Power and Energy Measurement

EIE 240 Electrical and Electronic Measurement
April 24, 2015

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Work, Energy and Power

- Work is an activity of force and movement in the direction of force (Joules)
- Energy is the capacity for doing work (Joules)
- Power is the rate of using energy (Watt)

$P = W / t$, Work done in the time

$\quad = E / t$, Energy transferred in the time

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Electrical Power

- DC: Instantaneous power at time t

$$P(t) = I(t)\,V(t)$$
$$= I^2(t)\,R$$
$$= V^2(t)\,/\,R$$

- AC: Mean power per one cycle

$$P_{avg} = 1/T \int_{t=0 \to T} P(t)\,dt$$
$$= I_{rms}^2\,R$$
$$= V_{rms}^2\,/\,R$$

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Apparent and True Power

- True power is the power used by the load in Watt unit (W),

$$P = I^2R = V^2/R$$

- Reactive power in Volt-Amps-Reactive unit (VAR),

$$Q = I^2X = V^2/X$$

- Apparent power is the power that is applied to the load in Volt-Amps unit (VA),
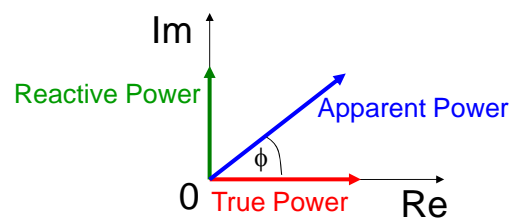
$$S = I^2Z = V^2/Z$$

where impedance Z = R + jX

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Power Factor

$$\cos \phi = \text{True Power} / \text{Apparent Power}$$

- Pure resistance, $\cos(0°) = 1 \Rightarrow$ perfect
- Pure inductance, $\cos(90°) = 0$
- Pure capacitance, $\cos(-90°) = 0$

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Power Factor (Cont'd)
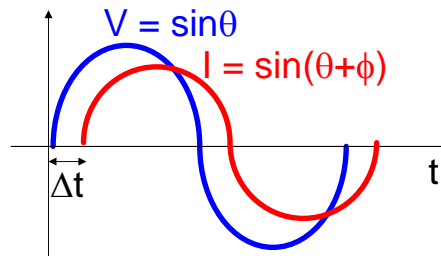
$$P = I V \cos\phi$$

- Up to a kind of loads being powered
- In phase or $\phi = 0$ (resistor)
- Lagging or current lags voltage, $\phi > 0$ (inductor)
- Leading or current leads voltage, $\phi < 0$ (capacitor)
- Lower power factor (higher $|\phi|$), higher losses
- Need to improve power factor

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Power Factor (Cont'd)

$I = \sin\omega(t+\Delta t)$

$\quad = \sin(\omega t+\omega\Delta t)$

$\quad = \sin(\theta+\phi)$

$V = \sin\theta$

$I = \sin(\theta+\phi)$

Δt t

$\phi = 0°$ ➔ $I = \sin(\theta)$, resistor

$\phi = 90°$ ➔ $I = \sin(\theta+90°)$

$\quad\quad\quad\quad = \cos(\theta)$, inductor

$\phi = -90°$ ➔ $I = \sin(\theta-90°)$

$\quad\quad\quad\quad = -\cos(\theta)$, capacitor

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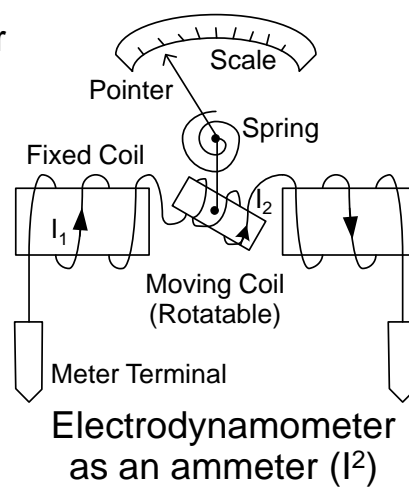
Wattmeter

- To measure the true power
- Two fixed coil is used to provide a magnetic field that is proportional to a current passing through it, instead of using a permanent magnet.
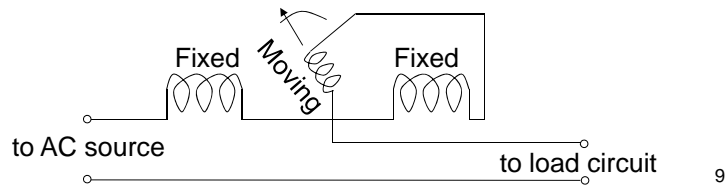
B   I

Right Hand's Rule

Scale

Pointer

Spring

Fixed Coil

I_1 I_2

Moving Coil
(Rotatable)

Meter Terminal

Electrodynamometer
as an ammeter (I^2)

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Wattmeter (Cont'd)

- Fixed coils, $B \propto I_1$
- Moving coil, Torgue $\propto B\, I_2$

$$\theta = K\, I_1\, I_2$$
$$= K\, I^2 \qquad\qquad , I = I_1 = I_2$$
$$\propto I^2 \quad \rightarrow \text{ammeter (non-linear scale)}$$

Fixed $\quad$ Moving $\quad$ Fixed

to AC source

to load circuit
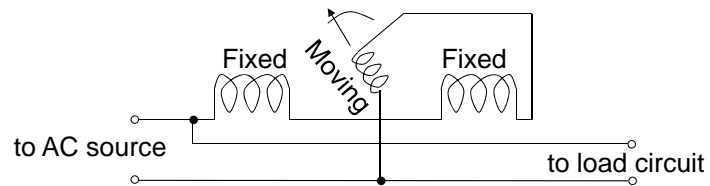
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Wattmeter (Cont'd)

- Full-scale deflection current 5-100 mA
- For larger FSD up to 20 A, it is shunted by a low resistance.
- Electrodynamometer can be used for both DC and AC (rms) measurement.

Fixed $\quad$ Moving $\quad$ Fixed
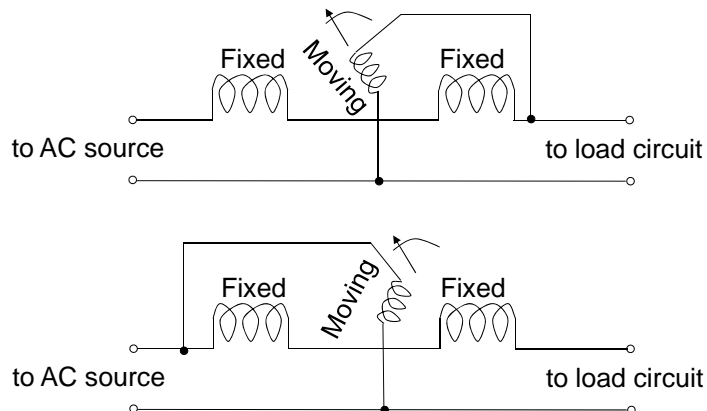
Shunt
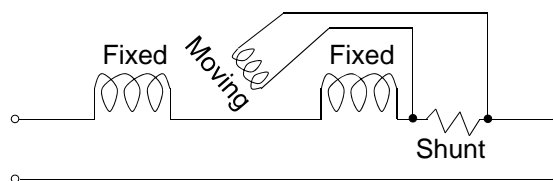
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Wattmeter (Cont'd)

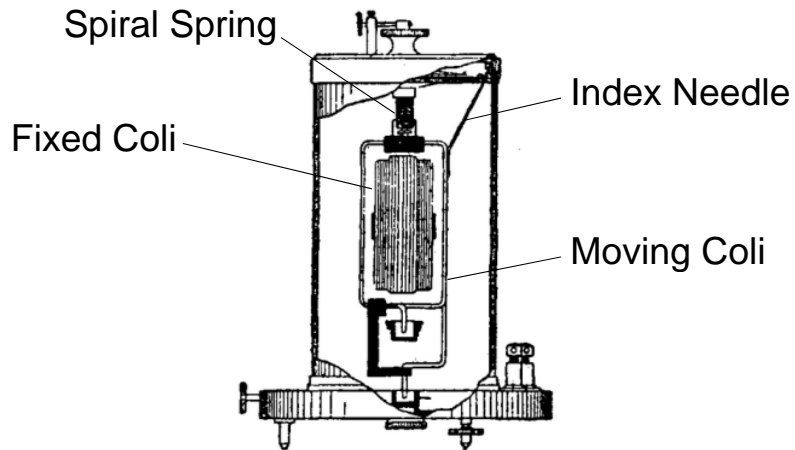- Electrodynamometer as a voltmeter
  $\theta \propto V^2$

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Wattmeter (Cont'd)

- Electrodynamometer as a wattmeter
  $\theta \propto IV$
- Loading error!

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Wattmeter (Cont'd)

- Electrodynamometer

Spiral Spring

Index Needle

Fixed Coli

Moving Coli
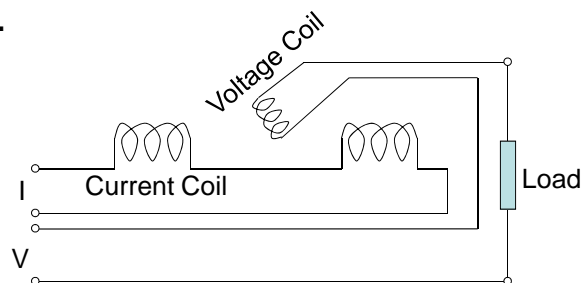
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Single-Phase Wattmeter

- Based on the principles of the dynamometer
- The current passing through moving coil and load is proportional to the potential difference across the load.
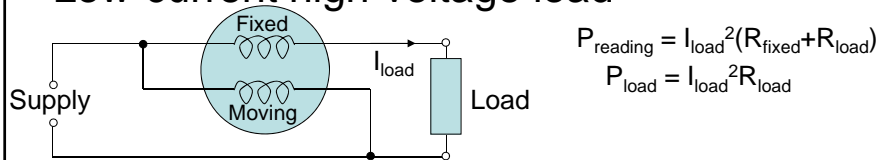- $\theta = K\, I_1\, I_2$
  $= K\, I\, (V/R)$
  $= (K/R)\, (IV)$
  $\propto P$

Voltage Coil

Current Coil

Load

I

V

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Single-Phase Wattmeter (Cont'd)

- Low-current high-voltage load

$$P_{reading} = I_{load}^2(R_{fixed}+R_{load})$$
$$P_{load} = I_{load}^2 R_{load}$$

Wattmeter reads high due to the potential drop across the fixed coil (if neglect the reactance).

- High-current low-voltage load
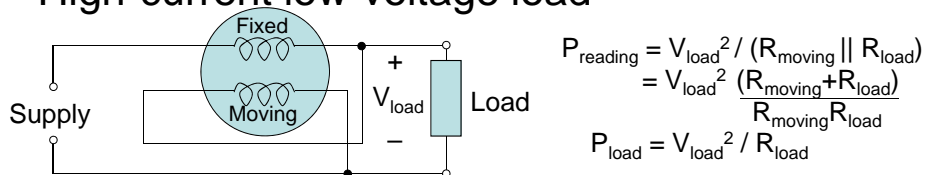
$$P_{reading} = V_{load}^2 / (R_{moving} \| R_{load})$$
$$= V_{load}^2 \frac{(R_{moving}+R_{load})}{R_{moving}R_{load}}$$
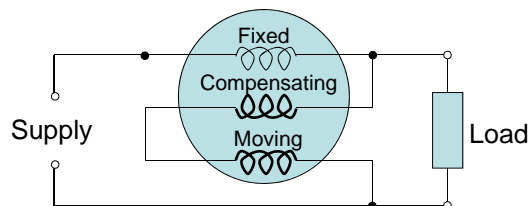$$P_{load} = V_{load}^2 / R_{load}$$

Wattmeter reads high due to the current through the movable coil (if neglect the reactance).

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# Single-Phase Wattmeter (Cont'd)

- Compensated wattmeter

To reduce the systematic error, the compensating coil current is, however, in the opposite direction to the load current and cancels out the proportion of the magnetic flux due to the moving coil current (the same number of turns).

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Crossed-Coil Power Factor Meter

- Two moving coils are crossed and connected in series to an inductor and resistor.
- For PF = 1, the coil with R will be in phase with the line current (moving), while the coil with L will be out of phase by 90° (no moving).
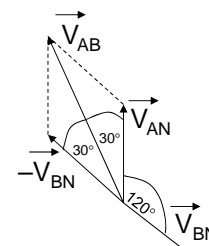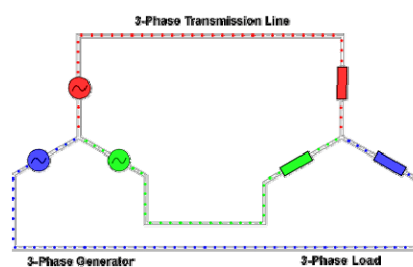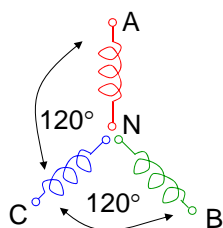- For PF = 0, coil L moves and coil R does not move.
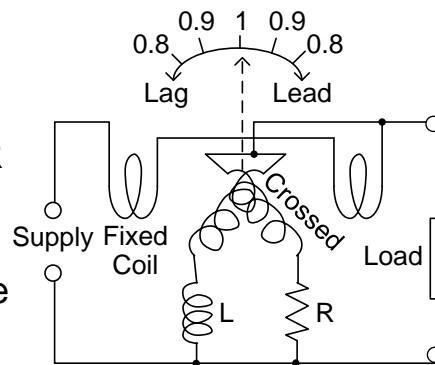
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Three-Phase Power Distribution

The three-phase system uses less conductor material to transmit electric power that the others.

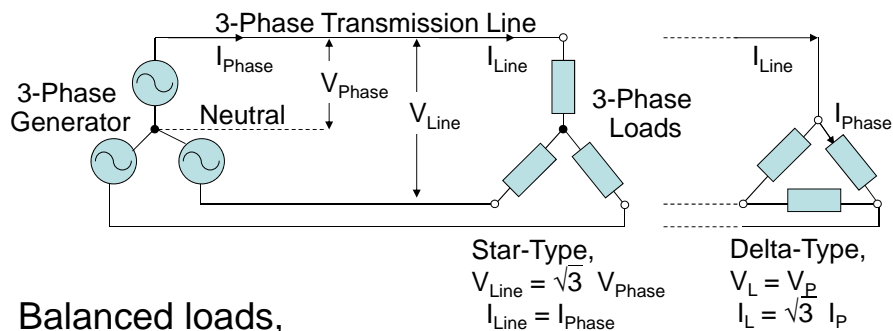Y-Type, $\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN}$

$|\vec{V}_{AB}| = |\vec{V}_{AN}| \cos30° + |\vec{V}_{BN}| \cos30°$

$= \sqrt{3} \ |\vec{V}_{AN}|$, $|\vec{V}_{AN}| = |\vec{V}_{BN}|$

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Three-Phase Loads

Star-Type,
$V_{Line} = \sqrt{3} \ V_{Phase}$
$I_{Line} = I_{Phase}$

Delta-Type,
$V_L = V_P$
$I_L = \sqrt{3} \ I_P$

Balanced loads,

$P_1 = P_2 = P_3 = P = I_L \ V_P \cos\phi$

$P_{total} = P \times 3 = 3 \ I_L \ V_P \cos\phi$

$\qquad\qquad = 3 \ I_L \ (V_L \ / \ \sqrt{3} \ ) \cos\phi$

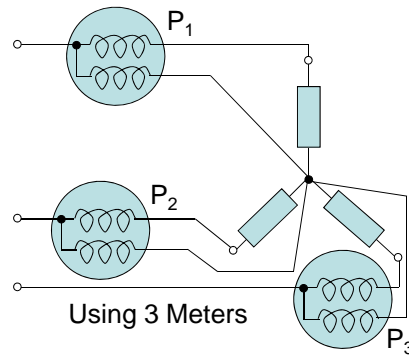$\qquad\qquad = \sqrt{3} \ I_L \ V_L \cos\phi$

Similarly, $Q = \sqrt{3} \ I_L \ V_L \sin\phi$

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Three-Phase Wattmeter

- For three-phase system with balanced load, just using single-phase wattmeter, $P \times 3$

Current Coil

Voltage Coil

Reading
$P = \sqrt{3} \ /3 \ I_L V_L \cos\phi$
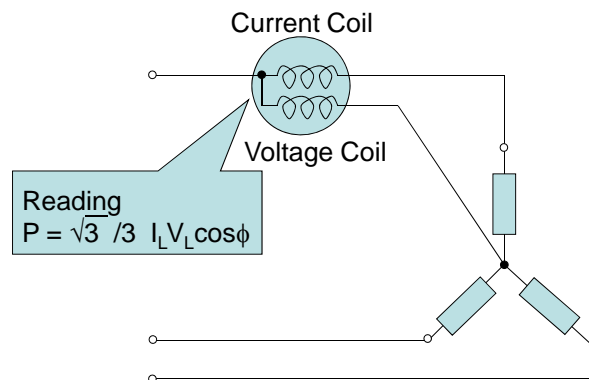
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Unbalanced-Load 3-Phase Wattmeter

Using 3 Meters

By using 3 wattmeters,

$$P_{total} = P_1 + P_2 + P_3$$

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Balanced-Load 3-Phase Wattmeter
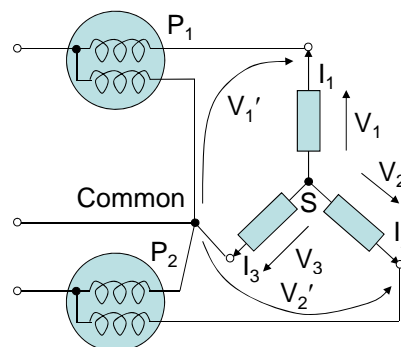
Brondel's theory,
$$P = I_1V_1 + I_2V_2 + I_3V_3$$

Kirchoff's law,
$$V_1 = V_3 + V_1'$$
$$V_2 = V_3 + V_2'$$

Therefore,
$$P = (I_1+I_2+I_3)V_3 \nearrow 0$$
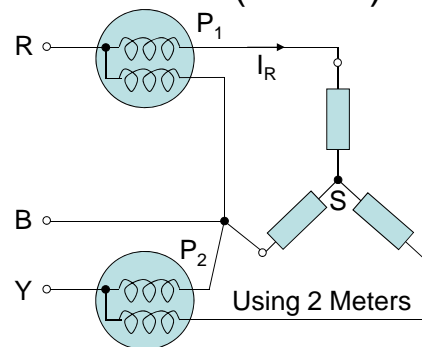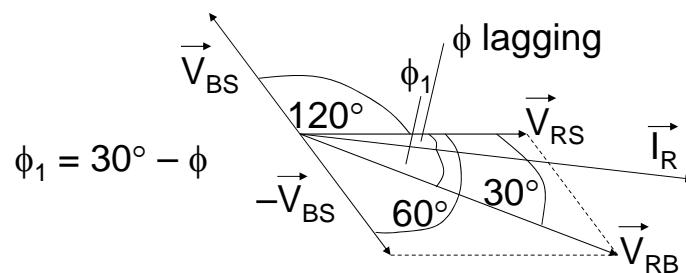$$\quad + I_1V_1' + I_2V_2'$$
$$= P_1 + P_2$$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

By using 2 wattmeters,

$P_1 = I_R V_{RB} \cos\phi_1$

$\vec{V}_{RB} = \vec{V}_{RS} - \vec{V}_{BS}$

Balanced, $|\vec{V}_{RS}| = |\vec{V}_{BS}|$

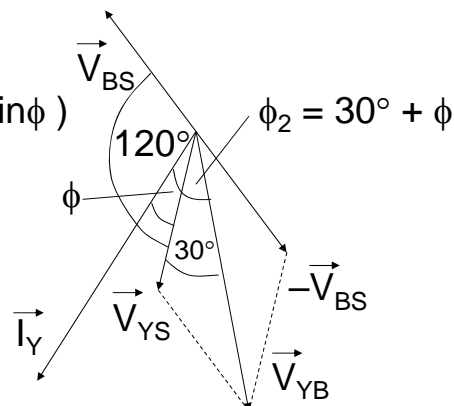$\phi_1 = 30° - \phi$

R ○

B ○

Y ○

P_1

I_R

S

P_2

Using 2 Meters

$\vec{V}_{BS}$

ϕ lagging

ϕ_1

120°

$\vec{V}_{RS}$

$\vec{I}_R$

$-\vec{V}_{BS}$

60°

30°

$\vec{V}_{RB}$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

$P_1 = I_L V_L \cos(30° - \phi)$
$\quad = I_L V_L (\sqrt{3} /2 \cos\phi + 1/2 \sin\phi)$

Similarly,
$P_2 = I_L V_L (\sqrt{3} /2 \cos\phi - 1/2 \sin\phi)$

$\vec{V}_{BS}$

$\phi_2 = 30° + \phi$

120°

ϕ

30°

$-\vec{V}_{BS}$

$\vec{I}_Y$

$\vec{V}_{YS}$

$\vec{V}_{YB}$

Note: $\cos(\theta \mp \phi) = \cos\theta \cos\phi \pm \sin\theta \sin\phi$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

Total Power,

$$P_{total} - P_1 + P_2$$
$$= \sqrt{3} \, I_L V_L \cos\phi$$

The same as using 1 meter for balanced load system

and $P_1 - P_2 = I_L V_L \sin\phi$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

Reactive Power,
$$Q = \sqrt{3} \, I_L V_L \sin\phi = \sqrt{3} \, (P_1 - P_2)$$

$$(P_1 - P_2)/(P_1 + P_2) = I_L V_L \sin\phi \, / \, \sqrt{3} \, I_L V_L \cos\phi$$
$$= \tan\phi \, / \, \sqrt{3}$$
$$\tan\phi = \sqrt{3} \, (P_1 - P_2) \, / \, (P_1 + P_2)$$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

$$\sin^2\phi + \cos^2\phi = 1$$

$$\tan^2\phi + 1 = 1/\cos^2\phi$$

$$3(P_1-P_2)^2/(P_1+P_2)^2 + 1 = 1/\cos^2\phi$$

Power factor,
$$\cos\phi = \sqrt{1 / [1 + 3(P_1-P_2)^2/(P_1+P_2)^2]}$$

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Watt-Hour-Meter

To measure the electrical energy supplied to industrial and domestic consumers in unit (kW-hr)

Generated Torque by Eddy Current = k_g I V $\cos\phi$
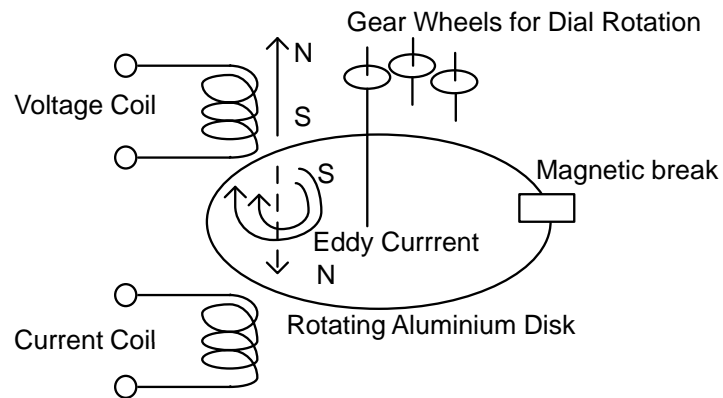
Magnetic Breaking Torque = k_b N

where N is number of revolution per unit time
(gearing to a mechanical counter to count watt-hours)

$$k_b N = k_g I V \cos\phi$$

$$N = (k_g/k_b) IV\cos\phi$$

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Watt-Hour-Meter (Cont'd)

Gear Wheels for Dial Rotation

N

Voltage Coil

S

S

Magnetic break

Eddy Currrent

N

Current Coil

Rotating Aluminium Disk

Therefore, number of revolution refers to the energy used by loads.

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