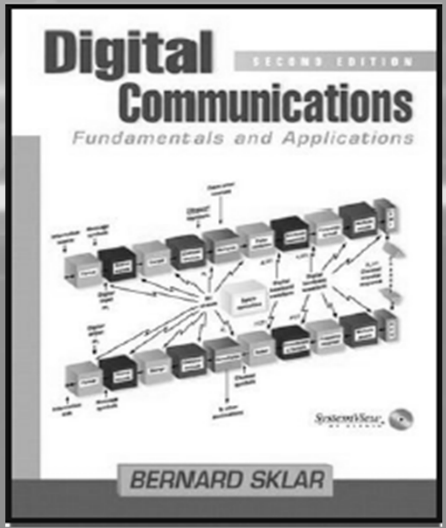


ENE 467
Digital Communications

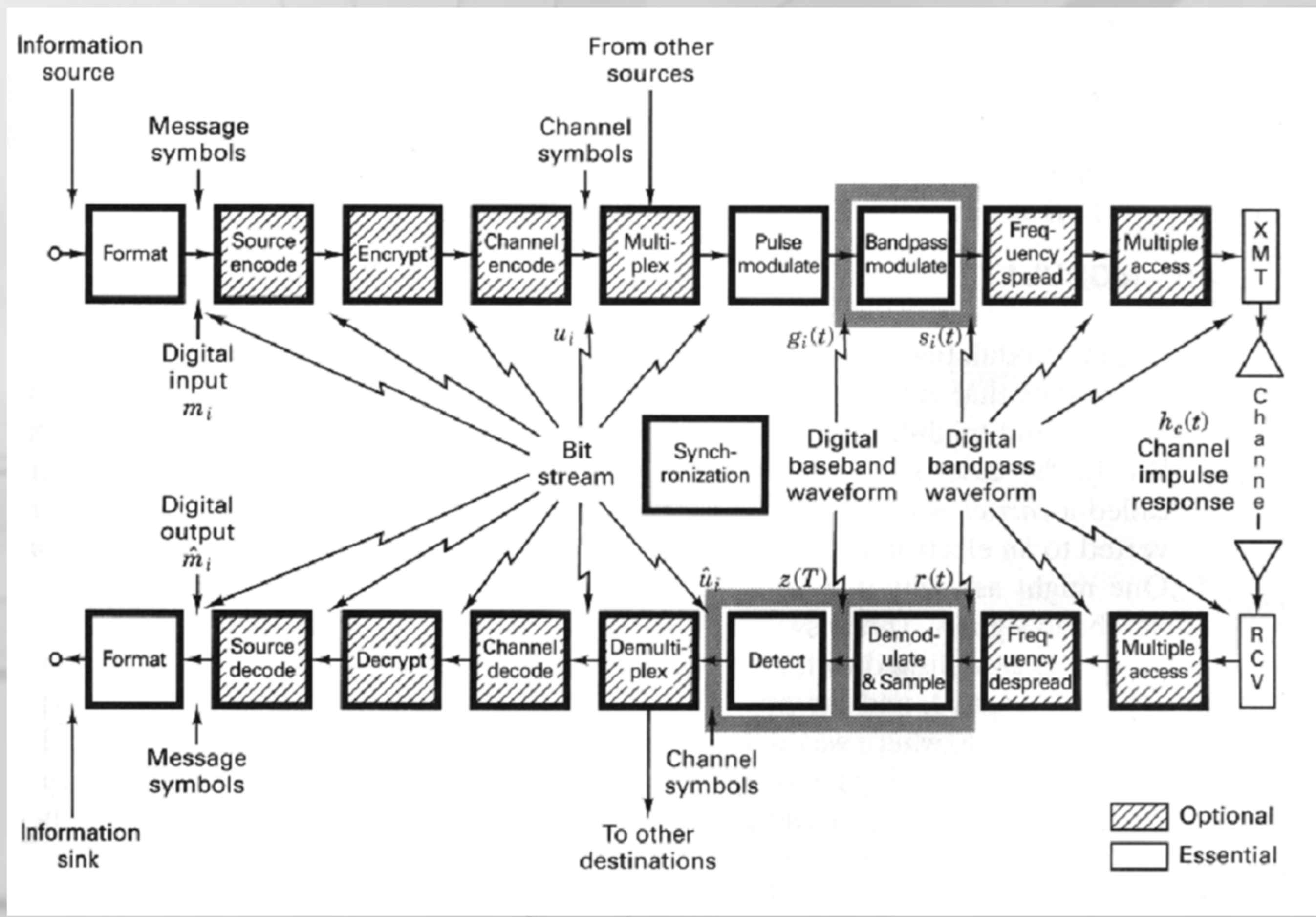
TEACHING BY

ASST. PROF. SUWAT PATTARAMALAI, PH.D.

4. Band-pass Modulation and Demodulation

- Outcome
  - Can explain the reason to modulation
  - Can design techniques for digital band-pass modulation (PSK, FSK, ASK, APK)
  - Can design detecting techniques for Gaussian noise channel
  - Can design Coherent and Non-coherent detector for PSK and FSK
  - Can explain Complex envelope
  - Can explain Error performance of binary system and M-ary signaling
  - Can explain Symbol error performance for M-ary systems

4. Band-pass Modulation and Demodulation



4. Band-pass Modulation and Demodulation

- Why modulate?
  - Process that digital symbols are transformed into waveforms that are compatible with the characteristics of the channel
  - Baseband modulation: the waveforms are shaped pulses
  - Band-pass modulation: the waveforms shaped pulses modulate a sinusoid (carrier)
  - Transmission electromagnetic (EM) fields through antennas
  - Size of antenna depends on wavelength of waveform (about wavelength/4)
  - For 3000 Hz baseband signal needs antenna length about 25 kilometers
  - For 900 MHz carrier, the antenna length is about 8 cm.
  - Modulation used for separate the different signals in frequency-division multiplexing, spread-spectrum modulation

Digital Bandpass Modulation Techniques

carrier wave is

$$s(t) = A(t)\cos\theta(t)$$

$$\theta(t) = \omega_0 t + \phi(t)$$

$$s(t) = A(t)\cos[\omega_0 t + \phi(t)]$$

$$\omega = 2\pi f$$

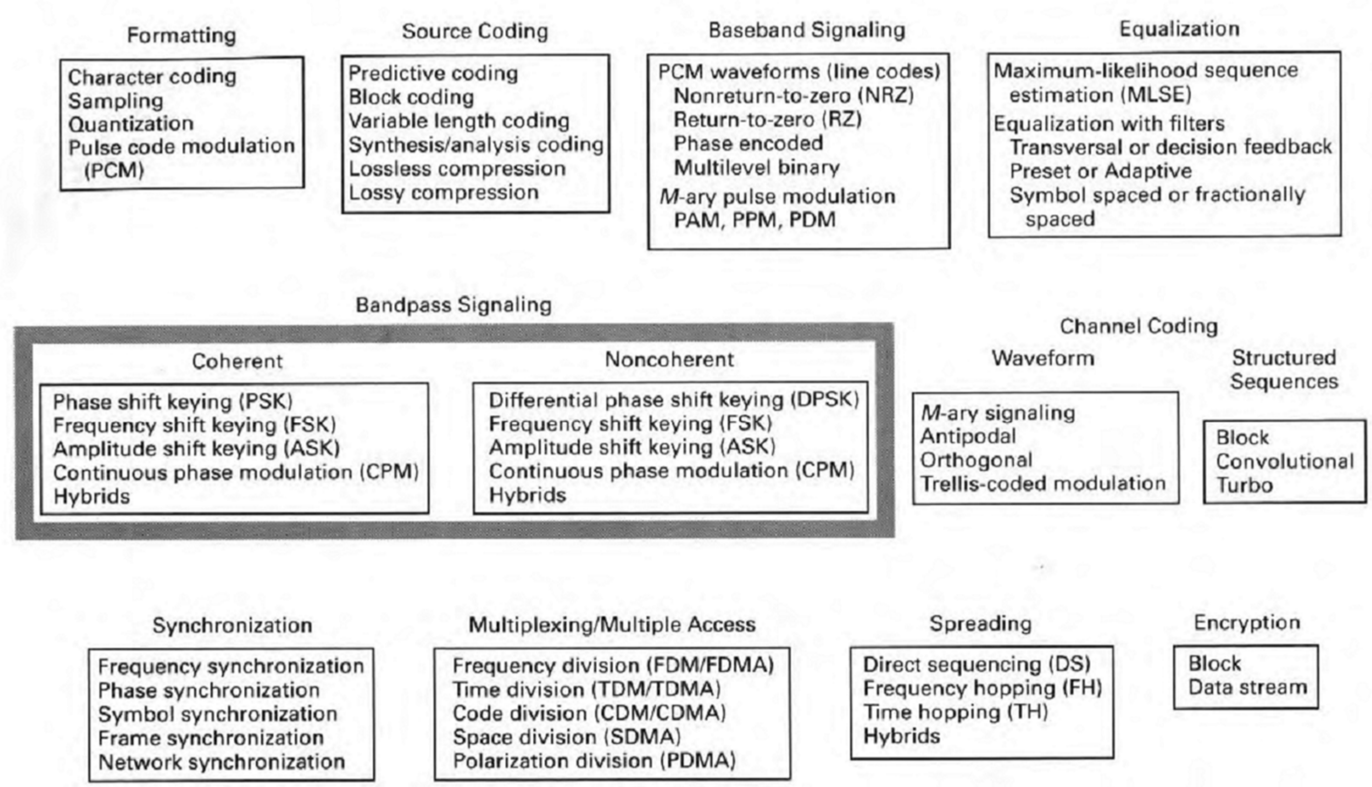


Figure 4.1 Basic digital communication transformations.

Digital Bandpass Modulation Techniques

- Phasor Representation of a Sinusoid

$$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$

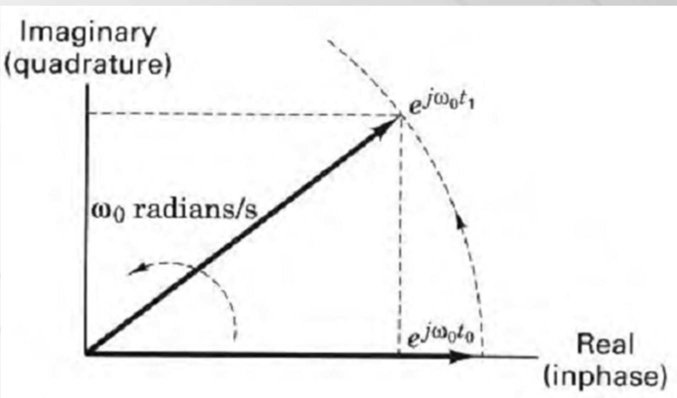


Figure 4.2 Phasor representation of a sinusoid.

amplitude modulated (AM)

$$s(t) = \mathrm{Re}\left\{e^{j\omega_0 t}\left(1 + \frac{e^{j\omega_m t}}{2} + \frac{e^{-j\omega_m t}}{2}\right)\right\}$$

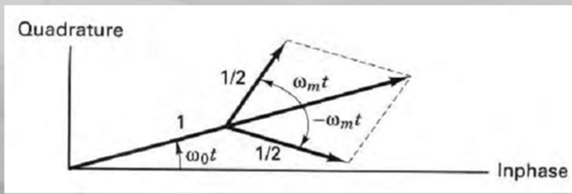


Figure 4.3 Amplitude modulation.

frequency modulating (FM)

narrowband FM (NFM)

$$s(t) = \mathrm{Re}\left\{e^{j\omega_0 t}\left(1 - \frac{\beta}{2}e^{-j\omega_m t} + \frac{\beta}{2}e^{j\omega_m t}\right)\right\}$$

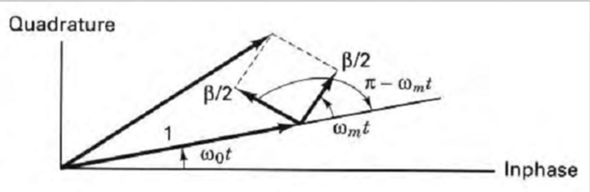


Figure 4.4 Narrowband frequency modulation.

4. Band-pass Modulation and Demodulation

- Phase Shift Keying

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_0 t + \phi_i(t)\right] \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

where the phase term, $\phi_i(t)$, will have M discrete values, typically given by

$$\phi_i(t) = \frac{2\pi i}{M} \qquad i = 1, \ldots, M$$

- Frequency Shift Keying

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_i t + \phi\right) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

- Amplitude Shift Keying

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos\left(\omega_0 t + \phi\right) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

- Amplitude Phase Keying

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos\left[\omega_0 t + \phi_i(t)\right] \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

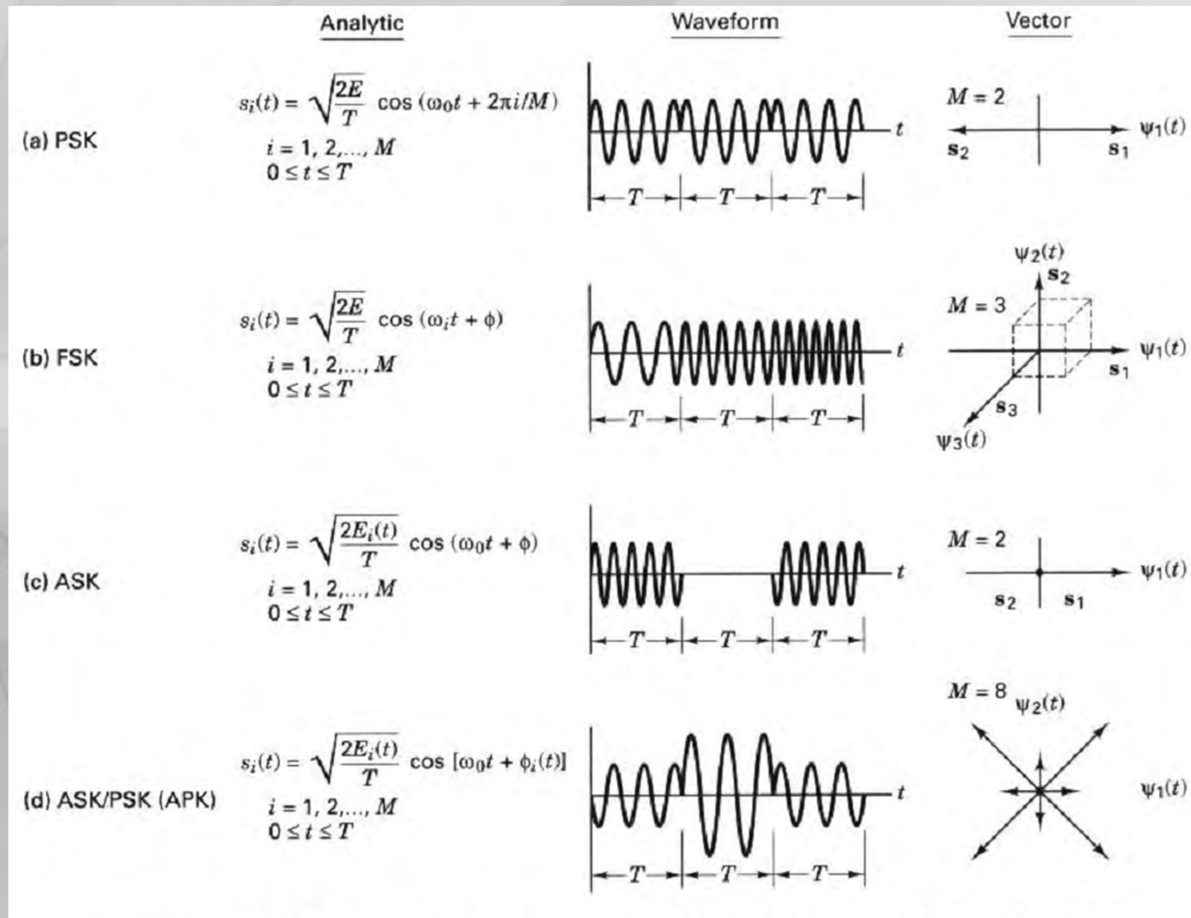


Figure 4.5 Digital modulations. (a) PSK. (b) FSK. (c) ASK. (d) ASK/PSK (APK).

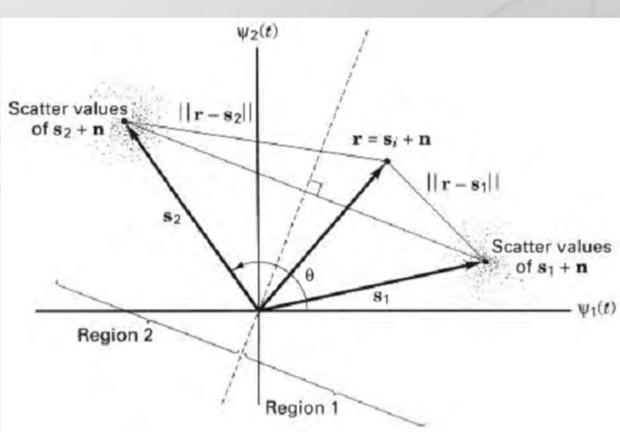
Detection of signal in Gaussian noise

- Decision regions

Correlation Receiver

$$z_i(T) = \int_0^T r(t)s_i(t)\,dt \qquad i = 1, \dots, M$$

Figure 4.6 Two-dimensional signal space, with arbitrary equal-amplitude vectors $\mathbf{s}_1$ and $\mathbf{s}_2$.



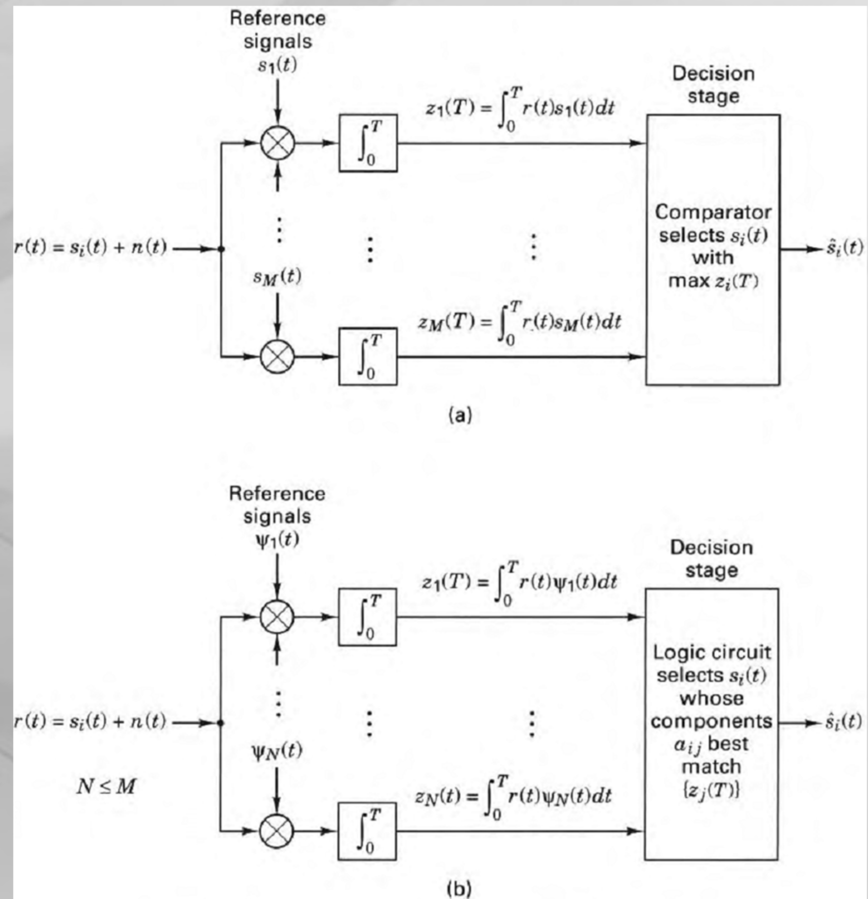


Figure 4.7 (a) Correlator receiver with reference signals $\{s_i(t)\}$. (b) Correlator receiver with reference signals $\{\psi_j(t)\}$.

Detection of signal in Gaussian noise

- Binary Detection

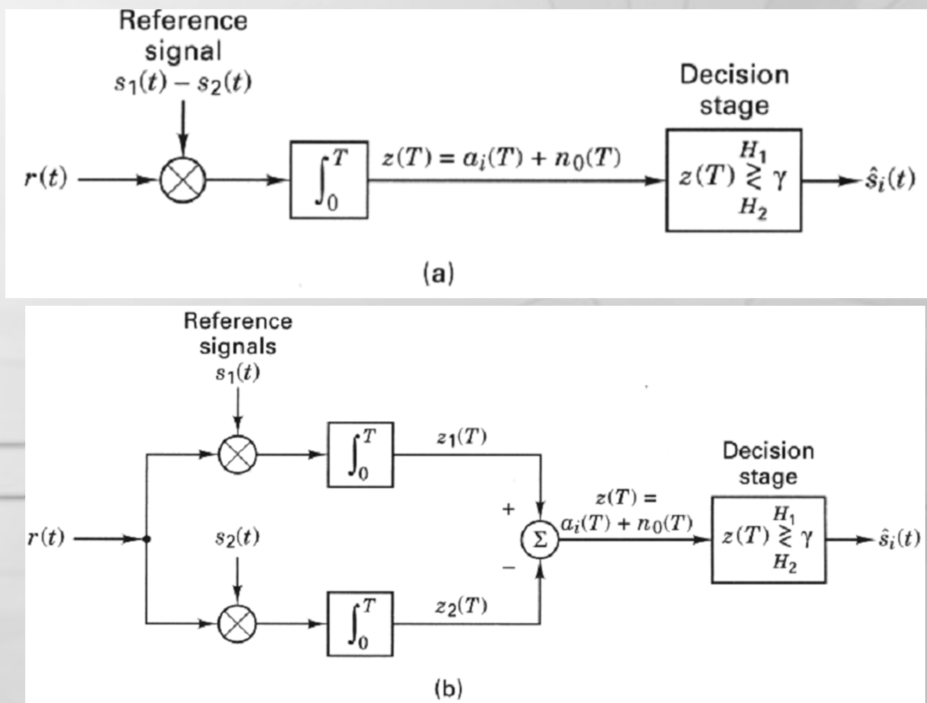


Figure 4.8 Binary correlator receiver. (a) Using a single correlator. (b) Using two correlators.

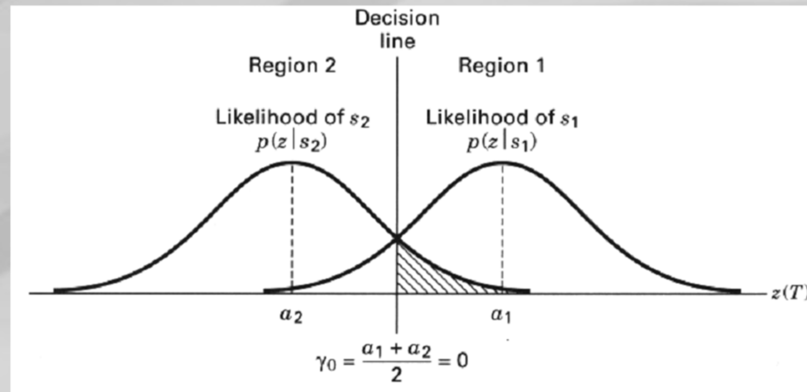


Figure 4.9 Conditional probability density functions: $p(z/s_1)$, $p(z/s_2)$.

$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_1}{\sigma_0}\right)^2\right]$$

$$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_2}{\sigma_0}\right)^2\right]$$

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1 + a_2}{2} = \gamma_0$$

Coherent Detection

- Coherent Detection of PSK

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi) \qquad 0 \le t \le T$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi + \pi)$$

$$= -\sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi) \qquad 0 \le t \le T$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \qquad \text{for } 0 \le t \le T$$

$$s_i(t) = a_{i1}\psi_1(t)$$

$$s_1(t) = a_{11}\psi_1(t) = \sqrt{E}\,\psi_1(t)$$

$$s_2(t) = a_{21}\psi_1(t) = -\sqrt{E}\,\psi_1(t)$$

$$\mathbf{E}\{z_1|s_1\} = \mathbf{E}\left\{\int_0^T \sqrt{E}\,\psi_1^2(t) + n(t)\psi_1(t)\,dt\right\}$$

$$\mathbf{E}\{z_2|s_1\} = \mathbf{E}\left\{\int_0^T -\sqrt{E}\,\psi_1^2(t) + n(t)\psi_1(t)\,dt\right\}$$

$$\mathbf{E}\{z_1|s_1\} = \mathbf{E}\left\{\int_0^T \frac{2}{T}\sqrt{E}\cos^2\omega_0 t + n(t)\sqrt{\frac{2}{T}}\cos\omega_0 t\,dt\right\} = \sqrt{E}$$

and

$$\mathbf{E}\{z_2|s_1\} = \mathbf{E}\left\{\int_0^T -\frac{2}{T}\sqrt{E}\cos^2\omega_0 t + n(t)\sqrt{\frac{2}{T}}\cos\omega_0 t\,dt\right\} = -\sqrt{E}$$

Coherent Detection

- Coherent Detection of MPSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_0 t - \frac{2\pi i}{M}\right) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t$$

$$\begin{aligned} s_i(t) &= a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array} \\ &= \sqrt{E} \cos\left(\frac{2\pi i}{M}\right)\psi_1(t) + \sqrt{E} \sin\left(\frac{2\pi i}{M}\right)\psi_2(t) \end{aligned}$$

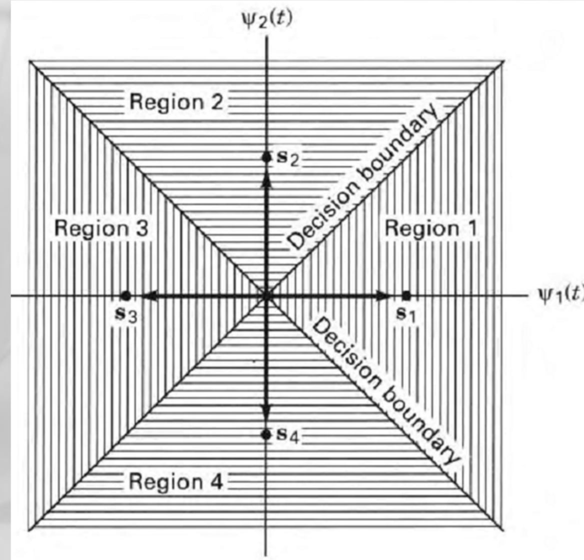


Figure 4.11 Signal space and decision regions for a QPSK system.

$$r(t) = \sqrt{\frac{2E}{T}} (\cos \phi_i \cos \omega_0 t + \sin \phi_i \sin \omega_0 t) + n(t) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array} \qquad (4.34)$$

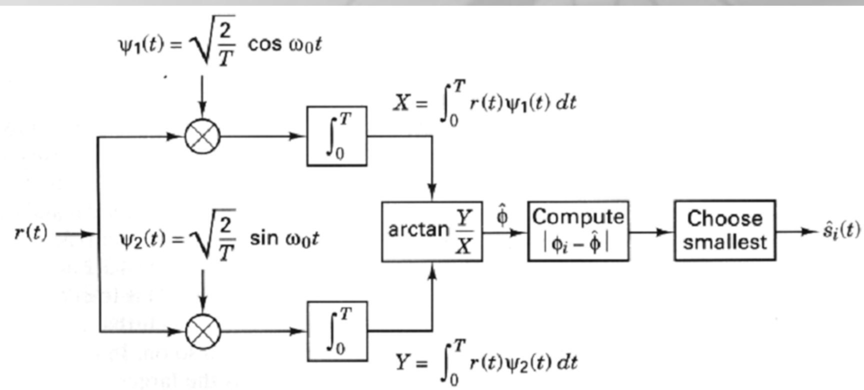


Figure 4.12 Demodulator for MPSK signals.

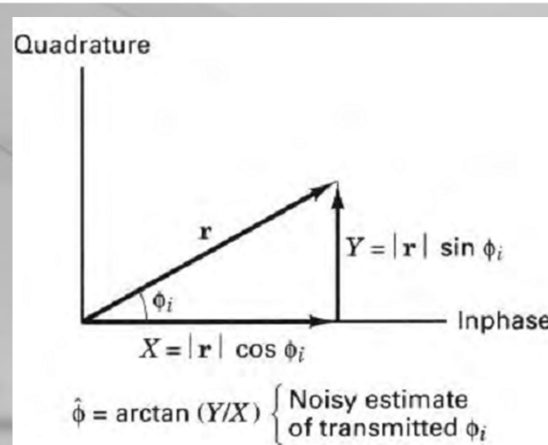


Figure 4.13 In-phase and quadrature components of the received signal vector **r**.

Coherent Detection

- Coherent Detection of FSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \dots, M \end{array}$$

$$\psi_j(t) = \sqrt{\frac{2}{T}} \cos \omega_j t \qquad j = 1, \dots, N$$

$$a_{ij} = \int_0^T \sqrt{\frac{2E}{T}} \cos(\omega_i t) \sqrt{\frac{2}{T}} \cos \omega_j t \, dt$$

$$a_{ij} = \begin{cases} \sqrt{E} & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$d(\mathbf{s}_i, \mathbf{s}_j) = \|\mathbf{s}_i - \mathbf{s}_j\| = \sqrt{2E} \qquad \text{for } i \ne j$$

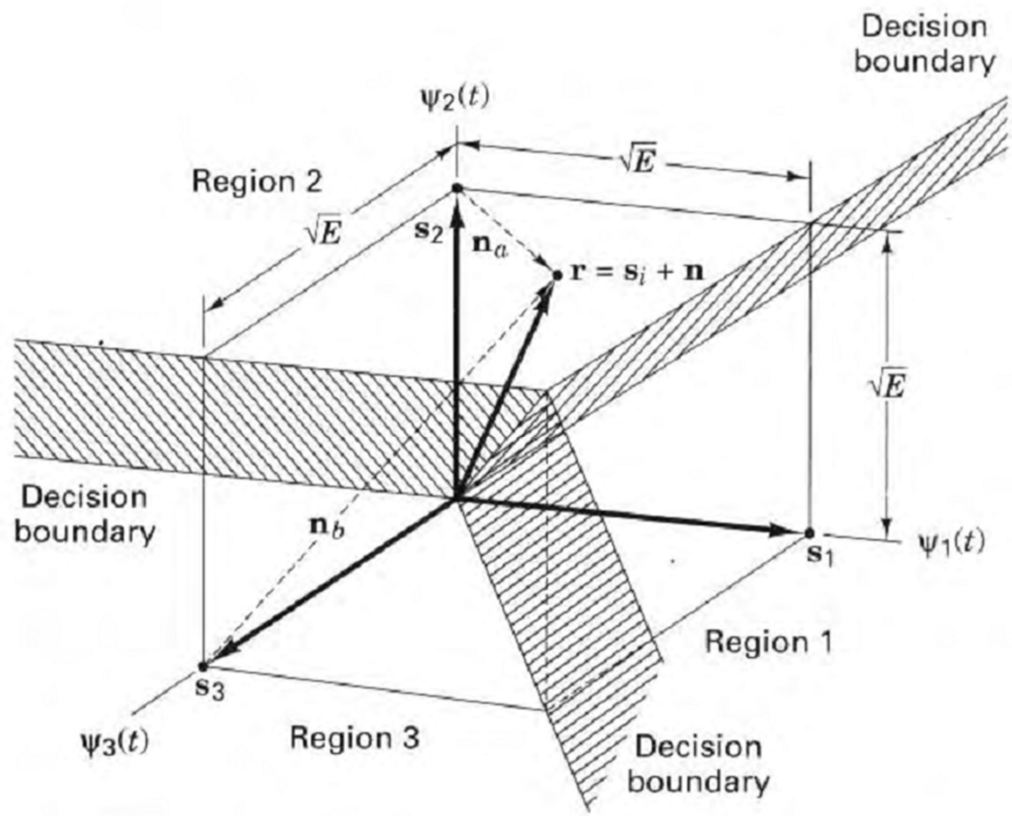


Figure 4.14 Partitioning the signal space for a 3-ary FSK signal.

Noncoherent Detection

- Detection of Differential PSK (DPSK): Differential encoding data

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \theta_i(t)] \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

$$r(t) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \theta_i(t) + \alpha] + n(t) \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \ldots, M \end{array}$$

$$[\theta_k(T_2) + \alpha] - [\theta_j(T_1) + \alpha] = \theta_k(T_2) - \theta_j(T_1) = \phi_i(T_2)$$

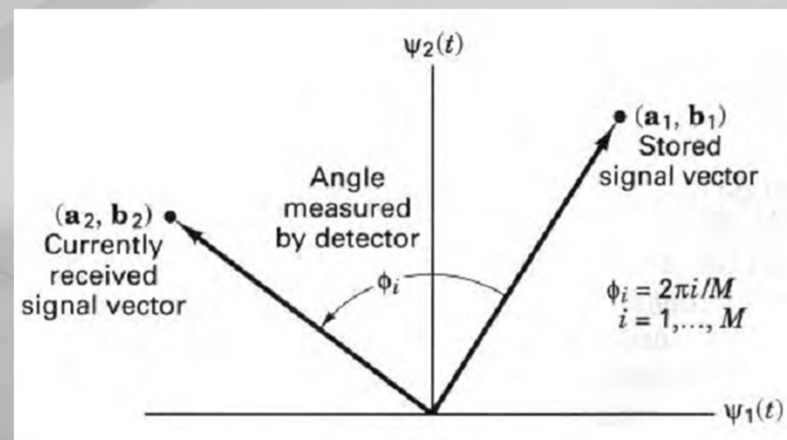


Figure 4.16 Signal space for DPSK.

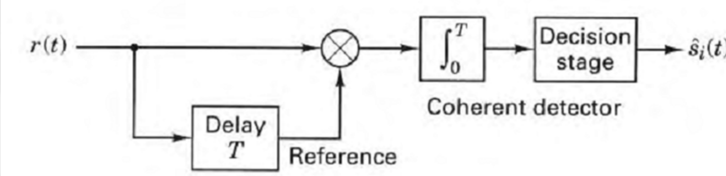
Noncoherent Detection

- Binary Differential PSK: encode $c(k) = c(k-1) \oplus m(k)$ or $c(k) = \overline{c(k-1) \oplus m(k)}$

symbol $\oplus$ represents modulo-2 addition

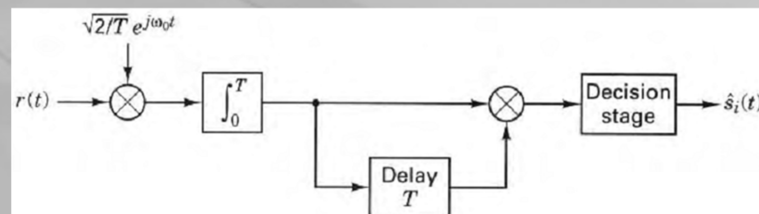
Sample index, k	0	1	2	3	4	5	6	7	8	9	10
Information message, $m(k)$		1	1	0	1	0	1	1	0	0	1
Differentially encoded message (first bit arbitrary), $c(k)$	1	1	1	0	0	1	1	1	0	1	1
Corresponding phase shift, $\theta(k)$	π	π	π	0	0	π	π	π	0	π	π

(a)



Detected message, $\hat{m}(k)$ 1 1 0 1 0 1 1 0 0 1

(b)



(c)

Figure 4.17 Differential PSK (DPSK). (a) Differential encoding. (b) Differentially coherent detection. (c) Optimum differentially coherent detection.

Noncoherent Detection

- Noncoherent Detection of FSK

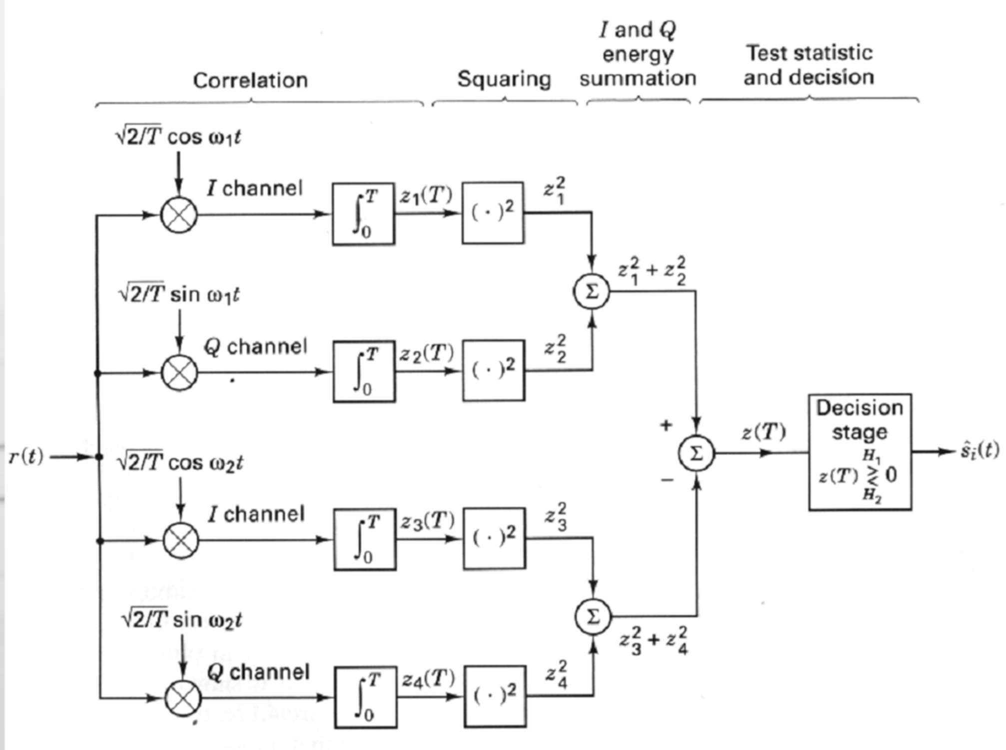


Figure 4.18 Quadrature receiver.

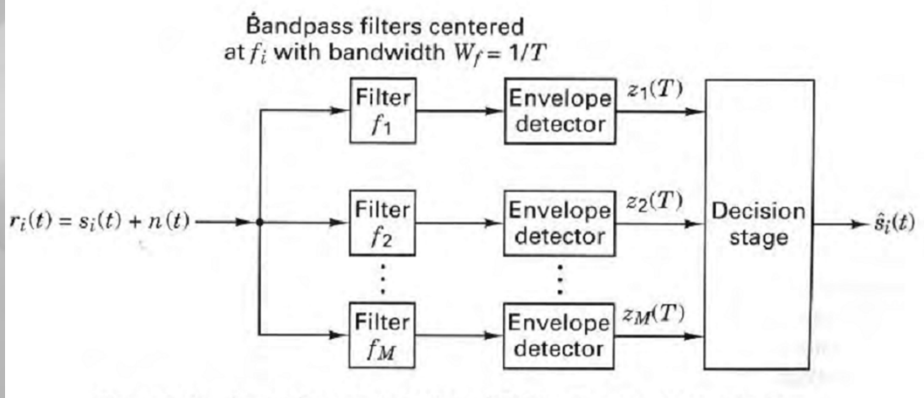


Figure 4.19 Noncoherent detection of FSK using envelope detectors.

Required Tone Spacing for Noncoherent FSK

- Minimum Tone Spacing and Bandwidth

$$s_i(t) = (\cos 2\pi f_i t)\,\mathrm{rect}\,(t/T)$$

$$\text{where } \mathrm{rect}\,(t/T) = \begin{cases} 1 & \text{for } -T/2 \le t \le T/2 \\ 0 & \text{for } |t| > T/2 \end{cases}$$

$$\mathcal{F}\{s_i(t)\} = T\,\mathrm{sinc}\,(f - f_i)T$$

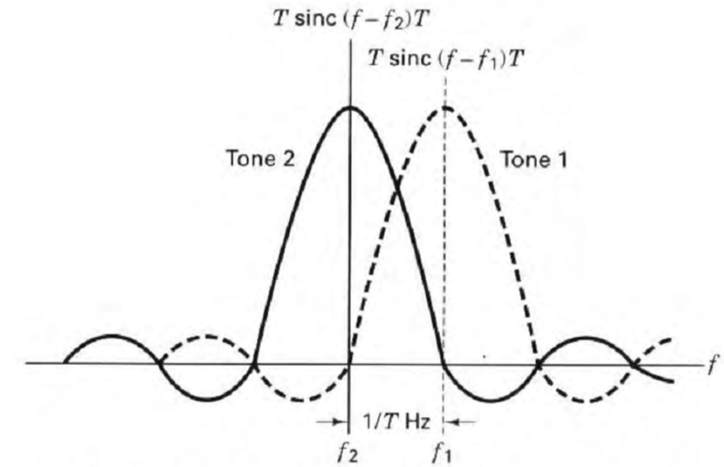


Figure 4.20 Minimum tone spacing for noncoherently detected orthogonal FSK signaling.

corresponds to a minimum tone separation of $1/T$

M-ary case, the bandwidth of noncoherently detected orthogonal MFSK is equal to M/T.

the minimum tone spacing is reduced to $1/2T$ when coherent detection is used

Complex Envelope

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_0 t}\}$$

where $g(t)$ is known as the *complex envelope*, expressed as

$$g(t) = x(t) + jy(t) = |g(t)|e^{j\theta(t)} = R(t)e^{j\theta(t)}$$

The magnitude of the complex envelope is then

$$R(t) = |g(t)| = \sqrt{x^2(t) + y^2(t)}$$

and its phase is

$$\theta(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

$$\begin{aligned} s(t) &= \mathrm{Re}\{[x(t) + jy(t)][\cos\omega_0 t + j\sin\omega_0 t]\} \\ &= x(t)\cos\omega_0 t - y(t)\sin\omega_0 t \end{aligned}$$

- Quadrature Implementation of a Modulator

$$g_k = x_k + jy_k = 0.707A + j0.707A$$

$$\begin{aligned} s(t) &= \mathrm{Re}\{g_k e^{j\omega_0 t}\} \\ &= \mathrm{Re}\{(x_k + jy_k)(\cos\omega_0 t + j\sin\omega_0 t)\} \\ &= x_k\cos\omega_0 t - y_k\sin\omega_0 t \\ &= 0.707A\cos\omega_0 t - 0.707A\sin\omega_0 t \\ &= A\cos\left(\omega_0 t + \frac{\pi}{4}\right) \end{aligned}$$

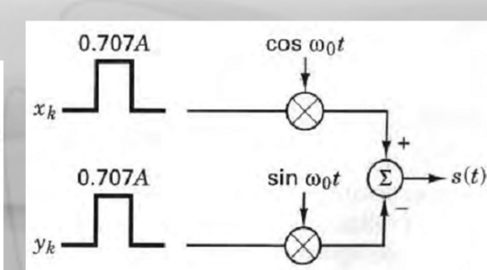


Figure 4.21 Quadrature type modulator.

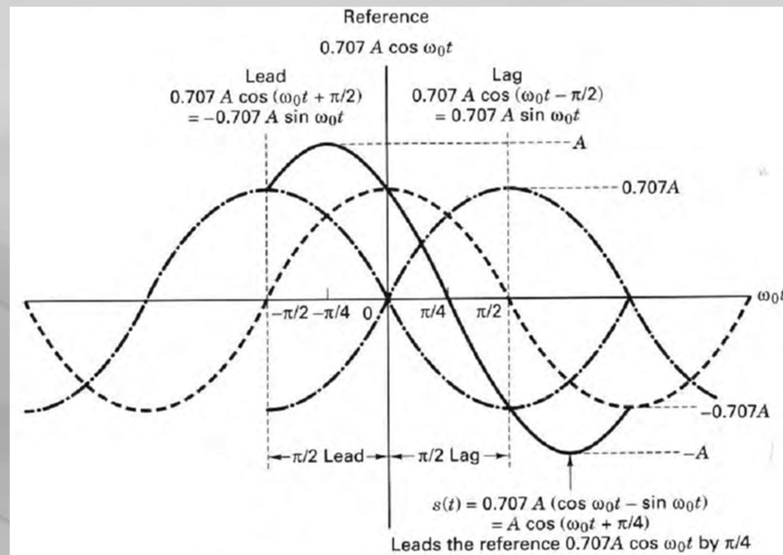


Figure 4.22 Lead/Lag relationships of sinusoids.

Complex Envelope

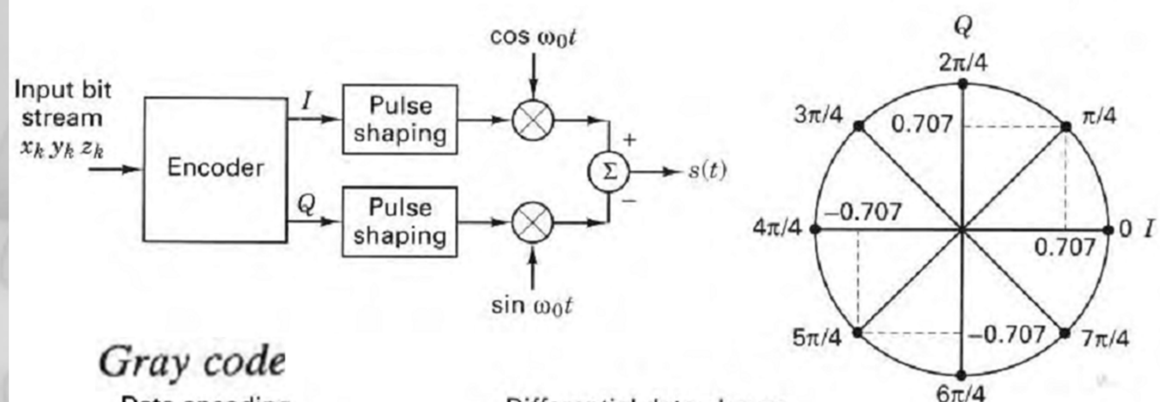
- D8PSK Modulator

$$\phi_k = \Delta\phi_k + \phi_{k-1}$$

$$\begin{aligned} s(t) &= \mathrm{Re}\{(x_k + jy_k)(\cos\omega_0 t + j\sin\omega_0 t)\} \\ &= x_k\cos\omega_0 t - y_k \sin\omega_0 t \end{aligned}$$

for time $k = 2$

$$\begin{aligned} s(t) &= -0.707\cos\omega_0 t + 0.707\sin\omega_0 t \\ &= \sin\left(\omega_0 t - \frac{\pi}{4}\right) \end{aligned}$$



Gray code

Data encoding

x_k	y_k	z_k	$\Delta\phi_k$
0	0	0	0
0	0	1	$\pi/4$
0	1	1	$2\pi/4$
0	1	0	$3\pi/4$
1	1	0	$4\pi/4$
1	1	1	$5\pi/4$
1	0	1	$6\pi/4$
1	0	0	$7\pi/4$

Differential data phasor

$\phi_k = \phi_{k-1} + \Delta\phi_k$

Let $\phi_0 = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$
$x_k\, y_k\, z_k$	110	001	110	010
$\Delta\phi_k$:	π	$\pi/4$	π	$3\pi/4$
ϕ_k:	π	$5\pi/4$	$\pi/4$	π
I:	−1	−0.707	0.707	−1
Q:	0	−0.707	0.707	0

Figure 4.23 Quadrature implementation of a D8PSK modulator.

Complex Envelope

- D8PSK Demodulator

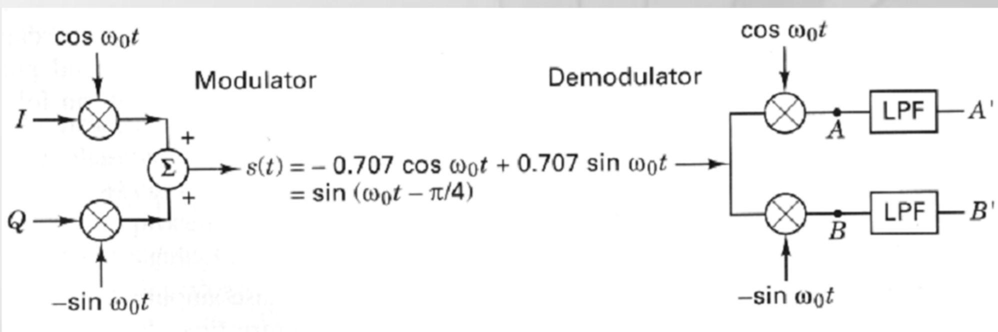


Figure 4.24 Modulator/demodulator example.

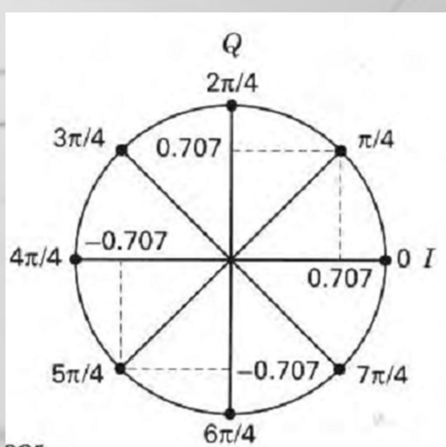
$$A = (-0.707 \cos \omega_0 t + 0.707 \sin \omega_0 t) \cos \omega_0 t$$
$$= -0.707 \cos^2 \omega_0 t + 0.707 \sin \omega_0 t \cos \omega_0 t$$

$$A = \frac{-0.707}{2}(1 + \cos 2\omega_0 t) + \frac{0.707}{2} \sin 2\omega_0 t$$

$$A' = -0.707 \text{ (times a scale factor)}$$

$$B = (-0.707 \cos \omega_0 t + 0.707 \sin \omega_0 t)(-\sin \omega_0 t)$$
$$= \frac{0.707}{2} \sin 2\omega_0 t - \frac{0.707}{2}(1 - \cos 2\omega_0 t)$$

$$B' = -0.707 \text{ (times a scale factor)}$$



earlier time $k = 1$

the signal phase to be π

$$\Delta\phi_{k=2} = \phi_{k=2} - \phi_{k=1} = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$$

$$x_2 \, y_2 \, z_2 = 001$$

Error Performance for Binary Systems

- Probability of Bit Error for Coherently Detected BPSK
  - Probability of symbol error, P_E
  - Probability of bit error, P_B

$$\left.\begin{aligned} s_1(t) &= \sqrt{E}\,\psi_1(t) \\ s_2(t) &= -\sqrt{E}\,\psi_1(t) \end{aligned}\right\} 0 \le t \le T$$

$$\begin{array}{ll} s_1(t) & \text{if } z(T) > \gamma_0 = 0 \\ s_2(t) & \text{otherwise} \end{array}$$

$$P_B = \int_{(a_1-a_2)/2\sigma_0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$$

$$Q(X) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) du$$

$$P_B = \int_{\sqrt{2E_b/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$
$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Example 4.4 Bit Error Probability for BPSK Signaling

Find the bit error probability for a BPSK system with a bit rate of 1 Mbit/s. The received waveforms $s_1(t) = A \cos \omega_0 t$ and $s_2(t) = -A \cos \omega_0 t$, are coherently detected with a matched filter. The value of A is 10 mV. Assume that the single-sided noise power spectral density is $N_0 = 10^{-11}$ W/Hz and that signal power and energy per bit are normalized relative to a 1-Ω load.

Solution

$$A = \sqrt{\frac{2E_b}{T}} = 10^{-2}\ \text{V} \qquad T = \frac{1}{R} = 10^{-6}\ \text{s}$$

Thus,

$$E_b = \frac{A^2}{2} T = 5 \times 10^{-11}\ \text{J} \quad \text{and} \quad \sqrt{\frac{2E_b}{N_0}} = 3.16$$

Also,

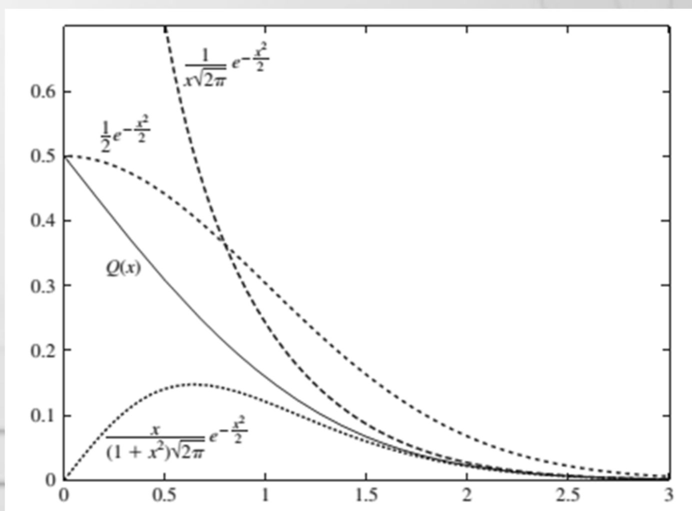
$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(3.16)$$

Using Table B.1 or Equation (3.44), we obtain

$$P_B = 8 \times 10^{-4}$$

Error Performance for Binary Systems

- Q function



Plot of $Q(x)$ and its upper and lower bounds.

From the last two bounds we conclude that for large x we have

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

TABLE 2.3–1
Table of Q Function Values

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0	0.500000	1.8	0.035930	3.6	0.000159	5.4	3.3320×10^{-8}
0.1	0.460170	1.9	0.028717	3.7	0.000108	5.5	1.8990×10^{-8}
0.2	0.420740	2	0.022750	3.8	7.2348×10^{-5}	5.6	1.0718×10^{-8}
0.3	0.382090	2.1	0.017864	3.9	4.8096×10^{-5}	5.7	5.9904×10^{-9}
0.4	0.344580	2.2	0.013903	4	3.1671×10^{-5}	5.8	3.3157×10^{-9}
0.5	0.308540	2.3	0.010724	4.1	2.0658×10^{-5}	5.9	1.8175×10^{-9}
0.6	0.274250	2.4	0.008198	4.2	1.3346×10^{-5}	6	9.8659×10^{-10}
0.7	0.241960	2.5	0.006210	4.3	8.5399×10^{-6}	6.1	5.3034×10^{-10}
0.8	0.211860	2.6	0.004661	4.4	5.4125×10^{-6}	6.2	2.8232×10^{-10}
0.9	0.184060	2.7	0.003467	4.5	3.3977×10^{-6}	6.3	1.4882×10^{-10}
1	0.158660	2.8	0.002555	4.6	2.1125×10^{-6}	6.4	7.7689×10^{-11}
1.1	0.135670	2.9	0.001866	4.7	1.3008×10^{-6}	6.5	4.0160×10^{-11}
1.2	0.115070	3	0.001350	4.8	7.9333×10^{-7}	6.6	2.0558×10^{-11}
1.3	0.096800	3.1	0.000968	4.9	4.7918×10^{-7}	6.7	1.0421×10^{-11}
1.4	0.080757	3.2	0.000687	5	2.8665×10^{-7}	6.8	5.2309×10^{-12}
1.5	0.066807	3.3	0.000483	5.1	1.6983×10^{-7}	6.9	2.6001×10^{-12}
1.6	0.054799	3.4	0.000337	5.2	9.9644×10^{-8}	7	1.2799×10^{-12}
1.7	0.044565	3.5	0.000233	5.3	5.7901×10^{-8}	7.1	6.2378×10^{-13}

Error Performance for Binary Systems

- Probability of Bit Error for Coherently Detected BPSK , DBPSK

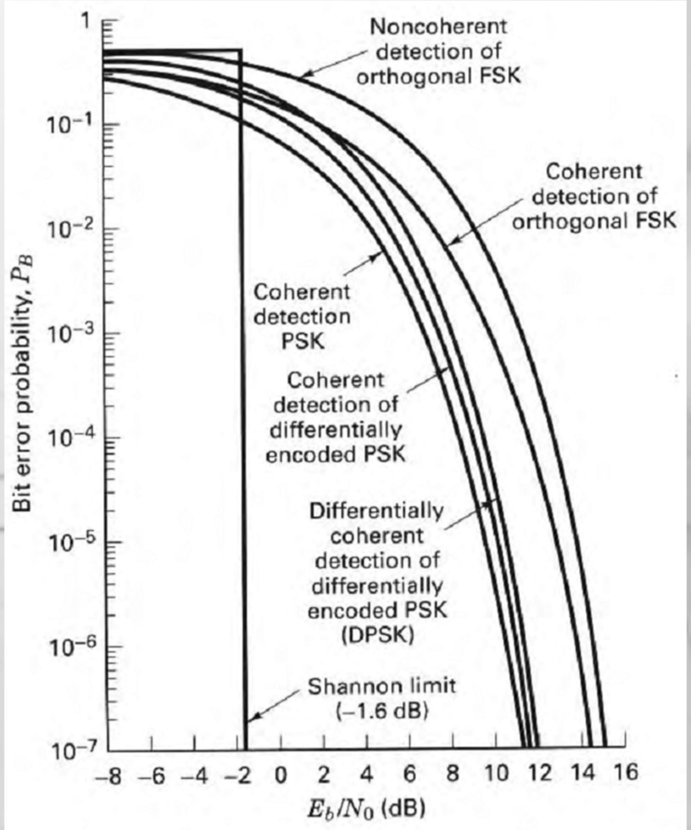


Figure 4.25 Bit error probability for several types of binary systems.

BPSK

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Differentially Encoded Binary PSK

$$P_B = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

Coherent Binary Orthogonal FSK

$$P_B = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp\left(-\frac{u^2}{2}\right) du = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Noncoherent Binary Orthogonal FSK

$$P_B = \frac{1}{2} \exp\left(-\frac{A^2T}{4N_0}\right) = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

Binary DPSK

$$P_B = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$$

Error Performance for Binary Systems

- Probability of Bit Error for Coherently Detected BPSK , DBPSK

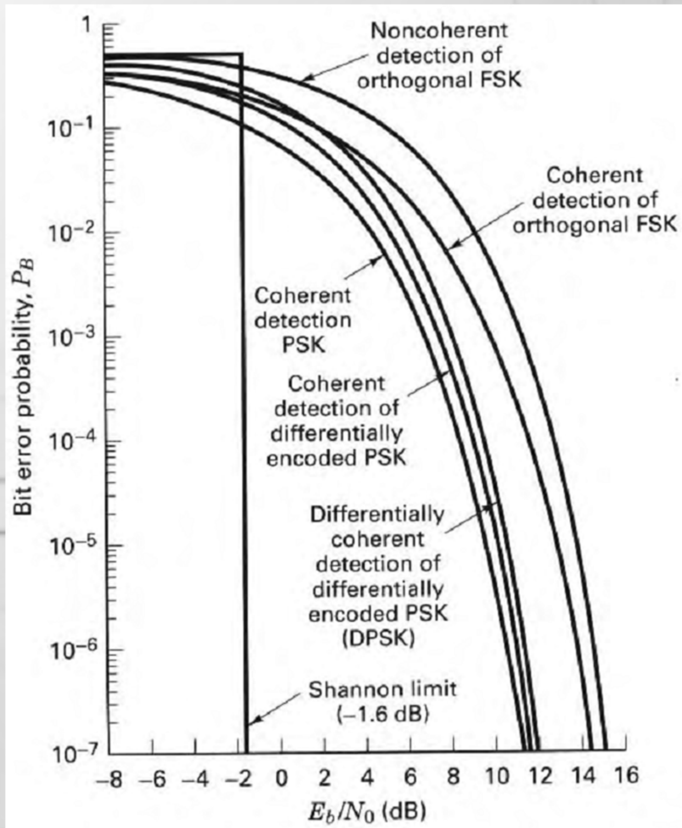


Figure 4.25 Bit error probability for several types of binary systems.

TABLE 4.1 Probability of Error for Selected Binary Modulation Schemes

Modulation	P_B
PSK (coherent)	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
DPSK (differentially coherent)	$\frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$
Orthogonal FSK (coherent)	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
Orthogonal FSK (noncoherent)	$\frac{1}{2}\exp\left(-\frac{1}{2}\frac{E_b}{N_0}\right)$

M-ary Signaling and Performance

Ideal Probability of Bit Error Performance

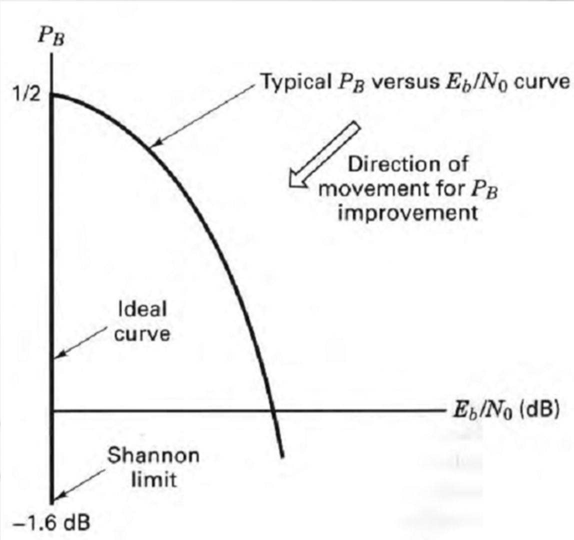


Figure 4.27 Ideal P_B versus E_b/N_0 curve.

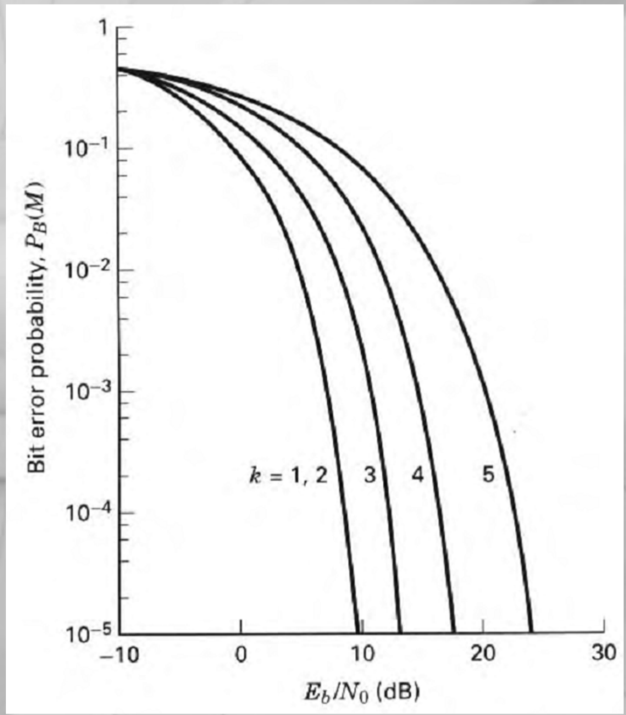


Figure 4.29 Bit error probability for coherently detected multiple phase signaling.

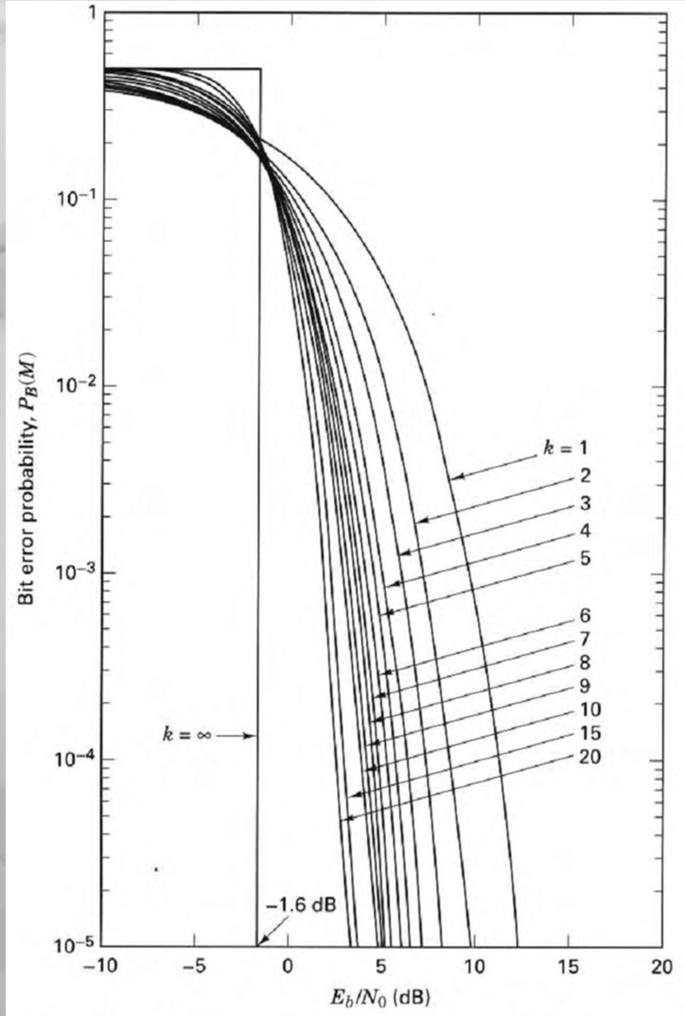


Figure 4.28 Bit error probability for coherently detected *M*-ary orthogonal signaling. (Reprinted from W. C. Lindsey and M. K. Simon, *Telecommunication Systems Engineering,* Prentice Hall, Inc., Englewood Cliffs, N.J., 1973, courtesy of W. C. Lindsey and Marvin K. Simon).

M-ary Signaling and Performance

Vectorial View of MPSK Signaling

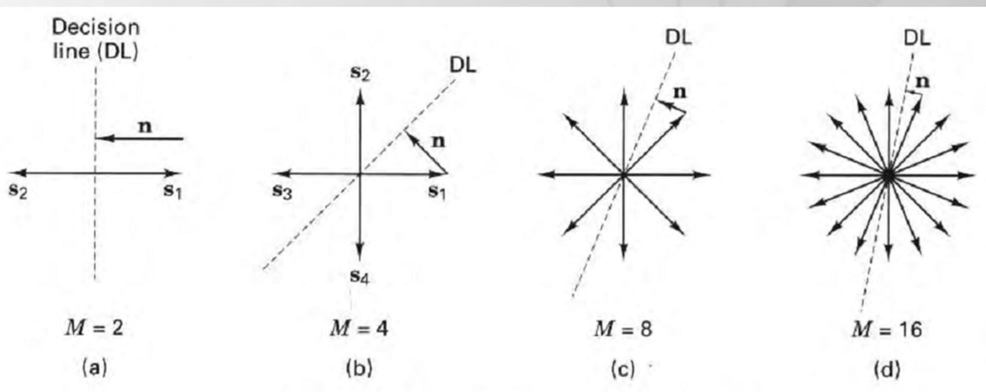


Figure 4.30 MPSK signal sets for $M = 2, 4, 8, 16$.

BPSK and QPSK Have the Same Bit Error Probability

BPSK

$$\frac{E_b}{N_0} = \frac{S}{N}\left(\frac{W}{R}\right)$$

QPSK

$$\frac{E_b}{N_0} = \frac{S/2}{N_0}\left(\frac{W}{R/2}\right) = \frac{S}{N_0}\left(\frac{W}{R}\right)$$

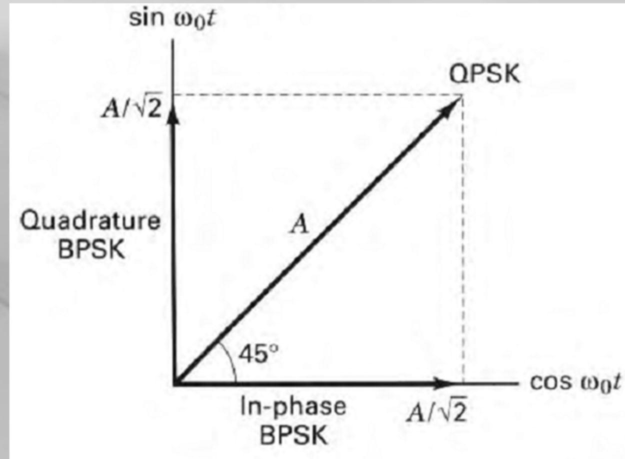


Figure 4.31 In-phase and quadrature BPSK components of QPSK signaling.

M-ary Signaling and Performance

Vectorial View of MFSK Signaling

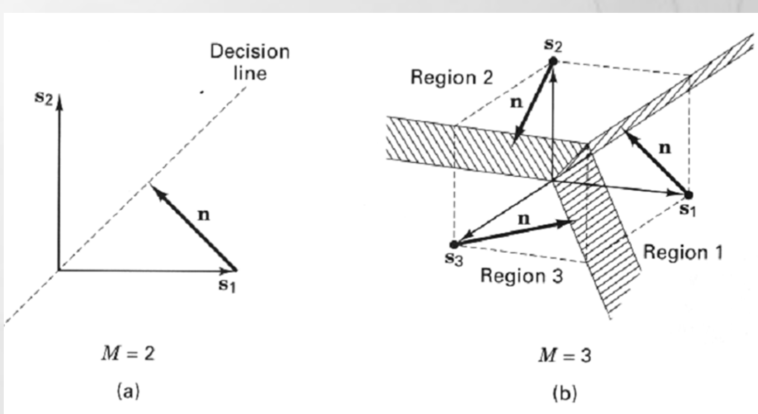


Figure 4.32 MFSK signal sets for $M = 2, 3$.

$$\frac{E_b}{N_0} = \frac{S}{N}\left(\frac{WT}{\log_2 M}\right) = \frac{S}{N}\left(\frac{WT}{k}\right)$$

$$WT \approx 1$$

$$\frac{E_b}{N_0} \approx \frac{S}{N}\left(\frac{1}{k}\right)$$

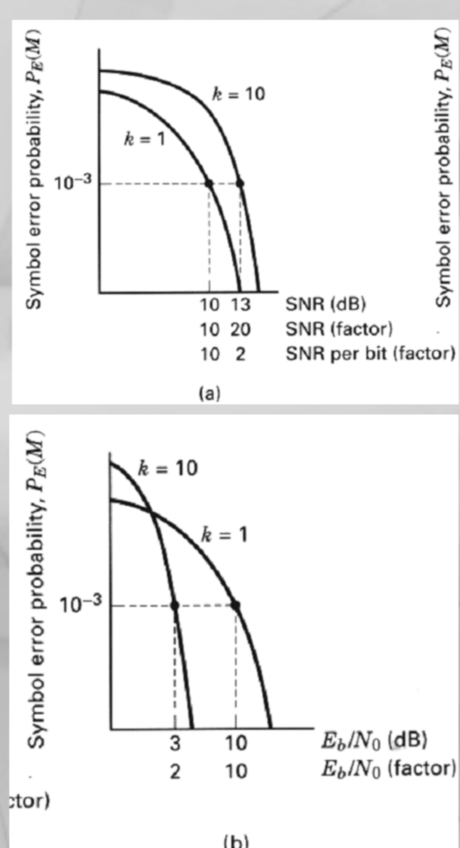


Figure 4.34 Mapping P_E versus SNR into P_E versus E_b/N_0 for orthogonal signaling. (a) Unnormalized. (b) Normalized.

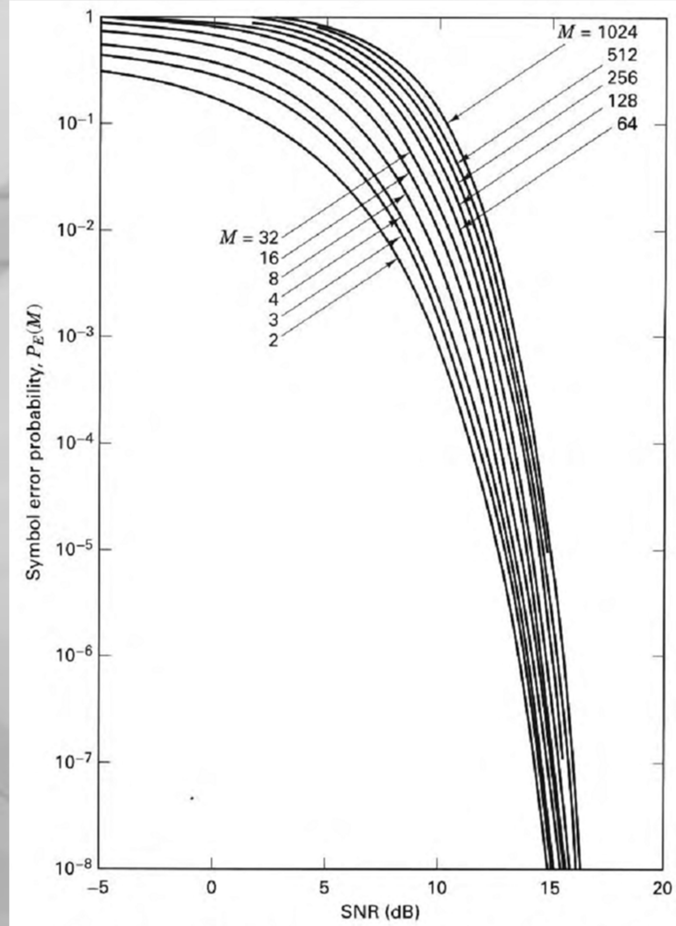


Figure 4.33 Symbol error probability versus SNR for coherent FSK signaling. (From Bureau of Standards, *Technical Note 167,* March 1963.) (Reprinted from *Central Radio Propagation Laboratory Technical Note 167,* March 25, 1963, Fig. 1, p. 5, courtesy of National Bureau of Standards.)

Symbol Error Performance for M-ary System (M>2)

Probability of Symbol Error for MPSK

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$$

Probability of Symbol Error for DPSK (for large E_s/N_0)

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{\sqrt{2}M}\right)$$

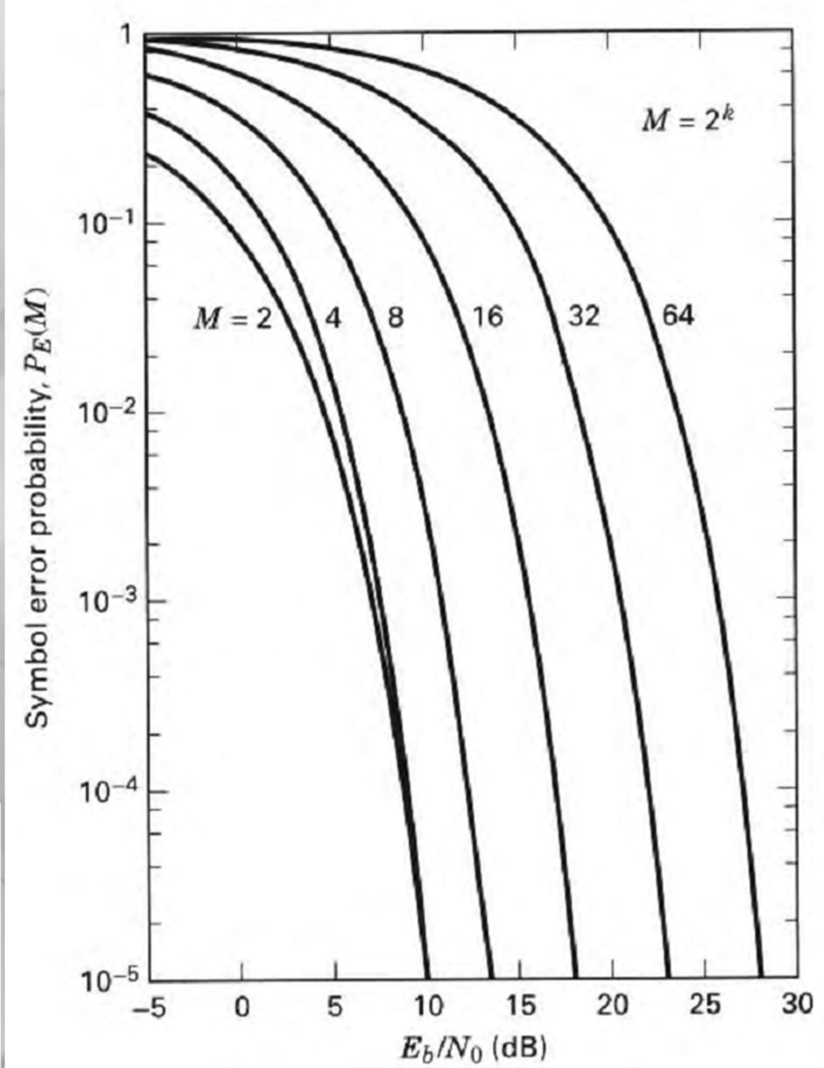


Figure 4.35 Symbol error probability for coherently detected multiple phase signaling. (Reprinted from W. C. Lindsey and M. K. Simon, *Telecommunication Systems Engineering,* Prentice-Hall, Inc. Englewood Cliffs, N.J., 1973, courtesy of W. C. Lindsey and Marvin K. Simon.)

Symbol Error Performance for M-ary System (M>2)

Probability of Symbol Error for MFSK

coherently detected

$$P_E(M) \le (M-1)\, Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

noncoherently detected

$$P_E(M) < \frac{M-1}{2} \exp\left(-\frac{E_s}{2N_0}\right)$$

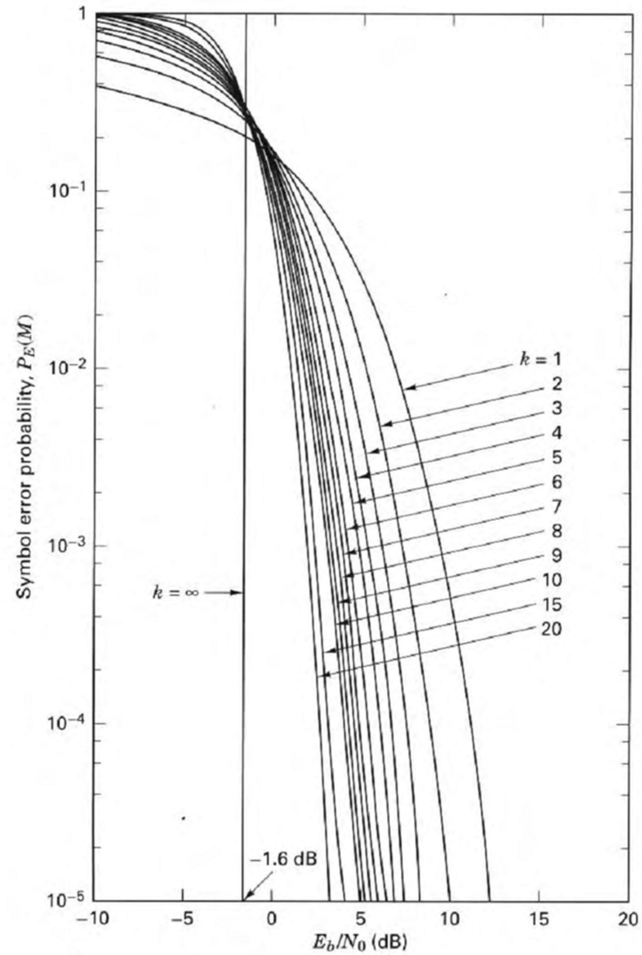


Figure 4.36 Symbol error probability for coherently detected *M*-ary orthogonal signaling. (Reprinted from W. C. Lindsey and M. K. Simon, *Telecommunication Systems Engineering,* Prentice-Hall, Inc., Englewod Cliffs, N.J., 1973, courtesy of W. C. Lindsey and Marvin K. Simon.)

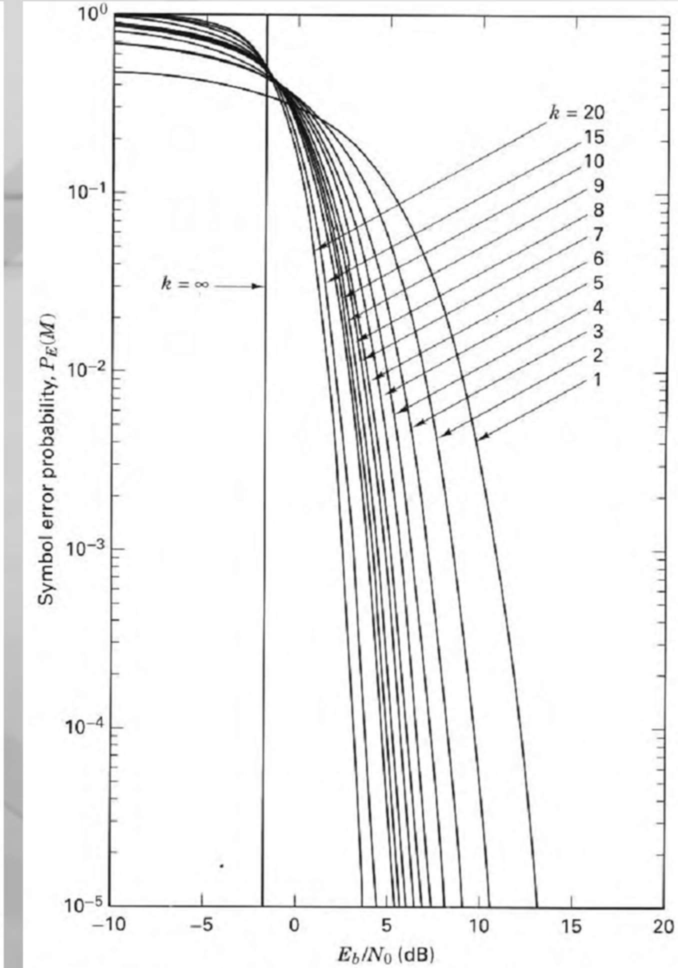


Figure 4.37 Symbol error probability for noncoherently detected *M*-ary orthogonal signaling. (Reprinted from W. C. Lindsey and M. K. Simon, *Telecommunication Systems Engineering,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1973, courtesy of W. C. Lindsey and Marvin K. Simon.)

Bit Error Performance for M-ary System (M>2)

Bit Error Probability versus Symbol Error Probability for Orthogonal Signals

It can be shown [9] that the relationship between probability of bit error (P_B) and probability of symbol error (P_E) for an M-ary orthogonal signal set is

$$\frac{P_B}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1} \qquad (4.112)$$

In the limit as k increases, we get

$$\lim_{k\to\infty} \frac{P_B}{P_E} = \frac{1}{2}$$

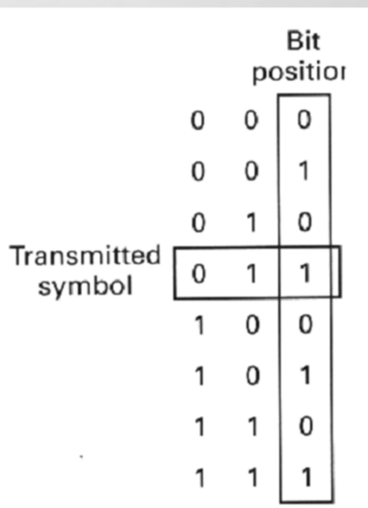


Figure 4.38 Example of P_B versus P_E.

Bit Error Probability Versus Symbol Error Probability for Multiple Phase Signaling

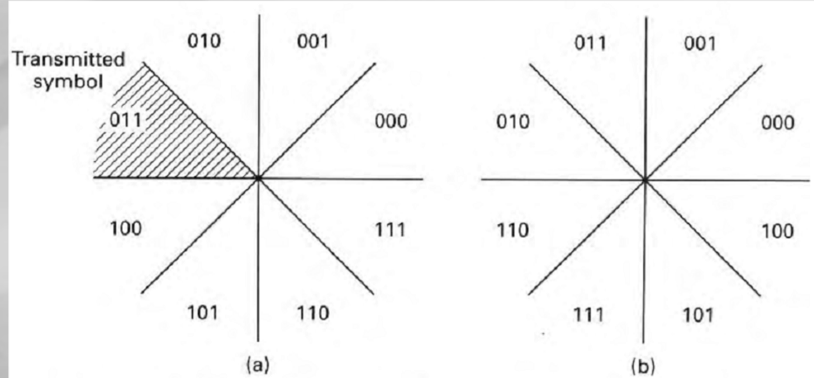


Figure 4.39 Binary-coded versus Gray-coded decision regions in an MPSK signal space. (a) Binary coded. (b) Gray coded.

$$P_B \approx \frac{P_E}{\log_2 M} \qquad (\text{for } P_E \ll 1)$$

Conclusion

We have catalogued some basic bandpass digital modulation formats, particularly phase shift keying (PSK) and frequency shift keying (FSK). We have considered a geometric view of signal vectors and noise vectors, particularly antipodal and orthogonal signal sets. This geometric view allows us to consider the detection problem in the light of an orthogonal signal space and signal regions. This view of the space, and the effect of noise vectors causing transmitted signals to be received in the incorrect region, facilitates the understanding of the detection problem and the performance of various modulation and demodulation techniques. In Chapter 9 we reconsider the subjects of modulation and demodulation, and we investigate some bandwidth-efficient modulation techniques.

Problems and Questions

- Problems

4.3. If a system's main performance criterion is bit error probability, which of the following two modulation schemes would be selected for an AWGN channel? Show computations.

Binary noncoherent orthogonal FSK with $E_b/N_0 = 13$ dB

Binary coherent PSK with $E_b/N_0 = 8$ dB

4.4. The bit stream

1 0 1 0 1 0 1 1 1 1 0 1 0 1 0 1 0 0 0 0 1 1 1 1

is to be transmitted using DPSK modulation. Show four different differentially encoded sequences that can represent the data sequence above, and explain the algorithm that generated each.

4.5. **(a)** Calculate the minimum required bandwidth for a noncoherently detected orthogonal binary FSK system. The higher-frequency signaling tone is 1 MHz and the symbol duration is 1 ms.

(b) What is the minimum required bandwidth for a noncoherent MFSK system having the same symbol duration?

4.8. Find the optimum (minimum probability of error) threshold γ_0, for detecting the equally likely signals $s_1(t) = \sqrt{2E/T} \cos \omega_0 t$ and $s_2(t) = \sqrt{\frac{1}{2} E/T} \cos (\omega_0 t + \pi)$ in AWGN, using a correlator receiver as shown in Figure 4.7b. Assume a reference signal of $\psi_1(t) = \sqrt{2/T} \cos \omega_0 t$.

Problems and Questions

- Questions

4.1. At what location in the system is E_b/N_0 defined? (See Section 4.3.2.)

4.2. Amplitude- or phase-shift keying is visualized as a constellation of points or phasors on a plane. Why can't we use a similarly simple visualization for orthogonal signaling such as FSK? (See Section 4.4.4.)

4.3. In the case of MFSK signaling, what is the minimum tone spacing that insures signal *orthogonality?* (See Section 4.5.4.)

4.4. What benefits are there in using *complex notation* for representing sinusoids? (See Sections 4.2.1 and 4.6.)

4.5. Digital modulation schemes fall into one of the two classes with opposite behavior characteristics: *orthogonal* signaling, and *phase/amplitude* signaling. Describe the behavior of each class. (See Sections 4.8.2.)

4.6. Why do binary phase shift keying (BPSK) and quaternary phase shift keying (QPSK) manifest the same bit-error-probability relationship? (See Section 4.8.4.)

4.7. In the case of multiple-phase shift keying (MPSK), why does *bandwidth efficiency* improve with higher dimensional signaling? (See Sections 4.8.2 and 4.8.3.)

4.8. In the case of orthogonal signaling such as MFSK, why does *error-performance* improve with higher dimensional signaling? (See Section 4.8.5.)

4.9. The use of a Gray code for assigning bits to symbols, represents one of the few cases in digital communications where a benefit can be achieved *free-of-charge.* Explain why there is no cost. (See Section 4.9.4.)