

EIE 211 : Electronic Devices and  
Circuit Design II  
Lecture  $4x^2(dx/dt)$ : CMOS Cascode

**Table 5.3** Small-Signal Equivalent-Circuit Models for the MOSFET

*Small-Signal Parameters*

**NMOS transistors**

- Transconductance:

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{OV}}$$

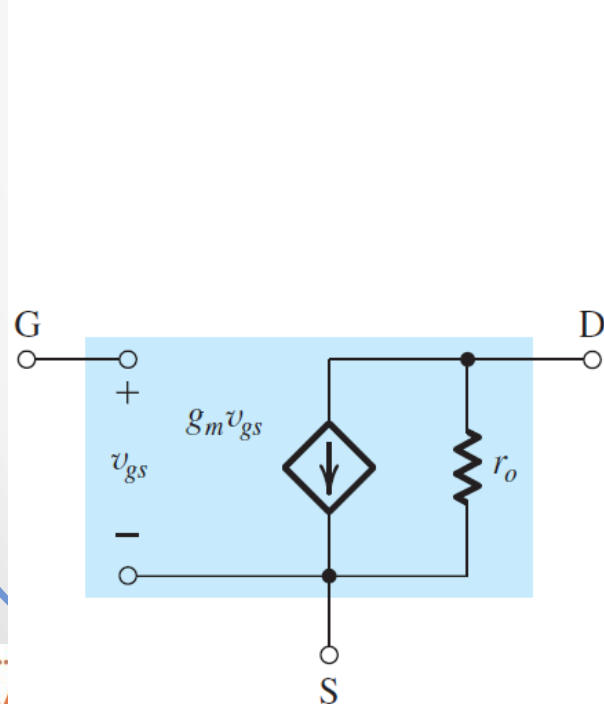
- Output resistance:

$$r_o = V_A / I_D = 1 / \lambda I_D$$

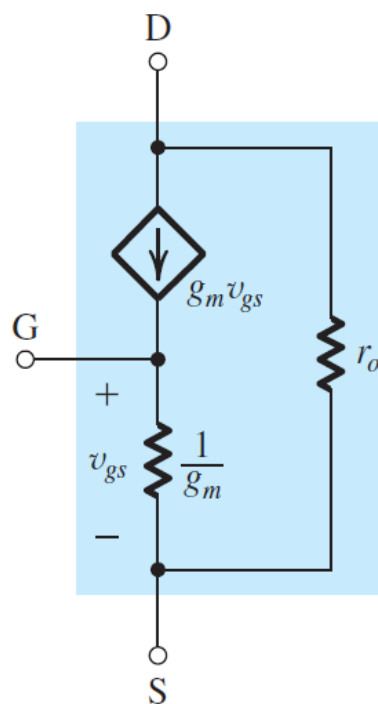
**PMOS transistors**

Same formulas as for NMOS *except* using  $|V_{OV}|$ ,  $|V_A|$ , and replacing  $\mu_n$  with  $\mu_p$ .

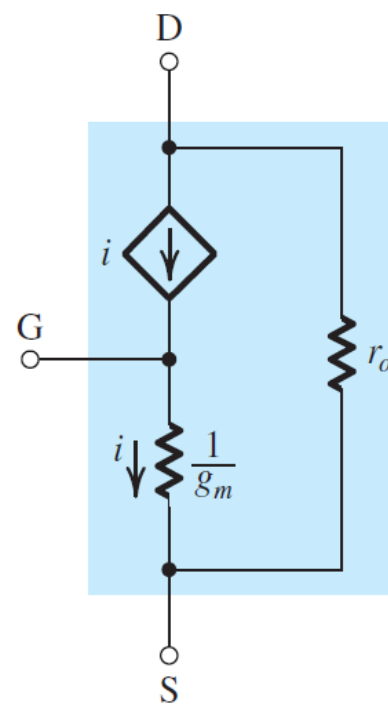
*Small-Signal Equivalent Circuit Models*



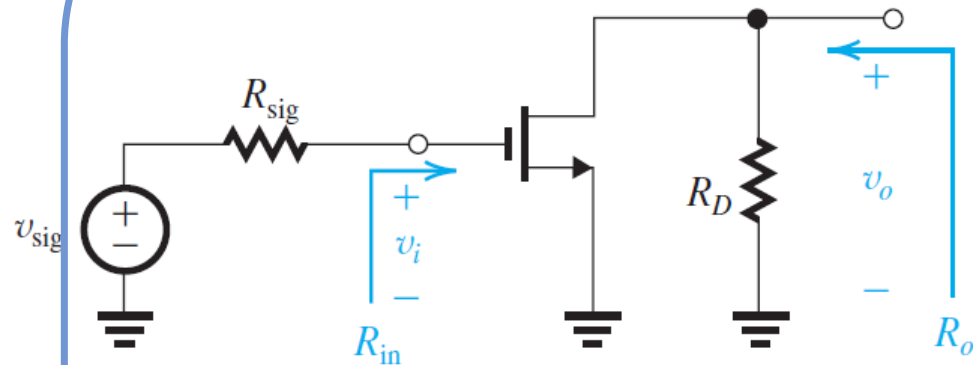
Hybrid- $\pi$  model



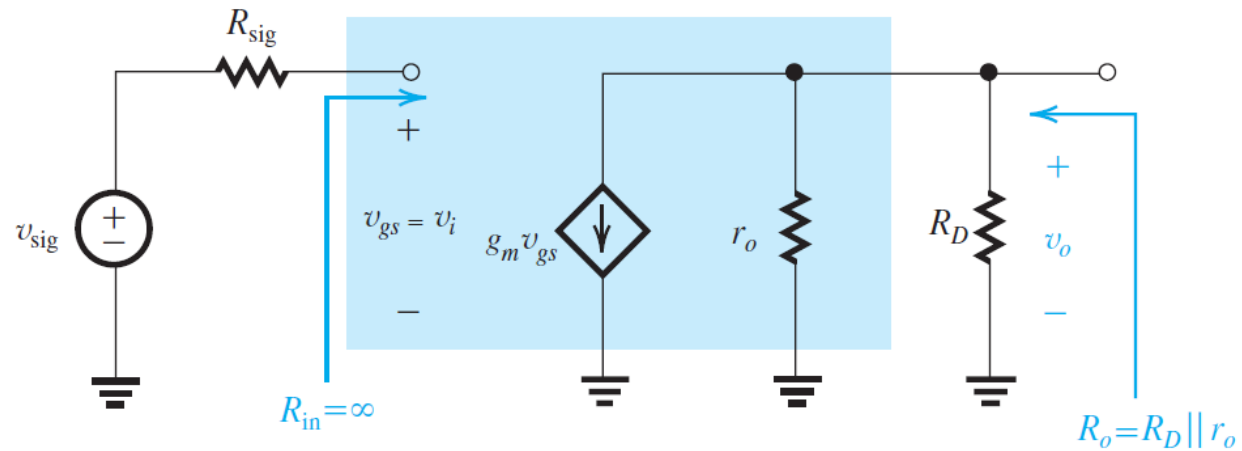
T models



# Common Source Amplifier



(a)



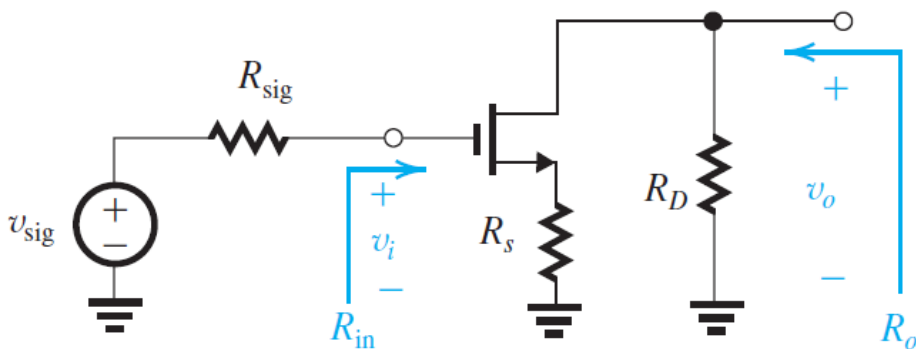
(b)

$$R_{in} = \infty$$

$$R_o = R_D \parallel r_o$$

$$G_v = A_v = -g_m(R_D \parallel R_L \parallel r_o)$$

# Common Source Amplifier w/ Degeneration Resistor



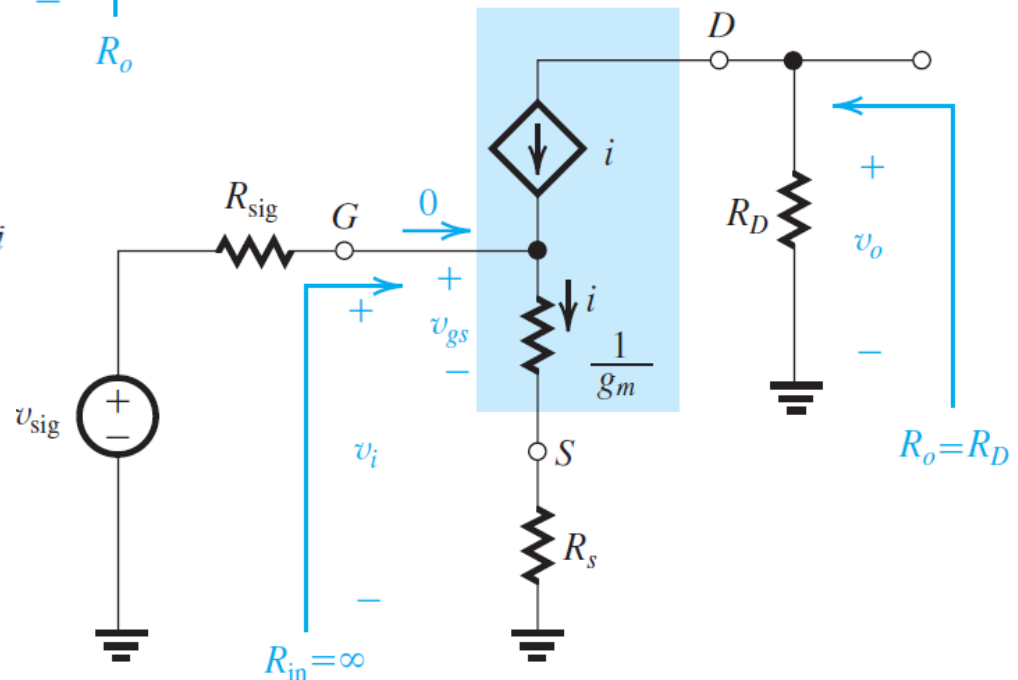
$$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_s} = \frac{v_i}{1 + g_m R_s}$$

$$i = \frac{v_i}{1/g_m + R_s} = \left( \frac{g_m}{1 + g_m R_s} \right) v_i$$

$$v_o = -i R_D$$

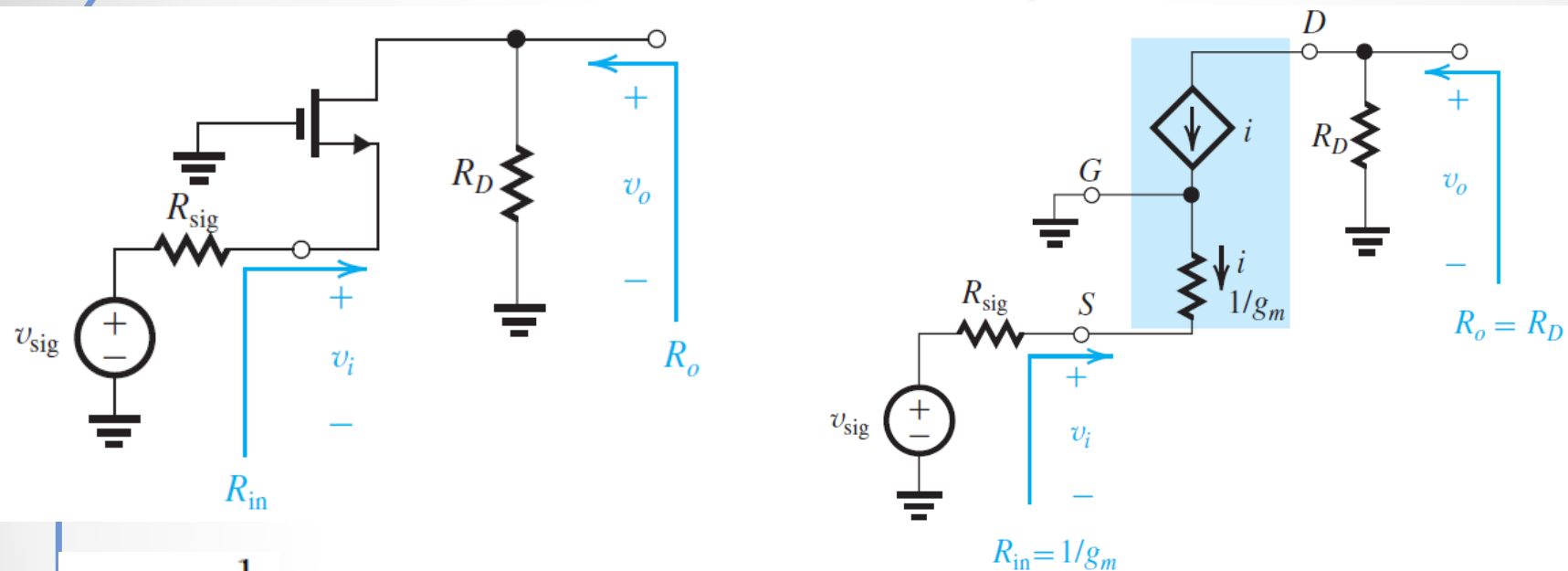
$$A_{vo} = \frac{v_o}{v_i} = -\frac{R_D}{1/g_m + R_s}$$

$$A_{vo} = -\frac{g_m R_D}{1 + g_m R_s}$$



$$\text{Voltage gain from gate to drain} = -\frac{\text{Total resistance in drain}}{\text{Total resistance in source}}$$

# Common Gate Amplifier



$$R_{in} = \frac{1}{g_m}$$

$$v_o = -iR_D$$

$$i = -\frac{v_i}{1/g_m}$$

$$A_{vo} \equiv \frac{v_o}{v_i} = g_m R_D$$

$$R_o = R_D$$

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1/g_m}{1/g_m + R_{sig}}$$

$$G_v = \frac{1/g_m}{R_{sig} + 1/g_m} [g_m (R_D \parallel R_L)] = \frac{(R_D \parallel R_L)}{R_{sig} + 1/g_m}$$

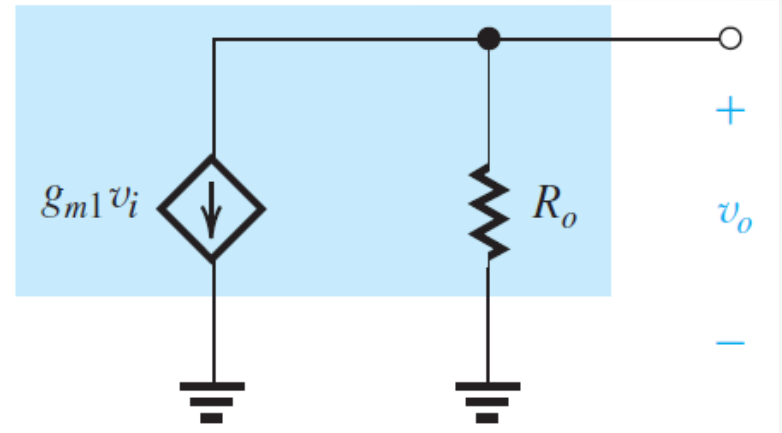
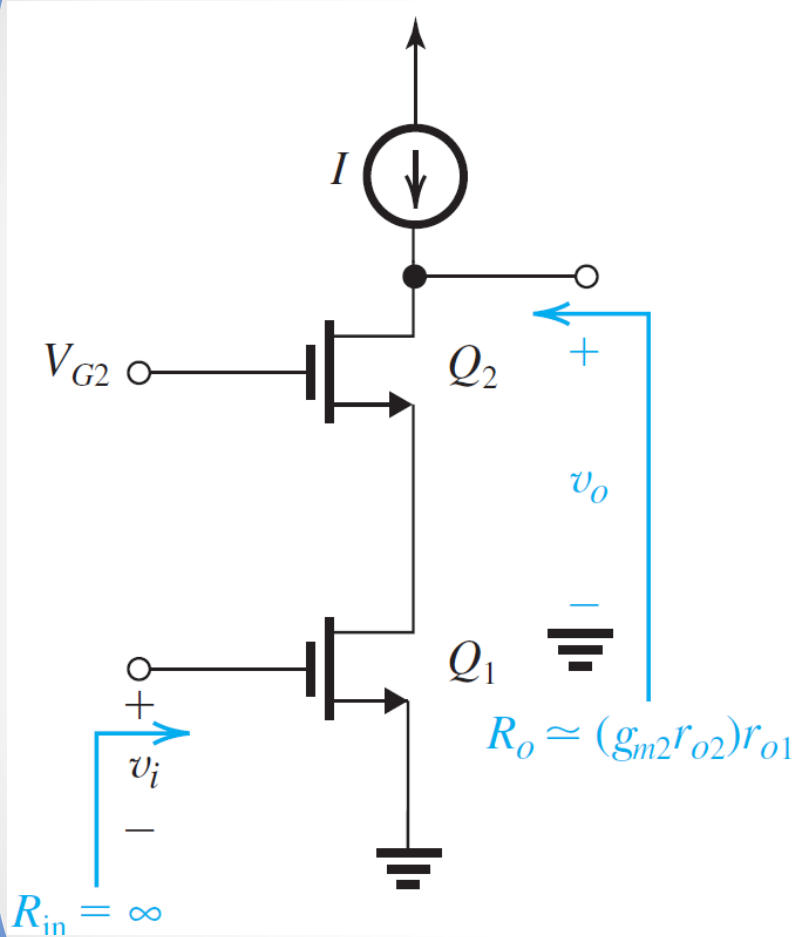
**Table 5.4** Characteristics of MOSFET Amplifiers

<i>Amplifier type</i>	<i>Characteristics<sup>a, b</sup></i>				
	$R_{in}$	$A_{vo}$	$R_o$	$A_v$	$G_v$
Common source (Fig. 5.45)	$\infty$	$-g_m R_D$	$R_D$	$-g_m(R_D \parallel R_L)$	$-g_m(R_D \parallel R_L)$
Common source with $R_s$ (Fig. 5.47)	$\infty$	$-\frac{g_m R_D}{1 + g_m R_s}$	$R_D$	$\frac{-g_m(R_D \parallel R_L)}{1 + g_m R_s}$ $-\frac{R_D \parallel R_L}{1/g_m + R_s}$	$-\frac{g_m(R_D \parallel R_L)}{1 + g_m R_s}$ $-\frac{R_D \parallel R_L}{1/g_m + R_s}$
Common gate (Fig. 5.48)	$\frac{1}{g_m}$	$g_m R_D$	$R_D$	$g_m(R_D \parallel R_L)$	$\frac{R_D \parallel R_L}{R_{sig} + 1/g_m}$
Source follower (Fig. 5.50)	$\infty$	1	$\frac{1}{g_m}$	$\frac{R_L}{R_L + 1/g_m}$	$\frac{R_L}{R_L + 1/g_m}$

**TABLE 6.5** Characteristics of BJT Amplifiers<sup>a, b, c</sup>

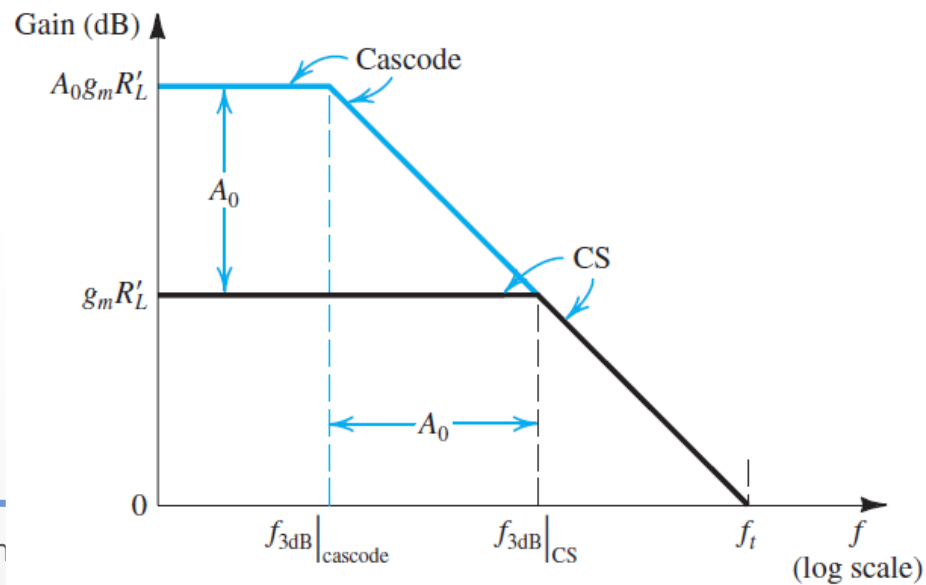
	$R_{in}$	$A_{vo}$	$R_o$	$A_v$	$G_v$
Common emitter (Fig. 6.50)	$(\beta + 1)r_e$	$-g_m R_C$	$R_C$	$-g_m(R_C \parallel R_L)$ $-\alpha \frac{R_C \parallel R_L}{r_e}$	$-\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)r_e}$
Common emitter with $R_e$ (Fig. 6.52)	$(\beta + 1)(r_e + R_e)$	$-\frac{g_m R_C}{1 + g_m R_e}$	$R_C$	$\frac{-g_m(R_C \parallel R_L)}{1 + g_m R_e}$ $-\alpha \frac{R_C \parallel R_L}{r_e + R_e}$	$-\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)(r_e + R_e)}$
Common base (Fig. 6.53)	$r_e$	$g_m R_C$	$R_C$	$g_m(R_C \parallel R_L)$ $\alpha \frac{R_C \parallel R_L}{r_e}$	$\alpha \frac{R_C \parallel R_L}{R_{sig} + r_e}$
Emitter follower (Fig. 6.55)	$(\beta + 1)(r_e + R_L)$	1	$r_e$	$\frac{R_L}{R_L + r_e}$	$\frac{R_L}{R_L + r_e + R_{sig}/(\beta + 1)}$ $G_{vo} = 1$ $R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$

# MOS Cascode

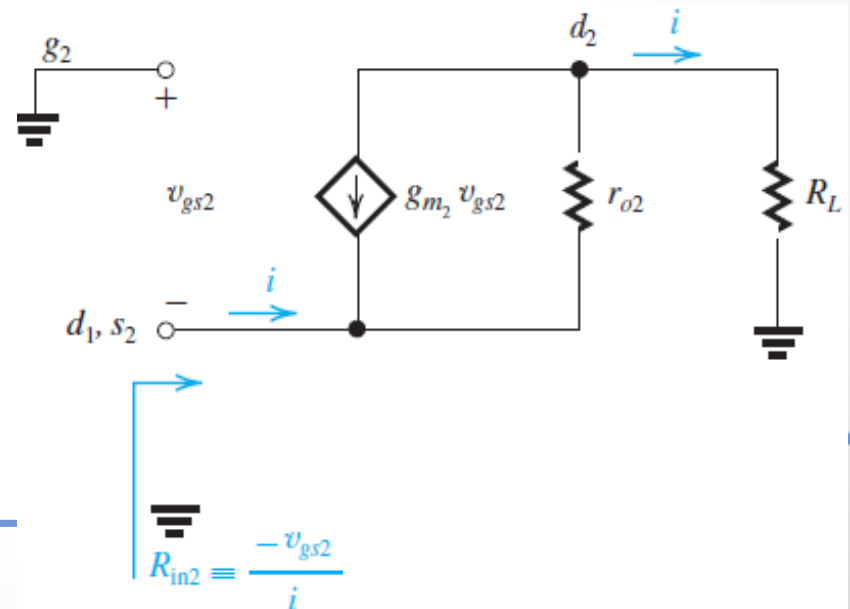
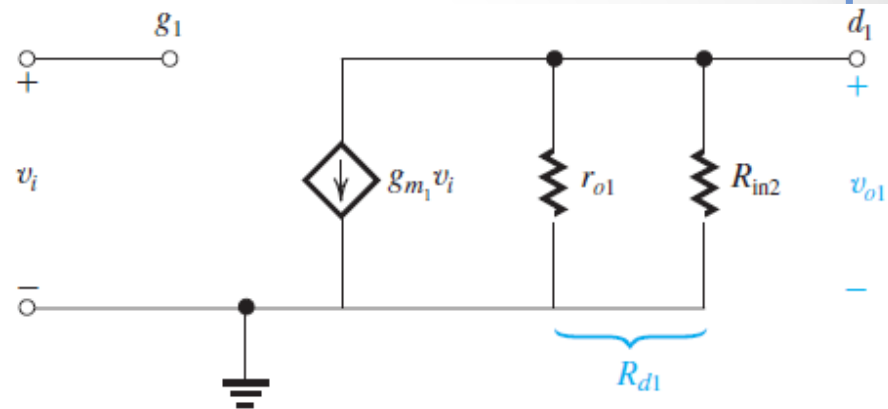
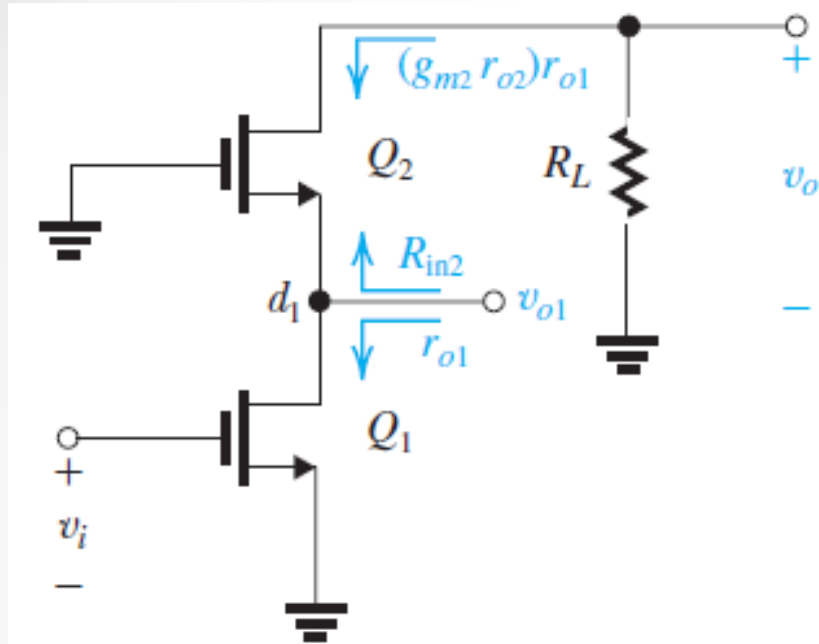




	Common Source	Cascode
Circuit		
DC Gain	$-g_m R'_L$	$-A_0 g_m R'_L$
$f_{3dB}$	$\frac{1}{2\pi(C_L + C_{gd})R'_L}$	$\frac{1}{2\pi(C_L + C_{gd})A_0 R'_L}$
$f_t$	$\frac{g_m}{2\pi(C_L + C_{gd})}$	$\frac{g_m}{2\pi(C_L + C_{gd})}$



# MOS Cascode: resistance

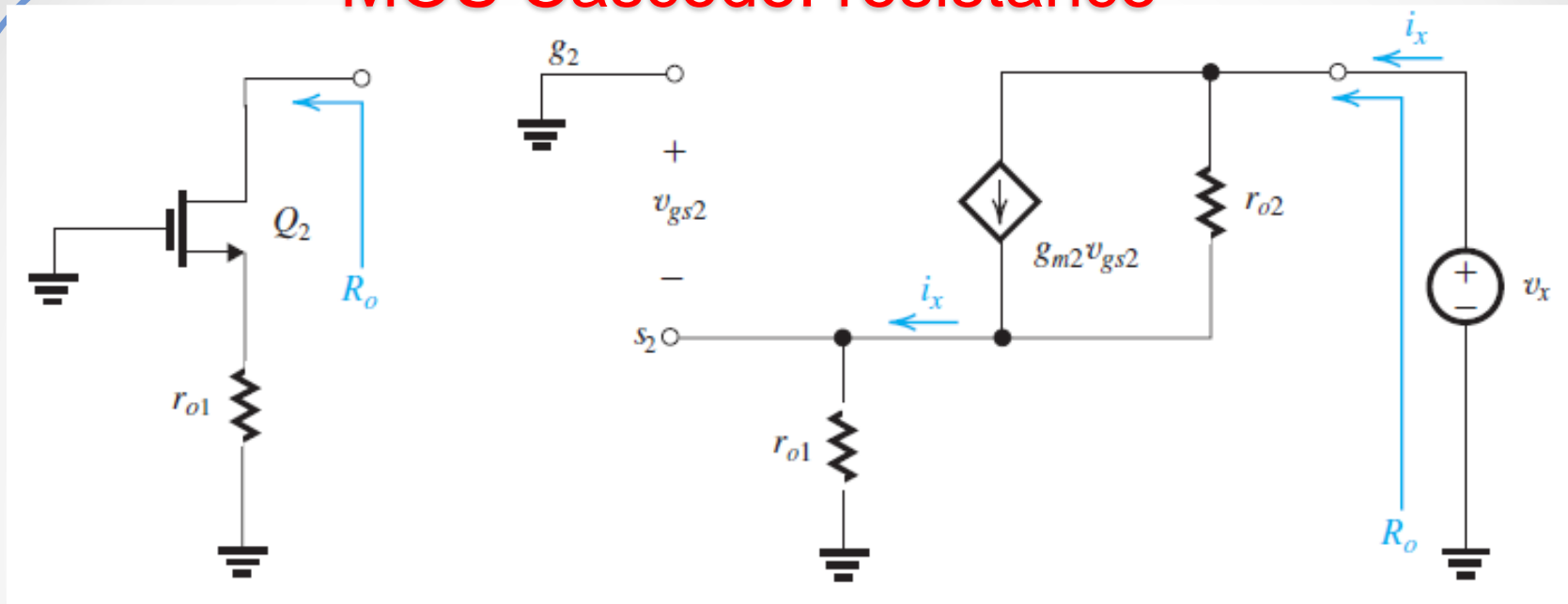


$$-v_{gs2} = (i + g_{m2}v_{gs2})r_{o2} + iR_L$$

$$R_{in2} \equiv -v_{gs2}/i$$

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2}r_{o2}}$$

# MOS Cascode: resistance



$$R_o \equiv \frac{v_x}{i_x}$$

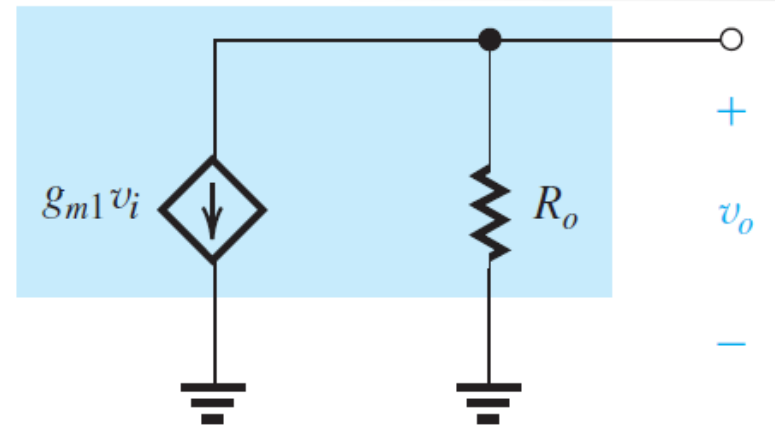
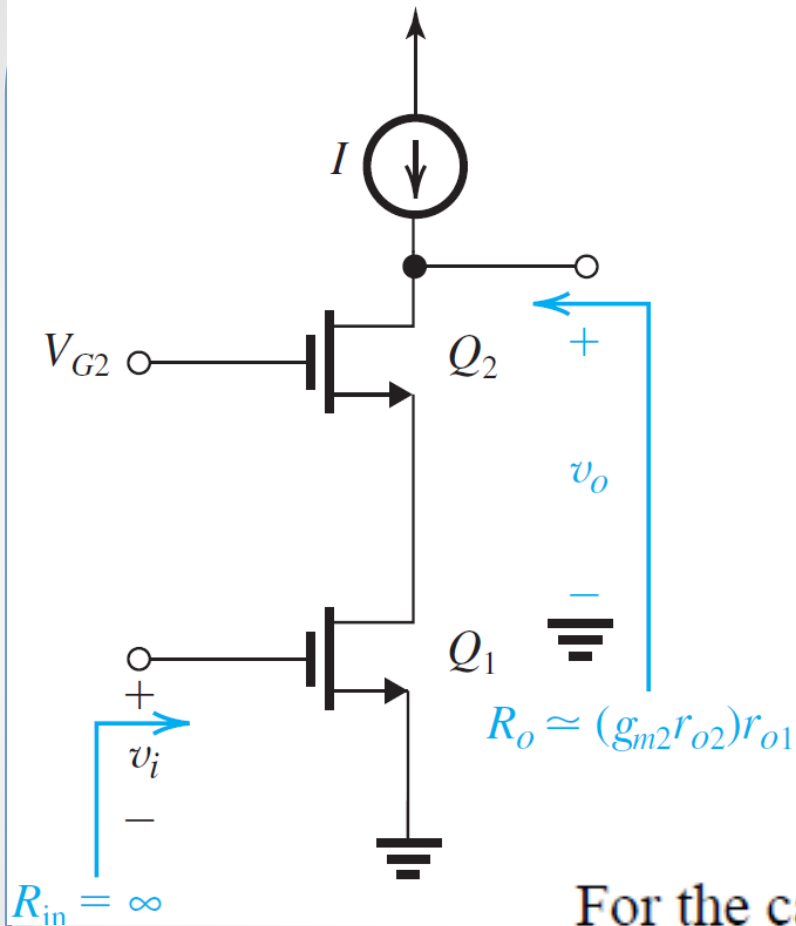
$$v_x = (i_x - g_{m2}v_{gs2})r_{o2} + i_x r_{o1}$$

$$-v_{gs2} = i_x r_{o1}$$

$$v_x = i_x(r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1})$$

$$R_o = r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1} \simeq (g_{m2}r_{o2})r_{o1}$$

# MOS Cascode: voltage gain



$$A_{vo} = \frac{v_o}{v_i} = -g_{m1}R_o$$

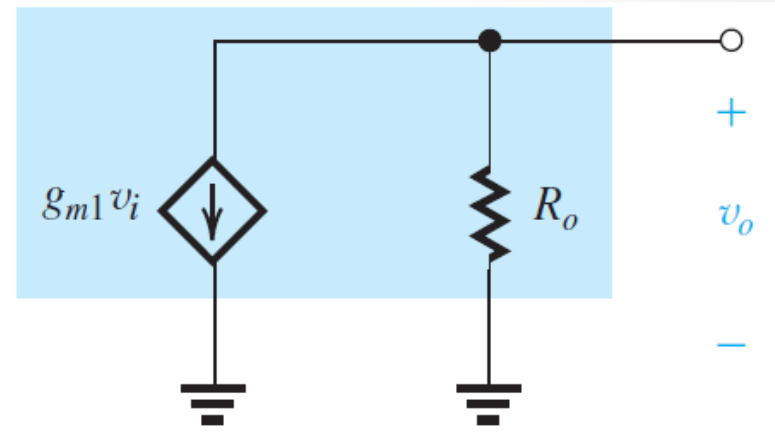
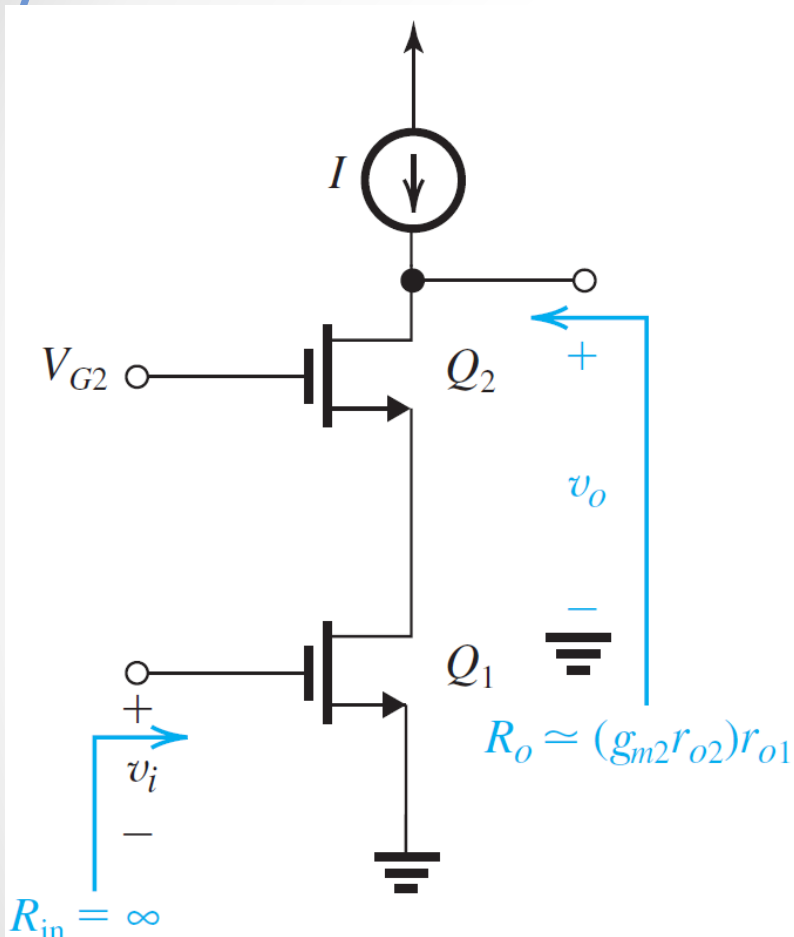
$$A_{vo} = -g_{m1} \times (g_{m2}r_{o2})r_{o1}$$

$$A_{vo} = -(g_{m1}r_{o1})(g_{m2}r_{o2})$$

For the case  $g_{m1} = g_{m2} = g_m$  and  $r_{o1} = r_{o2} = r_o$ ,

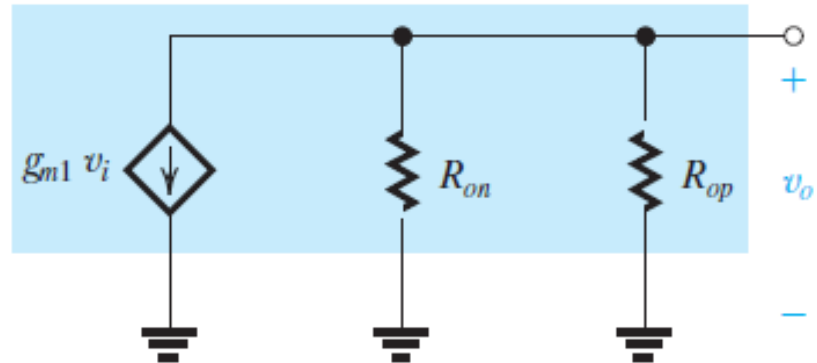
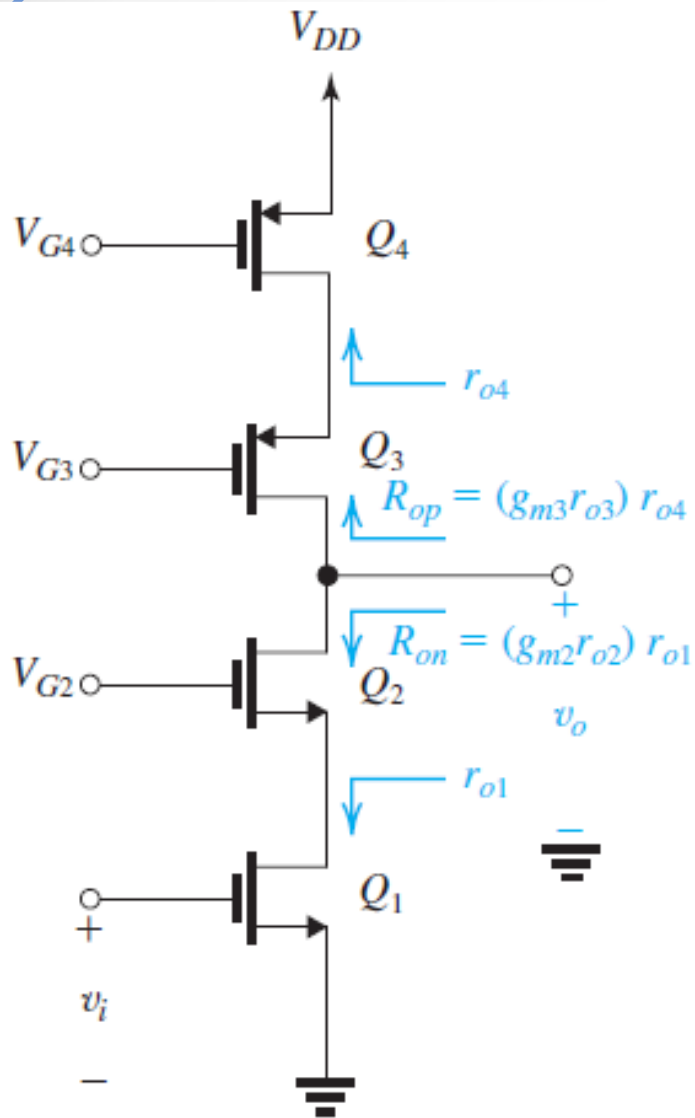
$$A_{vo} = -(g_m r_o)^2 = -A_0^2$$

# MOS Cascode: voltage gain



If there's a load resistance  $R_L$  connected to the output, the voltage gain will be

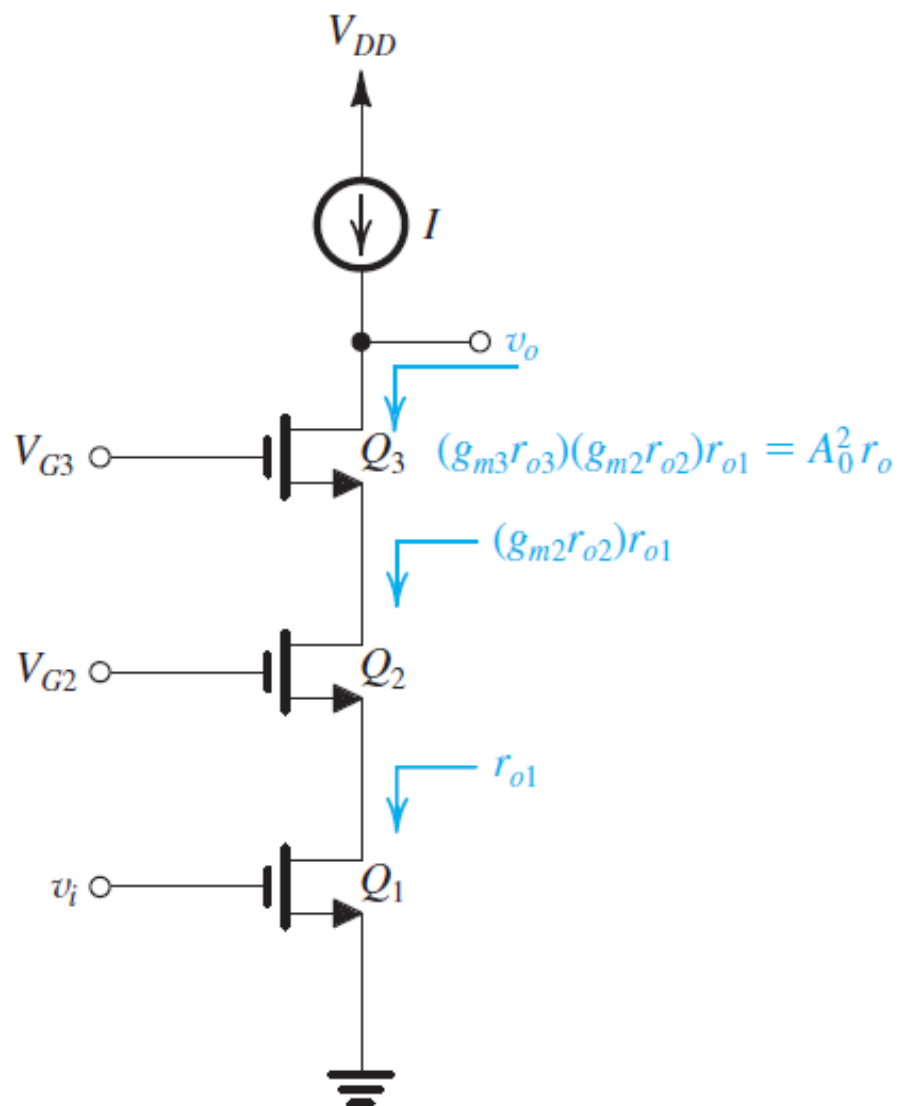
$$A_v = -g_{m1}(g_{m2}r_{o2}r_{o1} \parallel R_L)$$



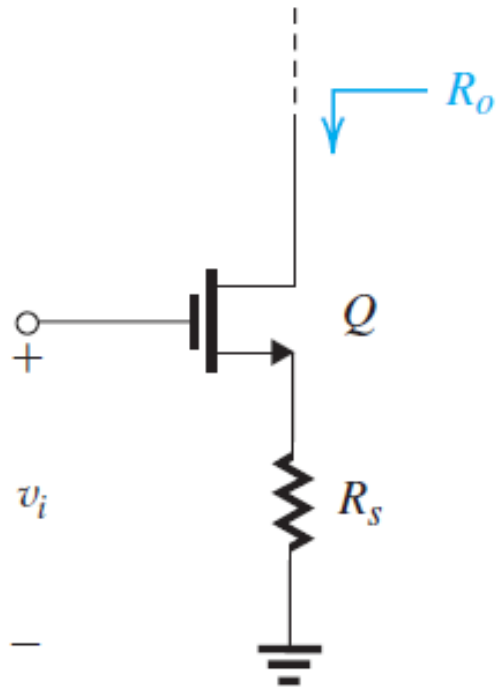
$$A_v = \frac{v_o}{v_i} = -g_{m1}[R_{on} \parallel R_{op}]$$

$$A_v = -g_{m1}\{[(g_{m2}r_{o2})r_{o1}] \parallel [(g_{m3}r_{o3})r_{o4}]\}$$

# MOS Double Cascode



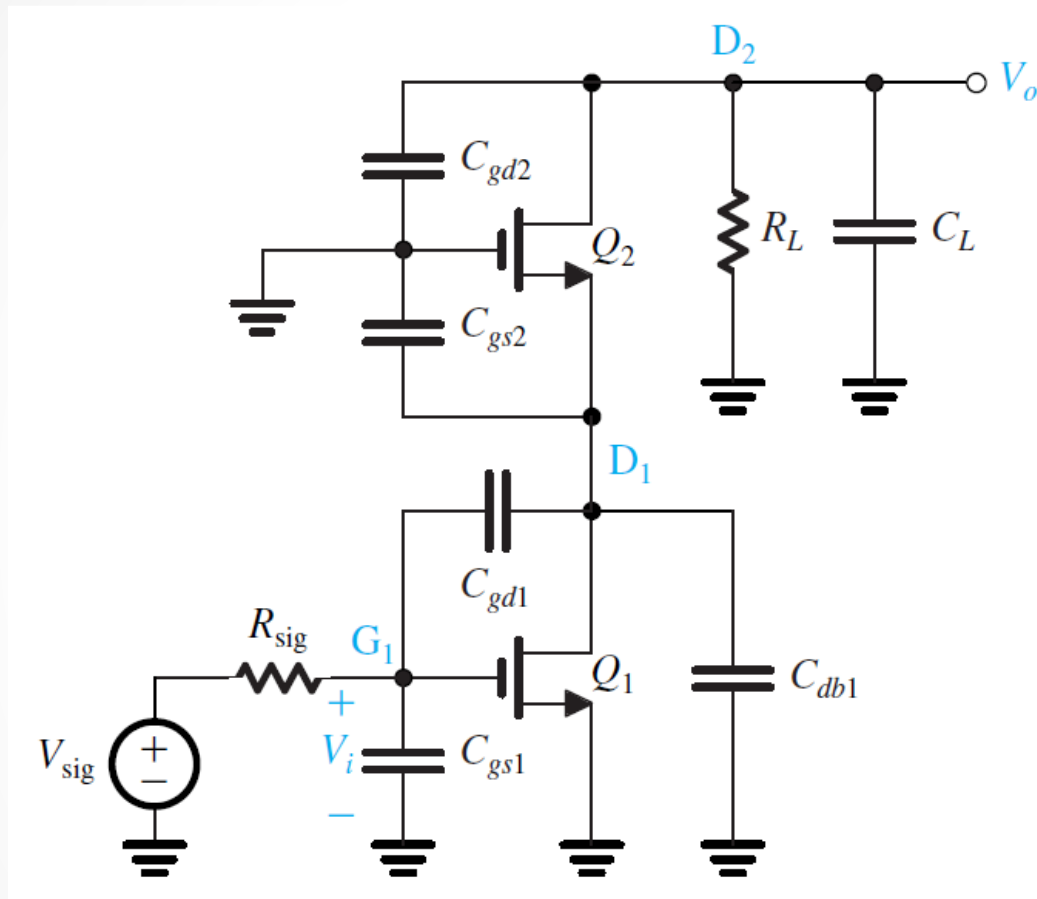
# MOS Cascode: output resistance of a CS amplifier with degeneration resistor



$$R_o = R_s + r_o + g_m r_o R_s$$
$$\approx (1 + g_m R_s) r_o$$



# MOS Cascode Frequency Response



# MOS Cascode Frequency Response

We are going to use “open-ckt” time constant method to find  $f_H$

1. Capacitance  $C_{gs1}$  sees a resistance  $R_{sig}$ .
2. Capacitance  $C_{gd1}$  sees a resistance  $R_{gd1}$ , which can be obtained by adapting the formula in Eq. (9.84) to

$$R_{gd1} = (1 + g_{m1}R_{d1})R_{sig} + R_{d1} \quad (9.116)$$

where  $R_{d1}$ , the total resistance at  $D_1$ , is given by

$$R_{d1} = r_{o1} \parallel R_{in2} = r_{o1} \parallel \frac{r_{o2} + R_L}{g_{m2}r_{o2}} \quad (9.117)$$

3. Capacitance  $(C_{db1} + C_{gs2})$  sees a resistance  $R_{d1}$ .
4. Capacitance  $(C_L + C_{gd2})$  sees a resistance  $(R_L \parallel R_o)$  where  $R_o$  is given by

$$R_o = r_{o2} + r_{o1} + (g_{m2}r_{o2})r_{o1}$$

$$\begin{aligned} \tau_H = & C_{gs1}R_{sig} + C_{gd1}[(1 + g_{m1}R_{d1})R_{sig} + R_{d1}] \\ & + (C_{db1} + C_{gs2})R_{d1} + (C_L + C_{gd2})(R_L \parallel R_o) \end{aligned}$$

$$f_H \simeq \frac{1}{2\pi\tau_H}$$





















COURTESY LEGO

## References

Microelectronic Circuits by Adel S. Sedra & Kenneth C. Smith. Saunders College Publishing

“Chapter 7: Frequency Response”, a lecture note by Prof. Yang Hua, Ph.D., Department of Electronic Engineering, Shanghai Jiao Tong University (SJTU), Shanghai, China