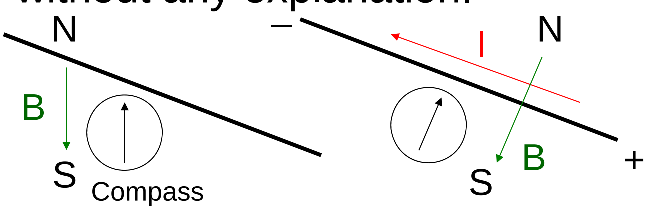
Electrical Indicating Instruments

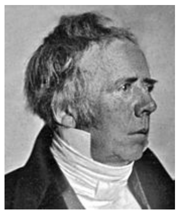
EIE 240 Electrical and Electronic Measurement
Class 4, February 6, 2015

1

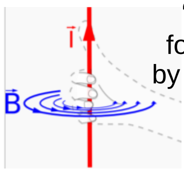
Analogue Meter's Concept

- Han Oersted, in 1820, noted his finding without any explanation.

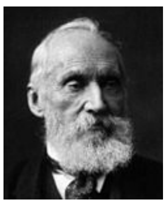


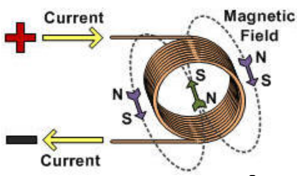


- Lord Kelvin made more sensitivity to a current.



"Magnetic Field for Line of Current by Biot Savart's Law"



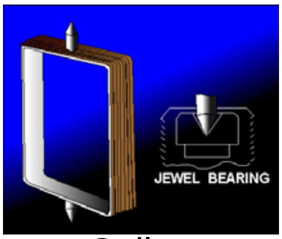


Right-hand Rule

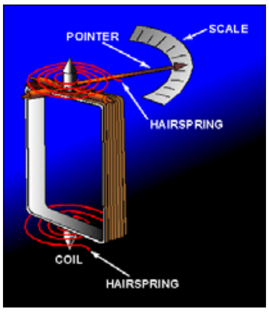
2

Electromechanical Instrument

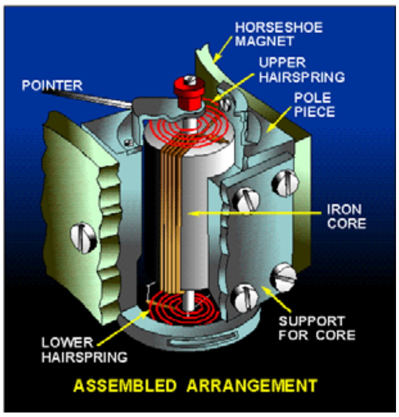
- Moving Coil Galvanometer
  http://youtu.be/_sD_5iyHl3s
- Permanent-Magnet Moving-Coil (PMMC) developed in 1881



Coil



Pointer

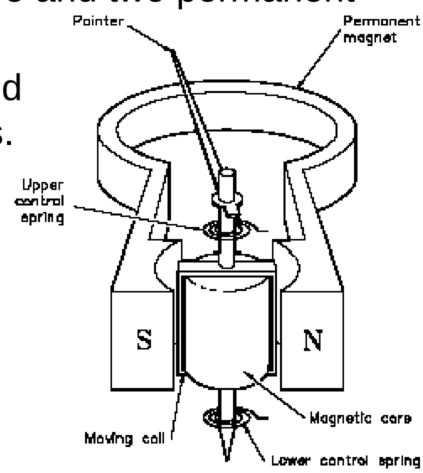


Permanent Magnet

3

- A wire coil is attached to a shaft that pivots on two jewel bearing.
- The coil can rotate in a space between a cylindrical soft-iron core and two permanent magnetic pole pieces.
- The rotation is opposed by two fine hairsprings.

Deflecting Force
= Controlling Force

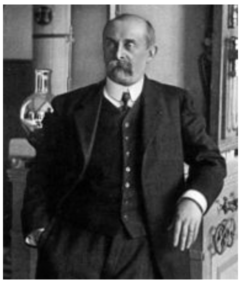


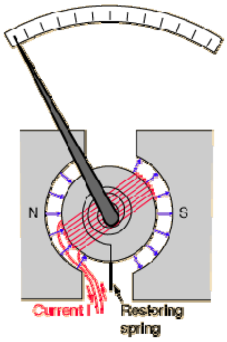
4

- Jacques D'Arsonval's Movement

  When a current is passed through the coil it rotates, the angle through which it rotates being proportional to the current (0.0000001 – 1 μA).

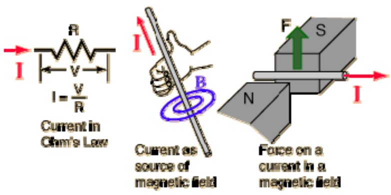
  The magnetic field is designed (magnetic pole piece's shape) that it is always at the right angles to the coil sides no matter what angle the coil has rotated through.

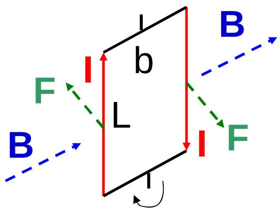




5

- Electric Current





Moving Coil

- Fleming's Left-Hand Rule

$F = L\, I \times B$

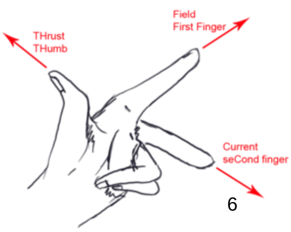
"Magnetic Force from a Straight Current-carrying Conductor"

Force (Newton)

Coil Side Length (Metre)

Current (Ampere)

Magnetic Field (Tesla)



6

- Torque (moment) is an angular force defined by linear force multiplied by a radius.

Damping Torque in Coil = F (b/2)

$= B\ I\ L\ b\ /\ 2$

$$Torque_{total} = 2\ (B\ I\ L\ b\ /\ 2) = B\ I\ L\ b$$

$$= B\ I\ A \qquad ,\ \text{Area}\ A = Lb$$

for N coils, $Torque_{total} = N\ B\ I\ A$

$$= K_{coil}\ I \qquad ,\ K_{coil} = NBA$$

7

- Controlling torque in springs

$$Torque_{spring} = K_s\ \theta$$

- Critical damping or balancing forces (Newton's 3rd Law)

Action = Reaction

$$Torque_{total} = Torque_{spring}$$

$$K_{coil}\ I = K_s\ \theta$$

$$\theta = (K_{coil}\ /\ K_s)\ I$$

- If the magnetic field is not uniform throughout the entire region, the scales are nonlinear!

8

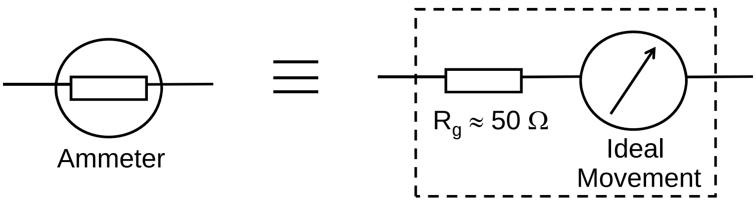
Galvanometer

- Galvanometer with a zero at the center of the scale used in DC instruments that can detect current flow in either direction
- Galvanometer with a zero at the left end of the scale indicates an upscale reading only for the proper way of connecting the meter into the circuit

9

Equivalent Ammeter Circuit

- The resistance of the meter coil and leads introduces a departure from the ideal ammeter behavior. The model usually used to describe an ammeter in equivalent circuit is a resistance R_g in series with an ideal ammeter (no resistance)



10

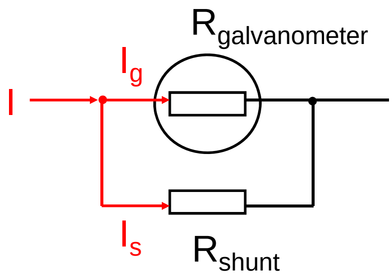
Full-Scale-Deflection Currents

- Current range is 10 μA – 20 mA
- Shunt resistor connected in parallel

$$V_s = V_g$$

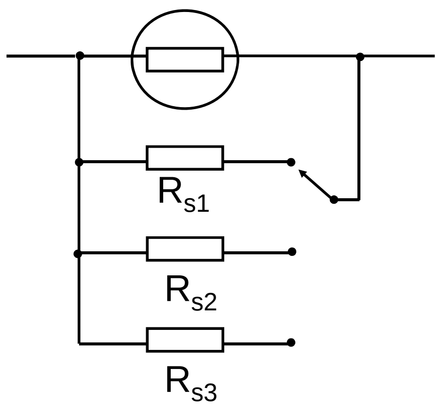
$$(I - I_g)R_s = I_g R_g$$

$$I = \underbrace{(R_s+R_g)/R_s}_{\text{Scaling factor}}\ I_g$$



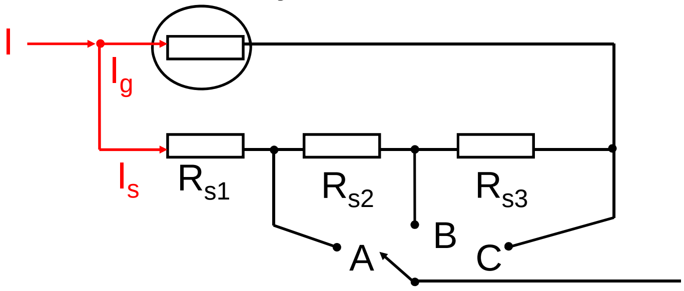
11

- Multi-range shunt can be made by switching into a circuit



12

- Universal shunt or “Ayrton shunt”

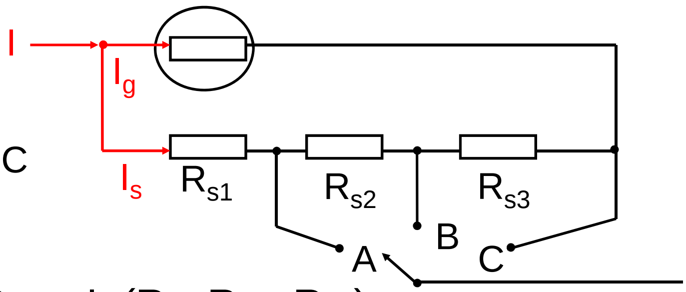


Ayrton shunt named after its inventor William E. Ayrton is a high-resistance shunt used in galvanometers to increase their range.

It eliminates the possibility of having a meter without a shunt which is a serious concern.

- Universal shunt or “Ayrton shunt” (Cont’d)

Range
A > B > C



A: $(I - I_g)R_{s1} = I_g\,(R_g+R_{s2}+R_{s3})$

$$I = (R_g+R_{s1}+R_{s2}+R_{s3})/R_{s1}\; I_g$$

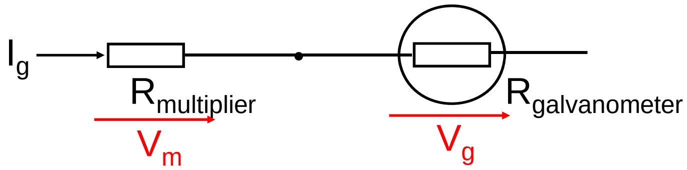
B: $(I - I_g)(R_{s1}+R_{s2}) = I_g\,(R_g+R_{s3})$

$$I = (R_g+R_{s1}+R_{s2}+R_{s3})/(R_{s1}+R_{s2})\; I_g$$

C: $$I = (R_g+R_{s1}+R_{s2}+R_{s3})/(R_{s1}+R_{s2}+R_{s3})\; I_g$$

Full-Scale-Deflection Voltages

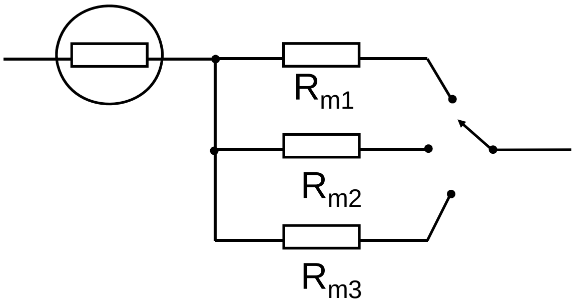
- Since V = IR, the response of a moving coil meter responding to a current is also proportional to the potential difference across the meter.
- However, because R_g and I_g are low, it can only be used for low voltages (≈ 0.05 V).
- Multiplier resistor can be connected in series



$$V = I_gR_m + I_gR_g = (R_m+R_g)\ I_g$$

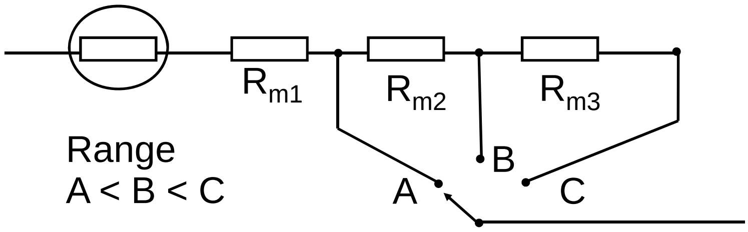
15

- Multi-range voltmeter



16

• Multi-range voltmeter with a chain arrangement

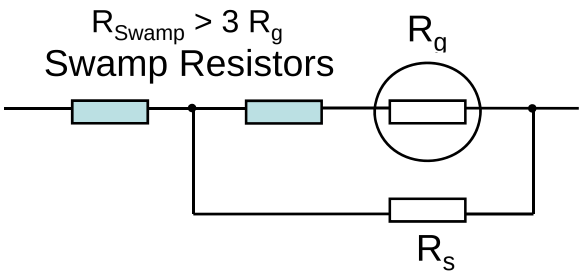


$$A: V = I_gR_g + I_gR_{m1} = (R_m+R_g)\ I_g$$
$$B: V = I_gR_g + I_gR_{m1} + I_gR_{m2} = (R_{m1}+R_{m2}+R_g)\ I_g$$
$$C: V = I_gR_g + I_gR_{m1} + I_gR_{m2} + I_gR_{m3}$$
$$= (R_{m1}+R_{m2}+R_{m3}+R_g)\ I_g$$

17

Temperature on Moving-Coil Meter

- Higher temperature of coil (tin copper wire), higher resistance, lower reading ⇒ decrease spring tension
- Measured value is decreased 0.2% for increasing of 1°C temperature.
- Swamp resistor (manganin wire), whose resistance changes slowly with temperature, connected in series
- However, the sensitivity is decreased.



18

Sensitivity on Moving-Coil Meter

- Sensitivity = Pointer Change / Input Change

$$S = \theta / I$$

or

$$S = 1 / I_{fsd}$$
$$= R / V_{fsd}$$

19

Ammeter & Voltmeter Loading

- Systematic loading error
  $\Rightarrow$ Calculation by using Thévenin's theorem in Lecture 3

20

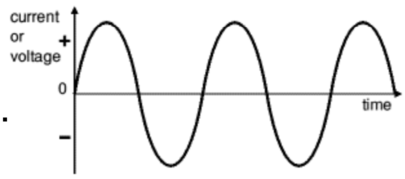
Sources and Loads

- An electrical circuit consists of nothing more than “sources” and “loads”.
- Source is to produces an electrical energy, e.g. a chemical battery, an electronic power supply, or a mechanical generator.
- Load is to be powered by that electrical energy, e.g. a light bulb, an electronic clock, an electric fan, or just a resistor.
- What sources are AC? And what kinds of loads do not care which way current flows through them?

21

Alternating Current

- A current flows one way, then the other way, continually reversing direction.
- The usual waveform of an AC power is a sine wave that the change is so regular. The average value is zero (integration in a period).



AC from a power supply

- Main electricity in Thailand has a frequency of 50 Hz (0.02 sec/cycle), and 60 Hz in US.
- Voltage is continually changing between positive (+) and negative (–). The effective voltage in Thailand is 220 V, and 120 V in US.

22

Why Use Alternating Current?

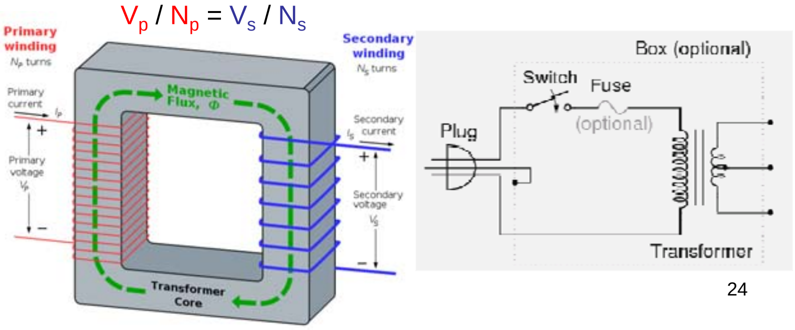
- In transmission of power, $P_{load} = IV$, the overhead wires are not a perfect conductor and exhibit some resistance, R. The absorbed energy is dissipated as waste heat. The lost power is $P_{lost} = I^2R$.
- At the same power, if the current is doubled (voltage reduced by half), $P_{load} = (2I)(V/2)$, the lost power is four times greater, $P_{lost} = (2I)^2R = 4I^2R$. To minimize that loss, we have to use much larger wires, and pay a high price for all that extra copper.

23

Why Use Alternating Current? (Cont'd)

- The solution is to use a transformer that can convert AC power at a higher/lower voltage with very slight losses. In practice, AC generators create electricity at a reasonable voltage, then we use transformers to step it up to very high levels (≈ 100,000 V) for long-distance transmission with a low current, and then use additional transformers to step it back down for local distribution to individual homes in safe value.

$V_p / N_p = V_s / N_s$



24

Averaging Power for AC

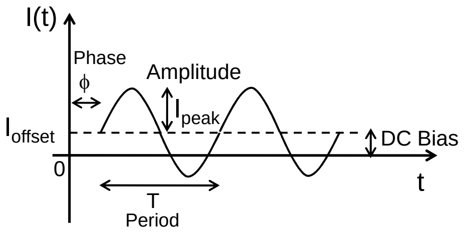
- $P(t) = I^2(t)\ R$ is just an instantaneous power at a time t.
- Mean power or the total energy converted in one cycle is

$$
\begin{aligned}
P &= 1/T \int_{t=0\to T} P\ dt \qquad , T = \text{period} \\
&= 1/T \int_{t=0\to T} I^2R\ dt \\
&= R\ [\ 1/T \int_{t=0\to T} I^2\ dt\ ] \\
&= R\ [\ \mathrm{sqrt}(1/T \int_{t=0\to T} I^2\ dt)\ ]^2 \\
&= R\ (I_{rms})^2
\end{aligned}
$$

25

Sine Wave

- $y = \sin(\theta)$
  $= \sin(\omega t)$
  $= \sin(2\pi t/T)$, $\omega = 2\pi/T$ for sinewave
  $= \sin(2\pi f t)$, $f = 1/T$
- $I(t) = I_{offset} + I_{peak}\sin(\omega t - \phi)$

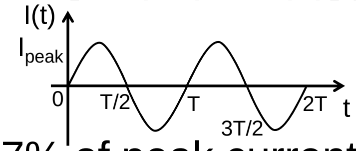


26

RMS or Effective Value

For only sinusoidal wave, $\omega = 2\pi/T$

$$
\begin{aligned}
I_{root\text{-}mean\text{-}square} &= \text{sqrt}(I^2_{average}) = \text{sqrt}(1/T\textstyle\int_{t=0\to T} I^2(t)dt) \\
&= \text{sqrt}(\ 1/T \textstyle\int_{t=0\to T} [\ I_{peak}\sin(\omega t)\ ]^2\ dt\ ) \\
&= \text{sqrt}(\ (I_{peak})^2/T \textstyle\int_{t=0\to T} \sin^2(\omega t)\ dt\ ) \\
&= \text{sqrt}(\ (I_{peak})^2/2T \textstyle\int_{t=0\to T} 1-\cos(2\omega t)\ dt\ ) \\
&= \text{sqrt}(\ (I_{peak})^2/2T\ \{\ T - [\sin(4\pi)-\sin(0)]\ \}\ ) \\
&= \text{sqrt}(\ (I_{peak})^2/2\ ) \\
&= I_{peak}\ /\ \text{sqrt}(2) \\
&= 0.707\ I_{peak} \quad \text{(70.7\% of peak current)}
\end{aligned}
$$

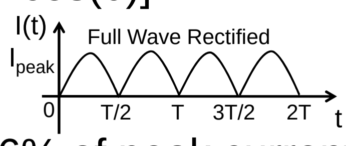


27

Positive Half-Cycle Averaging

For only sinusoidal wave, $T=2\pi$ and $\omega=2\pi/T=1$

$$
\begin{aligned}
I_{+\ half\ cycle} &= I_{average} = 1/(T/2) \textstyle\int_{t=0\to T/2} I(t)\ dt \\
&= 2/T \textstyle\int_{t=0\to T/2} I_{peak}\sin(\omega t)\ dt \\
&= 2I_{peak}/T \textstyle\int_{t=0\to T/2} \sin(\omega t)\ dt \\
&= 2I_{peak}/T\ [\ -\cos(\omega t)\ ]_{t=0\to T/2} \\
&= 2I_{peak}/T\ [\ -\cos(\omega T/2) + \cos(0)\ ] \\
&= 2I_{peak}/T\ [-\cos(\pi)+\cos(0)] \\
&= 4I_{peak}\ /\ T \\
&= 2I_{peak}\ /\ \pi \\
&= 0.636\ I_{peak} \quad \text{(63.6\% of peak current)}
\end{aligned}
$$

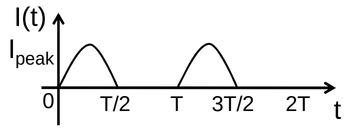


28

Half-Wave Rectified Sine

For only sinusoidal wave, $T=2\pi$ and $\omega=2\pi/T=1$

$$
\begin{aligned}
I_{+\text{ half wave}} &= 1/T \int_{t=0\to T} I(t)\,dt \\
&= 1/T \int_{t=0\to T/2} I_{peak}\sin(\omega t)\,dt \\
&= I_{peak}/T \int_{t=0\to T/2} \sin(\omega t)\,dt \\
&= I_{peak}/T\,[\,-\cos(\omega t)\,]_{t=0\to T/2} \\
&= 2I_{peak}/T \\
&= I_{peak}/\pi \\
&= 0.318\, I_{peak} \quad \text{(31.8\% of peak current)}
\end{aligned}
$$



29

Form Factor

- Form factor is a calibration constant or the ratio between average value and rms value,

  $$FF = Value_{rms} / Value_{avg}$$

  for sin, $FF = (I_{peak}/sqrt(2)) / (2I_{peak}/\pi)$

  $$= \pi / 2sqrt(2)$$

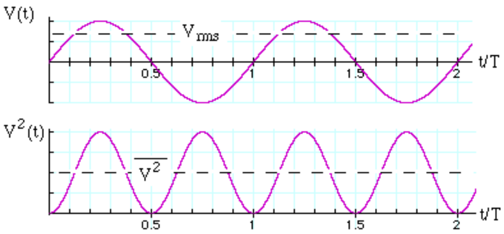
  $$= 1.11 \qquad (111\%)$$

- If the AC signal is not a pure sin wave, the meter still reads an average value of the rectified wave. However, the form factor no longer is 1.11. It is based on the shape of the signal.

30

for Sinusoidal Wave

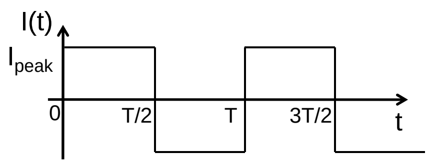
- $I_{rms} = 0.707\ I_{peak}$
- $V_{rms} = 0.707\ V_{peak}$
- $I_{+\ half\ cycle} = 0.636\ I_{peak}$
- $V_{+\ half\ cycle} = 0.636\ V_{peak}$
- $FF_{Sinusoidal} = 1.11$



31

for Square Wave

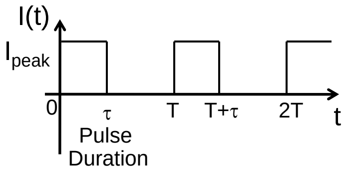
- $I_{rms} = sqrt(\ 1/(T/2) \int_{t=0\rightarrow T/2} I^2(t)\ dt\ )$
  $= sqrt(\ 2/T \int_{t=0\rightarrow T/2} I^2_{peak}\ dt\ )$
  $= sqrt(\ 2I^2_{peak}/T \int_{t=0\rightarrow T/2} dt\ )$
  $= sqrt(\ 2I^2_{peak}/T\ [\ (T/2) - (0)\ ]\ )$
  $= sqrt(\ I^2_{peak}\ )$
  $= I_{peak}$
- $V_{rms} = V_{peak}$
- $I_{+\ half\ cycle} = I_{peak}$
- $V_{+\ half\ cycle} = V_{peak}$
- $FF_{Square} = 1$



32

for Pulse Train

- $I_{rms} = \text{sqrt}(\ 1/T \int_{t=0\to\tau} I^2(t)\ dt\ )$
  $= \text{sqrt}(\ 1/T \int_{t=0\to\tau} I^2_{peak}\ dt\ )$
  $= \text{sqrt}(\ I^2_{peak}/T \int_{t=0\to\tau} dt\ )$
  $= \text{sqrt}(\ I^2_{peak}/T\ [\ \tau - 0\ ]\ )$
  $= \text{sqrt}(\tau/T)\ I_{peak}$
  $= \text{sqrt}(D)\ I_{peak}$, Duty cycle, $D = \tau/T$



- $V_{rms} = \text{sqrt}(D)\ V_{peak}$
- Average value for DC pulse train signal

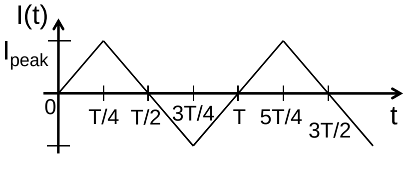
  $I_{avg} = 1/T \int_{t=0\to\tau} I_{peak}\ dt = D\ I_{peak}$

  $V_{avg} = D\ V_{peak}$

33

for Triangular Wave

- $I_{rms} = \text{sqrt}(\ 1/(T/4) \int_{t=0\to T/4} I^2(t)\ dt\ )$
  $= \text{sqrt}(\ 4/T \int_{t=0\to T/4} [\ (4I_{peak}/T)t\ ]^2\ dt\ )$
  $= \text{sqrt}(\ 4^3\ I^2_{peak}/T^3 \int_{t=0\to T/4} t^2\ dt\ )$
  $= \text{sqrt}(\ 4^3\ I^2_{peak}/T^3\ [\ (T/4)^3/3 - (0)^3/3\ ]\ )$
  $= \text{sqrt}(\ I^2_{peak} / 3\ )$
  $= \text{sqrt}(3)/3\ I_{peak}$
  $= 0.577\ I_{peak}$ (57.7% of peak current)
- $V_{rms} = 0.577\ V_{peak}$



34

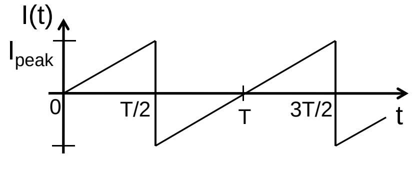
for Triangular Wave (Cont'd)

- $I_{+\text{ half cycle}} = 4/T \int_{t=0\to T/4} (4I_{peak}/T)t\ dt$
  $= 4^2\ I_{peak}/T^2 \int_{t=0\to T/4} t\ dt$
  $= 4^2\ I_{peak}/T^2\ [\ (T/4)^2/2 - (0)^2/2\ ]$
  $= I_{peak} / 2$
  $= 0.5\ I_{peak}$ (50% of peak current)
- $V_{+\text{ half cycle}} = 0.5\ V_{peak}$
- $FF_{Triangular} = 1.154$ (115.4%)

35

for Sawtooth Wave

- $I_{rms} = \text{sqrt}(\ 1/(T/2) \int_{t=0\to T/2} I^2(t)\ dt\ )$
  $= \text{sqrt}(\ 2/T \int_{t=0\to T/2} [\ (2I_{peak}/T)t\ ]^2\ dt\ )$
  $= \text{sqrt}(\ 2^3\ I^2_{peak}/T^3 \int_{t=0\to T/2} t^2\ dt\ )$
  $= \text{sqrt}(\ 2^3\ I^2_{peak}/T^3\ [\ (T/2)^3/3 - (0)^3/3\ ]\ )$
  $= \text{sqrt}(\ I^2_{peak} / 3\ )$
  $= \text{sqrt}(3)/3\ I_{peak}$
  $= 0.577\ I_{peak}$ (57.7% of peak current)
- $V_{rms} = 0.577\ V_{peak}$



36

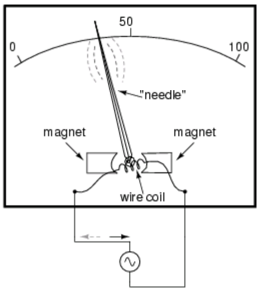
for Sawtooth Wave (Cont'd)

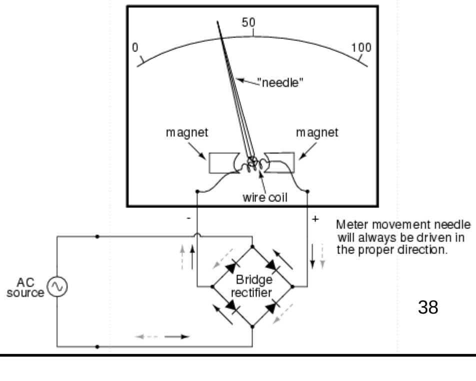
- $I_{+\text{ half cycle}} = 2/T \int_{t=0\to T/2} (2I_{peak}/T)t\, dt$
  $= 2^2\, I_{peak}/T^2 \int_{t=0\to T/2} t\, dt$
  $= 2^2\, I_{peak}/T^2\, [\, (T/2)^2/2 - (0)^2/2\,]$
  $= I_{peak} / 2$
  $= 0.5\, I_{peak}$ (50% of peak current)
- $V_{+\text{ half cycle}} = 0.5\, V_{peak}$
- $FF_{Sawtooth} = 1.154$ (115.4%)

37

AC on Moving-Coil Meter

- Alternating current = alternating torque
- Very low frequency → alternating pointer
- Higher frequencies → pointer is not moved.
- AC/DC converter is needed.

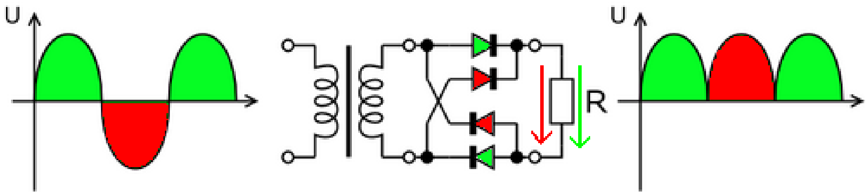




38

AC/DC Converter

- Full-wave rectifier circuit using bridge diodes
- Current flows only in one direction, anode to cathode, when diodes are forward biased.
- Inertia of moving coil → average value



- R → Galvanometer G

39

AC Calibration

Actually, AC meter responses to an average value of half-cycle wave. It can show root-mean-square values of sinusoidal wave by calibrating the scale by multiplying with form factor constant.

$$I_{rms} = FF \times I_{+\text{ half cycle}}$$
$$= 1.11 \times I_{+\text{ half cycle}}$$

40

Error for Other-Form Wave

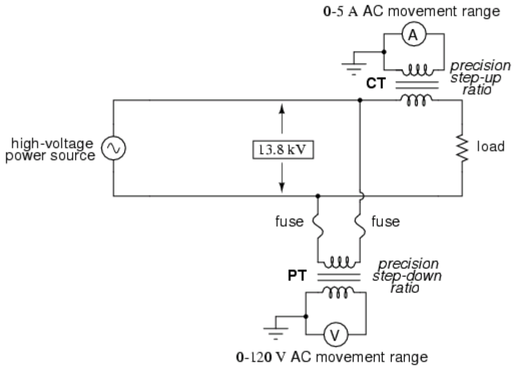
- If the input signal is not a sinusoidal wave form (FF $\neq$ 1.11), a reading error is,

$$\% \text{ Error} = (1.11 - FF) / FF \quad \times 100\%$$

41

Full-Scale-Deflection for AC Meter

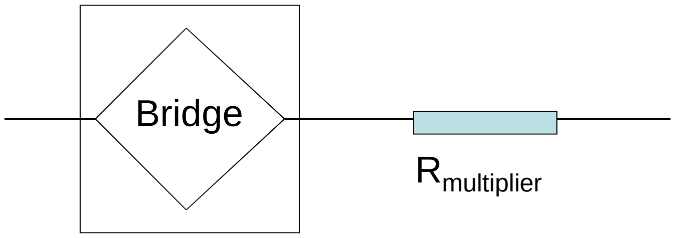
- Current transformer scales current down.
- Potential transformer scales voltage down.
- Each points in transformer can be tapped.



42

AC Ammeter and Voltmeter

- Shunt resistor connected in parallel or
- Multiplier resistance connected in series to the bridge



43

Sensitivity: AC Vs DC

- e.g. if the AC input is 10 V_{rms}

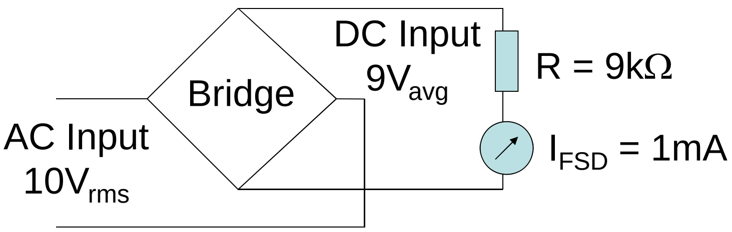
$$V_{peak} = 1.414 \times 10\text{ V}$$
$$= 14.14\text{ V}$$
$$V_{avg} = 0.636 \times 14.14\text{ V}$$
$$= 9\text{ V}$$

44

- Sensitivity = $V_{DC} / V_{AC} \quad \times 100\%$

  $= V_{avg} / V_{rms} \quad \times 100\%$

  $= 90\%$



45

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46