King Mongkut's University of Technology Thonburi
Final Exam of Second Semester, Academic Year 2007

COURSE CPE112 Discrete Mathematics for Computer Engineers
Computer Engineering Dept, 1st Yr.

Monday 10 March 2008 **09.00-12.00 h.**

Instructions

1. This examination contains 3 problems, 6 pages (including this cover page).
2. The answers must be written in the answer book.
3. Students are allowed to use calculators.
4. Students are allowed to use dictionaries or electronic dictionaries.
5. No books, notes, or any other documents can be taken into the examination room.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room. Students must not take the examination and the answers out of the examination room.

Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by

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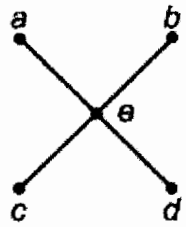
This examination is approved by the department of computer engineering

Associate Professor Dr. Suthep Madarasami

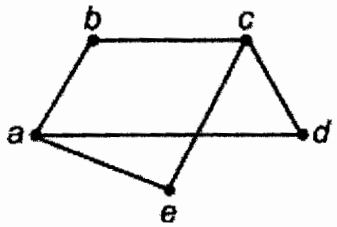
1. Graphs and Trees (54%)

1.1) Determine whether the following graph is bipartite. If it is bipartite, show the sets V_1 and V_2. (1% for each: total 5%)

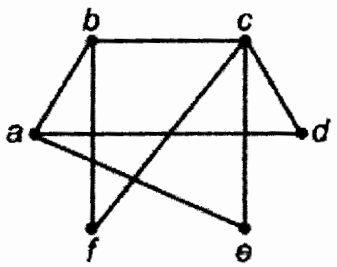
1.1.1)



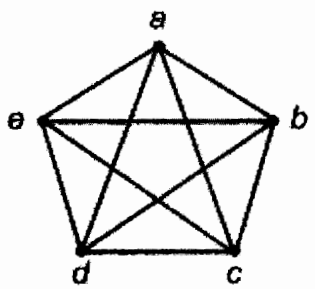
1.1.2)



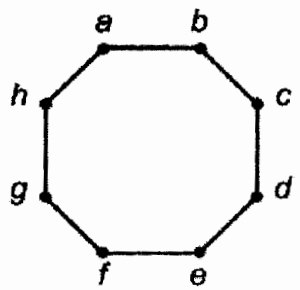
1.1.3)



1.1.4)



1.1.5)

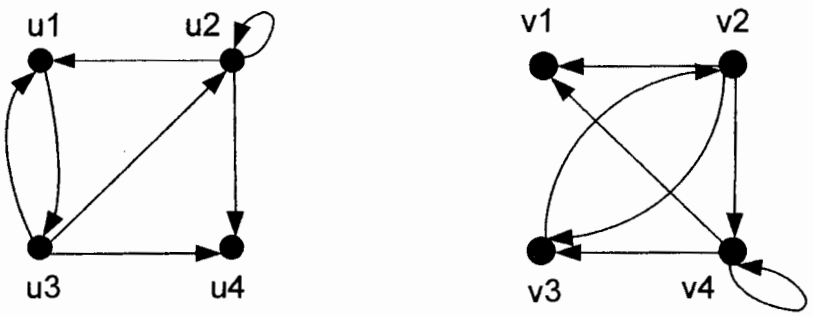


1.2) Determine whether the following pair of graphs is isomorphic. If it is isomorphic, show the function. (14%)

Definition The directed graph $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are isomorphic if there is one-to-one and onto function $f:V_1 \rightarrow V_2$ such that

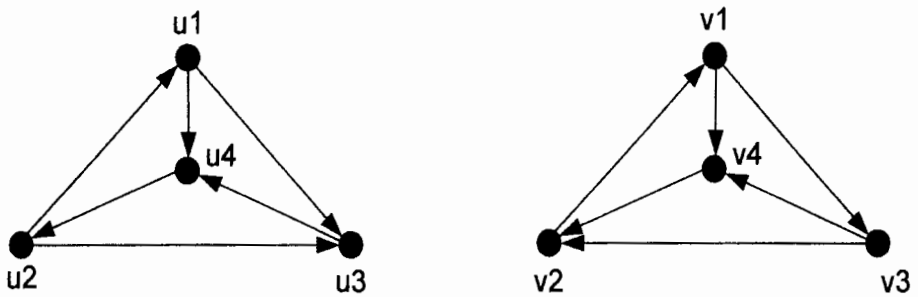
For all pairs of vertices *a* and *b* in V1, $(a,b) \in E_1$ iff $(f(a),f(b)) \in E_2$

1.2.1) (3%)

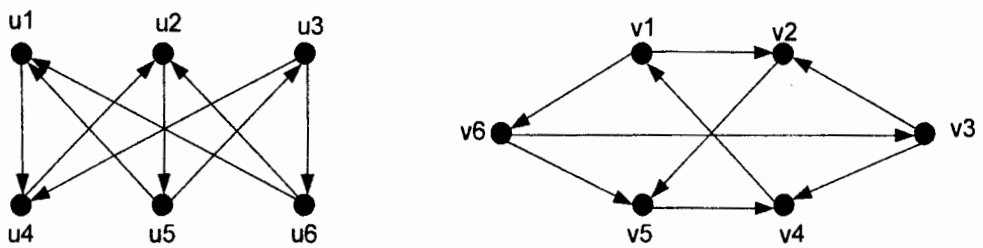


1.2.2) (3%)

1.2.3) (4%)

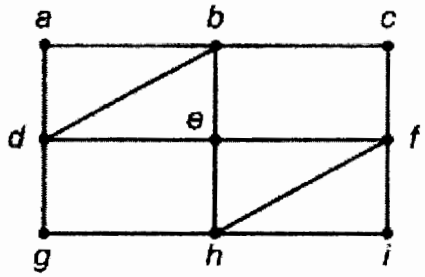


1.2.4) (4%)

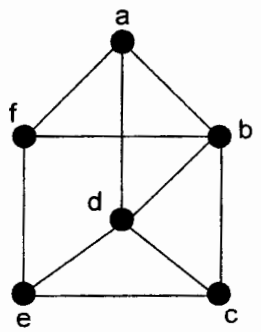


1.3) Euler Paths and Circuits

1.3.1) Determine whether the following graph has Euler circuit. If there is Euler circuit, show it. (2%)

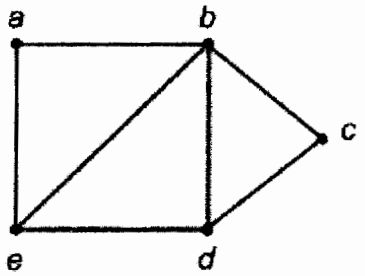


1.3.2) Determine whether the following graph has Euler circuit or path. If there is Euler circuit, show it. If there is Euler path, show it (2%)

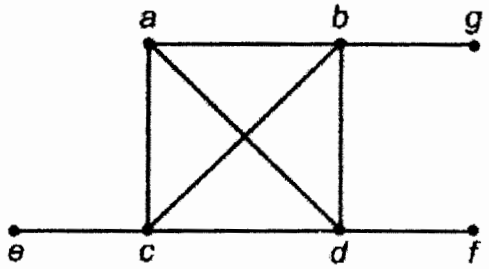


1.4) Hamilton paths and circuits

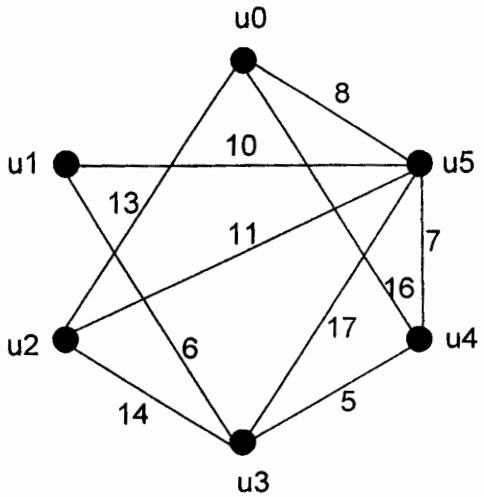
1.4.1) Determine whether the following graph has a Hamilton circuit. If it has, shows the Hamilton circuit. (2%)



1.4.2) Determine whether the following graph has a Hamilton path. If it has, shows the Hamilton path. (2%)



1.5) Find the shortest path from *u0* to *u3* in the following graph by using Dijkstra's algorithm. Show a selected edge at each iteration. For example, at 1st iteration, (*u0,u5*) is selected. (10%)



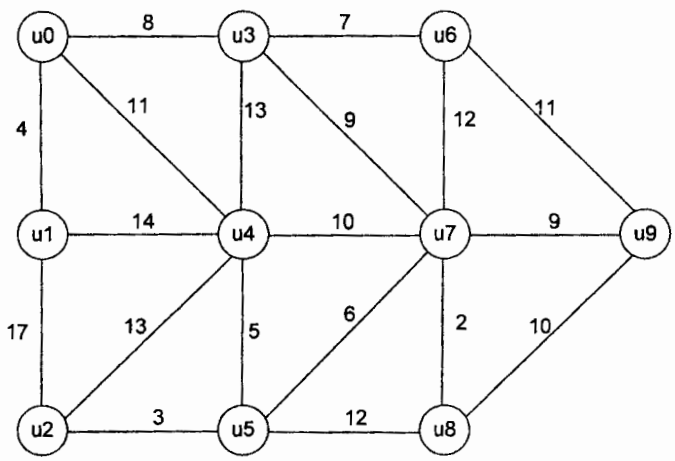
1.6) Tree properties

1.6.1) Consider a full 3-nary tree which has 30 nodes. How many number of edges does the tree have? (2%)

1.6.2) Consider a balanced but non-full 0-nary tree which has the height at 8. What is the maximal number of leaves of the tree? (2%)

1.6.3) Consider a balanced 6-ary tree which has 220 leaves. What is the height of the tree? (3%)

1.7) Use Prim's algorithm to find a minimum spanning tree of the following graph and the starting node is *u0*. Show a selected edge at each iteration. (10%).



2. Boolean Algebra (20%)

Find sum-of-product expansion of the following Boolean functions and then use Karnaugh Map technique to simplify them

2.1) $f(x,y,z)=xyz+\bar{x}yz+x\bar{y}\bar{z}+\bar{x}\bar{y}z$ (8%)

2.2) $f(x,y,z)=\bar{y}(\overline{xz})+yx+y\bar{z}$ (12%)

3. Automata (26%)

3.1) Determine whether 1011 is in the languages which are represented by the following regular expressions (5%)

3.1.1) 10*1*

3.1.2) 1*01(0 ∪ 1)

3.1.3) 0*(10 ∪ 11)*

3.1.4) 10*1(00)*

3.1.5) (10)* ∪ (11)*

3.2) Let Σ={*a, b*}. Write down regular expression for the following languages.

3.2.1) All strings that contain *aaaaa* as a substring. (3%)

3.2.2) All strings such that the first letter is the same as the last letter. For example, {aa, bb,...} (4%)

3.3) Let Σ={*a, b*}. Construct a DFA which accepts the language described by the regular expression *a(a ∪ b)*b* ? (14 %)