King Mongkut's University of Technology Thonburi
Final Examination of the 2^{nd} Semester, Academic Year 2553

Subject: CPE 222 Signals and Systems — **Computer Engineering Department 2^{nd} Yr.**
Wed. 2 March 2011 — 13:00 – 16:00

Instructions:

1. This exam has 6 questions (75 points), 4 pages (including the cover page).
2. This exam is closed-book, closed-note. Relevant tables and formulas are given in the last page.
3. A calculator is allowed.
4. Write down your solutions in the answer book. The solutions must be readable and have completed details to receive full credits.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room.

Students must not take the examination and the answers out of the examination room.
Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by
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This examination has been approved by the Department of Computer Engineering.

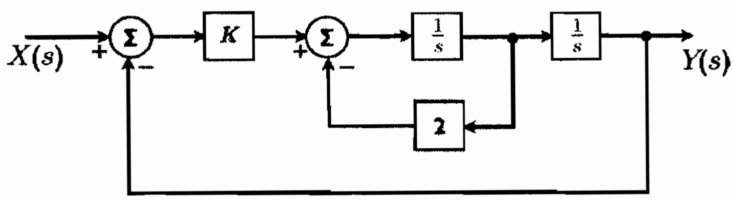
1. (15 points) Consider an LTIC with the following transfer function:

$$H(s) = \frac{s+5}{s^2+5s+6}$$

   (a) (5 points) If the input is $x(t) = e^{-3t}u(t)$ and all the initial conditions are zero, determine the system output $y(t)$.

   (b) (10 points) If the input is $x(t) = 2\cos(t+\frac{\pi}{3})u(t)$ and all the initial conditions are zero, determine the system output $y(t)$.

2. (15 points) Consider a feedback control system with the following block diagram:

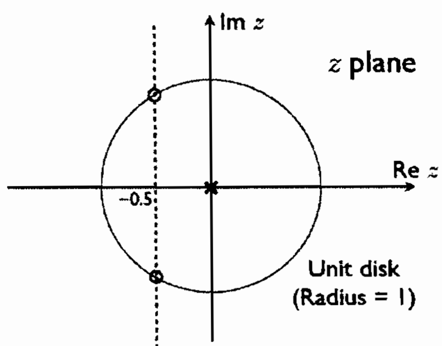


   (a) (5 points) Find the system transfer function in terms of K. What is the range of K for which the system is BIBO stable? Give a reason why.

   (b) (5 points) For $K = 5$, find the unit step response.

   (c) (5 points) What is the highest value of K that makes the unit step response non-oscillatory? Find the unit step response for such value of K.

3. (10 points) Consider an LTID system with two zeros and one pole on the z-plane as shown in the figure below:



   The response of this system is 1 for all n when the input is 1 for all n. Determine the impulse response of this system.

4. (10 points) Given the causal system function whose algebraic expression is

$$H(z) = \frac{z^3+2z^2+1}{z^3+3z^2+2},$$

   Determine the output when the input $x[n]$ are:

   (a) (5 points) $x[n] = 5$

   (b) (5 points) $x[n] = 0.5\cos(\pi n)$

5. (10 points) Consider an LTID system specified by the following equation:

$$y[n+2] - 0.3y[n+1] + 0.2y[n] = 3x[n+2] - 2x[n+1]$$

   (a) (5 points) Determine $y[n]$ if the input is $x[n] = u[n]$. What is the steady-state value of $y[n]$?

   (b) (5 points) Determine the steady-state response if the input is $x[n] = \cos(n - \frac{\pi}{3})u[n]$.

6. (15 points) Suppose you are to design a low-pass filter with the following specifications: Passband gain of 0.8, Passband frequency of 1 rad/s, Stopband gain of 0.2, and Stopband frequency of 3 rad/s.

   (a) (4 points) Determine a transfer function of the Butterworth filter that meets the above specifications.

   (b) (8 points) Using the impulse invariance method, determine the transfer function of a digital filter that realizes the Butterworth analog filter in part (a). Assume that the filter is band-limited to the frequency where the gain drops to about 1% of its maximum value.

   (c) (3 points) Determine the difference equation that describes the relationship between the output $y[n]$ and the sampled input $x[n]$ corresponding to the digital filter you obtained in part (b).

Transform Pairs and Formulas

Frequency Responses

LTIC with transfer function $H(s)$:

$$x(t) = Ae^{j\omega t} \Rightarrow y(t) = H(j\omega) \cdot Ae^{j\omega t}$$

$$x(t) = A\cos(\omega t) \Rightarrow y(t) = |H(j\omega)| \cdot A\cos(\omega t + \angle H(j\omega))$$

LTID with transfer function $H(z)$:

$$x[n] = Ae^{j\Omega} \Rightarrow y[n] = H(e^{j\Omega}) \cdot Ae^{j\Omega}$$

$$x[n] = A\cos(\Omega n) \Rightarrow y[n] = |H(e^{j\Omega})| \cdot A\cos(\Omega n + \angle H(e^{j\Omega}))$$

Transfer function of LTIC with zeros z_i, $1 \le i \le m$ and poles p_i, $1 \le i \le n$:

$$H(s) = K\frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}, \; K \text{ constant}$$

Transfer function of LTID with zeros z_i, $1 \le i \le m$ and poles p_i, $1 \le i \le n$:

$$H(z) = K\frac{\prod_{i=1}^{m}(z - z_i)}{\prod_{i=1}^{n}(z - p_i)}, \; K \text{ constant}$$

Laplace and Z transforms

$x(t)$	$X(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb}$
$\frac{dx}{dt}$	$sX(s) - x(0^-)$
$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$

$x[n]$	$X(z)$
$u[n]$	$\frac{z}{z-1}$
$a^n u[n]$	$\frac{z}{z-a}$
$na^n u[n]$	$\frac{az}{(z-a)^2}$
$r\|a\|^n\cos(\beta n+\theta)u[n]$	$\frac{0.5re^{j\theta}z}{z-a} + \frac{0.5re^{-j\theta}z}{z-a^*}, \; a = \|a\|e^{j\beta}$
$x[n-m]u[n]$	$\frac{1}{z^m}X(z) + \frac{1}{z^m}\sum_{k=1}^{m} x[-k]z^k$
$x[n+m]u[n]$	$z^mX(z) - z^m\sum_{k=1}^{m-1} x[k]z^{-k}$

Identities

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$-1 = e^{j\pi}$$

$$\pm j = e^{\pm j\pi/2}$$

$$a + jb = \sqrt{a^2+b^2}\,e^{j\tan^{-1}\frac{b}{a}}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2\cos(x)\sin(x)$$

$$\cos(A \mp B) = \cos(A)\cos(B) \pm \sin(A)\sin(B)$$

Bufferworth filter

$$2^{nd} \text{ order:} \quad H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2}$$

$$3^{rd} \text{ order:} \quad H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3} = \frac{\omega_c^3}{(s+\omega_c)(s^2 + \omega_c s + \omega_c^2)}$$

Magnitude response:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}}$$

Impulse Invariance Transformations

$$\frac{1}{s-\lambda} \Rightarrow \frac{Tz}{z - e^{\lambda T}}$$

$$\frac{1}{(s-\lambda)^2} \Rightarrow \frac{T^2 z e^{\lambda T}}{(z - e^{\lambda T})^2}$$

$$\frac{As + B}{s^2 + 2as + c} \Rightarrow \frac{Trz[z\cos\theta - e^{-aT}\cos(bT - \theta)]}{z^2 - (2e^{-aT}\cos bT)z + e^{-2aT}},$$

$$r = \sqrt{\frac{A^2c + B^2 - 2ABa}{c - a^2}}$$

$$b = \sqrt{c - a^2}$$

$$\theta = \tan^{-1}\left(\frac{Aa - B}{A\sqrt{c - a^2}}\right)$$