ENE 104
Electric Circuit Theory

Lecture 12:

Two-Port Networks

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Objectives : Ch17

Objectives include:

- Learning the distinction between one-port and two-port network
- Techniques for characterizing networks by **y**, **z**, **h**, and **t** parameters
- Transformation methods between **y**, **z**, **h**, and **t** parameters
- Performing circuit analysis using network parameters, including cascaded networks

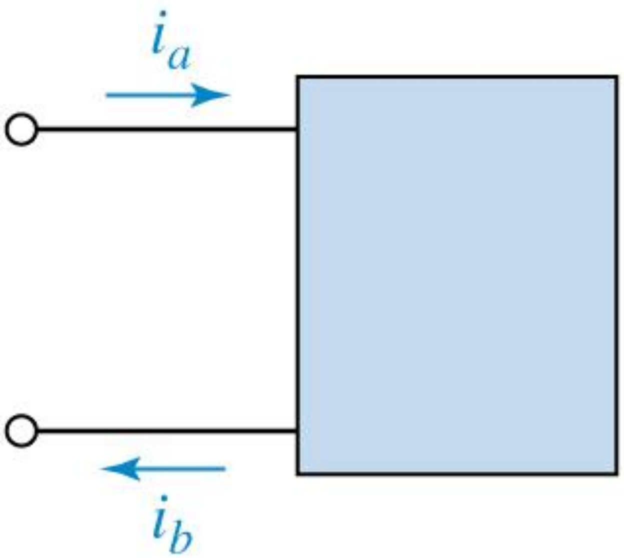
Introduction:

The circuit is linear, and the ability to measure voltages and currents:

It is possible to characterize such a network with a set of parameters that allow us to predict how the network will interact with other networks.

A multiport Network:

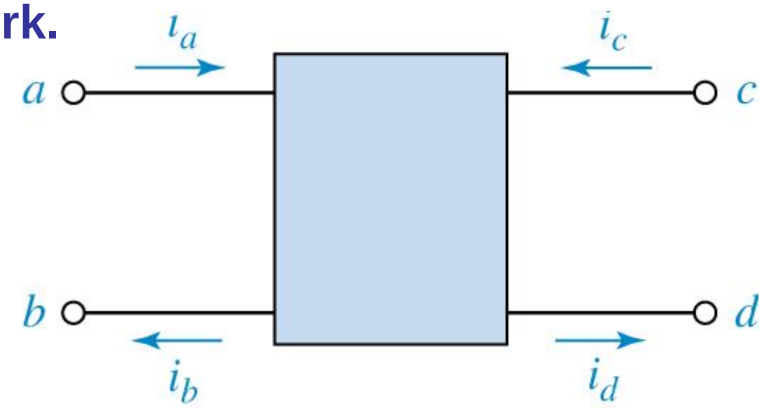
A one-port network.



i_a must be equal i_b

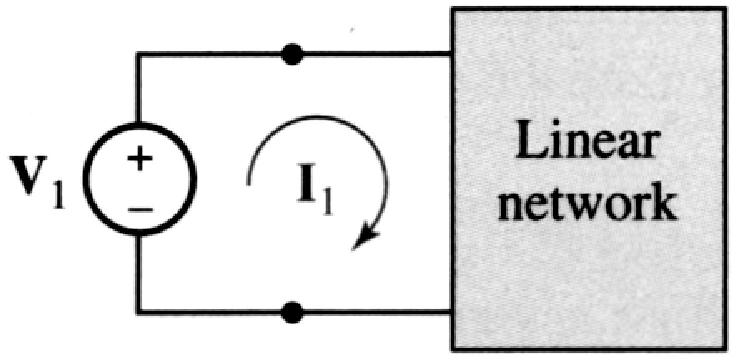
A two-port network.

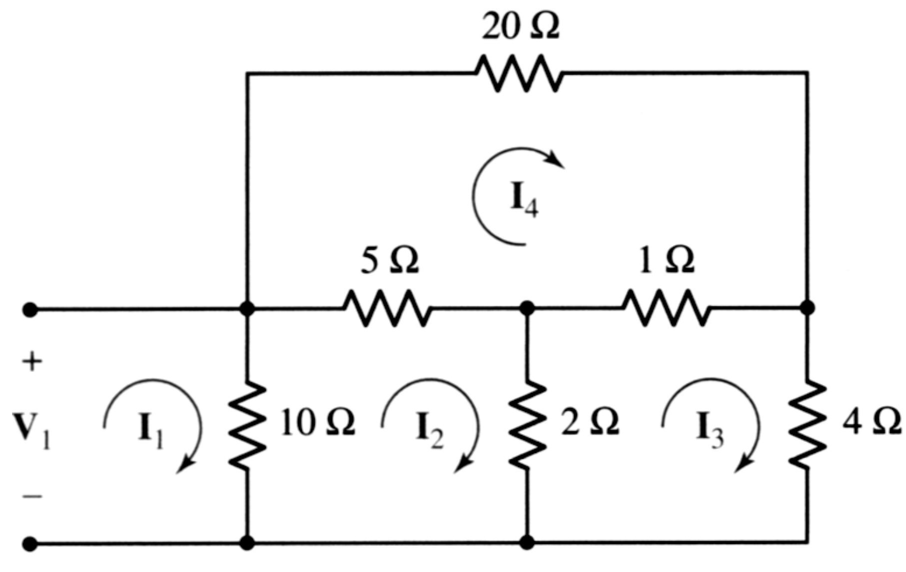
$$i_a = i_b$$



$$i_c = i_d$$

A multiport Network:





$$\mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2 + \mathbf{Z}_{13}\mathbf{I}_3 + \ldots + \mathbf{Z}_{1N}\mathbf{I}_N = \mathbf{V}_1$$

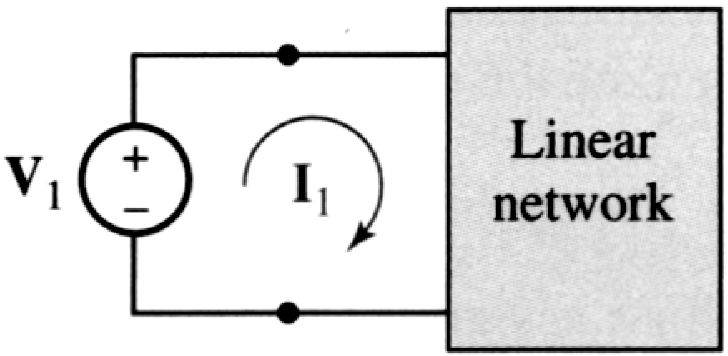
$$\mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2 + \mathbf{Z}_{23}\mathbf{I}_3 + \ldots + \mathbf{Z}_{2N}\mathbf{I}_N = \mathbf{V}_2$$

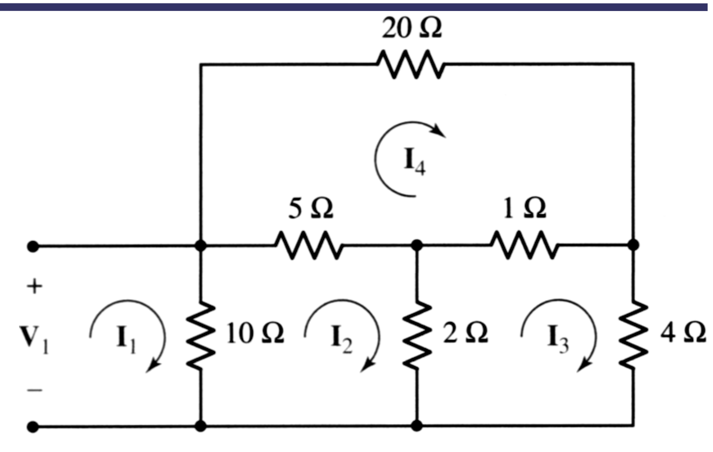
$$\mathbf{Z}_{31}\mathbf{I}_1 + \mathbf{Z}_{32}\mathbf{I}_2 + \mathbf{Z}_{33}\mathbf{I}_3 + \ldots + \mathbf{Z}_{3N}\mathbf{I}_N = \mathbf{V}_3$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\mathbf{Z}_{N1}\mathbf{I}_1 + \mathbf{Z}_{N2}\mathbf{I}_2 + \mathbf{Z}_{N3}\mathbf{I}_3 + \ldots + \mathbf{Z}_{NN}\mathbf{I}_N = \mathbf{V}_N$$

A multiport Network:





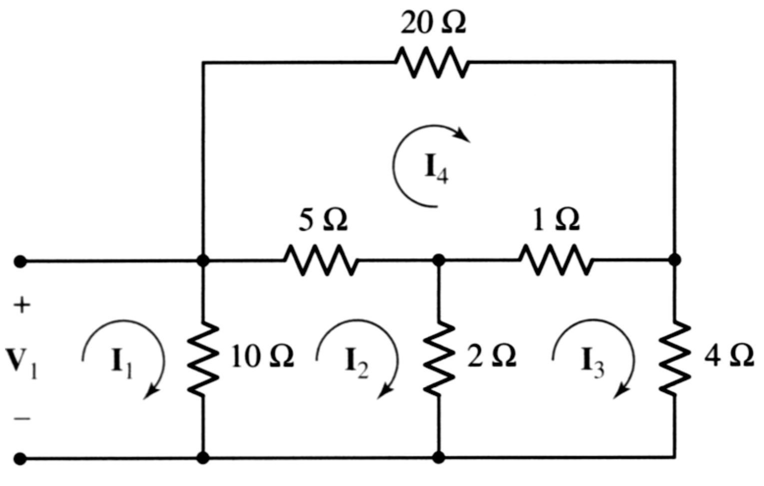
The circuit determinant:

$$\Delta_Z = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{vmatrix}$$

A multiport Network:

The Cramer's rule:

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{V}_1 & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \cdots & \mathbf{Z}_{1N} \\ 0 & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \cdots & \mathbf{Z}_{2N} \\ 0 & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \cdots & \mathbf{Z}_{3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \mathbf{Z}_{N2} & \mathbf{Z}_{N3} & \cdots & \mathbf{Z}_{NN} \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \cdots & \mathbf{Z}_{1N} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \cdots & \mathbf{Z}_{2N} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \cdots & \mathbf{Z}_{3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{Z}_{N1} & \mathbf{Z}_{N2} & \mathbf{Z}_{N3} & \cdots & \mathbf{Z}_{NN} \end{vmatrix}}$$

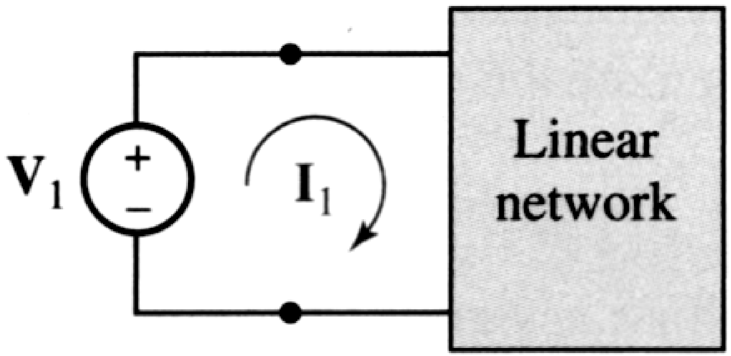


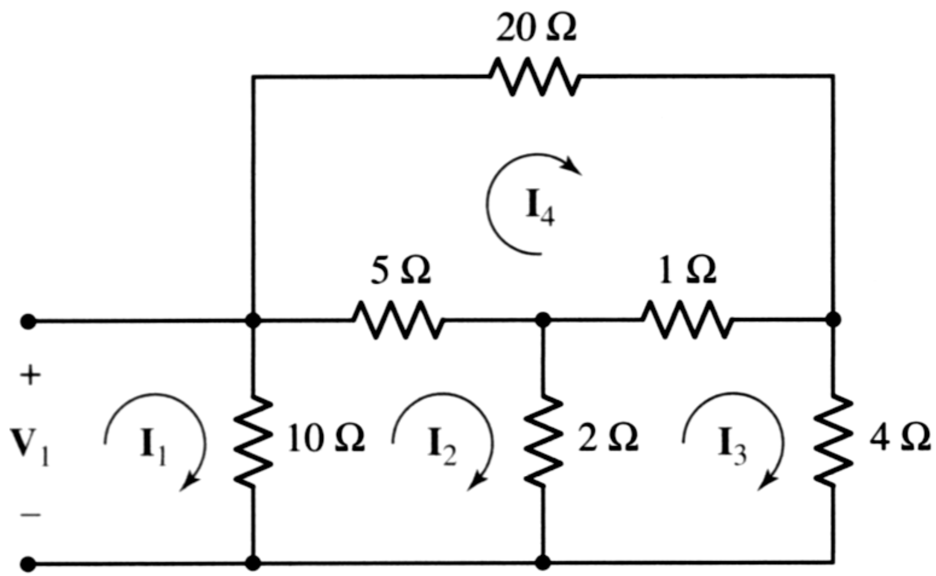
minor

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 \Delta_{11}}{\Delta_\mathbf{Z}}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\Delta_\mathbf{Z}}{\Delta_{11}}$$

Example:

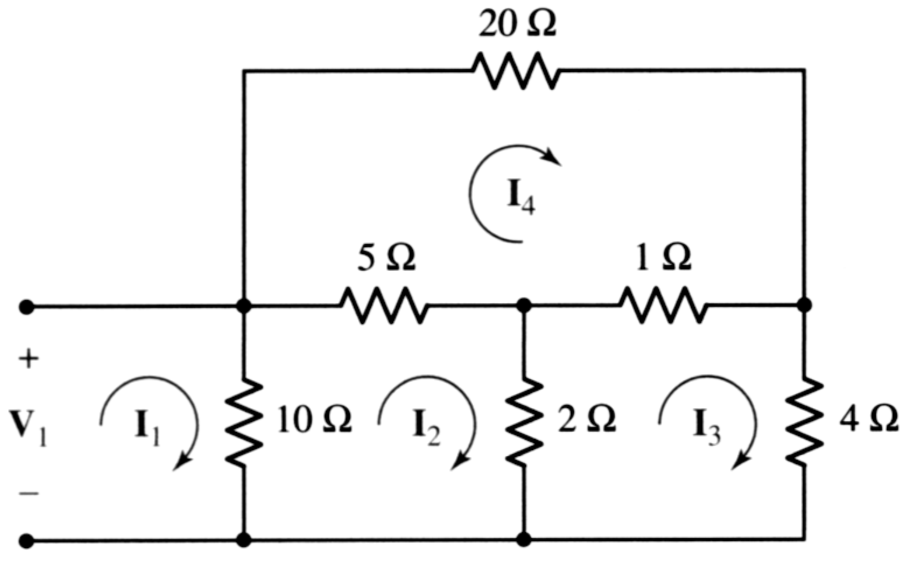




Calculate the input impedance:

Example:

By inspection:



$$
\begin{aligned}
\mathbf{V}_1 &= 10\mathbf{I}_1 - 10\mathbf{I}_2 \\
0 &= -10\mathbf{I}_1 + 17\mathbf{I}_2 - 2\mathbf{I}_3 - 5\mathbf{I}_4 \\
0 &= -2\mathbf{I}_2 + 7\mathbf{I}_3 - \mathbf{I}_4 \\
0 &= -5\mathbf{I}_2 - \mathbf{I}_3 + 26\mathbf{I}_4
\end{aligned}
$$

Example:

By inspection:

$$
\begin{aligned}
\mathbf{V}_1 &= 10\mathbf{I}_1 - 10\mathbf{I}_2 \\
0 &= -10\mathbf{I}_1 + 17\mathbf{I}_2 - 2\mathbf{I}_3 - 5\mathbf{I}_4 \\
0 &= -2\mathbf{I}_2 + 7\mathbf{I}_3 - \mathbf{I}_4 \\
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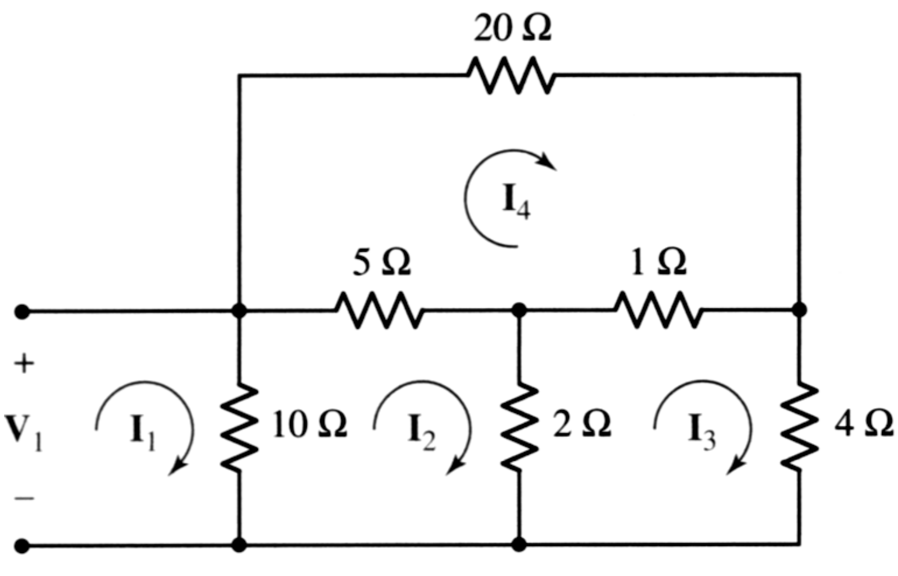
\end{aligned}
$$

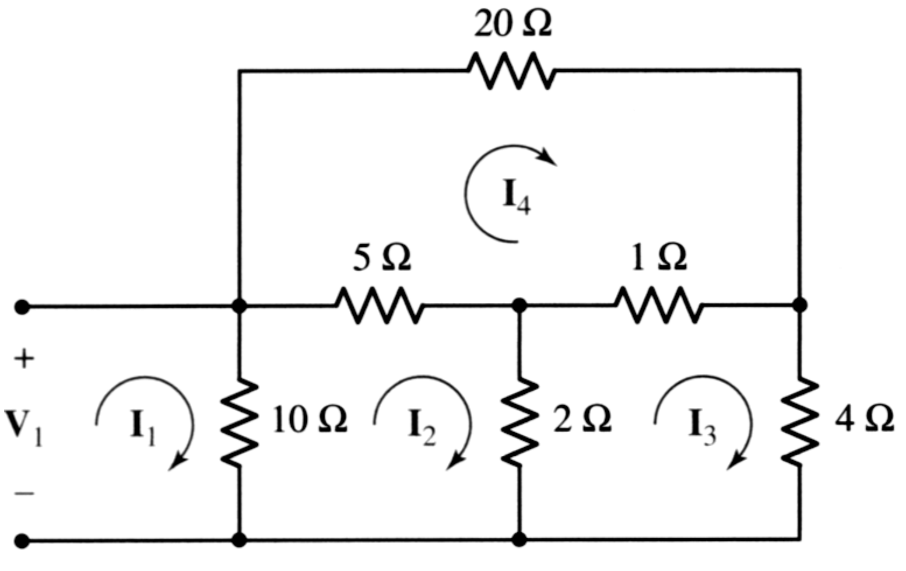


$$
\Delta_Z = \begin{vmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & -5 & -1 & 26 \end{vmatrix} = 9680\ \Omega^4
$$

Example:

By inspection:

$$\begin{aligned}
\mathbf{V}_1 &= 10\mathbf{I}_1 - 10\mathbf{I}_2 \\
0 &= -10\mathbf{I}_1 + 17\mathbf{I}_2 - 2\mathbf{I}_3 - 5\mathbf{I}_4 \\
0 &= -2\mathbf{I}_2 + 7\mathbf{I}_3 - \mathbf{I}_4 \\
0 &= -5\mathbf{I}_2 - \mathbf{I}_3 + 26\mathbf{I}_4
\end{aligned}$$

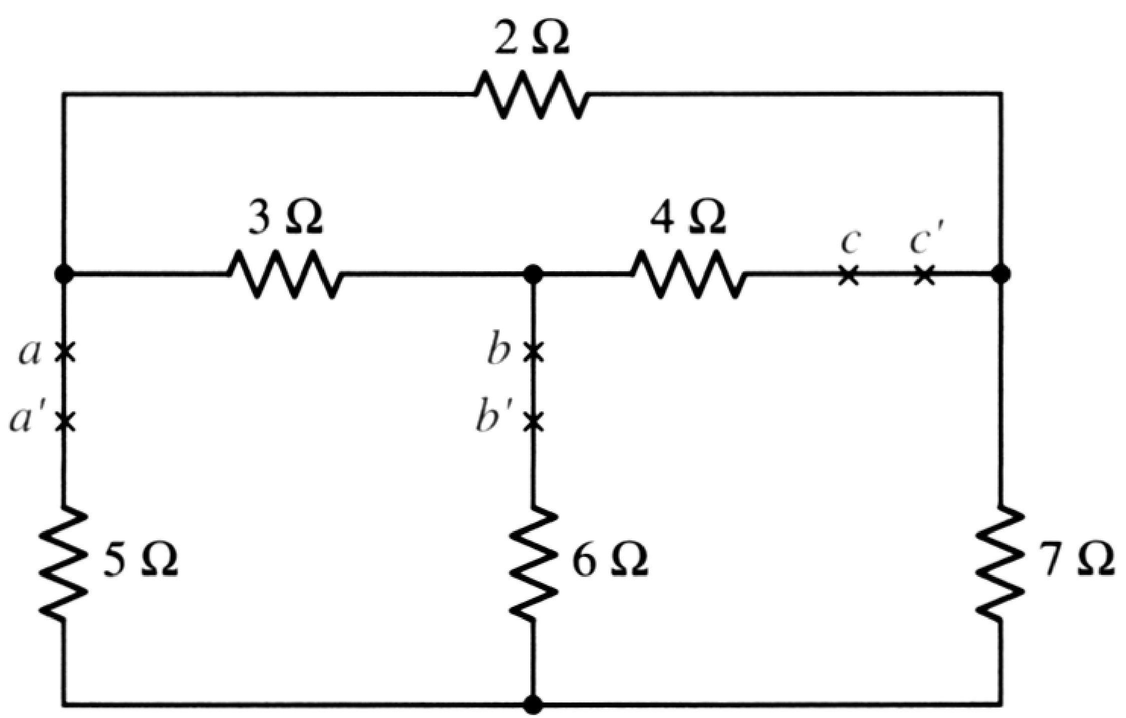


$$\Delta_{11} = \begin{vmatrix} 17 & -2 & -5 \\ -2 & 7 & -1 \\ -5 & -1 & 26 \end{vmatrix} = 2778$$

$$\mathbf{Z}_{in} = \frac{9680}{2778} = 3.485\ \Omega$$

Practice: 17.1

Find the input impedance of the network shown in Figure below if it is formed into a one-port network by breaking it at terminals: (a) a and a'; (b) b and b'; (c) c and c'.

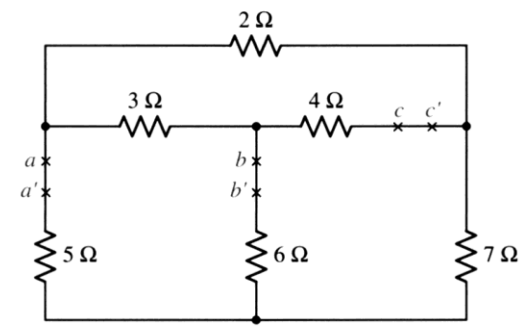


Practice: 17.1

- Assign clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ to the top, left-hand and right-hand meshes, respectively.

$$\begin{aligned}
\mathbf{V}_{cc'} &= 9\,\mathbf{I}_1 - 3\,\mathbf{I}_2 - 4\,\mathbf{I}_3 \\
\mathbf{V}_{aa'} - \mathbf{V}_{bb'} &= -3\,\mathbf{I}_1 + 14\,\mathbf{I}_2 - 6\,\mathbf{I}_3 \\
\mathbf{V}_{bb'} - \mathbf{V}_{cc'} &= -4\,\mathbf{I}_1 - 6\,\mathbf{I}_2 + 17\,\mathbf{I}_3
\end{aligned}$$

The circuit determinant is $\Delta_{\mathbf{z}} = \begin{vmatrix} 9 & -3 & -4 \\ -3 & 14 & -6 \\ -4 & -6 & 17 \end{vmatrix} = 1297.$

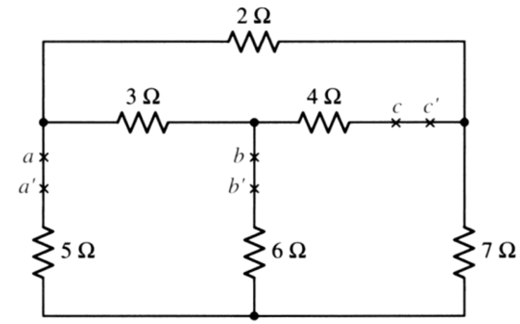


(a) $\mathbf{Z}_{aa'} = \mathbf{V}_{aa'} / \mathbf{I}_2$ with $\mathbf{V}_{bb'} = \mathbf{V}_{cc'} = 0$.

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 9 & 0 & -4 \\ -3 & \mathbf{V}_{aa'} & -6 \\ -4 & 0 & 17 \end{vmatrix}}{\Delta_{\mathbf{z}}} = \frac{\mathbf{V}_{aa'}}{\Delta_{\mathbf{z}}}\begin{vmatrix} 9 & -4 \\ -4 & 17 \end{vmatrix} = \frac{137}{1297}\mathbf{V}_{aa'}$$

Thus, $\mathbf{Z}_{aa'} = 1297/137 = \boxed{9.467\ \Omega.}$

Practice: 17.1



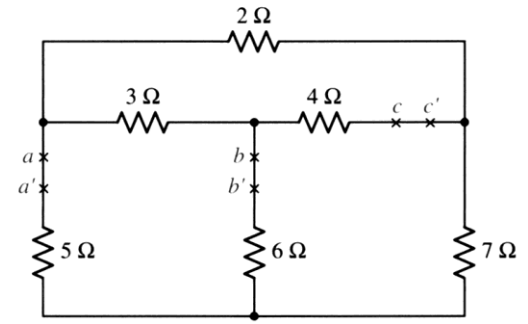
(b) $\mathbf{Z}_{bb'} = \dfrac{\mathbf{V}_{bb'}}{\mathbf{I}_3 - \mathbf{I}_2}$ with $\mathbf{V}_{aa'} = \mathbf{V}_{cc'} = 0.$

$$\Delta_z \cdot \mathbf{I}_3 = \begin{vmatrix} 9 & -3 & 0 \\ -3 & 14 & -\mathbf{V}_{bb'} \\ -4 & -6 & \mathbf{V}_{bb'} \end{vmatrix} = \mathbf{V}_{bb'}\begin{vmatrix} 9 & -3 \\ -4 & -6 \end{vmatrix} + \mathbf{V}_{bb'}\begin{vmatrix} 9 & -3 \\ -3 & 14 \end{vmatrix} = \mathbf{V}_{bb'}(-66+117) = 51\mathbf{V}_{bb'}$$

$$\Delta_z \cdot \mathbf{I}_2 = \begin{vmatrix} 9 & 0 & -4 \\ -3 & -\mathbf{V}_{bb'} & -6 \\ -4 & \mathbf{V}_{bb'} & 17 \end{vmatrix} = -\mathbf{V}_{bb'}\begin{vmatrix} 9 & -4 \\ -4 & 17 \end{vmatrix} - \mathbf{V}_{bb'}\begin{vmatrix} 9 & -4 \\ -3 & -6 \end{vmatrix} = -\mathbf{V}_{bb'}(137-66) = -71\mathbf{V}_{bb'}$$

$$\text{so } \mathbf{Z}_{bb'} = \frac{\Delta_z}{51+71} = \frac{1297}{122} = \boxed{10.63\ \Omega}$$

Practice: 17.1



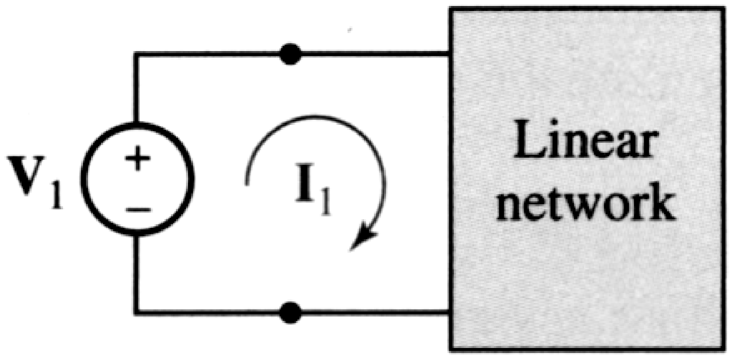
(c) $\mathbf{Z}_{cc'} = \dfrac{\mathbf{V}_{cc'}}{\mathbf{I}_1 - \mathbf{I}_3}$ with $\mathbf{V}_{aa'} = \mathbf{V}_{bb'} = 0.$

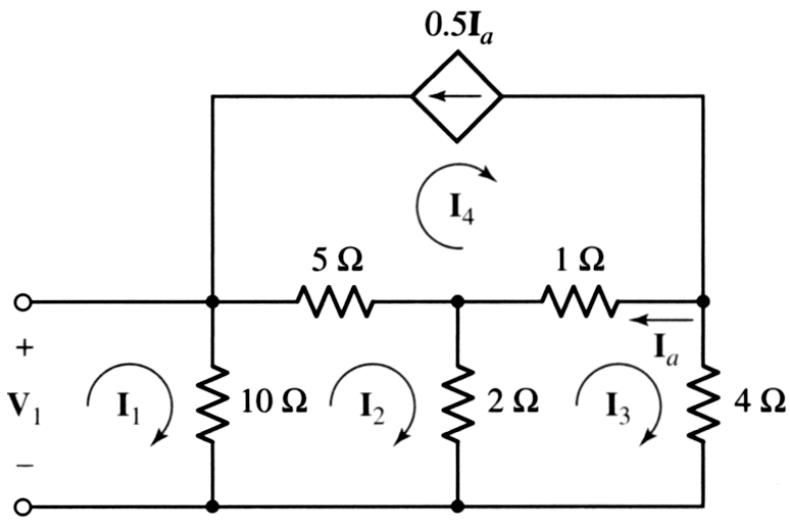
$$\Delta_z \cdot \mathbf{I}_1 = \begin{vmatrix} \mathbf{V}_{cc'} & -3 & -4 \\ 0 & 14 & -6 \\ -\mathbf{V}_{cc'} & -6 & 17 \end{vmatrix} = \mathbf{V}_{cc'}\begin{vmatrix} 14 & -6 \\ -6 & 17 \end{vmatrix} - \mathbf{V}_{cc'}\begin{vmatrix} -3 & -4 \\ 14 & -6 \end{vmatrix} = \mathbf{V}_{cc'}(202-74) = 128\mathbf{V}_{cc'}$$

$$\Delta_z \cdot \mathbf{I}_3 = \begin{vmatrix} 9 & -3 & \mathbf{V}_{cc'} \\ -3 & 14 & 0 \\ -4 & -6 & -\mathbf{V}_{cc'} \end{vmatrix} = \mathbf{V}_{cc'}\begin{vmatrix} -3 & 14 \\ -4 & -6 \end{vmatrix} - \mathbf{V}_{cc'}\begin{vmatrix} 9 & -3 \\ -3 & 14 \end{vmatrix} = \mathbf{V}_{cc'}(74-117) = -43\mathbf{V}_{cc'}$$

$$\text{so } \mathbf{Z}_{cc'} = \frac{\Delta_z}{128+43} = \frac{1297}{171} = \boxed{7.585\ \Omega}$$

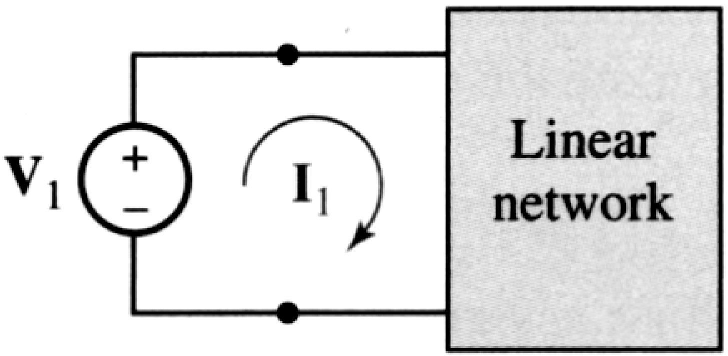
Example:

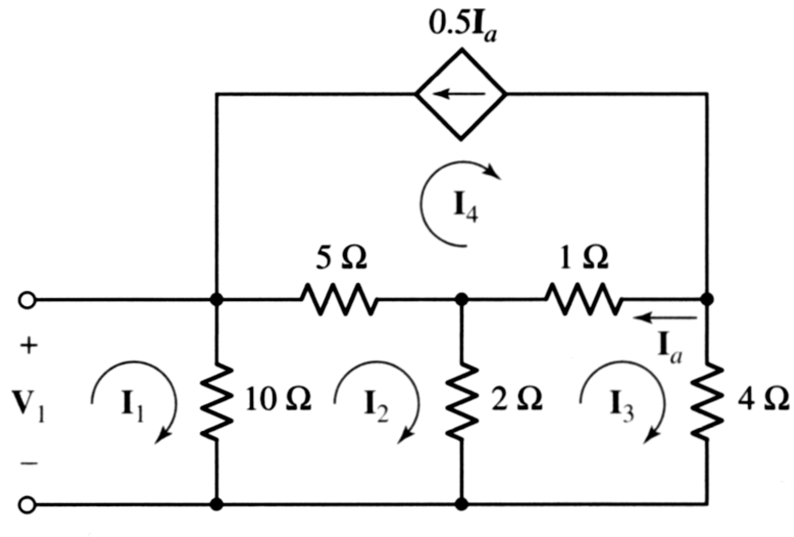




Calculate the input impedance:

Example:





$$
\begin{aligned}
10\mathbf{I}_1 - 10\mathbf{I}_2 &= \mathbf{V}_1 \\
-10\mathbf{I}_1 + 17\mathbf{I}_2 - 2\mathbf{I}_3 - 5\mathbf{I}_4 &= 0 \\
-2\mathbf{I}_2 + 7\mathbf{I}_3 - \mathbf{I}_4 &= 0
\end{aligned}
$$

and $\mathbf{I}_4 = -0.5\mathbf{I}_a = -0.5(\mathbf{I}_4 - \mathbf{I}_3)$

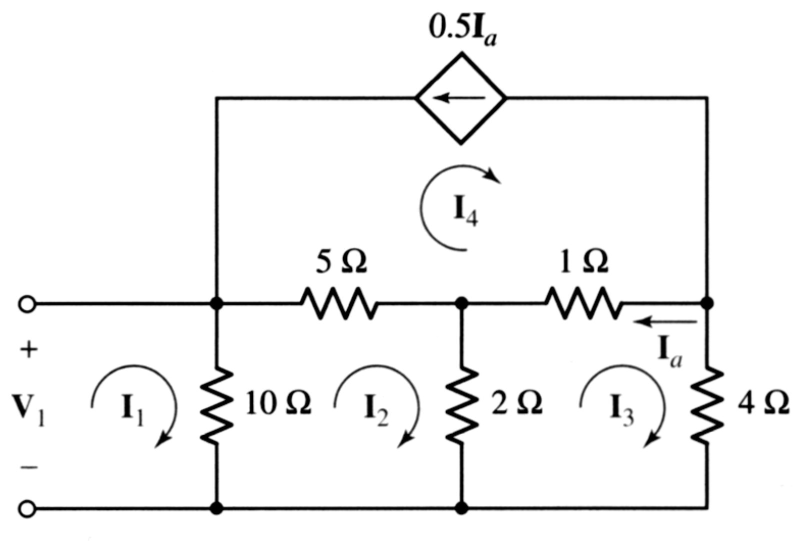
or

$$-0.5\mathbf{I}_3 + 1.5\mathbf{I}_4 = 0$$

Example:

$$\begin{aligned}
10\mathbf{I}_1 \quad -10\mathbf{I}_2 \qquad\qquad\qquad\qquad &= \mathbf{V}_1 \\
-10\mathbf{I}_1 \quad +17\mathbf{I}_2 \quad -2\mathbf{I}_3 \quad -5\mathbf{I}_4 &= 0 \\
-2\mathbf{I}_2 \quad +7\mathbf{I}_3 \quad -\mathbf{I}_4 &= 0
\end{aligned}$$

$$-0.5\mathbf{I}_3 + 1.5\mathbf{I}_4 = 0$$

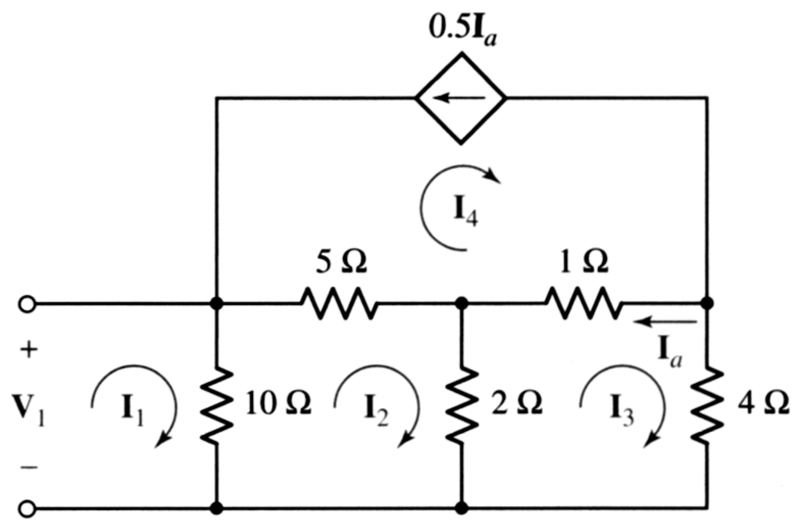


$$\Delta_Z = \begin{vmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & 0 & -0.5 & 1.5 \end{vmatrix} = 590 \ \Omega$$

Example:

$$\begin{array}{rrrrcr} 10\mathbf{I}_1 & -10\mathbf{I}_2 & & & = & \mathbf{V}_1 \\ -10\mathbf{I}_1 & +17\mathbf{I}_2 & -2\mathbf{I}_3 & -5\mathbf{I}_4 & = & 0 \\ & -2\mathbf{I}_2 & +7\mathbf{I}_3 & -\mathbf{I}_4 & = & 0 \end{array}$$

$$-0.5\mathbf{I}_3 + 1.5\mathbf{I}_4 = 0$$

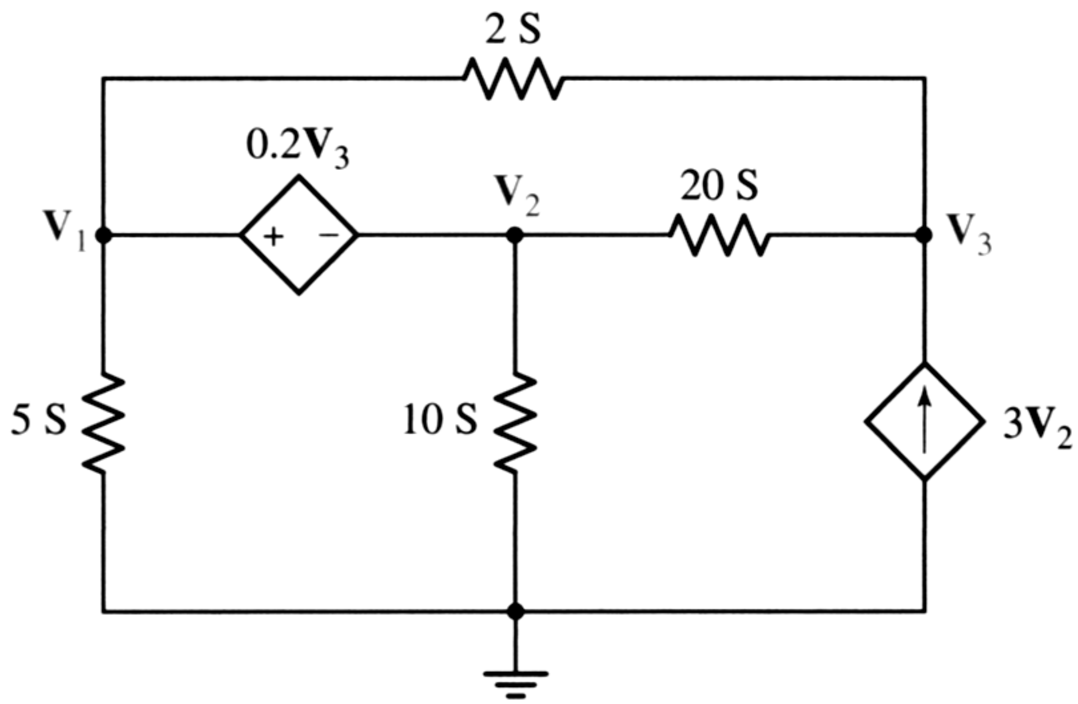


$$\Delta_{11} = \begin{vmatrix} 17 & -2 & -5 \\ -2 & 7 & -1 \\ 0 & -0.5 & 1.5 \end{vmatrix} = 159\ \Omega$$

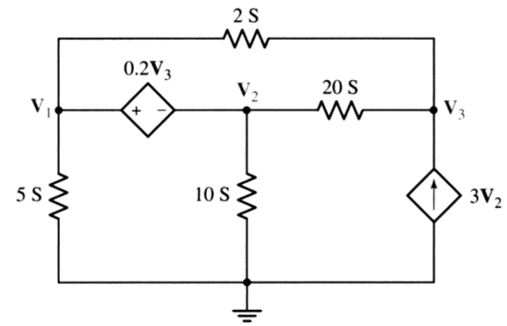
$$\mathbf{Z}_{in} = \frac{590}{159} = 3.711\ \Omega$$

Practice: 17.2

Write a set of nodal equations for the circuit of figure below, calculate Δ_Y, and then find the input admittance seen between: (a) node 1 and the reference node; (b) node 2 and the reference.



Practice: 17.2



At the 1,2 supernode: $0 = 5\mathbf{V}_1 + 10\mathbf{V}_2 + 2(\mathbf{V}_1 - \mathbf{V}_3) + 20(\mathbf{V}_2 - \mathbf{V}_3)$ [1]

At node 3: $3\,\mathbf{V}_2 = 20(\mathbf{V}_3 - \mathbf{V}_2) + 2(\mathbf{V}_3 - \mathbf{V}_1)$ [2]

And: $\mathbf{V}_1 - \mathbf{V}_2 = 0.2\mathbf{V}_3$ [3]

Simplifying,

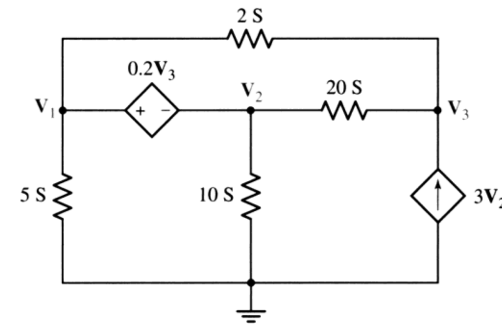
$7\,\mathbf{V}_1 + 30\mathbf{V}_2 - 22\mathbf{V}_3 = 0$ [1]

$-2\,\mathbf{V}_1 - 23\mathbf{V}_2 + 22\mathbf{V}_3 = 0$ [2]

$\mathbf{V}_1 - \mathbf{V}_2 - 0.2\mathbf{V}_3 = 0$ [3]

$$\Delta_{\mathbf{Y}} = \begin{vmatrix} 7 & 30 & -22 \\ -2 & -23 & 22 \\ 1 & -1 & -0.2 \end{vmatrix} = 284.2$$

Practice: 17.2



(a) If we inject a current **I** into node 1 by connecting a current source in parallel with the 5-S conductance, we obtain:

$$\mathbf{V}_1 = \frac{\begin{vmatrix} \mathbf{I} & 30 & -22 \\ 0 & -23 & 22 \\ 0 & -1 & -0.2 \end{vmatrix}}{\Delta_Y} = \frac{\mathbf{I}\begin{vmatrix} -23 & 22 \\ -1 & -0.2 \end{vmatrix}}{\Delta_Y} = \mathbf{I}\,\frac{26.6}{284.2}$$

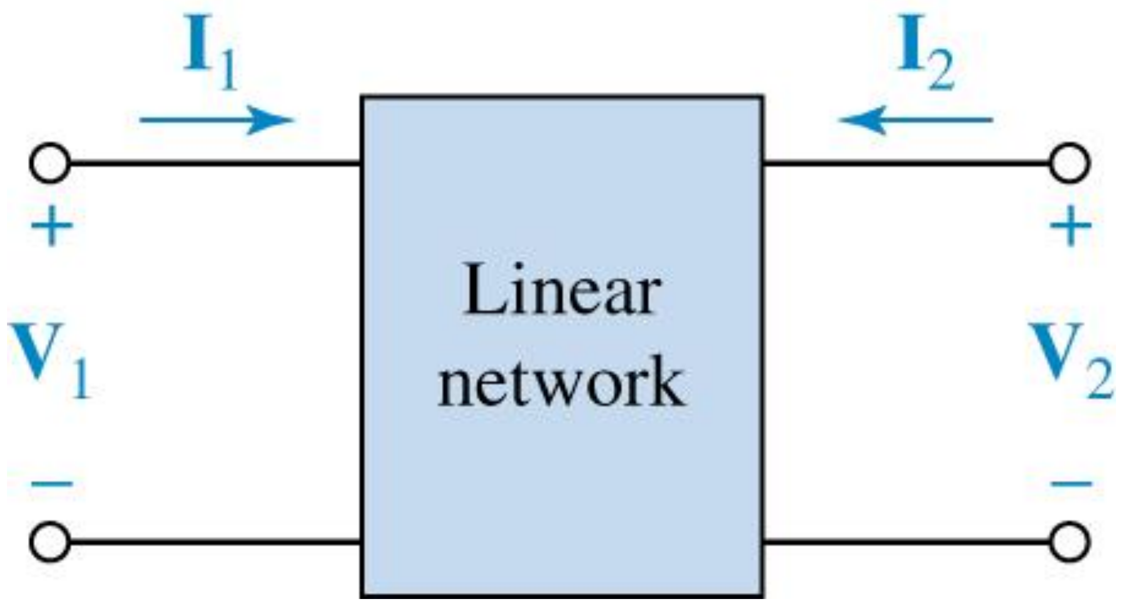
Thus, the input admittance is $\mathbf{I}/\mathbf{V}_1 = 284.2/26.6 = \boxed{10.68\text{ S.}}$

(b) We now connect the current source between node 2 and ground.

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 7 & \mathbf{I} & -22 \\ -2 & 0 & 22 \\ 1 & 0 & -0.2 \end{vmatrix}}{\Delta_Y} = \frac{-\mathbf{I}\begin{vmatrix} -2 & 22 \\ 1 & -0.2 \end{vmatrix}}{\Delta_Y} = -\mathbf{I}\,\frac{(-21.6)}{284.2}$$

Thus, the input admittance is $\mathbf{I}/\mathbf{V}_2 = 284.2/21.6 = \boxed{13.16\text{ S.}}$

Admittance Parameters:



A general two-port with terminal voltages and currents specified.

The two-port is composed of:

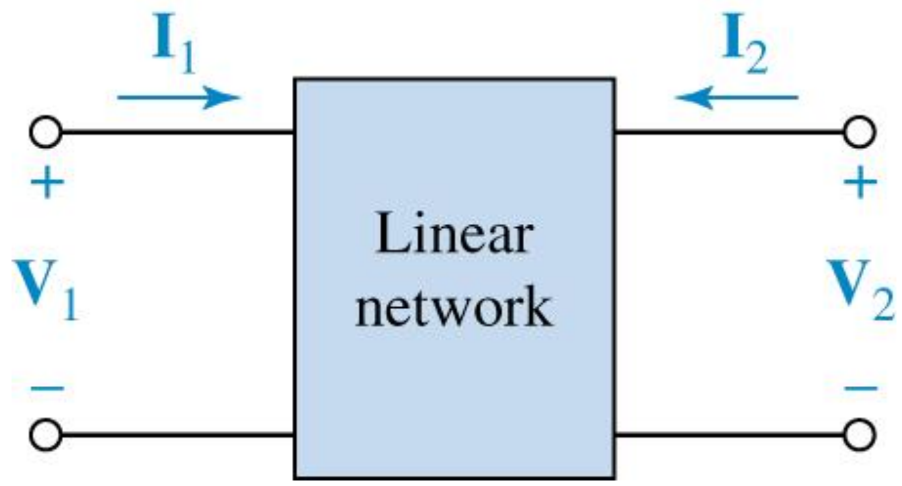
- **linear elements, no energy stored within the circuits**
- **possibly including dependent sources,**
- **but not containing any independent sources.**
- **all external connections must be made to either the input port or the output port.**

Admittance Parameters:

We may begin with the set of equations:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$



The matrix equation:

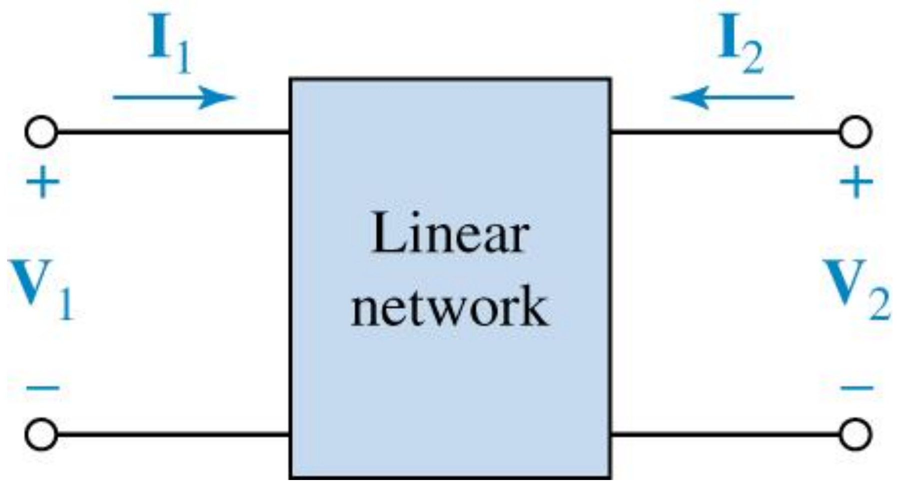
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Admittance Parameters:

We may begin with the set of equations:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$



The short-circuit admittance parameters

The short-circuit input admittance: $\mathbf{y}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$

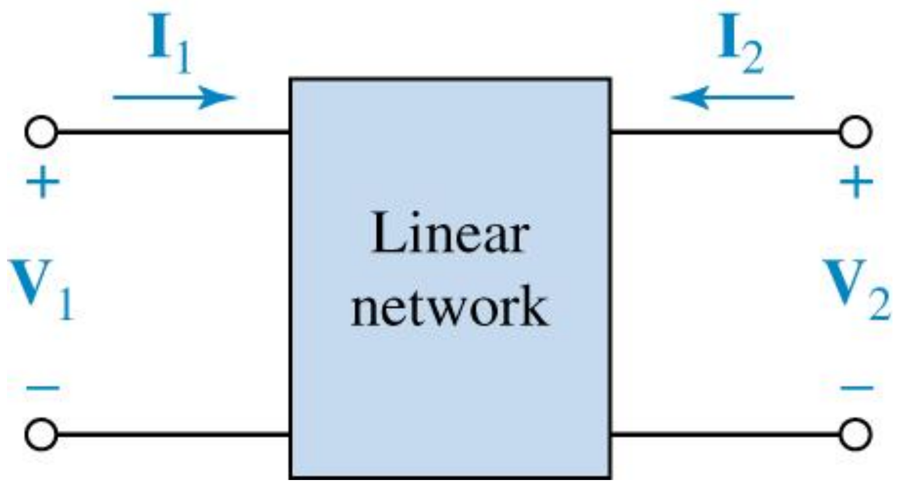
The short-circuit output admittance: $\mathbf{y}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$

Admittance Parameters:

We may begin with the set of equations:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$



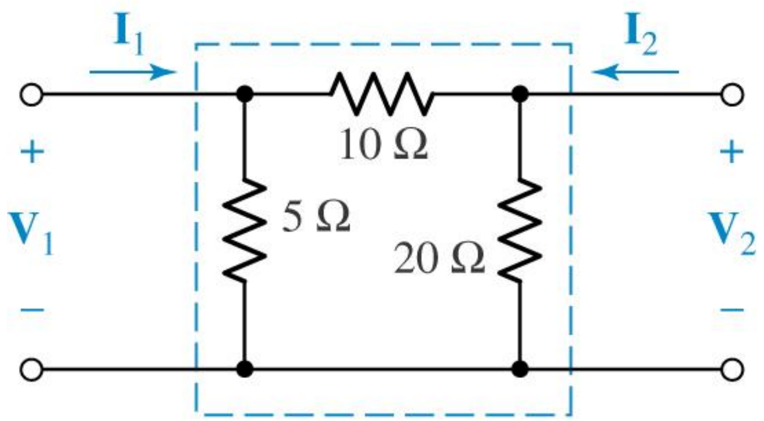
The short-circuit admittance parameters

The short-circuit transfer admittance:

$$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

Example:

Find the short-circuit admittance parameters:

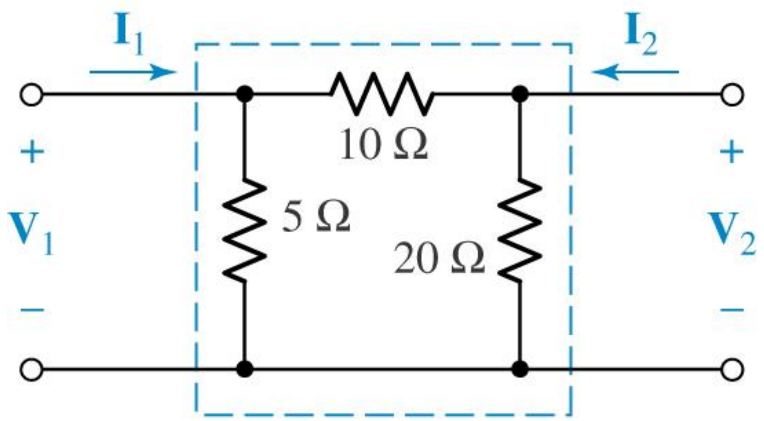


$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

Example:

Find the short-circuit admittance parameters:

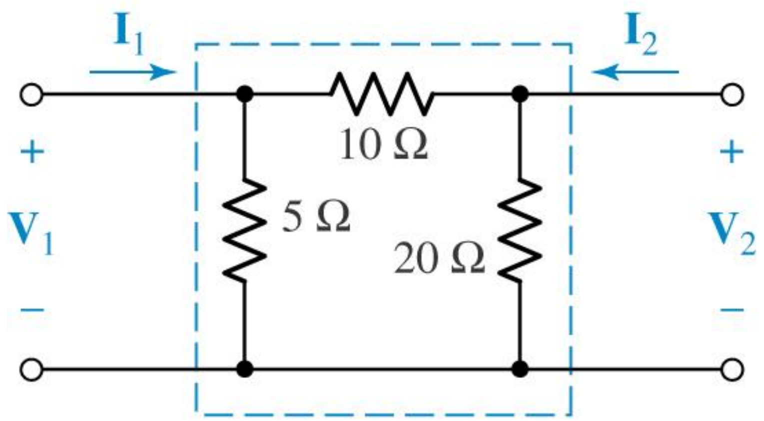


$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{15}{5 \cdot 10} = 0.3 \quad \text{S.}$$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{30}{10 \cdot 20} = 0.15 \quad \text{S.}$$

Example:

Find the short-circuit admittance parameters:

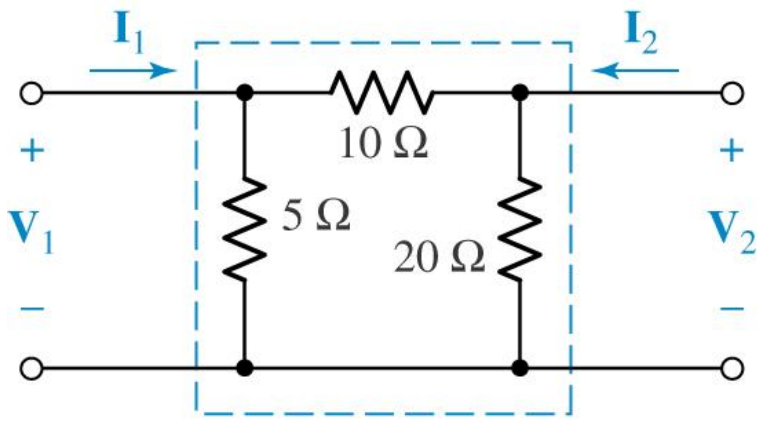


When $\mathbf{V}_1 = 0, \mathbf{I}_1 = \frac{-\mathbf{V}_2}{10} \quad \therefore \mathbf{y}_{12} = \left.\frac{-\mathbf{V}_2/10}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = -0.1 \quad \text{S.}$

When $\mathbf{V}_2 = 0, \mathbf{I}_2 = \frac{-\mathbf{V}_1}{10} \quad \therefore \mathbf{y}_{21} = \left.\frac{-\mathbf{V}_1/10}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = -0.1 \quad \text{S.}$

Example:

Find the short-circuit admittance parameters:



$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

Write the expressions:

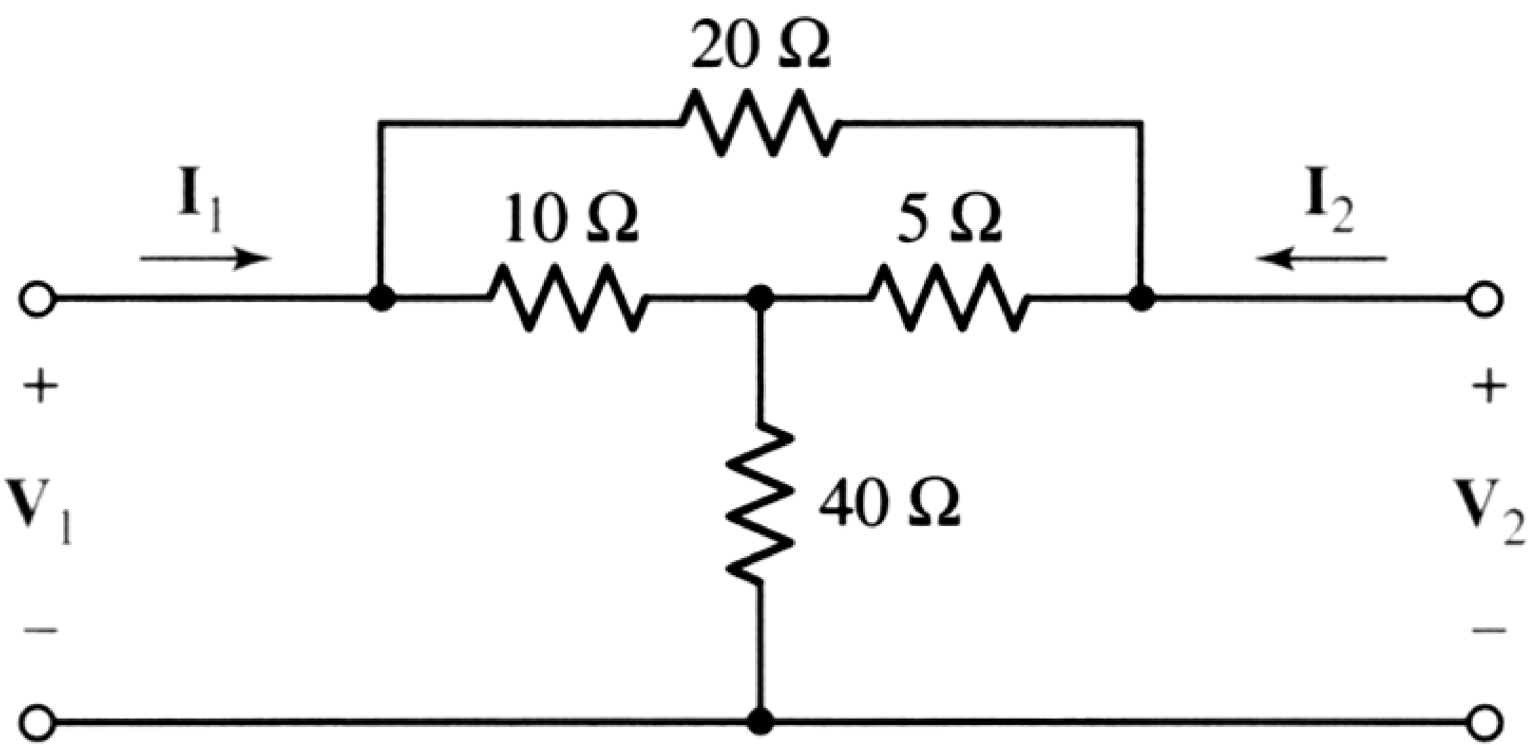
$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = 0.3\mathbf{V}_1 - 0.1\mathbf{V}_2$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 - \mathbf{V}_1}{10} + \frac{\mathbf{V}_2}{20} = -0.1\mathbf{V}_1 + 0.15\mathbf{V}_2$$

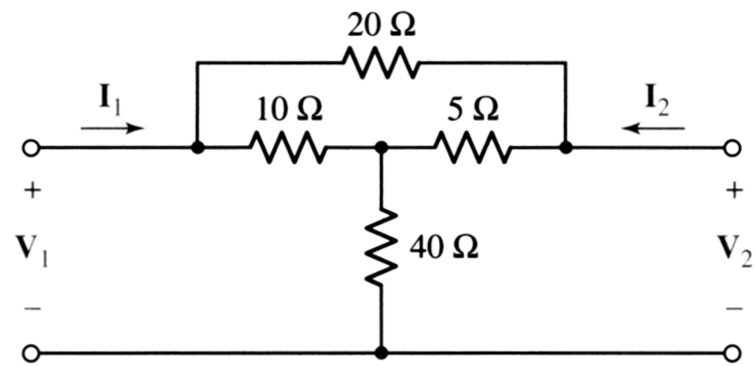
$$\Rightarrow \mathbf{y} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.15 \end{bmatrix}$$

Practice: 17.3

By applying the appropriate 1-V sources and short circuits to the circuit shown in figure below, find (a) $\mathbf{y}_{11}$; (b) $\mathbf{y}_{21}$; (c) $\mathbf{y}_{22}$; (d) $\mathbf{y}_{12}$



Practice: 17.3



(a) $\mathbf{y}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$ so short right terminals and apply $\mathbf{V}_1 = 1$ V.

$5\ \Omega \,||\, 40\ \Omega = 4.444\ \Omega$; $20\ \Omega \,||\, (10 + 4.444) = 8.387\ \Omega$.

Thus, $\mathbf{y}_{11} = 1/8.387 = \boxed{119.2\text{ mS.}}$

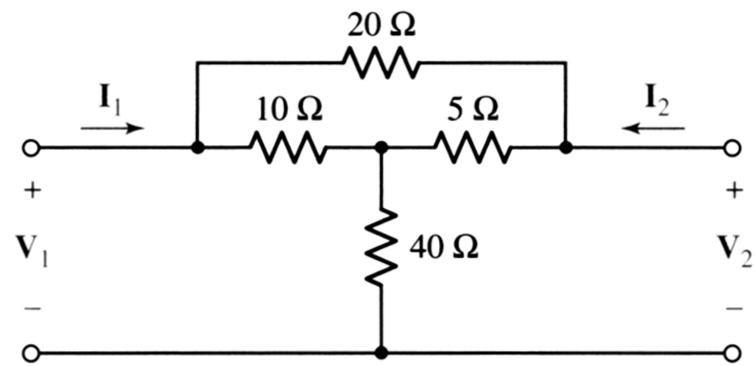
(b) $\mathbf{y}_{21} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$ so short right terminals and apply $\mathbf{V}_1 = 1$ V.

Define a clockwise current $\mathbf{I}_3$ in the top mesh.

Mesh 1: $-1 + 50\,\mathbf{I}_1 - 10\,\mathbf{I}_3 + 40\,\mathbf{I}_2 = 0$
Mesh 2: $-40\,\mathbf{I}_1 - 5\,\mathbf{I}_3 - 45\,\mathbf{I}_2 = 0$
Top mesh: $-10\,\mathbf{I}_1 + 35\,\mathbf{I}_3 + 5\,\mathbf{I}_2 = 0$

Solving, $\mathbf{I}_2 = \boxed{-111.5\text{ mS}}$

Practice: 17.3



(c) $\mathbf{y}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$ so short left terminals and apply $\mathbf{V}_2 = 1$ V.

$10\ \Omega \,||\, 40\ \Omega = 8\ \Omega;\ \ 20\ \Omega \,||\, (8+5) = 7.879\ \Omega.$

Thus, $\mathbf{y}_{22} = 1/7.879 = \boxed{126.9 \text{ mS.}}$

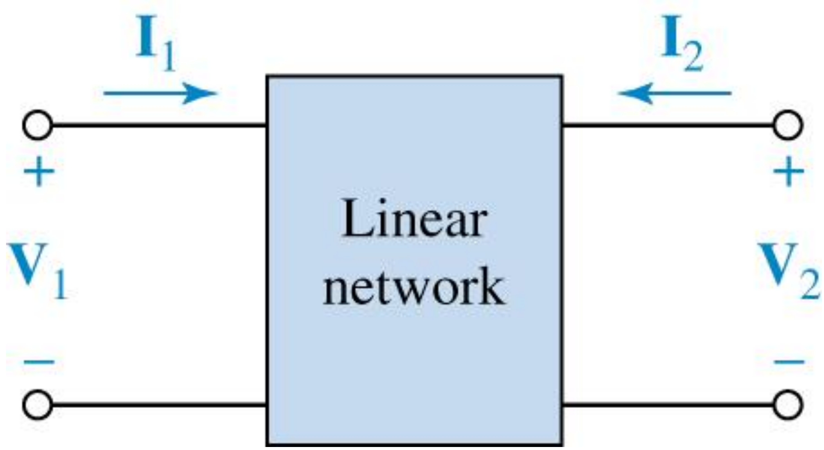
(d) $\mathbf{y}_{12} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$ so short left terminals and apply $\mathbf{V}_2 = 1$ V.

Mesh 1: $50\,\mathbf{I}_1 - 10\,\mathbf{I}_3 + 40\,\mathbf{I}_2 = 0$
Mesh 2: $-40\,\mathbf{I}_1 - 5\,\mathbf{I}_3 - 45\,\mathbf{I}_2 = -1$
Top mesh: $-10\,\mathbf{I}_1 + 35\,\mathbf{I}_3 + 5\,\mathbf{I}_2 = 0$

Solving, $\mathbf{I}_1 = \boxed{-111.5 \text{ mS.}}$

The Terminal Equations:

There are six different ways in which to combine the four variables:



$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\mathbf{V}_1 = \mathbf{a}_{11}\mathbf{V}_2 - \mathbf{a}_{12}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{a}_{21}\mathbf{V}_2 - \mathbf{a}_{22}\mathbf{I}_2$$

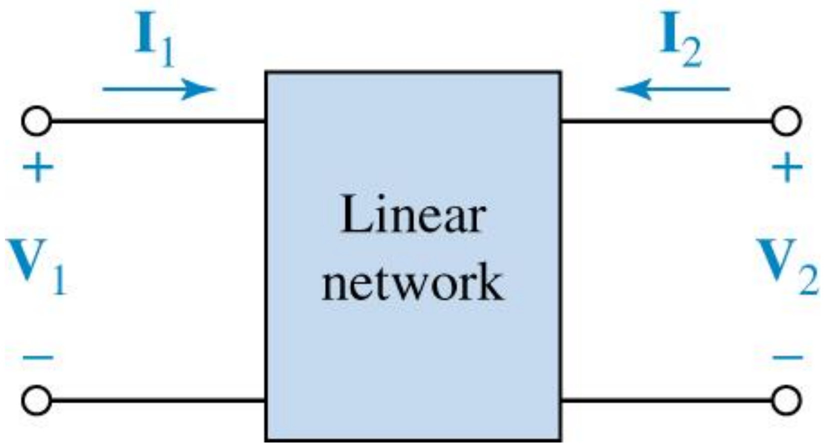
$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\mathbf{V}_2 = \mathbf{b}_{11}\mathbf{V}_1 - \mathbf{b}_{12}\mathbf{I}_1$$
$$\mathbf{I}_2 = \mathbf{b}_{21}\mathbf{V}_1 - \mathbf{b}_{22}\mathbf{I}_1$$

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

Admittance parameters:



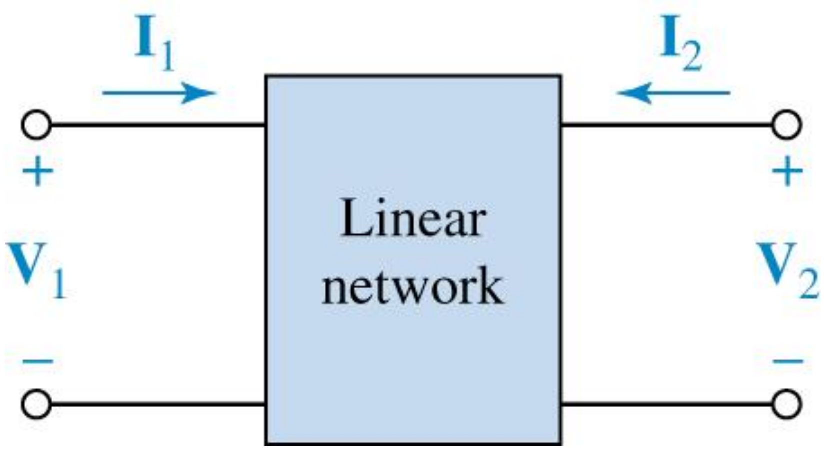
$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

impedance parameters:

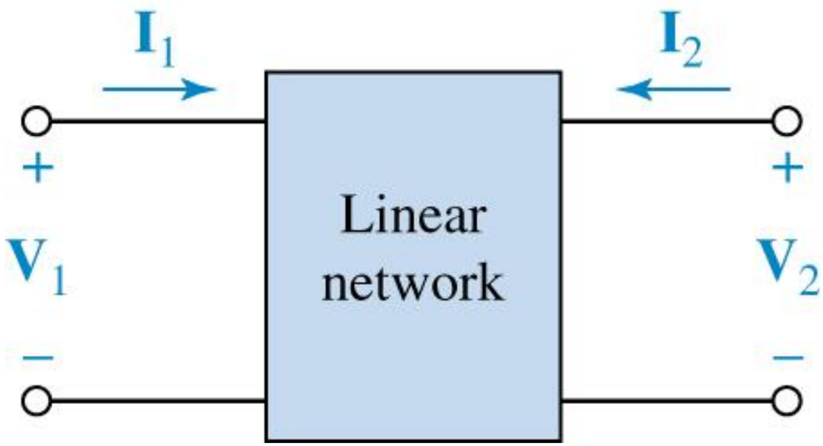


$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \qquad \mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}$$

transmission parameters:



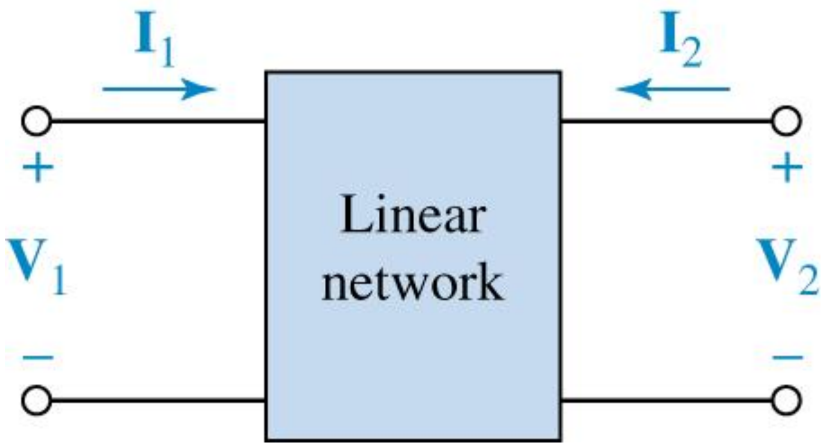
$$\mathbf{V}_1 = \mathbf{a}_{11}\mathbf{V}_2 - \mathbf{a}_{12}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{a}_{21}\mathbf{V}_2 - \mathbf{a}_{22}\mathbf{I}_2$$

$$\mathbf{a}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \qquad \mathbf{a}_{12} = \left.\frac{-\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \ \Omega$$

$$\mathbf{a}_{22} = \left.\frac{-\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \qquad \mathbf{a}_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \ \mathrm{S.}$$

Note: Hayt's use "t" instead of "a"

transmission parameters:

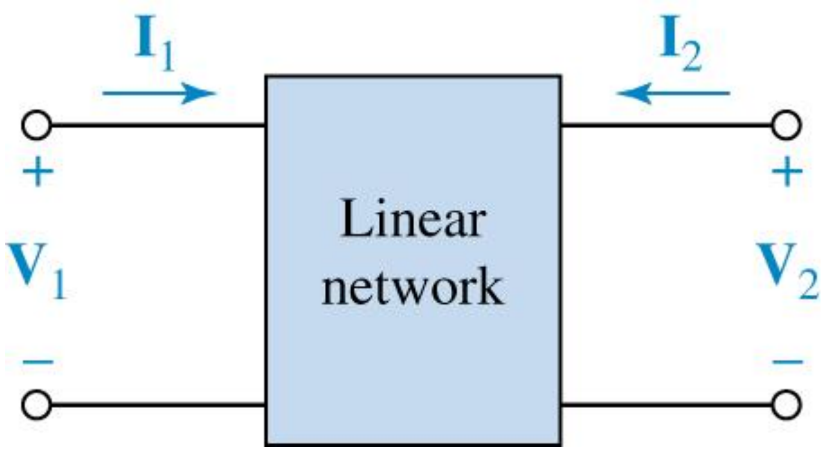


$$\mathbf{V}_2 = \mathbf{b}_{11}\mathbf{V}_1 - \mathbf{b}_{12}\mathbf{I}_1$$
$$\mathbf{I}_2 = \mathbf{b}_{21}\mathbf{V}_1 - \mathbf{b}_{22}\mathbf{I}_1$$

$$\mathbf{b}_{11} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0} \qquad \mathbf{b}_{12} = \left.\frac{-\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \ \Omega$$

$$\mathbf{b}_{22} = \left.\frac{-\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \qquad \mathbf{b}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0} \ \mathrm{S}.$$

hybrid parameters:

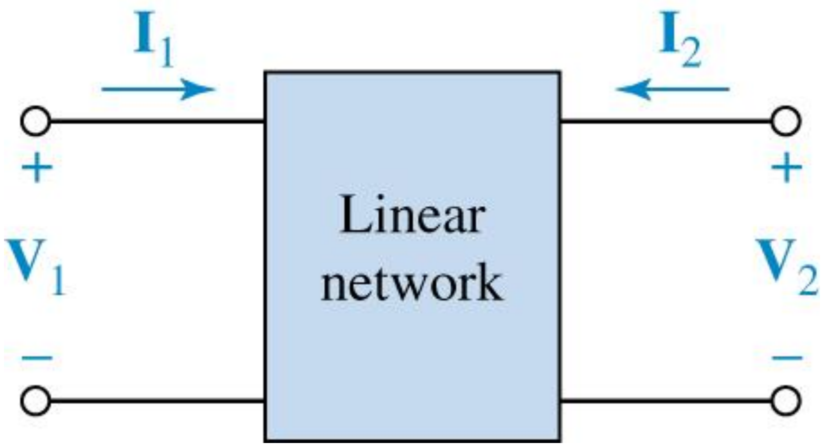


$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \quad \Omega \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \quad \mathrm{S}. \qquad \mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}$$

hybrid parameters:



$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$\mathbf{g}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0} \quad \text{S.} \qquad \mathbf{g}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{g}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0} \quad \Omega \qquad \mathbf{g}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}$$

Relationships Among ...:

To find the z-parameters as function of y-parameters,

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{\begin{vmatrix} \mathbf{I}_1 & \mathbf{y}_{12} \\ \mathbf{I}_2 & \mathbf{y}_{22} \end{vmatrix}}{\begin{vmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{vmatrix}} = \frac{\mathbf{y}_{22}}{\Delta\mathbf{y}}\mathbf{I}_1 - \frac{\mathbf{y}_{12}}{\Delta\mathbf{y}}\mathbf{I}_2 \qquad \mathbf{V}_2 = \frac{\begin{vmatrix} \mathbf{I}_1 & \mathbf{y}_{12} \\ \mathbf{I}_2 & \mathbf{y}_{22} \end{vmatrix}}{\Delta\mathbf{y}} = \frac{\mathbf{y}_{22}}{\Delta\mathbf{y}}\mathbf{I}_1 - \frac{\mathbf{y}_{12}}{\Delta\mathbf{y}}\mathbf{I}_2$$

Transformation between …:

Table 17.1 Transformations between **y**, **z**, **h**, and **t** parameters

	y		**z**		**h**		**t**	
y	y_{11}	y_{12}	$\frac{z_{22}}{\Delta_z}$	$\frac{-z_{12}}{\Delta_z}$	$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$	$\frac{t_{22}}{t_{12}}$	$\frac{-\Delta_t}{t_{12}}$
	y_{21}	y_{22}	$\frac{-z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$\frac{-1}{t_{12}}$	$\frac{t_{11}}{t_{12}}$
z	$\frac{y_{22}}{\Delta_y}$	$\frac{-y_{12}}{\Delta_y}$	z_{11}	z_{12}	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{t_{11}}{t_{21}}$	$\frac{\Delta_t}{t_{21}}$
	$\frac{-y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	z_{21}	z_{22}	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{t_{21}}$	$\frac{t_{22}}{t_{21}}$
h	$\frac{1}{y_{11}}$	$\frac{-y_{12}}{y_{11}}$	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	h_{11}	h_{12}	$\frac{t_{12}}{t_{22}}$	$\frac{\Delta_t}{t_{22}}$
	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	$\frac{-z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	h_{21}	h_{22}	$\frac{-1}{t_{22}}$	$\frac{t_{21}}{t_{22}}$
t	$\frac{-y_{22}}{y_{21}}$	$\frac{-1}{y_{21}}$	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$\frac{-\Delta_h}{h_{21}}$	$\frac{-h_{11}}{h_{21}}$	t_{11}	t_{12}
	$\frac{-\Delta_y}{y_{21}}$	$\frac{-y_{11}}{y_{21}}$	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$\frac{-h_{22}}{h_{21}}$	$\frac{-1}{h_{21}}$	t_{21}	t_{22}

For all parameter sets: $\Delta_p = p_{11}p_{22} - p_{12}p_{21}$.

Reciprocal Two-Port Circuits:

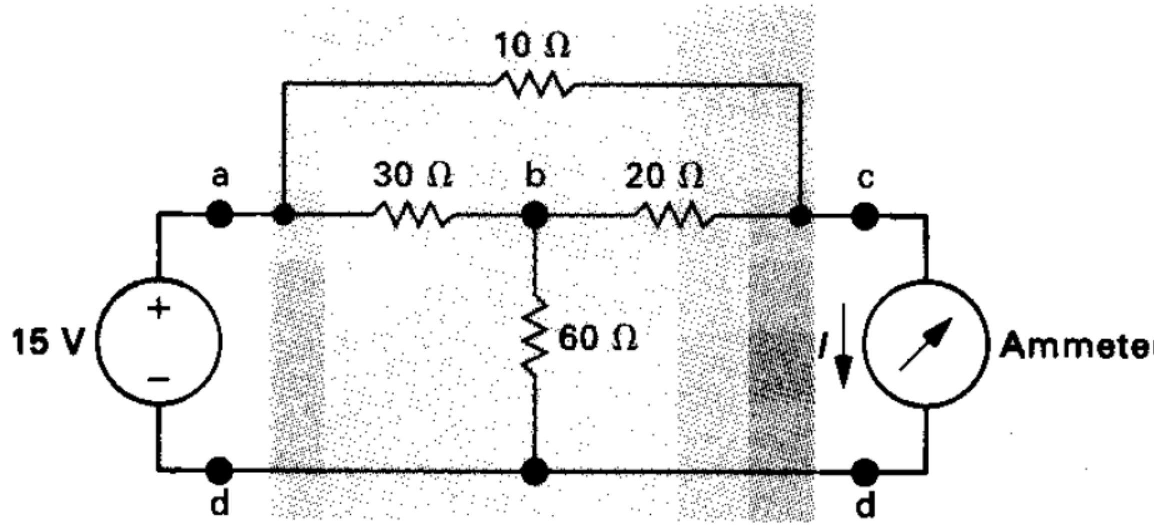
The following relationship exist,

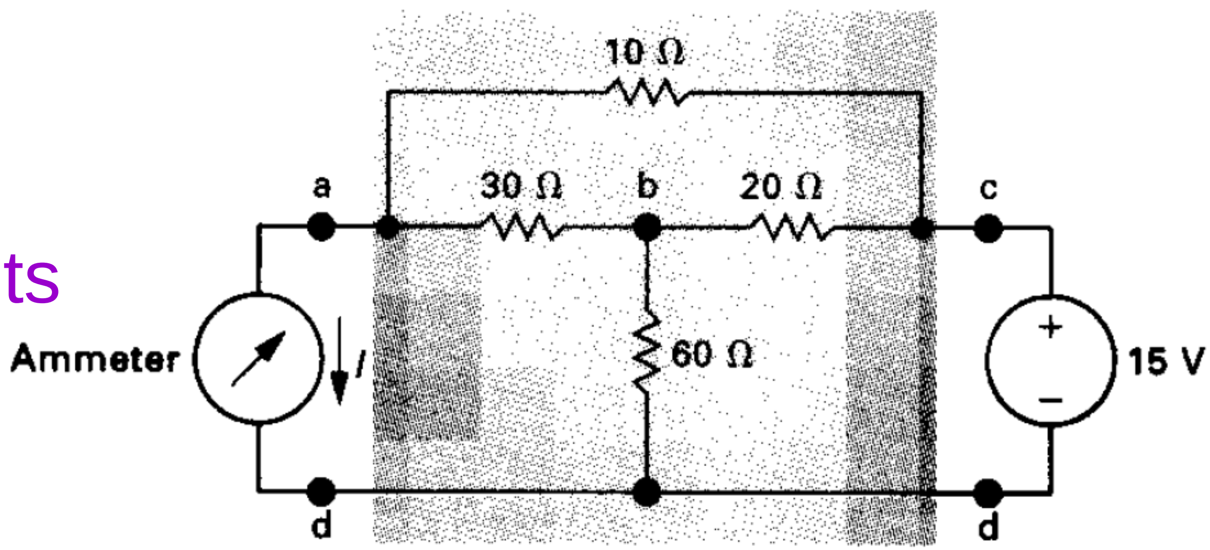
$$
\begin{aligned}
\mathbf{z}_{12} &= \mathbf{z}_{21}, \\
\mathbf{y}_{12} &= \mathbf{y}_{21}, \\
\mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21} &= \Delta\mathbf{a} = 1, \\
\mathbf{b}_{11}\mathbf{b}_{22} - \mathbf{b}_{12}\mathbf{b}_{21} &= \Delta\mathbf{b} = 1, \\
\mathbf{h}_{12} &= -\mathbf{h}_{21}, \\
\mathbf{g}_{12} &= -\mathbf{g}_{21}
\end{aligned}
$$

Reciprocal Two-Port Circuits:

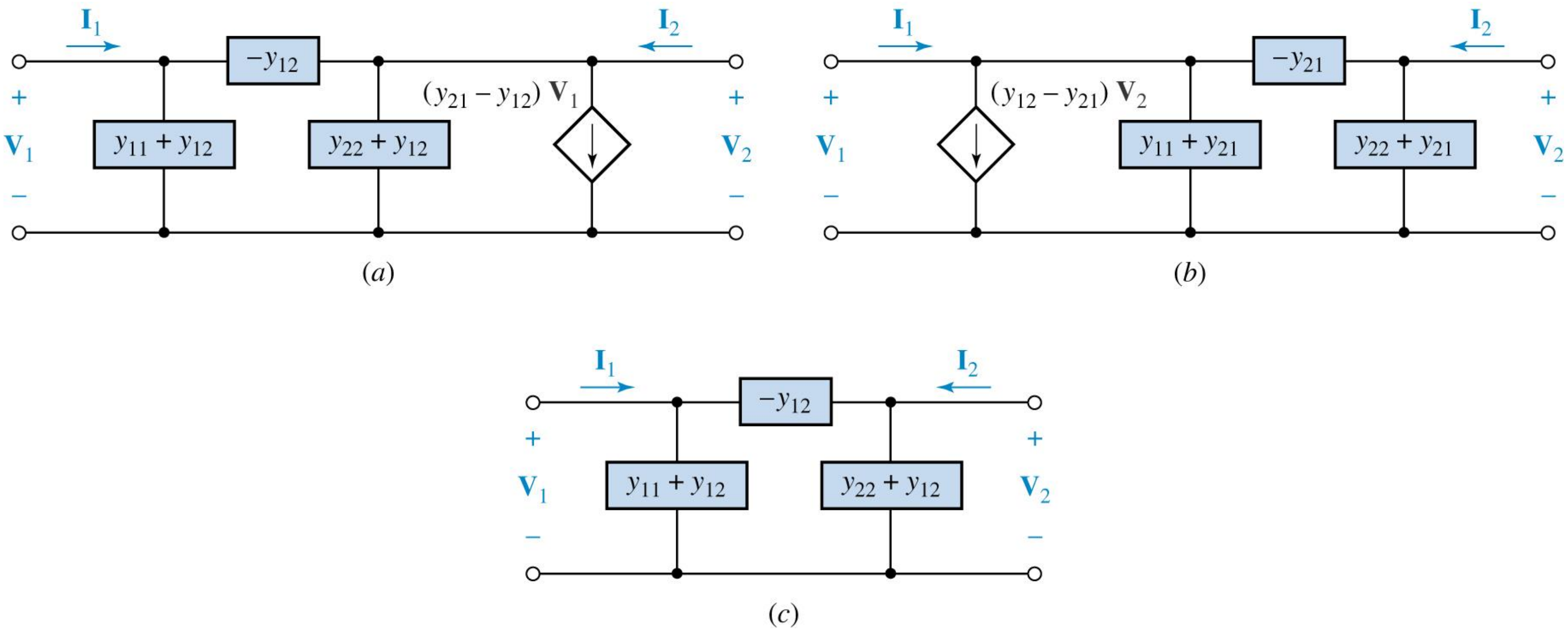
A reciprocal two-port circuit:

A reciprocal two-port circuit is symmetric if its ports can be interchanged without disturbing the values of the terminal currents and voltages.



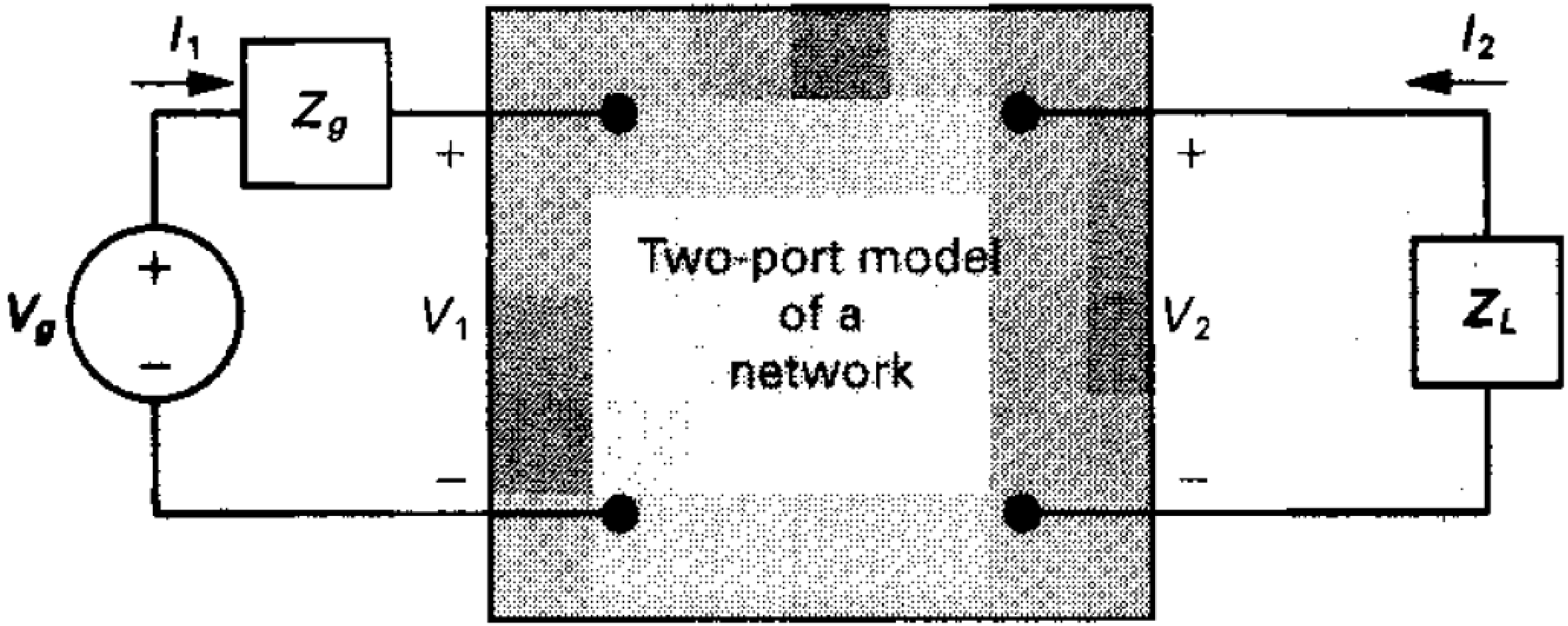


Some Equivalent Networks:



(*a*, *b*) Two-ports which are equivalent to any general linear two-port. The dependent source in part *a* depends on $\mathbf{V}_1$, and that in part *b* depends on $\mathbf{V}_2$. (c) An equivalent for a bilateral network.

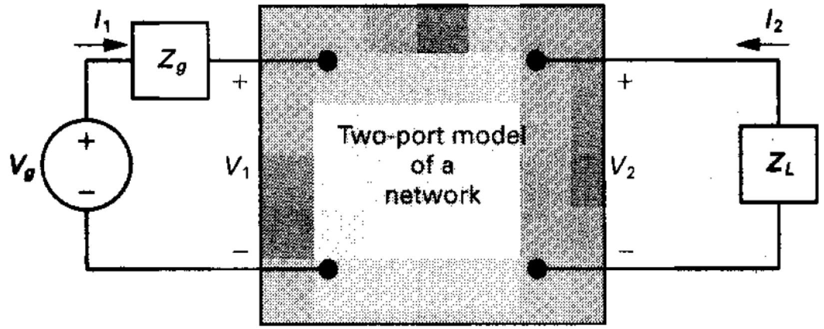
Analysis of the Terminated …:



Six characteristics of the terminated two-port circuits define its terminal behavior:

Analysis of the Terminated …:

Six characteristics of the terminated two-port circuits define its terminal behavior:

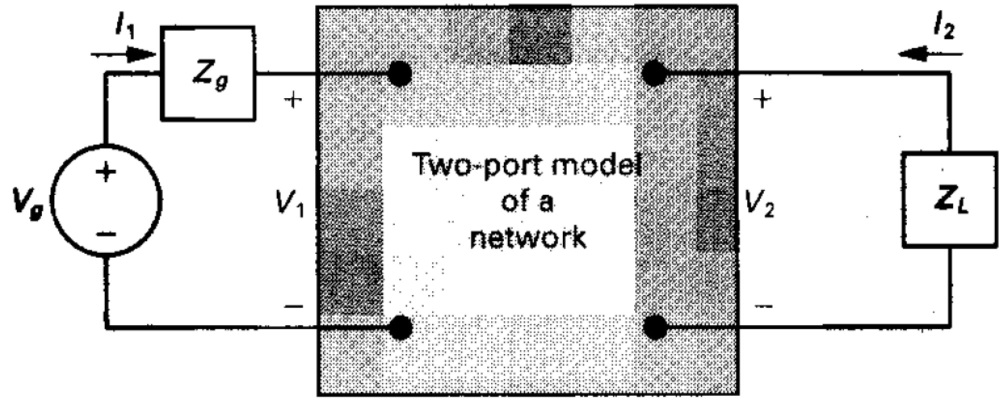


1. The input impedance
2. the output current
3. the Thevenin voltage and impedance
4. The current gain
5. The voltage gain $\dfrac{\mathbf{V}_2}{\mathbf{V}_1}$
6. The voltage gain $\dfrac{\mathbf{V}_2}{\mathbf{V}_g}$

Analysis of the Terminated …:

Six characteristics in terms of the z-parameters:

The input impedance:
(the impedance seen looking into port 1)



$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad \ldots{*}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad \ldots{**}$$

$$\mathbf{V}_1 = \mathbf{V}_g - \mathbf{I}_1\mathbf{Z}_g \qquad \ldots{***}$$

$$\mathbf{V}_2 = -\mathbf{I}_2\mathbf{Z}_L \qquad \ldots{****}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

$$\mathbf{V}_2 = -\mathbf{I}_2\mathbf{Z}_L = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\Rightarrow \mathbf{I}_2 = \frac{-\mathbf{z}_{21}\mathbf{I}_1}{\mathbf{Z}_L + \mathbf{z}_{22}}$$

$$sub \quad \mathbf{I}_2 \quad in \quad *$$

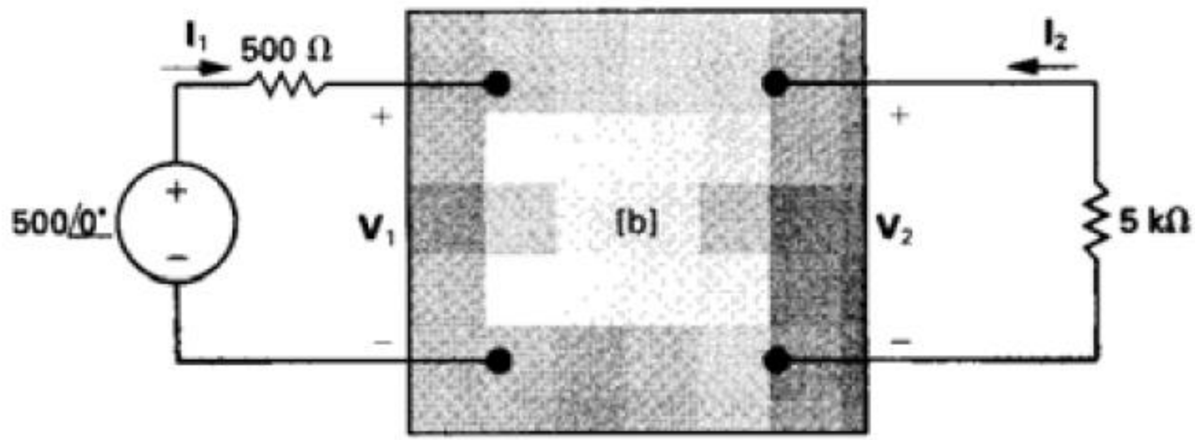
Analysis of the Terminated …:

z PARAMETERS
$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$
$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$
$V_{Th} = \frac{z_{21}}{z_{11} + Z_g}V_g$
$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$
$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$
$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$
$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$

Example:

The two-port circuit shown in Fig. is described in terms of its b parameters, the values of which are

$$b_{11} = -20, \quad b_{12} = -3000\ \Omega, \quad b_{21} = -2\text{ mS}, \quad \text{and} \quad b_{22} = -0.2.$$



a) Find the phasor voltage $\mathbf{V}_2$.

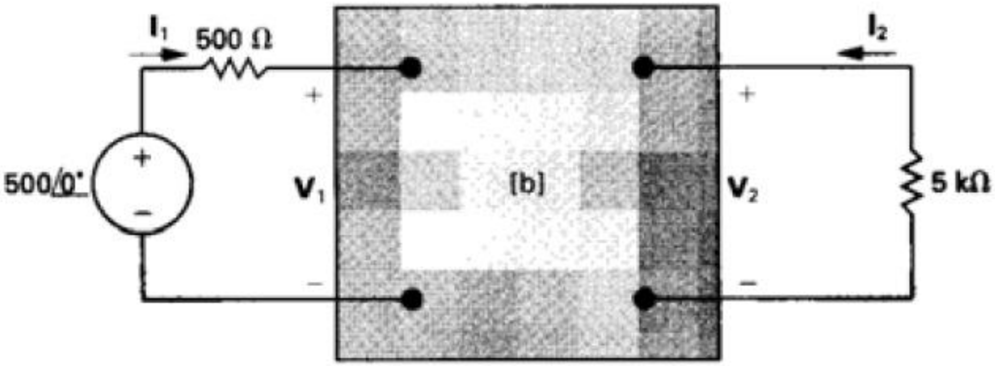
b) Find the average power delivered to the 5 kΩ load.

c) Find the average power delivered to the input port.

d) Find the load impedance for maximum average power transfer.

e) Find the maximum average power delivered to the load in (d).

Example:



a) Find the phasor voltage $\mathbf{V}_2$.

b) Find the average power delivered to the 5 kΩ load.

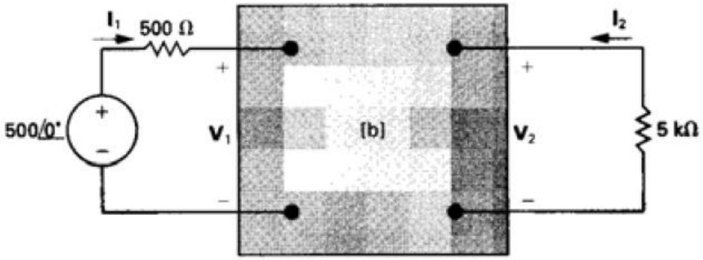
c) Find the average power delivered to the input port.

d) Find the load impedance for maximum average power transfer.

e) Find the maximum average power delivered to the load in (d).

b PARAMETERS
$Z_{in} = \dfrac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$
$I_2 = \dfrac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$
$V_{Th} = \dfrac{V_g \Delta b}{b_{22} + b_{21}Z_g}$
$Z_{Th} = \dfrac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$
$\dfrac{I_2}{I_1} = \dfrac{-\Delta b}{b_{11} + b_{21}Z_L}$
$\dfrac{V_2}{V_1} = \dfrac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$
$\dfrac{V_2}{V_g} = \dfrac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$

Example:



$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

$$= \frac{(-2)(5000)}{-3000 + (-20)500 + (-0.2)5000 + [-2 \times 10^{-3}(500)(5000)]}$$

$$= \frac{10}{19}.$$

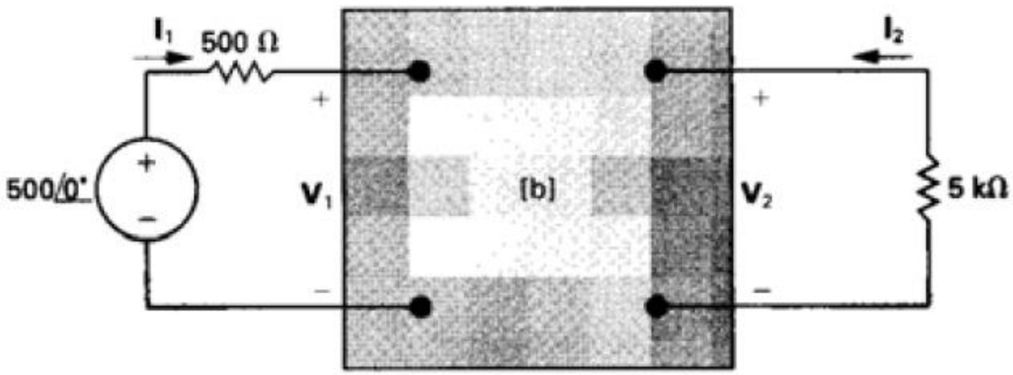
Then,

$$\mathbf{V}_2 = \left(\frac{10}{19}\right) 500 = 263.16\underline{/0^\circ} \text{ V}.$$

b) The average power delivered to the 5000 Ω load is

$$P_2 = \frac{263.16^2}{2(5000)} = 6.93 \text{ W}.$$

Example:



$$Z_{\text{in}} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$
$$= \frac{(-0.2)(5000) - 3000}{-2 \times 10^{-3}(5000) - 20}$$
$$= \frac{400}{3} = 133.33\ \Omega.$$

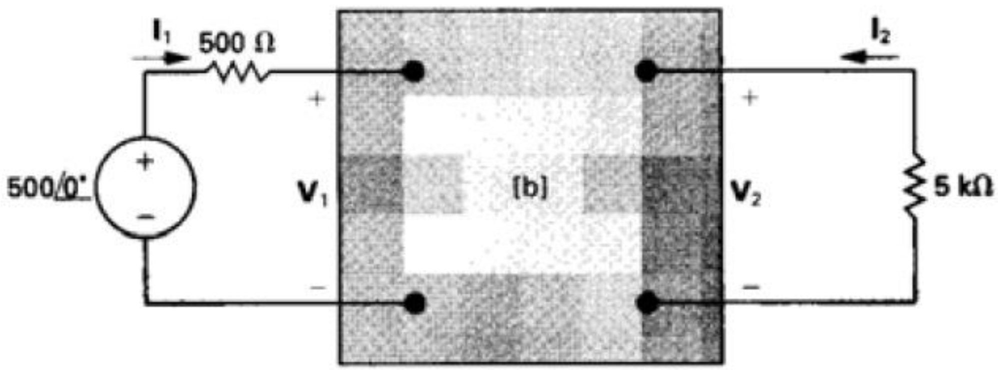
Now $\mathbf{I}_1$ follows directly:

$$\mathbf{I}_1 = \frac{500}{500 + 133.33} = 789.47\ \text{mA}.$$

The average power delivered to the input port is

$$P_1 = \frac{0.78947^2}{2}(133.33) = 41.55\ \text{W}.$$

Example:

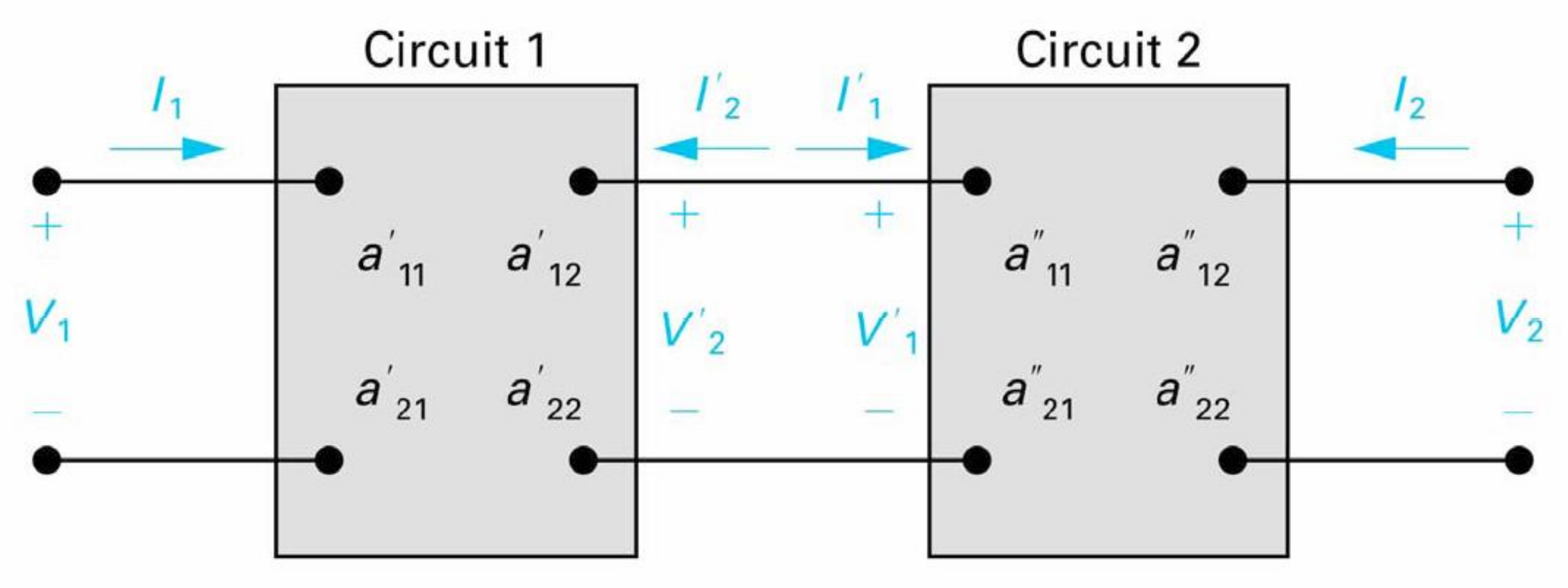


b) The average power delivered to the 5000 Ω load is

$$P_2 = \frac{263.16^2}{2(5000)} = 6.93 \text{ W}.$$

Interconnected Two-port Circuits:

The five basic interconnections of two-port circuit:
(a) Cascade



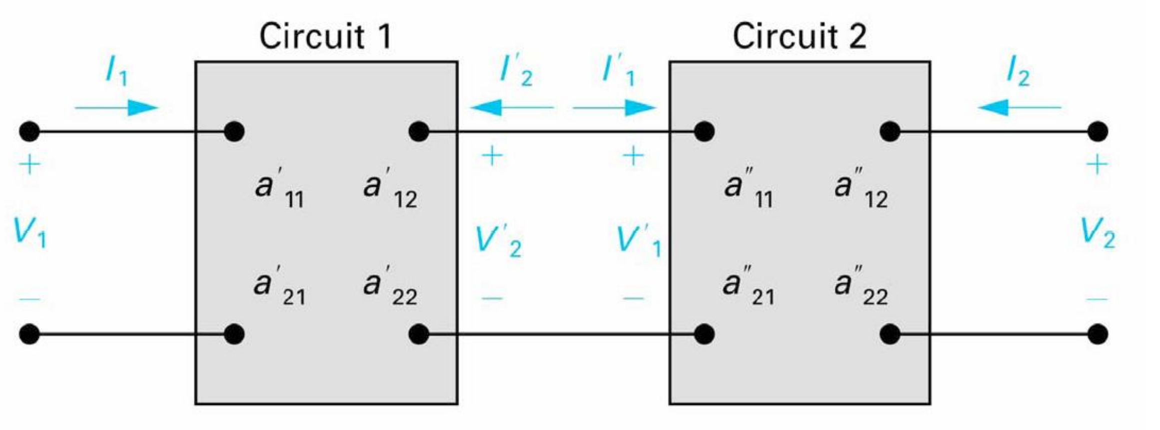
$$V_1 = a'_{11} V'_2 - a'_{12} I'_2,$$

$$I_1 = a'_{21} V'_2 - a'_{22} I'_2.$$

Interconnected Two-port Circuits:

(a) Cascade

$$V_1 = a'_{11}V'_2 - a'_{12}I'_2,$$
$$I_1 = a'_{21}V'_2 - a'_{22}I'_2.$$



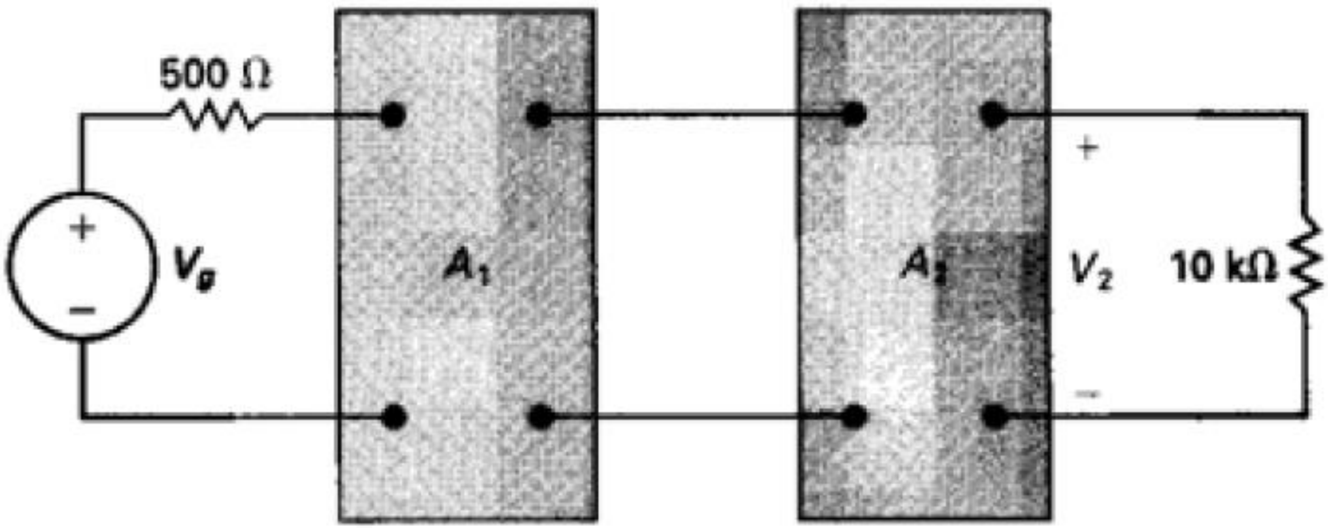
$$V_1 = a'_{11}V'_1 + a'_{12}I'_1,$$
$$I_1 = a'_{21}V'_1 + a'_{22}I'_1.$$

$$V'_1 = a''_{11}V_2 - a''_{12}I_2,$$
$$I'_1 = a''_{21}V_2 - a''_{22}I_2.$$

$$V_1 = (a'_{11}a''_{11} + a'_{12}a''_{21})V_2 - (a'_{11}a''_{12} + a'_{12}a''_{22})I_2,$$
$$I_1 = (a'_{21}a''_{11} + a'_{22}a''_{21})V_2 - (a'_{21}a''_{12} + a'_{22}a''_{22})I_2.$$

Example:

Two identical amplifiers are connected in cascade, as shown in Fig. 18.11. Each amplifier is described in terms of its h parameters. The values are $h_{11} = 1000\ \Omega$, $h_{12} = 0.0015$, $h_{21} = 100$, and $h_{22} = 100\ \mu S$. Find the voltage gain V_2/V_g.



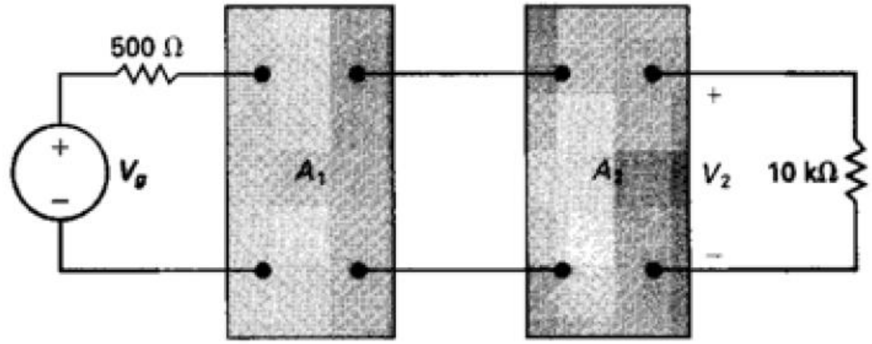
Example:

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{+0.05}{100} = 5 \times 10^{-4},$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{100} = -10\ \Omega,$$

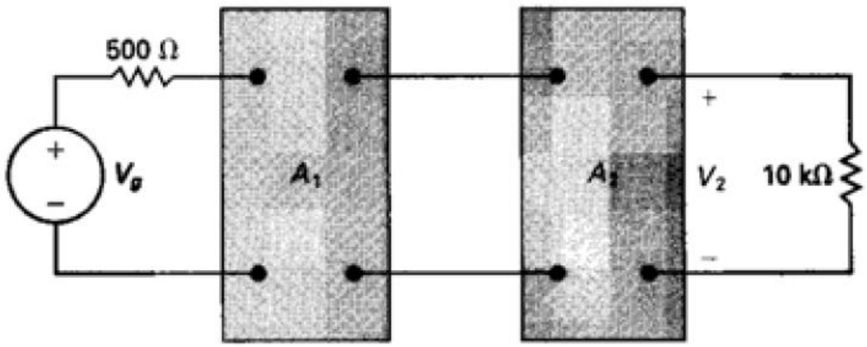
$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-100 \times 10^{-6}}{100} = -10^{-6}\ \text{S},$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{100} = -10^{-2}.$$



$$\begin{aligned} a_{11} &= a'_{11}a'_{11} + a'_{12}a'_{21} \\ &= 25 \times 10^{-8} + (-10)(-10^{-6}) = 10.25 \times 10^{-6}, \\ a_{12} &= a'_{11}a'_{12} + a'_{12}a'_{22} \\ &= (5 \times 10^{-4})(-10) + (-10)(-10^{-2}) = 0.095\ \Omega, \\ a_{21} &= a'_{21}a'_{11} + a'_{22}a'_{21} \\ &= (-10^{-6})(5 \times 10^{-4}) + (-0.01)(-10^{-6}) \\ &= 0.0095 \times 10^{-6}\ \text{S}, \\ a_{22} &= a'_{21}a'_{12} + a'_{22}a'_{22} \\ &= (-10^{-6})(-10) + (-10^{-2})^2 = 1.1 \times 10^{-4}. \end{aligned}$$

Example:



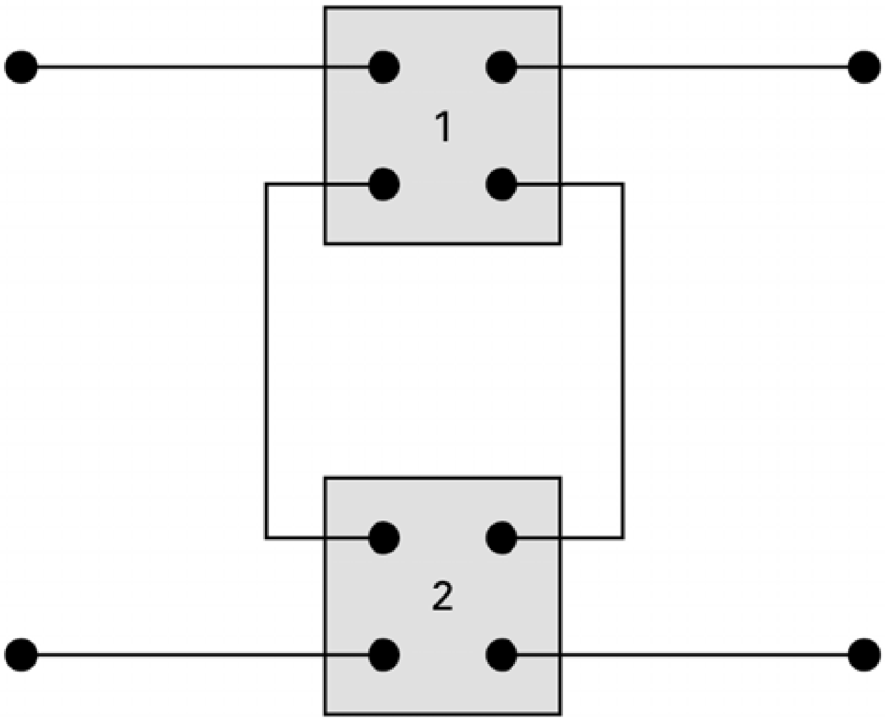
$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{10^4}{[10.25 \times 10^{-6} + 0.0095 \times 10^{-6}(500)]10^4 + 0.095 + 1.1 \times 10^{-4}(500)}$$

$$= \frac{10^4}{0.15 + 0.095 + 0.055} = \frac{10^5}{3} = 33{,}333.33.$$

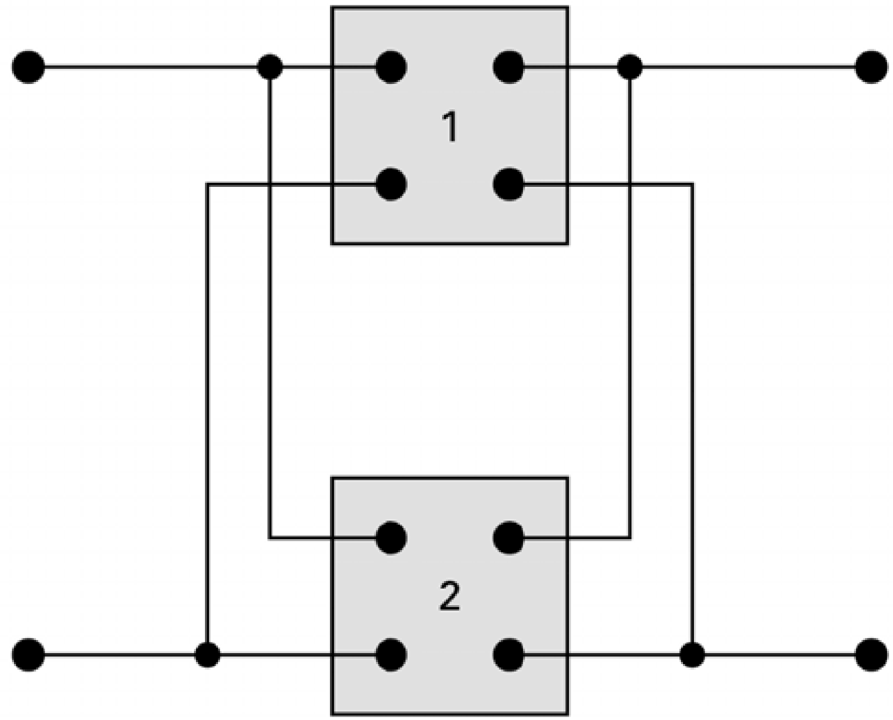
Interconnected Two-port Circuits:

(b) Series



(b)

(c) Parallel

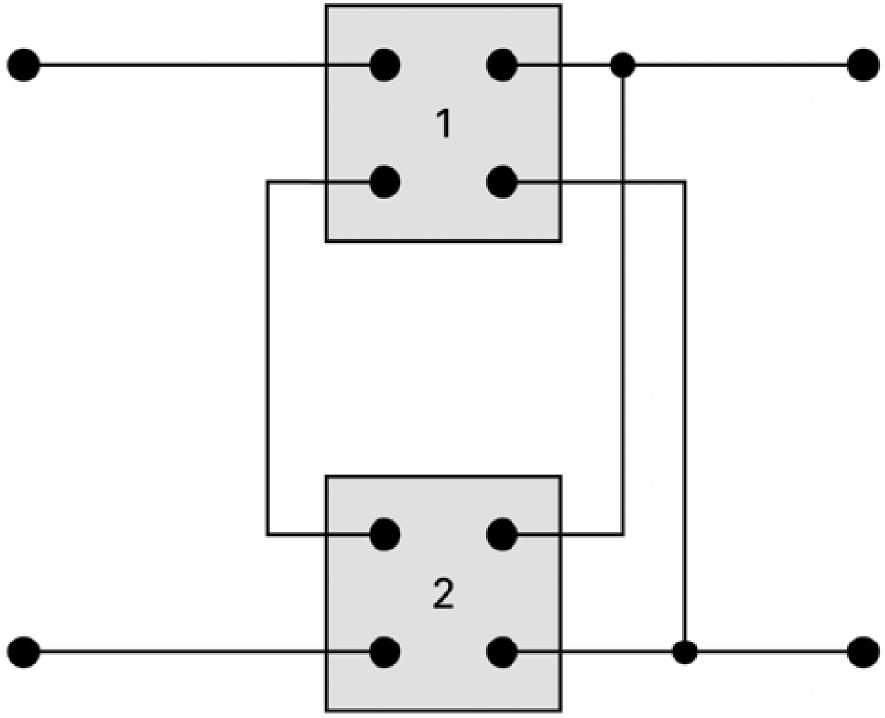


(c)

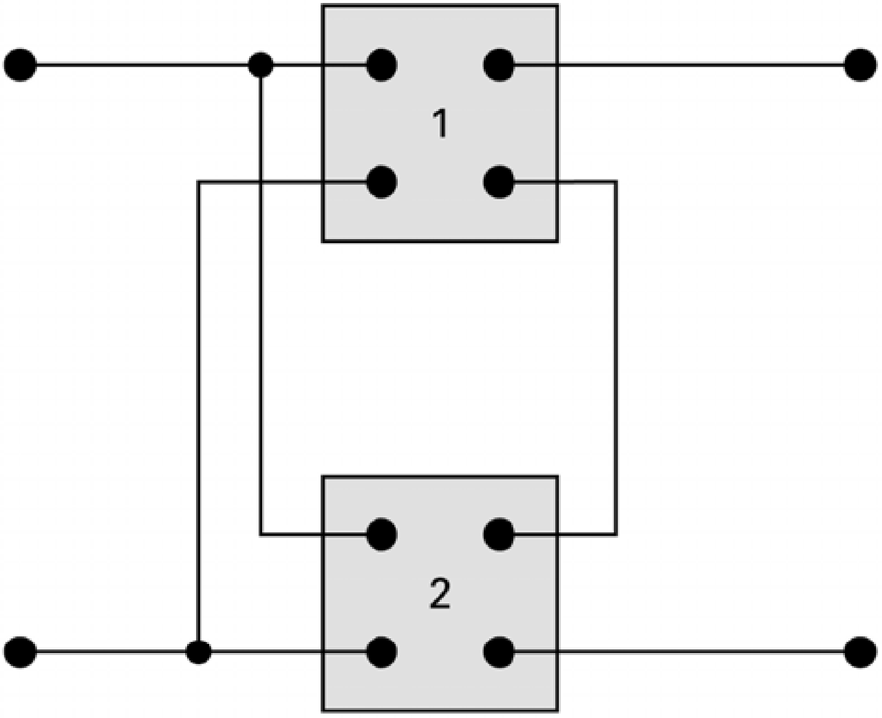
Interconnected Two-port Circuits:

(d) Series- Parallel

(e) Parallel- Series



(d)



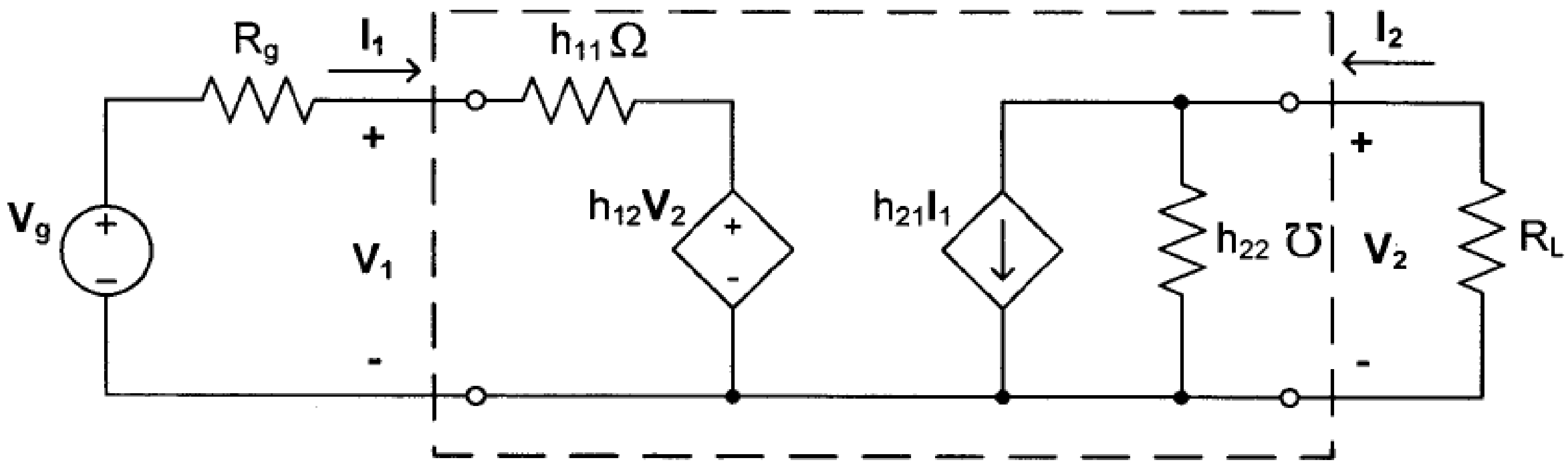
(e)

Example: Final 2/47

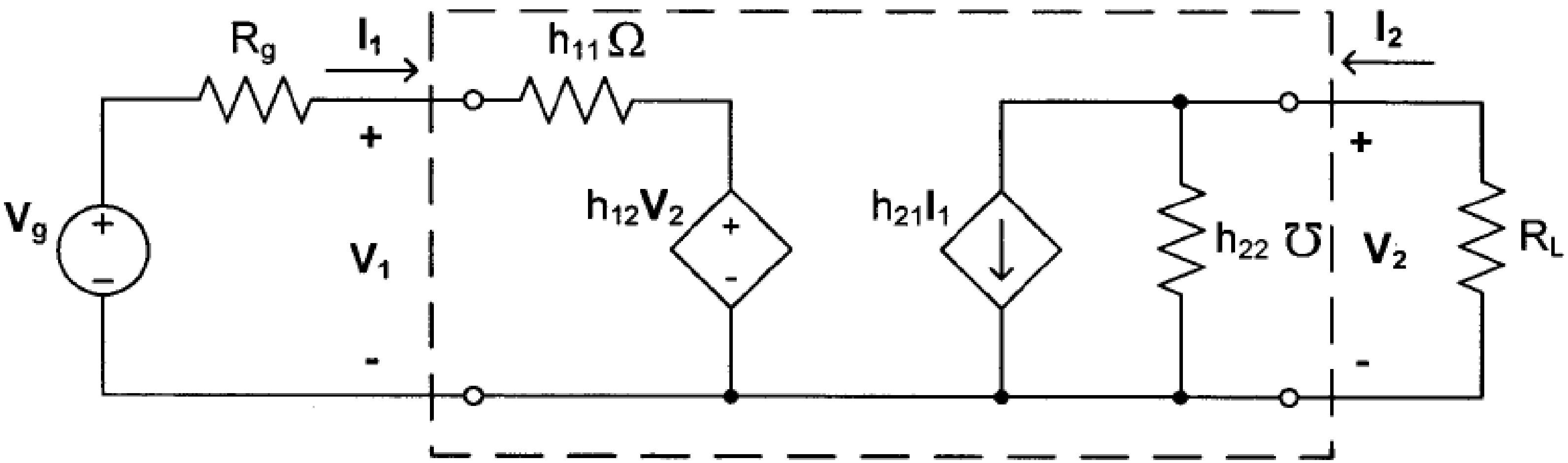
5.] จากวงจร (an equivalent circuit satisfied by the h-parameter equations) ตามรูป จงหา (20 คะแนน)

5.1.) The current gain, $A_i = \dfrac{I_2}{I_1}$

5.2.) The voltage gain, $A_v = \dfrac{V_2}{V_g}$



Example: Final 2/47



from $V_1 = h_{11} I_1 + h_{12} V_2 \quad \ldots (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \quad \ldots (2)$

$V_2 = -R_L I_2 \quad \ldots (3)$

$V_1 = V_g - I_1 R_g \quad \ldots (4)$

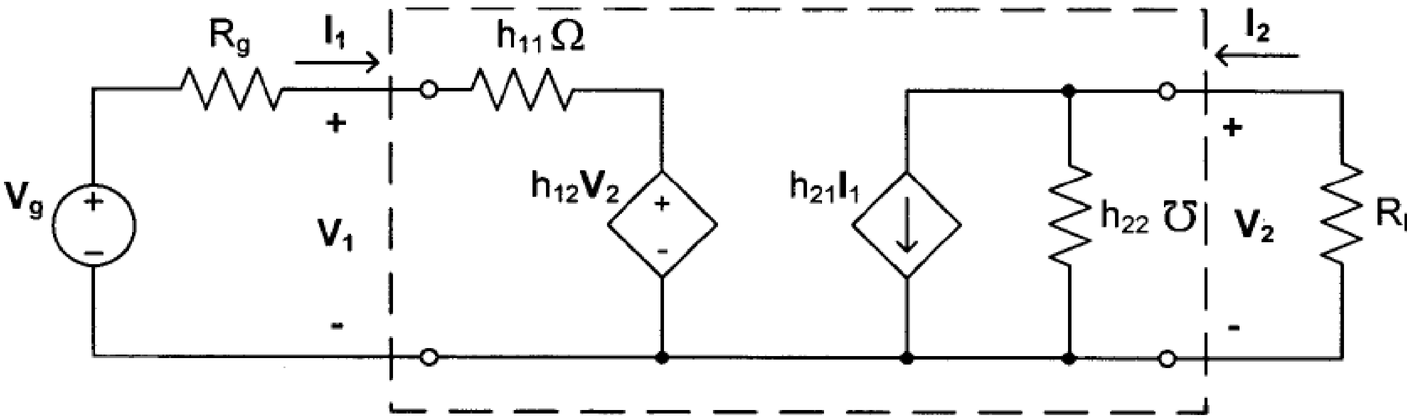
[5.1] แทน V_2 จาก (3) ใน (2);

$$I_2 = h_{21} I_1 + h_{22}(-R_L I_2)$$

$$(1 + h_{22} R_L)\, I_2 = h_{21} I_1$$

$$\therefore A_i = \frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22} R_L} \quad \ldots \text{Ans}$$

Example: Final 2/47



[5.2] จาก สมการ (1) & (4) ;

$$V_g - I_1 R_g = h_{11} I_1 + h_{12} V_2$$

$$V_g = (h_{11} + R_g) I_1 + h_{12} V_2 \quad \ldots (5)$$

จากสมการ (2) & (3) ;

$$I_2 = -\frac{V_2}{R_L} = h_{21} I_1 + h_{22} V_2$$

$$\therefore \quad I_1 = \left[\frac{-1}{h_{21} R_L} - \frac{h_{22}}{h_{21}} \right] V_2 \quad \ldots (6)$$

แทนค่า I_1 จาก (6) ลงใน สมการ (5) ;

$$V_g = (h_{11} + R_g) \left[\frac{-1}{h_{21} R_L} - \frac{h_{22}}{h_{21}} \right] V_2 + h_{22} V_2$$

$$\therefore \frac{V_g}{V_2} = \frac{-h_{11} - R_g - h_{11} h_{22} R_L - R_g h_{22} R_L + h_{22} h_{21} R_L}{h_{21} R_L}$$

$$\therefore \frac{V_2}{V_g} = \frac{h_{21} R_L}{-h_{11} - R_g - h_{11} h_{22} R_L - R_g h_{22} R_L + h_{22} h_{21} R_L}$$

Example: Final 2/46

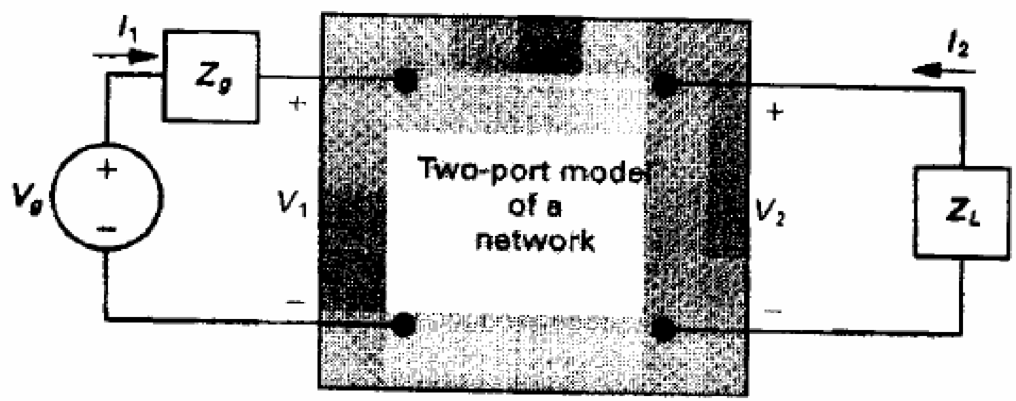
5.] จากวงจร the two-port network ตามรูป จงแสดงให้เห็นว่า $V_{Th} = \dfrac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$ และ

$$Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$ (20 คะแนน)

Note: $I_1 = y_{11}V_1 + y_{12}V_2$,

$I_2 = y_{21}V_1 + y_{22}V_2$ และ

$\Delta y = y_{11}y_{22} - y_{12}y_{21}$

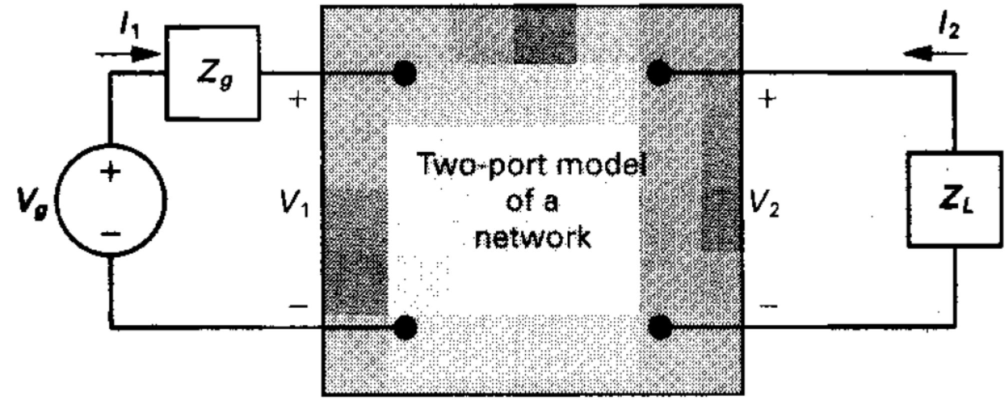


Example: Final 2/46

from: $I_1 = y_{11}V_1 + y_{12}V_2$ .. [1]

$I_2 = y_{21}V_1 + y_{22}V_2$ .. [2]

$V_1 = V_g - I_1 Z_g$ ... [3]



Find $V_{Th} = V_{oc} = V_2\big|_{I_2=0}$

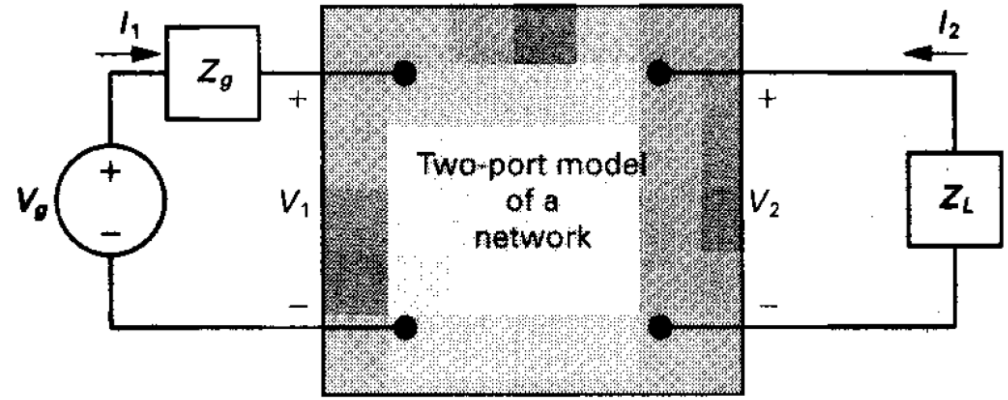
[1]; $0 = y_{11}V_1 + y_{12}V_2 - I_1$

[2]; $0 = y_{21}V_1 + y_{22}V_2$; $I_2 = 0$

[3]; $V_g = V_1 + I_1 Z_g$

$$\begin{bmatrix} 0 \\ 0 \\ V_g \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & -1 \\ y_{21} & y_{22} & 0 \\ 1 & 0 & Z_g \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \end{bmatrix}$$

Example: Final 2/46



$$\therefore V_2 = \frac{\begin{vmatrix} y_{11} & 0 & -1 \\ y_{21} & 0 & 0 \\ 1 & V_g & Z_g \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} & -1 \\ y_{21} & y_{22} & 0 \\ 1 & 0 & Z_g \end{vmatrix}}$$

$$= \frac{-y_{21} V_g}{+y_{22} + Z_g (y_{11} y_{22} - y_{12} y_{21})}$$

$$= \frac{-y_{21} V_g}{y_{22} + \Delta y Z_g} \quad \text{Q.E.D.}$$

find $Z_{th} = \left. \dfrac{V_2}{I_2} \right|_{V_g = 0}$

from [3] ; $V_1 = -I_1 Z_g$ ... [4]

แทนค่า V_1 ใน [4] ; $I_1 = y_{11}(-I_1 Z_g) + y_{12} V_2$

$(1 + y_{11} Z_g) I_1 = y_{12} V_2$

$I_1 = \left(\dfrac{y_{12}}{1 + y_{11} Z_g} \right) V_2$

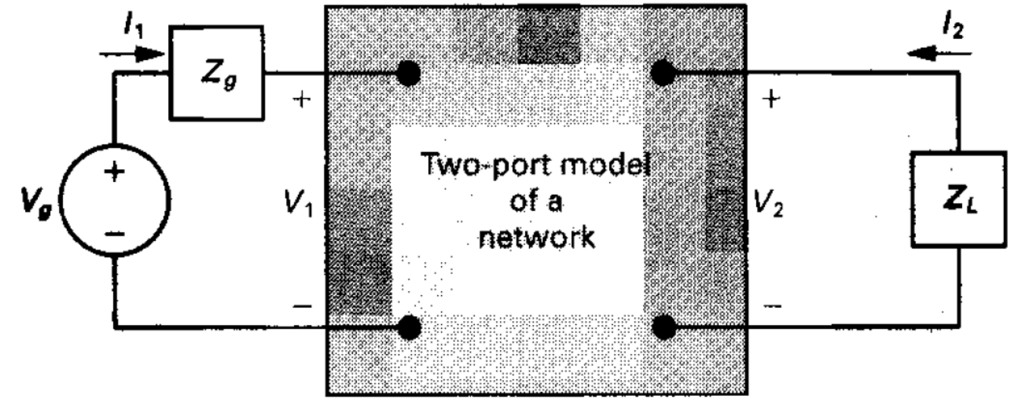
Example: Final 2/46

แทนค่า $V_1 = -I_1 Z_g$ ลงใน (2) ;

$$I_2 = y_{21}(-I_1 Z_g) + y_{22}V_2$$

$$= y_{21}\left(\frac{-y_{12}}{1+y_{11}Z_g}\right)Z_g + y_{22}V_2$$

$$= \left[\frac{-y_{12}y_{21}Z_g}{1+y_{11}Z_g} + y_{22}\right]V_2$$



$$\therefore \frac{V_2}{I_2} = \frac{1+y_{11}Z_g}{-y_{12}y_{21}Z_g + y_{22}(1+y_{11}Z_g)}$$

$$= \frac{1+y_{11}Z_g}{y_{22} + y_{11}y_{22}Z_g - y_{12}y_{21}Z_g}$$

$$= \frac{1+y_{11}Z_g}{y_{22} + \Delta y\, Z_g}$$

QED.

Reference:

•W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition. Copyright ©2002 McGraw-Hill. All rights reserved.

•James W. Nilsson, Susan A. Riedel, “Electric Circuits” Sixth edition, Addison Wesley, 2001