ENE 104
Electric Circuit Theory

Lecture 08:
The RLC Circuit (cont.)

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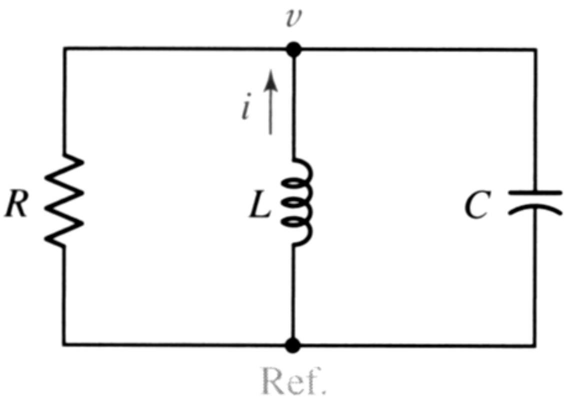
Objectives : Ch9

- the characteristic damping factor and resonant frequency for both series and parallel RLC circuits
- overdamped, critically damped, and underdamped response
- the complete response
- op amps

Summary:

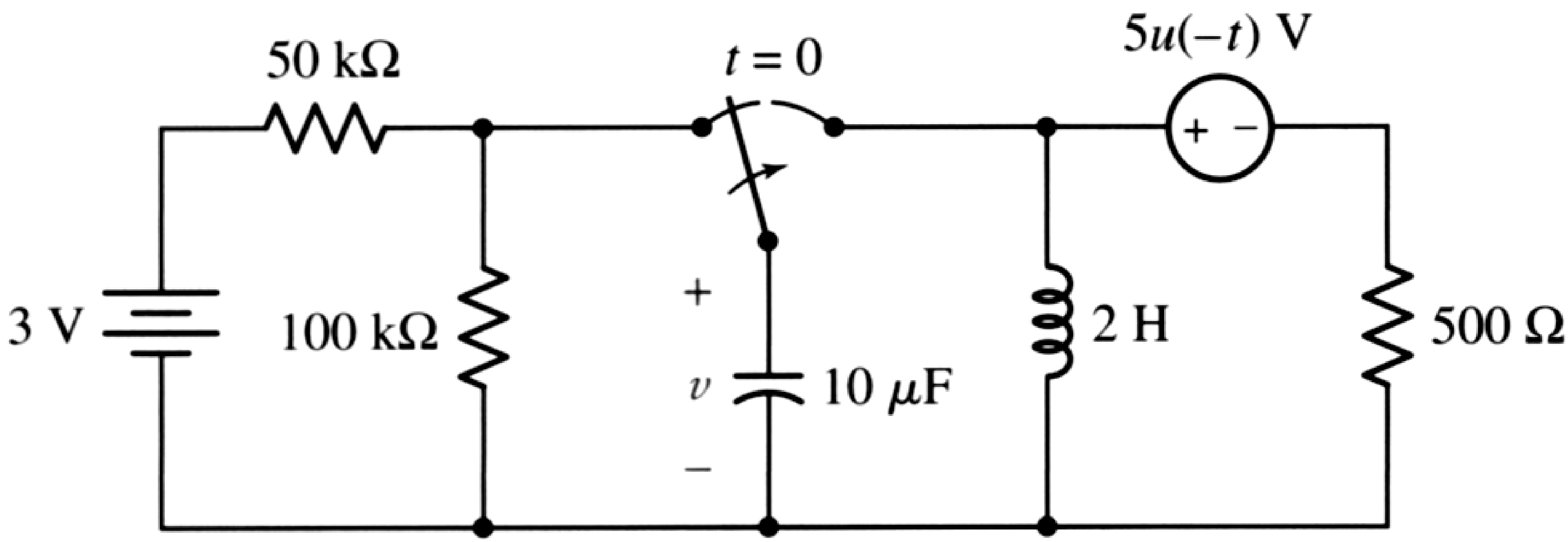


$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Damping	Natural Response Equations	Coefficient Equations Overdamped
Overdamped ($\alpha > \omega_0$)	$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$x(0) = A_1 + A_2$ $\left.\frac{dx}{dt}\right\vert_{t=0^+} = A_1 s_1 + A_2 s_2$
Critically damped ($\alpha = \omega_0$)	$x(t) = e^{-\alpha t}\left(B_1 t + B_2\right)$	$x(0) = B_2$ $\left.\frac{dx}{dt}\right\vert_{t=0^+} = B_1 - \alpha B_2$
Underdamped ($\alpha < \omega_0$)	$x(t) = e^{-\alpha t}(C_1 \cos \omega_d t + C_2 \sin \omega_d t)$ Note: $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$x(0) = C_1$ $\left.\frac{dx}{dt}\right\vert_{t=0^+} = -\alpha C_1 + \omega_d C_2$

Example:



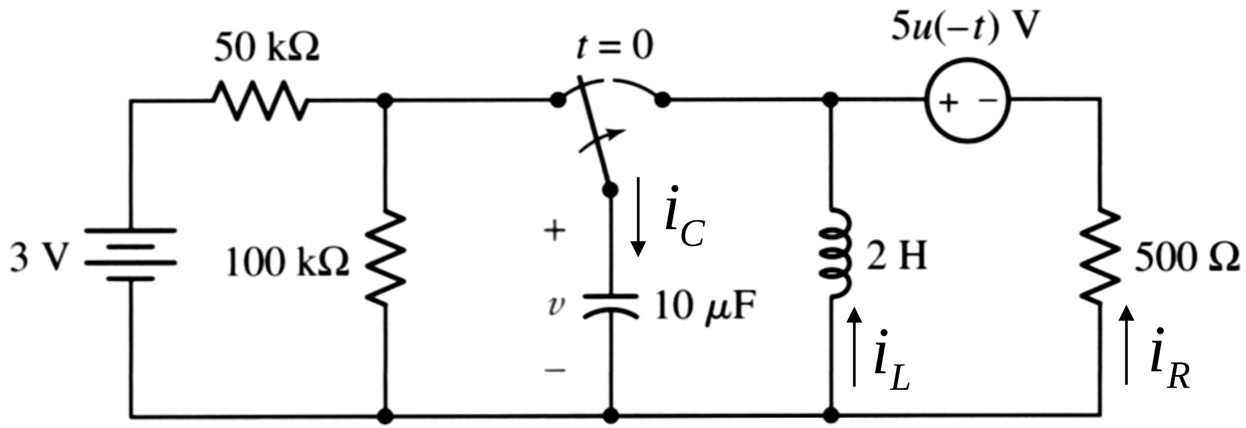
Find

- $\frac{dv}{dt}$ at $t = 0^+$
- $v(t)$ at $t = 1$ ms.
- t_0, the first value of t greater than zero at which $v = 0$

Example:

From

$$i_C(t) = C\frac{dv_C(t)}{dt}$$



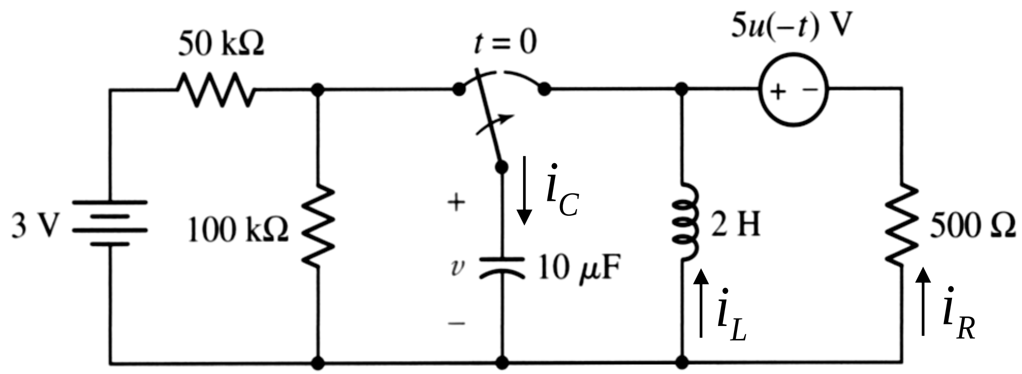
$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C} = \frac{i_L(0^+)+i_R(0^+)}{C} = \frac{i_L(0^-)+i_R(0^+)}{C}\, V\!/s.$$

$$i_L(0^-) = \frac{-5 \quad V.}{500\Omega} = -0.01 \quad A.$$

$$i_R(0^+) = \frac{-v_C(0^+)}{500\Omega} = \frac{-v_C(0^-)}{500\Omega} = \frac{-2V}{500\Omega} = -0.004 \quad A.$$

$$\longrightarrow \left.\frac{dv}{dt}\right|_{t=0^+} = -1400\, V\!/s.$$

Example: v(t=1ms.)



$$\alpha = \frac{1}{2RC}$$

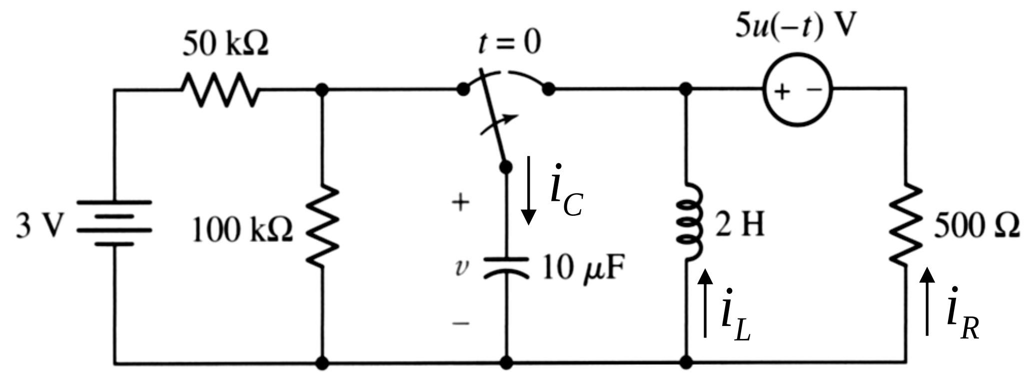
$$= \frac{1}{2 \cdot 500\Omega \cdot 10\mu F} = 100 \quad s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2H \cdot 10\mu F}} = 223.6 \quad rad/s$$

$$\boxed{\alpha < \omega_0} \longrightarrow v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

Example: v(t=1ms.)



$$\boxed{\alpha < \omega_0}$$

An underdamped C/T

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 200 \quad rad/s$$

$$\longrightarrow \quad v(t) = e^{-100t}(B_1 \cos 200t + B_2 \sin 200t)$$

At t = 0, $\Rightarrow \quad x(0) = B_1 \ldots (*) \qquad ; B_1 = v(0^+) = 2$

And evaluating the derivative at t = 0,

$$\Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0} = -\alpha B_1 + \omega_d B_2 \ldots (**)}$$

Example: v(t=1ms.)

$$v(t) = e^{-100t}(B_1 \cos 200t + B_2 \sin 200t) \qquad ; B_1 = 2$$

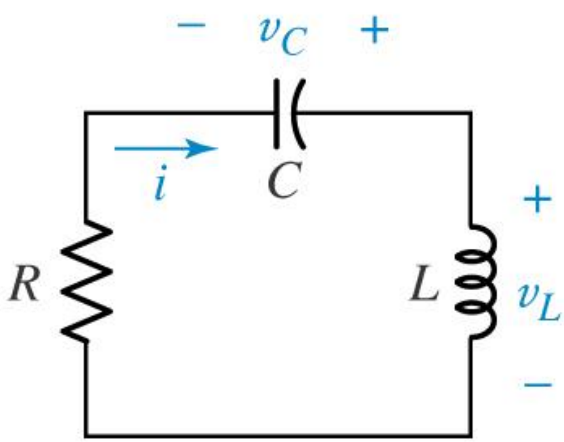
From $\quad i_C(t) = C\dfrac{dv_C(t)}{dt}$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -1400 = -100B_1 + 200B_2 \qquad \Rightarrow B_2 = -6$$

$$\Rightarrow v(t) = e^{-100t}(2\cos 200t - 6\sin 200t)$$

$$\therefore v(t = 1ms.) = 0.695 \quad V.$$

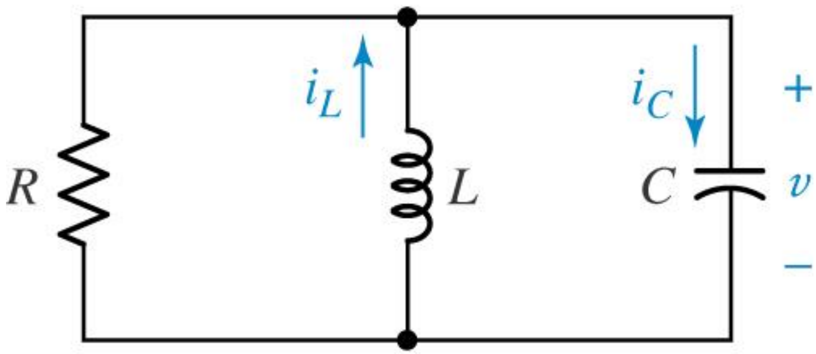
The Source-Free Series RLC C/T:



(a)

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{t_0}^{t} i\,dt' - v_C(t_0) = 0$$

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$



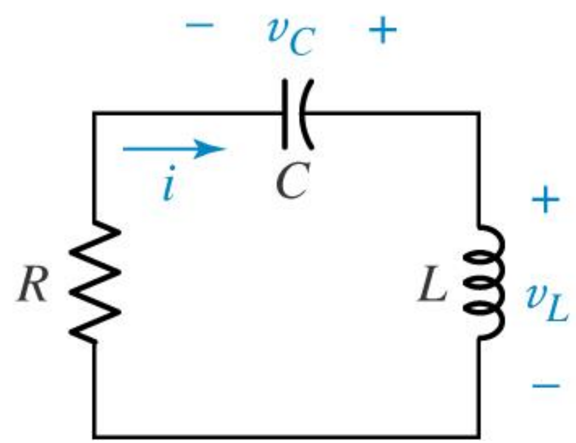
(b)

$$C\frac{dv}{dt} + \frac{1}{R}v + \frac{1}{L}\int_{t_0}^{t} v\,dt' - i_L(t_0) = 0$$

$$C\frac{d^2 v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

The Series RLC Response:

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$



The overdamped response:

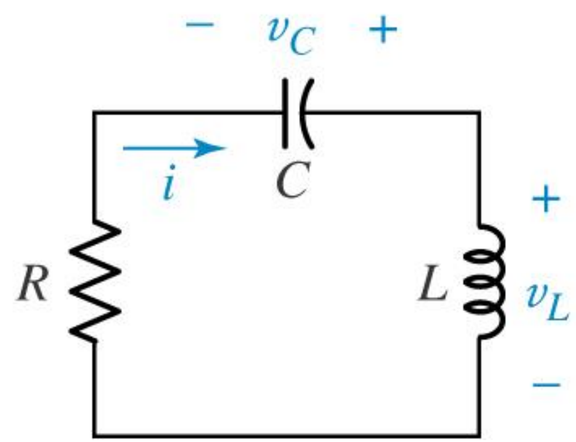
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where $$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

thus $$\alpha = \frac{R}{2L}$$ and $$\omega_0 = \frac{1}{\sqrt{LC}}$$

The Series RLC Response:

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$



The overdamped response:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

The critically damped response:

$$i(t) = e^{-\alpha t}(A_1 t + A_2)$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

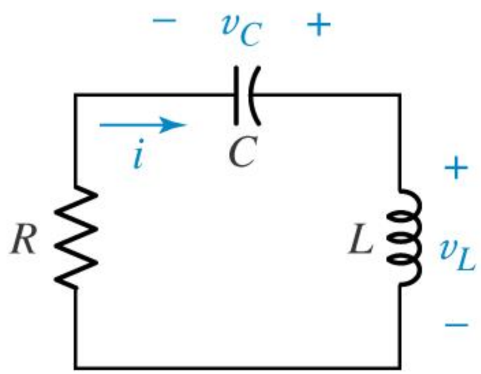
$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

The underdamped response:

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Summary:

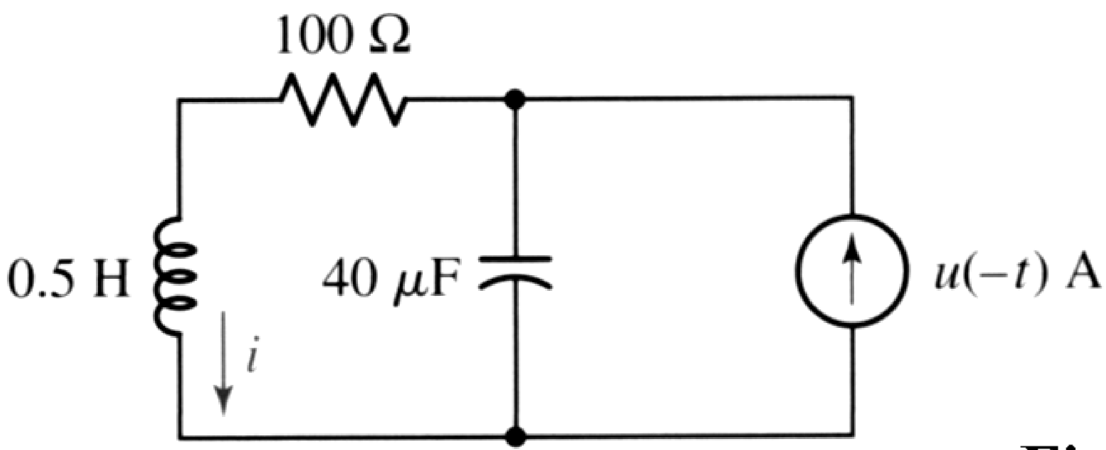


$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Damping	Natural Response Equations	Coefficient Equations Overdamped
Overdamped $(\alpha > \omega_0)$	$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$x(0) = A_1 + A_2$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = A_1 s_1 + A_2 s_2$
Critically damped $(\alpha = \omega_0)$	$x(t) = e^{-\alpha t}\left(B_1 t + B_2\right)$	$x(0) = B_2$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = B_1 - \alpha B_2$
Underdamped $(\alpha < \omega_0)$	$x(t) = e^{-\alpha t}(C_1 \cos \omega_d t + C_2 \sin \omega_d t)$ Note: $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$x(0) = C_1$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = -\alpha C_1 + \omega_d C_2$

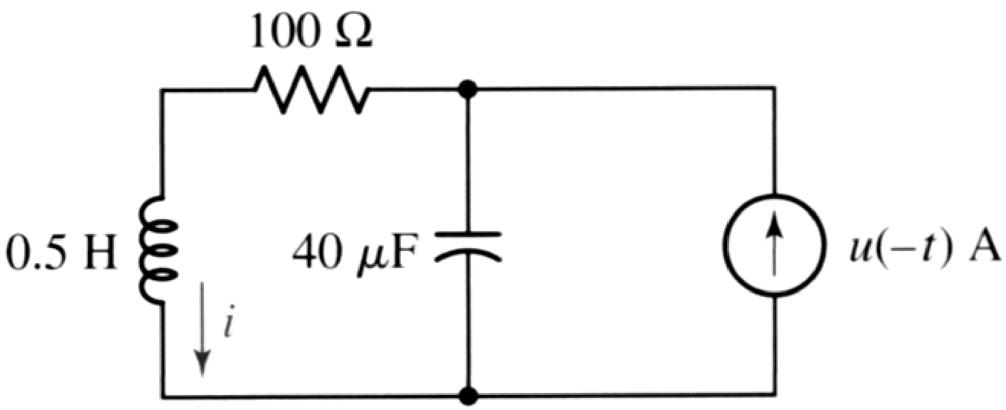
Practice: 9.5



Find

- α
- ω_0
- $i(0^+)$
- $\left.\dfrac{di}{dt}\right|_{t=0^+}$ at $t = 0^+$
- $i(12ms)$

Practice :

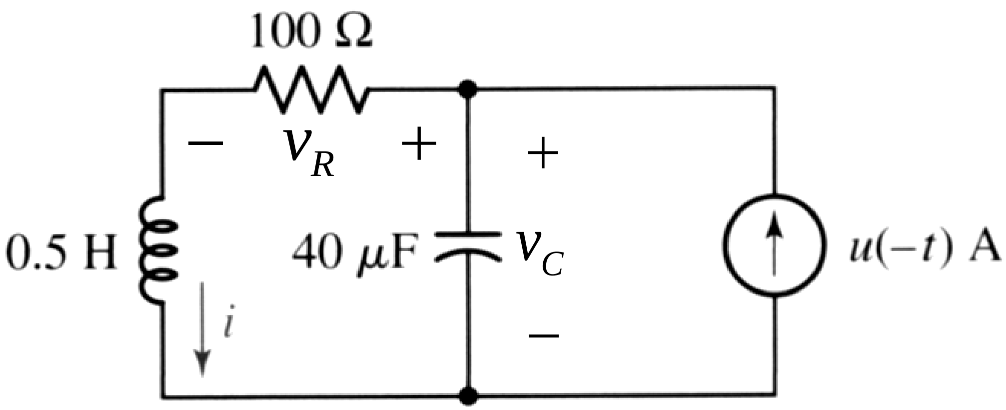


$$\alpha = \frac{R}{2L} = \frac{100\Omega}{2\cdot 0.5H} = 100 \quad s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5H\cdot 40\mu F}} = 223.6 \quad rad/s$$

$$i(0^+) = i_L(0^-) = 1 \quad A.$$

Practice :



$$\alpha = 100 \quad s^{-1}$$

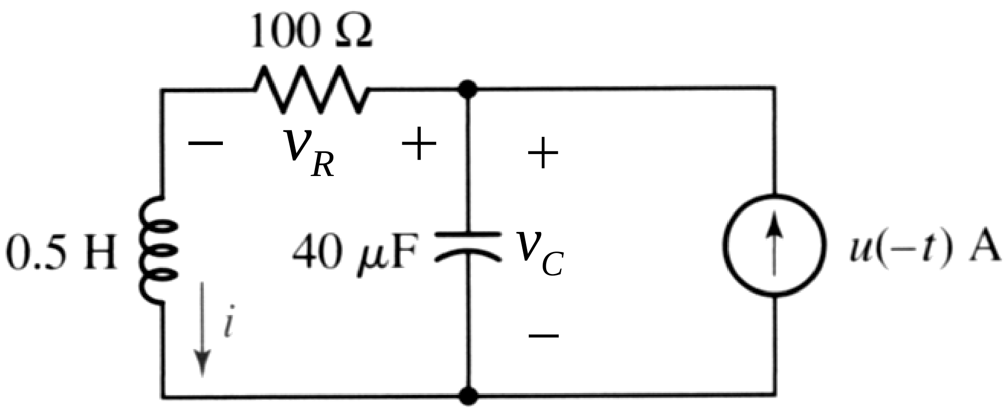
$$\omega_0 = 223.6 \quad rad/s$$

$$i(0^+) = 1 \quad A.$$

from $\quad v_L(t) = L\dfrac{di_L}{dt}$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{v_C(0^+) - v_R(0^+)}{L} = \frac{100-100}{L} = 0$$

Practice : i(t=12ms.)



$$\alpha = 100 \quad s^{-1}$$

$$\omega_0 = 223.6 \quad rad/s$$

$$i(0^+) = 1 \quad A.$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = 0$$

$$\boxed{\alpha < \omega_0}$$

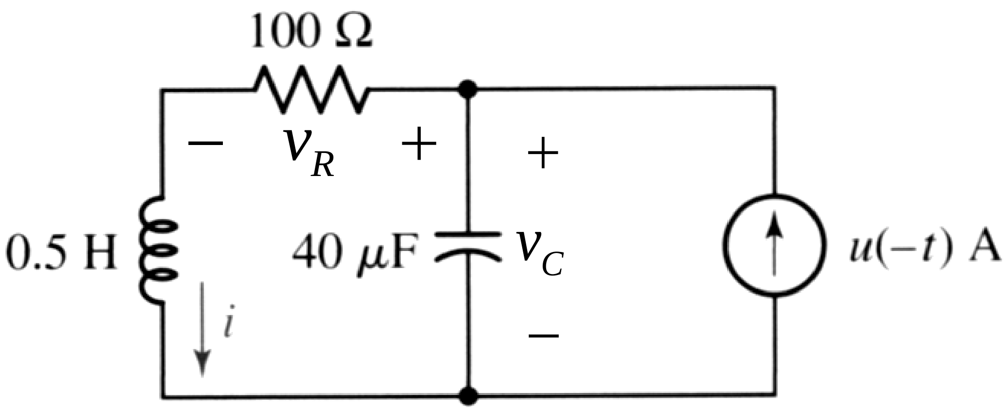
An underdamped C/T

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 200 \quad rad/s$$

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$\rightarrow \quad i(t) = e^{-100t}(B_1 \cos 200t + B_2 \sin 200t)$$

Practice : i(t=12ms.)



$$\alpha = 100 \quad s^{-1}$$

$$\omega_d = 200 \quad rad/s$$

$$i(0^+) = 1 \quad A.$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = 0$$

$\boxed{\alpha < \omega_0}$ An underdamped C/T

$$x(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

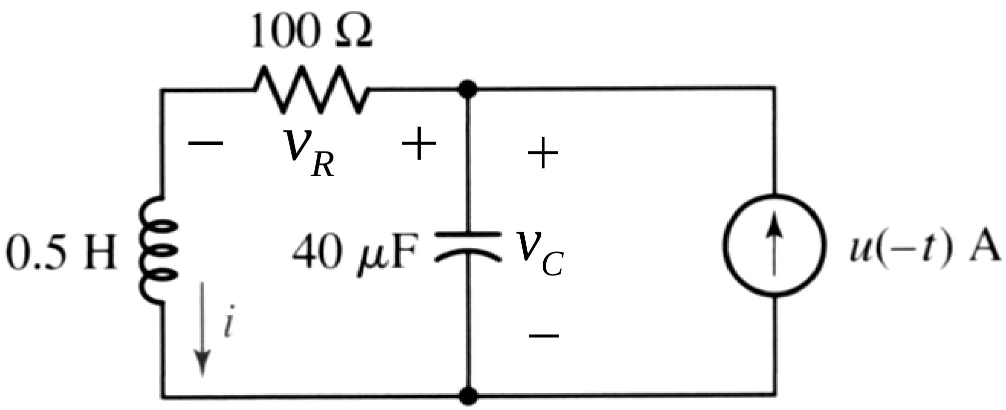
$$\Rightarrow \quad x(0) = B_1 ...(*)$$

$$\Rightarrow \quad \left.\frac{dx}{dt}\right|_{t=0} = -\alpha B_1 + \omega_d B_2 ...(**)$$

$$;\therefore B_1 = i(0) = 1$$

$$;\therefore B_2 = 0.5$$

Practice : i(t=12ms.)



$$\alpha = 100 \quad s^{-1}$$

$$\omega_d = 200 \quad rad/s$$

$$B_1 = 1, B_2 = 0.5$$

$\boxed{\alpha < \omega_0}$ An underdamped C/T

$$x(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$\longrightarrow \quad i(t) = e^{-100t}(\cos 200t + 0.5 \sin 200t)$$

$$\therefore i(12ms.) = -0.1204 \quad A.$$

The Complete Response:

The complete response

= a force response + a natural response

a force response: $v_f(t) = V_f$

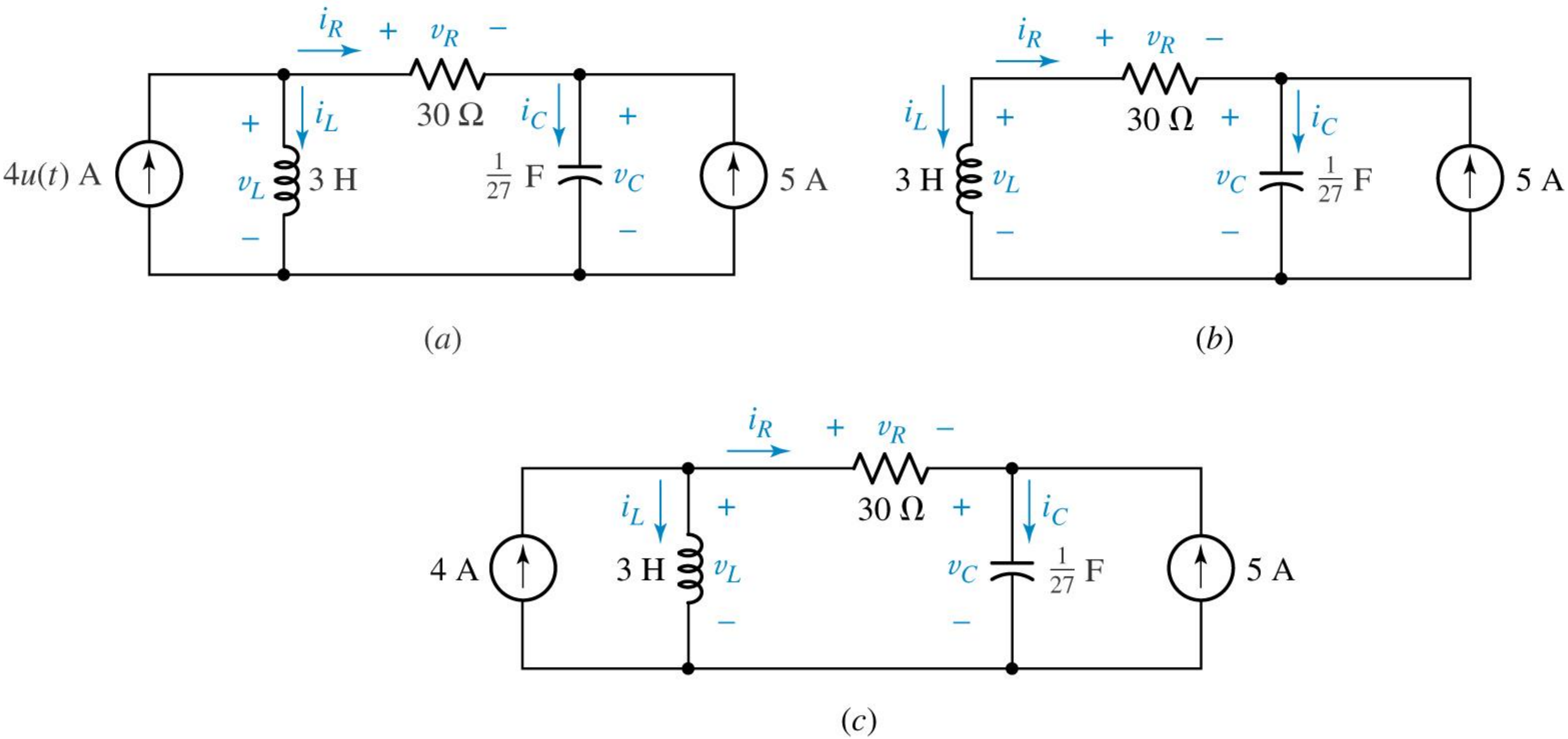
a natural response: $v_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

The complete response: $v(t) = V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Summary:

Damping	Step Response Equations	Coefficient Equations Overdamped
Overdamped $(\alpha > \omega_0)$	$x(t) = X_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$	$x(0) = X_f + A_1' + A_2'$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = A_1' s_1 + A_2' s_2$
Critically damped $(\alpha = \omega_0)$	$x(t) = X_f + e^{-\alpha t}\left(B_1' t + B_2'\right)$	$x(0) = X_f + B_2'$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = B_1' - \alpha B_2'$
Underdamped $(\alpha < \omega_0)$	$x(t) = X_f + e^{-\alpha t}(C_1' \cos \omega_d t + C_2' \sin \omega_d t)$ Note: $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$x(0) = X_f + C_1'$ $\left.\frac{dx}{dt}\right\|_{t=0^+} = -\alpha C_1' + \omega_d C_2'$

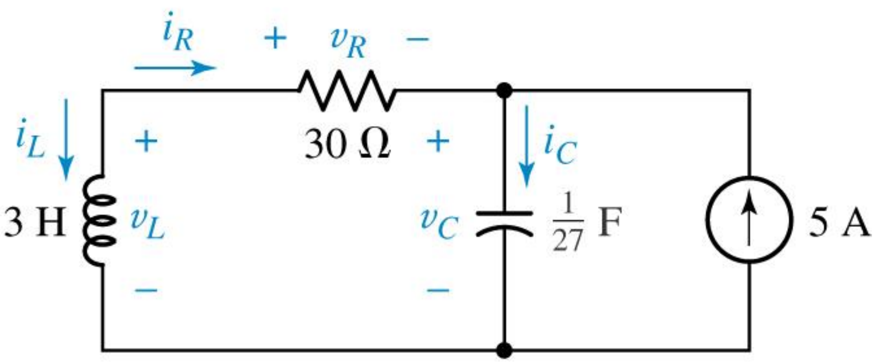
Example:

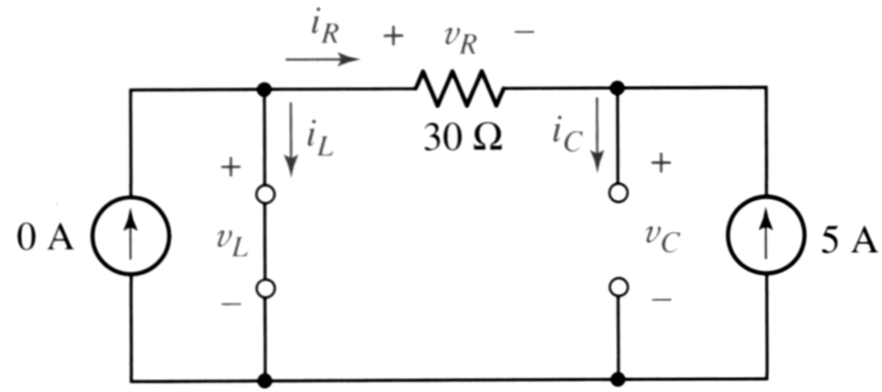


An RLC circuit that is used to illustrate several procedures by which the initial conditions may be obtained. The desired response is nominally taken to be $v_C(t)$.

Example:

t < 0





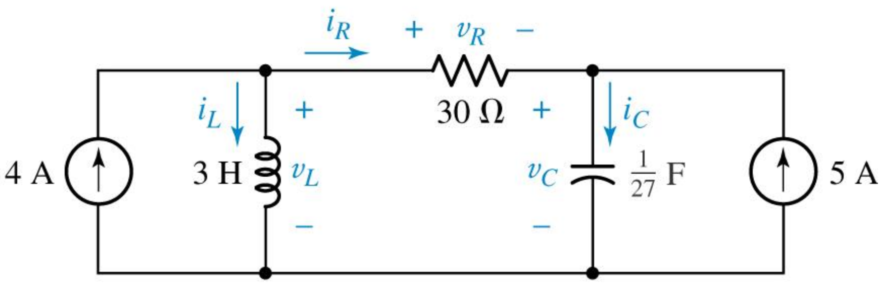
$$i_L(0^-) = 5A. \qquad v_C(0^-) = (5A.)\cdot(30\Omega) = 150V.$$

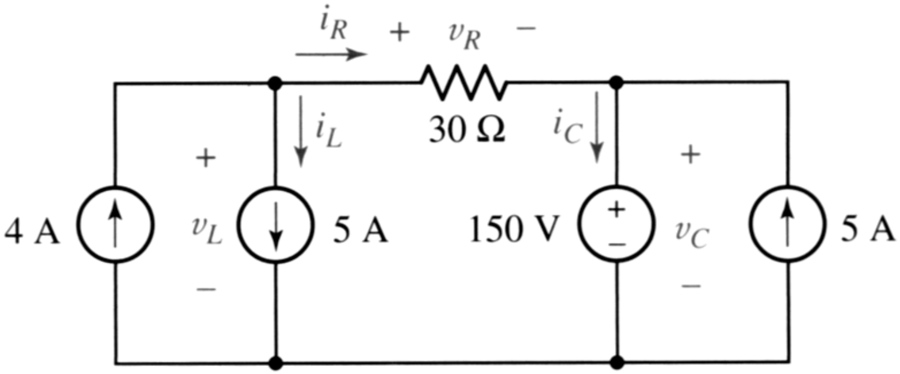
$$i_R(0^-) = -5A. \qquad v_R(0^-) = -150V.$$

$$i_C(0^-) = 0 \qquad v_L(0^-) = 0$$

Example:

$$t \geq 0$$





$$i_L(0^+) = i_L(0^-) = 5A.$$

$$i_R(0^+) = 4 - i_L(0^+) = -1A.$$

$$i_C(0^+) = 5 + i_R(0^+) = 4A.$$

$$v_C(0^+) = v_C(0^-) = 150V.$$

$$v_R(0^+) = i_R(0^+) \cdot 30\Omega = -30V.$$

$$v_L(0^+) = v_C(0^+) + v_R(0^+) = 120V.$$

Example:

from $$v_L(t) = L\frac{di_L}{dt}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{120}{3} = 40 \quad A/s.$$

from $$4 - i_L - i_R = 0 \Rightarrow 0 - \frac{di_L}{dt} - \frac{di_R}{dt} = 0$$

$$therefore \quad \left.\frac{di_R}{dt}\right|_{t=0^+} = -\frac{di_L}{dt} = -40 \quad A/s.$$

from $$5 - i_C + i_R = 0 \Rightarrow 0 - \frac{di_C}{dt} + \frac{di_R}{dt} = 0$$

$$therefore \quad \left.\frac{di_C}{dt}\right|_{t=0^+} = \frac{di_R}{dt} = -40 \quad A/s.$$

Example:

from $$i_C(t) = C\frac{dv_C}{dt}$$

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{4}{\frac{1}{27}} = 108 \quad V/s.$$

from $\cdots$

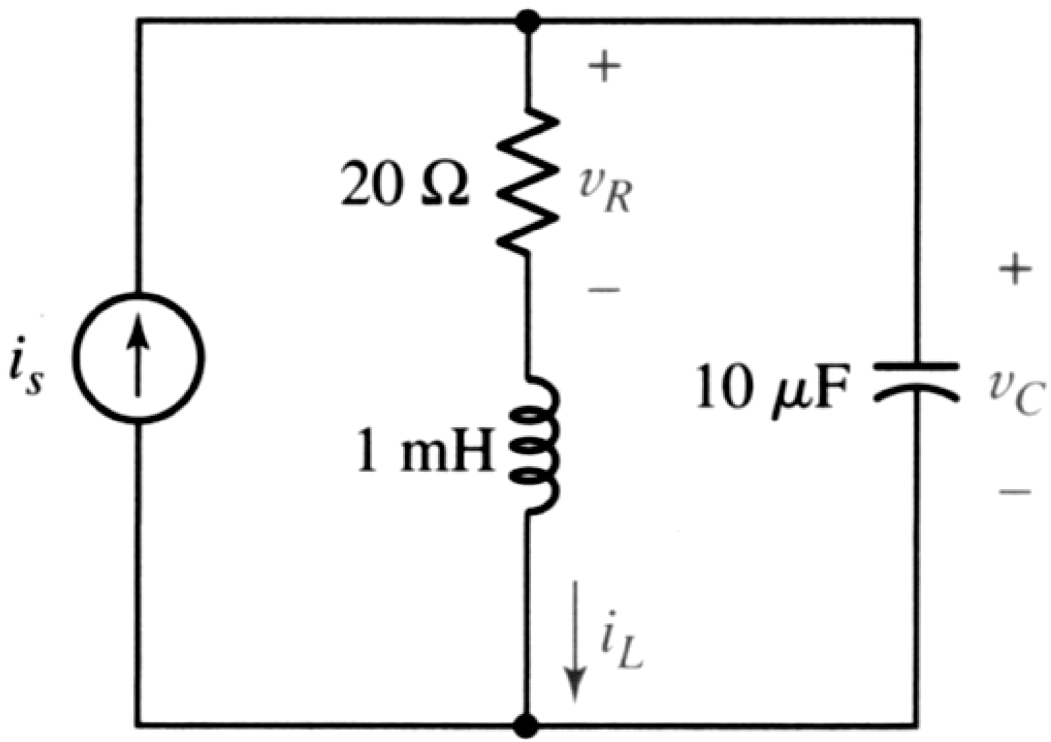
$$\left.\frac{dv_R}{dt}\right|_{t=0^+} = -1200 \quad A/s.$$

from $\cdots$

$$\left.\frac{dv_L}{dt}\right|_{t=0^+} = -1092 \quad A/s.$$

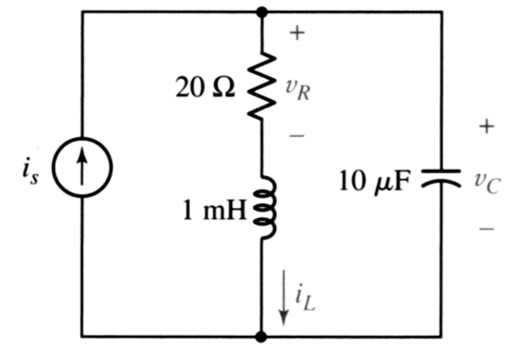
Practice: 9.6

Let $i_s = 10u(-t) - 20u(t)$ A. in Figure. Find (a) $i_L(0^-)$; (b) $v_C(0^+)$; (c) $v_R(0^+)$; (d) $i_{L,(\infty)}$; (e) $i_L(0.1ms)$.



Practice: 9.6

Let $i_s = 10u(-t) - 20u(t)$ A. in Figure. Find (a) $i_L(0^-)$; (b) $v_C(0^+)$; (c) $v_R(0^+)$; (d) $i_{L,(\infty)}$; (e) $i_L(0.1ms)$.



(a) $i_L(0^+) = i_L(0^-) = \underline{10\ \text{A}}$ $\quad (i_s = 10\ \text{A},\ t < 0)$

(b) $v_C(0^+) = v_C(0^-) = 20\ (10) = \underline{200\ \text{V}}$

(c) Since the inductor current cannot change in zero time, $v_R(0^+) = 20\,(10) = \underline{200\ \text{V}}$

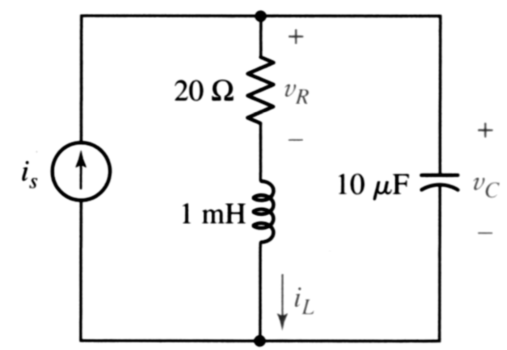
(d) $i_L(\infty) = \underline{-\ 20\ \text{A}}$ due to $i_s \to -\ 20$ A

(e) $\alpha = \dfrac{R}{2L} = 10\ 000\,\text{s}^{-1}$ and $\omega_o = 1/\sqrt{LC} = 10\ 000$ rad/s

$\therefore$ circuit is critically damped. So,

Practice: 9.6

Let $i_s = 10u(-t) - 20u(t)$ A. in Figure. Find (a) $i_L(0^-)$; (b) $v_C(0^+)$; (c) $v_R(0^+)$; (d) $i_{L,(\infty)}$; (e) $i_L(0.1ms)$.



$$i_L(t) = e^{-10^4 t}(A_1 t + A_2) - 20$$

Applying initial conditions,

$$i_L(0^+) = 10 = A_2 - 20 \qquad [1]$$

and so $A_2 = 30$

$$\frac{di_L}{dt} = -10^4 e^{-10^4 t}(A_1 t + A_2) + A_1 e^{-10^4 t}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -10^4 A_2 + A_1 = \frac{1}{L}\left[v_C(0^+) - v_R(0^+)\right]$$

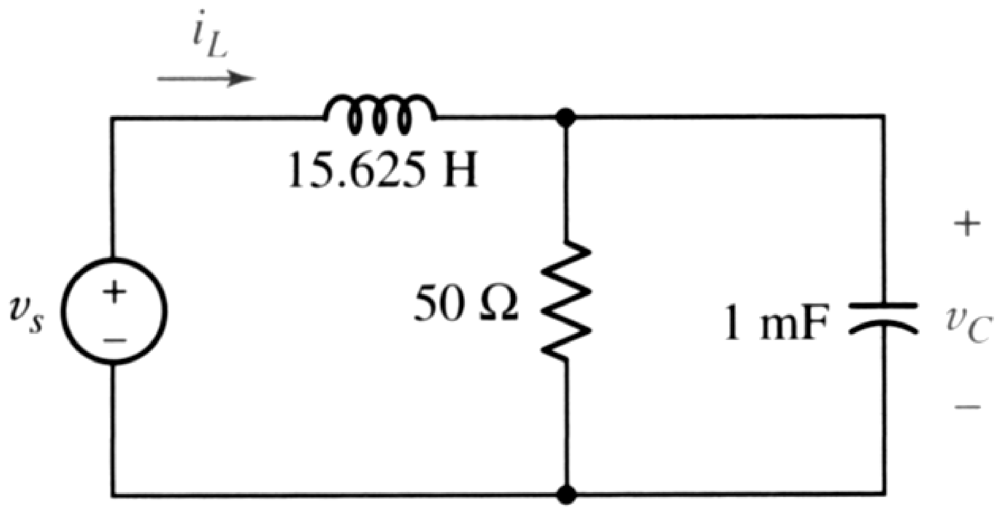
or $\quad A_1 - 30\times10^4 = 10^3(200-200)$

so $\quad A_1 = 30\times10^4$

Thus, $i_L(t) = e^{-10^4 t}(30\times10^4 t + 30) - 20$

$= \underline{2.073 \text{ A}}$

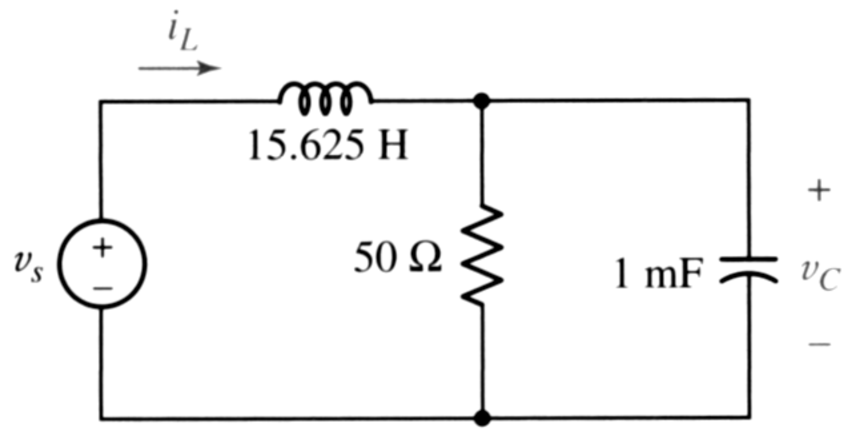
Practice: 9.7



Let $v_s = 10 + 20u(t)$ V.

Find:
- $i_L(0)$
- $v_C(0)$
- $i_{L,f}$
- $i_L(0.1s.)$

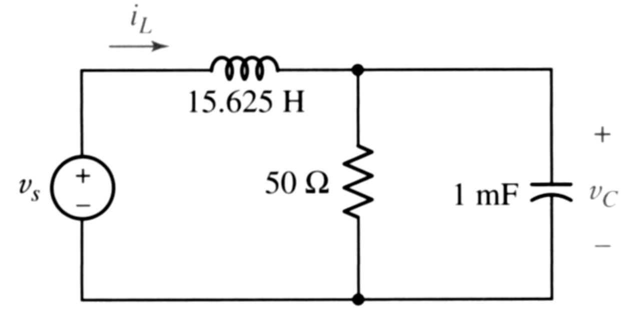
Practice :



Find:

- $i_L(0) = \dfrac{10V.}{50\Omega} = 0.2 \text{ A.}$
- $v_C(0) = i_L(0) \cdot 50\Omega = 10 \text{ V.}$
- $i_{L,f} = \dfrac{10+20}{50\Omega} = 0.6 \text{ A.}$
- $i_L(0.1s.) = \ldots$

Practice :



(d) $$\alpha = \frac{1}{2RC} = 10\,\mathrm{s}^{-1} \quad \text{and} \quad \omega_o = \frac{1}{\sqrt{LC}} = 8 \text{ rad/s}$$

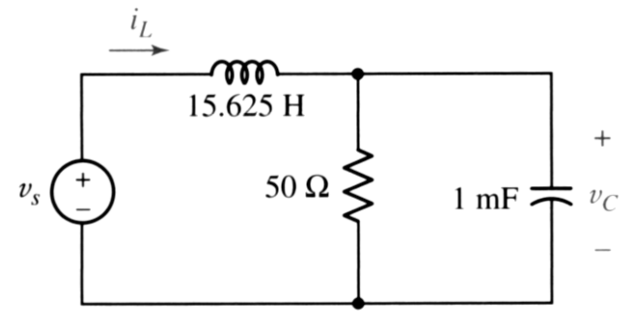
Thus, the circuit is overdamped and

$$i_L(t) = Ae^{s_1 t} + Be^{s_2 t} + 0.6$$

where $s_1 = -4\,\mathrm{s}^{-1}$ and $s_2 = -16\,\mathrm{s}^{-1}$

$$i_L(0) = 0.2 = A + B + 0.6 \qquad [1]$$

Practice :



$$\frac{di_L}{dt} = -4Ae^{-4t} - 16Be^{-16t}$$

$$\text{so} \quad \left.\frac{di_L}{dt}\right|_{t=0^+} = -4A - 16B = \frac{1}{L}\left[-v_C(0) + 30\right] \quad \text{or} \quad -4A - 16B = \frac{20}{15.625} \qquad [2]$$

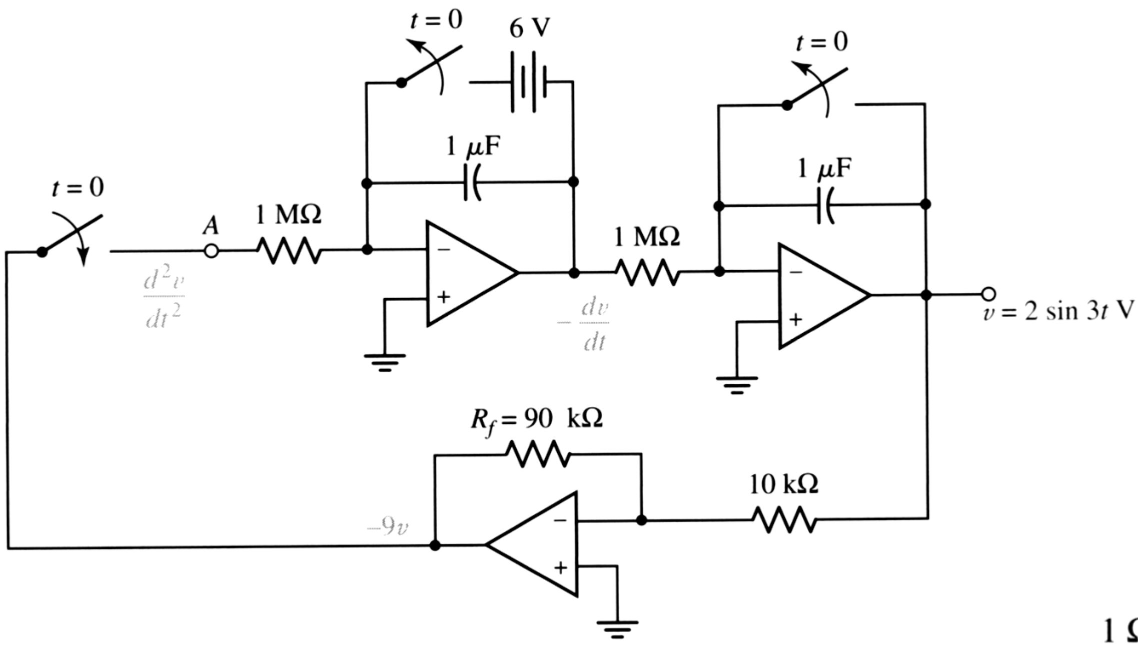
Solving Eqns. [1] and [2] simultaneously, $\text{A} = -\ 0.4267$ and $\text{B} = 0.02667$

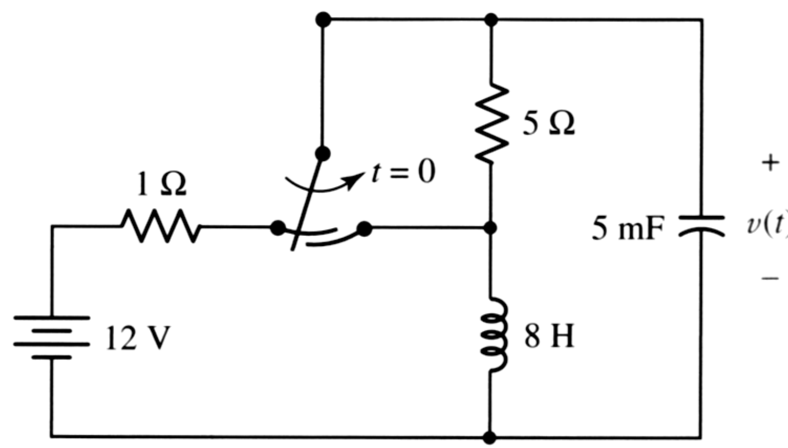
so that $i_L(t) = -426.7e^{-4t} + 26.67e^{-16t} + 600 \text{ mA}$

Thus, $i_L(0.1) = \underline{319.4 \text{ mA}}$

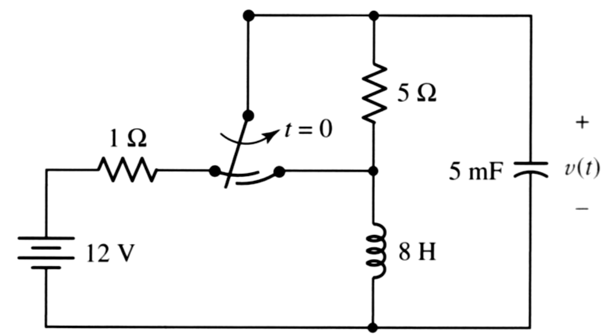
Practice: 9.8

Give new values for R_f and the two initial voltages in the circuit on the left if the output represents the voltage $v(t)$ in the circuit on the right.





Practice: 9.8



$$i_L(0^-) = \frac{12}{1+5} = 2\text{ A} \qquad = i_L(0^+)$$

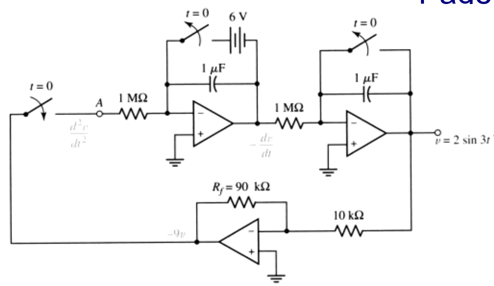
$$v(0^-) = 12\frac{5}{1+5} = 10\text{ V} \qquad = v(0^+)$$

- define i flowing out of "+" reference of $v(t)$.

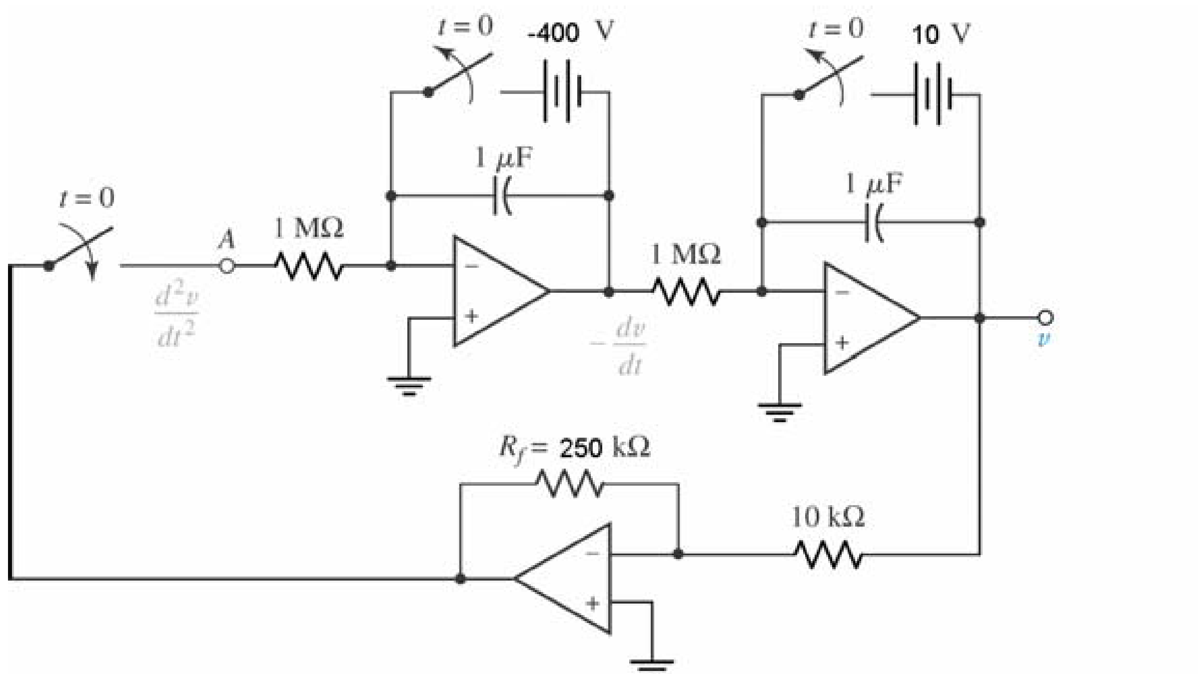
$$v = L\frac{di}{dt} \quad \text{and} \quad i = -C\frac{dv}{dt} \qquad \text{Thus,}\ v = -LC\frac{d^2v}{dt^2} \quad \text{or} \quad \frac{d^2v}{dt^2} = -25v$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{-i_L(0^+)}{C} = \frac{-2}{0.005} = -400\text{ V}$$

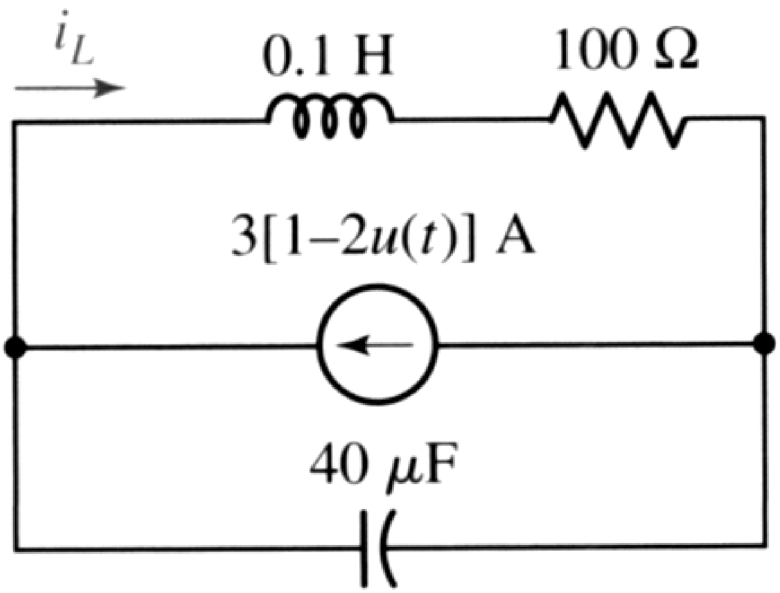
Practice: 9.8



so an initial voltage of +400 V is required where – 6 V was needed previously. At the $v(t)$ node, an initial voltage of +10 V is required where 0 V was previously needed. Previously a gain of –9 was obtained using R_1 = 10 kΩ and R_f = 90 kΩ. Now we require a gain of – 25, so replace R_f with 250 kΩ.



Example: Ex 42 Page 300



Find - $i_L(t)$

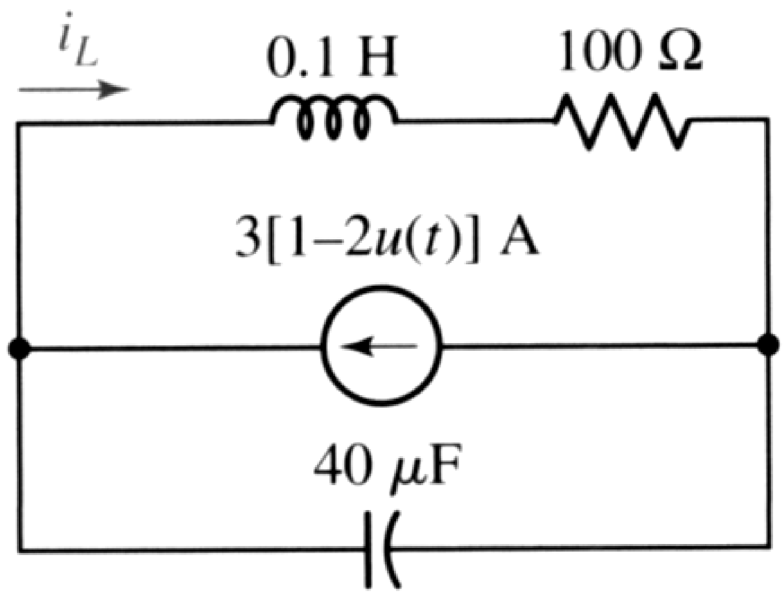
- At what instant of time after t = 0 is

$$i_L(t) = 0?$$

Example: Ex 42 Page 300

Series RLC:

$$\alpha = \frac{R}{2L} = \frac{100\Omega}{2 \cdot 0.1H} = 500 \quad s^{-1}$$



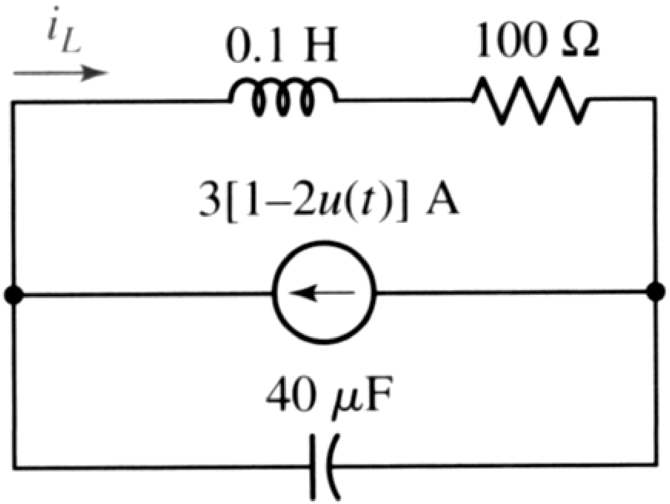
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1H \cdot 40\mu F}} = 500 \quad rad/s$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$



$$x(t) = X_f + e^{-\alpha t}\left(B_1^{'}t + B_2^{'}\right)$$

$$x(0) = X_f + B_2^{'}$$

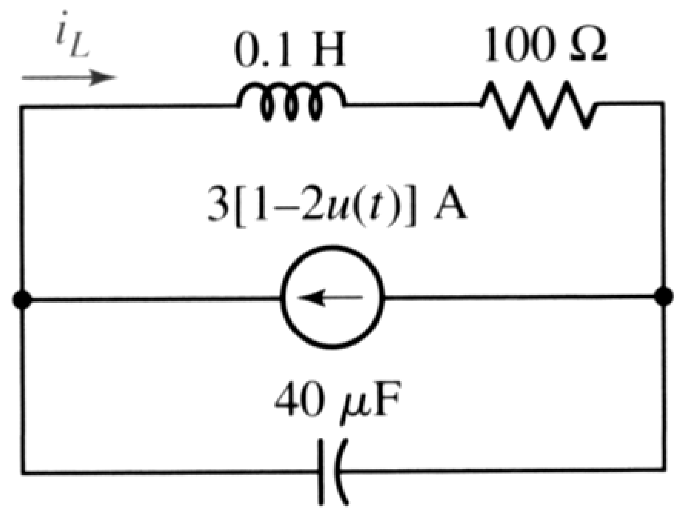
$$\left.\frac{dx}{dt}\right|_{t=0^+} = B_1^{'} - \alpha B_2^{'}$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$



$$i_L(t) = I_f + e^{-\alpha t}\left(B_1' t + B_2'\right)$$

$$i_L(0) = I_f + B_2'$$

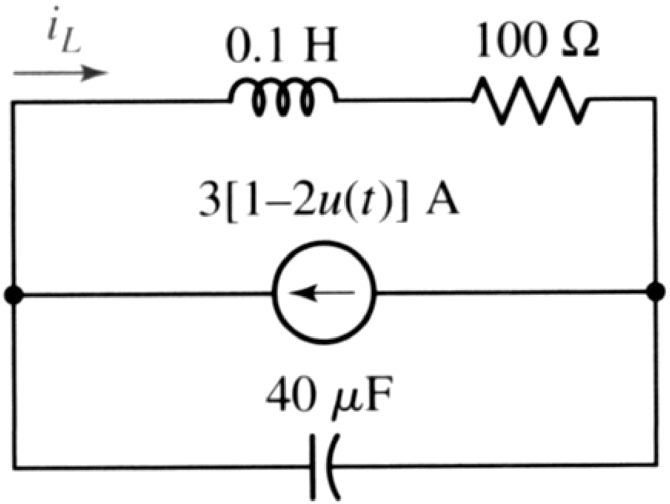
$$\left.\frac{di_L}{dt}\right|_{t=0^+} = B_1' - \alpha B_2'$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$



$$i_L(t) = I_f + e^{-\alpha t}\left(B_1' t + B_2'\right)$$

$$i_L(0) = I_f + B_2'$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = B_1' - \alpha B_2'$$

$$i_L(0) = 3$$

$$I_f = 3(1-2) = -3$$

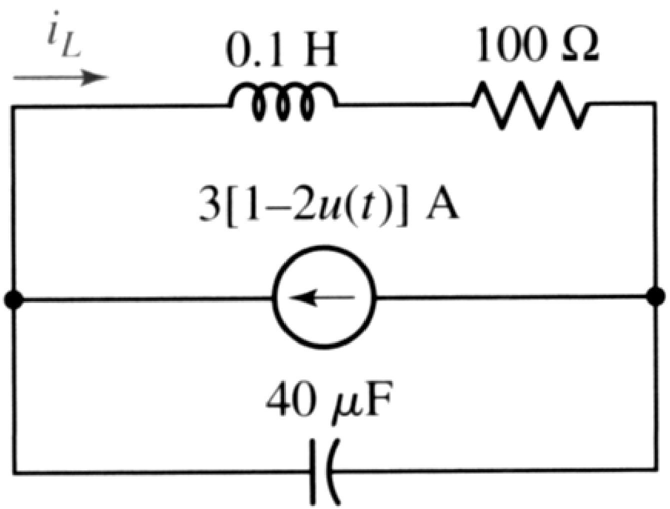
$$v_C(0) = 300$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$



$$i_L(t) = I_f + e^{-\alpha t}\left(B_1' t + B_2'\right)$$

$$i_L(0) = I_f + B_2'$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = B_1' - \alpha B_2'$$

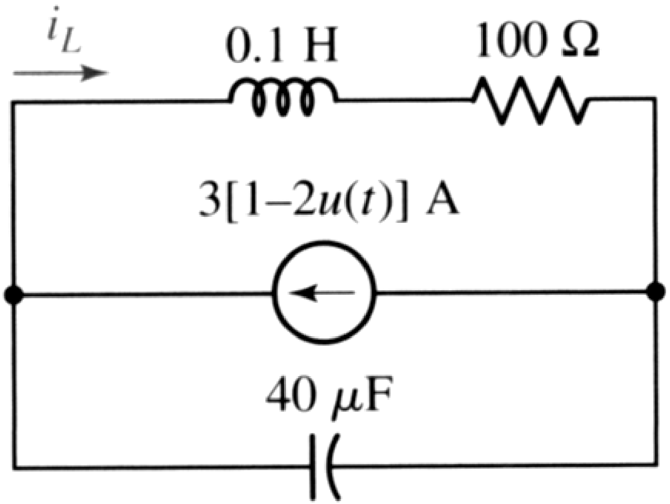
$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{v_C(0^+) - v_R(0^+)}{L} = \frac{300-300}{L} = 0$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$



$$i_L(t) = I_f + e^{-\alpha t}\left(B_1' t + B_2'\right)$$

$$i_L(0) = I_f + B_2'$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = B_1' - \alpha B_2'$$

$$B_2' = 6$$

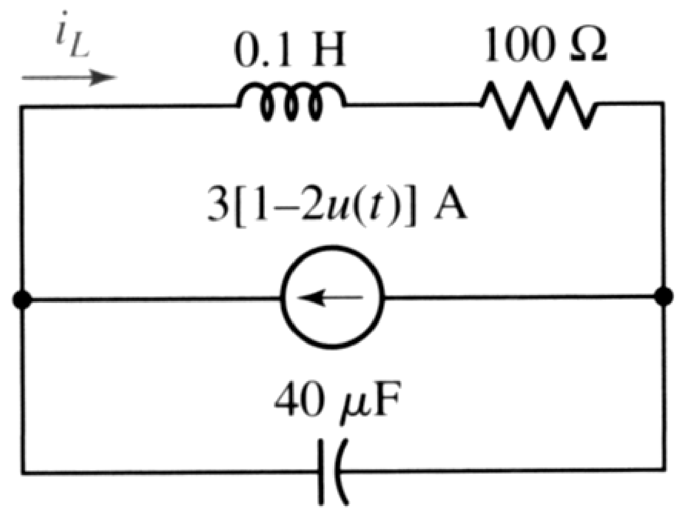
$$B_1' = 3000$$

Example: Ex 42 Page 300

Series RLC:

Critically damped

$$(\alpha = \omega_0)$$

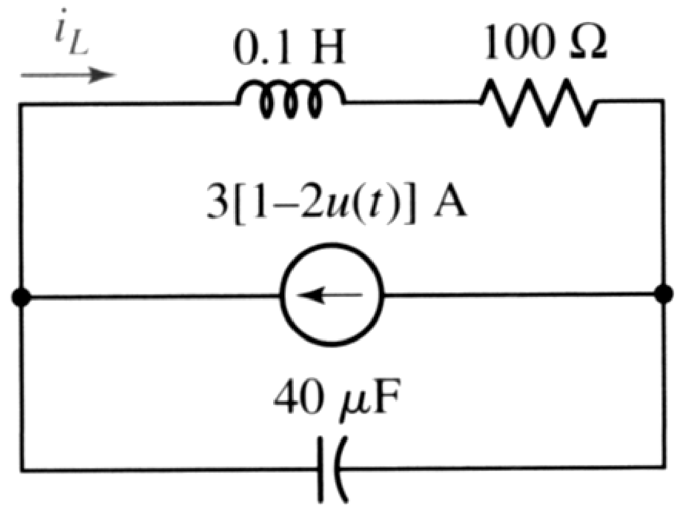


$$i_L(t) = -3 + e^{-500t}(3000t + 6), t > 0$$

$$\therefore i_L(t) = 3u(-t) + \left\{-3 + e^{-500t}(3000t + 6)\right\}u(t)$$

Example: Ex 42 Page 300

Find - At what instant of time after t = 0 is $i_L(t) = 0$?

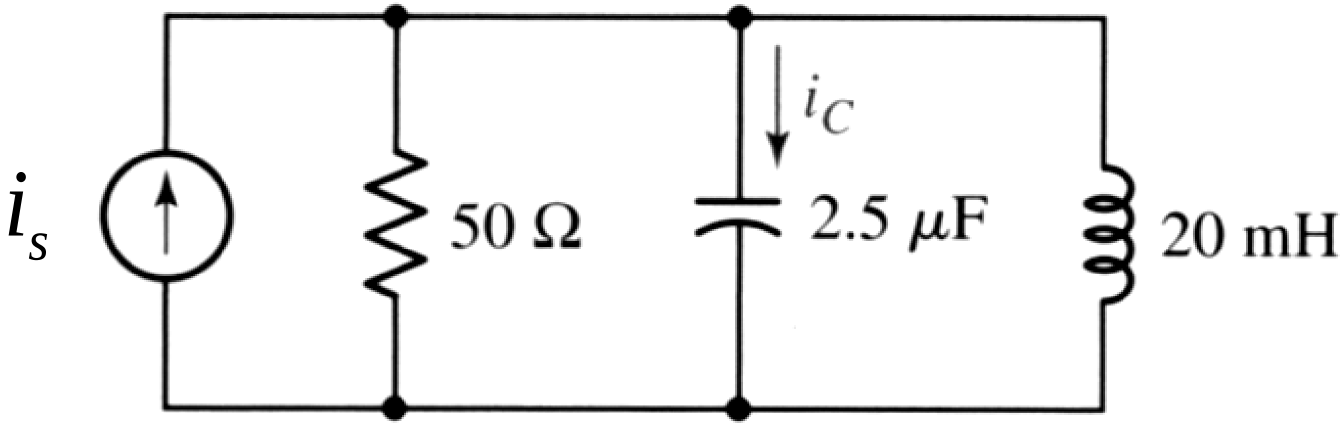


From $$i_L(t) = -3 + e^{-500t}\left(3000t + 6\right), t > 0$$

$$i_L(t) = -3 + e^{-500t}\left(3000t + 6\right) = 0$$

$$\Rightarrow t_0 = 3.357ms.$$

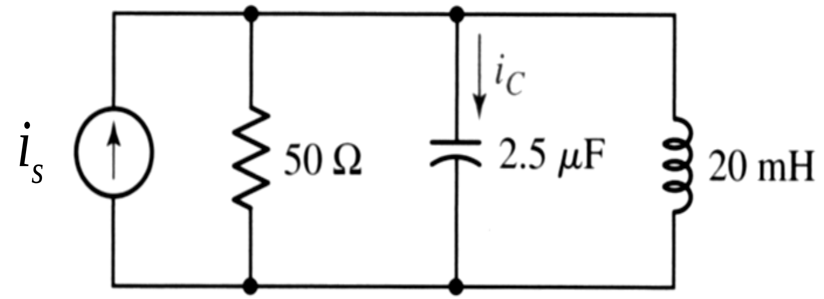
Example: Ex 43 Page 300



With i_s =2[1+u(t)] A. Find $i_C(t)$

Example: Ex 43 Page 300

Parallel RLC:



$$\alpha = \frac{1}{2RC}$$

$$= \frac{10^6}{2 \cdot 50\Omega \cdot 2.5} = 4000 \quad s^{-1}$$

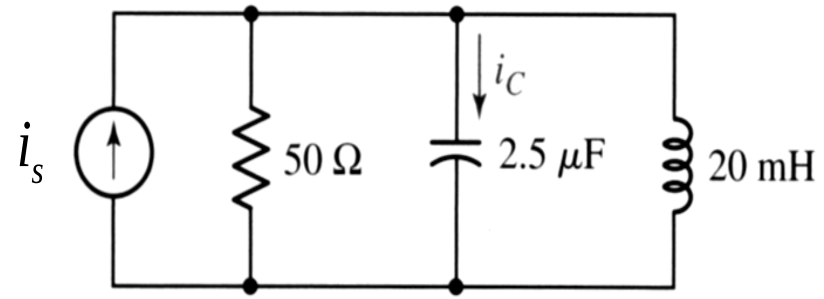
$$\omega_0^2 = \frac{1}{LC}$$

$$= \frac{1}{20mH \cdot 2.5\mu F} = 20 \times 10^6$$

Example: Ex 43 Page 300

Parallel RLC:
Underdamped

$$(\alpha < \omega_0)$$



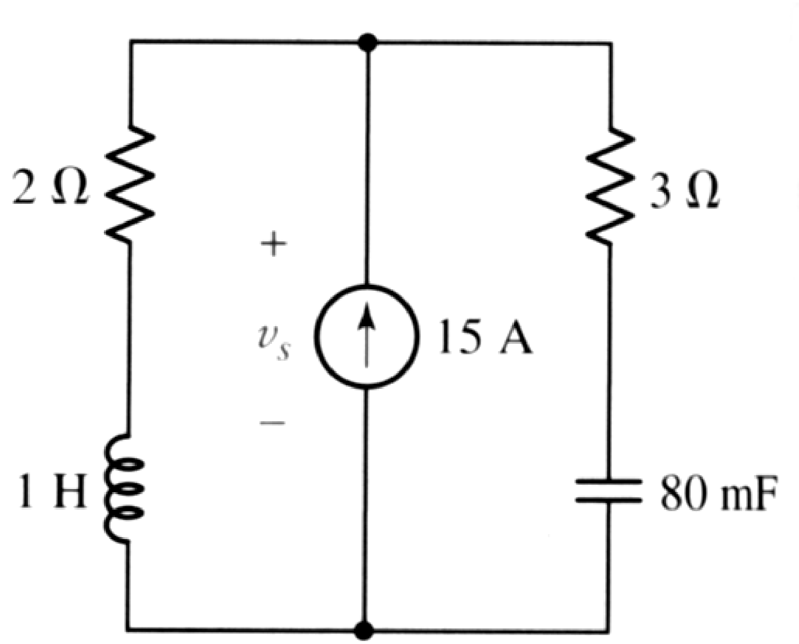
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2000$$

$$i_L(0) = 2$$

$$I_{C,f} = 0$$

$$v_C(0) = 0$$

Example: Final 2/46 (Ex49 p300)



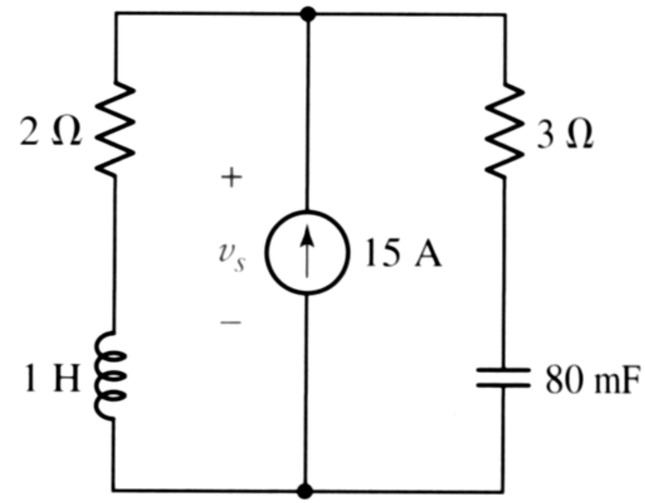
ในวงจร ตามรูป แหล่งจ่ายกระแส เปลี่ยนค่าจาก 15 A. เป็น 22 A. ทันทีทันใด ที่เวลา t = 0 จงหา

$$v_s(0^-), v_s(0^+), v_s(\infty), and \quad v_s(3.4s.)$$

Example: Final 2/46 (Ex49 p300)

$$v_s(0^-) = 15A \cdot 2\Omega = 30$$

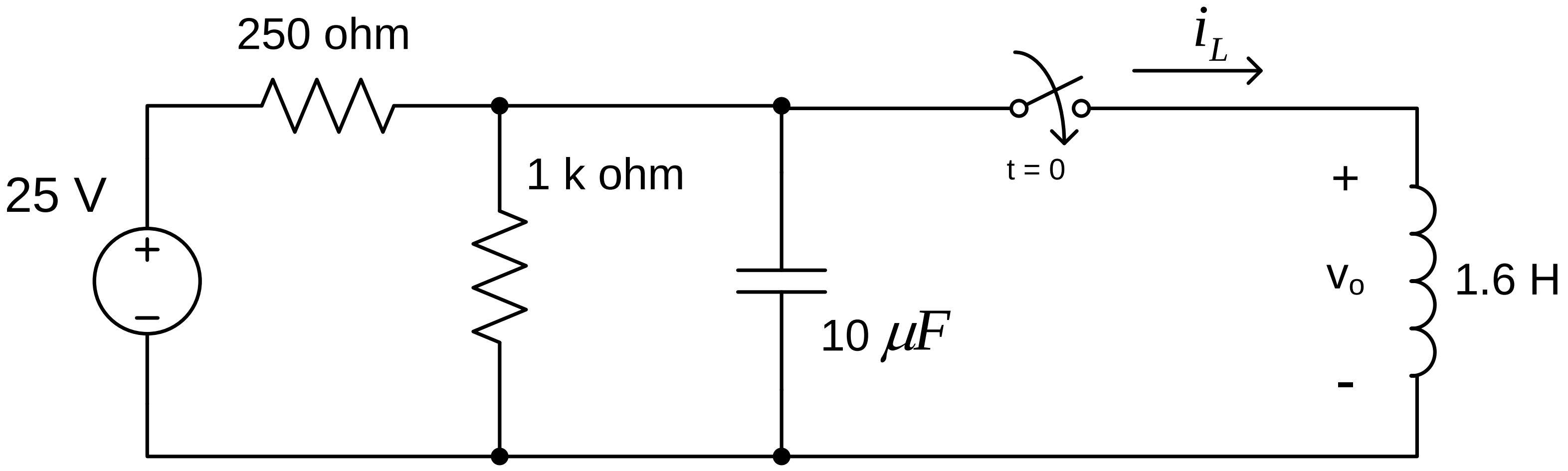
$$\begin{aligned} v_s(0^+) &= i_C(0^+)\cdot 3\Omega + v_C(0^+) \\ &= (22 - i_L(0^+))\cdot 3\Omega + v_s(0^+) \\ &= 51 \end{aligned}$$



$$v_s(\infty) = 22A \cdot 2\Omega = 44$$

$$v_s(3.4s.) = \ldots$$

Example: Final 2/47



Find

$$i_L(t = 0^+) \qquad \frac{dv_o(0^+)}{dt}$$

$$v_o(t = 0^+) \qquad i_L(t)$$

$$\frac{di_L(0^+)}{dt} \qquad v_o(t)$$

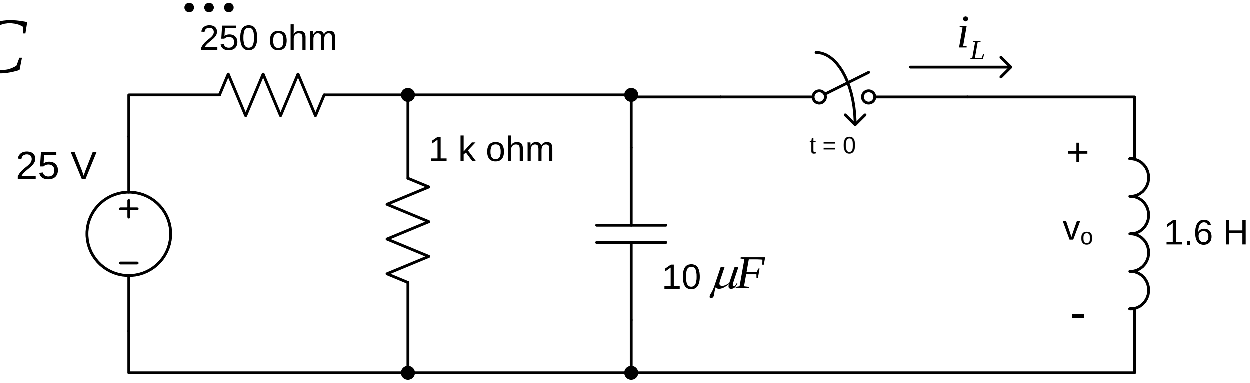
Example: Final 2/47

$$i_L(t=0^+) = i_L(0^-) = 0$$

$$v_o(t=0^+) = v_C(0^-) = \frac{1000}{1000+250}\cdot 25 = 20$$

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{20}{1.6} = 12.5$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{C} = \ldots$$

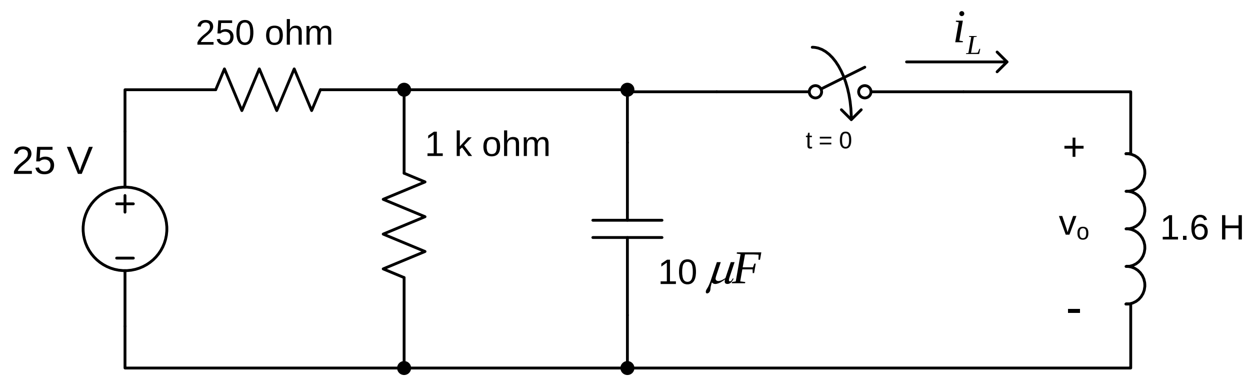


Example: Final 2/47

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{C} = \ldots$$

From $$\frac{v_C(0^+) - 25}{250} + \frac{v_C(0^+)}{1000} + i_C(0^+) = 0$$

$$\Rightarrow i_C(0^+) = 0$$



Ex:

สำหรับวงจร RLC แบบขนานที่มี voltage response

$$v(t) = D_1 t e^{-4000t} + D_2 e^{-4000t} \qquad , t \geq 0$$

ค่า initial current ในตัว inductor มีค่า 5 mA, และ ค่า initial voltage ในตัวเก็บประจุมีค่า 25 V. โดยที่ค่า inductance ของตัว inductor มีขนาด 5 H. ให้หา

a) ค่าของ R, C, D_1, D_2
b) $i_C(t)$ สำหรับ $t > 0^+$

Ex:

[a] $$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (4000)^2$$

$$\therefore \quad C = \frac{1}{(16 \times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_\text{R}(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_\text{C}(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5 \times 10^{-3}}{12.5 \times 10^{-9}} = -6 \times 10^5$$

$$\therefore \quad D_1 = -6 \times 10^5 + 4000(25) = -5 \times 10^5$$

Ex:

[b]

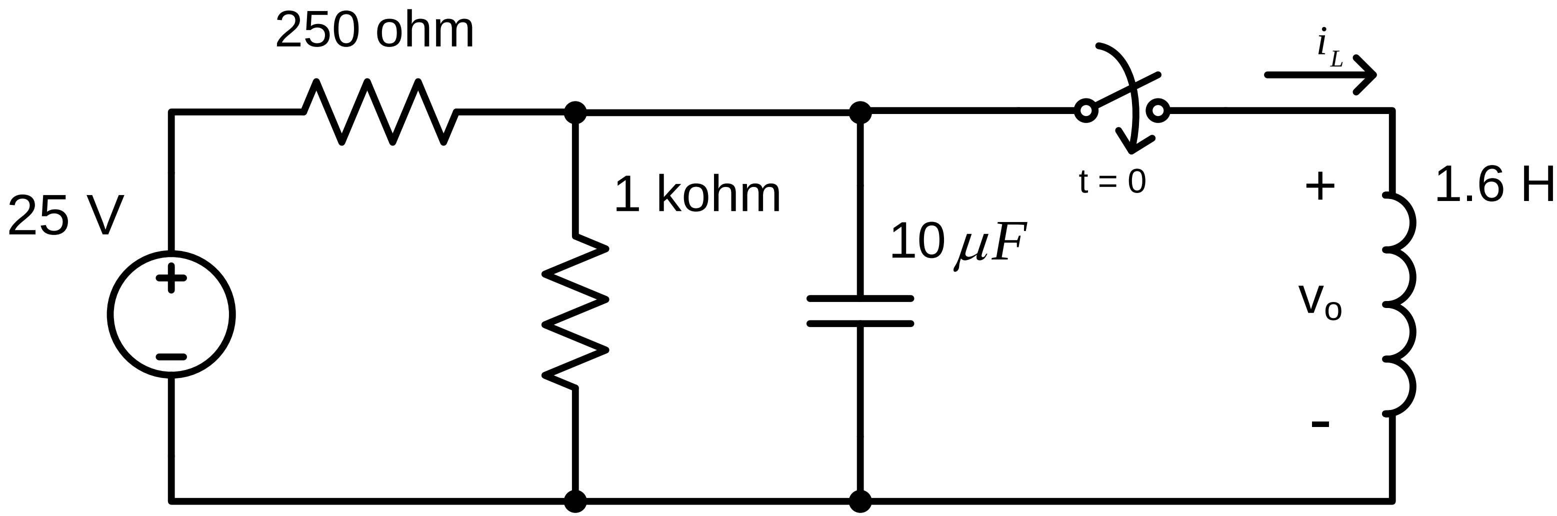
$$v = -5 \times 10^5 t e^{-4000t} + 25 e^{-4000t}$$

$$\frac{dv}{dt} = [20 \times 10^8 t - 6 \times 10^5] e^{-4000t}$$

$$i_C = C\frac{dv}{dt} = 12.5 \times 10^{-9} [20 \times 10^8 t - 6 \times 10^5] e^{-4000t}$$

$$= (25{,}000t - 7.5) e^{-4000t}\ \text{mA}, \qquad t > 0$$

Ex:

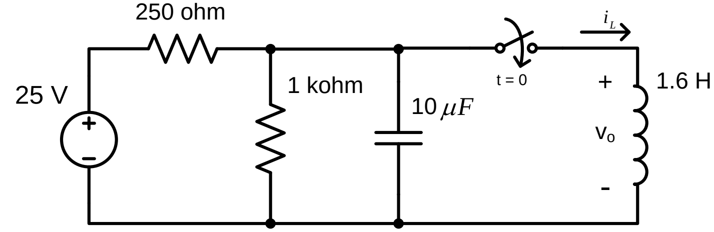


ให้หา

a) $v_o(t) \quad , t \geq 0^+$

b) $i_L(t) \quad , t \geq 0$

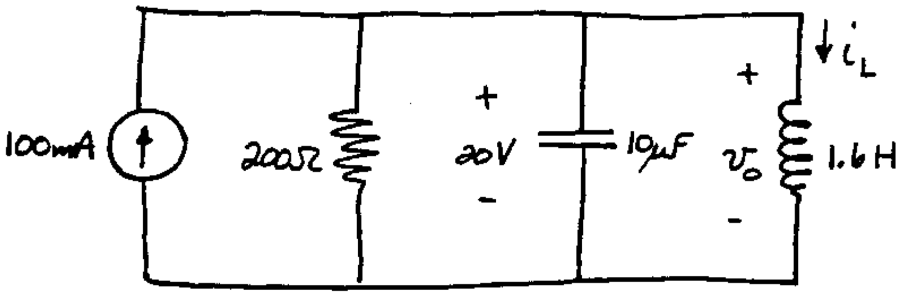
Ex:



$$v_o(0^-) = v_o(0^+) = \frac{1000}{1250}(25) = 20\text{ V}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$t > 0$



$$-100 + \frac{20}{0.2} + i_C(0^+) + 0 = 0; \qquad \therefore\ i_C(0^+) = 0$$

$$\frac{1}{2RC} = \frac{10^6}{(400)(10)} = 250\text{ nepers}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{10(1.6)} = 62{,}500$$

$$\therefore\ \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

$$v_o = V_f + D_1' te^{-250t} + D_2' e^{-250t}$$

$$V_f = 0$$

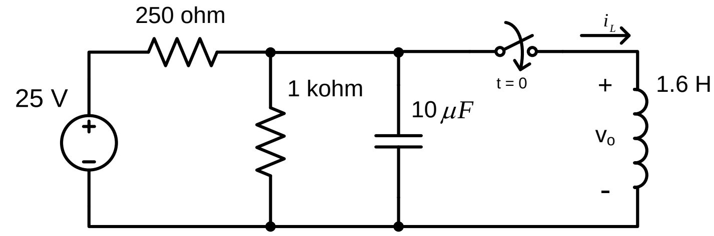
$$\frac{dv_o(0)}{dt} = -250D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 250D_2' = 5000\text{ V/s}$$

$$\therefore\ v_o = 5000te^{-250t} + 20e^{-250t}\text{ V}, \quad t \geq 0^+$$

Ex:



[b] $$i_L = I_f + D_3' t e^{-250t} + D_4' e^{-250t}$$

$$i_L(0^+) = 0; \qquad I_f = 100\,\text{mA}; \qquad \frac{di_L(0^+)}{dt} = \frac{20}{1.6} = 12.5\ \text{A/s}$$

$$\therefore\quad 0 = 100 + D_4'; \qquad D_4' = -100\,\text{mA};$$

$$-250D_4' + D_3' = 12.5; \qquad D_3' = -12.5\,\text{A/s}$$

$$\therefore\quad i_L = 100 - 12{,}500te^{-250t} - 100e^{-250t}\ \text{mA} \qquad t \geq 0$$

Hw:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.