction Capacitances

- Source-body depletion-layer capacitance

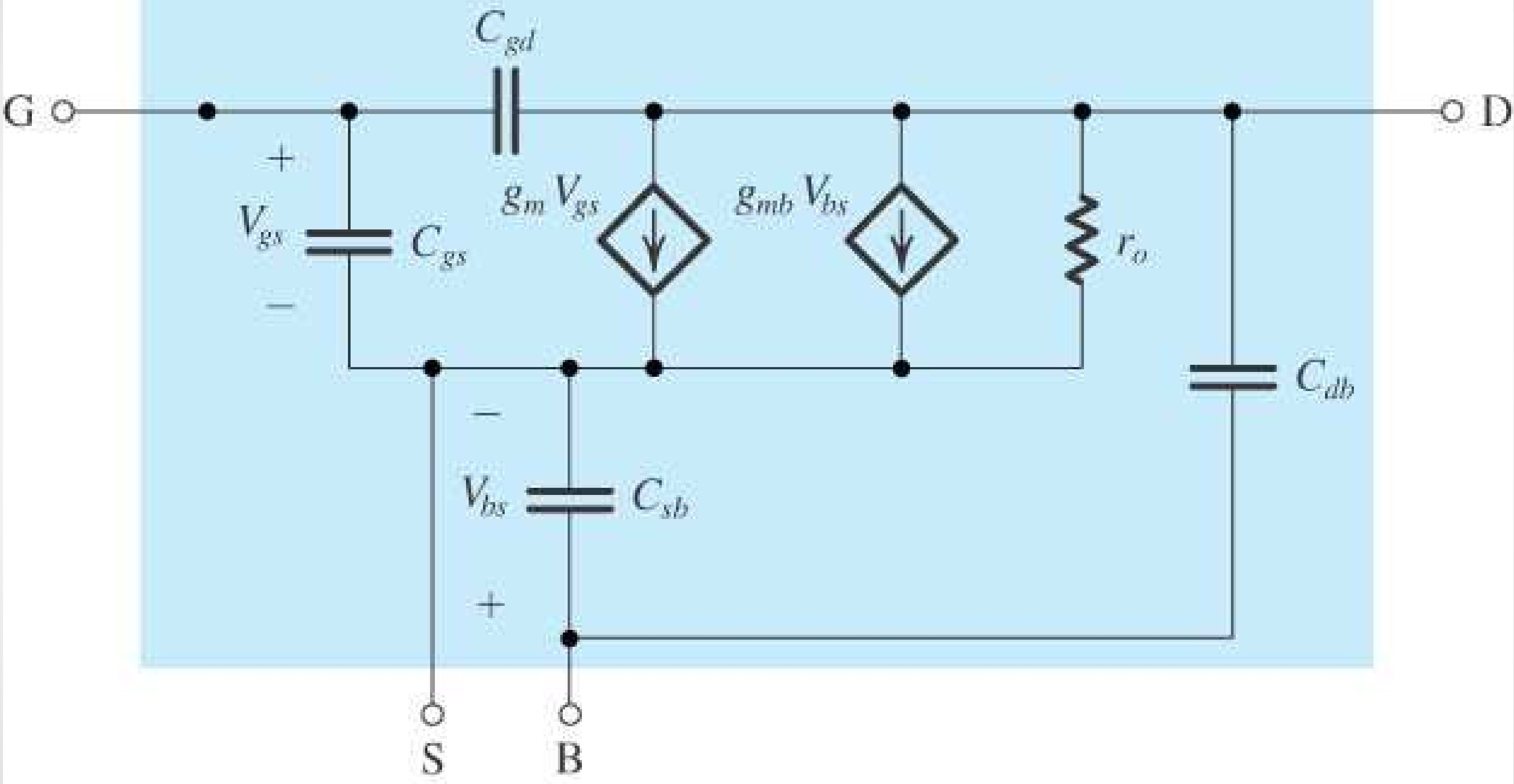
$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}}$$

- drain-body depletion-layer capacitance

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}}$$

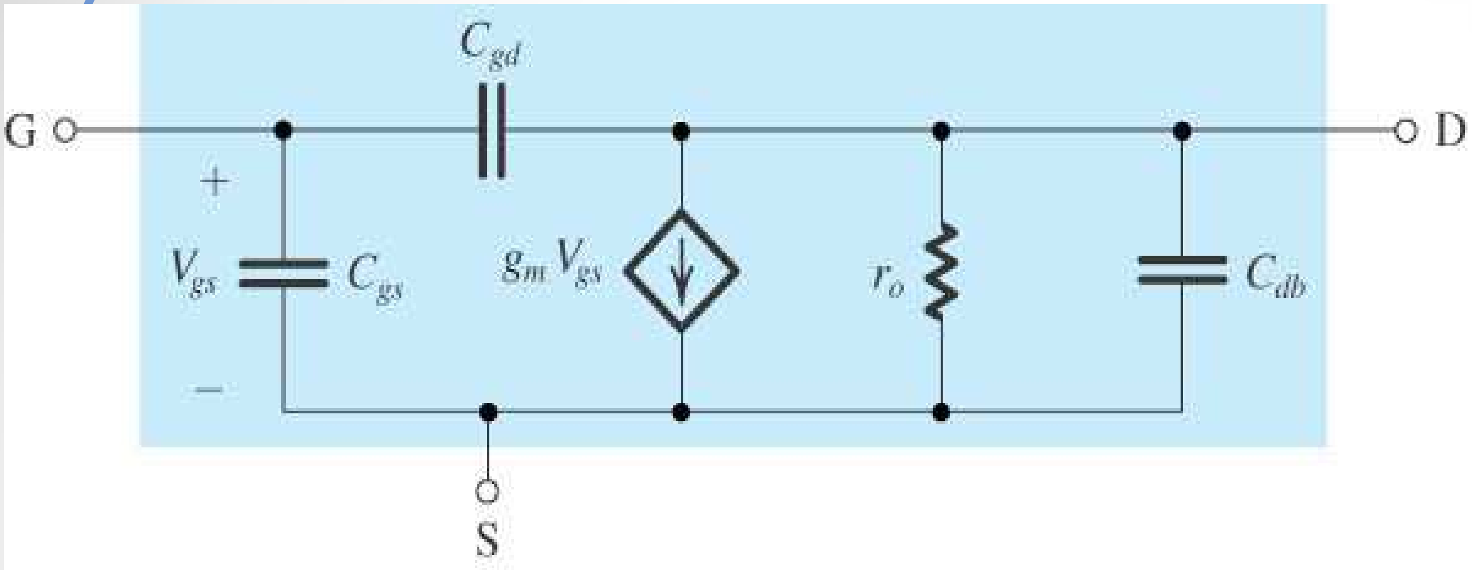
- V_0 is the junction build-in potential

High Frequency MOSFET Model

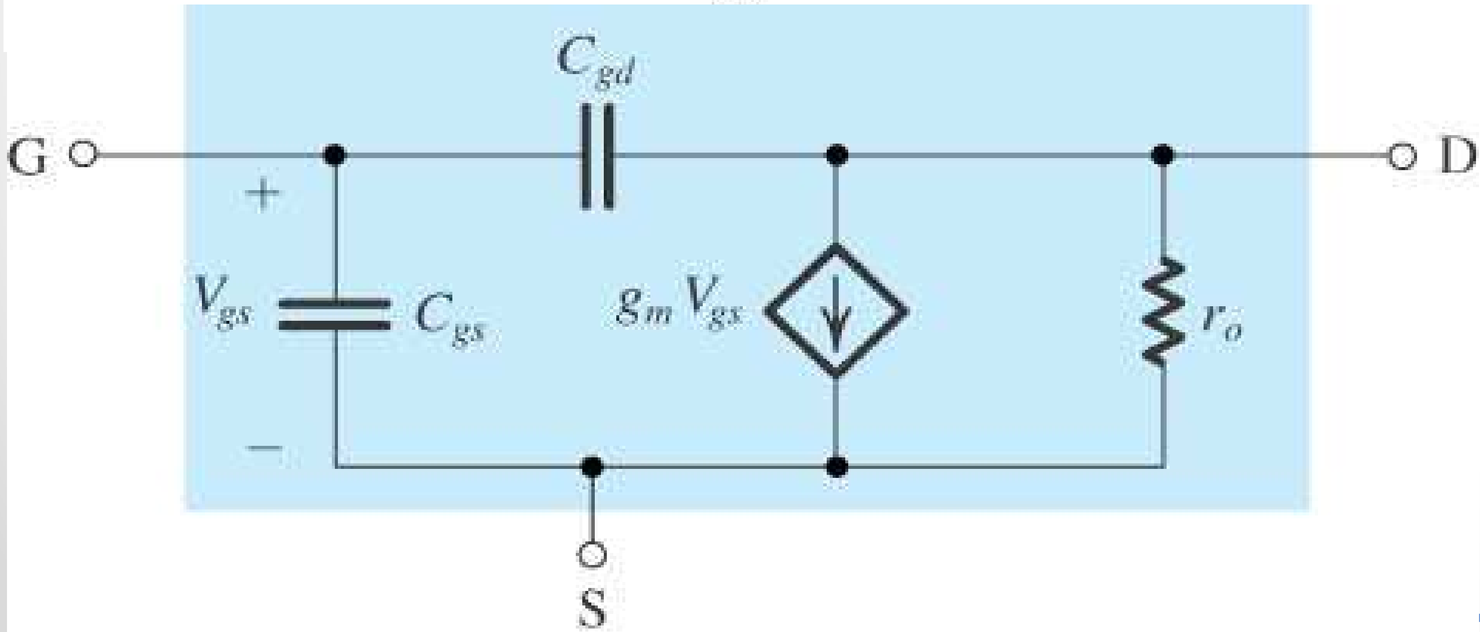


(a)

High Frequency MOSFET Model



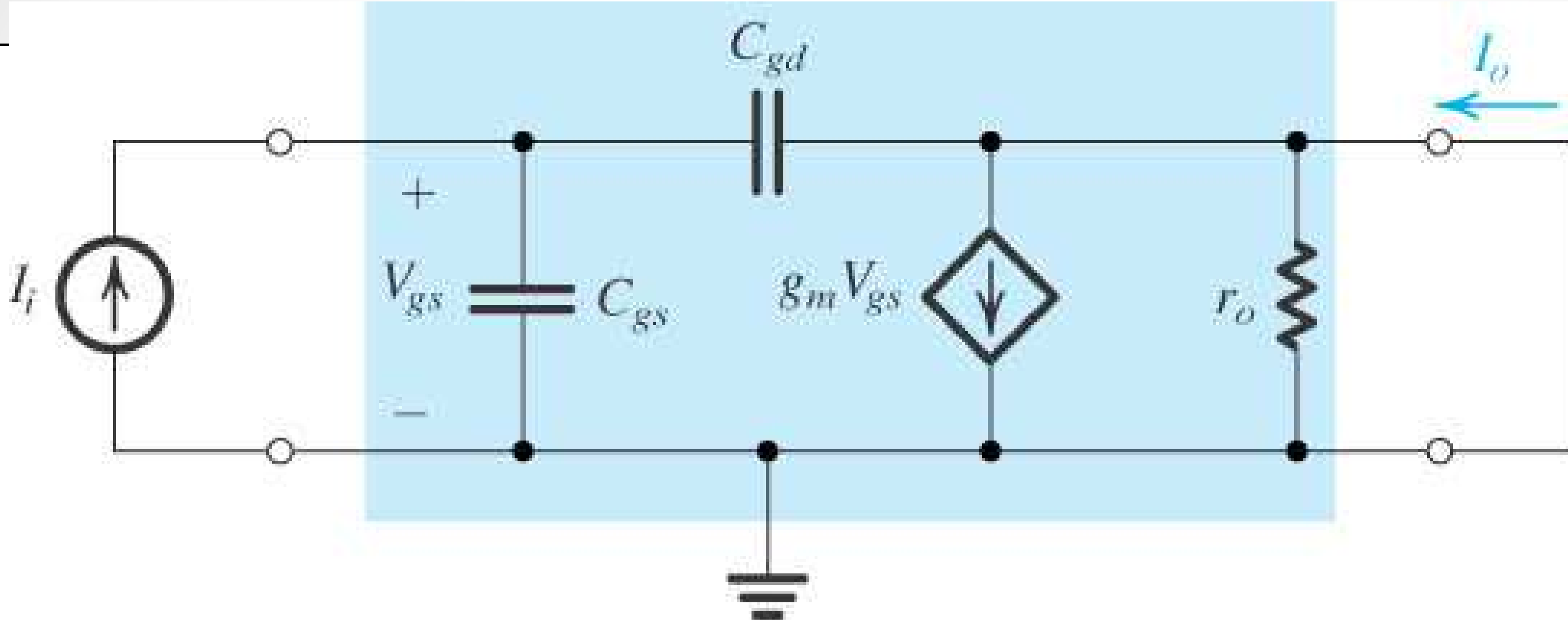
(b) The equivalent circuit for the case in which the source is connected to the substrate (body).



(c)

(c) The equivalent circuit model of (b) with C_{db} neglected (to simplify analysis).

The MOSFET Unity-Gain Frequency



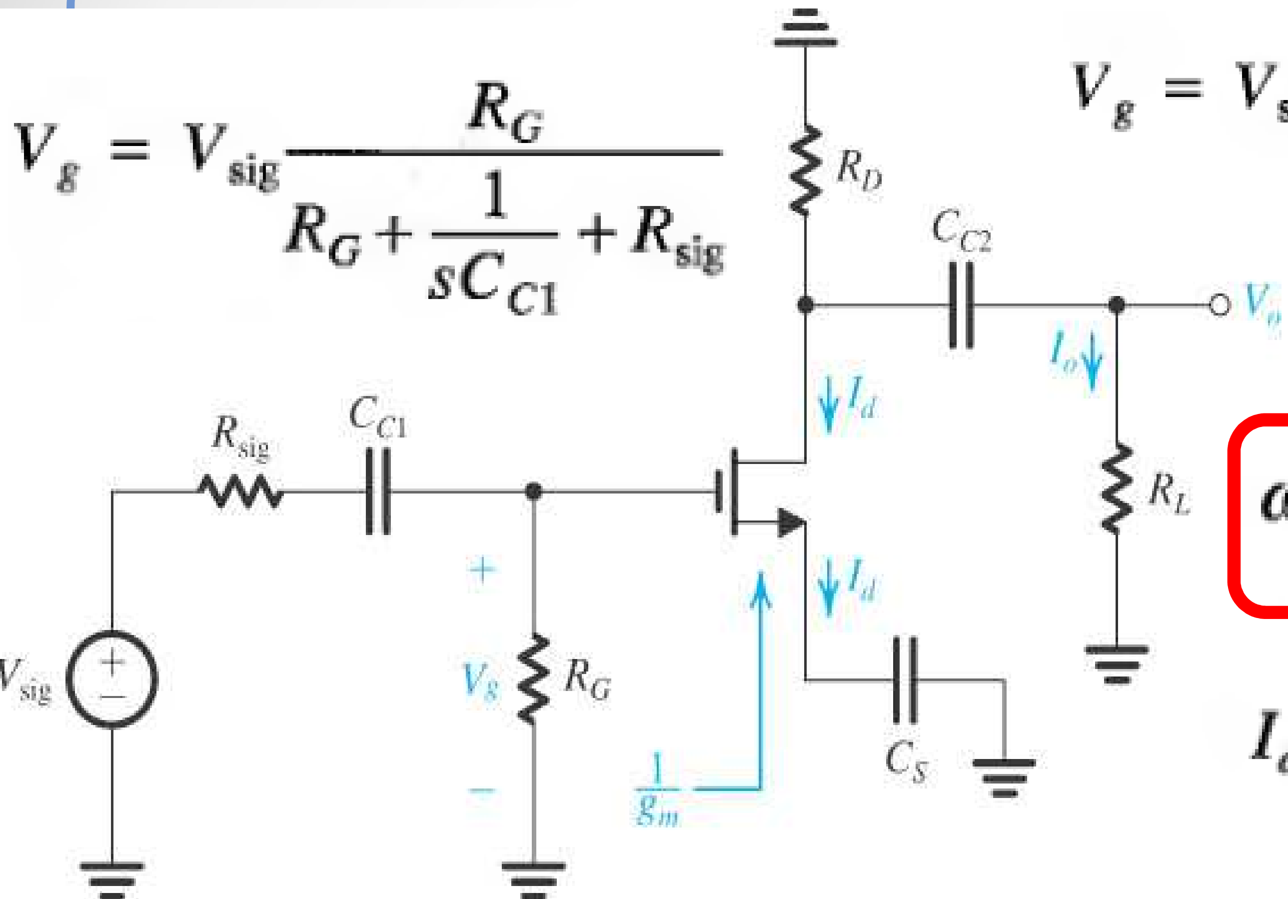
- Current gain

$$\frac{I_o}{I_i} = \frac{g_m}{s(C_{gs} + C_{gd})}$$

- Unity-gain frequency $$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

Low Frequency Response of the CS and CE Amplifiers

$$V_g = V_{sig}\frac{R_G}{R_G + \dfrac{1}{sC_{C1}} + R_{sig}}$$



$$V_g = V_{sig}\frac{R_G}{R_G + R_{sig}}\frac{s}{s + \dfrac{1}{C_{C1}(R_G + R_{sig})}}$$

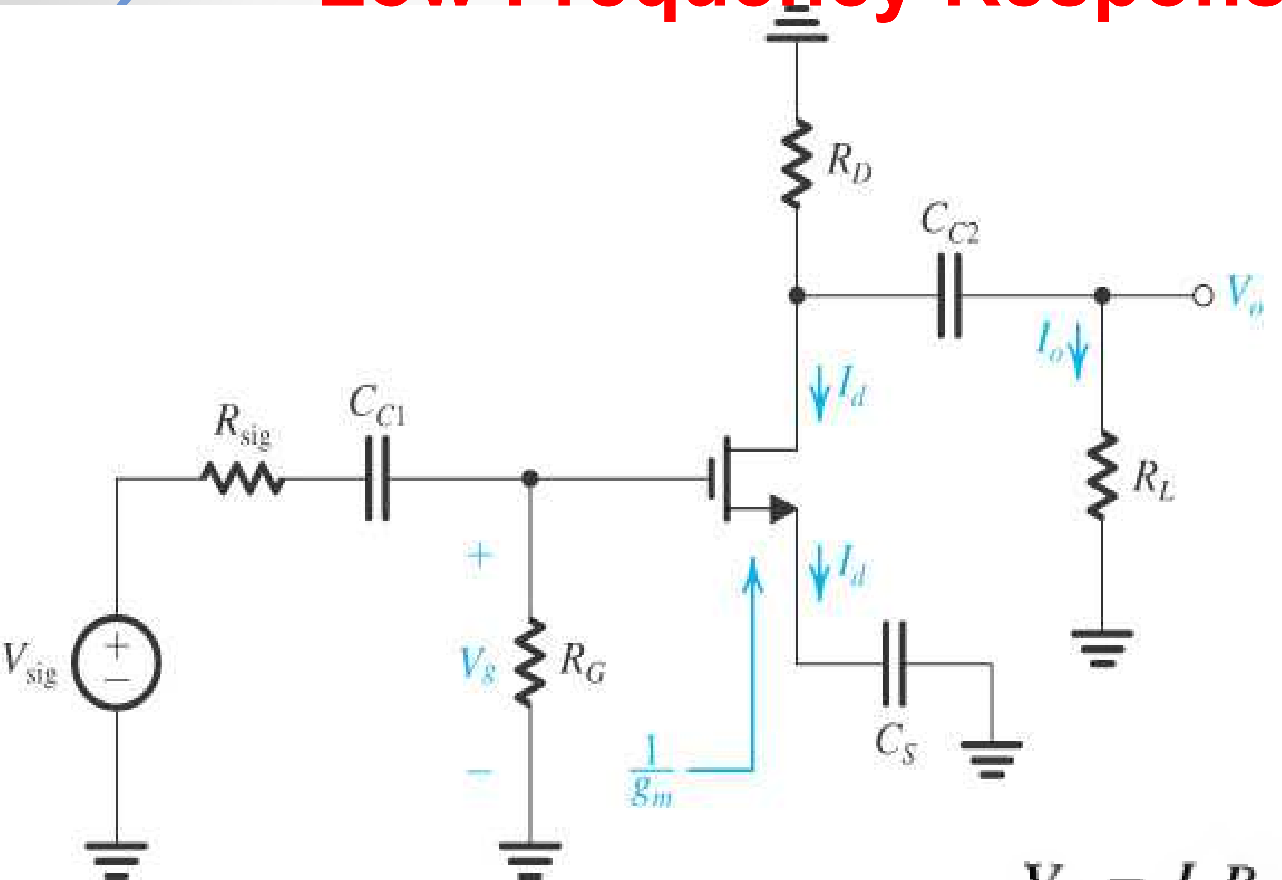
$$\omega_{P1} = \omega_0 = \frac{1}{C_{C1}(R_G + R_{sig})}$$

$$I_d = \frac{V_g}{\dfrac{1}{g_m} + \dfrac{1}{sC_S}}$$

$$I_d = g_m V_g \frac{s}{s + \dfrac{g_m}{C_S}}$$

$$\omega_{P2} = \frac{g_m}{C_S}$$

Low Frequency Response of the CS

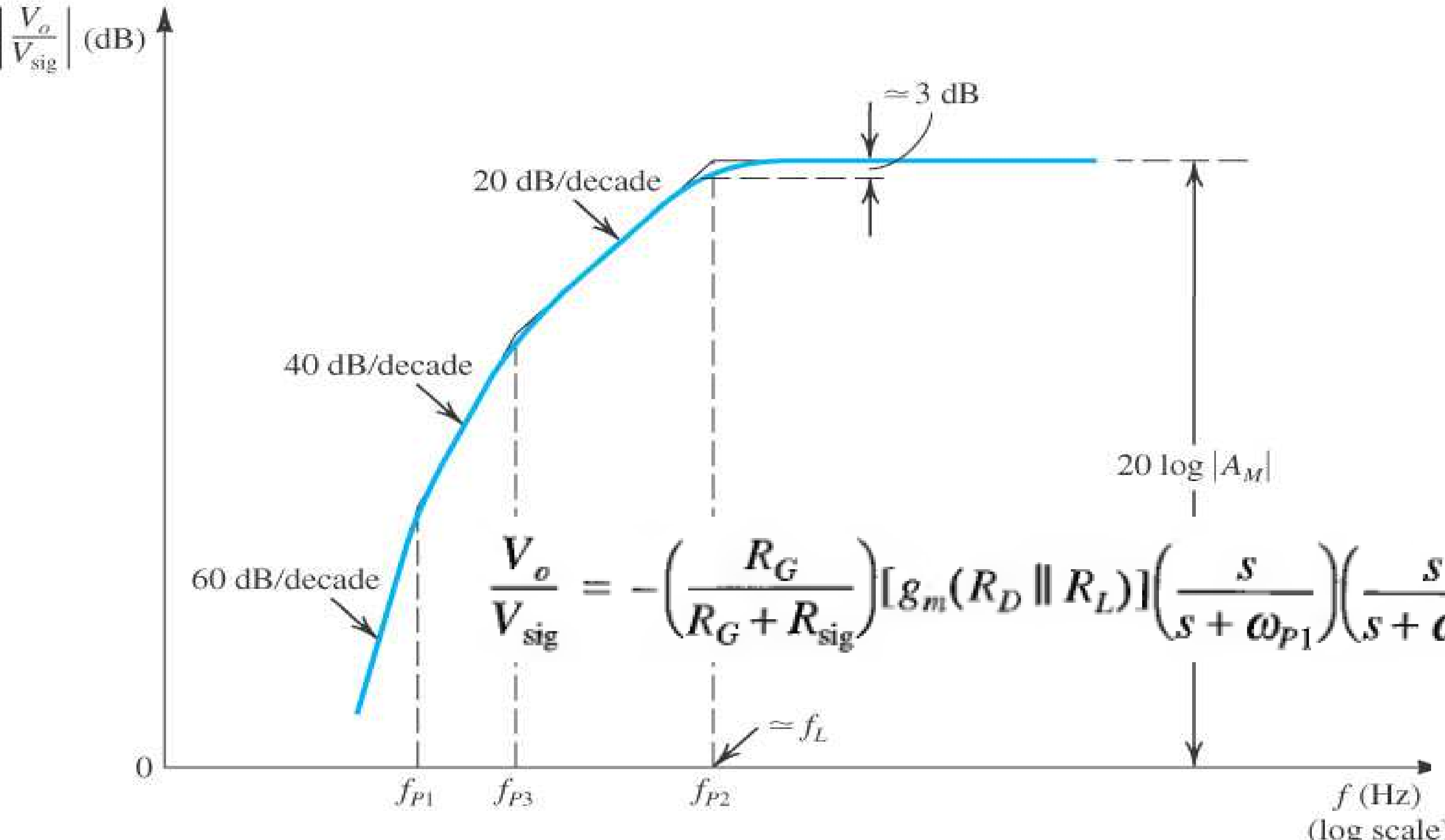


$$\omega_{P3} = \frac{1}{C_{C2}(R_D + R_L)}$$

$$I_o = -I_d \frac{R_D}{R_D + \frac{1}{sC_{C2}} + R_L}$$

$$V_o = I_o R_L = -I_d \frac{R_D R_L}{R_D + R_L} \frac{s}{s + \frac{1}{C_{C2}(R_D + R_L)}}$$

$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right)[g_m(R_D \parallel R_L)]\left(\frac{s}{s + \omega_{P1}}\right)\left(\frac{s}{s + \omega_{P2}}\right)\left(\frac{s}{s + \omega_{P3}}\right)$$



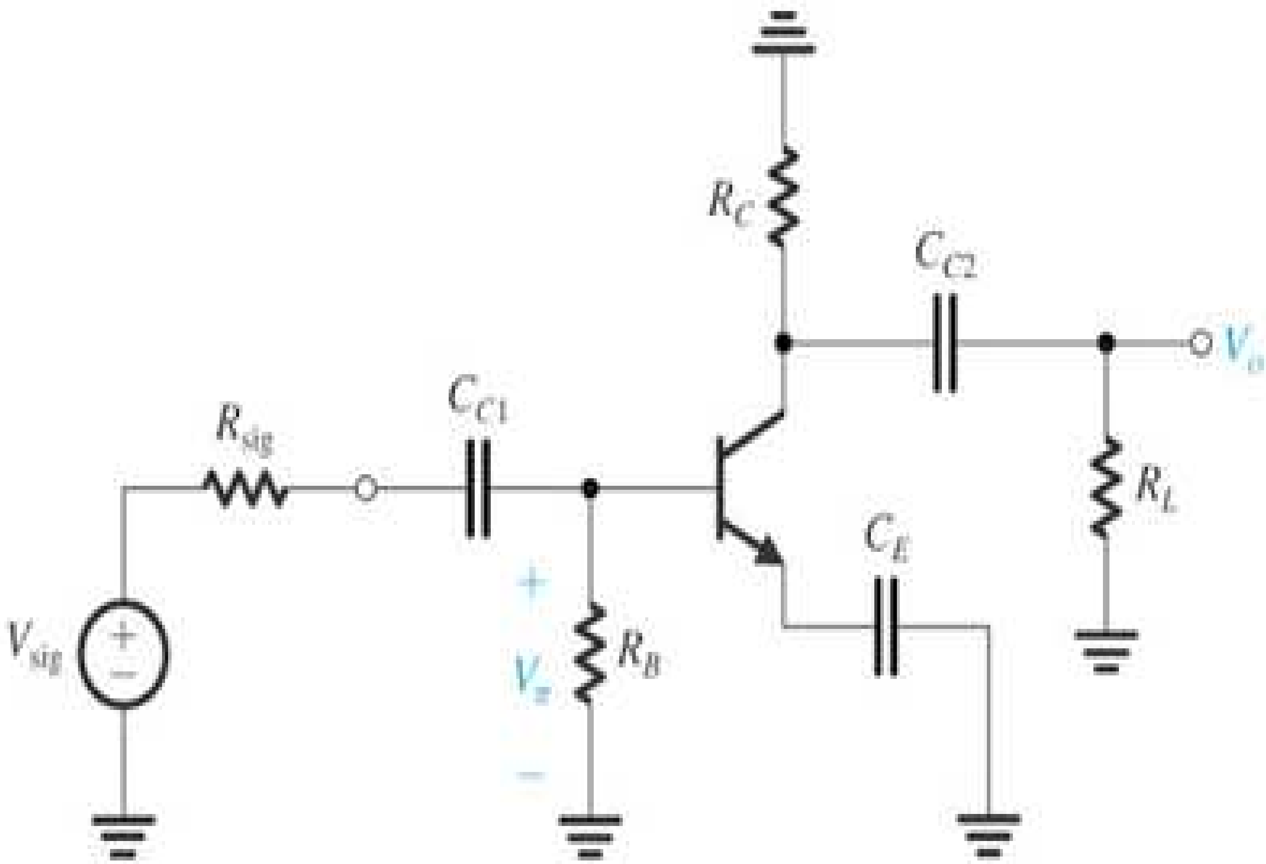
$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right)[g_m(R_D \parallel R_L)]\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

$$f_L \cong f_{P2}$$

The procedure to find quickly the time constant:

1. Reduce Vsig to zero
2. Consider each capacitor separately; that is, assume that the other capacitors are as perfect short circuits
3. For each capacitor, find the total resistance seen between its terminals

Analysis of CE Amplifier



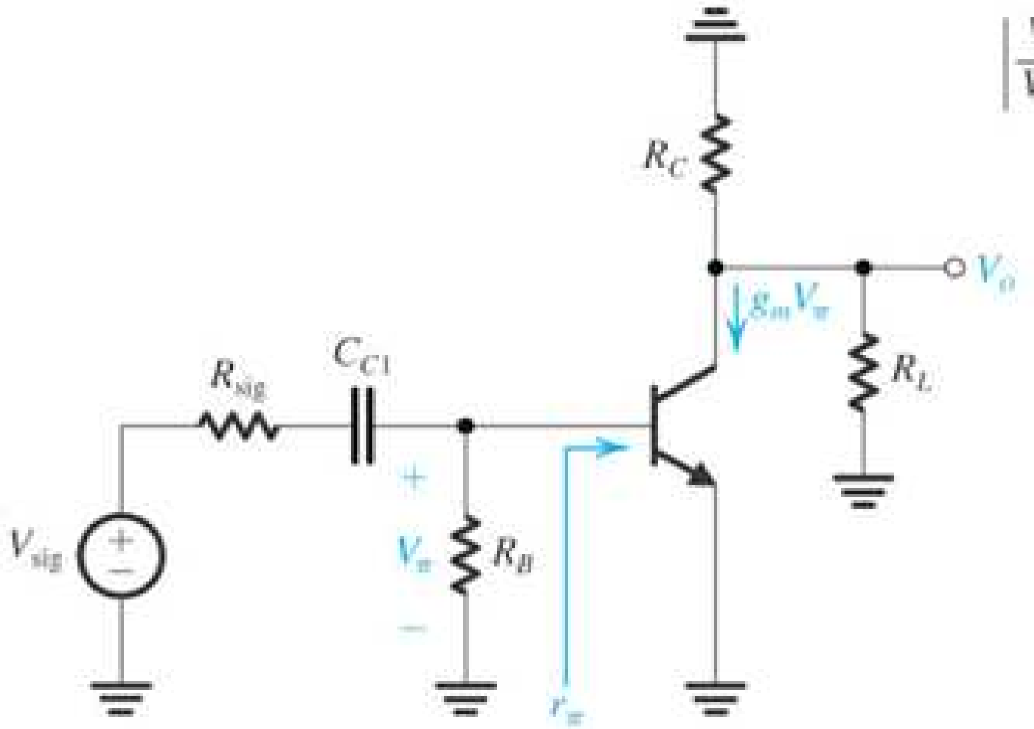
(a)

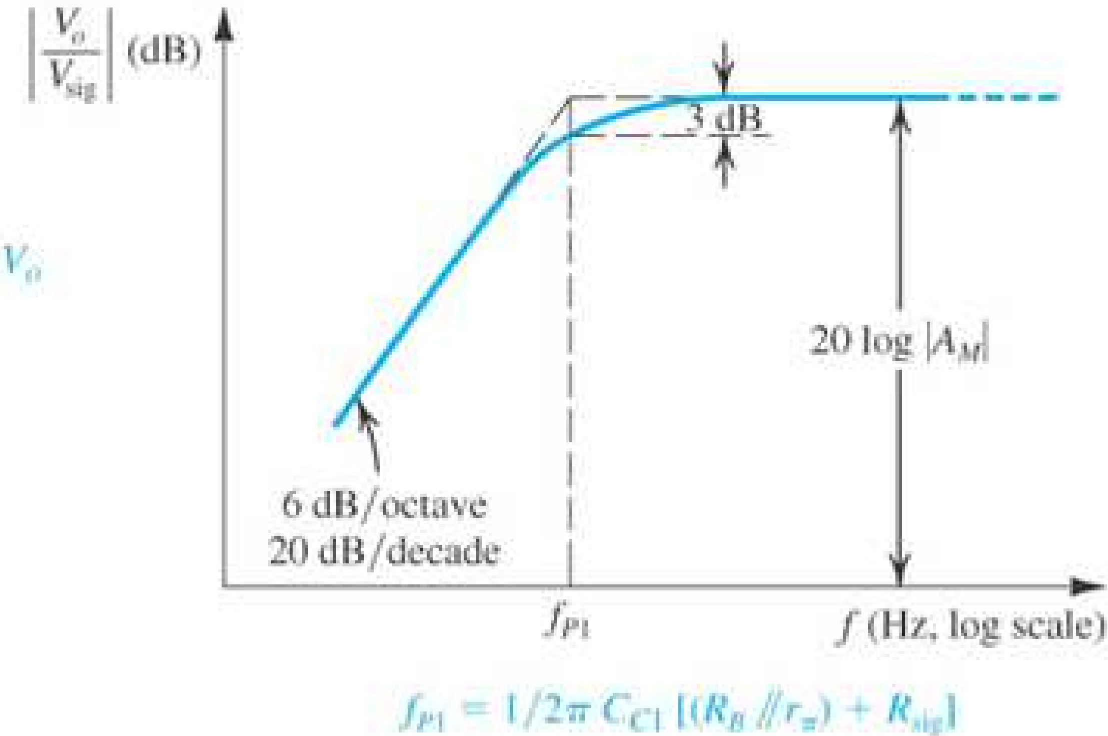
Low frequency small signal analysis:

1) Eliminate the DC source

2) Ignore C_π and C_μ and r_o

3) Ignore r_x, which is much smaller than r_π



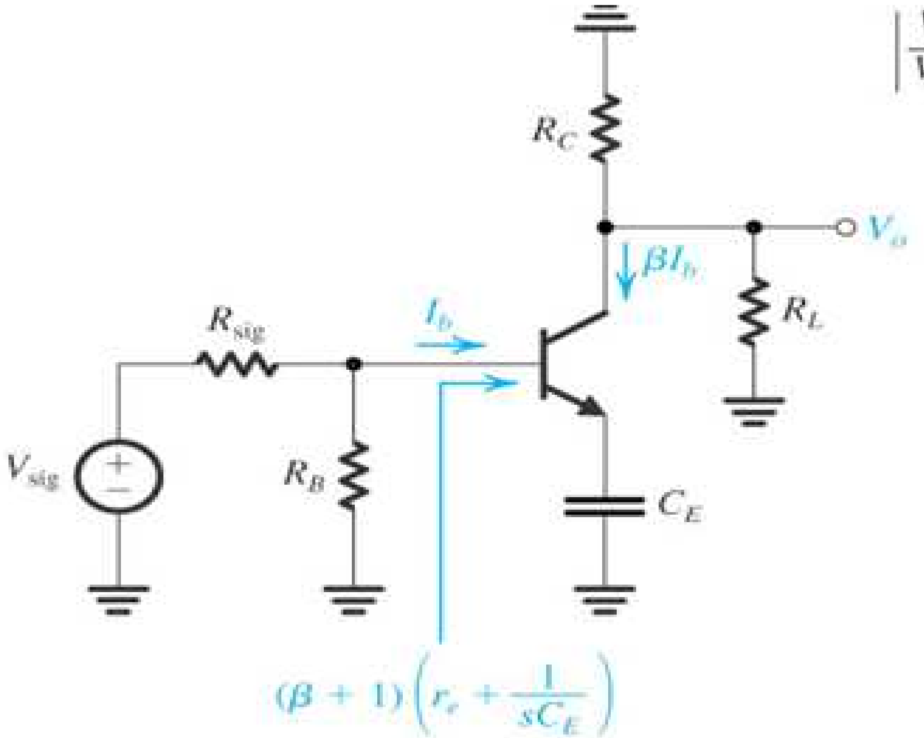


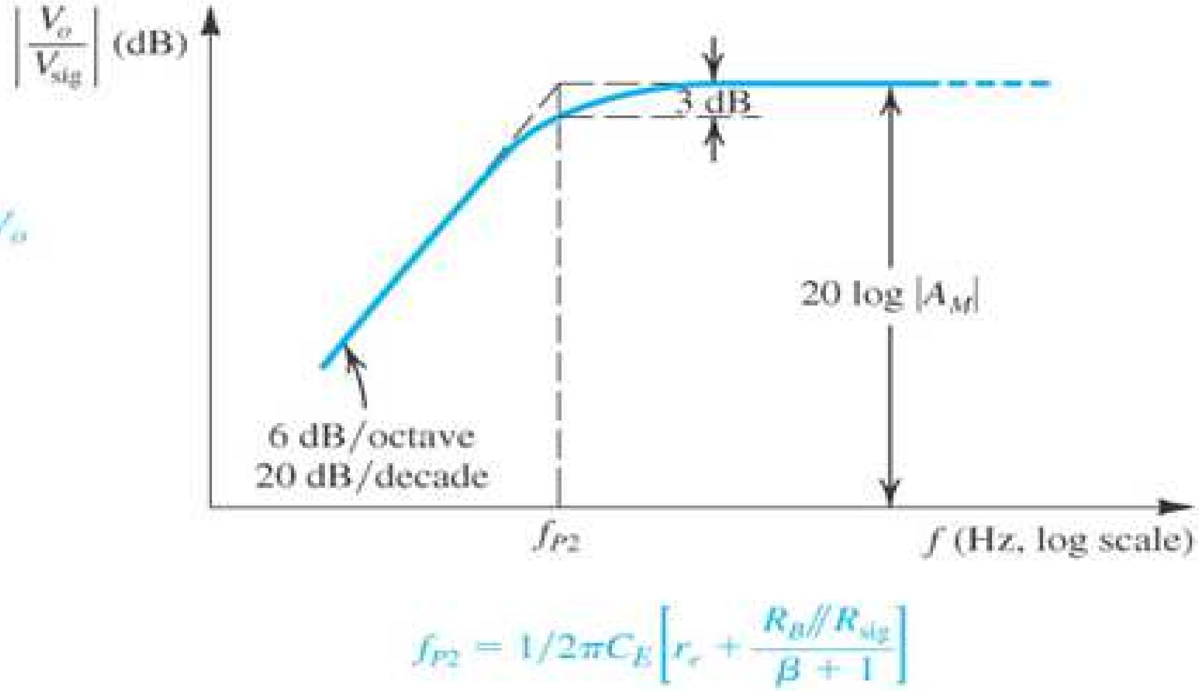
(b) the effect of C_{C1} is determined with C_E and C_{C2} assumed to be acting as perfect short circuits;

$$V_\pi = V_{sig} \frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{sig} + \frac{1}{sC_{C1}}}$$

$$\frac{V_o}{V_{sig}} = -\frac{(R_B \| r_\pi)}{(R_B \| r_\pi) + R_{sig}} g_m (R_C \| R_L) \left[\frac{s}{s + \frac{1}{C_{C1}[(R_B \| r_\pi) + R_{sig}]}} \right]$$

$$V_o = -g_m V_\pi (R_C \| R_L) \qquad \omega_{P1} = \frac{1}{C_{C1}[(R_B \| r_\pi) + R_{sig}]}$$





(c)

(c) the effect of *CE* is determined with *CC*1 and *CC*2 assumed to be acting as perfect short circuits

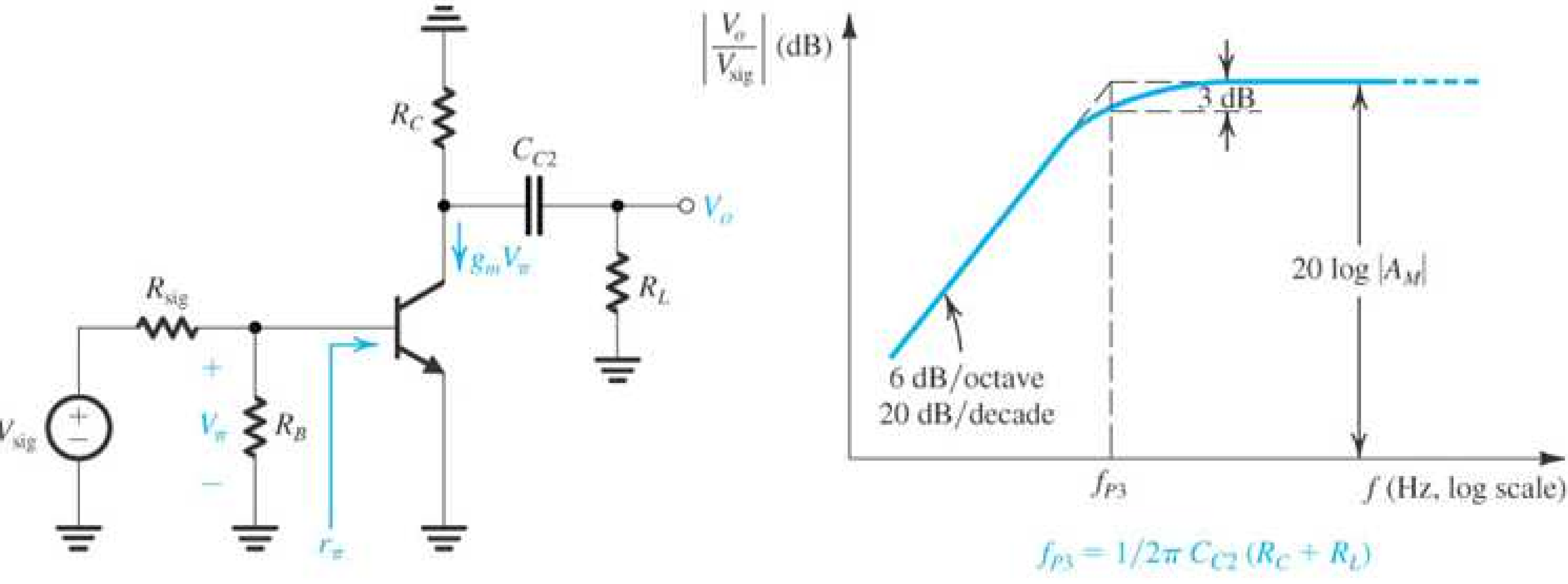
$$I_b = V_{sig}\frac{R_B}{R_B + R_{sig}}\frac{1}{(R_B \parallel R_{sig}) + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)}$$

$$V_o = -\beta I_b (R_C \parallel R_L)$$

$$= -\frac{R_B}{R_B + R_{sig}}\frac{\beta(R_C \parallel R_L)}{(R_B \parallel R_{sig}) + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)}V_{sig}$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}}\frac{\beta(R_C \parallel R_L)}{(R_B \parallel R_{sig}) + (\beta + 1)r_e}\frac{s}{s + \left[1/C_E\left(r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}\right)\right]}$$

$$\omega_{P2} = \frac{1}{C_E\left[r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}\right]}$$



(d)

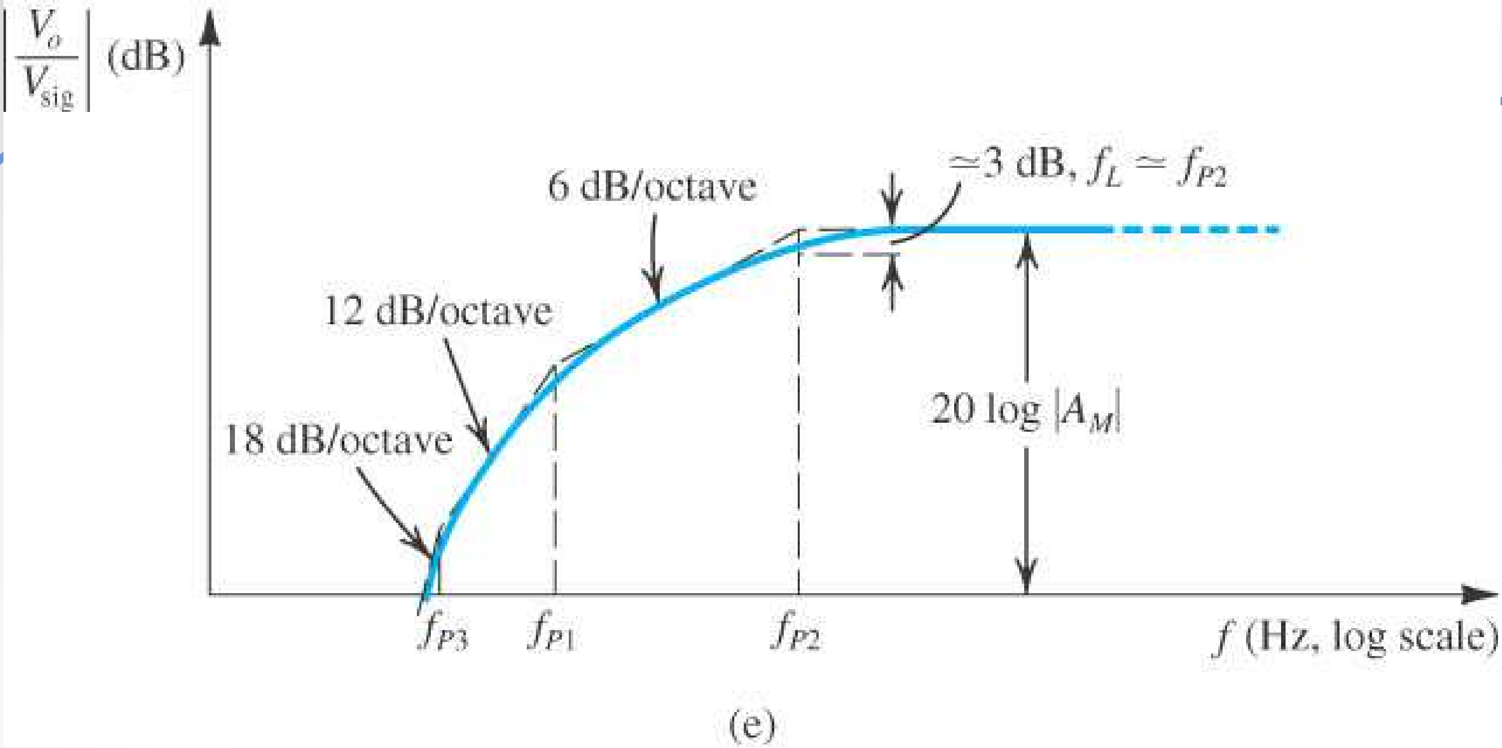
(d) the effect of C_{C2} is determined with C_{C1} and C_E assumed to be acting as perfect short circuits;

$$V_\pi = V_{sig}\frac{R_B \parallel r_\pi}{(R_B \parallel r_\pi) + R_{sig}}$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B \parallel r_\pi}{(R_B \parallel r_\pi) + R_{sig}} g_m (R_C \parallel R_L)\left[\frac{s}{s + \frac{1}{C_{C2}(R_C + R_L)}}\right]$$

$$V_o = -g_m V_\pi \frac{R_C}{R_C + \frac{1}{sC_{C2}} + R_L} R_L$$

$$\omega_{P3} = \frac{1}{C_{C2}(R_C + R_L)}$$



(e)

(e) sketch of the low-frequency gain under the assumptions that C_{C1}, C_E, and C_{C2} do not interact and that their break (or pole) frequencies are widely separated.

$$\frac{V_o}{V_{sig}} = -A_M\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

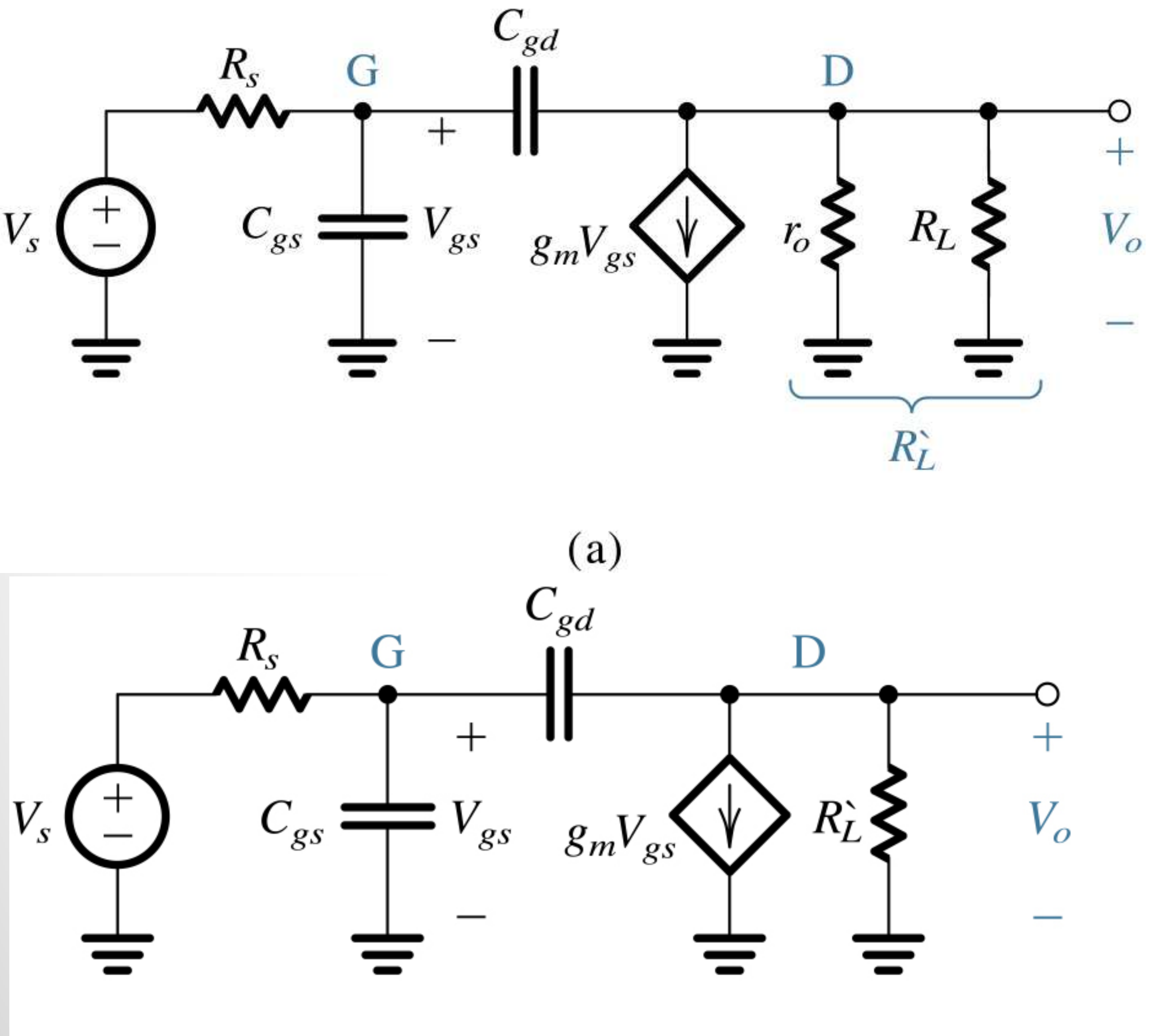
$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_E} + \frac{1}{C_{C2}R_{C2}}\right]$$

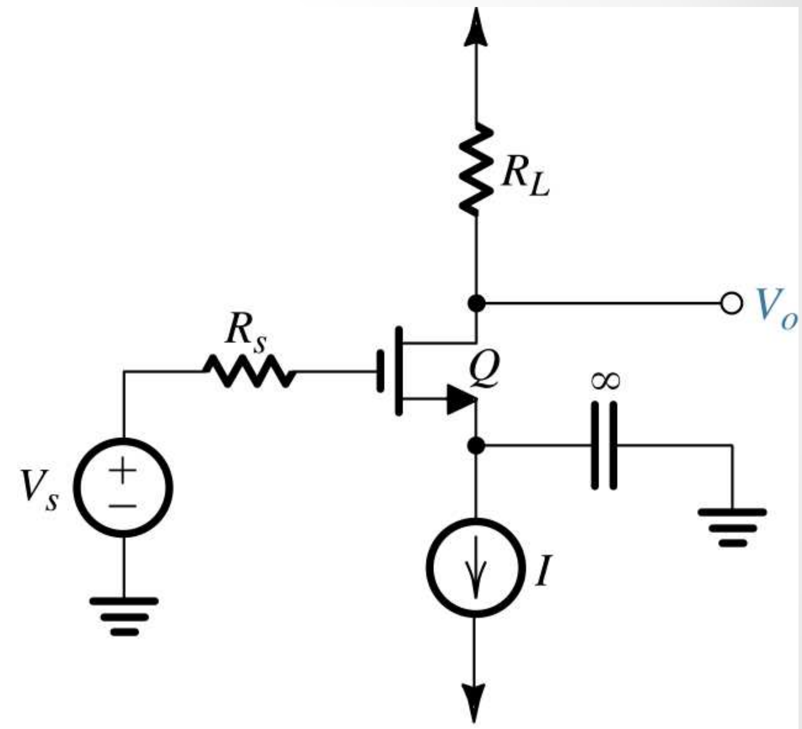
$$f_L = f_{P1} + f_{P2} + f_{P3}$$

High Frequency Response of the CS and CE Amplifiers

- Miller's theorem.
- Analysis of the high frequency response.
  - Using Miller's theorem.
  - Using open-circuit time constants.

High Frequency Equivalent Circuit Model of the CS Amplifier

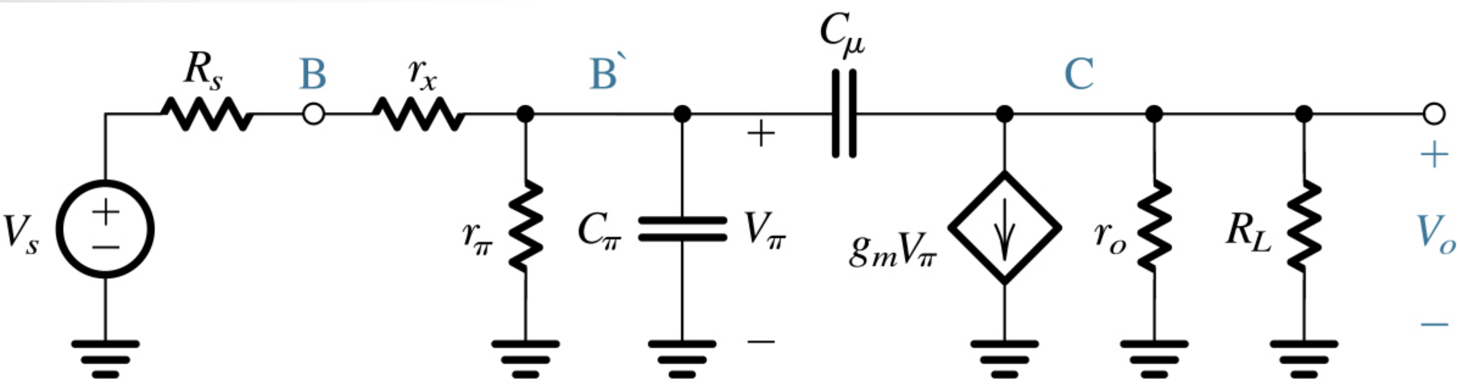




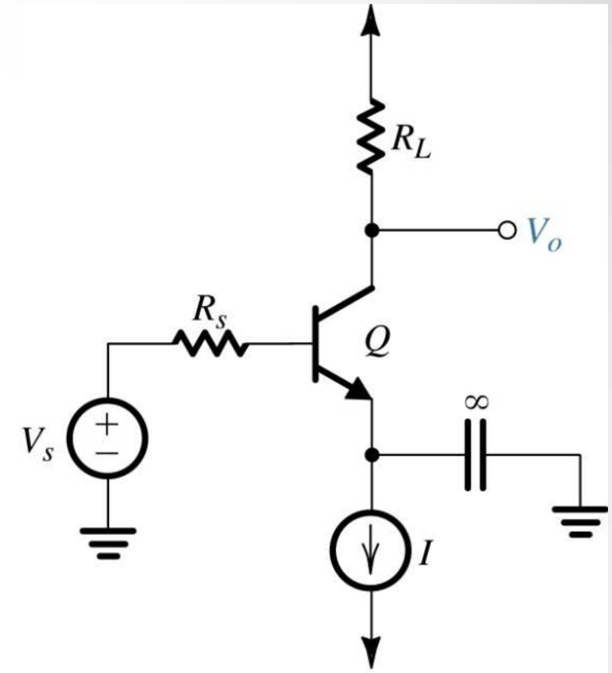
(a)

Fig. 7.16 (a) Equivalent circuit for analyzing the high-frequency response of the amplifier circuit of Fig. 7.15(a). Note that the MOSFET is replaced with its high-frequency equivalent-circuit. (b) A slightly simplified version of (a) by combining R_L and r_o into a single resistance $R'_L = R_L // r_o$.

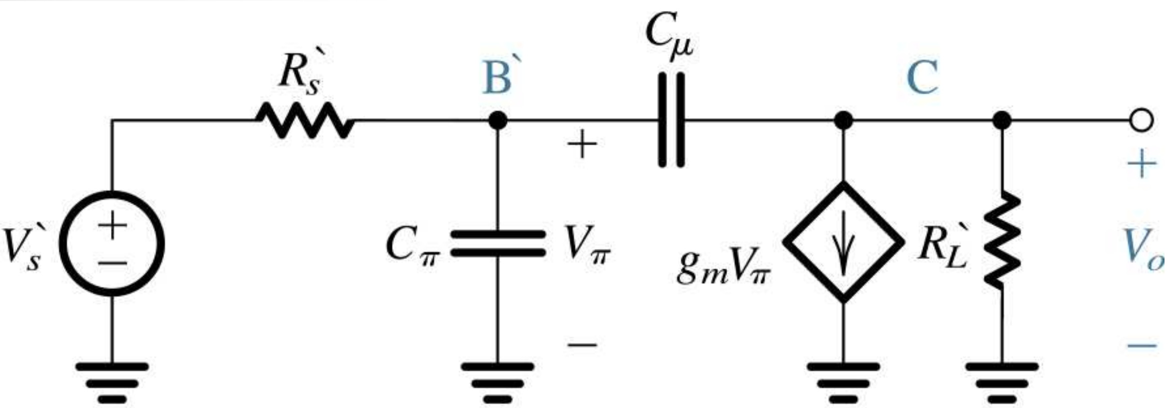
High Frequency Equivalent Circuit Model of the CE Amplifier



(a)



(b)



(b)

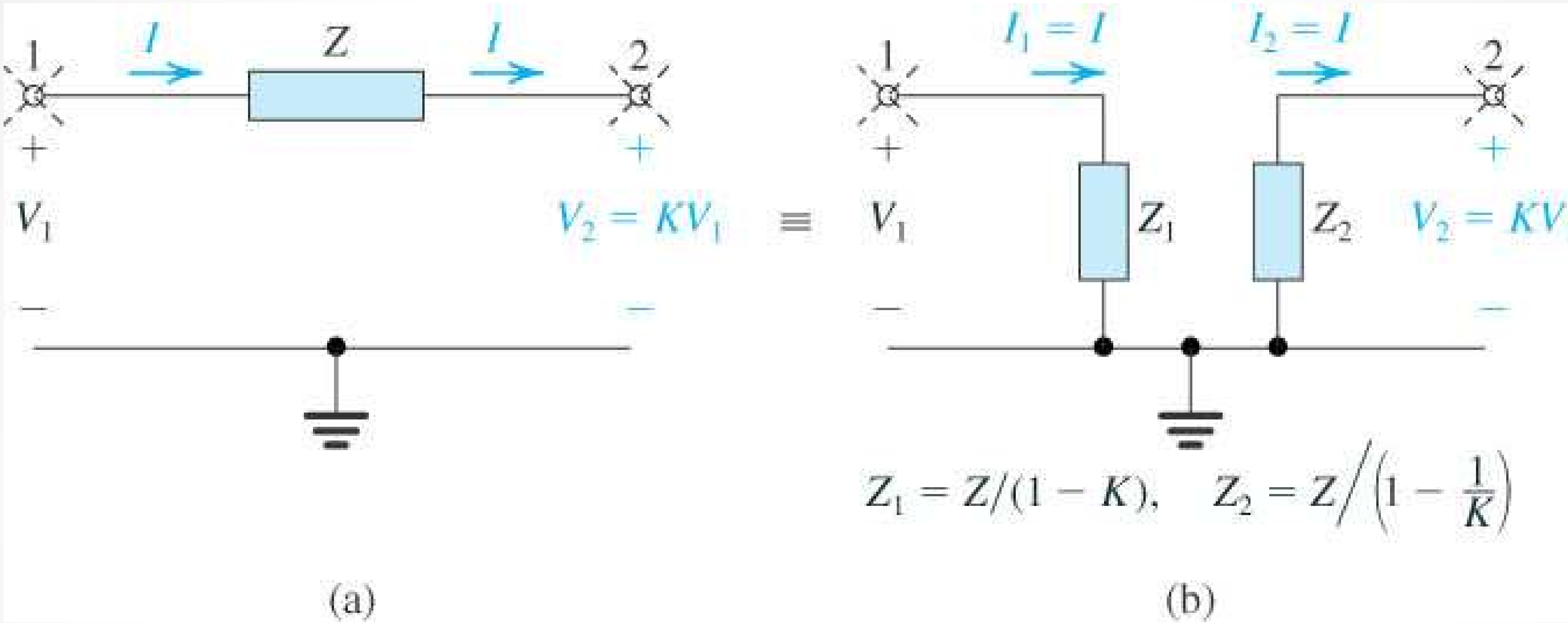
$$V_s' = V_s \frac{r_\pi}{R_s + r_s + r_\pi}$$

$$R_s' = \left(R_s + r_x\right) // r_\pi$$

$$R_L' = R_L // r_o$$

Fig. . 7.17 (a) Equivalent circuit for the analysis of the high-frequency response of the common-emitter amplifier of Fig. 7.15(b). Note that the BJT is replaced with its hybrid-Π high-frequency equivalent circuit. (b) An equivalent but simpler version of the circuit in (a),

Miller's Theorem



Impedance Z can be replaced by two impedances:

$$I_1 = \frac{V_1}{Z_1} = I = \left(\frac{V_1 - KV_1}{Z}\right)$$

Z_1 connected between node 1 and ground

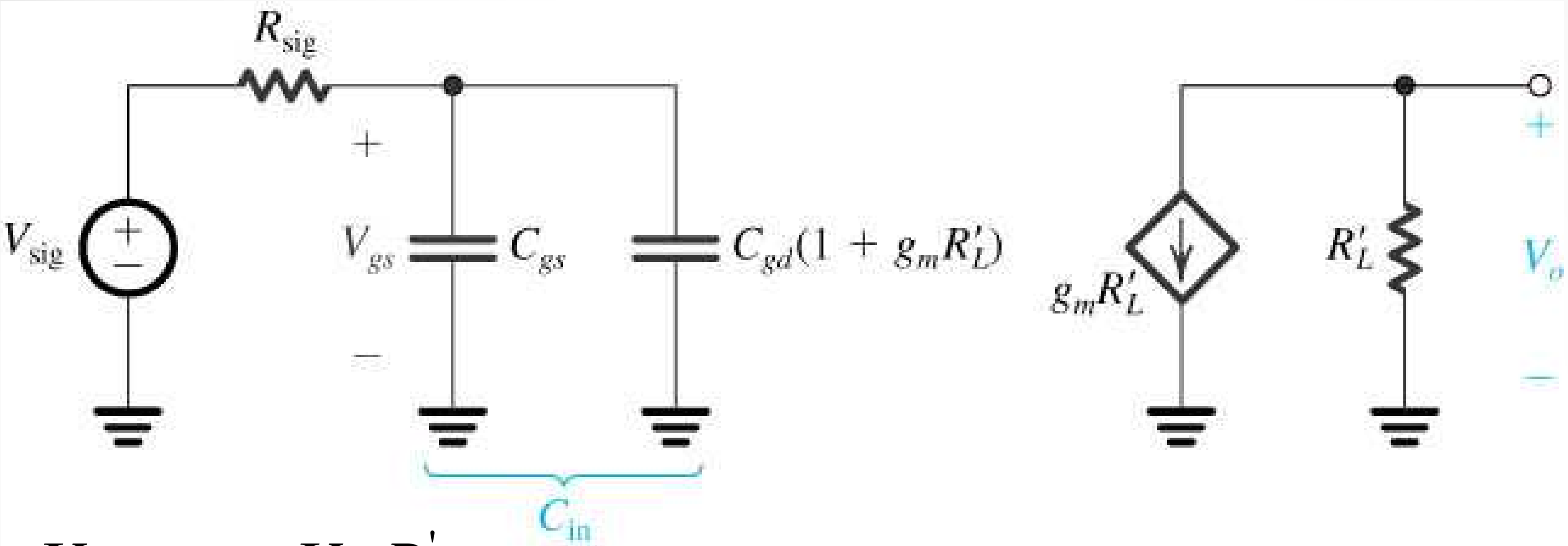
Z_2 connected between node 2 and ground

$$I_2 = \frac{0 - V_2}{Z_2} = \frac{0 - KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z}$$

Miller's Theorem

- The miller equivalent circuit is valid as long as the conditions that existed in the network when K was determined are not changed.
- Miller theorem can be used to determining the input impedance and the gain of an amplifier; it cannot be applied to determine the output impedance.

Analysis Using Miller's Theorem



$$V_o \approx -g_m V_{gs} R_L^{'}$$

Neglecting the current through Cgd

$$C_{eq} = C_{gd}(1 + g_m R_L^{'})$$

$$C_T = C_{gs} + C_{gd}(1 + g_m R_L^{'})$$

- Approximate equivalent circuit obtained by applying Miller's theorem.
- This model works reasonably well when R_{sig} is large.
- The high-frequency response is dominated by the pole formed by R_{sig} and C_T.

Analysis Using Miller's Theorem

- Using miller's theorem the bridge capacitance C_{gd} can be replaced by two capacitances which connected between node G and ground, node D and ground.
- The upper 3dB frequency is only determined by this pole.

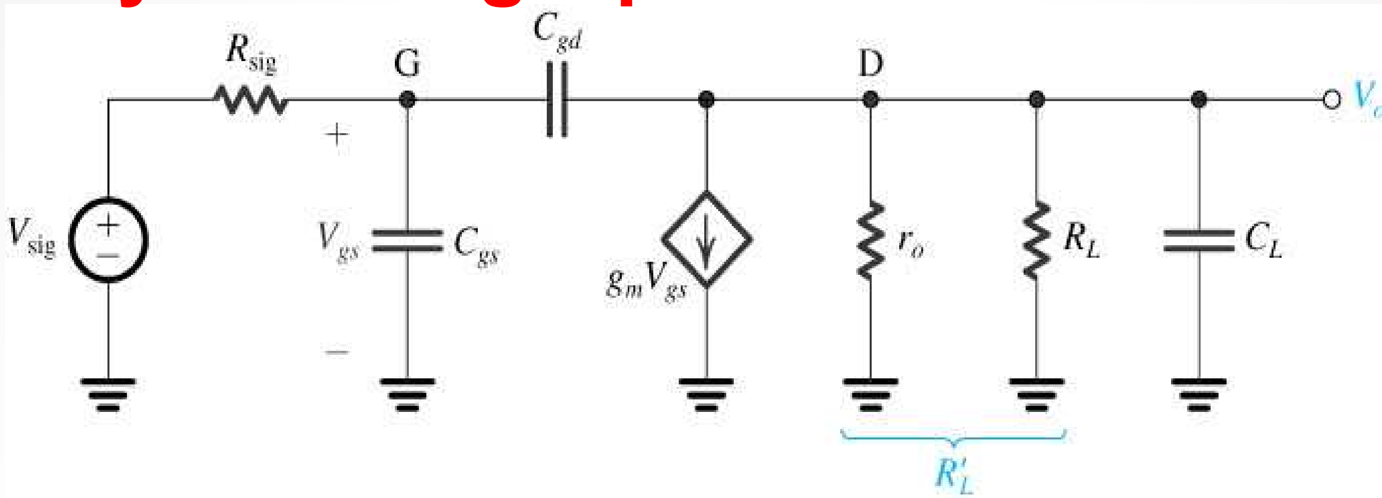
$$\frac{V_o}{V_{sig}} \cong \frac{A_M}{1 + \frac{s}{\omega_H}}$$

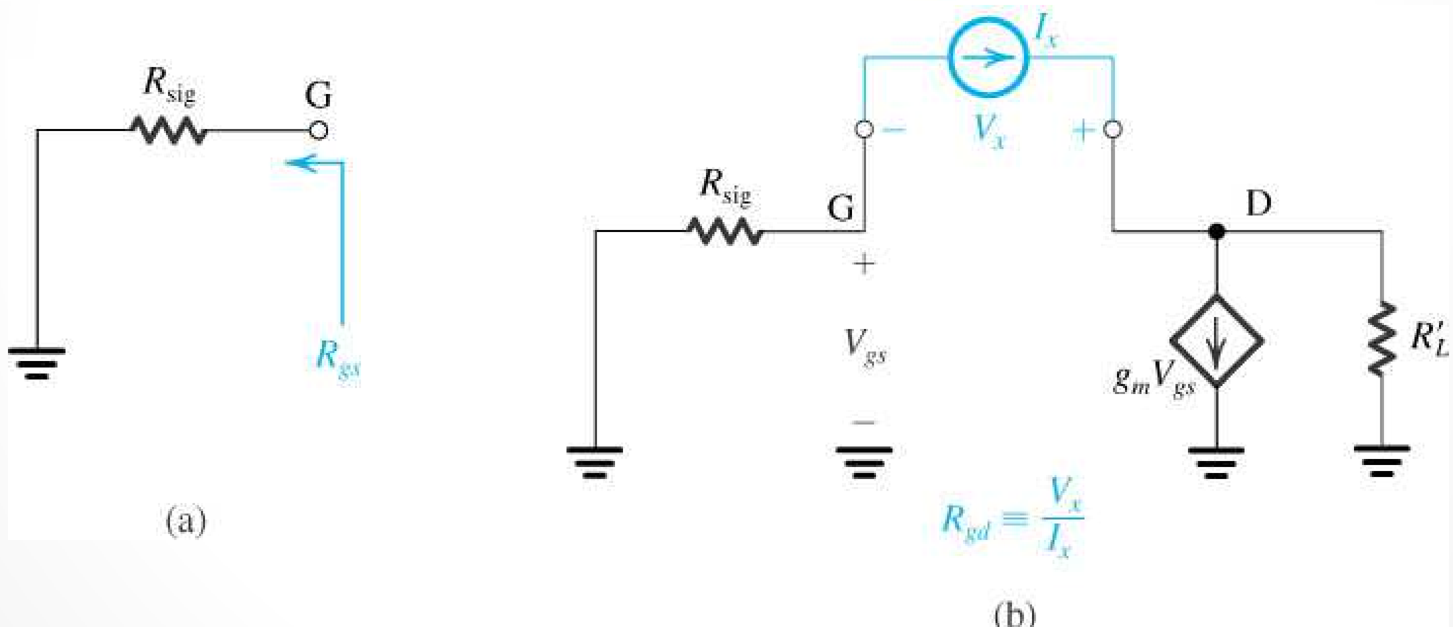
$$A_M = -g_m R_L'$$

$$C_T = C_{gs} + C_{gd}(1 + g_m R_L^{'})$$

$$f_H = \frac{1}{2\pi C_T R_{sig}}$$

Analysis Using Open-Ckt Time Constants

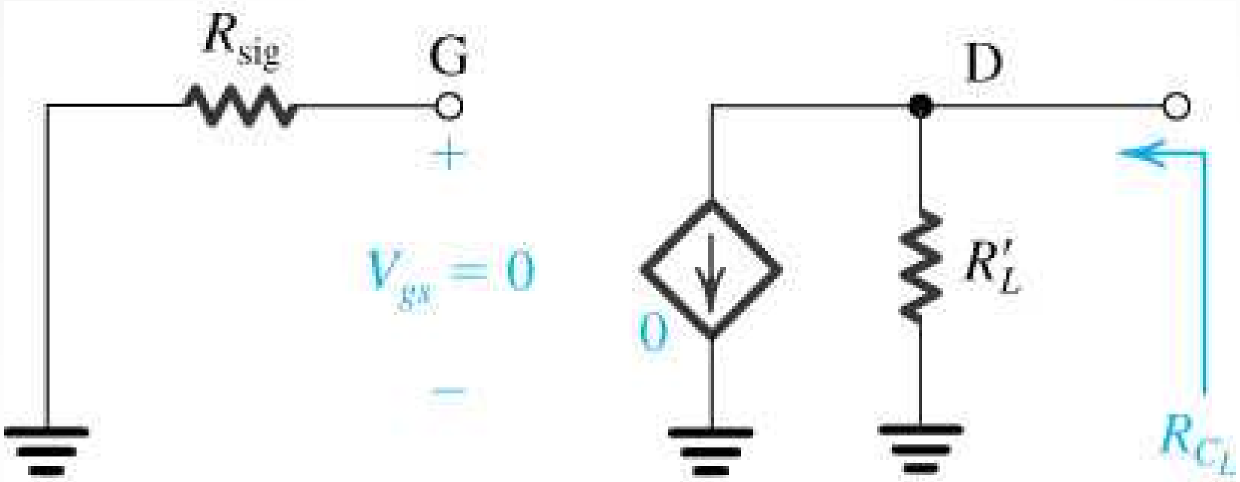




$$R_{gs} = R_{sig}$$

$$R_{gd} = R_{sig}(1 + g_m R_L') + R_L'$$

Analysis Using Open-Ckt Time Constants



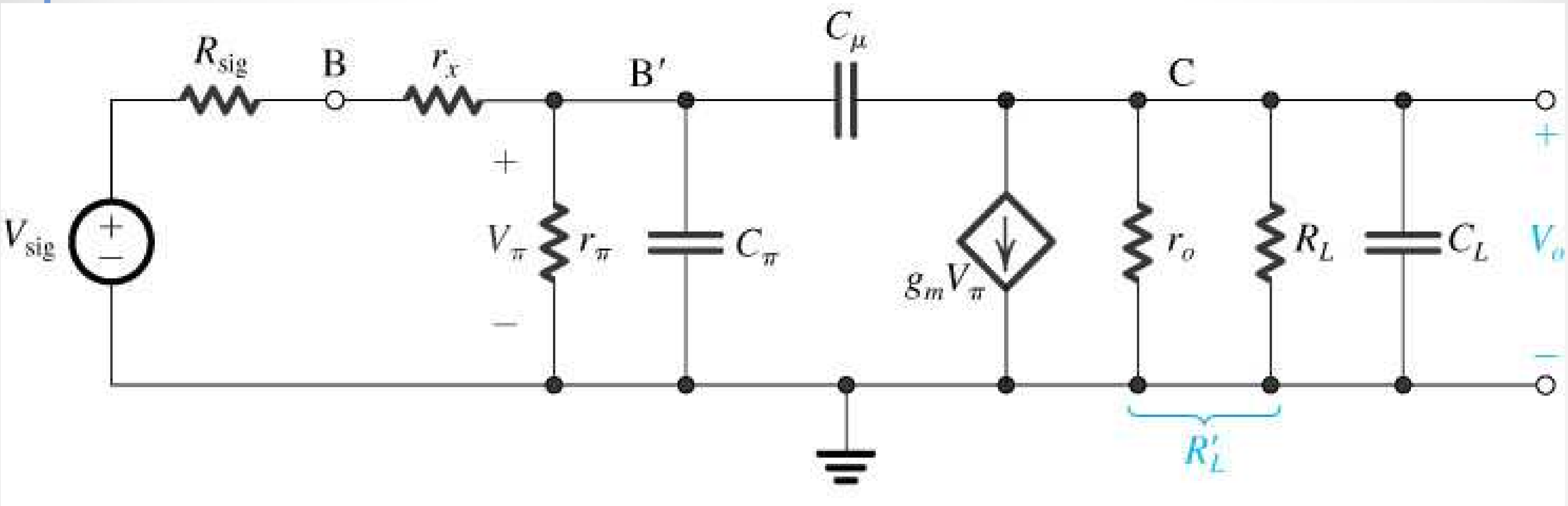
$$R_{C_L} = R_L'$$

(c)

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$$
$$= C_{gs}R_{sig} + C_{gd}[R_{sig}(1 + g_m R_L') + R_L'] + C_L R_L'$$

$$f_H \cong \frac{1}{2\pi\tau_H}$$

High Frequency Equivalent Circuit of the CE Amplifier



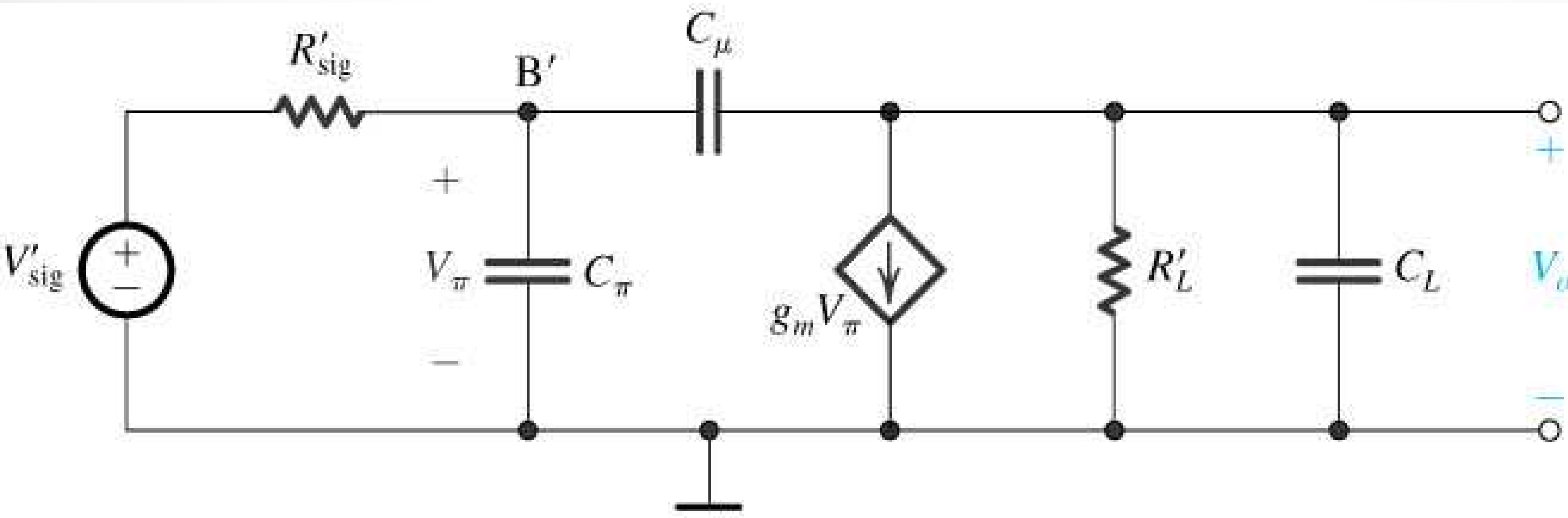
(a)

Thevenin theorem:

$$V'_{sig} = V_{sig} \frac{r_\pi}{R_{sig} + r_x + r_\pi}$$

$$R'_{sig} = r_\pi \,\|\, (R_{sig} + r_x)$$

Equivalent Circuit with Thévenin Theorem Employed



$$A_M = -\frac{r_\pi}{R_{sig} + r_x + r_\pi}(g_m R'_L)$$ (b)

Using the method of open-ckt time constants yields

$$\tau_H = C_\pi R_\pi + C_\mu R_\mu + C_L C_{C_L}$$

$$= C_\pi R'_{sig} + C_\mu[(1 + g_m R'_L)R'_{sig} + R'_L] + C_L R'_L$$

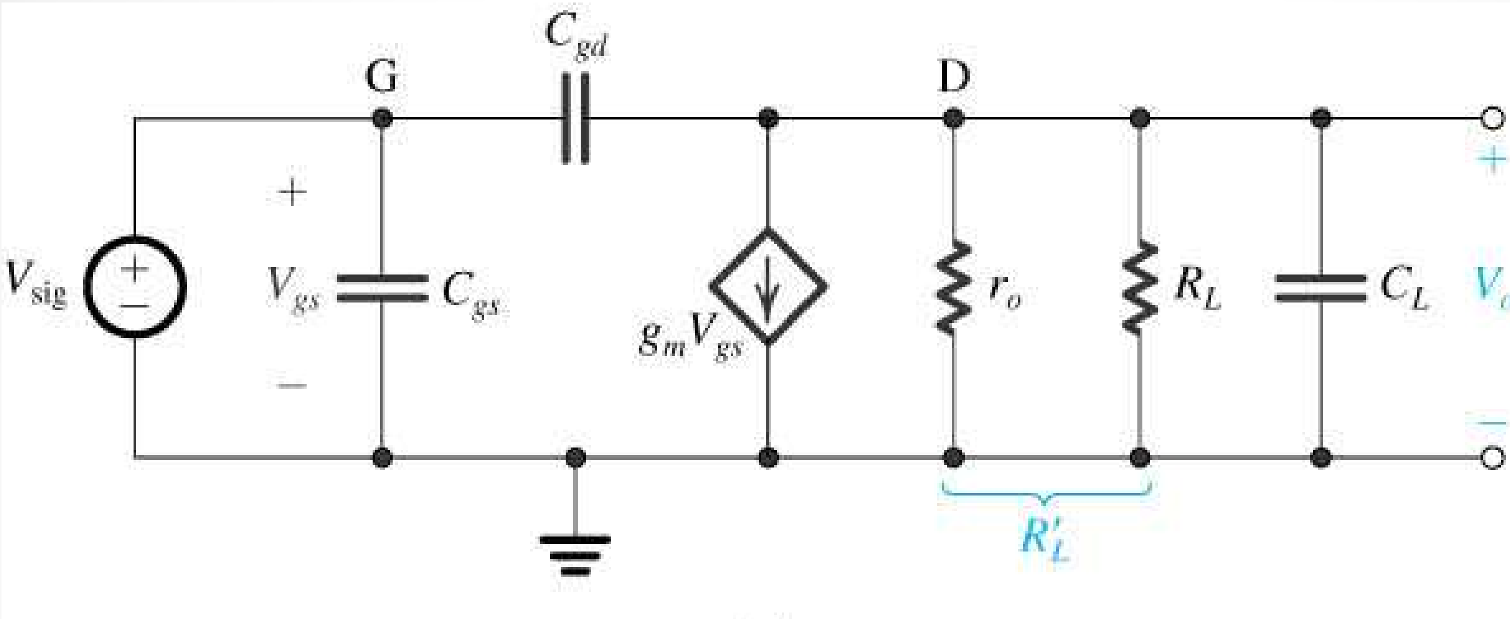
$$f_H \cong \frac{1}{2\pi\tau_H}$$

Using Miller's theorem we obtain

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$f_H \cong \frac{1}{2\pi C_{in} R'_{sig}}$$

The Situation When Rsig Is Low



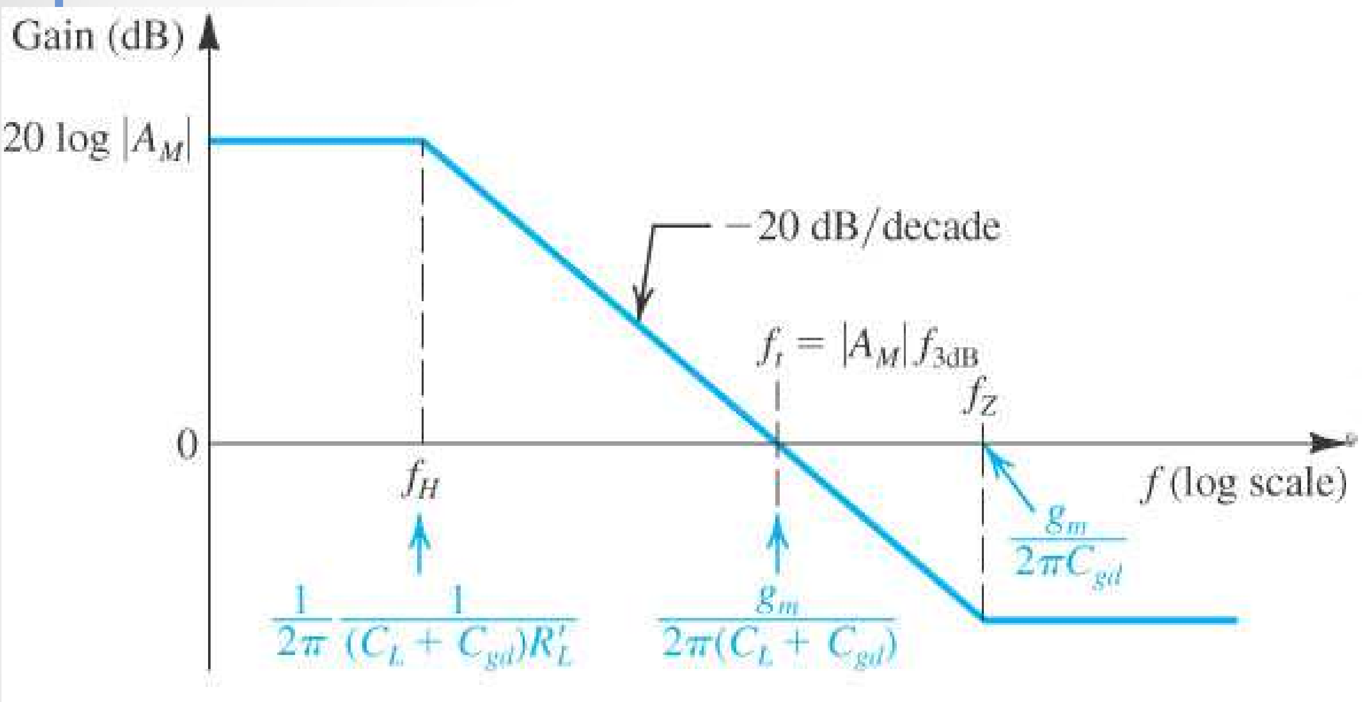
(a)

High-frequency equivalent circuit of a CS amplifier fed with a signal source having a very low (effectively zero) resistance.

$$\frac{V_o}{V_{sig}} = \frac{(-g_m R_L')[1 - s(C_{gd}/g_m)]}{1 + s(C_L + C_{gd})R_L'} \qquad f_H = \frac{1}{2\pi(C_L + C_{gd})R_L'}$$

The Situation When Rsig Is Low

fT, which is equal to the gain-bandwidth product



(c)

$$f_t = |A_M| f_H$$

$$= g_m R'_L \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

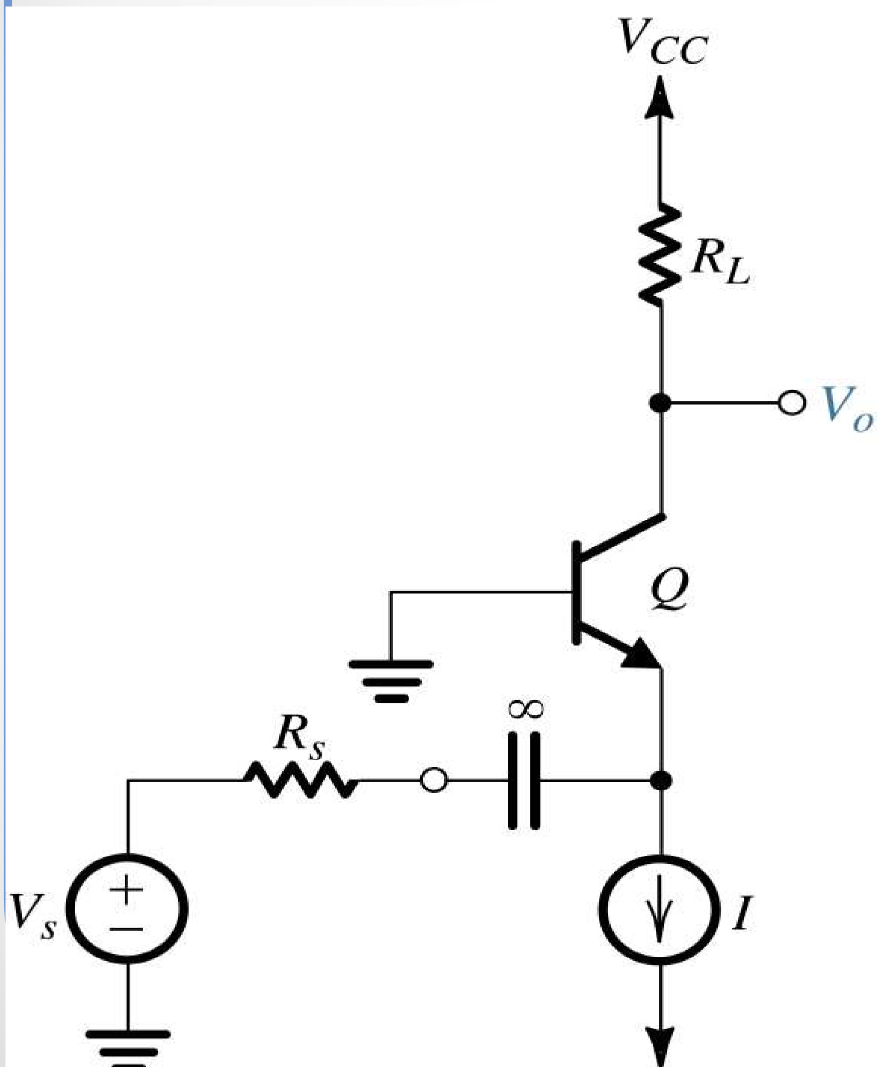
$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})R'_L}$$

Bode plot for the gain of the circuit in (a).

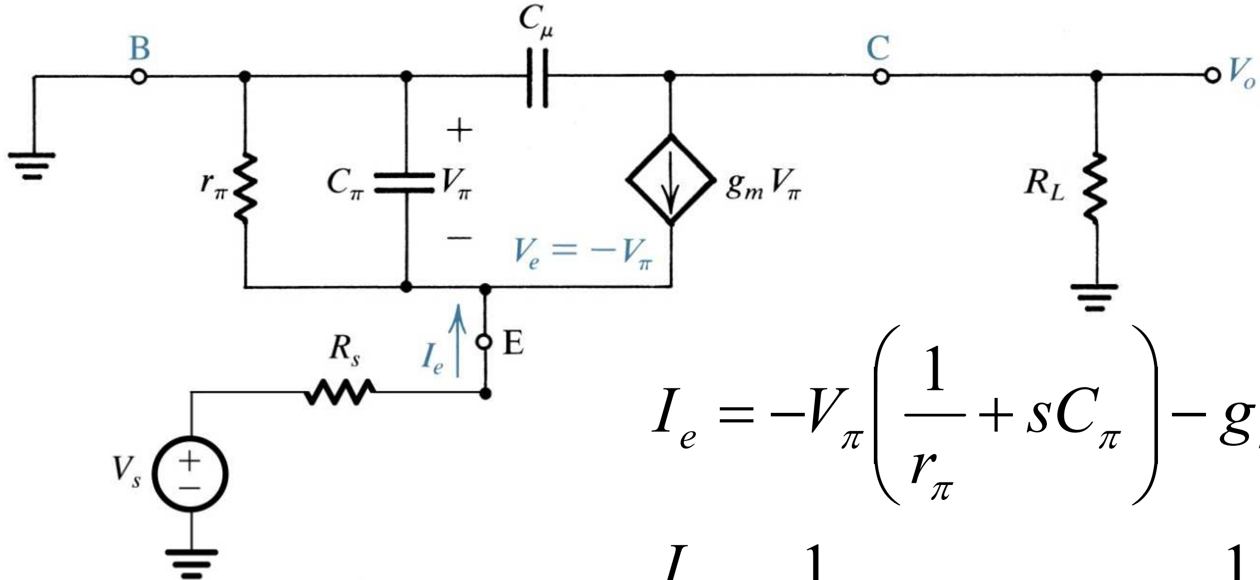
The Situation When R_{sig} Is Low

- The high frequency gain will no longer be limited by the interaction of the source resistance and the input capacitance.
- The high frequency limitation happens at the amplifier output.
- To improve the 3-dB frequency, we shall reduce the equivalent resistance seen through G(B) and D(C) terminals.

Frequency Response of the CG and CB Amplifier

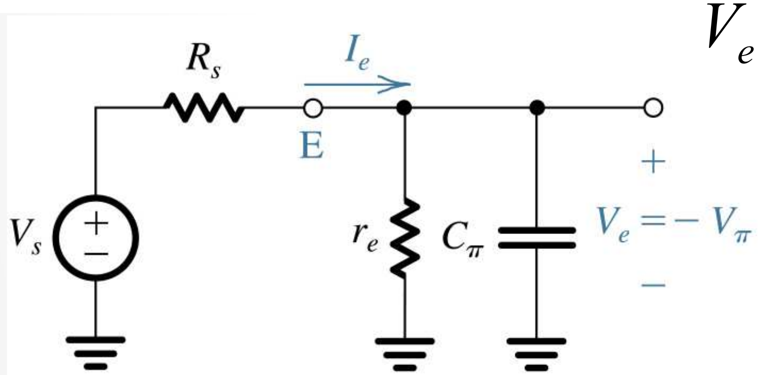


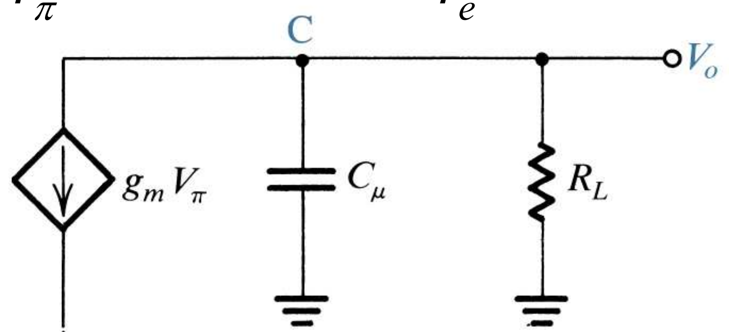
- High-frequency response of the CS and CE Amplifiers is limited by the Miller effect Introduced by feedback Ceq.
- To extend the upper frequency limit of a transistor amplifier stage one has to reduce or eliminate the Miller C multiplication.



$$I_e = -V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right) - g_m V_\pi = V_e\left(\frac{1}{r_\pi} + g_m + sC_\pi\right)$$

$$\frac{I_e}{V_e} = \frac{1}{r_\pi} + g_m + sC_\pi = \frac{1}{r_e} + sC_\pi$$





$$\omega_{p1} = \frac{1}{C_\pi\left(r_e // R_s\right)} \quad \text{for CG amplifier } \omega_{p1} = \frac{1}{C_{gs}\left(1/g_m // R_s\right)}$$

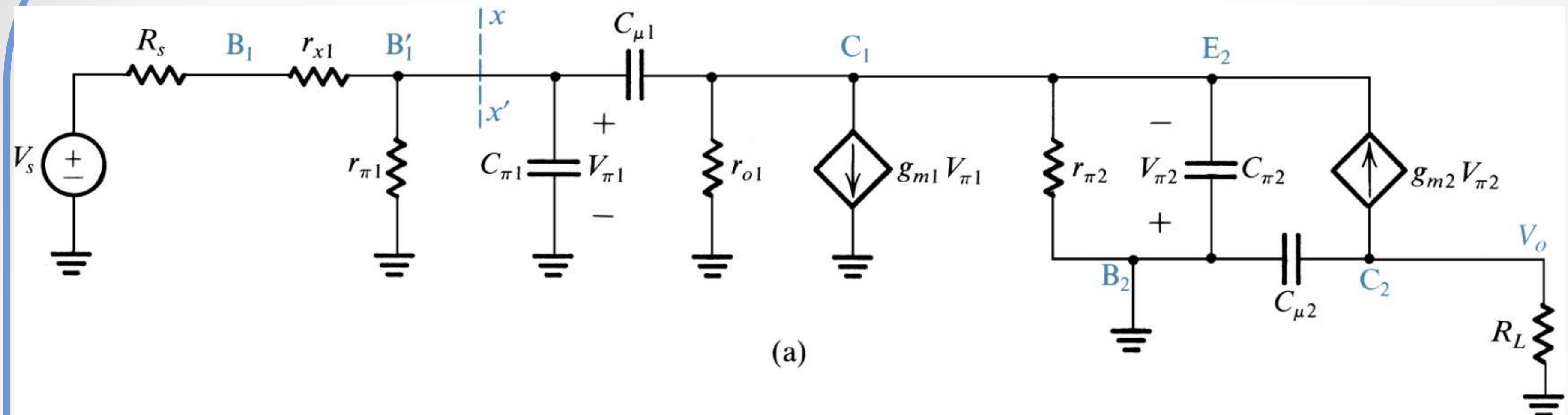
$$\omega_{p2} = \frac{1}{C_\mu R_L} \quad \text{for CG amplifier } \omega_{p2} = \frac{1}{C_{gd} R_L}$$

CB amplifier has a much higher upper-cutoff frequency than that of CE amplifier

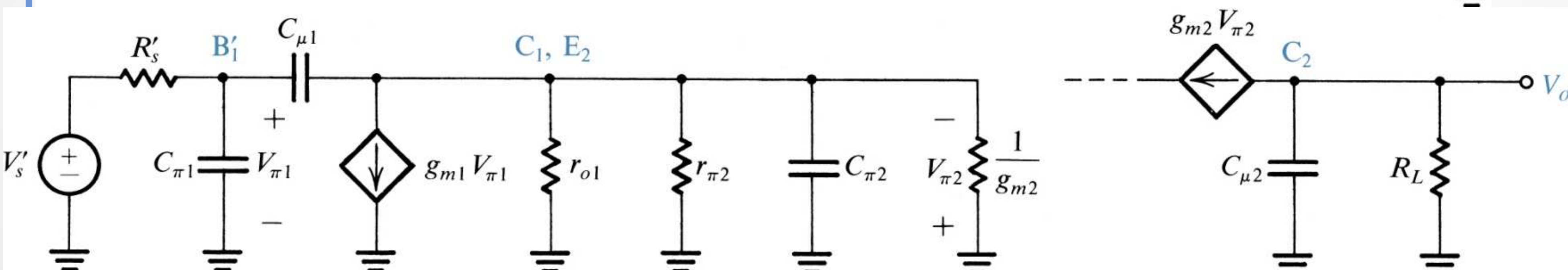
Comparison between CG (CB) and CS (CE)

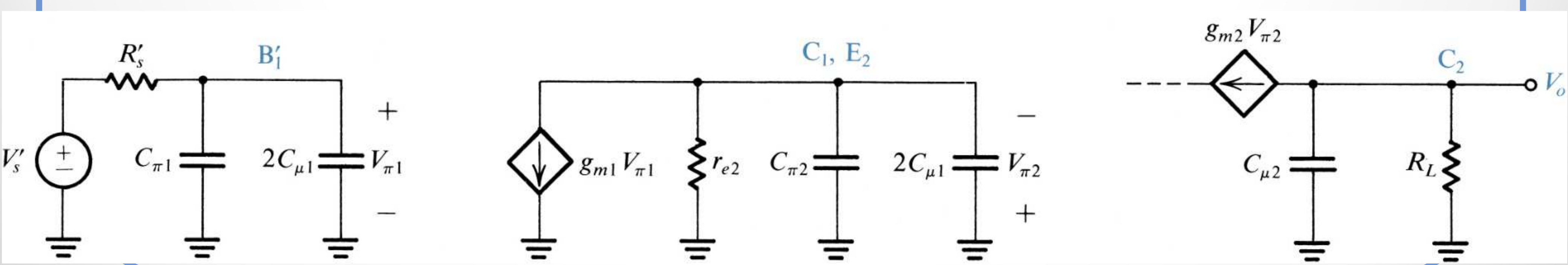
- Open-circuit voltage gain for CG(CB) almost equals to the one for CS(CE)
- Much smaller input resistance and much larger output resistance
- CG(CB) amplifier is not desirable in voltage amplifier but suitable as current buffer.
- Superior high frequency response because of the absence of Miller's effects
- Cascode amplifier is the significant application for CG(CB) circuit

Frequency Response of the BJT Cascode



(a)





Frequency Response of the BJT Cascode

- The cascode configuration combines the advantages of the CE and CB circuits

$$\omega_1 = \frac{1}{R_S^{'}\left(C_{\pi 1} + 2C_{\mu 1}\right)} \approx \omega_H$$

$$\omega_2 = \frac{1}{C_{\pi 2} r_{e2}}$$

$$\omega_3 = \frac{1}{C_{\mu 2} R_L}$$

$$A_M = \frac{V_o}{V_S} = -g_m R_L \frac{r_\pi}{r_\pi + r_x + R_S}$$

Questions:

（1） the reason for gain decreasing of high-frequency response__， the reason for gain decreasing of low-frequency response___。

A. coupling capacitors and bypass capacitors

B. diffusion capacitors and junction capacitors

C. linear characteristics of semiconductors

D. the quiescent point is not proper

（2）when signal frequency is equal to f_L or f_H， the gain of of amplifier decreases ___compare to that of midband frequency。

A.0.5times B.0.7times C.0.9times

that is decreasing ___。

A.3dB B.4dB C.5dB

（3）The CE amplifier circuit， when $f = f_L$， the phase is _。

A.＋45° B.－90° C.－135°

when $f = f_H$， the phase is __。

A.－45° B.－135° C.－225°


Always give 100% at work
12% Monday
23% Tuesday
40% Wednesday
20% Thursday
5% Friday

References

Microelectronic Circuits by Adel S. Sedra & Kenneth C. Smith. Saunders College Publishing

"Chapter 7: Frequency Response", a lecture note by Prof. Yang Hua, Ph.D., Department of Electronic Engineering, Shanghai Jiao Tong University (SJTU), Shanghai, China