Component Measurements

EIE 240 Electrical and Electronic Measurement
Lecture 8, March 27, 2015

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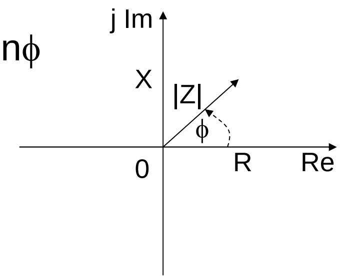
Component RLC

- Impedance = $\dfrac{\text{Potential Difference Phasor}}{\text{Current Phasor}}$
- Impedance = Resistance + j Reactance , $j = \sqrt{-1}$

$$\begin{aligned} Z &= R + j\,X \\ &= |Z|\cos\phi + j\,|Z|\sin\phi \\ &= |Z|e^{j\phi} \\ &= |Z|\angle\phi \end{aligned}$$



where $|Z| = \sqrt{R^2 + X^2}$

$\phi = \tan^{-1}(X/R)$

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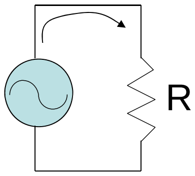
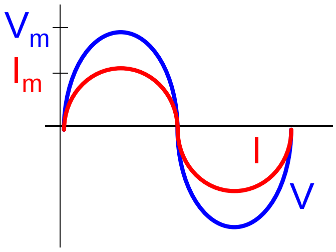
AC Response

- Resistor

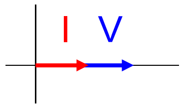
$I_m \sin\omega t$

$V_m \sin\omega t$





Resistance
V = R I
= R I_m sinωt
= V_m sinωt
(V and I in phase)



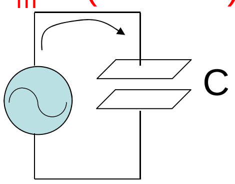
Phasor Diagram

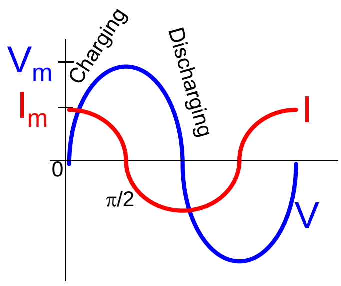
AC Response (Cont'd)

- Capacitor is a passive component storing the energy in an electric field charged by the voltage across the dielectric.

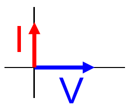
$I = C\ dV/dt$
$= I_m \sin(\omega t + \pi/2)$

$V = V_m \sin(\omega t)$





Capacitive Reactance
$I = dq/dt = C\ dV/dt$
$V = 1/C \int I_m \sin\omega t\ dt + V_0$
$= -1/\omega C\ I_m \cos\omega t$
$= 1/\omega C\ I_m \sin(\omega t - \pi/2)$
$= X_C I_m \sin(\omega t - \pi/2)$
$= V_m \sin(\omega t - \pi/2)$
(V lags I by 90°)



Phasor Diagram

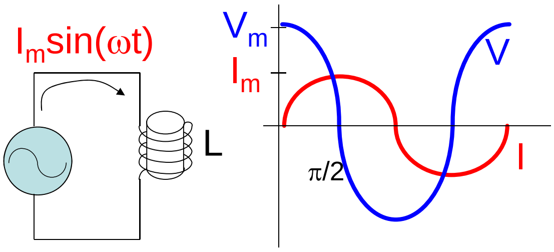
$X_C = 1/\omega C = 1/2\pi f C = \tau/2\pi C$, Higher f, Lower X_C (Lowpass Filter)

AC Response (Cont'd)

- Inductor is a passive component storing the energy in a magnetic field induced by the electric current passing through it.

$V = L\, dI/dt$

$\quad = V_m \sin(\omega t+\pi/2)$



Inductive Reactance

$V = d(N\phi)/dt$

$\quad = L\, d(I_m \sin\omega t)/dt$

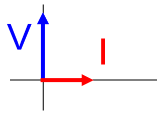
$\quad = \omega L\, I_m \cos\omega t$

$\quad = \omega L\, I_m \sin(\omega t+\pi/2)$

$\quad = X_L I_m \sin(\omega t+\pi/2)$

$\quad = V_m \sin(\omega t+\pi/2)$

(V leads I by 90°)



Phasor Diagram

$X_L = \omega L = 2\pi f L = 2\pi L/\tau$, , Higher f, Higher X_L (Highpass Filter)

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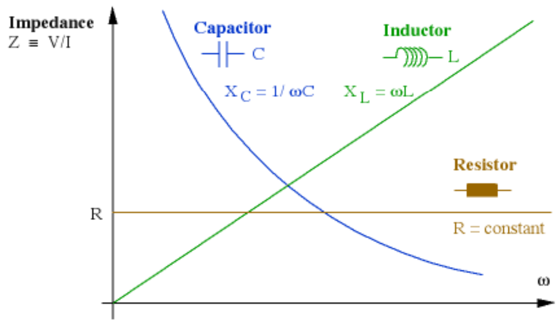
Frequency Response

- The resonance of a series RLC circuit occurs when the inductive and capacitive reactances are equal in magnitude but cancel each other because they are 180° apart in phase, ($Z = \sqrt{R^2 + (X_L - X_C)^2} = R$) and $1 / 2\pi f_0 C = 2\pi f_0 L$

$$f_0 = 1 / 2\pi\sqrt{LC}$$



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Q-Factor and D-Factor

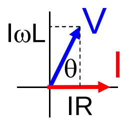
- Q-factor is to express the quality of component in ability to store and release energy or quality of L ➔ L+R,

  Q = Energy Stored / Power Loss

  = Reactance / Resistance

  = ωL / R

  = $\tan\theta$



- D-factor is for a dissipation of C ➔ C + R,

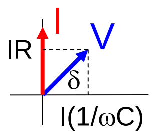
  D = 1/Q

  = Power Loss / Energy Stored

  = R / ($1/\omega C$)

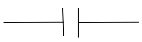
  = ωRC

  = $\tan\delta$

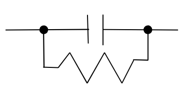


Capacitor Model

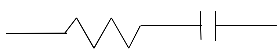
- An ideal capacitor stores but does not dissipate energy.



- Because the dielectric separating the capacitor plates are not a perfect insulator, it causes a small leakage current flowing through the capacitors ➔ parallel model.
  $D = V^2/R \,/\, V^2/X_C = 1/\omega RC$

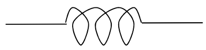


- Plate loss due to the resistances of the plates and leads can become quite significant in higher frequency case ➔ series model.
  $D = I^2R \,/\, I^2X_C = \omega RC$

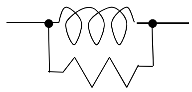


Inductor Model

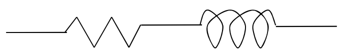
- An ideal inductor stores but does not dissipate energy.



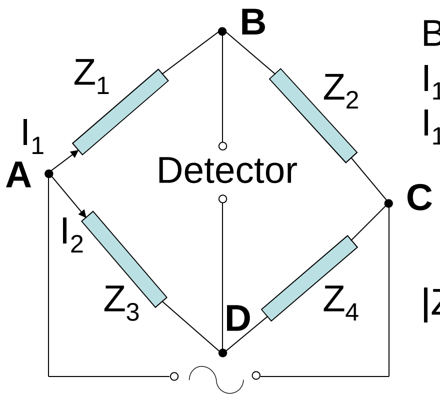
- Time-varying current in a ferromagnetic inductor, which causes a time-varying magnetic field in its core, causes energy losses in the core material that are dissipated as heat ➔ parallel model.
  $Q = V^2/X_L \,/\, V^2/R = R/\omega L$



- Resistance of the wire ➔ series model.
  $Q = I^2X_L \,/\, I^2R = \omega L/R$



AC Bridge



Wheatstone Bridge

Balanced Bridge,

$I_1 |Z_1| \angle\phi_1 = I_2 |Z_3| \angle\phi_3$

$I_1 |Z_2| \angle\phi_2 = I_2 |Z_4| \angle\phi_4$

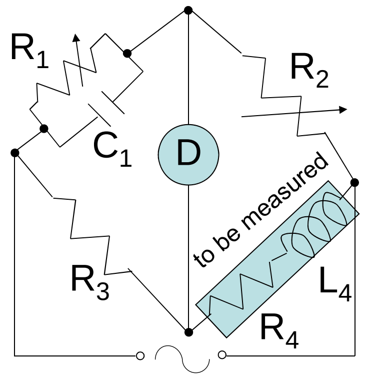
$$|Z_1|/|Z_2| \angle\phi_1-\phi_2 = |Z_3|/|Z_4| \angle\phi_3-\phi_4$$

$$\frac{R_1+jX_1}{R_2+jX_2} = \frac{R_3+jX_3}{R_4+jX_4}$$

Inductance Measurement

There is no pure components, e.g. an inductor can be considered to be a pure inductance (L_4) in series with a pure resistance (R_4).

Maxwell-Wien Bridge (for medium Q = 1-10)



Impedances,

$1/Z_1 = 1/R_1 + 1/(1/j\omega C_1)$

$Z_1 = R_1 / (1+j\omega R_1 C_1)$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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Maxwell-Wien Bridge (Cont'd)

Balanced bridge,

$$Z_1 / Z_2 = Z_3 / Z_4$$
$$Z_4 = Z_2 Z_3 / Z_1$$
$$R_4 + j\omega L_4 = R_2 R_3 (1+j\omega R_1 C_1)/R_1$$
$$= R_2 R_3/R_1 + j\omega R_2 R_3 C_1$$

Real part: $R_4 = R_2 R_3/R_1$

Imagination part: $L_4 = R_2 R_3 C_1$

The balancing is independent of frequency.

Adjust R_1 and R_2 to get the bridge balanced (Null)

$$Q = \omega L_4/R_4 = \omega(R_2 R_3 C_1)/(R_2 R_3/R_1) = \omega R_1 C_1$$

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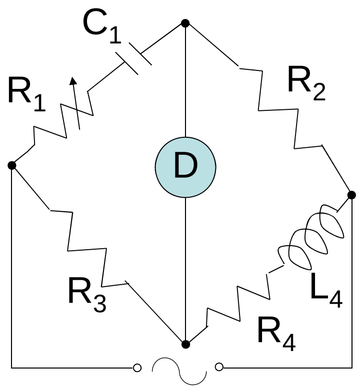
Hay Bridge

For high $Q \geq 10$



Impedances,

$Z_1 = R_1 + 1/j\omega C_1$

$= R_1 - j/\omega C_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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Hay Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 R_3 / (R_1 - j/\omega C_1)$$

$$(R_1R_4 + L_4/C_1) + j\,(\omega R_1 L_4 - R_4/\omega C_1) = R_2R_3$$

Imagination part: $\omega R_1 L_4 = R_4/\omega C_1$

$$L_4 = R_4 / \omega^2 R_1 C_1$$

Real part: $R_1R_4 + L_4/C_1 = R_2R_3$

$$R_1R_4 + R_4/\omega^2 R_1 C_1^2 = R_2R_3$$

$$R_4\,(R_1 + 1/\omega^2 R_1 C_1^2) = R_2R_3$$

$$R_4\,(\omega^2 R_1^2 C_1^2 + 1)/(\omega^2 R_1 C_1^2) = R_2R_3$$

$$R_4 = (\omega^2 R_1 R_2 R_3 C_1^2) / (\omega^2 R_1^2 C_1^2 + 1)$$

$$L_4 = (R_2 R_3 C_1) / (\omega^2 R_1^2 C_1^2 + 1)$$

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Hay Bridge (Cont'd)

$Q = \omega L_4/R_4$
$= 1/\omega R_1 C_1$

Therefore, $L_4 = (R_2 R_3 C_1) / ((1/Q^2) + 1)$
$\approx R_2 R_3 C_1$ if $Q \geq 10$

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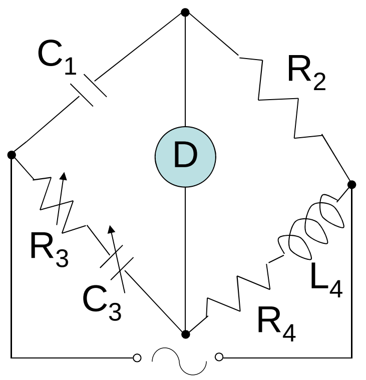
Owen Bridge



Impedances,
$Z_1 = 1/j\omega C_1$
$Z_2 = R_2$
$Z_3 = R_3 - j/\omega C_3$
$Z_4 = R_4 + j\omega L_4$

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Owen Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$

$$R_4+j\omega L_4 = R_2\,(R_3 - j/\omega C_3)\; j\omega C_1$$

$$= R_2C_1/C_3 + j\omega R_2R_3C_1$$

Real part: $R_4 = R_2C_1/C_3$

Imagination part: $L_4 = R_2R_3C_1$

The balancing is independent of frequency.

$Q = \omega L_4/R_4 = \omega R_2R_3C_1C_3 / R_2C_1 = \omega R_3C_3$

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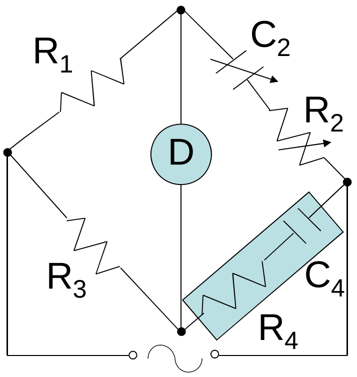
Series Capacitance Bridge

Capacitor can be considered to be a pure capacitance in series with, or sometimes in parallel with, a pure resistance.



Impedances,

$Z_1 = R_1$

$Z_2 = R_2 - j/\omega C_2$

$Z_3 = R_3$

$Z_4 = R_4 - j/\omega C_4$

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Series Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 - j/\omega C_4 = (R_2 - j/\omega C_2)\, R_3 / R_1$$

$$= R_2R_3/R_1 - j(R_3/\omega C_2 R_1)$$

Real part: $R_4 = R_2R_3/R_1$

Imagination part: $C_4 = C_2R_1/R_3$

Used for low D = 0.001-0.1

$D = 1/Q = \omega R_4 C_4 = \omega R_2 C_2$

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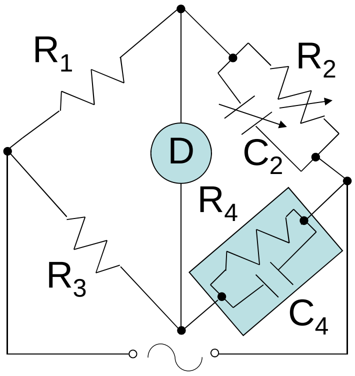
Parallel Capacitance Bridge

Used for D = 0.05-50



Impedances,

$Z_1 = R_1$

$Z_2 = 1 / (1/R_2 + j\omega C_2)$

$= R_2/(1+j\omega C_2 R_2)$

$Z_3 = R_3$

$Z_4 = R_4/(1+j\omega C_4 R_4)$

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Parallel Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$

$$R_4 / (1+j\omega C_4R_4) = R_2R_3 / (1+j\omega C_2R_2)R_1$$

$$R_1R_4 + j\omega C_2R_1R_2R_4 = R_2R_3 + j\omega C_4R_2R_3R_4$$

Real part: $R_1R_4 = R_2R_3$

$$R_4 = R_2R_3/R_1$$

Imagination part: $C_2R_1R_2R_4 = C_4 R_2R_3R_4$

$$C_4 = C_2R_1/R_3$$

$D = 1 / \omega R_4C_4 = 1 / \omega R_2C_2$

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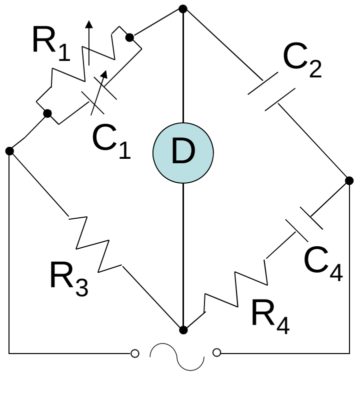
Schering Bridge

Used for very low D



Impedances,

$Z_1 = R_1/(1+j\omega C_1R_1)$

$Z_2 = 1/j\omega C_2$

$Z_3 = R_3$

$Z_4 = R_4 – j/\omega C_4$

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Schering Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 - j/\omega C_4 = R_3(1+j\omega C_1 R_1) / j\omega C_2 R_1$$

$$= (\omega C_1 R_1 R_3 - jR_3) / \omega C_2 R_1$$

$$= R_3 C_1/C_2 - j(R_3/\omega R_1 C_2)$$

Real part: $R_4 = R_3 C_1/C_2$

Imagination part: $1/C_4 = R_3/R_1 C_2$

$$C_4 = C_2 R_1/R_3$$

$D = \omega R_4 C_4 = \omega R_1 C_1$

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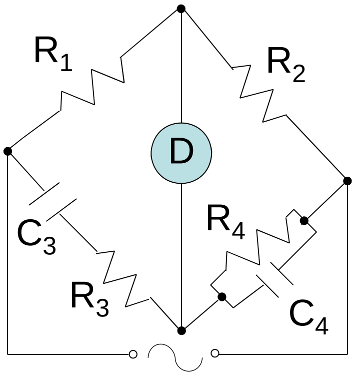
Wien Bridge

Used as frequency-dependent circuit



Impedances,

$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = R_3 - j/\omega C_3$

$Z_4 = R_4/(1+j\omega C_4 R_4)$

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Wien Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$
$$R_4 / (1+j\omega C_4R_4) = R_2(R_3 - j/\omega C_3) / R_1$$
$$R_1R_4/R_2 = R_3 + R_4C_4/C_3 + j(\omega C_4R_3R_4 - 1/\omega C_3)$$

Imagination part: $\omega C_4R_3R_4 = 1/\omega C_3$

$$C_4R_4 = 1/\omega^2C_3R_3$$

Real part: $R_1R_4/R_2 = R_3 + R_4C_4/C_3$

$$R_4 = (R_3R_2C_3 + R_2R_4C_4)/ R_1C_3$$
$$= (R_3R_2C_3 + R_2/\omega^2C_3R_3) / R_1C_3$$
$$= R_2(\omega^2C_3^2R_3^2 + 1) / (\omega^2C_3^2R_1R_3)$$

and $C_4 = 1/\omega^2C_3R_3R_4$

$$= C_3R_1 / R_2(\omega^2C_3^2R_3^2 + 1)$$

$D = 1 / \omega R_4C_4 = \omega R_3C_3$

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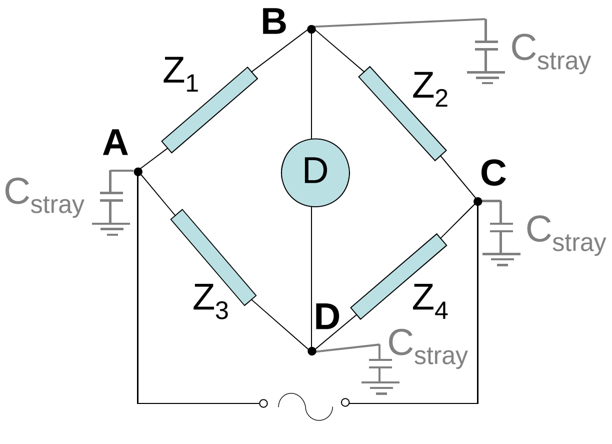
Stray Impedance

There are stray capacitances between the various element and the ground and it mat affect bridge balance.



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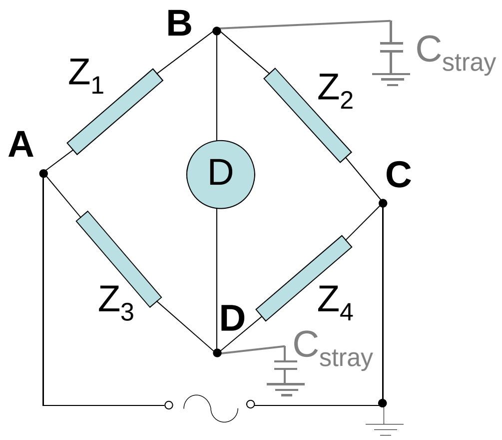
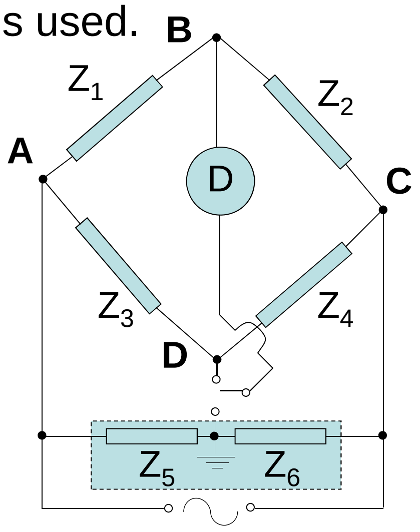
Stray Impedance (Cont'd)

The stray capacitances can be reduced by earthing one side of AC supply.



Stray Impedance (Cont'd)

The minimize stray capacitances between the detector terminals and earth, Wagner earth is used.



To ensuring that the points B and D of a balanced bridge are at ground potential

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References

- http://www.faqs.org/docs/electric/DC/DC_8.html
- http://avstop.com/ac/Aviation_Maintenance_Technician_Handbook_General/10-74.html
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- MAGNET LAB – Wheatstone Bridge webpage: http://www.magnet.fsu.edu/education/tutorials/java/wheatstonebridge/index.html
- Electronics Demonstrations webpage: http://www.falstad.com/circuit/e-index.html

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