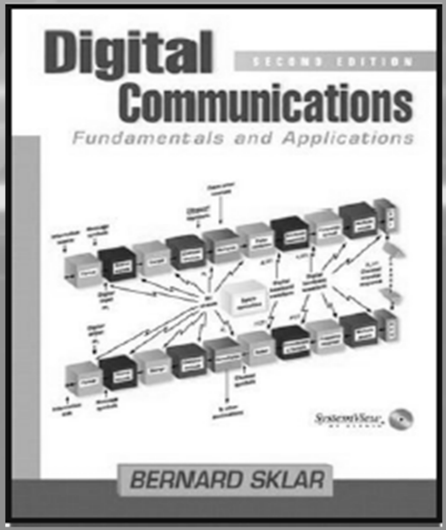


ENE 467
Digital Communications

TEACHING BY

ASST. PROF. SUWAT PATTARAMALAI, PH.D.

Why Digital?, Definition, Basic

- Outcome
  - Know the reason of using digital signal
  - Know and can explain the different between digital and analog
  - Can explain block diagram of a typical digital communication system
  - Understand types and properties of signals (deterministic, periodic, analog, energy, Spectra Density, Autocorrelation
  - Understand Noise in communication systems and random signals (random variable, random process, Ergodicity, and power spectral density
  - Understand and know the output from signal transmission through Linear Systems (Impulse response, frequency transfer function, etc.)
  - Know the definition of bandwidth (Baseband and band-pass and dilemma of bandwidth)

Why Digital?

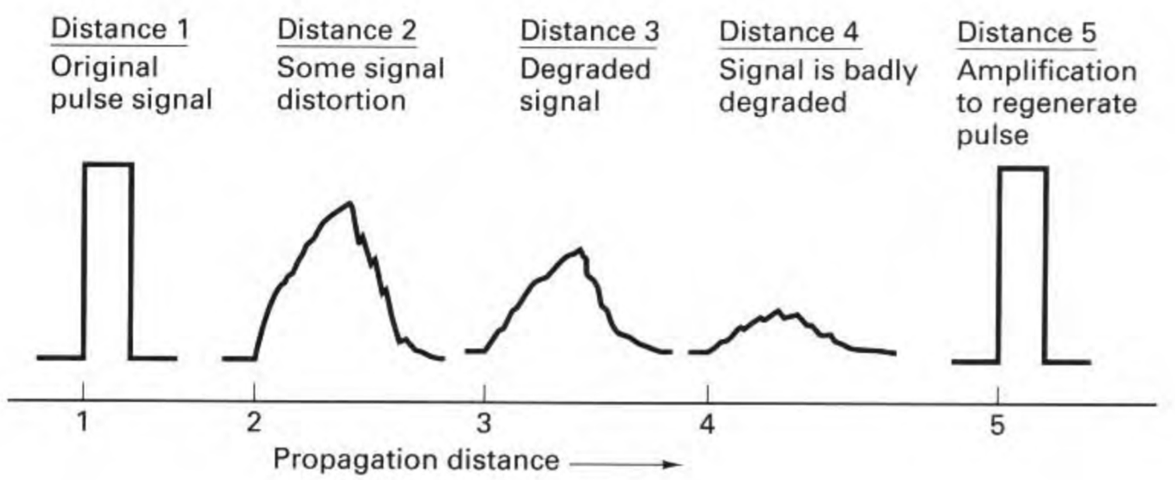
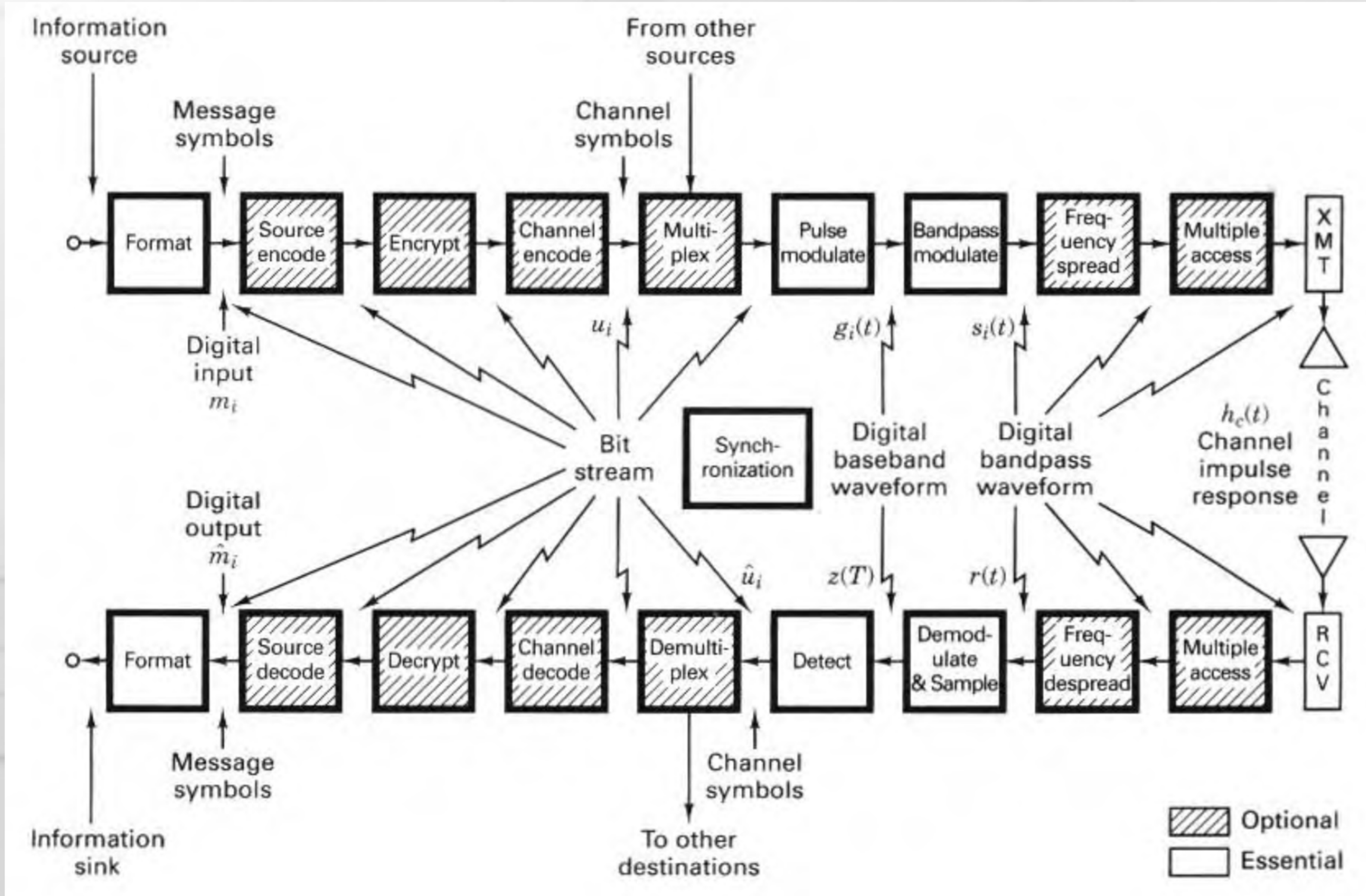


Figure 1.1 Pulse degradation and regeneration.

- Pulse signal is affected by
  - Non-ideal frequency transfer function (impulse response)
  - Unwanted electrical noise or other interference

Digital vs Analog

- Once the analog signal is distorted, it can not be removed.
- Digital is more reliable at low cost than analog
- Digital is more flexible implementation than analog hardware
- Digital signals can be combined with TDM simpler than analog with FDM
- Group of bits called packets (Data communication)
- Digital system is very signal processing intensive compare with analog
- Digital also need a significant share resources to synchronization at various level
- Non graceful degradation: when SNR drops below threshold quality is very poor suddenly



Block diagram of a typical digital communication system

Basic signal processing functions (Transformation)

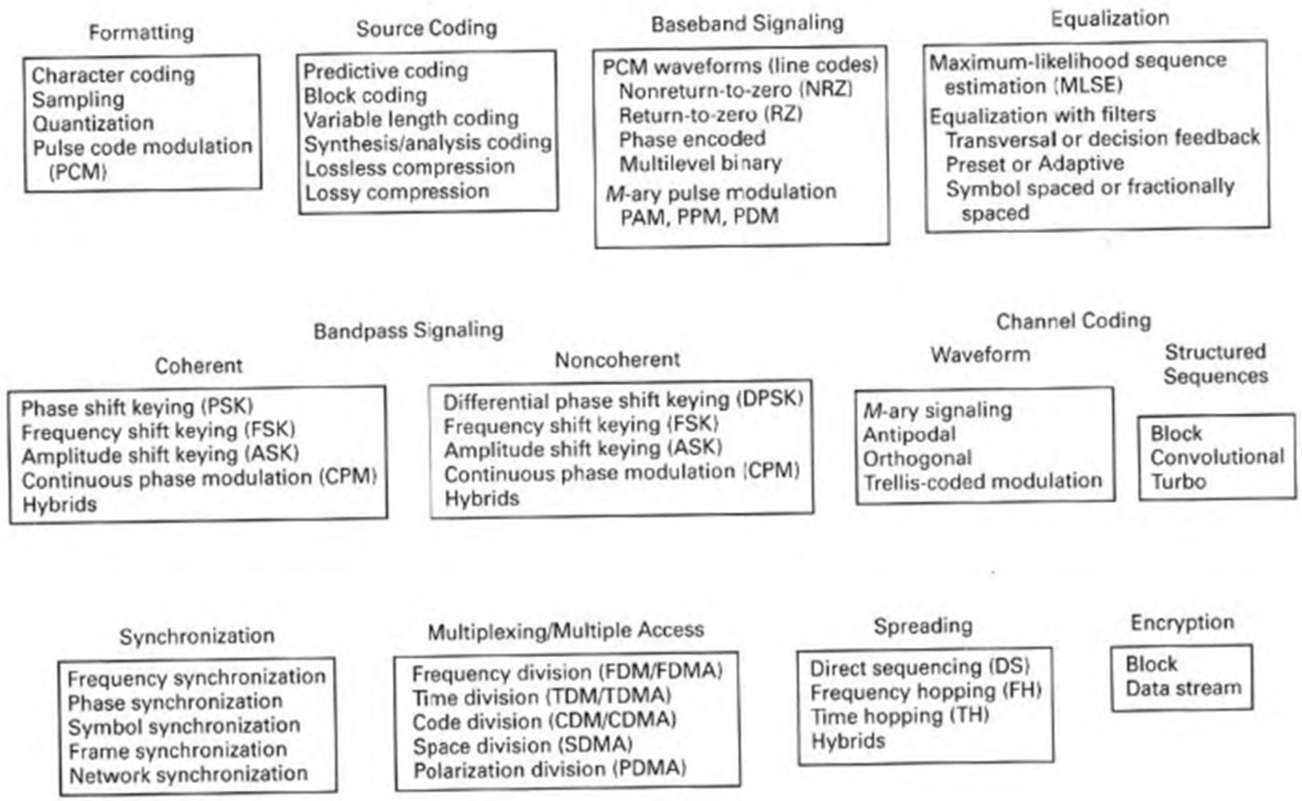


Figure 1.3 Basic digital communication transformations.

Basic Digital Communication Nomenclature

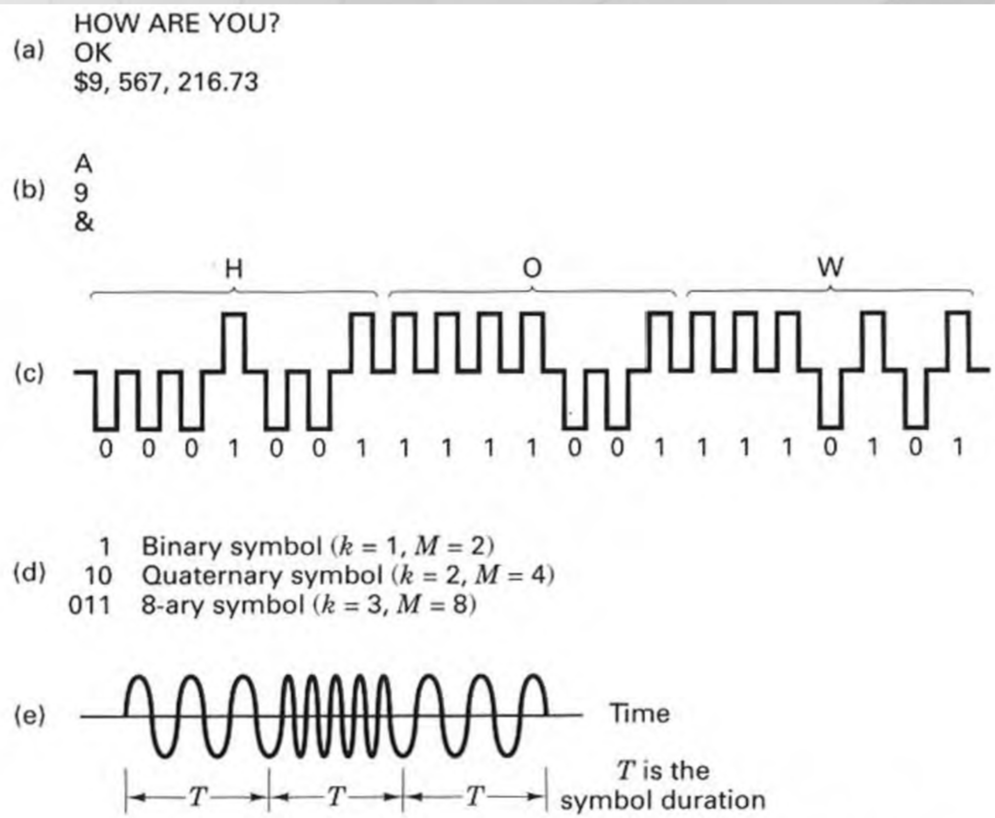


Figure 1.4 Nomenclature examples. (a) Textual messages. (b) Characters. (c) Bit stream (7-bit ASCII). (d) Symbols m_i, $i = 1, \ldots, M$, $M = 2^k$. (e) Bandpass digital waveform $s_i(t)$, $i = 1, \ldots, M$.

Signals

- Deterministic and random signals
- Periodic and non periodic signals
- Analog and discrete signals
- Energy and power signals
- Unit impulse function

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\delta(t) = 0 \qquad \text{for } t \neq 0$$

$$\delta(t) \text{ is unbounded at } t = 0$$

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$$

Spectral density

- Energy spectral density

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

- Power spectral density

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Example 1.1 Average Normalized Power

(a) Find the average normalized power in the waveform, $x(t) = A \cos 2\pi f_0 t$, using time averaging.

(b) Repeat part (a) using the summation of spectral coefficients.

Solution

(a) Using Equation (1.17a), we have

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t\ dt$$

$$= \frac{A^2}{2T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 4\pi f_0 t)\, dt$$

$$= \frac{A^2}{2T_0} (T_0) = \frac{A^2}{2}$$

Autocorrelation

- Autocorrelation of an Energy Signal

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)\,dt \qquad \text{for } -\infty < \tau < \infty$$

- Properties:

1. $R_x(\tau) = R_x(-\tau)$	symmetrical in τ about zero
2. $R_x(\tau) \le R_x(0)$ for all τ	maximum value occurs at the origin
3. $R_x(\tau) \leftrightarrow \psi_x(f)$	autocorrelation and ESD form a Fourier transform pair, as designated by the double-headed arrows
4. $R_x(0) = \int_{-\infty}^{\infty} x^2(t)\,dt$	value at the origin is equal to the energy of the signal

- Autocorrelation of an Periodic (Power) Signal

$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)x(t+\tau)\,dt \qquad \text{for } -\infty < \tau < \infty$$

- Properties:

1. $R_x(\tau) = R_x(-\tau)$	symmetrical in τ about zero
2. $R_x(\tau) \le R_x(0)$ for all τ	maximum value occurs at the origin
3. $R_x(\tau) \leftrightarrow G_x(f)$	autocorrelation and PSD form a Fourier transform pair
4. $R_x(0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt$	value at the origin is equal to the average power of the signal

Random signals

- Random variables
  - Distribution function
  - Probability density function

$$F_X(x) = P(X \le x)$$

1. $0 \le F_X(x) \le 1$
2. $F_X(x_1) \le F_X(x_2)$ if $x_1 \le x_2$
3. $F_X(-\infty) = 0$
4. $F_X(+\infty) = 1$

$$p_X(x) = \frac{dF_X(x)}{dx}$$

$$\begin{aligned} P(x_1 \le X \le x_2) &= P(X \le x_2) - P(X \le x_1) \\ &= F_X(x_2) - F_X(x_1) \\ &= \int_{x_1}^{x_2} p_X(x)\, dx \end{aligned}$$

1. $p_X(x) \ge 0.$
2. $\int_{-\infty}^{\infty} p_X(x)\, dx = F_X(+\infty) - F_X(-\infty) = 1.$

  - Ensemble averages
    - Mean value

$$m_X = \mathbf{E}\{X\} = \int_{-\infty}^{\infty} x p_X(x)\, dx$$

    - Nth moment

$$\mathbf{E}\{X^n\} = \int_{-\infty}^{\infty} x^n p_X(x)\, dx$$

    - variance

$$\mathrm{var}(X) = \mathbf{E}\{(X - m_X)^2\} = \int_{-\infty}^{\infty} (x - m_X)^2 p_X(x)\, dx = \mathbf{E}\{X^2\} - m_X^2$$

Random signals

- Random processes
  - Mean

$$\mathbf{E}\{X(t_k)\} = \int_{-\infty}^{\infty} x p_{X_k}(x)\, dx = m_X(t_k)$$

  - Autocorrelation

$$R_X(t_1, t_2) = \mathbf{E}\{X(t_1)X(t_2)\}$$

  - Wide-Sense Stationary (WSS) if

$$\mathbf{E}\{X(t)\} = m_X = \text{a constant}$$

$$R_X(t_1, t_2) = R_X(t_1 - t_2)$$

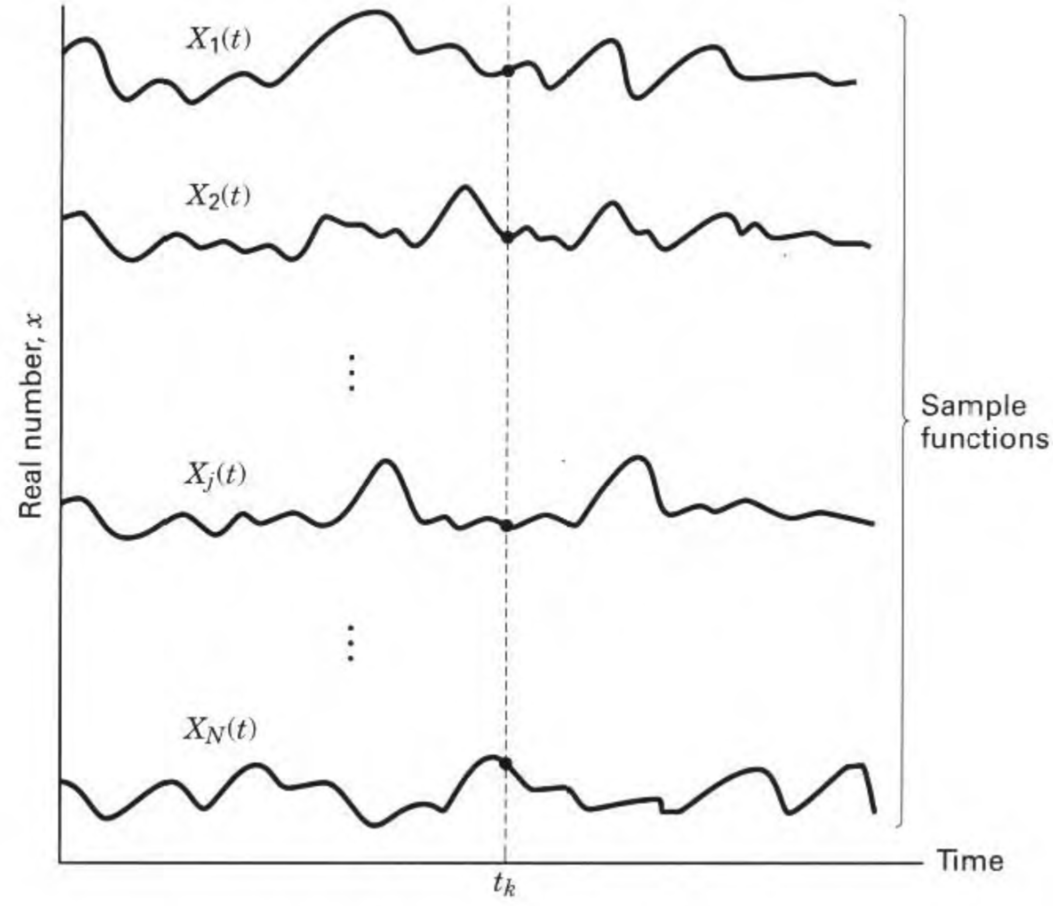


Figure 1.5 Random noise process.

Random signals

- Ergodic process if
  - time average = ensemble average
- Ergodic in mean

$$m_X = \lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} X(t)\, dt$$

- Ergodic in autocorrelation

$$R_X(\tau) = \lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} X(t)X(t+\tau)\, dt$$

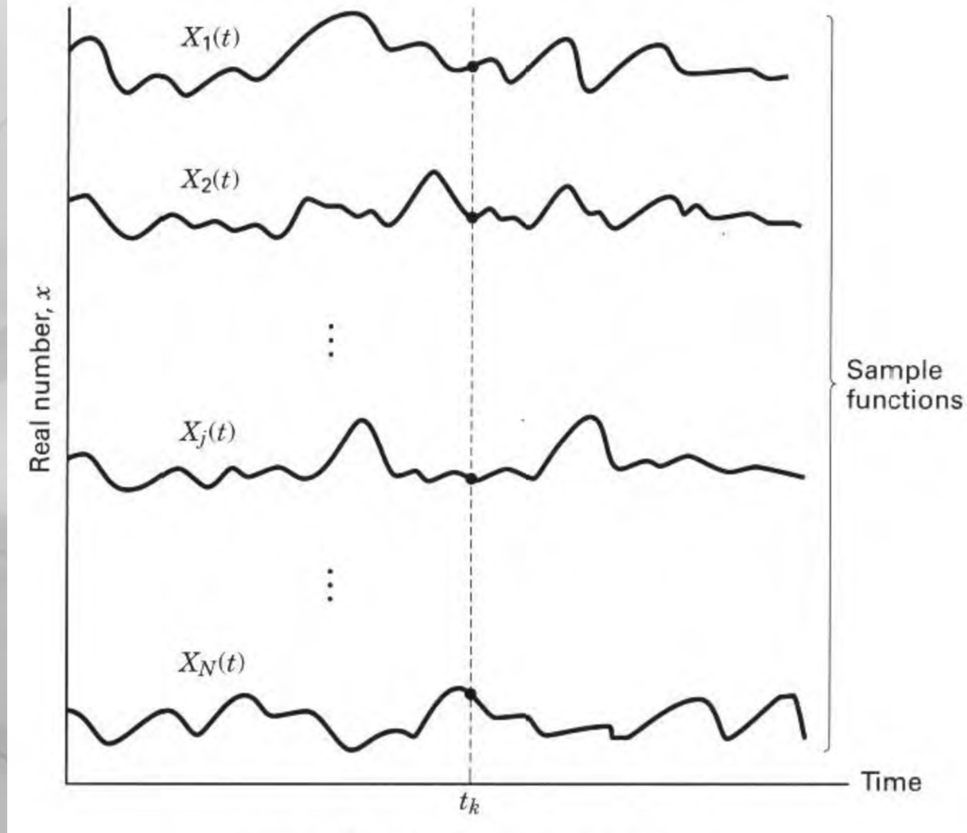
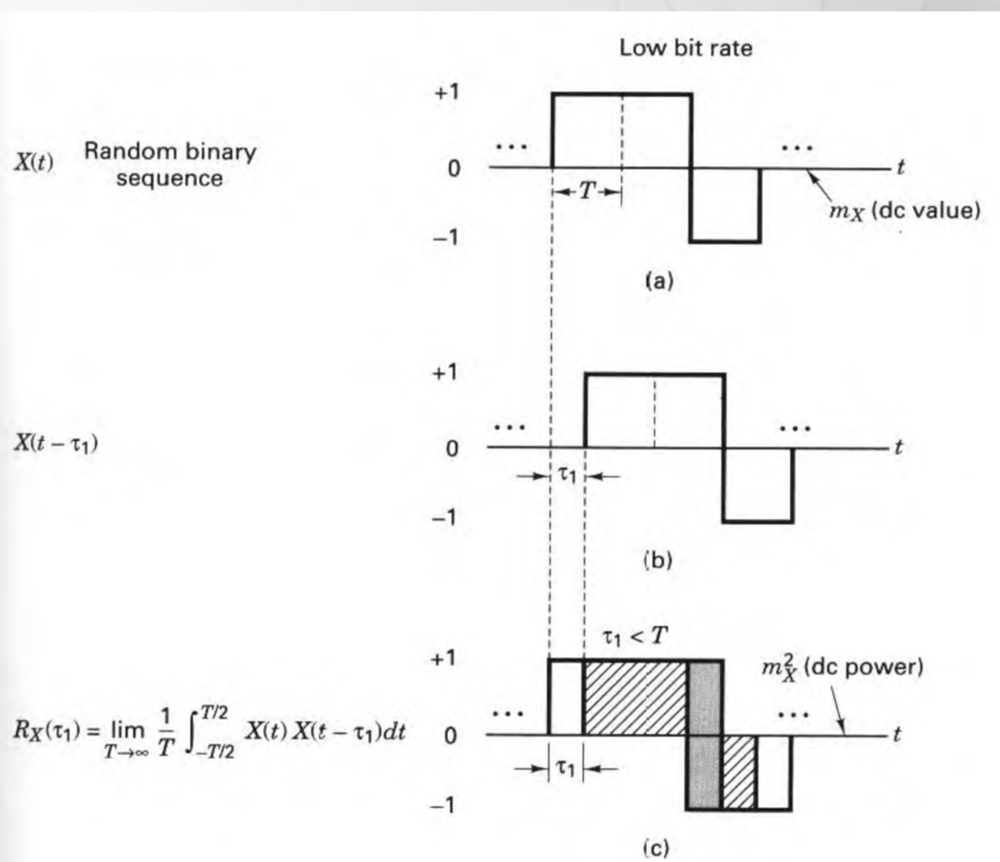


Figure 1.5 Random noise process.

Ergodic process and electrical parameters

1. The quantity $m_X = \mathbf{E}\{X(t)\}$ is equal to the dc level of the signal.
2. The quantity m_X^2 is equal to the normalized power in the dc component.
3. The second moment of $X(t)$, $\mathbf{E}\{X^2(t)\}$, is equal to the total average normalized power.
4. The quantity $\sqrt{\mathbf{E}\{X^2(t)\}}$ is equal to the root-mean-square (rms) value of the voltage or current signal.
5. The variance σ_X^2 is equal to the average normalized power in the time-varying or ac component of the signal.
6. If the process has zero mean (i.e., $m_X = m_X^2 = 0$), then $\sigma_X^2 = \mathbf{E}\{X^2\}$ and the variance is the same as the mean-square value, or the variance represents the total power in the normalized load.
7. The standard deviation σ_X is the rms value of the ac component of the signal.
8. If $m_X = 0$, then σ_X is the rms value of the signal.

Autocorrelation and power spectral density



$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & \text{for } |\tau| \le T \\ 0 & \text{for } |\tau| > T \end{cases}$$

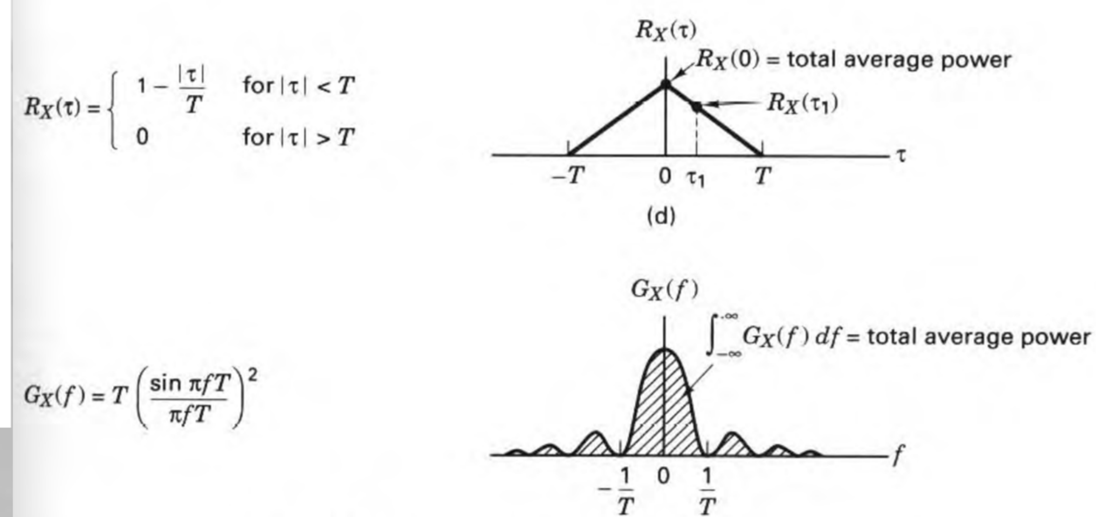
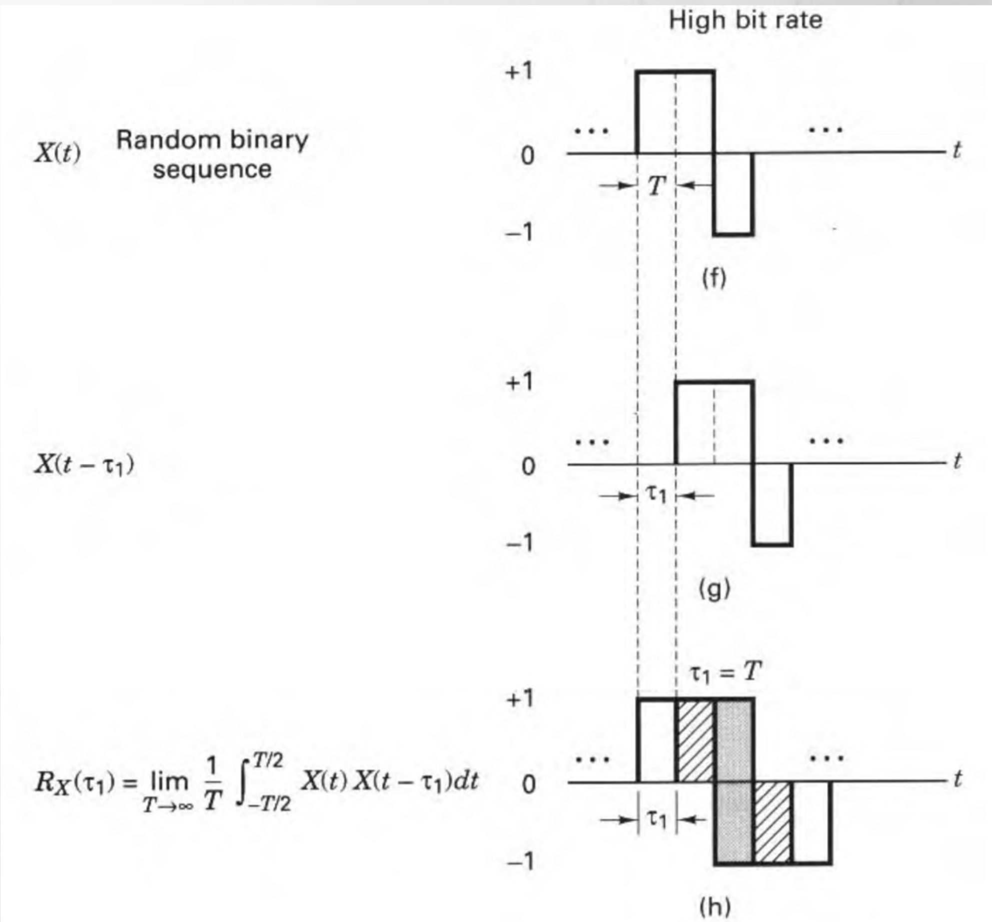


Figure 1.6 Autocorrelation and power spectral density.

Autocorrelation and power spectral density



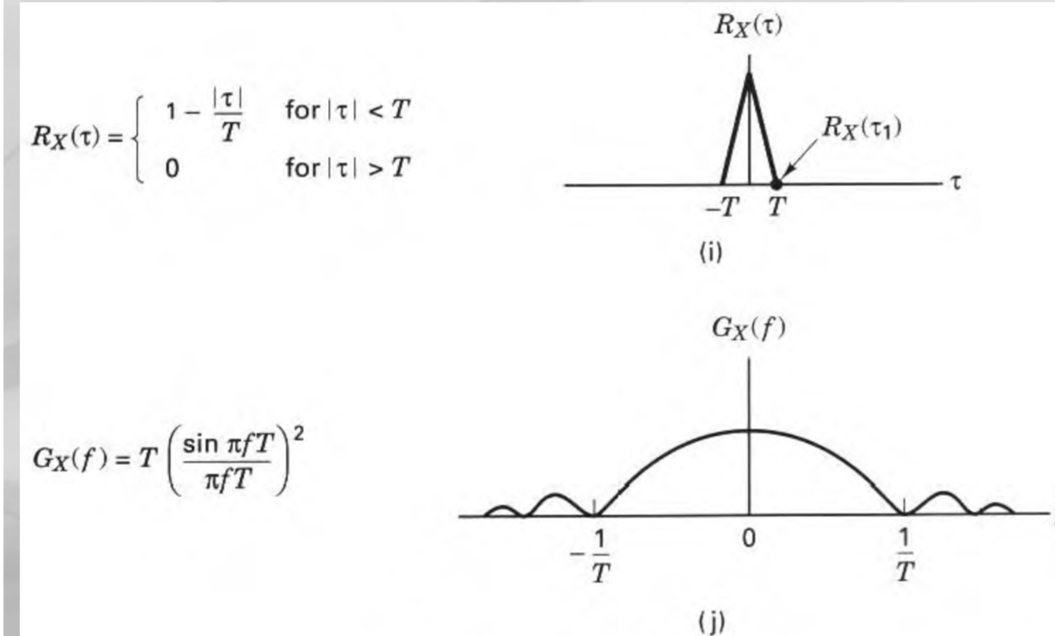


Figure 1.6 continued

$$G_X(f) = T\left(\frac{\sin \pi f T}{\pi f T}\right)^2 = T\,\mathrm{sinc}^2 fT$$

Noise in communication

$$z = a + n$$

- Gaussian
  - PDF

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a}{\sigma}\right)^2\right]$$

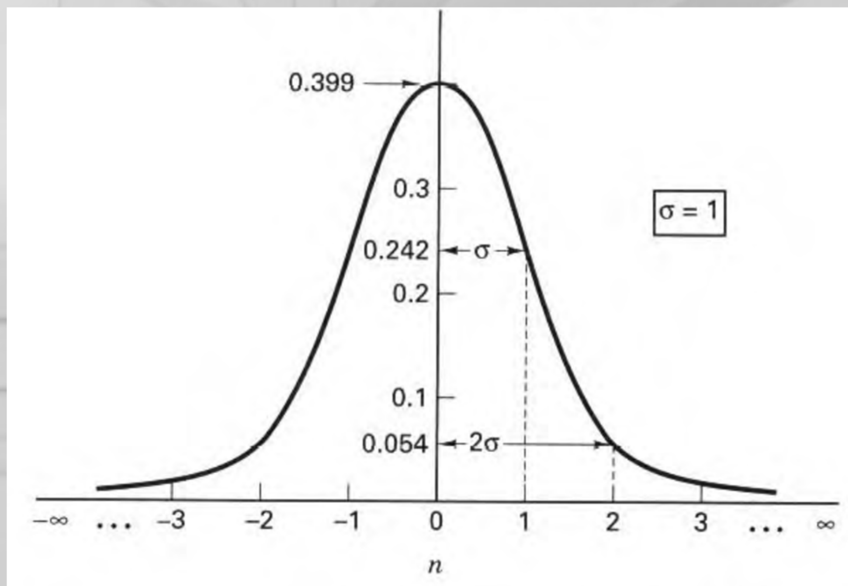


Figure 1.7 Normalized ($\sigma = 1$) Gaussian probability density function.

Noise in communication

- White noise
  - PSD

$$G_n(f) = \frac{N_0}{2} \qquad \text{watts/hertz}$$

  - Autocorrelation function

$$R_n(\tau) = \mathcal{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\,\delta(\tau)$$

  - Average power

$$P_n = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df = \infty$$

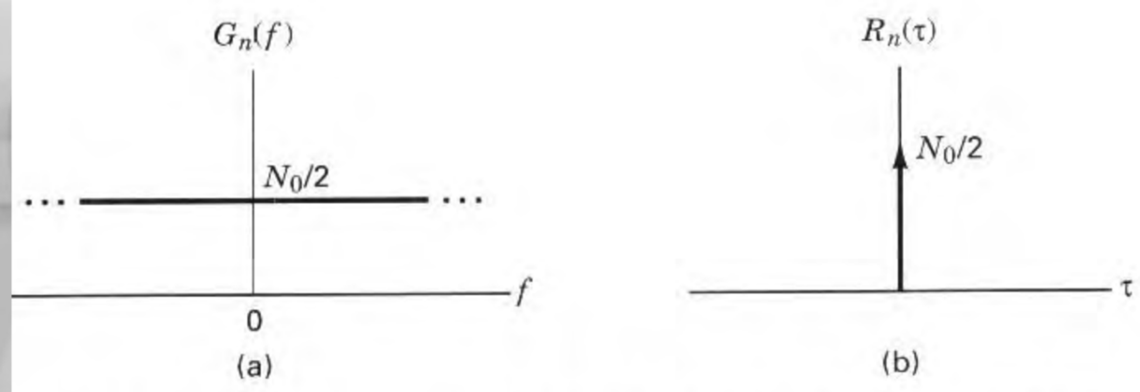
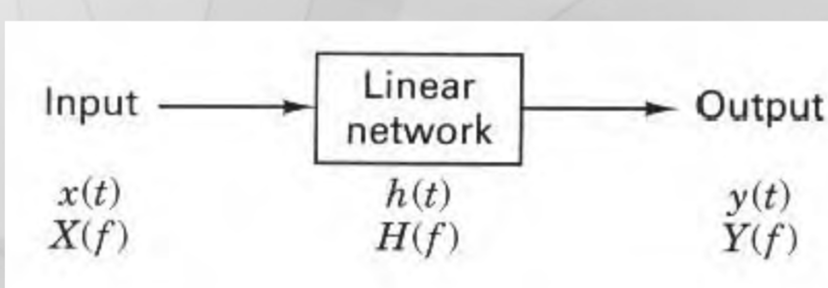


Figure 1.8 (a) Power spectral density of white noise. (b) Autocorrelation function of white noise.

Signal transmission through linear system

- Linear system



- Impulse response $h(t) = y(t) \qquad \text{when } x(t) = \delta(t)$
- output

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau$$

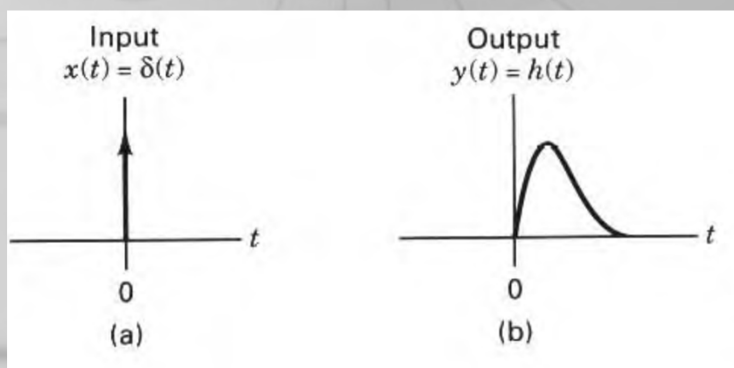
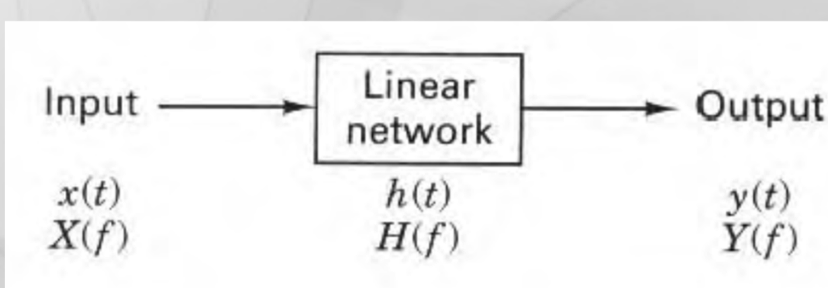


Figure 1.10 (a) Input signal $x(t)$ is a unit impulse function. (b) Output signal $y(t)$ is the system's impulse response $h(t)$.

Signal transmission through linear system

- Linear system



$$H(f) = \frac{Y(f)}{X(f)}$$

- Frequency transfer function

$$Y(f) = X(f)H(f)$$

$$H(f) = |H(f)|\, e^{j\theta(f)}$$

$$\theta(f) = \tan^{-1}\frac{\mathrm{Im}\,\{H(f)\}}{\mathrm{Re}\,\{H(f)\}}$$

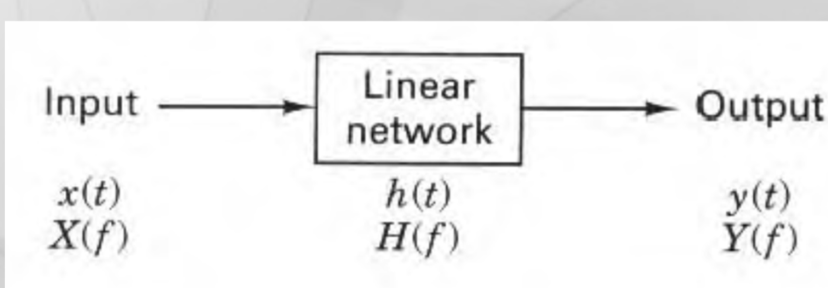
- Example:

$$x(t) = A\cos 2\pi f_0 t$$

$$y(t) = A\,|H(f_0)|\cos\left[2\pi f_0 t + \theta(f_0)\right]$$

Signal transmission through linear system

- Linear system



- Distortionless transmission

$$y(t) = Kx(t - t_0)$$

$$Y(f) = KX(f)e^{-j2\pi f t_0}$$

$$H(f) = Ke^{-j2\pi f t_0}$$

- Envelope delay

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$$

$$t_0 \text{ (seconds)} = \frac{\theta \text{ (radians)}}{2\pi f \text{ (radians/second)}}$$

Signal transmission through linear system

- Ideal Filter

$$H(f) = |H(f)|\, e^{-j\theta(f)}$$

where

$$|H(f)| = \begin{cases} 1 & \text{for } |f| < f_u \\ 0 & \text{for } |f| \geq f_u \end{cases}$$

and

$$e^{-j\theta(f)} = e^{-j2\pi f t_0}$$

$$\begin{aligned} h(t) = \mathscr{F}^{-1}\{H(f)\} &= \int_{-\infty}^{\infty} H(f) e^{j2\pi f t}\, df \\ &= \int_{-f_u}^{f_u} e^{-j2\pi f t_0} e^{j2\pi f t}\, df \\ &= \int_{-f_u}^{f_u} e^{j2\pi f (t - t_0)}\, df \\ &= 2f_u \frac{\sin 2\pi f_u (t - t_0)}{2\pi f_u (t - t_0)} \\ &= 2f_u \operatorname{sinc} 2f_u (t - t_0) \end{aligned}$$

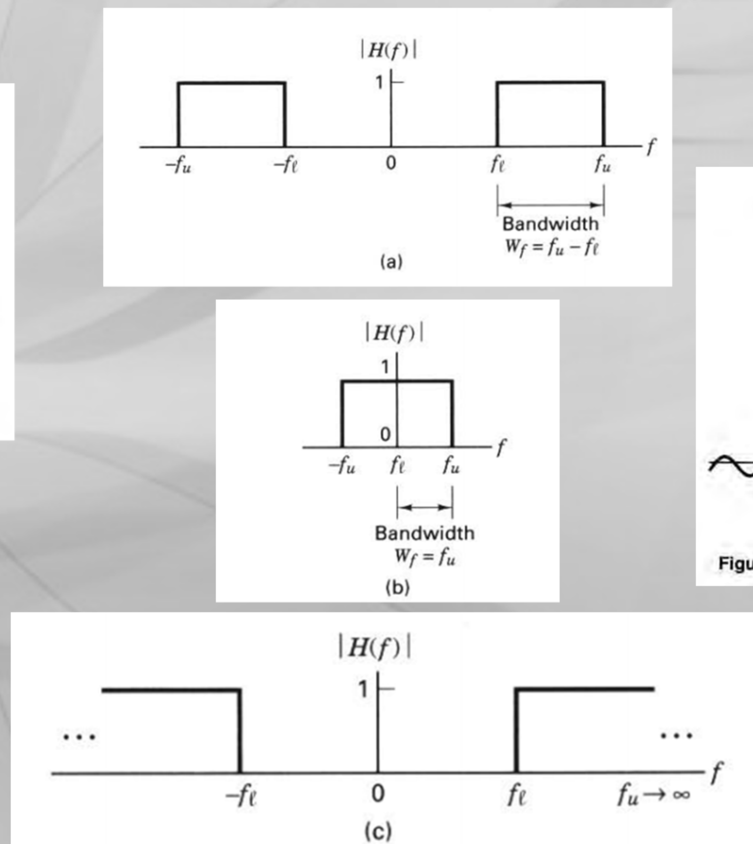


Figure 1.11 Ideal filter transfer function. (a) Ideal bandpass filter. (b) Ideal low-pass filter. (c) Ideal high-pass filter.

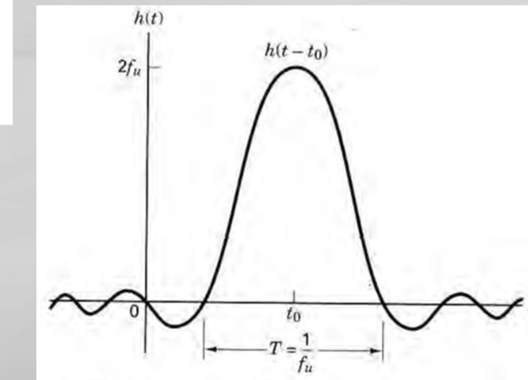


Figure 1.12 Impulse response of the ideal low-pass filter.

Signal transmission through linear system

- Realizable Filter

$$H(f) = \frac{1}{1 + j2\pi f \Re C} = \frac{1}{\sqrt{1 + (2\pi f \Re C)^2}} e^{-j\theta(f)}$$

$$|H_n(f)| = \frac{1}{\sqrt{1 + (f/f_u)^{2n}}} \qquad n \geq 1$$

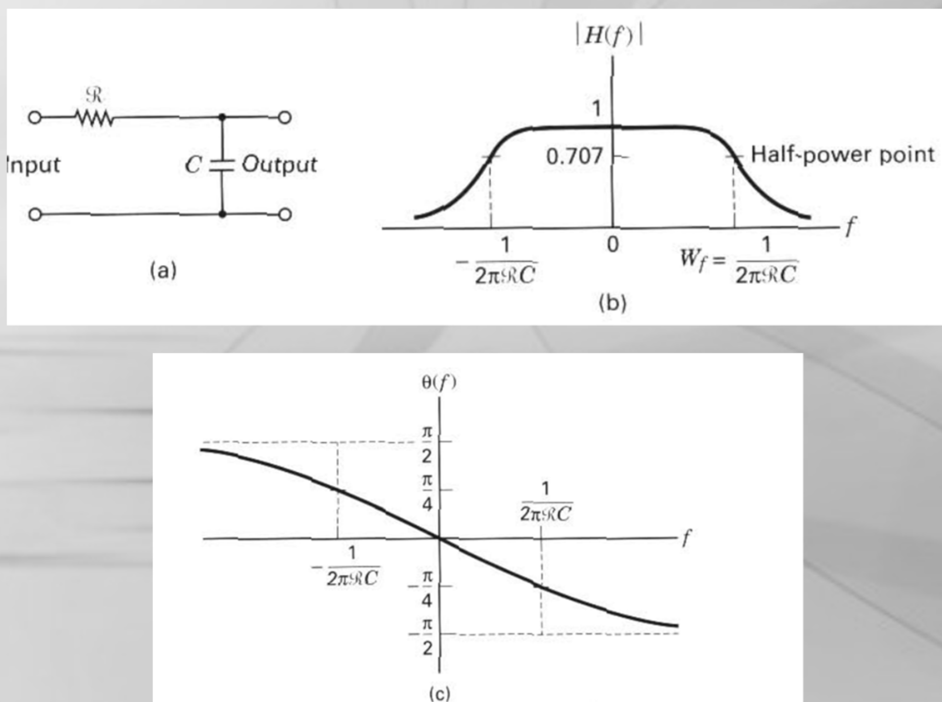


Figure 1.13 ℜC filter and its transfer function. (a) ℜC filter. (b) Magnitude characteristic of the ℜC filter. (c) Phase characteristic of the ℜC filter.

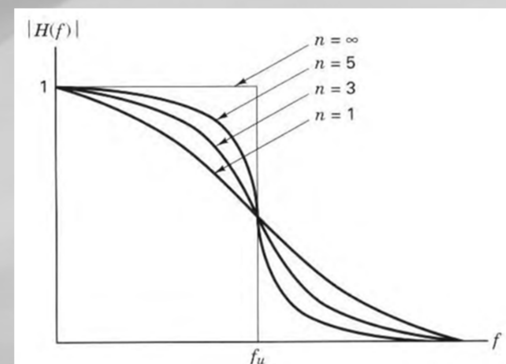


Figure 1.14 Butterworth filter magnitude response.

Signal transmission through linear system

- Signals, Circuits, and Spectra

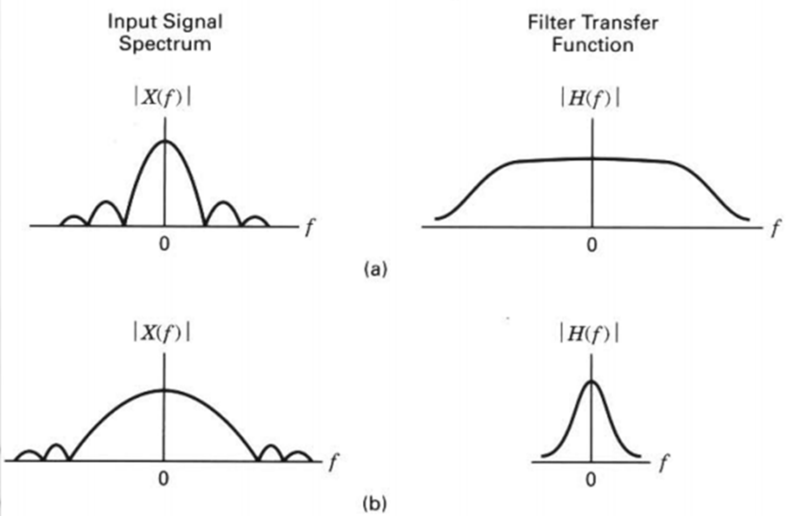
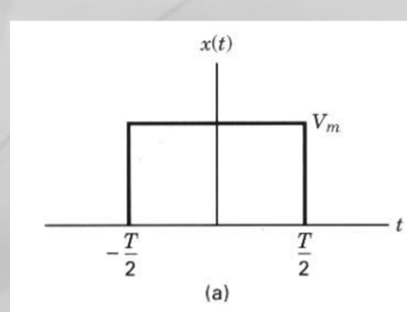


Figure 1.15 Spectral characteristics of the input signal and the circuit contribute to the spectral characteristics of the output signal. (a) Case 1: Output bandwidth is constrained by input signal bandwidth. (b) Case 2: Output bandwidth is constrained by filter bandwidth.



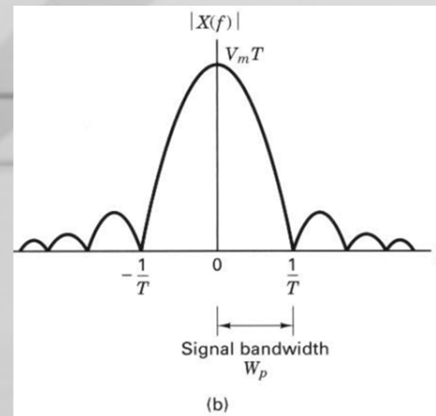
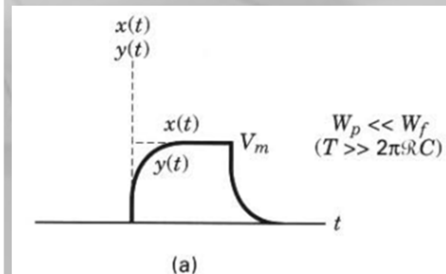


Figure 1.16 (a) Ideal pulse. (b) Magnitude spectrum of the ideal pulse.



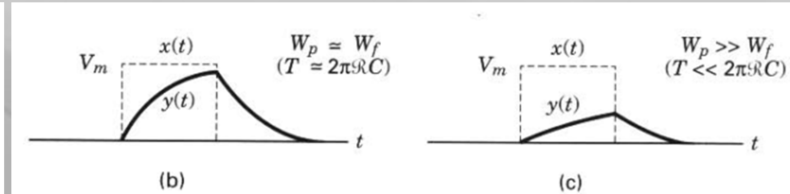
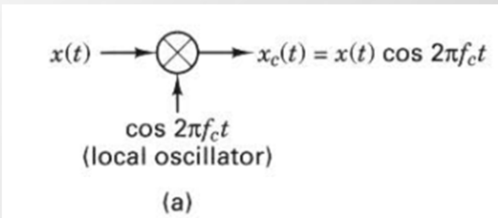
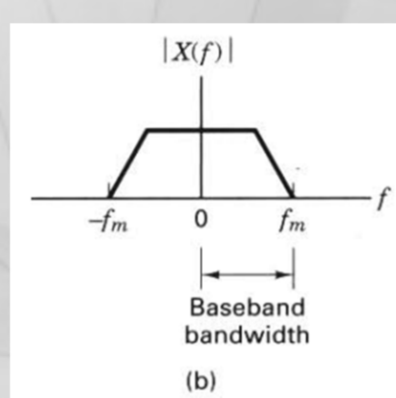


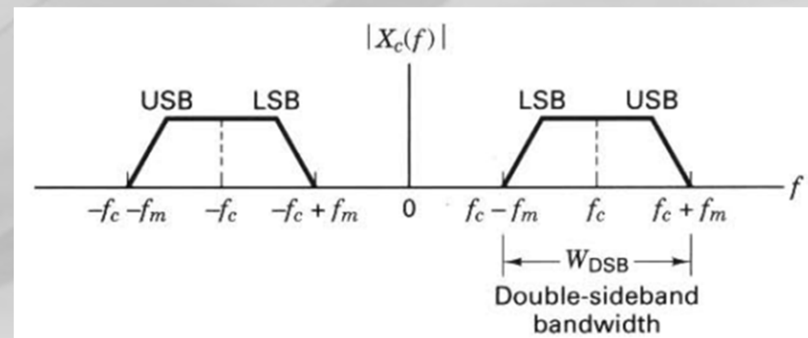
Figure 1.17 Three examples of filtering an ideal pulse. (a) Example 1: Good-fidelity output. (b) Example 2: Good-recognition output. (c) Example 3: Poor-recognition output.

Bandwidth of Digital Data

- Bandwidth



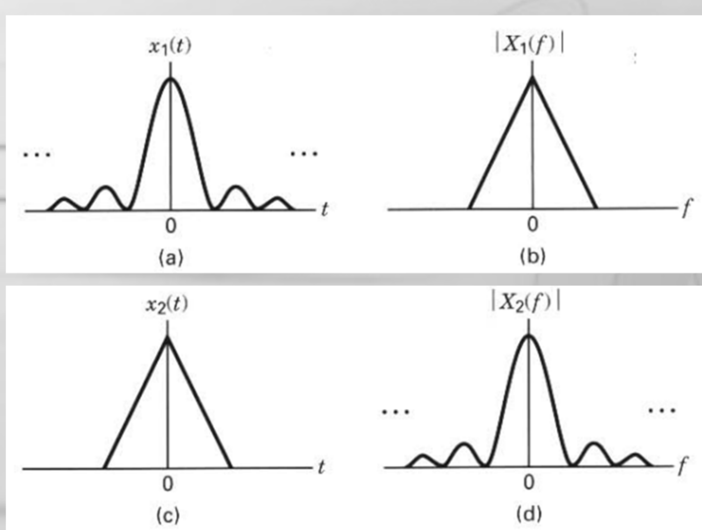




$f_c \gg f_m$

$W_{DSB} = 2f_m$

Figure 1.18 Comparison of baseband and double-sideband spectra. (a) Heterodyning. (b) Baseband spectrum. (c) Double-sideband spectrum.



Bandwidth Dilemma

Figure 1.19 (a) Strictly bandlimited signal in the time domain. (b) In the frequency domain. (c) Strictly time limited signal in the time domain. (d) In the frequency domain.

Bandwidth of Digital Data

- Bandwidth

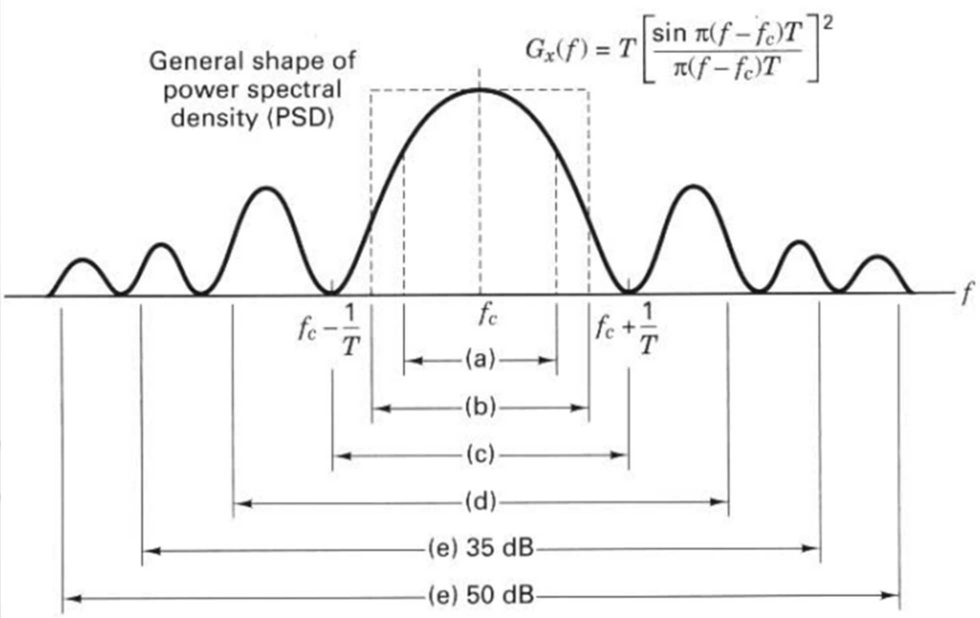


Figure 1.20 Bandwidth of digital data. (a) Half-power. (b) Noise equivalent. (c) Null to null. (d) 99% of power. (e) Bounded PSD (defines attenuation outside bandwidth) at 35 and 50 dB.

(a) *Half-power bandwidth.* This is the interval between frequencies at which $G_x(f)$ has dropped to half-power, or 3 dB below the peak value.

(b) *Equivalent rectangular or noise equivalent bandwidth.* The noise equivalent bandwidth was originally conceived to permit rapid computation of output noise power from an amplifier with a wideband noise input; the concept can similarly be applied to a signal bandwidth. The noise equivalent bandwidth W_N of a signal is defined by the relationship $W_N = P_x/G_x(f_c)$, where P_x is the total signal power over all frequencies and $G_x(f_c)$ is the value of $G_x(f)$ at the band center (assumed to be the maximum value over all frequencies).

(c) *Null-to-null bandwidth.* The most popular measure of bandwidth for digital communications is the width of the main spectral lobe, where most of the signal power is contained. This criterion lacks complete generality since some modulation formats lack well-defined lobes.

(d) *Fractional power containment bandwidth.* This bandwidth criterion has been adopted by the Federal Communications Commission (FCC Rules and Regulations Section 2.202) and states that the occupied bandwidth is the band that leaves exactly 0.5% of the signal power above the upper band limit and exactly 0.5% of the signal power below the lower band limit. Thus 99% of the signal power is inside the occupied band.

(e) *Bounded power spectral density.* A popular method of specifying bandwidth is to state that everywhere outside the specified band, $G_x(f)$ must have fallen at least to a certain stated level below that found at the band center. Typical attenuation levels might be 35 or 50 dB.

(f) *Absolute bandwidth.* This is the interval between frequencies, outside of which the spectrum is zero. This is a useful abstraction. However, for all realizable waveforms, the absolute bandwidth is infinite.

Conclusion

CONCLUSION

In this chapter, the goals of the book have been outlined and the basic nomenclature has been defined. The fundamental concepts of time-varying signals, such as classification, spectral density, and autocorrelation, have been reviewed. Also, random signals have been considered, and white Gaussian noise, the primary noise model in most communication systems, has been characterized, statistically and spectrally. Finally, we have treated the important area of signal transmission through linear systems and have examined some of the realizable approximations to the ideal case. We have also established that the concept of an absolute bandwidth is an abstraction, and that in the real world we are faced with the need to choose a definition of bandwidth that is useful for our particular application. In the remainder of the book, each of the signal processing steps introduced in this chapter will be explored in the context of the typical system block diagram appearing at the beginning of each chapter.

Problems and Questions

- Problems

1.2. Determine the energy spectral density of a square pulse $x(t) = \text{rect}(t/T)$, where $\text{rect}(t/T)$ equals 1, for $-T/2 \le t \le T/2$, and equals 0, elsewhere. Calculate the normalized energy E_x in the pulse.

1.4. Using time averaging, find the average normalized power in the waveform $x(t) = 10 \cos 10t + 20 \cos 20t$.

1.12. The Fourier transform of a signal $x(t)$ is defined by $X(f) = \text{sinc}\, f$, where the sinc function is as defined in Equation (1.39). Find the autocorrelation function, $R_x(\tau)$, of the signal $x(t)$.

1.15. The two-sided power spectral density, $G_x(f) = 10^{-6} f^2$, of a waveform $x(t)$ is shown in Figure P1.2.

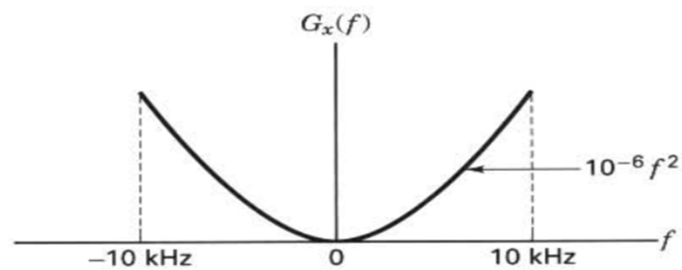


Figure P1.2

(a) Find the normalized average power in $x(t)$ over the frequency band from 0 to 10 kHz.
(b) Find the normalized average power contained in the frequency band from 5 to 6 kHz.

Problems and Questions

- Questions

1.1. How does the plot of a signal's autocorrelation function reveal its bandwidth occupancy? (See Section 1.5.4.)

1.2. What two requirements must be fulfilled in order to insure distortionless transmission through a linear system? (See Section 1.6.3.)

1.3. Define the parameter *envelope delay* or *group delay*. (See Section 1.6.3.)

1.4. What mathematical dilemma is the cause for there being several different definitions of bandwidth? (See Section 1.7.2.)