King Mongkut's University of Technology Thonburi
Final Exam of Second Semester, Academic Year 2013

COURSE CPE112 Discrete Mathematics for Computer Engineers
Computer Engineering Dept, 1st Yr.

Thursday 15 May 2014 **09.00-12.00 h.**

Instructions

1. This examination contains 3 main problems, 9 pages (including this cover page).
2. The answers must be written in these exam sheets.
3. Students are allowed to use calculators.
4. No books, notes, or any other documents can be taken into the examination room.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room. Students must not take the examination and the answers out of the examination room.

Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by

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This examination is approved by the department of computer engineering

1. Relations (30 points)

The relation R_1 on {1, 2, 3} is given

$R_1 = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$

1.1) Represent the relation R_1 with a matrix, with the elements of this set listed in increasing order. (2 points)

1.2) Represent the relation R_1 with a directed graph. (2 points)

1.3) Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R_2 on set A, where

$$R_2 = \{(a, b) \mid a + b \leq 4\}$$ (2 points)

1.4) Represent the relation R_2 with a matrix, with the elements of this set listed in increasing order. (2 points)

1.5) Is R_1 an equivalence relation? Explain (4 points)

Definition: A relation on a set A is called an *equivalence relation* if it is reflexive, symmetric and transitive.

1.6) Is R_2 an equivalence relation? Explain (4 points)

1.7) Determine whether the relations represented by the zero-one matrices are equivalence relations. (3 points for each, total = 6 points)

1.7.1)

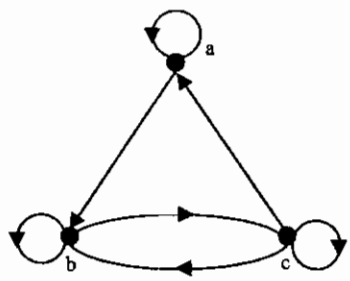
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

1.7.2)

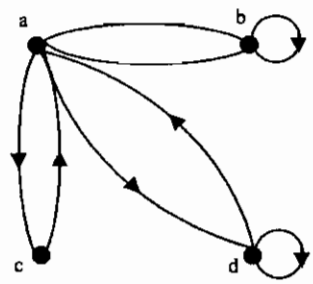
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

1.8) Determine whether the relations represented by the directed graphs are reflexive, symmetric, anti-symmetric, and/or transitive? (4 points for each, total = 8 points)

1.8.1)



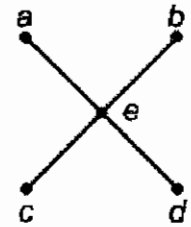
1.8.2)



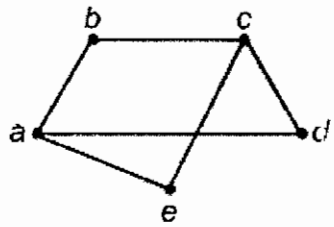
2. Graphs and Trees (40%)

2.1) Determine whether the following graph is bipartite. If it is bipartite, show the sets V_1 and V_2. (2 points for each, total = 10 points)

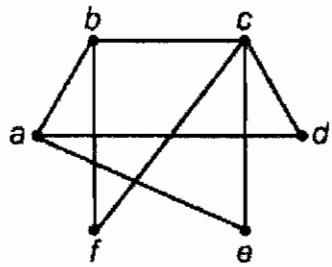
2.1.1)



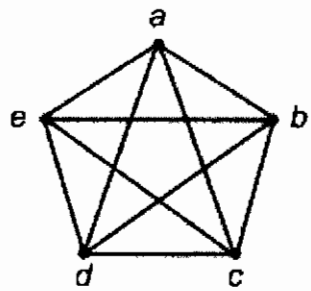
2.1.2)



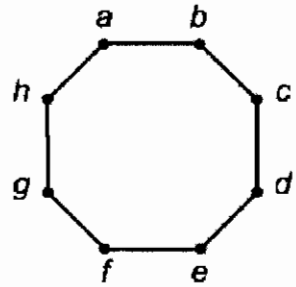
2.1.3)



2.1.4)



2.1.5)

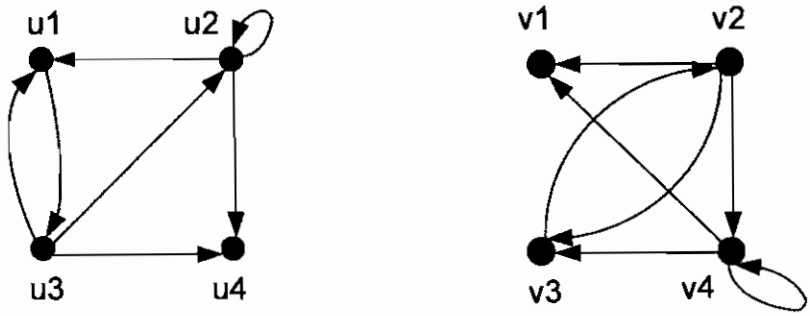


2.2) Determine whether the following pair of graphs is isomorphic. If it is isomorphic, show the function.

Definition The directed graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is one-to-one and onto function f: $V_1 \rightarrow V_2$ such that

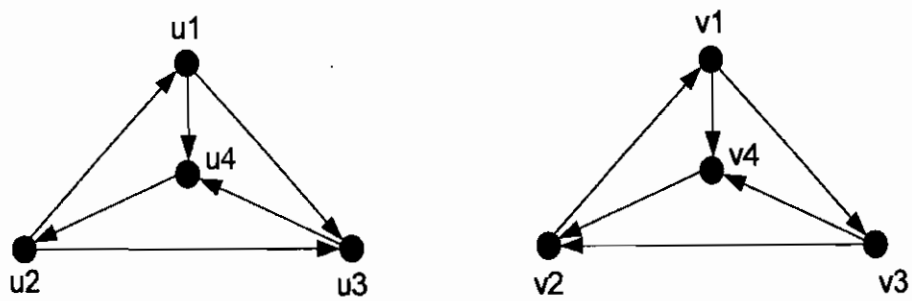
For all pairs of vertices *a* and *b* in V_1, $(a,b) \in E_1$ iff $(f(a), f(b)) \in E_2$

2.2.1) (2 points)



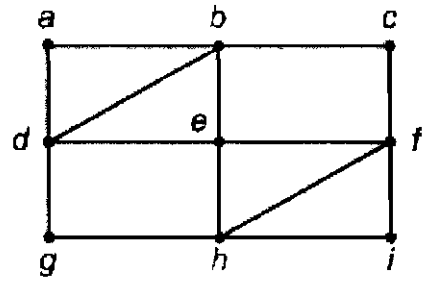
2.2.2) (2 points)

2.2.3) (4 points)

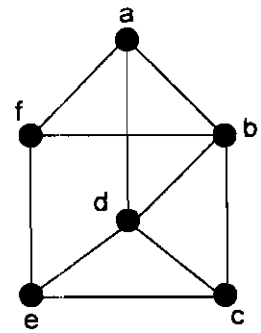


2.3) Euler Paths and Circuits (4 points for each, total = 8 points)

2.3.1) Determine whether the following graph has Euler circuit. If there is Euler circuit, show it.

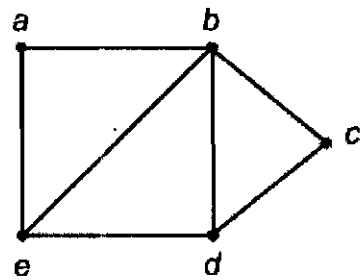


2.3.2) Determine whether the following graph has Euler circuit or path. If there is Euler circuit, show it. If there is Euler path, show it

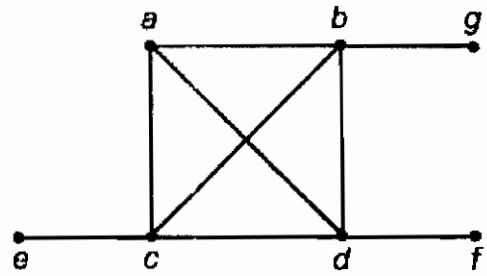


2.4) Hamilton paths and circuits (2 points for each, total = 4 points)

2.4.1) Determine whether the following graph has a Hamilton circuit. If it has, shows the Hamilton circuit.



2.4.2) Determine whether the following graph has a Hamilton path. If it has, shows the Hamilton path.



2.5) Tree properties (10 points)

2.5.1) Consider a full 3-nary tree which has 30 nodes. How many number of edges does the tree have? (2 points)

2.5.2) Consider a balanced but non-full 4-nary tree which has the height at 8. What is the maximal number of leaves of the tree? (3 points)

2.5.3) Consider a balanced 6-ary tree which has 220 leaves. What is the height of the tree? (5 points)

3. Discrete Probability (30 points)

Conditional Probability:

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{5/16}{1/2} = \frac{5}{8}.$$

Byes' Theorem:

$$p(F|E) = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\overline{F})p(\overline{F})}$$

3.1) What is the probability that two people chosen at random were born on the same day of the week? (4 points)

3.2) What is the probability that in a group of 10 people chosen at random, there are at least two born on the same day of the week? (3 points)

3.3) Let E and F are events in a sample space and p(E) = 1/3, p(F) = 1/2, and p(E | F) = 2/5. Find p(F | E) (3 points)

3.4) A bit string of length four is generated at random so that each of the 16 bit strings of length 4 is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 1? (10 points)

3.5) Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids? (10 points)