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King Mongkut's University of Technology Thonburi

Midterm Examination, 1/2016

Seat No.

Subject: EIE 208 Electrical Engineering Mathematics ✓

For: 2nd yr. students, Dept. of Electronic and Telecommunication Engineering

Thur., Septempber 22, 2016 1:00pm – 4:00pm

Instructions:-

1. This exam consists of 10 pages (including this page) for 7 problems with the total score of 100.
2. This exam is ***closed books***. Textbooks and documents related to the subject are not allowed.
3. *Answer* each problem on the exam itself (use the back pages for extra spaces).
4. complying the university rules is allowed.
5. A ***dictionary*** is not allowed.
6. Do not bring any exam papers and answer sheets outside the exam room.

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment with university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Open Minds ... No Cheating! GOOD LUCK!!!

This exam is designed by Asst. Prof. Dr. Pinit Kumhom (Ext. 9075, 9070)

This examination has been approved by the committees of the ENE department.

R. Silapunt

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(Assoc. Prof. Dr. Rardchawadee Silapunt)

Head of Electronic and Telecommunication Engineering Department

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Name-Surname: ____________ **Student No.:** ____________

Prob. No.	1	2	3	4	5	6	7	8	9	10	Total
Full Score	10	15	15	16	14	10	20				100
Recieved Score											

Problem 1 [Math. and Engineering] (10 points) Describe the importance of mathematics in engineering and how mathematics is applied to engineering.

Problem 2 [Signals] (15 points)

2.1 (5 points) Decide whether or not each of the following statements is true (T) or false (F). If it is false, make the correction. ***Answering true (T) for a false statement or answering fasle (F) without correcting the statement will result in a negative score of -0.5 point. If you do not sure, leave it blank.***

____ 1. A discrete-time signal can be generated by sampling a continuous signal periodically with the sampling frequency at least 10 times the maximum frequency of the signal.

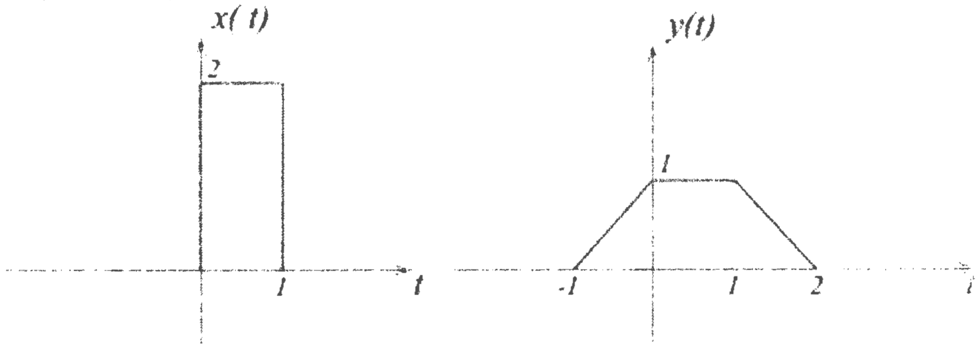
____ 2. A discrete-time signal can be described by a function of variable k, $k = 0, 1, \ldots$, where k represents the sampling points.

____ 3 A sinusoidal signal can be completely specified by its amplitude and frequency.

____ 4 If $x(t)$ is a periodic signal, then $x(t) = x(t + kT)$, where T is a positive real number, $k = 0, \pm 1, \pm 2, \ldots$

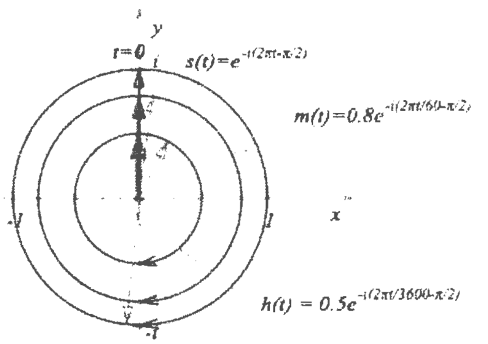
____ 5 A complex signal $z(t) = z_0 + re^{j2\pi t}, 0 \leq t \leq 1$ plots complex points on the circle with radius r centered at 0 for one cycle.

2.2 (10 points) Given a signal $x(t)$ and $y(t)$ shown below, plot $x(-2t + 2)$, $y(0.25t - 1)$ and $x(t)y(-t - 1)$



Problem 3.1 [Signals] (10 points) Plot the trajectory of the complex function $z(t) = 1 + e^{-i\pi/3}e^{i100\pi t}$ for $0 \le t < 20ms$ on the z-plane, and plot its real and imaginary parts.

Problem 3.2 [Signals] (5 points) The trajectories of a clock's second, minute, and hour pointers can be described by $s(t)$, $m(t)$, and $h(t)$ given below, respectively. Provided that the clock starts from 00:00:00 ($t = 0$), where all pointers are set at the number 12 ($\theta = \pi/2$), what is the angle of each pointer measured from the x-axis in degree at the time 1:12:24pm (Hint: $t = 13 \cdot 3600 + 12 \cdot 60 + 24$ s)?



$$
\begin{aligned}
s(t) &= e^{-i(2\pi t/60-\pi/2)} \\
m(t) &= 0.8e^{-i(2\pi t/3600-\pi/2)} \\
h(t) &= 0.5e^{-i(2\pi t/(12\cdot 3600)-\pi/2)}
\end{aligned}
$$

Problem 4 [Complex numbers] (16 points) Manipulate the following complex numbers so that they can be expressed in (1) rectangular form, $z = x + iy$, and (2) polar form using exponential function, $z = re^{i\theta}$, where θ is the argument of z in the principal branch. Show how your results are obtained.

4.1

$$z = \frac{4e^2 e^{-i\pi/6}}{\sqrt{12} - 2i}$$

4.2 $z = (\pi - i\pi)(1 - \pi)(\sqrt{2} + \sqrt{2}i)$

4.3

$$z = \frac{(\pi + \sqrt{3}i\pi)e^{i\pi/2}}{\sqrt{3} - i}$$

4.4

$$z = \frac{e^{-2}\sin\pi/6}{-1-i} - \frac{e^{-2}e^{i\pi/2}\cos\pi/6}{1+i}$$

Problem 5 [Complex Roots] (14 points) Find all the solutions of the following equation and sketch them on the complex plane.

Hint: Use the quadractic equation, and for a complex constant $c = re^{i\theta}$

$$z_k = \sqrt[n]{r}e^{i\theta/n}e^{i2\pi k/n} \;\; (k = 0, 1, \ldots, n-1)$$

where $z_k, k = 0, 1, 2, \ldots, n-1$ are all the n^{th} root of c.

$(z^4+16)(z^2+2iz-4) = 0$

Problem 6 [Analytic Functions] (10 points) Find the domain that make the following complex function analytic. (Hint: Use the Cauchy-Riemann Equations, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$)

6.1 $f(z) = 3x^2 - 3y^2 + 3y - 2x + i(-3x + 6xy - 2y)$

6.2 $f(z) = e^{x^2-y^2}[\cos(2xy) + i\sin(2xy)]$

Problem 7.1 [Rational Function] (10 points) Express the rational function

$$R(z) = \frac{z+1}{(z-1)^2(4z^2+4)}$$

in partial fraction form using the following theorem.

Theorem (Partial Fraction)

If

$$R_{m,n}(z) = \frac{p_m(z)}{q_n(z)} = \frac{a_0 + a_1 z + a_2 z^2 + \cdots + a_m z^m}{b_n(z-c_1)^{d_1}(z-c_2)^{d_2}\cdots(z-c_r)^{d_r}} \tag{1}$$

is a rational function whose *denominator degree* $n = d_1 + d_2 + \cdots + d_r$ *exceeds* its *numerator degree* m, then $R_{m,n}$ has a *partial decomposition* of the form

$$\begin{aligned} R_{m,n}(z) \quad = \quad & \frac{A_0^{(1)}}{(z-c_1)^{d_1}} + \frac{A_1^{(1)}}{(z-c_1)^{d_1-1}} + \cdots + \frac{A_{d_1-1}^{(1)}}{(z-c_1)} \\ & + \frac{A_0^{(2)}}{(z-c_2)^{d_1}} + \frac{A_1^{(2)}}{(z-c_2)^{d_2-1}} + \cdots + \frac{A_{d_2-1}^{(2)}}{(z-c_2)} \\ & + \cdots + \frac{A_0^{(r)}}{(z-c_r)^{d_r}} + \cdots + \frac{A_{d_r-1}^{(r)}}{(z-c_1)} \end{aligned} \tag{2}$$

where $\{A_s^{(j)}\}$ are constants (The c_k's are assumed *distinct*), and for each c_j with multiplicity d_j,

$$A_s^{(j)} = \lim_{z \to c_j} \frac{1}{s!}\frac{d^s}{dz^s}\left[(z-c_j)^{d_j} R_{m,n}(z)\right] \quad (s = 0, 1, \ldots d_j - 1) \tag{3}$$

Problem 7.2 [Complex Integration] (10 points) Find the integeral $\int_\Gamma f(z)dz$, where $f(z)$ is the function $R(z)$ in **Problem 7.1** along the contour Γ shown in the Figure below.