Seat Number

King Mongkut's University of Technology Thonburi
Final Examination
Semester 1 – Academic Year 2017

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3rd Yr (Inter. Program)
Exam Date: Thursday December 7, 2017 **Time:** 9.00am-12.00pm

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Key 20/12/2017.

Instructions:-

1. This exam consists of 4 problems with a total of 15 pages, including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rule is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds … No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment within university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Question No.	1	2	3	4	TOTAL
Full Score	25	25	25	25	100
Graded Score					

Name ______________________________ **Student ID** ______________

This examination is designed by
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This examination has been approved by the committees of the ENE department.

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Problem 1: Probability Distributions [25 points]

Random variables V, W, X, and Y have the following probability distributions:

$$\mathbb{P}\{V=k\} = \begin{cases} \frac{e^{-10}10^k}{k!}, & k=0,1,2,3,\ldots \\ 0, & \text{otherwise} \end{cases}$$

$$f_W(t) = \frac{1}{\sqrt{6\pi}} e^{-\frac{1}{6}(t+1)^2}, \qquad -\infty < t < \infty$$

$$f_X(t) = \begin{cases} 2e^{-2t}, & t>0 \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{P}\{Y=k\} = \begin{cases} \binom{5}{k} 0.3^k 0.7^{5-k}, & k=0,1,2,\ldots,5 \\ 0, & \text{otherwise.} \end{cases}$$

Answer the questions below. You <u>do not</u> need to justify your answers.

(a) [5 points] Is V discrete or continuous? What is the distribution of V? Circle a correct answer form each column.

Answer: V is a $\left\{ \begin{array}{c} \text{discrete} \\ \text{continuous} \end{array} \right\}$ random variable that has $\left\{ \begin{array}{c} \text{a normal} \\ \text{a Poisson} \\ \text{a uniform} \\ \text{a binomial} \\ \text{an exponential} \end{array} \right\}$ distribution.

(b) [5 points] Is W discrete or continuous? What is the distribution of W? Circle a correct answer form each column.

Answer: W is a $\left\{ \begin{array}{c} \text{discrete} \\ \text{continuous} \end{array} \right\}$ random variable that has $\left\{ \begin{array}{c} \text{a normal} \\ \text{a Poisson} \\ \text{a uniform} \\ \text{a binomial} \\ \text{an exponential} \end{array} \right\}$ distribution.

(c) [5 points] What are the parameters of random variable X? Circle a correct answer.

(A) The number of trials $n = 5$ and the probability of success $p = 0.3$

(B) The mean $\mu = 1$ and the standard deviation $\sigma = 3$

(C) The rate parameter $\lambda = -2$

(D) The mean $\mu = 10$

(E) None of the above

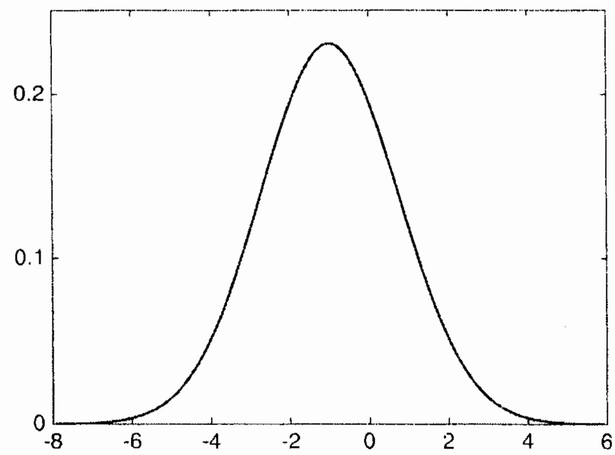
(d) [5 points] What are the expected value and the variance of random variable Y? Fill in the blanks.

Answer:

Expected value = ________________________________

Variance = ________________________________

(e) [5 points] What is the graph below? Circle a correct answer from each column.



Answer: This graph is the $\left\{ \begin{array}{c} \text{probability mass function (pmf)} \\ \text{probability density function (pdf)} \\ \text{cumulative distribution function (cdf)} \end{array} \right\}$

of random variable $\left\{ \begin{array}{c} V \\ W \\ X \\ Y \end{array} \right\}$.

Problem 2: University Courses [25 points]

For a certain university, let $X =$ the number of courses for which a randomly-selected student is registered. Suppose that X has the following probability mass function (pmf):

x	0	1	2	3	4
$p_X(x)$	0.1	0.1	0.3	0.3	0.2

(a) [6 points] Find the probability that a randomly-selected student registers for two or more courses.

(b) [6 points] Plot the cumulative distribution function (cdf) of X.

(c) [7 points] At this university, every student pays a flat-rate fee of 5000 baht, plus a fee of 1000 baht per each course for which the student registers. A student who registers for X courses will pay the total fee of

$$W = 5000 + 1000X$$

baht to the university.

Find the standard deviation of W.

(d) [6 points] At another university, let Y = the number of courses for which a randomly-selected student is registered. Suppose that Y is independent of X and has the following pmf:

y	0	1	2	3
$p_Y(y)$	0	0.2	0.3	0.5

Compute the probability $\mathbb{P}\{X = 1 \text{ or } Y = 2\}$.

Problem 3: Reaction Time [25 points]

The reaction time (in seconds) to a certain stimulus is a continuous random variable with the probability density function (pdf)

$$f_X(x) = \begin{cases} \frac{4}{3} \cdot \frac{1}{x^2}, & 1 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

(a) [6 points] Obtain the cumulative distribution function (cdf).

(b) [5 points] What is the probability that reaction time is between 2 and 3 sec?

(c) [6 points] Compute the expected reaction time.

(d) [8 points] If an individual takes more than 2 sec to react, a light comes on and stays on either until one further second has elapsed or until the person reacts (whichever happens first). Determine the expected amount of time that the light remains lit.

Problem 4: Dinner [25 points]

A restaurant offers three fixed-price dinners costing 100 baht, 200 baht, and 300 baht. For a randomly-selected couple dinning at the restaurant, let X = the cost of the man's dinner and Y = the cost of the woman's dinner. The joint probability mass function (pmf) of X and Y is given in the following table.

$p_{X,Y}(x,y)$		y = 100	200	300
	100	0.05	0.05	0.10
x	200	0.05	0.10	0.35
	300	0	0.20	0.10

(a) [6 points] What is the probability that the man's dinner costs at most 200 baht and the woman's dinner costs at most 200 baht?

(b) [6 points] Fill in the blanks below for the marginal pmf of X and the marginal pmf of Y. You do not need to justify the answers.

$\mathbb{P}\{X = 100\} =$ ______________ $\mathbb{P}\{Y = 100\} =$ ______________

$\mathbb{P}\{X = 200\} =$ ______________ $\mathbb{P}\{Y = 200\} =$ ______________

$\mathbb{P}\{X = 300\} =$ ______________ $\mathbb{P}\{Y = 300\} =$ ______________

(c) [5 points] Given that the man's dinner costs 200 baht, what is the conditional probability that the woman's dinner costs 100 baht?

(d) [8 points] What is the expected total cost of dinner for the two people?

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Formula Sheet

$$p_V(v) = \mathbb{P}\{V = v\} \geq 0, \text{ for every } v$$

$$\sum_{\text{all possible values } v\text{'s}} p_V(v) = \sum_{\text{all possible values } v\text{'s}} \mathbb{P}\{V = v\} = 1$$

$$F_V(v) = \mathbb{P}\{V \leq v\} = \sum_{i \text{ such that } i \leq v} \mathbb{P}\{V = i\}$$

The cdf of any rv—either discrete or continuous—has these properties: $\lim_{v \to -\infty} F_V(v) = 0$; $\lim_{v \to \infty} F_V(v) = 1$; and $x \leq y \implies F_V(x) \leq F_V(y)$.

$$\mathbb{E}\{X\} = \mu_X = \sum_{x \in \mathcal{D}} x \cdot p_X(x) = \sum_{x \in \mathcal{D}} x \cdot \mathbb{P}\{X = x\}$$

$$\mathbb{E}\{h(X)\} = \sum_{x \in \mathcal{D}} h(x) \cdot p_X(x) = \sum_{x \in \mathcal{D}} h(x) \cdot \mathbb{P}\{X = x\}$$

$$\mathbb{E}\{aX + b\} = a\mathbb{E}\{X\} + b$$

$$\mathbb{V}\{X\} = \sum_{x \in \mathcal{D}} (x - \mu)^2 \cdot \mathbb{P}\{X = x\} = \mathbb{E}\left\{\left(X - \mathbb{E}\{X\}\right)^2\right\}$$

$$\sigma_X = \sqrt{\sigma_X^2}$$

$$\mathbb{V}\{X\} = \left[\sum_{x \in \mathcal{D}} x^2 \mathbb{P}\{X = x\}\right] - \mu^2 = \mathbb{E}\{X^2\} - \left[\mathbb{E}\{X\}\right]^2$$

$$\mathbb{V}\{aX + b\} = a^2 \mathbb{V}\{X\}$$

Binomial distribution with parameters n and p:

$$\mathbb{P}\{X = x\} = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{E}\{X\} = np, \quad \mathbb{V}\{X\} = np(1-p)$$

Poisson distribution with parameter μ:

$$\mathbb{P}\{X = x\} = \begin{cases} \frac{e^{-\mu} \cdot \mu^x}{x!}, & x = 0, 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{E}\{X\} = \mathbb{V}\{X\} = \mu$$

$$\mathbb{P}\{a \le X \le b\} = \int_a^b f_X(x)dx \qquad \mathbb{P}\{X \ge c\} = \int_c^\infty f_X(x)dx$$

$$\mathbb{P}\{X \le c\} = \int_{-\infty}^c f_X(x)dx \qquad f_X(x) \ge 0, \text{ for each real number } x$$

$$\int_{-\infty}^\infty f_X(x)dx = 1$$

If X is a continuous rv and if c is a real number, then $\mathbb{P}\{X = c\} = 0$.

A uniform pdf on an interval $[a, b]$: $f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$

$\mathbb{E}\{X\} = \frac{b-a}{2}$, $\mathbb{V}\{X\} = \frac{(b-a)^2}{12}$

An exponential pdf with rate λ: $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

$\mathbb{E}\{X\} = \frac{1}{\lambda}$, $\mathbb{V}\{X\} = \frac{1}{\lambda^2}$

A normal or Gaussian pdf with mean μ and variance σ^2: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $-\infty < x < \infty$

$$F_X(x) = \mathbb{P}\{X \le x\} = \int_{-\infty}^x f_X(t)dt$$

$$\mathbb{P}\{X \le c\} = \mathbb{P}\{X < c\} = F_X(c)$$

$$\mathbb{P}\{X \ge c\} = \mathbb{P}\{X > c\} = 1 - F_X(c)$$

$$\mathbb{P}\{a \le X \le b\} = \mathbb{P}\{a \le X < b\} = \mathbb{P}\{a < X \le b\} = \mathbb{P}\{a < X < b\} = F_X(b) - F_X(a)$$

$$f_X(x) = \frac{d}{dx}F_X(x) \qquad \mathbb{E}\{X\} = \int_{-\infty}^\infty x f_X(x)dx$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax)$$

$$\int udv = uv - \int vdu \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \qquad \int xe^{-ax^2}dx = -\frac{1}{2a}e^{-ax^2}$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \qquad \int x^2e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax}$$

$$\int \frac{1}{x}dx = \ln x$$

$$\mathbb{E}\{h(X)\} = \int_{-\infty}^{\infty} h(x) \cdot f_X(x)dx$$

$$\mathbb{V}\{X\} = \mathbb{E}\left\{\left(X - \mathbb{E}\{X\}\right)^2\right\} = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)dx$$

$$p(x,y) = p_{X,Y}(x,y) = \mathbb{P}\{X = x \text{ and } Y = y\} = \mathbb{P}\{X = x, Y = y\}$$

$$\mathbb{P}\{(X,Y) \in A\} = \sum\sum\nolimits_{(x,y)\in A} p_{X,Y}(x,y), \text{ for any set } A \subseteq \mathbb{R}^2$$

$$p_X(x) = \sum_{y \text{ such that } p_{X,Y}(x,y)>0} p_{X,Y}(x,y)$$

$$p_Y(y) = \sum_{x \text{ such that } p_{X,Y}(x,y)>0} p_{X,Y}(x,y)$$

$$p_{Y|X}(y|x) = \frac{p_{X,Y}(x,y)}{p_X(x)}, \text{ for a given } x \text{ and for } -\infty < y < \infty$$

$$\mathbb{P}\{(X,Y) \in A\} = \iint_A f_{X,Y}(x,y)dxdy, \text{ for any set } A \subseteq \mathbb{R}^2$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy, \text{ for } -\infty < x < \infty$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx, \text{ for } -\infty < y < \infty$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}, \text{ for a given } x \text{ and for } -\infty < y < \infty$$

$$\mathbb{E}\{h(X,Y)\} = \sum\sum\nolimits_{\text{all possible values } (x,y)\text{'s}} h(x,y)p_{X,Y}(x,y)$$

$$\mathbb{E}\{h(X,Y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)f_{X,Y}(x,y)dxdy$$

$$\mathbb{E}\{aX + bY\} = a\mathbb{E}\{X\} + b\mathbb{E}\{Y\}$$