เลขที่นั่งสอบ..........

มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี

การสอบกลางภาคเรียน 1 / 2550

วิชา MEE 462 Vibrations นักศึกษาวิศวกรรมเครื่องกลปีที่ 4

สอบวันที่ 8 สิงหาคม พ.ศ. 2550 เวลา 9.00 น.- 12.00น.

ผศ. มนัสพงษ์ ชมอุตม์

ผศ.ดร. สาทิสส์ ทรงชน

ดร.อนันทวิทย์ ตู้จินดา

ผู้ออกข้อสอบ

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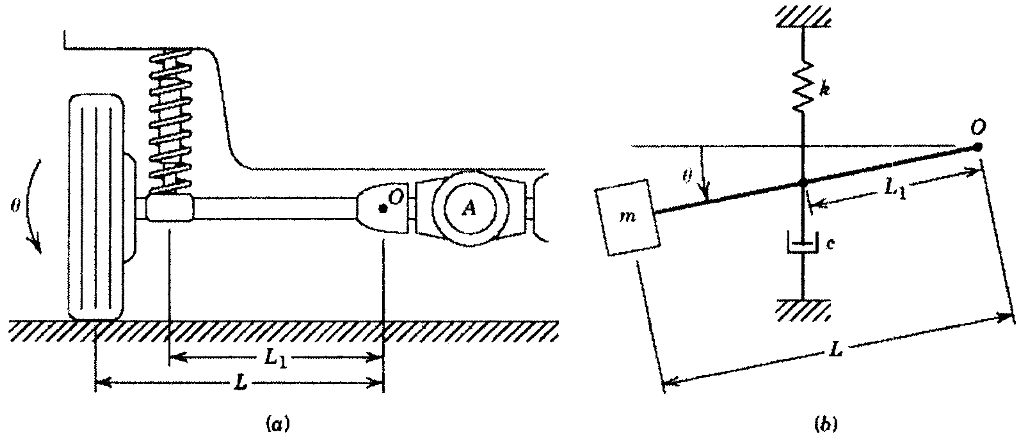
คำเตือน

1. ข้อสอบทั้งหมดมี 5 ข้อ ทำลงในข้อสอบและมีสูตรให้อยู่สองแผ่นท้ายข้อสอบ
2. ห้ามนำเอกสารทุกชนิดเข้าห้องสอบ
3. อนุญาตให้นำเครื่องคิดเลขเข้าห้องสอบ

1.) Given the system shown in the figure 1



(a) A swing-axle suspension. (b) Suspension model.

Figure 1

1.1) Draw free body diagrams of the suspension model (5 marks)

1.2) Use the appropriate form of Newton's law to derive the differential equation governing the motion (7 marks)

1.3) Determine the damping ratio (ζ) (4 marks)

1.4) Determine the damped natural frequency if $0<\zeta<1$ (4 marks)

2.) Consider the system shown in the figure 2. Determine free response of systems for 2.1 and 2.2 with 10 mm. initial displacement and zero initial velocity

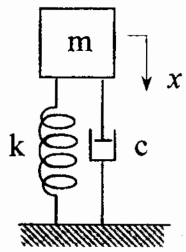


Figure 2

2.1) Given $k = 5000N/m$, $m = 10kg$ and $c = 224Ns/m$ (10 marks)

2.2) Given $k = 5000N/m$, $m = 10kg$ and $c = 492Ns/m$ (10 marks)

3.) Figure 3 shows a system being driven by base excitation through a damping element. Assume that the base displacement is sinusoidal: $y(t) = Y \sin \omega t$

3.1) Draw free body diagrams (3 marks)

3.2) Use the appropriate form of Newton's law to derive the differential equation governing the motion (4 marks)

3.3) Derive the expression for X ,the steady-state amplitude of motion of the mass m (10 marks)

3.4) Derive the expression for F_t ,the steady-state amplitude of the force transmitted to the support. (3 marks)

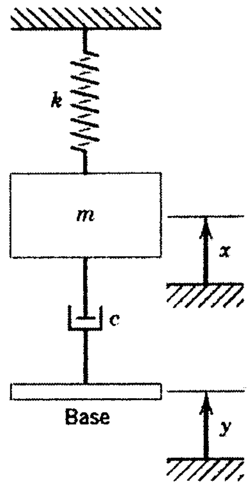


Figure 3

4.) A beam which is pivoted at point O and supported on the right end by a spring and on the left end by a dashpot is shown in Figure 4. A moment $M(t) = 10\,u_s(t)$, where $u_s(t)$ is the unit step function, is applied to the beam. The spring stiffness, *k*, is 1600 N/m and the damping coefficient, *c*, is 96 Ns/m. Assuming that the beam is rigid, has the moment of inertia about the point O, I_0, of 1 kg.m^2 and has the length, *L*, of 0.25 m and the length, *D*, of 0.75 m. Note: the rotation angle, θ, is measured with respect to the static equilibrium position (i.e. the effect of the beam's weight can be ignored).

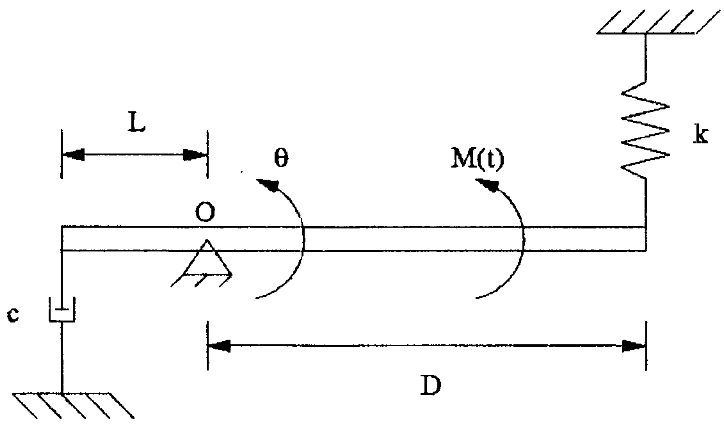


Figure 4: A beam is excited by a step moment.

4.1) Draw a free body diagram of the above system. (2 marks)

4.2) Derive the equation of motion. (3 marks)

4.3)Use the Laplace transform and the initial conditions $\theta(0) = \dot{\theta}(0) = 0$ to obtain the general expression for the step response of the above system. (8 marks)

4.4) Sketch the response of the system from t = 0 s to t = 1 s. (3 marks)

4.5) Estimate the maximum rotation of the response. (4 marks)

5.) A mass-dashpot-spring system shown in Figure 5 is excited by a unit impulse, $\delta(t)$. The displacement x is measured with respect to the fixed ground when the system is in static equilibrium. The values of the mass, the damping coefficient and the spring stiffness are 1 kg, 20 Ns/m and 100 N/m, respectively.

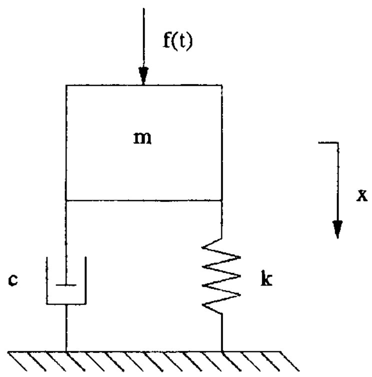


Figure 5

5.1) Draw a free body diagram of the above system. (1 marks)

5.2) Derive the equation of motion (1 marks)

5.3) Use the Laplace transform and the initial conditions $x(0) = \dot{x}(0) = 0$ to obtain the general expression for the impulse response of the above system. (6 marks)

5.4) Sketch the response of the system from t = 0 s to t = 1 s. (4 marks)

5.5) Find the time taken before the response reduces to 20% of the maximum response. (5 marks)

5.6) If the damping decreases, sketch what the response would look like and explain why it should take that form? (3 marks)

Free response of the 1-DOF:

$$x(t)=\frac{e^{-\zeta\omega_n t}}{2\sqrt{\zeta^2-1}}\left\{\left[\frac{\dot{x}_0}{\omega_n}+x_0(\zeta+\sqrt{\zeta^2-1})\right]e^{\omega_n t\sqrt{\zeta^2-1}}+\right.$$

$$\left.\left[-\frac{\dot{x}_0}{\omega_n}+x_0(-\zeta+\sqrt{\zeta^2-1})\right]e^{-\omega_n t\sqrt{\zeta^2-1}}\right\} \quad \textit{if overdamped}$$

$$x(t)=e^{-\omega_n t}[x_0+(\dot{x}_0+\omega_n x_0)t] \quad \textit{if critically damped}$$

$$x(t)=\frac{\sqrt{(\dot{x}_0+\zeta\omega_n x_0)^2+(x_0\omega_d)^2}}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t+\phi) \quad \textit{if underdamped}$$

$$\phi=\tan^{-1}\left(\frac{x_0\omega_d}{\dot{x}_0+\zeta\omega_n x_0}\right), \qquad \omega_d=\omega_n\sqrt{1-\zeta^2}$$

ก) Harmonic excitation: $F(t)=F_o\cos\omega_{dr}t$

$$x_p=\frac{X_o}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\cos(\omega_{dr}t-\phi)$$

$$\phi=\tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

$$r=\frac{\omega_{dr}}{\omega_n} \qquad X_o=\frac{F_o}{k}$$

ข) Base Excitation: $y=Y\sin\omega_b t$

$$x_p=Y\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}\cos(\omega_b t-\phi_1-\phi_2)$$

$$\phi_1=\tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) \qquad \phi_2=\tan^{-1}\left(\frac{1}{2\zeta r}\right)$$

ค) Rotating Unbalance: $F(t)=m_0 e\omega_r^2\sin\omega_r t$

$$x_p=\frac{m_0 e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\sin(\omega_r t-\phi)$$

Table 5.2-1 Laplace Transform Pairs

	F(s)	F(t), t ≥ 0
1.	1	$\delta(t)$, unit impulse at t = 0
2.	$\frac{1}{s}$	$u_s(t)$, unit step
3.	$\frac{n!}{s^{n+1}}$	t^n
4.	$\frac{1}{s+a}$	e^{-at}
5.	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$
6.	$\frac{a}{s(s+a)}$	$1-e^{-at}$
11.	$\frac{b}{s^2+b^2}$	$\sin bt$
12.	$\frac{s}{s^2+b^2}$	$\cos bt$
13.	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
14.	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-bt}\cos bt$
15.	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t \qquad \zeta<1$
16.	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi) \qquad \zeta<1$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}+\pi \qquad (third\ quadrant)$

17 $sF(s) - f(0) \Leftrightarrow \frac{df}{dt}$

18 $s^2F(s) - sf(0) - \left.\frac{df}{dt}\right|_{t=0} \Leftrightarrow \frac{d^2f}{dt^2}$

19 $G(s) = e^{-sD}F(s) \Leftrightarrow g(t) = \begin{cases} 0 & t<D \\ f(t-D) & t \geq D \end{cases}$