ENE 104
Electric Circuit Theory

Lecture 04:
Useful Circuit Analysis Techniques

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Objectives :

- Superposition
- Source transformation
- the Thevenin equivalent of any network
- the Norton equivalent of any network
- the load resistance that will result in maximum power transfer
- a Δ-connected network, a Y-connected network
- a dc sweep in PSpice

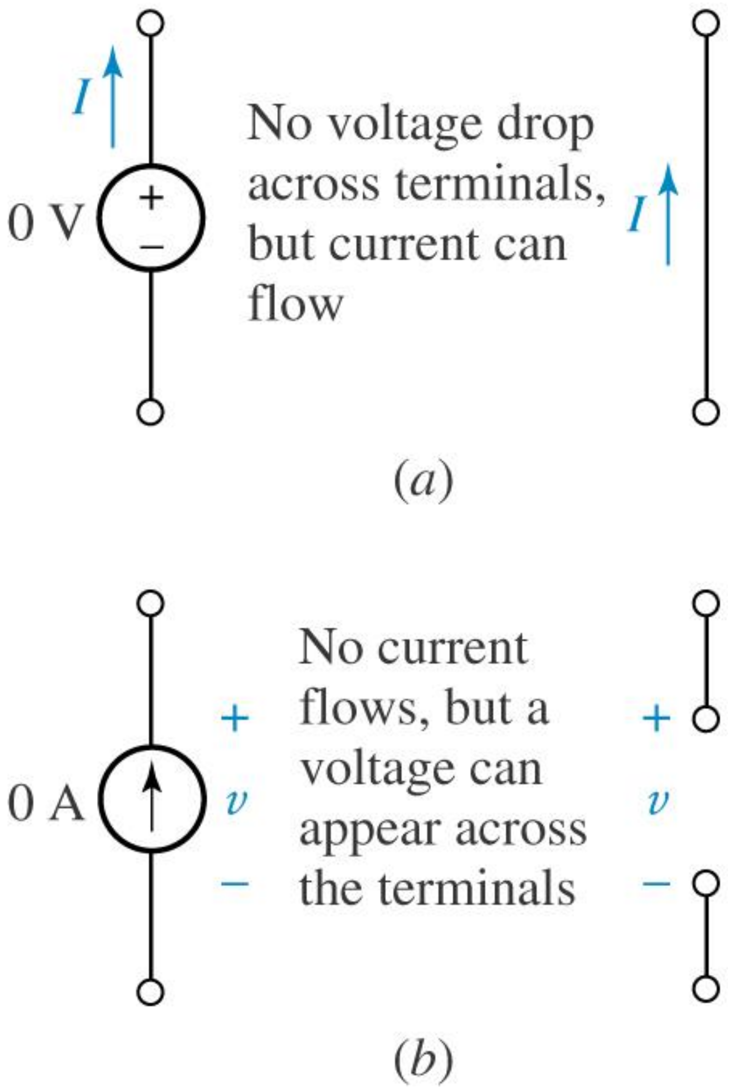
Ch5-Useful Techniques:

Learning methods of simplifying the analysis of more complicated circuits.

The principle of superposition:

The response in a linear circuit having more than one independent source can be obtained by adding the responses caused by the separate independent sources acting alone.

The Superposition Principle:

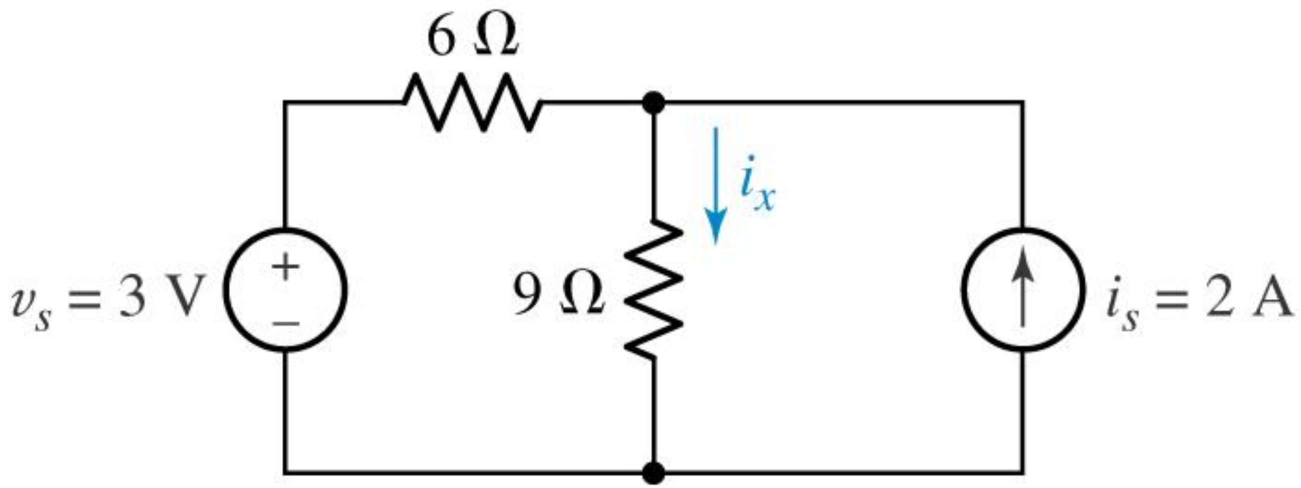


(a) **A voltage source set to zero acts like a short circuit.**

(b) **A current source set to zero acts like an open circuit.**

Example 5.1:

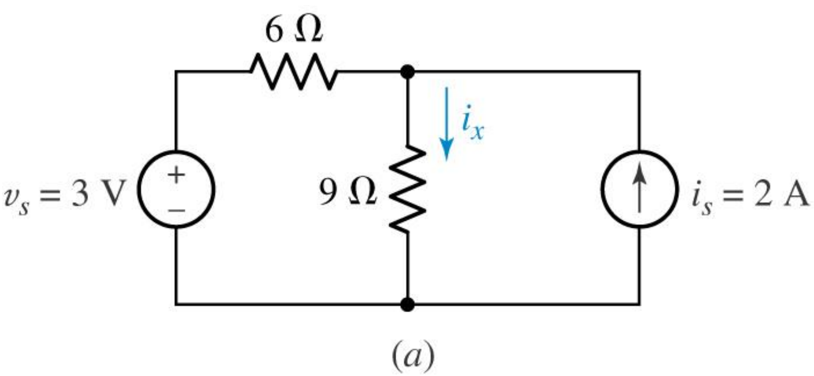
Use superposition to find the current i_x.



(a)

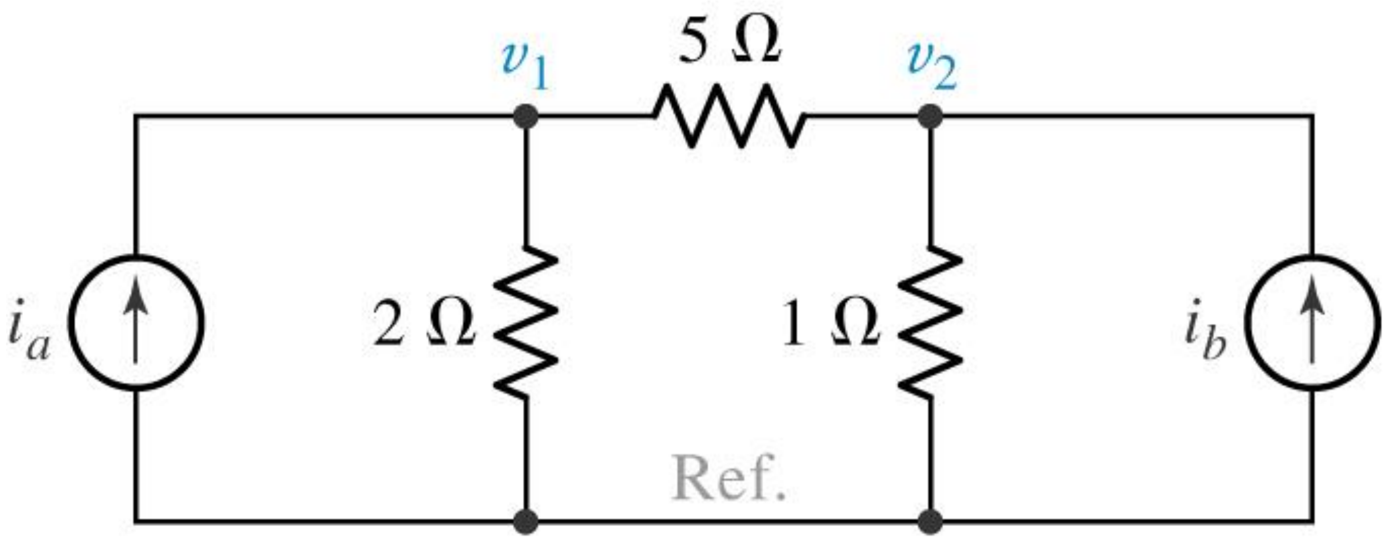
Example:

Use superposition to find the current i_x.



$$i_x = i_x' + i_x'' = \frac{3}{6+9} + 2\left(\frac{6}{6+9}\right) = 0.2 + 0.8 = 1.0 \text{ A.}$$

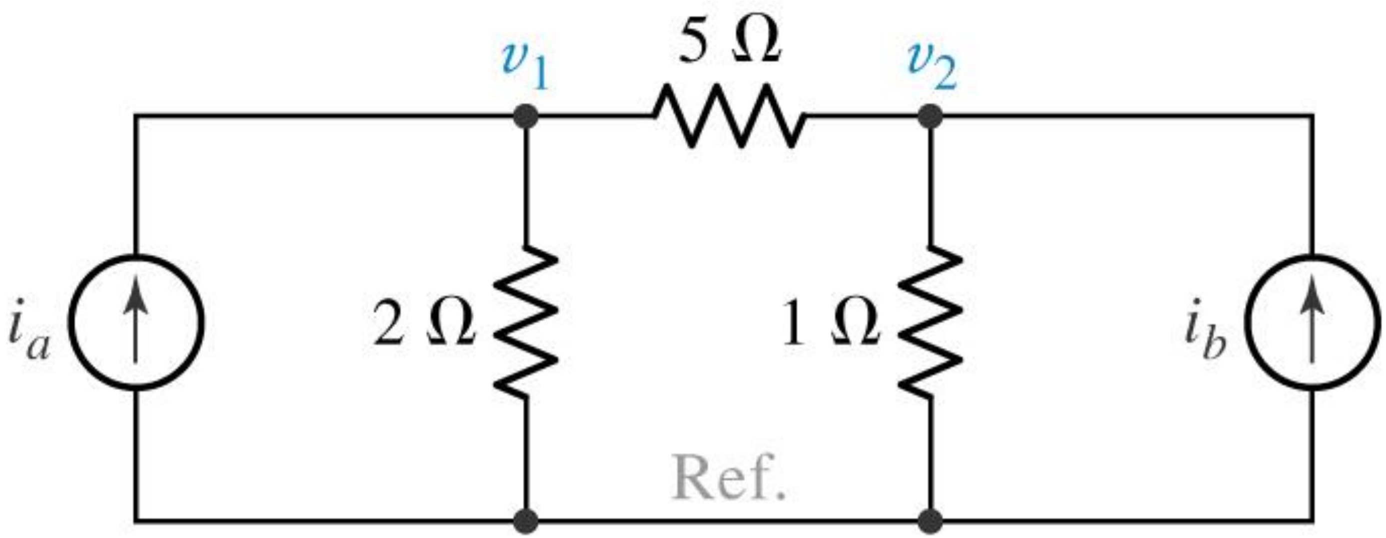
The Superposition Principle:



Sources are often called forcing functions.

→ responses

The Superposition Principle:



Nodal equations:

$$0.7v_1 - 0.2v_2 = i_a \qquad \ldots [1]$$

$$-0.2v_1 + 1.2v_2 = i_b \qquad \ldots [2]$$

The Superposition Principle:

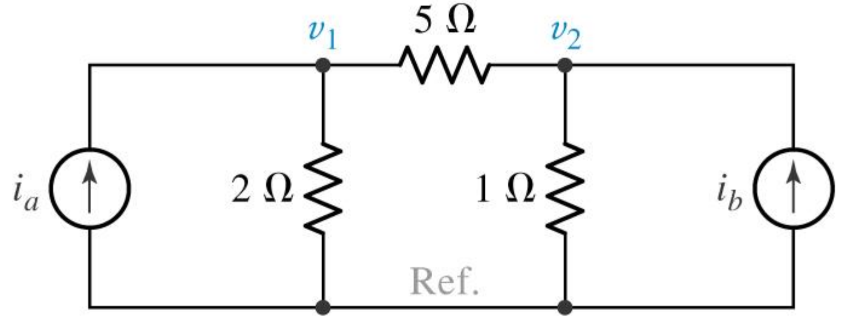
Change the two forcing functions:

$$0.7v_{1x} - 0.2v_{2x} = i_{ax} \qquad \dots [3]$$

$$-0.2v_{1x} + 1.2v_{2x} = i_{bx} \qquad \dots [4]$$

$$0.7v_{1y} - 0.2v_{2y} = i_{ay} \qquad \dots [5]$$

$$-0.2v_{1y} + 1.2v_{2y} = i_{by} \qquad \dots [6]$$



$$(0.7v_{1x} + 0.7v_{1y}) - (0.2v_{2x} + 0.2v_{2y}) = (i_{ax} + i_{ay}) \qquad \dots [7]$$

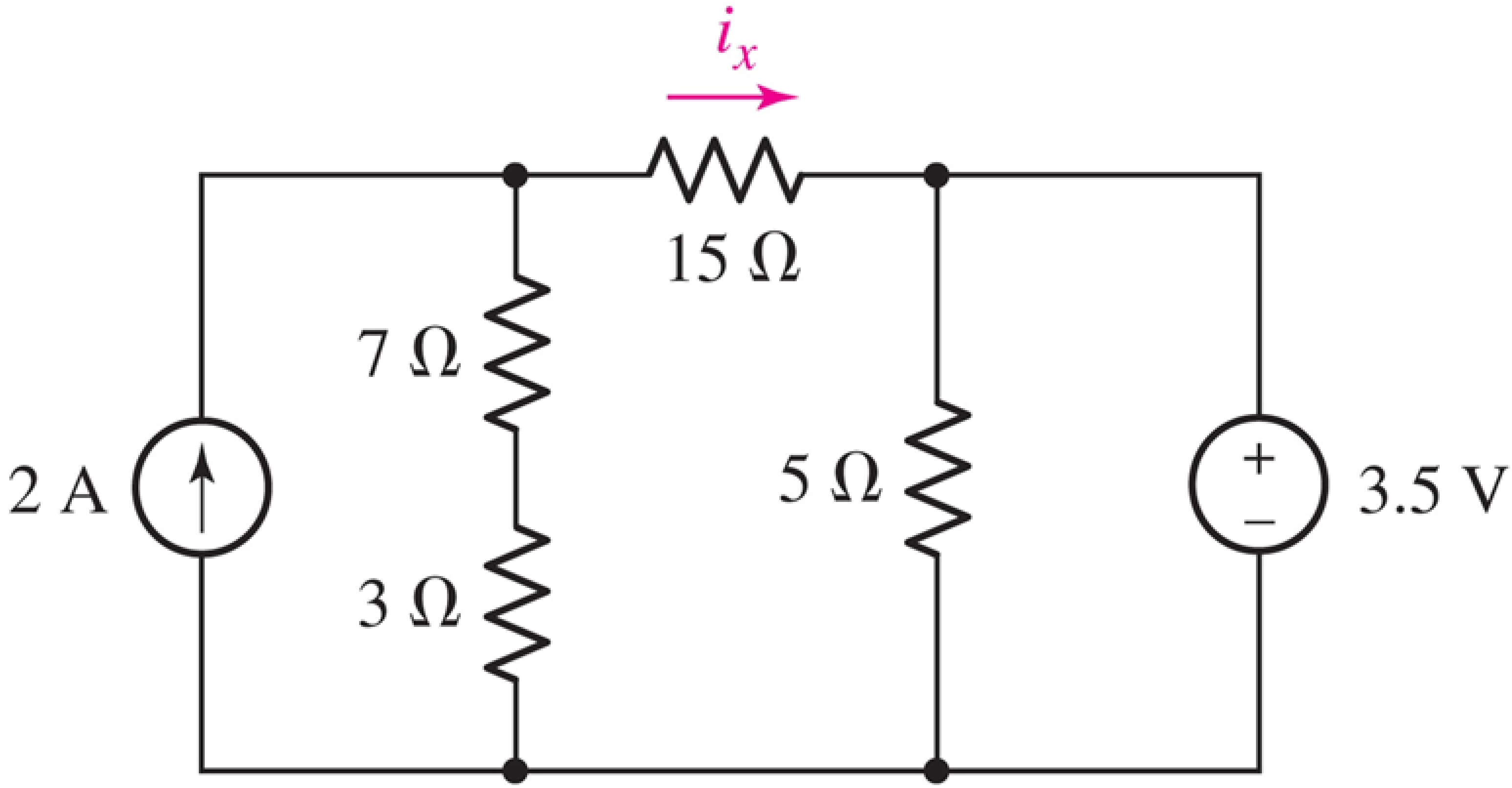
$$0.7v_1 - 0.2v_2 = i_a \qquad \dots [1]$$

$$-(0.2v_{1x} + 0.2v_{1y}) + (1.2v_{2x} + 1.2v_{2y}) = (i_{bx} + i_{by}) \qquad \dots [8]$$

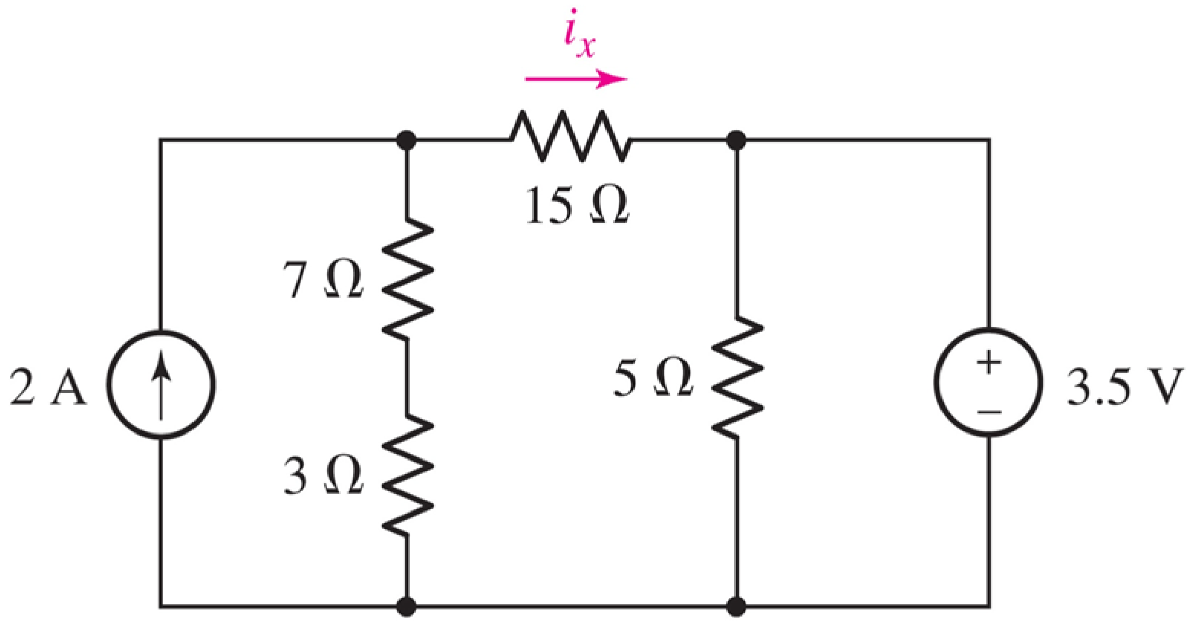
$$-0.2v_1 + 1.2v_2 = i_b \qquad \dots [2]$$

Practice: 5.1

Use superposition to find the current i_x.



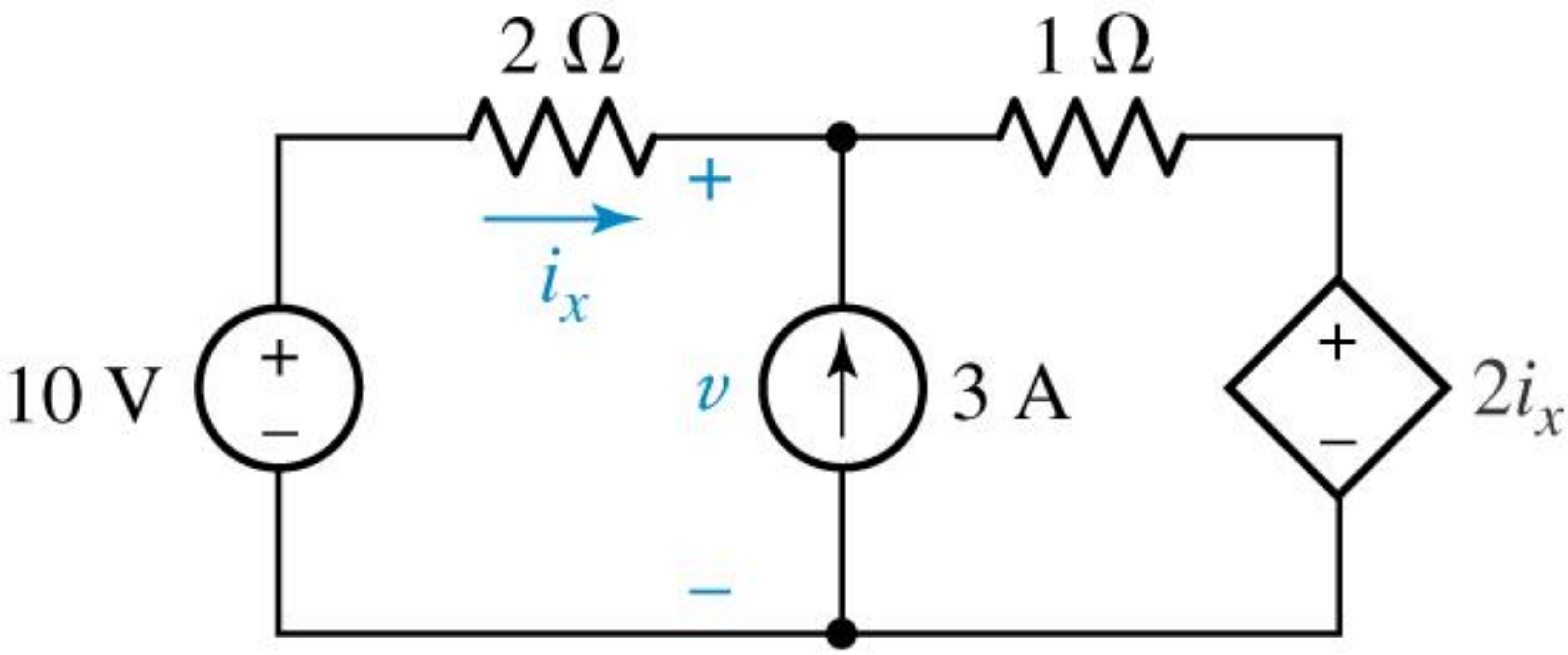
Practice: 5.1



$$i_x = i_x\big|_{2A} + i_x\big|_{3.5V} = \frac{10}{(10+15)}(2A) + \frac{-3.5V}{25\Omega}$$

Example 5.3:

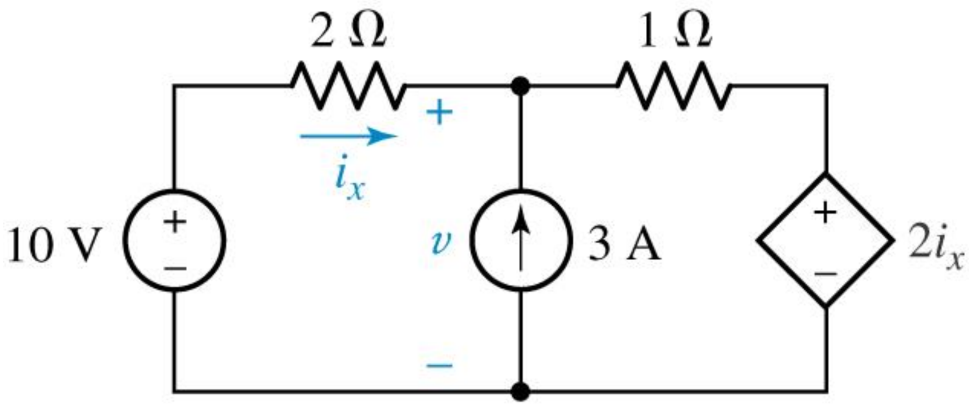
Use superposition to find the current i_x.



(a)

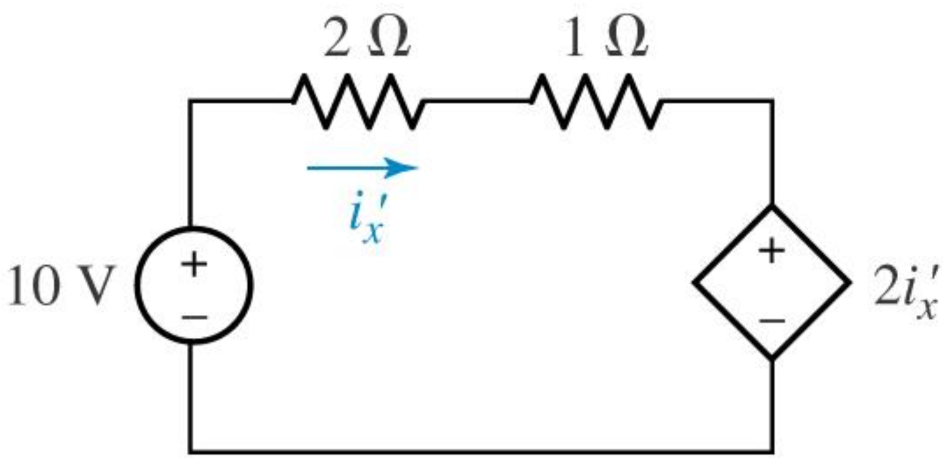
Example:

Use superposition to find the current *i_x*.

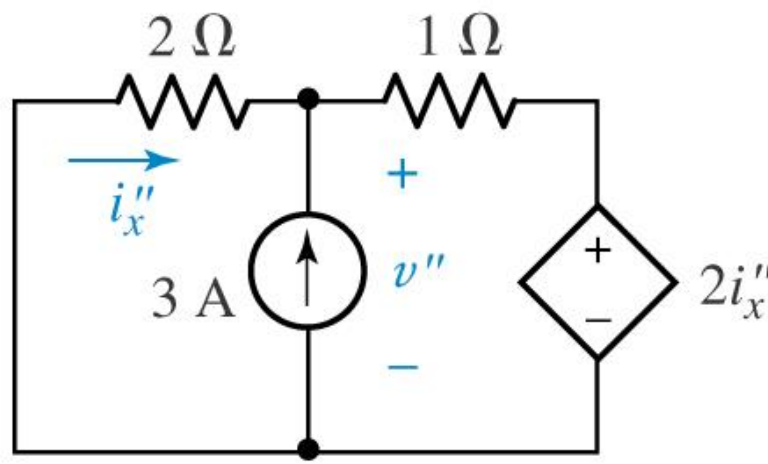


(a)

$$i_x = i_x^{'} + i_x^{''}$$



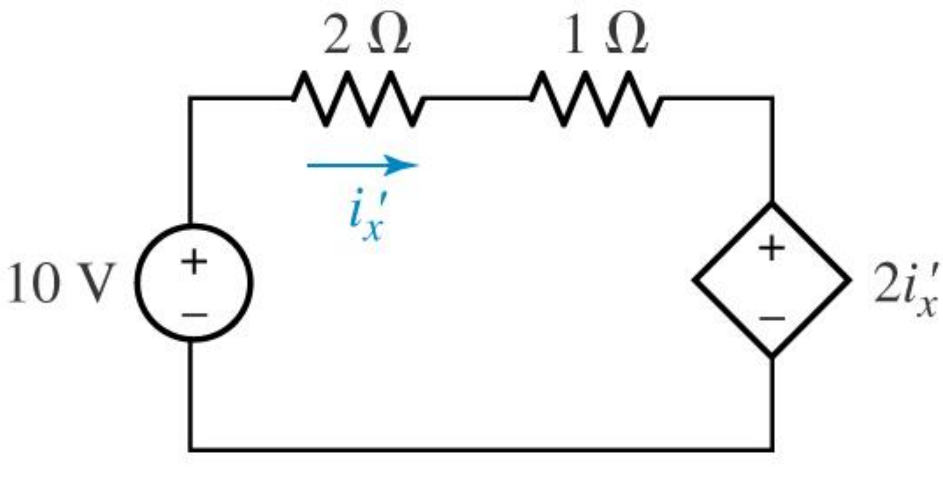
(b)



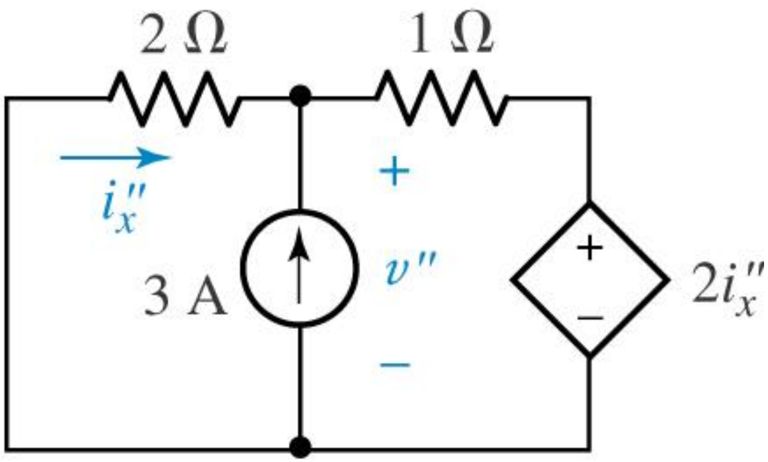
(c)

Example:

Use superposition to find the current i_x.



(b)



(c)

Figure b:

$$-10 + 2i_x' + 1i_x' + 2i_x' = 0$$

$$\therefore i_x' = 2\text{ A.}$$

Figure c:

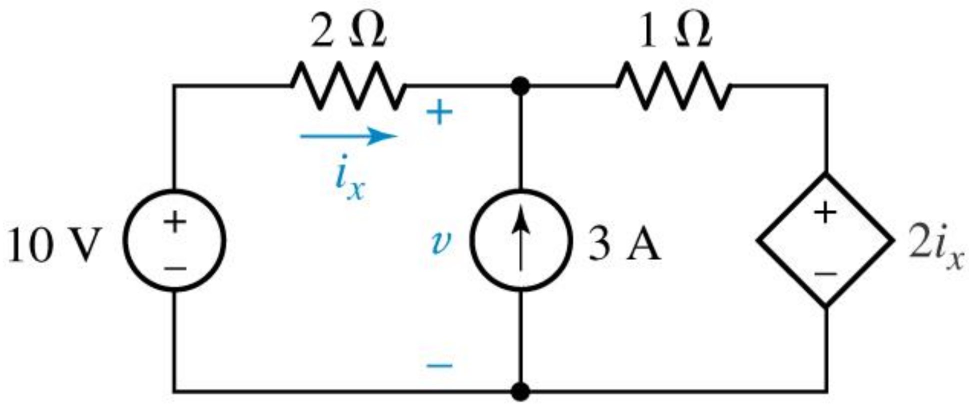
$$\frac{v''}{2} + \frac{v'' - 2i_x''}{1} = 3 \qquad \ldots [1]$$

And

$$v'' = -2i_x'' \qquad \ldots [2]$$

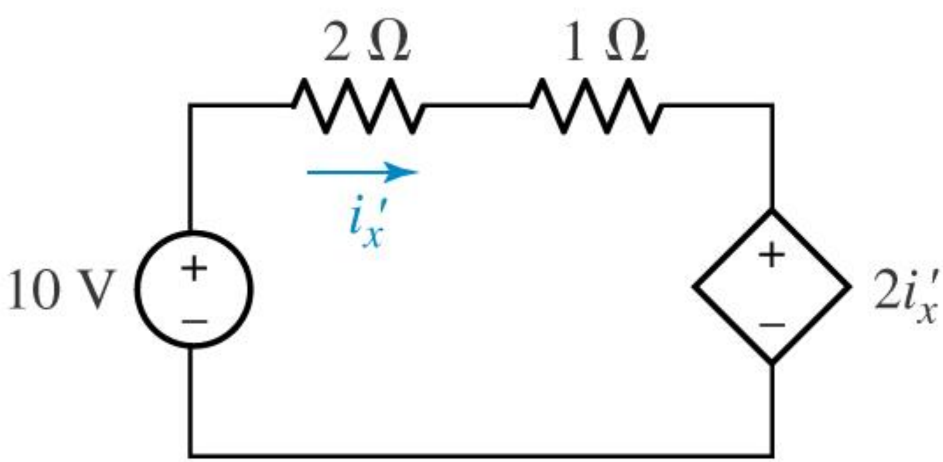
Example:

Use superposition to find the current i_x.

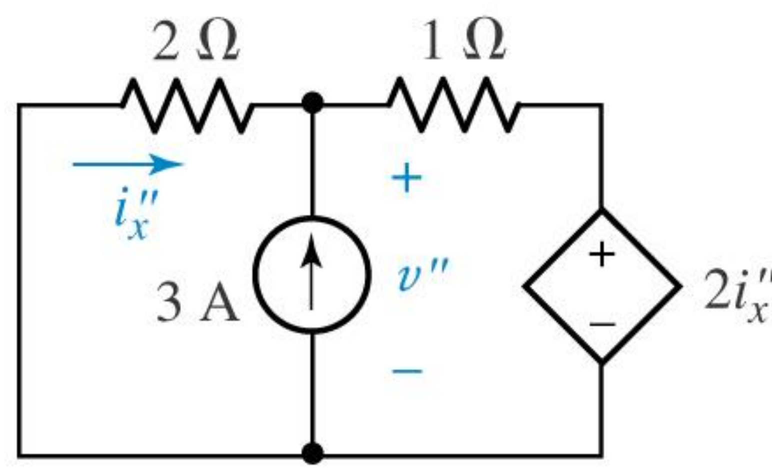


(a)

$$
\begin{aligned}
i_x &= i_x' + i_x'' \text{ A.} \\
&= 2 + (-0.6) \text{ A.} \\
&= 1.4 \text{ A.}
\end{aligned}
$$



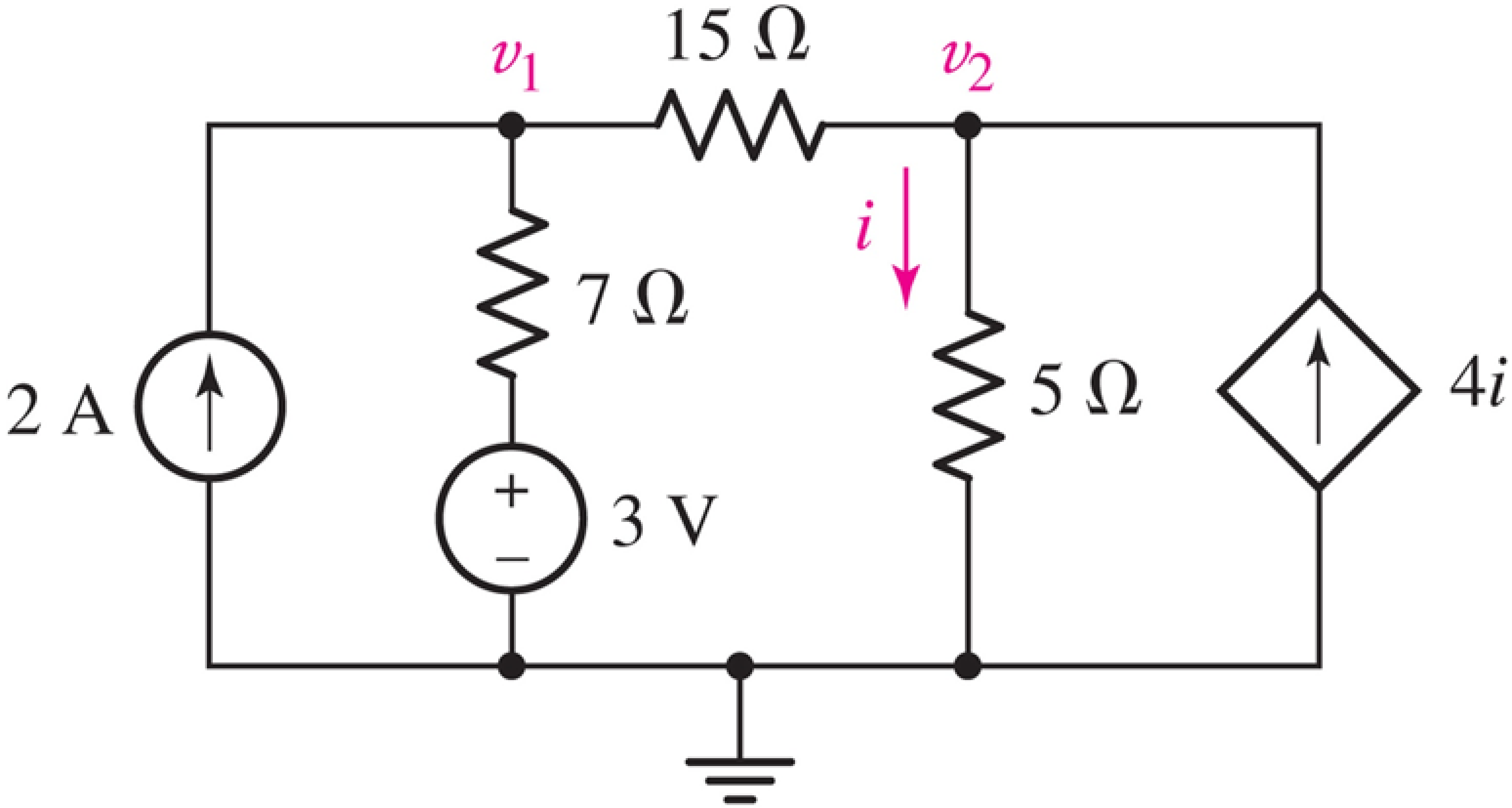
(b)



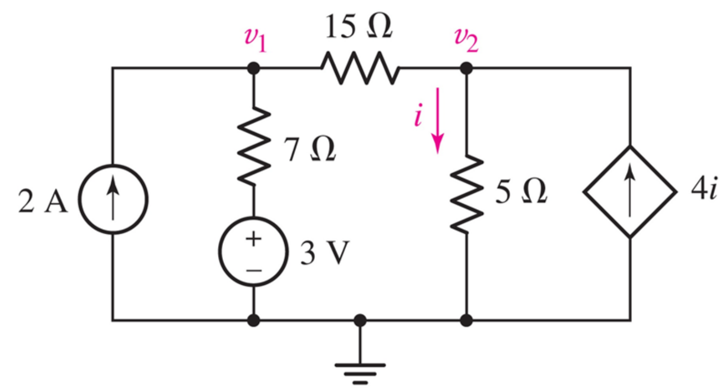
(c)

Practice: 5.2

Use superposition to obtain the voltage across each current source.



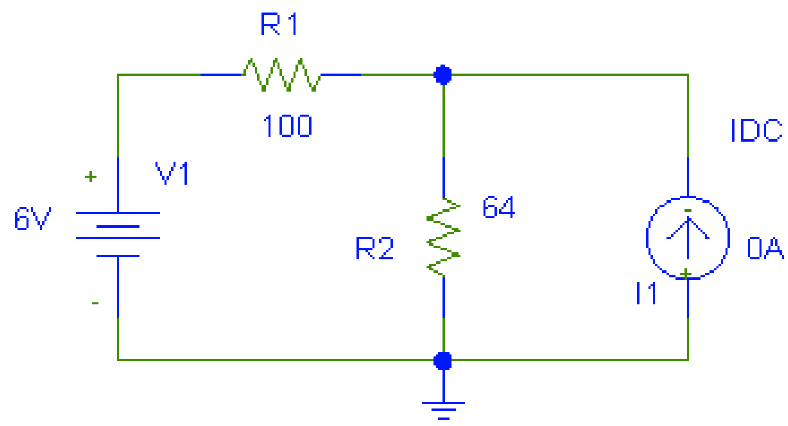
Practice: 5.2

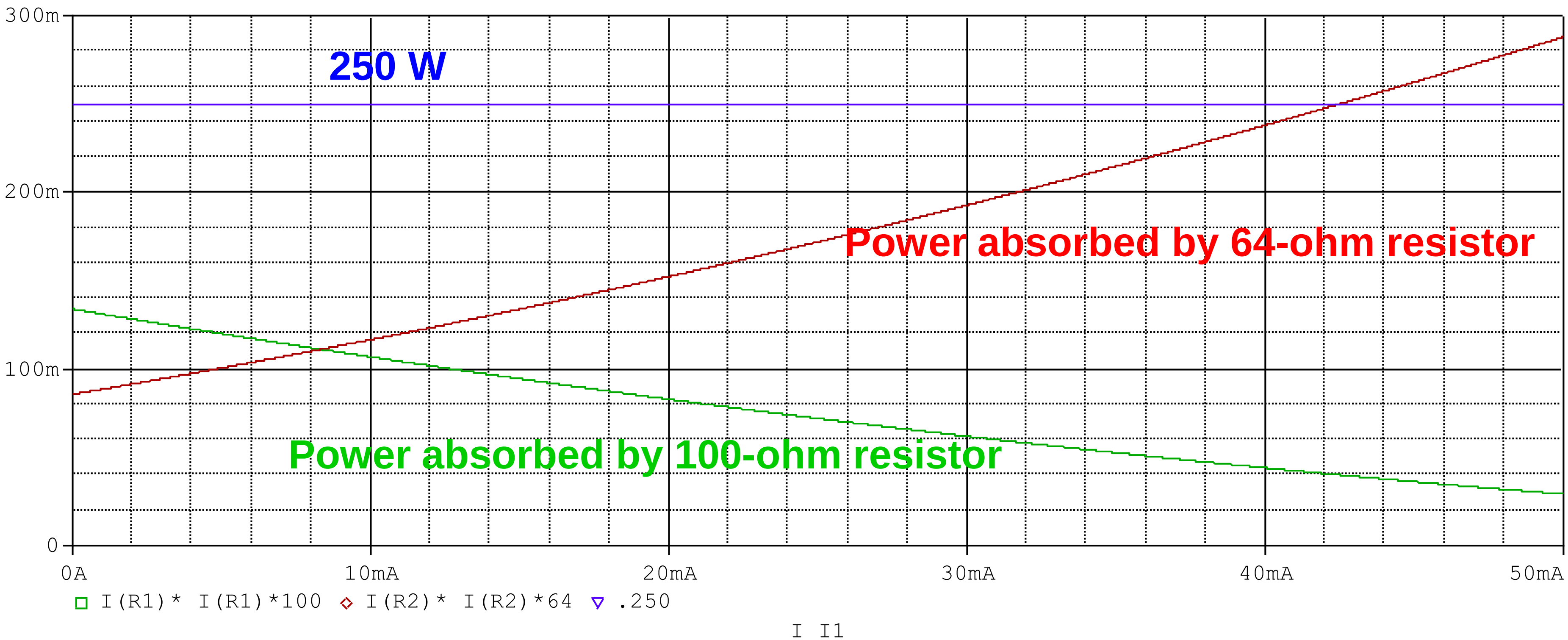


$$v_1\big|_{2A} = \frac{5}{(22+5)}(4i)(7\Omega) = \frac{5}{(22+5)}\left(4\frac{v_2}{5\Omega}\right)(7\Omega)$$

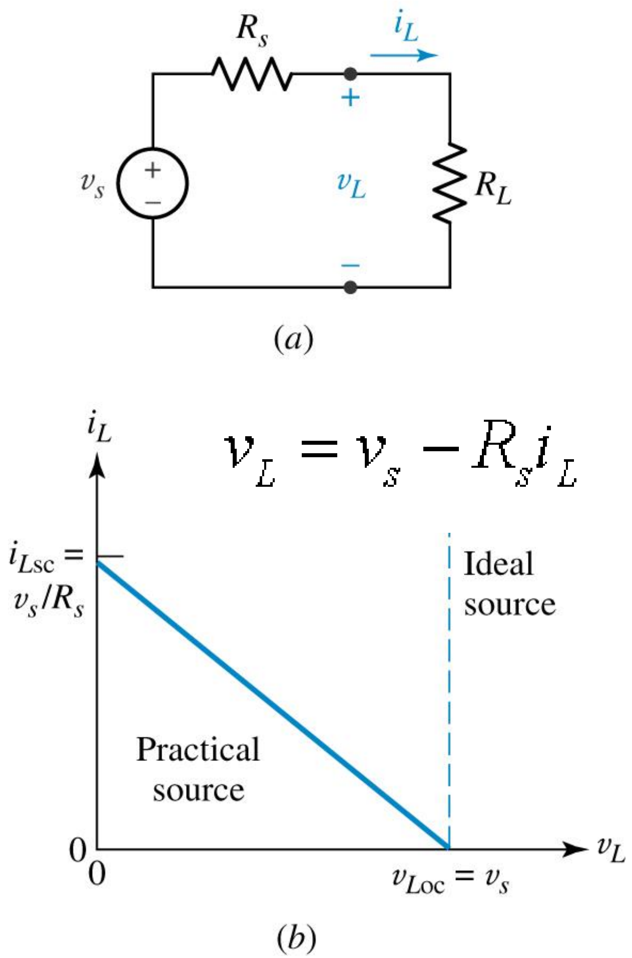
$$v_2\big|_{2A} = \frac{5}{(22+5)}(4i)(5\Omega) = \frac{5}{(22+5)}\left(4\frac{v_2}{5\Omega}\right)(5\Omega)$$

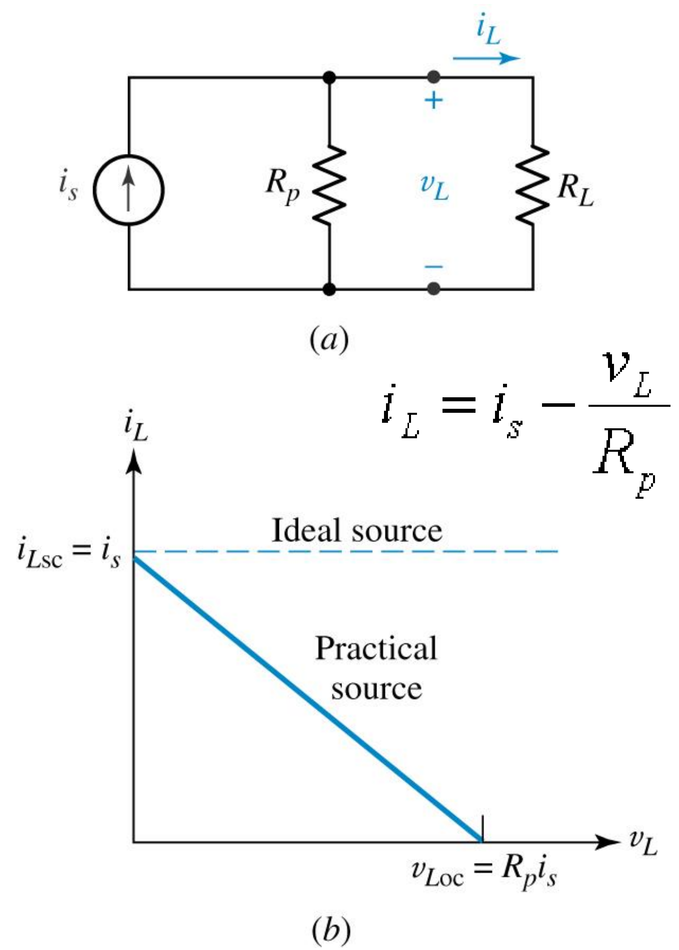
Pspice: dc sweep





Source Transformation:

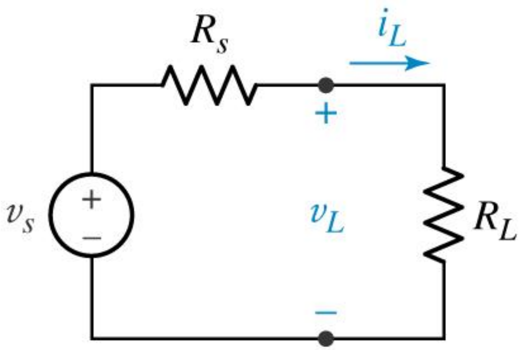


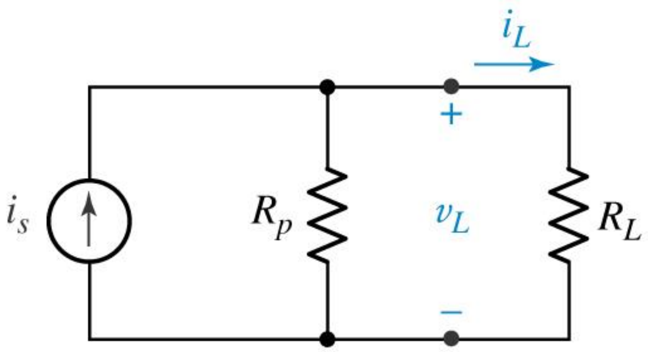


(a) A general practical voltage source connected to a load resistor R_L. (b) The terminal characteristics compared to an ideal source.

(a) A general practical current source connected to a load resistor R_L. (b) The terminal characteristics compared to an ideal source.

Equivalent Sources:





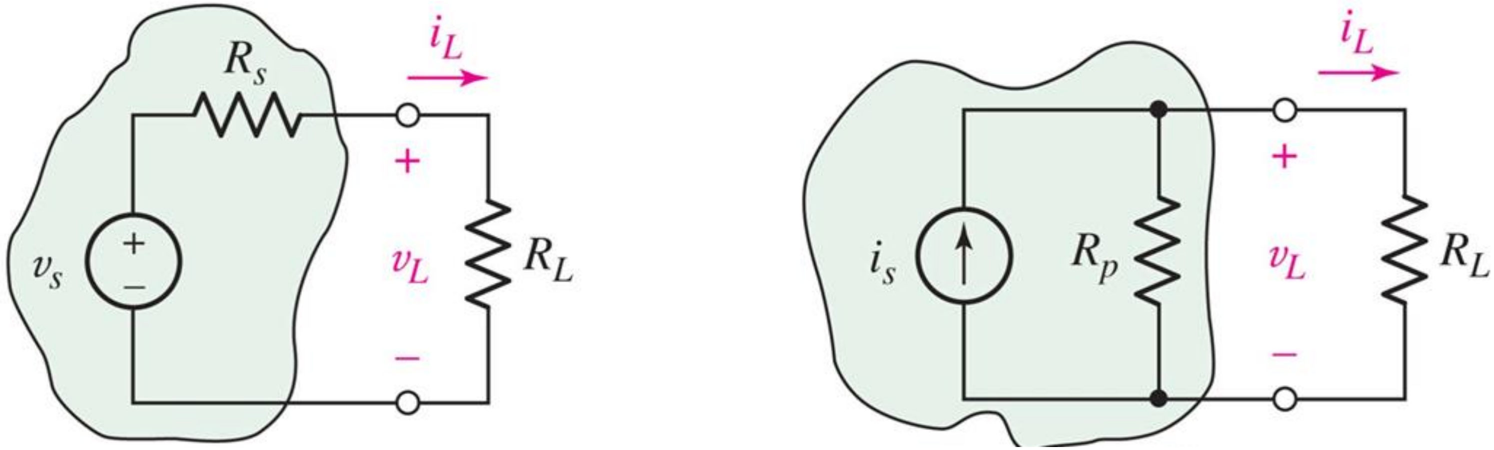
$$v_L = v_s \frac{R_L}{R_s + R_L}$$

$$= \left(\frac{v_s}{R_s + R_L} \right) \cdot R_L$$

$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) \cdot R_L$$

$$= \left(\frac{i_s R_p}{R_p + R_L} \right) \cdot R_L$$

Equivalent Sources:



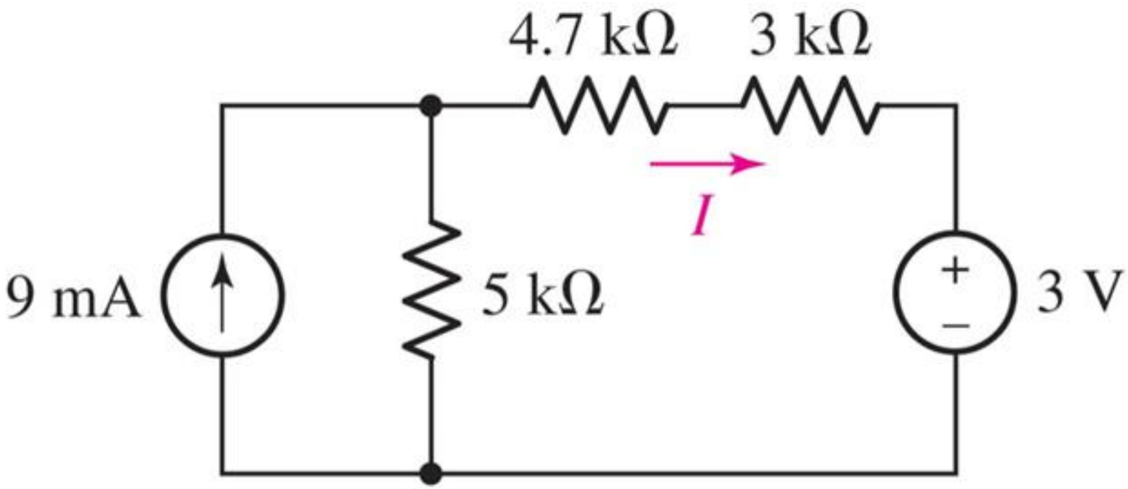
The two practical sources are electrically equivalent, If

$$R_s = R_p$$

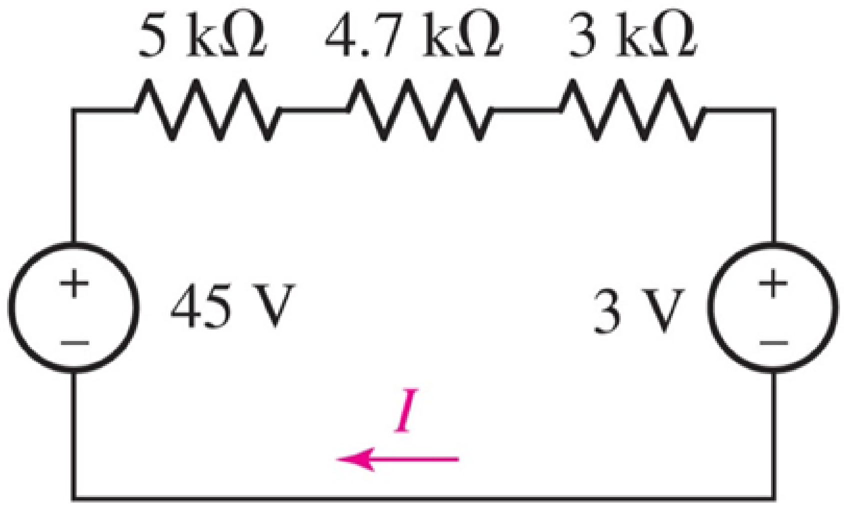
And

$$v_s = R_p i_s = R_s i_s$$

Example 5.4:



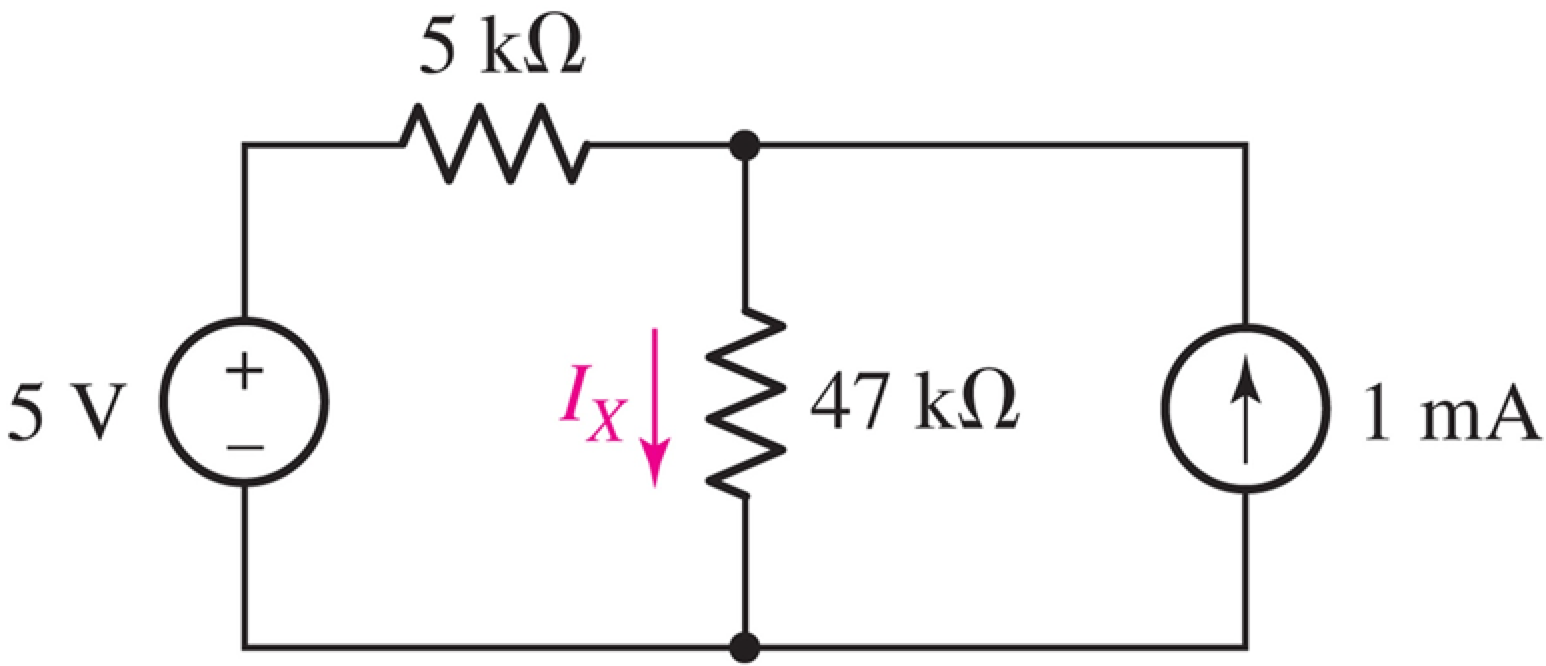
(a)



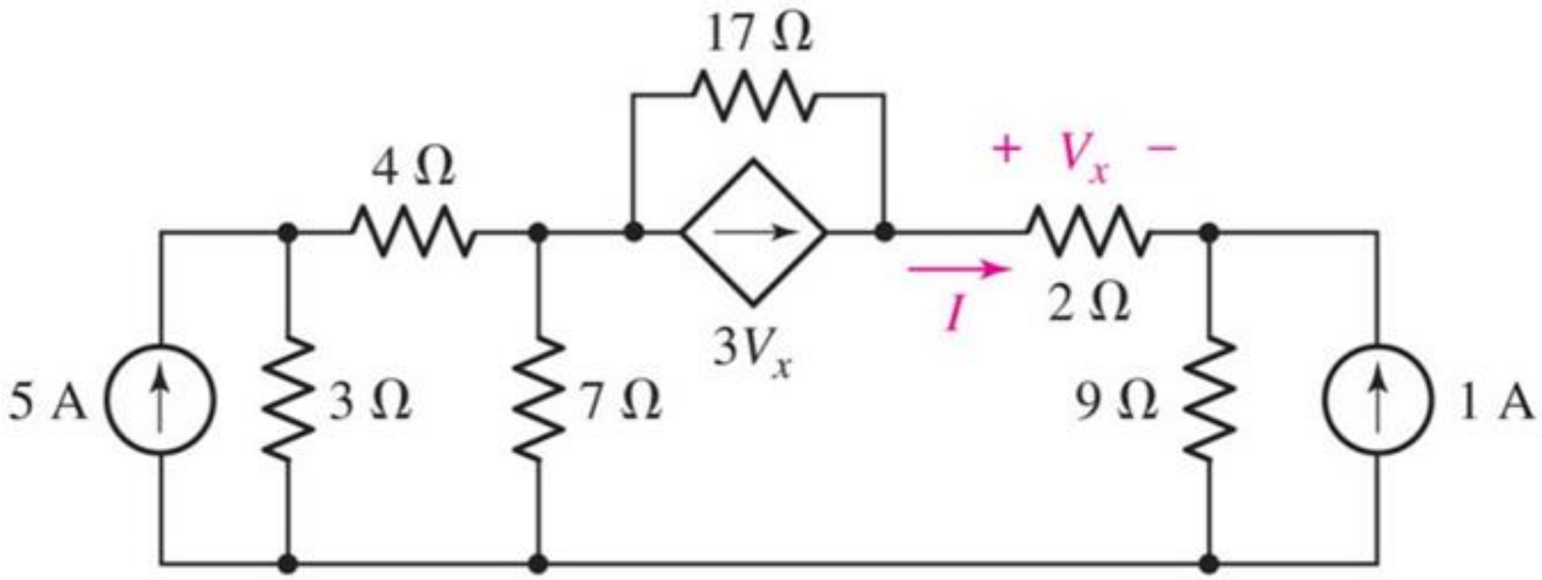
(b)

Practice: 5.3

compute the current i_x after performing a source transformation on the voltage source

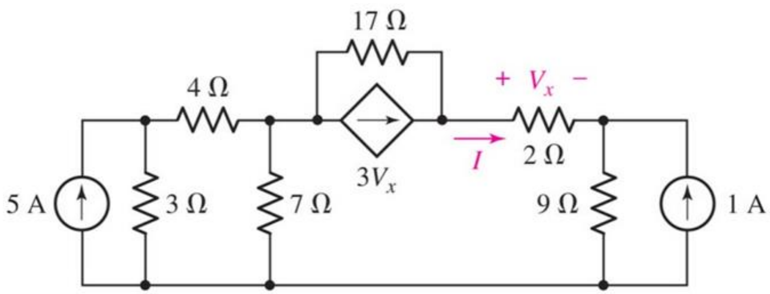


Example 5.5:

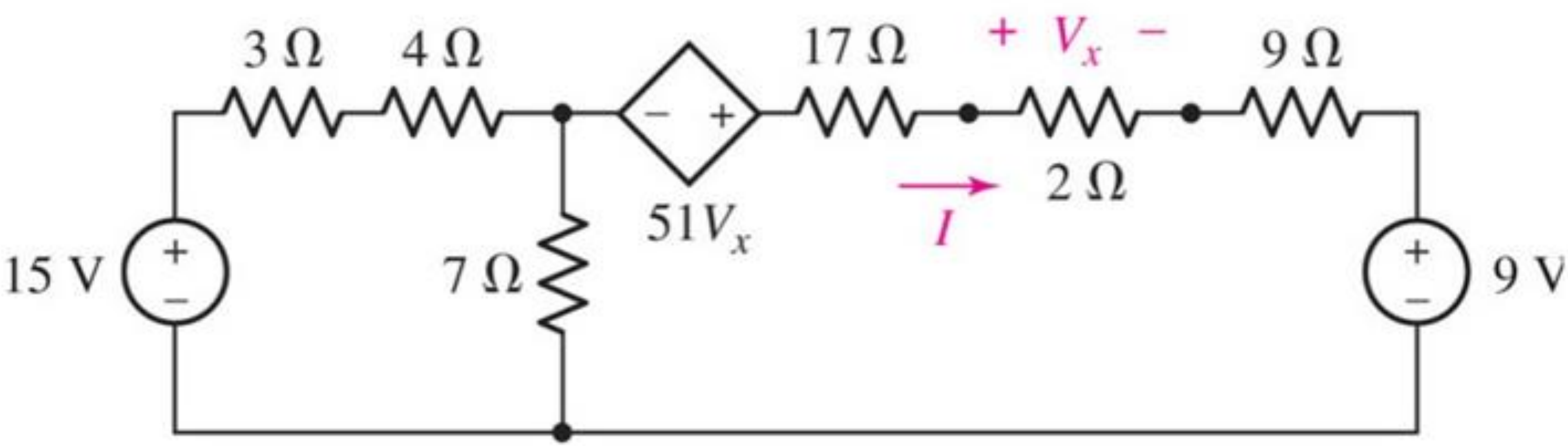


(a)

Example:

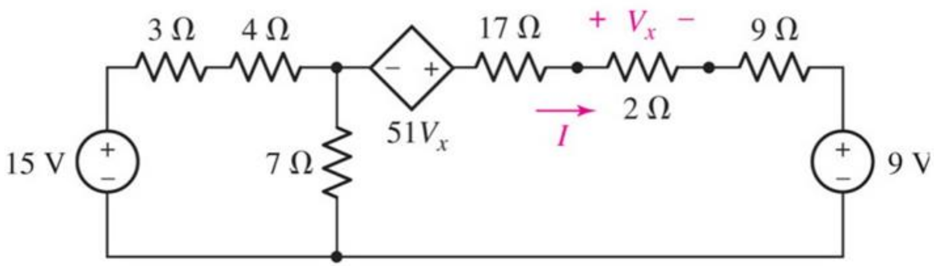


(a)



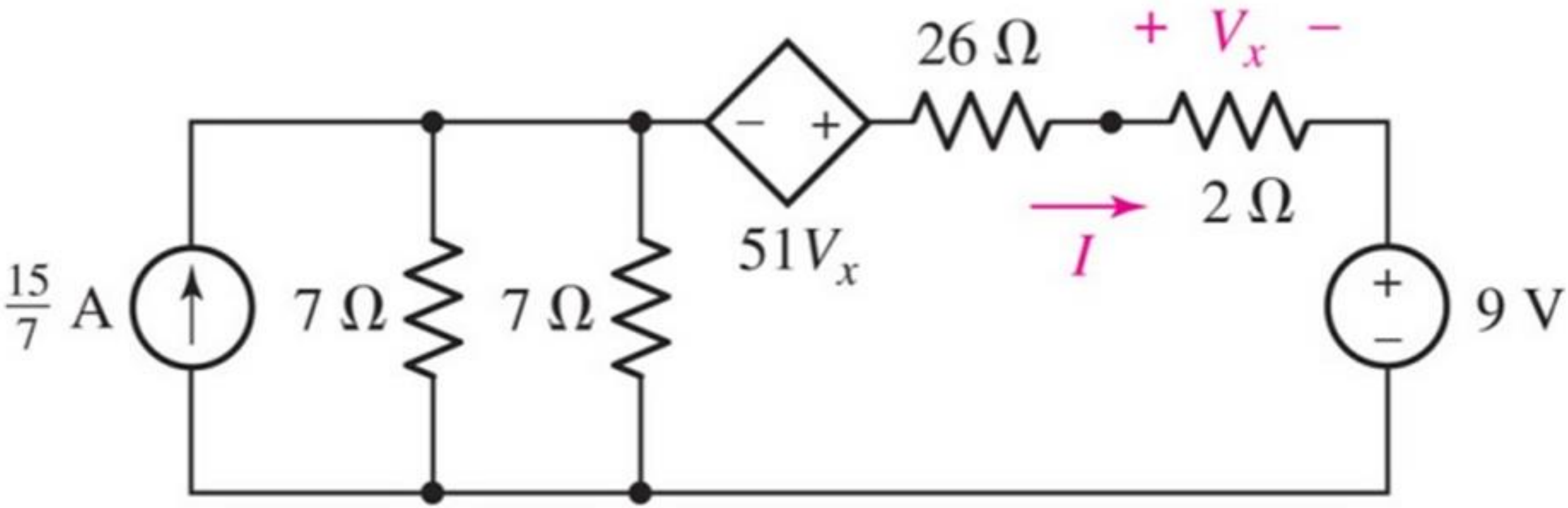
(b)

Example:



3 Ω
4 Ω
17 Ω
+ V_x −
9 Ω
2 Ω
I
51V_x
15 V
7 Ω
9 V

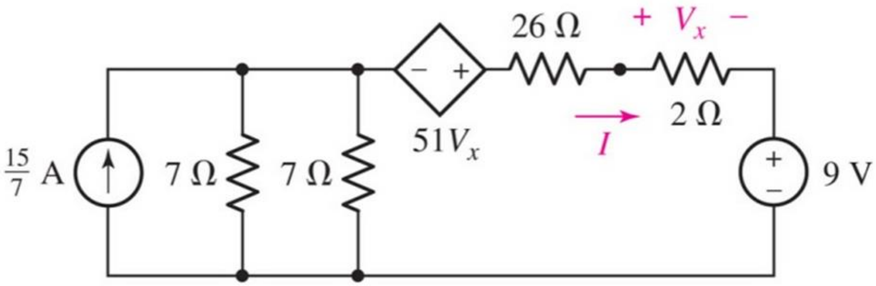
(b)



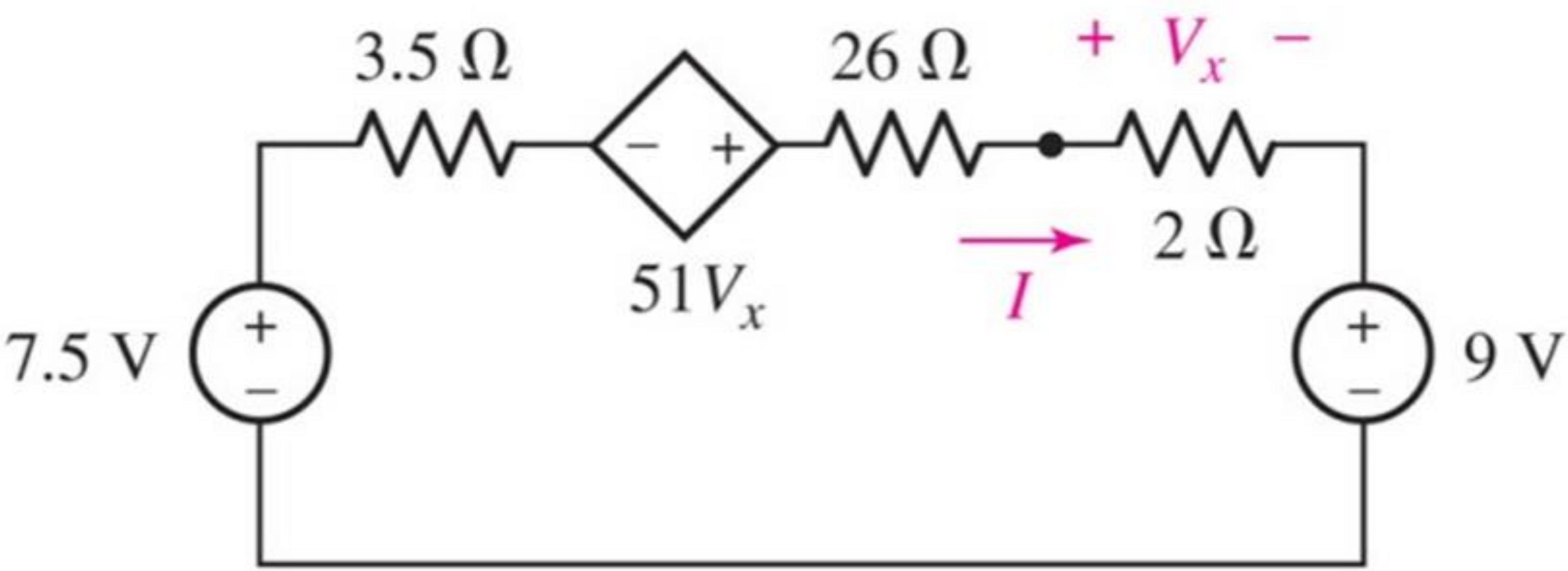
26 Ω
+ V_x −
2 Ω
I
51V_x
$\frac{15}{7}$ A
7 Ω
7 Ω
9 V

(c)

Example:

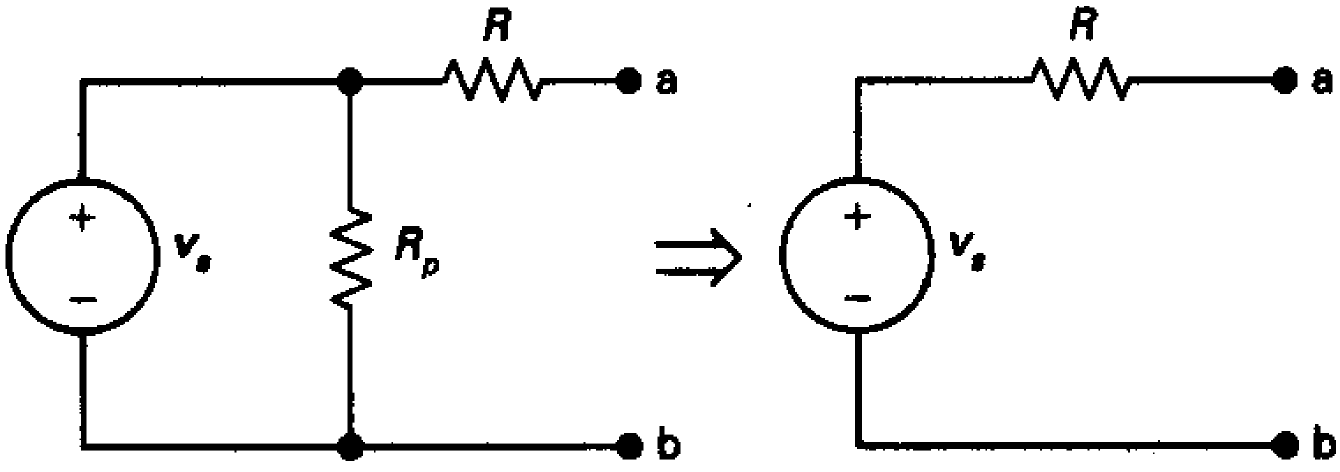


(c)

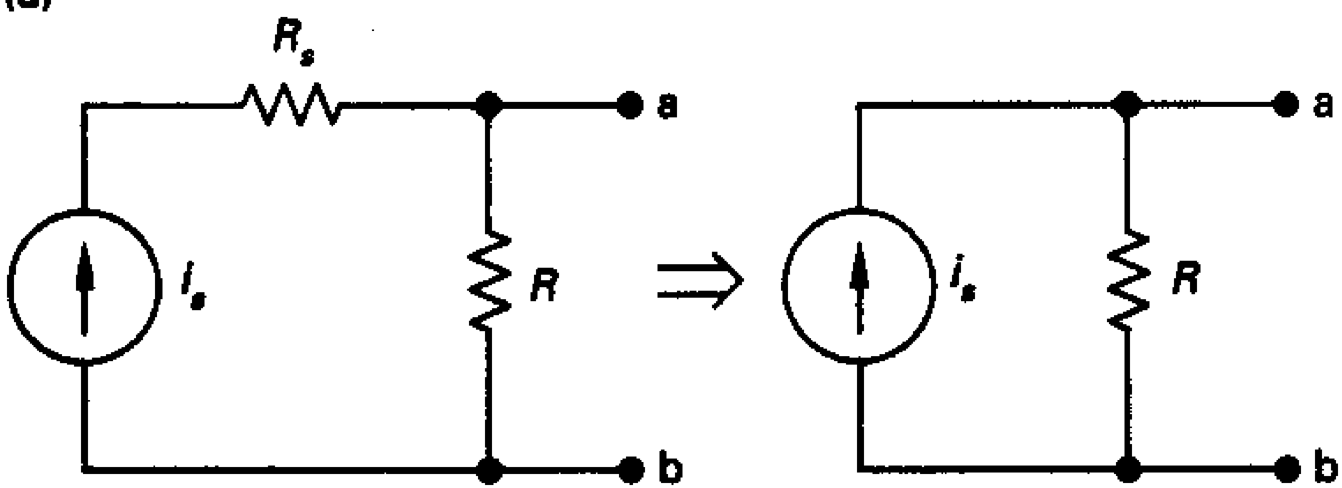


(d)

The equivalents:



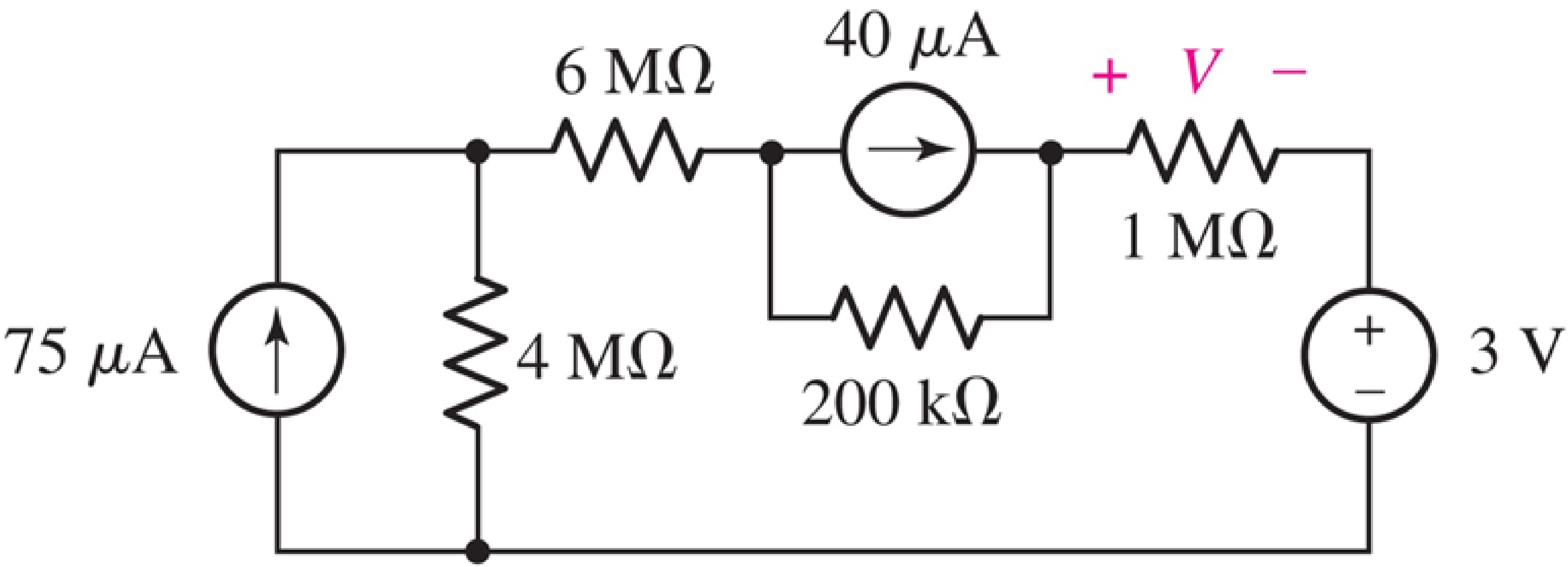
(a)



(b)

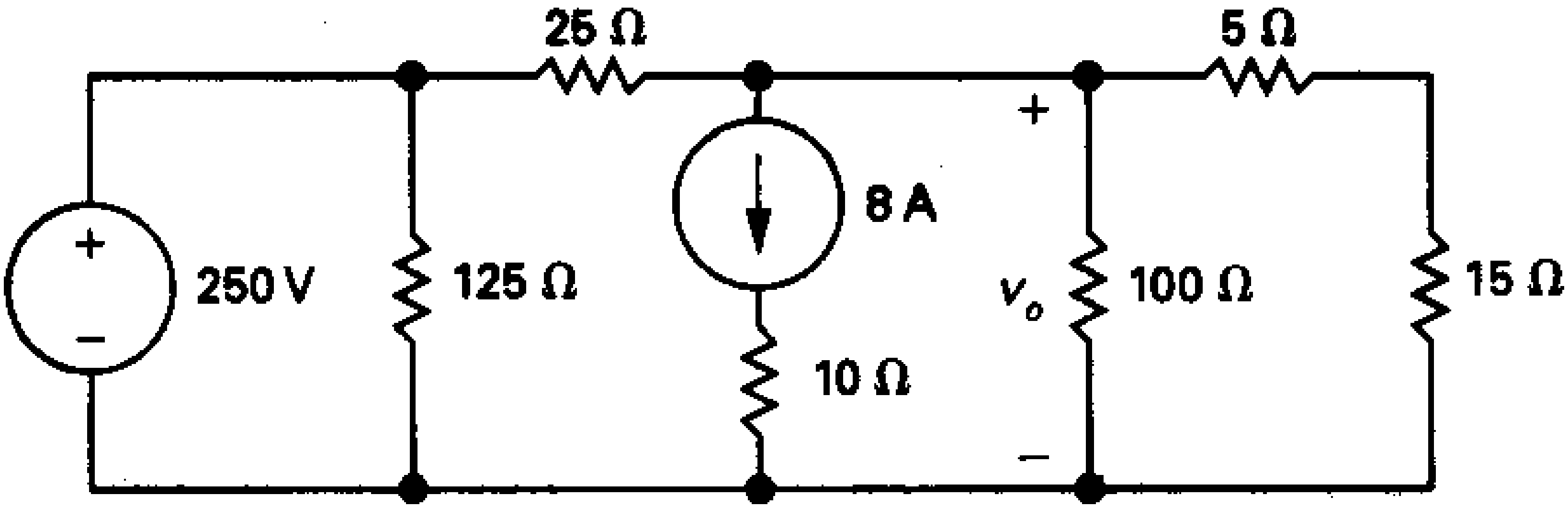
Practice: 5.4

Compute the voltage V



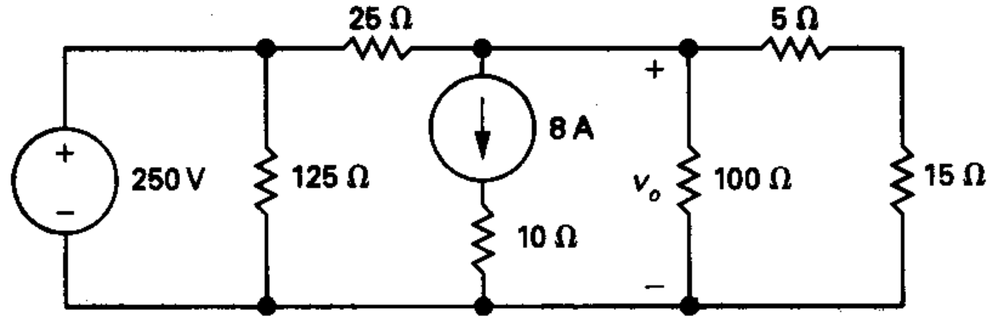
Example:

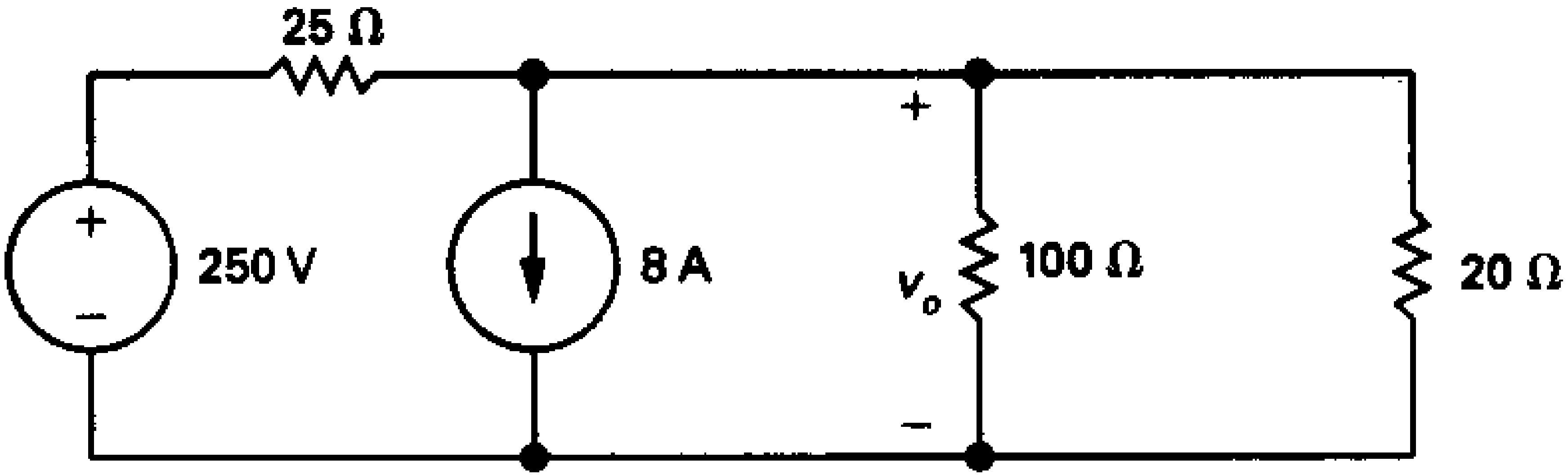
Find the voltage v_o



Example:

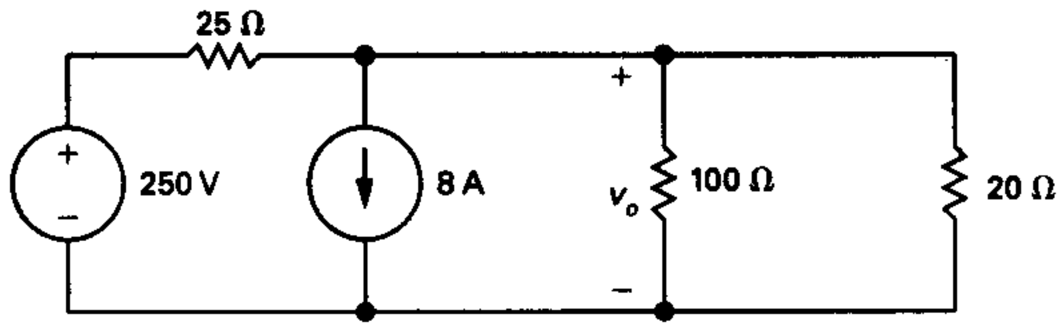
Find the voltage v_o

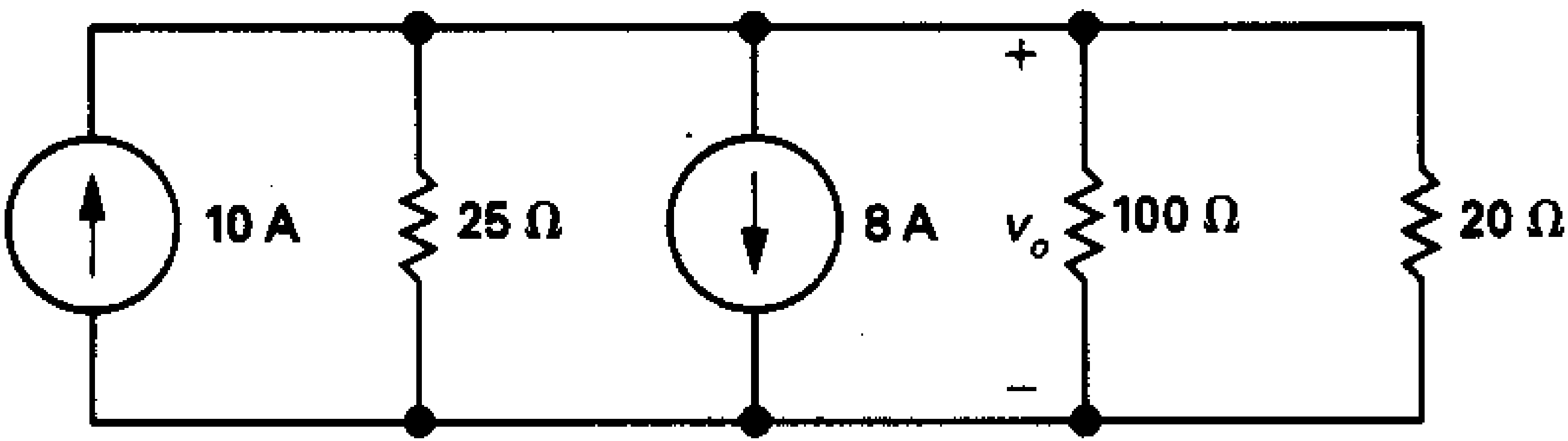




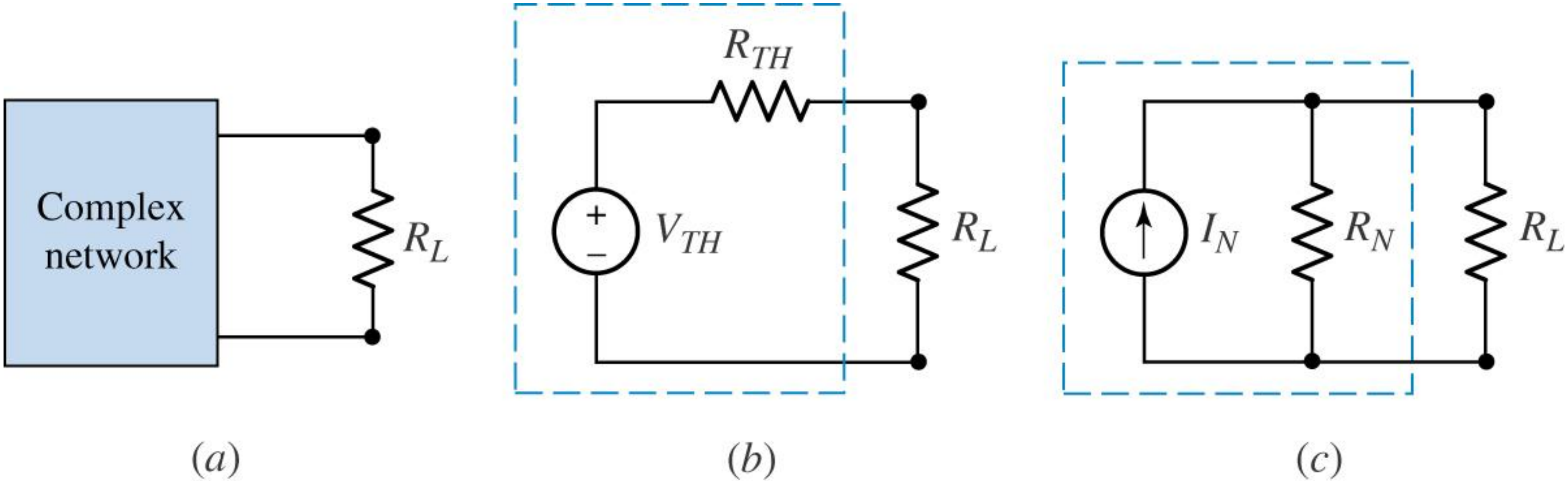
Example:

Find the voltage v_o





Thevenin and Norton Equi. :



(a) A complex network including a load resistor R_L.

(b) A Thévenin equivalent network connected to R_L.

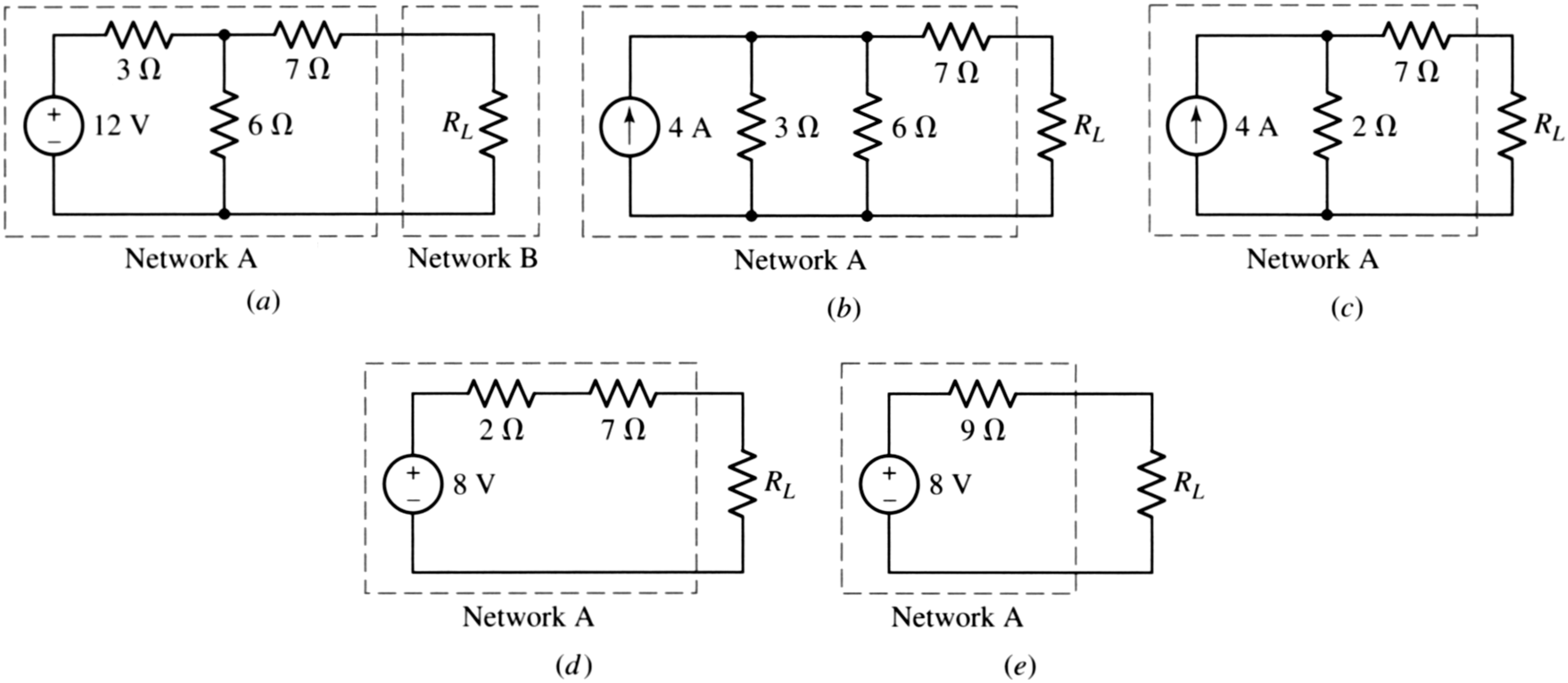
(c) A Norton equivalent network connected to R_L.

Thevenin's theorem:

Given any linear circuit, rearrange it in the form of two networks *A* and *B* connected by two wires. Define a voltage v_{oc} as the open-circuit voltage which appears across the terminals of *A* when *B* is disconnected. Then all currents and voltages in *B* will remain unchanged if all *independent* voltage and current sources in *A* are "killed" or "zeroed out," and an independent voltage source v_{oc} is connected, with proper polarity, in series with the dead (inactive) *A* network.

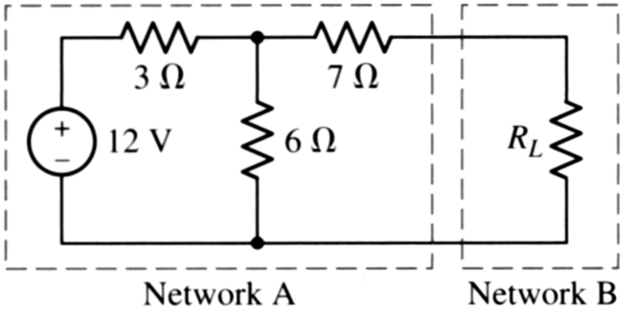
Example 5.6:

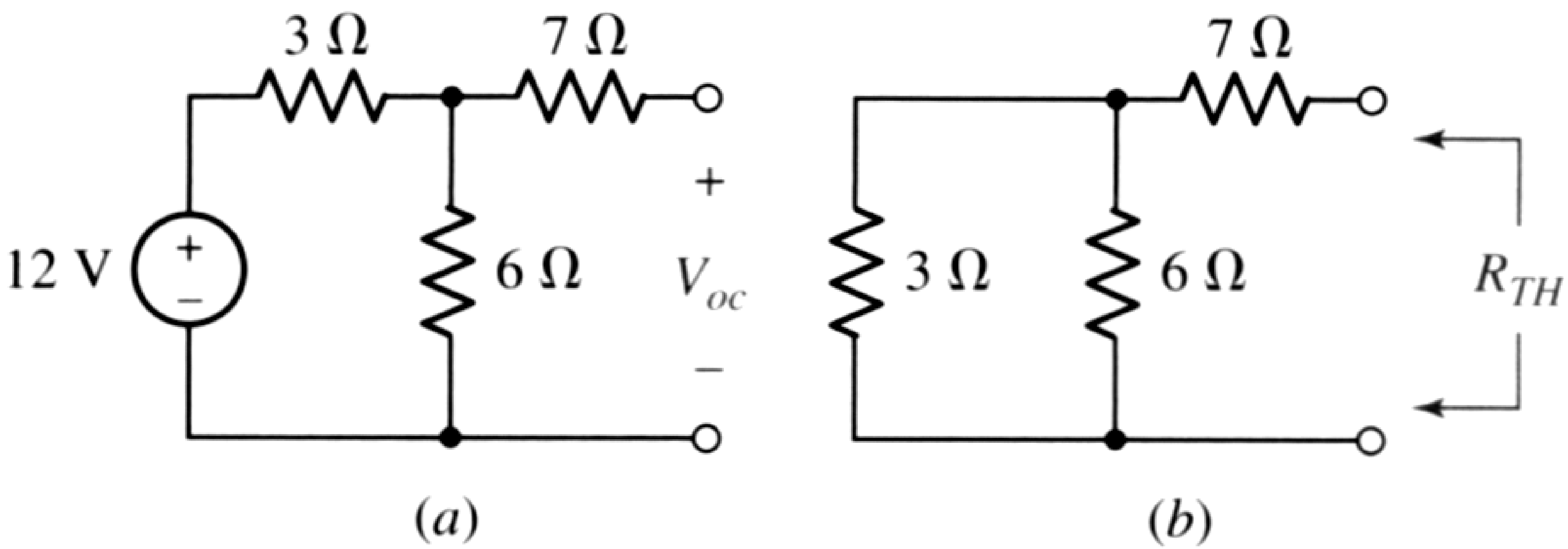
Determine the Thevenin equivalent.



Example:

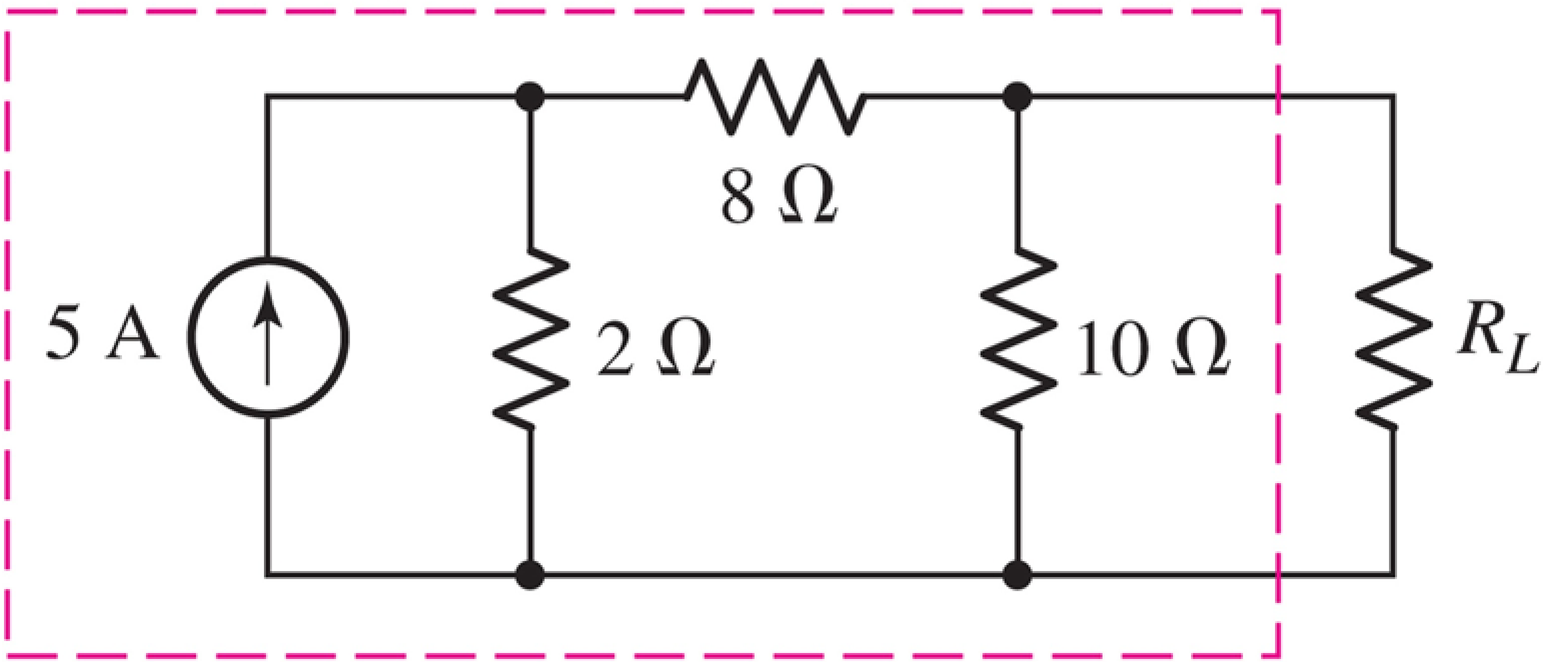
Determine the Thevenin equivalent. (use Theory)





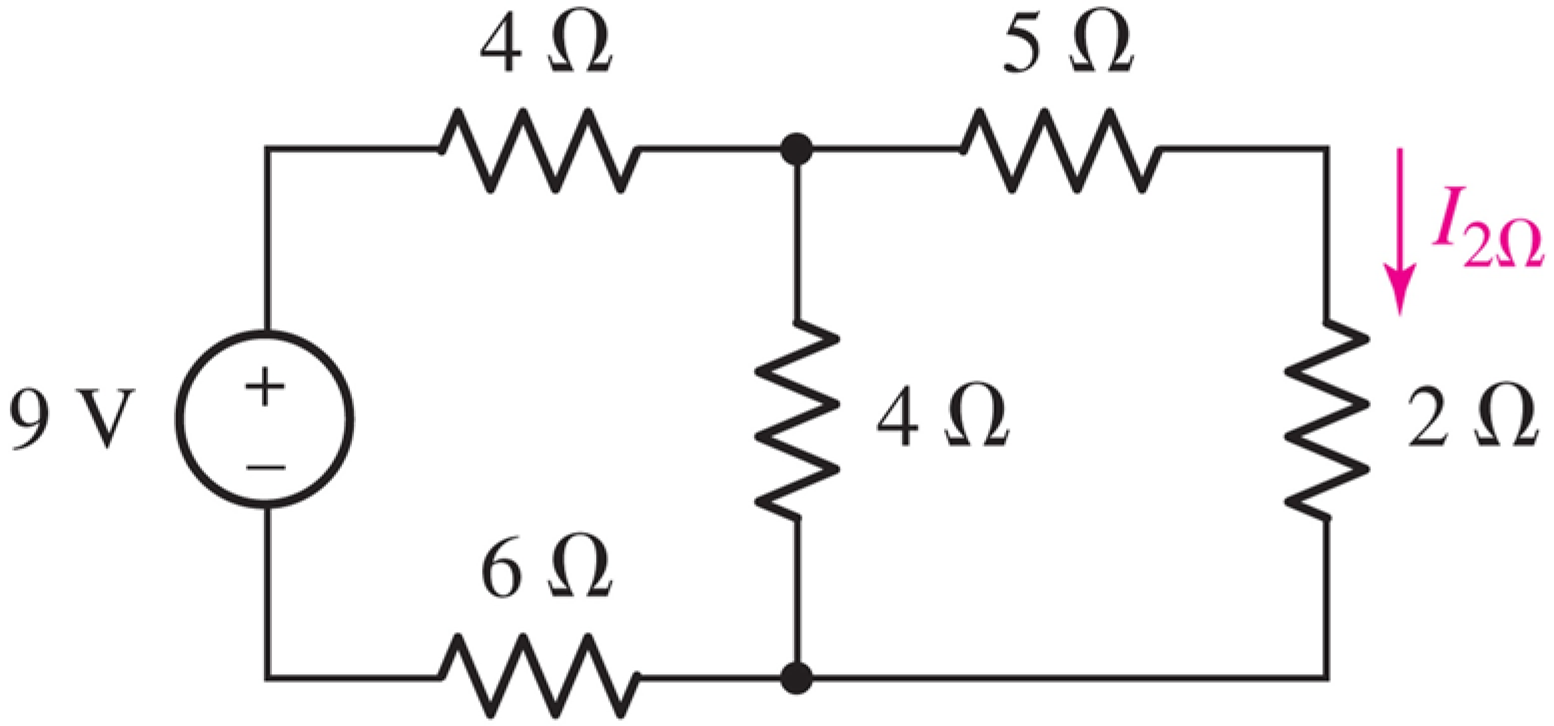
Practice: 5.5

Using repeated source transformations, determine the Norton equivalent of the highlighted network



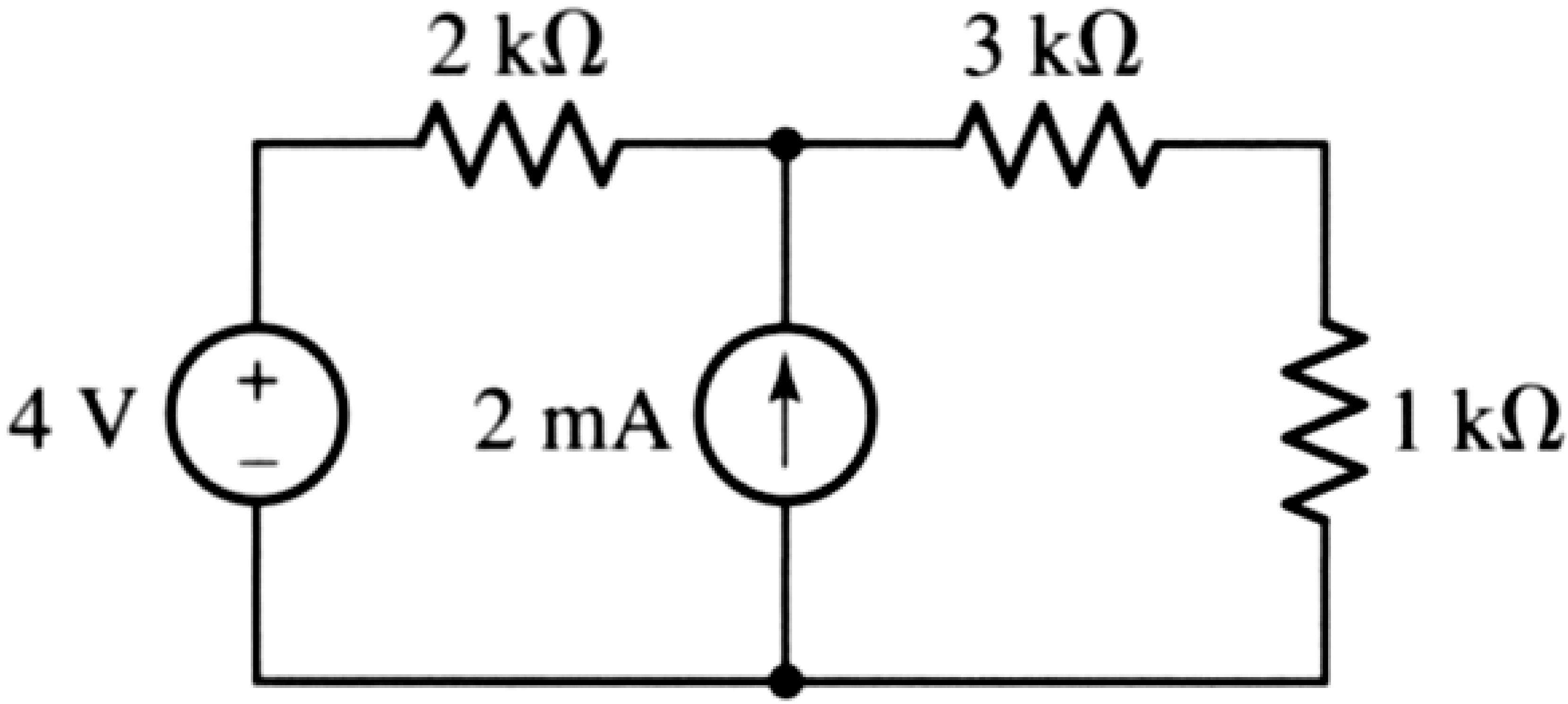
Practice: 5.6

Use Thevenin's theorem to find the current through the 2-Ω resistor



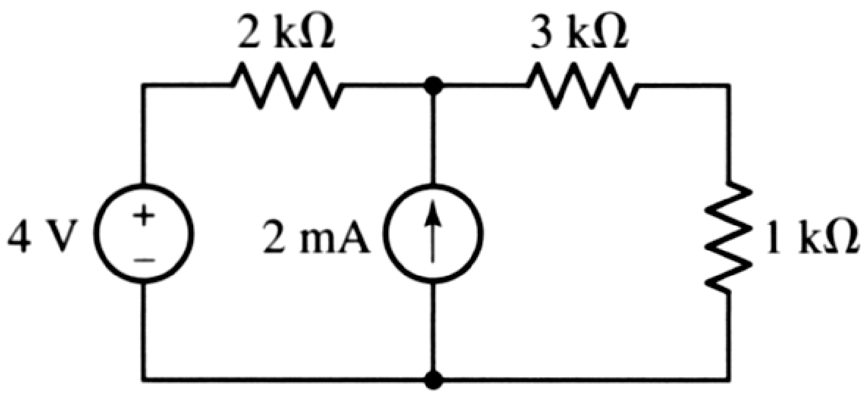
Example 5.8:

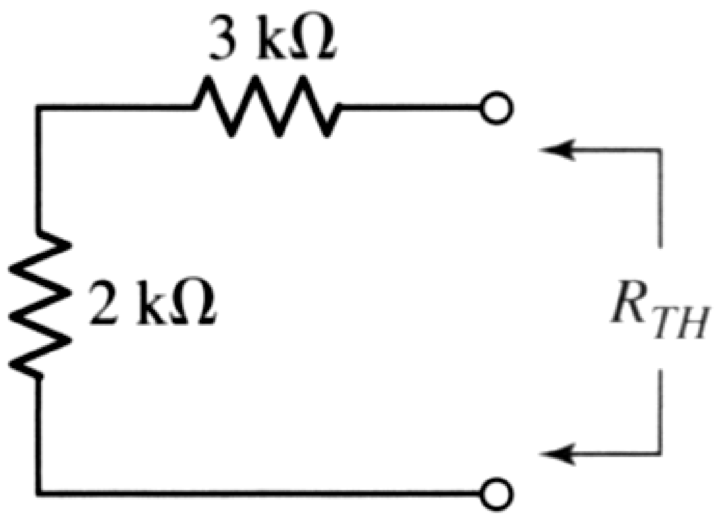
Determine the Thevenin equivalent for the network faced by 1-kohm .

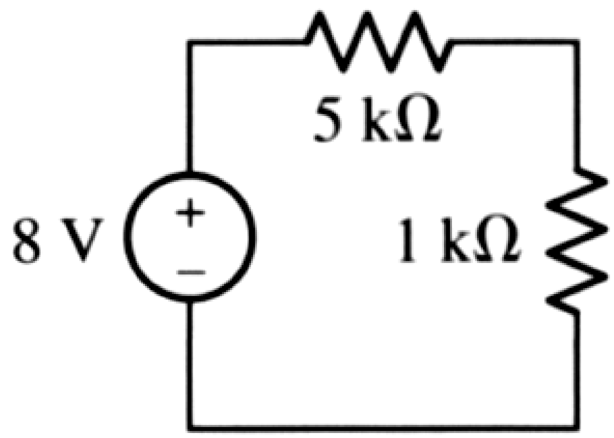


Example:

Determine the Thevenin equivalent.





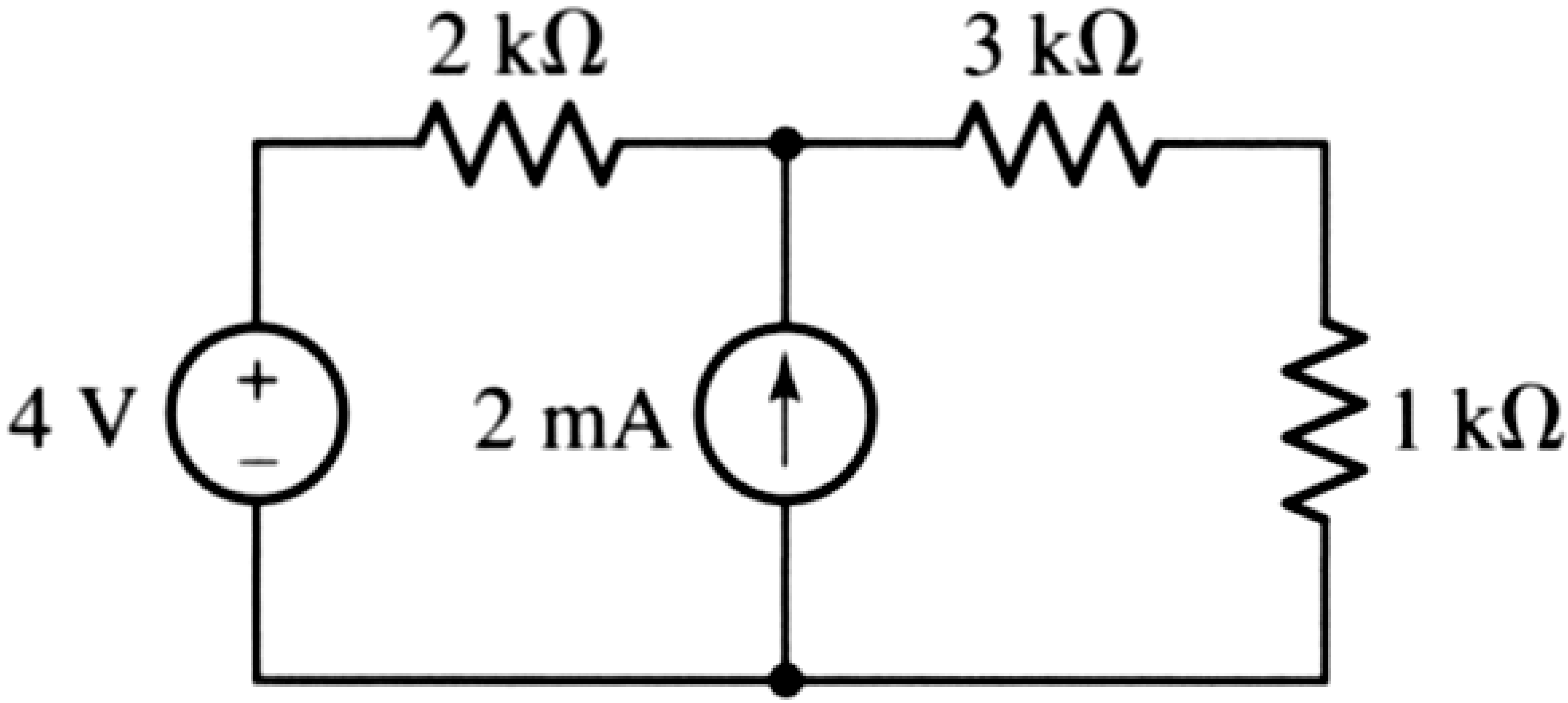


Norton’s theorem:

Given any linear circuit, rearrange it in the form of two networks *A* and *B* connected by two wires. If either network contains a dependent source, its control variable must be in that same network. Define a current i_{sc} as the short circuit current that appears when *B* is disconnected and the terminals of *A* are short-circuited. Then all currents and voltages in *B* will remain unchanged if all *independent* voltage and current sources in *A* are “killed” or “zeroed out,” and an independent current source i_{sc} is connected, with proper polarity, in parallel with the dead (inactive) *A* network

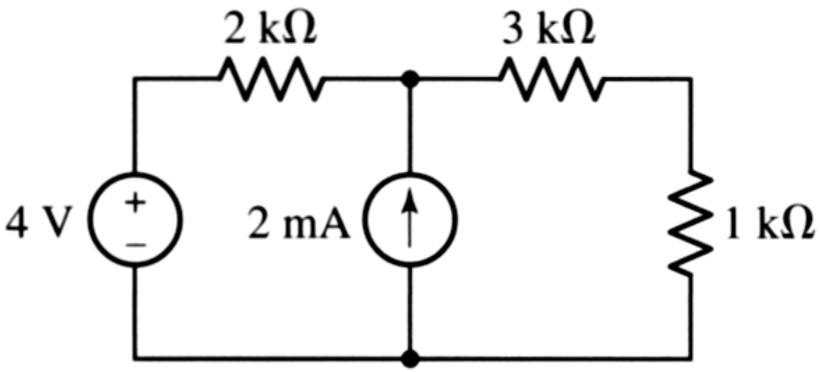
Example:

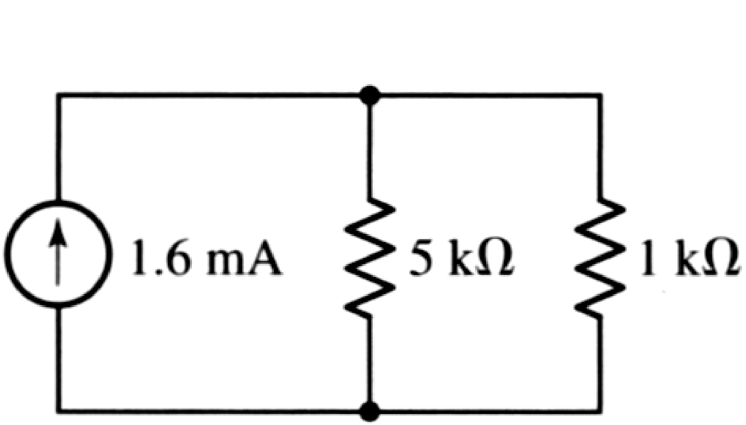
Determine the Norton equivalent for the network faced by 1-kohm .

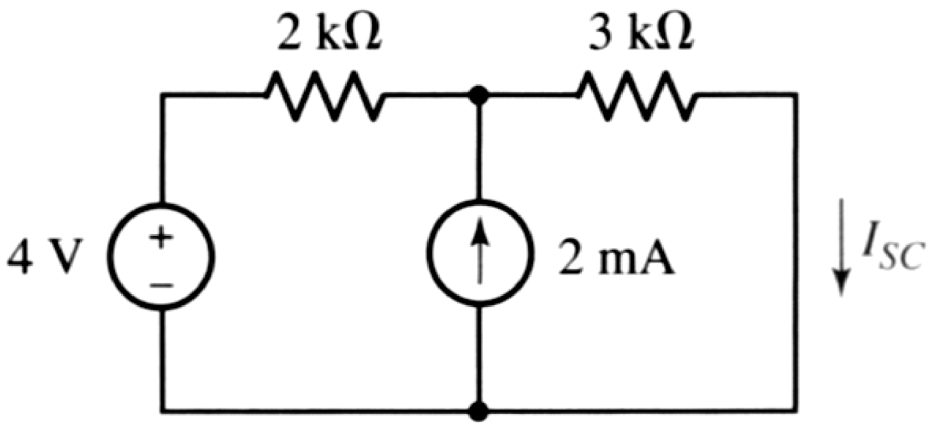


Example:

Determine the Norton equivalent.

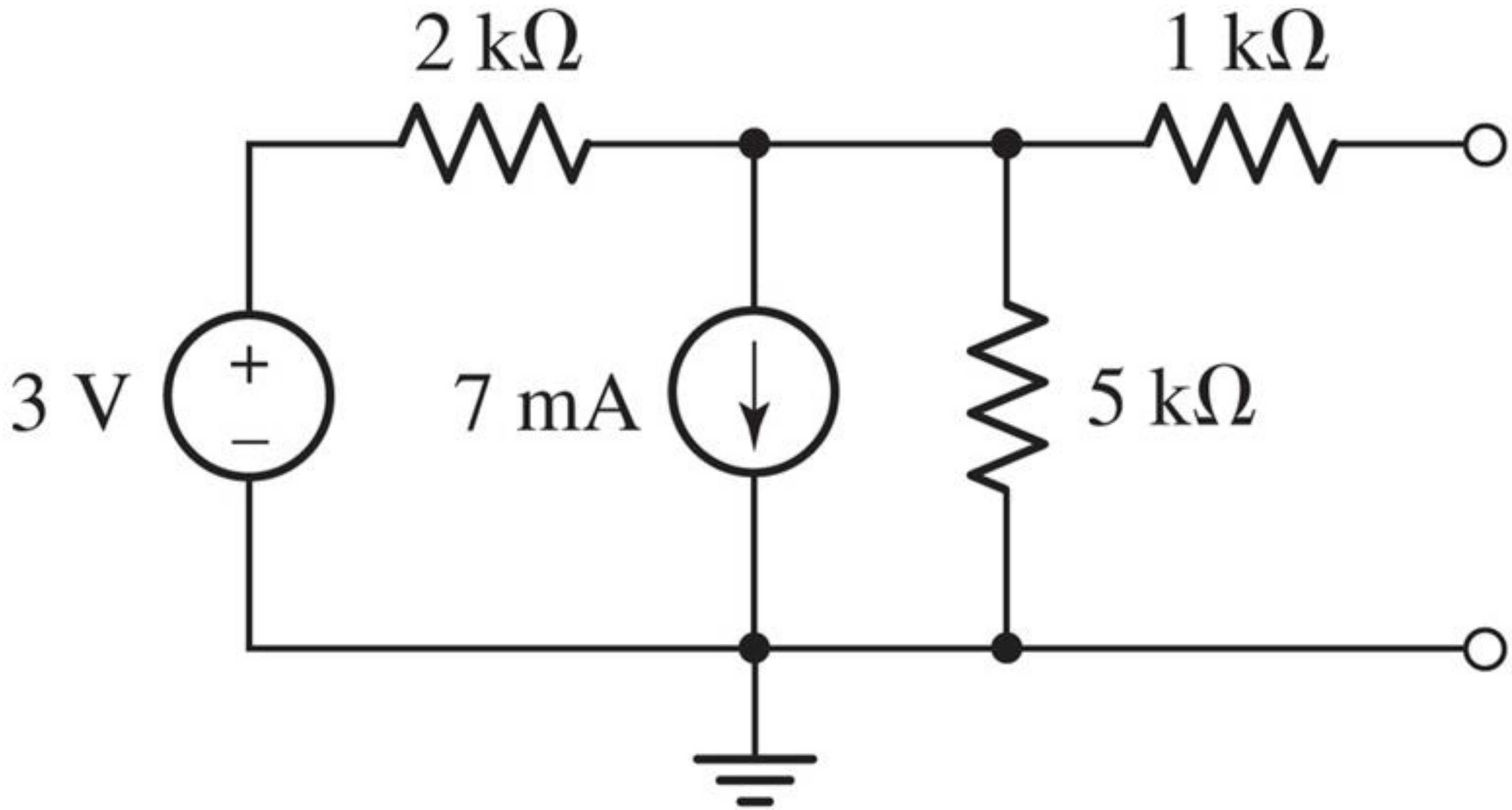






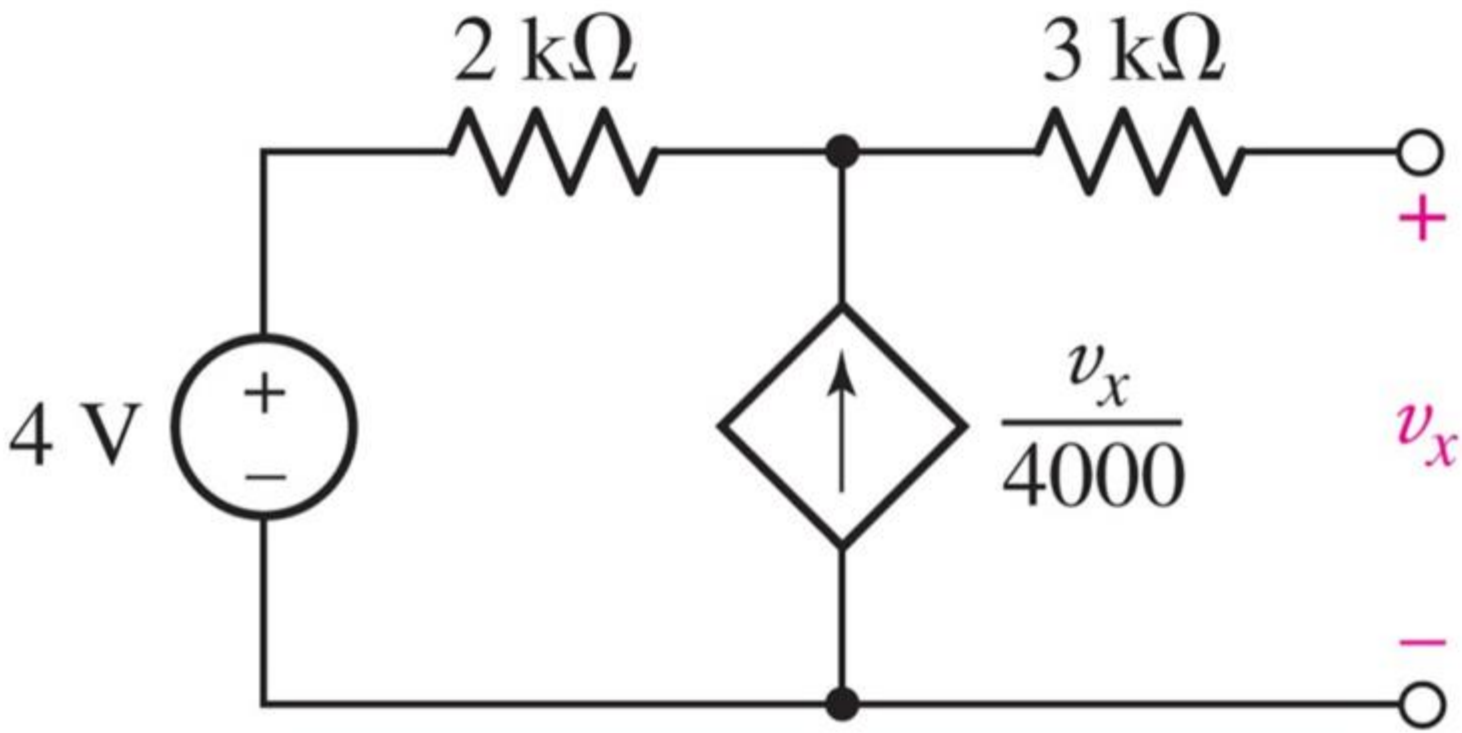
Practice: 5.7

Determine the Thevenin and Norton equivalents



Example 5.9:

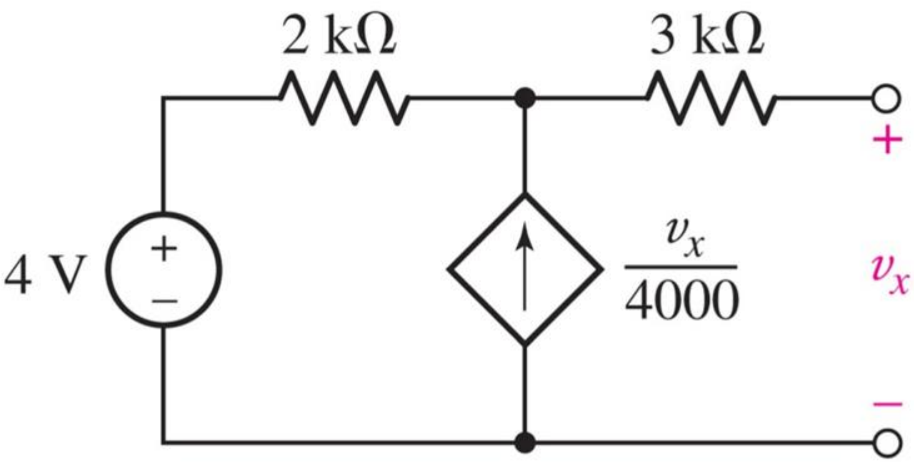
Determine the Thevenin equivalent.



(a)

Example:

Determine the Thevenin equivalent.



(a)

$$\text{Find } V_{oc}: \; V_{oc} = V_x$$

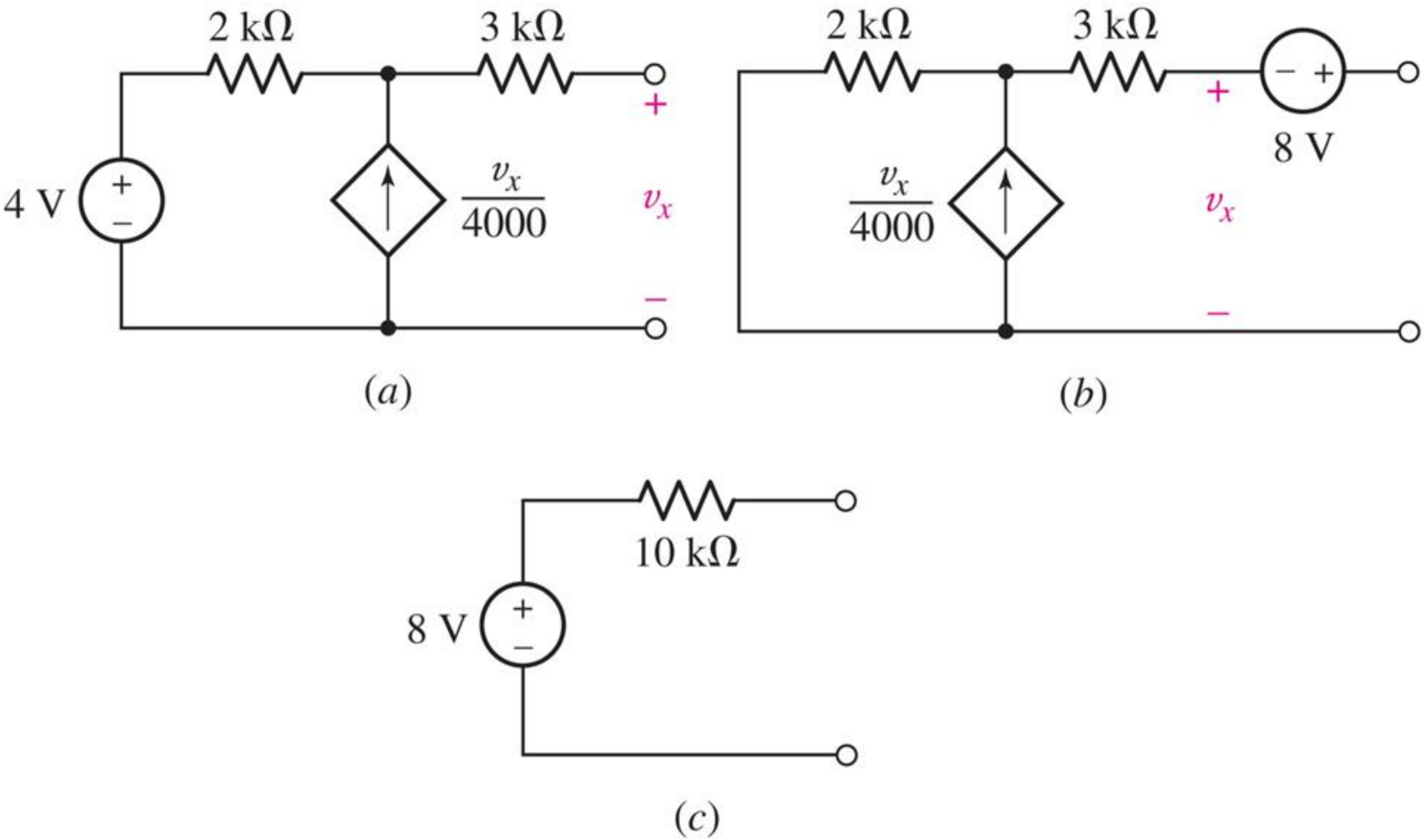
$$-4 + 2k\left(\frac{-V_x}{4000}\right) + 3k(0) + V_x = 0$$

$$\therefore V_x = 8\,\text{V}.$$

$$\text{Find } I_{sc}: \; I_{sc} = \frac{4}{(2k + 3k)} = 0.8\,\text{mA}.$$

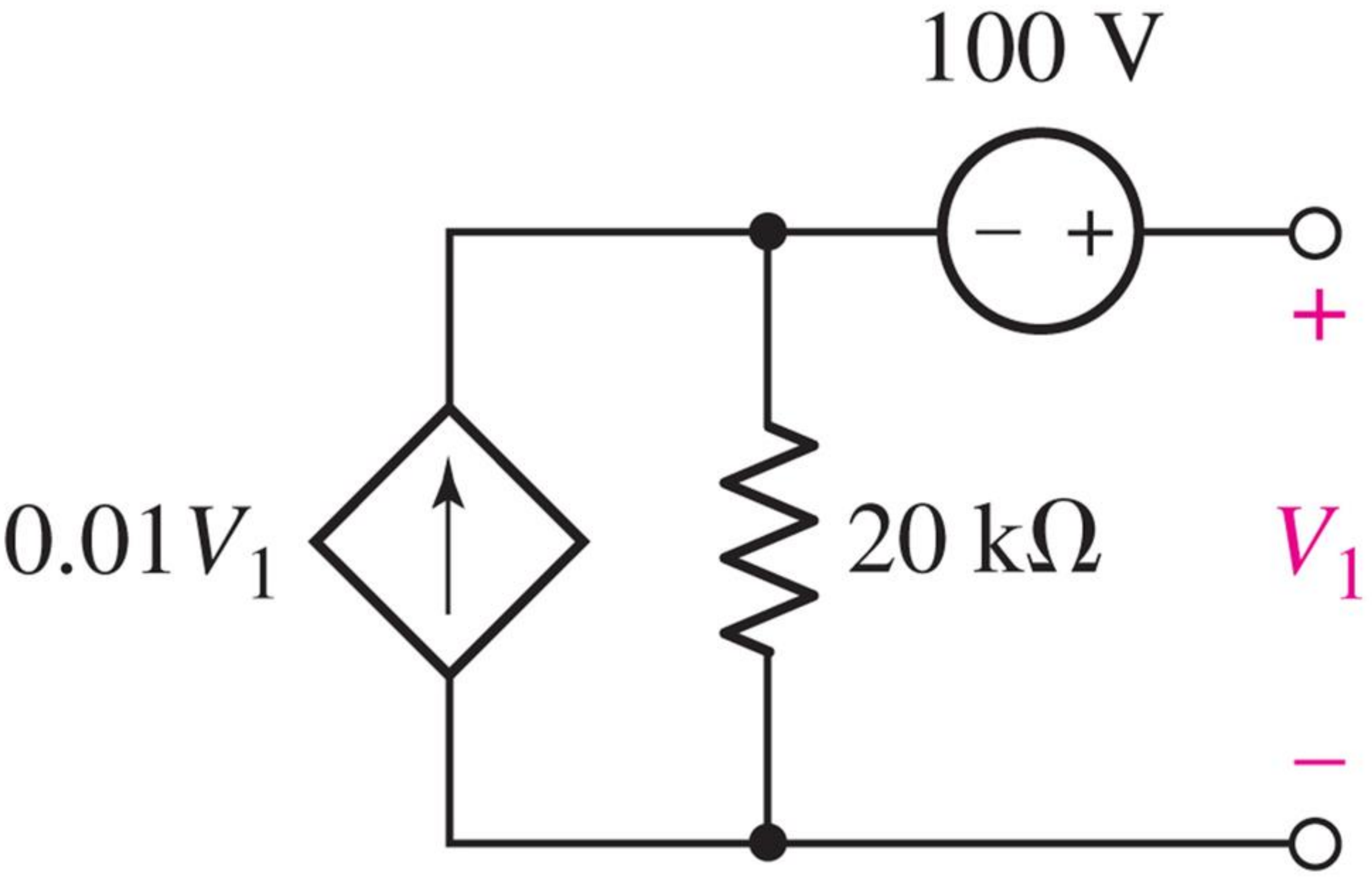
$$\therefore R_{TH} = \frac{V_{oc}}{I_{SC}} = \frac{8}{0.8 \times 10^{-3}} = 10k\Omega$$

Example:



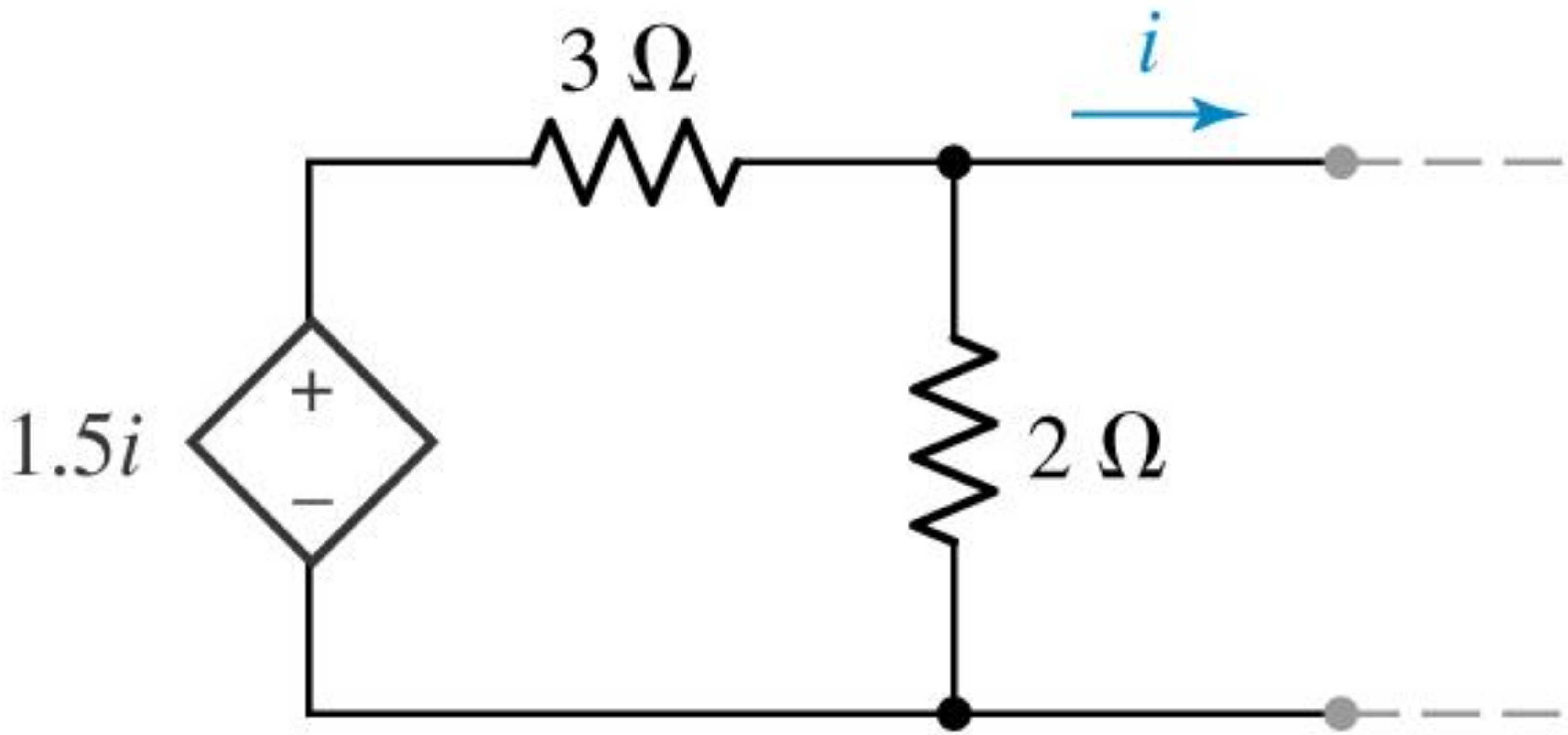
Practice: 5.8

Determine the Thevenin equivalent



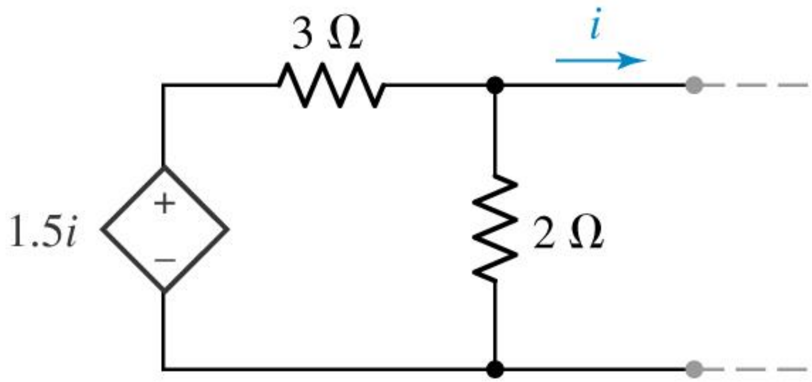
Example 5.10:

Find the Thévenin equivalent of the circuit shown.

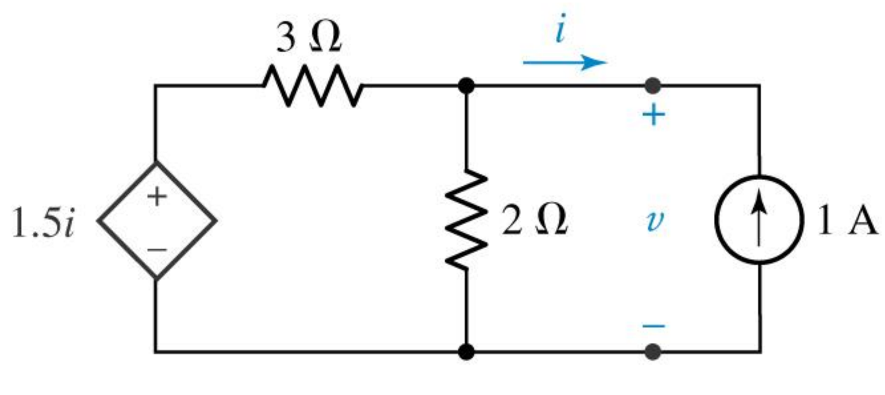


Example:

Find the Thévenin equivalent of the circuit shown.



$$\text{Find } V_{oc} : \because i = 0; \therefore V_{oc} = 0$$



Apply a 1-A. source,

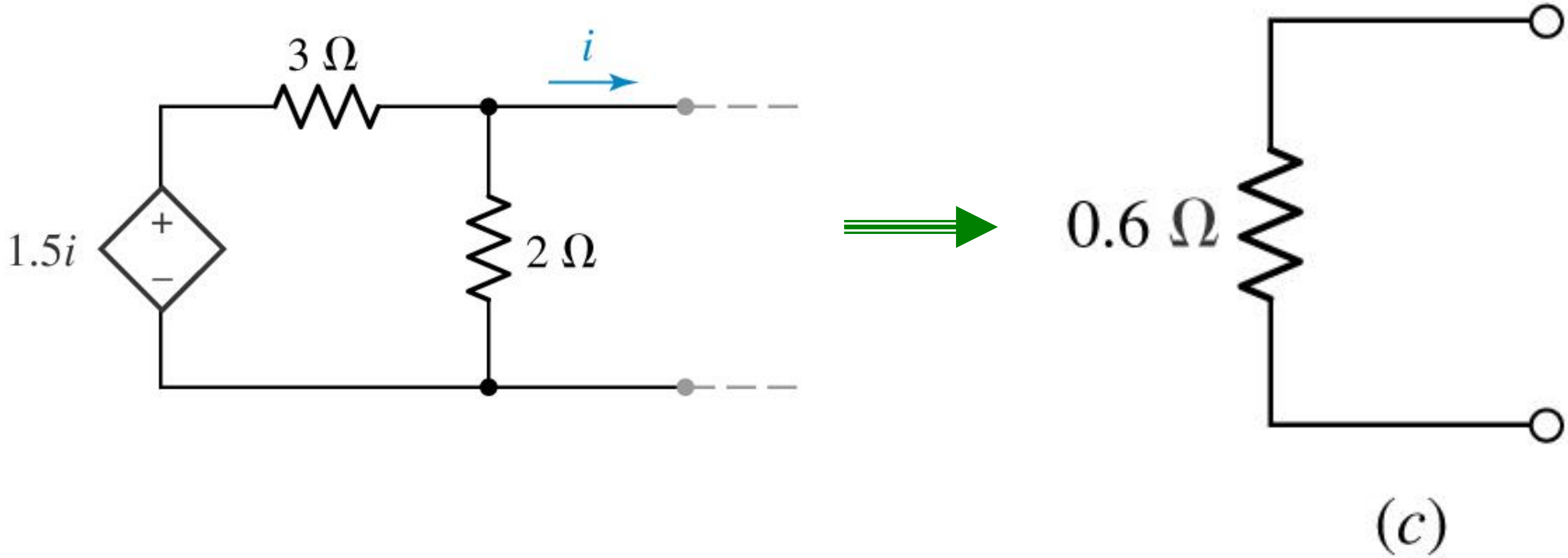
$$\frac{v_{test} - 1.5(-1)}{3} + \frac{v_{test}}{2} - 1 = 0$$

$$\therefore v_{test} = 0.6 \text{ V}.$$

$$\text{Thus } R_{TH} = \frac{v_{test}}{1A.} = 0.6\Omega$$

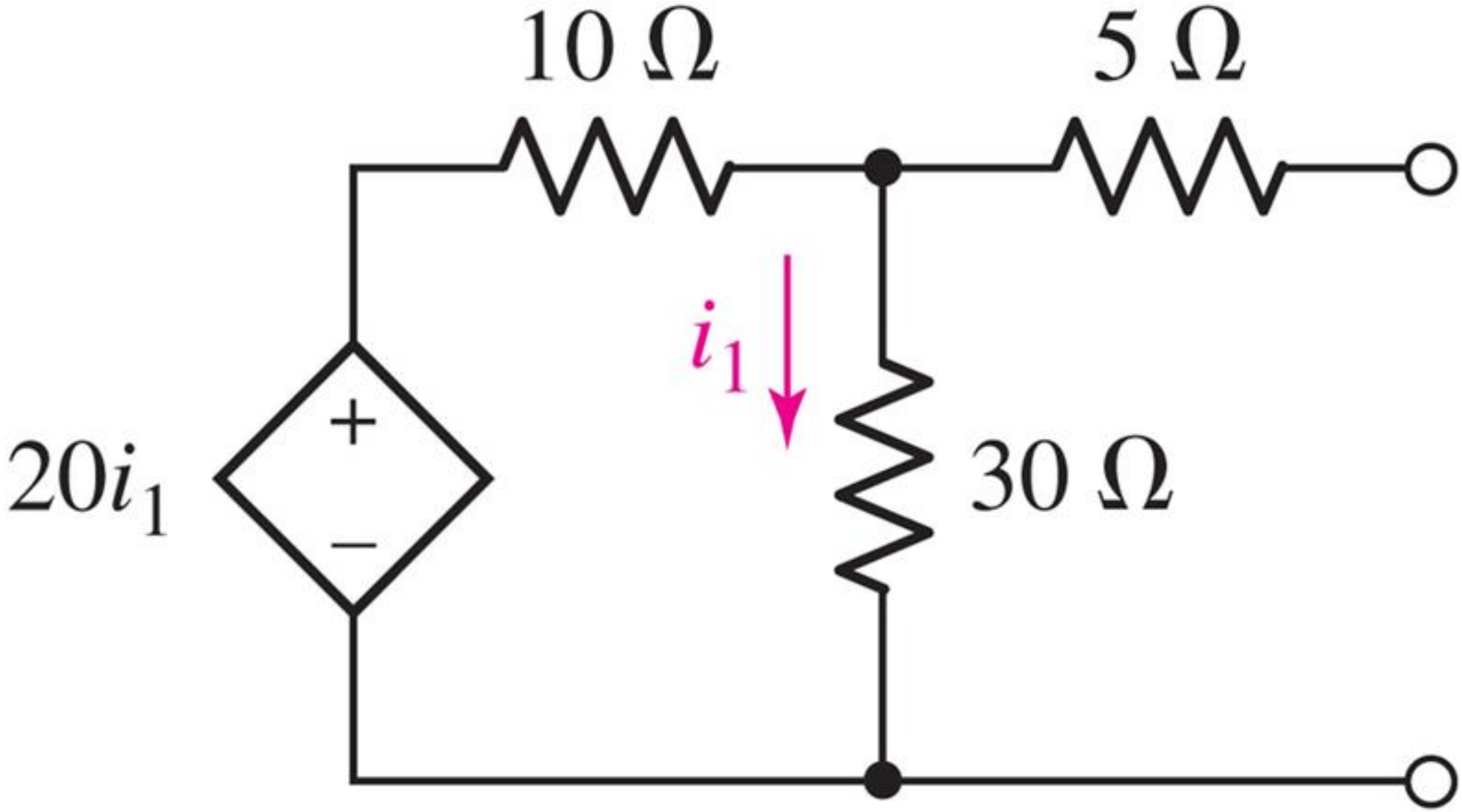
Example:

the Thévenin equivalent



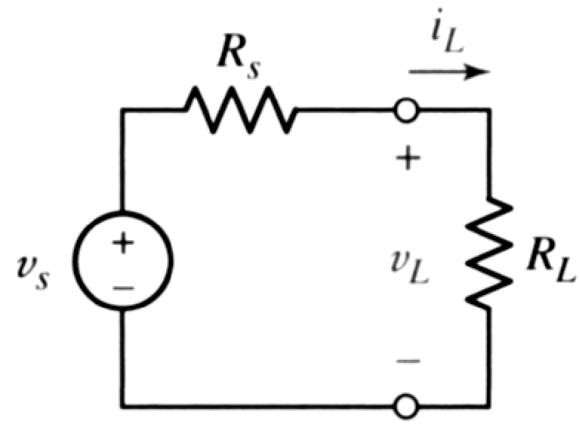
Practice: 5.9

Determine the Thevenin equivalent



Maximum Power Transfer:

The power delivered to the load R_L is



$$p_L = i_L^2 R_L = \frac{v_s^2 R_L}{(R_s + R_L)^2}$$

To find the value of R_L that absorbed a maximum power from the given practical source, we differentiate with respect to R_L :

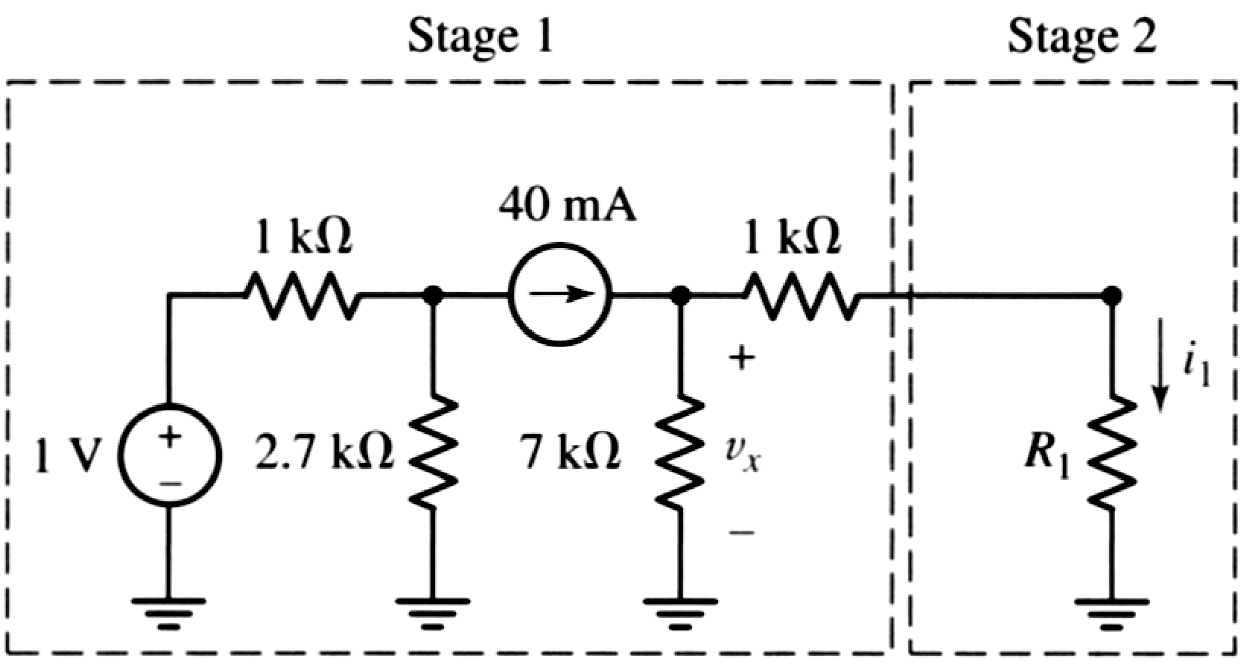
$$\frac{dp_L}{dR_L} = \frac{(R_s + R_L)^2 v_s^2 - v_s^2 R_L 2(R_s + R_L)}{(R_s + R_L)^4} = 0$$

Or

$$R_s = R_L$$

Example:

Ex55p141: Select R1 so that maximum power is transferred from stage 1 to stage 2



Example:

Ex55p141: Select R1 so that maximum power is transferred from stage 1 to stage 2

55. Thévenize the left-hand network, assigning the nodal voltage V_x at the free end of right-most 1-kΩ resistor.

A single nodal equation: $40\times10^{-3} = \dfrac{V_x\big|_{oc}}{7\times10^{3}}$

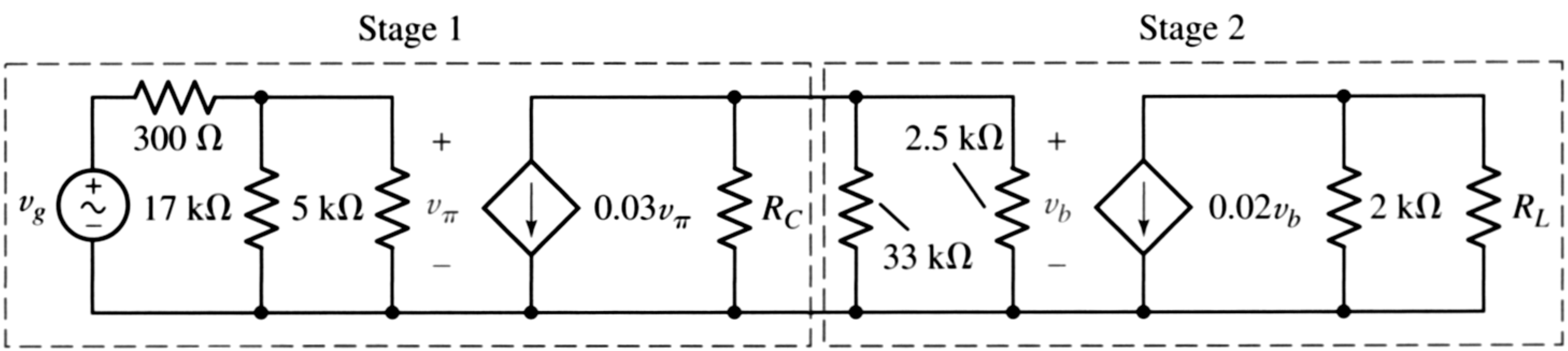
So $V_{TH} = V_x\big|_{oc} = 280\text{ V}$

$R_{TH} = 1\text{ k} + 7\text{ k} = 8\text{ k}\Omega$

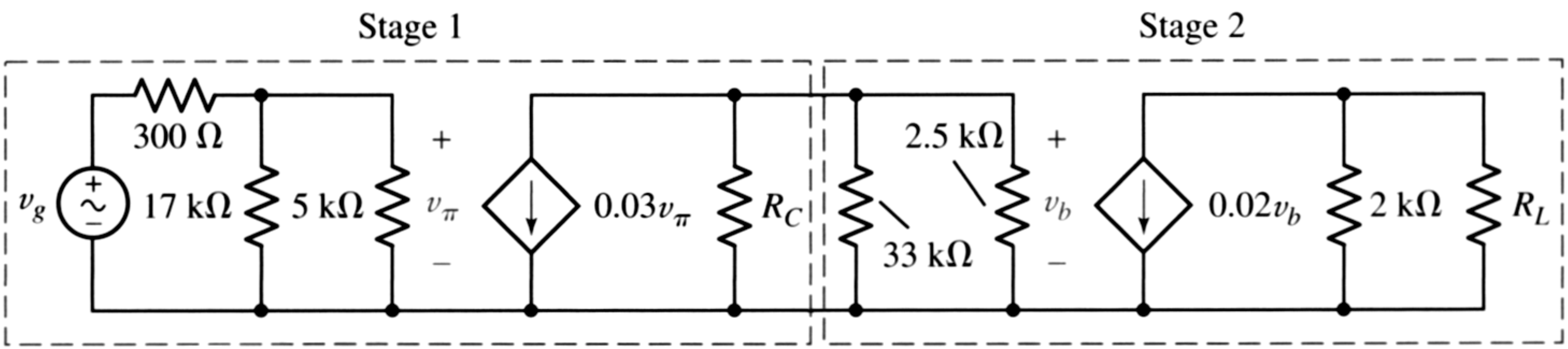
$\boxed{\text{Select } R_1 = R_{TH} = 8\text{ k}\Omega.}$

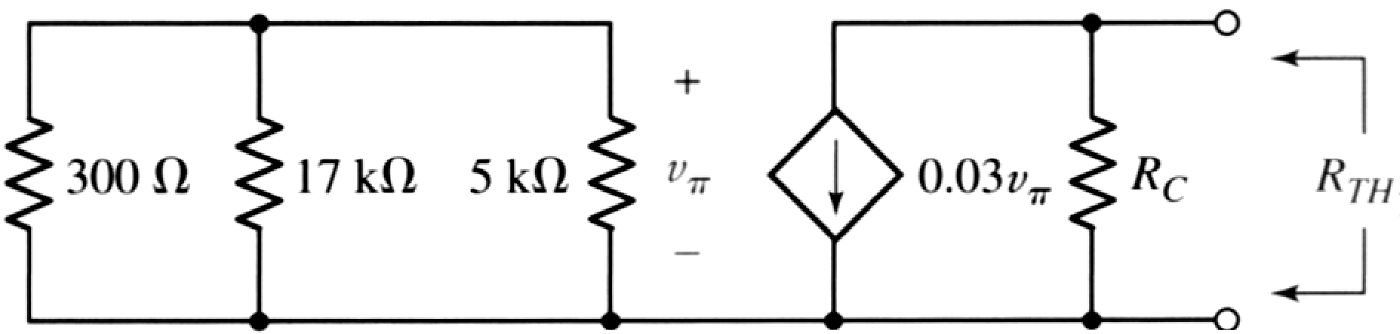
Example 5.11:

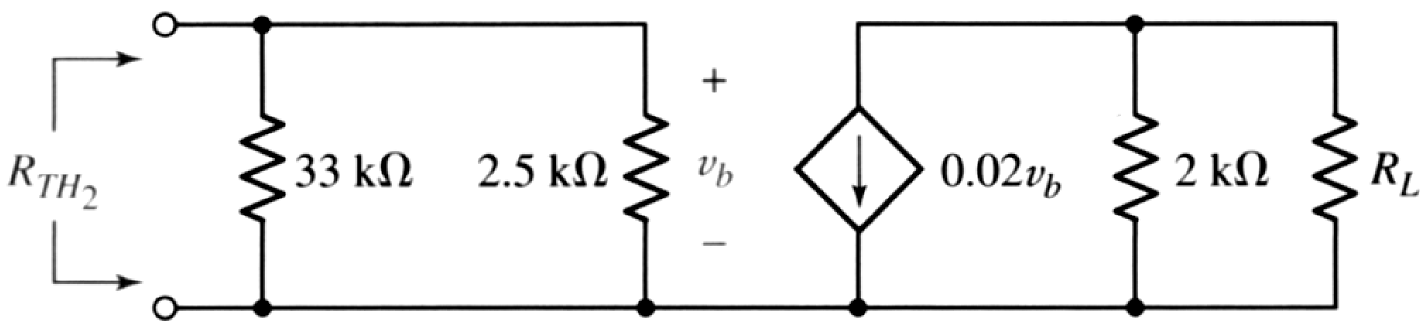
the circuit shown is a model of a two-stage bipolar junction transistor amplifier. Determine the value of RC required for the first stage to deliver maximum power to the second stage.



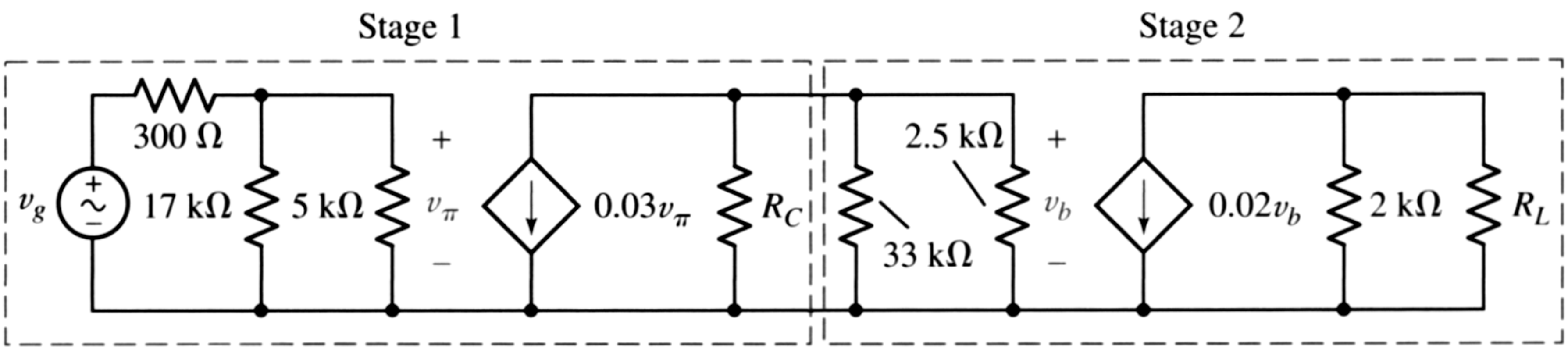
Example:


Stage 1
Stage 2
300 Ω
v_g
17 kΩ
5 kΩ
+
v_π
−
$0.03v_\pi$
R_C
2.5 kΩ
33 kΩ
+
v_b
−
$0.02v_b$
2 kΩ
R_L

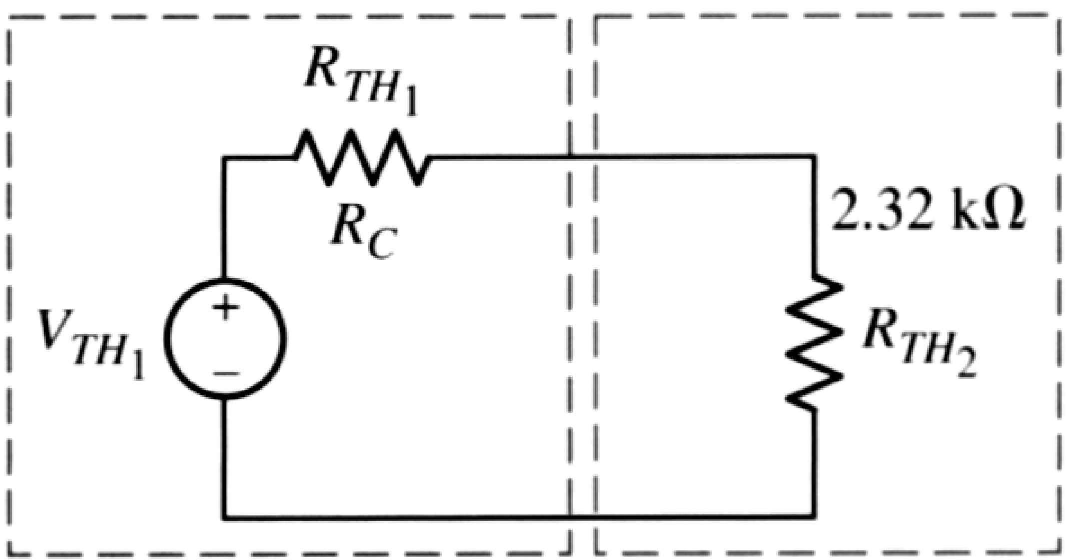

300 Ω
17 kΩ
5 kΩ
+
v_π
−
$0.03v_\pi$
R_C
R_{TH_1}


R_{TH_2}
33 kΩ
2.5 kΩ
+
v_b
−
$0.02v_b$
2 kΩ
R_L

Example:



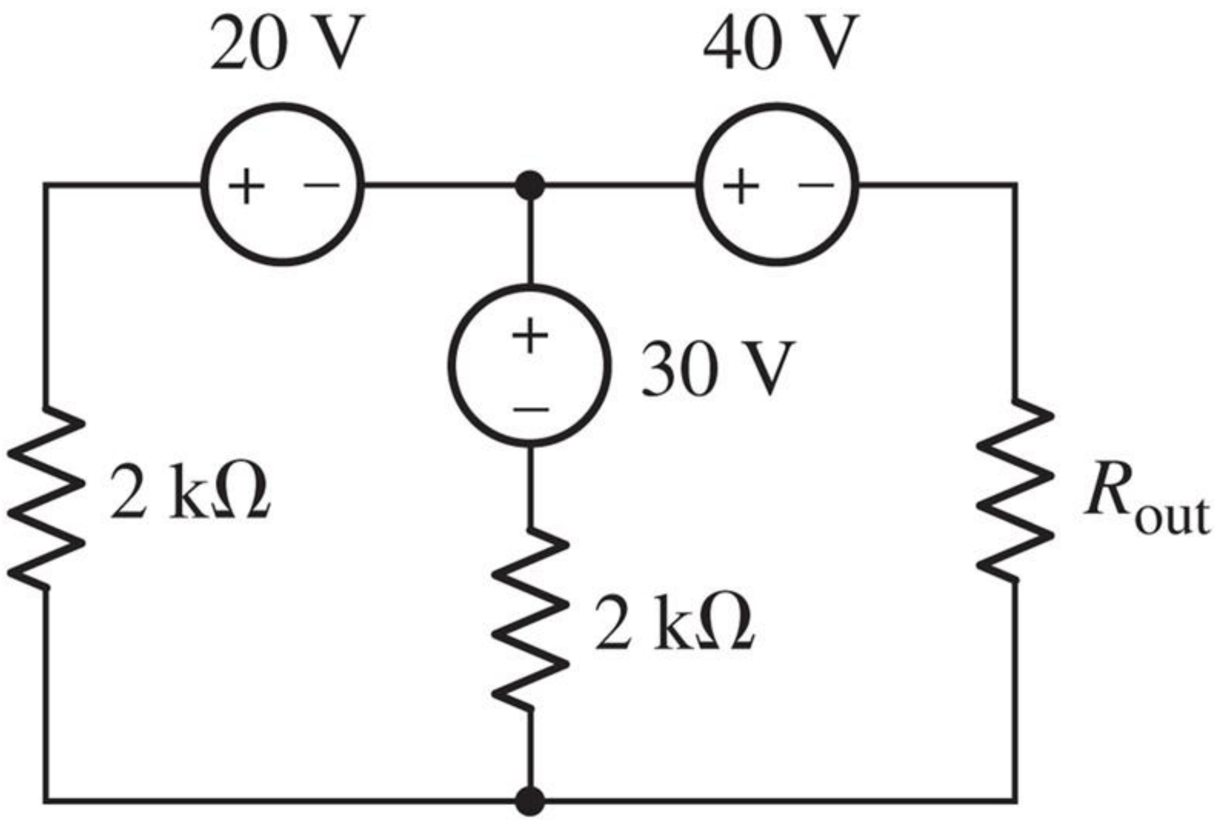
Stage 1
Stage 2
300 Ω
v_g
17 kΩ
5 kΩ
v_π
$0.03v_\pi$
R_C
2.5 kΩ
33 kΩ
v_b
$0.02v_b$
2 kΩ
R_L



R_{TH_1}
R_C
V_{TH_1}
2.32 kΩ
R_{TH_2}

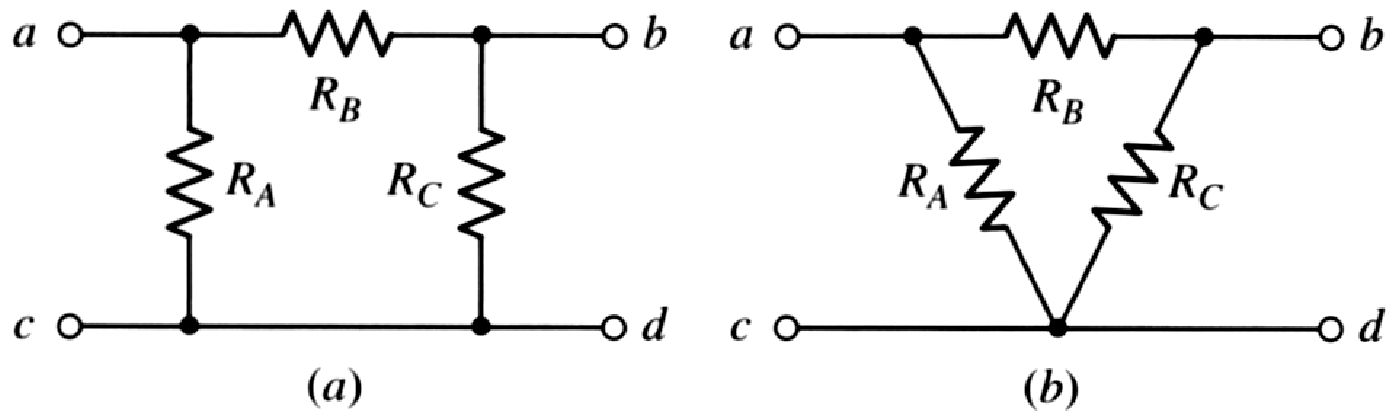
Practice: 5.10

(a) If Rout = 3kΩ, find the power delivered to it
(b) What is the maximum power that can be delivered to any Rout
(c) What two different values of Rout will have exactly 20 mW delivered too them?

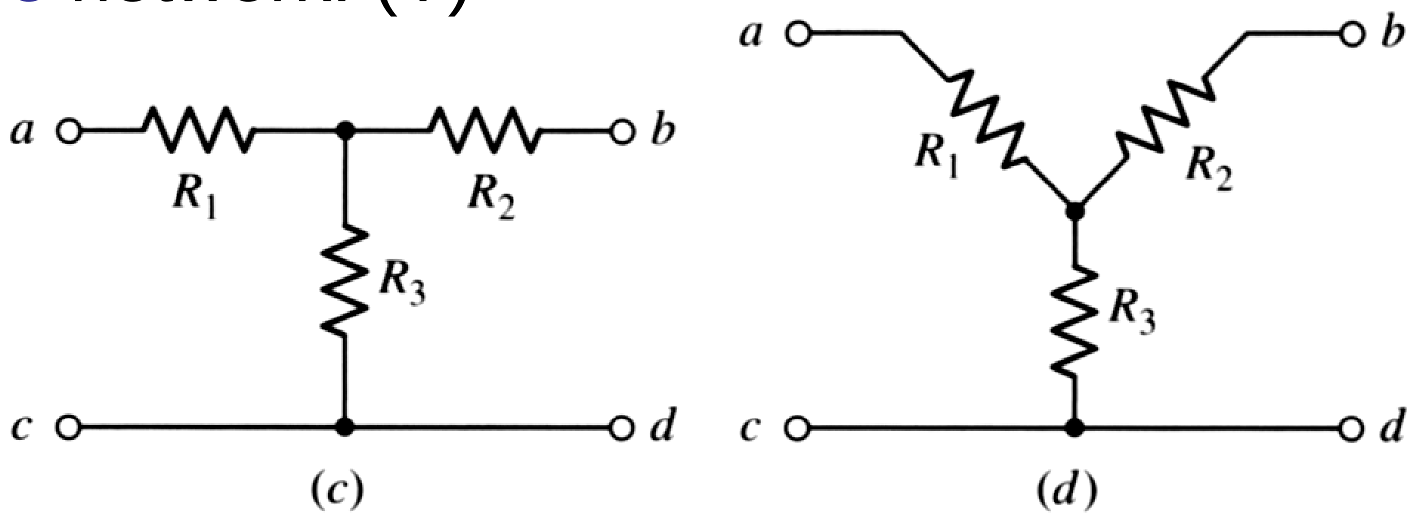


Delta-Wye Conversion:

A delta network: (Δ)



A Wye network: (Y)



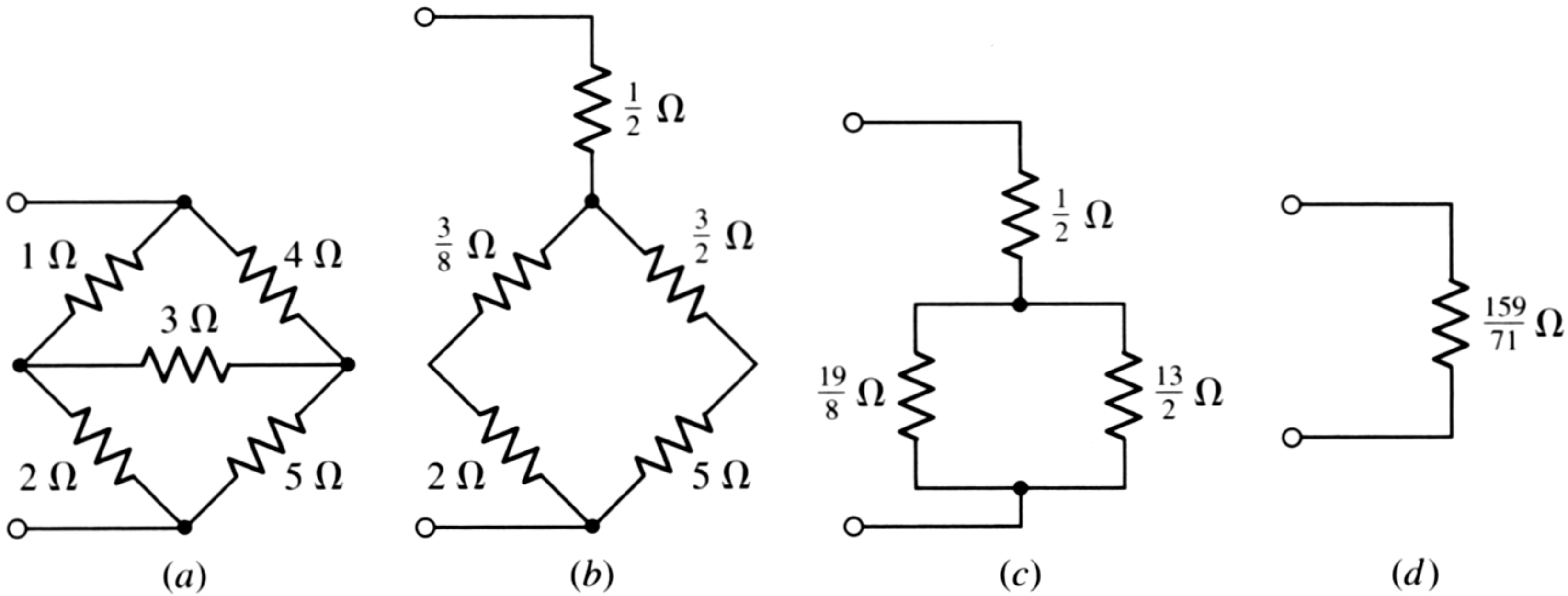
Delta-Wye Conversion:

$$R_{ab} = R_1 + R_2$$
$$= R_B // (R_A + R_C) \ \ldots (1)$$
$$R_{bc} = R_2 + R_3$$
$$= R_C // (R_A + R_B) \ \ldots (2)$$
$$R_{ac} = R_1 + R_3$$
$$= R_A // (R_B + R_C) \ \ldots (2)$$

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$
$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

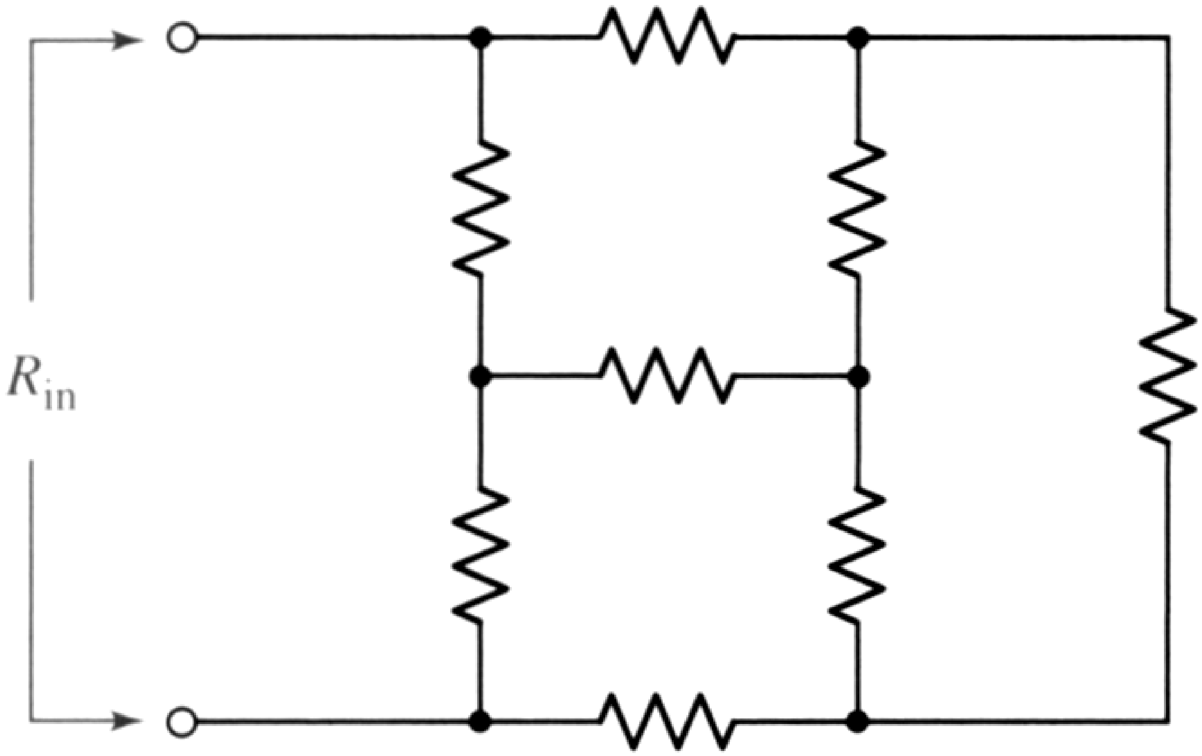
$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$
$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$
$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C}$$

Example 5.12:


1 Ω
4 Ω
3 Ω
2 Ω
5 Ω
(a)
$\frac{1}{2}$ Ω
$\frac{3}{8}$ Ω
$\frac{3}{2}$ Ω
2 Ω
5 Ω
(b)
$\frac{1}{2}$ Ω
$\frac{19}{8}$ Ω
$\frac{13}{2}$ Ω
(c)
$\frac{159}{71}$ Ω
(d)

Practice: 5.11

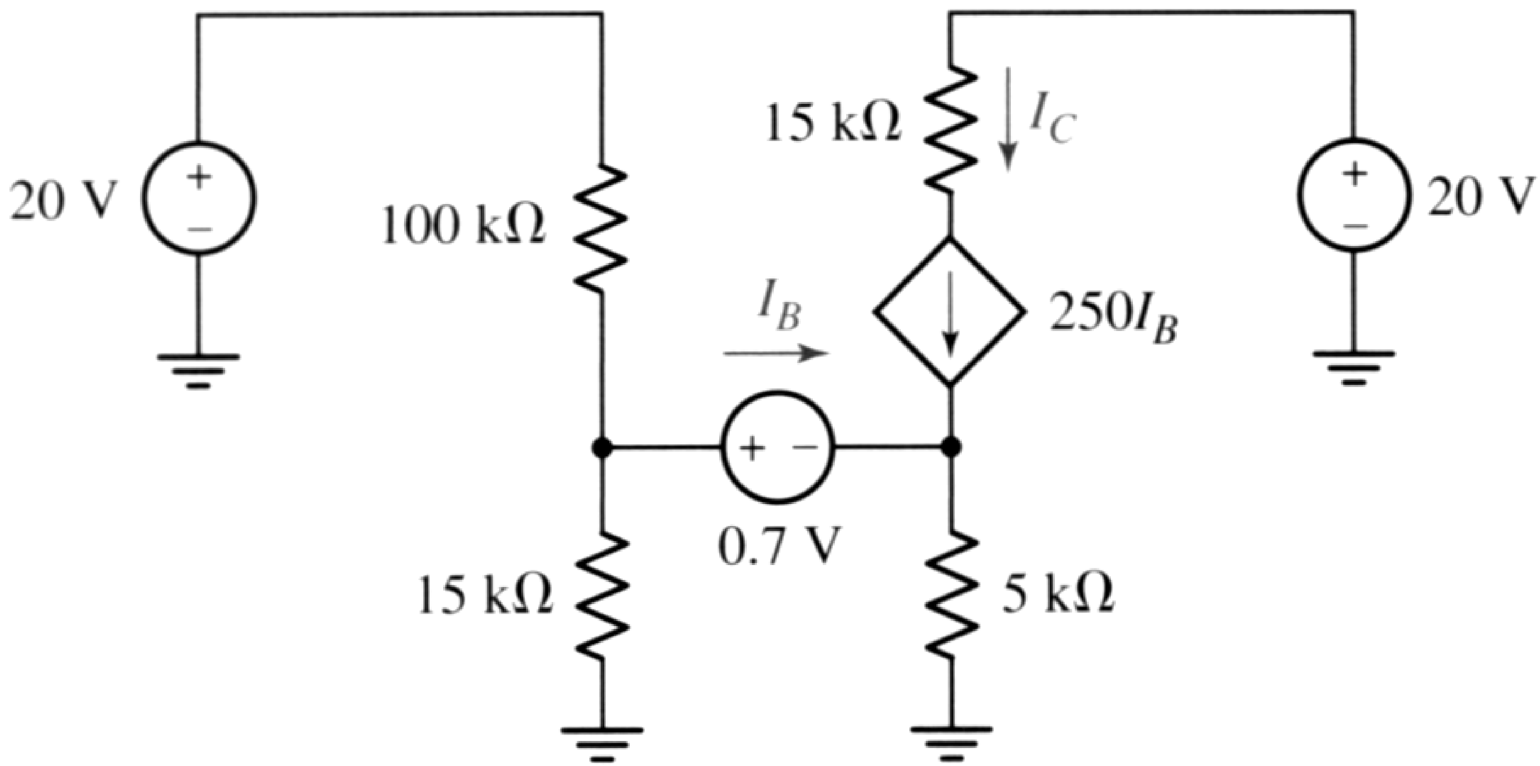
Use the technique of Y-Δ conversion to find the thévenin equivalent resistance of the circuit



Each R is 10 Ω

Example:

Ch5-66, page 143, Sixth Edition



Homework:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.