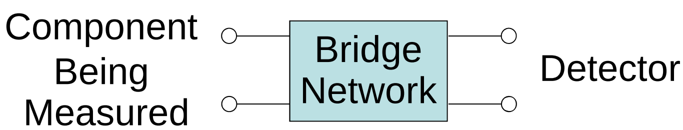
Bridge Method

EIE 240 Electrical and Electronic Measurement
Class 7, March 13, 2015

Bridge Method

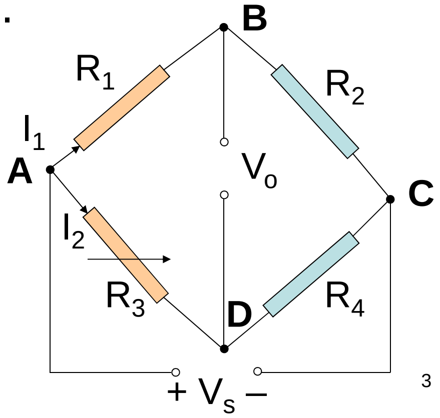
- Diode bridge is an arrangement of four or more diodes for AC/DC full-wave rectifier.
- Component bridge methods are used for measurement of resistance, capacitance, inductance, etc.
- The network will be balanced when the detector reading becomes zero.



Wheatstone Bridge

- Wheatstone bridge was invented by Samuel Hunter Christie in 1833 and improved and popularized by Sir Charles Wheatstone in 1843.
- DC supply, V_s
- Output voltage, V_o

Sir Charles Wheatstone (1802 – 1875)



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Wheatstone Bridge (Cont'd)

- When $V_o = 0$ V, the potential at B must equal to the potential at D

$$I_1R_1 = I_2R_3$$
$$I_1R_2 = I_2R_4$$

Hence $\quad I_1R_1 = I_2R_3 = (I_1R_2/R_4)\ R_3$

$$\mathbf{R_1/R_2 = R_3/R_4}$$

- The balance condition is independent of V_S

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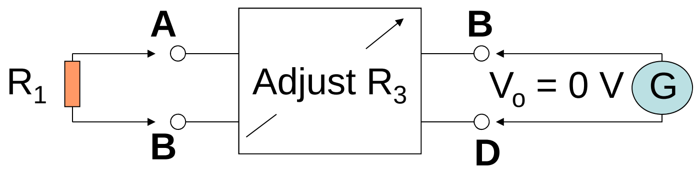
Wheatstone Bridge (Cont'd)

- R_1 is the input resistance to be measured by comparing to accurately known resistors (standard).
- R_2 and R_4 are known-fixed resistances.
- R_3 can be adjusted to give the zero potential difference condition.



Wheatstone Bridge

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Wheatstone Bridge (Cont'd)

- Change in R_1, change R_3
- The precision is about 1 Ω to 1 MΩ.
- The accuracy is mainly up to the known resistors and the sensitivity of the null detector (± 0.1 to 0.2%).
- Error comes from changes in resistances of the bridge arms by changes in temperatures or thermoelectric EMF in contacts.

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Sensitivity of the Bridge

- If no galvanometer at the output,

$$V_{AB} = V_s\, R_1/(R_1+R_2)$$
$$V_{AD} = V_s\, R_3/(R_3+R_4)$$

Thus, $V_o = V_{AB} - V_{AD}$

$$\mathbf{V_o} = V_s\,(\, \mathbf{R_1}/(\mathbf{R_1}+R_2) - R_3/(R_3+R_4)\,)$$

- The relationship between V_o and R_1 is non-linear
- $V_o = 0$ V when $R_1/R_2 = R_3/R_4$

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Sensitivity of the Bridge (Cont'd)

- Changing R_1 to $R_1+\Delta R_1$ gives a change of V_o to $V_o+\Delta V_o$

$$V_o+\Delta V_o=V_s((R_1+\Delta R_1)/((R_1+\Delta R_1)+R_2) - R_3/(R_3+R_4))$$

Then
$$(V_o+\Delta V_o)-V_o = V_s\left(\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2} - \frac{R_3}{R_3+R_4}\right) - V_s\left(\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right)$$

$$\Delta V_o = V_s\left(\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2} - \frac{R_1}{R_1+R_2}\right)$$

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Sensitivity of the Bridge (Cont'd)

- If small changes $\Delta R_1 << R_1$ then the sensitivity of Wheatstone bridge can be computed from,

$$\Delta V_o \approx \Delta R_1 V_s/(R_1+R_2)$$

$$\Delta V_o/\Delta R_1 \approx V_s/(R_1+R_2)$$

- Higher R_1 to be measured, lower sensitivity

- Amplifier can be used to amplify V_o

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Unbalanced Bridge

- If there is a galvanometer, R_g, between the two output terminals, the current I_g can be determined by Thévenin equivalent circuit.

$$V_{th} = V_o = V_s \left(R_1/(R_1+R_2) - R_3/(R_3+R_4) \right)$$

- Short voltage source, then Thévenin resistance is $R_1//R_2 + R_3//R_4$

$$R_{th} = R_1R_2/(R_1+R_2) + R_3R_4/(R_3+R_4)$$

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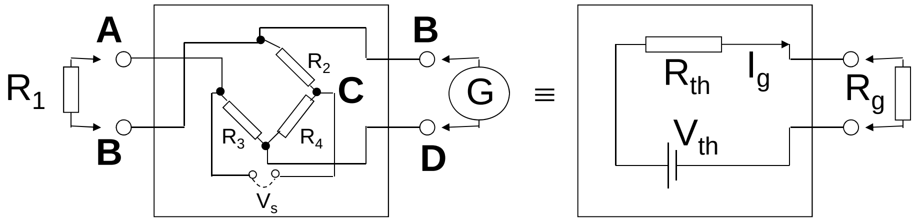
Unbalanced Bridge (Cont'd)



For unbalanced bridge,

$$I_g = V_{th} / (R_{th} + R_g)$$

and

$$V_g = I_gR_g$$
$$= V_{th}R_g/(R_{th}+R_g)$$

If balanced or $V_g = 0$ V $\Rightarrow$ like no movement

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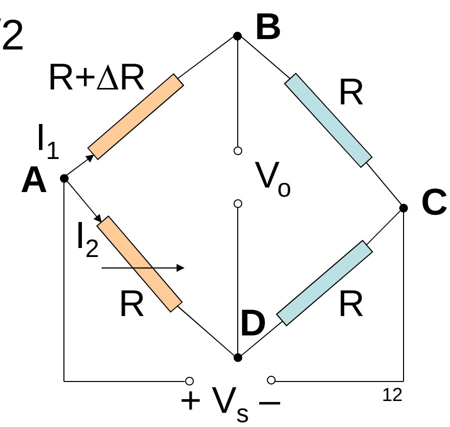
Slightly Unbalanced Bridge

- If $R_2 = R_3 = R_4 = R$ and $R_1 = R+\Delta R$
- $V_{AB} = V_s\,(R+\Delta R)/(R+\Delta R+R)$
  $= V_s\,(R+\Delta R)/(2R+\Delta R)$
- $V_{AD} = V_s\, R/(R+R) = V_s/2$
- $V_o = V_{AB} - V_{AD}$
  $= V_s\left(\dfrac{(R+\Delta R)}{(2R+\Delta R)} - \dfrac{1}{2}\right)$
  $= V_s\Delta R\, /\, (4R+2\Delta R)$
- If $\Delta R < 5\%$ of R
  $V_o \approx V_s\Delta R\, /\, 4R$



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Slightly Unbalanced Bridge (Cont'd)

- For Thévenin's equivalent circuit,

$V_{th} = V_o$
$= V_s\Delta R / 4R$

$R_{th} = (R+\Delta R)//R + R//R$
$= R(R+\Delta R)/(2R+\Delta R) + R/2$
$\approx R/2 + R/2 = R$

$I_g = V_{th} / (R_{th} + R_g)$
$V_g = I_gR_g$

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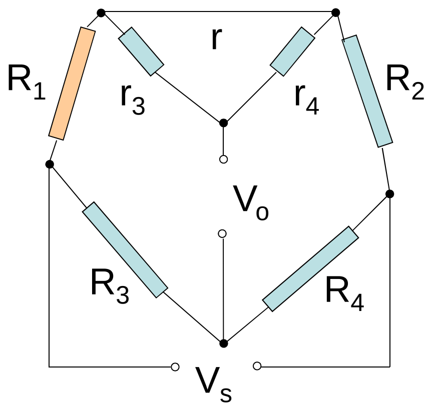
Kelvin Double Bridge

- A modification of Wheatstone bridge for low resistance measurement ($R_1 < 1\Omega$)
- Because non-perfect wire resistances affect the measurement.



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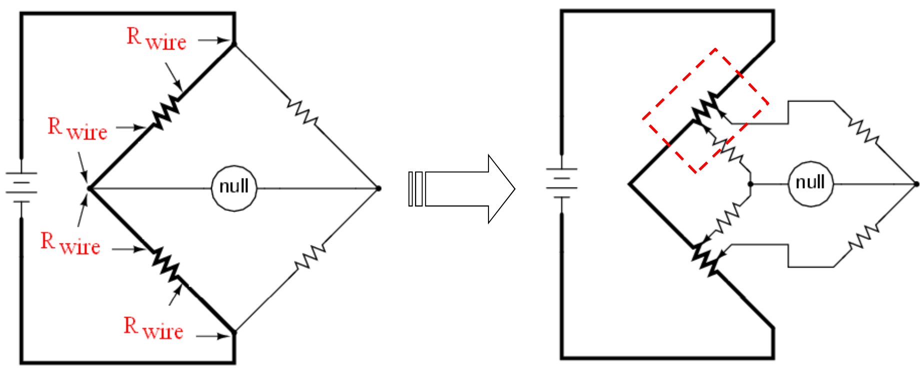
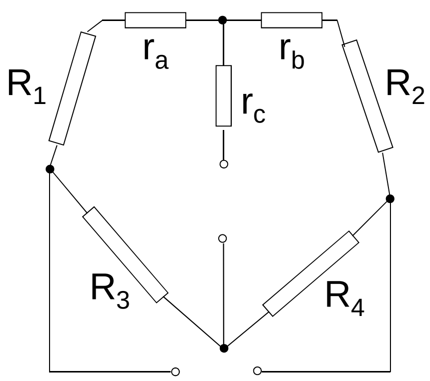
- Using four-terminal resistors (two for voltage supply and 2 for current supply)



- The yoke r is connected to R_1 and R_2
- The relationship between r_3, r_4, R_3 and R_4

$$\mathbf{R_3/R_4 = r_3/r_4}$$

- Using the delta-star transformation, the equivalent circuit



$r_a = r_3 r\ /\ (r_3+r_4+r)$
$r_b = r_4 r\ /\ (r_3+r_4+r)$

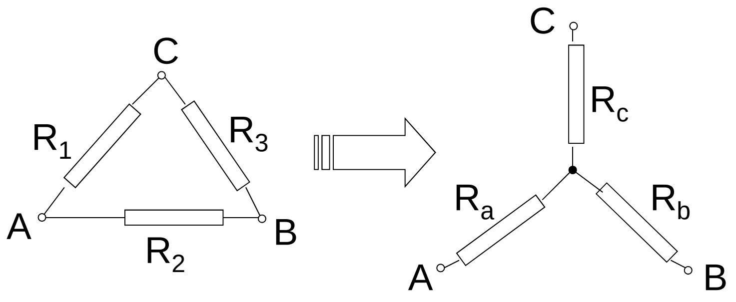
Note

- Δ-Y Transformation

$R_a = R_1R_2 / (R_1+R_2+R_3)$

$R_b = R_2R_3 / (R_1+R_2+R_3)$

$R_c = R_1R_3 / (R_1+R_2+R_3)$



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- The balance condition is the same as Wheatstone bridge (Null V_o = 0 V)

$(R_1+r_a) / (R_2+r_b) = R_3 / R_4$

$$
\begin{aligned}
R_1 &= R_3(R_2+r_b)/R_4 - r_a \\
&= R_3R_2/R_4 + R_3r_b/R_4 - r_a \\
&= R_3R_2/R_4 + R_3r_4r/(r_3+r_4+r)R_4 \\
&\quad - r_3r/(r_3+r_4+r) \\
&= R_3R_2/R_4 \\
&\quad + r_4r(R_3/R_4 - r_3/r_4) / (r_3+r_4+r) \\
R_1 &= R_3R_2/R_4
\end{aligned}
$$

Therefore $R_1/R_2 = R_3/R_4 = r_3/r_4$

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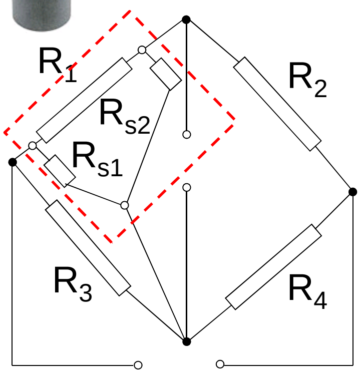
High Resistance Bridge

- For very high resistance, e.g. 1,000 MΩ, there is leakage currents over the surface of the insulated post.



- Using three-terminal resistors (parallel with 2 leakage resistances)
- $R_{s1} >> R_3$ and $R_{s1} // R_3$ to avoid the leakage effect
- R_{s2} may affect the detector sensitivity

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Bridge Compensation

- The resistance of long leads will be affected by changes in temperatures
- To avoid this, 3 leads are required to connect to the coils
- They are all the same length and resistance

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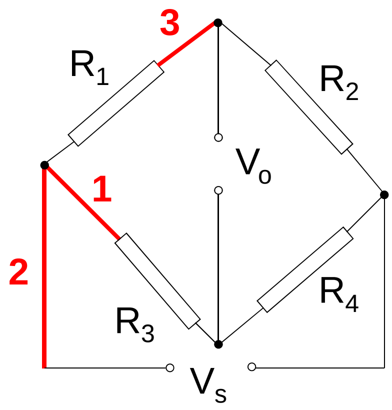
Bridge Compensation (Cont'd)

- Any changes in lead resistance will affect all 3 leads equally and occur in 2 arms of bridge and will cancel out.



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Bridge Controlled Circuits

- The bridge can be used as an error detector in a control circuit, using the potential difference at the output of the bridge that is sensitive to any physical parameters.
- Passive circuit elements such as strain gauges, temperature sensitive resistors (thermistors) or photo resistors are used as one arm of Wheatstone bridge.
- A change in the elements (pressure, heat or light) causes the bridge to be unbalanced.

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References

- Hotek Technologies, Inc webpage : http://www.hotektech.com/
- Yokogawa webpage: http://tmi.yokogawa.com/us/
- MAGNET LAB – Wheatstone Bridge webpage: http://www.magnet.fsu.edu/education/tutorials/java/wheatstonebridge/index.html