King Mongkut's University of Technology Thonburi

Midterm Examination 2/2007

CPE ~~222~~ ๒๐๐ Signals and Systems	**Computer Engineering Department**
Date: December 21, 2007	**Time: 1:00 – 4:00 p.m.**

Instructions:

1. **Calculator and Ruler with mathematical formula are allowed** in the examination room.
2. **Books, documents, and notes are not allowed** in the examination room.
3. **Do not take the examination sheets out** of the examination room.
4. This examination has **7 pages (5 problems, 80 marks)**.

1. a). Determine whether these c-t systems are **causal**, **linear**, **time invariant** and have **memory**. Justify your answers. *(7 marks)*

   i) $y(t) = |x(t)| = \begin{cases} x(t) & \text{when } x(t) \geq 0 \\ -x(t) & \text{when } x(t) < 0 \end{cases}$

   ii) $y(t) = \int_0^t (t-\tau)x(\tau)d\tau$

   b). Evaluate the following integral: $\int_{-\infty}^{\infty} \delta(t-1)(t^2 + \cos \pi t)dt$ *(3 marks)*

   c). Consider three d-t systems S1, S2, and S3 whose respective responses to a complex exponential input $e^{j\pi n/2}$ are specified as:

   S1: $e^{j\pi n/2} \Rightarrow e^{j\pi n/2}u[n]$,
   S2: $e^{j\pi n/2} \Rightarrow e^{j3\pi n/2}$,
   S3: $e^{j\pi n/2} \Rightarrow 2e^{j5\pi n/2}$.

   Determine whether the given information is sufficient to conclude that each system is definitely LTI. *(5 marks)*

2. Consider an LTI system which is **causal** and has its characteristics defined by **the difference equation**: $y[n] = 0.25y[n-1] + x[n]$. Determine the response of this system when the input $x[n] = \delta[n-1]$ *(10 marks)*

3. The response of an LTI system to the input signal $x(t) = \delta(t)$ is:

   $$y(t) = \delta(t+2) + 2\delta(t+1).$$

   Determine the response of this system when the input signal is:

   $$x(t) = \begin{cases} t+1 & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$ *(10 marks)*

4. Consider an LTI system defined as: $y(t) = \int_{t-1}^{t} x(\tau)d\tau$, where x(t) is the input signal and y(t) is the output signal. Determine:

   a) The impulse response and the frequency response of this system, *(6 marks)*

   b) The response of this system when the input signal is as defined in Fig. P4, *(12 marks)*

   c) The response of this system when the input is:

   $$x(t) = \cos(\pi t) + \sin(2\pi t + \pi/4).$$ *(12 marks)*

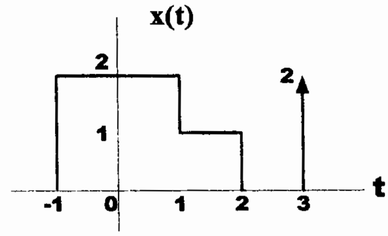

Figure P4

5. Determine the impulse response of a system having the frequency response defined as: *(15 marks)*

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

Note:

Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\Omega_0 t} \quad \text{and} \quad X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\Omega_0 t}dt$$

Discrete-Time Fourier Series:

$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\omega_0 n} \quad \text{and} \quad X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\omega_0 n}$$

Fourier Transform:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad \text{and} \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Discrete-Time Fourier Transform:

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n} \quad \text{and} \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

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TABLE 1 PROPERTIES OF THE FOURIER TRANSFORM

Property	Aperiodic Signal	Fourier Transform
	$x(t)$, $y(t)$, $h(t)$	$X(j\Omega)$, $Y(j\Omega)$, $H(j\Omega)$
Linearity	$ax(t) + by(t)$	$aX(j\Omega) + bY(j\Omega)$
Time Shifting	$x(t-t_0)$	$e^{-j\Omega t_0} X(j\Omega)$
Frequency Shifting	$e^{j\Omega_0 t} x(t)$	$X(j(\Omega-\Omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\Omega)$
Time Reversal	$x(-t)$	$X(-j\Omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\Omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\Omega)Y(j\Omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)Y(j(\Omega-\theta))d\theta$
Differentiation in Time	$\frac{dx(t)}{dt}$	$j\Omega X(j\Omega)$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\Omega}X(j\Omega) + \pi X(0)\delta(\Omega)$
Differentiation in Frequency	$t\,x(t)$	$j\frac{d}{d\Omega}X(j\Omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\Omega) = X^*(-j\Omega) \\ \Re e\{X(j\Omega)\} = \Re e\{X(-j\Omega)\} \\ \Im m\{X(j\Omega)\} = -\Im m\{X(-j\Omega)\} \\ \|X(j\Omega)\| = \|X(-j\Omega)\| \\ \sphericalangle X(j\Omega) = \sphericalangle X(-j\Omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\Omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\Omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = Ev\{x(t)\}$ [$x(t)$ real]	$\Re e\{X(j\Omega)\}$

$x_o(t) = Od\{x(t)\}$ [x(t) real]	$j\Im m\{X(j\Omega)\}$

Parseval's Relation for Aperiodic Signal

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2\,d\Omega$$

TABLE 2 BASIC FOURIER TRANSFORM PAIRS

Signal	**Fourier Transform**	**Fourier Series Coefficients (if periodic)**
$\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - k\Omega_0)$	a_k
$e^{j\Omega_0 t}$	$2\pi\delta(\Omega-\Omega_0)$	$a_1 = 1$ $a_k = 0$, Otherwise
$\cos(\Omega_0 t)$	$\pi[\delta(\Omega-\Omega_0) + \delta(\Omega+\Omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, Otherwise
$\sin(\Omega_0 t)$	$\frac{\pi}{j}[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, Otherwise
$x(t) = 1$	$2\pi\delta(\Omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (This is the Fourier series representation for any choice at $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1 & \lvert t\rvert \le T_1 \\ 0 & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2\sin k\Omega_0 T_1}{k} \delta(\Omega - k\Omega_0)$	$\frac{\Omega_0 T_1}{\pi} \sin c\left(\frac{k\Omega_0 T_1}{\pi}\right) =$ $\frac{\sin k\Omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k

$e^{-at}u(t), \quad Re\{a\} > 0$	$\frac{1}{a + j\Omega}$	
$x(t) = \begin{cases} 1 & \|t\| < T_1 \\ 0 & \|t\| > T_1 \end{cases}$	$\frac{2\sin\Omega T_1}{\Omega}$	
$\frac{\sin Wt}{\pi t}$	$X(j\Omega) = \begin{cases} 1 & \|\Omega\| < W \\ 0 & \|\Omega\| > W \end{cases}$	
$\delta(t)$	1	
$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$	
$(t+1)\, e^{-at}u(t), \quad Re\{a\} > 0$	$\frac{1}{(a + j\Omega)^2}$	
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t), \; Re\{a\} > 0$	$\frac{1}{(a + j\Omega)^n}$	

TABLE 3 BASIC DISCRETE-TIME-FOURIER TRANSFORM PAIRS

Signal	**Fourier Transform**	**Fourier Series Coefficients (if periodic)**
$\sum_{k=<N>} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1 & k = m, m \pm N, m \pm 2N.. \\ 0 & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$\cos(\omega_0 n)$	$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$

		$a_k = \begin{cases} \frac{1}{2} & k = m, m \pm N, m \pm 2N.. \\ 0 & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$\sin(\omega_0 n)$	$\frac{\pi}{j}\sum_{l=-\infty}^{\infty}\{\delta(\omega-\omega_0-2\pi l)-\delta(\omega+\omega_0-2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2j} & k = m, m \pm N, m \pm 2N \\ 0 & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$x[n] = 1$	$2\pi\sum_{l=-\infty}^{\infty}\delta(\omega-2\pi l)$	$a_k = \begin{cases} 1 & k = 0, \pm N, \pm 2N \ldots \\ 0 & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1 & \lvert n \rvert \le N_1 \\ 0 & N_1 < \lvert n \rvert \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi\sum_{l=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[2\pi/N)(N_1+1/2)}{N\sin[2\pi k/2N]}$, $k \ne 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1+1}{N}, k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{\infty}\delta[n-kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$	
$x[n] = \begin{cases} 1 & \lvert n \rvert \le N_1 \\ 0 & \lvert n \rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1+1/2)}{\sin(\omega/2)}$	
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1 & 0 \le \lvert\omega\rvert \le W \\ 0 & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(e^{j\omega})$ periodic with period 2 .	

$\delta[n]$	1	
$u[n]$	$\frac{1}{1-e^{j\omega}}+$ $\sum_{k=-\infty}^{\infty}\pi\delta(\omega-2\pi k)$	
$(n+1)a^n u[n], \quad \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^2}$	
$\frac{(n+r-1)!}{n!(r-1)!}a^n u[n], \quad \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^r}$	