Capacitance Measurement II

EIE 240 Electrical and Electronic Measurement
Class 9, April 3, 2015

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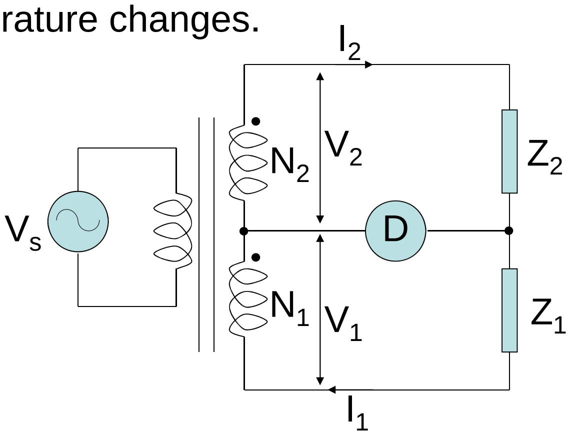
Transformer Ratio Bridges

Not only varying the impedances, but bridge can be also balanced by varying the turns ratio of a transformer. There is a small number of standard resistors and capacitors and no effect of temperature changes.



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Single Ratio Transformer Bridge

Tap a transformer $\Rightarrow$ voltage divider of V_s

$V_1 = kN_1 = I_1Z_1 \Rightarrow I_1 = kN_1 / Z_1$
$V_2 = kN_2 = I_2Z_2 \Rightarrow I_2 = kN_2 / Z_2$

To balance the bridge or no current through the detector, D = Null

$I_1 = I_2 \quad \Rightarrow \quad Z_1 / Z_2 = N_1 / N_2$

Impedance Ratio   Turn Ratio

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Single Ratio Transformer Bridge (Cont'd)

- Resistance Measurement

  Z_1 = Unknown resistor R_x

  Z_2 = Standard resistor R_s

$$R_x = R_s \frac{N_1}{N_2}$$

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Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement

Z_1 = Unknown $C_x \parallel R_x$ (leakage resistance)

$= 1/(1/R_x + j\omega C_x)$

$= R_x / (1+j\omega R_x C_x)$

Z_2 = Standard $C_s \parallel R_s$

$= R_s / (1+j\omega R_s C_s)$

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Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement (Cont'd)

Balanced, $Z_1 / Z_2 = N_1 / N_2$

$$1/Z_1 = (N_2/N_1)\, 1/Z_2$$

$$(1+j\omega R_x C_x)/R_x = (N_2/N_1)\,(1+j\omega R_s C_s)/R_s$$

$$1/R_x + j\omega C_x = (N_2 / N_1 R_s) + j\omega C_s N_2/N_1$$

Real part: $R_x = R_s N_1/N_2$

Imagination part: $C_x = C_s N_2/N_1$

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Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement

Z_1 = Unknown $L_x \parallel R_x$

$= 1/(1/R_x + 1/j\omega L_x)$

$= 1/(1/R_x - j/\omega L_x)$

Z_2 = Standard $C_s \parallel R_s$

$= 1/(1/R_s + j\omega C_s)$

$= 1/(1/R_s - j\omega C_s)$, Reversed Current

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Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement (Cont'd)

Balanced, $1/Z_1 = (N_2/N_1)\ 1/Z_2$

$1/R_x - j/\omega L_x = (N_2/N_1)\ (1/R_s - j\omega C_s)$

Real part: $R_x = R_s N_1/N_2$

Imagination part: $1/\omega L_x = (N_2/N_1)\omega C_s$

$L_x = N_1 / N_2 \omega^2 C_s$

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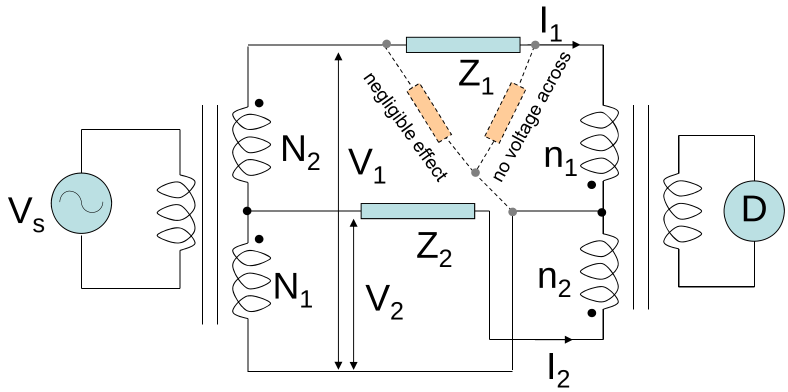
Double Ratio Transformer Bridge

To measure the impedance of components *in Situ.*



$I_1 = V_1 / Z_1 = k(N_1+N_2)/Z_1$
$I_2 = V_2 / Z_2 = kN_2/Z_2$

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Double Ratio Transformer Bridge (Cont'd)

Balanced, null current or zero magnetic flux,

$$n_1 I_1 = n_2 I_2$$

$$n_1(N_1+N_2)/Z_1 = n_2 N_2/Z_2$$

$$Z_1 = Z_2 \frac{n_1(N_1+N_2)}{n_2 N_2}$$

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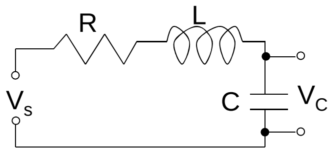
Q-Meter

RLC Series Resonance

$Z = R + j\omega L + 1/j\omega C$

$= R + j(\omega L - 1/\omega C)$



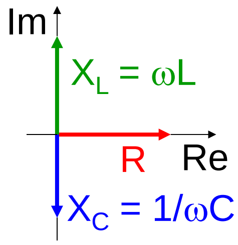
Resonant frequency

(When the voltage across C is a maximum.)

$\omega_0 L = 1/\omega_0 C$

$\omega_0^2 = 1/LC$

$\omega_0 = 1/\sqrt{LC}$



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Q-Meter (Cont'd)

$I_0 = V_s/R$

$V_C = I_0 X_C$

$= (V_s/R)(1/\omega_0 C)$

$= (1/\omega_0 RC)\ V_s$

$= Q\ V_s$

$\propto Q$

where Q = Reactance/Resistance

$= \omega_0 L / R$

$= \omega_0(1/\omega_0^2 C) / R$

$= 1 / \omega_0 RC$

(Unloaded)

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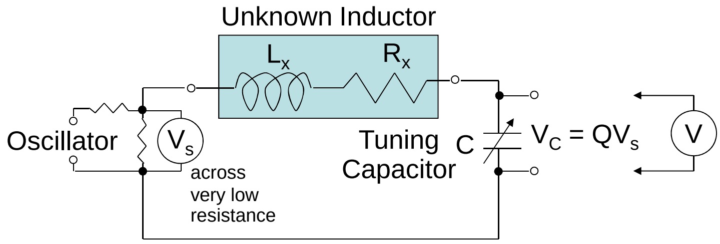
Q-Meter (Cont'd)



Tuning to resonance, $\omega_0 = 1/\sqrt{LC}$

$$L_x = 1 / \omega_0^2 C$$

$$R_x = 1 / Q\omega_0 C$$

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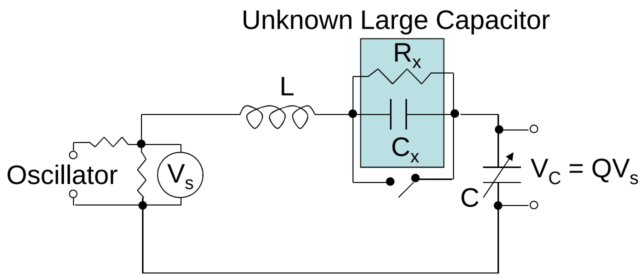
Q-Meter: Low Impedance Measurement



Short circuit $\Rightarrow$ tuning $C = C_1$, $L_1 = L$, R_1

$$Q_1 = 1/\omega_0 R_1 C_1$$

$$R_1 = 1/\omega_0 C_1 Q_1$$

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Q-Meter: Low Impedance Measurement (Cont'd)

Remove short circuit $\Rightarrow$ then tuning $C = C_2$

$$C_2 \ \& \ C_x = C_1,\ L_2 = L,\ R_2 = R_1 \ \& \ R_x$$

Series capacitors, $C_1 = 1 / (1/C_x + 1/C_2)$

$$= C_2C_x / (C_x+C_2)$$

$$C_1C_x + C_1C_2 = C_2C_x$$

$$C_x = C_1C_2 / (C_2 - C_1)$$

$Q_2 = 1/\omega_0R_2C_2$

$R_2 = 1/\omega_0C_2Q_2$

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Q-Meter: Low Impedance Measurement (Cont'd)

$R_1 = R_2 - R_x$

$R_x = R_2 - R_1$, Leakage resistance

$= 1/\omega_0C_2Q_2 - 1/\omega_0C_1Q_1$

$= (C_1Q_1 - C_2Q_2) / (\omega_0C_1C_2Q_1Q_2)$

$Q_x = 1/\omega_0R_xC_x$

$= (\omega_0C_1C_2Q_1Q_2)(C_2-C_1) / \omega_0(C_1Q_1 - C_2Q_2)C_1C_2$

$= Q_1Q_2(C_2-C_1) / (C_1Q_1-C_2Q_2)$

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Q-Meter: Low Impedance Measurement (Cont'd)

If the unknown component is an inductor,

$$L_x = 1/\omega_0^2 C_x$$
$$= (C_2 - C_1) / \omega_0^2 C_1 C_2$$

If the unknown component is a pure resistor (no reactance),

$$R_x = (C_1Q_1 - C_2Q_2) / (\omega_0 C_1 C_2 Q_1 Q_2)$$
$$= (Q_1 - Q_2) / \omega_0 C_1 Q_1 Q_2 \qquad , C_1 = C_2$$

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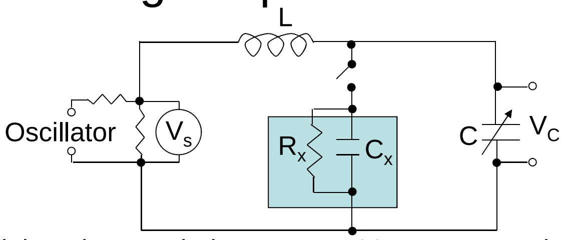
Q-Meter: High Impedance Measurement



For high resistance, inductance > 100mH, or capacitance < 400 pF

Open circuit and tune $C = C_1$, $L_1 = L$, R_1

Then short circuit and tune $C = C_2$

$$C_2 \,//\, C_x = C_1,\ L_2 = L,\ R_2 = R_x \,//\, R_1$$

$$C_x + C_2 = C_1 \quad \Rightarrow \quad C_x = C_1 - C_2$$

$$R_2 = R_x \| R_1 = R_x R_1/(R_x+R_1) \Rightarrow R_x = R_1R_2/(R_1 - R_2)$$

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Q-Meter: High Impedance Measurement (Cont'd)

Parallel RLC $\Rightarrow$ $Q = \omega_0 RC$ (loaded)

$R_1 = Q_1/\omega_0 C_1$ and $R_2 = Q_2/\omega_0 C_2$

Therefore

$$R_x = Q_1 Q_2 \omega_0 C_1 C_2 / \omega_0^2 C_1 C_2 (Q_1 C_2 - Q_2 C_1)$$
$$= Q_1 Q_2 / \omega_0 (Q_1 C_2 - Q_2 C_1)$$
$$Q_x = \omega_0 R_x C_x$$
$$= \omega_0 Q_1 Q_2 (C_1 - C_2) / \omega_0 (Q_1 C_2 - Q_2 C_1)$$
$$= Q_1 Q_2 (C_1 - C_2) / (Q_1 C_2 - Q_2 C_1)$$

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Q-Meter: High Impedance Measurement (Cont'd)

For unknown inductance,

$$L_x = 1/\omega_0^2 C_x$$
$$= 1/\omega_0^2 (C_1 - C_2)$$

For pure resistance,

$$R_x = Q_1 Q_2 / \omega_0 (Q_1 C_2 - Q_2 C_1)$$
$$= Q_1 Q_2 / \omega_0 C_1 (Q_1 - Q_2) \qquad , C_1 = C_2$$

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