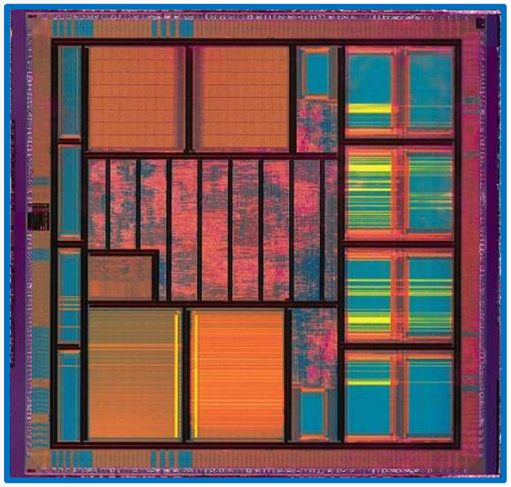
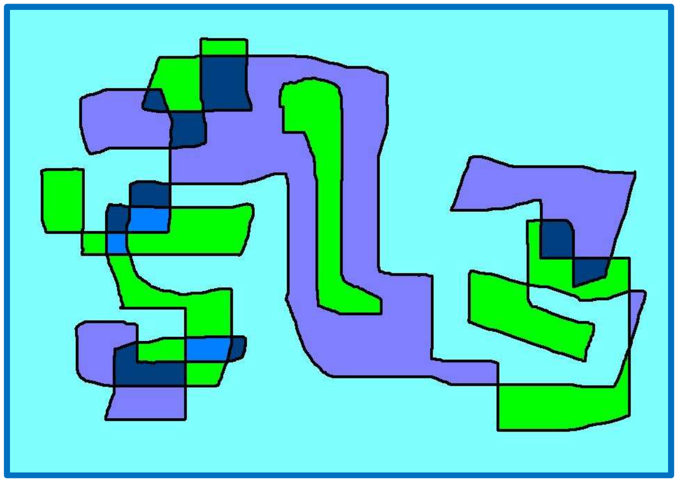
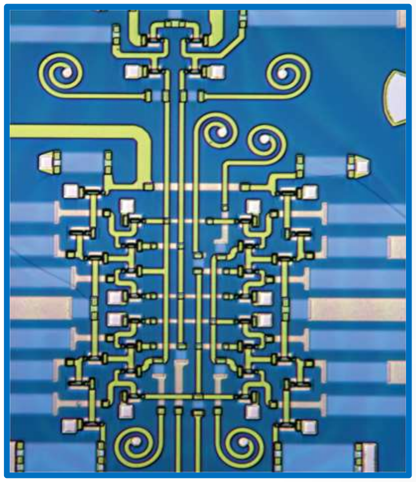






ENE/EIE 211: Electronic Devices and Circuit Design II

Lectures 9 &10 : Two-port Networks & Feedback

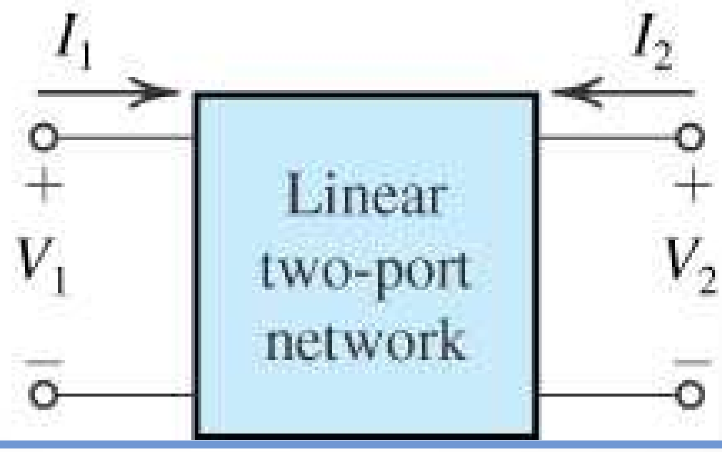
Two-Port Network Parameters

Characterization of linear, two-port networks

Before we begin a discussion on the topic of oscillators, we need to study feedback. However, in order to understand how the feedback works, we also need to first learn the two-port network parameters.

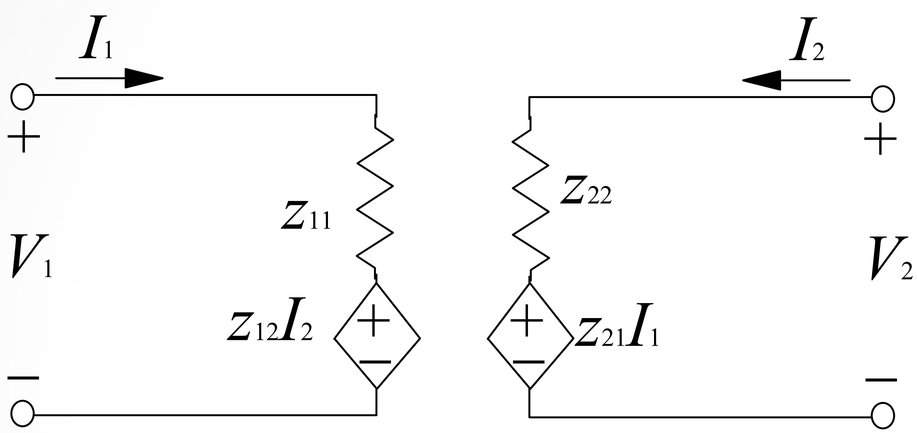
A two-port network has four port variables: V_1, I_1, V_2 and I_2. If the two-port network is linear, we can use two of the variables as excitation variables and the other two as response variables. For example, the network can be excited by a voltage V_1 at port 1 and a voltage V_2 at port 2, and the two current I_1 and I_2 can be measured to represent the network response.

There are four parameter sets commonly used in electronics. They are the admittance (y), the impedance (z), the hybrid (h) and the inverse-hybrid (g) parameters, respectively.



Two-Port Network (z-parameters)

(Open-Circuit Impedance)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

At port 1

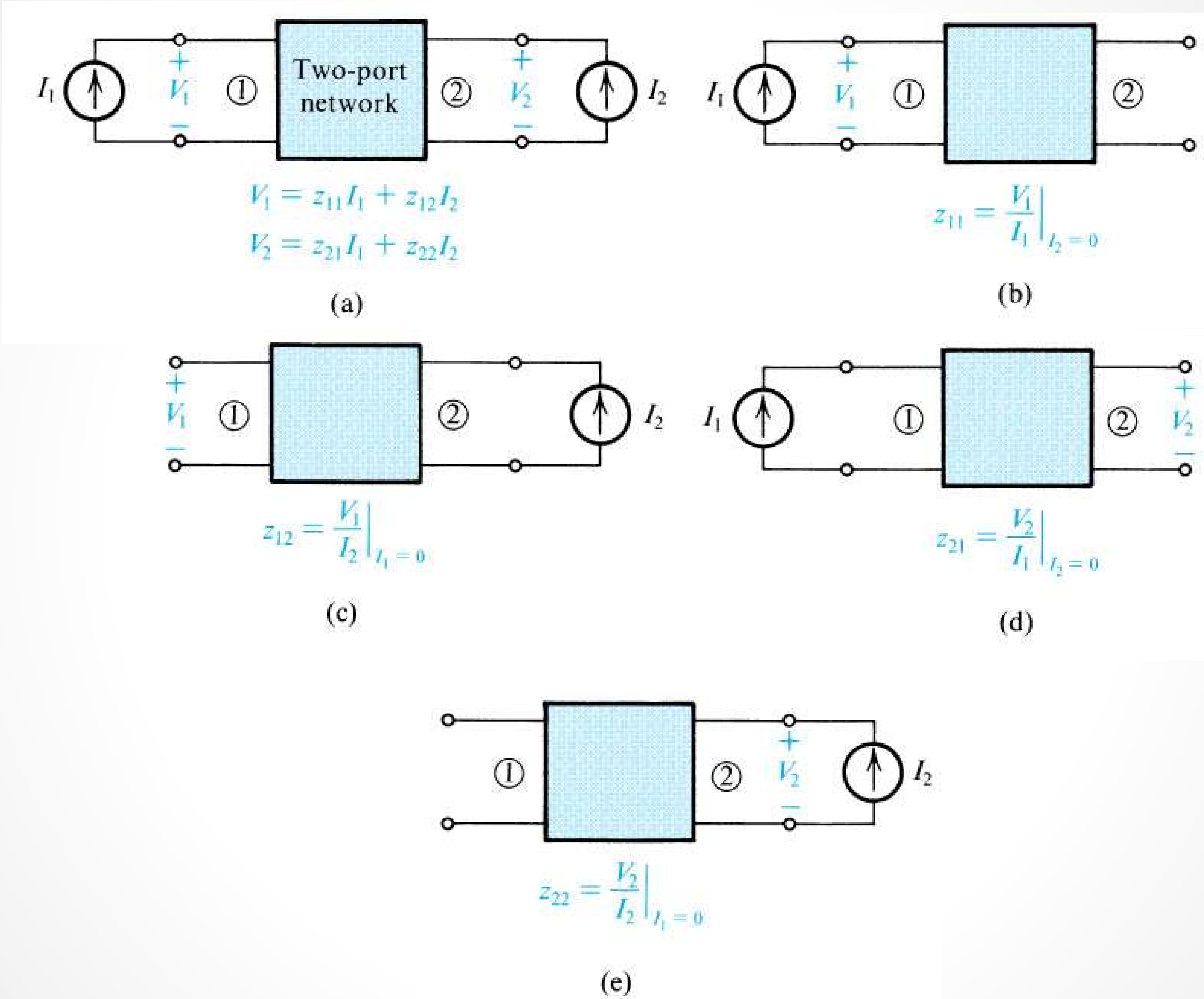
Open-circuit input impedance $z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2 = 0}$

Open-circuit reverse transimpedance $z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1 = 0}$

At port 2

Open-circuit forward transimpedance $z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2 = 0}$

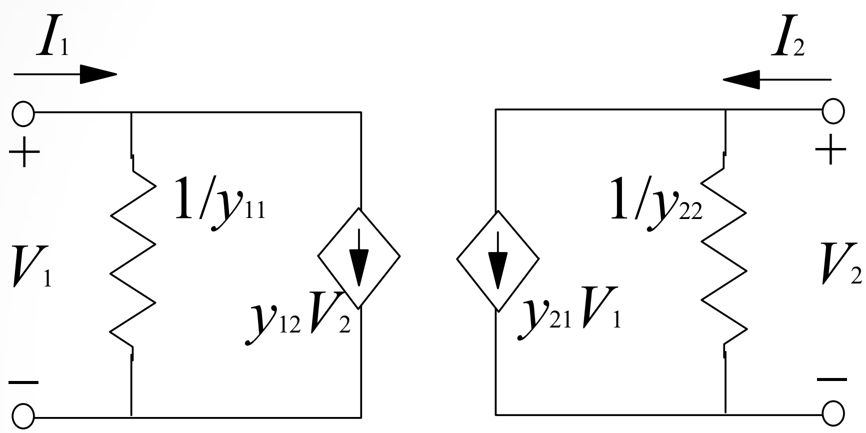
Open-circuit output impedance $z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1 = 0}$



I_1
V_1
①
Two-port network
②
V_2
I_2
$V_1 = z_{11}I_1 + z_{12}I_2$
$V_2 = z_{21}I_1 + z_{22}I_2$
(a)
$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$
(b)
$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$
(c)
$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$
(d)
$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$
(e)

Two-Port Network (*y*-parameters)

(Short-Circuit Admittance)

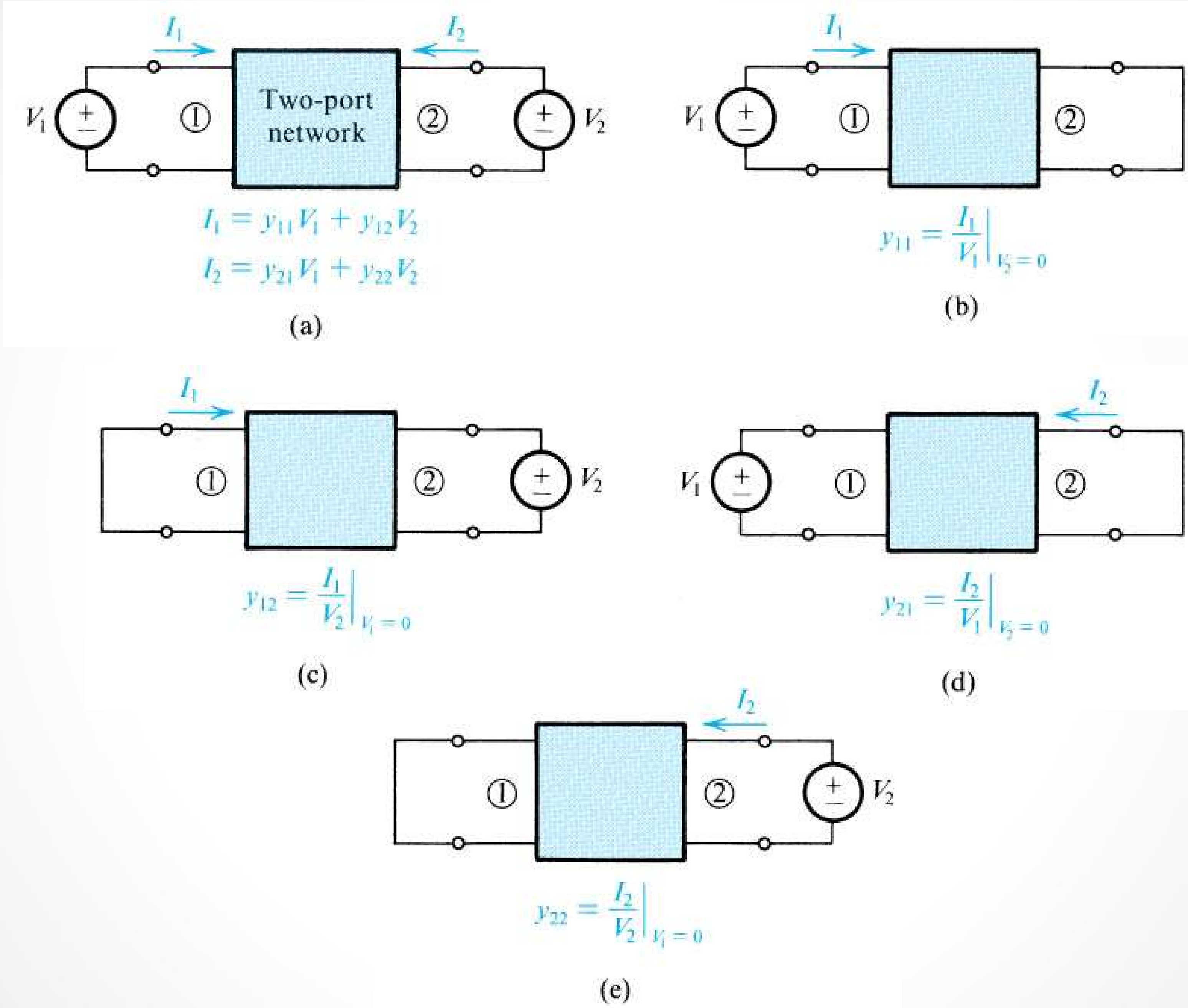


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

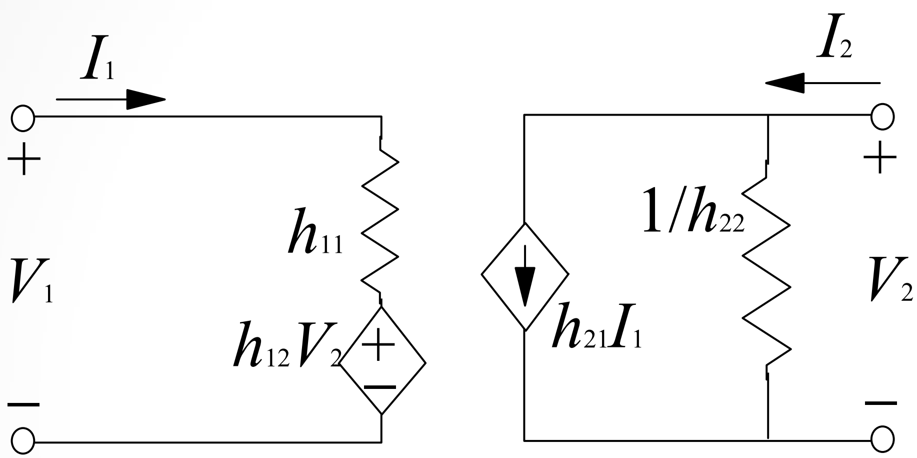
At port 1	At port 2
Short-circuit input admittance $y_{11} = \left.\dfrac{I_1}{V_1}\right\|_{V_2 = 0}$	Short-circuit forward transadmittance $y_{21} = \left.\dfrac{I_2}{V_1}\right\|_{V_2 = 0}$
Short-circuit reverse transadmittance $y_{12} = \left.\dfrac{I_1}{V_2}\right\|_{V_1 = 0}$	Short-circuit output admittance $y_{22} = \left.\dfrac{I_2}{V_2}\right\|_{V_1 = 0}$



I_1
I_2
V_1
V_2
①
②
Two-port network
$I_1 = y_{11}V_1 + y_{12}V_2$
$I_2 = y_{21}V_1 + y_{22}V_2$
(a)
$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$
(b)
$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$
(c)
$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$
(d)
$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$
(e)

Two-Port Network (*h*-parameters)

(hybrid)



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

At port 1

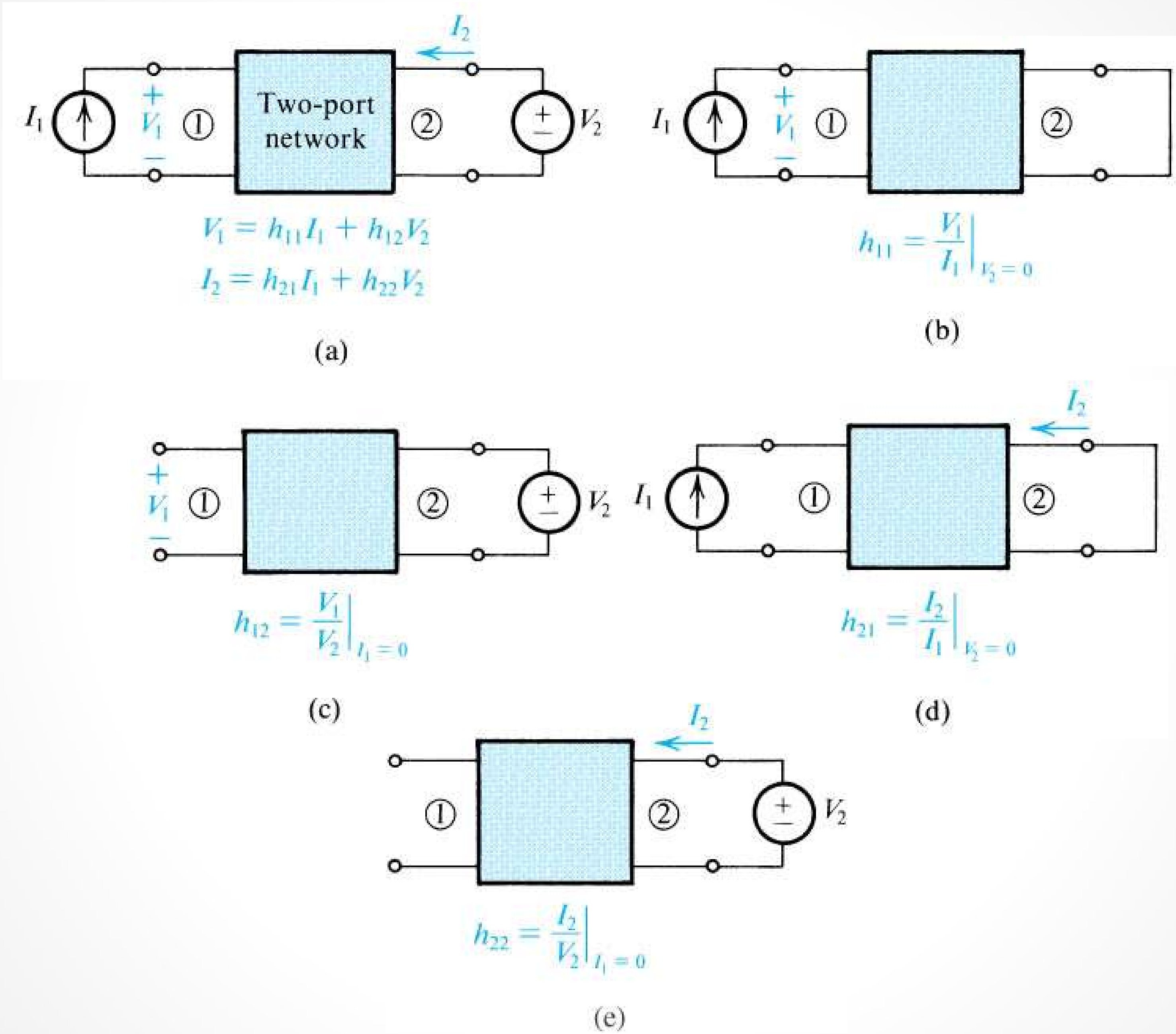
Short-circuit input impedance $h_{11} = \left.\frac{V_1}{I_1}\right| V_2 = 0$

Open-circuit reverse voltage gain $h_{12} = \left.\frac{V_1}{V_2}\right| I_1 = 0$

At port 2

Short-circuit forward current gain $h_{21} = \left.\frac{I_2}{I_1}\right| V_2 = 0$

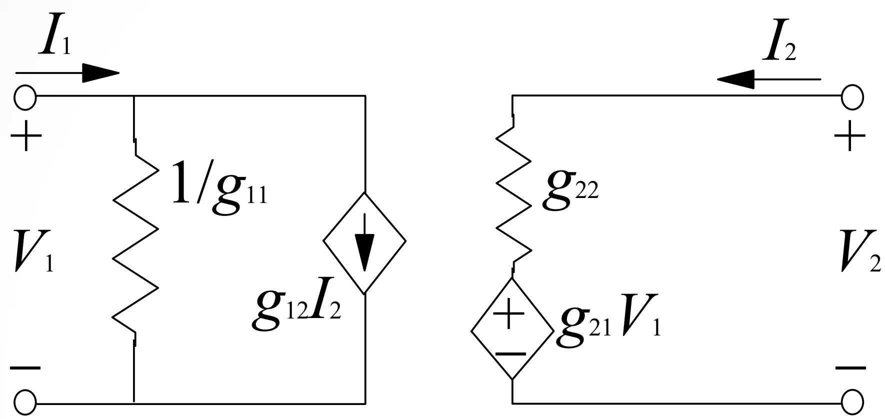
Open-circuit output admittance $h_{22} = \left.\frac{I_2}{V_2}\right| I_1 = 0$



Two-port network
I_1 V_1 ① ② I_2 V_2
$V_1 = h_{11}I_1 + h_{12}V_2$
$I_2 = h_{21}I_1 + h_{22}V_2$
(a)
$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$
(b)
$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$
(c)
$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$
(d)
$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$
(e)

Two-Port Network (*g*-parameters)

(inverse-hybrid)



$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

At port 1

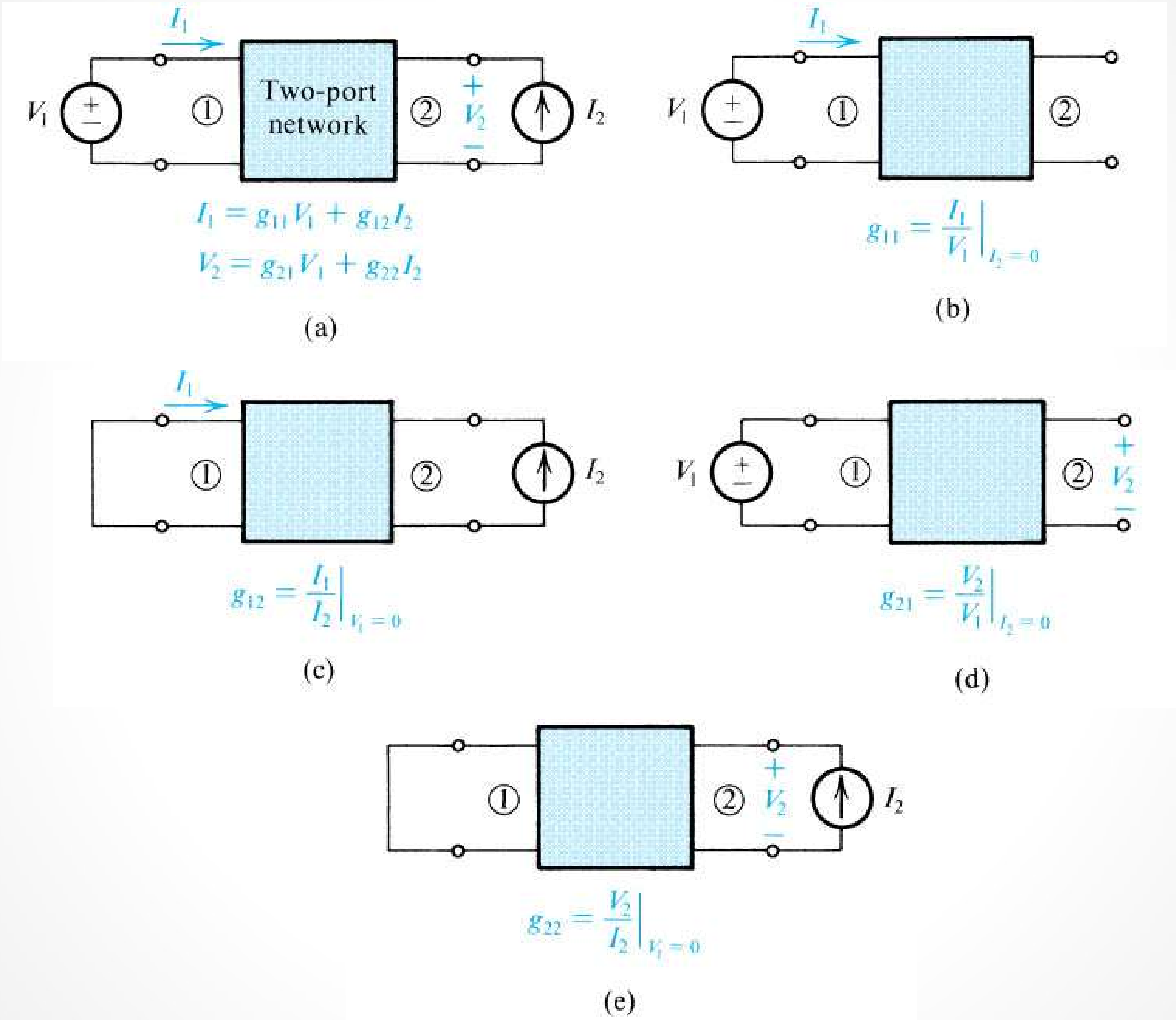
Open-circuit input admittance $$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2 = 0}$$

Short-circuit reverse current gain $$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1 = 0}$$

At port 2

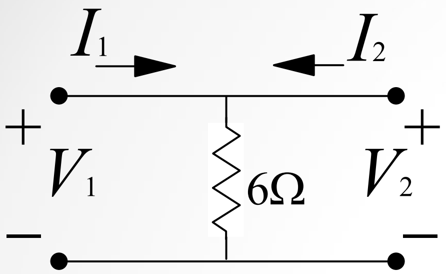
Open-circuit forward current gain $$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2 = 0}$$

Short-circuit output impedance $$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1 = 0}$$



I_1
V_1
+
−
①
Two-port network
②
+
V_2
−
I_2
$I_1 = g_{11}V_1 + g_{12}I_2$
$V_2 = g_{21}V_1 + g_{22}I_2$
(a)
I_1
V_1
①
②
$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$
(b)
I_1
①
②
I_2
$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$
(c)
V_1
①
②
+
V_2
−
$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$
(d)
①
②
+
V_2
−
I_2
$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$
(e)

z-parameter examples

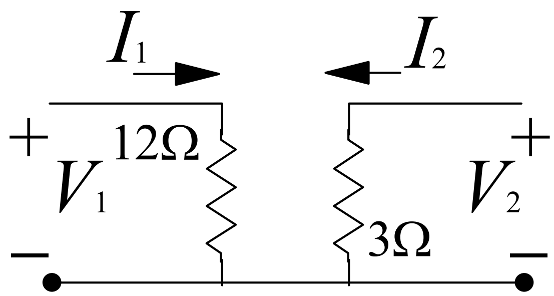


$$Z_{11} = 6\Omega \quad Z_{22} = 6\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = 6\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = 6\Omega$$

$$[Z] = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$$

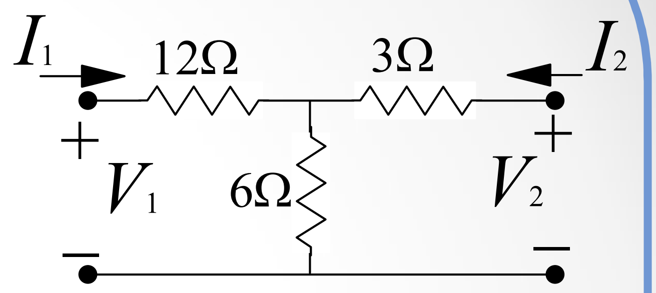


$$Z_{11} = 12\Omega \quad Z_{22} = 3\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = 0\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = 0\Omega$$

$$[Z] = \begin{bmatrix} 12 & 0 \\ 0 & 3 \end{bmatrix}$$



$$Z_{11} = 18\Omega \quad Z_{22} = 9\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = \frac{6I_2}{I_2} = 6\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{6I_1}{I_1} = 6\Omega$$

$$[Z] = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

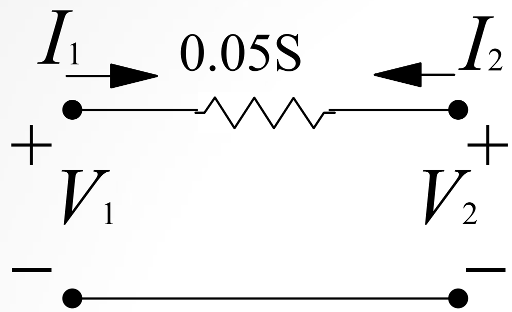
Note: (1) *z*-matrix in the last circuit = sum of two former z-matrices

(2) *z*-parameters is normally used in analysis of **series-series** circuits

(3) $Z_{12} = Z_{21}$ (reciprocal circuit)

(4) $Z_{12} = Z_{21}$ and $Z_{11} = Z_{22}$ (symmetrical and reciprocal circuit)

y-parameter examples

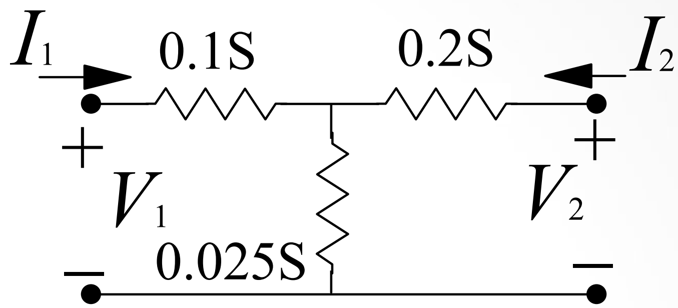


$$y_{11} = 0.05S \quad y_{22} = 0.05\text{S}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-0.05V_2}{V_2} = -0.05\text{S}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-0.05V_1}{V_1} = -0.05\text{S}$$

$$[y] = \begin{bmatrix} 0.05 & -0.05 \\ -0.05 & 0.05 \end{bmatrix}$$



$$y_{11} = \left(\frac{1}{0.1} + \frac{1}{0.2+0.025}\right)^{-1} = 0.0692\text{S}$$

$$y_{22} = \left(\frac{1}{0.2} + \frac{1}{0.1+0.025}\right)^{-1} = 0.0769\text{S}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

But $I_2 = y_{22}V_2 = 0.0769V_2$

$$\frac{-I_1}{0.1} = \frac{I_2 + I_1}{0.025}$$

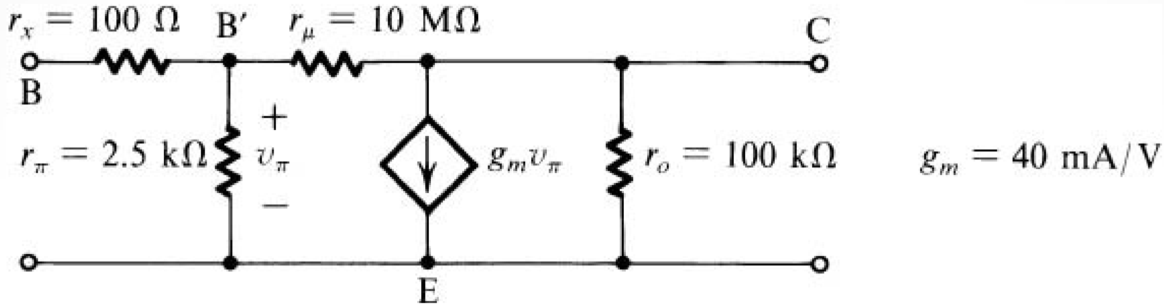
$$\Rightarrow \quad I_1 = -0.8I_2 = -0.0615V_2$$

$$y_{12} = -0.0615\text{S}$$

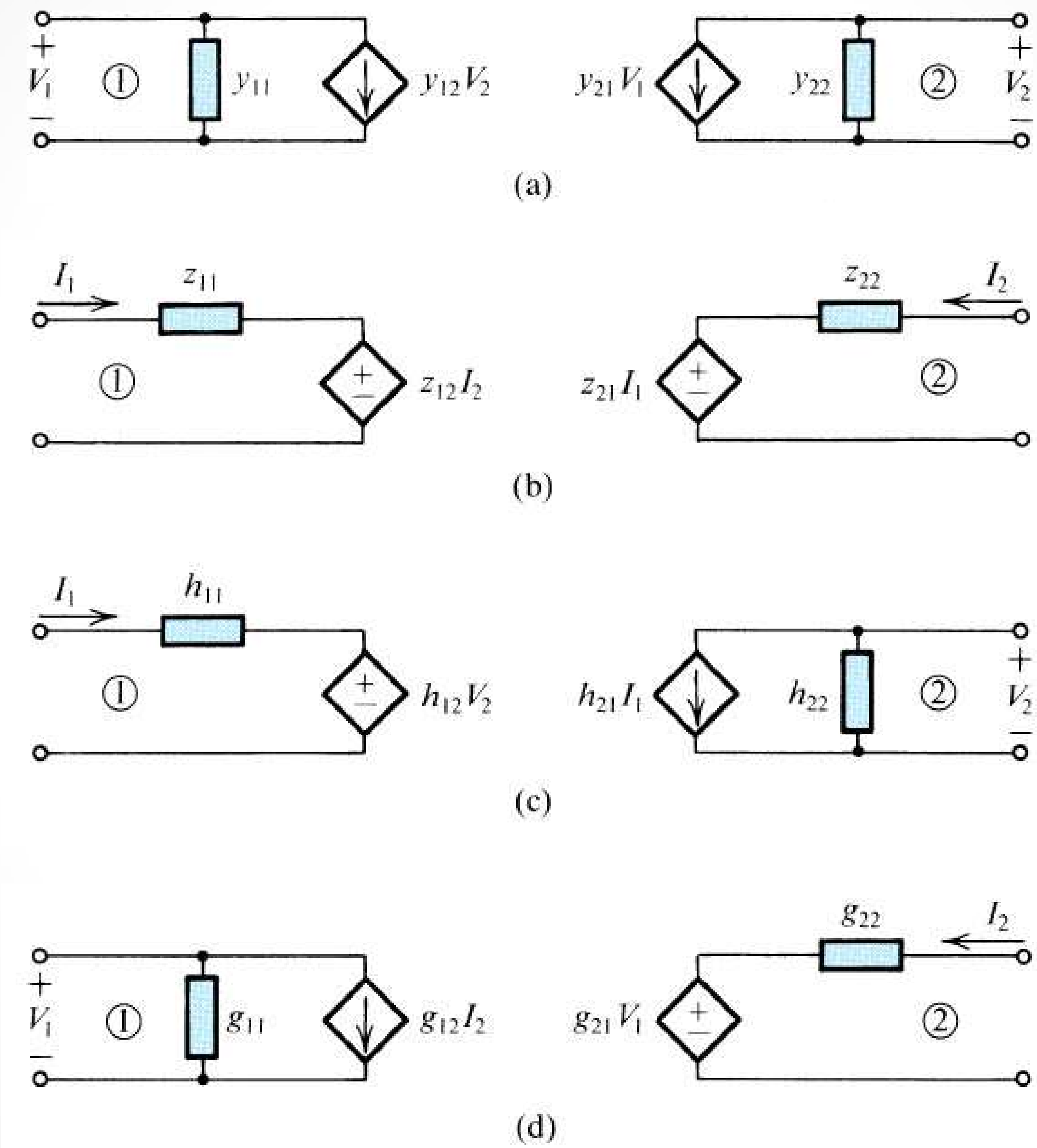
By reciprocal, $y_{21} = y_{12} = -0.0615\text{S}$

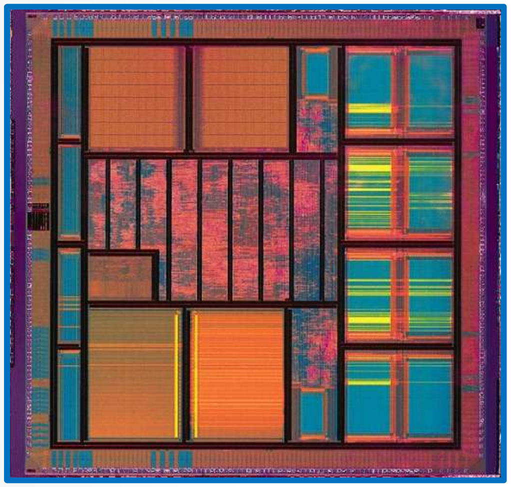
$$[y] = \begin{bmatrix} 0.0692 & -0.0615 \\ -0.0615 & 0.0769 \end{bmatrix}$$

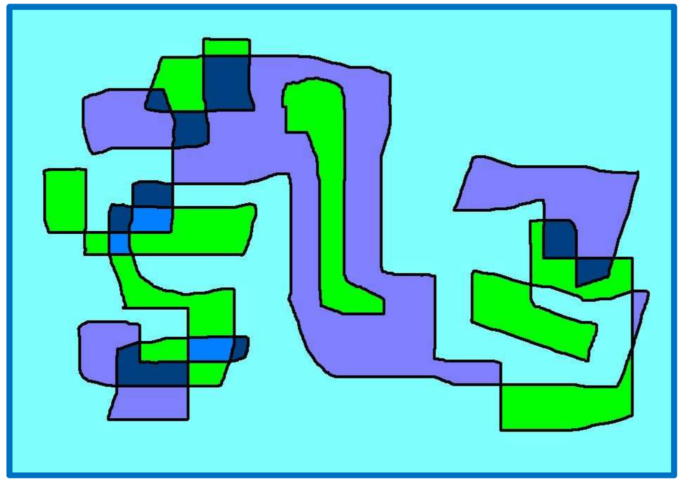
Example: figure below shows the small-signal equivalent-ckt model of a transistor. Calculate the values of the h parameters.

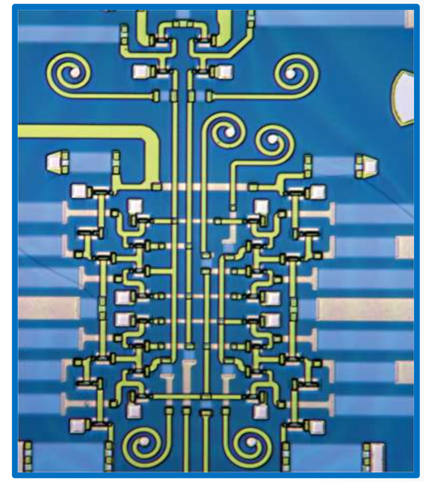


Summary: Equivalent-Circuit Representation





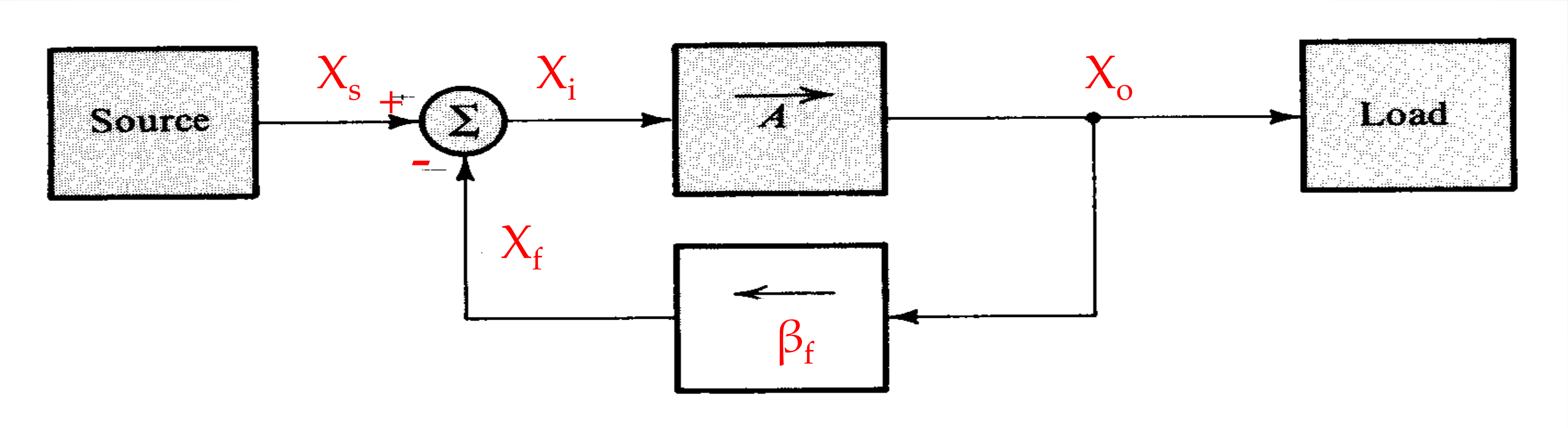




ENE/EIE 211: Electronic Devices and Circuit Design II

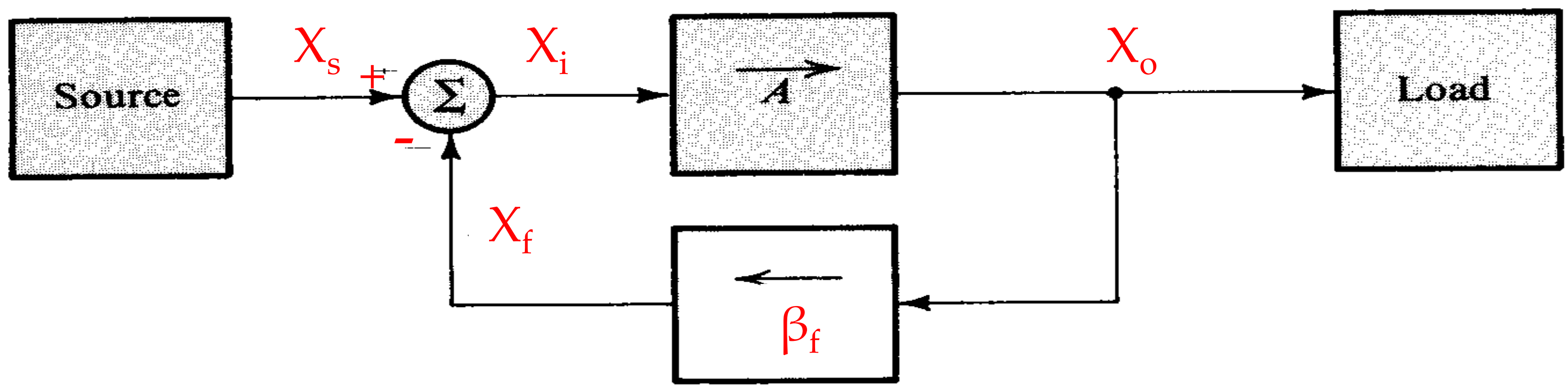
Feedback

Feedback



- **What is feedback?** Taking a portion of the signal arriving at the load and feeding it back to the input.
- **What is negative feedback?** Adding the feedback signal to the input so as to partially cancel the input signal to the amplifier.
- **Doesn't this reduce the gain?** Yes, this is the price we pay for using feedback.
- **Why use feedback?** Provides a series of benefits, such as improved bandwidth, that outweigh the costs in lost gain and increased complexity in amplifier design.

Feedback Amplifier Analysis



$X_f = \beta_f X_o$ where β_f is called the feedback factor

$X_o = AX_i$ where A is the amplifier's gain, e.g. voltage gain

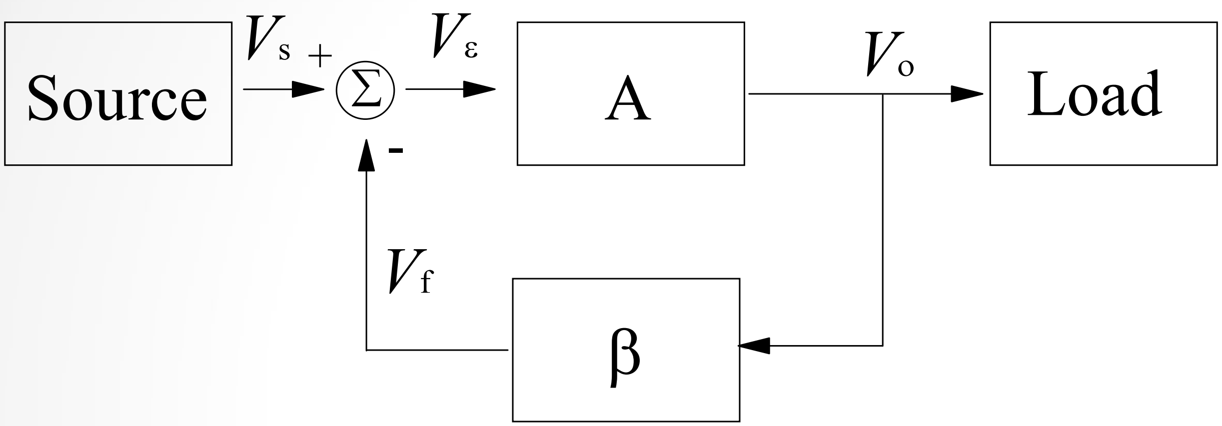
$X_i = X_s - X_f$ where X_i is the net input signal to the basic amplifier,

X_s = the signal from the source

The amplifier's gain with feedback is given by

$$\boxed{A_f = \frac{X_o}{X_s} = \frac{AX_i}{X_i + X_f} = \frac{A}{1 + \dfrac{X_f}{X_i}} = \frac{A}{1 + \dfrac{\beta_f X_o}{X_i}} = \frac{A}{1 + \beta_f A} < A}$$

Summary: General Feedback Structure



A : Open Loop Gain
$A = V_o / V_\varepsilon$

β : feedback factor
$\beta = V_f / V_o$

$$V_\varepsilon = V_s - V_f$$

$$V_f = \beta \cdot V_o$$

$$V_\varepsilon = V_S - \beta \cdot V_o$$

$$V_o = A \cdot V_\varepsilon$$

Close loop gain : $A_f = \dfrac{V_o}{V_s} = \dfrac{A}{1 + A\beta} = \dfrac{1}{\beta}\left(\dfrac{T}{1+T}\right)$

Loop Gain : $T = A \cdot \beta$

Amount of feedback : $1 + A \cdot \beta$

Note : $A_f\big|_{A\to\infty} = \dfrac{1}{\beta}$

The product Aβ must be positive for the feedback network to be the negative feedback network.

Advantages of Negative Feedback

* **Gain desensitivity** - less variation in amplifier gain with changes in β (current gain) of transistors due to dc bias, temperature, fabrication process variations, etc.
* **Bandwidth extension** - extends dominant high and low frequency poles to higher and lower frequencies, respectively.

$$\omega_{Hf} = \left(1+\beta_f A\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A\right)}$$

* **Noise reduction** - improves signal-to-noise ratio
* **Improves amplifier linearity** - reduces distortion in signal due to gain variations due to transistors
* **Impedance Control** - control input and output impedances by applying appropriate feedback topologies

* **Cost of these advantages:**
  - ↘ Loss of gain, may require an added gain stage to compensate.
  - ↘ Added complexity in design

Gain Desensitivity

Feedback can be used to desensitize the closed-loop gain to variations in the basic amplifier. Let's see how.

Assume β is constant. Taking differentials of the closed-loop gain equation gives…

$$A_f = \frac{A}{1+A\beta} \qquad\qquad dA_f = \frac{dA}{(1+A\beta)^2}$$

Divide by A_f

$$\frac{dA_f}{A_f} = \frac{dA}{(1+A\beta)^2}\frac{1+A\beta}{A} = \frac{1}{1+A\beta}\frac{dA}{A}$$

This result shows the effects of variations in *A* on A_f is mitigated by the feedback amount. 1+*A*β is also called the desensitivity amount

We will see through examples that feedback also affects the input and resistance of the amplifier (increases R_i and decreases R_o by 1+Aβ factor)

Bandwidth Extension

We've mentioned several times in the past that we can trade gain for bandwidth. Finally, we see how to do so with feedback… Consider an amplifier with a high-frequency response characterized by a single pole and the expression:

Apply negative feedback β and the resulting closed-loop gain is:

$$A(s) = \frac{A_M}{1 + s/\omega_H}$$

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{A_M/(1 + A_M\beta)}{1 + s/\omega_H(1 + A_M\beta)}$$

- Notice that the midband gain reduces by $(1+A_M\beta)$ while the 3-dB roll-off frequency increases by $(1+A_M\beta)$

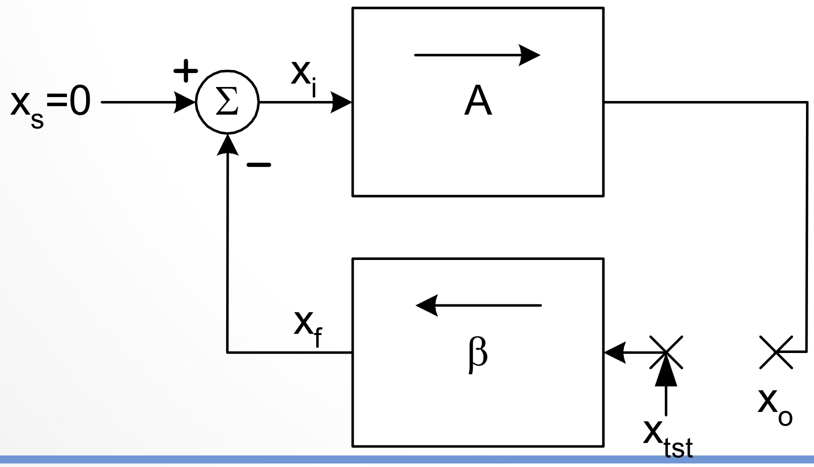
Finding Loop Gain

Generally, we can find the loop gain with the following steps:

- Break the feedback loop anywhere (at the output in the ex. below)
- Zero out the input signal x_s
- Apply a test signal to the input of the feedback circuit
- Solve for the resulting signal x_o at the output

  If x_o is a voltage signal, x_{tst} is a voltage and measure the open-circuit voltage

  If x_o is a current signal, x_{tst} is a current and measure the short-circuit current
- The negative sign comes from the fact that we are apply negative feedback



$$x_f = \beta x_{tst}$$

$$x_i = 0 - x_f$$

$$x_o = Ax_i = -Ax_f = -\beta A x_{tst}$$

$$\text{loop gain} = -\frac{x_o}{x_{tst}} = \beta A$$

Basic Types of Feedback Amplifiers

- * There are four types of feedback amplifiers. Why?
  - ↘ Output sampled can be a current or a voltage
  - ↘ Quantity fed back to input can be a current or a voltage
  - ↘ Four possible combinations of the type of output sampling and input feedback
- * One particular type of amplifier, e.g. voltage amplifier, current amplifier, etc. is used for each one of the four types of feedback amplifiers.
- * Feedback factor β_f is a different type of quantity, e.g. voltage ratio, resistance, current ratio or conductance, for each feedback configuration.
- * Before analyzing the feedback amplifier's performance, need to start by recognizing the type or configuration.
- * Terminology used to name types of feedback amplifier, e.g. Series-shunt
  - ↘ First term refers to nature of feedback connection at the input.
  - ↘ Second term refers to nature of sampling connection at the output.

Basic Feedback Topologies

Depending on the input signal (voltage or current) to be amplified and form of the output (voltage or current), amplifiers can be classified into four categories. Depending on the amplifier category, one of four types of feedback structures should be used.

(Type of Feedback)	**(Type of Sensing)**
(1) Series (Voltage)	Shunt (Voltage)
(2) Series (Voltage)	Series (Current)
(3) Shunt (Current)	Shunt (Voltage)
(4) Shunt (Current)	Series (Current)

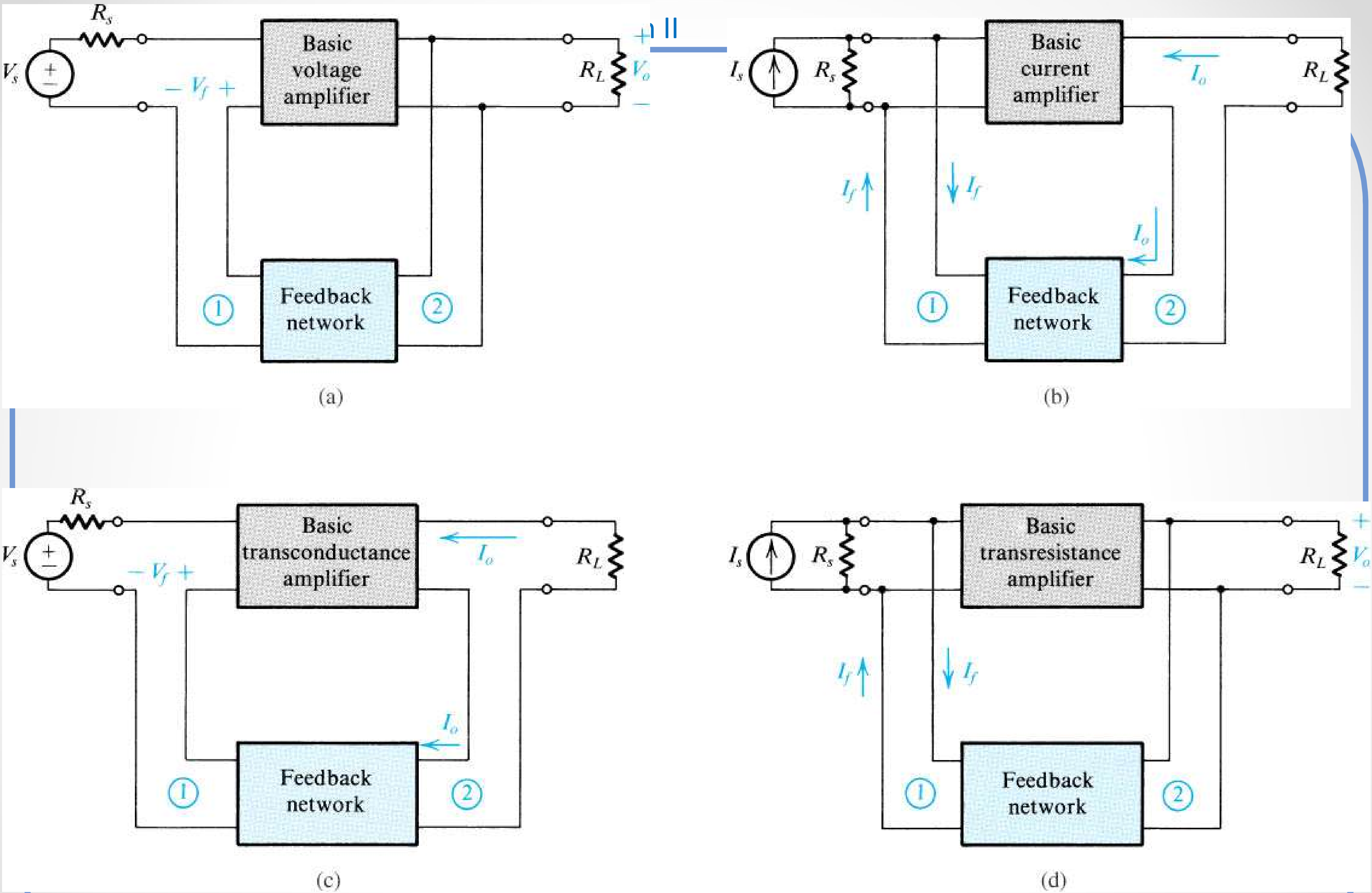
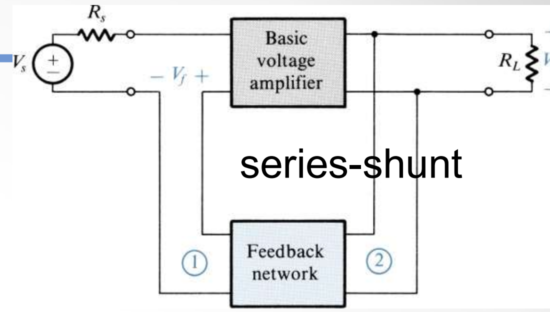
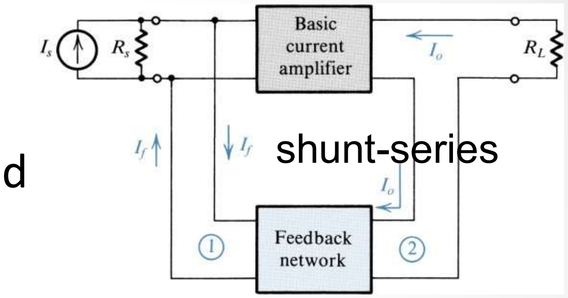


Figure 8.4 The four basic feedback topologies: **(a)** voltage-mixing voltage-sampling (series–shunt) topology; **(b)** current-mixing current-sampling (shunt–series) topology; **(c)** voltage-mixing current-sampling (series–series) topology; **(d)** current-mixing voltage-sampling (shunt–shunt) topology.

Basic Feedback Topologies



Depending on the input signal (voltage or current) to be amplified and form of the output (voltage or current), amplifiers can be classified into four categories. Depending on the amplifier category, one of four types of feedback structures should be used (series-shunt, series-series, shunt-shunt, or shunt-series)



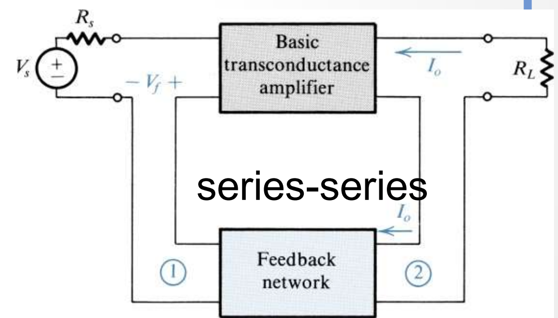
Voltage amplifier – voltage-controlled voltage source

Requires high input impedance, low output impedance

Use series-shunt feedback (voltage-voltage feedback)

Current amplifier – current-controlled current source

Use shunt-series feedback (current-current feedback)

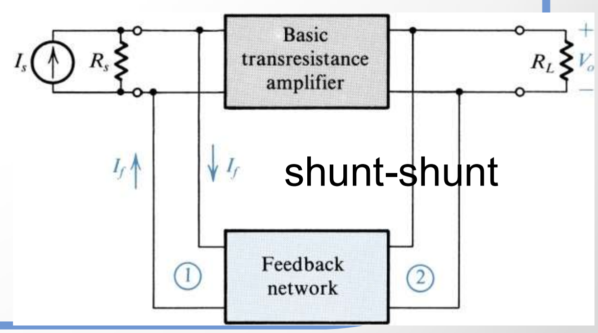


Transconductance amplifier – voltage-controlled current source

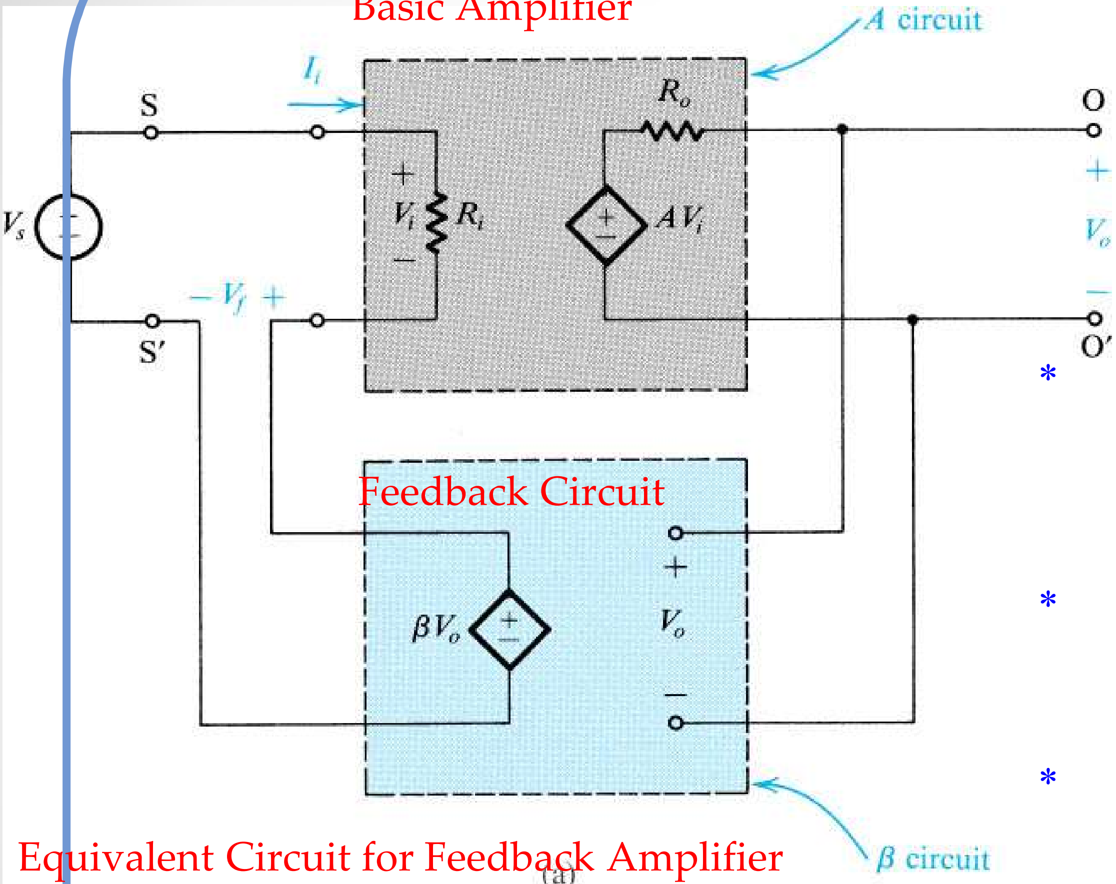
Use series-series feedback (current-voltage feedback)

Transimpedance amplifier – current-controlled voltage source

Use shunt-shunt feedback (voltage-current feedback)

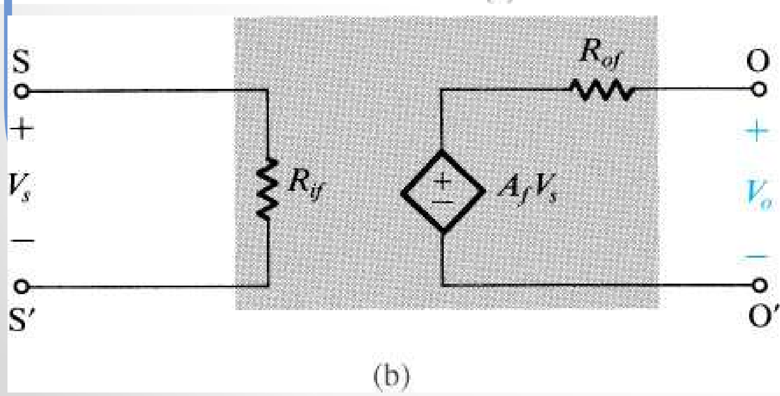


Series-Shunt Feedback Amplifier - Ideal Case



(a)

Equivalent Circuit for Feedback Amplifier



(b)

* Assumes feedback circuit does not load down the basic amplifier A, i.e. doesn't change its characteristics

- Doesn't change gain A
- Doesn't change pole frequencies of basic amplifier A
- Doesn't change R_i and R_o

* For the feedback amplifier as a whole, feedback does change the midband voltage gain from A to A_f

$$A_f = \frac{A}{1+\beta_f A}$$

* Does change input resistance from R_i to R_{if}

$$R_{if} = R_i\left(1+\beta_f A\right)$$

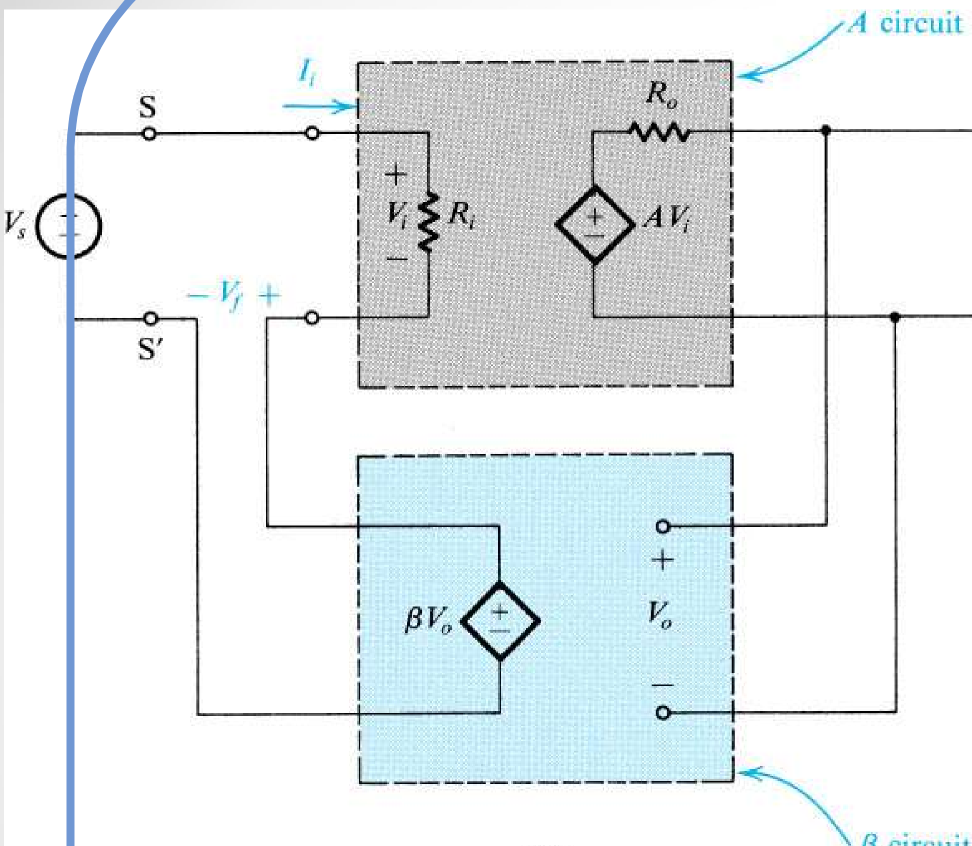
* Does change output resistance from R_o to R_{of}

$$R_{of} = \frac{R_o}{1+\beta_f A}$$

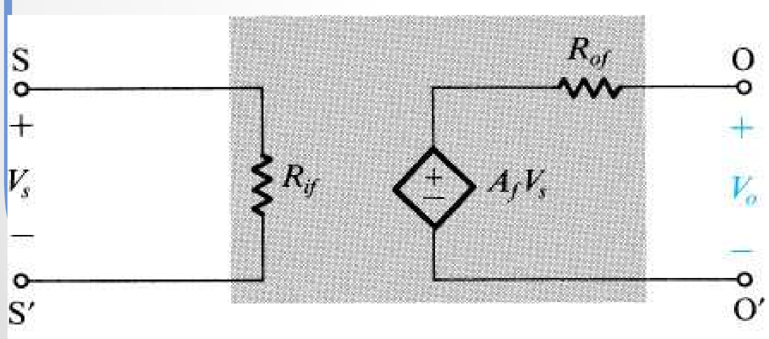
* Does change low and high frequency 3dB frequencies

$$\omega_{Hf} = \left(1+\beta_f A\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A\right)}$$

Series-Shunt Feedback Amplifier - Ideal Case



(a)



(b)

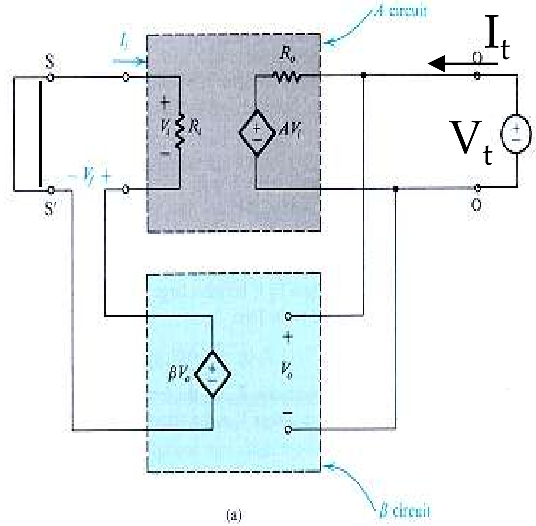
Midband Gain

$$A_{Vf}=\frac{V_o}{V_s}=\frac{A_V V_i}{V_i+V_f}=\frac{A_V}{1+\frac{V_f}{V_i}}=\frac{A_V}{1+\frac{\beta_f V_o}{V_i}}=\frac{A_V}{1+\beta_f A_V}$$

Input Resistance

$$R_{if}=\frac{V_s}{I_i}=\frac{V_i+V_f}{I_i}=\frac{V_i+\beta_f V_o}{\left(V_i/R_i\right)}=R_i\left(1+\beta_f A_V\right)$$

Output Resistance



$$I_t=\frac{V_t-A_V V_i}{R_o}$$

But $V_s=0$ *so* $V_i=-V_f$

and $V_f=\beta_f V_o=\beta_f V_t$ *so*

$$I_t=\frac{V_t-A_V\left(-V_f\right)}{R_o}=\frac{V_t+A_V\left(\beta_f V_t\right)}{R_o}=\frac{V_t\left(1+A_V\beta_f\right)}{R_o}$$

$$so\ \ R_{of}=\frac{V_t}{I_t}=\frac{R_o}{\left(1+A_V\beta_f\right)}$$

Series-Shunt Feedback Amplifier - Ideal Case

Low Frequency Pole

$$\text{For } A(\omega)=\frac{A_o}{1+\frac{\omega_L}{s}} \quad \text{then } A_f(\omega)=\frac{A(\omega)}{1+\beta_f A(\omega)}=\frac{\left(\frac{A_o}{1+\frac{\omega_L}{s}}\right)}{\left[1+\beta_f\frac{A_o}{1+\frac{\omega_L}{s}}\right]}=\frac{A_o}{\left[1+\frac{\omega_L}{s}+\beta_f A_o\right]}=\frac{\left(\frac{A_o}{1+\beta_f A_o}\right)}{\left[1+\frac{\omega_L}{1+\beta_f A_o}\left(\frac{1}{s}\right)\right]}=\frac{A_{fo}}{\left(1+\frac{\omega_{Lf}}{s}\right)}$$

$$\text{where} \quad A_{fo}=\frac{A_o}{1+\beta_f A_o} \qquad \boxed{\omega_{Lf}=\frac{\omega_L}{1+\beta_f A_o}}$$

← Low 3dB frequency lowered by feedback.

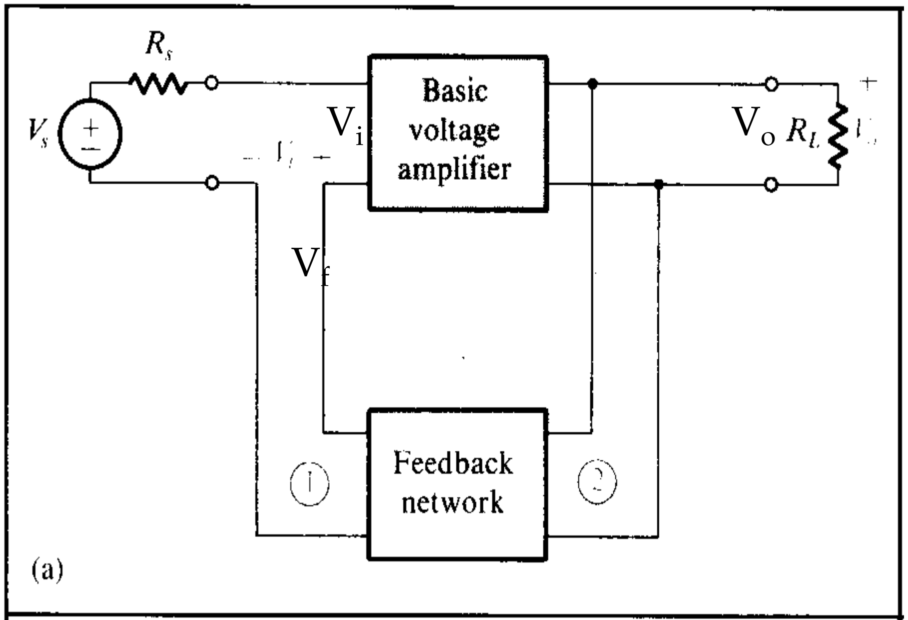
High Frequency Pole

$$\text{For } A(\omega)=\frac{A_o}{1+\frac{s}{\omega_H}} \quad \text{then } A_f(\omega)=\frac{A(\omega)}{1+\beta_f A(\omega)}=\frac{\left(\frac{A_o}{1+\frac{s}{\omega_H}}\right)}{\left[1+\beta_f\frac{A_o}{1+\frac{s}{\omega_H}}\right]}=\frac{A_o}{\left[1+\frac{s}{\omega_H}+\beta_f A_o\right]}=\frac{\left(\frac{A_o}{1+\beta_f A_o}\right)}{\left[1+\frac{s}{\omega_H\left(1+\beta_f A_o\right)}\right]}=\frac{A_{fo}}{\left(1+\frac{s}{\omega_{Hf}}\right)}$$

$$\text{where} \quad A_{fo}=\frac{A_o}{1+\beta_f A_o} \qquad \boxed{\omega_{Hf}=\omega_H\left(1+\beta_f A_o\right)}$$

← Upper 3dB frequency raised by feedback.

Practical Feedback Networks



* How do we take these loading effects into account?

* **Feedback networks consist of a set of resistors**
  - **Simplest case (only case considered here)**
  - **In general, can include C's and L's (not considered here)**
  - **Transistors sometimes used (gives variable amount of feedback) (not considered here)**
* **Feedback network needed to create V_f feedback signal at input (desirable)**
* **Feedback network has parasitic (loading) effects including:**
* **Feedback network loads down amplifier input**
  - **Adds a finite series resistance**
  - **Part of input signal V_s lost across this series resistance (undesirable), so V_i reduced**
* **Feedback network loads down amplifier output**
  - **Adds a finite shunt resistance**
  - **Part of output current lost through this shunt resistance so not all output current delivered to load R_L (undesirable)**

Equivalent Network for Feedback Network

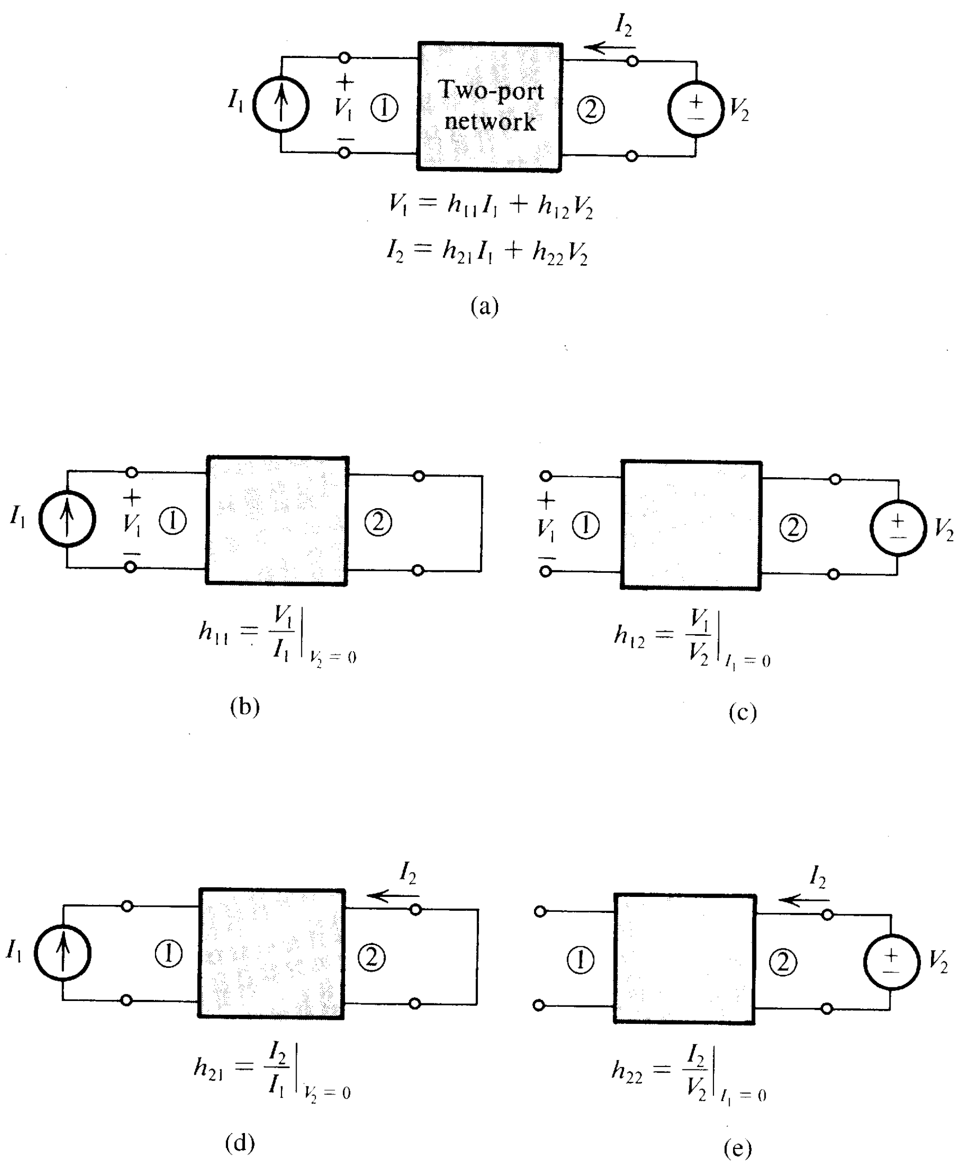
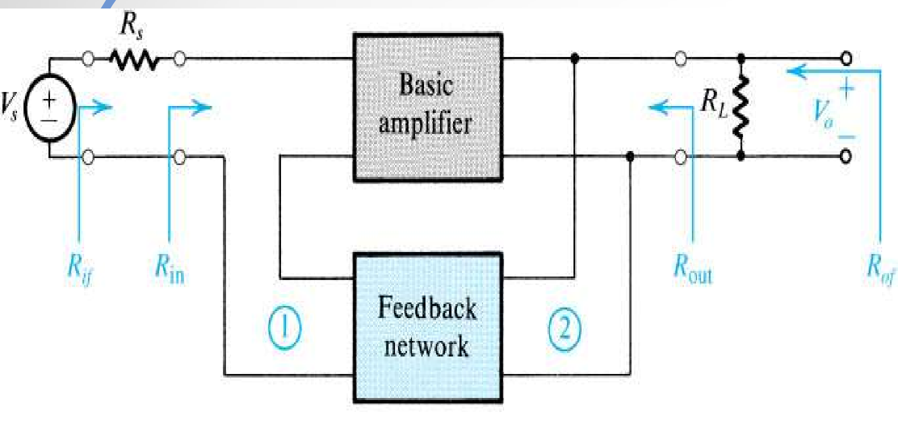


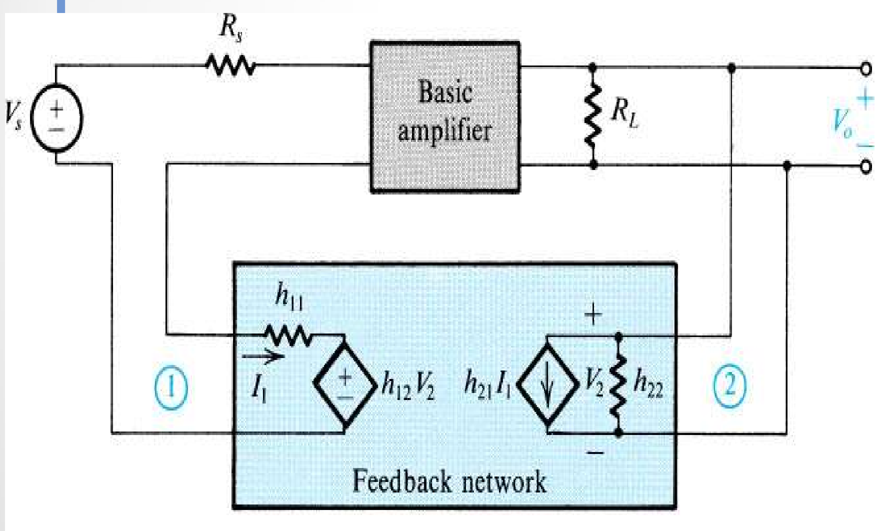
Fig. B.4 Definition and conceptual measurement circuits for h parameters.

- * Need to find an equivalent network for the feedback network including feedback effect and loading effects.
- * Feedback network is a two port network (input and output ports)
- * Can represent with h-parameter network (This is the best for this particular feedback amplifier configuration)
- * h-parameter equivalent network has FOUR parameters
- * h-parameters relate input and output currents and voltages
- * Two parameters chosen as independent variables. For h-parameter network, these are input current I_1 and output voltage V_2
- * Two equations relate other two quantities (output current I_2 and input voltage V_1) to these independent variables
- * Knowing I_1 and V_2, can calculate I_2 and V_1 if you know the h-parameter values
- * h-parameters can have units of ohms, 1/ohms or no units (depends on which parameter)

Series-Shunt Feedback Amplifier - Practical Case



(a)



(b)

$$R_{in} = R_{if} - R_s \qquad R_{out} = 1/(\frac{1}{R_{of}} - \frac{1}{R_L})$$

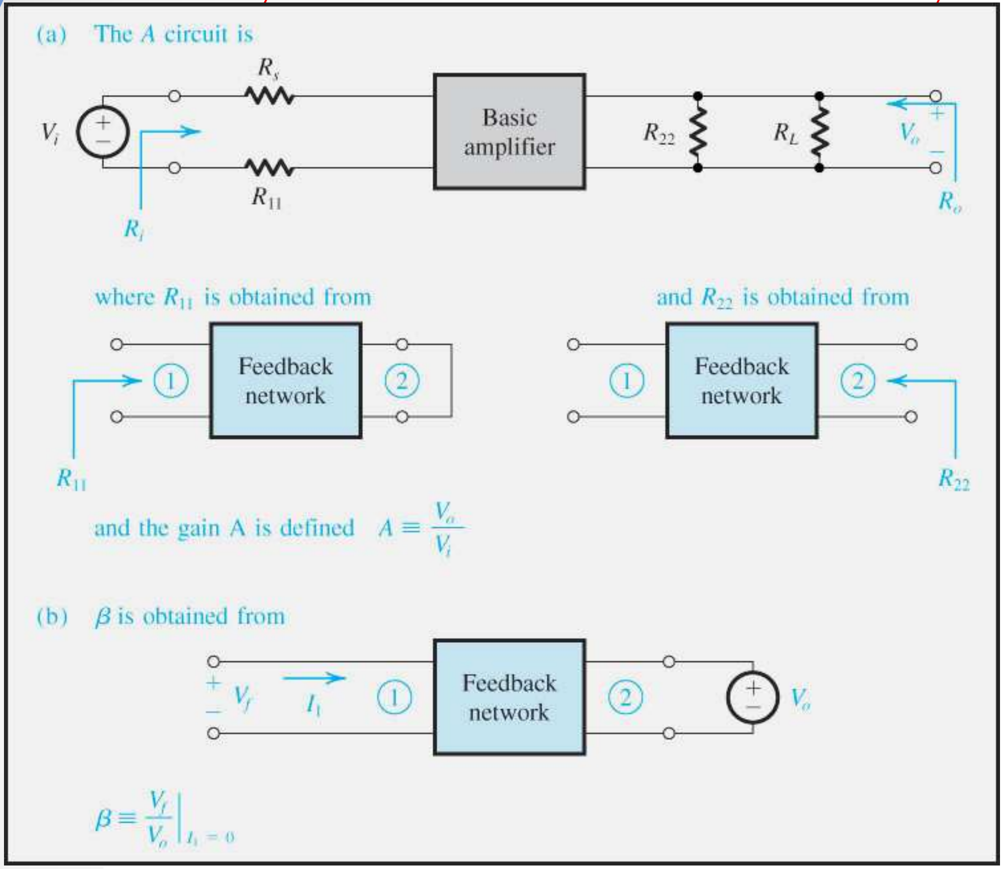
* Feedback network consists of a set of resistors
* These resistors have loading effects on the basic amplifier, i.e they change its characteristics, such as the gain
* Can use h-parameter equivalent circuit for feedback network
  - Feedback factor β_f given by h_{12} since
  $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{V_f}{V_o} = \beta_f$$
  - Feedforward factor given by h_{21} (neglected)
  - h_{22} gives feedback network loading on output
  - h_{11} gives feedback network loading on input
* Can incorporate loading effects in a modified basic amplifier. Basic gain of amplifier A_V becomes a new, modified gain A_V' (incorporates loading effects).
* Can then use feedback analysis from the ideal case.

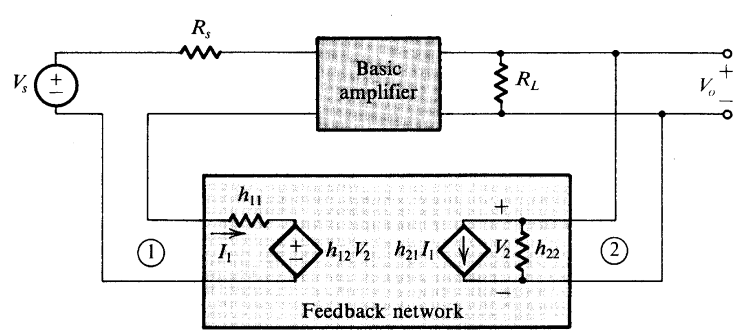
$$A_{Vf} = \frac{A_V'}{1+\beta_f A_V'} \qquad R_{if} = R_i(1+\beta_f A_V') \qquad R_{of} = \frac{R_o}{(1+A_V'\beta_f)}$$

$$\omega_{Hf} = (1+\beta_f A_V')\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{(1+\beta_f A_V')}$$

Series-Shunt Feedback Amplifier - Practical Case

Summary of Feedback Network Analysis

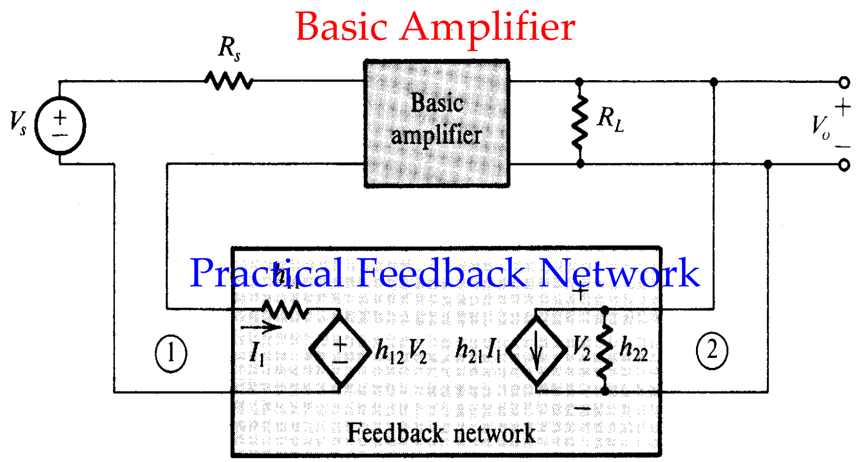




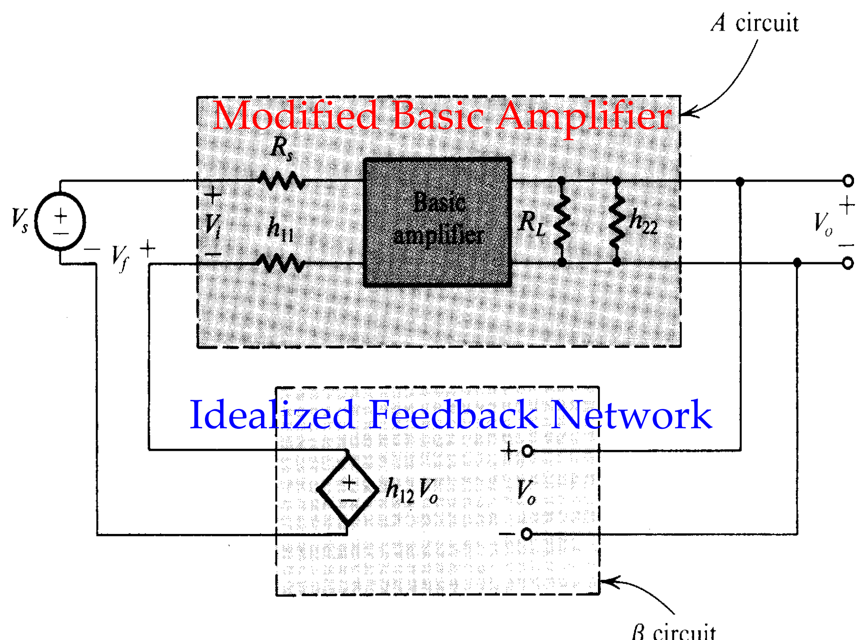
(b)

* **How do we determine the h-parameters for the feedback network?**
* **For the input loading term h_{11}**
  - **Turn off the feedback signal by setting $V_o = 0$.**
  - **Then evaluate the resistance seen looking into port 1 of the feedback network (also called R_{11} here).**
* **For the output loading term h_{22}**
  - **Open circuit the connection to the input so $I_1 = 0$.**
  - **Find the resistance seen looking into port 2 of the feedback network (also called R_{22} here).**
* **To obtain the feedback factor β_f (also called h_{12})**
  - **Apply a test signal V_o' to port 2 of the feedback network and evaluate the feedback voltage V_f (also called V_1 here) for $I_1 = 0$.**
  - **Find β_f from $\beta_f = V_f/V_o$'**

Summary of Approach to Analysis



(b)



(c)

Evaluate modified basic amplifier
(including loading effects of feedback network)

- **Including h_{11} at input**
- **Including h_{22} at output**
- **Including loading effects of source resistance**
- **Including load effects of load resistance**

Analyze effects of idealized feedback network using feedback amplifier equations derived

$$A_{Vf} = \frac{A_V'}{1+\beta_f A_V'}$$

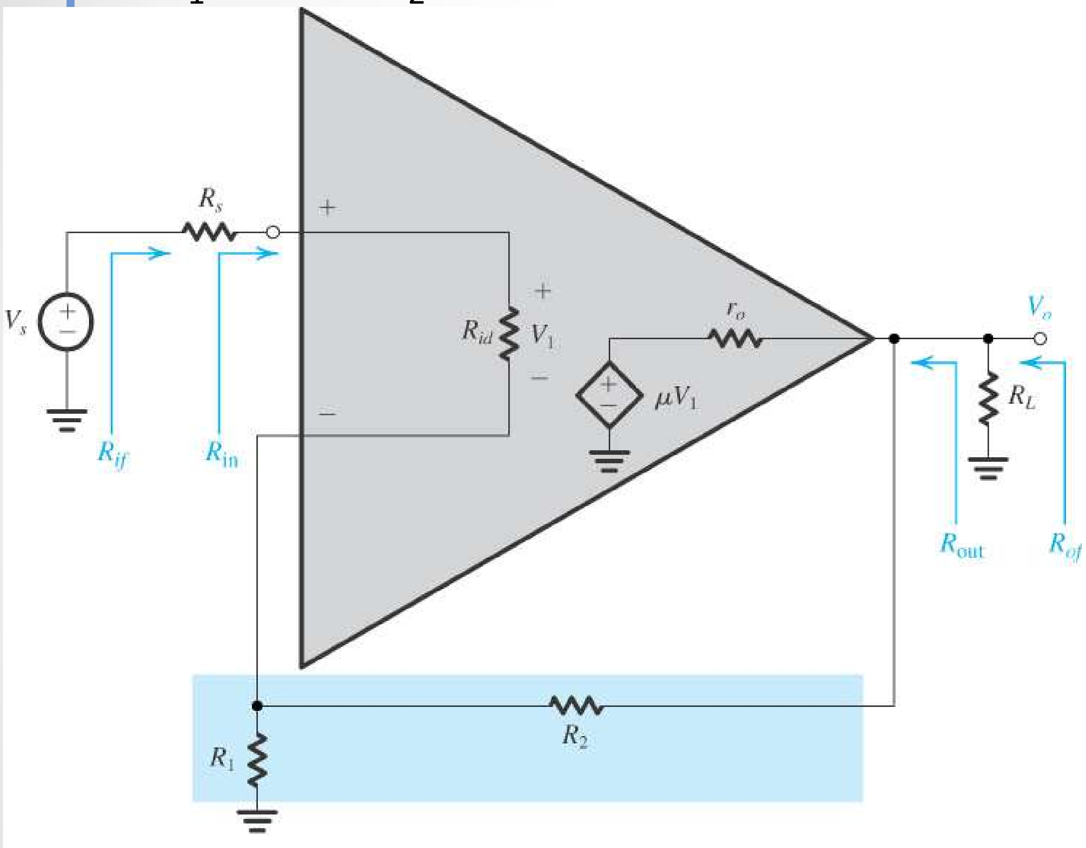
$$R_{if} = R_i'(1+\beta_f A_V') \qquad R_{of} = \frac{R_o'}{(1+A_V'\beta_f)}$$

$$\omega_{Hf} = (1+\beta_f A_V')\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{(1+\beta_f A_V')}$$

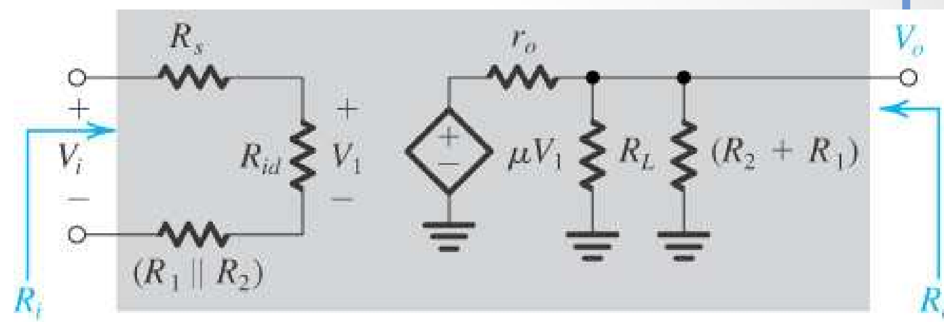
Note

- **A_v' is the modified voltage gain including the effects of h_{11} , h_{22} , R_S and R_L.**
- **R_i', R_o' are the modified input and output resistances including the effects of h_{11} , h_{22} , R_S and R_L.**

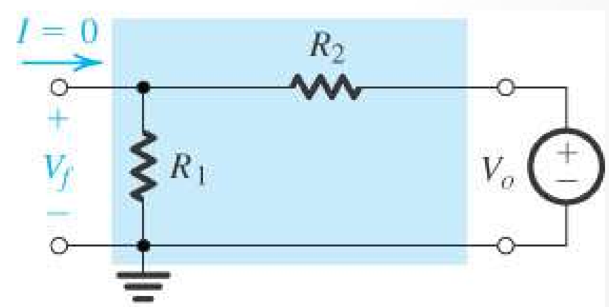
Example: Find expression for A, β, the closed-loop gain Vo/Vs, the input resistance Rin, and the output resistance Rout. Given μ = 104, R_{id} =100 kΩ, Ro = 1 kΩ, R_L = 2 kΩ, R_1 = 1 kΩ, R_2 = 1MΩ and Rs = 10 kΩ.



(a)

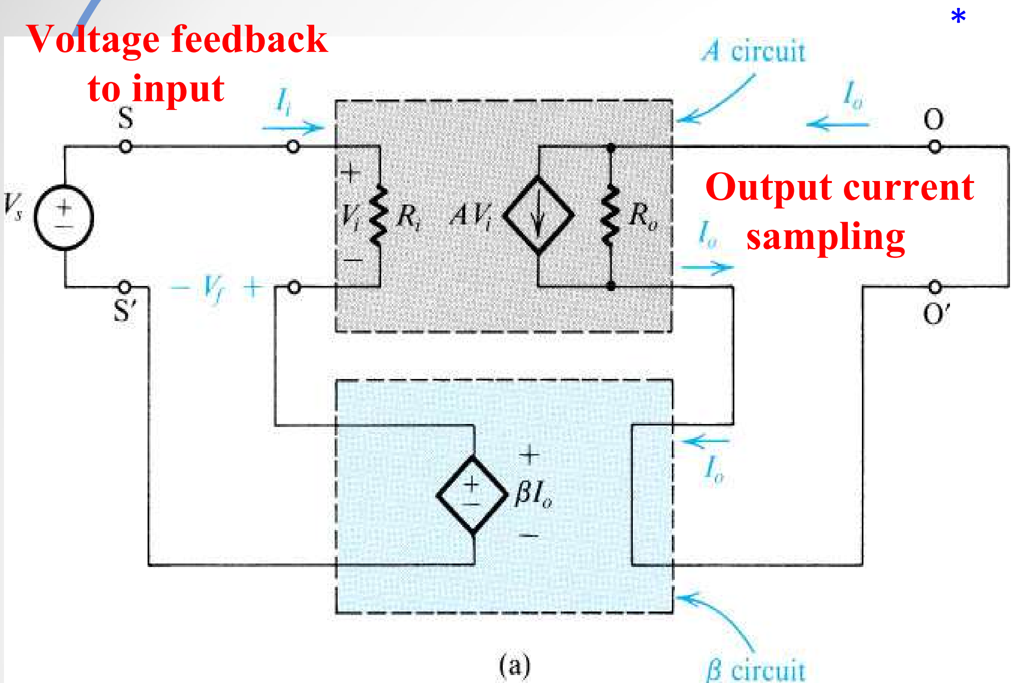


(b)

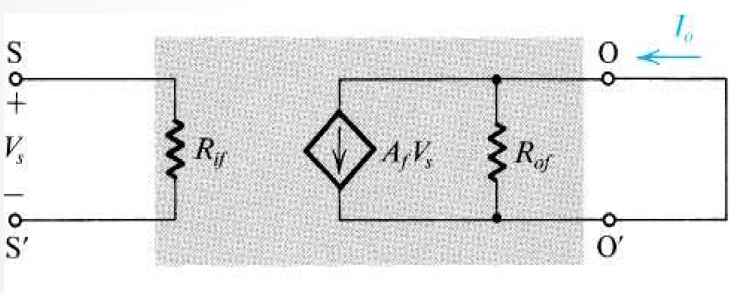


(c)

Series-Series Feedback Amplifier - Ideal Case



(a)



(b)

* Feedback circuit does not load down the basic amplifier A, i.e. doesn't change its characteristics

- Doesn't change gain A
- Doesn't change pole frequencies of basic amplifier A
- Doesn't change R_i and R_o

For this configuration, the appropriate gain is the TRANSCONDUCTANCE GAIN $A = A_{Co} = I_o/V_i$

For the feedback amplifier as a whole, feedback changes midband transconductance gain from A_{Co} to A_{Cfo}

$$A_{Cfo} = \frac{A_{Co}}{1+\beta_f A_{Co}}$$

* Feedback changes input resistance from R_i to R_{if}

$$R_{if} = R_i\left(1+\beta_f A_{Co}\right)$$

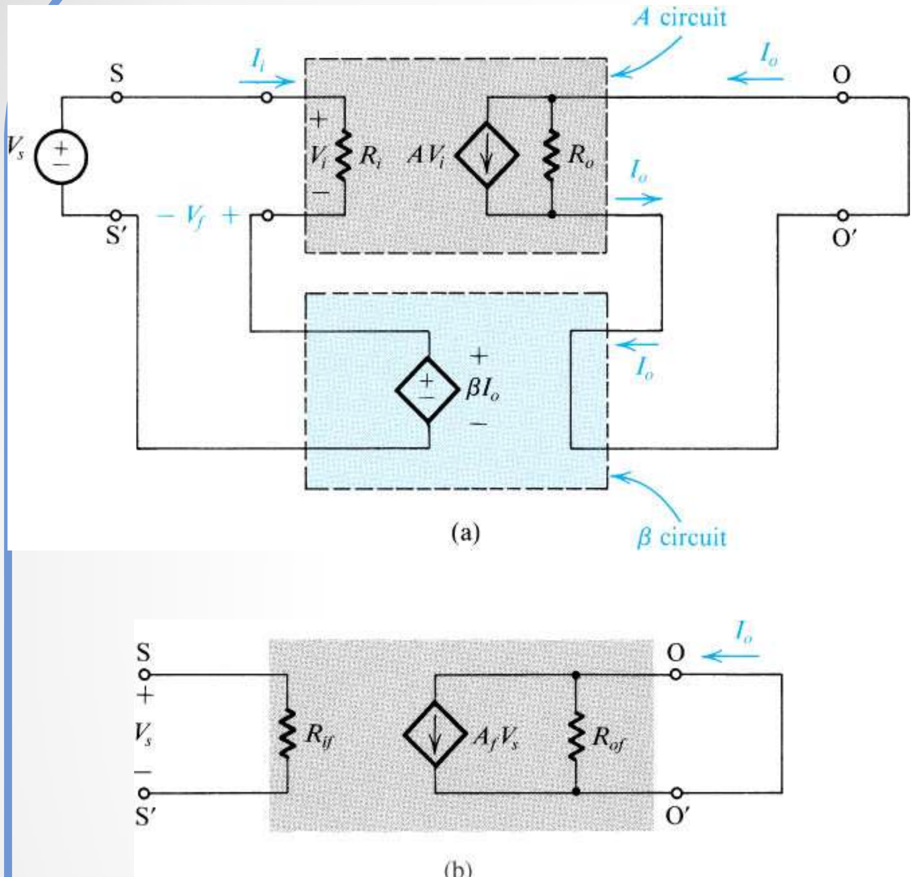
* Feedback changes output resistance from R_o to R_{of}

$$R_{of} = R_o\left(1+\beta_f A_{Co}\right)$$

* Feedback changes low and high frequency 3dB frequencies

$$\omega_{Hf} = \left(1+\beta_f A_{Co}\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A_{Co}\right)}$$

Series-Series Feedback Amplifier - Ideal Case



The series–series feedback amplifier: (a) ideal structure and (b) equivalent circuit.

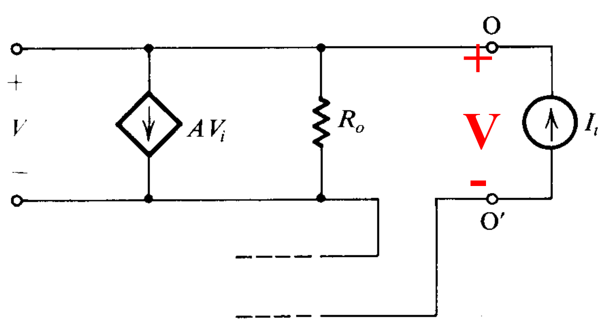


Fig. 8.14 Measuring the output resistance R_{of} of the series–series feedback amplifier.

Gain (Transconductance Gain)

$$A_{Cfo}=\frac{I_o}{V_s}=\frac{A_{Co}V_i}{V_i+V_f}=\frac{A_{Co}}{1+\frac{V_f}{V_i}}=\frac{A_{Co}}{1+\frac{\beta_f I_o}{V_i}}=\frac{A_{Co}}{1+\beta_f A_{Co}}$$

Input Resistance

$$R_{if}=\frac{V_s}{I_i}=\frac{V_i+V_f}{I_i}=\frac{V_i+\beta_f I_o}{\left(V_i/R_i\right)}=R_i\left(1+\beta_f A_{Co}\right)$$

Output Resistance

$$R_{of}=\frac{V}{I_t}$$

$$But\ \ V_s=0\ \ so\ \ V_i=-V_f$$

$$and\ \ V_f=\beta_f I_o=\beta_f I_t\ \ so\ \ V_i=-\beta_f I_t$$

$$V=\left(I_t-A_{Co}V_i\right)R_o=\left(I_t-A_{Co}\left(-\beta_f I_t\right)\right)R_o$$

$$=I_t\left(1+\beta_f A_{Co}\right)R_o$$

$$so\ \ R_{of}=\frac{V}{I_t}=R_o\left(1+\beta_f A_{Co}\right)$$

Equivalent Network for Feedback Network

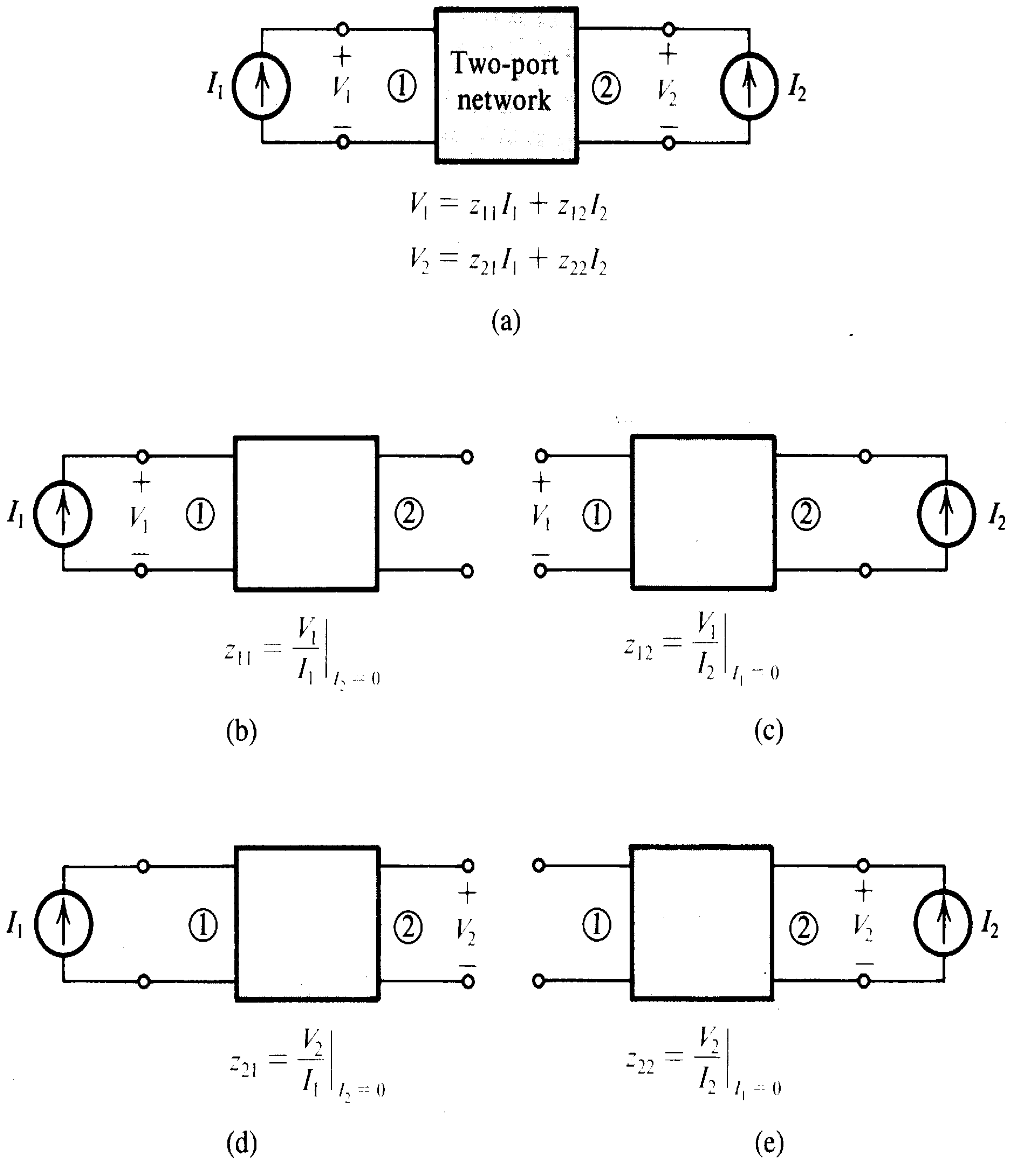
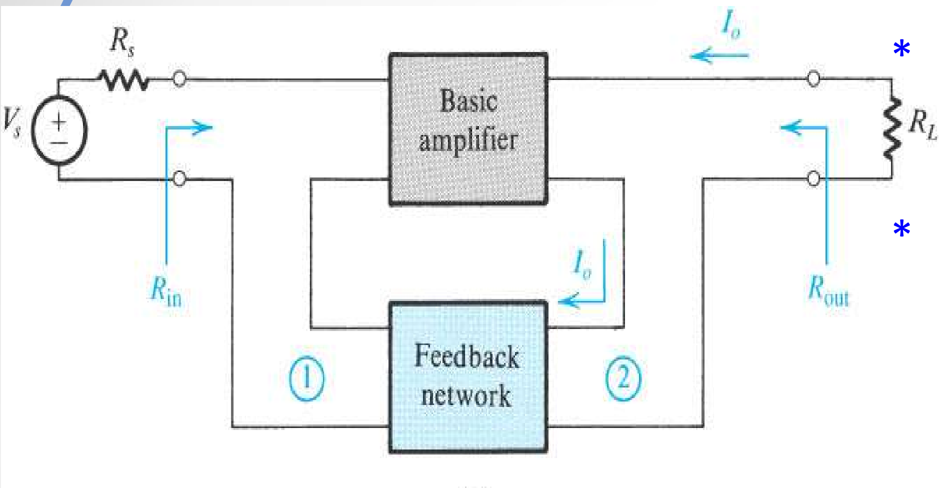


Fig. B.3 Definition and conceptual measurement circuits for z parameters.

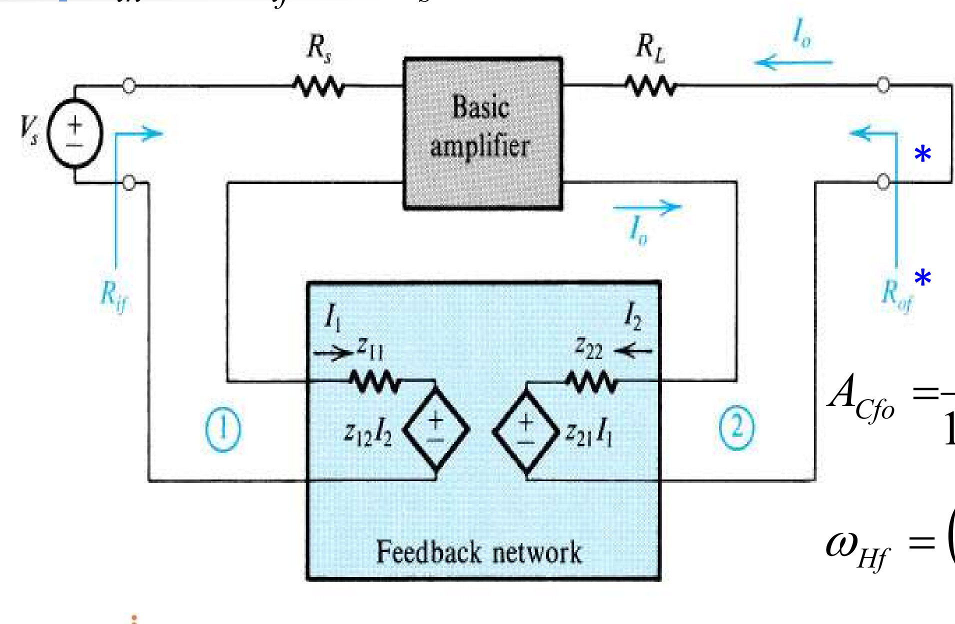
- * **Feedback network is a two port network (input and output ports)**
- * **Can represent with Z-parameter network (This is the best for this feedback amplifier configuration)**
- * **Z-parameter equivalent network has FOUR parameters**
- * **Z-parameters relate input and output currents and voltages**
- * **Two parameters chosen as independent variables. For Z-parameter network, these are input and output currents I_1 and I_2**
- * **Two equations relate other two quantities (input and output voltages V_1 and V_2) to these independent variables**
- * **Knowing I_1 and I_2, can calculate V_1 and V_2 if you know the Z-parameter values**
- * **Z-parameters have units of ohms !**

Series-Series Feedback Amplifier - Practical Case



(a)

$$R_{in} = R_{if} - R_s \quad \text{and} \quad R_{out} = R_{of} - R_L$$



(b)

- Feedback network consists of a set of resistors
- These resistors have loading effects on the basic amplifier, i.e they change its characteristics, such as the gain
- Can use z-parameter equivalent circuit for feedback network
  - Feedback factor β_f given by z_{12} since

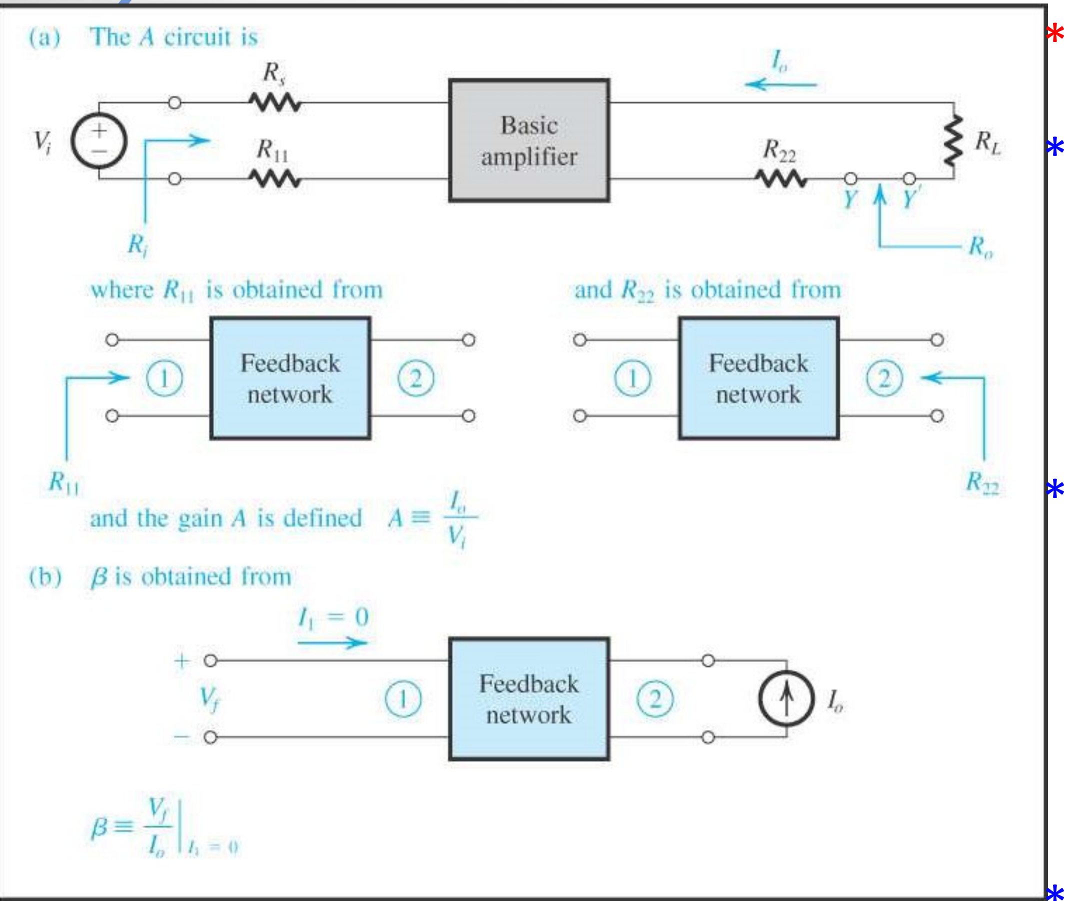
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_f}{I_o} = \beta_f$$

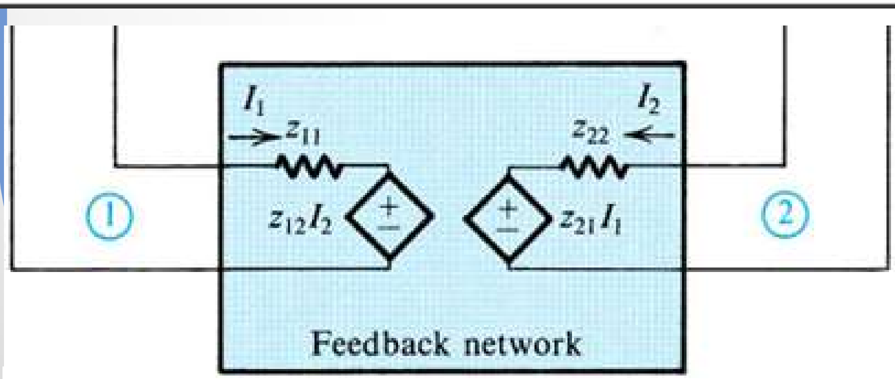
  - Feedforward factor given by z_{21} (neglected)
  - z_{22} gives feedback network loading on output
  - z_{11} gives feedback network loading on input
- Can incorporate loading effects in a modified basic amplifier. Gain A_{Co} becomes a new, modified gain A_{Co}'.
- Can then use analysis from ideal case

$$A_{Cfo} = \frac{A_{Co}'}{1+\beta_f A_{Co}'} \qquad R_{if} = R_i'(1+\beta_f A_{Co}') \qquad R_{of} = R_o'(1+\beta_f A_{Co}')$$

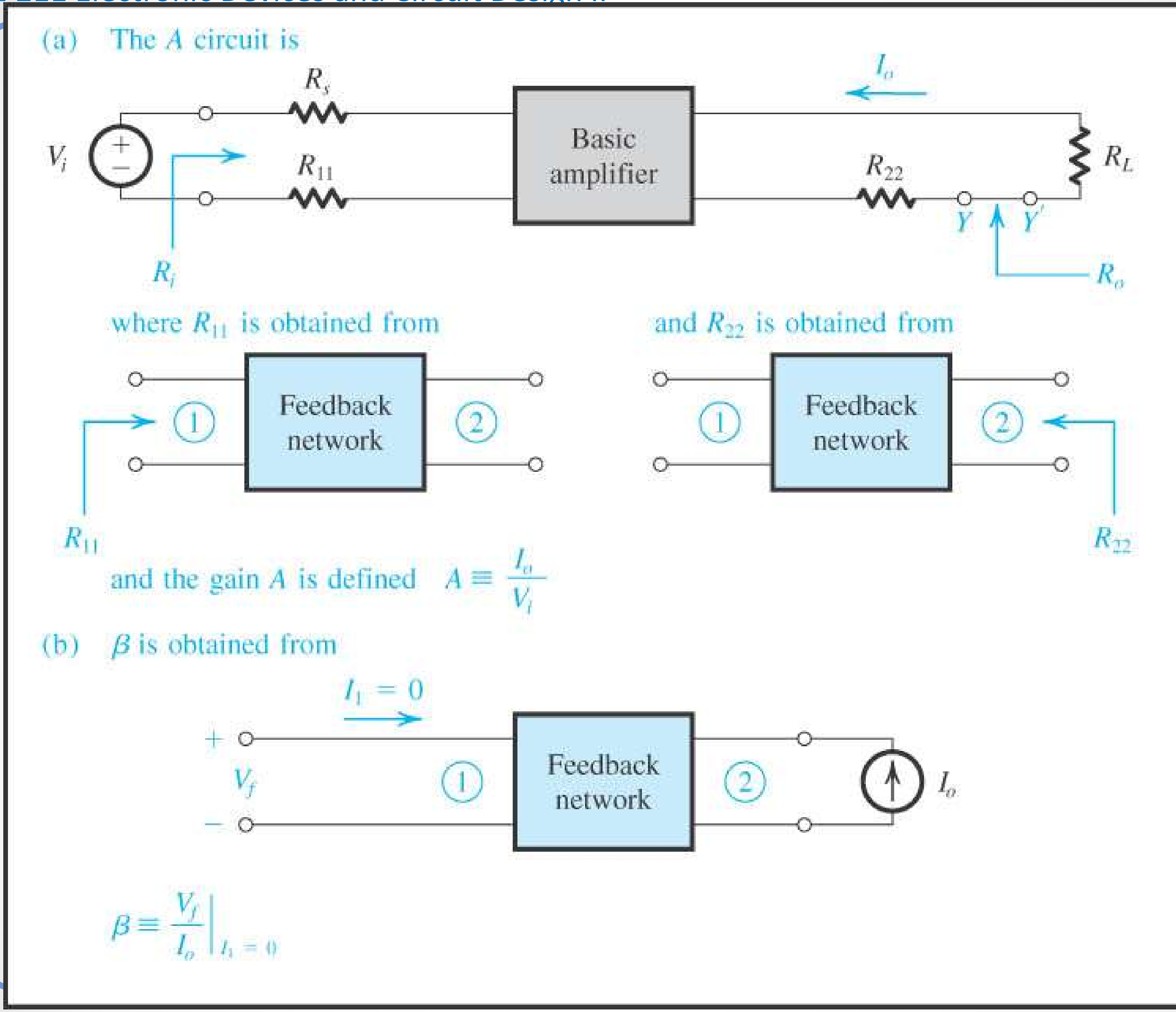
$$\omega_{Hf} = (1+\beta_f A_{Co}')\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{(1+\beta_f A_{Co}')}$$

Series-Series Feedback Amplifier - Practical Case

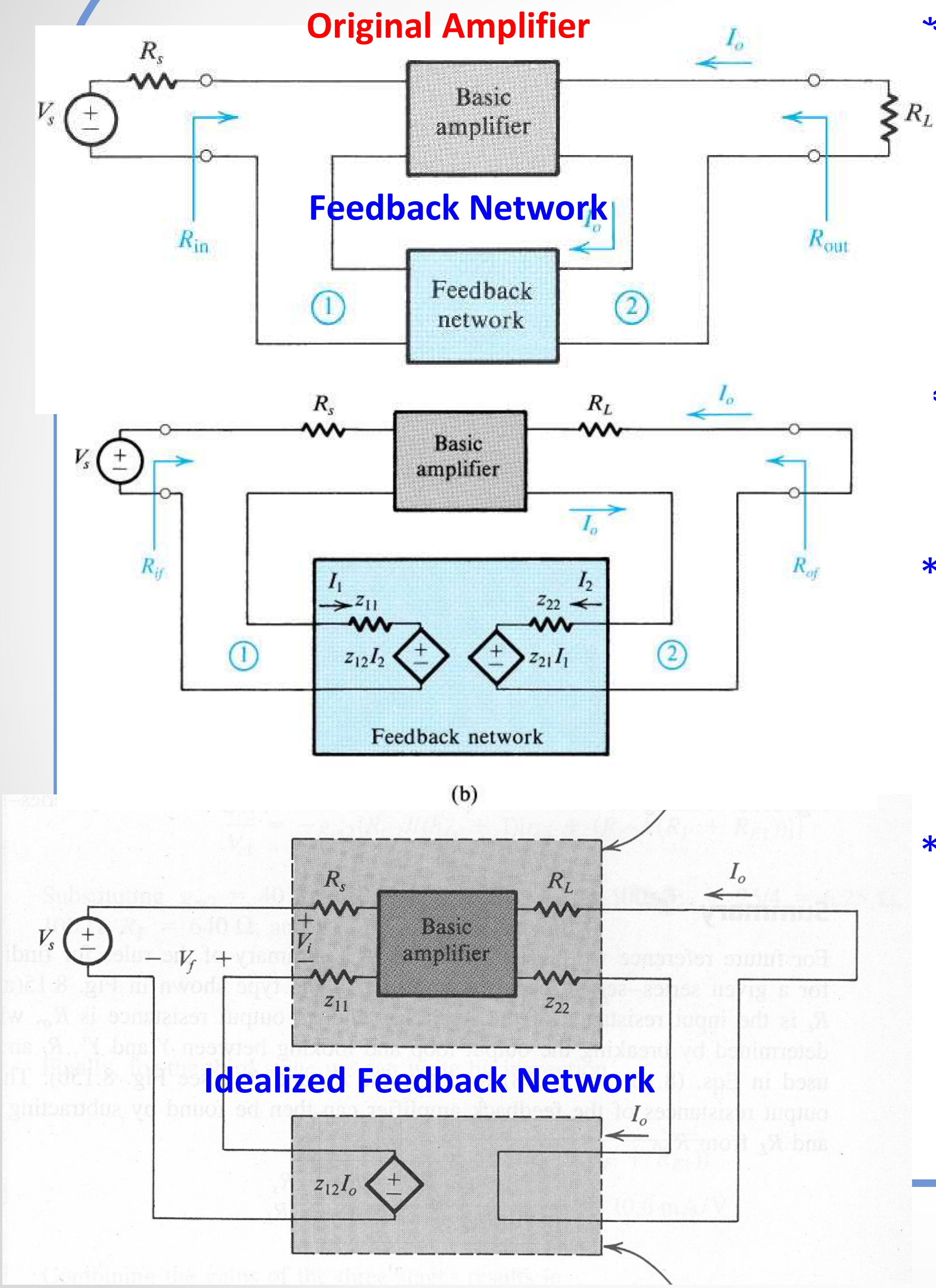




* How do we determine the z-parameters for the feedback network?
* For the input loading term z_{11}
  - We turn off the feedback signal by setting $I_o = 0$ ($I_2 = 0$).
  - We then evaluate the resistance seen looking into port 1 of the feedback network ($R_{11} = z_{11}$).
* For the output loading term z_{22}
  - We open circuit the connection to the input so $I_1 = 0$.
  - We find the resistance seen looking into port 2 of the feedback network ($R_{22} = z_{22}$).
* To obtain the feedback factor β_f (also called z_{12})
  - We apply a test signal I_o' to port 2 of the feedback network and evaluate the feedback voltage V_f (also called V_1 here) for $I_1 = 0$.
  - Find β_f from $\beta_f = V_f/I_o'$


(a) The A circuit is
R_s
I_o
V_i
Basic amplifier
R_{11}
R_{22}
R_L
Y
Y'
R_i
R_o
where R_{11} is obtained from
and R_{22} is obtained from
Feedback network
1
2
R_{11}
R_{22}
and the gain A is defined $A \equiv \frac{I_o}{V_i}$
(b) β is obtained from
$I_1 = 0$
+
V_f
−
Feedback network
1
2
I_o
$\beta \equiv \left. \frac{V_f}{I_o} \right|_{I_1 = 0}$

Series-Series Feedback Amplifier - Practical Case



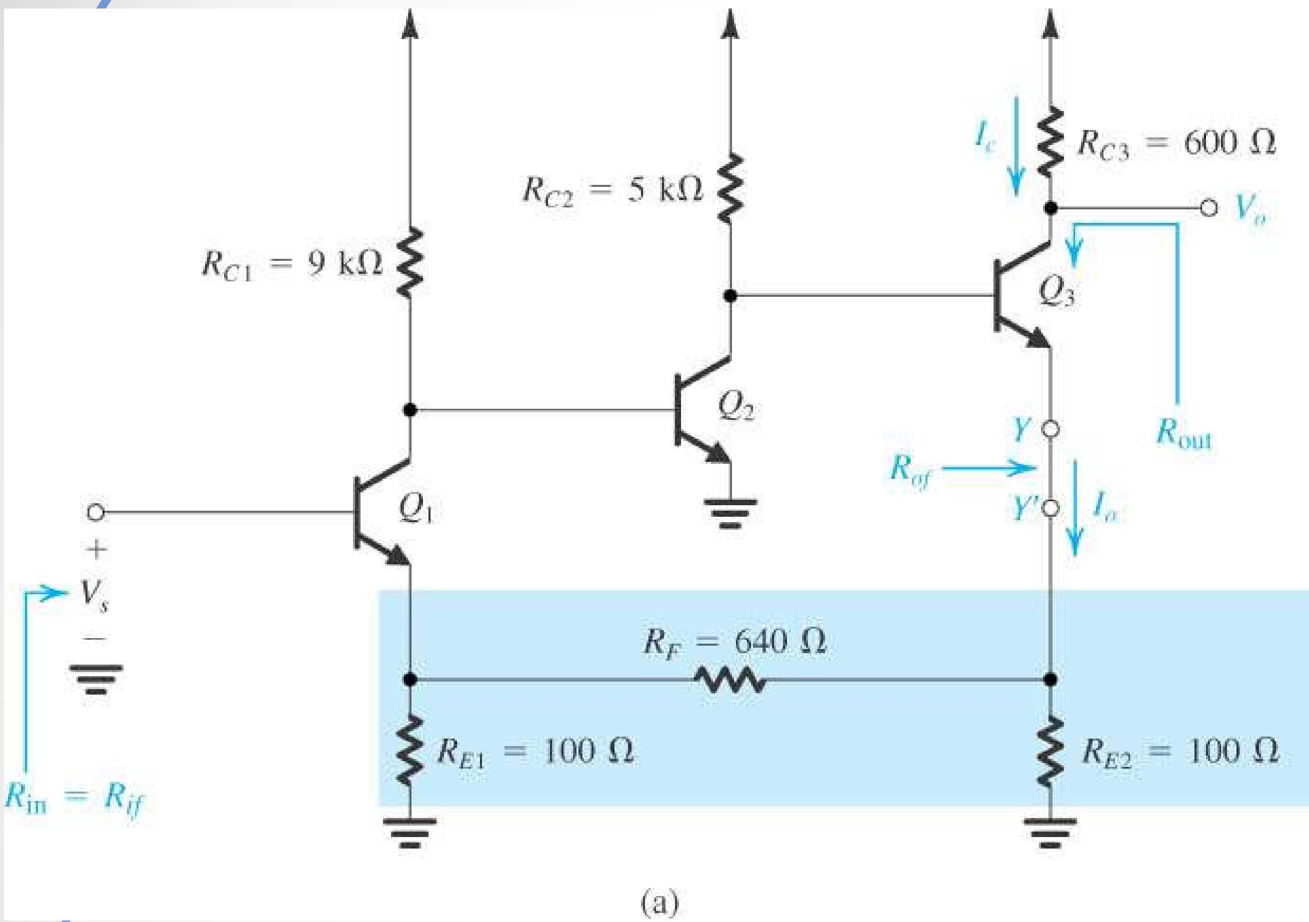
- **Modified basic amplifier (including loading effects of feedback network)**
  - Including z_{11} at input
  - Including z_{22} at output
  - Including loading effects of source resistance
  - Including load effects of load resistance
- **Now have an idealized feedback network, i.e. produces feedback effect, but without loading effects**
- **Can now use feedback amplifier equations derived**

$$A_{Cfo} = \frac{A_{Co}'}{1+\beta_f A_{Co}'} \qquad R_{if} = R_i'\left(1+\beta_f A_{Co}'\right) \qquad R_{of} = R_o'\left(1+\beta_f A_{Co}'\right)$$

$$\omega_{Hf} = \left(1+\beta_f A_{Co}'\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A_{Co}'\right)}$$

- **Note**
  - A_{Co}' is the modified transconductance gain including the loading effects of z_{11} , z_{22} , R_S and R_L.
  - R_i' and R_o' are modified input and output resistances including loading effects.

Example: A Feedback Triple



(a)

- **Three stage amplifier**
- **Each stage a CE amplifier**
- **Transistor parameters Given: $\beta_1 = \beta_2 = \beta_3 = 100$, $r_{x1} = r_{x2} = r_{x3} = 0$**
- **Coupled by capacitors, dc biased separately**
- **DC analysis (given):**

$$I_{C1} = 0.60\ mA,\ g_{m1} = \frac{I_{C1}}{V_T} = 23\ mA/V,$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 4.3K$$

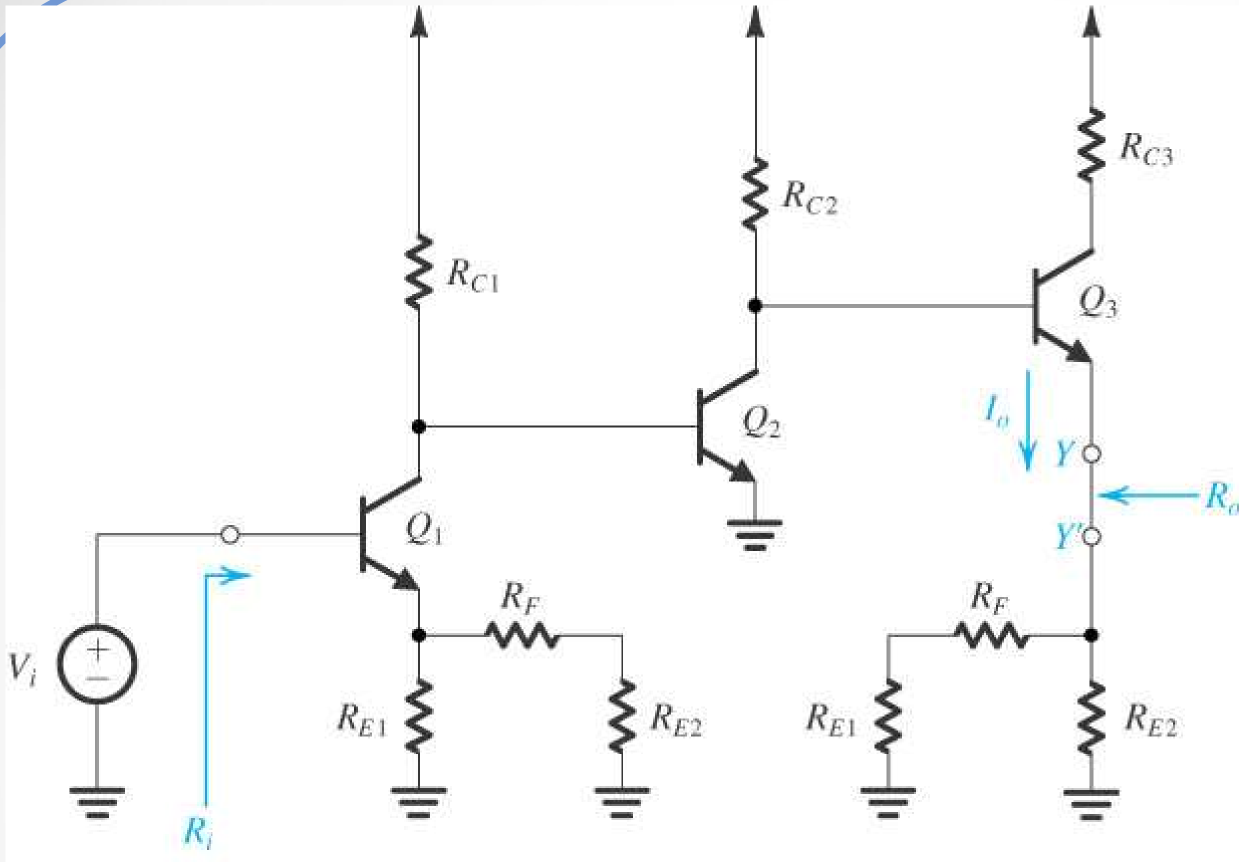
$$I_{C2} = 1.0\ mA,\ g_{m2} = \frac{I_{C2}}{V_T} = 39\ mA/V,$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = 2.6K$$

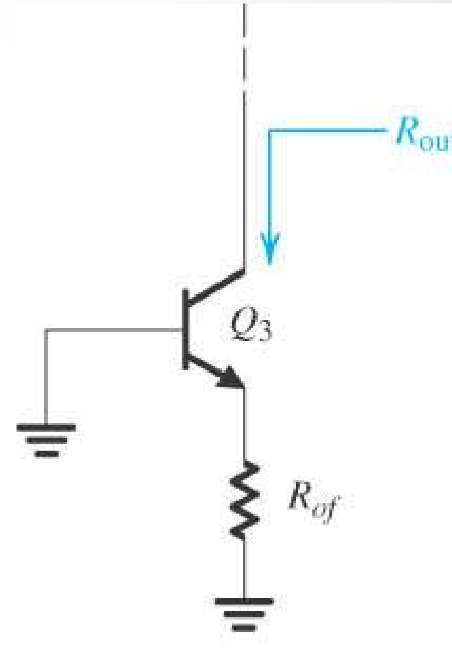
$$I_{C3} = 4.0\ mA,\ g_{m3} = \frac{I_{C3}}{V_T} = 156\ mA/V,$$

$$r_{\pi 3} = \frac{\beta_3}{g_{m3}} = 0.64K$$

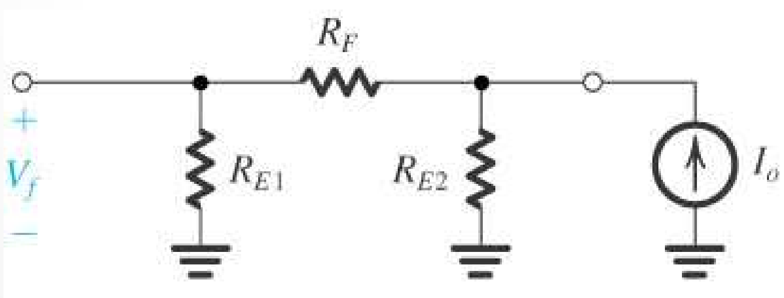
Note: Biasing resistors for each stage are not shown for simplicity in the analysis.


R_C1
R_C2
R_C3
Q_1
Q_2
Q_3
I_o
Y
Y'
R_o
R_F
R_F
R_E1
R_E2
R_E1
R_E2
V_i
R_i

(b)

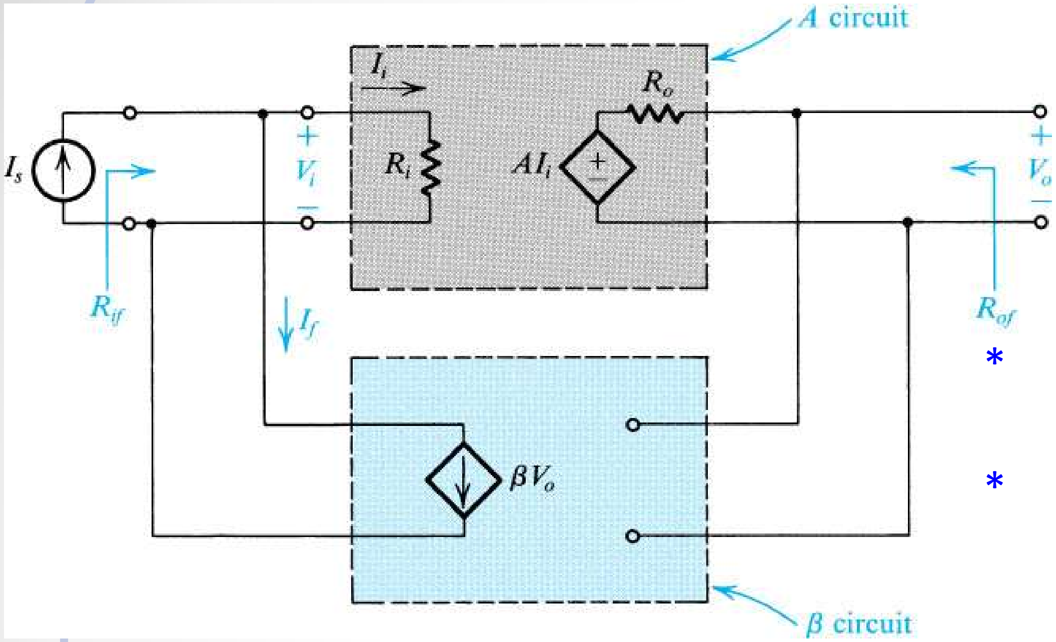

R_out
Q_3
R_of

(d)


R_F
+
V_f
−
R_E1
R_E2
I_o

(c)

Shunt-Shunt Feedback Amplifier - Ideal Case



* Feedback circuit does not load down the basic amplifier A, i.e. doesn't change its characteristics
  - Doesn't change gain A
  - Doesn't change pole frequencies of basic amplifier A
  - Doesn't change R_i and R_o
* For this configuration, the appropriate gain is the TRANSRESISTANCE GAIN $A = A_{Ro} = V_o/I_i$
* For the feedback amplifier as a whole, feedback changes midband transresistance gain from A_{Ro} to A_{Rfo}

$$A_{Rfo} = \frac{A_{Ro}}{1+\beta_f A_{Ro}}$$

* Feedback changes input resistance from R_i to R_{if}

$$R_{if} = \frac{R_i}{\left(1+\beta_f A_{Ro}\right)}$$

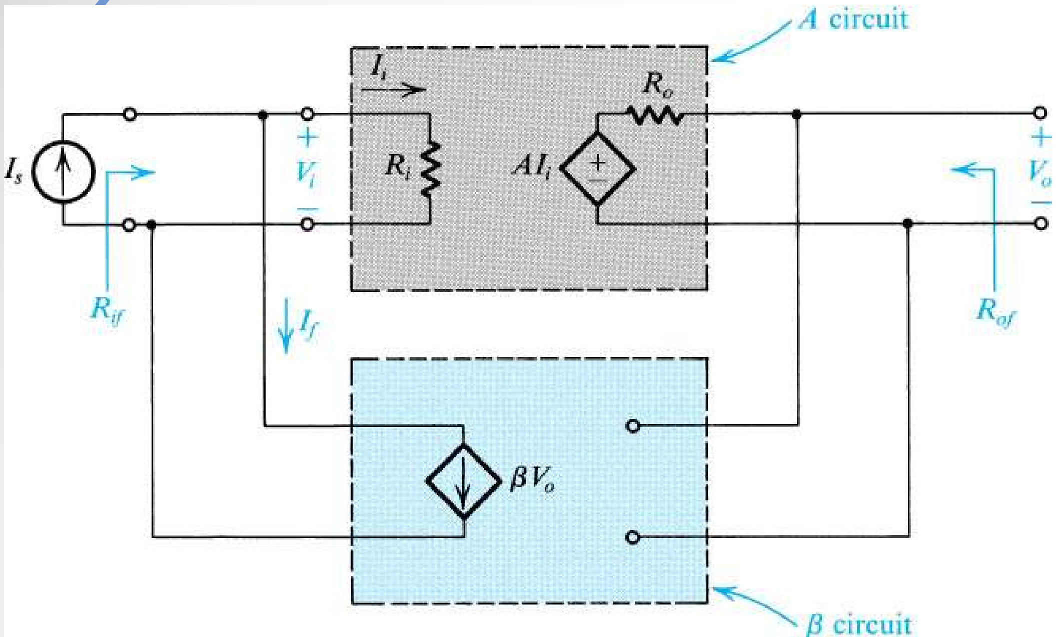
* Feedback changes output resistance from R_o to R_{of}

$$R_{of} = \frac{R_o}{\left(1+\beta_f A_{Ro}\right)}$$

* Feedback changes low and high frequency 3dB frequencies

$$\omega_{Hf} = \left(1+\beta_f A_{Ro}\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A_{Ro}\right)}$$

Shunt-Shunt Feedback Amplifier - Ideal Case



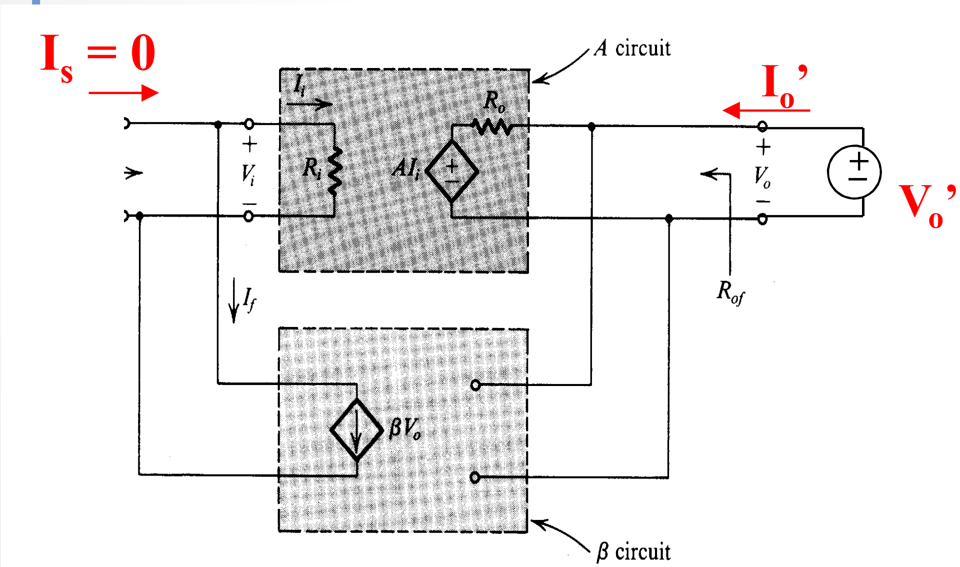


Fig. 8.18 Ideal structure for the shunt–shunt feedback amplifier.

Gain

$$A_{Rfo} = \frac{V_o}{I_s} = \frac{A_{Ro} I_i}{I_i + I_f} = \frac{A_{Ro}}{1 + \frac{I_f}{I_i}} = \frac{A_{Ro}}{1 + \frac{\beta_f V_o}{I_i}} = \frac{A_{Ro}}{1 + \beta_f A_{Ro}}$$

Input Resistance

$$R_{if} = \frac{V_s}{I_s} = \frac{V_s}{I_i + I_f} = \frac{V_s}{I_i + \beta_f V_o}$$

$$= \frac{V_s}{I_i \left(1 + \beta_f \frac{V_o}{I_i}\right)} = \frac{R_i}{(1 + \beta_f A_{Ro})}$$

Output Resistance

$$R_{of} = \frac{V_o'}{I_o'} = \frac{I_o' R_o + A_{Ro} I_i}{I_o'} = R_o + A_{Ro} \frac{I_i}{I_o'}$$

$$But \quad I_s = 0 \quad so \quad I_i = -I_f$$

$$and \quad I_f = \beta_f V_o' \quad so \quad I_i = -\beta_f V_o'$$

$$\frac{I_i}{I_o'} = \frac{-\beta_f V_o'}{I_o'} = -\beta_f R_{of}$$

$$R_{of} = R_o + A_{Ro}\left(-\beta_f R_{of}\right)$$

$$so \quad R_{of} = \frac{R_o}{(1 + \beta_f A_{Ro})}$$

Equivalent Network for Feedback Network

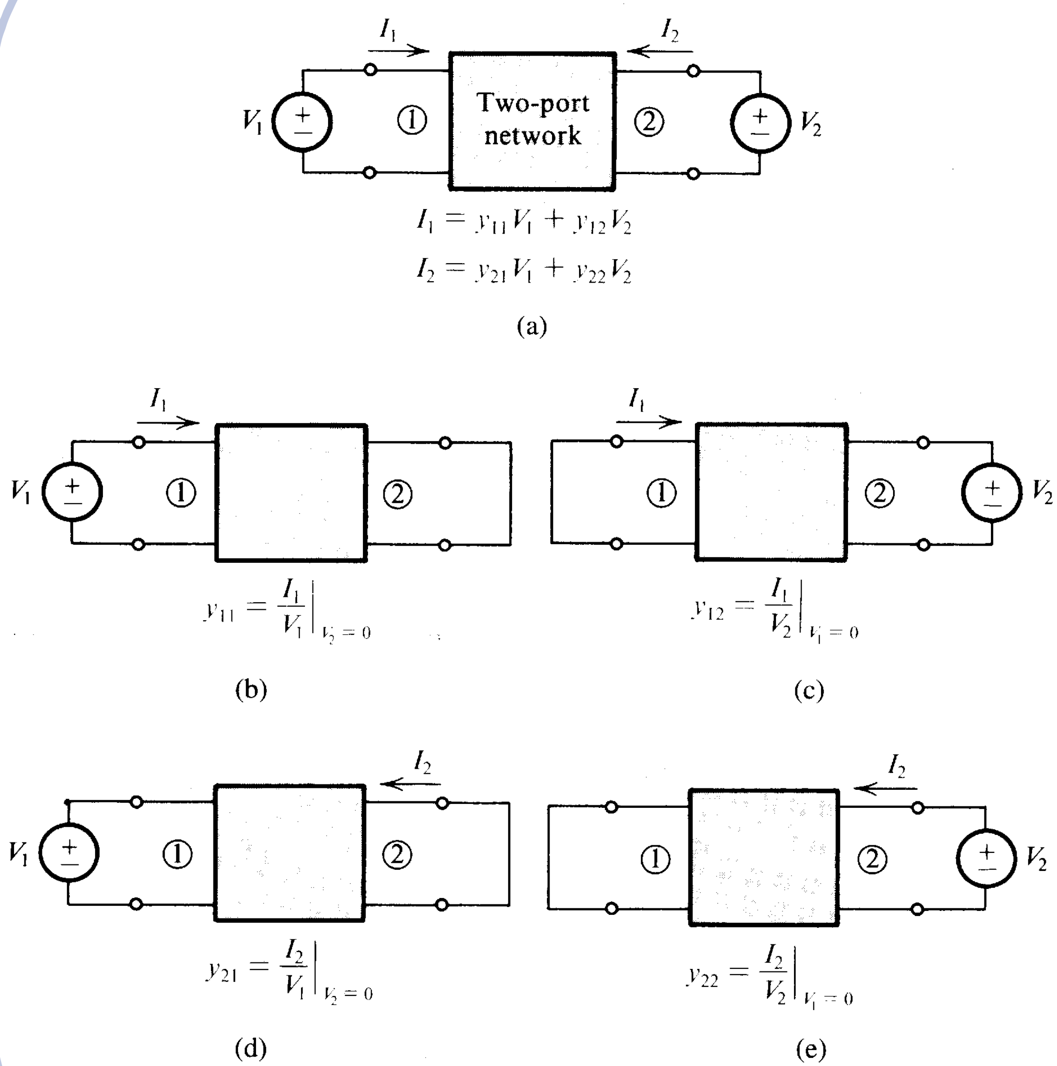
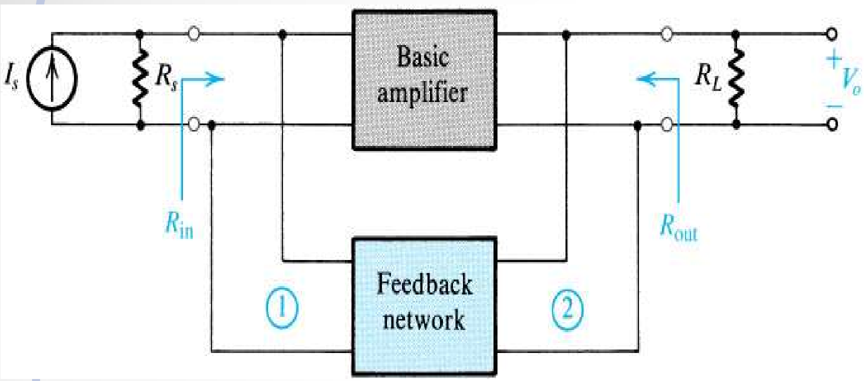


Fig. B.2 Definition and conceptual measurement circuits for y parameters.

- * Feedback network is a two port network (input and output ports)
- * Can represent with Y-parameter network (This is the best for this feedback amplifier configuration)
- * Y-parameter equivalent network has FOUR parameters
- * Y-parameters relate input and output currents and voltages
- * Two parameters chosen as independent variables. For Y-parameter network, these are input and output voltages V_1 and V_2
- * Two equations relate other two quantities (input and output currents I_1 and I_2) to these independent variables
- * Knowing V_1 and V_2, can calculate I_1 and I_2 if you know the Y-parameter values
- * Y-parameters have units of conductance (1/ohms=siemens) !

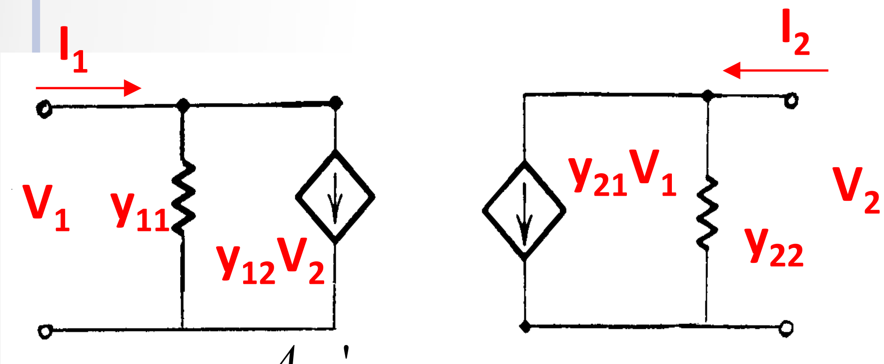
Shunt-Shunt Feedback Amplifier - Practical Case



- * Feedback network consists of a set of resistors
- * These resistors have loading effects on the basic amplifier, i.e they change its characteristics, such as the gain
- * Can use y-parameter equivalent circuit for feedback network
  - ↘ Feedback factor β_f given by y_{12} since

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{I_f}{V_o} = \beta_f$$

  - ↘ Feedforward factor given by y_{21} (neglected)
  - ↘ y_{22} gives feedback network loading on output
  - ↘ y_{11} gives feedback network loading on input
- * Can incorporate loading effects in a modified basic amplifier. Gain A_{Ro} becomes a new, modified gain A_{Ro}'.
- * Can then use analysis from ideal case



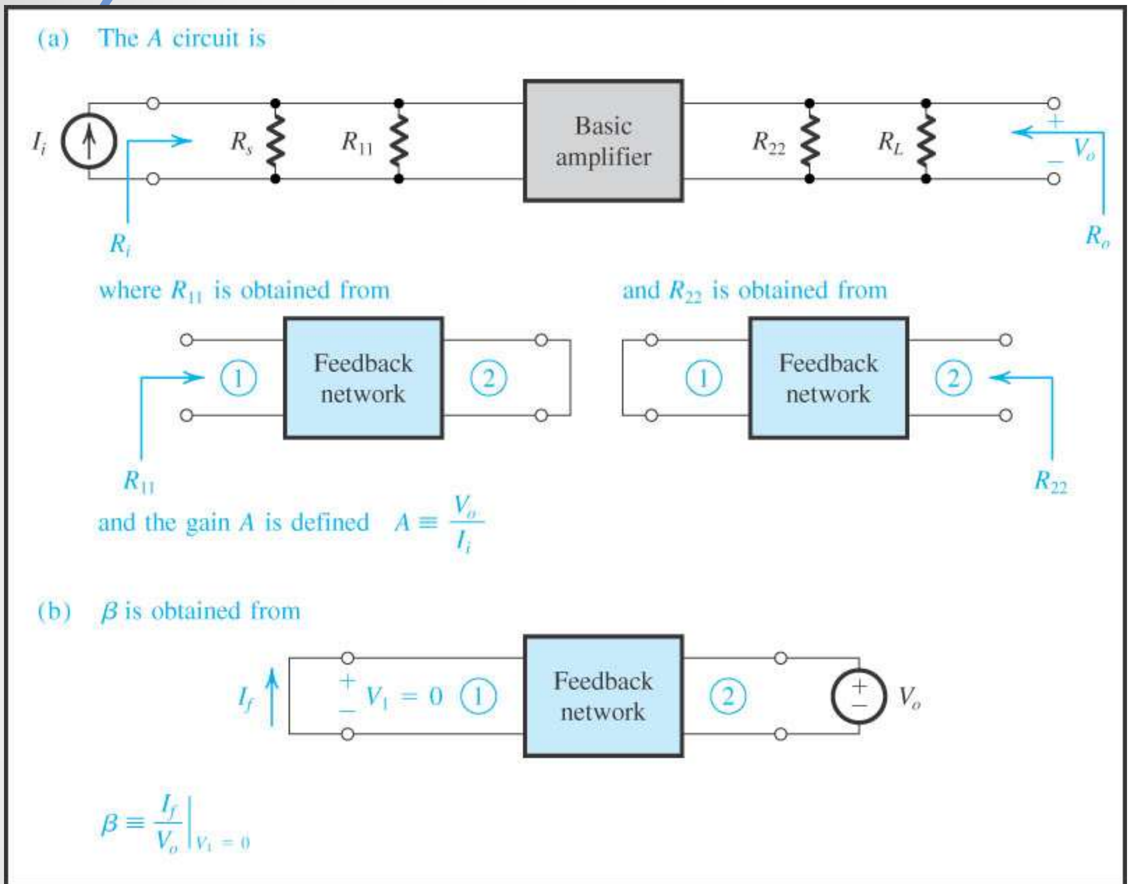
$$A_{Rfo} = \frac{A_{Ro}'}{1+\beta_f A_{Ro}'}$$

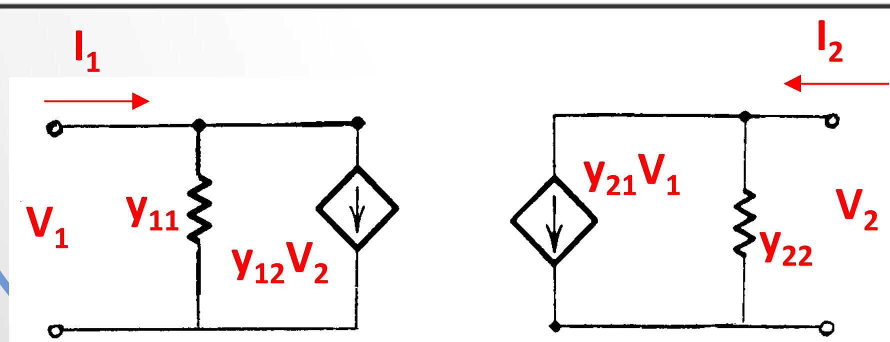
$$R_{if} = \frac{R_i}{(1+\beta_f A_{Ro}')} \qquad R_{of} = \frac{R_o}{(1+\beta_f A_{Ro}')}$$

$$\omega_{Hf} = (1+\beta_f A_{Ro}')\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{(1+\beta_f A_{Ro}')}$$

$$R_{in} = 1/(\frac{1}{R_{if}} - \frac{1}{R_s}) \qquad R_{out} = 1/(\frac{1}{R_{of}} - \frac{1}{R_L})$$

Shunt-Shunt Feedback Amplifier - Practical Case





* How do we determine the y-parameters for the feedback network?

* For the input loading term y_{11}

- We turn off the feedback signal by setting $V_o = 0$ ($V_2 = 0$).
- We then evaluate the resistance seen looking into port 1 of the feedback network ($R_{11} = y_{11}$).

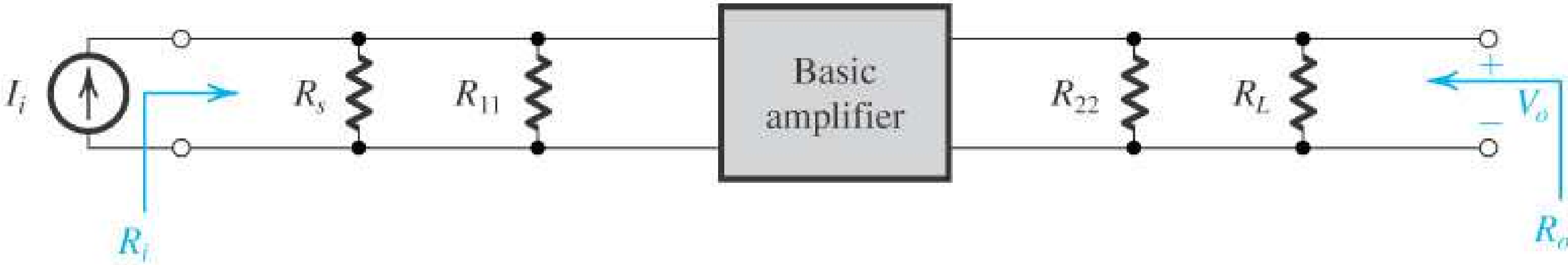
* For the output loading term y_{22}

- We short circuit the connection to the input so $V_1 = 0$.
- We find the resistance seen looking into port 2 of the feedback network.

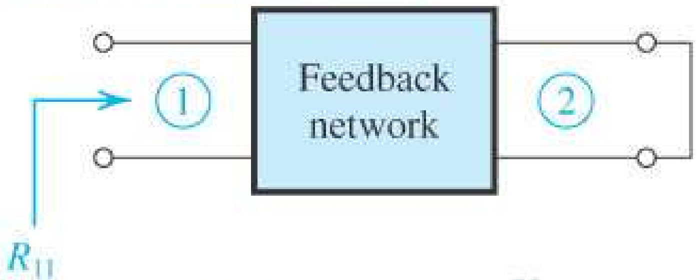
* To obtain the feedback factor β_f (also called y_{12})

- We apply a test signal V_o' to port 2 of the feedback network and evaluate the feedback current I_f (also called I_1 here) for $V_1 = 0$.
- Find β_f from $\beta_f = I_f/V_o'$

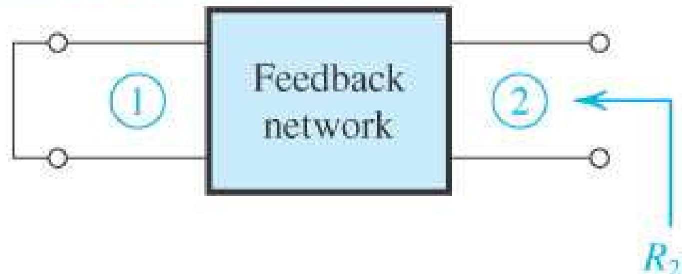
(a) The A circuit is



where R_{11} is obtained from



and R_{22} is obtained from

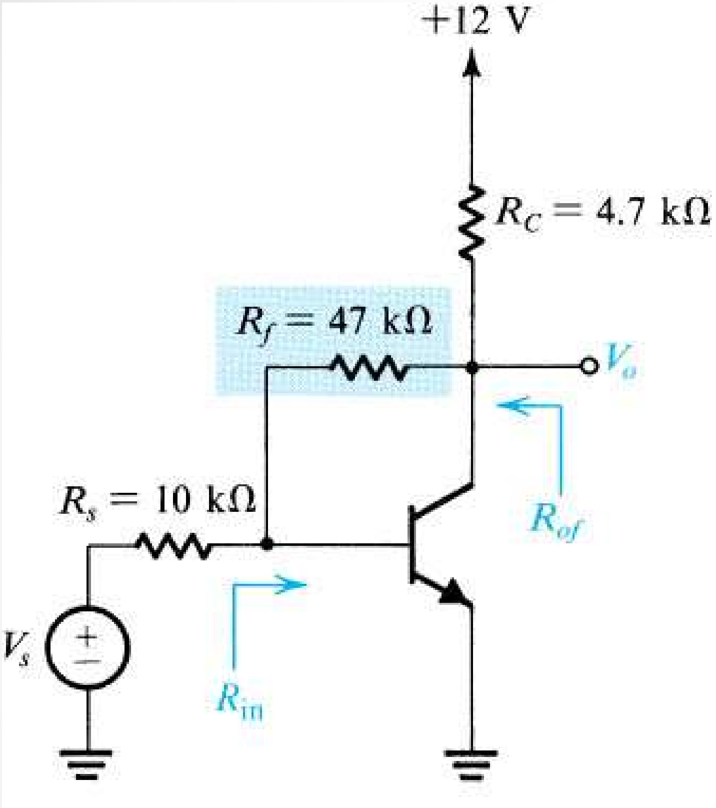


and the gain A is defined $A \equiv \dfrac{V_o}{I_i}$

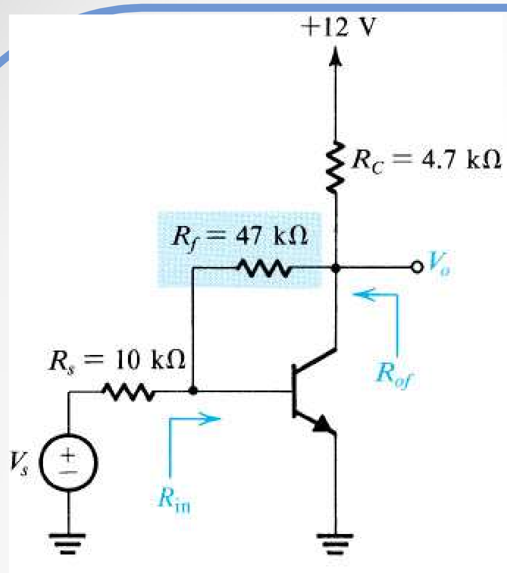
(b) β is obtained from

$$\beta \equiv \left.\frac{I_f}{V_o}\right|_{V_1 = 0}$$

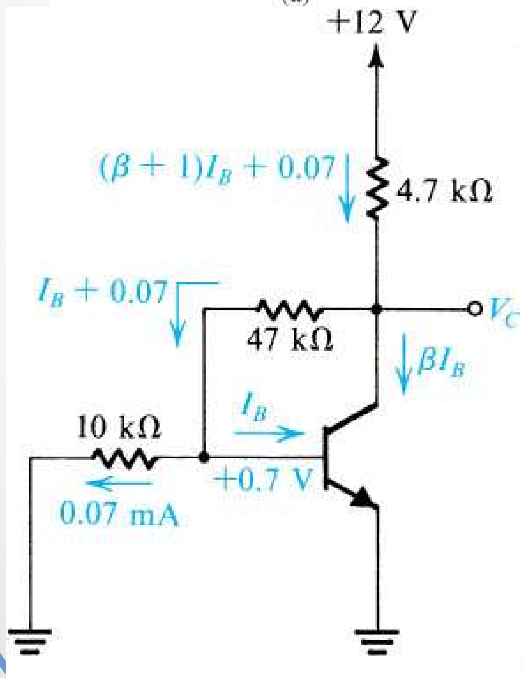
Example: Analyze the circuit below to determine the small-signal voltage gain V_o/V_s, R_{in} and R_{out}. The transistor has β = 100



(a)



(a)



(b)

- **Single stage CE amplifier**
- **Transistor parameters. Given: β =100, r_x= 0**
- **No coupling or emitter bypass capacitors**
- **DC analysis:**

$$V_{BE,active} = 0.7V$$

$$I_{10K} = \frac{0.7V}{10K} = 0.07\ mA = 70\mu A \qquad I_{47K} = I_B + 0.07\ mA$$

$$I_C = \beta I_B \qquad I_{4.7K} = I_C + (I_B + 0.07\ mA) = (\beta + 1)I_B + 0.07\ mA$$

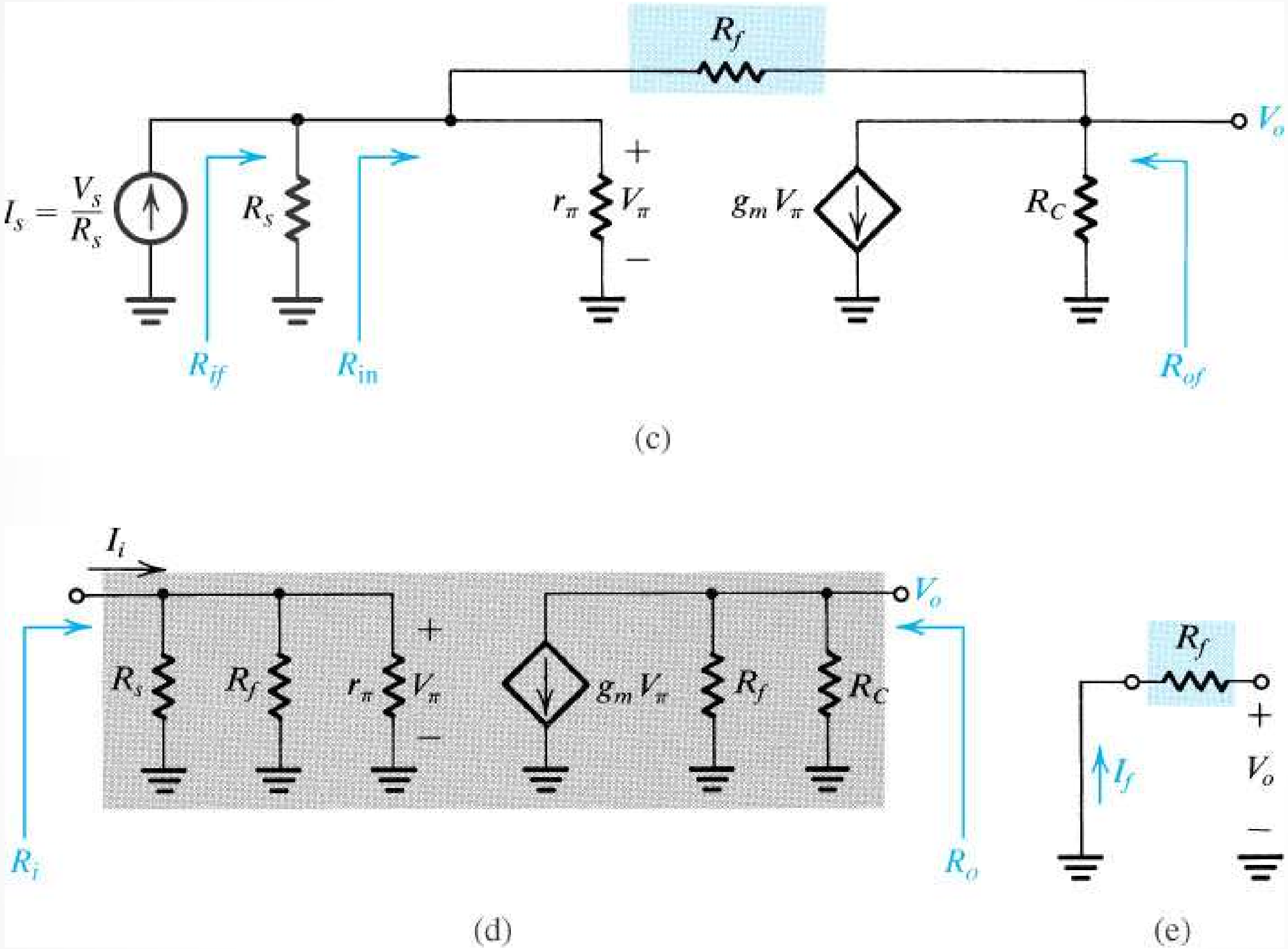
$$12V = I_{4.7K} 4.7K + (I_B + 0.07\ mA)47K + 0.7V$$

$$12V = ((\beta + 1)I_B + 0.07\ mA)4.7K + (I_B + 0.07\ mA)47K + 0.7V$$

$$12V - 0.33V - 3.3V = (101(4.7K) + 47K)I_B$$

$$I_B = \frac{8.37V}{522K} = 0.016\ mA = 16\ \mu A \qquad I_C = \beta I_B = 100(0.016\ mA) = 1.6\ mA$$

$$g_m = \frac{I_C}{V_T} = \frac{1.6\ mA}{0.0256\ V} = 63\ mA/V \qquad r_\pi = \frac{\beta}{g_m} = \frac{100}{63\ mA/V} = 1.6\ K$$



(c)

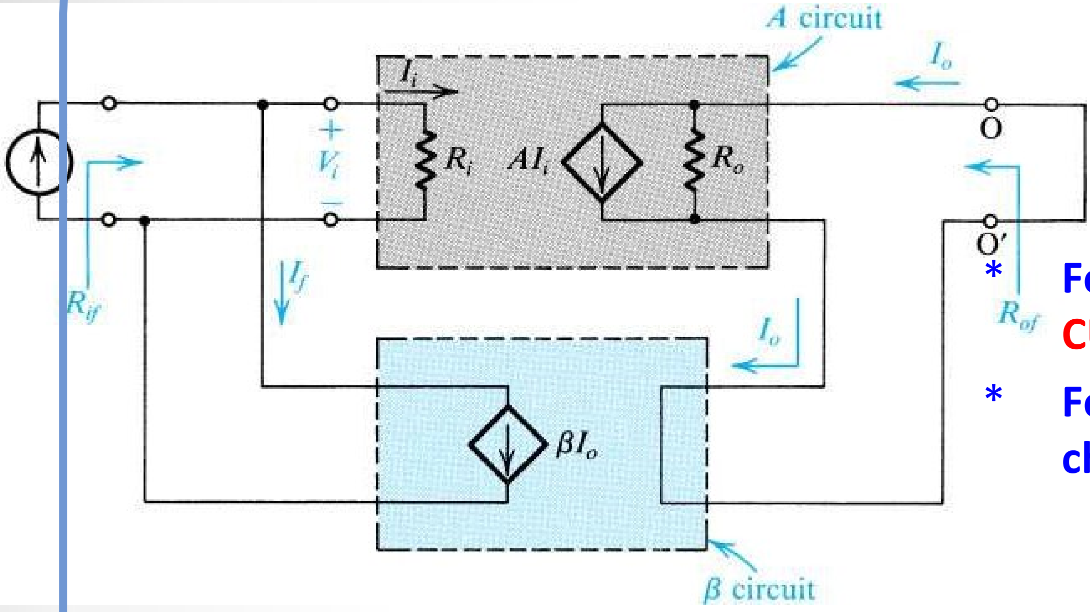
(d)

(e)

Shunt-Series Feedback Amplifier - Ideal Case

Current feedback

Current sampling



- * Feedback circuit does not load down the basic amplifier A, i.e. doesn't change its characteristics
  - Doesn't change gain A
  - Doesn't change pole frequencies of basic amplifier A
  - Doesn't change R_i and R_o
- * For this configuration, the appropriate gain is the CURRENT GAIN $A = A_{Io} = I_o/I_i$
- * For the feedback amplifier as a whole, feedback changes midband current gain from A_{Io} to A_{Ifo}

$$A_{Ifo} = \frac{A_{Io}}{1+\beta_f A_{Io}}$$

- * Feedback changes input resistance from R_i to R_{if}

$$R_{if} = \frac{R_i}{(1+\beta_f A_{Io})}$$

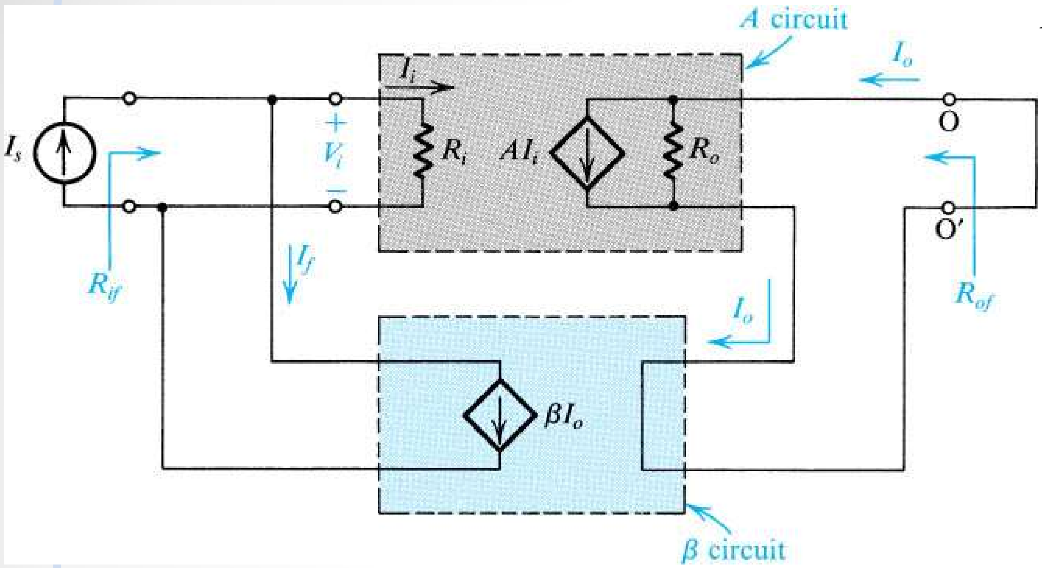
- * Feedback changes output resistance from R_o to R_{of}

$$R_{of} = R_o(1+\beta_f A_{Io})$$

- * Feedback changes low and high frequency 3dB frequencies

$$\omega_{Hf} = (1+\beta_f A_{Io})\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{(1+\beta_f A_{Io})}$$

Shunt-Series Feedback Amplifier - Ideal Case



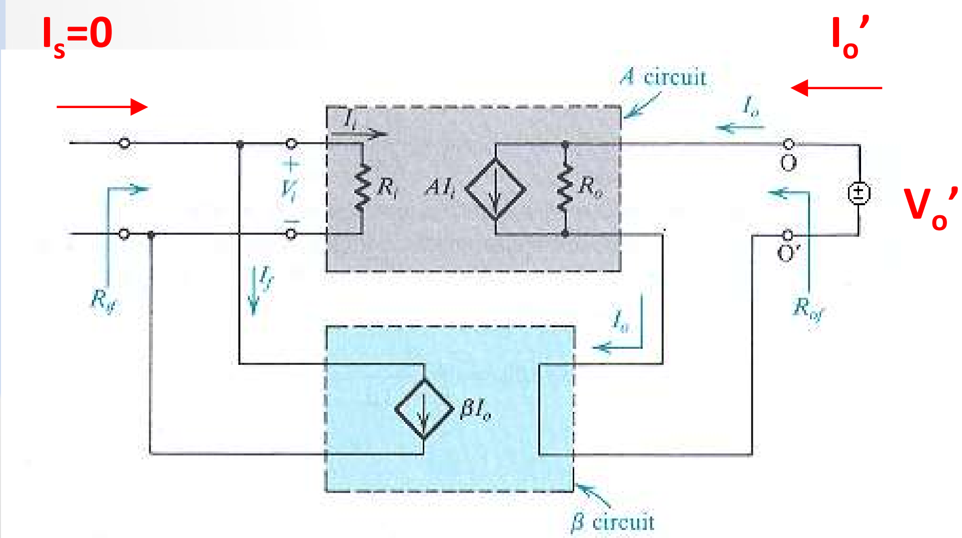


Fig. 8.22 Ideal structure for the shunt–series feedback amplifier.

Gain

$$A_{Ifo} = \frac{I_o}{I_s} = \frac{A_{Io} I_i}{I_i + I_f} = \frac{A_{Io}}{1 + \frac{I_f}{I_i}} = \frac{A_{Io}}{1 + \frac{\beta_f I_o}{I_i}} = \frac{A_{Io}}{1 + \beta_f A_{Io}}$$

Input Resistance

$$R_{if} = \frac{V_s}{I_s} = \frac{V_s}{I_i + I_f} = \frac{V_s}{I_i + \beta_f I_o}$$

$$= \frac{V_s}{I_i\left(1 + \beta_f \frac{I_o}{I_i}\right)} = \frac{R_i}{\left(1 + \beta_f A_{Io}\right)}$$

Output Resistance

$$R_{of} = \frac{V_o'}{I_o'} = \frac{R_o\left(I_o' - A_{Io} I_i\right)}{I_o'} = R_o\left(1 - A_{Io}\frac{I_i}{I_o'}\right)$$

$$But \;\; I_s = 0 \;\; so \;\; I_i = -I_f$$

$$and \;\; I_f = \beta_f I_o' \;\; so \;\; I_i = -\beta_f I_o'$$

$$\frac{I_i}{I_o'} = \frac{-\beta_f I_o'}{I_o'} = -\beta_f$$

$$R_{of} = R_o\left(1 + \beta_f A_{Io}\right)$$

Equivalent Network for Feedback Network

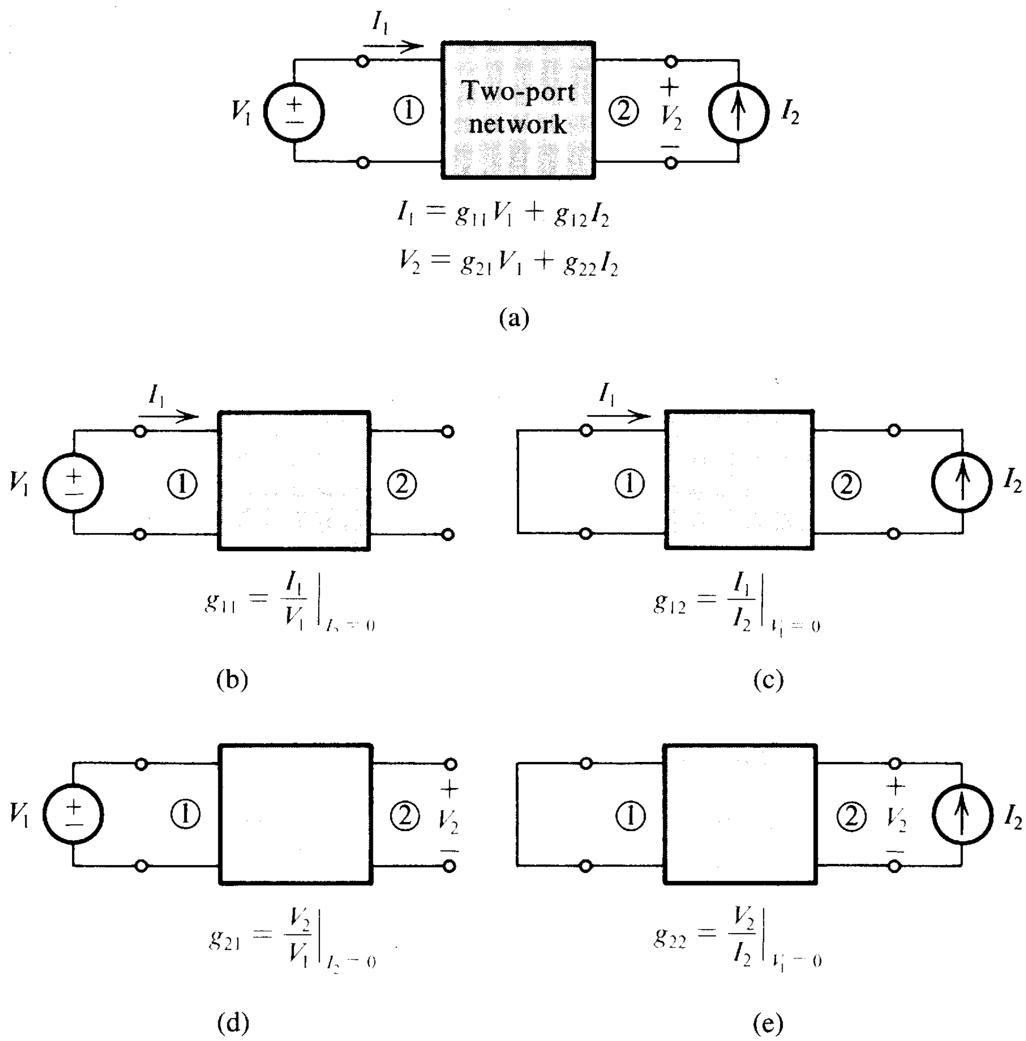
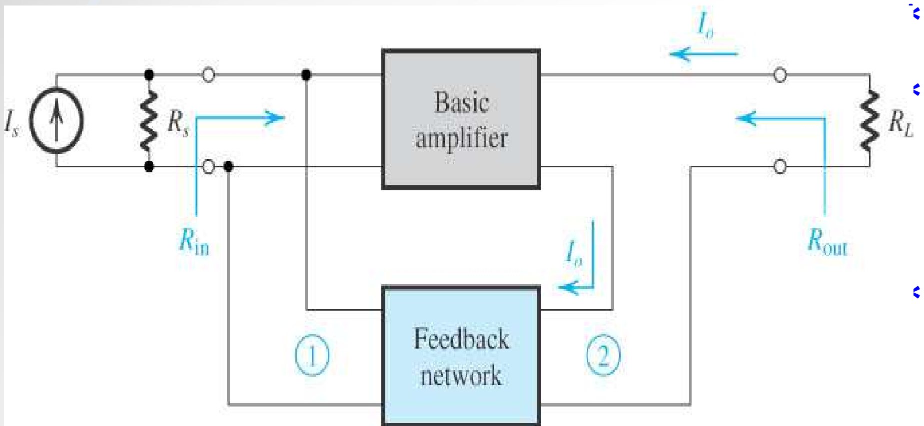
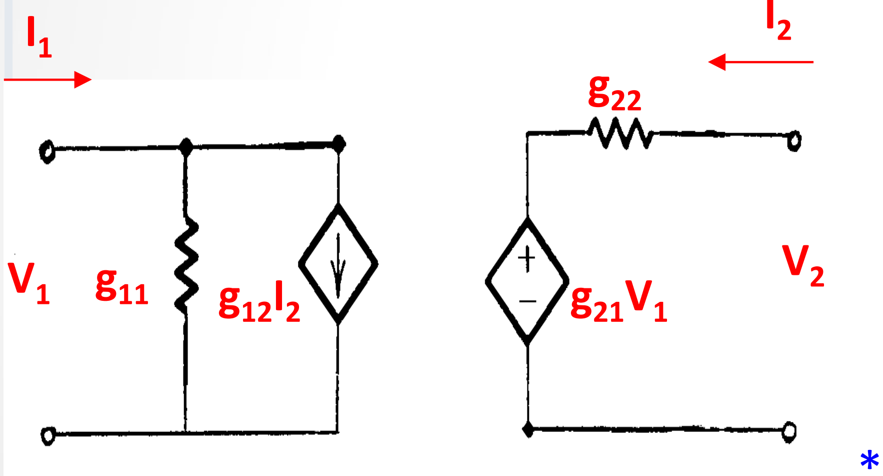


Fig. B.5 Definition and conceptual measurement circuits for g parameters.

- Feedback network is a two port network (input and output ports)
- Can represent with g-parameter network (This is the best for this feedback amplifier configuration)
- G-parameter equivalent network has FOUR parameters
- G-parameters relate input and output currents and voltages
- Two parameters chosen as independent variables. For G-parameter network, these are input voltages V_1 and the output current I_2
- Two equations relate other two quantities (input current I_1 and output V_2) to these independent variables
- Knowing V_1 and I_2, can calculate I_1 and V_2 if you know the G-parameter values
- G-parameters have various units of ohms, conductance (1/ohms=siemens) and no units !

Shunt-Series Feedback Amplifier - Practical Case





- Feedback network consists of a set of resistors
- These resistors have loading effects on the basic amplifier, i.e they change its characteristics, such as the gain
- Can use g-parameter equivalent circuit for feedback network
  - Feedback factor β_f given by g_{12} since

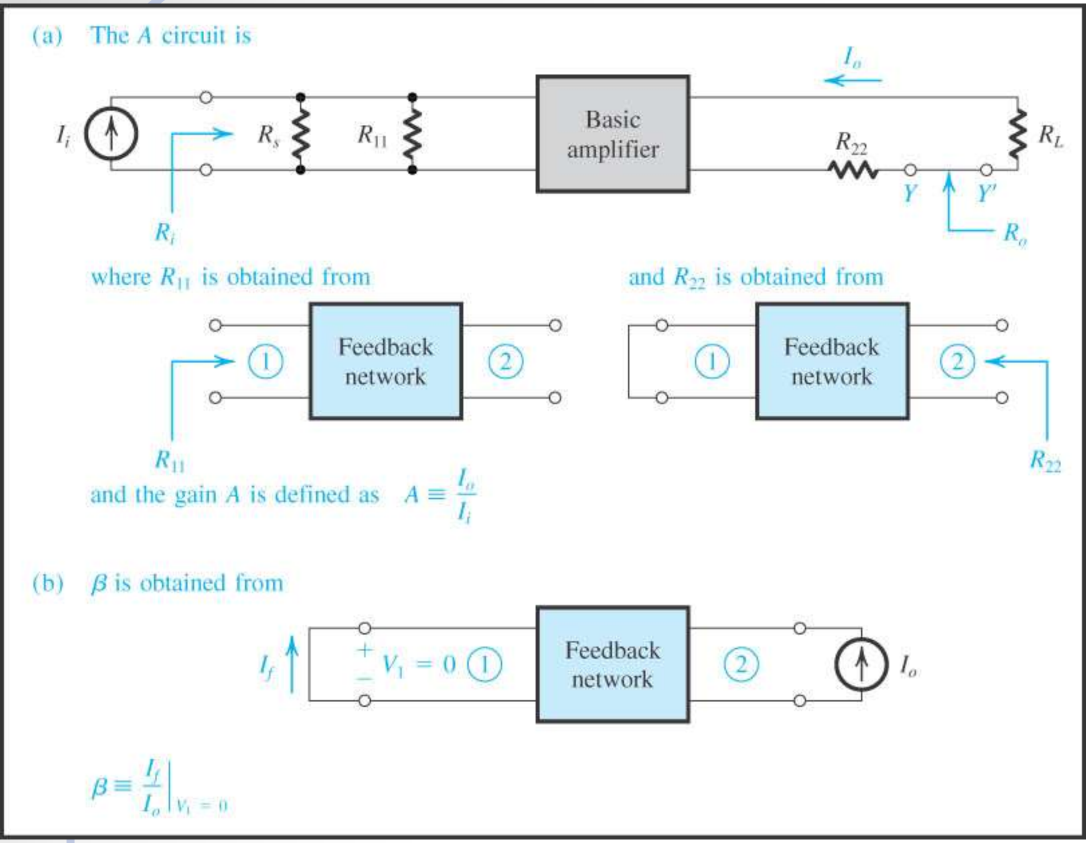
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{I_f}{I_o} = \beta_f$$

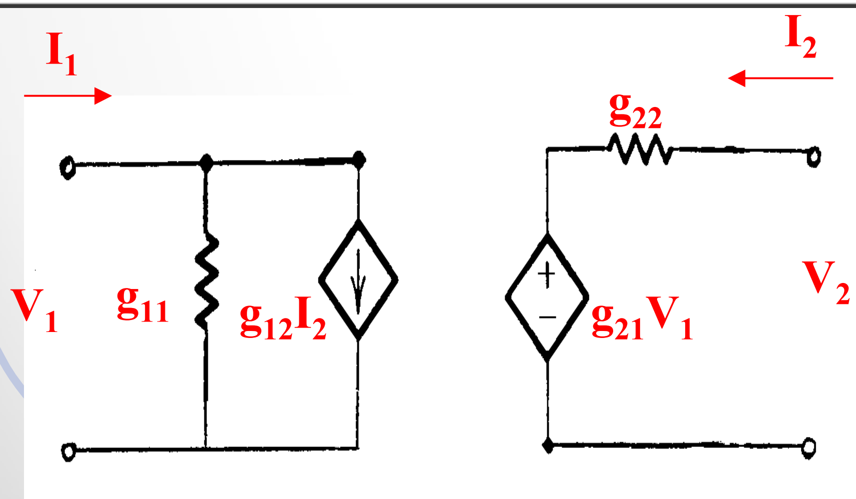
  - Feedforward factor given by g_{21} (neglected)
  - g_{22} gives feedback network loading on output
  - g_{11} gives feedback network loading on input
- Can incorporate loading effects in a modified basic amplifier. Gain A_{Io} becomes a new, modified gain A_{Io}'.
- Can then use analysis from ideal case

$$\omega_{Hf} = \left(1+\beta_f A_{Io}'\right)\omega_H \qquad \omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A_{Io}'\right)}$$

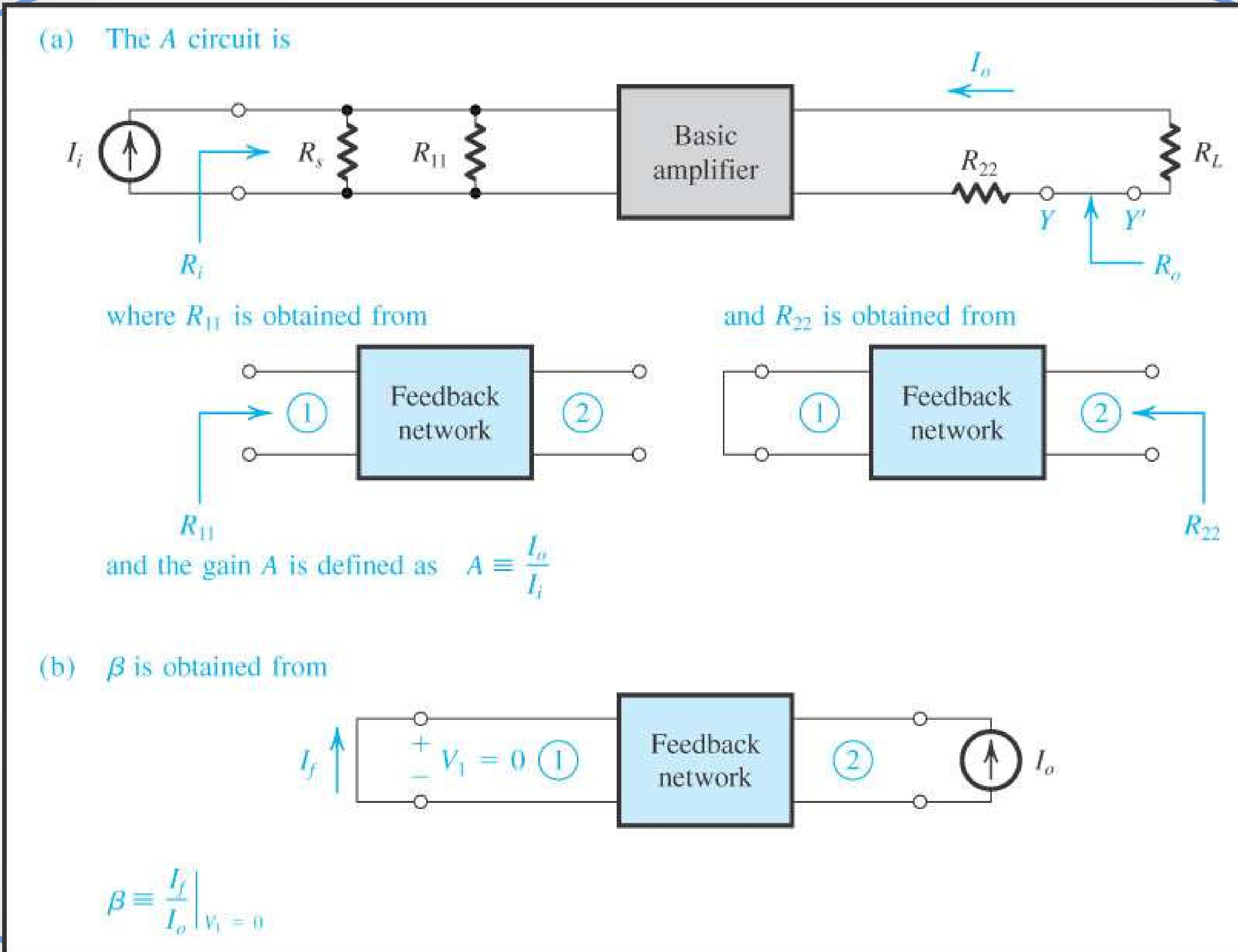
$$R_{in} = 1/\left(\frac{1}{R_{if}} - \frac{1}{R_s}\right) \qquad R_{out} = R_{of} - R_L$$

$$A_{Ifo} = \frac{A_{Io}'}{1+\beta_f A_{Io}'} \qquad R_{if} = \frac{R_i}{\left(1+\beta_f A_{Io}'\right)} \qquad R_{of} = R_o\left(1+\beta_f A_{Io}'\right)$$

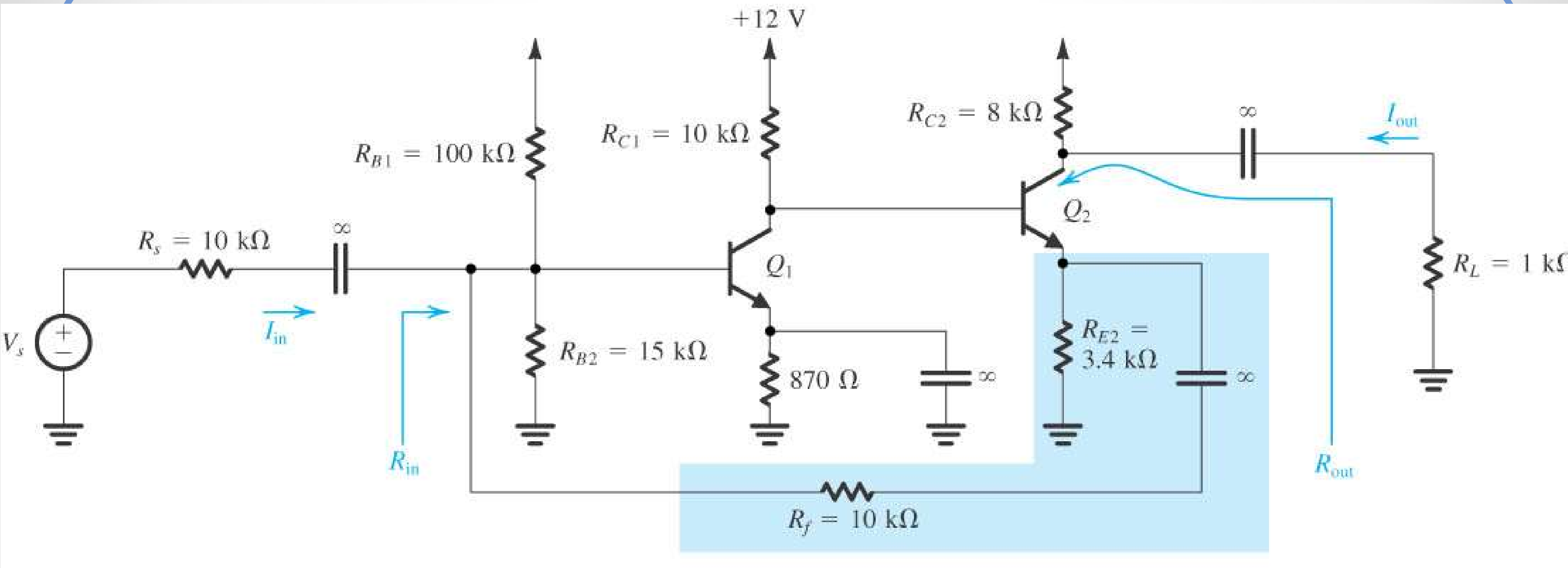




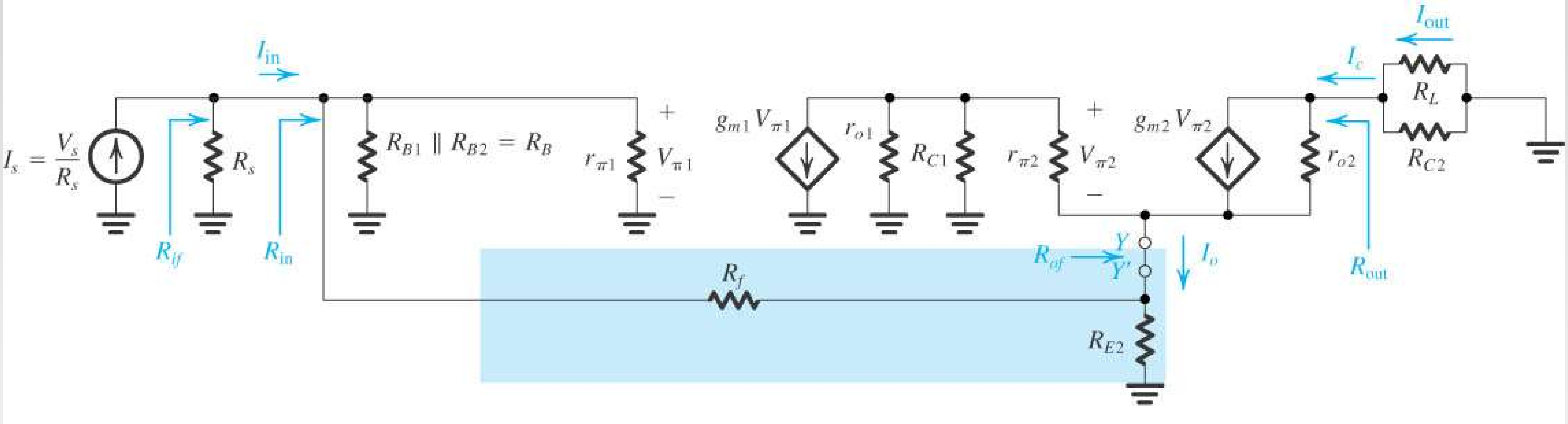
* **How do we determine the g-parameters for the feedback network?**
* **For the input loading term g_{11}**
  - **We turn off the feedback signal by setting $I_o = I_2 = 0$.**
  - **We then evaluate the resistance seen looking into port 1 of the feedback network.**
* **For the output loading term g_{22}**
  - **We short circuit the connection to the input so $V_1 = 0$.**
  - **We find the resistance seen looking into port 2 of the feedback network.**
* **To obtain the feedback factor β_f (also called g_{12})**
  - **We apply a test signal I_o' to port 2 of the feedback network and evaluate the feedback current I_f (also called I_1 here) for $V_1 = 0$.**
  - **Find β_f from $\beta_f = I_f/I_o$'**


(a) The A circuit is
I_o
I_i
R_s
R_{11}
Basic amplifier
R_{22}
R_L
Y
Y'
R_i
R_o
where R_{11} is obtained from
and R_{22} is obtained from
Feedback network
1
2
R_{11}
R_{22}
and the gain A is defined as $A \equiv \frac{I_o}{I_i}$
(b) β is obtained from
I_f
$V_1 = 0$
I_o
$\beta \equiv \left.\frac{I_f}{I_o}\right|_{V_1 = 0}$

Example: From this ckt, find I_{out}/I_{in}, R_{in} and R_{out}, assuming β = 100, V_A = 75 V

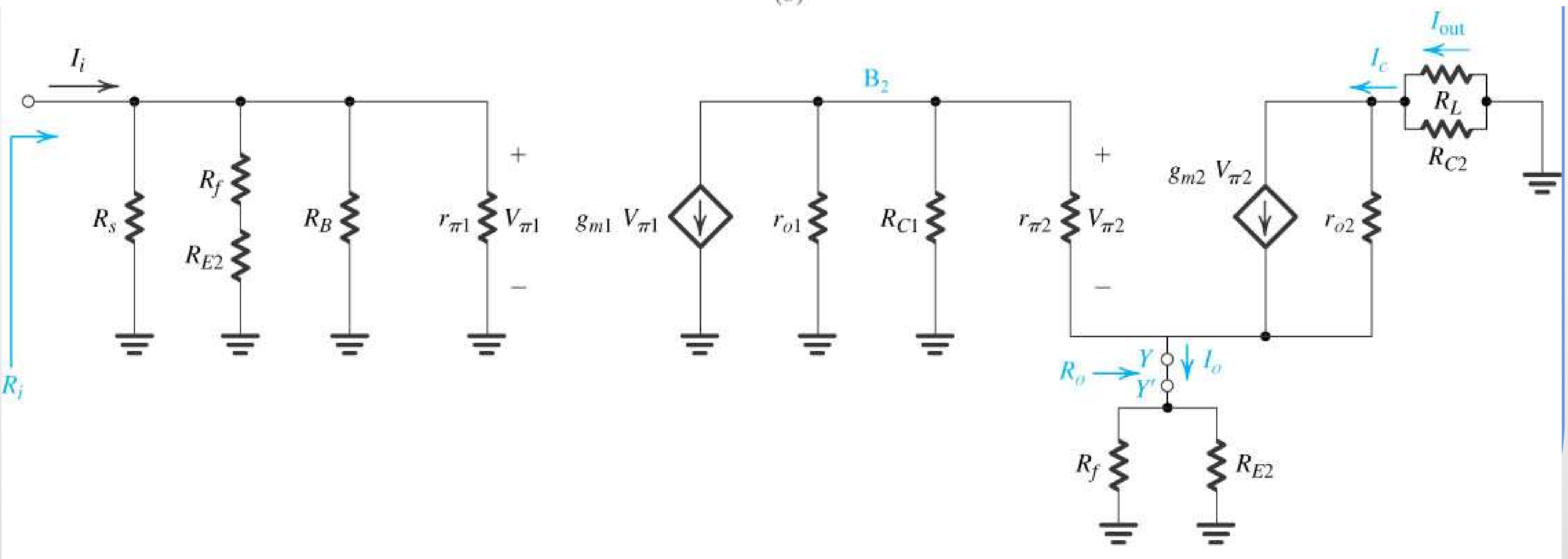


(a)



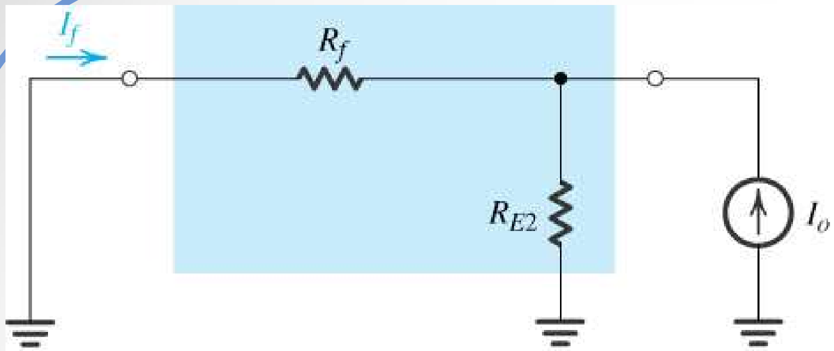
I_{in}
$I_s = \frac{V_s}{R_s}$
R_s
R_{if}
R_{in}
$R_{B1} \parallel R_{B2} = R_B$
$r_{\pi1}$
$V_{\pi1}$
$g_{m1}V_{\pi1}$
r_{o1}
R_{C1}
$r_{\pi2}$
$V_{\pi2}$
$g_{m2}V_{\pi2}$
r_{o2}
I_{out}
I_c
R_L
R_{C2}
R_{out}
R_{of}
Y
Y'
I_o
R_f
R_{E2}

(b)

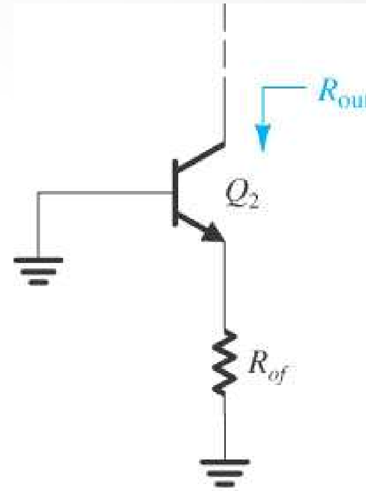


I_i
R_i
R_s
R_f
R_{E2}
R_B
$r_{\pi1}$
$V_{\pi1}$
$g_{m1}V_{\pi1}$
r_{o1}
R_{C1}
B_2
$r_{\pi2}$
$V_{\pi2}$
$g_{m2}V_{\pi2}$
r_{o2}
I_{out}
I_c
R_L
R_{C2}
R_o
Y
Y'
I_o
R_f
R_{E2}

(c)

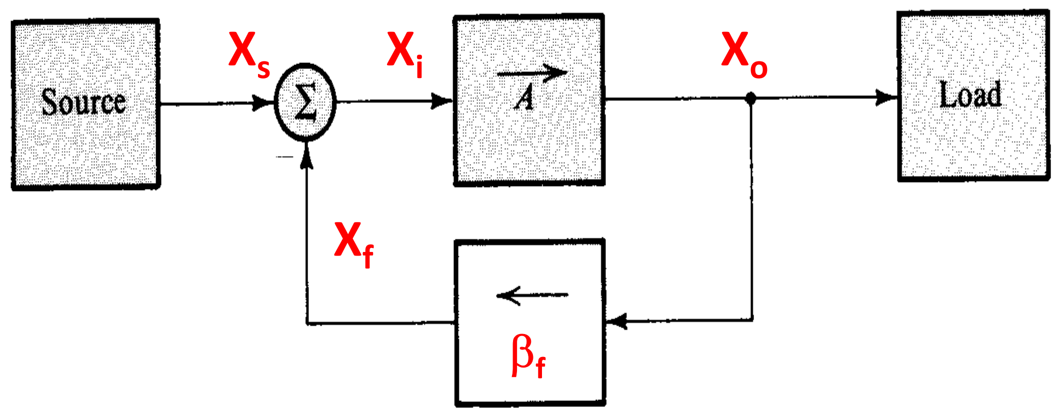


(d)



(e)

Summary of Feedback Amplifier Analysis



- Calculate low frequency poles and zeroes.
- Determine dominant (highest) low frequency pole ω_L including loading effects of feedback network
- Calculate new dominant low frequency pole ω_{Lf} .

$$\omega_{Lf} = \frac{\omega_L}{\left(1+\beta_f A_o\right)}$$

- Calculate high frequency poles and zeroes.
- Determine dominant (lowest) high frequency pole ω_H including loading effects of feedback network
- Calculate new dominant high frequency pole ω_{Hf} .

$$\omega_{Hf} = \left(1+\beta_f A_o\right)\omega_H$$

* Identify the amplifier configuration by:
  - Output sampling
    - I_o = series configuration
    - V_o = shunt configuration
  - Feedback to input
    - I_o = shunt configuration
    - V_o = series configuration
* Calculate loading effects of feedback network
  - On input
  - On output
* Calculate appropriate midband gain A' (modified by loading effects of feedback network)
* Calculate feedback factor β_f.
* Calculate midband gain with feedback A_f.

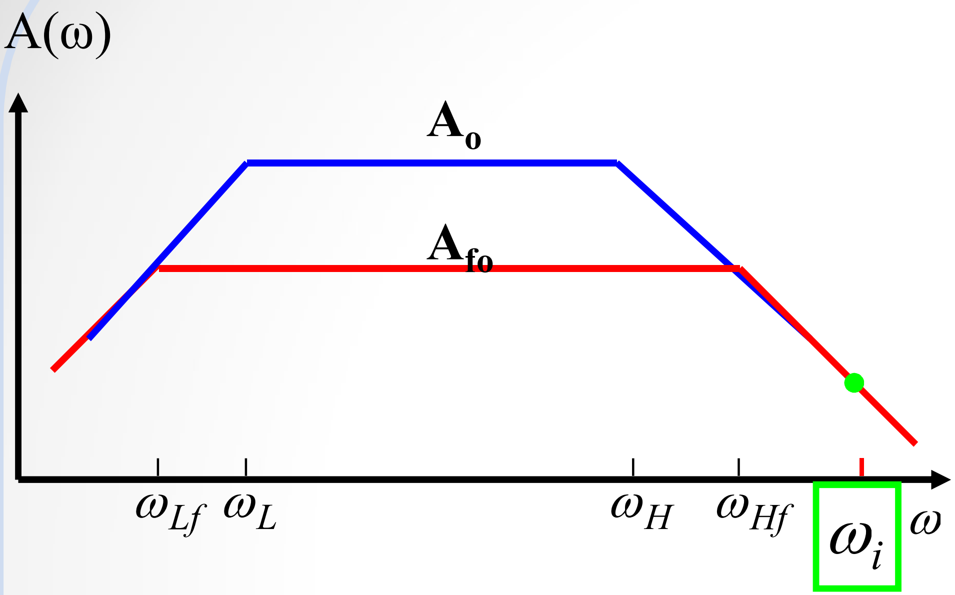
$$A_{fo} = \frac{A_o}{1+\beta_f A_o}$$

Summary of Feedback Amplifier Analysis

Table 8.1 SUMMARY OF RELATIONSHIPS FOR THE FOUR FEEDBACK-AMPLIFIER TOPOLOGIES

Feedback Amplifier	x_i	x_o	x_f	x_s	A	β	A_f	Source Form	Loading of Feedback Network Is Obtained: At Input	Loading of Feedback Network Is Obtained: At Output	To Find β, Apply to Port 2 of Feedback Network	Z_{if}	Z_{of}	Refer to Figs.
Series–shunt (voltage amplifier)	V_i	V_o	V_f	V_s	$\frac{V_o}{V_i}$	$\frac{V_f}{V_o}$	$\frac{V_o}{V_s}$	Thévenin	By short-circuiting port 2 of feedback network	By open-circuiting port 1 of feedback network	A voltage and find the open-circuit voltage at port 1	$Z_i(1 + A\beta)$	$\frac{Z_o}{1 + A\beta}$	8.4(a) 8.8 8.10 8.11
Shunt–series (current amplifier)	I_i	I_o	I_f	I_s	$\frac{I_o}{I_i}$	$\frac{I_f}{I_o}$	$\frac{I_o}{I_s}$	Norton	By open-circuiting port 2 of feedback network	By short-circuiting port 1 of feedback network	A current and find the short-circuit current at port 1	$\frac{Z_i}{1 + A\beta}$	$Z_o(1 + A\beta)$	8.4(b) 8.22 8.23 8.24
Series–series (transconductance amplifier)	V_i	I_o	V_f	V_s	$\frac{I_o}{V_i}$	$\frac{V_f}{I_o}$	$\frac{I_o}{V_s}$	Thévenin	By open-circuiting port 2 of feedback network	By open-circuiting port 1 of feedback network	A current and find the open-circuit voltage at port 1	$Z_i(1 + A\beta)$	$Z_o(1 + A\beta)$	8.4(c) 8.13 8.15 8.16
Shunt–shunt (transresistance amplifier)	I_i	V_o	I_f	I_s	$\frac{V_o}{I_i}$	$\frac{I_f}{V_o}$	$\frac{V_o}{I_s}$	Norton	By short-circuiting port 2 of feedback network	By short-circuiting port 1 of feedback network	A voltage and find the short-circuit current at port 1	$\frac{Z_i}{1 + A\beta}$	$\frac{Z_o}{1 + A\beta}$	8.4(d) 8.18 8.19 8.20

Feedback Amplifier Stability



Amplifier becomes unstable (oscillates) if at some frequency ω_i we have

$$|\beta(\omega_i)A(\omega_i)| = 1$$

$$and \quad \phi(\omega_i) = \tan^{-1}\left[\frac{\operatorname{Im}\{\beta(\omega_i)A(\omega_i)\}}{\operatorname{Re}\{\beta(\omega_i)A(\omega_i)\}}\right] = -180^0 = -\pi \ radians$$

since

$$\beta(\omega_i)A(\omega_i) = |\beta(\omega_i)A(\omega_i)|e^{j\phi(\omega_i)} = e^{-j\pi} = -1$$

so

$$A_f(\omega_i) = \frac{A(\omega_i)}{1+\beta(\omega_i)A(\omega_i)} = \frac{A(\omega_i)}{1-1} \to \infty$$

* **Feedback analysis**
  - **Midband gain with feedback** $A_{fo} = \dfrac{A_o}{1+\beta_f A_o}$
  - **New low and high 3dB frequencies**
    $\omega_{Hf} = (1+\beta_f A_o)\omega_H \qquad \omega_{Lf} = \dfrac{\omega_L}{(1+\beta_f A_o)}$
  - **Modified input and output resistances, e.g.**
    $R_{if} = \dfrac{R_i}{(1+\beta_f A_{Io}')} \qquad R_{of} = R_o(1+\beta_f A_{Io}')$

* **Amplifier's frequency characteristics**

$$A(s) = A_M F_H(s) F_L(s)$$

* **Feedback amplifier's gain**

$$A_f(s) = \frac{A(s)}{1+\beta_f(s)A(s)} \to A_f(\omega) = \frac{A(\omega)}{1+\beta_f(\omega)A(\omega)}$$

* **Define Loop Gain as $\beta_f A$**

$$\beta(\omega)A(\omega) = |\beta(\omega)A(\omega)|e^{j\phi(\omega)}$$

  - **Magnitude** $|\beta(\omega)A(\omega)|$
  - **Phase** $\phi(\omega) = \tan^{-1}\left[\dfrac{\operatorname{Im}\{\beta(\omega)A(\omega)\}}{\operatorname{Re}\{\beta(\omega)A(\omega)\}}\right]$

Transfer Function of the Feedback Amplifier

The closed-loop transfer function is given by

$$A_f(s) = \frac{A(s)}{1+\beta_f(s)A(s)} \rightarrow A_f(j\omega) = \frac{A(j\omega)}{1+\beta_f(j\omega)A(j\omega)}$$

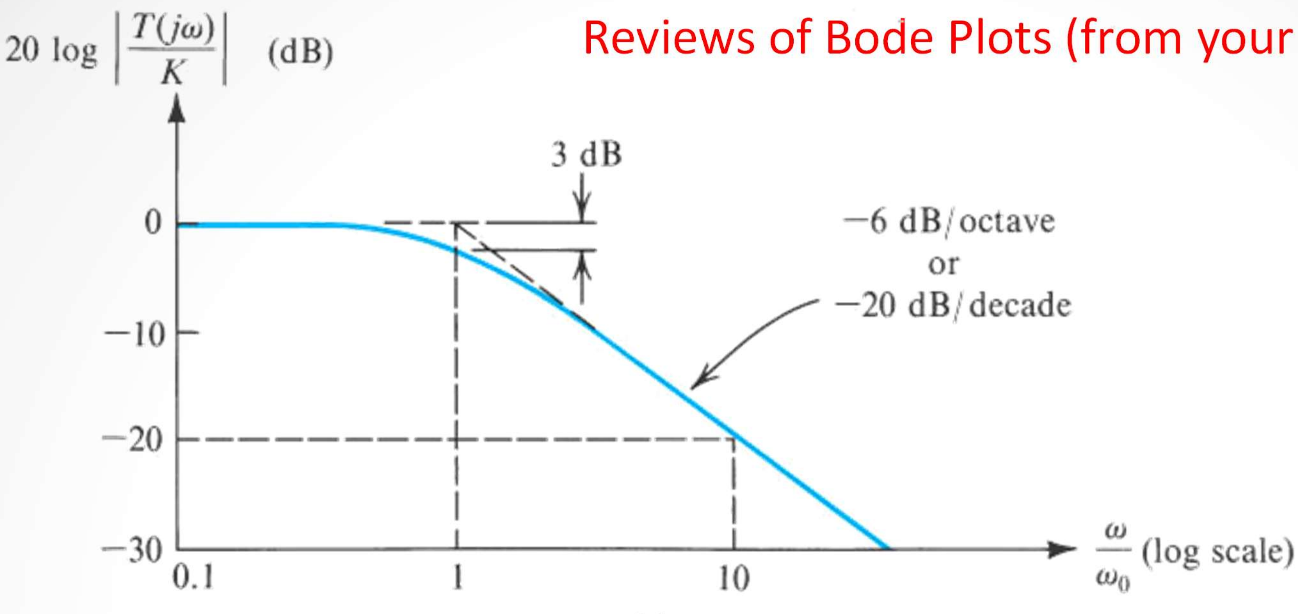
The loop gain L(jω) is a complex number that can be represented by its magnitude and phase,

$$L(j\omega) = \beta(j\omega)A(j\omega) = \left|\beta(j\omega)A(j\omega)\right| e^{j\phi(\omega)}$$

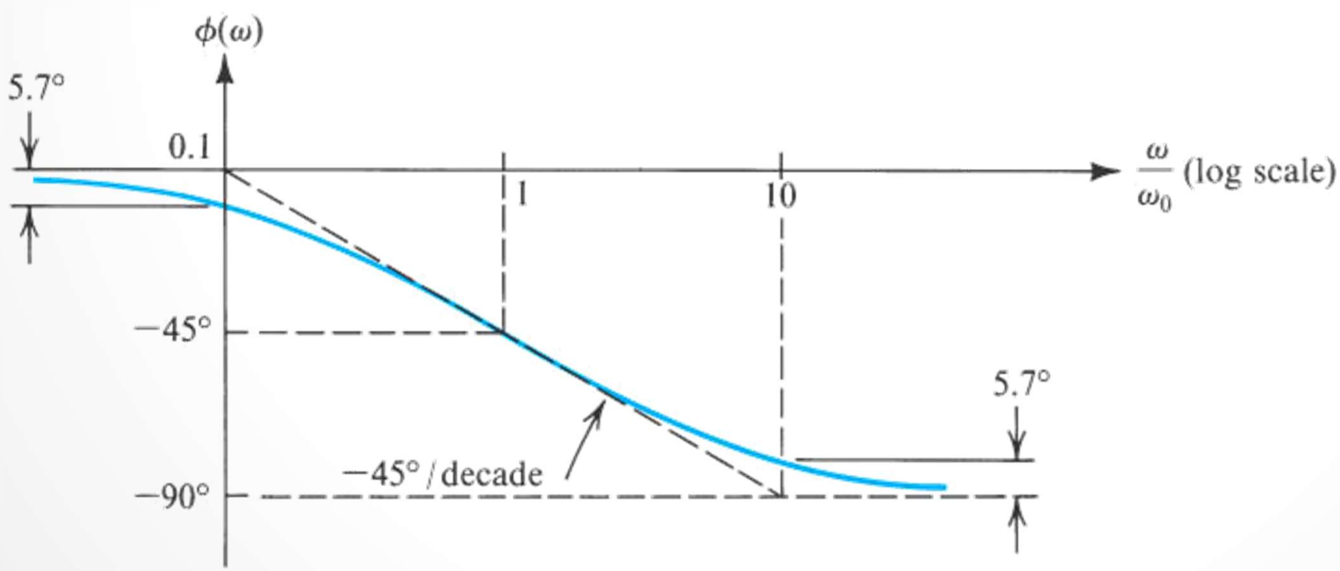
It is the manner in which the loop gain varies with frequency that determines the stability or instability of the feedback amplifier.

- The freq at which $\phi(\omega)$ becomes 180°, or ω_{180}, the loop gain A(jω)β(jω) will be a negative real number. The feedback becomes positive. Thus, |1+ A(jω)β(jω)| < 1, which makes |Af(jω)| > |A(jω)|. Nevertheless, the feedback is stable.
- If at ω_{180}, |A(jω)β(jω)| = 1, then |1+ A(jω)β(jω)| = 0. The amplifier will have an output for zero input; this is by definition as oscillator. The amplifier output produces sustained oscillation.
- If at ω_{180}, |A(jω)β(jω)| > 1, then the oscillation will grow in amplitude until some nonlinearity reduces the magnitude of the loop gain to exactly unity, at which point sustained oscillation will be obtained.

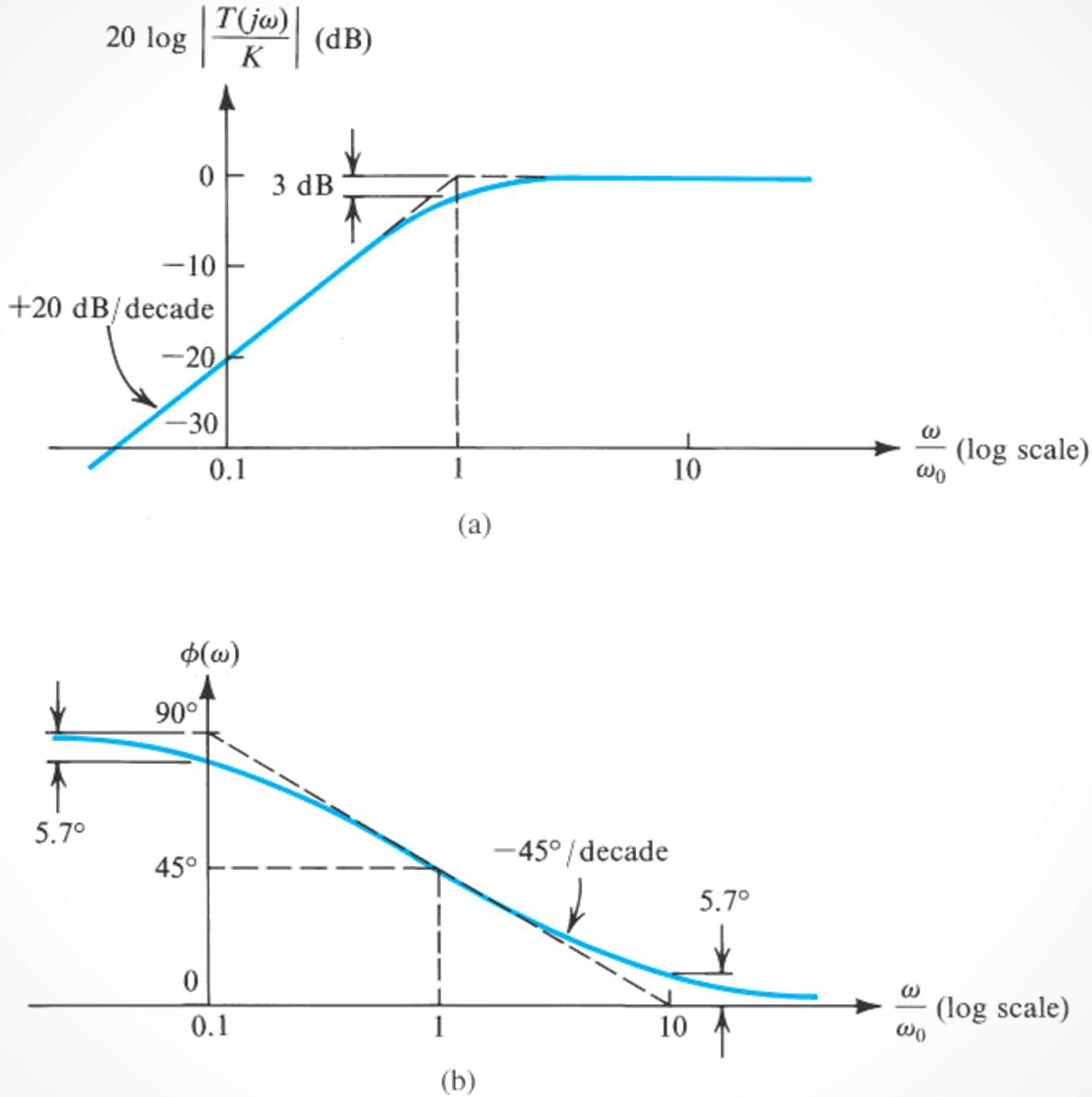
Reviews of Bode Plots (from your past life)



(a)



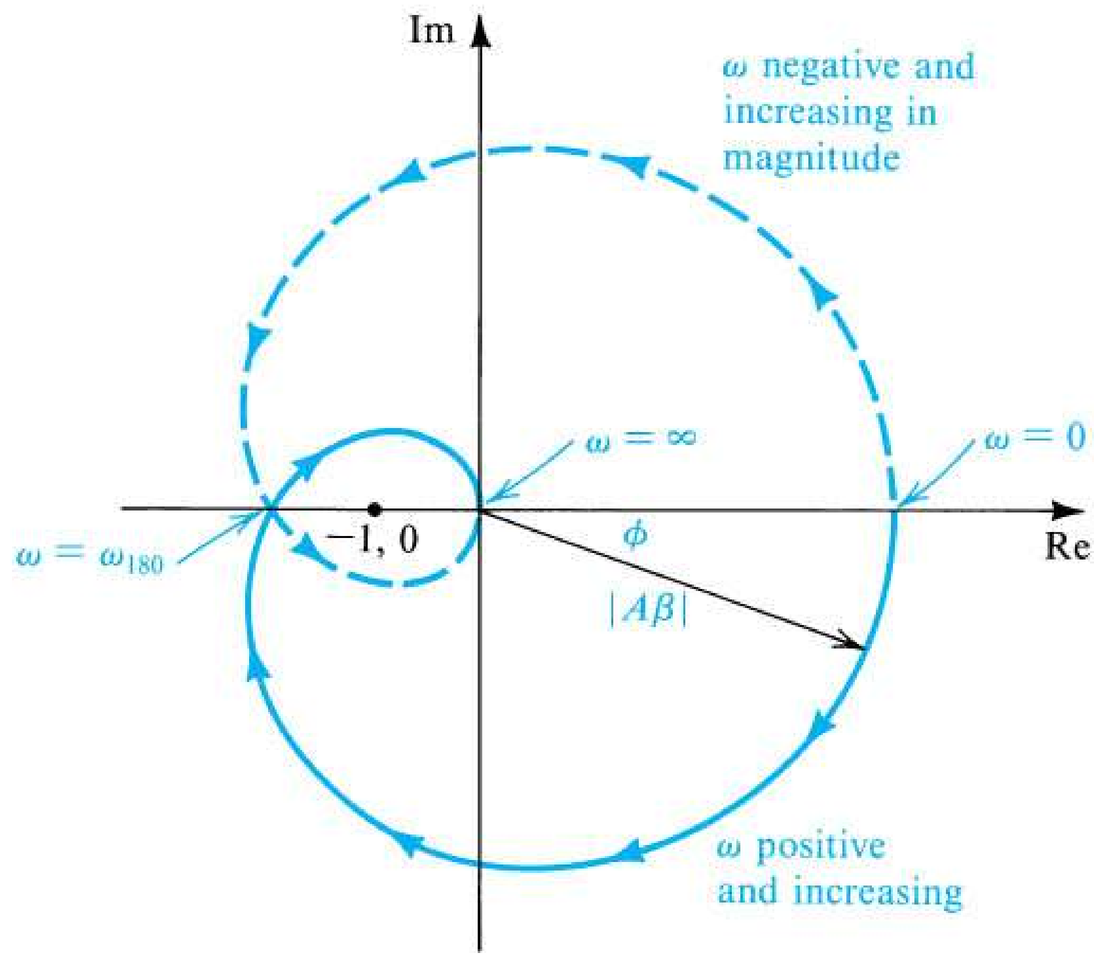
(b)



$20 \log \left| \frac{T(j\omega)}{K} \right|$ (dB)
0
3 dB
−10
+20 dB/decade
−20
−30
0.1
1
10
$\frac{\omega}{\omega_0}$ (log scale)
(a)
$\phi(\omega)$
90°
5.7°
45°
−45°/decade
5.7°
0
0.1
1
10
$\frac{\omega}{\omega_0}$ (log scale)
(b)

The Nyquist Plot

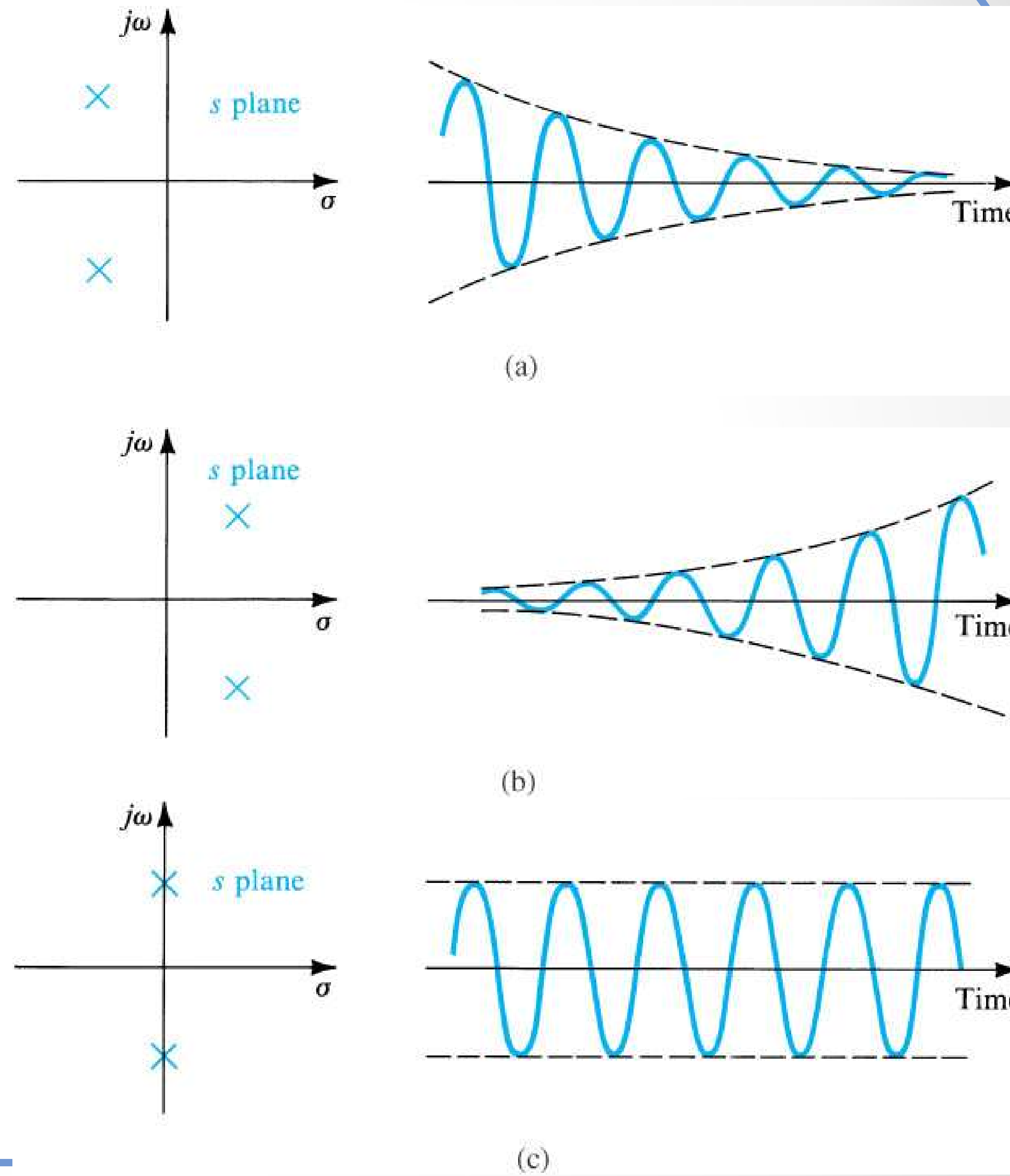
The Nyquist plot is a formalized approach for testing for stability based on the discussion previously. It is simply a polar plot of loop gain with frequency used as a parameter. Note that the radial distance is $|A\beta|$ and the angle is the phase angle ϕ.



- The solid line is for positive freq.
- Since the loop gain has a magnitude that is an even function of freq and a phase that is an odd function of freq, the $A\beta$ plot for negative freq is a mirror image through the Re axis.
- If the Nyquist plot encircles the point (-1,0), then the amplifier will be unstable. (This is sometimes referred to as the Nyquist Criterion.)

The effect of feedback on amplifier poles

1. Pole location and stability



2. Poles of the feedback amplifier

The feedback amplifier poles are obtained by solving the characteristic equation of the feedback loop:

$$\boxed{1 + A(s)\beta(s) = 0}$$

3. Amplifier with a single-pole response

Let the open loop gain of an amplifier be given by $A(s) = \dfrac{A_0}{1 + s/\omega_P}$

The closed loop gain will be given by $A_f(s) = \dfrac{A_0/(1 + A_0\beta)}{1 + s/\omega_P(1 + A_0\beta)}$

Thus, the feedback moves the pole along the negative real axis to a freq ω_{pf}:

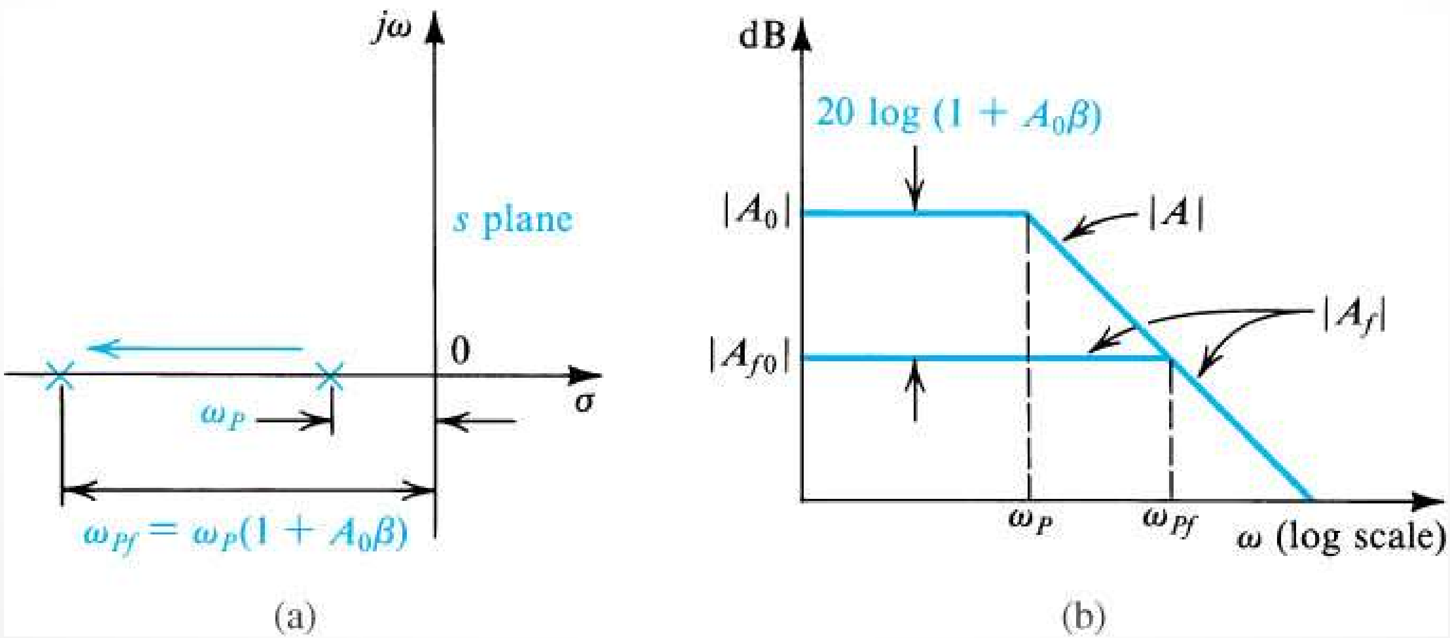
$$\omega_{Pf} = \omega_P(1 + A_0\beta)$$

From the Bode Plot, we see that at low freq, the difference between |A| and |Af| is 20log(1 + $A_o\beta$), but the two curves coincide at high freq. For $\omega \gg \omega_P(1 + A_0\beta)$,

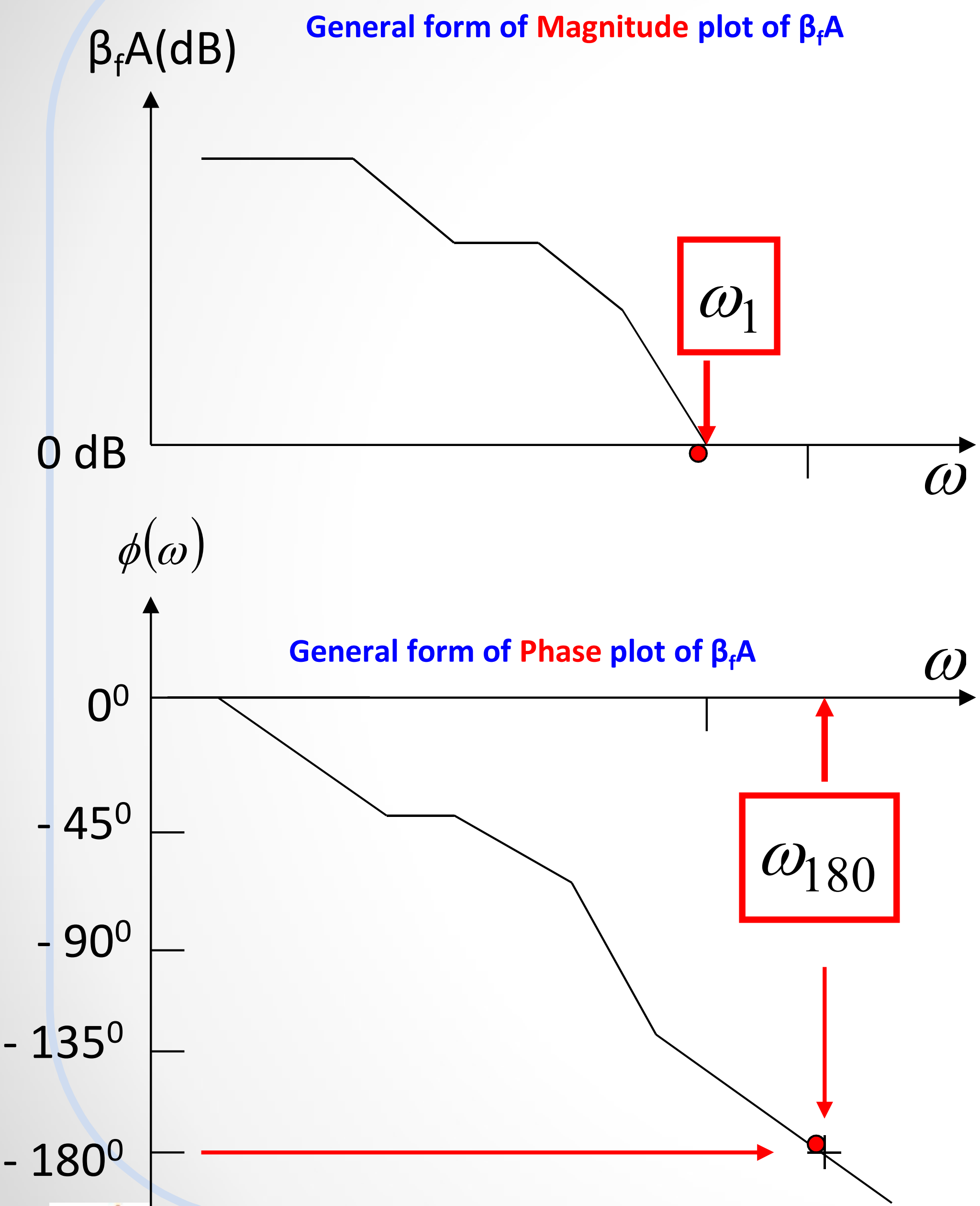
$$A_f(s) \simeq \frac{A_0\omega_P}{s} \simeq A(s)$$

Practically speaking, at such high freq, the loop gain is much smaller than unity and the feedback is ineffective.

- Applying a negative feedback results in extending its bandwidth at the expense of reduction in gain.
- Since the pole of the closed-loop amplifier never enters the right half of the s-plane, the single pole amplifier is stable for any value of β. Thus, this amplifier is said to be unconditionally stable.
- This is because the phase-lag associated with a single-pole response can never be greater than 90º. Thus the loop gain never achieves the 180º phase shift required for the feedback to become positive.



Feedback Amplifier Instability



* **Amplifier with one pole**

- **Phase shift of -90^0 is the maximum**

$$A(\omega) = \frac{A_o}{1 + j\dfrac{\omega}{\omega_{HP1}}}$$

- **Cannot get -180^0 phase shift.**
- **No instability problem**

* **Amplifier with two poles**

- **Phase shift of -180^0 the maximum**

$$A(\omega) = \frac{A_o}{\left(1 + j\dfrac{\omega}{\omega_{HP1}}\right)\left(1 + j\dfrac{\omega}{\omega_{HP2}}\right)}$$

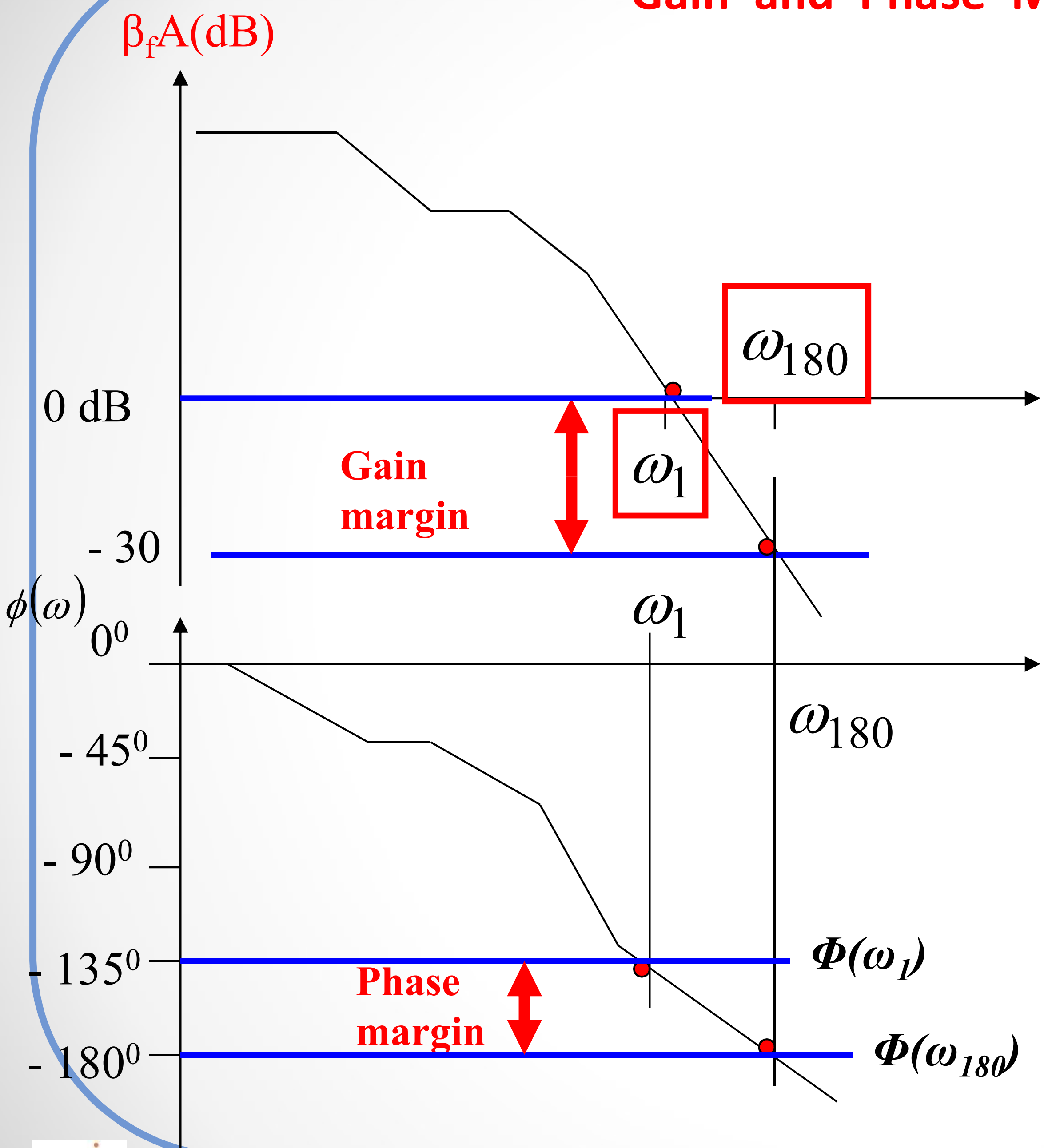
- **Can get -180^0 phase shift !**

* **For stability analysis, we define two important frequencies**

- **ω_1 is where magnitude of $\beta_f A$ goes to unity (0 dB)**
- **ω_{180} is where phase of $\beta_f A$ goes to -180^o**
- ***Instability problem if $\omega_1 = \omega_{180}$***

$$A_f(\omega_1 = \omega_{180}) = \frac{A(\omega_1)}{1 + \beta(\omega_1)A(\omega_1)} = \frac{A(\omega_1)}{1 - 1} \rightarrow \infty$$

Gain and Phase Margins



* ***Gain and phase margins measure how far amplifier is from the instability condition***

* **Phase margin**

$$\boxed{\text{Phase margin} = \phi(\omega_1) - \phi(\omega_{180})}$$

Example

$$\text{Phase margin} = \phi(\omega_1) - \phi(\omega_{180})$$

$$= -135^0 - (-180^0) = +45^0$$

* **Gain margin**

$$\boxed{\text{Gain margin} = \beta A(\omega_1) - \beta A(\omega_{180})}$$

Example

$$\text{Gain margin} = \beta A(\omega_1) - \beta A(\omega_{180})$$

$$= 0 \ dB - (-30 \ dB) = +30 \ dB$$

* **What are adequate margins?**

- **Phase margin = 50^0 (min)**
- **Gain margin = 10 dB (min)**

References

Microelectronic Circuits by Adel S. Sedra & Kenneth C. Smith. Saunders College Publishing

Microelectronic Circuit Design by Richard C. Jaeger. The McGraw-Hill Companies, Inc. 2011

Microelectronics Circuit Analysis and Design by Donald Neamen, The McGraw-Hill Companies, Inc. 2010

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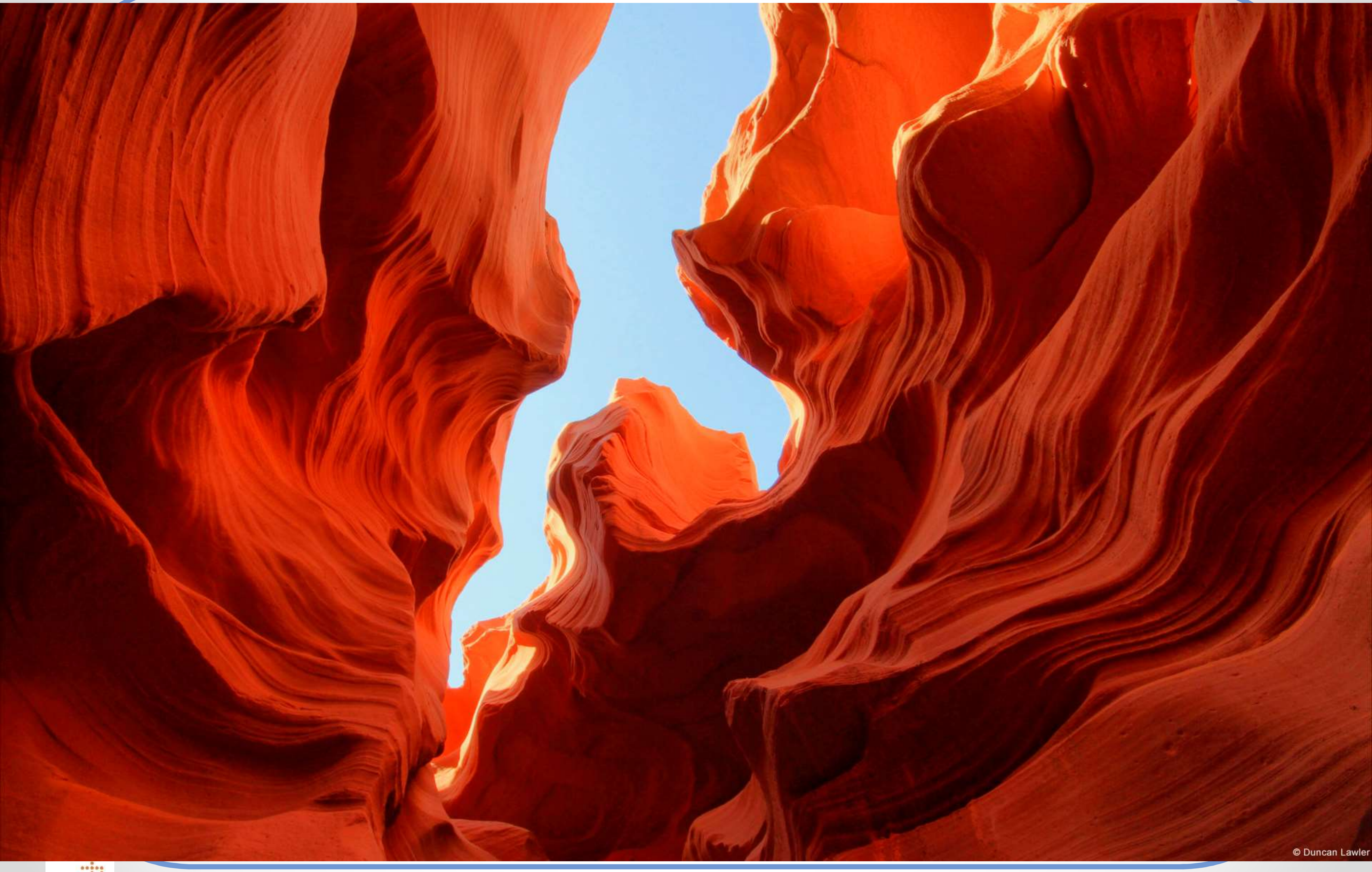
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