ENE 104
Electric Circuit Theory

Lecture 03:
Basic Nodal and Mesh Analysis

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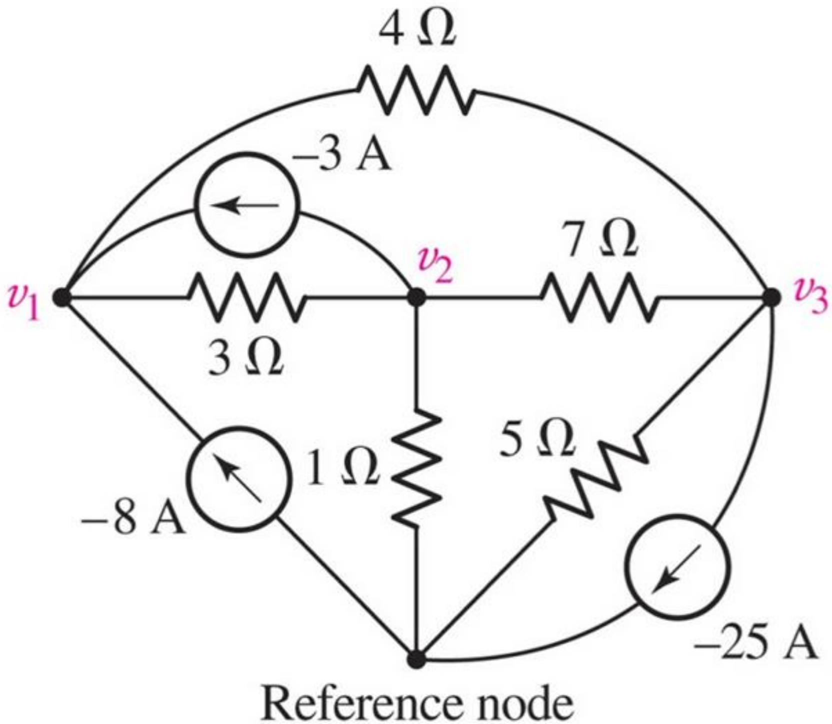
Objectives :

- Implementation of the **nodal analysis**
- Implementation of the **mesh analysis**
- **Supernodes** and **Supermeshes**
- Basic computer-aided circuit analysis

Example: 4.2

find the nodes voltages:



A KCL equation for node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

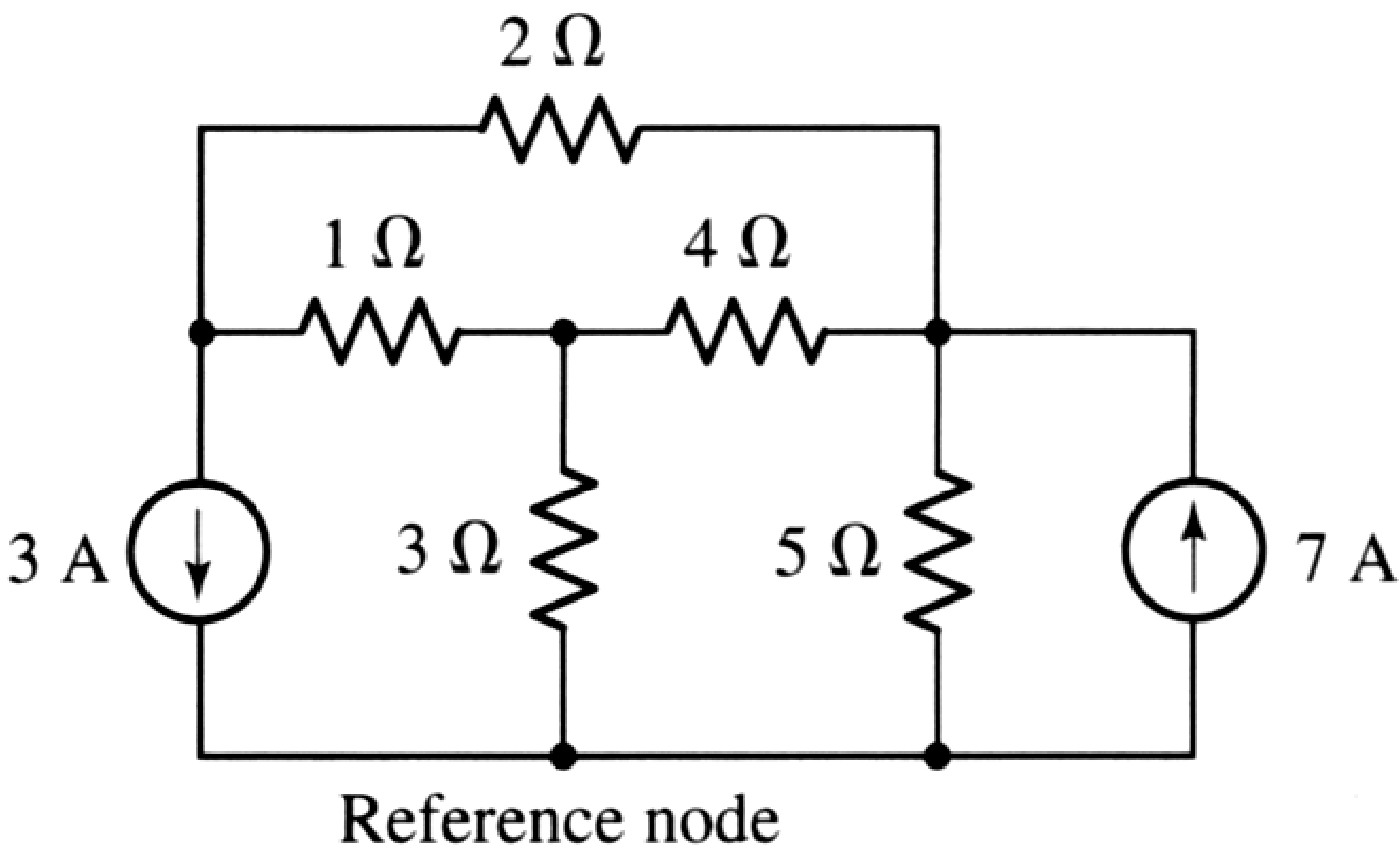
Node 2:

$$\frac{v_2 - v_3}{7} + \frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) = 0$$

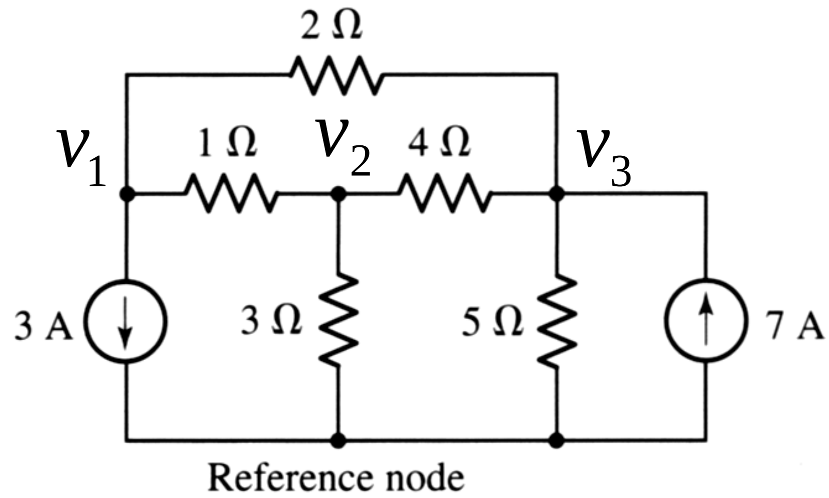
Node 3:

$$\frac{v_3 - v_2}{7} + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

Practice: 4.2



Practice: 4.2



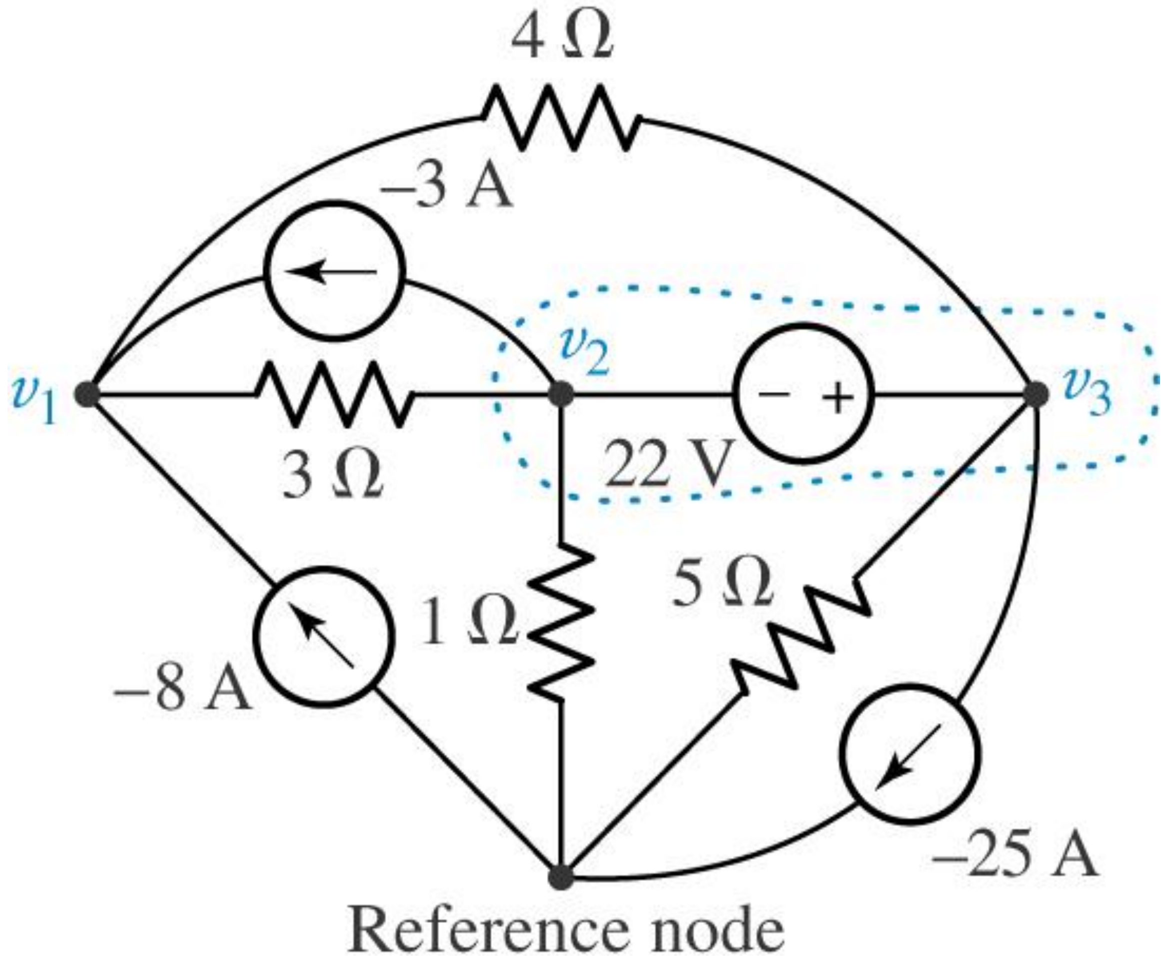
Node 1: $$3+\frac{v_1-v_3}{2\Omega}+\frac{v_1-v_2}{1\Omega}=0$$

Node 2: $$\frac{v_2-v_1}{1\Omega}+\frac{v_2}{3\Omega}+\frac{v_2-v_3}{4\Omega}=0$$

Node 3: $$-7+\frac{v_3-v_1}{2\Omega}+\frac{v_3-v_2}{4\Omega}+\frac{v_3}{5\Omega}=0$$

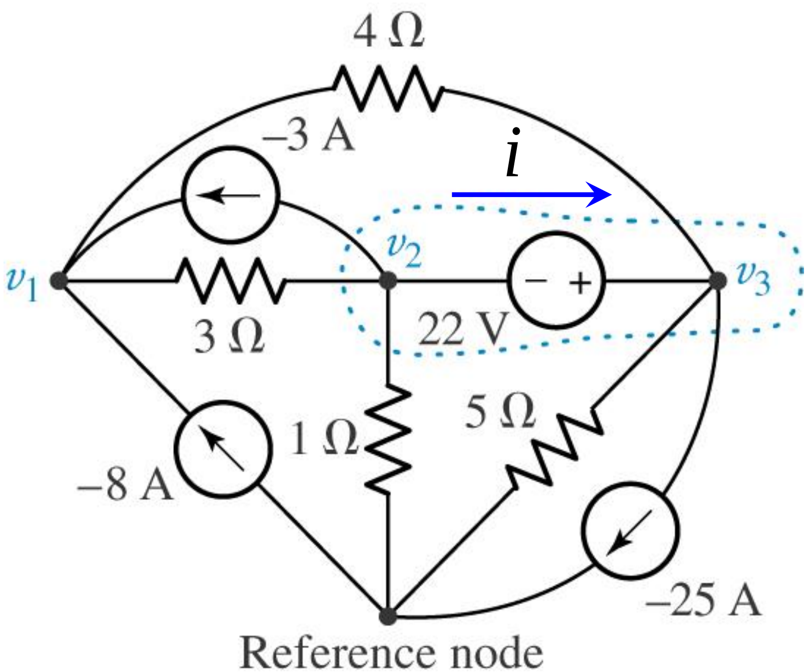
The Supernode:

find the nodes voltages:



The Supernode:

find the nodes voltages:



A KCL equation for

Node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

Node 2:

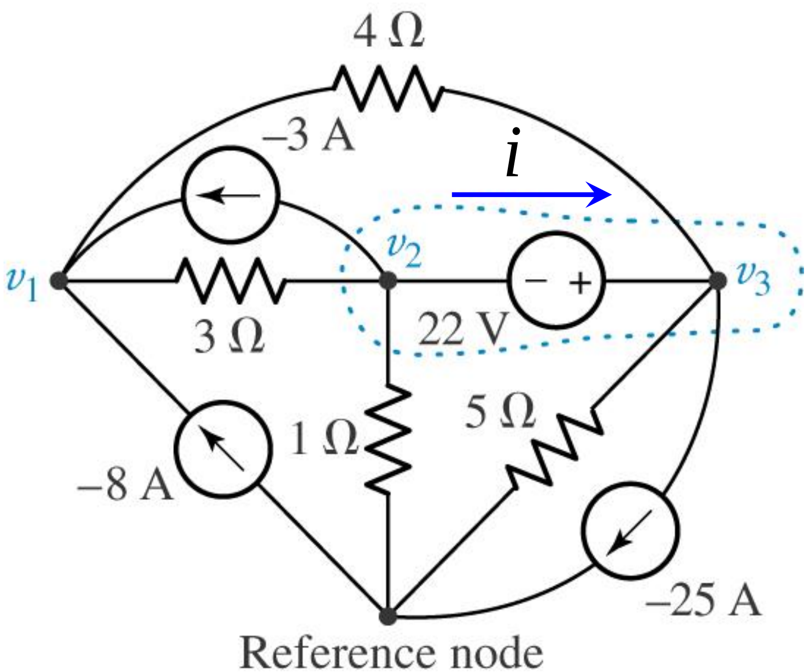
$$i + \frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) = 0$$

Node 3:

$$-i + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

The Supernode:

find the nodes voltages:



A KCL equation for Node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

The Supernode (Node 2+3):

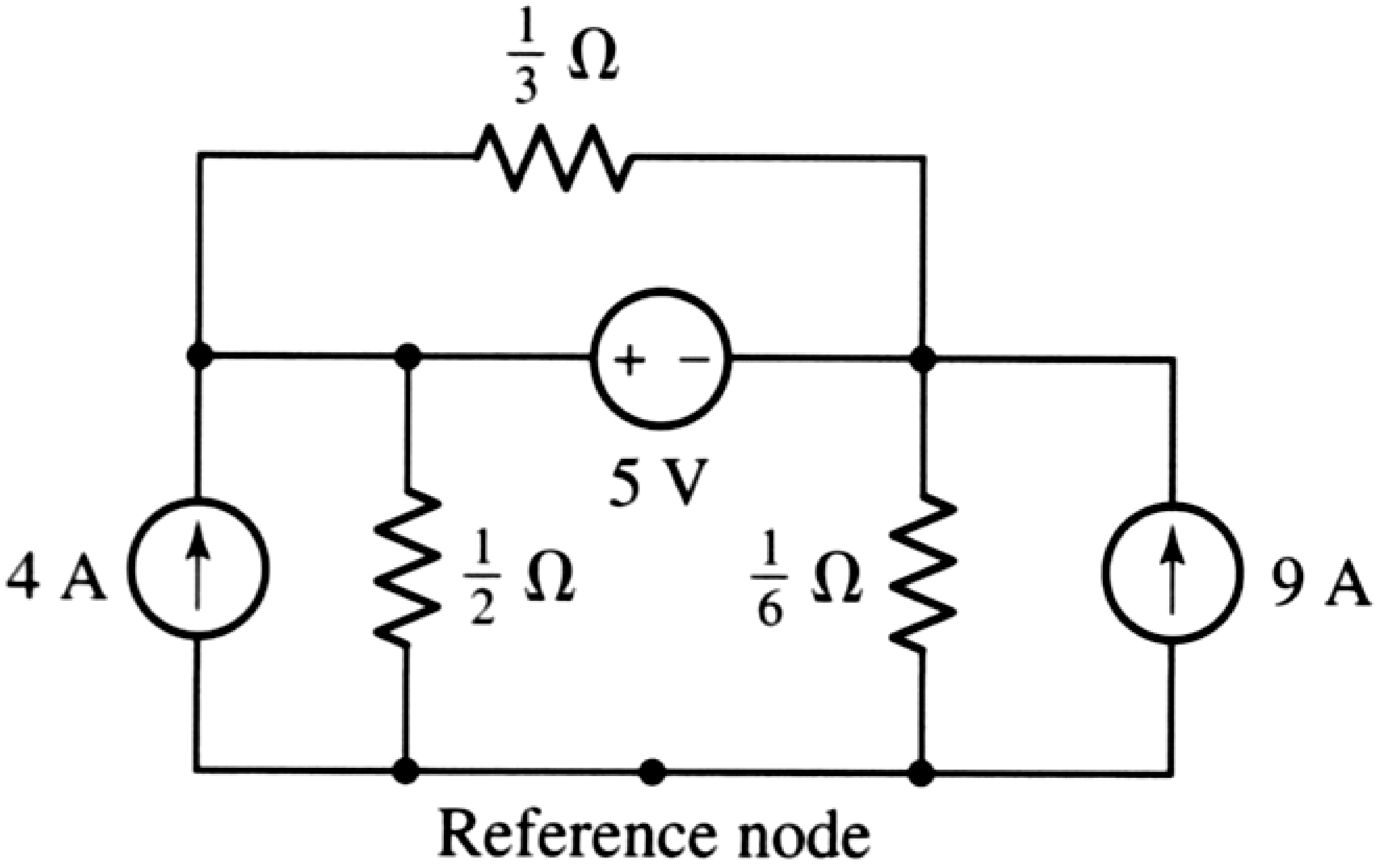
$$\frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

And one additional equation:

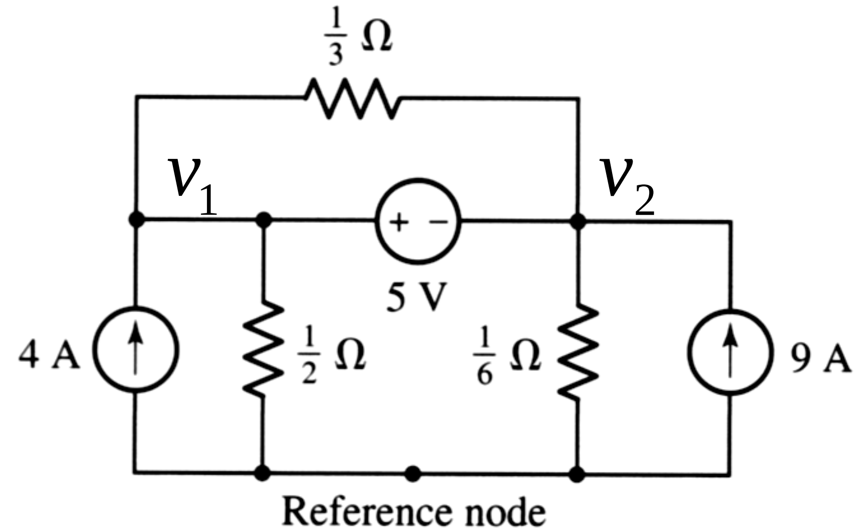
$$v_3 - v_2 = 22$$

Practice: 4.3

Compute the voltage across each current source



Practice: 4.3



Node 1:

$$-4+\frac{v_1-v_2}{\frac{1}{3}\Omega}+i+\frac{v_1}{\frac{1}{2}\Omega}=0$$

Node 2:

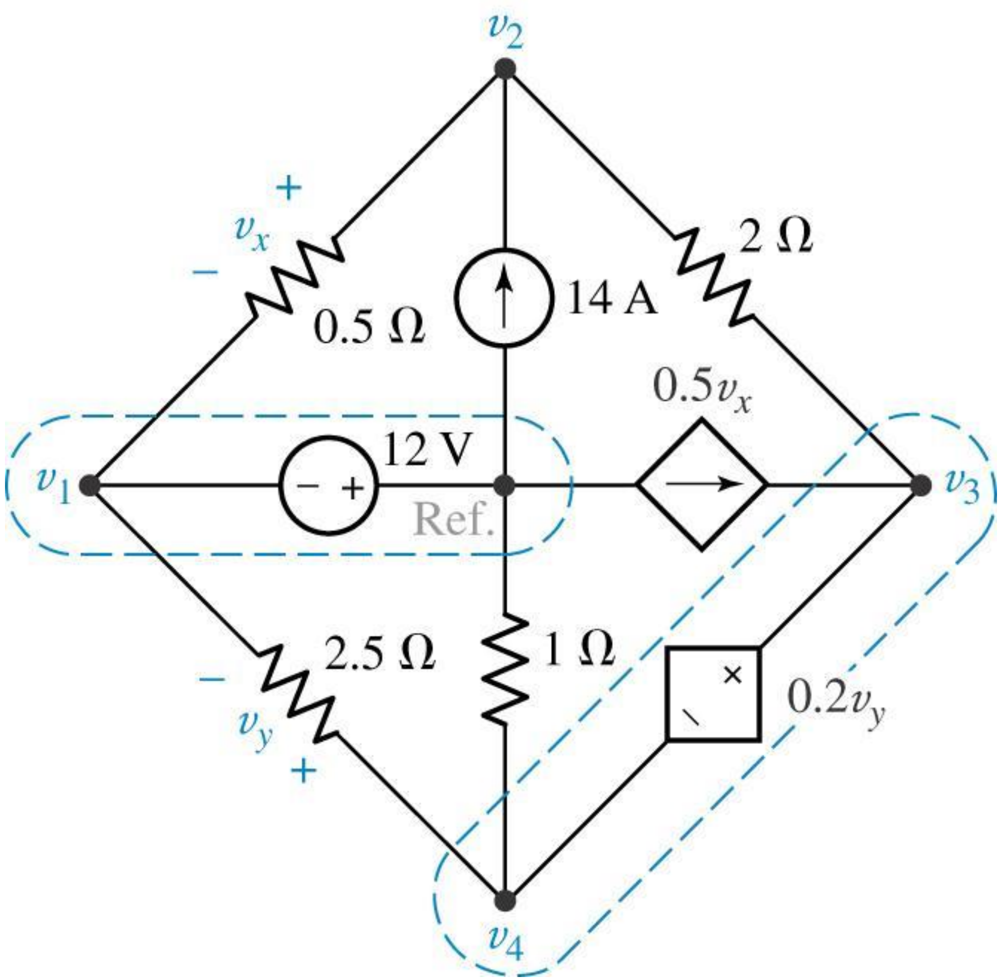
$$\frac{v_2-v_1}{\frac{1}{3}\Omega}-i+\frac{v_2}{\frac{1}{6}\Omega}-9=0$$

Supernode 1-2:

$$-4+\frac{v_1-v_2}{\frac{1}{3}\Omega}+\frac{v_1}{\frac{1}{2}\Omega}+\frac{v_2-v_1}{\frac{1}{3}\Omega}+\frac{v_2}{\frac{1}{6}\Omega}-9=0$$

Example: 4.4

Determine the node-to-reference voltages in the circuit below.



Example:

Determine the node-to-reference voltages in the circuit below.

Node 1:

$v_1 = -12\ \text{V}.$

Node 2:

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} - 14 = 0$$

The 3-4 supernode:

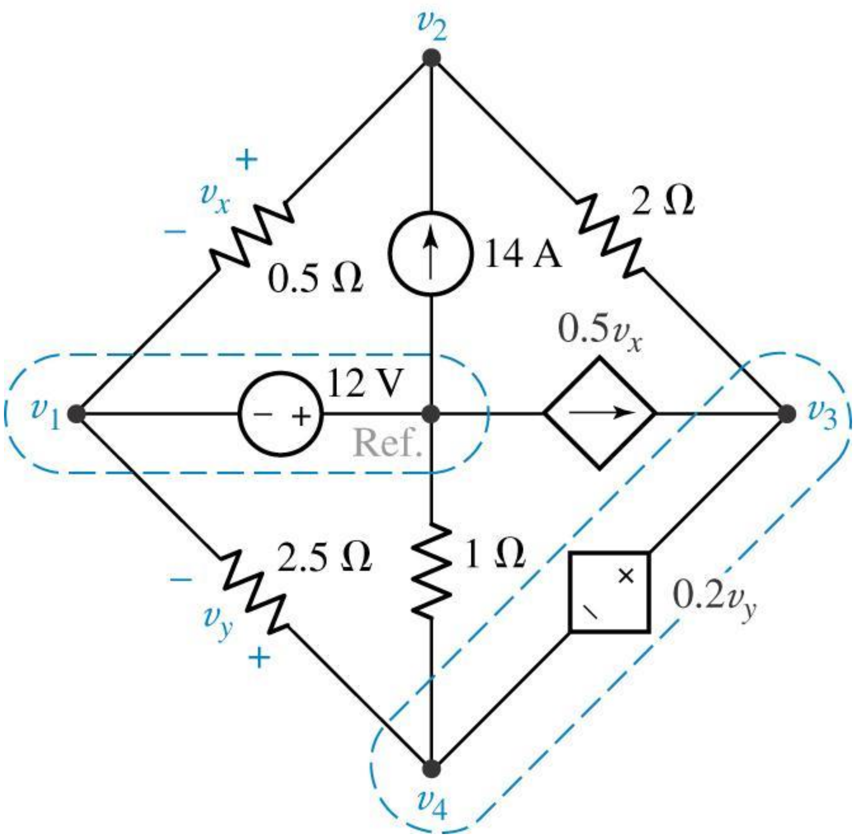
$$\frac{v_3 - v_2}{2} - 0.5v_x + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} = 0$$

$$\frac{v_3 - v_2}{2} - 0.5(v_2 - v_1) + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} = 0$$

Need an additional equation:

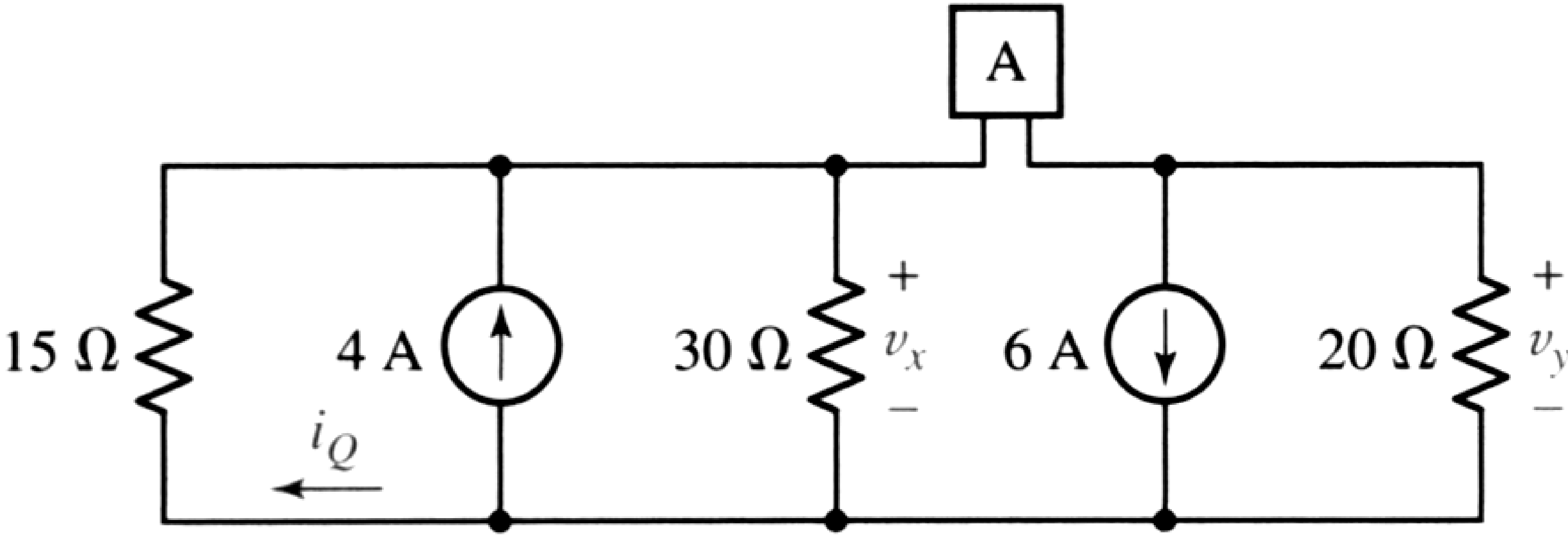
$v_3 - v_4 = 0.2v_y$

$v_3 - v_4 = 0.2(v_4 - v_1)$



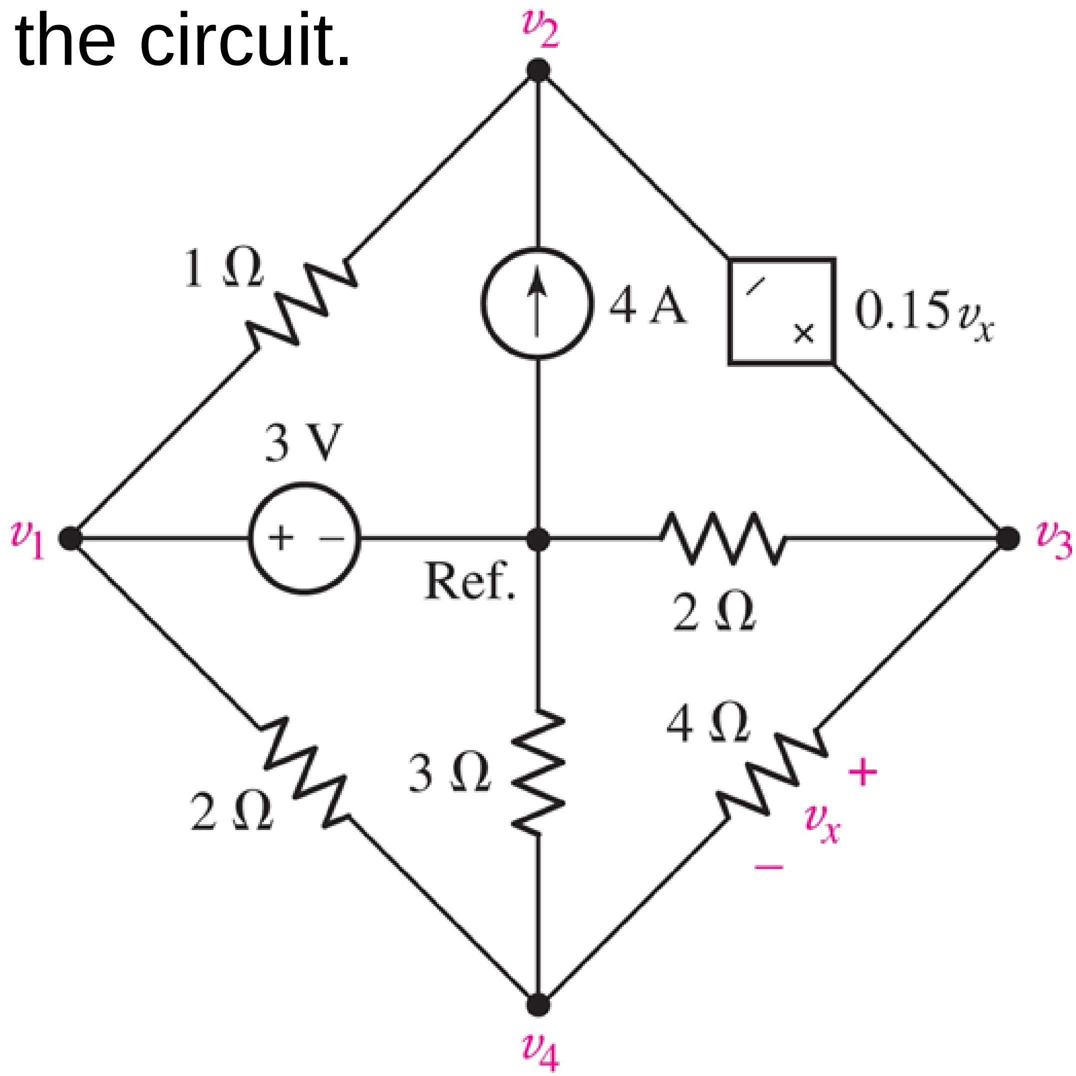
Practice: 4.4

Use nodal analysis to find v_x, if element A is (a) a 25 Ω; (b) a 5-A current source, arrow pointing right; (c) a 10-V voltage source, positive terminal on the right; (d) a short circuit



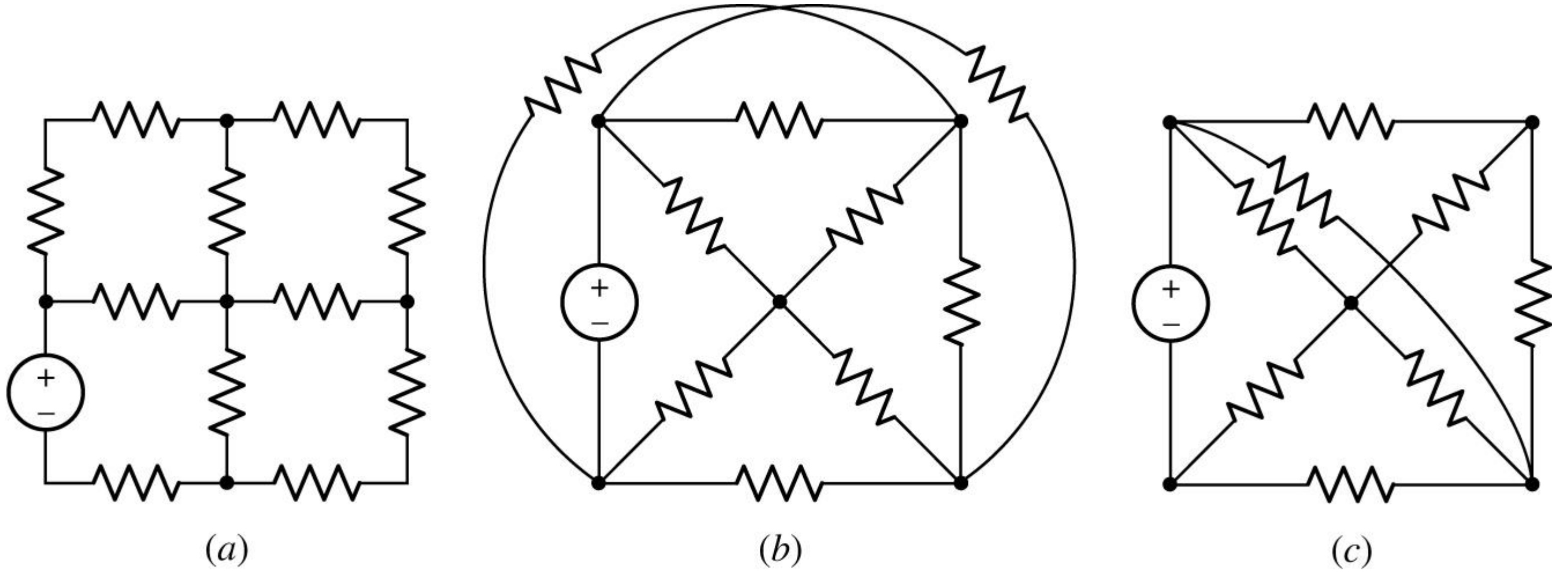
Practice:

Use nodal analysis to determine the nodal voltage in the circuit.



Mesh Analysis:

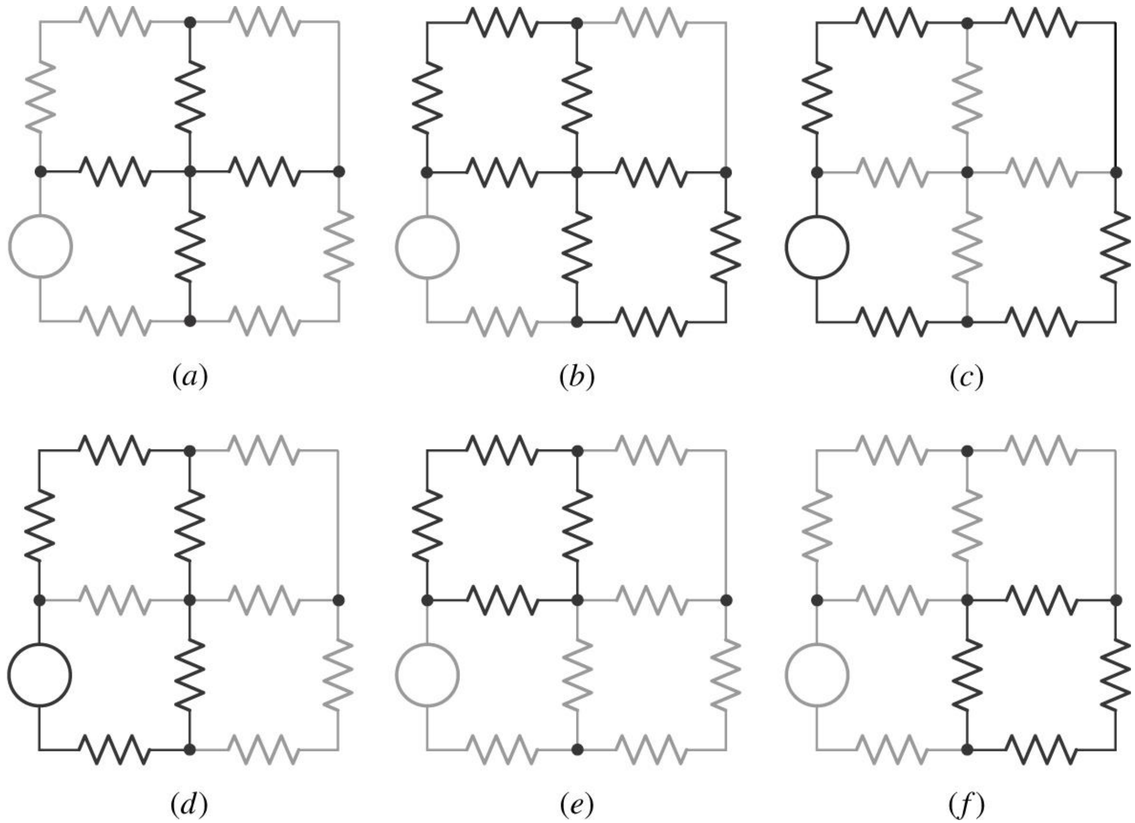
Mesh analysis is applicable only to those networks which are planar.



Examples of planar and nonplanar networks

Mesh is a loop that doesn't contain any other loops.

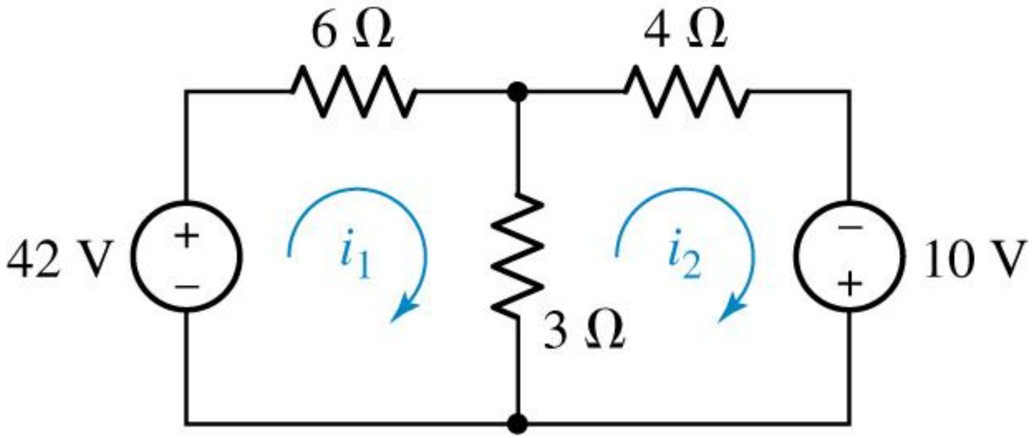
Path, Closed path, and Loop:



(a) The set of branches identified by the heavy lines is neither a path nor a loop. (b) The set of branches here is not a path, since it can be traversed only by passing through the central node twice. (c) This path is a loop but not a mesh, since it encloses other loops. (d) This path is also a loop but not a mesh. (e, f) Each of these paths is both a loop and a mesh.

Example: 4.6

Determine the two mesh currents, i_1 and i_2, in the circuit below.



For the left-hand mesh,

$$-42 + 6\,i_1 + 3\,(i_1 - i_2) = 0$$

For the right-hand mesh,

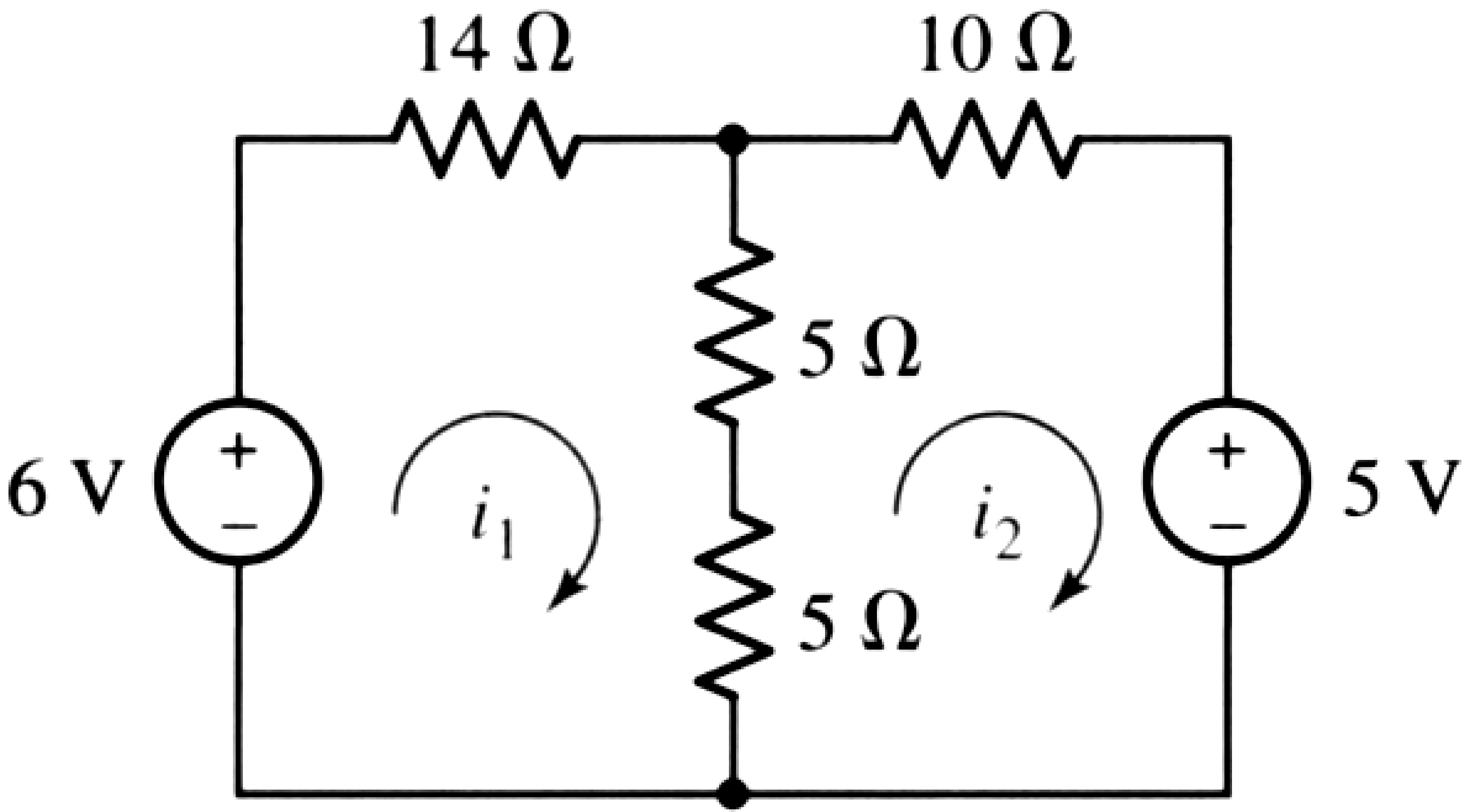
$$3\,(i_2 - i_1) + 4\,i_2 - 10 = 0$$

Solving, we find that i_1 = 6 A and i_2 = 4 A.

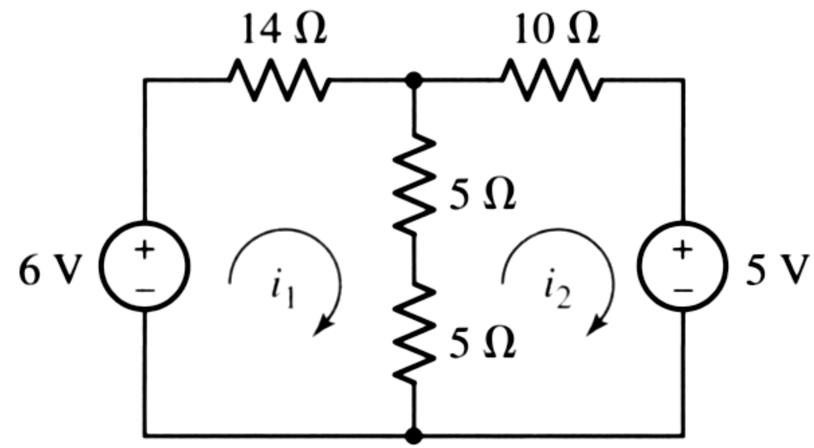
(The current flowing downward through the 3-Ω resistor is therefore $i_1 - i_2$ = 2 A.)

Practice: 4.5

Determine i_1 and i_2



Practice: 4.5

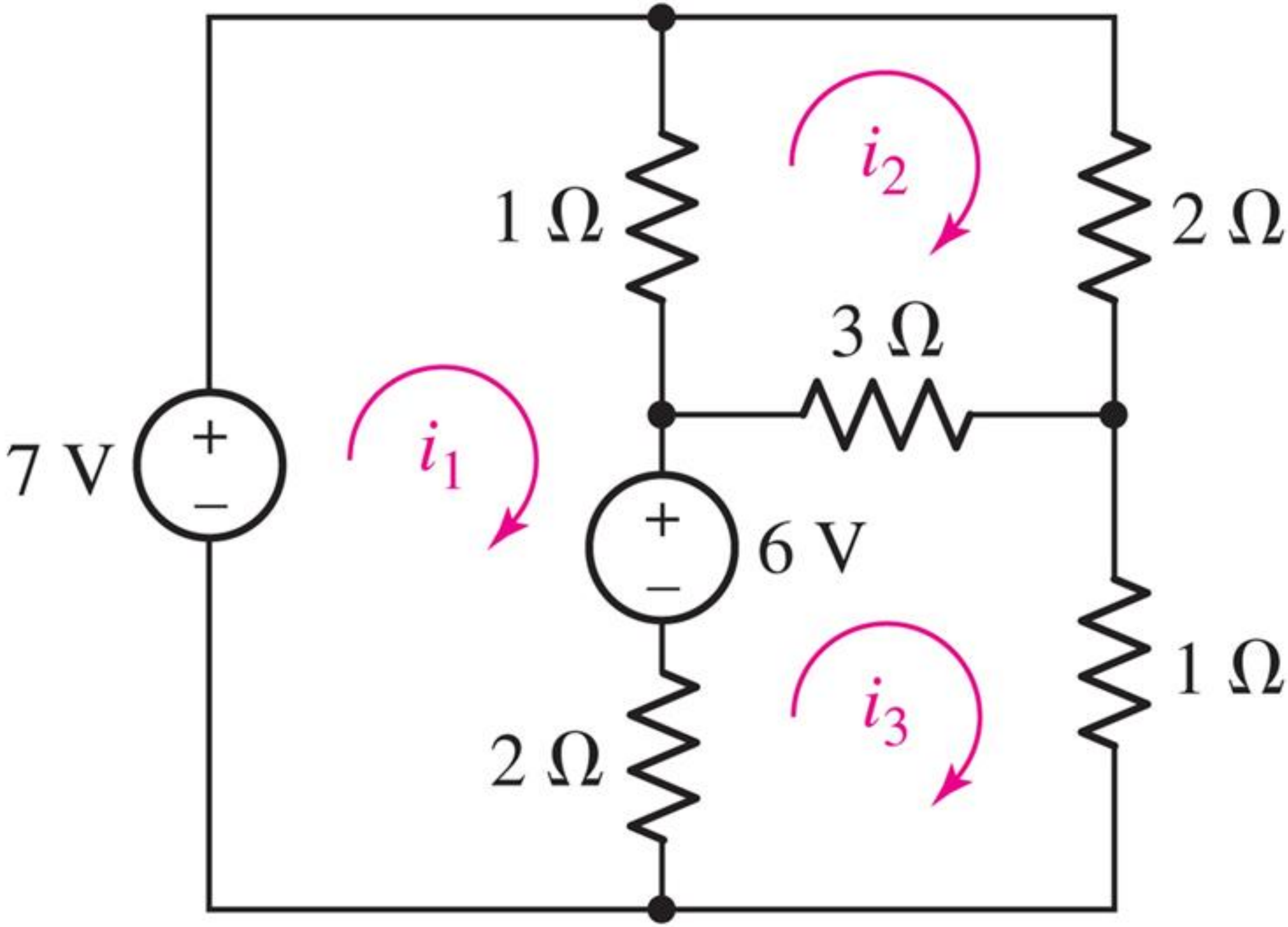


Mesh 1: $$-6+14i_1+10(i_1-i_2)=0$$

Mesh 2: $$10(i_2-i_1)+10i_2+5=0$$

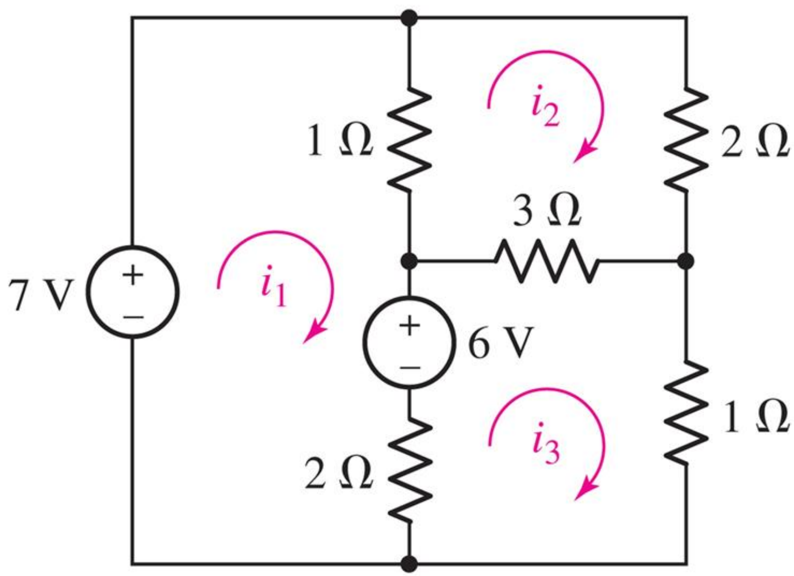
Example: 4.7

determine the three mesh currents



Example:

determine the three mesh currents



Mesh i_1:

$$-7+1(i_1-i_2)+6+2(i_1-i_3)=0$$

Mesh i_2:

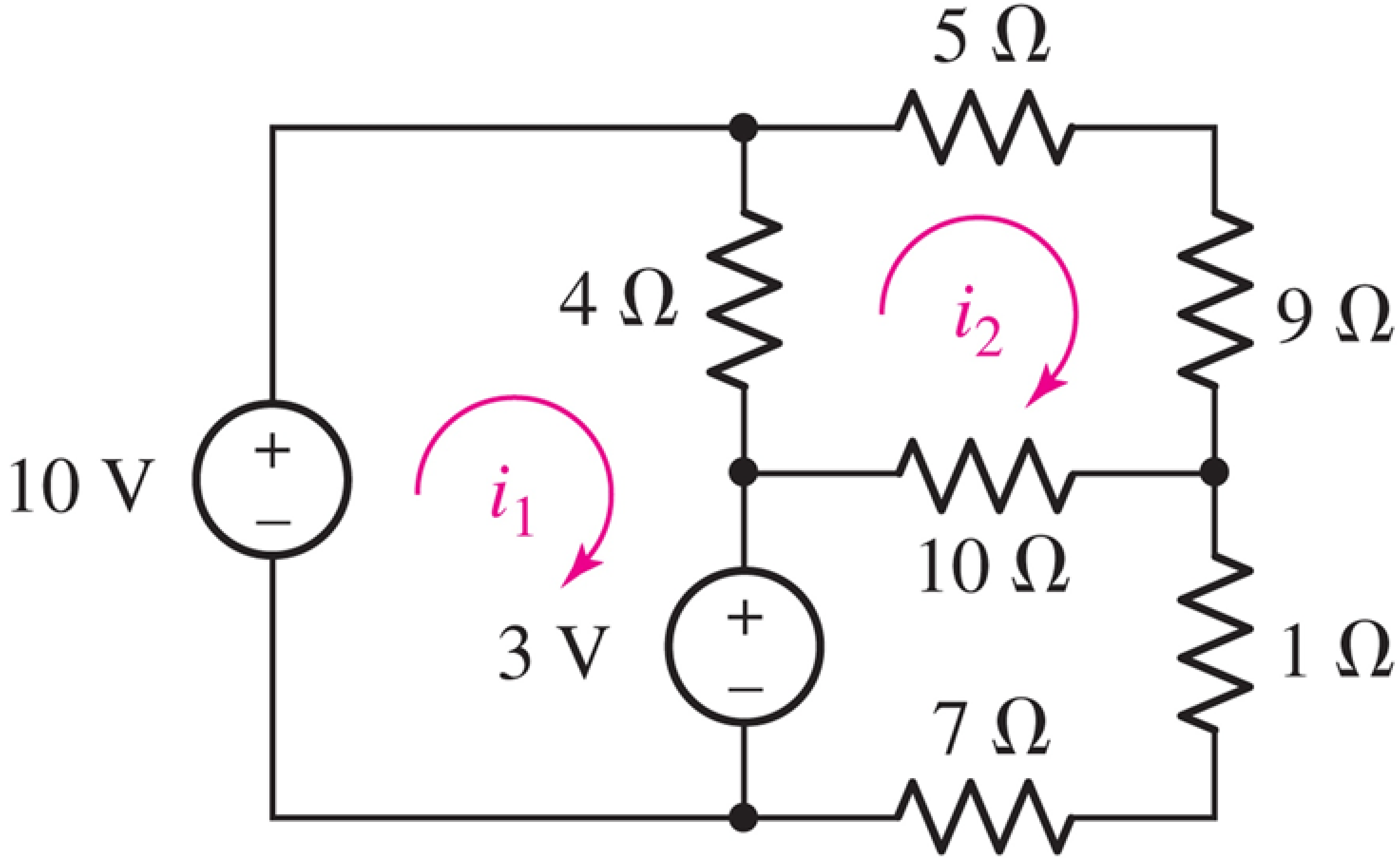
$$1(i_2-i_1)+2i_2+3(i_2-i_3)=0$$

Mesh i_3:

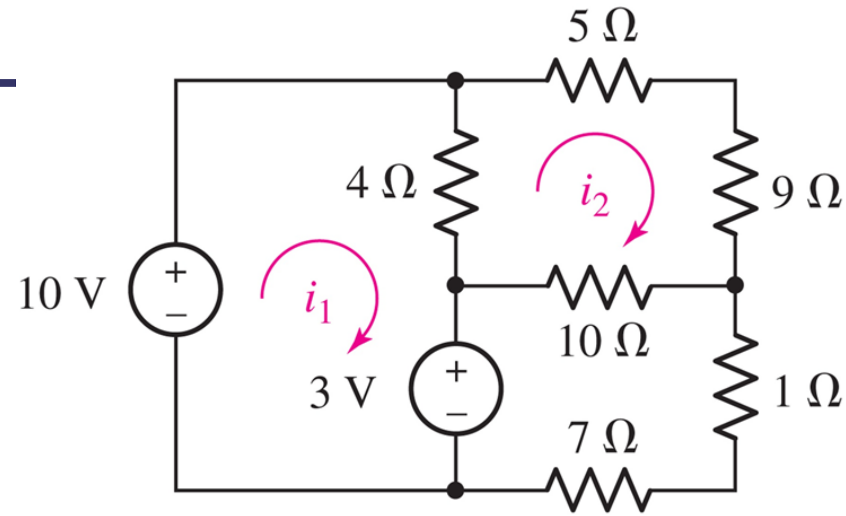
$$2(i_3-i_1)-6+3(i_3-i_2)+1i_3=0$$

Practice: 4.6

Determine i_1 and i_2



Practice: 4.6



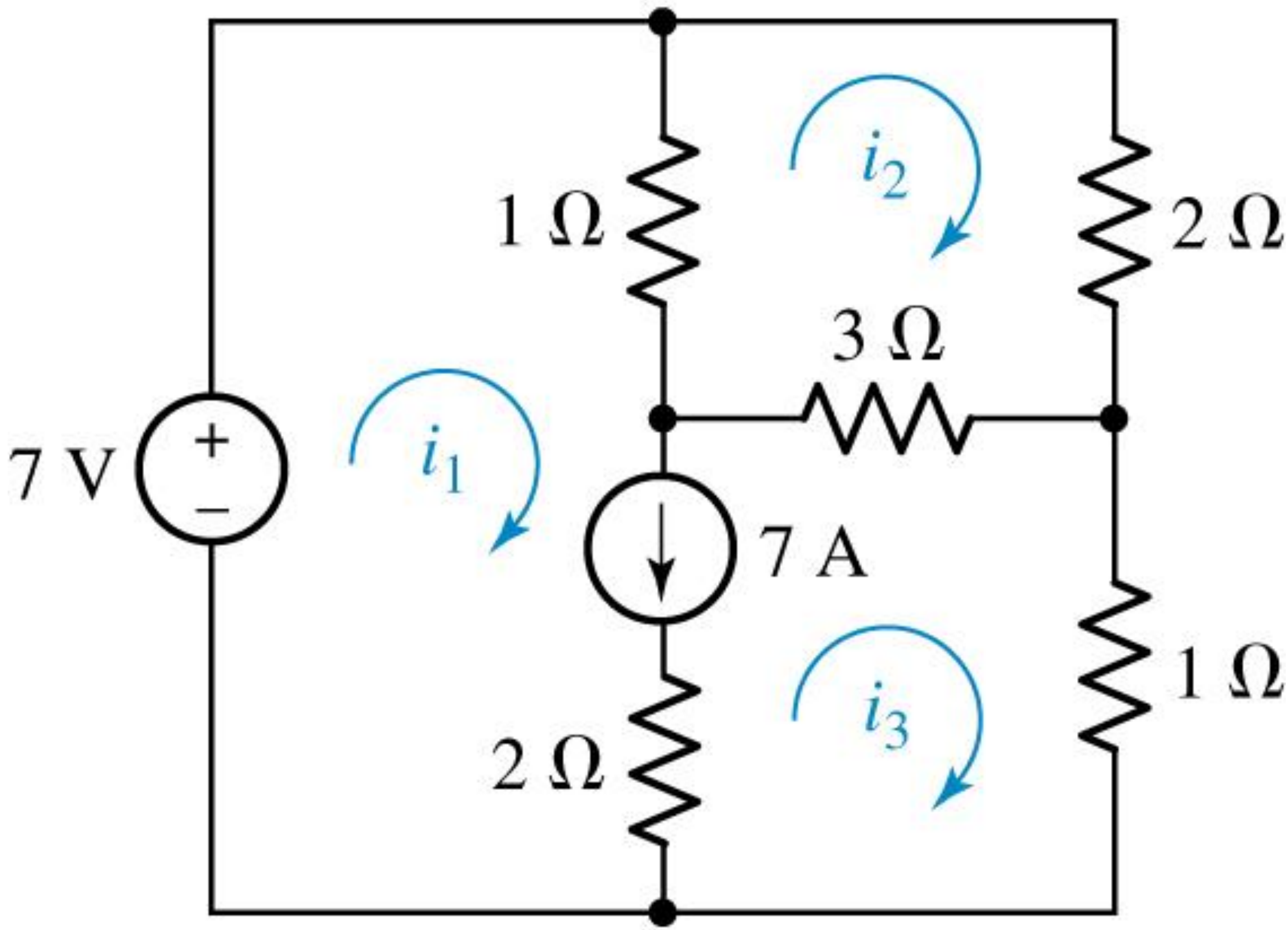
Mesh 1: $$-10+4(i_1-i_2)+3=0$$

Mesh 2: $$4(i_2-i_1)+5i_2+9i_2+10(i_2-i_3)=0$$

Mesh 3: $$-3+10(i_3-i_2)+8i_3=0$$

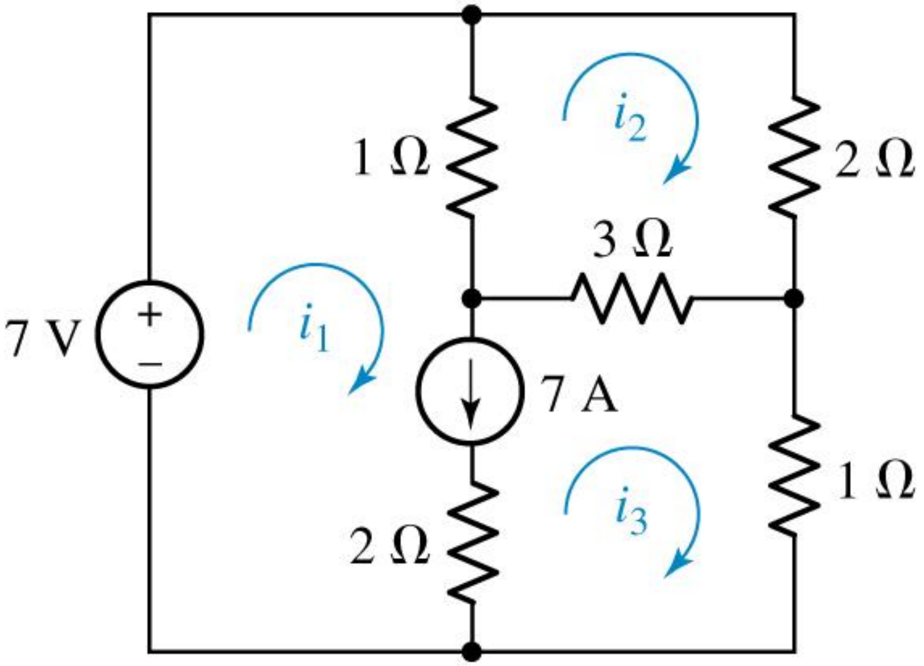
The Supermesh:

Find the three mesh currents in the circuit below.



The Supermesh:

Find the three mesh currents in the circuit below.



Apply KVL:

Mesh i_1:

$$-7 + 1(i_1 - i_2) + v + 2(i_1 - i_3) = 0$$

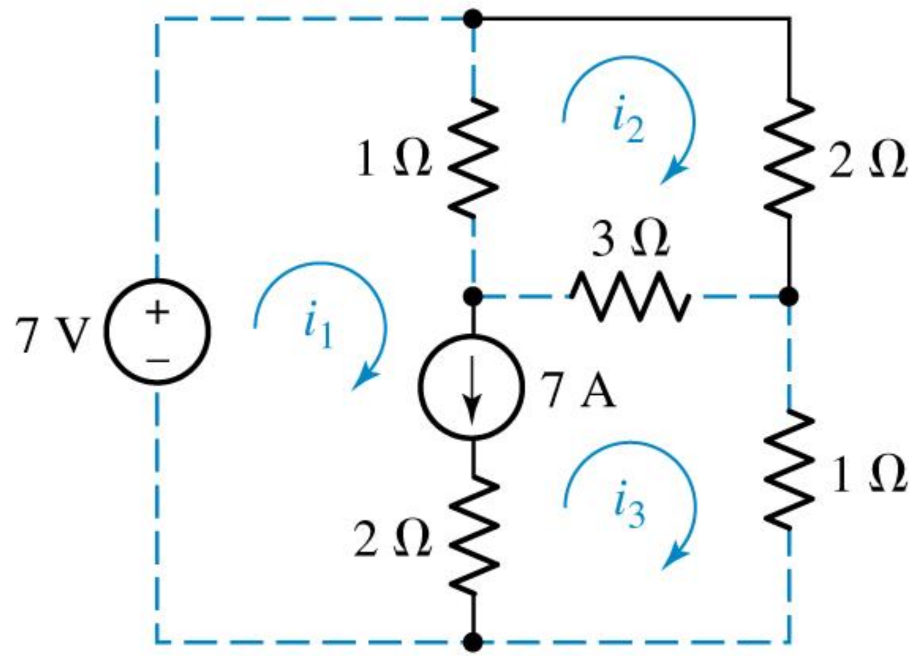
Mesh i_2 :

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Mesh i_3:

$$2(i_3 - i_1) - v + 3(i_3 - i_2) + 1i_3 = 0$$

The Supermesh:



A supermesh (i_1, i_3):

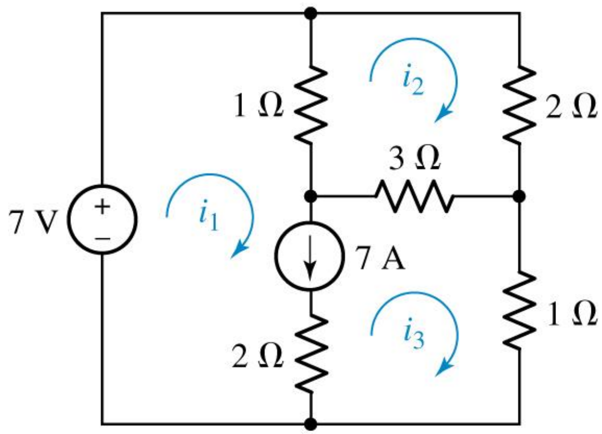
$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Mesh: i_2

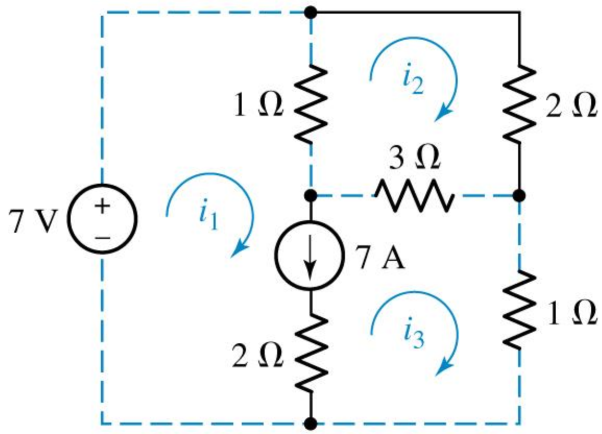
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

The Supermesh:

Find the three mesh currents in the circuit below.



(a)



(b)

Creating a "supermesh" from meshes 1 and 3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1\,i_3 = 0 \qquad [1]$$

Around mesh 2:

$$1(i_2 - i_1) + 2\,i_2 + 3(i_2 - i_3) = 0 \qquad [2]$$

Finally, we relate the currents in meshes 1 and 3:

$$i_1 - i_3 = 7 \qquad [3]$$

Rearranging,

$$i_1 - 4\,i_2 + 4\,i_3 = 7 \qquad [1]$$

$$-i_1 + 6\,i_2 - 3\,i_3 = 0 \qquad [2]$$

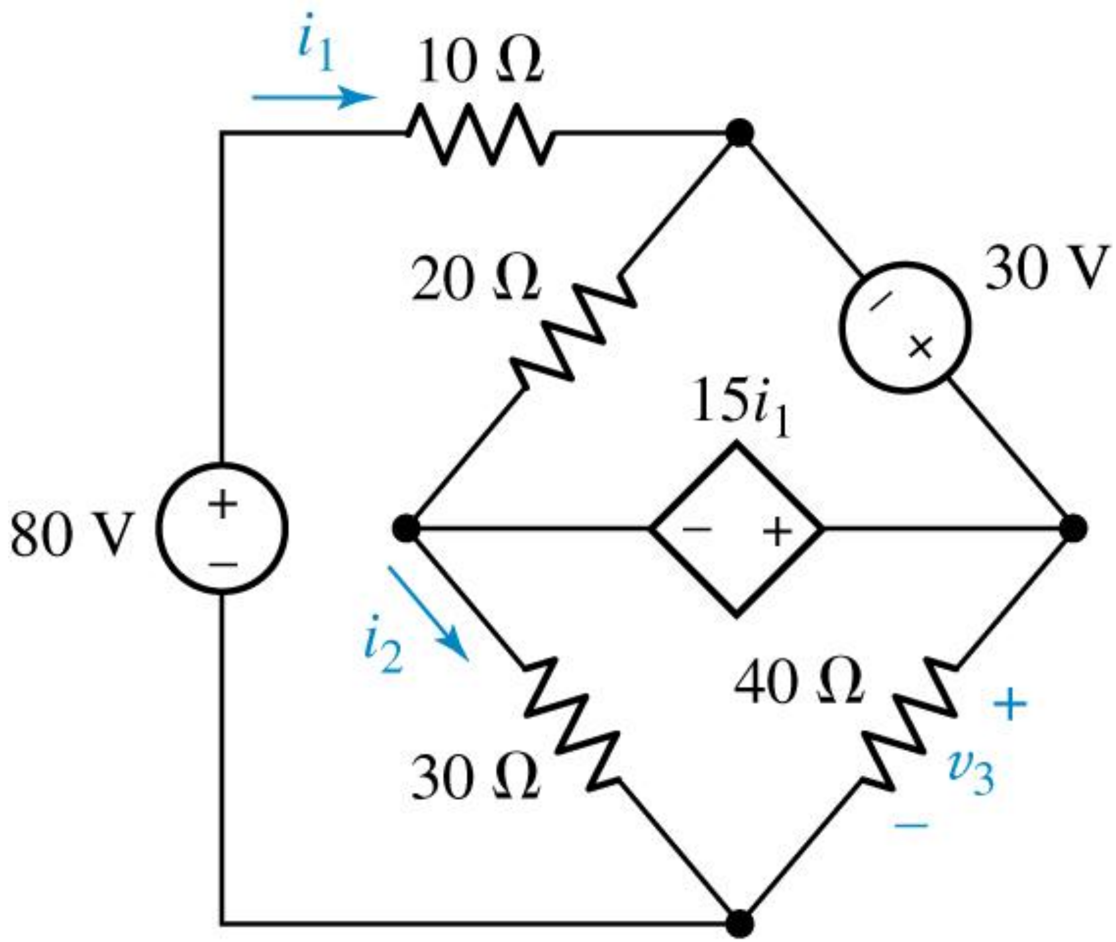
$$i_1 - i_3 = 7 \qquad [3]$$

Solving,

$i_1 = 9$ A, $i_2 = 2.5$ A, and $i_3 = 2$ A.

Practice: 4.8

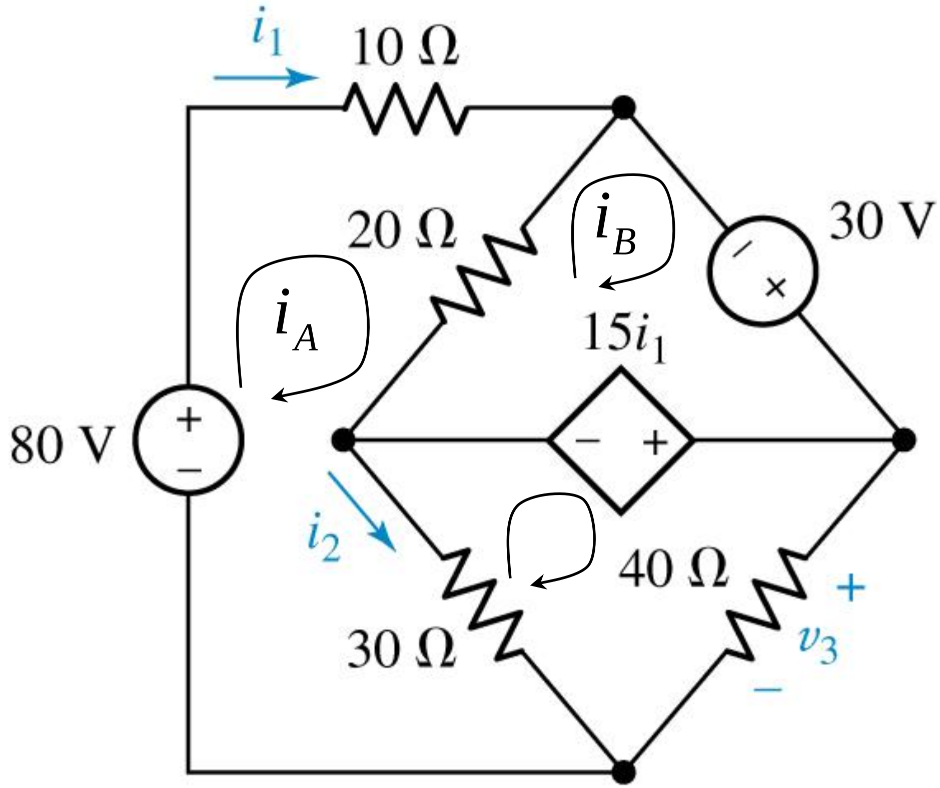
Find the voltage v_3 in the circuit below.



Practice: 4.8

Find the voltage v_3 in the circuit below.

$\text{Mesh}\, i_A:$



Practice: 4.8

Find the voltage v_3 in the circuit below.

Mesh i_A:

$$-80 + 10i_A + 20(i_A - i_B) + 30(i_A - i_C) = 0$$

Mesh i_B:

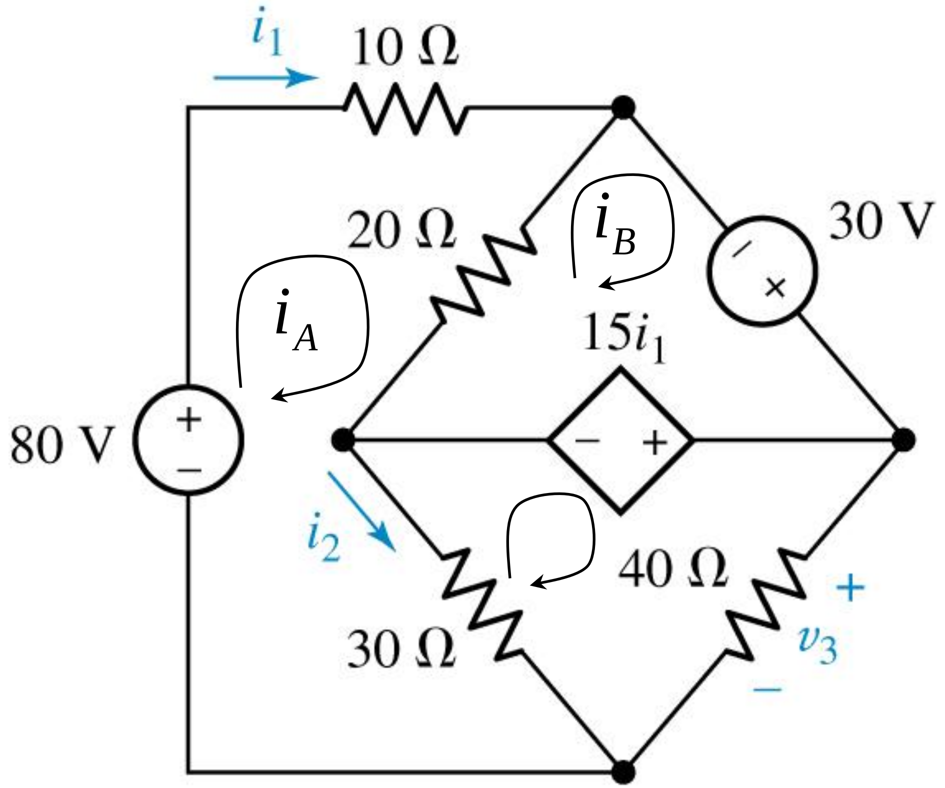
$$20(i_B - i_A) - 30 + 15i_1 = 0$$

$$20(i_B - i_A) - 30 + 15i_A = 0$$

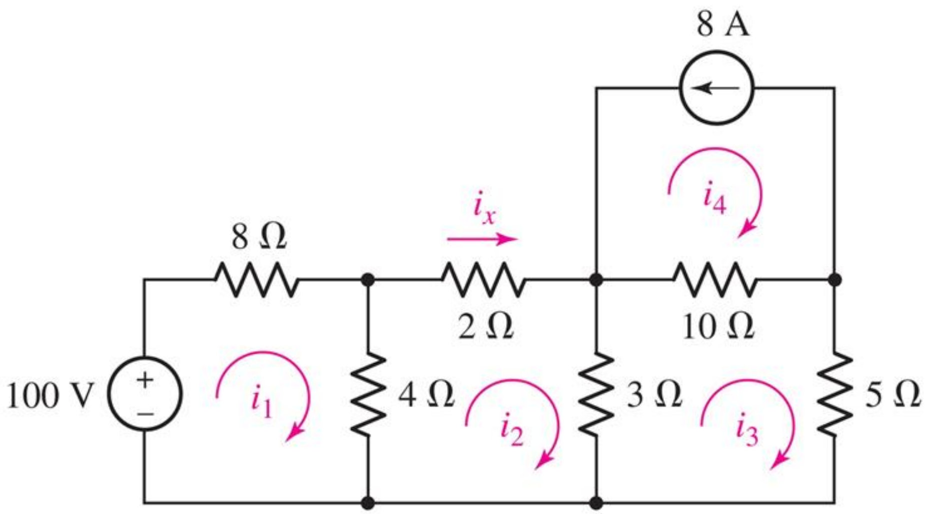
Mesh i_C:

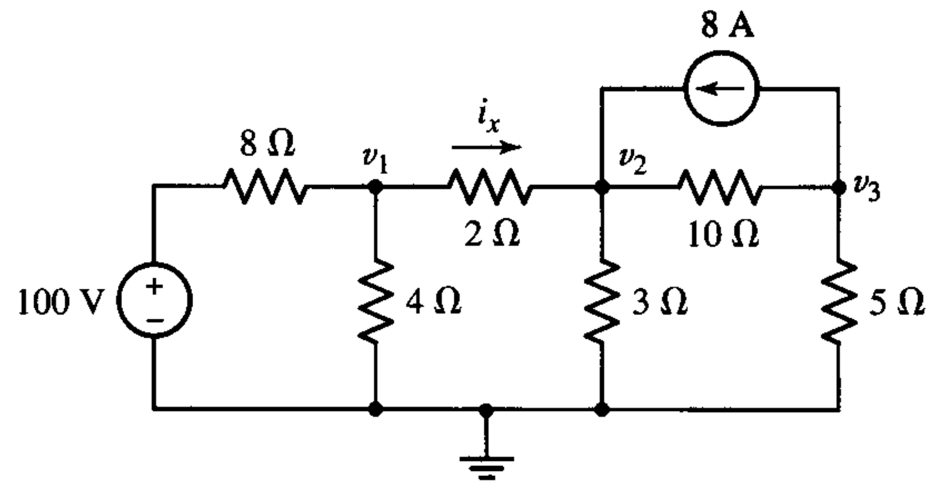
$$30(i_C - i_A) - 15i_1 + 40i_C = 0$$

$$30(i_C - i_A) - 15i_A + 40i_C = 0$$



A comparison:





A comparison:

Node v_1 :

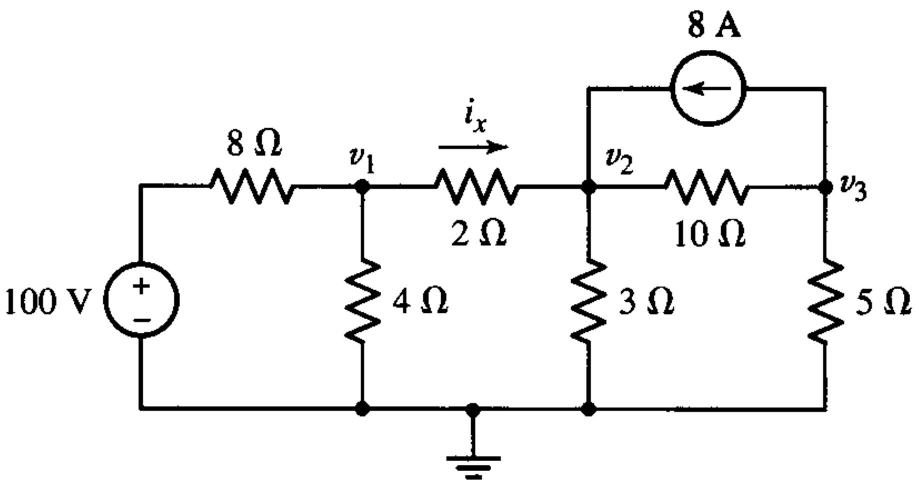
$$\frac{v_1 - 100}{8} + \frac{v_1}{4} + \frac{v_1 - v_2}{2} = 0$$

Node v_2 :

$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + \frac{v_2 - v_3}{10} - 8 = 0$$

Node v_3 :

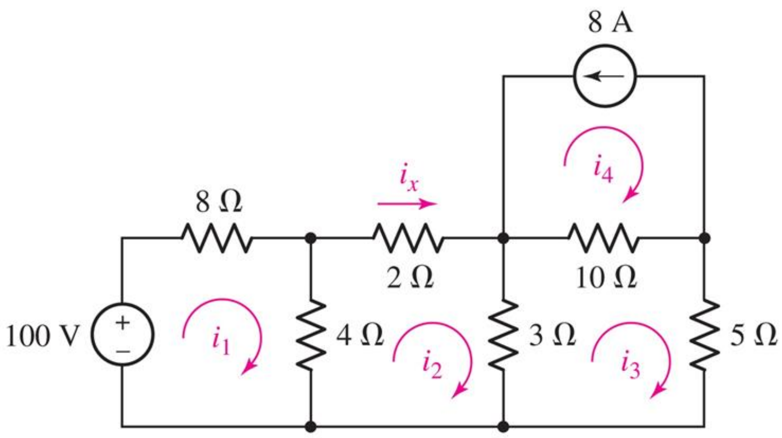
$$8 + \frac{v_3 - v_2}{10} + \frac{v_3}{5} = 0$$



Solving; $v_1 = 25.89$ V., $v_2 = 20.31$

$$\therefore i_x = \frac{v_1 - v_2}{2} = 2.79 \text{ A.}$$

A comparison:



Mesh i_1 :

$$-100 + 8i_1 + 4(i_1 - i_2) = 0$$

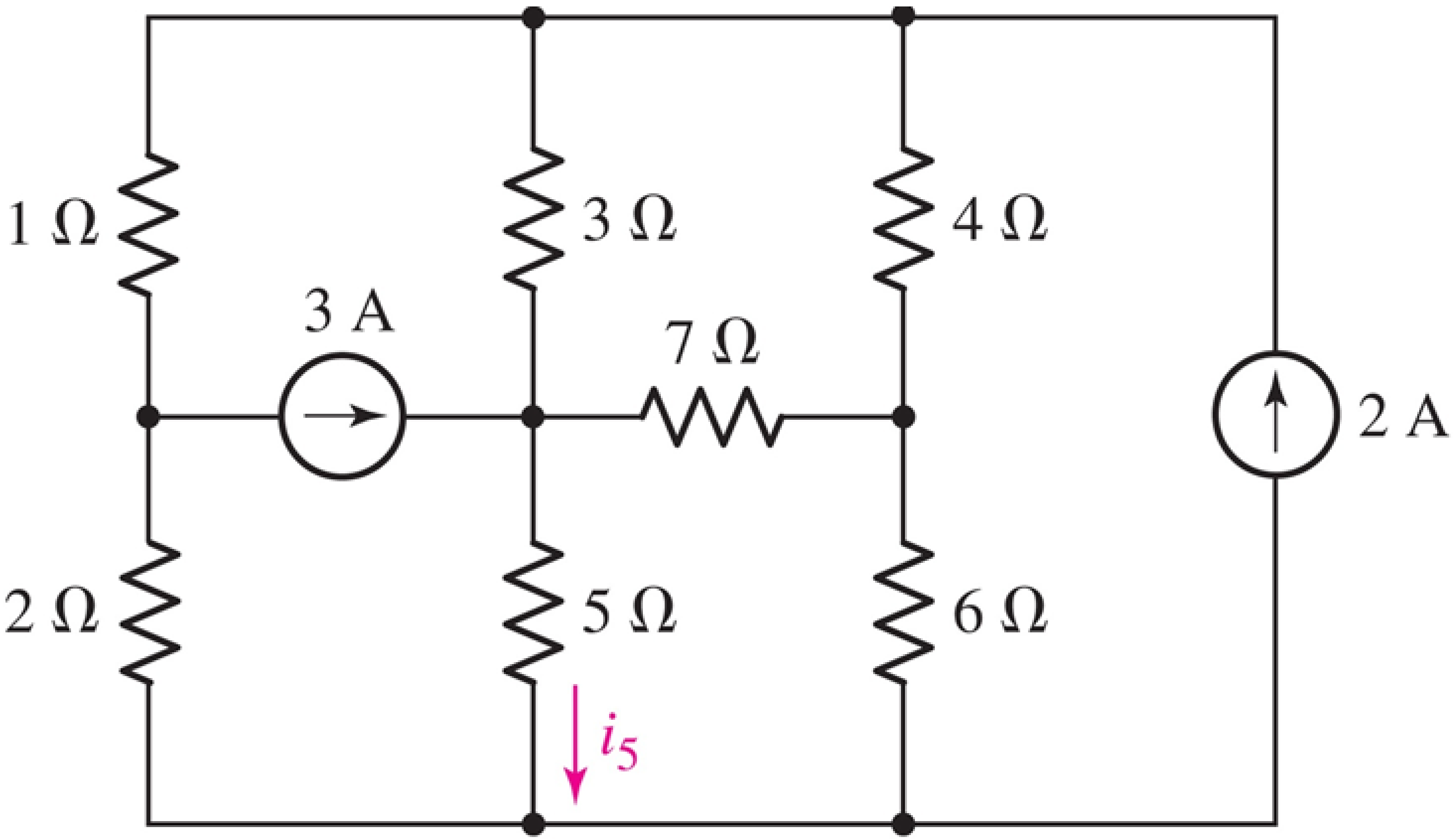
Mesh i_2 :

$$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Mesh i_3 :

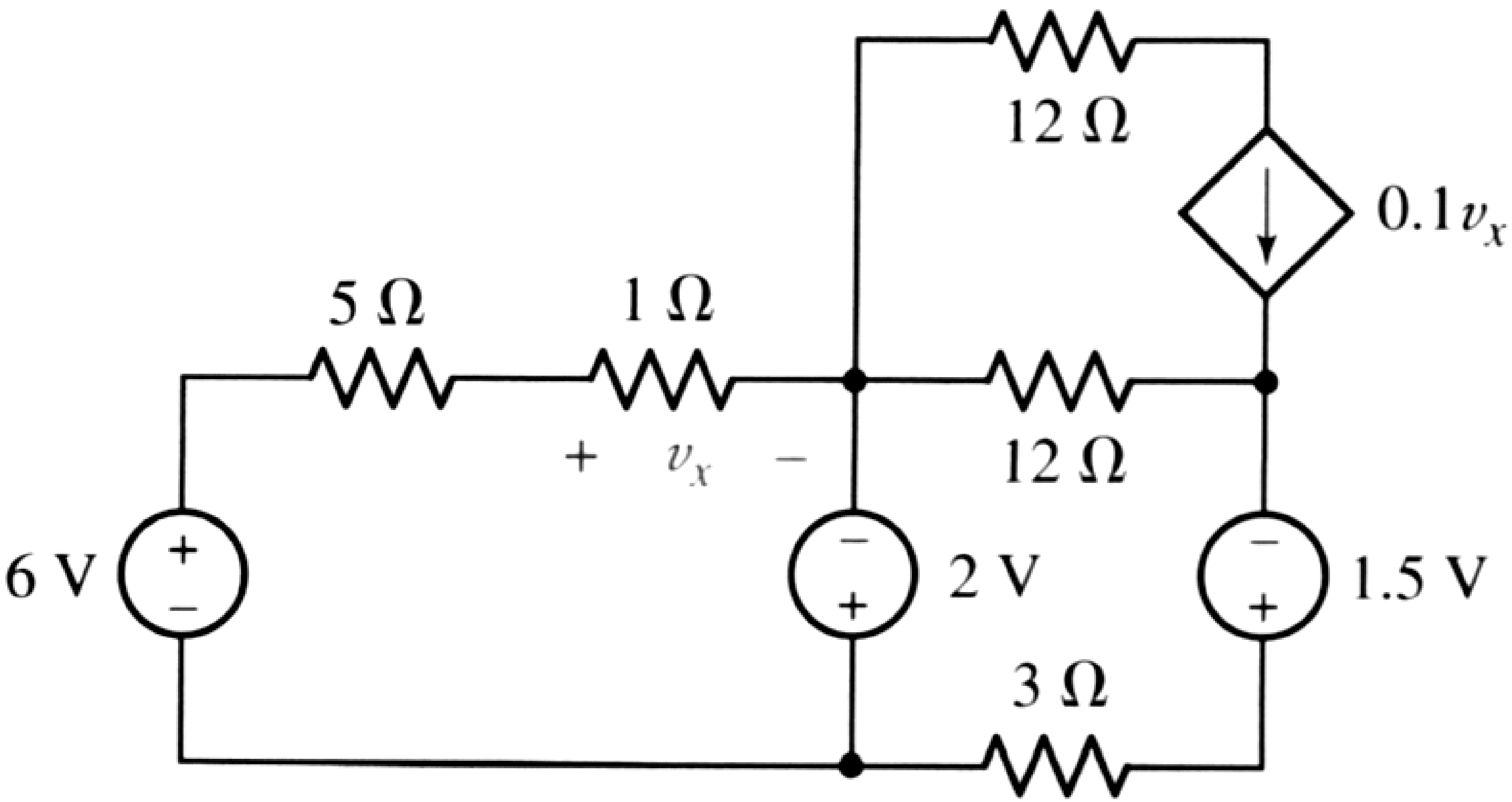
$$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0$$

A comparison:

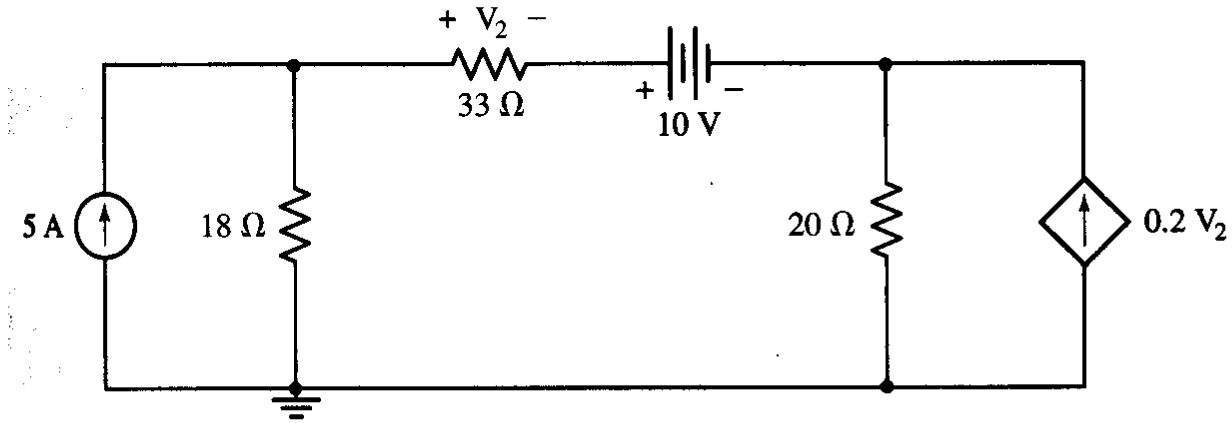


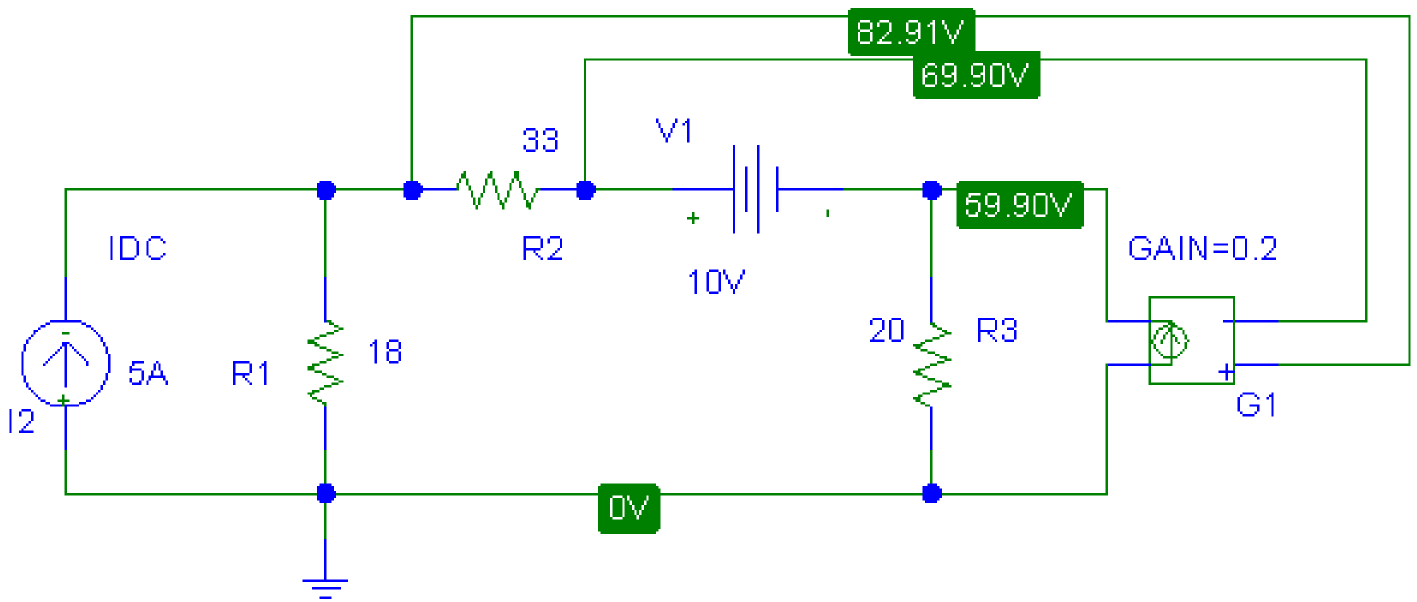
Ex:

Ch4-26 p.94 Determine each mesh current in the circuit

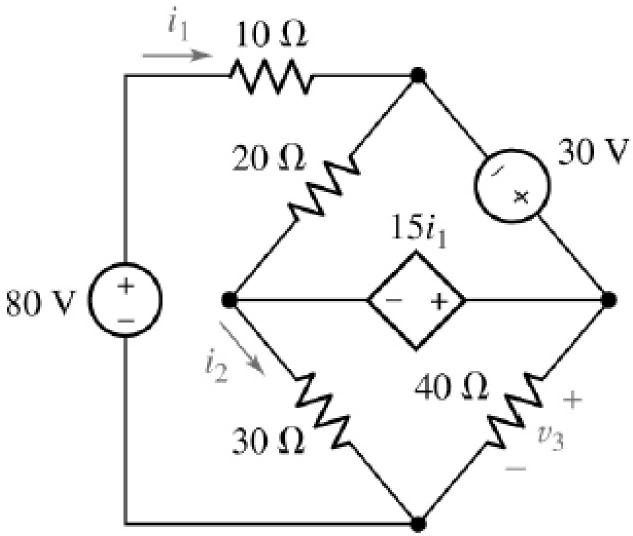


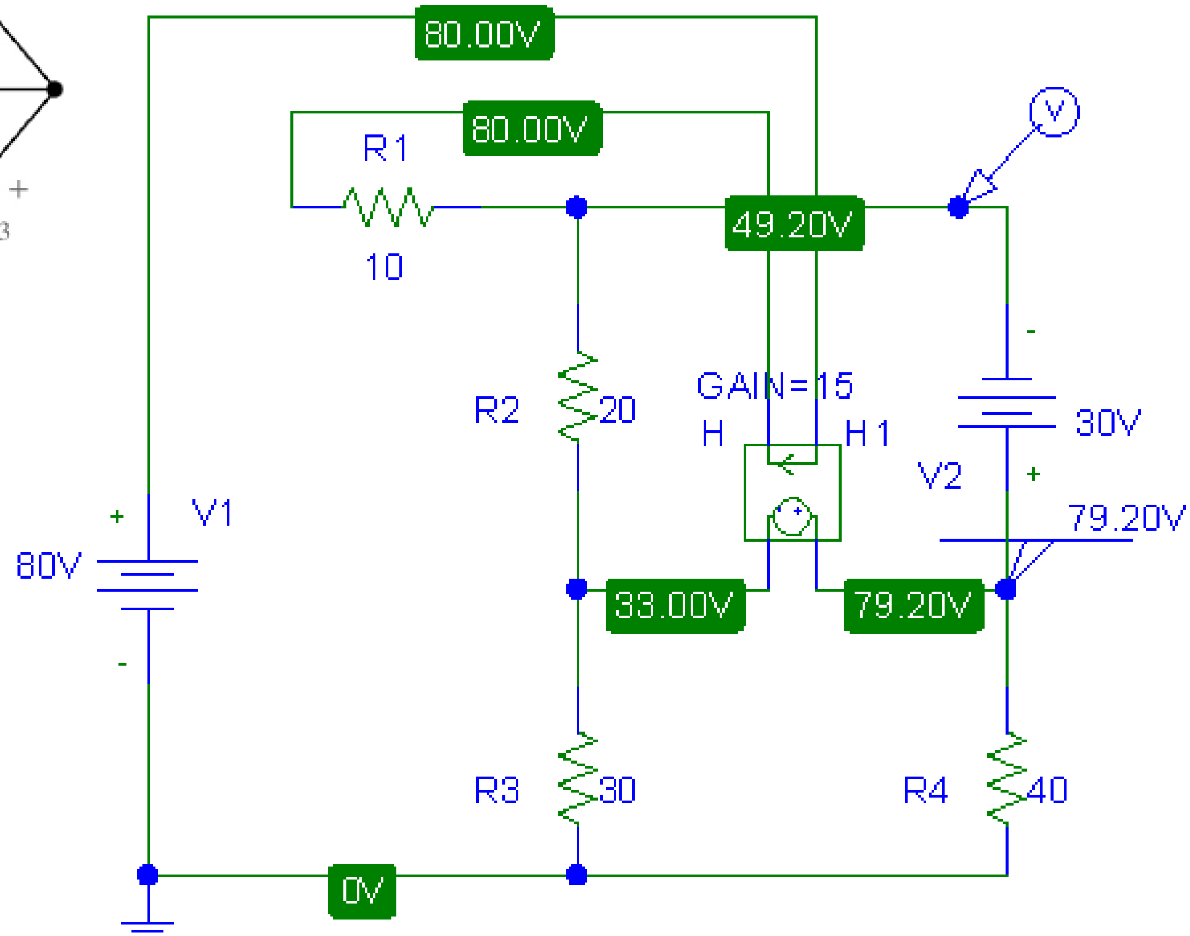
Computer-Aided:





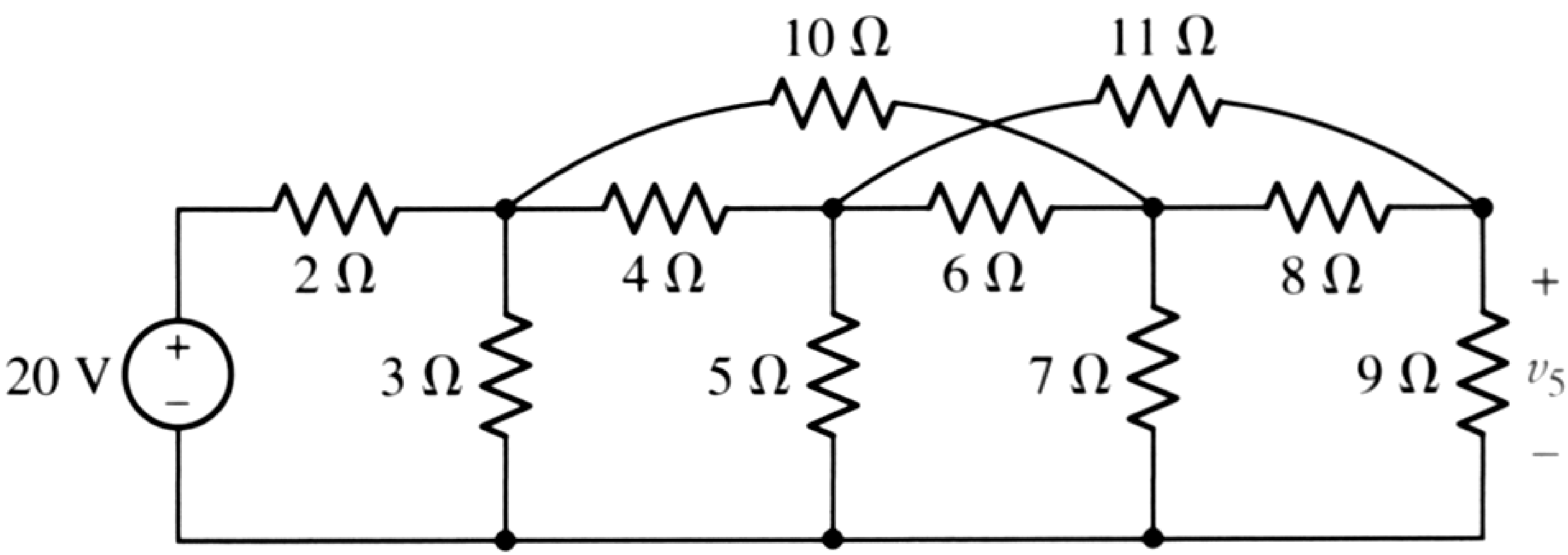
Computer-Aided:





Example:

Ch4.58 p.100 Write an appropriate input deck for SPICE to find v_5 in the circuit. Submit a printout of the output file, with the solution highlight



Homework:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.