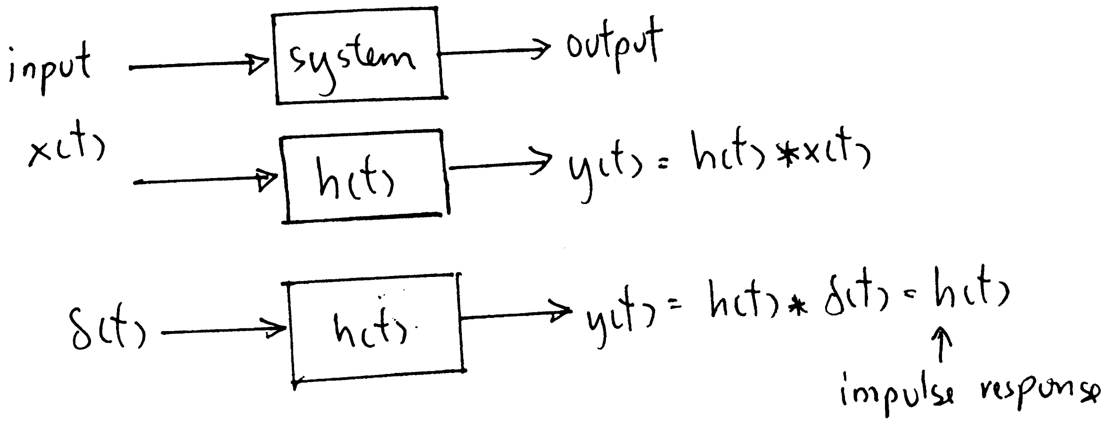
ENE208 convolution and LTI systems (continued)

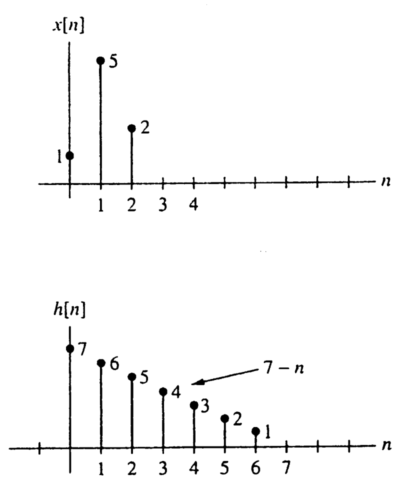
Recall that convolution method is used to determine the output of a system when the input is known.

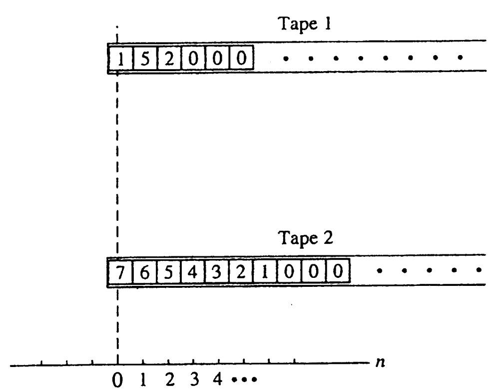


Example: consider the problem of finding the output of an LTI system having the impulse response

$$h[n] = \begin{cases} 0 & n < 0 \\ 7-n & 0 \leq n \leq 6 \\ 0 & 6 < n \end{cases}$$

when the input is $x[n] = \delta[n] + 5\delta[n-1] + 2\delta[n-2]$





From $y[n] = \sum_{m=-\infty}^{\infty} h[n-m]x[m]$

To perform the convolution, fix the value of n ($n=4$) and multiply the corresponding values of $h[n-m]$ and $x[m]$ for every m. Then sum the product $h[n-m]x[m]$ over m, as indicated by the convolution sum. You can imagine this operation being performed on either the plots or the tapes.

$\therefore$ $y[0] = 7\times 1 = 7$

$y[1] = 6\times 1 + 7\times 5 = 41$

$y[2] = 5\times 1 + 6\times 5 + 7\times 2 = 49$

$y[3] = 4\times 1 + 5\times 5 + 6\times 2 = 41$ $\cdots$

etc.

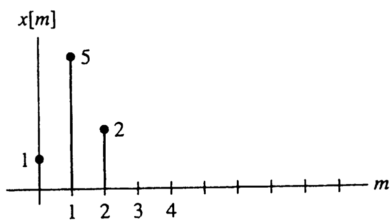
For $n<0$ and for $n>9$, the product $h[n-m]x[m]$ equals to 0 for all m.

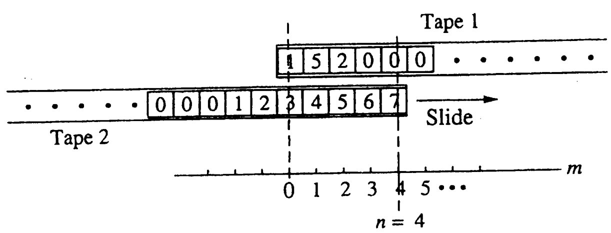
$\therefore$ $y[n] = \sum_{m=-\infty}^{\infty} h[n-m]x[m] = \sum_{m=-\infty}^{\infty} 0 = 0$ for $n<0$ and for $n>9$

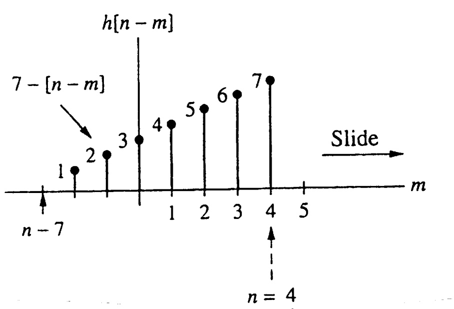
The sequence $y[n]$ is given by

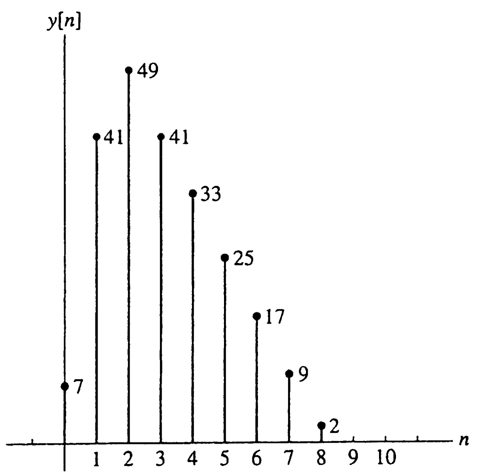
$$\{y[n]\} = \{\ldots, 0, 0, \underset{\uparrow}{7}, 41, 49, 41, 33, 25, 17, 9, 2, 0, 0, \ldots\}$$

where $\uparrow$ indicates the $n=0$ element.



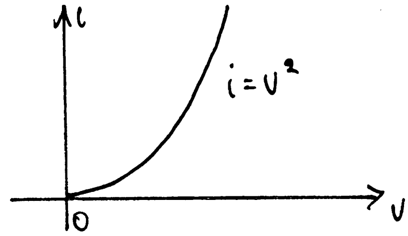






(c)

Example The current-voltage characteristic of a diode can often be approximated by $i(t) = v^2(t)$ for positive voltages as shown. Determine whether or not the system is linear.



Soln If we call $v(t)$ the input and $i(t)$ the output, then the transformation L is given by

$$i(t) = L\{v(t)\} = v^2(t) \quad , v > 0$$

First assume that $v(t) = v_1(t) + v_2(t)$ where both voltages are positive. then

$$L[v(t)] = L[v_1(t) + v_2(t)] = v_1^2(t) + 2v_1(t)\, v_2(t) + v_2^2(t)$$
$$= L[v_1(t)] + L[v_2(t)] + 2v_1(t)\, v_2(t)$$

The system is not additive because of the term $2v_1(t)v_2(t)$

Next assume that $v(t) = a v_1(t)$ Then $L[v(t)] = L[a v_1(t)] = a^2 L[v_1(t)]$ so that the system is not homogeneous either. Since the system is neither additive nor homogeneous, it is certainly not linear.

Example A first-order system is described by

$$y(k) = 0.2\,x(k) + 0.8\,y(k-1) \quad , k > 0$$
$$y(-1) = 0$$

The input is x and the output is y. Is the system linear?

Soln For an input x_1, the response is y_1 and for the input x_2, the response is y_2, each is given by

$$y_1(k) = 0.2\,x_1(k) + 0.8\,y_1(k-1) \quad , \quad y_1(-1) = 0$$
$$y_2(k) = 0.2\,x_2(k) + 0.8\,y_2(k-1) , \quad y_2(-1) = 0$$

now suppose that the input is the sum $x_1(k) + x_2(k)$ for all $k>0$. If we add the above two equations we obtain

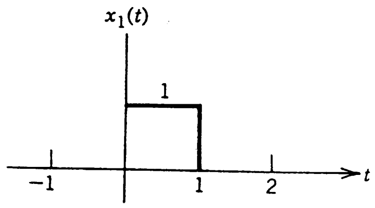
$$y_1(k) + y_2(k) = 0.2[x_1(k) + x_2(k)] + 0.8[y_1(k-1) + y_2(k-1)]$$

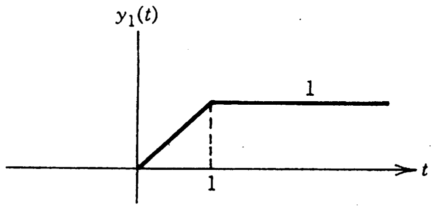
which indicates that the system is additive and therefore linear.

Example: show that the system $y(t) = \int_{-\infty}^{t} x(\lambda)\,d\lambda$ is time-invariant while

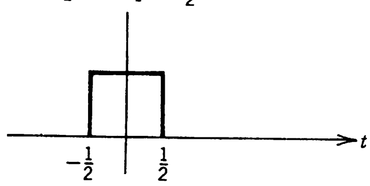
$$y(t) = \int_{0}^{t} x(\lambda)\,d\lambda \text{ is not}$$

Sol^n Let us compare the continuous-time systems as shown.
Suppose the system input is the square pulse $x_1(t)$ shown. The response of both systems is $y_1(t)$. But now let us shift the input to the left so that $x_2(t)$ is the input, where $x_2(t) = x_1(t+\frac{1}{2})$. Now the response of the first system differs from that of the other one.

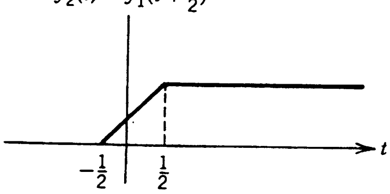




$x_2(t) = x_1(t + \frac{1}{2})$



$y_2(t) = y_1(t + \frac{1}{2})$



$y_2(t) \neq y_1(t + \frac{1}{2})$