Measurement and Testing

EIE 240 Electrical and Electronic Measurements (2/2011)
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Measurement

- **Measurement** is to determine the value or size of some quantity, e.g. a voltage or a current.
- **Analogue measurement** gives a response to a continuous quantity.
- **Digital measurement** is for the quantity at sampled times and quantized values.
- **Comparison measurement** is to compare the quantity with standards, e.g. null method.

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Testing

- **Testing** is to measure to ensure that a product conforms to its specification.
- **Manual testing** proceeded by human
- **Automatic testing** for reducing human error and increasing the performance.

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Error of a Measurement

- Errors are present in every experiment! If an experiment is well designed and carefully performed, the errors can be reduced to an acceptable level (their effects are not significant).
- Error = Measured Value – True Value
- Percentage Error = $\dfrac{\text{Error}}{\text{True Value}} \times 100\%$
- Degree of Uncertainty = $\pm$ %Error

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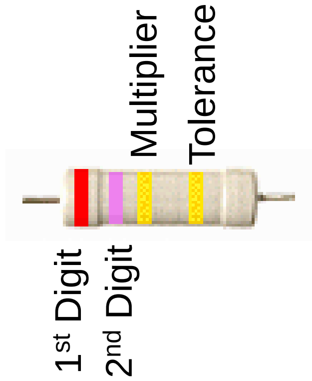
Error of a Measurement (2)

e.g. Resistor codes

None		(±20%)
Silver	-2	(±10%)
Gold	-1	(±5%)
Black	0	
Brown	1	(±1%)
Red	2	(±2%)
Orange	3	
Yellow	4	
Green	5	(±0.5%)
Blue	6	(±0.25%)
Violet	7	(±0.1%)
Grey	8	(±0.05%)
White	9	



$27\times10^{-1}\ \Omega\ \pm5\%$

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Random Error

- **Random error** is unpredictable for a successive reading of the same quantity.
- **Operating error** from the measurement situation leading to small variations.
- **Environmental error** such as a temperature or a humidity.
- **Stochastic error** e.g. electrical noise.

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Systematic Error

- **Systematic error** remains constant with repeated measurements.
- **Construction error** from manufacture of an instruments
- **Calibration error** from an incorrect setting.
- **Approximation error** e.g. for a linear scales
- **Ageing error** for the old instrument.
- **Loading error** for inserting a quantity affecting its value.

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Human Error

- **Human error** is the mistake made by humans in using instruments and taking the readings.
- **Misreading** of the operator.
- **Calculation error** of the operator.
- **Incorrect instruments** chosen by the operator.
- **Incorrect adjustment** of any conditions.

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Accuracy

- Accuracy refers to how closely a measured value agrees with the correct value.

- Error = Measured Value – Expected Value

$$e = x_{measured} - x_{expected}$$

- Percent Error = (Error / Expected Value)×100

$$\%e = |\,(x_{measured} - x_{expected}) / x_{expected}\,| \times 100$$

- Accuracy = 100 – Percent Error

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Precision

- Precision refers to how closely individual measurements agree with each other.

- Deviation = Measured Value – Average Value

$$d = x_{measured} - x_{average}$$

- Percent Deviation = (Deviation / Average Value) ×100

$$\%d = |\,(x_{measured} - x_{average}) / x_{average}\,| \times 100$$

- Precision = 100 – Percent Deviation

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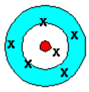
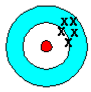
Accuracy Vs Precision

e.g. When a meter is said to be accurate to 1%, this means that a reading taken anywhere along one of its scale will not be in error by more than 1% of the full-scale value.



Accurate
(the average is accurate)
but not precise



Precise
but not accurate
(calibration needed)

Accurate
and precise

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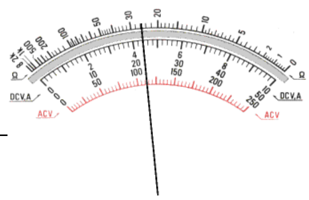
Resolution and Sensitivity

- Resolution is the significance of the least significant digits, e.g. the range of ammeter is 199 mA with a resolution of 0.1 mA. The range would be
  000.0, 000.1, 000.2, …, 199.9 mA
  or 3½ meter (the most significant digits can only be either a 0 or 1.)



- Sensitivity = $\dfrac{\text{Change in the Output}}{\text{Change in the Input}}$

e.g. $\dfrac{\text{Change in instrument scale reading}}{\text{Change in the quantity being measured}}$

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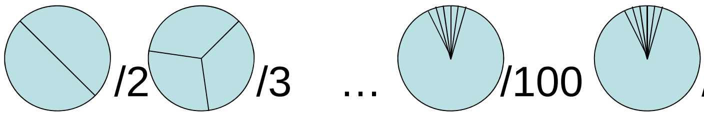
Statistical Evaluation

- Mean, $\overline{X} = \Sigma_{i=1 \to N} X_i / N$ $\Rightarrow$ the best value
- Deviation = $X_i - \overline{X}$
- Mean Deviation = $\Sigma_{i=1 \to N} |X_i - \overline{X}| / N$
- Standard Deviation,

  $\sigma = \sqrt{\Sigma_{i=1 \to N} (X_i - \mu)^2 / N}$ for a population

  s.d. = $\sqrt{\Sigma_{i=1 \to N} (X_i - \overline{X})^2 / N-1}$ for a sample (<30)

  e.g. a cake

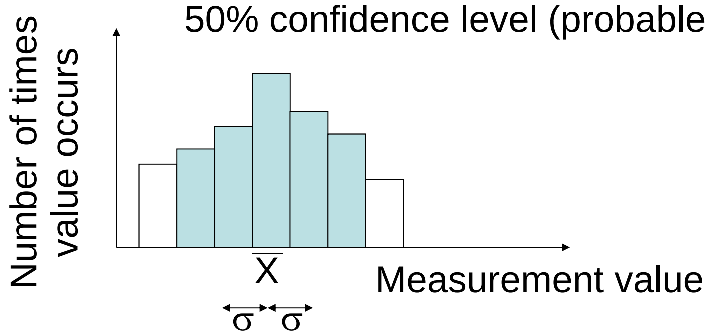


- Variance, σ^2

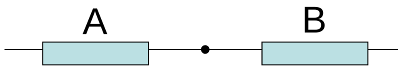
Normal Distribution (Gaussian)

Histogram

50% confidence level (probable error)



Summation of Error



$100\Omega \pm 10\%$ $\quad$ $200\Omega \pm 5\%$

$90\Omega \leftrightarrow 110\Omega$ $\quad$ $190\Omega \leftrightarrow 210\Omega$

$$X = A + B$$

$$X \pm \Delta X = A \pm \Delta A + B \pm \Delta B$$

$$= (A+B) \pm (\Delta A + \Delta B)$$

$$= (100\Omega + 200\Omega) \pm (10+10)$$

$$= 300\Omega \pm 20$$

$$280\Omega \leftrightarrow 320\Omega \quad \Rightarrow \quad \text{Extreme!}$$

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Summation of Error (Cont'd)

In case of summation,

$$X = A + B$$

$$X \pm \Delta X = A \pm \Delta A + B \pm \Delta B$$

$$= (A+B) \pm \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

$$= 300\Omega \pm 14.14$$

$$285.86\Omega \leftrightarrow 314.14\Omega \quad \Rightarrow \quad \text{Better}$$

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Summation of Error (Cont'd)

In case of multiplying,

$$X = AB$$

$$X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \cancelto{0}{\Delta A \Delta B}$$

$$\Delta X = \pm A\Delta B \pm B\Delta A$$

$$\Delta X/X = (\pm A\Delta B \pm B\Delta A) / AB \ \% \quad \Rightarrow \quad \%\text{Error}$$

$$= \pm \Delta B/B \pm \Delta A/A \ \%$$

$$= \pm (\Delta B/B + \Delta A/A) \ \%$$

$$= \pm \sqrt{(\Delta B/B)^2 + (\Delta A/A)^2} \ \%$$

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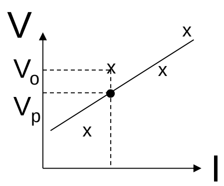
Linear Regression

$V = IR \quad \Rightarrow \quad$ Linear $y=mx+c$

Error = $V_p - V_o$

Minimum $\Sigma(V_p - V_o)^2$



Prediction Vs Observation
(Best Fitted?)

$$Y_i = \beta_0 + \beta_1 X_i + \cancelto{0}{\varepsilon_{random}} \quad , i=1,2,\ldots,N \text{ points}$$

$$\varepsilon = \Sigma_i [\, Y_i - (\beta_0 + \beta_1 X_i) \,]^2$$

$$\partial\varepsilon/\partial\beta_1 = -2\ \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i \,]\, X_i$$

$$= -2\ \Sigma_i [\, Y_i X_i - \beta_0 X_i - \beta_1 X_i^2 \,] = 0$$

$$\partial\varepsilon/\partial\beta_0 = -2\ \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i \,] = 0$$

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Linear Regression (Cont'd)

$$\Sigma_i [\, Y_iX_i - \beta_0X_i - \beta_1X_i^2 \,] = \Sigma_i [\, Y_i - \beta_0 - \beta_1X_i \,]$$

$$\Sigma_iY_iX_i - \beta_0\Sigma_iX_i - \beta_1\Sigma_iX_i^2 = \Sigma_iY_i - N\beta_0 - \beta_1\Sigma_iX_i$$

$$\Sigma_iY_iX_i - \Sigma_iY_i = \beta_0(\Sigma_iX_i - N) + \beta_1(\Sigma_iX_i^2 - \Sigma_iX_i)$$

$$(\Sigma_iY_iX_i)/N - \bar{Y} = (\bar{Y} - \beta_1\bar{X})(\bar{X} - 1) + \beta_1[\, (\Sigma_iX_i^2)/N - \bar{X} \,]$$

$$(\Sigma_iY_iX_i)/N - \cancel{\bar{Y}} = \bar{X}\bar{Y} - \cancel{\bar{Y}} + \beta_1[(\Sigma_iX_i^2)/N - \cancel{\bar{X}} - \bar{X}^2 + \cancel{\bar{X}}]$$

$$(\Sigma_iY_iX_i) = N\bar{X}\bar{Y} + \beta_1[\, \Sigma_iX_i^2 - N\bar{X}^2 \,]$$

$$\beta_1 = \frac{\Sigma_iY_iX_i - N\bar{X}\bar{Y}}{\Sigma_iX_i^2 - N\bar{X}^2} \quad \text{and } \beta_0 = \bar{Y} - \beta_1\bar{X}$$

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Linear Regression (Cont'd)

For nonlinear equation,

$Y = X^n \;\Rightarrow\; (\log Y) = n\,(\log X)$ → Logarithm

$Y = a^X \;\Rightarrow\; (\log Y) = (\log a)\, X$ → Semi-log

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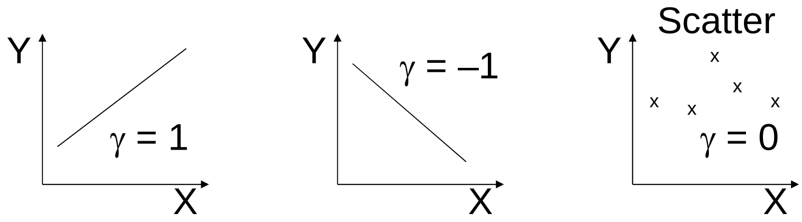
Correlation

$$\gamma = \frac{\text{Covariance}(X,Y)}{\sigma_X \sigma_Y} = \frac{1/N\ \Sigma(X - \bar{X})(Y - \bar{Y})}{\sigma_X \sigma_Y}$$



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