King Mongkut's University of Technology Thonburi
Final Exam of Second Semester, Academic Year 2007

COURSE CPE112 Discrete Mathematics for Computer Engineers
Computer Engineering Dept, 1st Yr. International Program

Monday 10 March 2008 **09.00-12.00 h.**

Instructions

1. This examination contains 4 problems, 9 pages (including this cover page).
2. Do all your work in these examination sheets.
3. Students are NOT allowed to use calculators.
4. Students are allowed to use paper-based dictionaries.
5. No books, notes, or any other documents can be taken into the examination room.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room.
Students must not take the examination and the answers out of the examination room.

Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by

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This examination is approved by the department of computer engineering

1. Relations (18 %)

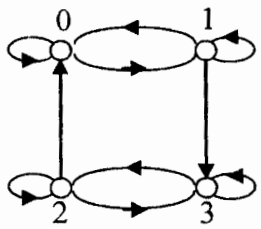
1.1) Consider the following relations on {0, 1, 2, 3}:

R1 = {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}

R2 = {(1, 1), (1, 2), (2, 1)}

R3 = {(a, b) | a divides b}

R4 = {(a, b) | $a \leq 2b$)}

R5 = 

1.1.1) Which of these relations are reflexive? (4 points)

1.1.2) Which of these relations are symmetric? (4 points)

1.1.3) Which of these relations are anti-symmetric? (4 points)

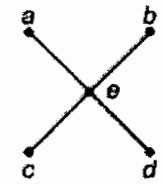
1.1.4) Which of these relations are transitive? (4 points)

1.1.5) Find R1 – R2 (2 points)

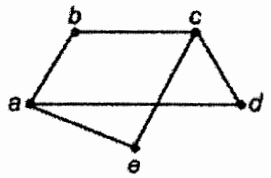
2. Graphs and Trees (30%)

2.1. Determine whether the following graph is bipartite. If it is bipartite, show the sets V_1 and V_2. (2% each: total 6%)

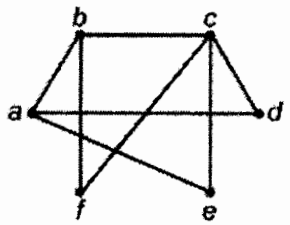
2.1.1)



2.1.2)



2.1.3)

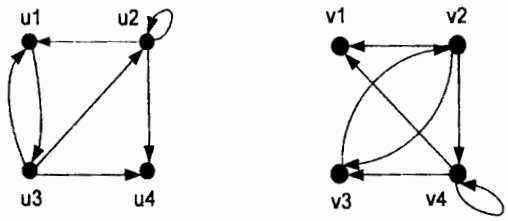


2.2. Determine whether the following pair of graphs is isomorphic. If it is isomorphic, show the function. (3% each: total 9%)

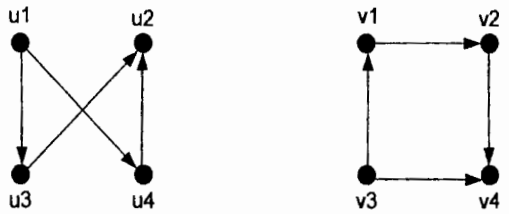
Definition The directed graph $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are isomorphic if there is one-to-one and onto function $f:V_1 \rightarrow V_2$ such that

For all pairs of vertices a and b in V_1, $(a,b) \in E_1$ iff $(f(a),f(b)) \in E_2$

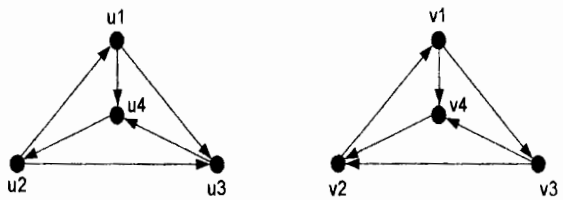
2.2.1)



2.2.2)

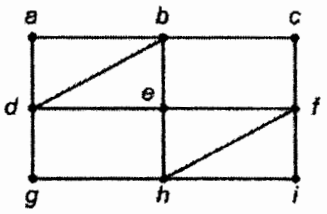


2.2.3)

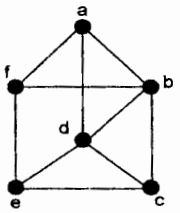


2.3. Euler Paths and Circuits (6%)

2.3.1) Determine whether the following graph has Euler circuit. If there is Euler circuit, show it. (3%)

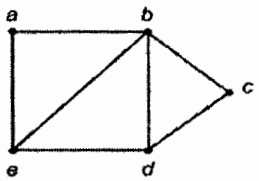


2.3.2) Determine whether the following graph has Euler circuit or path. If there is Euler circuit, show it. If there is Euler path, show it (3%)

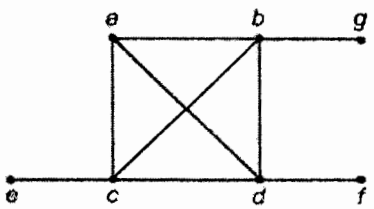


2.4. Hamilton paths and circuits (4%)

2.4.1) Determine whether the following graph has a Hamilton circuit. If it has, shows the Hamilton circuit. (2%)



2.4.2) Determine whether the following graph has a Hamilton path. If it has, shows the Hamilton path. (2%)



2.5) Tree properties (5%)

2.5.1) Consider a full 3-nary tree which has 30 nodes. How many number of edges does the tree have? (2%)

2.5.2) How many edges does a full binary tree with 500 internal vertices have? (3%)

3. Boolean Algebra (32%)

3.1) Find sum-of-product expansions of the following Boolean functions and then use Karnaugh Map technique to simplify them

3.1.1) $f(x,y,z) = xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z$ (3%)

3.1.2) $f(x,y,z) = \bar{y}(\bar{x}\bar{z}) + yx + y\bar{z}$ (3%)

3.1.3) $f(w,x,y,z) = \sum m(1,2,3,5,6,7,9,10,11,13)$ (4%)

3.2) Find the simplified sum of products of function F(w, x, y, z) that gives an output of 1 if a decimal digit, encoded using a binary coded decimal expression, is divisible by 3 or 4, and an output of 0 otherwise. Show your work with Karnaugh map (10%)
Hint: Consider Don't Care Condition

3.3) Fill in the table with values (in various based systems) of the given numbers. For example: If the value is given in base 2 = 1111. We can covert it to bases 8, 16 and 10 with the corresponding values equal to 17, F, and 15, respectively. (12%)

Base	**Number Representation**			
Base 2	10010.0	=	=	=
Base 8	=	27.3	=	=
Base 16	=	=	DF.B	=
Base 10	=	=	=	33.5
	Base 2	Base 8	Base 16	Base 10

4. Automata (20%)

4.1) Determine whether 1011 is in the languages which are represented by the following regular expressions (5%)

4.1.1) 10^*1^*

4.1.2) $1^*01(0 \cup 1)$

4.1.3) $0^*(10 \cup 11)^*$

4.1.4) $10^*1(00)^*$

4.1.5) $(10)^* \cup (11)^*$

4.2) Let $\Sigma=\{a, b\}$. Write down regular expression for the following languages.

4.2.1) All strings of length 2. (3 %)

4.2.2) All strings that contain *aaaaa* as a substring. (4%)

4.3) Let $\Sigma=\{a, b\}$. Construct a DFA which accepts the language described by the regular expression $a(a \cup b)^* b$? (8 %)