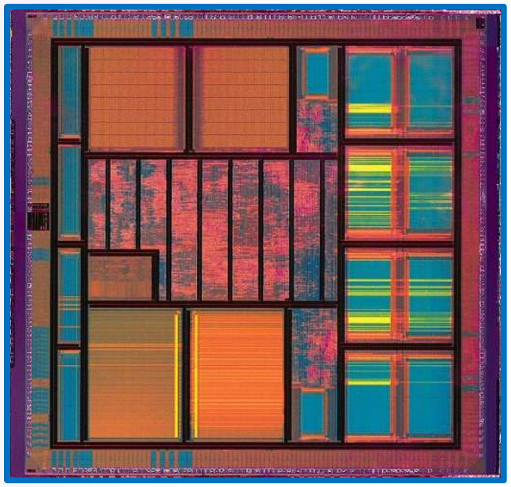
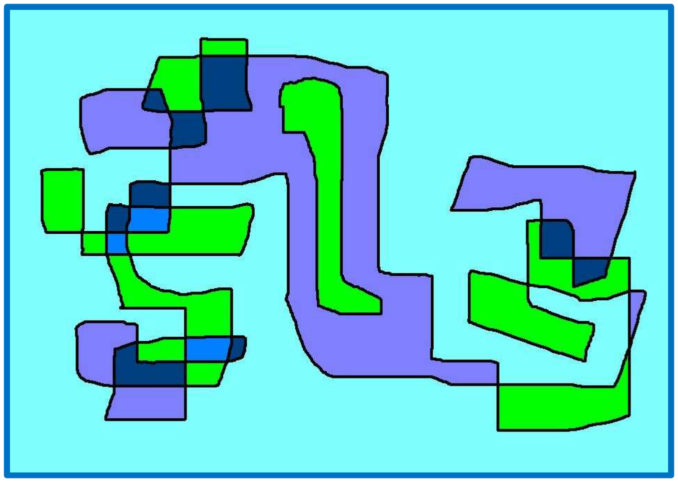
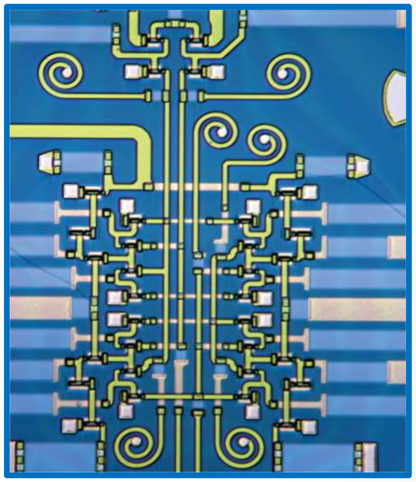






ENE/EIE 211: Electronic Devices and Circuit Design II

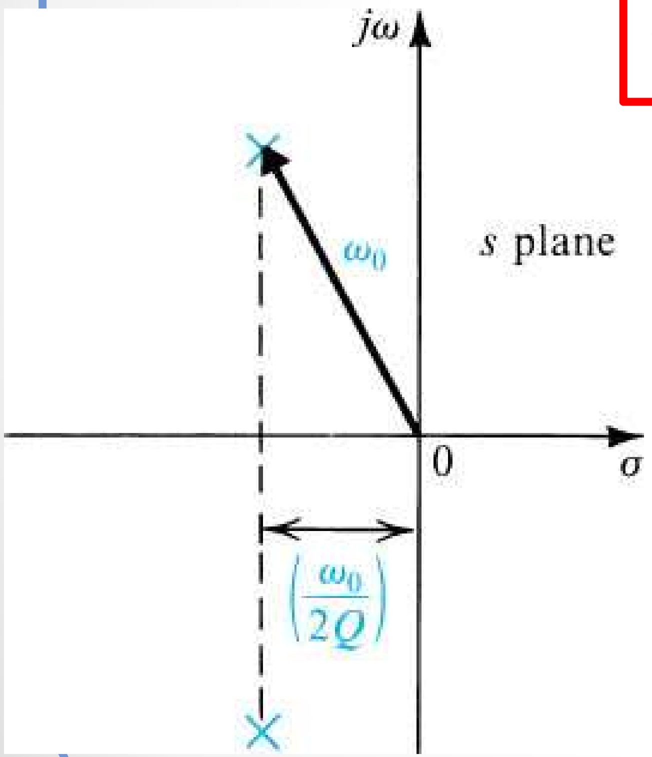
Lecture 8: Filters (cont.)

2. Second-Order Filters: the general second-order filter transfer function is

$$\boxed{T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2}}$$

where ω_o and Q determine the natural modes (poles) according to

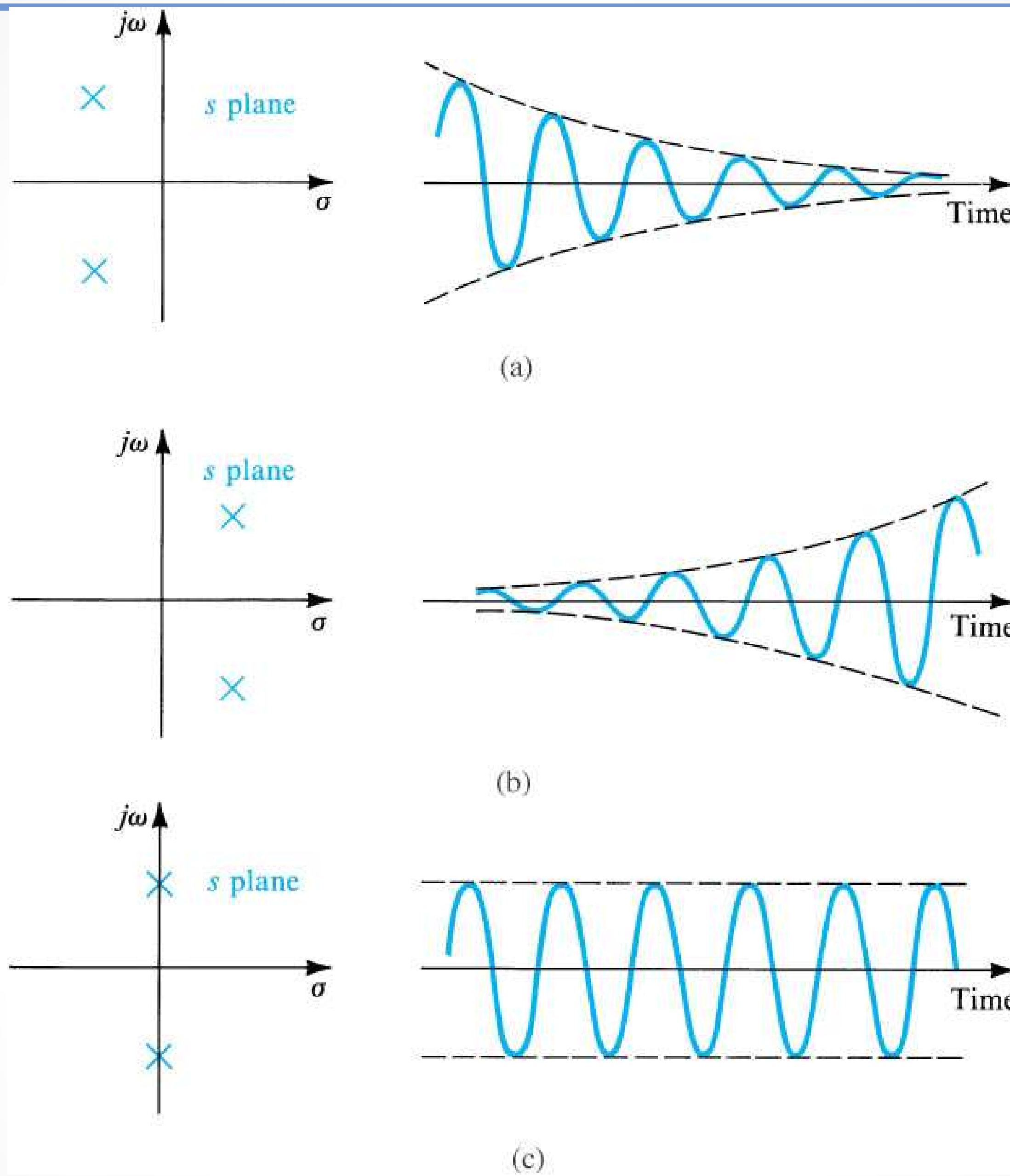
$$\boxed{p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1-(1/4Q^2)}}$$



The radial distance of the natural modes (from the origin) is equal to ω_o which is known as the pole frequency.

The parameter Q is called the pole quality factor or the pole Q. It determines the distance of the pole from the the jω axis: the higher the value of Q, the closer the poles are to the jω axis.

When Q = ∞, the poles are on the jω axis and can yield sustained oscillations in the circuit realization. When Q Is negative, poles are on the right half plane which also produces oscillations.



Relationship between pole location and transient response.

The numerator coefficients: a_0, a_1, a_2, determine the type of second-order filter function (i.e. LP, HP, etc.). Seven special cases of interest are shown on the next page.

In the low-pass (LP) case, the 2 transmission zeros are at s = ∞. The peak of the magnitude response occurs only for $Q > 1/\sqrt{2}$. The response obtained for $Q = 1/\sqrt{2}$ is Butterworth, or maximally flat response.

In the bandpass (BP) filter, the center frequency is equal to the pole freq ω_o. The selectivity is measured by its 3-dB bandwidth, which is $BW = \omega_2 - \omega_1 = \omega_0 / Q$ where

$$\omega_1, \omega_2 = \omega_0 \sqrt{1 + (1/4Q^2)} \pm \frac{\omega_0}{2Q}$$

As Q increases, the BW decreases and the filter becomes more selective.

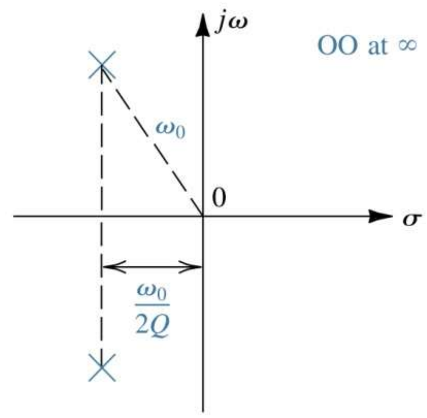
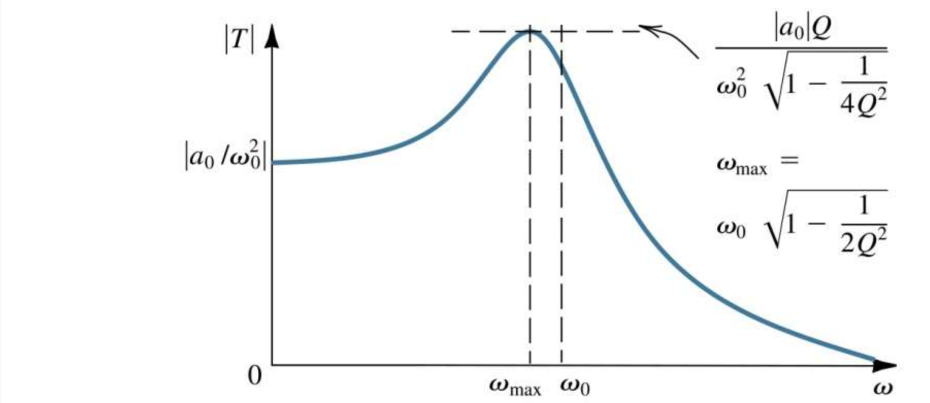
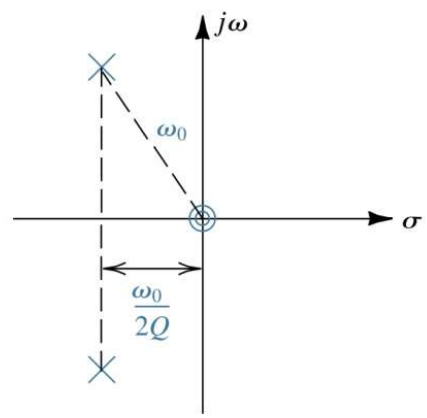
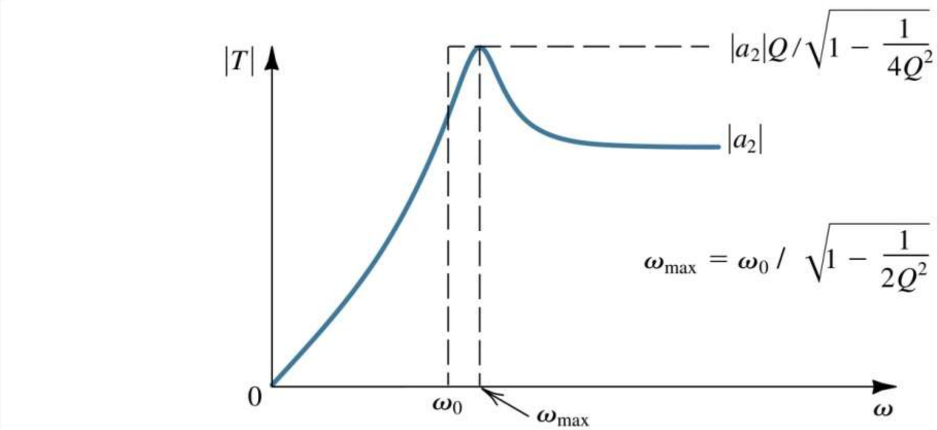
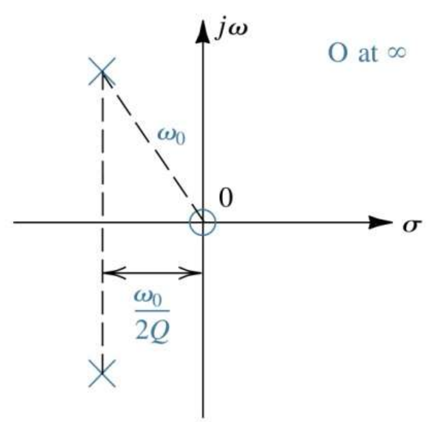
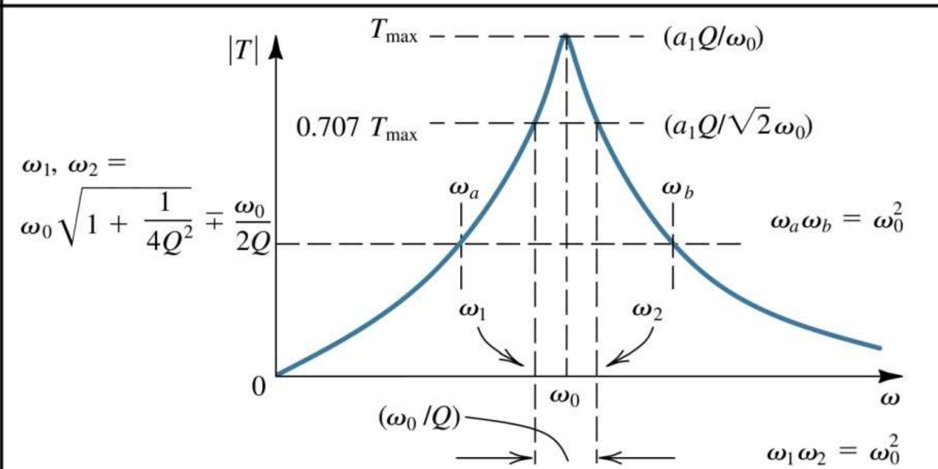
In the notch filter, the transmission zeros are on the jω axis at locations $\pm j\omega_n$, then the magnitude response exhibits zero transmission at $\omega = \omega_n$, which is known as the notch frequency. As shown, there are 3 different cases, depending on the relative magnitude of ω_o and ω_n.

In the all-pass (AP) filter, the two transmission zeros are in the right half of the s-plane, at the mirror-image locations of the poles. The magnitude response of the filter is constant over all frequencies; the flat gain in this case is equal to $|a_2|$. The frequency selectivity of the AP filter is in its phase response.

Example 1: For a maximally flat second-order low-pass filter ($Q = 1/\sqrt{2}$), show that at $\omega = \omega_o$, the magnitude response is 3 dB below the value at dc.

Example 2: Give the transfer function of a 2nd order bandpass filter with a center freq of 10^5 rad/s, a center-freq gain of 10 and a 3-dB bandwidth of 10^3 rad/s.

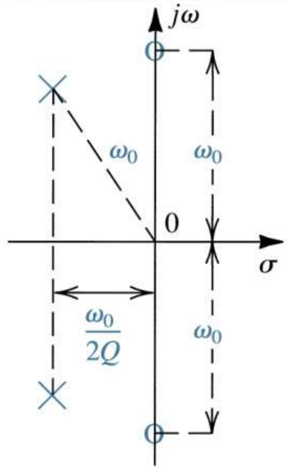
Second-Order Filter Functions (Fig. 12.16)

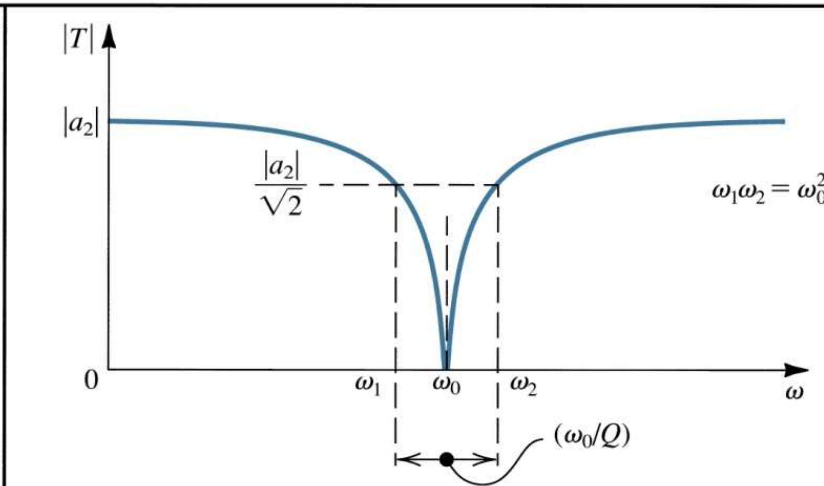
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

(d) Notch

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

dc gain = high-frequency gain = a_2





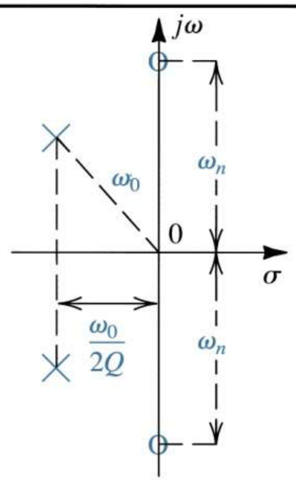
(e) Low-Pass Notch (LPN)

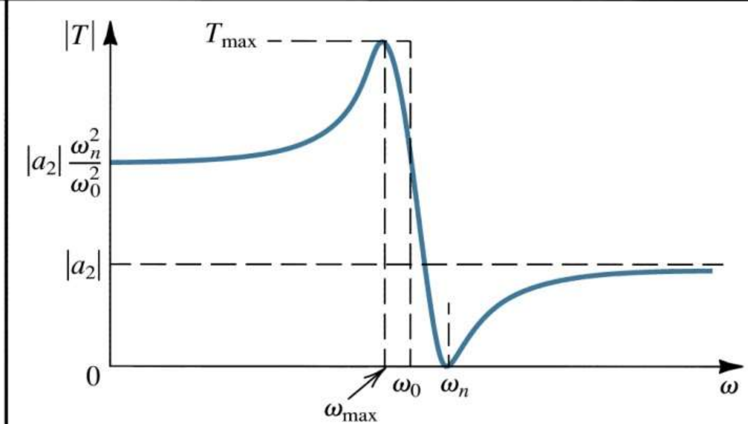
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \geqslant \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2





$$\omega_{\max} = \omega_0 \sqrt{\frac{\left(\frac{\omega_n^2}{\omega_0^2}\right)\left(1 - \frac{1}{2Q^2}\right) - 1}{\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1}}$$

$$T_{\max} = \frac{|a_2| \quad |\omega_n^2 - \omega_{\max}^2|}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$$

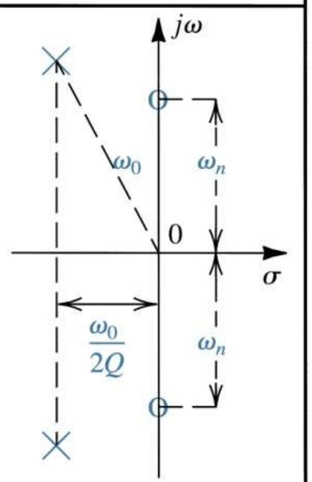
(f) High-Pass Notch (HPN)

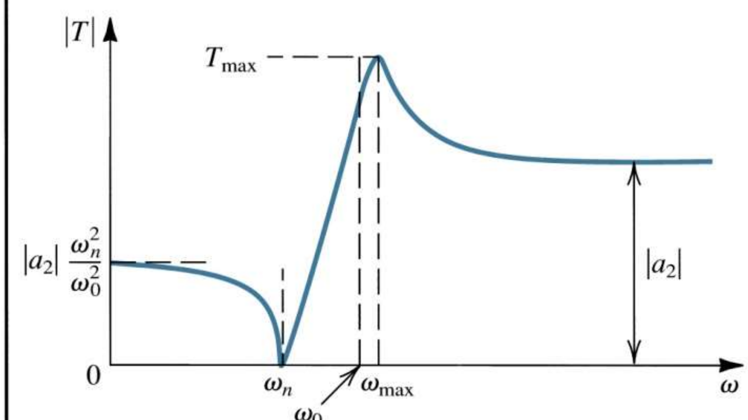
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \leqslant \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2

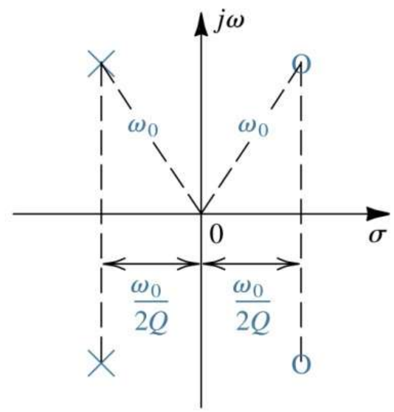


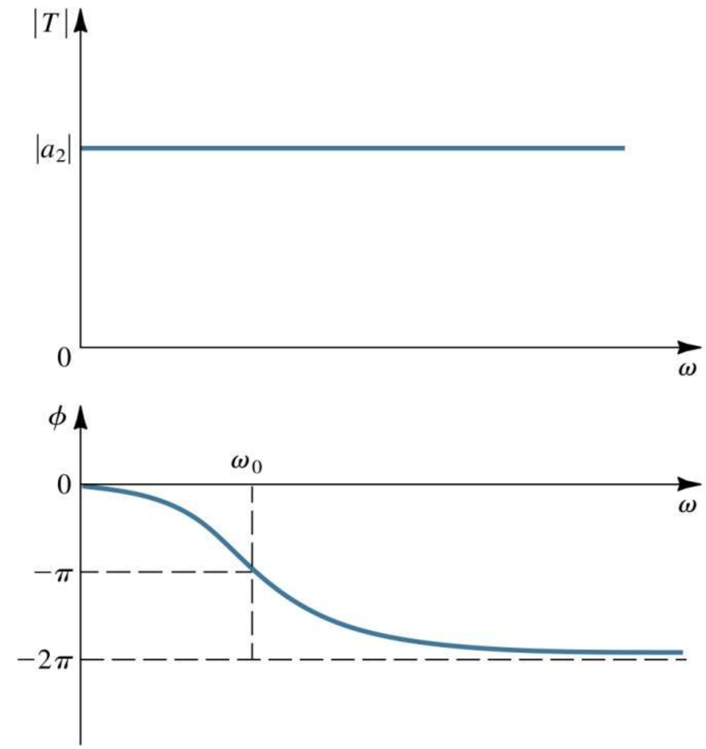


(g) All-Pass (AP)

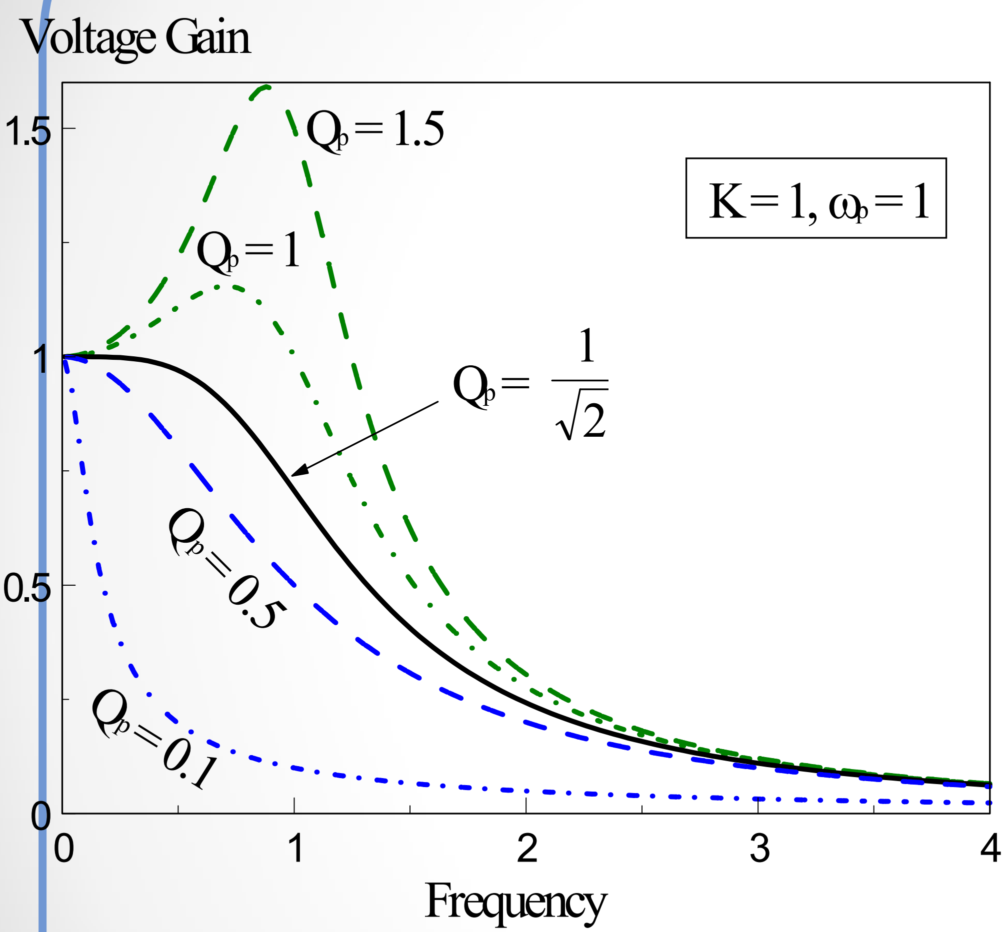
$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

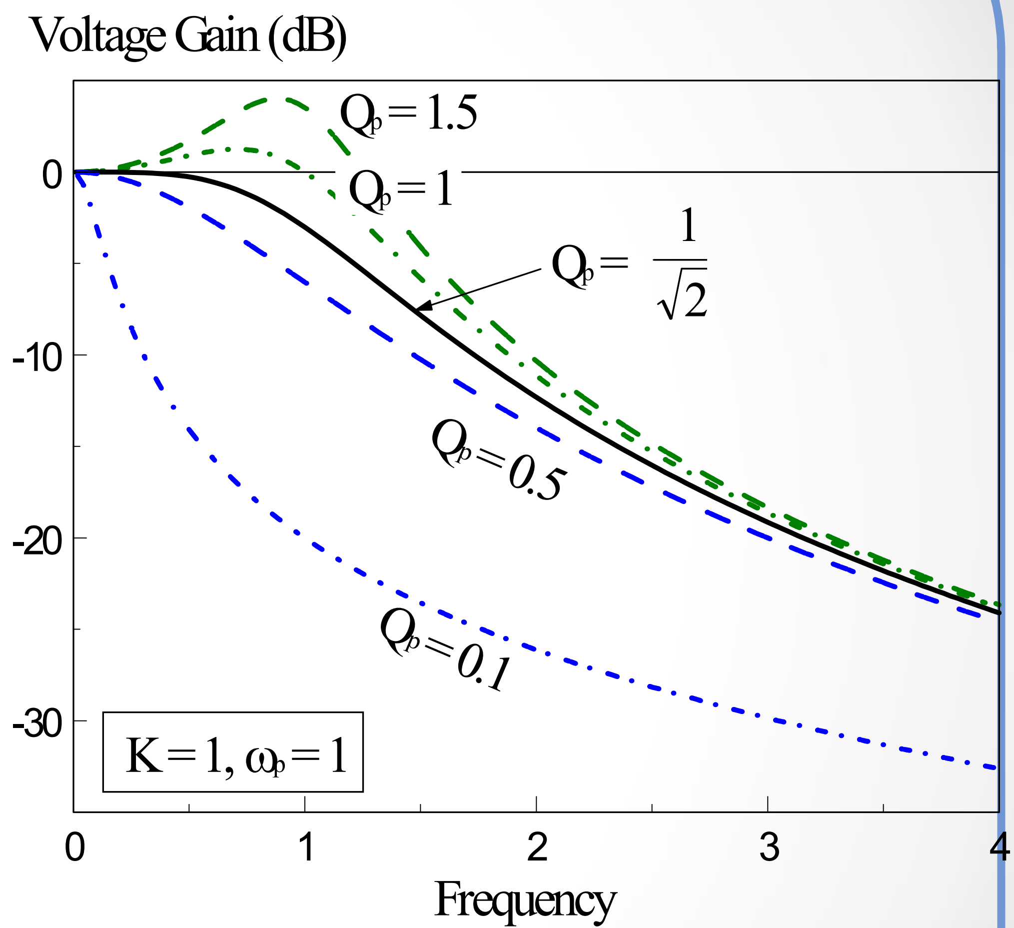
Flat gain $= a_2$





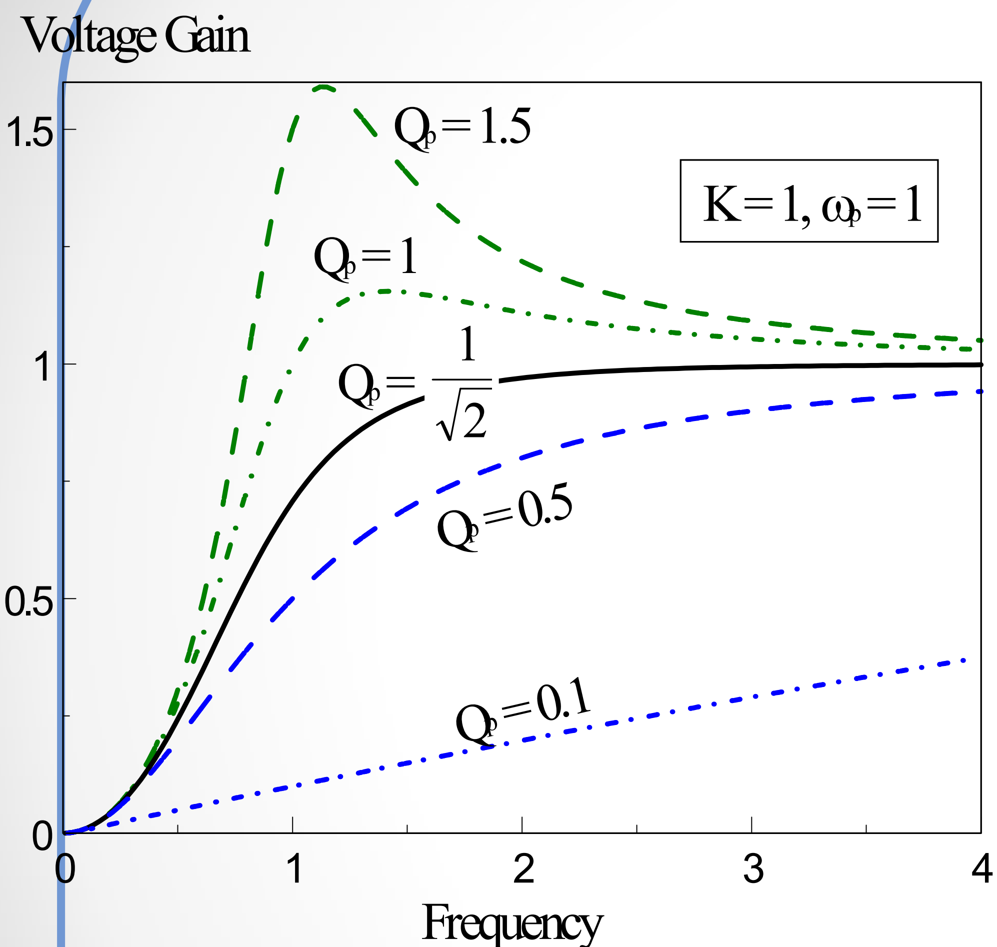
Low-Pass Filter

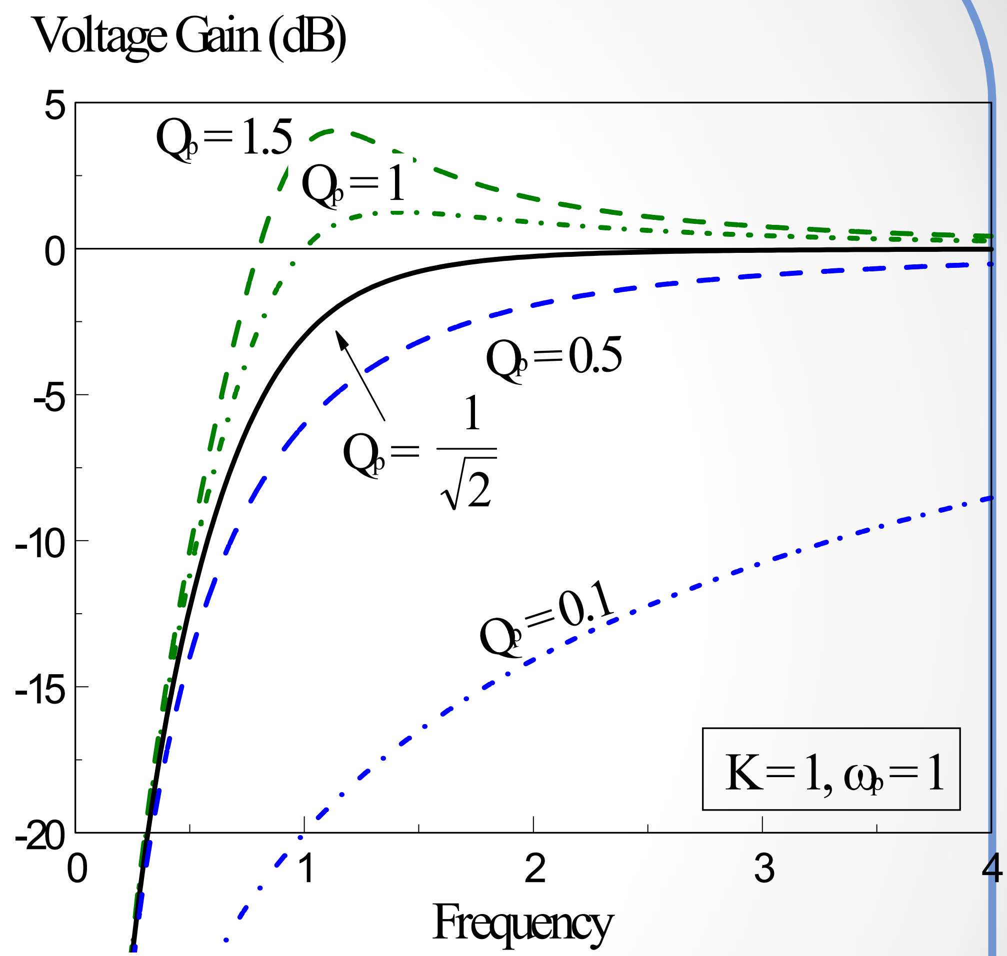




$$H(s) = \frac{1}{s^2 + \frac{s}{Q_p} + 1}$$

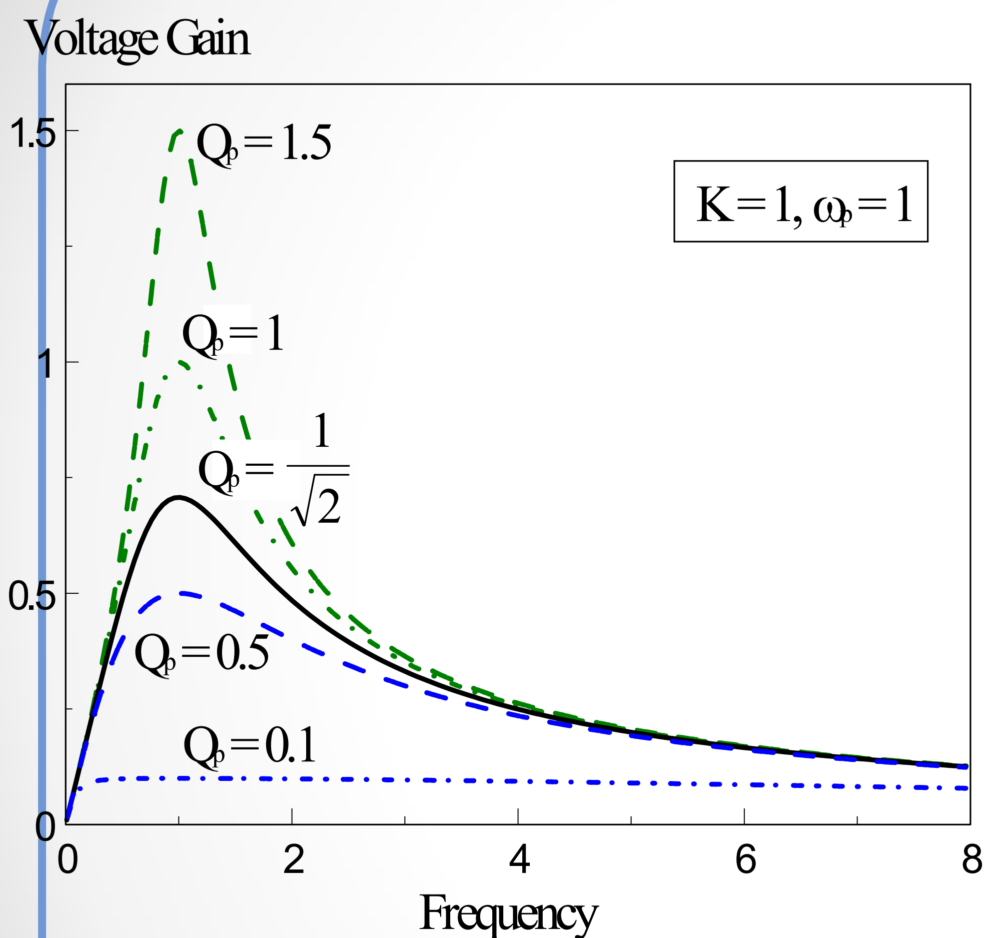
High-Pass Filter

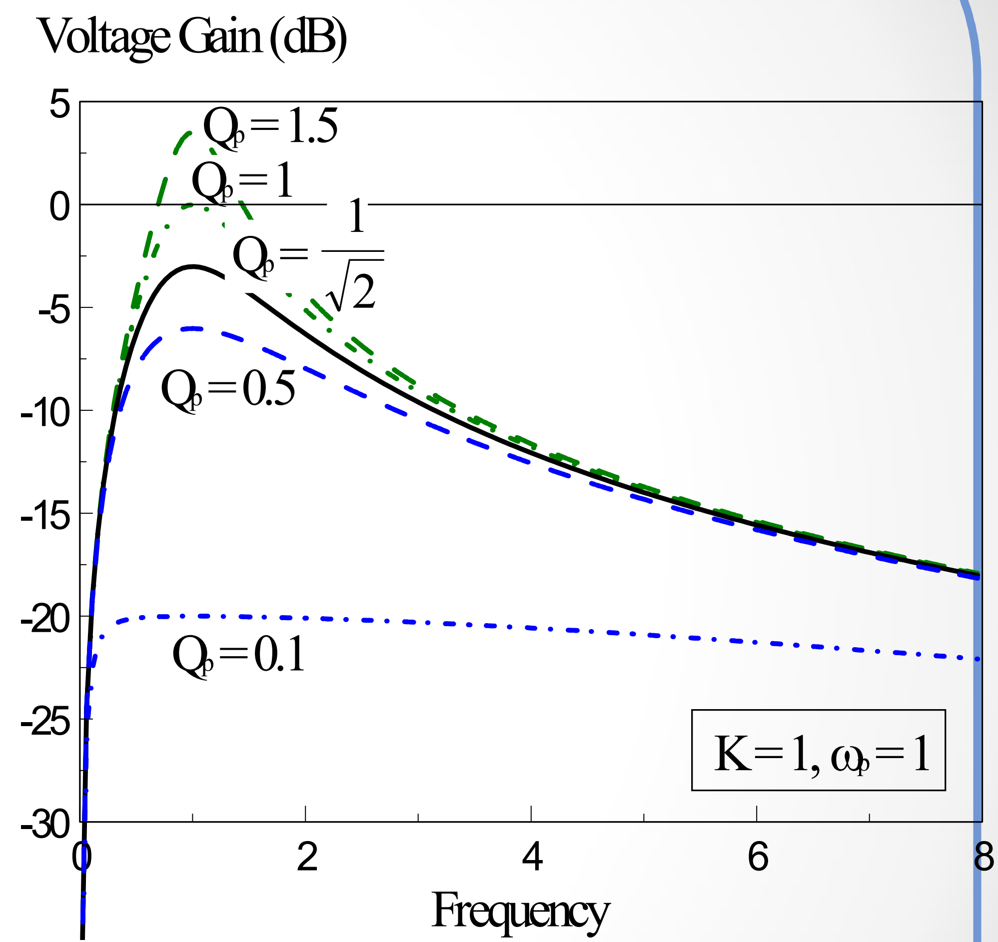




$$H(s) = \frac{s^2}{s^2 + \frac{s}{Q_p} + 1}$$

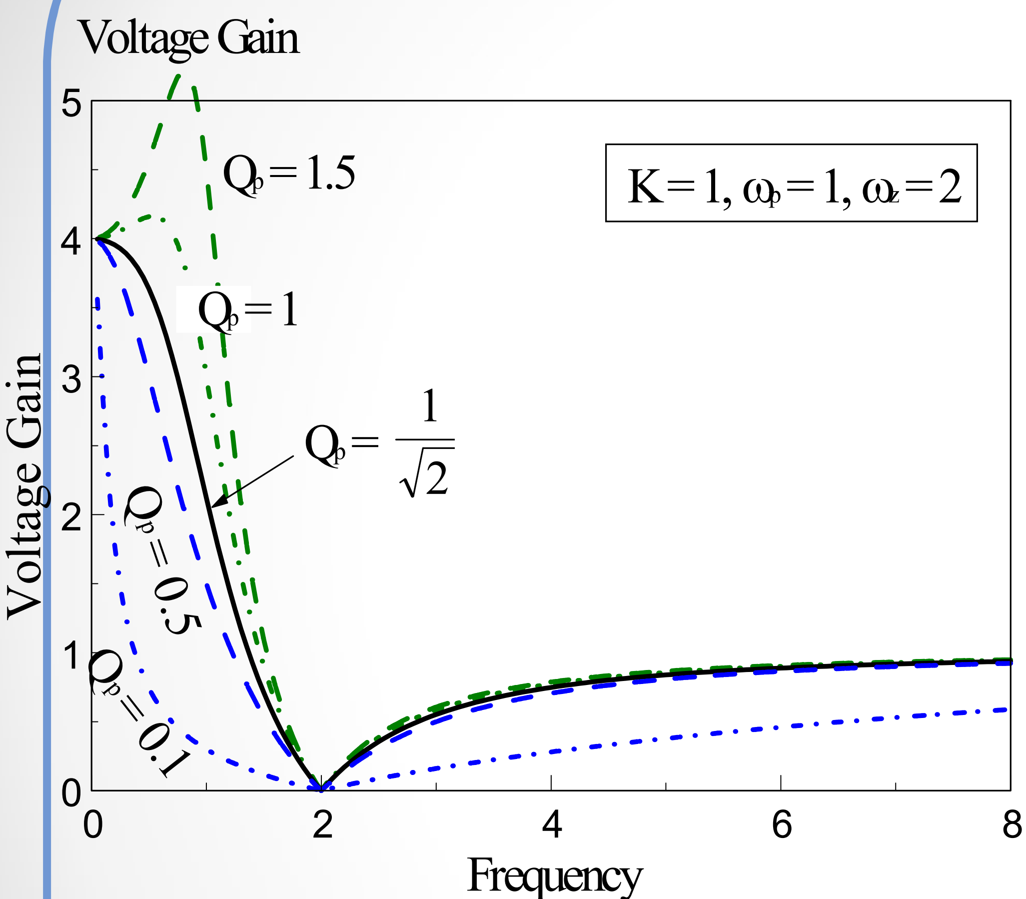
Band-Pass Filter

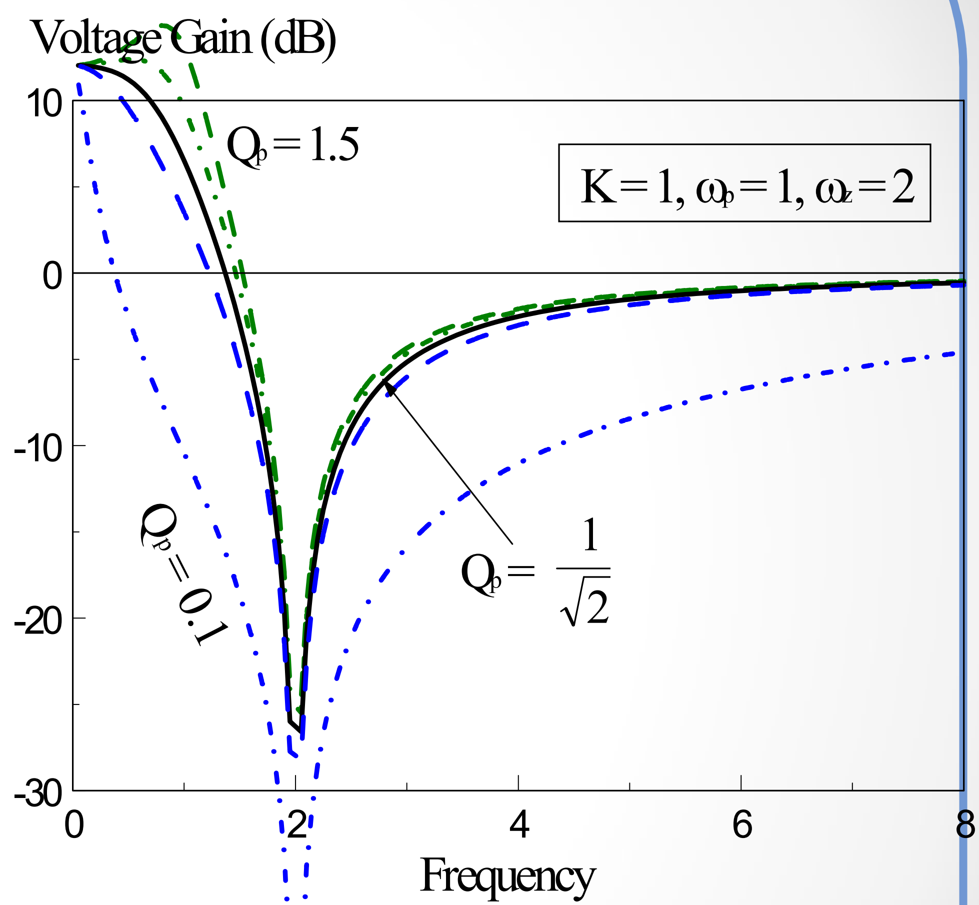




$$H(s) = \frac{s}{s^2 + \frac{s}{Q_p} + 1}$$

Band-Stop Filter



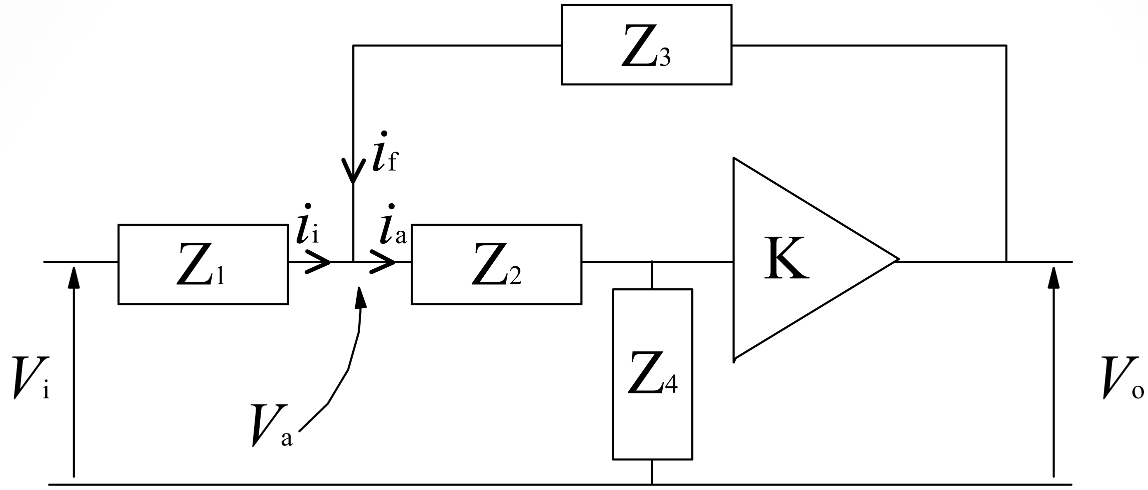


$$H(s) = \frac{s^2 + 2^2}{s^2 + \frac{s}{Q_p} + 1}$$

2^{nd} Order Active-Filter Implementation

- Voltage Controlled Voltage Source (**VCVS**) Positive Feedback (**Sallen-Key**)
- Infinite Gain Multiple Feedback (**IGMF**) Negative Feedback
- Antoniou Induction Replacement Circuit
- Two-integrator Loop Topology
- Kerwin-Huelsman-Newcomb or **KHN biquad**

Voltage Controlled Voltage Source (VCVS) Positive Feedback Active Filter (Sallen-Key)



By KCL at V_a: $$i_i + i_f - i_a = 0$$

where,

$$i_i = \frac{V_i - V_a}{Z_1}$$

$$i_f = \frac{V_o - V_a}{Z_3}$$

$$i_a = \frac{V_a}{Z_2 + Z_4}$$

Therefore, we get

$$\frac{V_i - V_a}{Z_1} + \frac{V_o - V_a}{Z_3} - \frac{V_a}{Z_2 + Z_4} = 0$$

Re-arrange into voltage group gives:

$$\frac{V_i}{Z_1} + \frac{V_o}{Z_3} - V_a\left(\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_2 + Z_4}\right) = 0 \qquad (1)$$

But, $$V_o = K i_a Z_4 = K \frac{Z_4 V_a}{Z_2 + Z_4} \qquad (2)$$

Substitute (2) into (1) gives

$$\frac{V_i}{Z_1} + \frac{V_o}{Z_3} - \frac{V_o\left(Z_2 + Z_4\right)}{K Z_4}\left(\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_2 + Z_4}\right) = 0$$

or

$$H = \frac{V_o}{V_i} = \frac{K}{\frac{Z_1}{Z_3}\left(1 - K\right) + \frac{Z_1 Z_2}{Z_3 Z_4} + 1 + \frac{1}{Z_4}\left(Z_1 + Z_2\right)} \qquad (3)$$

In admittance form:

$$H = \frac{K}{1 + Y_4\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right) + \frac{Y_3}{Y_1}\left(1 - K\right) + \frac{Y_3 Y_4}{Y_1 Y_2}} \qquad (4)$$

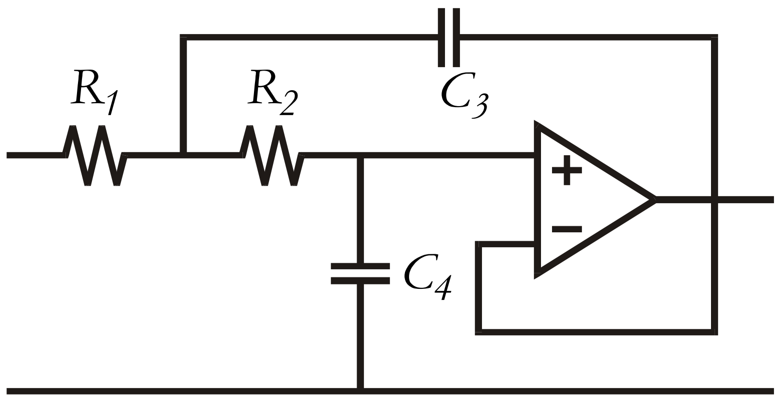
* This configuration is often used as a low-pass filter, so a specific example will be considered.

VCVS Low Pass Filter

$$H(s) = K\frac{1}{s^2 + as + b}$$

In order to obtain the above response, we let:

$$Z_1 = R_1 \quad Z_2 = R_2 \quad Z_3 = \frac{1}{j\omega C_3} = \frac{1}{sC_3} \quad Z_4 = \frac{1}{j\omega C_4} = \frac{1}{sC_4}$$



Then the transfer function (3) becomes:

$$H(s) = \frac{K}{1 + sC_4(R_1 + R_2) + sR_1C_3(1-K) + s^2R_1R_2C_3C_4} = \frac{K'}{s^2 + \frac{\omega_P}{Q_P}s + \omega_P^2} \quad (5)$$

we continue from equation (5),

$$H(s) = \frac{K}{s^2 R_1 R_2 C_3 C_4 + sC_4(R_1 + R_2) + sR_1 C_3 (1-K) + 1}$$

$$H(s) = \frac{K\dfrac{1}{R_1 R_2 C_3 C_4}}{s^2 + s\left[\dfrac{C_4(R_1 + R_2) + R_1 C_3 (1-K)}{R_1 R_2 C_3 C_4}\right] + \left[\dfrac{1}{R_1 R_2 C_3 C_4}\right]}$$

Equating the coefficient from equations (6) and (5), it gives:

$$\omega_P = \frac{1}{\sqrt{R_1 R_2 C_3 C_4}} = \frac{1}{\sqrt{R_1 C_3}} \frac{1}{\sqrt{R_2 C_4}} \qquad Q_P = \frac{1}{\sqrt{\dfrac{R_1 C_4}{R_2 C_3}} + \sqrt{\dfrac{R_2 C_4}{R_1 C_3}} + (1-K)\sqrt{\dfrac{R_1 C_3}{R_2 C_4}}}$$

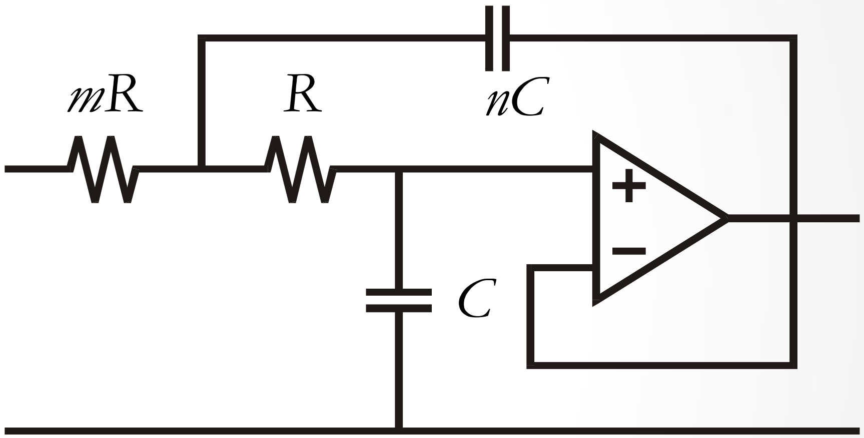
Now, K=1, equation (5) will then become,

$$H(s) = \frac{1}{1 + sC_4(R_1 + R_2) + s^2 R_1 R_2 C_3 C_4}$$

Simplified Design (VCVS filter)

$$Z_1 = mR \quad Z_2 = R \quad Z_3 = \frac{1}{j\omega(nC)} = \frac{1}{snC} \quad Z_4 = \frac{1}{j\omega C} = \frac{1}{sC}$$

$$H(s) = \frac{1}{1 + sRC(m+1) + s^2 nmR^2C^2}$$



Comparing with the low-pass response:

$$H(s) = K\frac{1}{s^2 + \frac{\omega_P}{Q_P}s + \omega_P^2}$$

It gives the following:

$$\omega_p = \frac{1}{RC\sqrt{nm}} \qquad Q_p = \frac{\sqrt{mn}}{m+1}$$

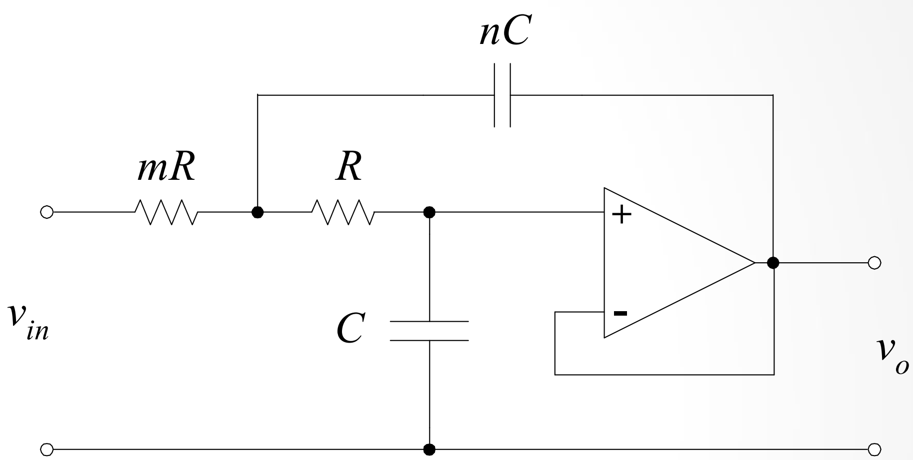
Example (VCVS low pass filter)

To design a low-pass filter with $f_O = 512\text{Hz}$ and $Q = \dfrac{1}{\sqrt{2}}$

Let m = 1

$$Q_P = \frac{\sqrt{mn}}{m+1} = \frac{\sqrt{1\times n}}{1+1} = \frac{\sqrt{n}}{2} = \frac{1}{\sqrt{2}}$$

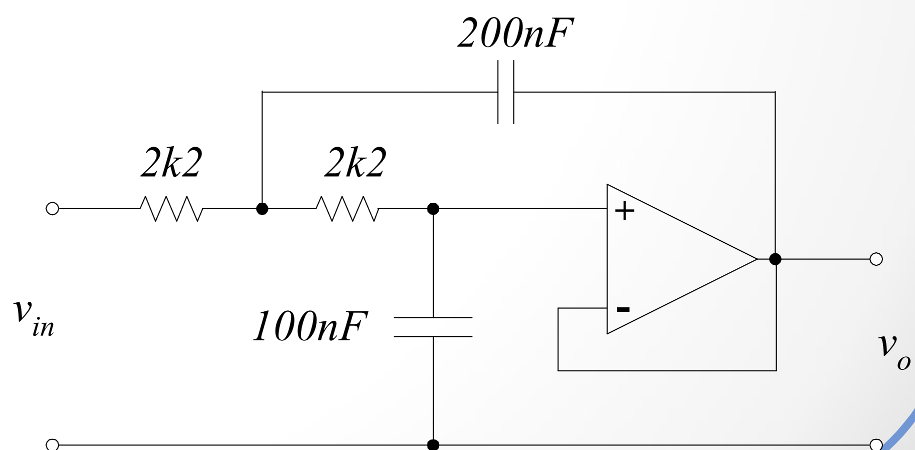
$\rightarrow$ n = 2



$$\omega_P = \frac{1}{RC\sqrt{mn}} = \frac{1}{RC\sqrt{1\times 2}} = \frac{1}{RC\sqrt{2}} = 2\pi(512Hz)$$

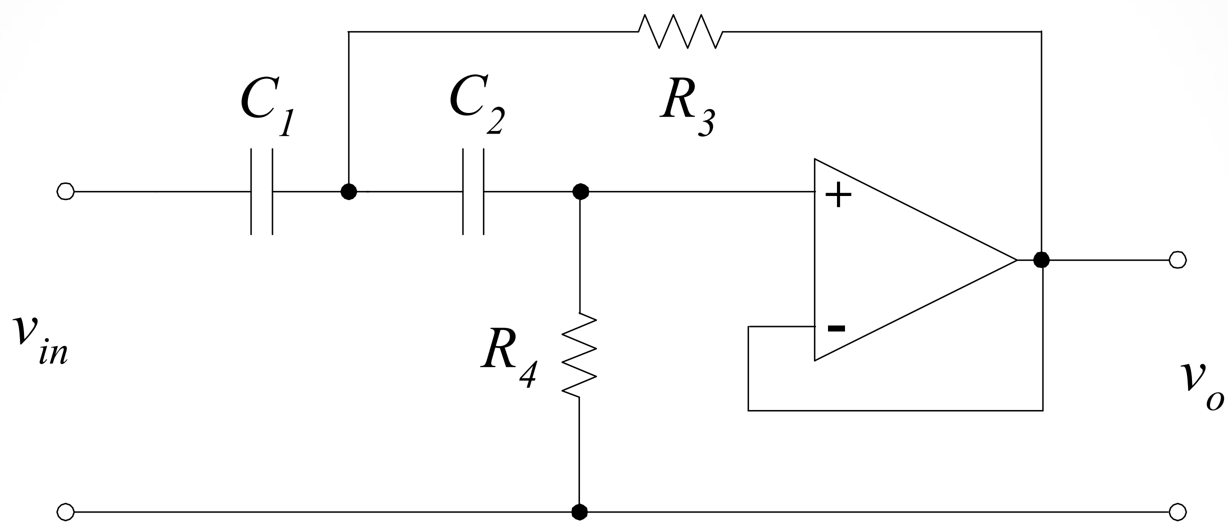
Choose $C = 100nF$

Then $R = 2{,}198\Omega \sim 2.2k\Omega$



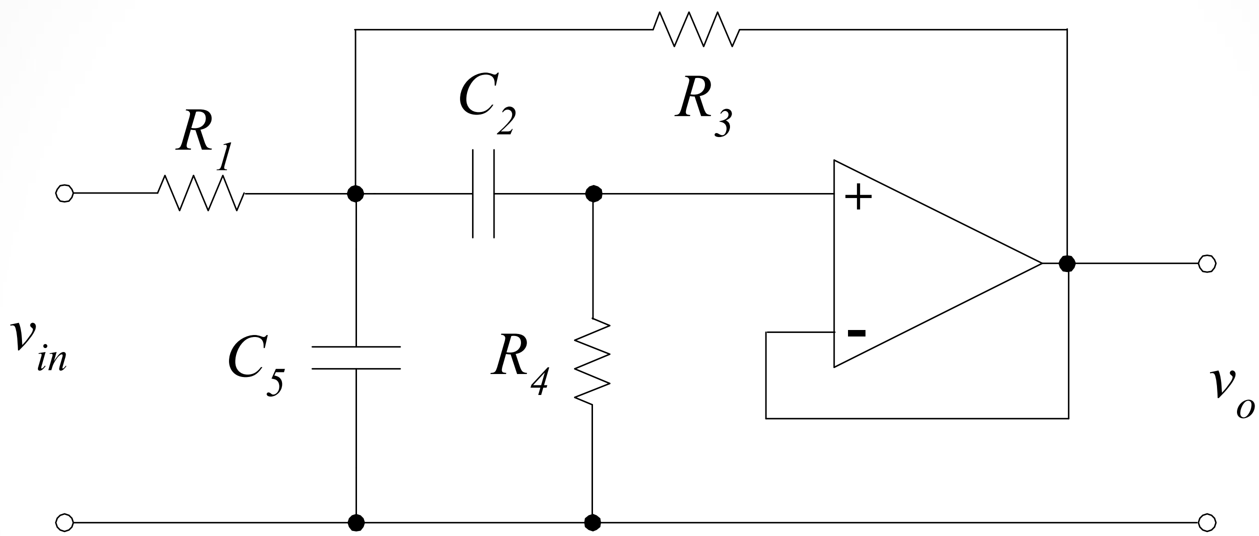
What happen if n = 1?

VCVS High Pass Filter



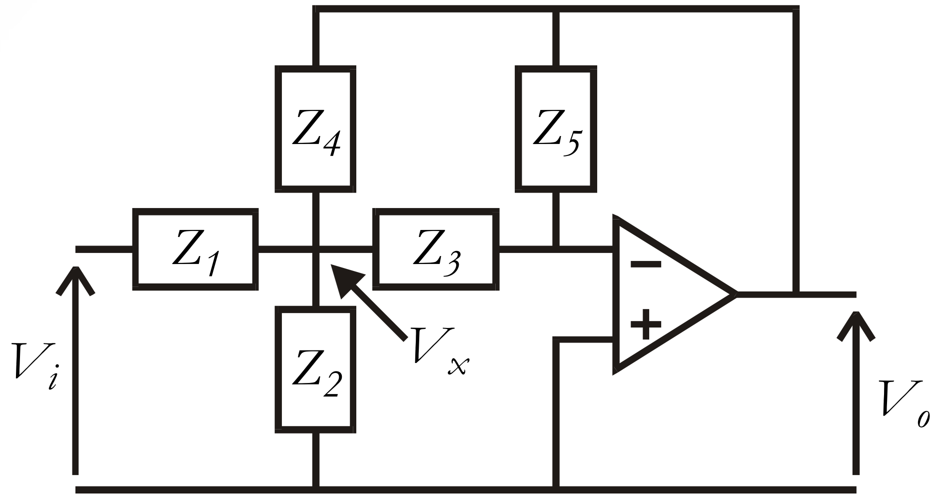
$$H(s)=\frac{KS^2}{s^2+s\left(\frac{1}{R_4C_1}+\frac{1}{R_4C_2}+\frac{1}{R_3C_1}(1-K)\right)+\frac{1}{R_3R_4C_1C_2}}=\frac{K'S^2}{s^2+\frac{\omega_P}{Q_P}s+\omega_P^2}$$

VCVS Band Pass Filter



$$H(s)=\frac{K\frac{s}{R_1C_5}}{s^2+\frac{s}{C_5}\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_3}(1-K)+\frac{C_5}{R_4C_2}\right)+\frac{R_1+R_3}{R_1R_3R_4C_2C_5}}=\frac{K'S}{s^2+\frac{\omega_P}{Q_P}s+\omega_P^2}$$

Infinite-Gain Multiple-Feedback (IGMF) Negative Feedback Active Filter



Note: because no current flows into v+, v- terminals of op-amp. Therefore, from KCL at node v- :
Vo/Z5+ Vx/Z3 = 0

$$v_i^+ = 0 \quad v_i^- = 0 \quad \Rightarrow \quad V_o = -\frac{Z_5}{Z_3}V_x \Rightarrow V_x = -\frac{Z_3}{Z_5}V_o \qquad (1)$$

By KCL at V_x,

$$\frac{V_i - V_x}{Z_1} = \frac{V_x}{Z_2} + \frac{V_x}{Z_3} + \frac{V_x - V_o}{Z_4} \qquad (2)$$

substitute (1) into (2) gives

$$\frac{V_i}{Z_1} + \frac{Z_3}{Z_1 Z_5}V_o = -\frac{Z_3}{Z_5 Z_2}V_o - \frac{V_o}{Z_5} - \frac{Z_3}{Z_4 Z_5}V_o - \frac{V_o}{Z_4} \qquad (3)$$

rearranging equation (3), it gives,

$$H = \frac{V_o}{V_i} = -\frac{\dfrac{1}{Z_1 Z_3}}{\dfrac{1}{Z_5}\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}+\dfrac{1}{Z_3}+\dfrac{1}{Z_4}\right)+\dfrac{1}{Z_3 Z_4}}$$

Or in admittance form:

$$H = \frac{V_o}{V_i} = -\frac{Y_1 Y_3}{Y_5\left(Y_1+Y_2+Y_3+Y_4\right)+Y_3 Y_4}$$

Filter \ Value	Z_1	Z_2	Z_3	Z_4	Z_5
LP	R_1	C_2	R_3	R_4	C_5
HP	C_1	R_2	C_3	C_4	R_5
BP	R_1	R_2	C_3	C_4	R_5

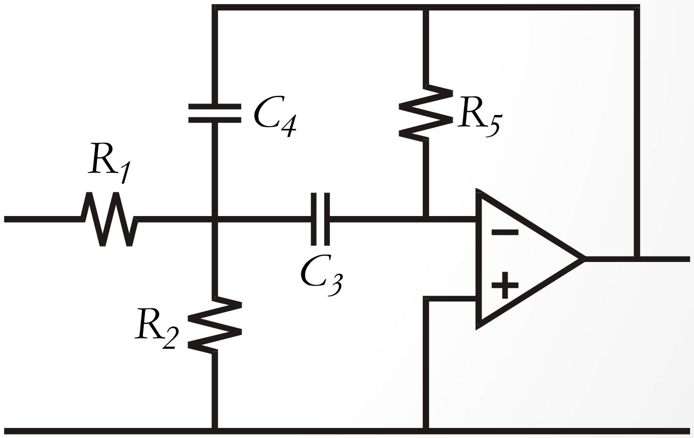
IGMF Band-Pass Filter

Band-pass: $$H(s) = K\frac{s}{s^2 + as + b}$$

To obtain the band-pass response, we let

$$Z_1 = R_1 \quad Z_2 = R_2 \quad Z_3 = \frac{1}{j\omega C_3} = \frac{1}{sC_3} \quad Z_4 = \frac{1}{j\omega C_4} = \frac{1}{sC_4} \quad Z_5 = R_5$$

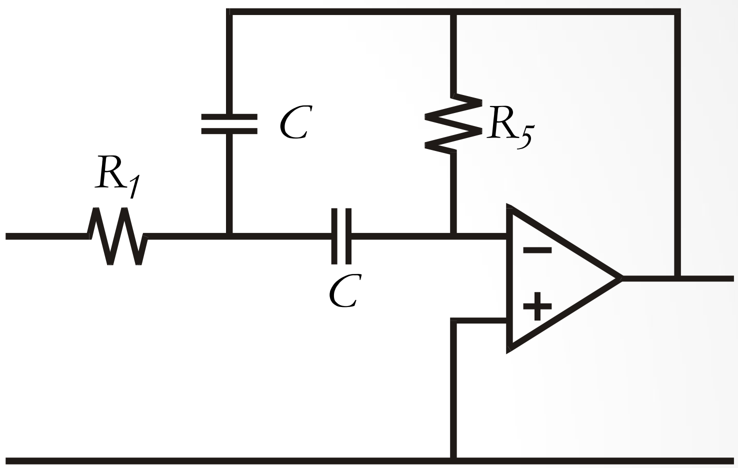
$$H(s) = -\frac{\dfrac{sC_3}{R_1}}{s^2 C_3 C_4 + s\dfrac{C_3 + C_4}{R_5} + \dfrac{1}{R_5}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)}$$



*This filter prototype has a very low sensitivity to component tolerance when compared with other prototypes.

Simplified design (IGMF filter)

$$H(s) = -\frac{\dfrac{sC}{R_1}}{\dfrac{1}{R_1R_5} + s\dfrac{2C}{R_5} + s^2C^2}$$



Comparing with the band-pass response

$$H(s) = K\frac{s}{s^2 + \dfrac{\omega_P}{Q_P}s + \omega_P^2}$$

Its gives,

$$\omega_p = \frac{1}{C\sqrt{R_1R_5}} \quad Q_p = \frac{1}{2}\sqrt{\frac{R_5}{R_1}} \quad \mathrm{K} = -\frac{1}{\mathrm{CR}_1} \quad H(j\omega_p) = -2Q^2$$

Example (IGMF band pass filter)

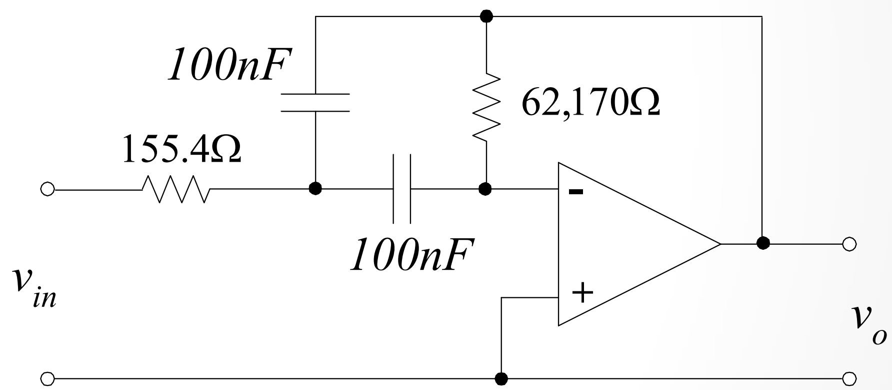
To design a band-pass filter with $f_O = 512\text{Hz}$ and $Q = 10$

$$\omega_P = \frac{1}{C\sqrt{R_1 R_5}} = 2\pi(512Hz)$$

$$C = 100nF \quad \rightarrow \quad R_1 R_5 = 9{,}662{,}741\Omega^2$$

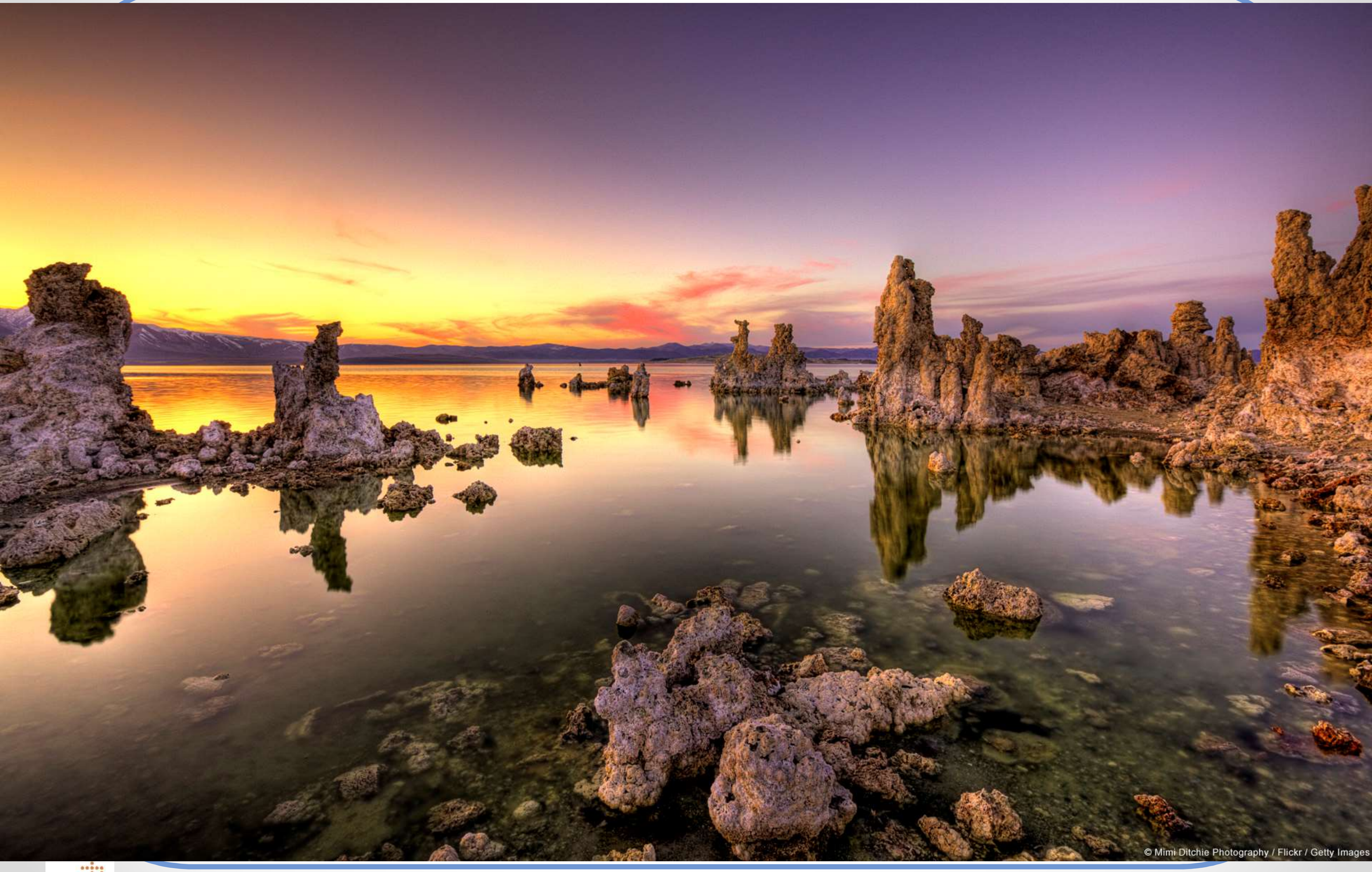
$$Q_P = \frac{1}{2}\frac{\sqrt{R_5}}{\sqrt{R_1}} = 10$$

$$\rightarrow R_1 = 155.4\Omega \qquad R_5 = 62{,}170\Omega$$



With similar analysis, we can choose the following values:

$$C = 10nF \quad R_1 = 1{,}554\Omega \quad \text{and} \quad R_5 = 621{,}700\Omega$$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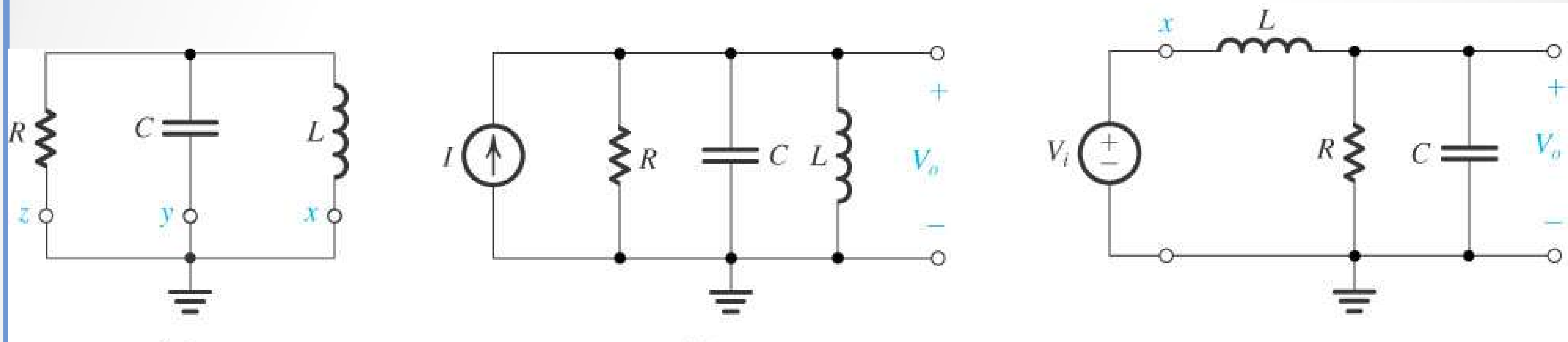
The Second-order LCR Resonators

The Resonator Natural Modes

The natural modes (poles) of the parallel resonance circuit can be determined by applying an excitation that does not change the natural structure of the ckt.



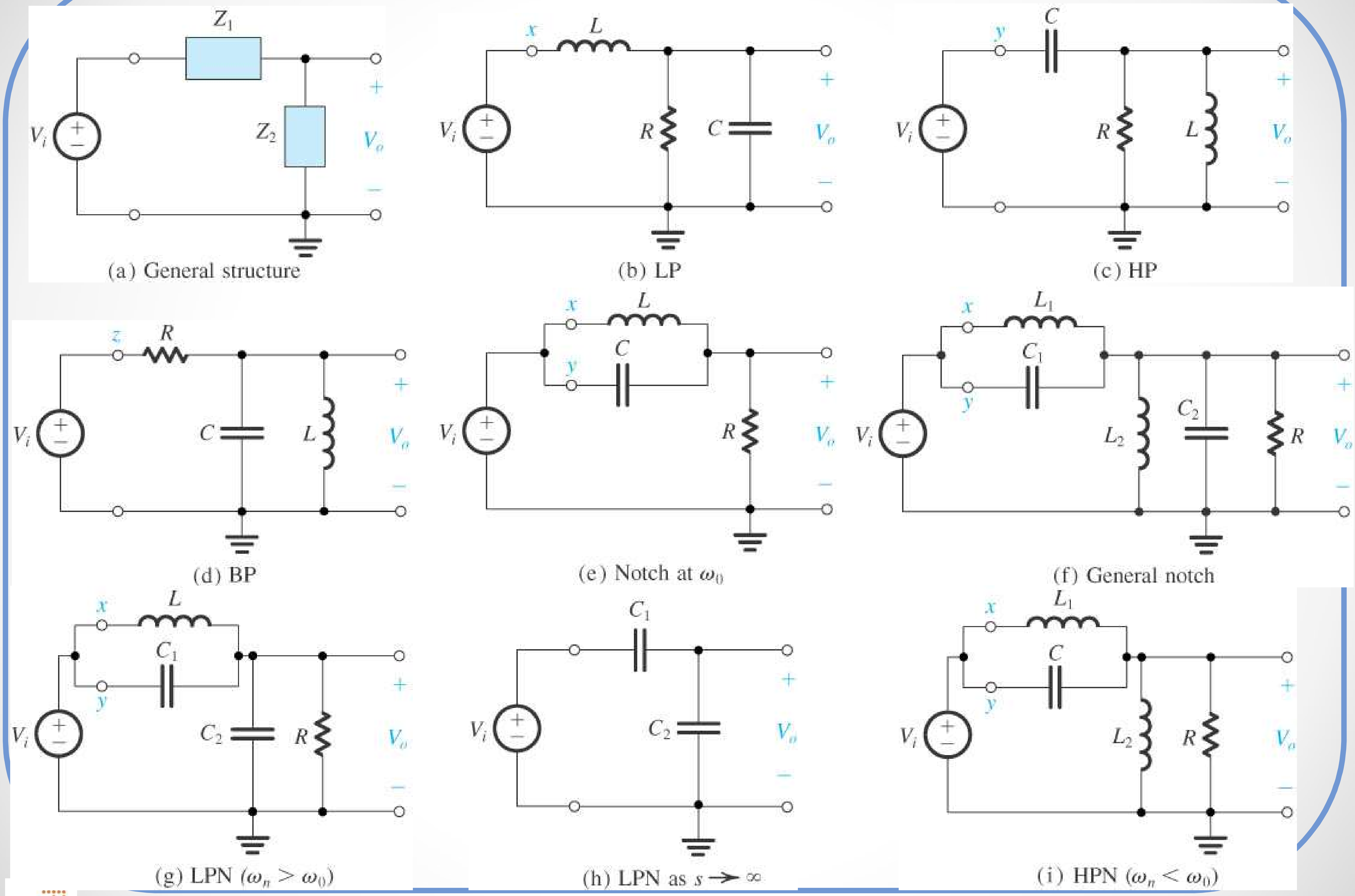
(a) (b) (c)

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{(1/sL) + sC + (1/R)} = \frac{s/C}{s^2 + s(1/CR) + (1/LC)}$$

Equating the denominator to the standard form $[s^2 + s(\omega_0/Q) + \omega_0^2]$, we'll get

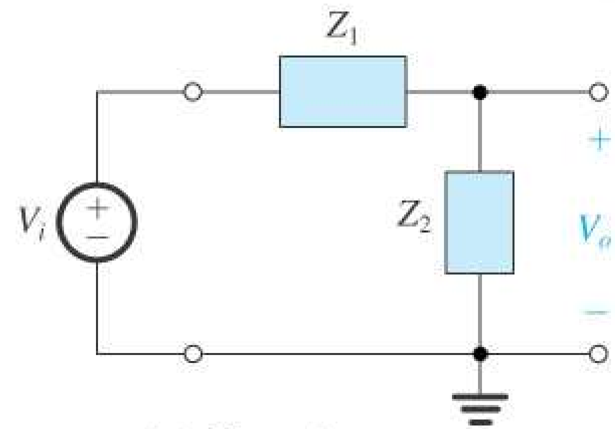
$\boxed{\omega_0^2 = 1/LC}$ $\boxed{\omega_0 = 1/\sqrt{LC}}$ $\boxed{\omega_0/Q = 1/CR}$ $\boxed{Q = \omega_0 CR}$

The Second-order LCR Resonators (Fig. 12.18)



The Second-order LCR Resonators

Realization of Transmission zeros



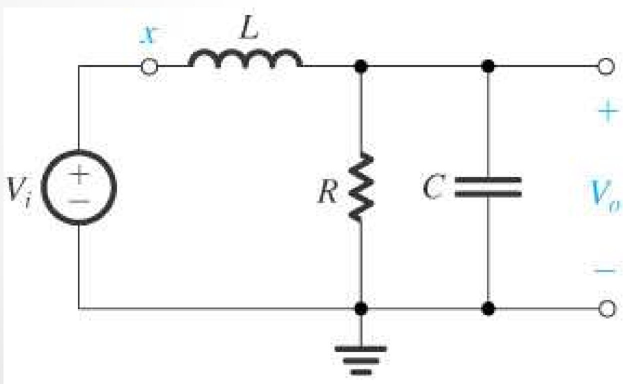
(a) General structure

The transmission zeros are:

- the values of s at which $Z_2(s)$ is zero, provided $Z_1(s)$ is not simultaneously zero, and
- the values of s at which $Z_1(s)$ is infinite, provided $Z_2(s)$ is not simultaneously infinite.

For example: the low-pass filter: $T(s) = \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)}$

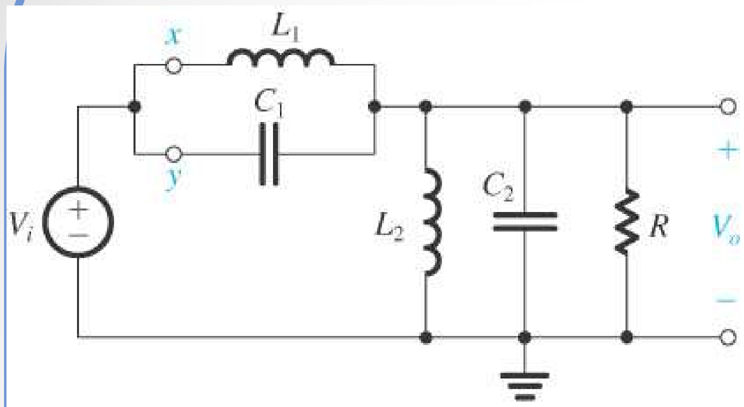
$$= \frac{1/LC}{s^2 + s(1/CR) + (1/LC)}$$



(b) LP

- 1/[sC+(1/R)] becomes zero at s = ∞
- sL becomes infinite at s = ∞
- So there are 2 zeros at s = ∞

Realization of the Notch function



(f) General notch

The impedance of the LC circuit becomes infinite at

$$\omega = \omega_0 = 1/\sqrt{LC}$$

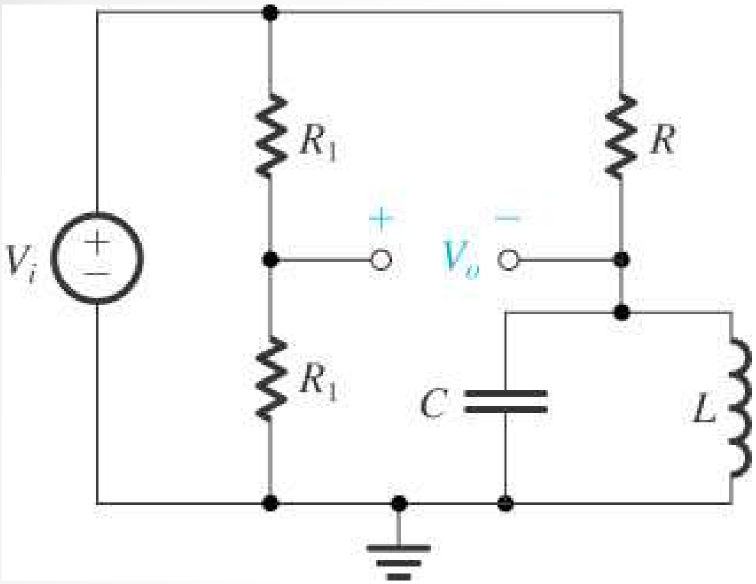
causing zero transmission at this frequency. The circuit has the corresponding transfer function

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

L_1 and C_1 are selected such that $L_1 C_1 = 1/\omega_n^2$. Thus the L_1C_1 tank circuit will introduce a pair of transmission zeros at +/- jω_n, provided the L_2C_2 tank is not resonant at ω_n. Apart from this restriction, the values of L_2, C_2 must be selected to ensure that the natural modes have not been altered; thus, $C_1 + C_2 = C$ and $L_1 \| L_2 = L$.

In other words, when Vi is replaced by a short ckt, the ckt should reduce to the original LCR resonator. Also, L_2 does not introduce a zero at s = 0 because at s = 0, L_1C_1 ckt also has a zero. In a similar way, C_2 does not introduce a zero at s = ∞.

Realization of the All-Pass function



$$T(s) = \frac{s^2 - (\omega_0 / Q)s + \omega_0^2}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

This transfer function can be written as

$$T(s) = 1 - \frac{2s(\omega_0 / Q)}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

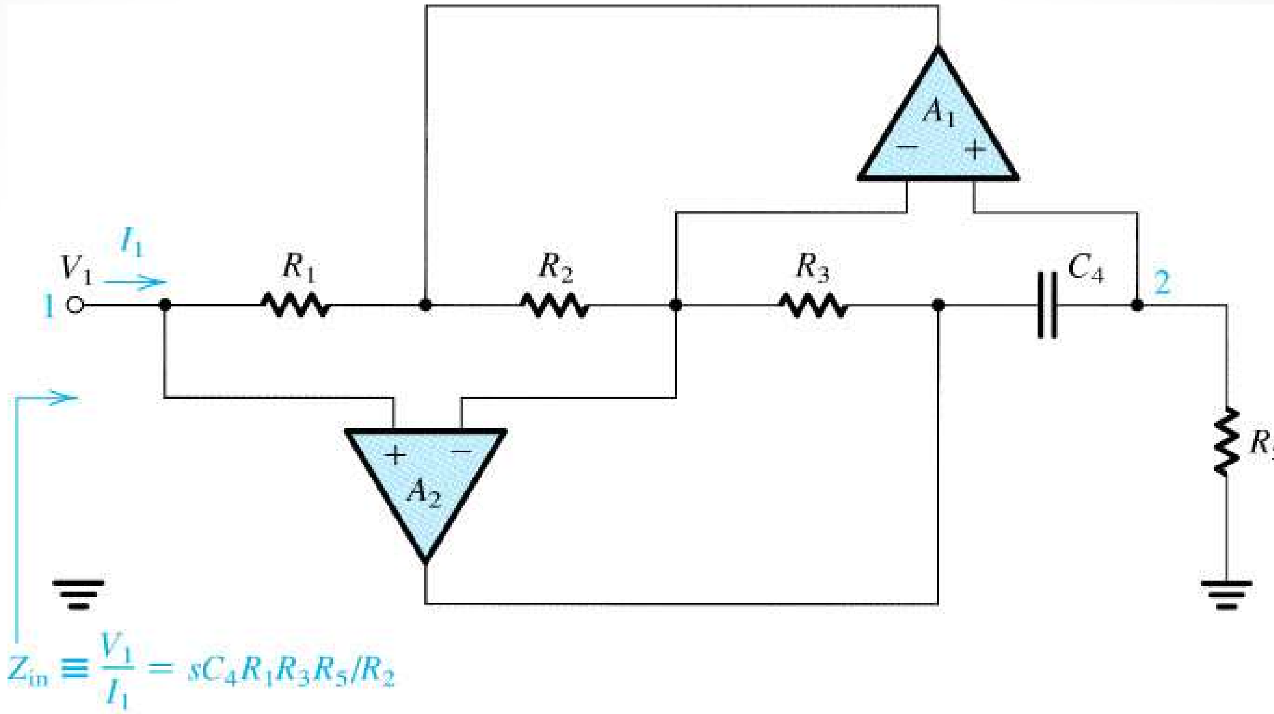
If we were to find the all-pass realization with a flat gain of 0.5, we'll get

$$T(s) = 0.5 - \frac{s(\omega_0 / Q)}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

This function can be realized using a voltage divider with a transmission ratio of 0.5 together with the bandpass circuit. But the circuit does not have a common ground terminal between the input and the output.

Second-Order Active Filters Based on Inductor Replacement

The Antoniou Induction Simulation Circuit



(a)

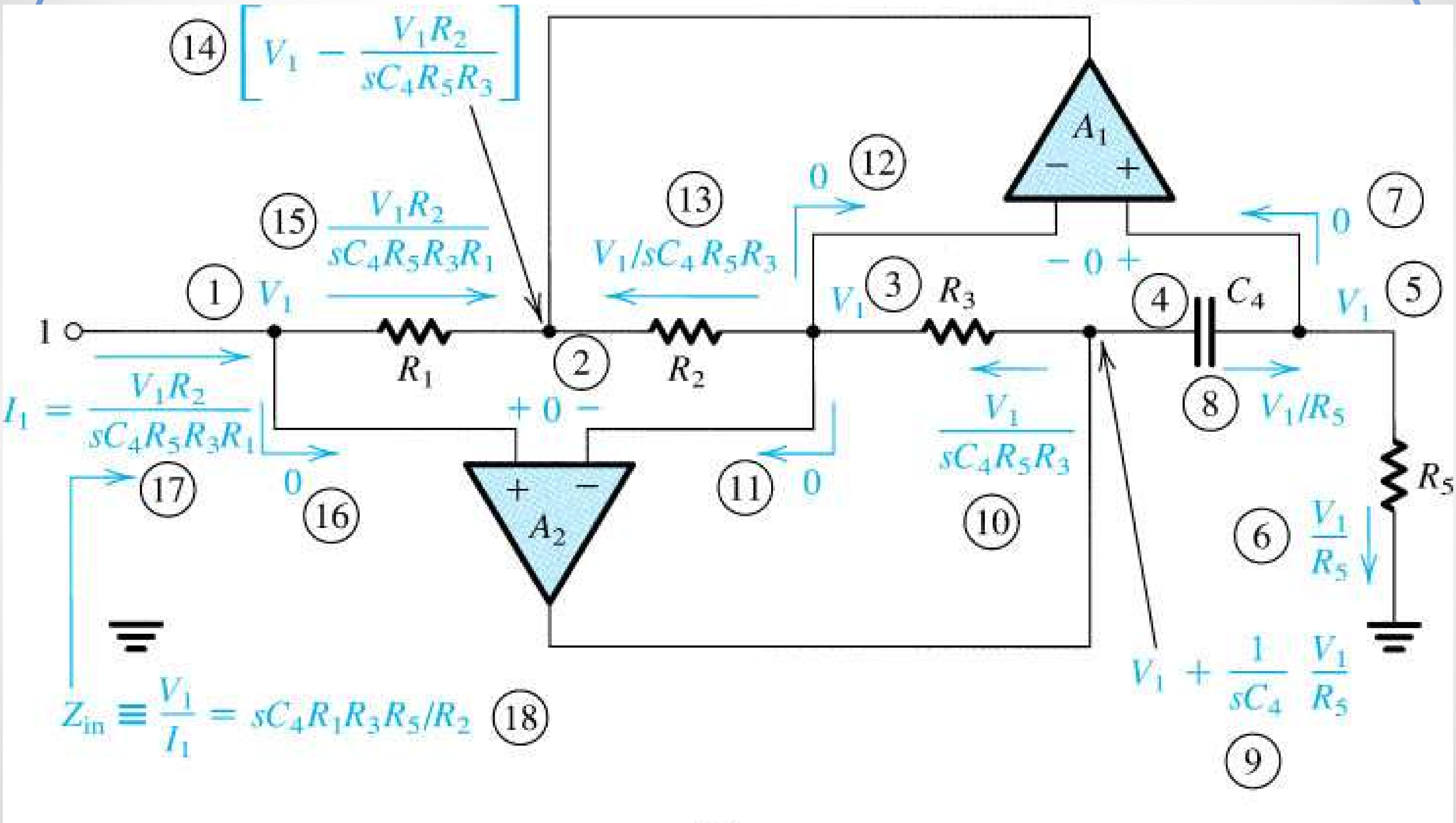
This ckt makes use of Op-amp RC as a substitute for L. It was invented by a Canadian engineer named Andreas Antoniou. Input impedance can be shown to be

$$Z_{in} = V_1 / I_1 = sC_4R_1R_3R_5 / R_2$$

which is that of an inductance given by

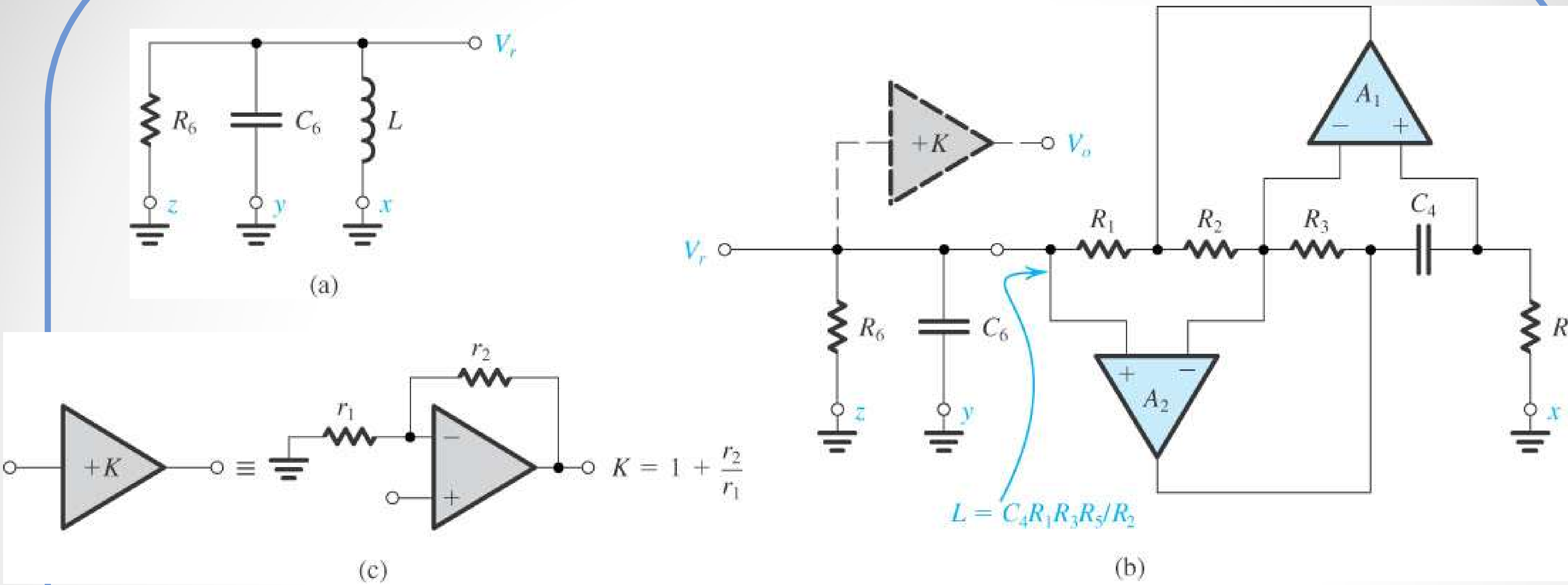
$$\boxed{L = C_4R_1R_3R_5 / R_2}$$

Analysis Steps



(b)

The Op Amp-RC Resonator (Fig. 12.21)



We replace the inductor L in ckt (a) with a simulated inductance realized by the Antoniou circuit. The result is ckt (b). It has a pole frequency at

$$\boxed{\omega_0 = 1/\sqrt{LC_6} = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}}$$

and

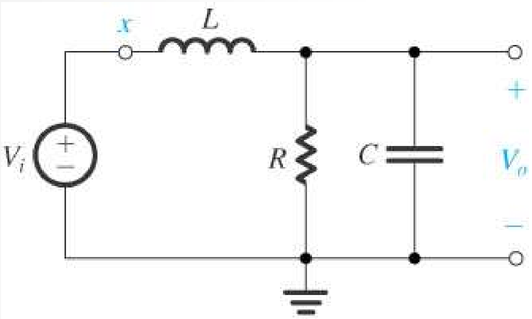
$$\boxed{Q = \omega_0 C_6 R_6 = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}}$$

If one selects $C_4 = C_6 = C$ and $R_1 = R_2 = R_3 = R_5 = R$, it will result in

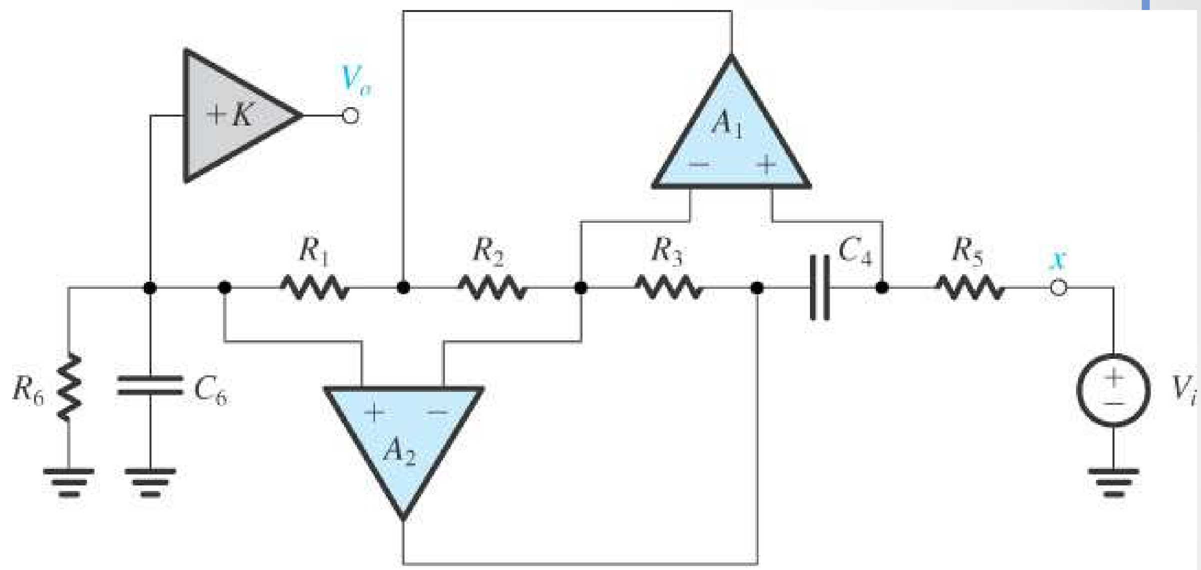
$\omega_0 = 1/CR$ and $Q = R_6/R$

Realization of the Various Filter Types

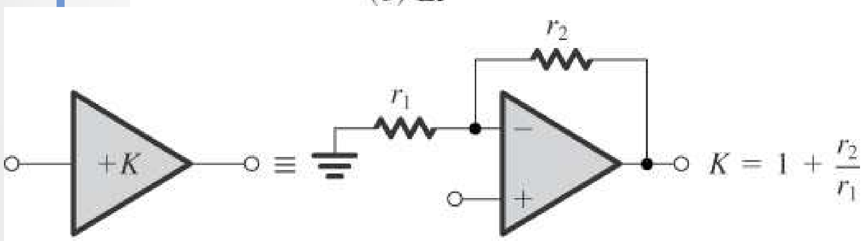
The op amp RC resonator can be used to generate ckt realizations for the various 2nd order filter functions by replacing L with the Antoniou ckt.



(b) LP



(a) LP



(c)

Connecting a load directly to output terminal may change the filter characteristics. This is why we need to use a buffer which is an amplifier of gain K. Not only does the amplifier K buffer the output of the buffer, but it also allows the designer to set the filter gain to any desired value by appropriately selecting the value of K.

Various examples are shown on the next page.

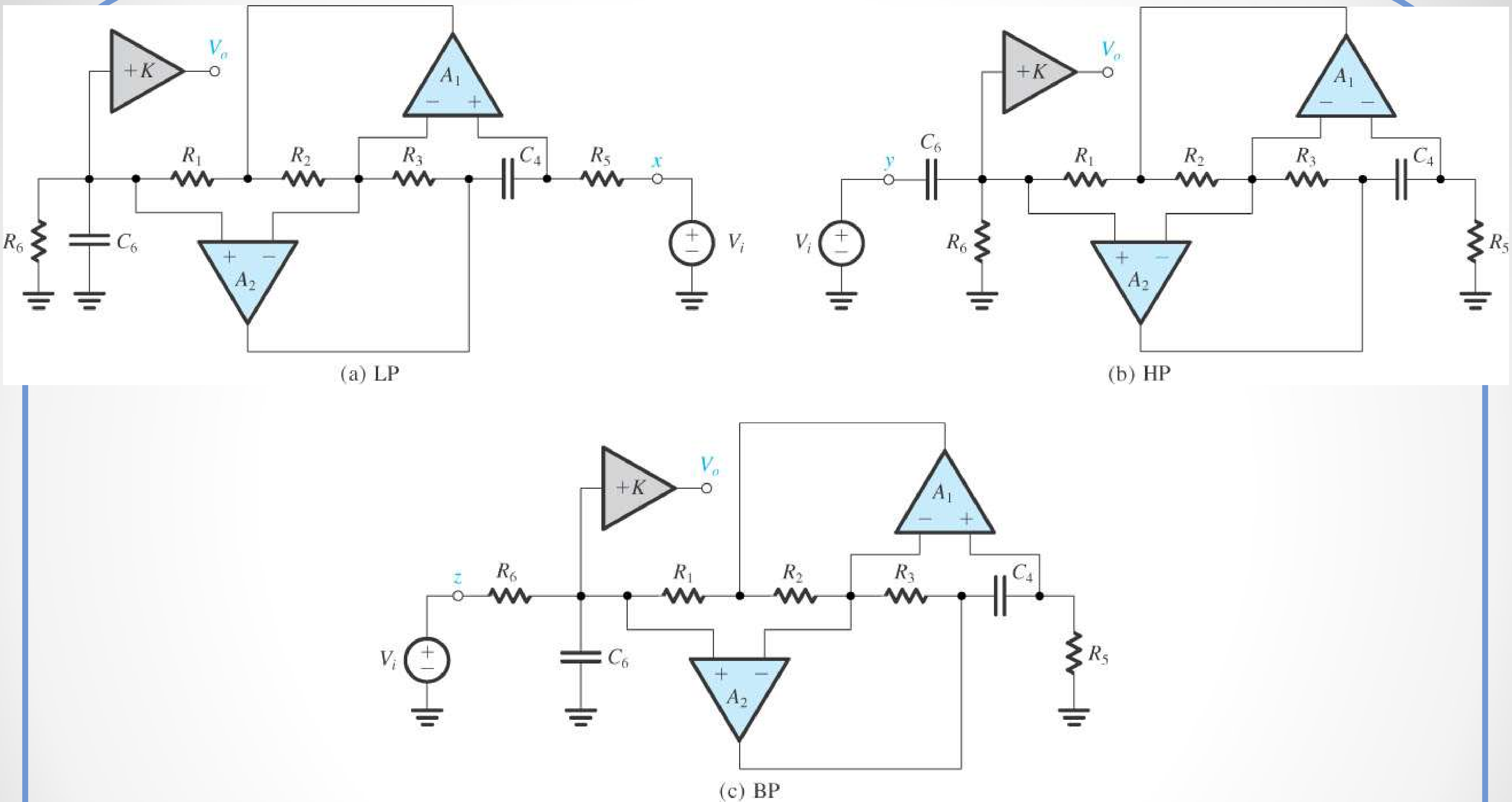


Figure 12.22 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 12.21(b): (a) LP, (b) HP, (c) BP,

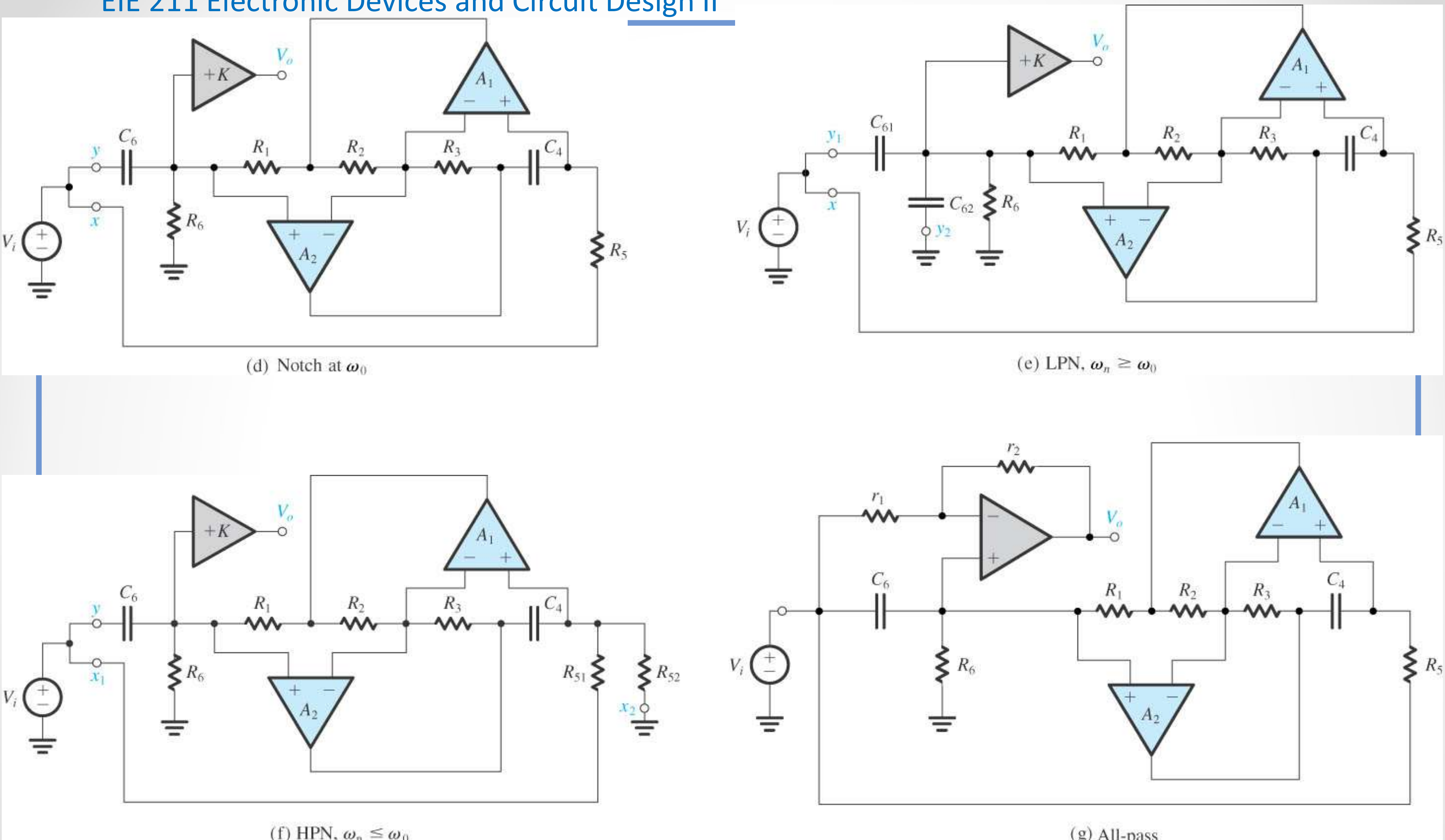


Figure 12.22 *(Continued)* (d) notch at w_0, (e) LPN, w_n ³ w_0, (f) HPN, w_n £ w_0, and (g) all pass. The circuits are based on the LCR circuits in Fig. 12.18. Design equations are given in Table 12.1.

Table 12.1 (also continues on the next slide…)

Circuit	Transfer Function and Other Parameters	Design Equations
Resonator Fig. 12.21(b)	$\omega_0 = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 12.22(a)	$T(s) = \dfrac{KR_2/C_4C_6R_1R_3R_5}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	$K =$ DC gain
High-pass (HP) Fig. 12.22(b)	$T(s) = \dfrac{Ks^2}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	$K =$ High-frequency gain
Bandpass (BP) Fig. 12.22(c)	$T(s) = \dfrac{Ks/C_6R_6}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	$K =$ Center-frequency gain
Regular notch (N) Fig. 12.22(d)	$T(s) = \dfrac{K[s^2 + (R_2/C_4C_6R_1R_3R_5)]}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	$K =$ Low- and high-frequency gain

Low-pass notch (LPN) Fig. 12.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	$K =$ DC gain $C_{61}+C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 12.22(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\dfrac{C_6}{C_4}\dfrac{R_2}{R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$	$K =$ High-frequency gain $\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5/[1-(\omega_n/\omega_0)^2]$
All-pass (AP) Fig. 12.22(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1/r_2) \quad$ Flat gain $= 1$	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$

Example: Design a 2nd order high pass active filter based on the inductor replacement

Second Order Active Filters based on the Two-Integrator-Loop Topology

To derive the **two-integrator loop biquadratic circuit**, or **biquad**, consider the high-pass transfer function

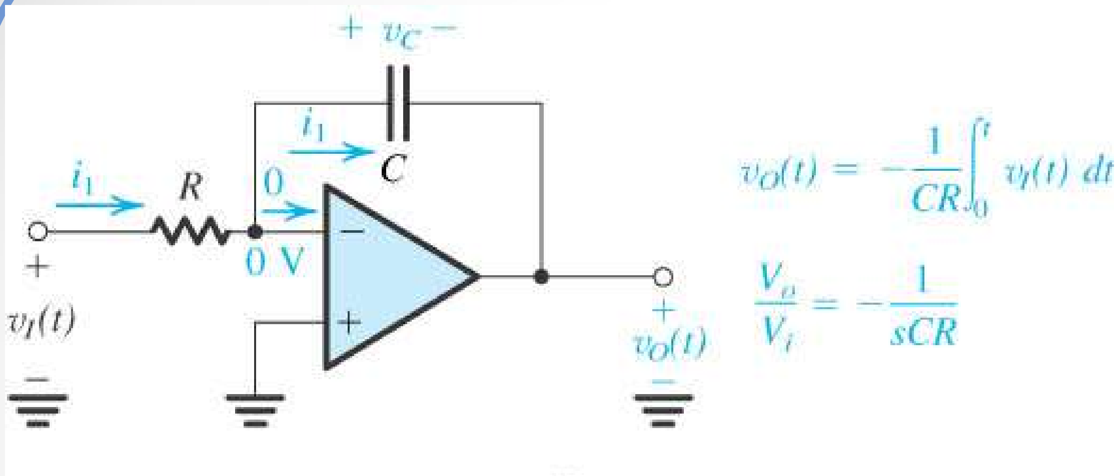
$$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_o}{s}V_{hp}\right) + \left(\frac{\omega_o^2}{s^2}V_{hp}\right) = KV_i$$

$$\boxed{V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_o}{s}V_{hp} - \frac{\omega_o^2}{s^2}V_{hp}}$$

We observe that the signal $(\omega_o/s)V_{hp}$ can be obtained by passing V_{hp} through an integrator with a time constant equal to $1/\omega_o$. Furthermore, passing the resulting signal through another identical integrator results in the signal $(\omega_o^2/s^2)V_{hp}$. The block diagram on the next page shows a two-integrator arrangement.

…Let's recall from lecture 3: The Inverting Integrator



(a)

$$\boxed{v_C(t) = V_C + \frac{1}{C}\int_0^t i_1(t)\,dt}$$

where V_C is the initial voltage on C at t = 0.

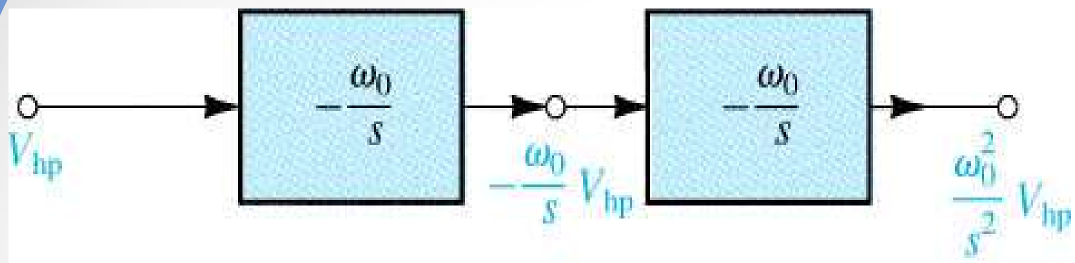
RC is called the integrator time-constant. Since there is a negative sign at the output voltage, this integrator ckt is said to be an inverting integrator or a Miller integrator.

The operation can be described alternatively in the freq domain by substituting $Z_1(s) = R$ and $Z_2(s) = 1/sC$ to obtain the transfer function:

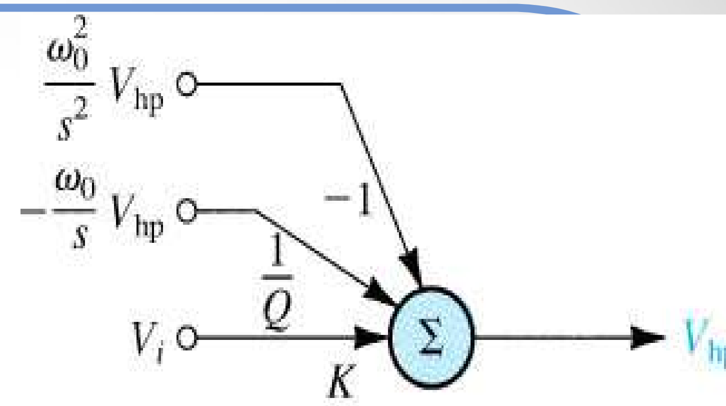
$$\boxed{\frac{V_O(s)}{V_i(s)} = -\frac{1}{sCR}}$$

Thus the integrator function has magnitude

$$\left|\frac{V_O(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\omega CR}$$ and phase $\phi = +90^O$



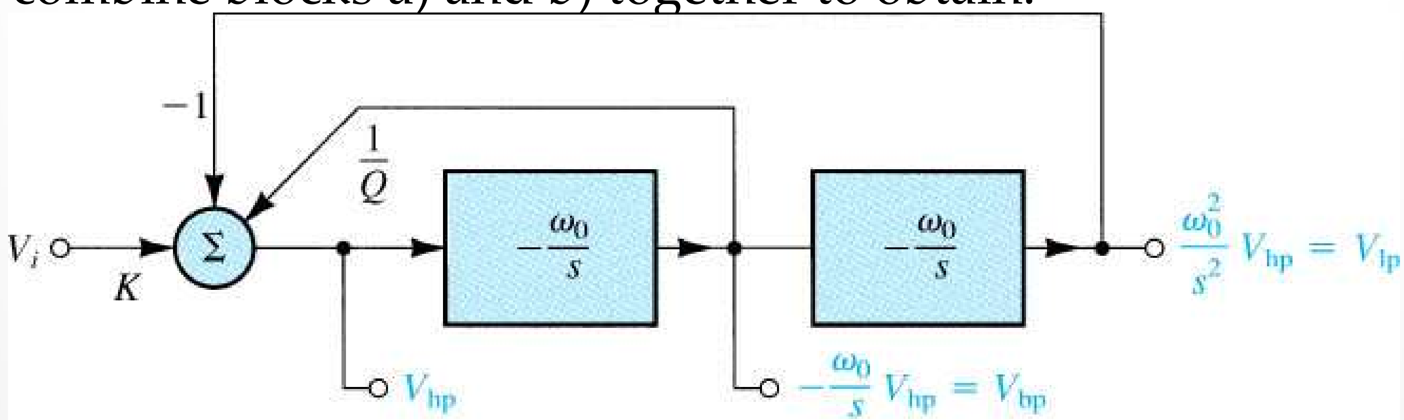
(a)



(b)

From $$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_o}{s}V_{hp} - \frac{\omega_o^2}{s^2}V_{hp}$$

It suggests that V_{hp} can be obtained by using the weighted summer in Fig b.
Now we combine blocks a) and b) together to obtain:



(c)

If we try to look at the Fig c. more carefully, we'll find that

$$\boxed{T_{hp} = \frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_o/Q) + \omega_o^2}}$$

And the signal at the output of the first integrator is $-(\omega_o/s)V_{hp}$, which is a band-pass function, with the center-frequency gain of $-KQ$,

$$\frac{(-\omega_o/s)V_{hp}}{V_i} = \boxed{-\frac{K\omega_o s}{s^2 + s(\omega_o/Q) + \omega_o^2} = T_{bp}(s)}$$

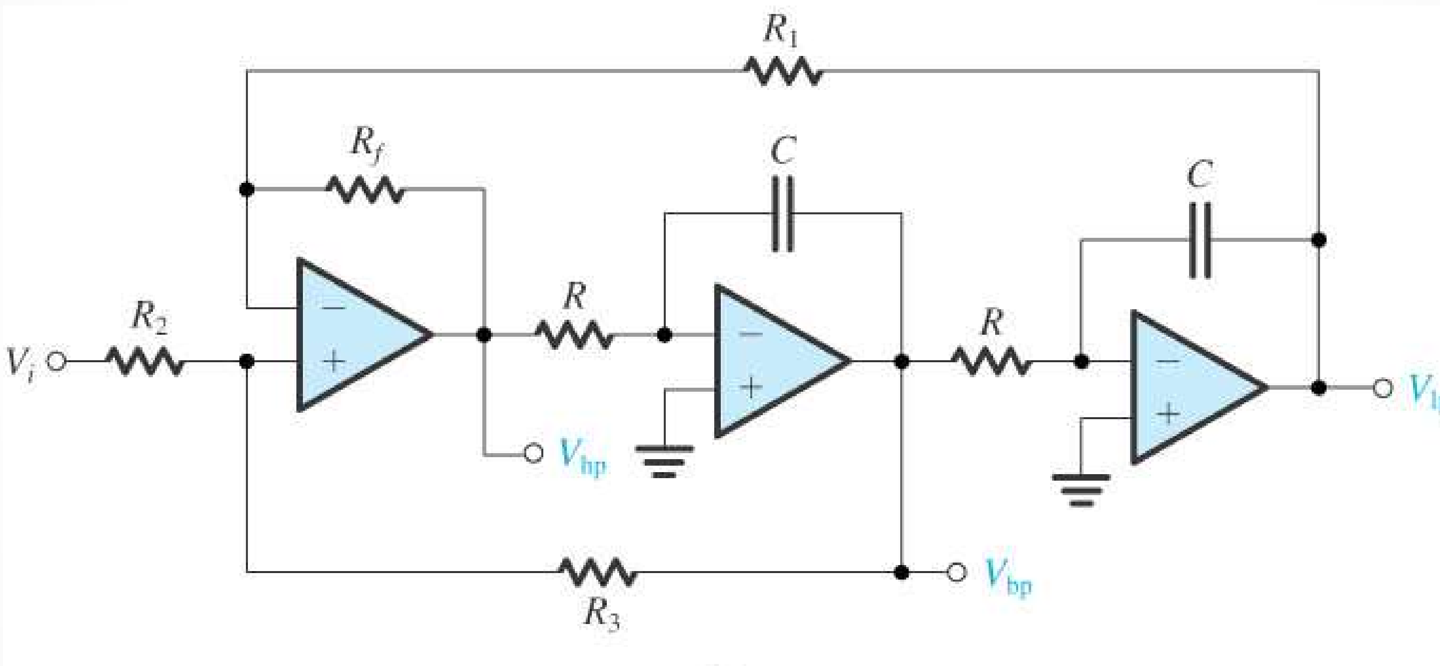
Therefore, the signal at the output of the first integrator is labeled V_{bp}. In, a similar way, the signal at the output of the second integrator is $(\omega_o^2/s^2)V_{hp}$, which is a low-pass function,

$$\frac{(\omega_o^2/s^2)V_{hp}}{V_i} = \boxed{\frac{K\omega_o^2}{s^2 + s(\omega_o/Q) + \omega_o^2} = T_{lp}(s)}$$

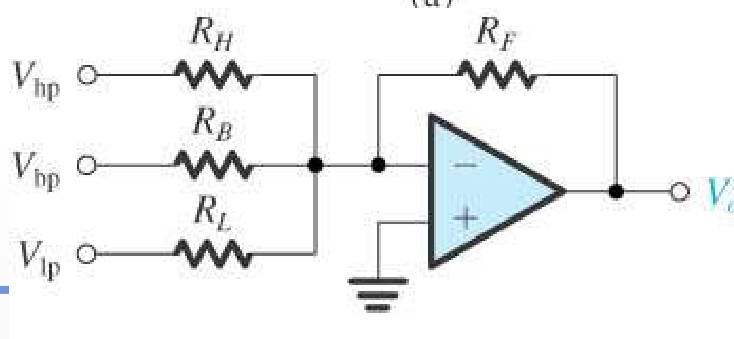
Thus, the output of the second integrator is labeled V_{lp}. Note that the dc gain of the low-pass filter is equal to K. Hence, the 2-integrator-loop biquad realizes 3 basic 2nd order filtering functions simultaneously, that's why it's called a universal active filter.

Circuit Implementation

We replace each integrator with a Miller integrator circuit having CR = 1/ωo and we replace the summer block with an op amp summing circuit that is capable of assigning both positive and negative weights to its inputs. The resulting ckt, known as the **Kerwin-Huelsman-Newcomb** or **KHN biquad**.



(a)



(b)

We can express the output of the summer V_{hp} in terms of its inputs, $V_{bp} = -(\omega_o/s)V_{hp}$ and $V_{lp} = (\omega_o^2/s^2)V_{hp}$, as

$$\boxed{V_{hp} = \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right)}$$

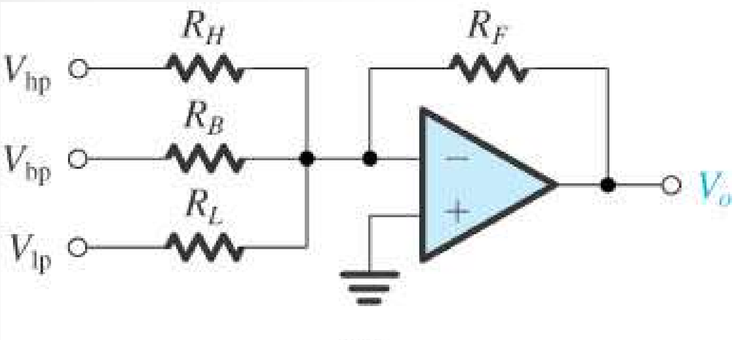
To determine all the parameters, we need to compare it to the original eq:

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

We can match them up, term by term, and will get:

$$R_f/R_1 = 1 \qquad R_3/R_2 = 2Q-1 \qquad K = 2-(1/Q)$$

The KHN biquad can be used to realize notch and all-pass functions by summing weighted versions of the three outputs, LP, BP, and HP as shown.



(b)

$$V_o = -\left(\frac{R_F}{R_H}V_{hp} + \frac{R_F}{R_B}V_{bp} + \frac{R_F}{R_L}V_{lp}\right)$$
$$= -V_i\left(\frac{R_F}{R_H}T_{hp} + \frac{R_F}{R_B}T_{bp} + \frac{R_F}{R_L}T_{lp}\right)$$

Substitute T_{hp}, T_{bp} and T_{lp} that we found previously, we'll get the overall transfer function

$$\boxed{\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}}$$

from which we can see that different transmission zeros can be obtained by the appropriate selection of the values of the summing resistors. For instance, a notch is obtained by selection $R_B = \infty$ and

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

Reference

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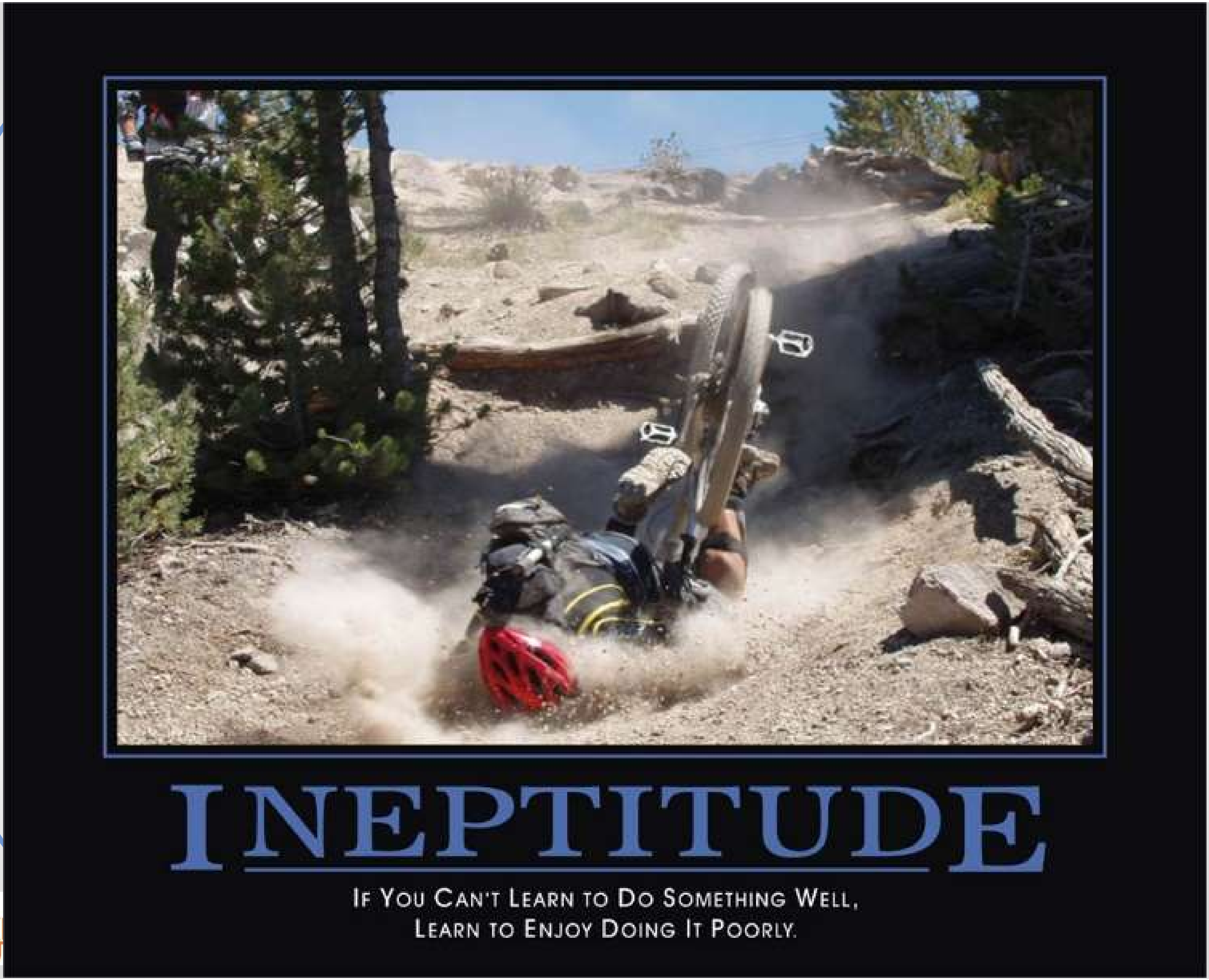
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Learn to Enjoy Doing It Poorly.


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