ENE 104
Electric Circuit Theory

Lecture 07:

The RLC Circuit

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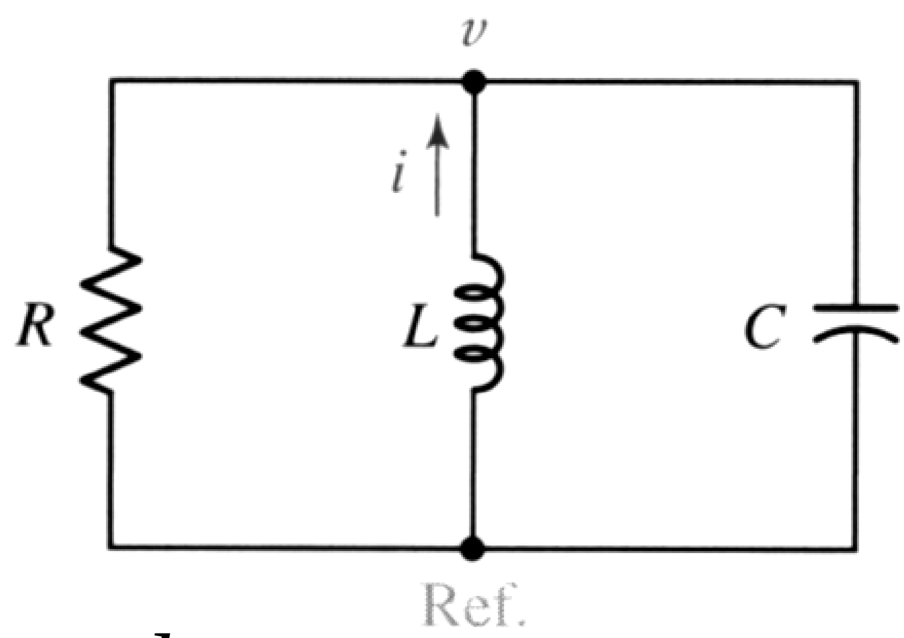
Objectives : Ch9

- the characteristic damping factor and resonant frequency for both series and parallel RLC circuits
- overdamped, critically damped, and underdamped response
- the complete response
- op amps

The Source-Free parallel Circuit:



The natural response:

$$i(0^+) = I_0$$

$$v(0^+) = V_0$$

The nodal equation:

$$\frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v dt' - i(t_0) + C\frac{dv}{dt} = 0$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

The Solution:

Assume: $$v = Ae^{st}$$

Then, $$CAs^2e^{st} + \frac{1}{R}Ase^{st} + \frac{1}{L}Ae^{st} = 0$$

Or, $$Ae^{st}\left(Cs^2 + \frac{1}{R}s + \frac{1}{L}\right) = 0$$

So, $$\underbrace{\left(Cs^2 + \frac{1}{R}s + \frac{1}{L}\right)}_{\text{the characteristic equation}} = 0$$

The Solution:

Two solutions: $$\left(Cs^2 + \frac{1}{R}s + \frac{1}{L}\right) = 0$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \longrightarrow v_1 = A_1 e^{s_1 t}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \longrightarrow v_2 = A_2 e^{s_2 t}$$

Satisfies,

$$C\frac{d^2 v_1}{dt^2} + \frac{1}{R}\frac{dv_1}{dt} + \frac{1}{L}v_1 = 0$$

$$C\frac{d^2 v_2}{dt^2} + \frac{1}{R}\frac{dv_2}{dt} + \frac{1}{L}v_2 = 0$$

The Solution:

$$\left.\begin{aligned} C\frac{d^2v_1}{dt^2}+\frac{1}{R}\frac{dv_1}{dt}+\frac{1}{L}v_1=0 \\ C\frac{d^2v_2}{dt^2}+\frac{1}{R}\frac{dv_2}{dt}+\frac{1}{L}v_2=0 \end{aligned}\right\} \quad C\frac{d^2(v_1+v_2)}{dt^2}+\frac{1}{R}\frac{d(v_1+v_2)}{dt}+\frac{1}{L}(v_1+v_2)=0$$

the general form of the natural response:

$$\boxed{v(t)=A_1e^{s_1t}+A_2e^{s_2t}}$$

Definition of Frequency Terms:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \begin{cases} s_1 = -\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 - \dfrac{1}{LC}} \\ s_2 = -\dfrac{1}{2RC} - \sqrt{\left(\dfrac{1}{2RC}\right)^2 - \dfrac{1}{LC}} \end{cases}$$

the resonant frequency: $\omega_0 = \dfrac{1}{\sqrt{LC}}$

the neper frequency, or
the exponential damping coefficient: $\alpha = \dfrac{1}{2RC}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Practice: 9.1

A parallel RLC circuit contains a 100-Ω resistor and has the parameter values $\alpha = 1000\ s^{-1}$ and $\omega_0 = 800$ rad/s. Find: (a) C; (b) L; (c) s_1; (d) s_2

$\alpha = 1000\ \text{s}^{-1}$ and $\omega_0 = 800$ rad/s, with $R = 100\ \Omega$.

(a) $$\alpha = \frac{1}{2RC} \quad \text{so} \quad C = \frac{1}{2R\alpha} = \underline{5\mu\text{F}}$$

(b) $$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{so} \quad L = \frac{1}{C\omega_o^2} = \underline{312.5\ \text{mH}}$$

(c) $$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = \underline{-\ 400\ \text{s}^{-1}}$$

(d) $$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = \underline{-\ 1600\ s^{-1}}$$

The Overdamped Parallel RLC:

$$\boxed{\alpha > \omega_0} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

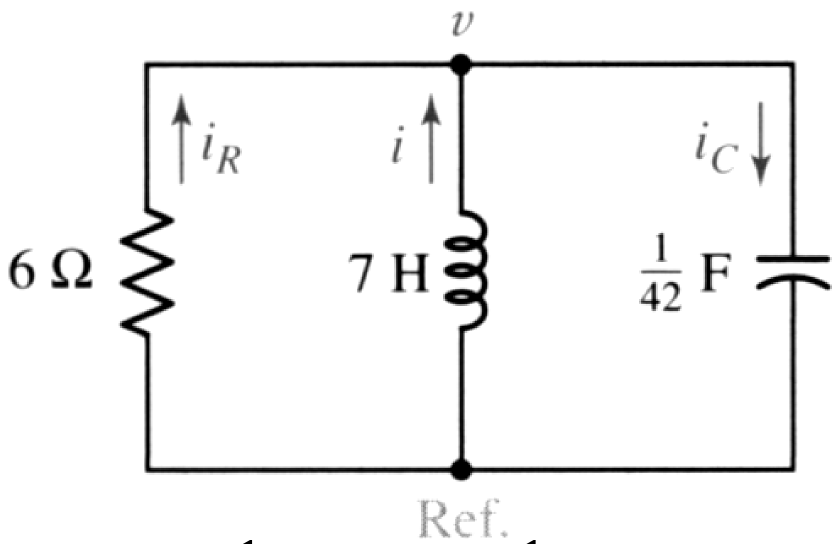
if $LC > 4R^2C^2$

then $\sqrt{\alpha^2 - \omega_0^2} < \alpha$

$$\left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right) < \left(-\alpha + \sqrt{\alpha^2 - \omega_0^2}\right) < 0$$

Both s_1 and s_2 are negative real number

Example:



The initial conditions:

$$i(0) = 10A.$$
$$v(0) = 0$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 6 \cdot {}^1\!/_{42}} = 3.5, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7 \cdot {}^1\!/_{42}}} = \sqrt{6}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -3.5 \pm \sqrt{3.5^2 - 6} = -1, -6$$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

Example: Finding A_1 and A_2

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

The initial conditions:

$$i(0) = 10 \quad A.$$
$$v(0) = 0 \quad V.$$

At t = 0,

$$v(0) = 0 = A_1 + A_2 \ldots (*) \quad \Rightarrow \quad \boxed{x(0) = A_1 + A_2 \ldots (*)}$$

And evaluating the derivative at t = 0^+,

$$\frac{dv}{dt} = -A_1 e^{-t} - 6A_2 e^{-6t}$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -A_1 - 6A_2 \ldots (**) \Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2 \ldots (**)}$$

Example: Finding A_1 and A_2

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

$$v(0) = 0 = A_1 + A_2 \ldots (*)$$

$$\left.\frac{dv}{dt}\right|_{t=0} = -A_1 - 6A_2 \ldots (**)$$

From $\quad i_C(t) = C\dfrac{dv_C(t)}{dt}$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C} = \frac{i(0^+) + i_R(0^+)}{C} = \frac{10+0}{1/42} = 420\, V/s.$$

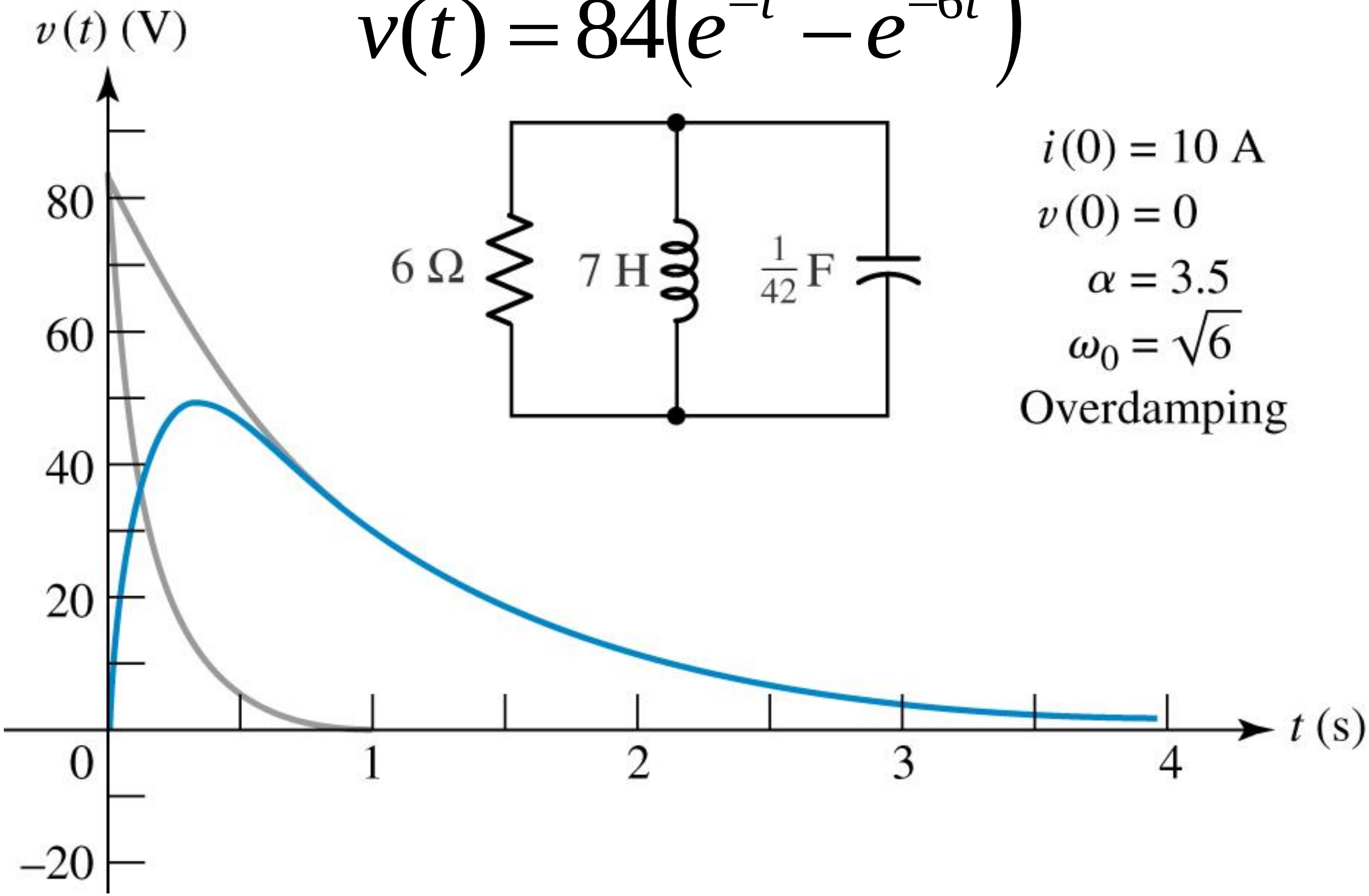
$$420 = -A_1 - 6A_2 \ldots (**)$$

$$\Rightarrow A_1 = 84, A_2 = -84$$

$$\therefore v(t) = 84e^{-t} - 84e^{-6t} = 84\left(e^{-t} - e^{-6t}\right)$$

Graphical Representation:

$$v(t) = 84\left(e^{-t} - e^{-6t}\right)$$



The response of the parallel network shown.

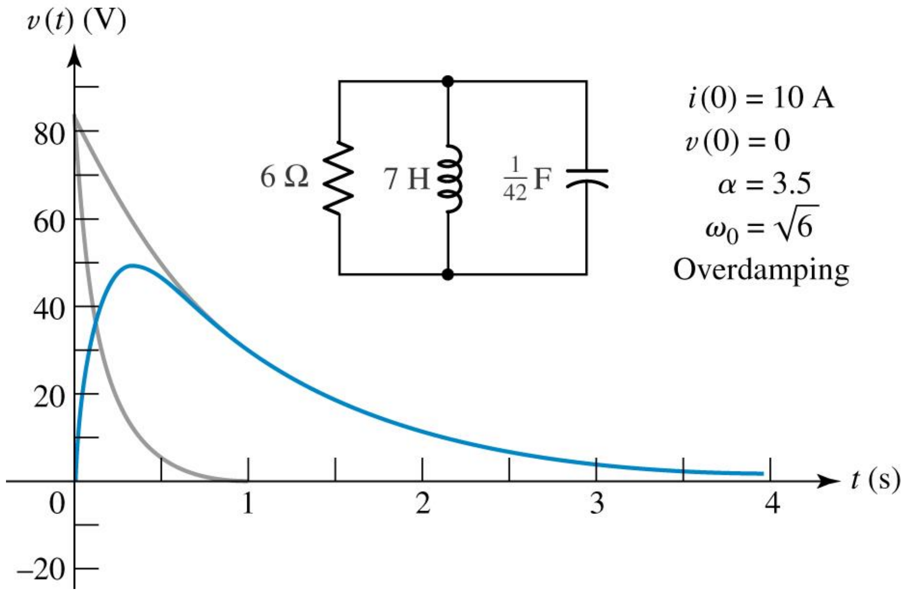
The settling time:

$$v(t) = 84\left(e^{-t} - e^{-6t}\right)$$

To determine maximum:

$$\frac{dv}{dt} = 84\left(-e^{-t} + 6e^{-6t}\right) = 0$$

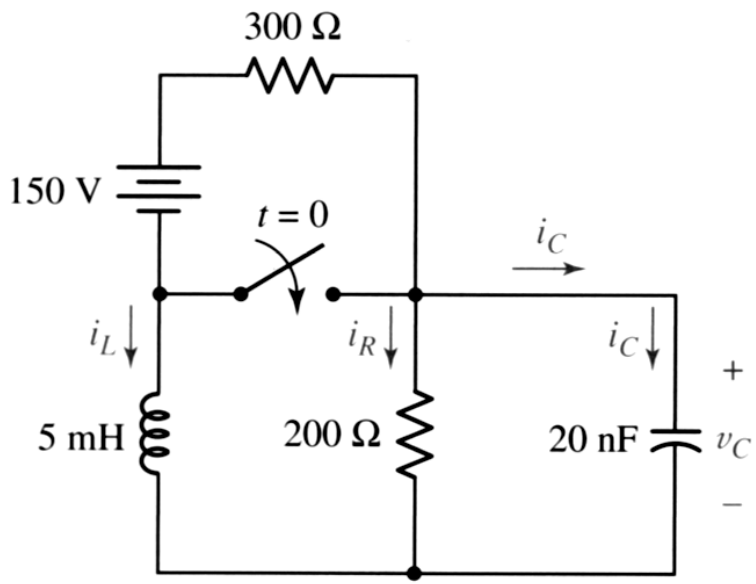
$$-e^{-t_m} + 6e^{-6t_m} = 0$$

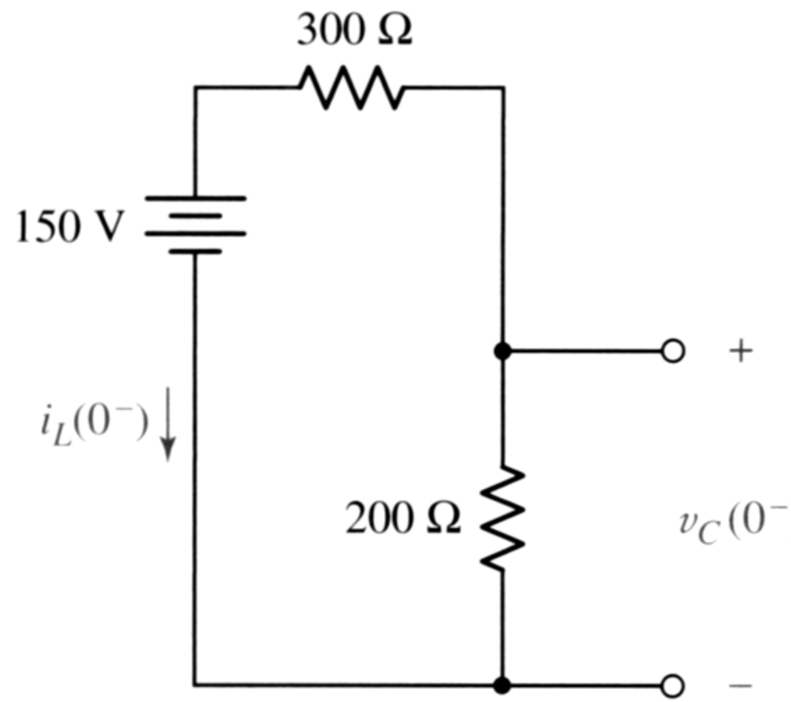


$$\Rightarrow t_m = 0.358 \quad s. \qquad \Rightarrow v(t_m) = 48.9 \quad V.$$

The settling time, t_s, is the time required for the response to drop to 1% (t_s =5.15 s.)

Example: find $v_C(t)$



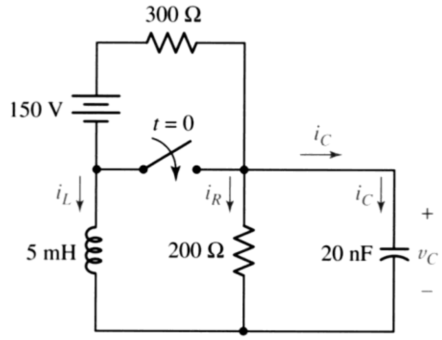


For t < 0, find the initial conditions;

$$i_L(0^-) = \frac{-150}{300\Omega + 200\Omega} = -0.3 \quad A. = i_L(0^+)$$

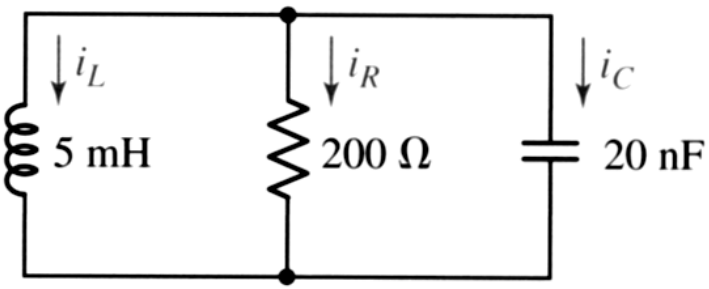
$$v_C(0^-) = -i_L(0^-) \cdot 200\Omega = 60 \quad V. = v_C(0^+)$$

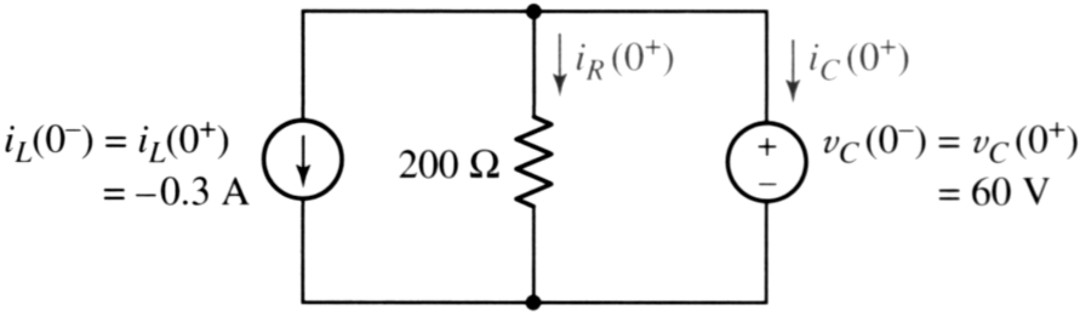
Example:



The initial conditions:

$$i_L(0^-) = i_L(0^+) = -0.3 \quad A.$$
$$v_C(0^-) = v_C(0^+) = 60 \quad V.$$





For t > 0;

$$\alpha = \frac{1}{2RC} = 125000 \quad s^{-1}, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 100000 \quad rad/s$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50000, \quad -200000 \quad s^{-1}$$

$$v_C(t) = A_1 e^{-50000t} + A_2 e^{-200000t}$$

Example: Finding A_1 and A_2

$$v_C(t) = A_1 e^{-50000t} + A_2 e^{-200000t}$$

The initial conditions:

$$i_L(0^-) = i_L(0^+) = -0.3 \quad A.$$
$$v_C(0^-) = v_C(0^+) = 60 \quad V.$$

At t = 0,

$$\Rightarrow \boxed{x(0) = A_1 + A_2 ...(*)}$$

$$v_C(0^+) = 60 = A_1 + A_2 ...(*)$$

And evaluating the derivative at t = 0,

$$\Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2 ...(**)}$$

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = -50000 A_1 - 200000 A_2 ...(**)$$

Example: Finding A_1 and A_2

$$v(t) = A_1 e^{-50000t} + A_2 e^{-200000t}$$

The initial conditions:

$$i_L(0^-) = i_L(0^+) = -0.3 \quad A.$$
$$v_C(0^-) = v_C(0^+) = 60 \quad V.$$

From $\quad i_C(t) = C\dfrac{dv_C(t)}{dt}$

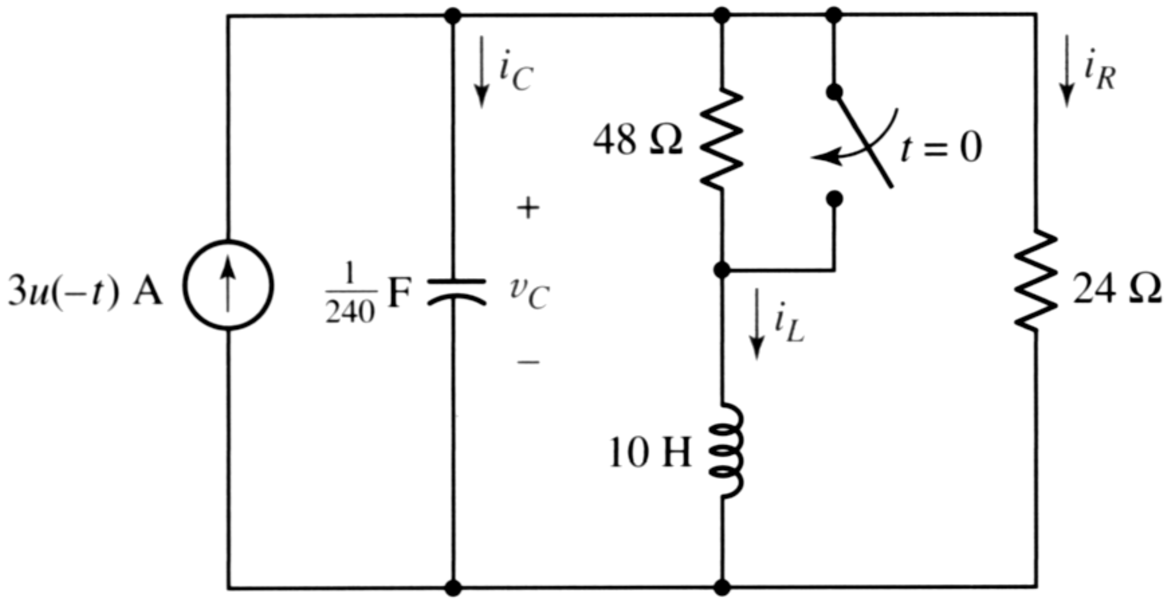
$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{-i_L(0^+) - i_R(0^+)}{C} = \frac{0.3 - {}^{60}\!/\!_{200\Omega}}{20\times 10^{-9}} = 0$$

$$0 = -50000A_1 - 200000A_2 \ldots (**)$$

$$\Rightarrow A_1 = 80, A_2 = -20$$

$$\therefore v_C(t) = 80e^{-50000t} - 20e^{-200000t} \; V. \quad , t > 0$$

Practice: 9.2 find …



$i_L(0^-) = \ldots \quad A.$

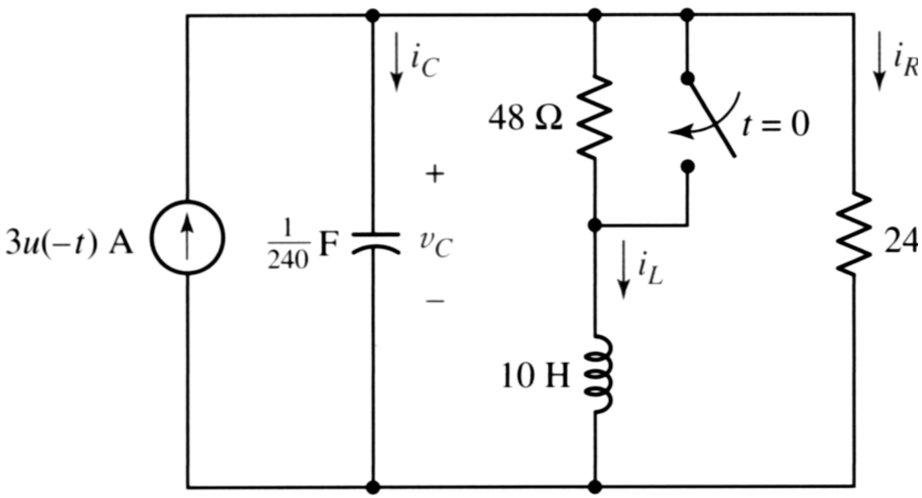
$v_C(0^-) = \ldots \quad V.$

$i_R(0^+) = \ldots \quad A.$

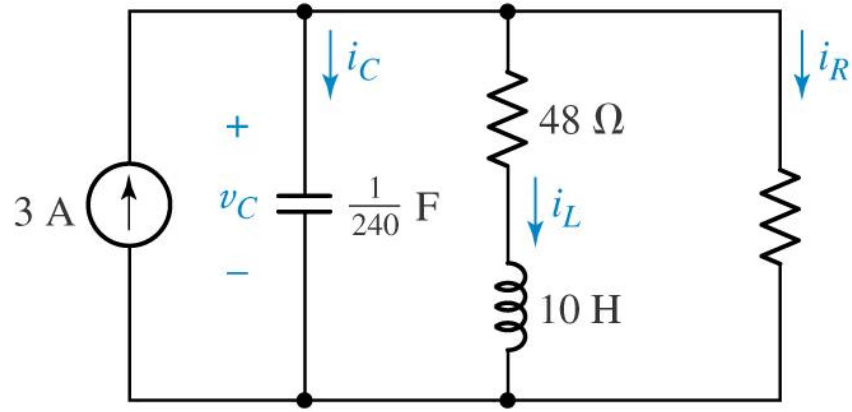
$i_C(0^+) = \ldots \quad A.$

$v_C(0.2) = \ldots \quad V.$

Practice : find …



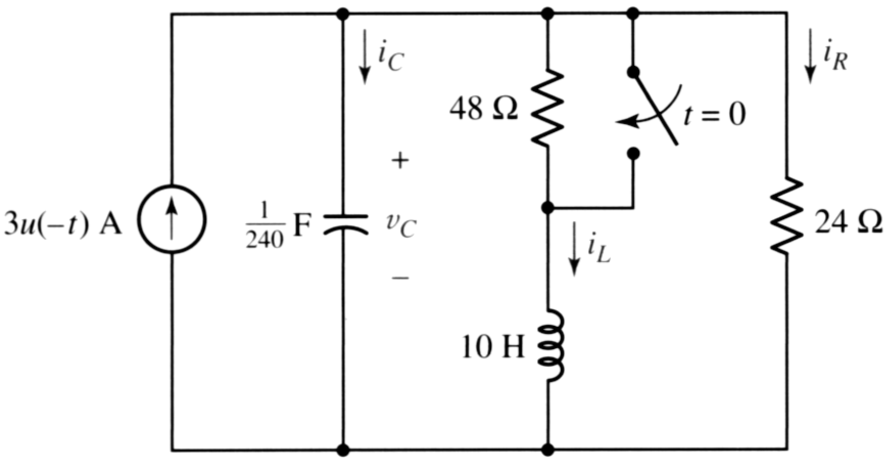
t < 0



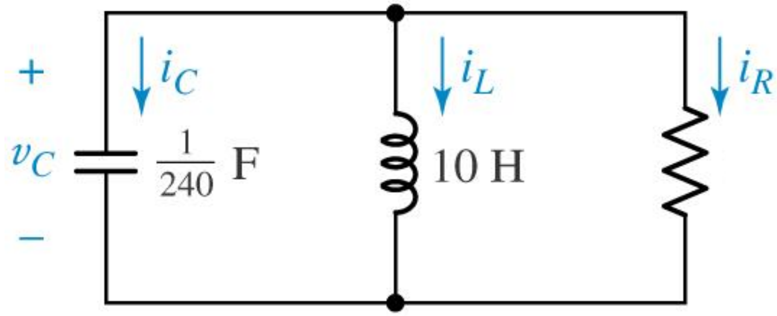
$$i_L(0^-) = 3 \cdot \frac{24\Omega}{24\Omega + 48\Omega} = 1 \quad A.$$

$$v_C(0^-) = i_L(0^-) \cdot 48\Omega = 48 \quad V.$$

Practice : find …



$t \geq 0$

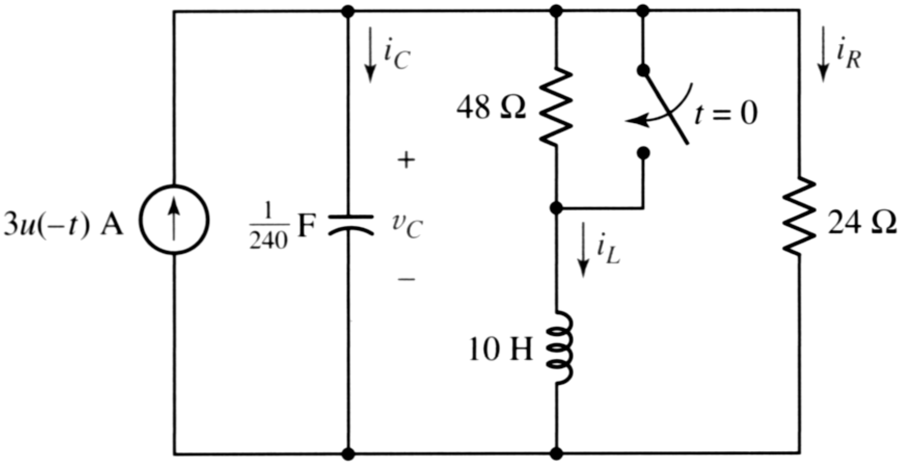


$$i_R(0^+) = \frac{v_C(0^+)}{24\Omega} = 2 \quad A.$$

$$i_C(0^+) = -i_R(0^+) - i_L(0^+) = -2 - 1 = -3 \quad A.$$

$$v_C(0.2) = \ldots \quad V.$$

Practice : find $v_C(t=0.2)$



The initial conditions:

$$i_L(0^-) = i_L(0^+) = 1 \quad A.$$

$$v_C(0^-) = v_C(0^+) = 48 \quad V.$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 24\Omega \cdot \frac{1}{240}F} \quad s^{-1}, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10H \cdot \frac{1}{240}F}} \quad rad/s$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad s^{-1}$$

Critical Damping:

$$\boxed{\alpha = \omega_0}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

or $LC = 4R^2C^2$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

thus $s_1 = s_2$

$$\alpha = \frac{1}{2RC}$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

Becomes: $$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \alpha^2 v = 0$$

The solution: $\Rightarrow$ $$\boxed{v(t) = e^{-\alpha t}\left(A_1 t + A_2\right)}$$

Critical Damping :

$$\alpha = \frac{1}{2RC} = \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7H \cdot \frac{1}{42} F}} = \sqrt{6} \quad rad/s$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha = -\sqrt{6}$$

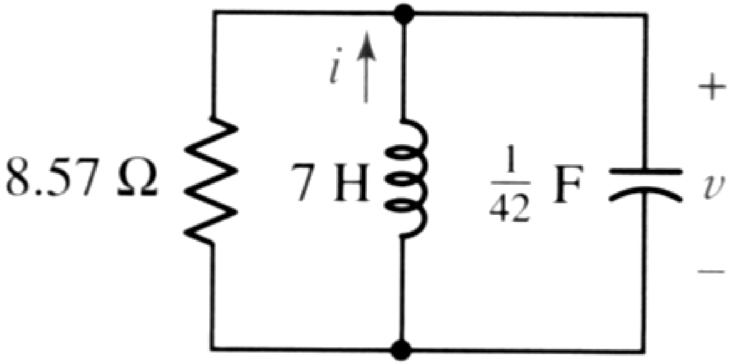
$$v(t) = e^{-\sqrt{6}t}\left(A_1 t + A_2\right)$$

$$6\Omega \rightarrow \frac{7\sqrt{6}}{2} \cong 8.57\Omega$$

The initial conditions:

$$i(0) = 10 \quad A.$$

$$v(0) = 0 \quad V.$$



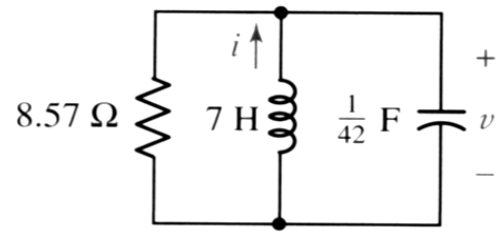
Critical Damping :

$$v(t) = e^{-\sqrt{6}t}\left(A_1 t + A_2\right)$$

The initial conditions:

$$i(0) = 10 \quad A.$$

$$v(0) = 0 \quad V.$$



At t = 0,

$$v(0) = 0 = A_2 \ldots (*)$$

$$\Rightarrow \boxed{x(0) = A_2 \ldots (*)}$$

And evaluating the derivative at t = 0,

$$\frac{dv}{dt} = -A_1 t\left(-\sqrt{6}\right)e^{-\sqrt{6}t} + A_1 e^{-\sqrt{6}t}$$

$$\left.\frac{dv}{dt}\right|_{t=0} = A_1 \ldots (**)$$

$$\Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0} = A_1 - \alpha A_2 \ldots (**)}$$

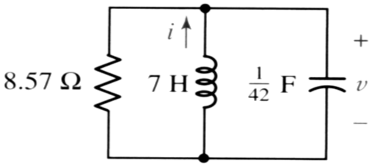
Critical Damping: Finding A_1 and A_2

$$v(t) = e^{-\sqrt{6}t}\left(A_1 t + A_2\right)$$

$$v(0) = 0 = A_2 \ldots (*)$$

$$\left.\frac{dv}{dt}\right|_{t=0} = A_1 \ldots (**)$$

From $$i_C(t) = C\frac{dv_C(t)}{dt}$$



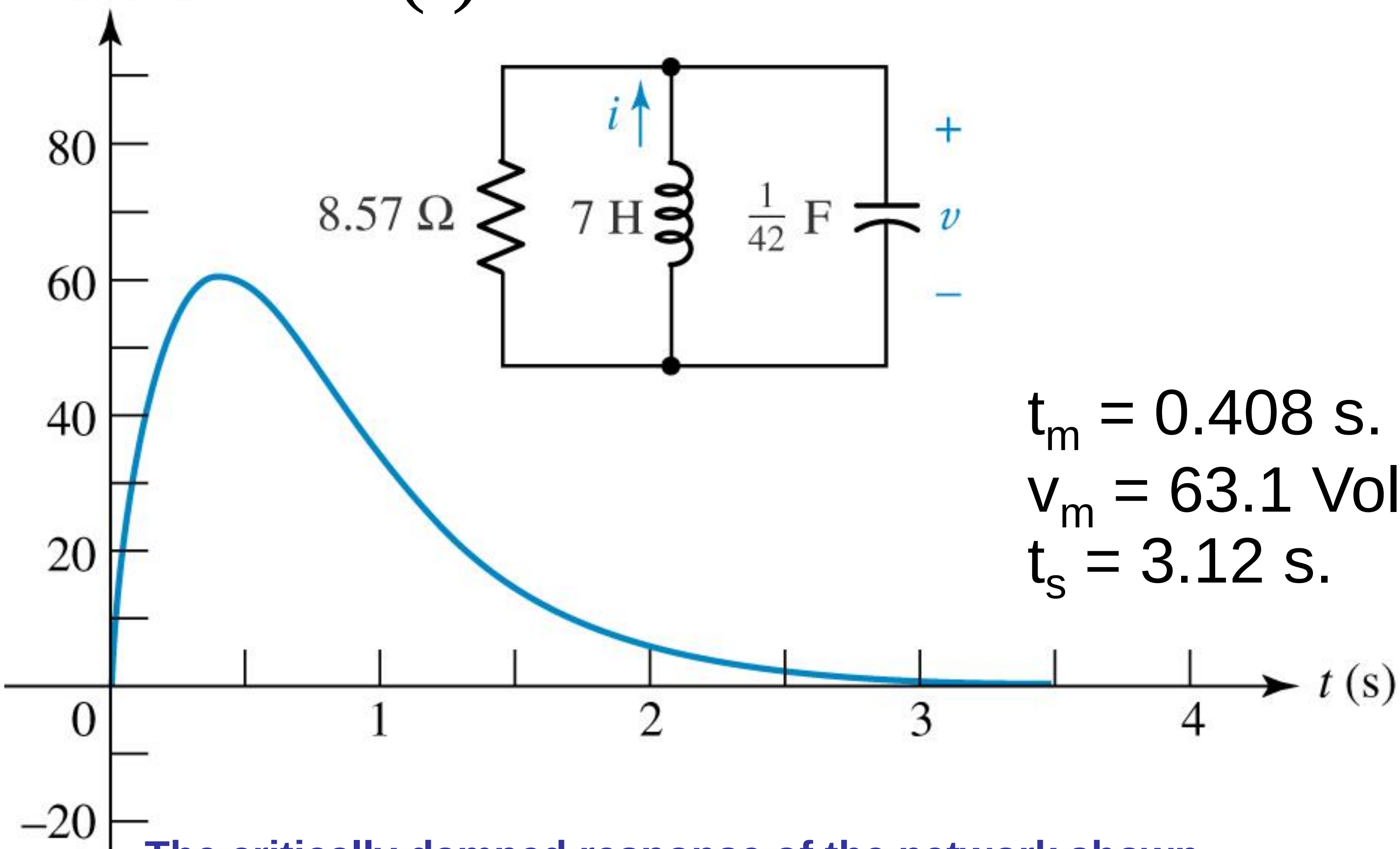
$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C} = \frac{i(0^+)+i_R(0^+)}{C} = \frac{10+0}{1/42} = 420\,V/s.$$

$$420 = A_1 \ldots (**)$$

$$\therefore v(t) = 420te^{-\sqrt{6}t} \quad V.$$

Critical Damping :

$$v(t) = 420te^{-\sqrt{6}t} \quad V.$$

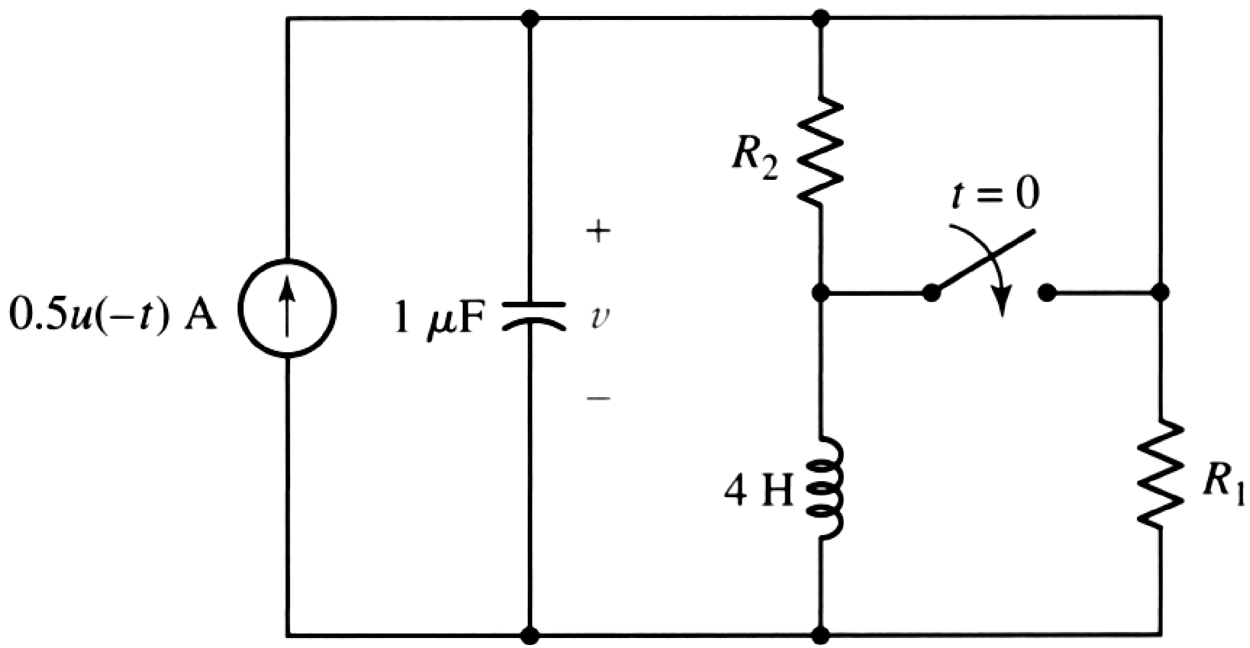


$t_m = 0.408$ s.
$v_m = 63.1$ Volts
$t_s = 3.12$ s.

The critically damped response of the network shown.

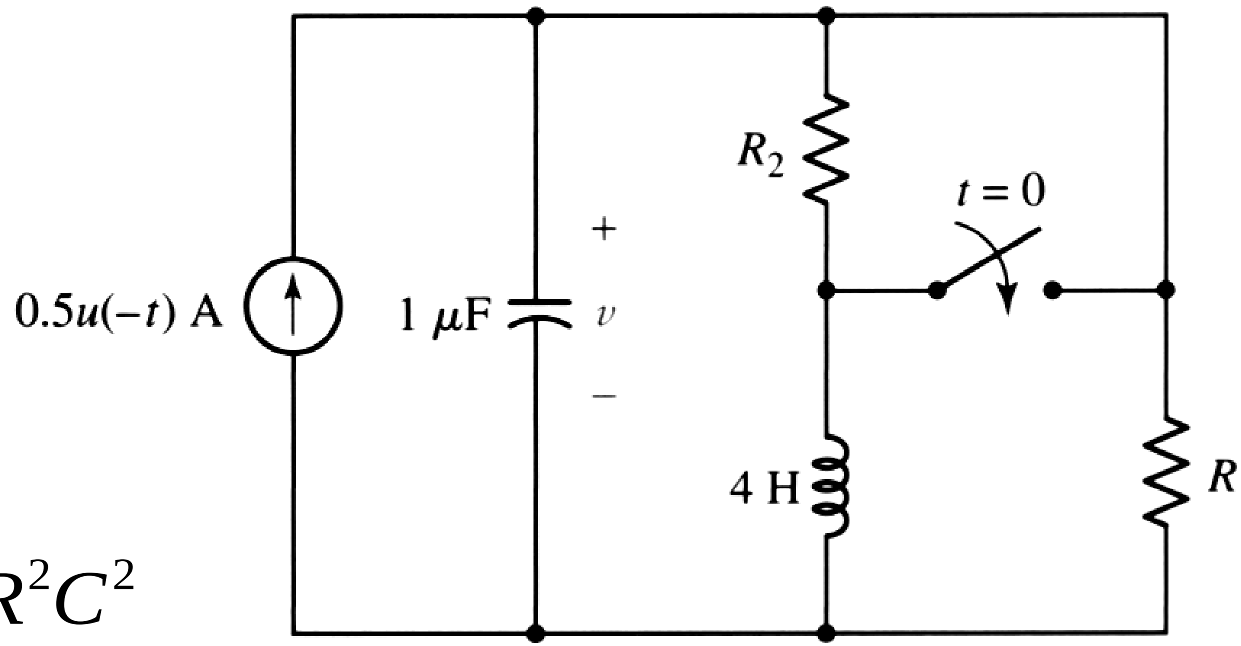
Practice: 9.3

- Choose R_1 so that the response after t = 0 will be critically damped
- Select R_2 to obtain v(0) = 100 V.
- Find v(t) at t = 1 ms.



Practice :

•Choose R_1 so that the response after t = 0 will be critically damped



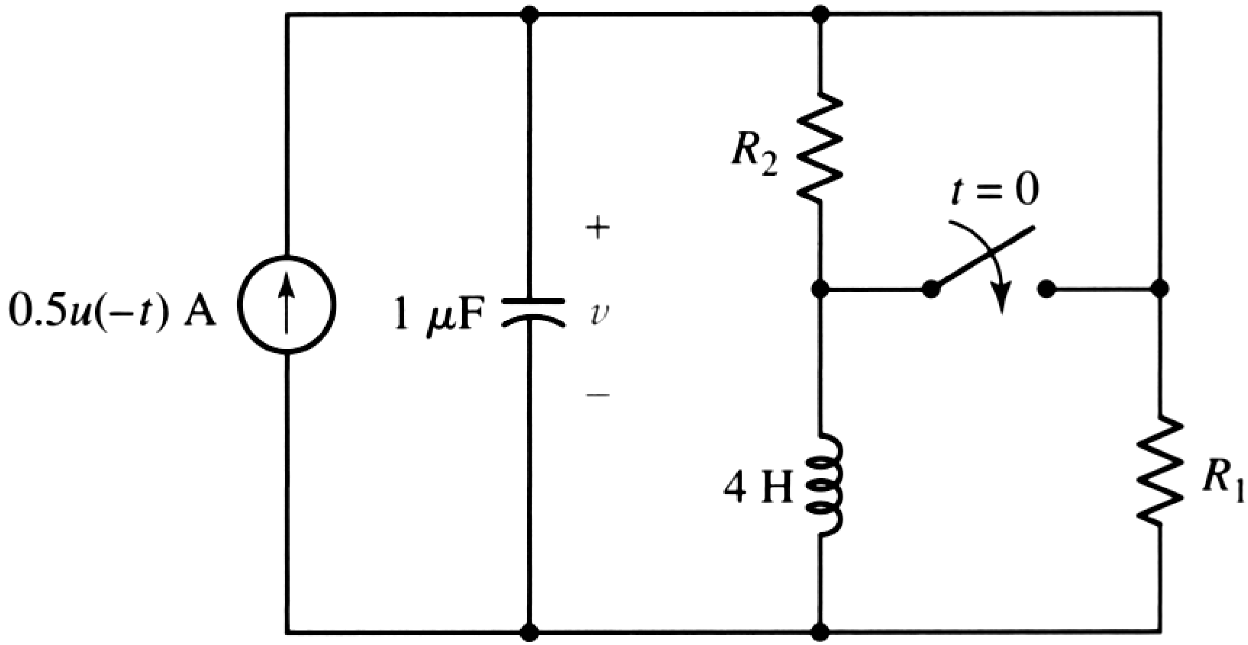
$$\boxed{\alpha = \omega_0}$$

$$LC = 4R^2C^2$$

$$\therefore R_1 = \sqrt{\frac{L}{4C}} = \sqrt{\frac{4H}{4 \cdot 1 \times 10^{-6} F}} = 1 \quad k\Omega$$

Practice :

- Select R_2 to obtain v(0) = 100 V.



$$(0.5A)\cdot\left(\frac{R_1}{R_1+R_2}\right)R_2 = 100 \quad V. \qquad ;R_1 = 1 \quad k\Omega$$

$$\Rightarrow \quad R_2 = 250 \quad \Omega$$

The Underdamped Parallel RLC:

$$\boxed{\alpha < \omega_0} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Then let $\sqrt{\alpha^2 - \omega_0^2} = \sqrt{-1} \cdot \sqrt{\omega_0^2 - \alpha^2} = j\sqrt{\omega_0^2 - \alpha^2}$

the natural resonant frequency: $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$$\begin{aligned} v(t) &= A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ &= e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) \\ &= e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right] \end{aligned}$$

$$\boxed{\therefore v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)}$$

Example: Underdamped

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 10.5\Omega \cdot {}^{1}\!/_{42} F} = 2 \quad s^{-1} \quad , \omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6} \quad rad/s$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{6 - 2^2} = \sqrt{2} \quad rad/s$$

$$v(t) \quad = \quad e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

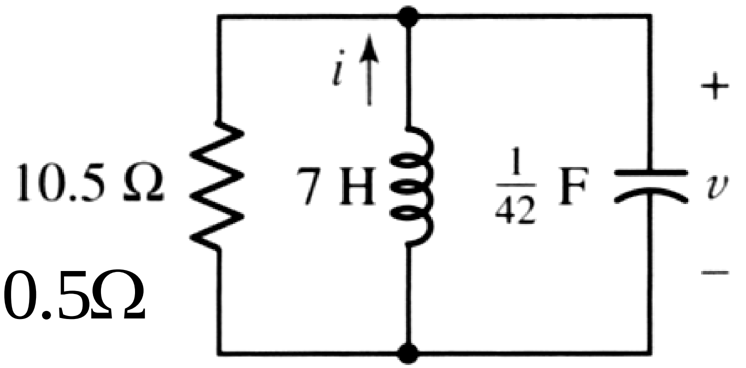
$$\therefore v(t) \quad = \quad e^{-2t}(B_1 \cos \sqrt{2}t + B_2 \sin \sqrt{2}t)$$

The initial conditions:

$$i(0) = 10 \quad A.$$

$$v(0) = 0 \quad V.$$

$$8.57\Omega \rightarrow 10.5\Omega$$



Example: Underdamped

$$v(t) = e^{-2t}(B_1 \cos\sqrt{2}t + B_2 \sin\sqrt{2}t)$$

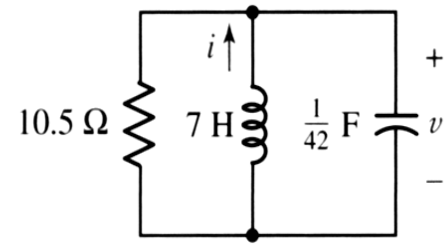
The initial conditions:

$$i(0) = 10 \quad A.$$
$$v(0) = 0 \quad V.$$

At t = 0,

$$v(0) = 0 = B_1 ...(*)$$

$$\Rightarrow \boxed{x(0) = B_1 ...(*)}$$



And evaluating the derivative at t = 0,

$$\frac{dv}{dt} = \sqrt{2}B_2 e^{-2t} \cos\sqrt{2}t - 2B_2 e^{-2t} \sin\sqrt{2}t$$

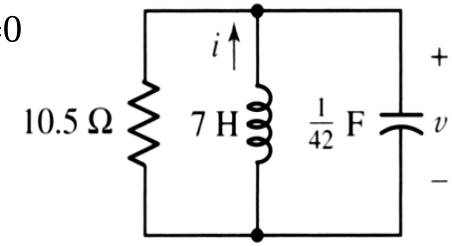
$$\left.\frac{dv}{dt}\right|_{t=0} = \sqrt{2}B_2 ...(**) \qquad \Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0} = -\alpha B_1 + \omega_d B_2 ...(**)}$$

Example: Underdamped

$$v(t) = e^{-2t}(B_1 \cos\sqrt{2}t + B_2 \sin\sqrt{2}t)$$

$$v(0) = 0 = B_1 \ldots (*)$$

$$\left.\frac{dv}{dt}\right|_{t=0} = \sqrt{2}B_2 \ldots (**)$$



From $$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C} = \frac{i(0^+) + i_R(0^+)}{C} = \frac{10+0}{1/42} = 420\, V/s.$$

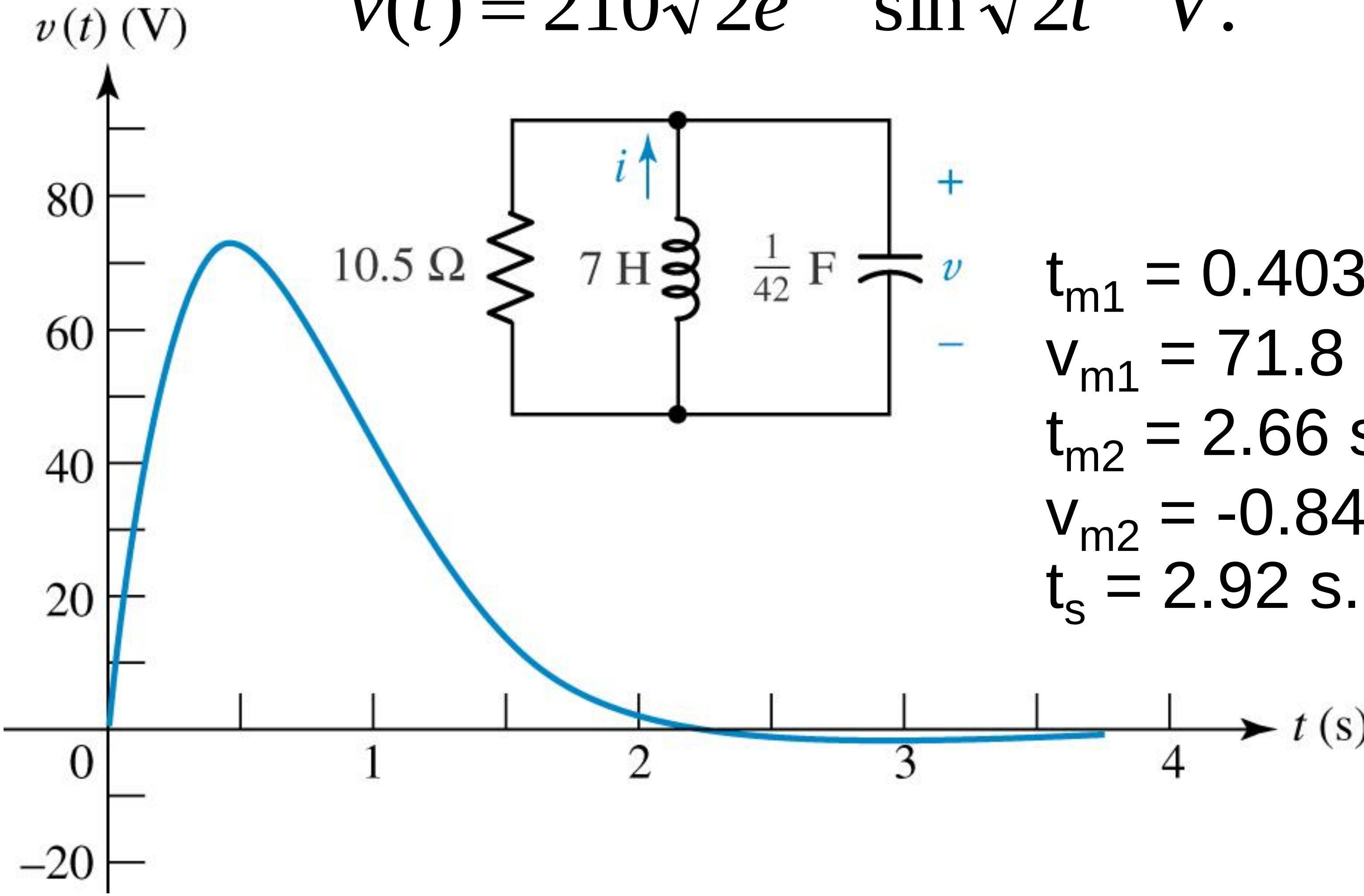
$$420 = \sqrt{2}B_2 \ldots (**)$$

$$\Rightarrow B_1 = 0, B_2 = 210\sqrt{2}$$

$$\therefore v(t) = 210\sqrt{2}e^{-2t} \sin\sqrt{2}t \quad V.$$

Example: Underdamped

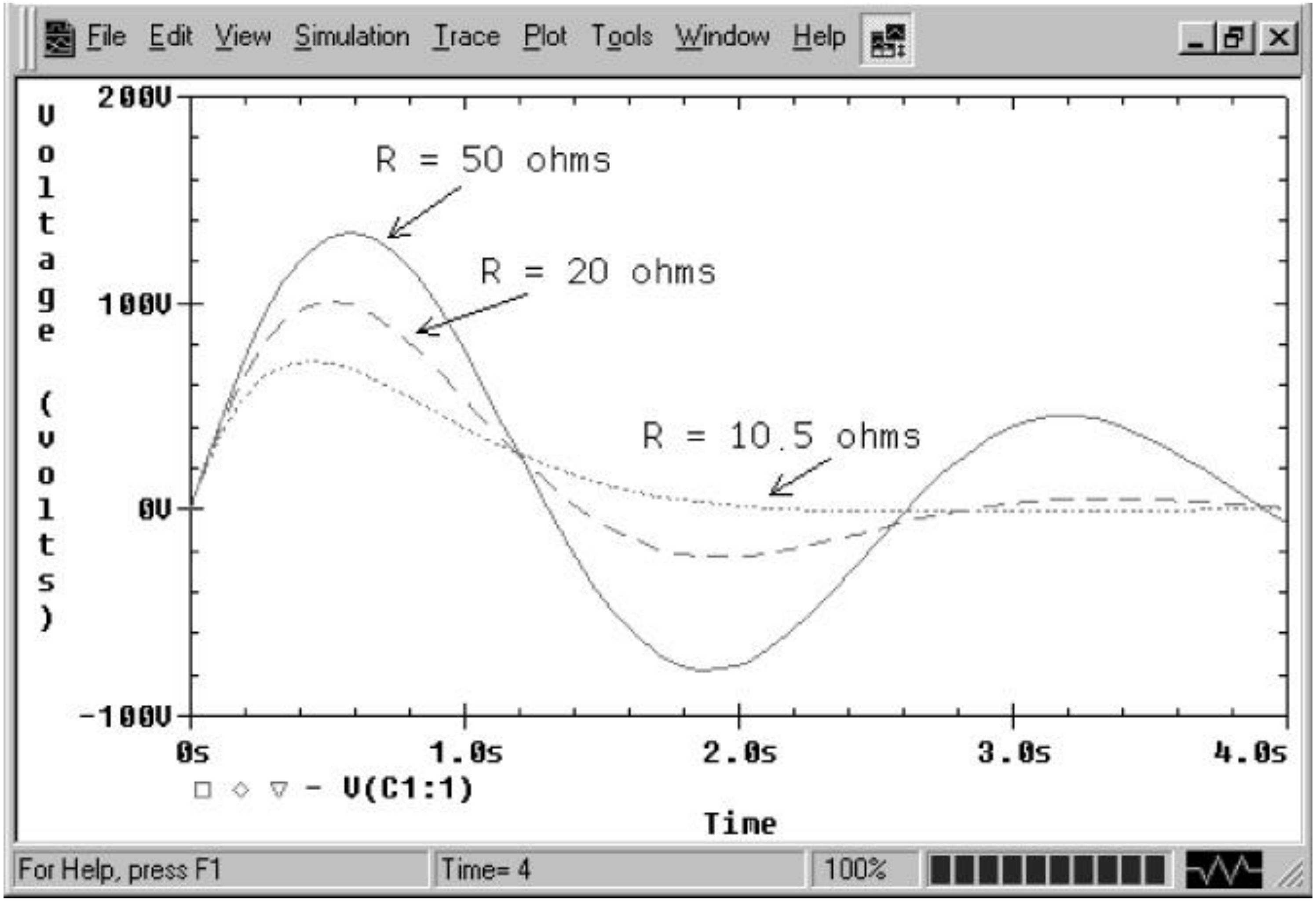
$$v(t) = 210\sqrt{2}e^{-2t}\sin\sqrt{2}t \quad V.$$



t_{m1} = 0.4035 s.
v_{m1} = 71.8 Volts
t_{m2} = 2.66 s.
v_{m2} = -0.845 Volts
t_s = 2.92 s.

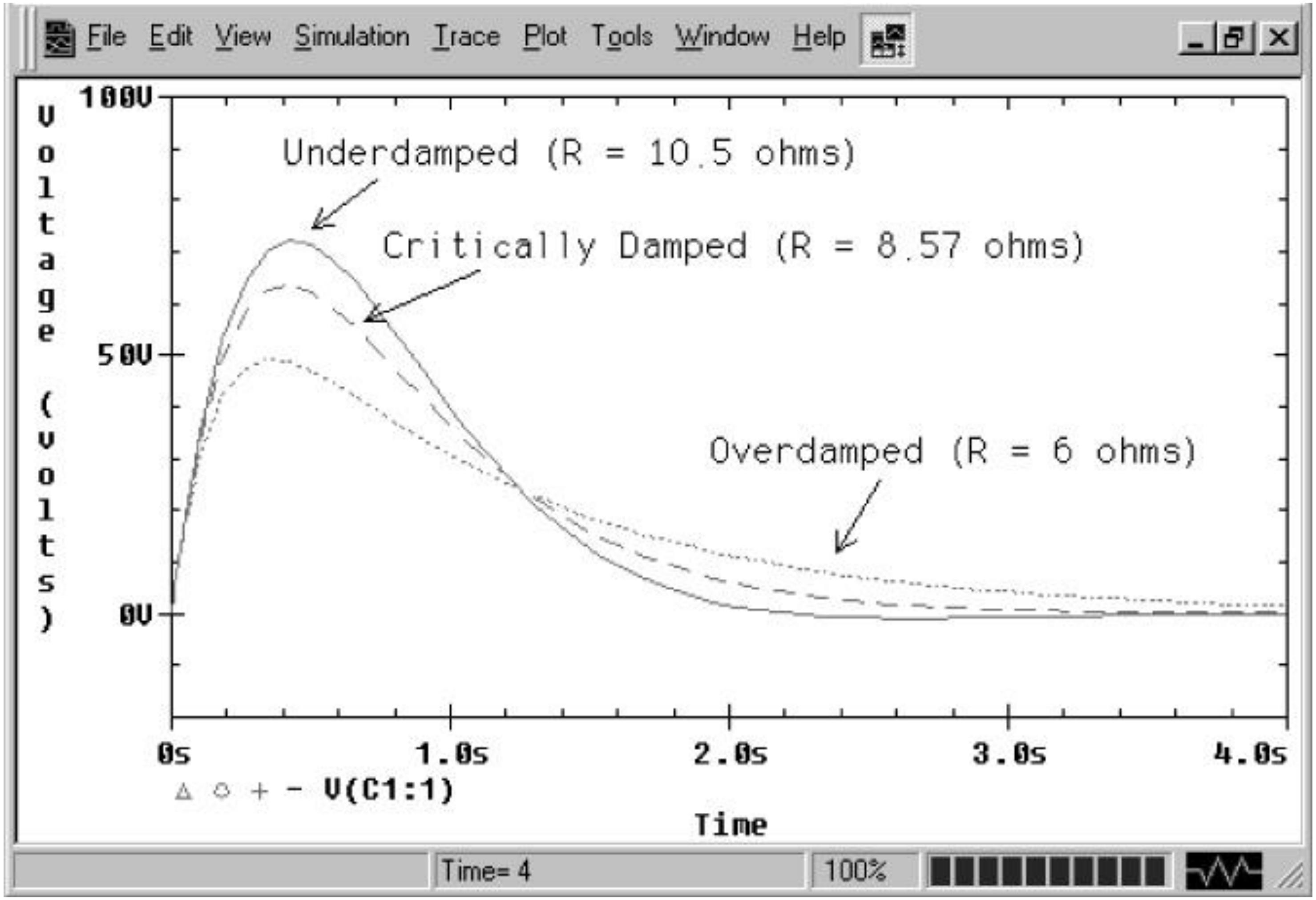
The underdamped response of the network shown.

Example:



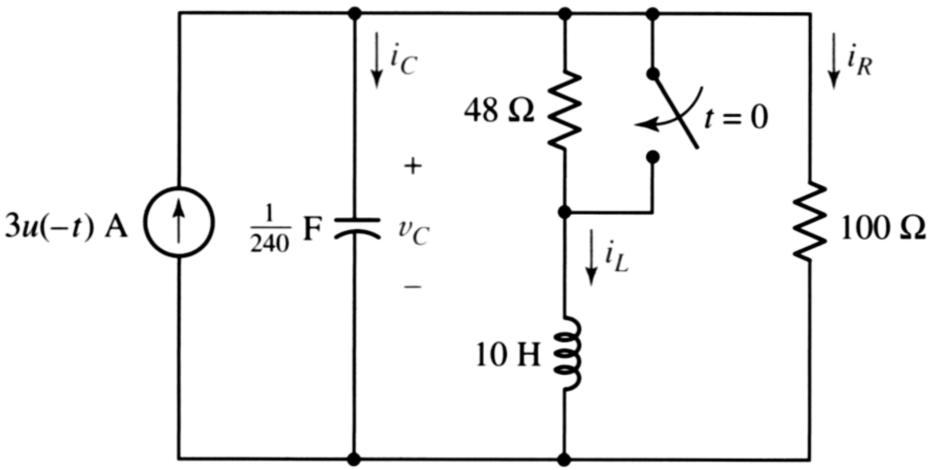
The response of the network for three different resistance values, showing an increase in the magnitude of oscillation.

Example:

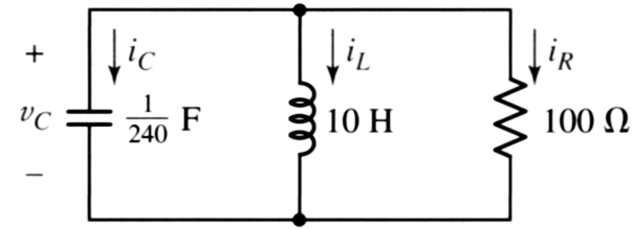


Simulated overdamped, critically damped, and underdamped voltage response for a parallel RLC network with L = 7 H and C = 1/42 F.

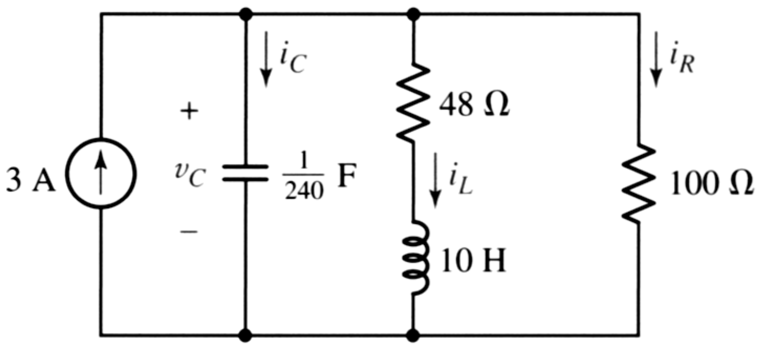
Example: find $i_L(t)$



(a)

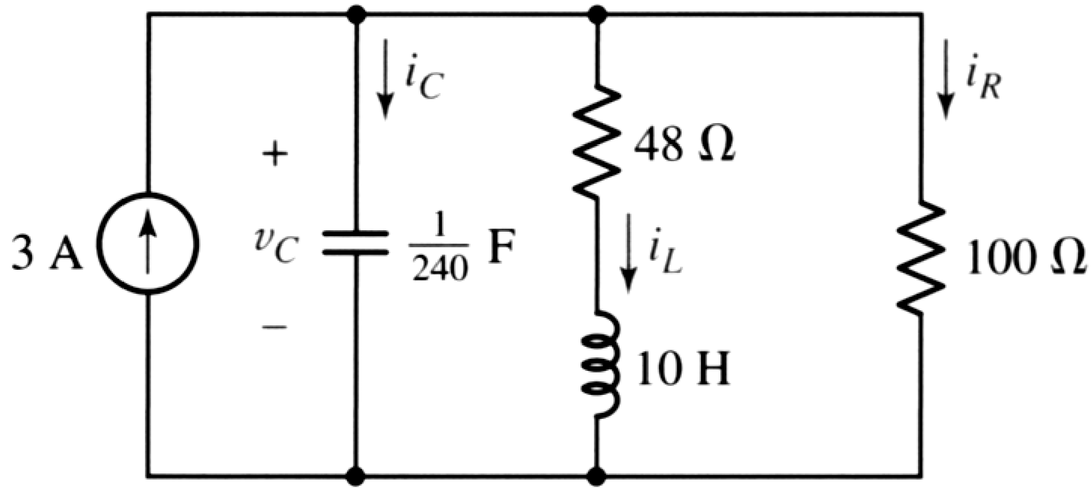


(b)



(c)

Example: find $i_L(t)$

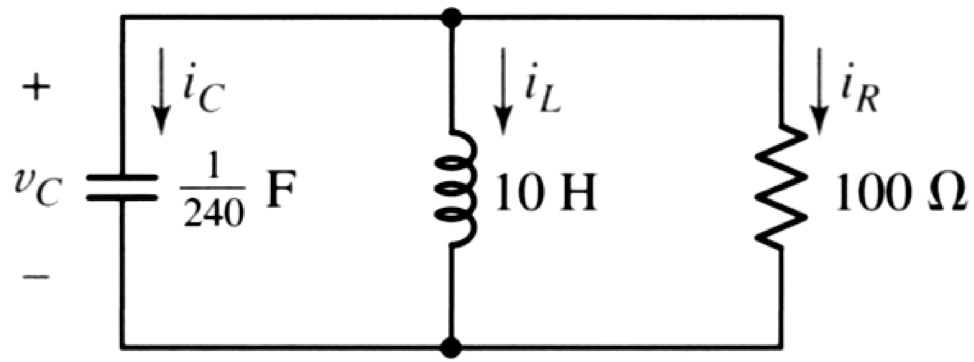


Determine the initial conditions:

$$i_L(0^-) = 3 \cdot \frac{100\Omega}{48\Omega + 100\Omega} = 2.027 \quad A.$$

$$v_C(0^-) = i_L(0^-) \cdot 48\Omega = 97.30 \quad V.$$

Example: find $i_L(t)$



$$\alpha = \frac{1}{2RC}$$

$$= \frac{1}{2 \cdot 100\Omega \cdot \frac{1}{240}F} = 1.2 \quad s^{-1}$$

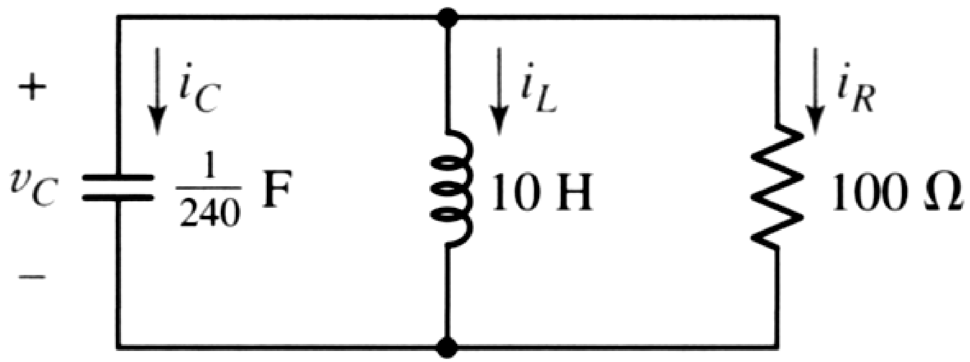
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10H \cdot \frac{1}{240}F}} = 4.899 \quad rad/s$$

$$\boxed{\alpha < \omega_0} \longrightarrow i_L(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

Example: find $i_L(t)$

$$\boxed{\alpha < \omega_0}$$



An underdamped C/T

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4.75 \quad rad/s$$

$$\longrightarrow \quad i_L(t) = e^{-1.2t}(B_1 \cos 4.75t + B_2 \sin 4.75t)$$

At t = 0, $\Rightarrow \boxed{x(0) = B_1 \ldots (*)}$ $\quad ; B_1 = i_L(0^+) = 2.027$

And evaluating the derivative at t = 0,

$$\Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0} = -\alpha B_1 + \omega_d B_2 \ldots (**)}$$

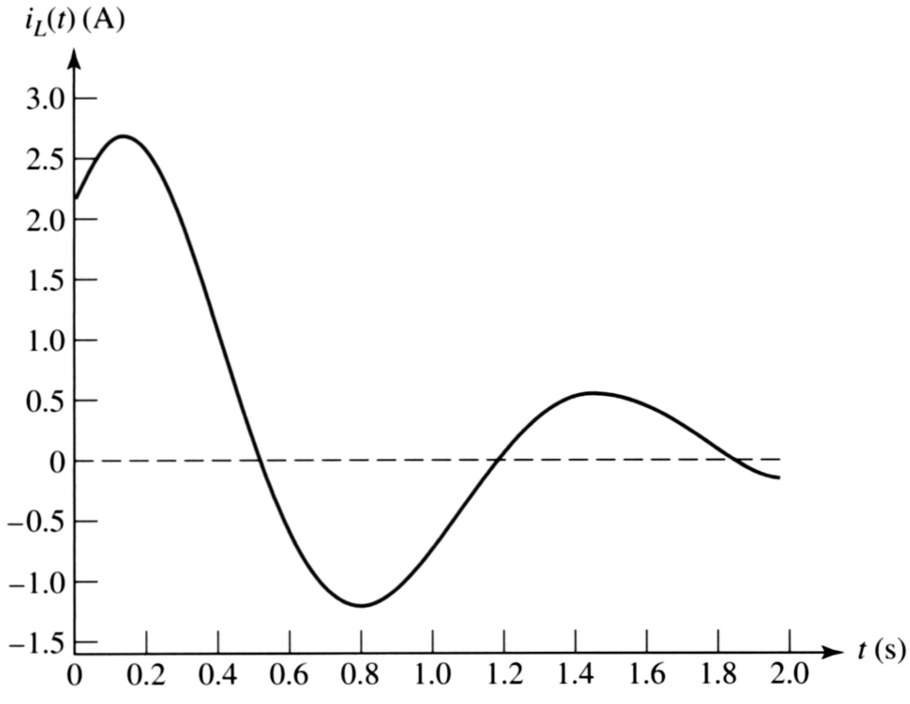
Example: find $i_L(t)$

from $$v_L(t) = L\frac{di_L}{dt}$$

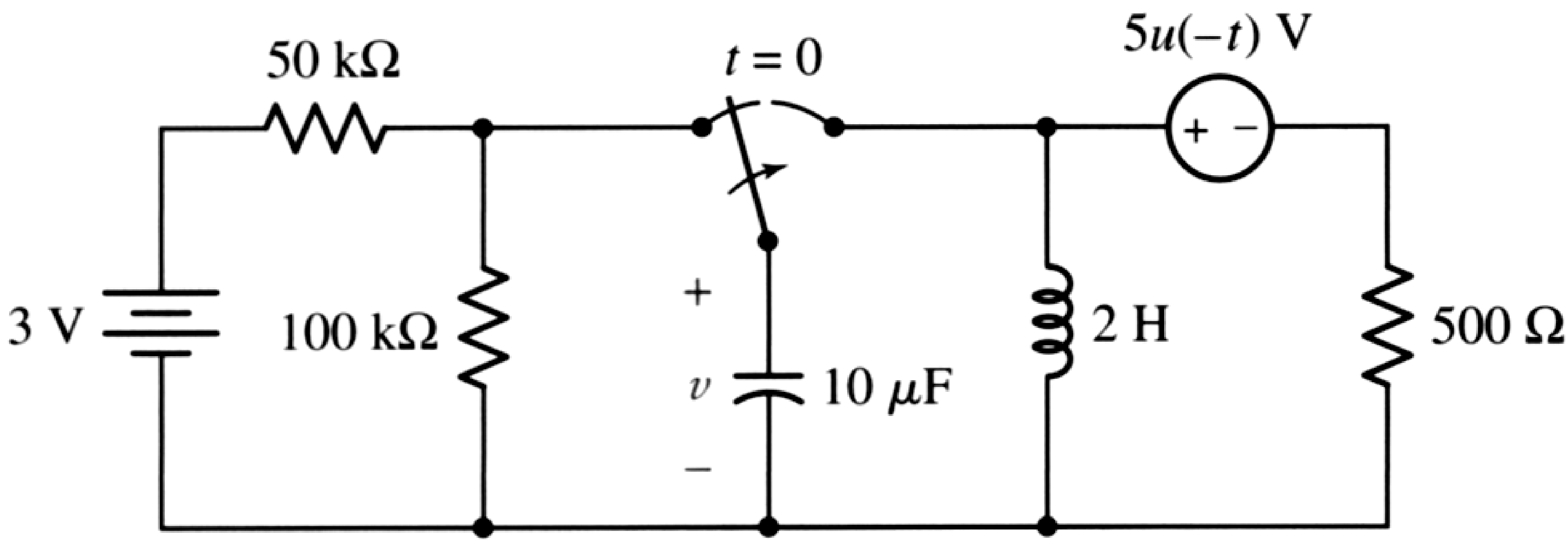
$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -1.2B_1 + 4.75B_2 = \frac{v_L(0^+)}{L} = \frac{v_C(0^+)}{L} = \frac{97.3}{10H}$$

$$\therefore B_2 = 2.561$$

$$\longrightarrow \quad i_L(t) = e^{-1.2t}(2.027\cos 4.75t + 2.561\sin 4.75t) \quad A.$$



Practice: 9.4



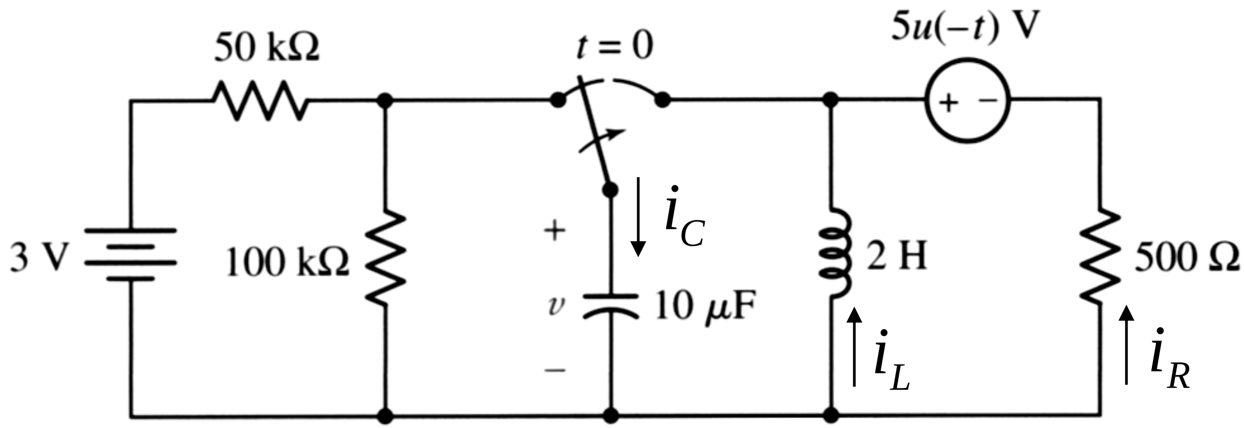
Find

- $\frac{dv}{dt}$ at $t = 0^+$
- $v(t)$ at $t = 1$ ms.
- t_0, the first value of t greater than zero at which $v = 0$

Practice :

From

$$i_C(t) = C\frac{dv_C(t)}{dt}$$



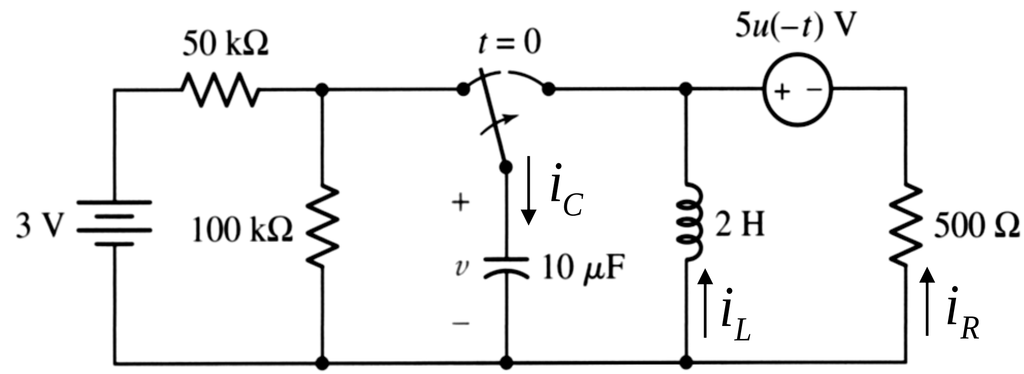
$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C} = \frac{i_L(0^+)+i_R(0^+)}{C} = \frac{i_L(0^-)+i_R(0^+)}{C} \,V/s.$$

$$i_L(0^-) = \frac{-5 \quad V.}{500\Omega} = -0.01 \quad A.$$

$$i_R(0^+) = \frac{-v_C(0^+)}{500\Omega} = \frac{-v_C(0^-)}{500\Omega} = \frac{-2V}{500\Omega} = -0.004 \quad A.$$

$$\longrightarrow \left.\frac{dv}{dt}\right|_{t=0^+} = -1400\,V/s.$$

Practice : v(t=1ms.)



$$\alpha = \frac{1}{2RC}$$

$$= \frac{1}{2 \cdot 500\Omega \cdot 10\mu F} = 100 \quad s^{-1}$$

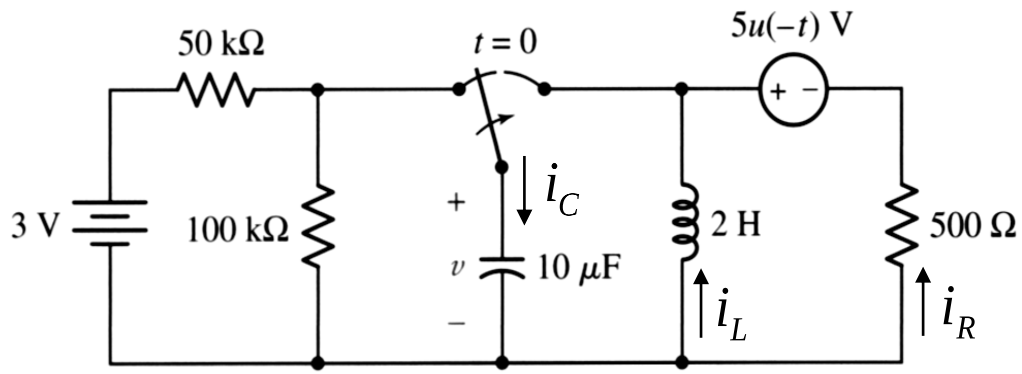
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2H \cdot 10\mu F}} = 223.6 \quad rad/s$$

$$\boxed{\alpha < \omega_0} \longrightarrow v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

Practice : v(t=1ms.)

$$\boxed{\alpha < \omega_0}$$



An underdamped C/T

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 200 \quad rad/s$$

$$\longrightarrow \quad v(t) = e^{-100t}(B_1 \cos 200t + B_2 \sin 200t)$$

At t = 0, $\Rightarrow \quad x(0) = B_1 \ldots (*) \qquad ; B_1 = v(0^+) = 2$

And evaluating the derivative at t = 0,

$$\Rightarrow \boxed{\left.\frac{dx}{dt}\right|_{t=0} = -\alpha B_1 + \omega_d B_2 \ldots (**)}$$

Practice : v(t=1ms.)

$$v(t) = e^{-100t}(B_1 \cos 200t + B_2 \sin 200t) \qquad ; B_1 = 2$$

From $i_C(t) = C\dfrac{dv_C(t)}{dt}$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -1400 = -100B_1 + 200B_2 \qquad \Rightarrow B_2 = -6$$

$$\Rightarrow v(t) = e^{-100t}(2\cos 200t - 6\sin 200t)$$

$$\therefore v(t = 1ms.) = 0.695 \quad V.$$

Homework:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.