

Power and Energy Measurement



EIE 240 Electrical and Electronic Measurement
Class 11, March 8, 2012

1

Work, Energy and Power

- Work is an activity of force and movement in the direction of force (Joules)
- Energy is the capacity for doing work (Joules)
- Power is the rate of using energy (Watt)

$$P = W / t \text{ , Work done in the time}$$
$$= E / t \text{ , Energy transferred in the time}$$

2

Electrical Power

- DC: Instantaneous power at time t

$$\begin{aligned}P(t) &= I(t) V(t) \\&= I^2(t) R \\&= V^2(t) / R\end{aligned}$$

- AC: Mean power per one cycle

$$\begin{aligned}P_{\text{avg}} &= 1/T \int_{t=0 \rightarrow T} P(t) dt \\&= I_{\text{rms}}^2 R \\&= V_{\text{rms}}^2 / R\end{aligned}$$

3

Apparent and True Power

- True power is the power used by the load in Watt unit (W),

$$P = I^2 R = V^2 / R$$

- Reactive power in Volt-Amps-Reactive unit (VAR),

$$Q = I^2 X = V^2 / X$$

- Apparent power is the power that is applied to the load in Volt-Amps unit (VA),

$$S = I^2 Z = V^2 / Z$$

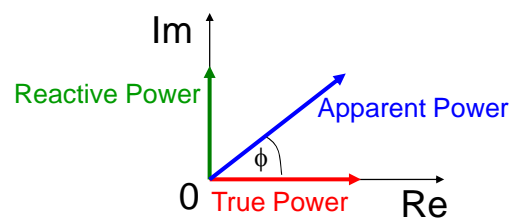
where impedance $Z = R + jX$

4

Power Factor

$$\cos \phi = \text{True Power} / \text{Apparent Power}$$

- Pure resistance, $\cos(0^\circ) = 1 \Rightarrow$ perfect
- Pure inductance, $\cos(90^\circ) = 0$
- Pure capacitance, $\cos(-90^\circ) = 0$



5

Power Factor (Cont'd)

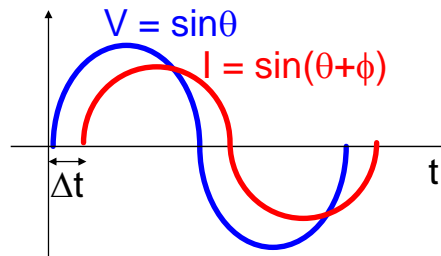
$$P = I V \cos \phi$$

- Up to a kind of loads being powered
- In phase or $\phi = 0$ (resistor)
- Lagging or current lags voltage, $\phi > 0$ (inductor)
- Leading or current leads voltage, $\phi < 0$ (capacitor)
- Lower power factor (higher $|\phi|$), higher losses
- Need to improve power factor

6

Power Factor (Cont'd)

$$\begin{aligned} I &= \sin\omega(t+\Delta t) \\ &= \sin(\omega t + \omega\Delta t) \\ &= \sin(\theta + \phi) \end{aligned}$$



- $\phi = 0^\circ \rightarrow I = \sin(\theta)$, resistor
- $\phi = 90^\circ \rightarrow I = \sin(\theta + 90^\circ)$
 $= \cos(\theta)$, inductor
- $\phi = -90^\circ \rightarrow I = \sin(\theta - 90^\circ)$
 $= -\cos(\theta)$, capacitor

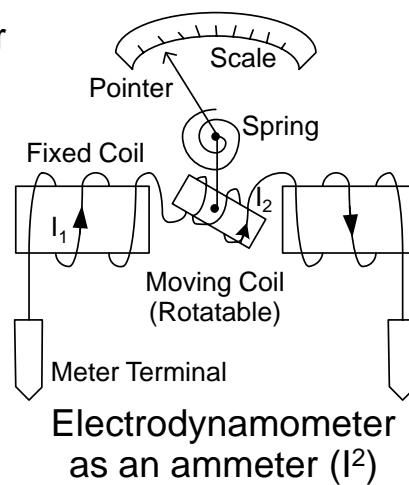
7

Wattmeter

- To measure the true power
- Two fixed coil is used to provide a magnetic field that is proportional to a current passing through it, instead of using a permanent magnet.



Right Hand's Rule

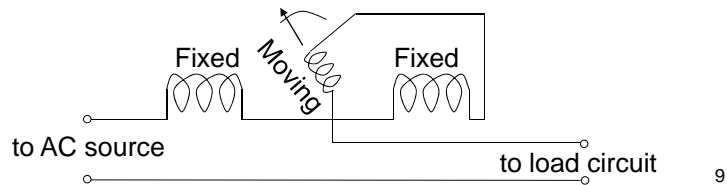


8

Wattmeter (Cont'd)

- Fixed coils, $B \propto I_1$
- Moving coil, Torque $\propto B I_2$

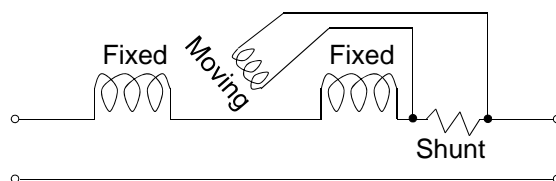
$$\begin{aligned}\theta &= K I_1 I_2 \\ &= K I^2, \quad I = I_1 = I_2 \\ &\propto I^2 \rightarrow \text{ammeter (non-linear scale)}\end{aligned}$$



9

Wattmeter (Cont'd)

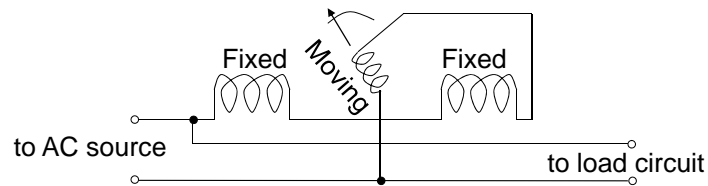
- Full-scale deflection current 5-100 mA
- For larger FSD up to 20 A, it is shunted by a low resistance.
- Electrodynamometer can be used for both DC and AC (rms) measurement.



10

Wattmeter (Cont'd)

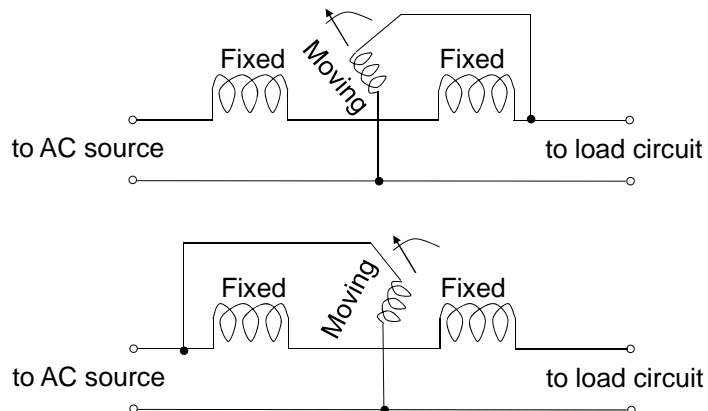
- Electrodynamometer as a voltmeter
 $\theta \propto V^2$



11

Wattmeter (Cont'd)

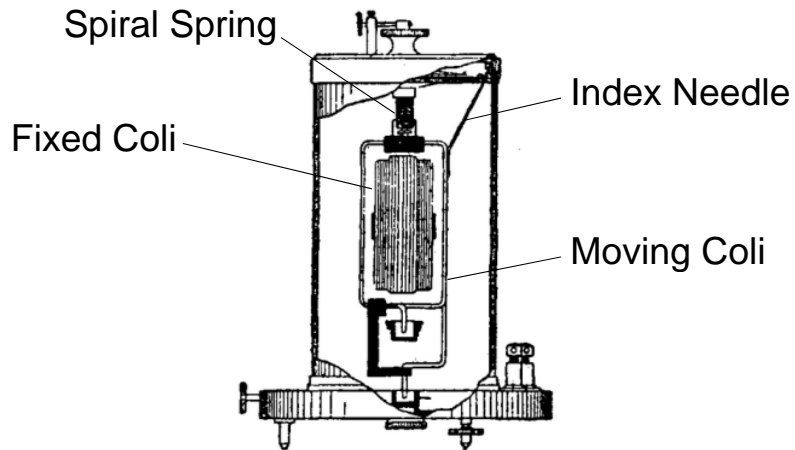
- Electrodynamometer as a wattmeter
 $\theta \propto IV$
- Loading error!



12

Wattmeter (Cont'd)

- Electrodynamometer

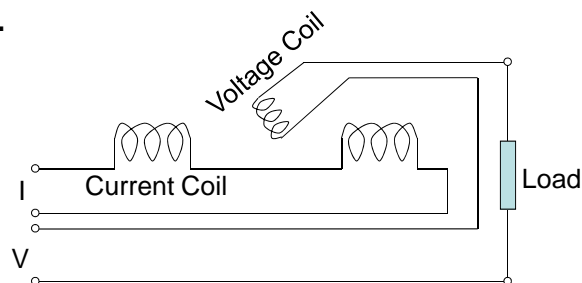


13

Single-Phase Wattmeter

- Based on the principles of the dynamometer
- The current passing through moving coil and load is proportional to the potential difference across the load.

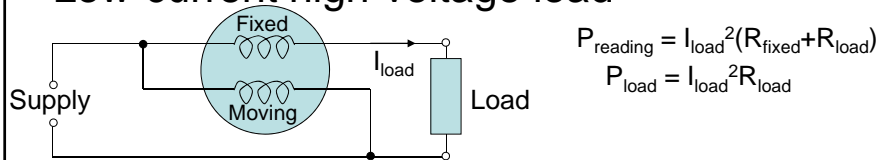
$$\begin{aligned}
 \theta &= K I_1 I_2 \\
 &= K I (V/R) \\
 &= (K/R) (IV) \\
 &\propto P
 \end{aligned}$$



14

Single-Phase Wattmeter (Cont'd)

- Low-current high-voltage load

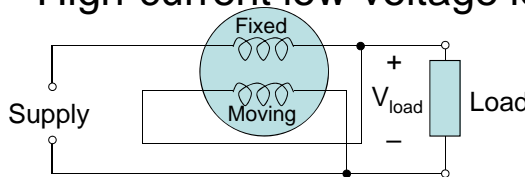


$$P_{\text{reading}} = I_{\text{load}}^2 (R_{\text{fixed}} + R_{\text{load}})$$

$$P_{\text{load}} = I_{\text{load}}^2 R_{\text{load}}$$

Wattmeter reads high due to the potential drop across the fixed coil (if neglect the reactance).

- High-current low-voltage load



$$P_{\text{reading}} = V_{\text{load}}^2 / (R_{\text{moving}} \parallel R_{\text{load}})$$

$$= V_{\text{load}}^2 \frac{R_{\text{moving}} R_{\text{load}}}{(R_{\text{moving}} + R_{\text{load}})}$$

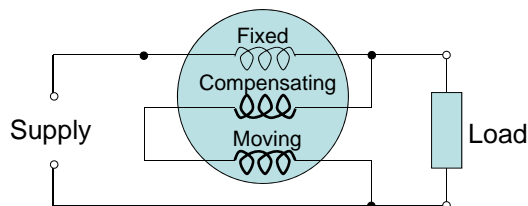
$$P_{\text{load}} = V_{\text{load}}^2 / R_{\text{load}}$$

Wattmeter reads high due to the current through the movable coil (if neglect the reactance).

15

Single-Phase Wattmeter (Cont'd)

- Compensated wattmeter

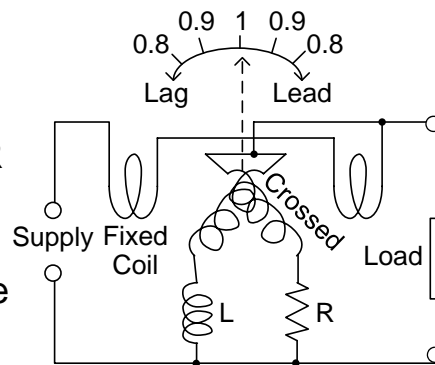


To reduce the systematic error, the compensating coil current is, however, in the opposite direction to the load current and cancels out the proportion of the magnetic flux due to the moving coil current (the same number of turns).

16

Crossed-Coil Power Factor Meter

- Two moving coils are crossed and connected in series to an inductor and resistor.
- For $PF = 1$, the coil with R will be in phase with the line current (moving), while the coil with L will be out of phase by 90° (no moving).
- For $PF = 0$, coil L moves and coil R does not move.



17

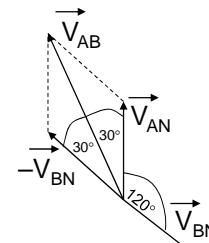
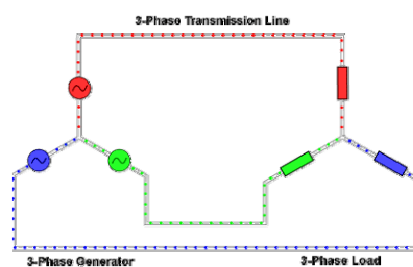
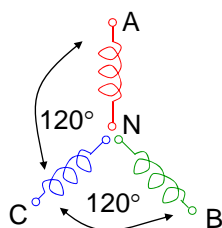
Three-Phase Power Distribution

The three-phase system uses less conductor material to transmit electric power than the others.

Y-Type, $\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN}$

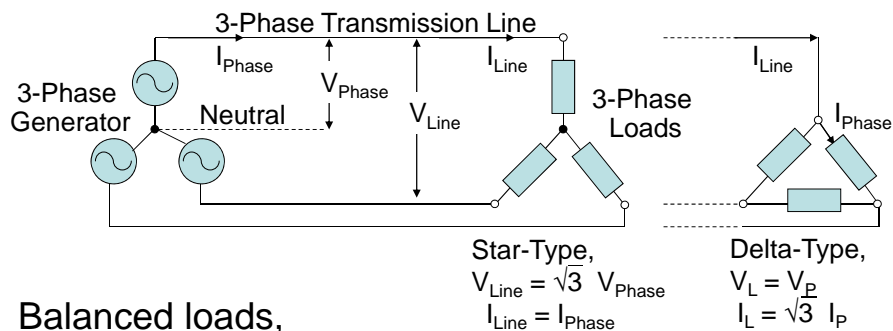
$$|\vec{V}_{AB}| = |\vec{V}_{AN}| \cos 30^\circ + |\vec{V}_{BN}| \cos 30^\circ$$

$$= \sqrt{3} |\vec{V}_{AN}|, \quad |\vec{V}_{AN}| = |\vec{V}_{BN}|$$



18

Three-Phase Loads



Balanced loads,

$$P_1 = P_2 = P_3 = P = I_L V_P \cos\phi$$

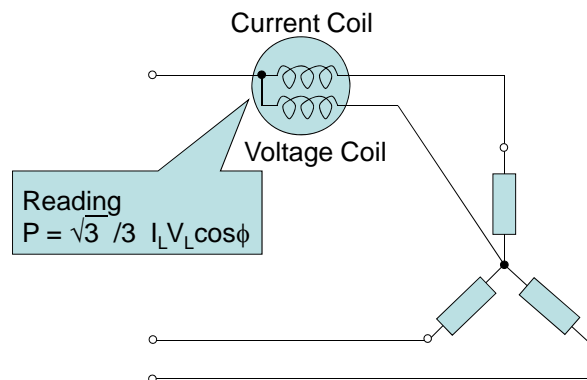
$$\begin{aligned} P_{total} &= P \times 3 = 3 I_L V_P \cos\phi \\ &= 3 I_L (V_L / \sqrt{3}) \cos\phi \\ &= \sqrt{3} I_L V_L \cos\phi \end{aligned}$$

$$\text{Similarly, } Q = \sqrt{3} I_L V_L \sin\phi$$

19

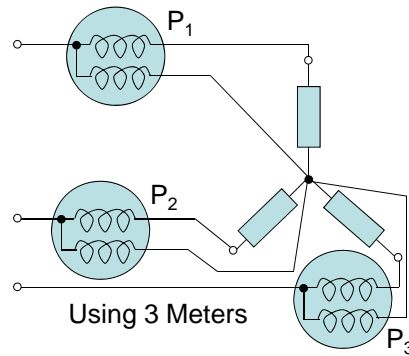
Three-Phase Wattmeter

- For three-phase system with balanced load, just using single-phase wattmeter, $P \times 3$



20

Unbalanced-Load 3-Phase Wattmeter



By using 3 wattmeters,

$$P_{\text{total}} = P_1 + P_2 + P_3$$

21

Balanced-Load 3-Phase Wattmeter

Brondel's theory,

$$P = I_1 V_1 + I_2 V_2 + I_3 V_3$$

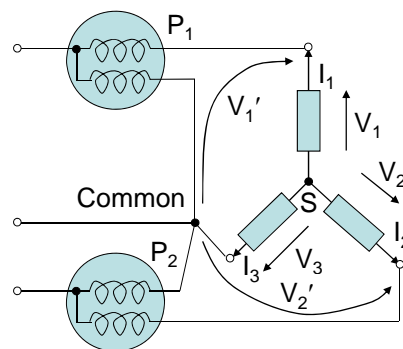
Kirchoff's law,

$$V_1 = V_3 + V_1'$$

$$V_2 = V_3 + V_2'$$

Therefore, $\rightarrow 0$

$$\begin{aligned} P &= (I_1 + I_2 + I_3) V_3 \\ &\quad + I_1 V_1' + I_2 V_2' \\ &= P_1 + P_2 \end{aligned}$$



22

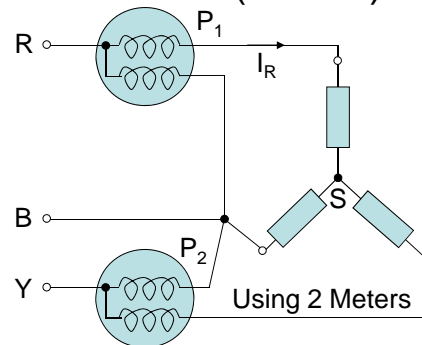
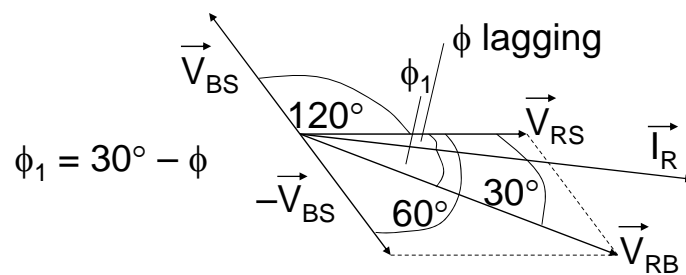
Balanced-Load 3-Phase Wattmeter (Cont'd)

By using 2 wattmeters,

$$P_1 = I_R V_{RB} \cos \phi_1$$

$$\vec{V}_{RB} = \vec{V}_{RS} - \vec{V}_{BS}$$

Balanced, $|\vec{V}_{RS}| = |\vec{V}_{BS}|$



23

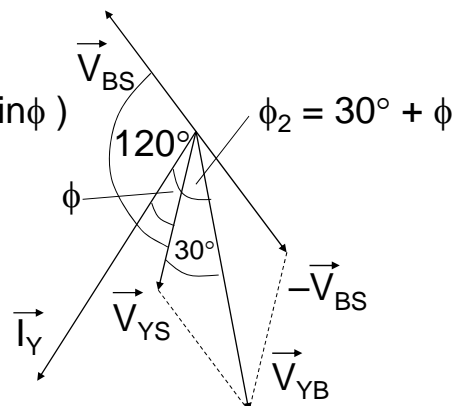
Balanced-Load 3-Phase Wattmeter (Cont'd)

$$P_1 = I_L V_L \cos(30^\circ - \phi)$$

$$= I_L V_L \left(\frac{\sqrt{3}}{2} \cos \phi + \frac{1}{2} \sin \phi \right)$$

Similarly,

$$P_2 = I_L V_L \left(\frac{\sqrt{3}}{2} \cos \phi - \frac{1}{2} \sin \phi \right)$$



Note: $\cos(\theta \mp \phi) = \cos \theta \cos \phi \pm \sin \theta \sin \phi$

24

Balanced-Load 3-Phase Wattmeter (Cont'd)

Total Power,

$$\begin{aligned} P_{\text{total}} &= P_1 + P_2 \\ &= \sqrt{3} I_L V_L \cos\phi \end{aligned}$$

The same as using 1 meter for balanced load system

$$\text{and } P_1 - P_2 = I_L V_L \sin\phi$$

25

Balanced-Load 3-Phase Wattmeter (Cont'd)

Reactive Power,

$$Q = \sqrt{3} I_L V_L \sin\phi = \sqrt{3} (P_1 - P_2)$$

$$\begin{aligned} (P_1 - P_2) / (P_1 + P_2) &= I_L V_L \sin\phi / \sqrt{3} I_L V_L \cos\phi \\ &= \tan\phi / \sqrt{3} \end{aligned}$$

$$\tan\phi = \sqrt{3} (P_1 - P_2) / (P_1 + P_2)$$

26

Balanced-Load 3-Phase Wattmeter (Cont'd)

$$\sin^2\phi + \cos^2\phi = 1$$

$$\tan^2\phi + 1 = 1/\cos^2\phi$$

$$3(P_1 - P_2)^2 / (P_1 + P_2)^2 + 1 = 1/\cos^2\phi$$

Power factor,

$$\cos\phi = \sqrt{1 / [1 + 3(P_1 - P_2)^2 / (P_1 + P_2)^2]}$$

27

Watt-Hour-Meter

To measure the electrical energy supplied to industrial and domestic consumers in unit (kW-hr)

Generated Torque by Eddy Current = $k_g I V \cos\phi$

Magnetic Breaking Torque = $k_b N$

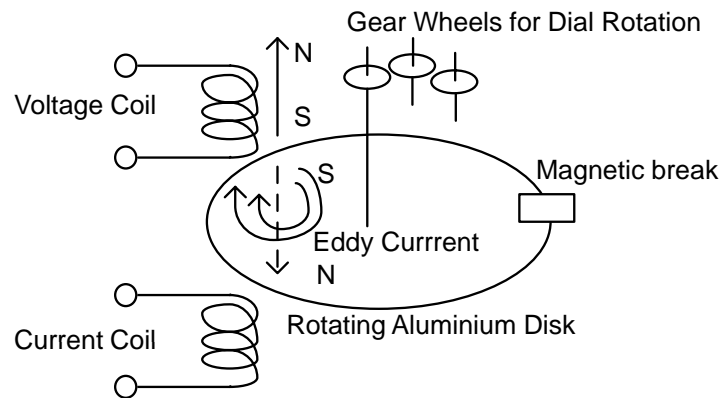
where N is number of revolution per unit time
(gearing to a mechanical counter to count watt-hours)

$$k_b N = k_g I V \cos\phi$$

$$N = (k_g/k_b) I V \cos\phi$$

28

Watt-Hour-Meter (Cont'd)



Therefore, number of revolution refers to the energy used by loads.