Component Measurements

EIE 240 Electrical and Electronic Measurement
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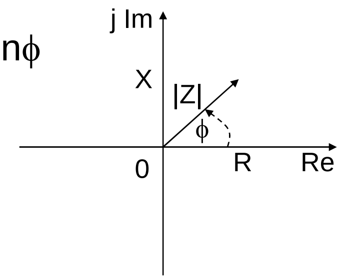
Component RLC

- Impedance = $\dfrac{\text{Potential Difference Phasor}}{\text{Current Phasor}}$
- Impedance = Resistance + j Reactance , $j = \sqrt{-1}$

$$
\begin{aligned}
Z &= R + jX \\
&= |Z|\cos\phi + j\,|Z|\sin\phi \\
&= |Z|e^{j\phi} \\
&= |Z|\angle\phi
\end{aligned}
$$

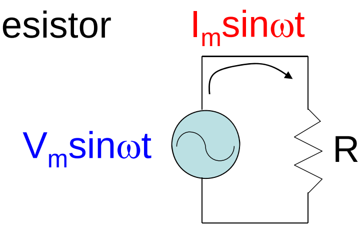


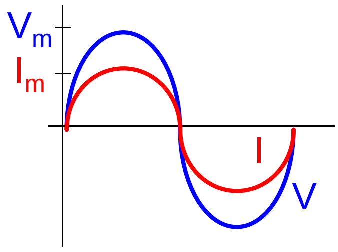
where $|Z| = \sqrt{R^2 + X^2}$

$\phi = \tan^{-1}(X/R)$

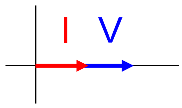
AC Response

- Resistor





Resistance
$V = R\,I$
$= R\,I_m \sin\omega t$
$= V_m \sin\omega t$
(V and I in phase)



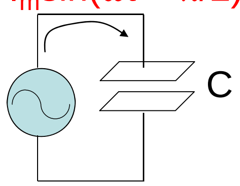
Phasor Diagram

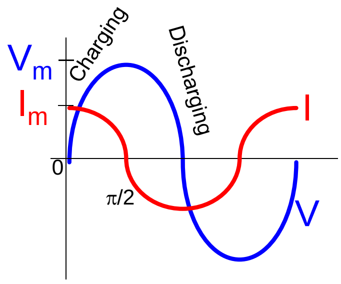
AC Response (Cont'd)

- Capacitor is a passive component storing the energy in an electric field charged by the voltage across the dielectric.

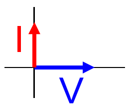
$I = C\,dV/dt$
$= I_m \sin(\omega t + \pi/2)$

$V = V_m \sin(\omega t)$





Capacitive Reactance
$I = dq/dt = C\,dV/dt$
$V = 1/C \int I_m \sin\omega t\, dt + V_0$
$= -1/\omega C\ I_m \cos\omega t$
$= 1/\omega C\ I_m \sin(\omega t - \pi/2)$
$= X_C I_m \sin(\omega t - \pi/2)$
$= V_m \sin(\omega t - \pi/2)$
(V lags I by 90°)



Phasor Diagram

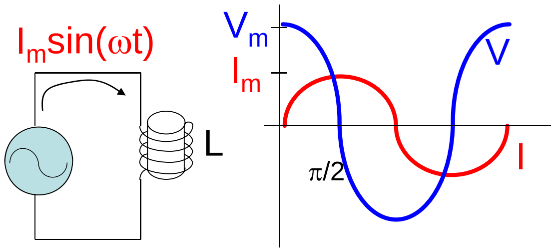
$X_C = 1/\omega C = 1/2\pi f C = \tau/2\pi C$, Higher f, Lower X_C (Lowpass Filter)

AC Response (Cont'd)

- Inductor is a passive component storing the energy in a magnetic field induced by the electric current passing through it.

$V = L\,dI/dt$

$\quad = V_m \sin(\omega t + \pi/2)$



Inductive Reactance

$V = d(N\phi)/dt$

$\quad = L\,d(I_m \sin\omega t)/dt$

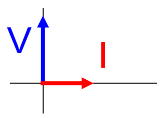
$\quad = \omega L\, I_m \cos\omega t$

$\quad = \omega L\, I_m \sin(\omega t + \pi/2)$

$\quad = X_L I_m \sin(\omega t + \pi/2)$

$\quad = V_m \sin(\omega t + \pi/2)$

(V leads I by 90°)



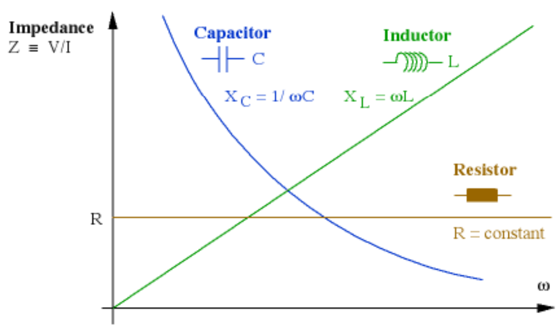
Phasor Diagram

$X_L = \omega L = 2\pi f L = 2\pi L/\tau$, , Higher f, Higher X_L (Highpass Filter)

Frequency Response

- The resonance of a series RLC circuit occurs when the inductive and capacitive reactances are equal in magnitude but cancel each other because they are 180° apart in phase, ($Z = \sqrt{R^2 + (X_L - X_C)^2} = R$) and $1 / 2\pi f_0 C = 2\pi f_0 L$

$$f_0 = 1 / 2\pi\sqrt{LC}$$



Q-Factor and D-Factor

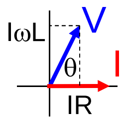
- Q-factor is to express the quality of component in ability to store and release energy or quality of L ➔ L+R,

  Q = Energy Stored / Power Loss

  = Reactance / Resistance

  = ωL / R

  = $\tan\theta$



- D-factor is for a dissipation of C ➔ C + R,

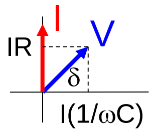
  D = 1/Q

  = Power Loss / Energy Stored

  = R / $(1/\omega C)$

  = ωRC

  = $\tan\delta$



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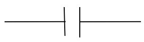
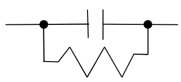
Capacitor Model

- An ideal capacitor stores but does not dissipate energy.



- Because the dielectric separating the capacitor plates are not a perfect insulator, it causes a small leakage current flowing through the capacitors ➔ parallel model.

  $D = V^2/R \,/\, V^2/X_C = 1/\omega RC$



- Plate loss due to the resistances of the plates and leads can become quite significant in higher frequency case ➔ series model.

  $D = I^2R \,/\, I^2X_C = \omega RC$

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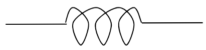
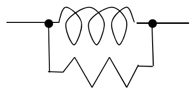
Inductor Model

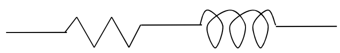
- An ideal inductor stores but does not dissipate energy.



- Time-varying current in a ferromagnetic inductor, which causes a time-varying magnetic field in its core, causes energy losses in the core material that are dissipated as heat ➔ parallel model.
  $Q = V^2/X_L \,/\, V^2/R = R/\omega L$



- Resistance of the wire ➔ series model.
  $Q = I^2X_L \,/\, I^2R = \omega L/R$



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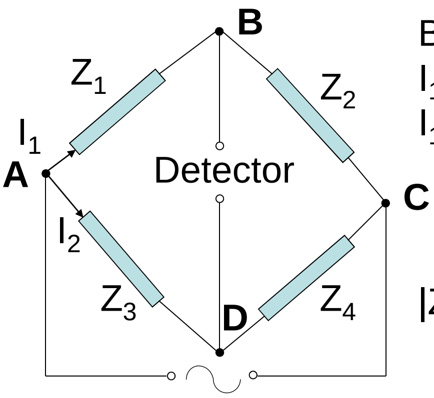
AC Bridge



Wheatstone Bridge

Balanced Bridge,

$I_1 |Z_1| \angle\phi_1 = I_2 |Z_3| \angle\phi_3$

$I_1 |Z_2| \angle\phi_2 = I_2 |Z_4| \angle\phi_4$

$|Z_1|/|Z_2| \angle\phi_1 - \phi_2 = |Z_3|/|Z_4| \angle\phi_3 - \phi_4$

$$\frac{R_1 + jX_1}{R_2 + jX_2} = \frac{R_3 + jX_3}{R_4 + jX_4}$$

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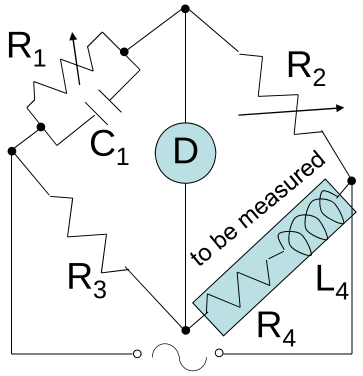
Inductance Measurement

There is no pure components, e.g. an inductor can be considered to be a pure inductance (L_4) in series with a pure resistance (R_4).

Maxwell-Wien Bridge (for medium Q = 1-10)



Impedances,

$1/Z_1 = 1/R_1 + 1/(1/j\omega C_1)$

$Z_1 = R_1 / (1+j\omega R_1 C_1)$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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Maxwell-Wien Bridge (Cont'd)

Balanced bridge,

$$Z_1 / Z_2 = Z_3 / Z_4$$

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 R_3 (1+j\omega R_1 C_1)/R_1$$

$$= R_2 R_3/R_1 + j\omega R_2 R_3 C_1$$

Real part: $R_4 = R_2 R_3/R_1$

Imagination part: $L_4 = R_2 R_3 C_1$

The balancing is independent of frequency.

Adjust R_1 and R_2 to get the bridge balanced (Null)

$$Q = \omega L_4/R_4 = \omega(R_2 R_3 C_1)/(R_2 R_3/R_1) = \omega R_1 C_1$$

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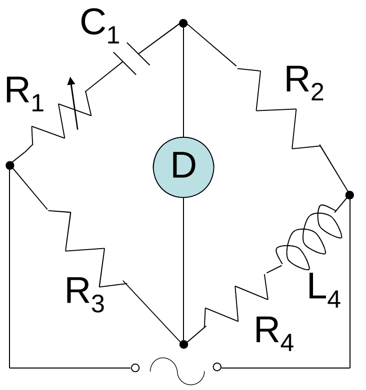
Hay Bridge

For high $Q \geq 10$



Impedances,

$Z_1 = R_1 + 1/j\omega C_1$

$= R_1 - j/\omega C_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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Hay Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 R_3 / (R_1 - j/\omega C_1)$$

$$(R_1R_4 + L_4/C_1) + j\,(\omega R_1 L_4 - R_4/\omega C_1) = R_2R_3$$

Imagination part: $\omega R_1 L_4 = R_4/\omega C_1$

$$L_4 = R_4 / \omega^2 R_1 C_1$$

Real part: $R_1R_4 + L_4/C_1 = R_2R_3$

$$R_1R_4 + R_4/\omega^2 R_1 C_1^2 = R_2R_3$$

$$R_4\,(R_1 + 1/\omega^2 R_1 C_1^2) = R_2R_3$$

$$R_4\,(\omega^2 R_1^2 C_1^2 + 1)/(\omega^2 R_1 C_1^2) = R_2R_3$$

$$R_4 = (\omega^2 R_1 R_2 R_3 C_1^2) / (\omega^2 R_1^2 C_1^2 + 1)$$

$$L_4 = (R_2 R_3 C_1) / (\omega^2 R_1^2 C_1^2 + 1)$$

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Hay Bridge (Cont'd)

$Q = \omega L_4/R_4$
$\quad = 1/\omega R_1C_1$

Therefore, $L_4 = (R_2R_3C_1) / ((1/Q^2) + 1)$
$\quad \approx R_2R_3C_1$ if $Q \geq 10$

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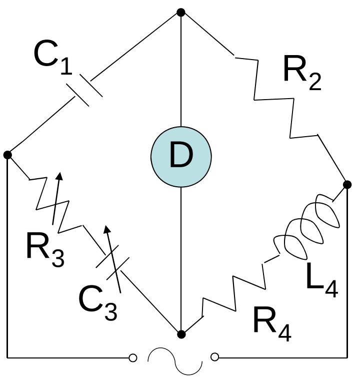
Owen Bridge



Impedances,
$Z_1 = 1/j\omega C_1$
$Z_2 = R_2$
$Z_3 = R_3 – j/\omega C_3$
$Z_4 = R_4 + j\omega L_4$

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Owen Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 (R_3 - j/\omega C_3)\ j\omega C_1$$

$$= R_2 C_1 / C_3 + j\omega R_2 R_3 C_1$$

Real part: $R_4 = R_2 C_1 / C_3$

Imagination part: $L_4 = R_2 R_3 C_1$

The balancing is independent of frequency.

$Q = \omega L_4 / R_4 = \omega R_2 R_3 C_1 C_3 / R_2 C_1 = \omega R_3 C_3$

References

- Electronics Demonstrations webpage: http://www.falstad.com/circuit/e-index.html