Sources of Error

EIE 240 Electrical and Electronic Measurement (2/2014)
Class 3, January 30, 2015

Errors in Electrical Measurements

- Systematic error ⇒ every times you measure e.g. loading or insertion of the measurement instrument
- Meter error ⇒ scaling (inaccurate marking), pointer bending, friction, no calibration
- Random error ⇒ temperature effect, noises (unwanted signals)
- Reading error ⇒ parallax, read a wrong scale
- Recording error ⇒ for many measured values

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Background

- Electric charge, $e^{-} = 1.6\times10^{-19}$ Coulombs
- Current, $I = dq/dt$ Coulomb/sec = Amps
- Voltage, the difference in electrical potential between two points (Joules/Coulomb)
- Power, $P = IV$ Watts
- Energy, $E = Pt$ Units (kW·hr)
- Ohm's Law, $V = IR$

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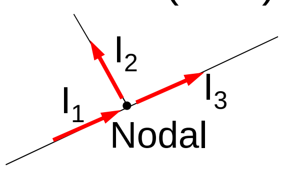
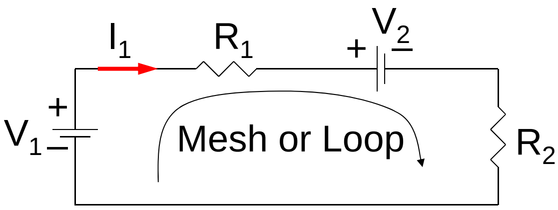
Background (Cont'd)

- Kirchhoff's Current Law (KCL)
  $I_1 - I_2 - I_3 = 0$



- Kirchhoff's Voltage Law (KVL)
  $-V_1 + I_1R_1 + V_2 + I_2R_2 = 0$



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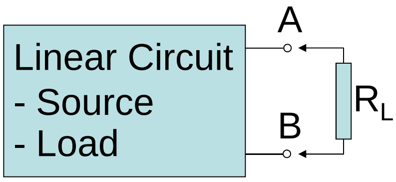
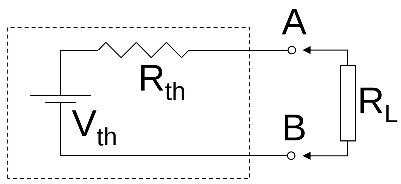
Background (Cont’d)

- Thévenin’s Theorem

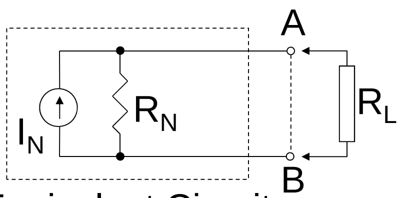




Equivalent Circuit

Open circuit to find V_{th}

- Norton’s Theorem
  Short circuit to find I_N



Equivalent Circuit

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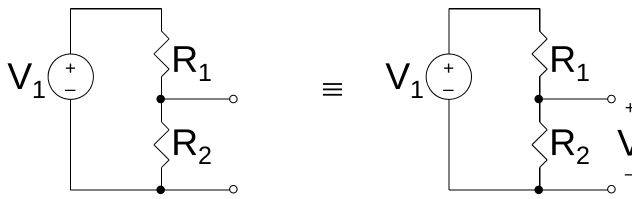
Background (Cont’d)

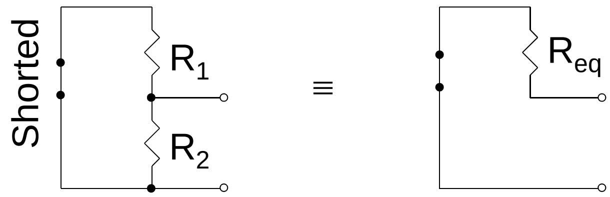
Example

No Load ⇒ Equivalent Source



$$V_{eq} = \frac{R_2}{R_1+R_2} V_1$$

No Source ⇒ Equivalent Load



$R_{eq} = R_1 \,||\, R_2$

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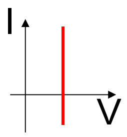
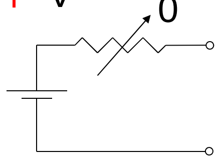
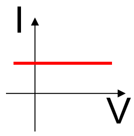
Background (Cont'd)

- Ideal Voltage Source Vs. Ideal Current Source

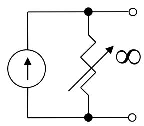


$I_\infty = V_{fixed} / R_0$





$V_\infty = I_{fixed} R_\infty$



- Superposition Theorem (for linear resistive network containing several sources)

  Voltage source ⇒ Short Circuit 0 V
  Current source ⇒ Open Circuit 0 A
  Vector summation of the individual voltage or current caused by each separate source

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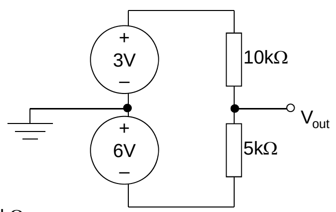
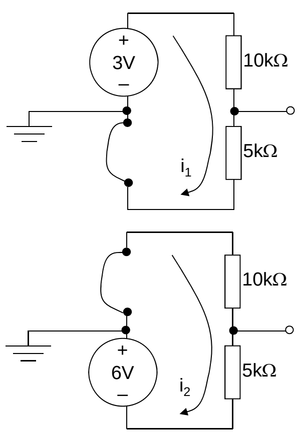
Background (Cont'd)

Example





$V_{out1} = 3V/(10k\Omega+5k\Omega) \cdot 5k\Omega = 1V$

$V_{out2} = -6V/(10k\Omega+5k\Omega) \cdot 10k\Omega = -4V$

$V_{out} = V_{out1} + V_{out2}$
$= 1V - 4V$
$= -3V$

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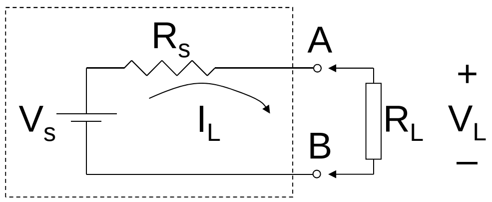
Background (Cont'd)

- The Maximum Power Transfer Theorem

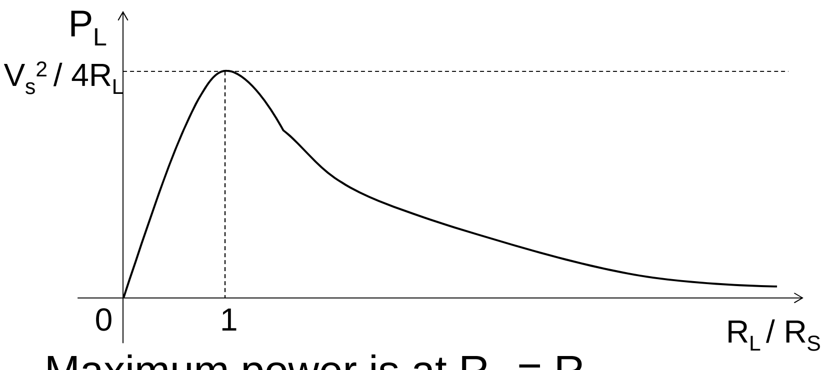


If V_s and R_s are fixed, when $I_L \uparrow$ then $V_L \downarrow$

$$P_L = (I_L)^2 R_L$$
$$= (V_s / (R_S+R_L))^2 R_L$$
$$= V_s^2 R_L / (R_S+R_L)^2$$
$$= [\, V_s^2 R_S / (R_S+R_L)^2 \,] \; (R_L / R_S)$$

Background (Cont'd)

- The Maximum Power Transfer Theorem



Maximum power is at $R_L = R_s$

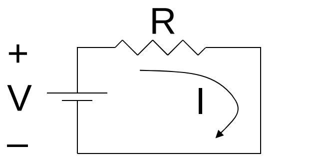
Therefore $V_L = V_s / 2$ and $I_L = V_s / 2R_L$

$$P_L = (I_L)^2 R_L = (V_s / 2R_L)^2 R_L = V_s^2 / 4R_L$$

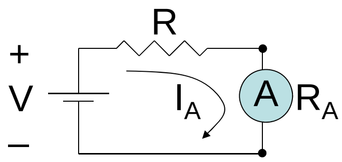
Ammeter Loading

For current measurement, open circuit to put an ammeter.

DC Circuit



$I = V/R$



$I_A = V/(R+R_A)$

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Ampmeter Loading (Cont'd)

Loading error, $I_A - I = V/(R+R_A) - V/R$

$$= \frac{VR - V(R+R_A)}{R(R+R_A)}$$

$$= \frac{-VR_A}{R(R+R_A)}$$

% Loading Error = $(I_A - I)/I \times 100\%$

$$= \frac{-R_A}{R+R_A} \times 100\%$$

It means if $R_A \uparrow$ then error $\uparrow$

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Ampmeter Loading (Cont'd)

- Using ammeters connected in series with the circuit being measured, it might not be practical or possible to redesign the meter for a lower input (lead-to-lead) resistance!

- However, we could select a value of shunt resistor to place in the circuit based on voltage drop, and it would be best to choose the lowest practical resistance.

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Ampmeter Loading (Cont'd)

- One ingenious way to reduce the impact that a current-measuring device has on a circuit is to use the circuit wire as part of the ammeter movement itself.
- All current-carrying wires produce a magnetic field, the strength of which is in direct proportion to the strength of the current.
- By building an instrument that measures the strength of that magnetic field, a no-contact ammeter can be produced.

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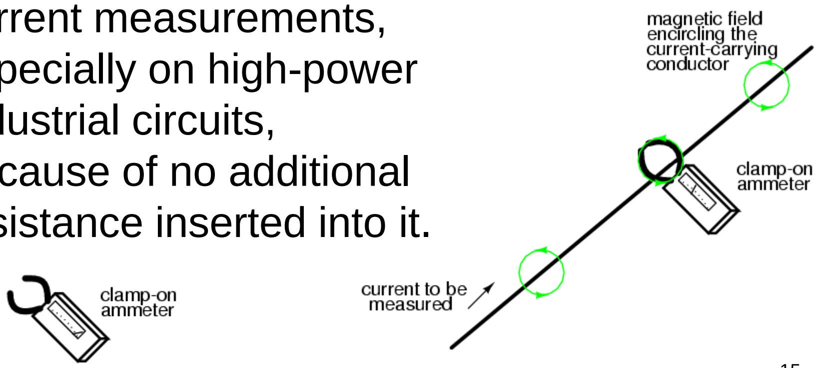
Ampmeter Loading (Cont'd)

- Ammeters of this design are called "clamp-on" meters because they have "jaws" which can be opened and secured around a circuit wire.
- Clamp-on ammeters make for quick and safe current measurements, especially on high-power industrial circuits, because of no additional resistance inserted into it.



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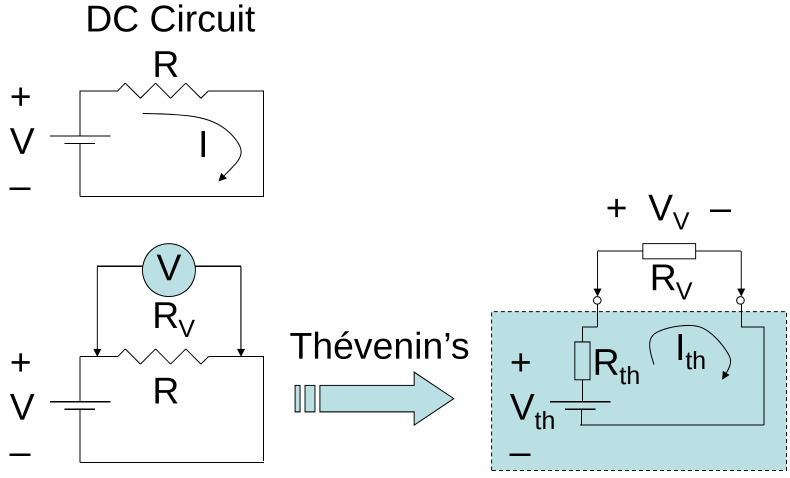
Voltmeter Loading

For voltage measurement, voltmeter is placed in parallel with the circuit element.



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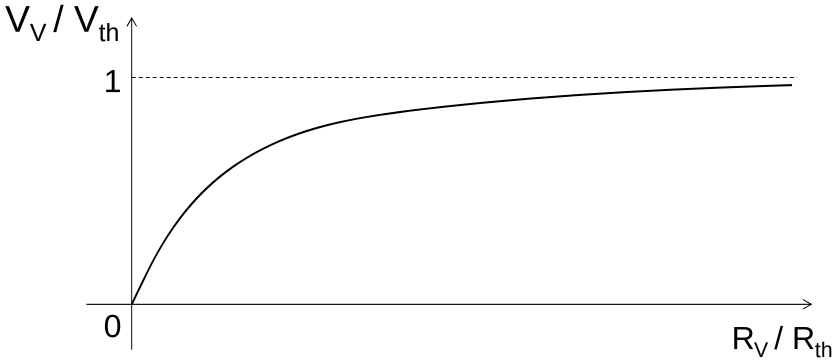
Voltmeter Loading (Cont'd)

$I_{th} = V_{th} / (R_{th} + R_V)$

$V_V = I_{th}R_V = V_{th}R_V / (R_{th} + R_V)$, Voltage Divider

$(V_V / V_{th}) = R_V / (R_{th} + R_V)$

$= [R_{th} / (R_{th} + R_V)] \; (R_V / R_{th})$



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Voltmeter Loading (Cont'd)

$$\text{Error} = V_V - V_{th}$$

$$= \left(\frac{R_V}{R_{th} + R_V} - 1 \right) V_{th}$$

$$\%\ \text{Error} = (V_V - V_{th})/V_{th} \times 100\%$$

$$= \frac{-R_{th}}{R_{th}+R_V} \times 100\%$$

It means if $R_V \uparrow$ then error $\downarrow$

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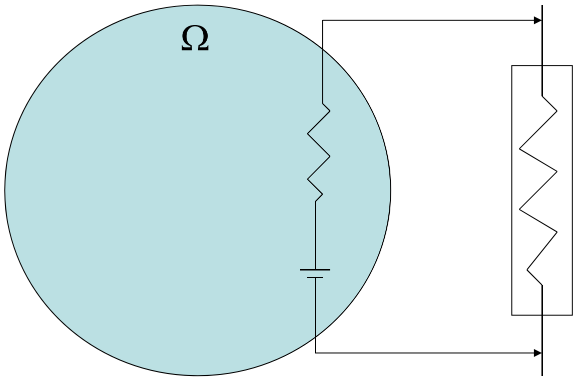
Ohmmeter Loading

Using a circuit within the instrument



Analog Ammeters & Voltmeters

- The users have to read the meter manual to find the basic accuracy specifications.
- Moving-coil-based analog meters are characterized by their full-scale deflection (f.s.d.) and effective resistance of the meters (few Ω - kΩ)
- Typical meters produce a f.s.d. current of 50 μA - 1 mA (this is not the ranges of the meters)

Percentage of FSD

- The accuracy of analog ammeters and voltmeters are quoted as a percentage of the full-scale deflection of pointer on the linear scale for all reading on the selected range.
- e.g. an ammeter having accuracy $\pm$2% f.s.d. within the range 0-10 A,
  Relative error $\Rightarrow \pm$ 2% of 10 A
  Absolute error $\Rightarrow \pm$ 0.2 A

- When a measured value is close to full scale, or at least above 2/3 of full scale, the published accuracy is meaningful.

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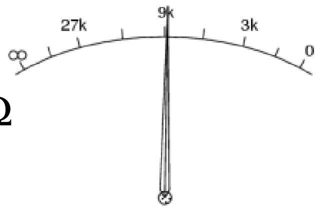
Percentage of Midscale

- Moving-coil-based ohmmeters is characterized by their midscale deflection of the pointer on nonlinear scale.
- What figure lies exactly between infinity and zero?
- e.g. $\pm$ 3% of midscale
  Relative error $\Rightarrow \pm$ 3% of 9 kΩ
  Absolute error $\Rightarrow \pm$ 0.27 kΩ



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Percentage of DMM Reading

- In the case of digital instruments, the accuracy is generally quoted as ± the percentage of the reading ± 1 digit.
- e.g. ± 0.5% Reading ± 1 Digit
  when you read a voltage 1.2 V with a digit of 0.03 V (depending on the range selected)

  error = ± 0.5% of 1.2 V ± 0.03 V

  = ± 0.036 V ⇒ absolute error

  = ± 3% of reading ⇒ relative error

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Potential Meter

- Before the moving coil meter, potentiometer was used in measuring the voltage in a circuit
- Calculated from a fraction of a known voltage (on resistive slide wire)
- Based on voltage divider (linearly)
- By means of a galvanometer (no deflection or null-balance)

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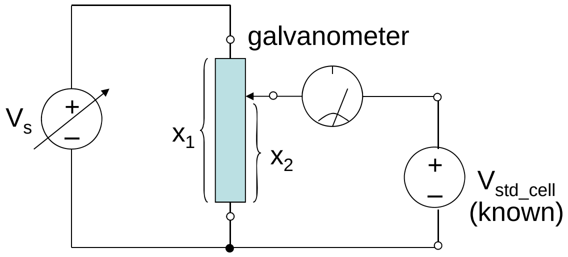
Calibration of Potentiometer



- The end of a uniform resistance wire is at x_1
- The sliding contact or wiper is then adjusted to x_2
- The standard electrochemical cell whose E.M.F. is known (e.g. 1.0183 volts for Weston cell)

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Calibration of Potentiometer (Cont'd)

- Adjust supply voltage V_s until the galvanometer shows zero
  (voltage on R_2 equals to the standard cell voltage or no current flows through the galvanometer)
- $x_2/x_1\ V_s = V_{std\_cell}$
  $V_s = x_1/x_2\ V_{std\_cell}$

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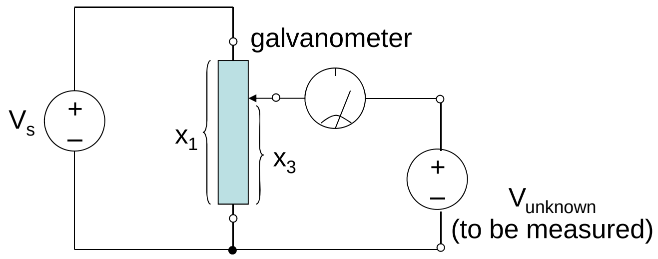
Measuring of Potentiometer



- Slide the wiper (change x_3) until the galvanometer shows zero again
- $V_{unknown} = x_3/x_1\ V_s$
  $= x_3/x_2\ V_{std\_cell}$

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Rheostat

- Variable resistor with two terminals

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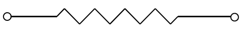
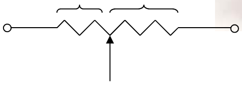
Loading for a Potentiometer

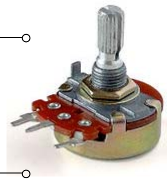
Resistor

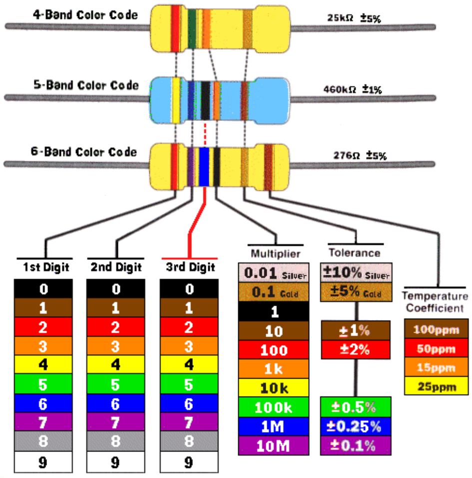
- Fixed



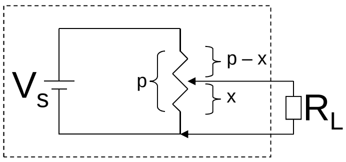
- Adjustable
  Potentiometer

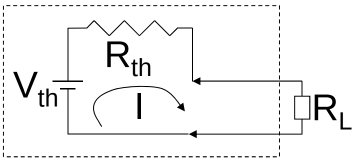






Loading of a Potentiometer (Cont'd)

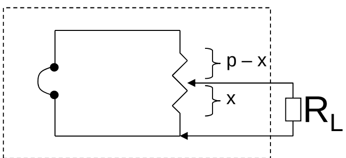




Thévenin's

Voltage Divider

$V_{th} = (x / p)\ V_s$



Superposition

$$\frac{1}{R_{th}} = \frac{1}{(x/p)R_p} + \frac{1}{((p-x)/p)R_p}$$

$$R_{th} = (x/p)R_p(1 - x/p)$$

Loading of a Potentiometer (Cont'd)

$$V_{th} = I\,(R_{th}+R_L)$$

$$(x/p)V_s = I\,[\,(x/p)(1 - x/p)R_p + R_L\,]$$

$$I = \frac{(x/p)V_s}{(x/p)(1 - x/p)R_p + R_L}$$

$$V_L = IR_L = \frac{(x/p)V_sR_L}{(x/p)(1 - x/p)R_p + R_L}$$

$$= \frac{(x/p)V_s}{(R_p/R_L)(x/p)(1 - x/p) + 1}$$

It means the relationship between V_L and x is nonlinear!

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Loading of a Potentiometer (Cont'd)

Error,

$$V_{th} - V_L = (x/p)V_s\left(1 - \frac{1}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

$$= (x/p)V_s\left(\frac{(R_p/R_L)(x/p)(1 - x/p)}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

If $R_L >> R_p$ then $R_p/R_L \approx 0$

Error $\approx (x/p)V_s(R_p/R_L)(x/p)(1 - x/p)$

$\approx V_s(R_p/R_L)[(x/p)^2 - (x/p)^3]$, $R_L \uparrow$ error $\downarrow$

% Error $\approx (R_p/R_L)[(x/p) - (x/p)^2] \times 100\%$

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Loading of a Potentiometer (Cont'd)

Let $d(\text{Error})/dx = V_s(R_p/R_L)[2x/p^2 – 3x^2/p^3] = 0$

We get $2x/p^2 = 3x^2/p^3$

$x/p = 2/3$ for the maximum

Therefore,

the max error $= V_s(R_p/R_L)[(2/3)^2 – (2/3)^3]$

$\approx 0.148\ V_s(R_p/R_L)$

and the max % error $\approx 22.2\ (R_p/R_L)$ %

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Potentiometric Voltage Measurement

- One ingenious solution to the problem of voltmeter loading is that of the *potentiometric* or *null-balance* instrument.
- In a potentiometric instrument, a precision adjustable voltage source is compared against the measured voltage, and a sensitive device called a *null detector* is used to indicate when the two voltages are equal.

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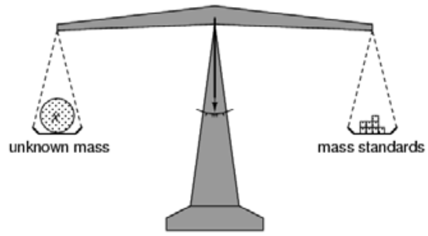
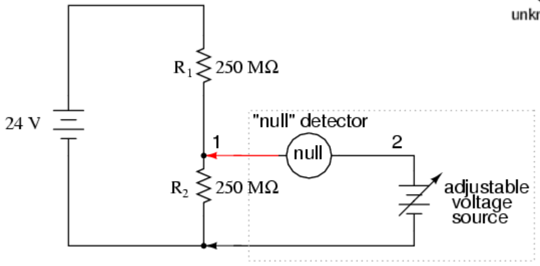
Potentiometric Voltage Measurement (Cont'd)

- When the voltages are equal, there will be zero current drawn from the circuit under test, and thus the measured voltage should be unaffected.





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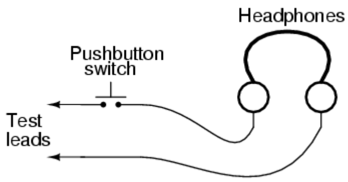
Null Detector

- An extremely simple type of sensitive null detector is a set of audio headphones.
- When a DC voltage is initially applied to a speaker, the resulting current through it will move the speaker cone and produce an audible "click."
- Another "click" sound will be heard when the DC source is disconnected.



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Reference

- http://en.wikipedia.org/wiki/Potentiometer
- http://www.allaboutcircuits.com/vol_1/chpt_8/5.html
- http://www.allaboutcircuits.com/vol_1/chpt_8/3.html

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