# Electrical and Electronic Measurements:

# Component Measurement

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# PART I

# AC Bridge Method

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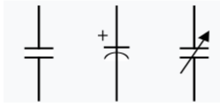
# Capacitor



Fixed Polarized Variable

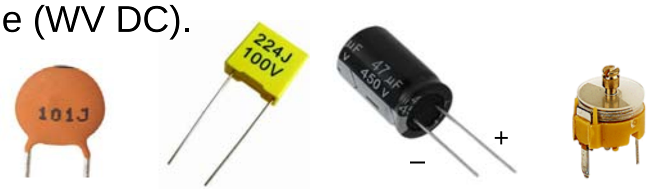
- Capacitor is a passive two-terminal component storing the energy in an electric field charged by the voltage across the dielectric.
- Capacitance is the ratio of the change in an electric charge in a system to the corresponding change in its electric potential. $C = \frac{q}{V}$
- The unit of capacitance is Farad (F), named after the English physicist Michael Faraday.
- However, the Farad is a very large unit of measurement to use on its own so sub-multiples of the Farad are generally used such as $\mu$F, nF and pF.



# Capacitor (Cont'd)

- All capacitors have a maximum voltage rating and when selecting a capacitor consideration must be given to the amount of voltage to be applied across the capacitor.
- The maximum amount of voltage that can be applied to the capacitor without damage to its dielectric material is generally given in the data sheets as working voltage (WV) or DC working voltage (WV DC).

**10**$\times 10^{1}$ pF
$\pm$ 5%




Ceramic type Film type Electrolytic type Trimmers

# Capacitance Identification

- Capacitance depends on the area of the plates, the distance between the plates, the type of dielectric material, and temperature.
- For text marking, the third digit represents the number of zeros to add to the end of the first two digits.
- The resulting number is the capacitance in pF.
- Tolerance

| A | B | C | D | E | F | G | H | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ±0.o5pF | ±0.1pF | ±0.25pF | ±0.5pF | ±0.5% | ±1% | ±2% | ±3% | ±5% | ±10% | ±15% | ±20% | ±30% |

| P | S | W | X | Z |
|---|---|---|---|---|
| –0%, +100% | –20%, +50% | –0%, +200% | –20%, +40% | –20%, +80% |

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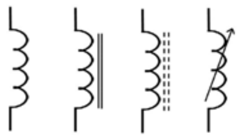
# Inductor

- Inductor is a passive two-terminal component storing the energy in a magnetic field induced by the electric current passing through it.



Air core Iron core Ferrite core Variable

- Inductance is the ratio of the voltage to the rate of change of current.
- An inductor typically consists of an insulated wire wound into a coil around a core.
- Inductance of a coil,

Number of turns

Magnetic permeability

Area of coil

$$L = \frac{\mu N^2 A}{l}$$

Length of coil

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# Inductor (Cont'd)

- The unit of inductance is the Henry (H) named for 19th century American scientist Joseph Henry.
- Inductors have values that typically range from 1 µH to 20 H.

$56 \times 10^{1}$ µH ± 5%

$10 \times 10^{0}$ µH ±10%


Air core

Iron core

Ferrite core

Variable core

Toroidal core

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# Inductance Identification

- For text marking, The third digit represents the number of zeros to add to the end of the first two digits.
- The resulting number is the inductance in µH.
- The fourth letter or alphabet represents the tolerance value.

| B | C | S | D | F | G | H | J | K | L | M | V | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ±0.15nH | ±0.2nH | ±0.3nH | ±0.5nH | ±1% | ±2% | ±3% | ±5% | ±10% | ±15% | ±20% | ±25% | ±30% |

- EIA (Electronic Industries Alliance) standard for four-color-band (µH)

| Band | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Meaning** | **1st Digit** | **2nd Digit** | **Multiplier (No. of zeros)** | **Tolerance %** |
| Gold | | | x 0.1 (divide by 10) | +/-5% |
| Silver | | | x 0.01 (divide by 100) | +/-10% |
| Black | 0 | 0 | x1 (No Zeros) | +/-20% |
| Brown | 1 | 1 | x10 (0) | |
| Red | 2 | 2 | x100 (00) | |
| Orange | 3 | 3 | x1000 (000) | |
| Yellow | 4 | 4 | x10000 (0,000) | |
| Green | 5 | 5 | | |
| Blue | 6 | 6 | | |
| Violet | 7 | 7 | | |
| Grey | 8 | 8 | | |
| White | 9 | 9 | | |

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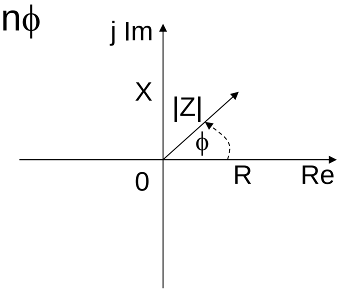
# Component RLC

- Impedance = $\dfrac{\text{Potential Difference Phasor}}{\text{Current Phasor}}$
- Impedance = Resistance + j Reactance , $j = \sqrt{-1}$

$$Z = R + jX$$
$$= |Z|\cos\phi + j\,|Z|\sin\phi$$
$$= |Z|e^{j\phi}$$
$$= |Z|\angle\phi$$




where $|Z| = \sqrt{R^2 + X^2}$

$\phi = \tan^{-1}(X/R)$

- Admittance, Y = 1/Z

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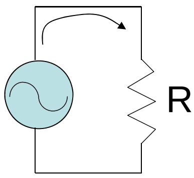
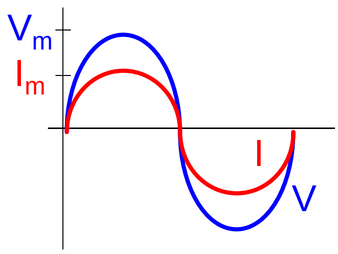
# AC Response

- Resistor

$I_m \sin\omega t$

$V_m \sin\omega t$



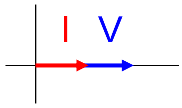

Resistance

$V = R\,I$

$= R\,I_m \sin\omega t$

$= V_m \sin\omega t$

(V and I in phase)




Phasor Diagram

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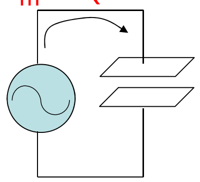
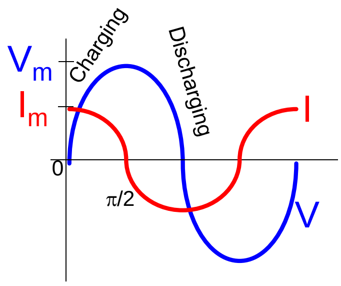
# AC Response (Cont'd)

$I = C\,dV/dt$
$= I_m \sin(\omega t + \pi/2)$

$V = V_m \sin(\omega t)$




Capacitive Reactance

$I = dq/dt = C\,dV/dt$

$V = 1/C \int I_m \sin\omega t\, dt + V_0$

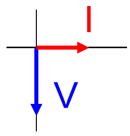
$= -1/\omega C\; I_m \cos\omega t$

$= 1/\omega C\; I_m \sin(\omega t - \pi/2)$

$= X_C I_m \sin(\omega t - \pi/2)$

$= V_m \sin(\omega t - \pi/2)$

(V lags I by 90°)




Phasor Diagram

$X_C = 1/\omega C = 1/2\pi f C = \tau/2\pi C$ , Higher f, Lower $X_C$ (Lowpass Filter)

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# AC Response (Cont'd)

- Note that capacitive reactance is an opposition to the change of voltage across an element.
- There are two choices in the literature for defining reactance for a capacitor. One is to use a uniform notion of reactance as the imaginary part of impedance, in which case the reactance of a capacitor is a negative number:
  $X_C = -\,1/\omega C$ and $Z_C = j\,X_C = -\,j/\omega C$ .
- Another choice is to define capacitive reactance as a positive number, however, one needs to remember to add a negative sign for the impedance of a capacitor:
  $X_C = 1/\omega C$ and $Z_C = -\,j\,X_C = 1/j\omega C$ .

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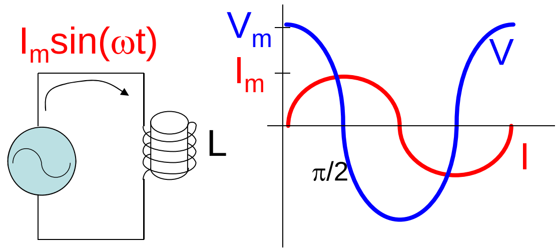
# AC Response (Cont'd)

$V = L\,dI/dt$

$= V_m \sin(\omega t + \pi/2)$




Inductive Reactance

$V = d(N\phi)/dt$

$= L\,d(I_m \sin\omega t)/dt$

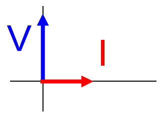
$= \omega L\, I_m \cos\omega t$

$= \omega L\, I_m \sin(\omega t + \pi/2)$

$= X_L I_m \sin(\omega t + \pi/2)$

$= V_m \sin(\omega t + \pi/2)$

(V leads I by 90°)




Phasor Diagram

$X_L = \omega L = 2\pi f L = 2\pi L/\tau$ , , Higher f, Higher $X_L$ (Highpass Filter)

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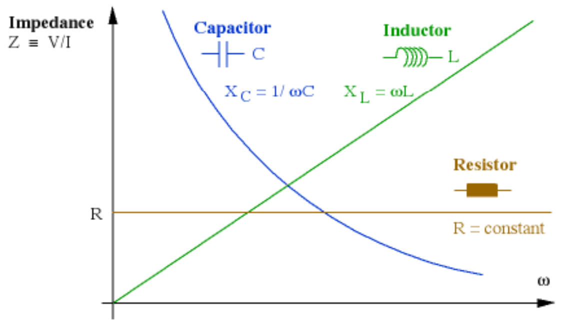
# Frequency Response

- The resonance of a series RLC circuit occurs when the inductive and capacitive reactances are equal in magnitude but cancel each other because they are 180° apart in phase, ($|Z| = \sqrt{R^2 + (X_L - X_C)^2} = R$) and $1 / 2\pi f_0 C = 2\pi f_0 L$

$$f_0 = 1 / 2\pi\sqrt{LC}$$

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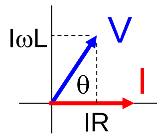
# Q-Factor and D-Factor

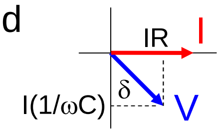
- Q-factor is to express the quality of component in ability to store and release energy or quality of L ➔ L+R,

  Q = Energy Stored / Power Loss
  = Reactance / Resistance
  = $\omega L$ / R
  = $\tan\theta$




- D-factor is for a dissipation of C ➔ C + R,

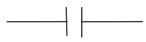
  D = 1/Q
  = Power Loss / Energy Stored
  = R / $(1/\omega C)$
  = $\omega RC$
  = $\tan\delta$



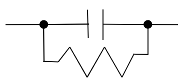



# Capacitor Model

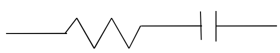
- An ideal capacitor stores but does not dissipate energy.



- Because the dielectric separating the capacitor plates are not a perfect insulator, it causes a small leakage current flowing through the capacitors ➔ parallel model.
  $D = V^2/R \,/\, V^2/X_C = 1/\omega RC$

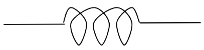


- Plate loss due to the resistances of the plates and leads can become quite significant in higher frequency case ➔ series model.
  $D = I^2R \,/\, I^2X_C = \omega RC$

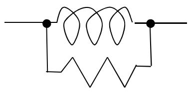


# Inductor Model

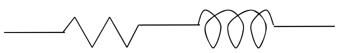
- An ideal inductor stores but does not dissipate energy.



- Time-varying current in a ferromagnetic inductor, which causes a time-varying magnetic field in its core, causes energy losses in the core material that are dissipated as heat ➔ parallel model.
  $Q = V^2/X_L \;/\; V^2/R = R/\omega L$

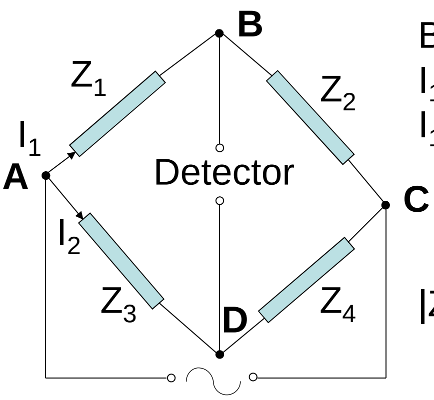


- Resistance of the wire ➔ series model.
  $Q = I^2X_L \;/\; I^2R = \omega L/R$





# AC Bridge




Wheatstone Bridge

Balanced Bridge,

$I_1 |Z_1| \angle\phi_1 = I_2 |Z_3| \angle\phi_3$

$I_1 |Z_2| \angle\phi_2 = I_2 |Z_4| \angle\phi_4$

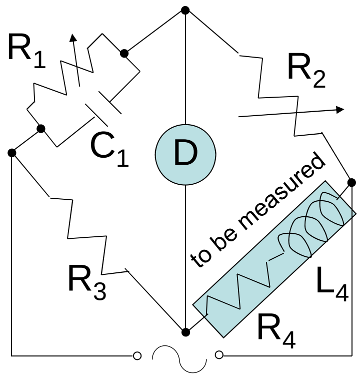
$|Z_1|/|Z_2| \angle\phi_1-\phi_2 = |Z_3|/|Z_4| \angle\phi_3-\phi_4$

$$\frac{R_1+jX_1}{R_2+jX_2} = \frac{R_3+jX_3}{R_4+jX_4}$$

# Inductance Measurement

There is no pure components, e.g. an inductor can be considered to be a pure inductance ($L_4$) in series with a pure resistance ($R_4$).

Maxwell-Wien Bridge (for medium Q = 1-10)




Impedances,

$1/Z_1 = 1/R_1 + 1/(1/j\omega C_1)$

$Z_1 = R_1 / (1+j\omega R_1 C_1)$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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# Maxwell-Wien Bridge (Cont'd)

Balanced bridge,

$$Z_1 / Z_2 = Z_3 / Z_4$$

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 R_3 (1+j\omega R_1 C_1)/R_1$$

$$= R_2 R_3/R_1 + j\omega R_2 R_3 C_1$$

James Clerk Maxwell (1831-1879), a Scottish scientist in the field of mathematical physics.

Real part: $R_4 = R_2 R_3/R_1$

Imagination part: $L_4 = R_2 R_3 C_1$

The balancing is independent of frequency.

Adjust $R_1$ and $R_2$ to get the bridge balanced (Null)

$$Q = \omega L_4/R_4 = \omega(R_2 R_3 C_1)/(R_2 R_3/R_1) = \omega R_1 C_1$$

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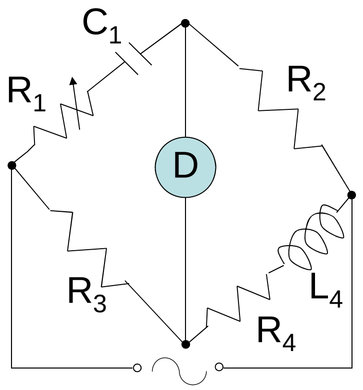
# Hay Bridge

For high $Q \geq 10$




Impedances,

$Z_1 = R_1 + 1/j\omega C_1$

$= R_1 - j/\omega C_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 + j\omega L_4$

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# Hay Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 R_3 / (R_1 - j/\omega C_1)$$

$$(R_1 R_4 + L_4/C_1) + j\,(\omega R_1 L_4 - R_4/\omega C_1) = R_2 R_3$$

Imagination part: $\omega R_1 L_4 = R_4/\omega C_1$

$$L_4 = R_4 / \omega^2 R_1 C_1$$

Real part: $R_1 R_4 + L_4/C_1 = R_2 R_3$

$$R_1 R_4 + R_4/\omega^2 R_1 C_1^2 = R_2 R_3$$

$$R_4\,(R_1 + 1/\omega^2 R_1 C_1^2) = R_2 R_3$$

$$R_4\,(\omega^2 R_1^2 C_1^2 + 1)/(\omega^2 R_1 C_1^2) = R_2 R_3$$

$$R_4 = (\omega^2 R_1 R_2 R_3 C_1^2) / (\omega^2 R_1^2 C_1^2 + 1)$$

$$L_4 = (R_2 R_3 C_1) / (\omega^2 R_1^2 C_1^2 + 1)$$

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# Hay Bridge (Cont'd)

$$Q = \omega L_4/R_4$$
$$= 1/\omega R_1 C_1$$

Therefore, $L_4 = (R_2R_3C_1) / ( (1/Q^2) + 1 )$
$\approx R_2R_3C_1$ if $Q \geq 10$

and $R_4 \approx \omega^2 R_1 R_2 R_3 C_1^2$

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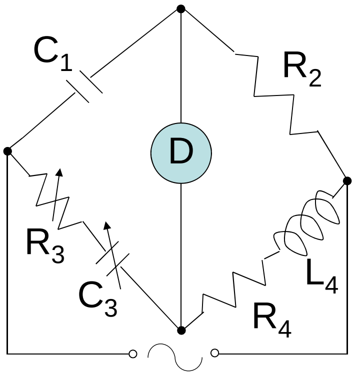
# Owen Bridge




Impedances,

$Z_1 = 1/j\omega C_1$
$Z_2 = R_2$
$Z_3 = R_3 - j/\omega C_3$
$Z_4 = R_4 + j\omega L_4$

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# Owen Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j\omega L_4 = R_2 (R_3 - j/\omega C_3)\, j\omega C_1$$

$$= R_2 C_1 / C_3 + j\omega R_2 R_3 C_1$$

Real part: $R_4 = R_2 C_1 / C_3$

Imagination part: $L_4 = R_2 R_3 C_1$

The balancing is independent of frequency.

$Q = \omega L_4 / R_4 = \omega R_2 R_3 C_1 C_3 / R_2 C_1 = \omega R_3 C_3$

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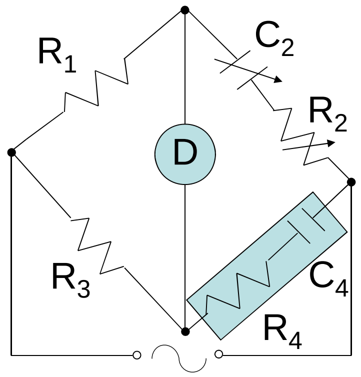
# Series Capacitance Bridge

Capacitor can be considered to be a pure capacitance in series with, or sometimes in parallel with, a pure resistance.




Impedances,

$Z_1 = R_1$

$Z_2 = R_2 - j/\omega C_2$

$Z_3 = R_3$

$Z_4 = R_4 - j/\omega C_4$

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# Series Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 - j/\omega C_4 = (R_2 - j/\omega C_2)\, R_3 / R_1$$

$$= R_2 R_3/R_1 - j(R_3/\omega C_2 R_1)$$

Real part: $R_4 = R_2 R_3/R_1$

Imagination part: $C_4 = C_2 R_1/R_3$

Used for low D = 0.001-0.1

$D = 1/Q = \omega R_4 C_4 = \omega R_2 C_2$

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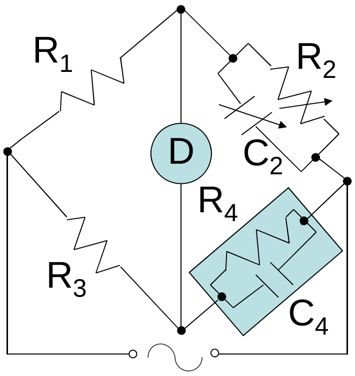
# Parallel Capacitance Bridge

Used for high D > 0.05




Impedances,

$Z_1 = R_1$

$Z_2 = 1 / (1/R_2 + j\omega C_2)$

$= R_2/(1+j\omega C_2 R_2)$

$Z_3 = R_3$

$Z_4 = R_4/(1+j\omega C_4 R_4)$

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## Parallel Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$

$$R_4 / (1+j\omega C_4R_4) = R_2R_3 / (1+j\omega C_2R_2)R_1$$

$$R_1R_4 + j\omega C_2R_1R_2R_4 = R_2R_3 + j\omega C_4R_2R_3R_4$$

Real part: $R_1R_4 = R_2R_3$

$$R_4 = R_2R_3/R_1$$

Imagination part: $C_2R_1R_2R_4 = C_4 R_2R_3R_4$

$$C_4 = C_2R_1/R_3$$

$D = 1/\omega R_4C_4 = 1/\omega R_2C_2$

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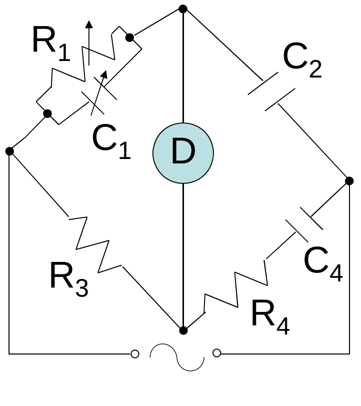
## Schering Bridge

Used for very low D




Impedances,

$Z_1 = R_1/(1+j\omega C_1R_1)$
$Z_2 = 1/j\omega C_2$
$Z_3 = R_3$
$Z_4 = R_4 – j/\omega C_4$

Harald Schering (25 November 1880 – 10 April 1959), a German physicist

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# Schering Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 - j/\omega C_4 = R_3(1+j\omega C_1 R_1) / j\omega C_2 R_1$$

$$= (\omega C_1 R_1 R_3 - jR_3) / \omega C_2 R_1$$

$$= R_3 C_1/C_2 - j(R_3/\omega R_1 C_2)$$

Real part: $R_4 = R_3 C_1/C_2$

Imagination part: $1/C_4 = R_3/R_1 C_2$

$$C_4 = C_2 R_1/R_3$$

$D = \omega R_4 C_4 = \omega R_1 C_1$

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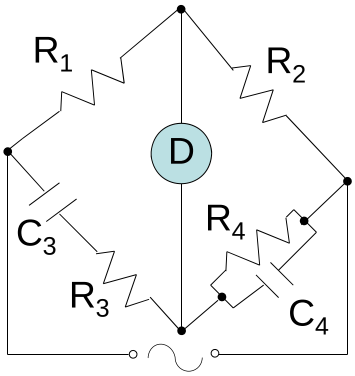
# Wien Bridge

Used as frequency-dependent circuit




Impedances,

$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = R_3 - j/\omega C_3$

$Z_4 = R_4/(1+j\omega C_4 R_4)$

Max Karl Werner Wien (25 December 1866 – 22 February 1938), a German physicist.

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# Wien Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 / (1+j\omega C_4 R_4) = R_2(R_3 - j/\omega C_3) / R_1$$

$$R_1 R_4/R_2 = R_3 + R_4 C_4/C_3 + j(\omega C_4 R_3 R_4 - 1/\omega C_3)$$

Imagination part: $\omega C_4 R_3 R_4 = 1/\omega C_3$

$$C_4 R_4 = 1/\omega^2 C_3 R_3$$

Real part: $R_1 R_4/R_2 = R_3 + R_4 C_4/C_3$

$$R_4 = (R_3 R_2 C_3 + R_2 R_4 C_4)/ R_1 C_3$$

$$= (R_3 R_2 C_3 + R_2/\omega^2 C_3 R_3) / R_1 C_3$$

$$= R_2(\omega^2 C_3^2 R_3^2 + 1) / (\omega^2 C_3^2 R_1 R_3)$$

and $C_4 = 1/\omega^2 C_3 R_3 R_4$

$$= C_3 R_1 / R_2(\omega^2 C_3^2 R_3^2 + 1)$$

$D = 1/\omega R_4 C_4 = \omega R_3 C_3$

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# Wien Bridge Oscillator

A type of electronic oscillator that generates sine waves with a large range of frequencies.

At balance,

Imagination part: $C_4 R_4 = 1/\omega^2 C_3 R_3$

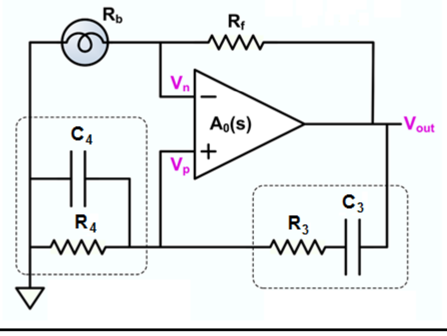
$$\omega^2 = 1 / R_3 R_4 C_3 C_4$$

$$f = 1/ 2\pi\sqrt{R_3 R_4 C_3 C_4}$$

If $R_3 = R_4 = R$

and $C_3 = C_4 = C$

Then $f = 1/ 2\pi RC$

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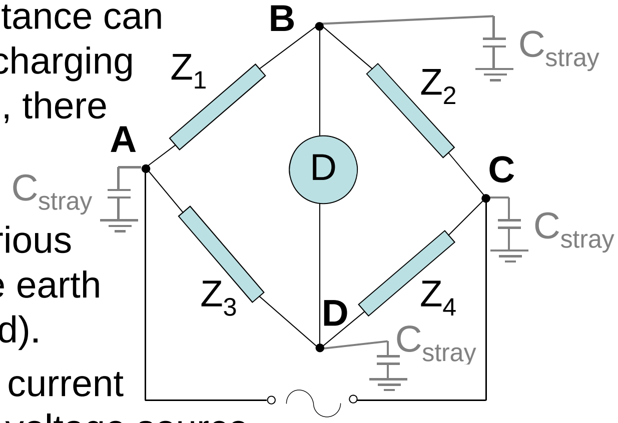
# Stray Impedance

- Because capacitance can conduct AC by charging and discharging, there are stray capacitances between the various element and the earth potential (ground).




- They form stray current paths to the AC voltage source and become significant when performing measurements at high frequency which may effect the bridge balanced condition.

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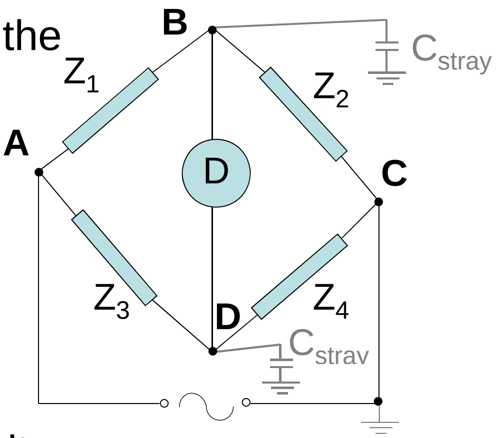
# Stray Impedance (Cont’d)

- One way to reduce the effect is to connect the null potential at ground potential, so that there will be no AC potential between the two.




- Thus no current through stray capacitances.

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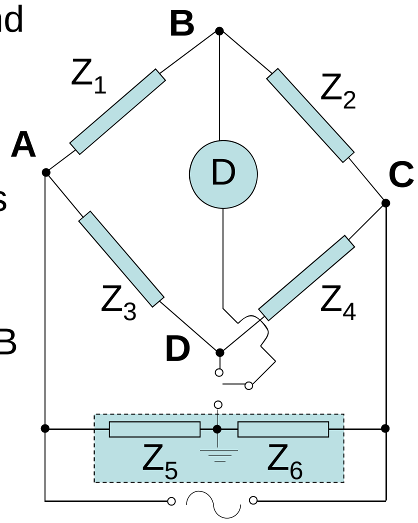
## Stray Impedance (Cont'd)

- Directly connecting to ground potential is not possible, the voltage divider circuit called Wagner earth is used to minimize stray capacitances between the detector terminals and earth.
- To ensuring that the points B and D of a balanced bridge are at ground potential

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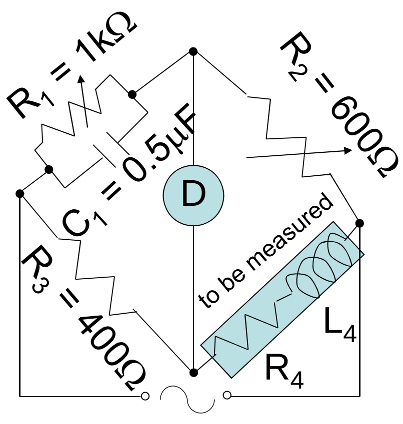
## Ex1. Maxwell-Wein Bridge




f = 1kHz

Impedances,

$\omega = 2\pi f$

$= 2\pi(1k)$

$= 2000\pi$ rad/s

$Z_1 = R_1 / (1+j\omega R_1 C_1)$

$= \dfrac{1k}{1+j(2000\pi)(1k)(0.5\mu)}$

$= 92 - j289 \quad \Omega$

$Z_2 = R_2 = 600 \quad \Omega$

$Z_3 = R_3 = 400 \quad \Omega$

$Z_4 = R_4 + j\omega L_4$

$= R_4 + j2000\pi L_4 \quad \Omega$

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Ex1. (Cont'd)

at balance,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 + j2000\pi L_4 = (600)(400) / (92 - j289)$$

$$= 240 + j754$$

$R_4 = 240 \quad \Omega$

$L_4 = 754 / 2000\pi$

$= 120 \quad mH$

$Q = \omega L_4 / R_4$

$= (2000\pi)(120m)/(240)$

$= 3.14$

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Ex1. (Cont'd)

check,

$R_4 = R_2 R_3 / R_1$

$= (600)(400) / (1k)$

$= 240 \quad \Omega$ CORRECT

$L_4 = R_2 R_3 C_1$

$= (600)(400)(0.5\mu)$

$=120 \quad mH$ CORRECT

$Q = \omega R_1 C_1$

$= (2000\pi)(1k)(0.5\mu)$

$= 3.14$ CORRECT

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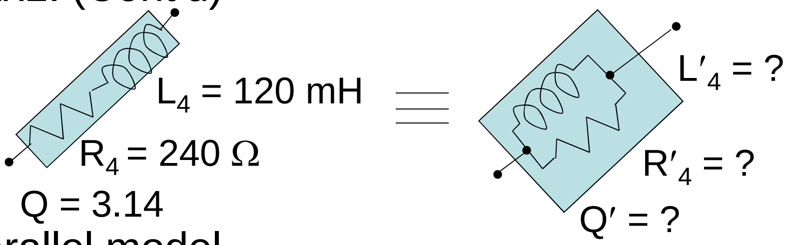
## Ex1. (Cont'd)




Parallel model,

$$Z'_4 = \frac{(R'_4)(j\omega L'_4)}{R'_4 + j\omega L'_4} = \frac{\omega^2(R'_4)(L'_4)^2 + j\omega(R'_4)^2(L'_4)}{(R'_4)^2 + \omega^2(L'_4)^2}$$

Series model,

$Z_4 = R_4 + j\omega L_4$

$\quad = (240) + j\omega(120m)$

$\quad = Z'_4$

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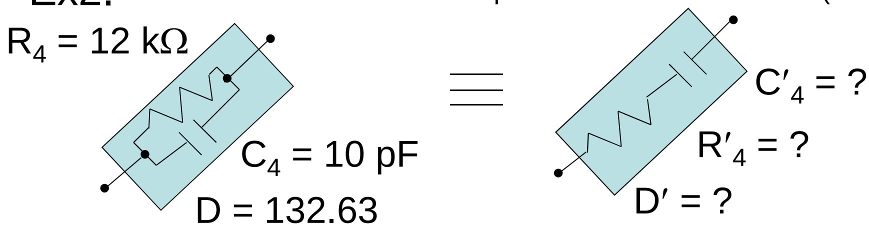
## Ex2.

Equivalent Series Resistance (ESR)




Parallel model at 10 kHz source,

$Z_4 = R_4/(1+j\omega C_4R_4)$

$\quad = (12k) / (1 + j2\pi(10k)(10p)(12k))$

$\quad = (12000) - j(90.47)$

Series model,

$Z'_4 = R'_4 - j/\omega C'_4 = Z_4$ therefore

$R'_4 = 12\ k\Omega$, $C'_4 = 0.18\ \mu F$, $D' = 135.72$

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# PART II

# Other Techniques

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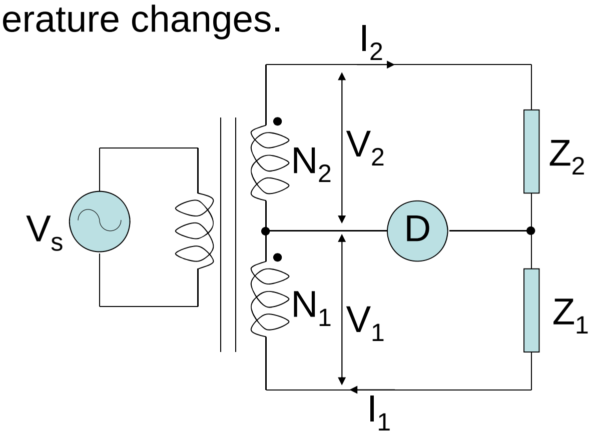
## Transformer Ratio Bridges

Not only varying the impedances, but bridge can be also balanced by varying the turns ratio of a transformer. There is a small number of standard resistors and capacitors and no effect of temperature changes.

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## Single Ratio Transformer Bridge

Tap a transformer $\Rightarrow$ voltage divider of $V_s$

$V_1 = kN_1 = I_1Z_1 \Rightarrow I_1 = kN_1 / Z_1$

$V_2 = kN_2 = I_2Z_2 \Rightarrow I_2 = kN_2 / Z_2$

To balance the bridge or no current through the detector, D = Null

$$I_1 = I_2 \quad \Rightarrow \quad \underbrace{Z_1 / Z_2}_{\text{Impedance Ratio}} = \underbrace{N_1 / N_2}_{\text{Turn Ratio}}$$

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## Single Ratio Transformer Bridge (Cont'd)

- Resistance Measurement

  $Z_1$ = Unknown resistor $R_x$

  $Z_2$ = Standard resistor $R_s$

$$R_x = R_s \frac{N_1}{N_2}$$

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## Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement

  $Z_1$ = Unknown $C_x \parallel R_x$ (leakage resistance)

  $= 1/(1/R_x + j\omega C_x)$

  $= R_x / (1+j\omega R_x C_x)$

  $Z_2$ = Standard $C_s \parallel R_s$

  $= R_s / (1+j\omega R_s C_s)$

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## Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement (Cont'd)

  Balanced, $Z_1 / Z_2 = N_1 / N_2$

  $$1/Z_1 = (N_2/N_1)\ 1/Z_2$$

  $$(1+j\omega R_x C_x)/R_x = (N_2/N_1)\ (1+j\omega R_s C_s)/R_s$$

  $$1/R_x + j\omega C_x = (N_2 / N_1 R_s) + j\omega C_s N_2/N_1$$

Real part: $R_x = R_s N_1/N_2$

Imagination part: $C_x = C_s N_2/N_1$

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## Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement

  $Z_1$ = Unknown $L_x \parallel R_x$

  $= 1/(1/R_x + 1/j\omega L_x)$

  $= 1/(1/R_x - j/\omega L_x)$

  $Z_2$ = Standard $C_s \parallel R_s$

  $= 1/(1/R_s + j\omega C_s)$

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## Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement (Cont'd)

  Balanced, $1/Z_1 = (N_2/N_1)\ 1/Z_2$

  $1/R_x - j/\omega L_x = (N_2/N_1)\ (1/R_s + j\omega C_s)$

Real part:

$$R_x = R_s N_1/N_2$$

Imagination part:

$$1/\omega L_x = -\ (N_2/N_1)\omega C_s$$ , Reversed Current

$$L_x = N_1 / N_2 \omega^2 C_s$$

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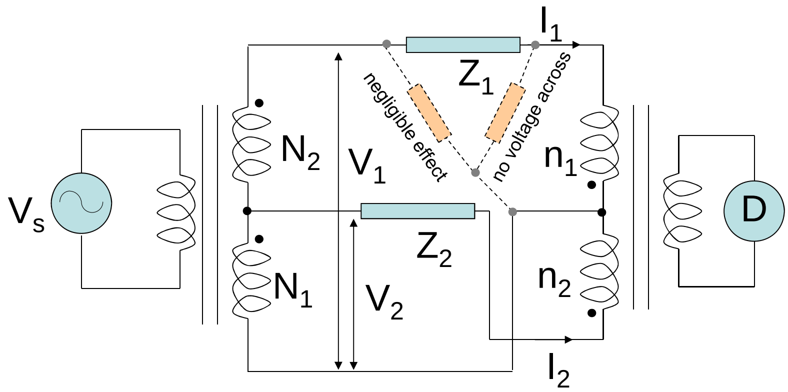
## Double Ratio Transformer Bridge

To measure the impedance of components *in Situ.*




$I_1 = V_1 / Z_1 = k(N_1+N_2)/Z_1$
$I_2 = V_2 / Z_2 = kN_2/Z_2$

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## Double Ratio Transformer Bridge (Cont'd)

Balanced, null current or zero magnetic flux,

$$n_1 I_1 = n_2 I_2$$

$$n_1(N_1+N_2)/Z_1 = n_2 N_2/Z_2$$

$$Z_1 = Z_2 \frac{n_1(N_1+N_2)}{n_2 N_2}$$

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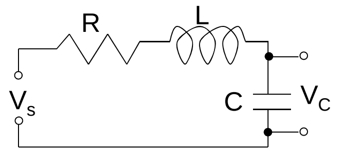
# Q-Meter

RLC Series Resonance

$Z = R + j\omega L + 1/j\omega C$

$\quad = R + j(\omega L - 1/\omega C)$



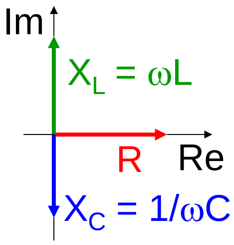

Resonant frequency

(When the voltage across C is a maximum.)

$\omega_0 L = 1/\omega_0 C$

$\omega_0^2 = 1/LC$

$\boxed{\omega_0 = 1/\sqrt{LC}}$






# Q-Meter (Cont'd)

$I_0 = V_s/R$

$V_C = I_0 X_C$

$\quad = (V_s/R)(1/\omega_0 C)$

$\quad = (1/\omega_0 RC)\, V_s$

$\quad = Q\, V_s$

$\quad \propto Q$

where Q = Reactance/Resistance

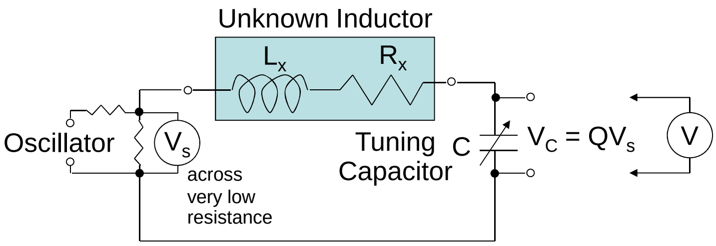
$\quad = \omega_0 L / R$

$\quad = \omega_0 (1/\omega_0^2 C) / R$

$\boxed{Q = 1 / \omega_0 RC}$

(Unloaded)

# Q-Meter (Cont'd)




Tuning to resonance, $\omega_0 = 1/\sqrt{LC}$

$L_x = 1 / \omega_0^2 C$

$R_x = 1 / Q\omega_0 C$

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# Q-Meter (Cont'd)

The circuit can be adjusted to the resonance by

- Preset the source frequency, and then vary the tuning capacitor for maximum value under this condition
- Preset the tuning capacitor and then adjust the source frequency
- This measures valus of Q in commonly regarded as the Q of the coil under test

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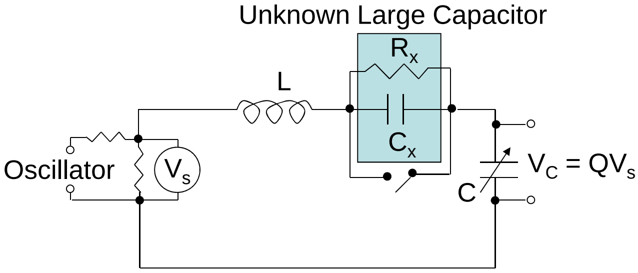
# Q-Meter: Low Impedance Measurement




Short circuit $\Rightarrow$ tuning $C = C_1$, $L_1 = L$, $R_1$

$Q_1 = 1/\omega_0 R_1 C_1$

$R_1 = 1/\omega_0 C_1 Q_1$

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# Q-Meter: Low Impedance Measurement (Cont'd)

Remove short circuit $\Rightarrow$ then tuning $C = C_2$

$C_2$ & $C_x = C_1$, $L_2 = L$, $R_2 = R_1$ & $R_x$

Series capacitors, $C_1 = 1 / (1/C_x + 1/C_2)$

$= C_2 C_x / (C_x + C_2)$

$C_1 C_x + C_1 C_2 = C_2 C_x$

$$\boxed{C_x = C_1 C_2 / (C_2 - C_1)}$$

$Q_2 = 1/\omega_0 R_2 C_2$

$R_2 = 1/\omega_0 C_2 Q_2$

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## Q-Meter: Low Impedance Measurement (Cont'd)

$R_2 = R_1 + R_x$

$R_x = R_2 - R_1$ , Leakage resistance

$= 1/\omega_0 C_2 Q_2 - 1/\omega_0 C_1 Q_1$

$$\boxed{R_x = (C_1 Q_1 - C_2 Q_2) / (\omega_0 C_1 C_2 Q_1 Q_2)}$$

$Q_x = 1/\omega_0 R_x C_x$

$$= \frac{(\omega_0 C_1 C_2 Q_1 Q_2)(C_2 - C_1)}{\omega_0 (C_1 Q_1 - C_2 Q_2) C_1 C_2}$$

$$\boxed{Q_x = Q_1 Q_2 (C_2 - C_1) / (C_1 Q_1 - C_2 Q_2)}$$

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## Q-Meter: Low Impedance Measurement (Cont'd)

If the unknown component is an inductor,

$L_x = 1/\omega_0^2 C_x$

$$\boxed{L_x = (C_2 - C_1) / \omega_0^2 C_1 C_2}$$

If the unknown component is a pure resistor (no reactance),

$R_x = (C_1 Q_1 - C_2 Q_2) / (\omega_0 C_1 C_2 Q_1 Q_2)$

$\boxed{R_x = (Q_1 - Q_2) / \omega_0 C_1 Q_1 Q_2}$ , $C_1 = C_2$

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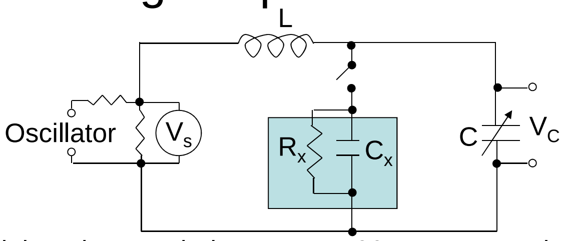
## Q-Meter: High Impedance Measurement




For high resistance, inductance > 100mH, or capacitance < 400 pF

Open circuit and tune $C = C_1$, $L_1 = L$, $R_1$

Then short circuit and tune $C = C_2$

$$C_2 \mathbin{//} C_x = C_1,\ L_2 = L,\ R_2 = R_x \mathbin{//} R_1$$

$C_x + C_2 = C_1 \Rightarrow \boxed{C_x = C_1 - C_2}$

$R_2 = R_x||R_1 = R_xR_1/(R_x+R_1) \Rightarrow R_x = R_1R_2/(R_1-R_2)$

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## Q-Meter: High Impedance Measurement (Cont'd)

Parallel RLC $\Rightarrow Q = \omega_0 RC$ (loaded)

$R_1 = Q_1/\omega_0 C_1$ and $R_2 = Q_2/\omega_0 C_2$

Therefore

$R_x = Q_1Q_2\omega_0C_1C_2 / \omega_0^2C_1C_2(Q_1C_2 - Q_2C_1)$

$\boxed{R_x = Q_1Q_2 / \omega_0(Q_1C_2 - Q_2C_1)}$

$Q_x = \omega_0 R_x C_x$

$= \omega_0 Q_1Q_2(C_1 - C_2) / \omega_0(Q_1C_2 - Q_2C_1)$

$\boxed{Q_x = Q_1Q_2(C_1 - C_2) / (Q_1C_2 - Q_2C_1)}$

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Q-Meter: High Impedance Measurement (Cont'd)

For unknown inductance,

$L_x = 1/\omega_0^2 C_x$

$\boxed{L_x = 1/\omega_0^2(C_1 - C_2)}$

For pure resistance,

$R_x = Q_1Q_2 / \omega_0(Q_1C_2 - Q_2C_1)$

$\boxed{R_x = Q_1Q_2 / \omega_0C_1(Q_1 - Q_2)}$ , $C_1 = C_2$



# References


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- Electronics Demonstrations webpage: http://www.falstad.com/circuit/e-index.html