Electrical Indicating Instruments: AC

EIE 240 Electrical and Electronic Measurement
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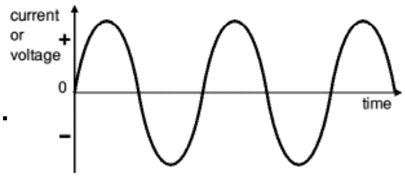
Sources and Loads

- An electrical circuit consists of nothing more than "sources" and "loads".
- Source is to produces an electrical energy, e.g. a chemical battery, an electronic power supply, or a mechanical generator.
- Load is to be powered by that electrical energy, e.g. a light bulb, an electronic clock, an electric fan, or just a resistor.
- What sources are AC? And what kinds of loads do not care which way current flows through them?

Alternating Current

- A current flows one way, then the other way, continually reversing direction.
- The usual waveform of an AC power is a sine wave that the change is so regular. The average value is zero (integration in a period).



AC from a power supply

- Main electricity in Thailand has a frequency of 50 Hz (0.02 sec/cycle), and 60 Hz in US.
- Voltage is continually changing between positive (+) and negative (–). The effective voltage in Thailand is 220 V, and 120 V in US.

3

Why Use Alternating Current?

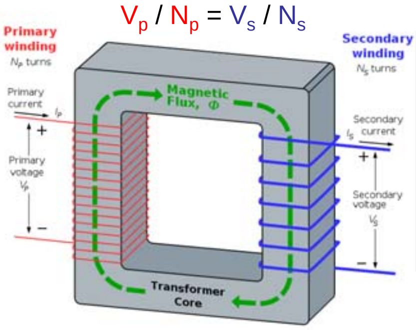
- In transmission of power, $P_{load} = IV$, the overhead wires are not a perfect conductor and exhibit some resistance, R. The absorbed energy is dissipated as waste heat. The lost power is $P_{lost} = I^2R$.
- At the same power, if the current is doubled (voltage reduced by half), $P_{load} = (2I)(V/2)$, the lost power is four times greater, $P_{lost} = (2I)^2R = 4I^2R$. To minimize that loss, we have to use much larger wires, and pay a high price for all that extra copper.

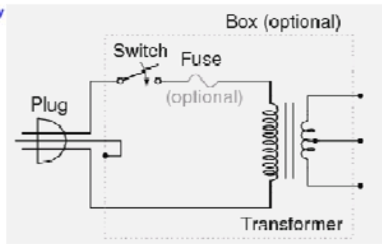
4

Why Use Alternating Current? (2)

- The solution is to use a transformer that can convert AC power at a higher/lower voltage with very slight losses. In practice, AC generators create electricity at a reasonable voltage, then we use transformers to step it up to very high levels (≈ 100,000 V) for long-distance transmission with a low current, and then use additional transformers to step it back down for local distribution to individual homes in safe value.

$V_p / N_p = V_s / N_s$





5

Averaging Power for AC

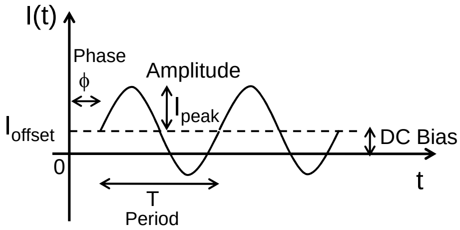
- $P(t) = I^2(t)\, R$ is just an instantaneous power at a time t.
- Mean power or the total energy converted in one cycle is

$$
\begin{aligned}
P &= 1/T \int_{t=0\to T} P\, dt \qquad , T = \text{period} \\
&= 1/T \int_{t=0\to T} I^2 R\, dt \\
&= R\, [\, 1/T \int_{t=0\to T} I^2\, dt\,] \\
&= R\, [\, \mathrm{sqrt}(1/T \int_{t=0\to T} I^2\, dt)\,]^2 \\
&= R\, (I_{rms})^2
\end{aligned}
$$

6

Sine Wave

- $y = \sin(\theta)$
  $= \sin(\omega t)$
  $= \sin(2\pi t/T)$, $\omega = 2\pi/T$ for sinewave
  $= \sin(2\pi f t)$, $f = 1/T$
- $I(t) = I_{offset} + I_{peak}\sin(\omega t - \phi)$



7

RMS or Effective Value

For only sinusoidal wave, $\omega = 2\pi/T$

$I_{root\text{-}mean\text{-}square} = \text{sqrt}(I^2_{average}) = \text{sqrt}(1/T\int_{t=0\to T} I^2(t)dt)$

$= \text{sqrt}(\ 1/T \int_{t=0\to T} [\ I_{peak}\sin(\omega t)\ ]^2\ dt\ )$

$= \text{sqrt}(\ (I_{peak})^2/T \int_{t=0\to T} \sin^2(\omega t)\ dt\ )$

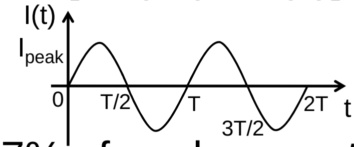
$= \text{sqrt}(\ (I_{peak})^2/2T \int_{t=0\to T} 1-\cos(2\omega t)\ dt\ )$

$= \text{sqrt}(\ (I_{peak})^2/2T\ \{\ T - [\sin(4\pi)-\sin(0)]\ \}\ )$

$= \text{sqrt}(\ (I_{peak})^2/2\ )$

$= I_{peak} / \text{sqrt}(2)$

$= 0.707\ I_{peak}$ (70.7% of peak current)

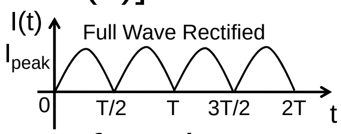


8

Positive Half-Cycle Averaging

For only sinusoidal wave, $T=2\pi$ and $\omega=2\pi/T=1$

$$I_{+\text{ half cycle}} = I_{average} = 1/(T/2) \int_{t=0\to T/2} I(t)\,dt$$
$$= 2/T \int_{t=0\to T/2} I_{peak}\sin(\omega t)\,dt$$
$$= 2I_{peak}/T \int_{t=0\to T/2} \sin(\omega t)\,dt$$
$$= 2I_{peak}/T\,[\,-\cos(\omega t)\,]_{t=0\to T/2}$$
$$= 2I_{peak}/T\,[\,-\cos(\omega T/2) + \cos(0)\,]$$
$$= 2I_{peak}/T\,[-\cos(\pi)+\cos(0)]$$
$$= 4I_{peak} / T$$
$$= 2I_{peak} / \pi$$
$$= 0.636\, I_{peak} \quad \text{(63.6\% of peak current)}$$

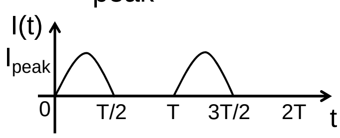


9

Half-Wave Rectified Sine

For only sinusoidal wave, $T=2\pi$ and $\omega=2\pi/T=1$

$$I_{+\text{ half wave}} = 1/T \int_{t=0\to T} I(t)\,dt$$
$$= 1/T \int_{t=0\to T/2} I_{peak}\sin(\omega t)\,dt$$
$$= I_{peak}/T \int_{t=0\to T/2} \sin(\omega t)\,dt$$
$$= I_{peak}/T\,[\,-\cos(\omega t)\,]_{t=0\to T/2}$$
$$= 2I_{peak}/T$$
$$= I_{peak} / \pi$$
$$= 0.318\, I_{peak} \quad \text{(31.8\% of peak current)}$$



10

Form Factor

- Form factor is a calibration constant or the ratio between average value and rms value,

  $$FF = Value_{rms} / Value_{avg}$$

  for sin, $FF = (I_{peak}/sqrt(2)) / (2I_{peak}/\pi)$

  $= \pi / 2sqrt(2)$

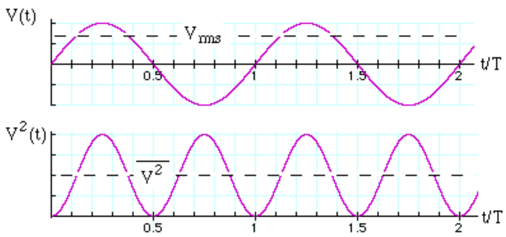
  $= 1.11$ (111%)

- If the AC signal is not a pure sin wave, the meter still reads an average value of the rectified wave. However, the form factor no longer is 1.11. It is based on the shape of the signal.

11

for Sinusoidal Wave

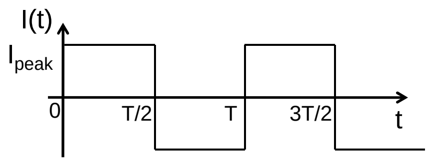
- $I_{rms} = 0.707\ I_{peak}$
- $V_{rms} = 0.707\ V_{peak}$
- $I_{+\text{ half cycle}} = 0.636\ I_{peak}$
- $V_{+\text{ half cycle}} = 0.636\ V_{peak}$
- $FF_{Sinusoidal} = 1.11$



12

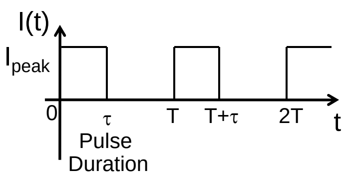
for Square Wave

- I_{rms} = sqrt(1/(T/2) $\int_{t=0\rightarrow T/2} I^2(t)\,dt$)
  = sqrt(2/T $\int_{t=0\rightarrow T/2} I^2_{peak}\,dt$)
  = sqrt($2I^2_{peak}$/T $\int_{t=0\rightarrow T/2} dt$)
  = sqrt($2I^2_{peak}$/T [(T/2) – (0)])
  = sqrt(I^2_{peak})
  = I_{peak}
- $V_{rms} = V_{peak}$
- $I_{+\text{ half cycle}} = I_{peak}$
- $V_{+\text{ half cycle}} = V_{peak}$
- $FF_{Square} = 1$



for Pulse Train

- I_{rms} = sqrt(1/T $\int_{t=0\rightarrow\tau} I^2(t)\,dt$)
  = sqrt(1/T $\int_{t=0\rightarrow\tau} I^2_{peak}\,dt$)
  = sqrt(I^2_{peak}/T $\int_{t=0\rightarrow\tau} dt$)
  = sqrt(I^2_{peak}/T [τ - 0])
  = sqrt(τ/T) I_{peak}
  = sqrt(D) I_{peak} , Duty cycle, D = τ/T

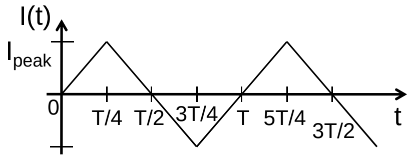


- V_{rms} = sqrt(D) V_{peak}
- Average value for DC pulse train signal

  I_{avg} = 1/T $\int_{t=0\rightarrow\tau} I_{peak}\,dt = D\,I_{peak}$
  $V_{avg} = D\,V_{peak}$

for Triangular Wave

- $I_{rms} = \text{sqrt}(\ 1/(T/4) \int_{t=0 \to T/4} I^2(t)\ dt\ )$
  $= \text{sqrt}(\ 4/T \int_{t=0 \to T/4} [\ (4I_{peak}/T)t\ ]^2\ dt\ )$
  $= \text{sqrt}(\ 4^3\ I^2_{peak}/T^3 \int_{t=0 \to T/4} t^2\ dt\ )$
  $= \text{sqrt}(\ 4^3\ I^2_{peak}/T^3\ [\ (T/4)^3/3 - (0)^3/3\ ]\ )$
  $= \text{sqrt}(\ I^2_{peak}\ /\ 3\ )$
  $= \text{sqrt}(3)/3\ I_{peak}$
  $= 0.577\ I_{peak}$ (57.7% of peak current)
- $V_{rms} = 0.577\ V_{peak}$



15

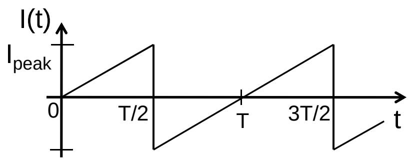
for Triangular Wave (Cont'd)

- $I_{+\ half\ cycle} = 4/T \int_{t=0 \to T/4} (4I_{peak}/T)t\ dt$
  $= 4^2\ I_{peak}/T^2 \int_{t=0 \to T/4} t\ dt$
  $= 4^2\ I_{peak}/T^2\ [\ (T/4)^2/2 - (0)^2/2\ ]$
  $= I_{peak}\ /\ 2$
  $= 0.5\ I_{peak}$ (50% of peak current)
- $V_{+\ half\ cycle} = 0.5\ V_{peak}$
- $FF_{Triangular} = 1.154$ (115.4%)

16

for Sawtooth Wave

- $I_{rms} = \text{sqrt}(1/(T/2) \int_{t=0\to T/2} I^2(t)\, dt)$
  $= \text{sqrt}(2/T \int_{t=0\to T/2} [(2I_{peak}/T)t]^2\, dt)$
  $= \text{sqrt}(2^3\, I^2_{peak}/T^3 \int_{t=0\to T/2} t^2\, dt)$
  $= \text{sqrt}(2^3\, I^2_{peak}/T^3 [(T/2)^3/3 - (0)^3/3])$
  $= \text{sqrt}(I^2_{peak} / 3)$
  $= \text{sqrt}(3)/3\ I_{peak}$
  $= 0.577\ I_{peak}$ (57.7% of peak current)
- $V_{rms} = 0.577\ V_{peak}$

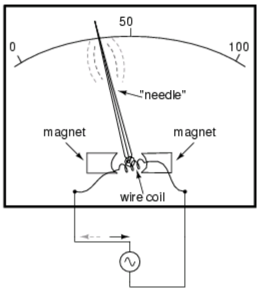


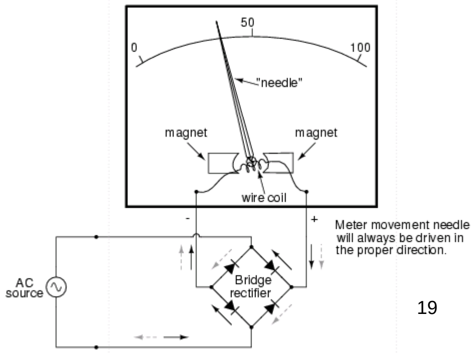
for Sawtooth Wave (Cont'd)

- $I_{+\text{ half cycle}} = 2/T \int_{t=0\to T/2} (2I_{peak}/T)t\, dt$
  $= 2^2\, I_{peak}/T^2 \int_{t=0\to T/2} t\, dt$
  $= 2^2\, I_{peak}/T^2 [(T/2)^2/2 - (0)^2/2]$
  $= I_{peak} / 2$
  $= 0.5\ I_{peak}$ (50% of peak current)
- $V_{+\text{ half cycle}} = 0.5\ V_{peak}$
- $FF_{Sawtooth} = 1.154$ (115.4%)

AC on Moving-Coil Meter

- Alternating current = alternating torque
- Very low frequency → alternating pointer
- Higher frequencies → pointer is not moved.
- AC/DC converter is needed.

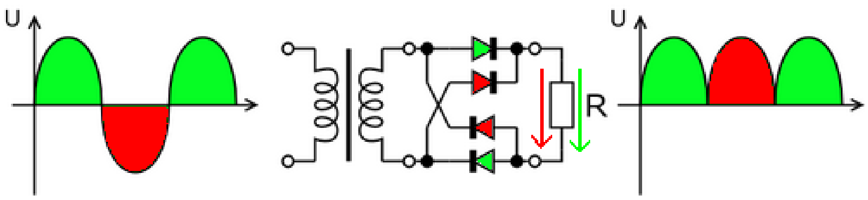




19

AC/DC Converter

- Full-wave rectifier circuit using bridge diodes
- Current flows only in one direction, anode to cathode, when diodes are forward biased.
- Inertia of moving coil → average value



- R → Galvanometer G

20

AC Calibration

Actually, AC meter responses to an average value of half-cycle wave. It can show root-mean-square values of sinusoidal wave by calibrating the scale by multiplying with form factor constant.

$$
\begin{aligned}
I_{rms} &= FF \times I_{+\text{ half cycle}} \\
&= 1.11 \times I_{+\text{ half cycle}}
\end{aligned}
$$

21

Error for Other-Form Wave

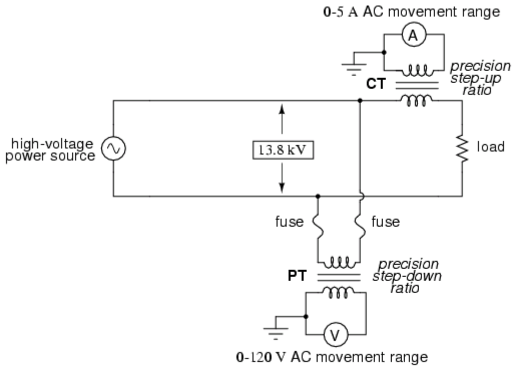
- If the input signal is not a sinusoidal wave form ($FF \neq 1.11$), a reading error is,

$$\text{Error} = (1.11 - FF) / FF \quad \times 100\%$$

22

Full-Scale-Deflection for AC Meter

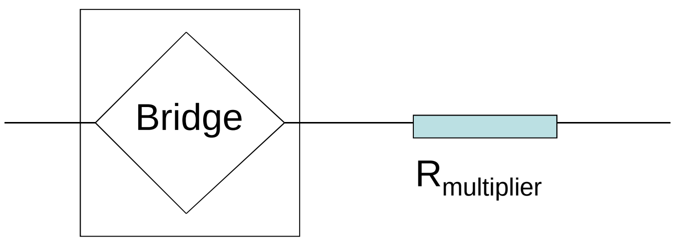
- Current transformer scales current down.
- Potential transformer scales voltage down.
- Each points in transformer can be tapped.



23

AC Ammeter and Voltmeter

- Shunt resistor connected in parallel or
- Multiplier resistance connected in series to the bridge



24

Sensitivity: AC Vs DC

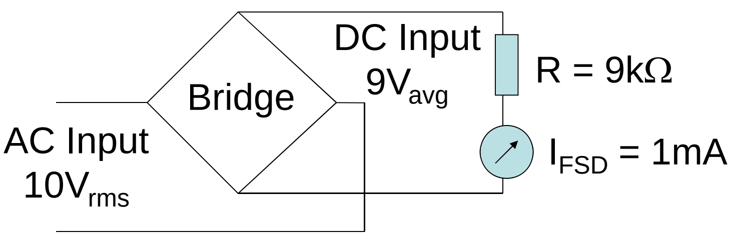
- e.g. if the AC input is 10 V_{rms}

$$V_{peak} = 1.414 \times 10\ V$$
$$= 14.14\ V$$
$$V_{avg} = 0.636 \times 14.14\ V$$
$$= 9\ V$$

25

- Sensitivity = $V_{DC} / V_{AC} \times 100\%$

$$= V_{avg} / V_{rms} \times 100\%$$
$$= 90\%$$



26

References

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- Wikipedia – Alternating Current webpage: http://en.wikipedia.org/wiki/Alternating_current
- Wikipedia – Main Electricity by Country webpage: http://en.wikipedia.org/wiki/Mains_electricity_by_country
- Wikipedia – Transformer webpage: http://en.wikipedia.org/wiki/Transformer
- All About Circuits – AC voltmeters and ammeters webpage: http://www.allaboutcircuits.com/vol_2/chpt_12/1.html