ENE 104
Electric Circuit Theory

Lecture 09:
Sinusoidal Steady-State Analysis

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Objectives : Ch10

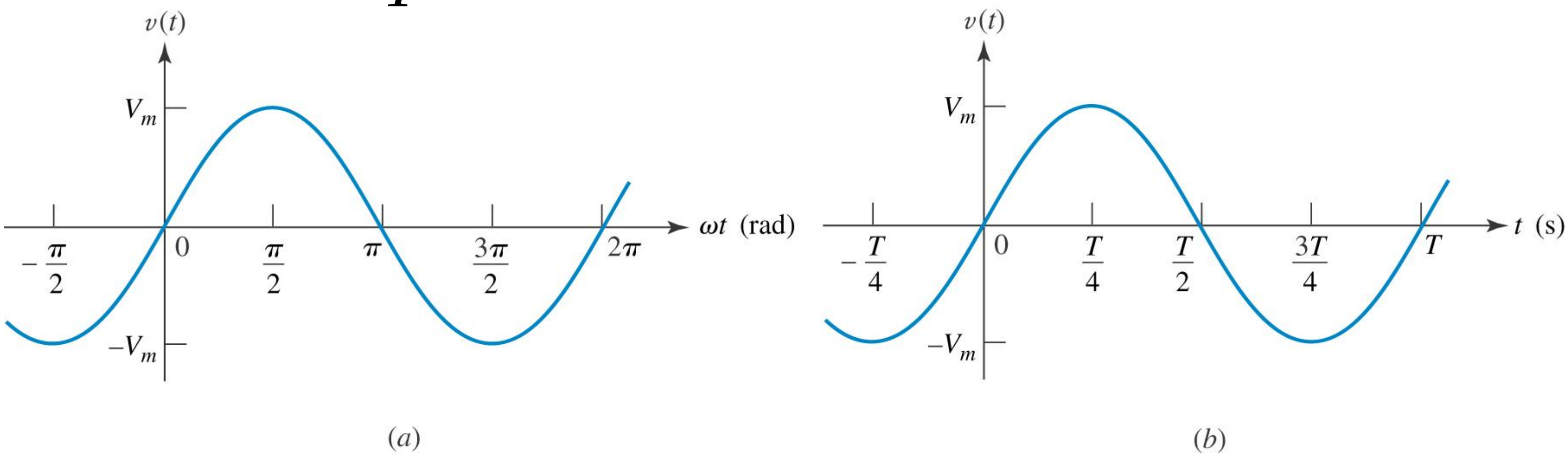
- sinusoidal functions
- impedance and admittance
- use phasors to determine the forced response of a circuit subjected to sinusoidal excitation
- Apply techniques using phasors

Introduction:

Characteristics of Sinusoids

$$v(t) = V_m \sin \omega t$$

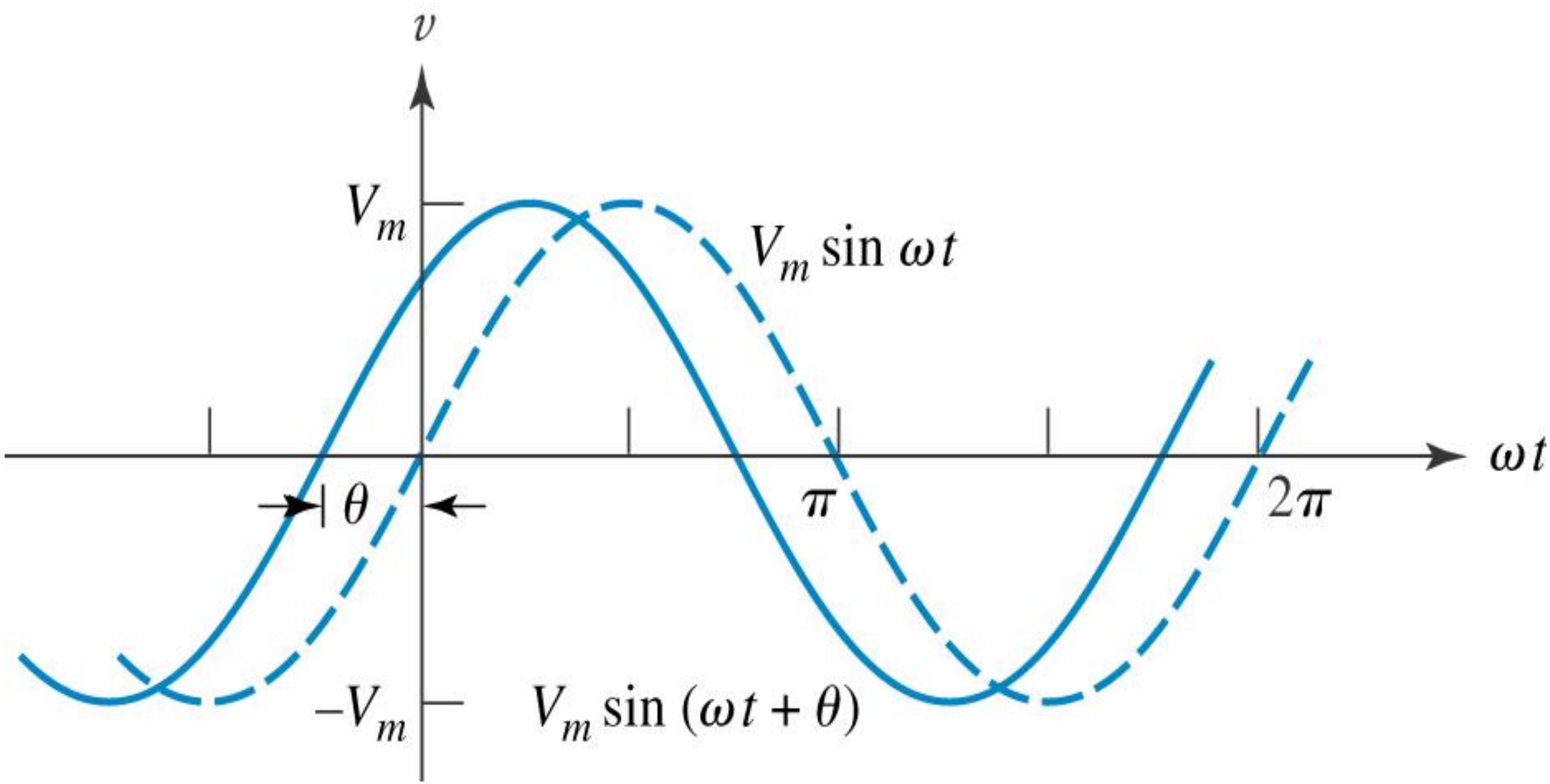
$$f = \frac{1}{T} \qquad \omega T = 2\pi \qquad \omega = 2\pi f$$



The sinusoidal function $v(t) = V_m \sin \omega t$ is plotted

(a) versus ωt and (b) versus t.

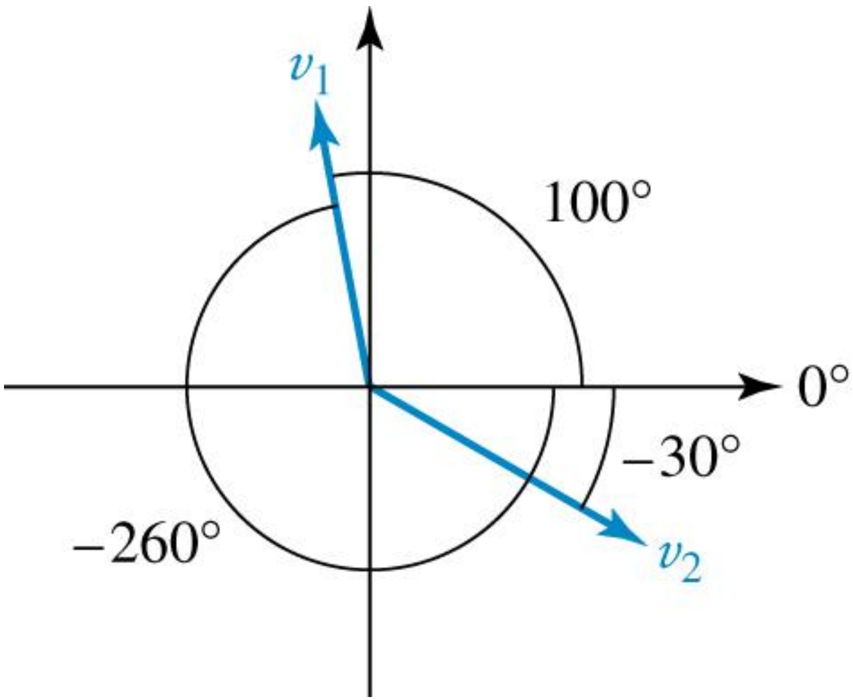
Lagging and leading:



The sine wave $V_m \sin(\omega t + \theta)$ leads $V_m \sin \omega t$ by θ rad.

Lagging and leading:

Converting Sines to Cosines



$$
\begin{aligned}
v_1 &= V_{m1}\cos(5t + 10°) \\
&= V_{m1}\sin(5t + 90° + 10°) \\
&= V_{m1}\sin(5t + 100°)
\end{aligned}
$$

$$v_2 = V_{m2}\sin(5t - 30°)$$

In electrical engineering, the phase angle is commonly given in degrees, rather than radian.

Lagging and leading:

Two sinusoidal waves whose phase are to be compared must:

1. Both be written as sine waves, or both as cosine waves.
2. Both be written with positive amplitudes.
3. Each have the same frequency.

Normally, the difference in phase between two sinusoids is expressed by that angle which is less than or equal to 180 degree in magnitude

Practice: 10.1

Find the angle by which i_1 lags v_1 if $v_1 = 120\cos(120\pi t - 40^o)$ V and i_1 equals (a) $2.5\cos(120\pi t + 20^o)$ A; (b) $1.4\,sin(120\pi t - 70^o)$ A; (c) $A\cos 100t + B sin 100t = C cos(100t + \phi)$.

$$v_1 = 120\cos(120\pi t - 40^\circ)$$

(a) $2.5\cos(120\pi t + 20^\circ)$ lags v_1 by $-40 - 20 = \underline{-60^\circ}$

(b) $1.4\sin(120\pi t - 70^\circ) = 1.4\cos(120\pi t - 160^\circ)$ lags v_1 by $-40 - (-160) = \underline{120^\circ}$

(c) $-0.8\cos(120\pi t - 110^\circ) = 0.8\cos(120\pi t + 70^\circ)$ lags v_1 by $-40 - 70 = \underline{-110^\circ}$

Practice: 10.2

Find A, B, C, and ϕ if $40\cos(100t - 40°) - 20\sin(100t + 170°) = A cos 100t + B sin 100t = C cos(100t + \phi)$.

Practice: 10.2

$$40\cos(100t - 40^\circ) - 20\sin(100t + 170^\circ)$$
$$= A\cos 100t + B\sin 100t$$
$$= C\cos(100t + \theta)$$

$$40\cos(100t - 40^\circ) = 40\left[\cos 100t \cdot \cos(-40^\circ) - \sin 100t \cdot \sin(-40^\circ)\right]$$
$$= 30.64\cos 100t + 25.71\sin 100t$$
$$-20\sin(100t + 170^\circ) = -20\left[\cos 170^\circ \cdot \sin 100t + \sin 170^\circ \cdot \cos 100t\right]$$
$$= 19.70\sin 100t - 3.473\cos 100t$$

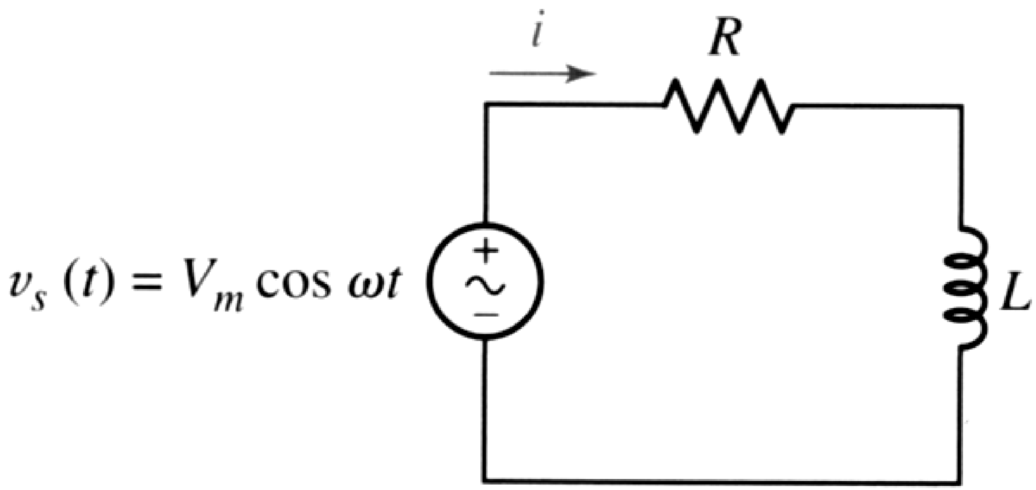
Thus, $A = 30.64 - 3.473 = \underline{27.17}$
$B = 25.71 + 19.70 = \underline{45.41}$

$$C = \sqrt{A^2 + B^2} = \underline{52.92}$$
$$\theta = \tan^{-1}\left(\frac{-B}{A}\right) = \underline{-59.11^\circ}$$

Forces Response:

Forces Response to sinusoidal Functions:

The Steady-State Response



$$L\frac{di}{dt} + Ri = V_m \cos \omega t$$

Forces Response:

The force response will have the general form

$$i(t) = I_1 \cos \omega t + I_2 \sin \omega t$$

Substitute, $$L\frac{di}{dt} + Ri = V_m \cos \omega t$$

$$L(-I_1 \omega \sin \omega t + I_2 \omega \cos \omega t) + R(I_1 \cos \omega t + I_2 \sin \omega t) = V_m \cos \omega t$$

Obtain,

$$(-LI_1\omega + RI_2)\sin \omega t + (LI_2\omega + RI_1 - V_m)\cos \omega t = 0$$

Forces Response:

Thus,

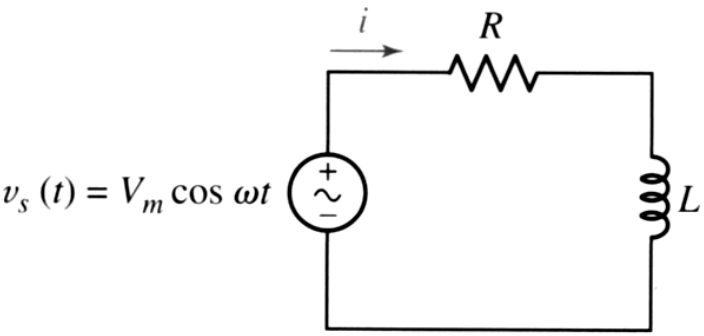
$$-LI_1\omega + RI_2 = 0$$

And $$LI_2\omega + RI_1 - V_m = 0$$

Leads to,

$$I_1 = \frac{RV_m}{R^2 + \omega^2 L^2}, \quad I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

Forces Response:



Thus the force response is obtained

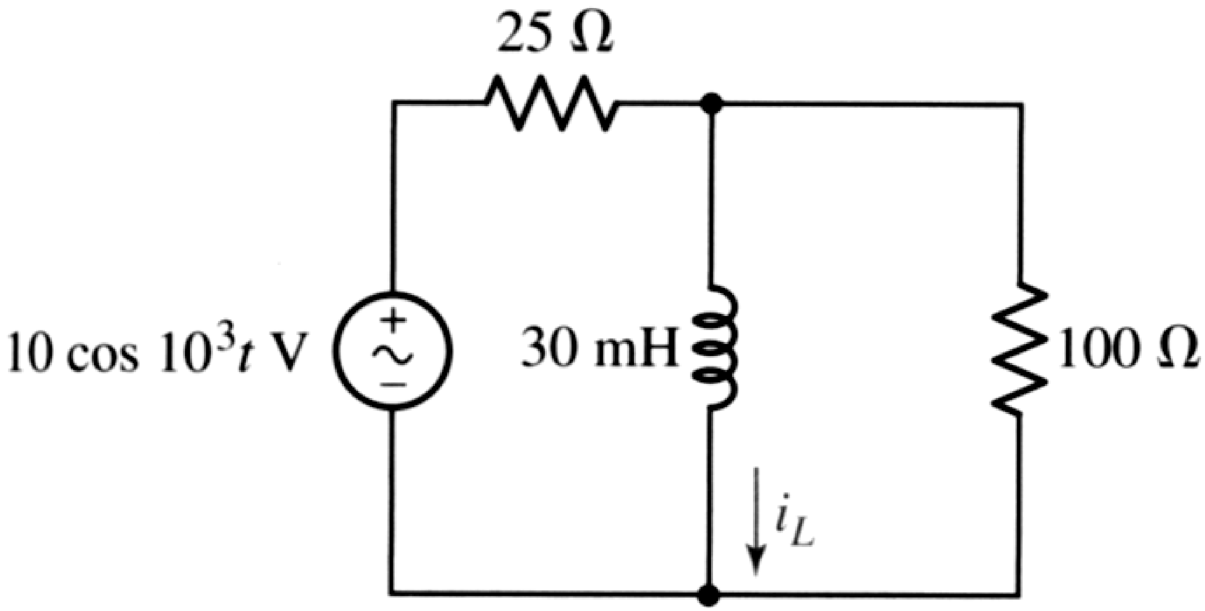
$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t$$

The alternate form:

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R} \right)$$

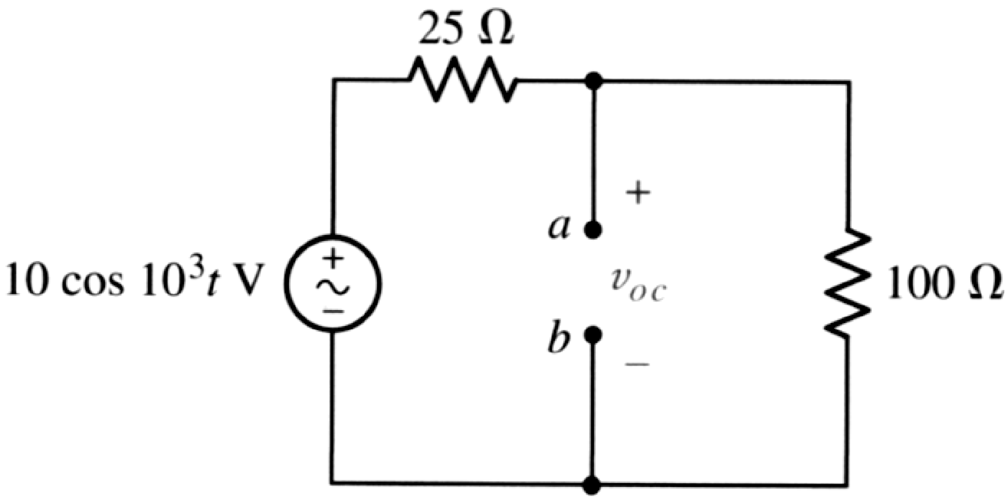
Example:

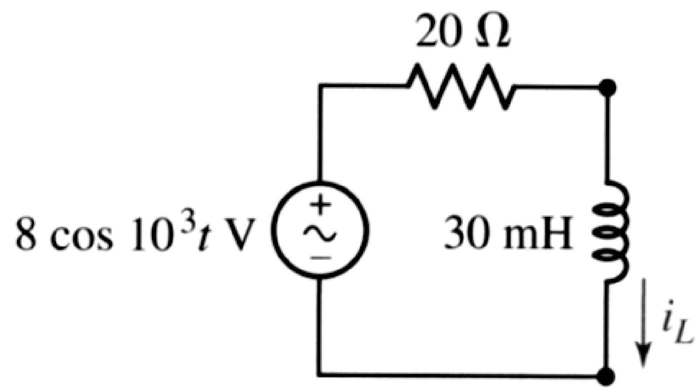
Find the current i_L



Example:

Find the current i_L

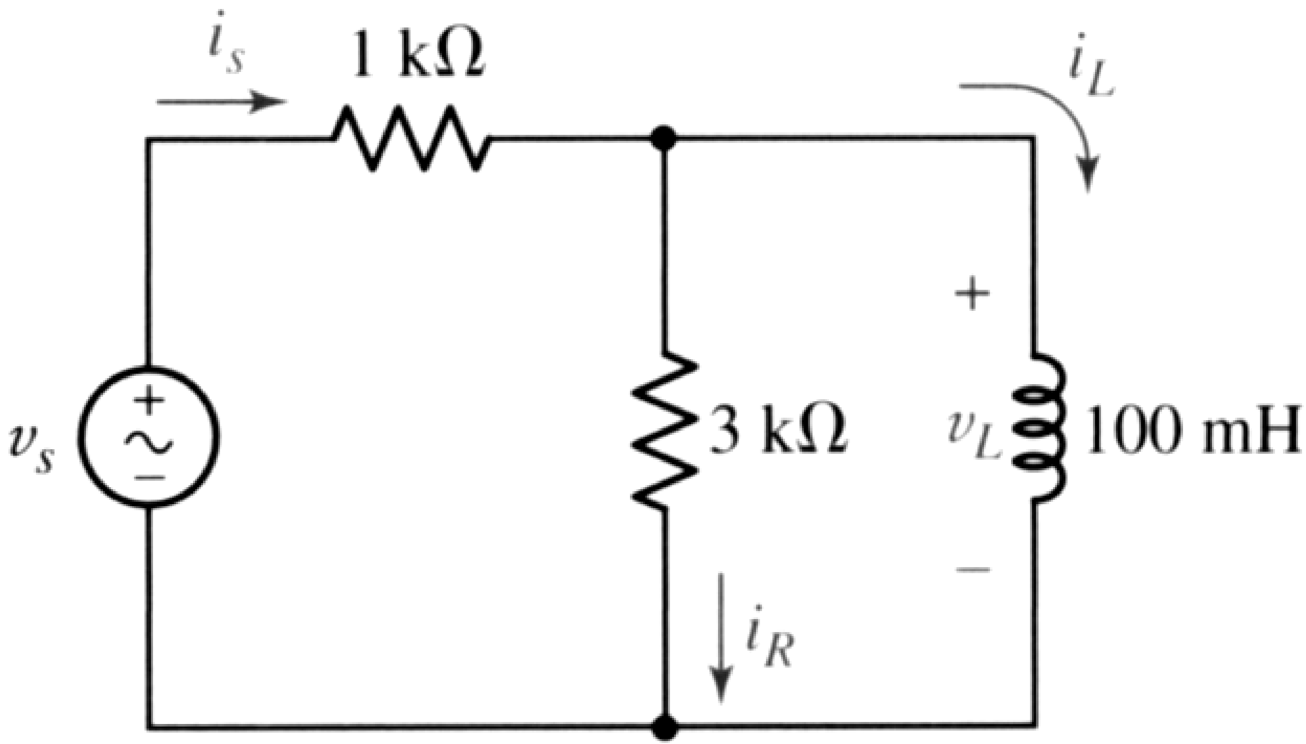




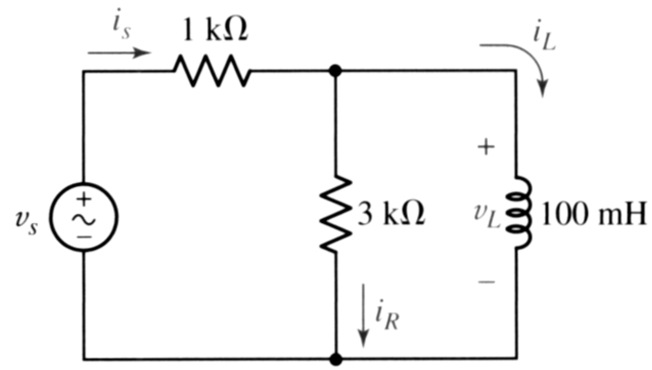
$$
\begin{aligned}
i(t) &= \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) \\
&= \frac{8}{\sqrt{20^2+\left(10^3\times 30\times 10^{-3}\right)^2}}\cos\left(10^3 t - \tan^{-1}\frac{\left(10^3\times 30\times 10^{-3}\right)}{20}\right) \\
&= 222\cos\left(10^3 t - 56.3^\circ\right) mA.
\end{aligned}
$$

Practice: 10.3

Let $v_s = 40cos8000t$ V in the circuit. Use Thevenin's theorem where it will do the most good, and find the value at t = 0 for (a) i_L; (b) v_L; (c) i_R; (d) i_s



Practice: 10.3



- Removing the inductor temporarily,

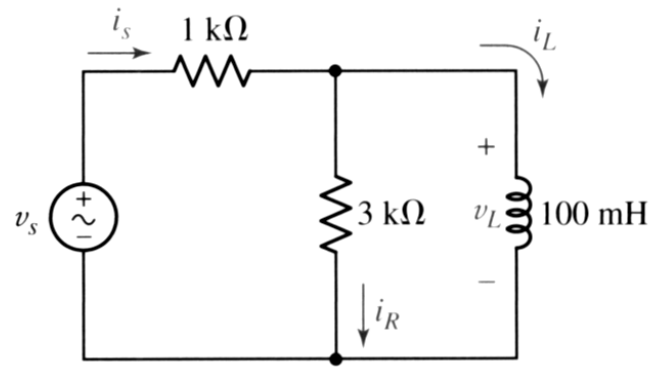
$$v_{oc} = 40\cos 8000t\left(\frac{3}{1+3}\right) = 30\cos 8000t \text{ V}$$

and $R_{TH} = 1\text{ k}\Omega // 3\text{ k}\Omega = 750\,\Omega$

$$\text{Thus, } i_L(t) = \frac{30}{\sqrt{750^2+800^2}}\cos\left(8000t - \tan^{-1}\frac{800}{750}\right)$$

$$= 27.36\cos(8000t - 46.85^\circ)\text{ mA}$$

Practice: 10.3



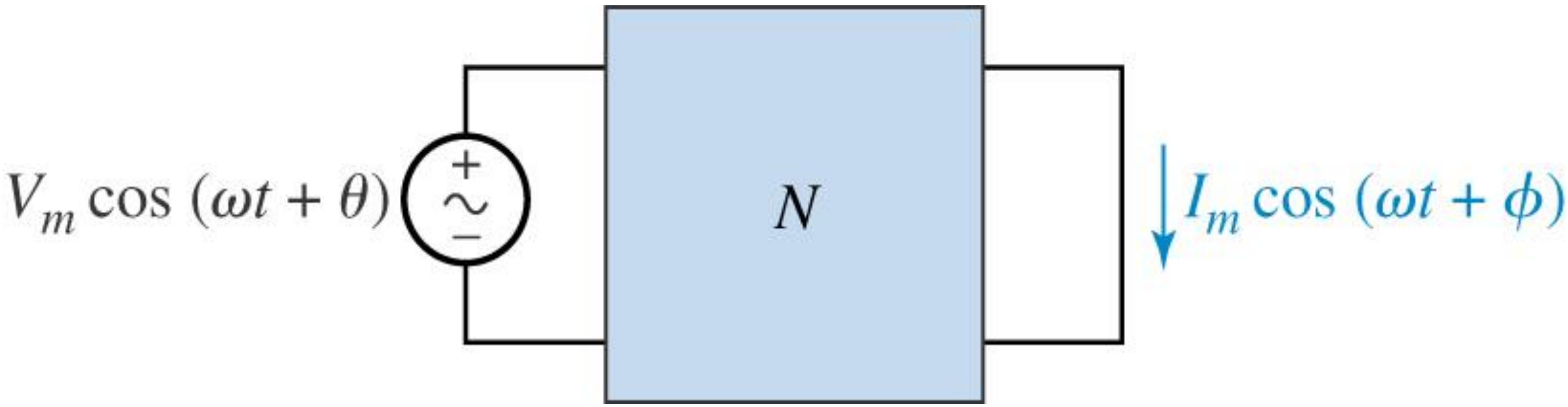
(a) $i_L(0) = 27.36\cos(-46.85°) = \underline{18.71\ \text{mA}}$

(b) $v_L(t) = L\dfrac{di_L}{dt} = (-100\times10^{-3})(0.02736)(8000)\sin(8000t - 46.85°)$ so $v_L(0) = \underline{15.97\ \text{V}}$

(c) $i_R(0) = \dfrac{v_L(0)}{3000} = \underline{5.323\ \text{mA}}$

(d) $i_S(0) = i_L(0) + i_R(0) = \underline{24.03\ \text{mA}}$

The Complex Forcing Fn:



Shifting the phase of the forcing function by 90 degree

$$V_m \cos(\omega t + \theta - 90^\circ) = V_m \sin(\omega t + \theta)$$

A corresponding response

$$I_m \cos(\omega t + \phi - 90^\circ) = I_m \sin(\omega t + \phi)$$

The Complex Forcing Fn:

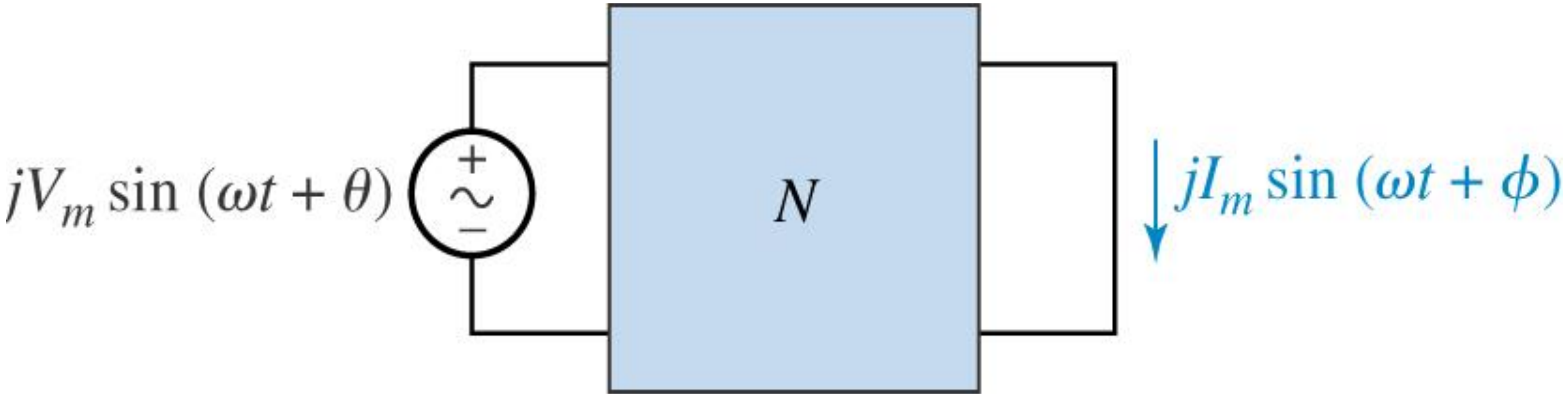
Imaginary Sources Lead to … Imaginary Response

Apply

$$jV_m \sin(\omega t + \theta)$$

The response is

$$jI_m \sin(\omega t + \phi)$$



The Complex Forcing Fn:

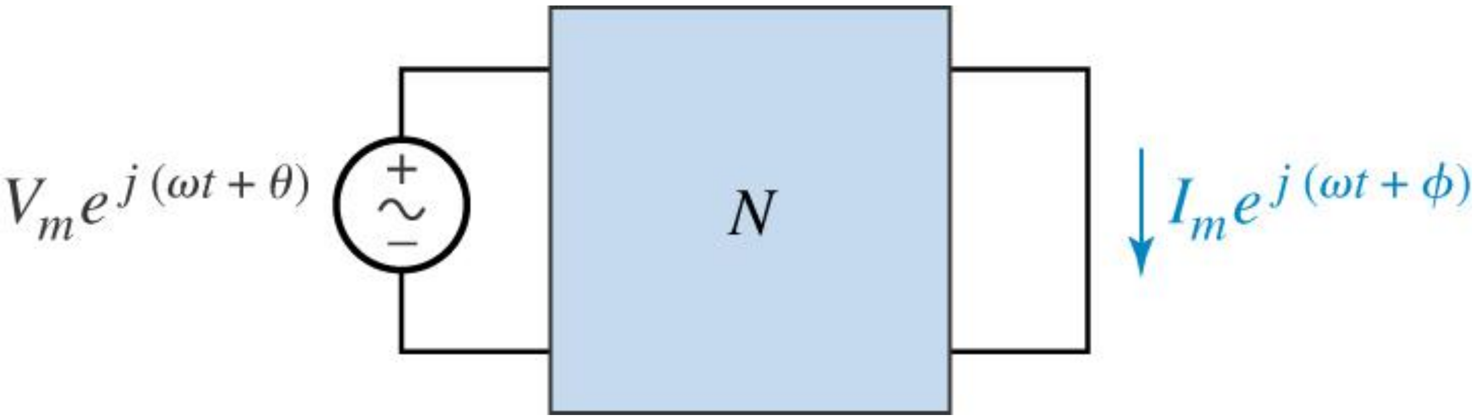
Applying a Complex Forcing Function

Apply

$$V_m \cos(\omega t + \theta) + jV_m \sin(\omega t + \theta) \xrightarrow{\text{Euler's identity}} V_m e^{j(\omega t + \theta)}$$

Produce a response

$$I_m \cos(\omega t + \phi) + jI_m \sin(\omega t + \phi) \xrightarrow{\text{Euler's identity}} I_m e^{j(\omega t + \phi)}$$



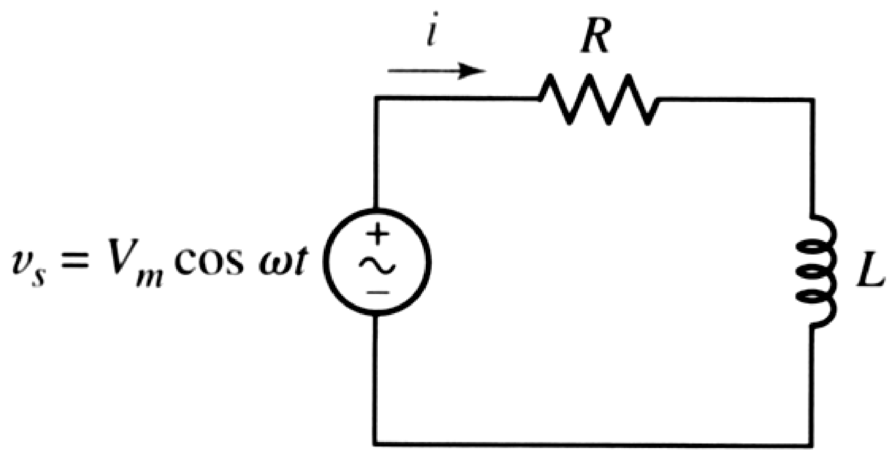
The Complex Forcing Fn:

An Algebraic Alternative to Differential Equations

Note: $\cos \omega t = \mathrm{Re}\left\{e^{j\omega t}\right\}$

The necessary complex source is

$$V_m e^{j\omega t}$$



The complex response is

$$I_m e^{j(\omega t+\phi)}$$

The Complex Forcing Fn:

The differential equation from the circuit

$$Ri + L\frac{di}{dt} = v_s$$

Insert complex expressions for v_s and i

$$RI_m e^{j(\omega t+\phi)} + L\frac{d}{dt}\left(I_m e^{j(\omega t+\phi)}\right) = V_m e^{j\omega t}$$

$$RI_m e^{j(\omega t+\phi)} + j\omega L\left(I_m e^{j(\omega t+\phi)}\right) = V_m e^{j\omega t}$$

$$RI_m e^{j\phi} + j\omega L I_m e^{j\phi} = V_m$$

The Complex Forcing Fn:

Factor the left side:

$$I_m e^{j\phi}(R + j\omega L) = V_m$$

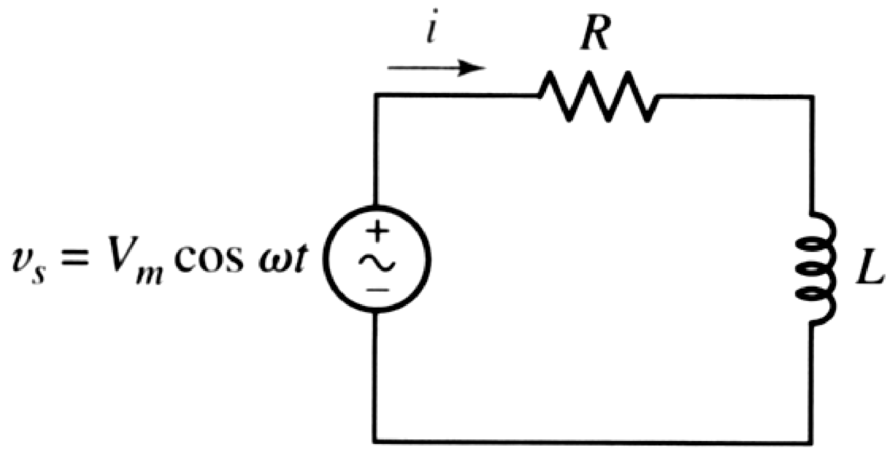
Rearrange:

$$I_m e^{j\phi} = \frac{V_m}{R + j\omega L}$$

Expressing the right side in polar form

$$I_m e^{j\phi} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{j(\tan^{-1} \omega L / R)}$$

The Complex Forcing Fn:



Thus

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$$

And

$$\phi = -\tan^{-1} \frac{\omega L}{R}$$

$$I_m \angle \phi,$$

$$V_m / \sqrt{R^2 + \omega^2 L^2} \angle -\tan^{-1} \omega L / R$$

We find that

$$i(t) = I_m \cos(\omega t + \phi) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R} \right)$$

Practice: 10.4

Evaluate and express the result in rectangular form: (a) $[(2\angle 30°)(5\angle -110°)](1+2j)$; (b) $(5\angle -200°) + 4\angle 20°$.

Evaluate and express the result in polar form: (c) $(2-j7)/(3-j)$; (d) $8-j4+[(5\angle 80°)/(2\angle 20°)]$.

Practice: 10.4

(a) $$\left(2\angle 30^\circ\right)\left(5\angle -110^\circ\right)\left(1+j2\right)=\left(2\angle 30^\circ\right)\left(5\angle -110^\circ\right)\left(2.236\angle 63.43^\circ\right)$$
$$=2\times 5\times 2.236\angle(30^\circ-110^\circ+63.43^\circ)$$
$$=22.36\angle -16.57^\circ=\underline{21.43-j6.377}$$

(b) $$5\angle -200^\circ+4\angle 20^\circ$$
$$=-4.698+j1.710+3.759+j1.368$$
$$=\underline{-0.939+j3.078}$$

(c) $$\frac{2-j7}{3-j}=\frac{7.280\angle -74.05^\circ}{3.162\angle -18.43^\circ}=\underline{2.302\angle -55.62^\circ}$$

(d) $$8-j4+\frac{5\angle 80^\circ}{2\angle 20^\circ}=8-j4+2.5\angle 60^\circ$$
$$=8-j4+1.25+j2.165$$
$$=9.25-j1.835=\underline{9.43\angle -11.22^\circ}$$

Practice: 10.5

If the use of the passive sign convention is specified, find the (a) complex voltage that results when the complex current $4e^{j800t}$ A is applied to the series combination of a 1-mF capacitor and a 2-Ω resistor; (b) complex current that results when the complex voltage $100e^{j2000t}$ V is applied to the parallel combination of a 1-mH inductor and a 50-Ω resistor.

Practice: 10.5

$$\text{(a)} \qquad v_{\text{combination}} = \frac{1}{10^{-3}} \int_{-\infty}^{t} 4\, e^{j800t'} dt' + 2 \times 4\, e^{j800t}$$

$$= \frac{4}{j800 \times 10^{-3}} e^{j800t} + 8\, e^{j800t}$$

$$= -j5\, e^{j800t} + 8\, e^{j800t} = (8 - j5)\, e^{j800t}$$

$$= 9.434\, e^{-j32.0°}\, e^{j800t}$$

$$= \underline{9.434\, e^{j(800t - 32°)} \text{ V}}$$

Practice: 10.5

(b) $$i_{\text{source}} = \frac{1}{10\times 10^{-3}}\int_{-\infty}^{t} 100\,e^{j2000t'}dt' + \frac{100}{50}e^{j2000t}$$

$$= \frac{10000}{j2000}e^{j2000t} + 2\,e^{j2000t}$$

$$= (2 - j5)\,e^{j2000t} = 5.385\,e^{-j68.2^\circ}e^{j2000t}$$

$$= \underline{5.385\;e^{\,j(2000t\,-\,68.2^\circ)}\ \text{A}}$$

The Phasor:

A phasor transformation:

$$i(t) = I_m \cos(\omega t + \phi)$$

$$\downarrow$$

$$i(t) = \mathrm{Re}\{I_m e^{j(\omega t + \phi)}\}$$

$$\downarrow$$

$$\mathbf{I} = I_m e^{j\phi}$$

$$\downarrow$$

$$\mathbf{I} = I_m \angle \phi$$

$$v(t) = V_m \cos(\omega t) \longrightarrow V_m \angle 0^\circ$$

$$i(t) = I_m \cos(\omega t + \phi) \longrightarrow I_m \angle \phi$$

Practice: 10.6

Transform each of the following functions of time into phasor form: (a) $-5\sin(580t - 110°)$; (b) $3\cos 600\text{t} - 5\sin(600t + 110°)$; (c)$8co\,s(4t - 30°) + 4\sin(4t - 100°)$.
Hint: First convert each into a single cosine function with a positive magnitude

Practice: 10.6

(a)

$$-5\sin(580t-110°)=5\cos(580t-110°+180°-90°)$$
$$=5\cos(580t-20°)\Rightarrow \underline{5\angle -20°}$$

(b)

$$3\cos 600t-5\sin(600t+110°)$$
$$=3\cos 600t-5\cos(600t+110°-90°)$$
$$=3\cos 600t-5\cos 600t\cos 20°+5\sin 600t\sin 20°$$
$$=3\cos 600t-4.698\cos 600t+1.71\sin 600t$$
$$=-1.698\cos 600t+1.71\sin 600t$$
$$=-2.41\cos(600t+45.2°)$$
$$=2.41\cos(600t-134.8°)\Rightarrow \underline{2.41\angle -134.8°}$$

Practice: 10.6

(c) $8\cos(4t-30^\circ)+4\sin(4t-100^\circ)$

$$= 8\cos(4t-30^\circ)+4\cos(4t-190^\circ)$$

$$= 8\cos 4t\cos(-30^\circ)-8\sin 4t\sin(-30^\circ)+4\cos 4t\cos(-190^\circ)-4\sin 4t\sin(-190^\circ)$$

$$= (6.928-3.939)\cos 4t+(4-0.6946)\sin 4t$$

$$= 2.989\cos 4t+3.305\sin 4t$$

$$= 4.456\cos(4t-47.87^\circ) \Rightarrow \underline{4.456\angle -47.87^\circ}$$

Practice: 10.7

Let ω = 2000 rad/s and t = 1 ms. Find the instantaneous value of each of the currents given here in phasor form: (a) $j10$ A; (b) $20 + j10$ A; (c) $20 + j(10\angle 20°)$ A.

Practice: 10.7

$\omega = 2000$ rad/s and $t = 1$ ms

(a) $j10\text{ A} = 10\angle{+90^\circ}\text{ A}$ so $i(t) = 10\cos(2000t + 90^\circ)\text{ A}$ and $i(10^{-3}) = \underline{-9.093\text{ A}}$

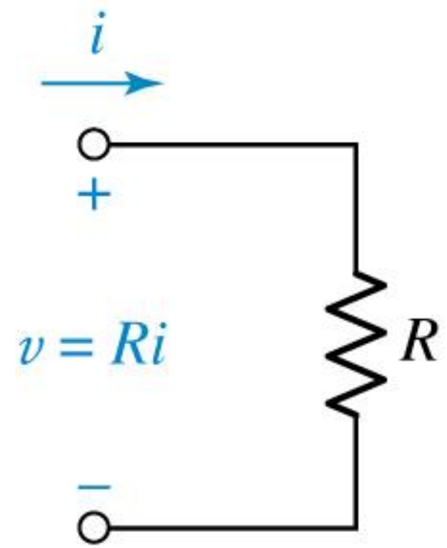
(b) $20 + j10\text{ A} = 22.36\angle 26.57^\circ$ so $i(t) = 22.36\cos(2000t + 26.57^\circ)\text{ A}$
and $i(10^{-3}) = \underline{-17.42\text{ A}}$

(c)
$$
\begin{aligned}
20 + j(10\angle 20^\circ)\text{ A} &= 20 + (1\angle 90^\circ)(10\angle 20^\circ)\text{ A} \\
&= 20 + 10\angle 110^\circ\text{ A} \\
&= 19.06\angle 29.54^\circ\text{ A}
\end{aligned}
$$
so $i(t) = 19.06\cos(2000t + 29.54^\circ)\text{ A}$ and $i(10^{-3}) = \underline{-15.45\text{ A}}$

Phasor Relationship:

The Resistors

$$v(t) = Ri(t)$$



Apply the complex voltage

$$v(t) = V_m e^{j(\omega t+\theta)} = V_m \cos(\omega t+\theta) + jV_m \sin(\omega t+\theta)$$

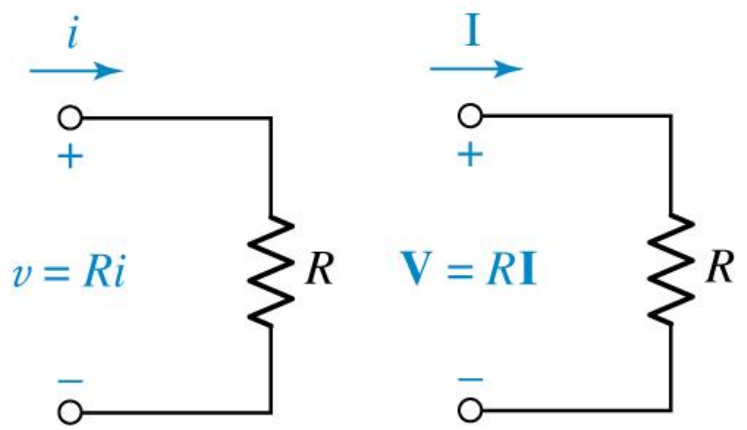
Assume the complex current response

$$i(t) = I_m e^{j(\omega t+\phi)} = I_m \cos(\omega t+\phi) + jI_m \sin(\omega t+\phi)$$

Phasor Relationship:

So that

$$V_m e^{j(\omega t+\theta)} = R I_m e^{j(\omega t+\phi)}$$



Dividing throughout by $e^{j\omega t}$

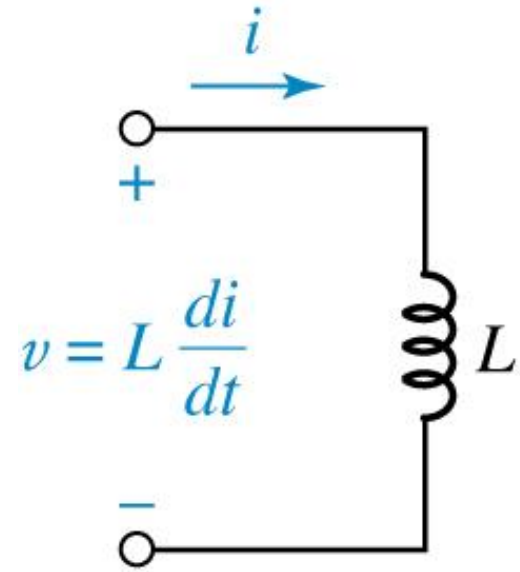
$$V_m e^{j\theta} = R I_m e^{j\phi}$$

In polar form: $\mathbf{V_m \angle \theta = R I_m \angle \phi}$

Thus, $\mathbf{V} = \mathbf{R}\mathbf{I}$

Phasor Relationship:

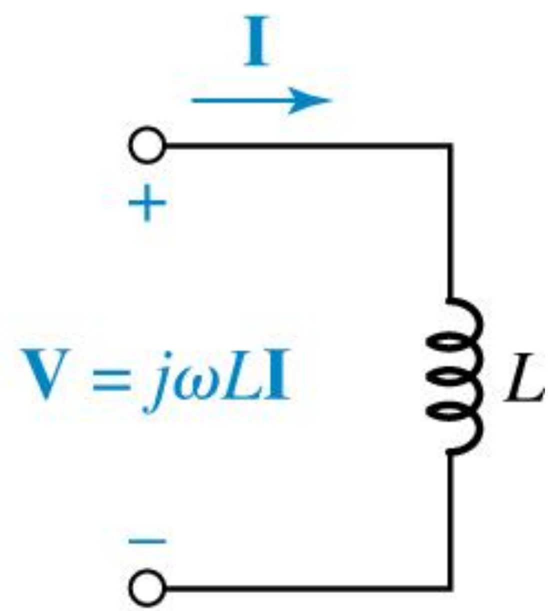
The Inductor: $v(t) = L\dfrac{di}{dt}$



$$V_m e^{j(\omega t+\theta)} = L\frac{d}{dt} I_m e^{j(\omega t+\phi)}$$

$$= j\omega L I_m e^{j(\omega t+\phi)}$$

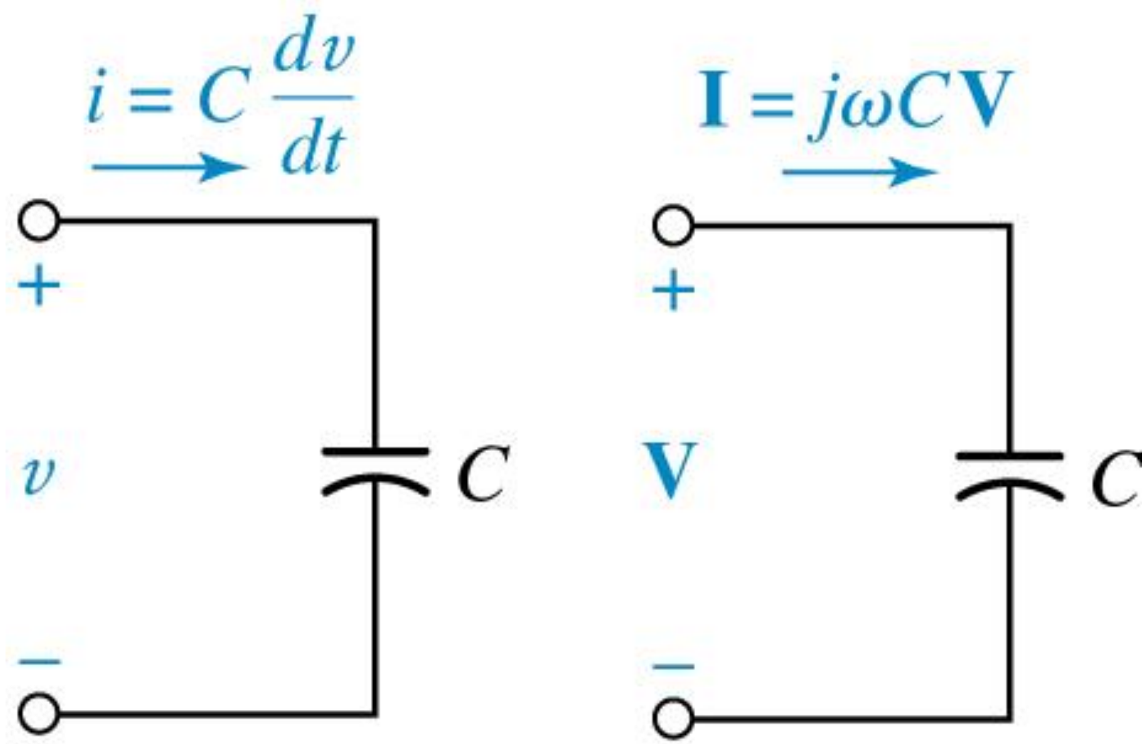
Dividing throughout by $e^{j\omega t}$



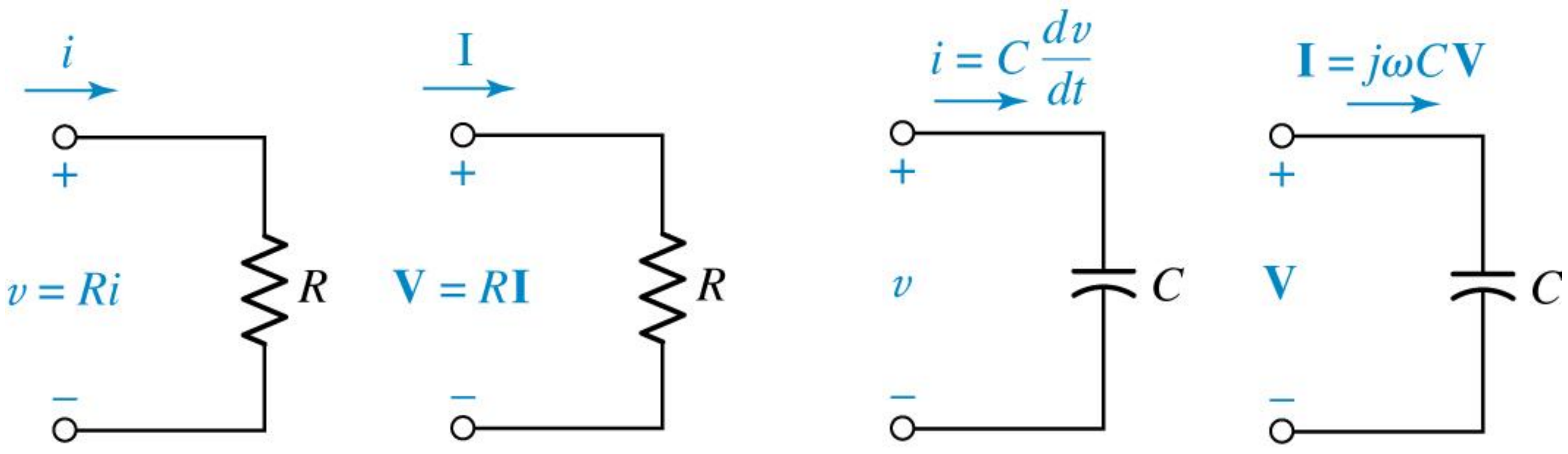
$$V_m e^{j\theta} = j\omega L I_m e^{j\phi}$$

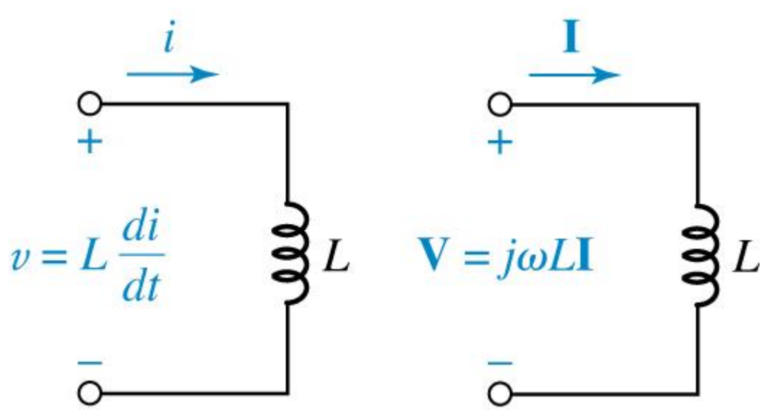
Phasor Relationship:

The Capacitor: $i(t) = C\dfrac{dv}{dt}$



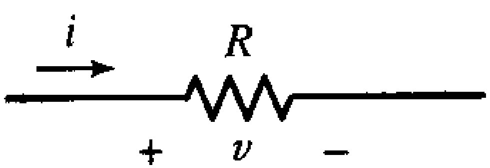
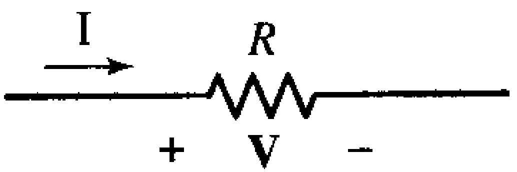
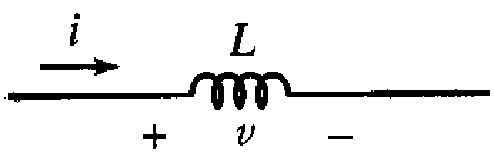
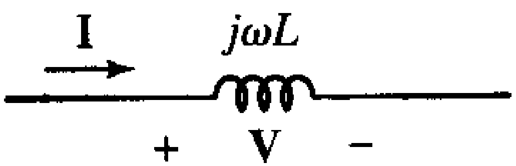
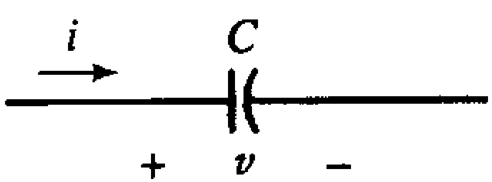
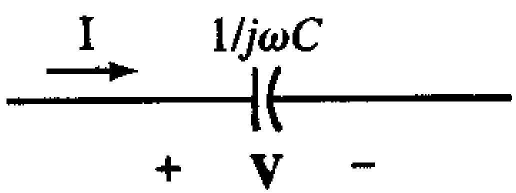
Phasor Relationship:





Phasor Relationship:

Table 10.1 | Comparison of time-domain and frequency-domain voltage-current expressions

Time domain		Frequency domain	
	$v = Ri$	$\mathbf{V} = R\mathbf{I}$	
	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$	
	$v = \dfrac{1}{C}\displaystyle\int i\,dt$	$\mathbf{V} = \dfrac{1}{j\omega C}\mathbf{I}$	

Phasor:

Kirchhoff's Laws Using Phasors

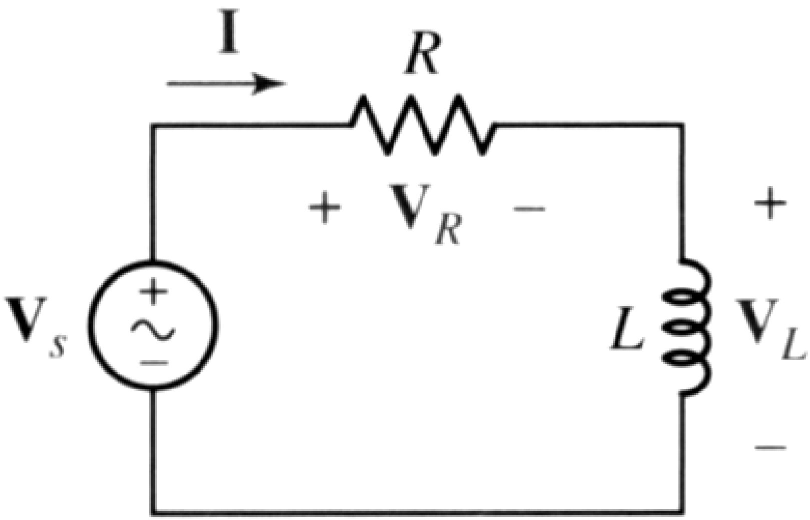
In the time domain

$$v_1(t) + v_2(t) + \ldots + v_N(t) = 0$$

We can obtain

$$\mathrm{V}_1 + \mathrm{V}_2 + \ldots + \mathrm{V}_N = 0$$

Phasor:



$$\mathbf{V}_R + \mathbf{V}_L = \mathbf{V}_s$$

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s$$

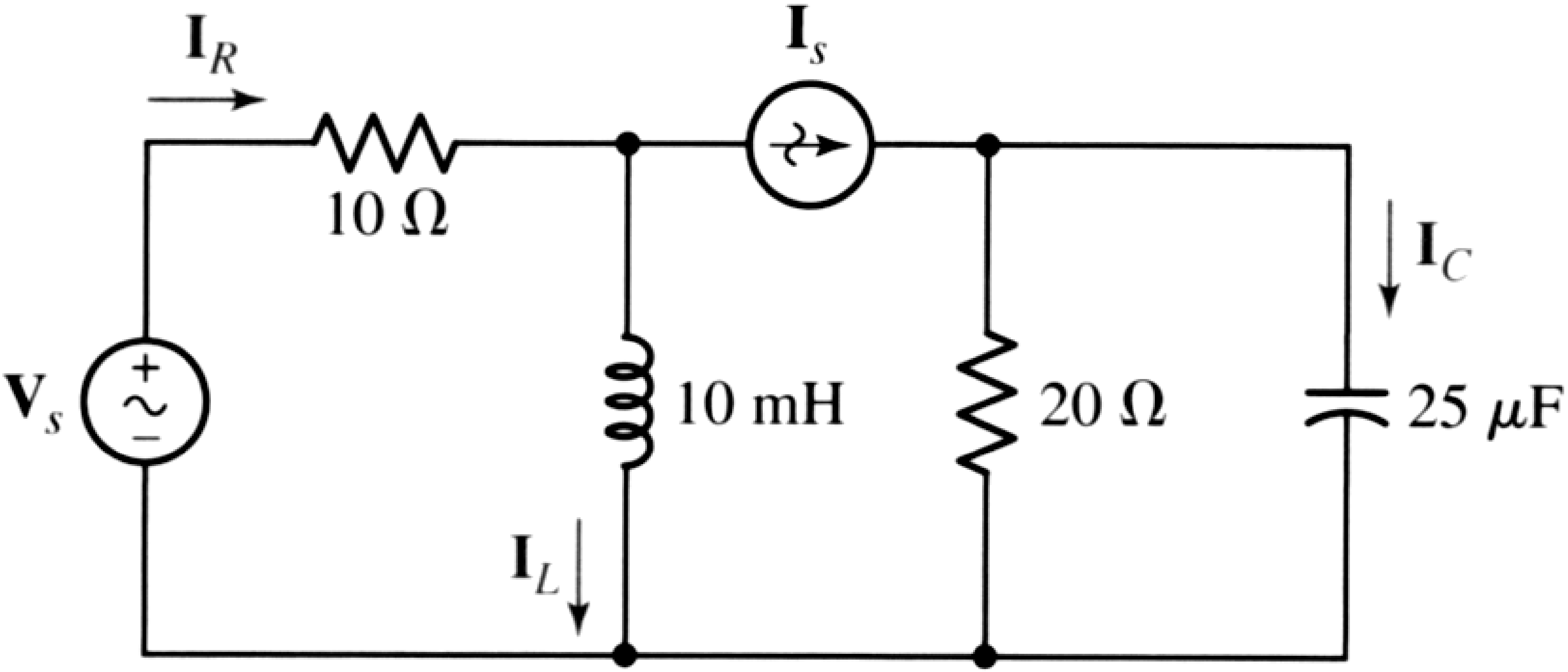
$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L}$$

$$\mathbf{I} = \frac{V_m \angle 0^\circ}{R + j\omega L}$$

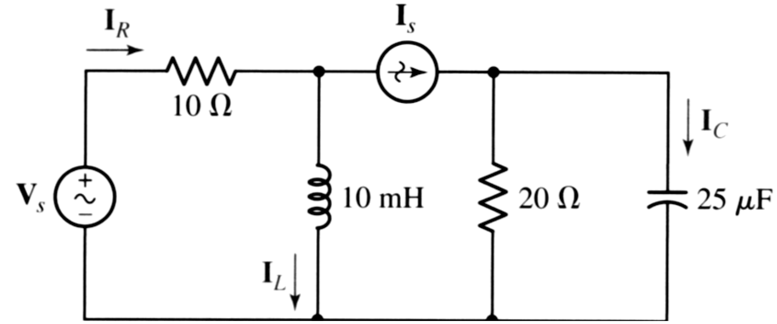
$$\mathbf{I} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle(-\tan^{-1}(\omega L / R))$$

Practice: 10.8

In the circuit, let ω = 1200 rad/s, $\mathbf{I}_C = 1.2\angle 28°$ A, and $\mathbf{I}_L = 3\angle 53°$ A. Find (a) $\mathbf{I}_s$; (b) $\mathbf{V}_s$; (c) $i_R(t)$.



Practice: 10.8



The inductor is represented by a $j(10 \times 10^{-3})\,(1200) = j12\ \Omega$ impedance and the capacitor by a $\dfrac{-j}{(1200)(25\times10^{-6})} = -j33.33\,\Omega$ impedance.

(a) The voltage across the 20–Ω resistor is then

$(1.2\angle 28°)(-j33.33) = 40\angle -62°$ V and the current through it is $\dfrac{40\angle -62°}{20} = 2\angle -62°$ A

Thus, $\mathbf{I}_s = 2\angle -62° + 1.2\angle 28°$ (by KCL)

$= \underline{2.332\angle{-31.04°}\ \text{A}}$

(b)
$$\begin{aligned}\mathbf{V}_s &= 10\mathbf{I}_R + (j12)\mathbf{I}_L \\ &= 10(\mathbf{I}_L + \mathbf{I}_S) + (12\angle 90°)\mathbf{I}_L \\ &= 10(3\angle 53° + 2.332\angle -31.04°) + (12\angle 90°)(3\angle 53°) \\ &= \underline{34.86\angle 74.55°\ \text{V}}\end{aligned}$$

(c) $\mathbf{I}_R = \mathbf{I}_L + \mathbf{I}_S = 3\angle 53° + 2.332\angle -31.04° = 3.986\angle 17.42°$

$\therefore\ i_R(t) = \underline{3.986\cos(1200t + 17.42°)\ \text{A}}$

Impedance: **Z**

From

$$\mathbf{V} = R\mathbf{I} \qquad \mathbf{V} = j\omega L\mathbf{I} \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

phasor voltage/phasor current ratio as impedance

$$\frac{\mathbf{V}}{\mathbf{I}} = R \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

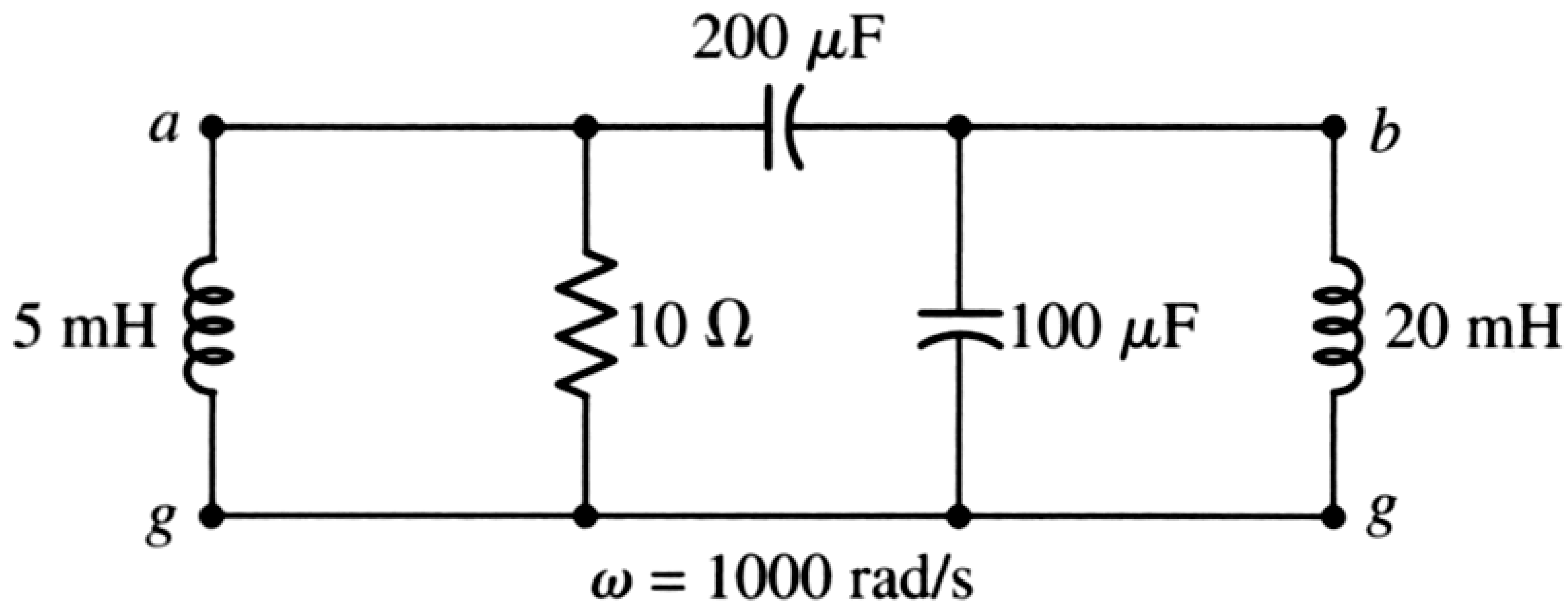
$$\boxed{\begin{aligned} \mathbf{Z}_R &= R \\ \mathbf{Z}_L &= j\omega L \\ \mathbf{Z}_C &= \frac{1}{j\omega C} \end{aligned}}$$

A resistance

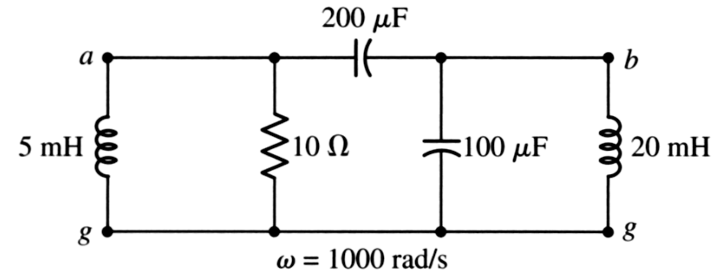
A reactance

Practice: 10.9

With reference to the network shown, find the input impedance $\mathbf{Z}_{in}$ that would be measured between terminals: (a) a and g; (b) b and g; (c) a and b



Practice: 10.9



$$5\ \text{mH} \rightarrow j5\,\Omega;\ 20\ \text{mH} \rightarrow j20\,\Omega$$

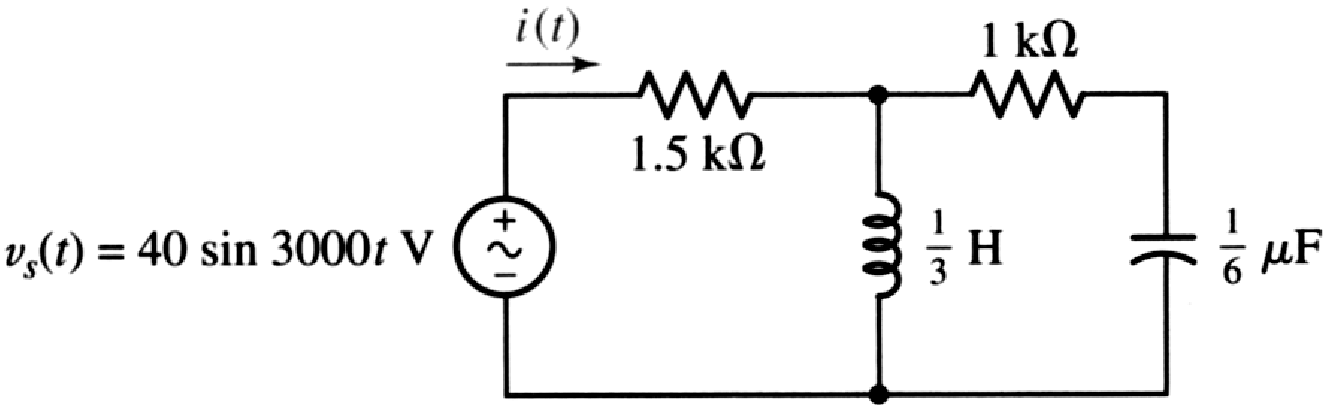
$$200\,\mu F \rightarrow -j5\,\Omega;\ 100\,\mu F \rightarrow -j10\,\Omega$$

(a)
$$\begin{aligned}\mathbf{Z}_{in}(a,g) &= (j5 // 10) // [-j5 + (-j10 // j20)] \\ &= (2 + j4) // [-j25] \\ &= \underline{2.809 + j4.494\ \Omega}\end{aligned}$$

(b)
$$\begin{aligned}\mathbf{Z}_{in}(b,g) &= (j20 // - j10) // [-j5 + (10 // j5)] \\ &= -j20 // (2 - j) = \underline{1.798 - j1.124\ \Omega}\end{aligned}$$

(c)
$$\begin{aligned}\mathbf{Z}_{in}(a,b) &= -j5 // [(j5 // 10) + (-j10 // j20)] \\ &= -j5 // (2 + j4 + (-j20)] \\ &= -j5 // (2 - j16) = \underline{0.1124 - j3.820\ \Omega}\end{aligned}$$

Example: find i(t)

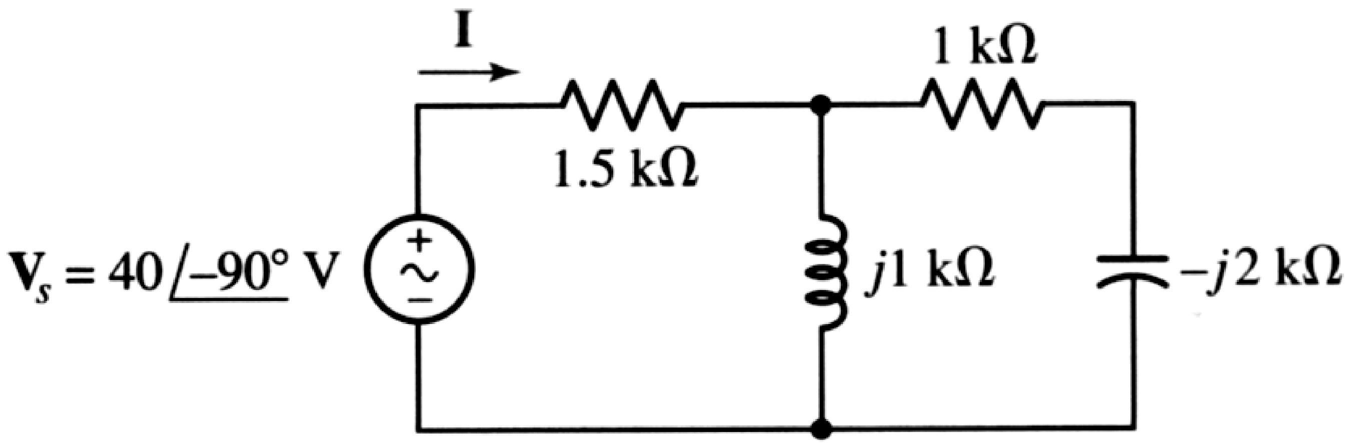


$$40\sin(3000t) = 40\cos(3000t - 90^\circ) \rightarrow 40\angle{-90^\circ}$$

$$j\omega L = j \cdot 3000 \cdot \frac{1}{3} = j1k \quad \Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{3000 \cdot \frac{1}{6}\mu} = -j2k \quad \Omega$$

Example:



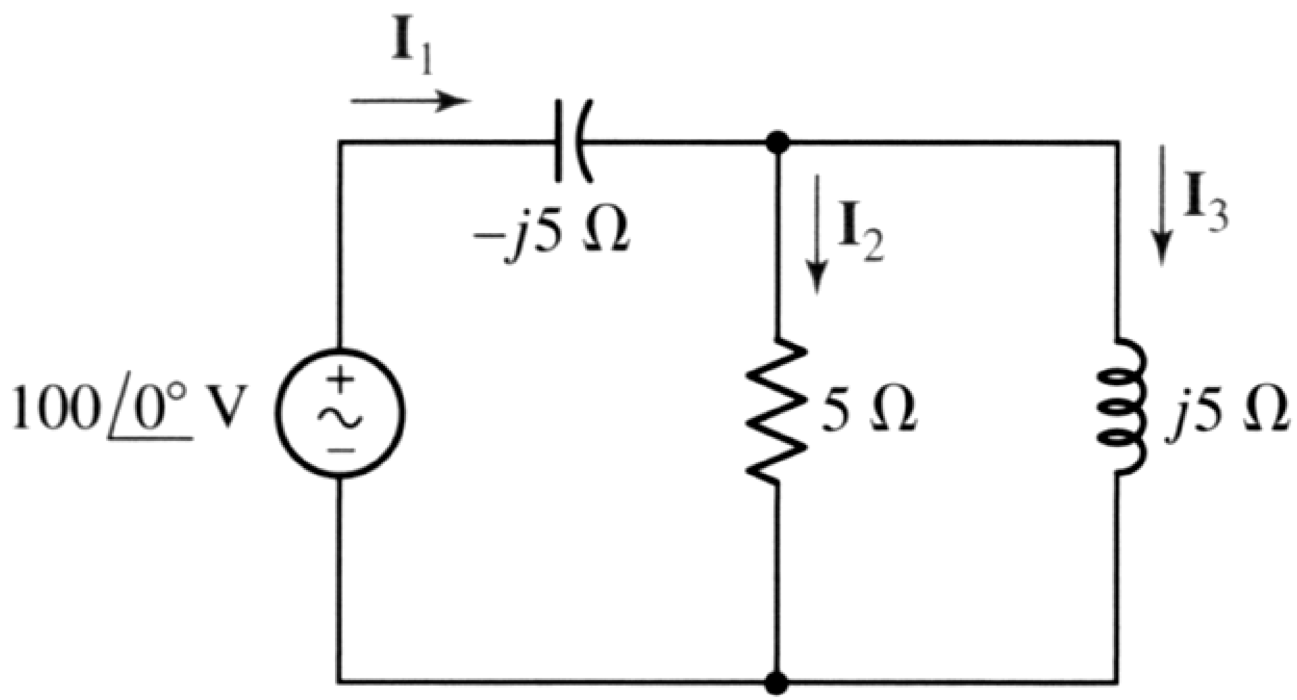
$$Z_{total} = 1.5 + \frac{(j)\cdot(1-2j)}{j+1-2j}$$
$$= 2 + j1.5 = 2.5\angle 36.87^\circ \quad k\Omega$$

$$I = \frac{V_s}{Z_{total}} = \frac{40\angle -90^\circ}{2.5\angle 36.87^\circ} mA = 16\angle -126.9^\circ \quad mA$$

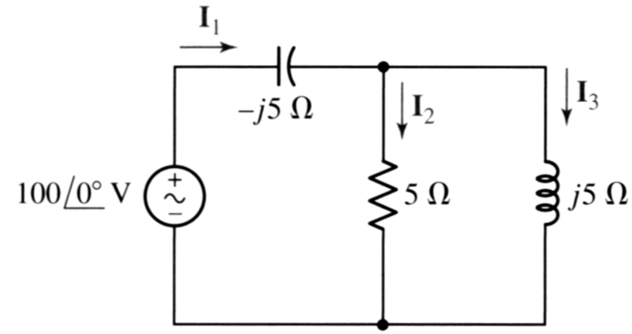
$$\rightarrow 16\cos(3000t - 126.9^\circ)mA$$

Practice: 10.10

In the frequency-domain circuit, find (a) $\mathbf{I}_1$; (b) $\mathbf{I}_2$; (c) $\mathbf{I}_3$



Practice: 10.10



(a) $$\mathbf{I}_1 = \frac{100}{-j5 + 5//j5} = \frac{100}{2.5 - j2.5} = \frac{100}{3.536\angle -45^\circ}$$

$$= \underline{28.28\angle 45^\circ \text{ A}}$$

(b) $$\mathbf{I}_2 = \mathbf{I}_1 \frac{j5}{5 + j5} = 28.28\angle 45^\circ \left(\frac{5\angle 90^\circ}{7.071\angle 45^\circ} \right)$$

$$= \underline{20\angle 90^\circ \text{ A}}$$

(c) $$\mathbf{I}_3 = \mathbf{I}_1 - \mathbf{I}_2 = 28.28\angle 45^\circ - 20\angle 90^\circ$$

$$= 20\angle -0.009^\circ \text{ A} \approx \underline{20\angle 0^\circ \text{ A}}$$

Admittance: **Y**

Admittance

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$$

$$\mathbf{Y} = \frac{1}{\mathbf{Z}}$$

$$\mathbf{Y} = G + jB = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX}$$

G = the conductance
B = the susceptance
R = the resistance
X = the reactance
… immittance

Practice: 10.11

Determine the admittance (in rectangular form) of (a) an impedance $\mathbf{Z} = 1000 + j400\ \Omega$; (b) a network consisting of the parallel combination of an 800-Ω resistor, a 1-mH inductor, and a 2-nF capacitor, if $\omega = 1$ Mrad/s; (c) a network consisting of the series combination of an 800-Ω resistor, a 1-mH inductor, and a 2-nF capacitor, if $\omega = 1$ Mrad/s

Practice: 10.11

(a) $\mathbf{Z} = 1000 + j400\,\Omega = 1077\angle 21.80^\circ\,\Omega$

$$\therefore\ \mathbf{Y} = \frac{1}{\mathbf{Z}} = 928.5\angle -21.8^\circ\,\mu S$$

$$= \underline{862.1 - j\,344.8\ \mu S}$$

(b) at $\omega = 10^6$ rad/s, 1 mH $\rightarrow j\,10^6\ \Omega$, 2 nF $\rightarrow -j500\ \Omega$

$$\mathbf{Y} = \frac{1}{800} + \frac{1}{j106} - \frac{1}{j500} = \underline{1.25 + j2\text{ mS}}$$

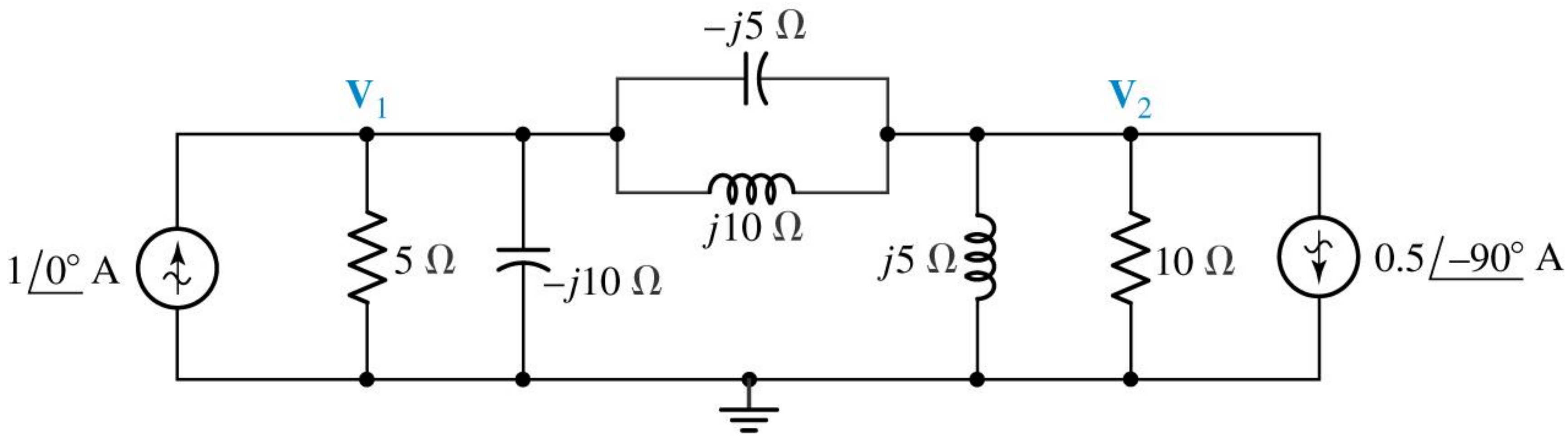
(c) $\mathbf{Z} = 800 + j10^6 - j500 = 999.5\times 10^3\angle 89.95^\circ\,\Omega$

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = 1.0005\angle -89.95^\circ\,\mu\text{S}$$

$$= \underline{800.8 - j10^6\text{ pS}}$$

Nodal and Mesh Analysis:

Example: find $v_1(t)$ and $v_2(t)$

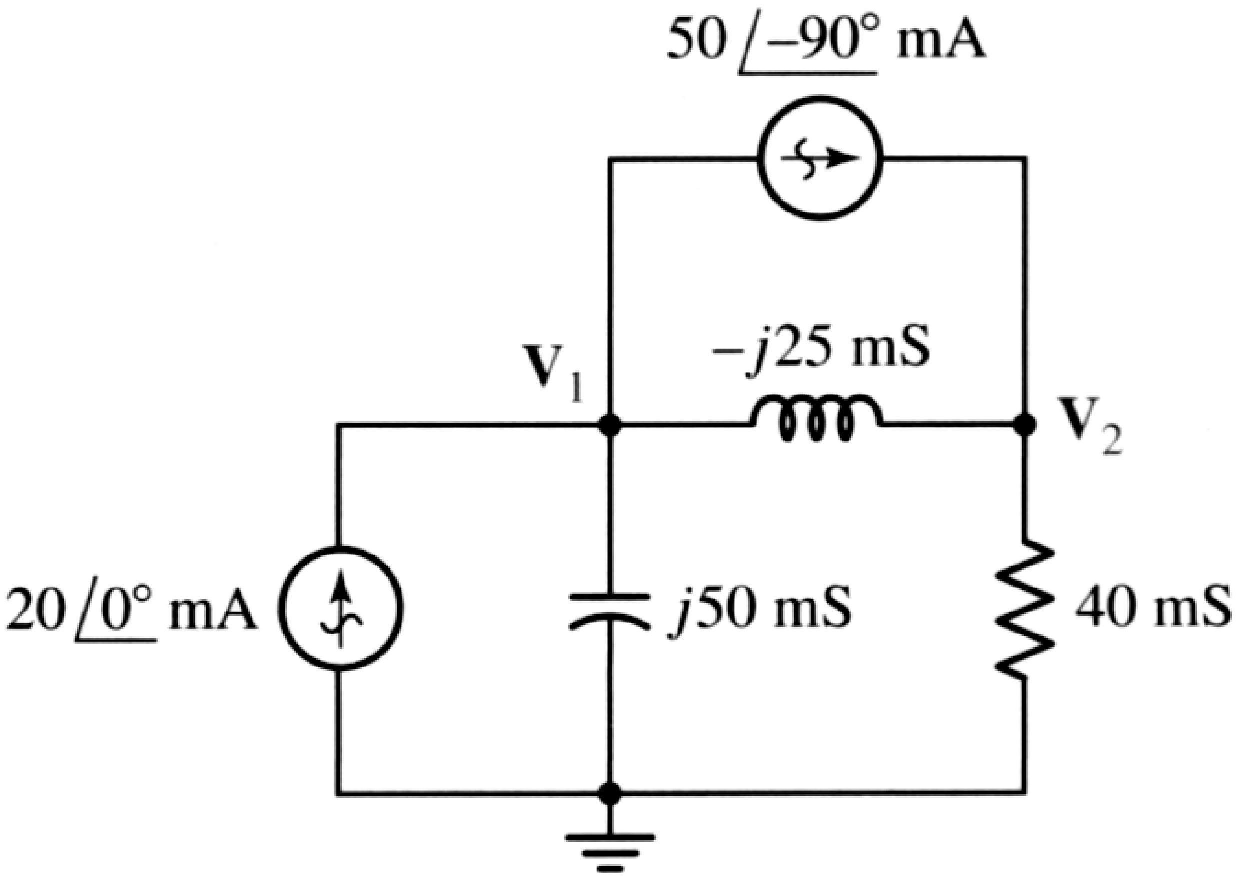


$$\frac{V_1}{5}+\frac{V_1}{-j10}+\frac{V_1-V_2}{-j5}+\frac{V_1-V_2}{j10}=1\angle 0^\circ \quad =1+j0$$

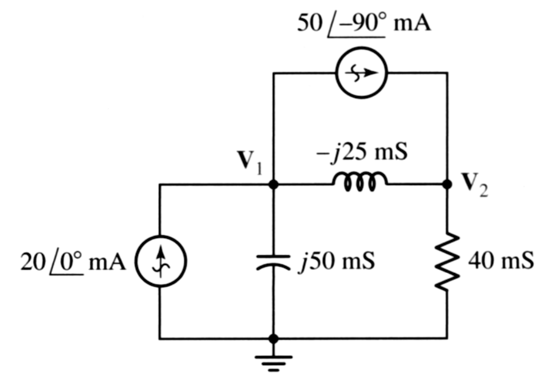
$$\frac{V_2-V_1}{-j5}+\frac{V_2-V_1}{j10}+\frac{V_2}{j5}+\frac{V_2}{10}=-(0.5\angle 0^\circ) \quad = j0.5$$

Practice: 10.12

Use nodal analysis on the circuit to find $\mathbf{V}_1$ and $\mathbf{V}_2$.



Practice: 10.12



$$-50\angle -90^\circ + 20 = j50\mathbf{V}_1 - j25(\mathbf{V}_1 - \mathbf{V}_2) \qquad [1]$$

$$50\angle -90^\circ = 40\,\mathbf{V}_2 - j25\,(\mathbf{V}_2 - \mathbf{V}_1) \qquad [2]$$

- rewrite, grouping terms:

$$20 + j50 = j25\mathbf{V}_1 + j25\mathbf{V}_2 \qquad [1]$$

$$-j50 = j25\mathbf{V}_1 + (40 - j25)\mathbf{V}_2 \qquad [2]$$

- Solving,

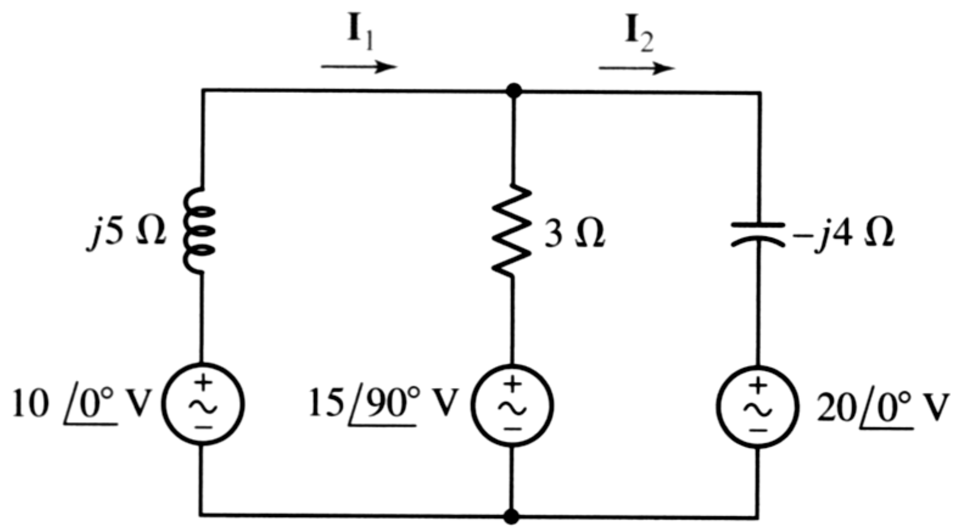
$$\mathbf{V}_1 = 0.9756 + j0.4195 = \underline{1.062\angle 23.27^\circ \text{ V}}$$

and

$$\mathbf{V}_2 = 1.024 - j1.2195 = 1.593\angle\underline{-49.97^\circ \text{ V}}$$

Nodal and Mesh Analysis:

Example: find I_1 and I_2

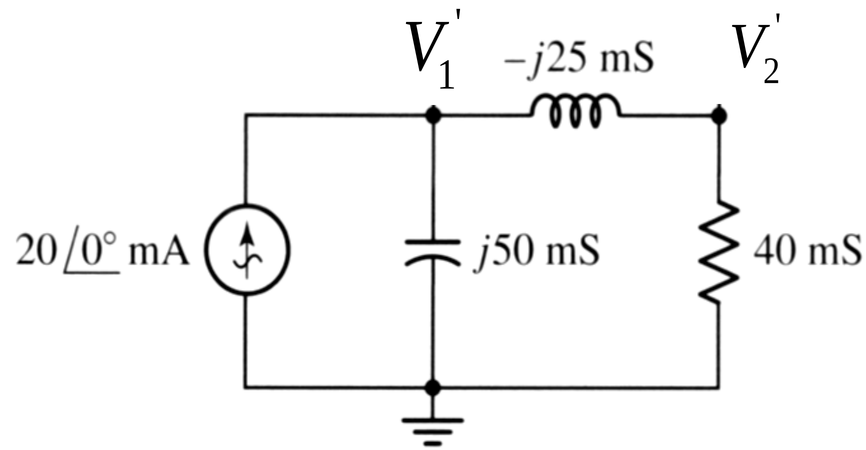


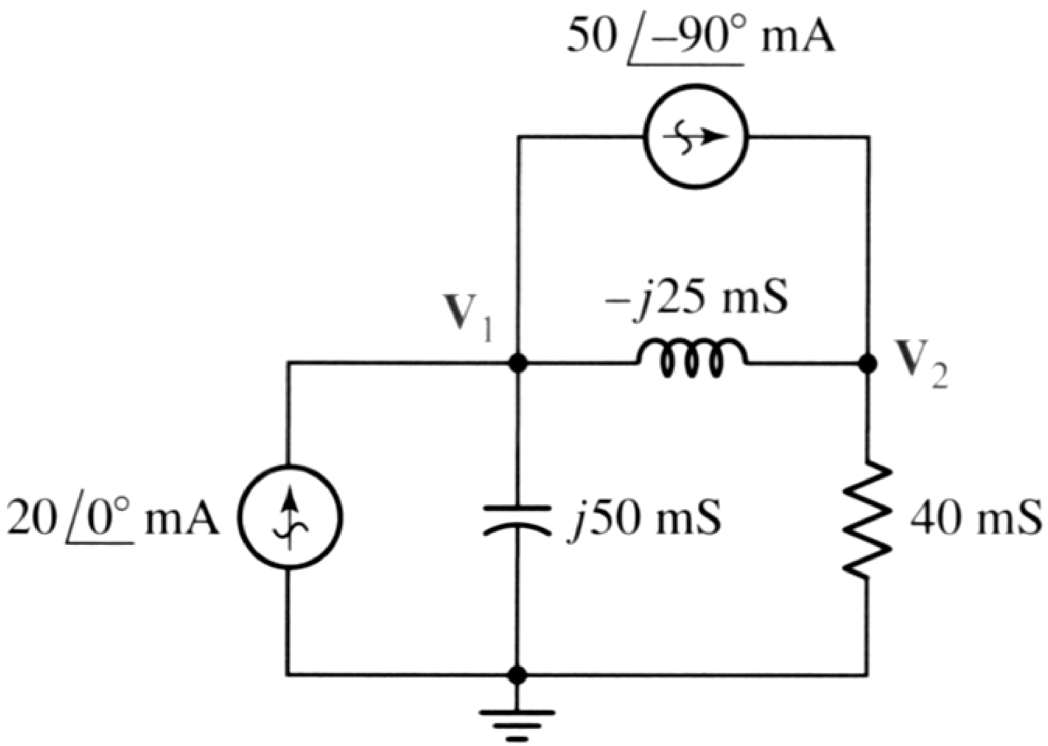
$$-10\angle 0^\circ + j5(I_1) + 3(I_1 - I_2) + 15\angle 90^\circ = 0$$

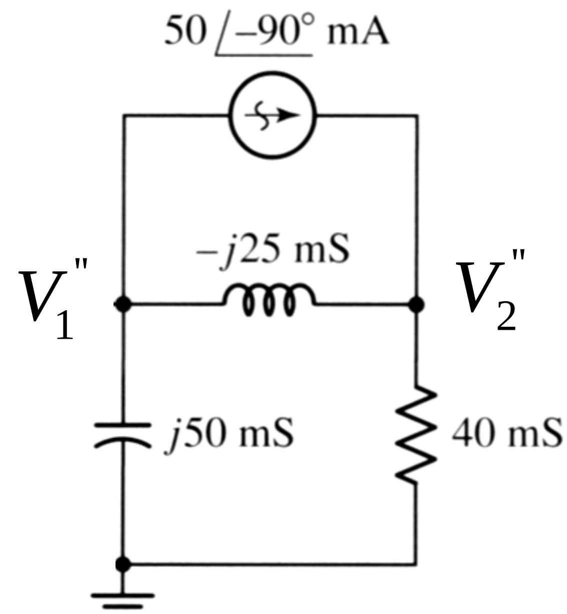
$$-15\angle 90^\circ + 3(I_2 - I_1) - j4(I_2) + 20\angle 0^\circ = 0$$

Superposition:

Example: find V_1

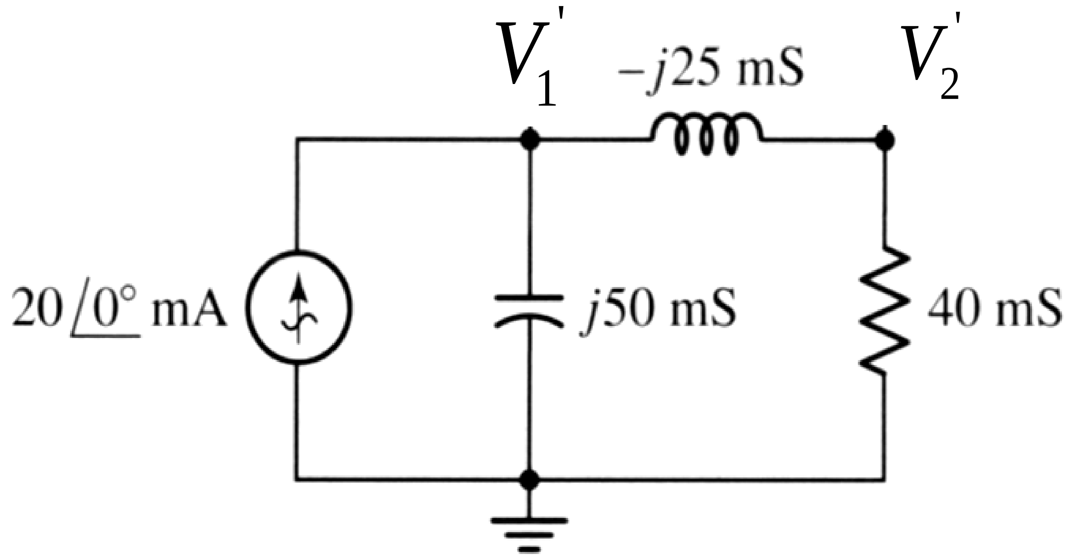






Superposition:

Example: find V_1



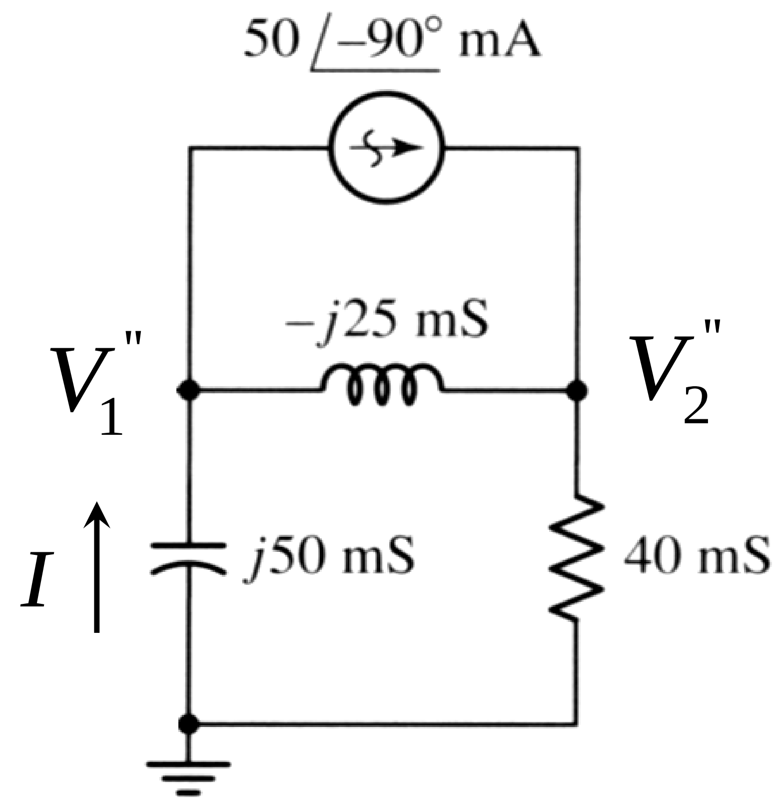
$$V_1' = \frac{I}{Y} = \frac{20}{j50 + \dfrac{(40)\cdot(-j25)}{40 - j25}}$$

$$= 0.1951 - j0.5561$$

Superposition:

Example: find V_1

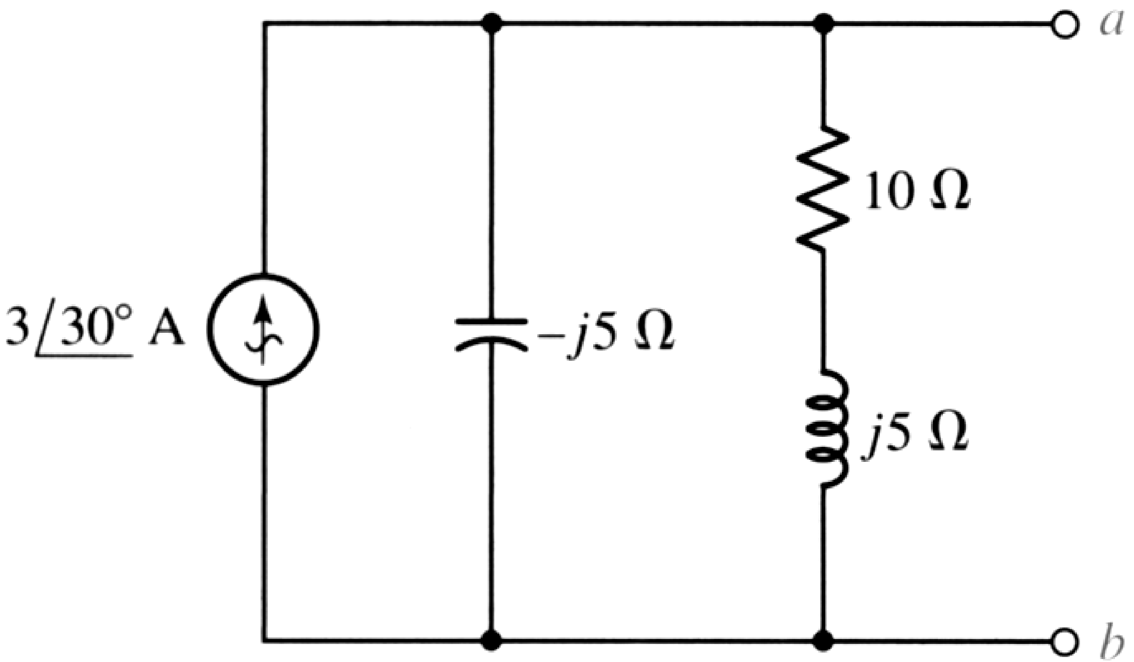
$$V_1'' = \frac{-I}{j50m}$$



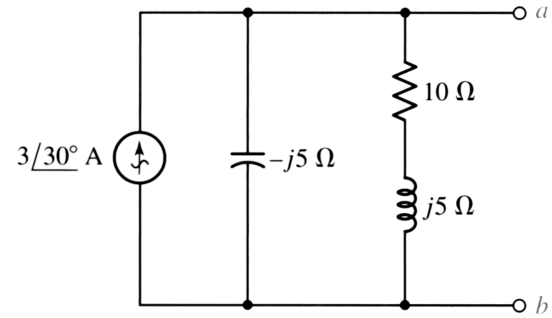
$$I = \frac{\left(\dfrac{1}{-j25m}\right)}{\left(\dfrac{1}{40m}+\dfrac{1}{j50m}\right)+\left(\dfrac{1}{-j25m}\right)} \cdot \left(50 \angle -90^\circ \ m\right)$$

Practice: 10.15

For the circuit, find the (a) open-circuit voltage $\mathbf{V}_{ab}$; (b) downward current in a short circuit between a and b; (c) Thevenin-equivalent impedance $\mathbf{Z}_{ab}$ in parallel with the current source.



Practice: 10.15



(a) $$\mathbf{V}_{ab} = (3\angle 30^\circ)\left[-j5 // (10 + j5)\right] = \underline{16.77\angle{-33.43^\circ}\ \text{V}}$$

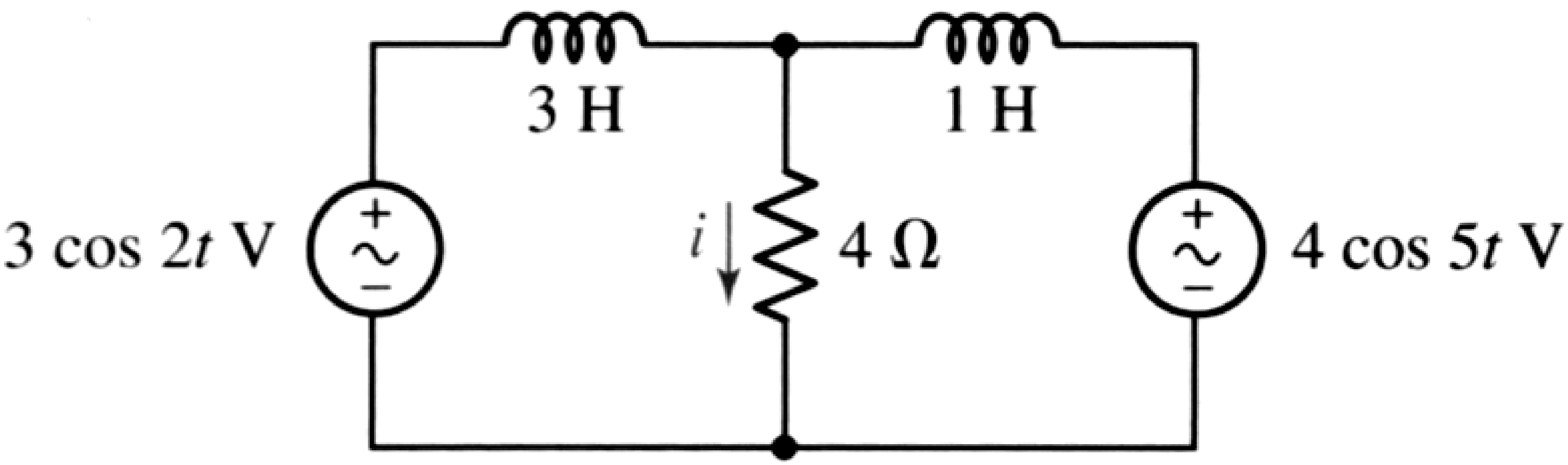
(b) $$\mathbf{Z}_{ab} = -j5 // (10 + j5) = 5.59\angle -63.43^\circ\ \Omega$$

$$\mathbf{I}_{SC} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = 3\angle 30^\circ\ \text{A} = \underline{2.598 + j1.5\ \text{A}}$$

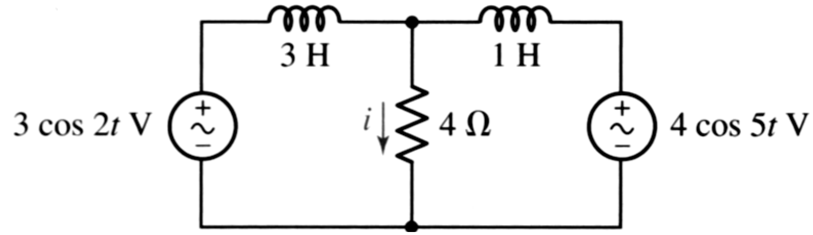
(c) $$\mathbf{Z}_{ab} = 5.59\angle -63.43^\circ\ \Omega = \underline{2.5 - j5\ \Omega}$$

Practice: 10.16

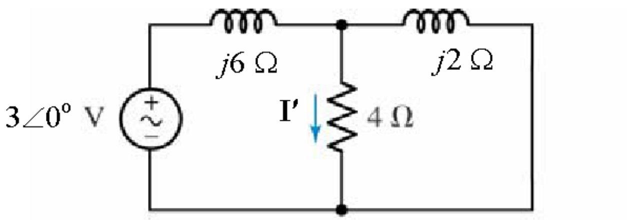
Determine the current i through the 4-Ω resistor



Practice: 10.16



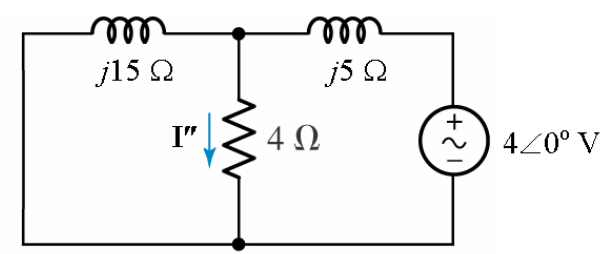
- Since the two sources do not operate at the same frequency, we must use superposition in the time domain.



$$\mathbf{V}'_{4\Omega} = 3\frac{(4\,//\,j2)}{j6+(4\,//\,j2)} = 3\frac{0.8+j1.6}{0.8+j7.6} = 0.7022\angle -20.56° \text{ V}$$

$$\mathbf{I}' = \frac{1}{4}\mathbf{V}'_{4\Omega} = 175.6\angle -20.56° \text{ mA}$$

so $i'(t) = 175.6\cos(2t - 20.56°)$ mA



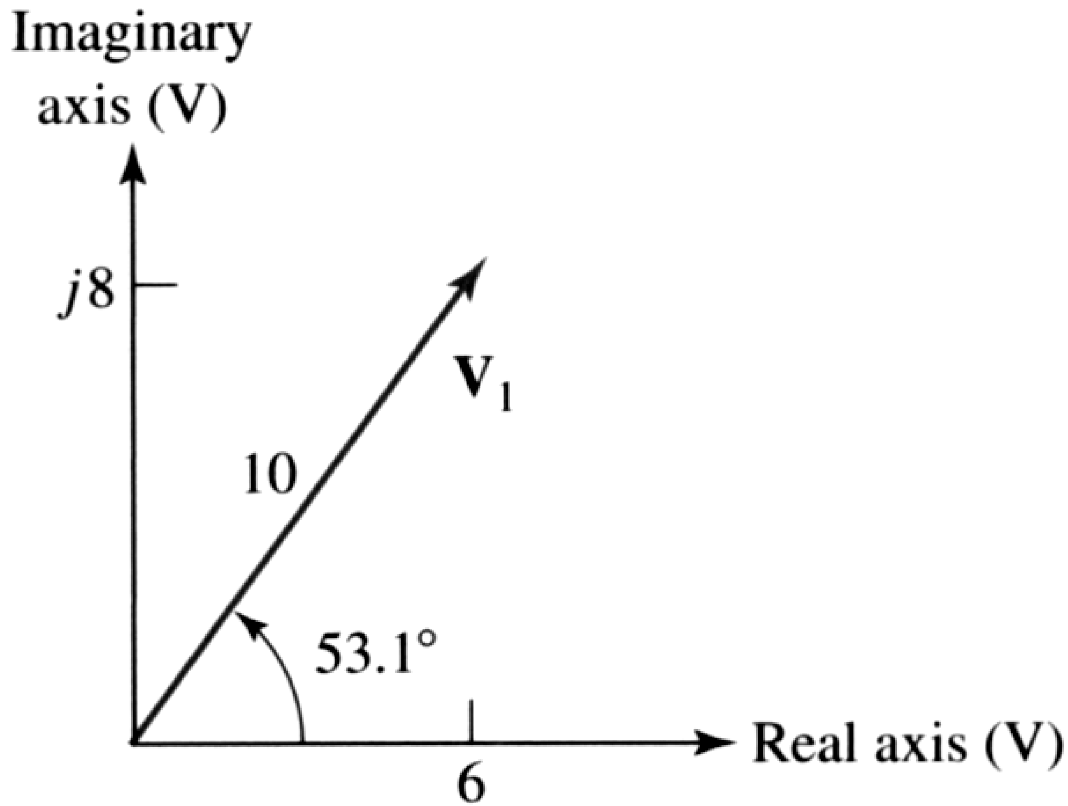
$$\mathbf{V}''_{4\Omega} = 4\frac{(4\,//\,j15)}{j5+(4\,//\,j15)} = 4\frac{(3.734+j0.9959)}{3.734+j5.996}\text{V} = 2.188\angle -43.15° \text{ V}$$

$$\mathbf{I}'' = \frac{1}{4}\mathbf{V}''_{4\Omega} = 547.1\angle -43.15° \text{ A}$$

so $i''(t) = 547.1\cos(5t - 43.15°)$ mA

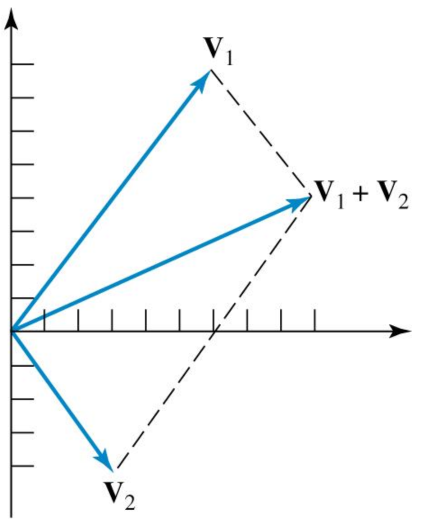
and since $i(t) = i'(t) + i''(t)$, $i(t) = \underline{175.6 \cos (2t - 20.56°) + 547.1 \cos (5t - 43.15°) \text{ mA}}$

Phasor Diagram:



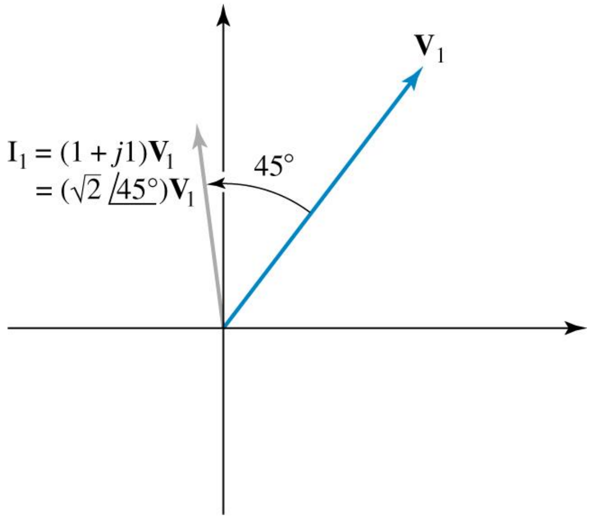
$$V_1 = 6 + j8 = 10\ \underline{/\ 53.1^\circ}$$

Phasor Diagram:



(*a*)

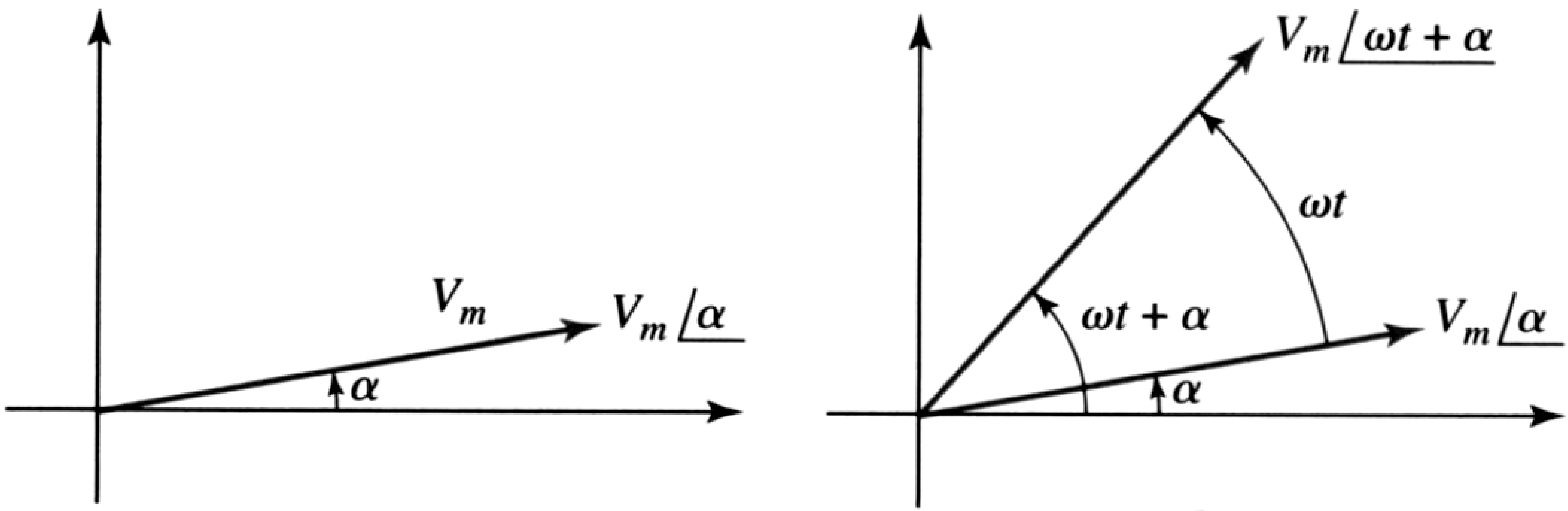
(a) A phasor diagram showing the sum of $\mathbf{V}_1 = 6 + j8$ V and $\mathbf{V}_2 = 3 - j4$ V, $\mathbf{V}_1 + \mathbf{V}_2 = 9 + j4$ V = $9.85\angle 24.0^o$ V.



(*b*)

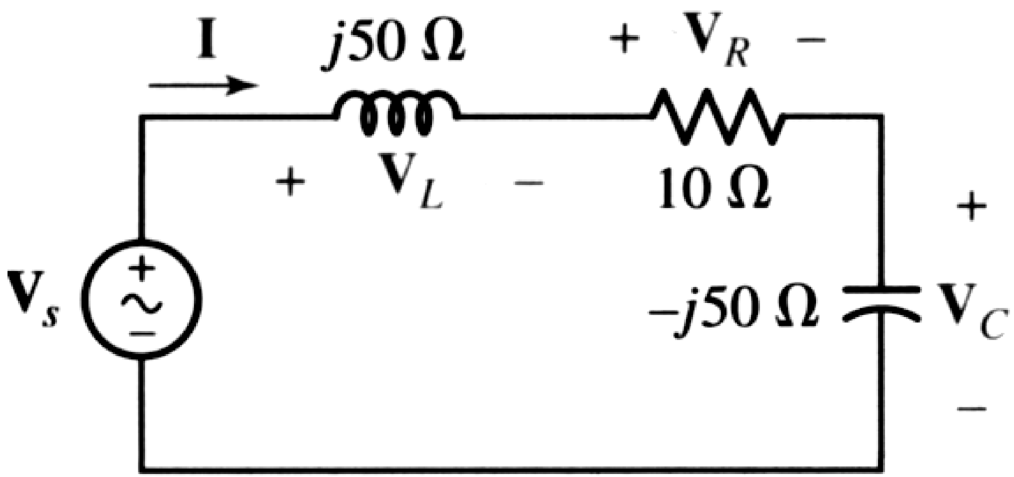
(b) (b) The phasor diagram shows $\mathbf{V}_1$ and $\mathbf{I}_1$, where $\mathbf{I}_1 = \mathbf{Y}\mathbf{V}_1$ and $\mathbf{Y} = 1 + j$ S = $1.4\angle 45^o$ S. The current and voltage amplitude scales are different.

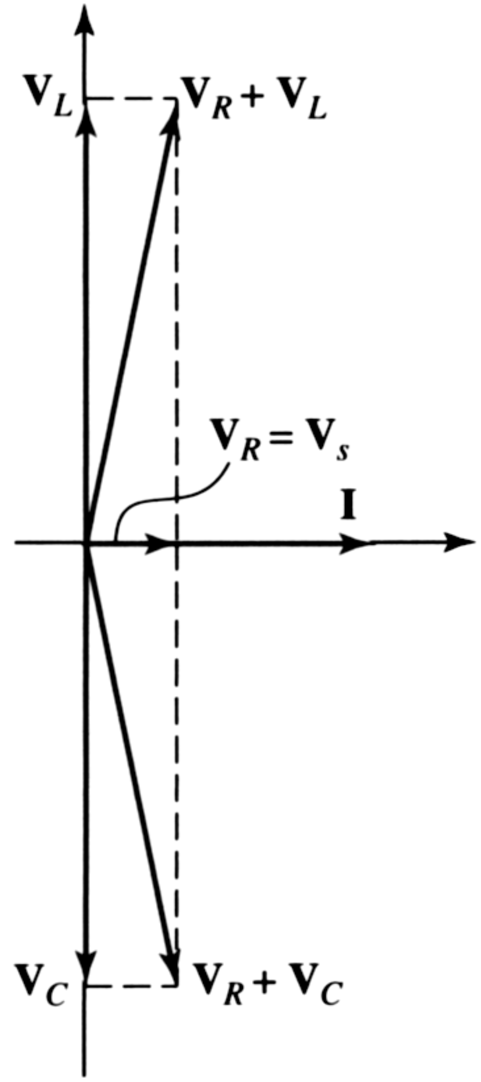
Phasor Diagram:



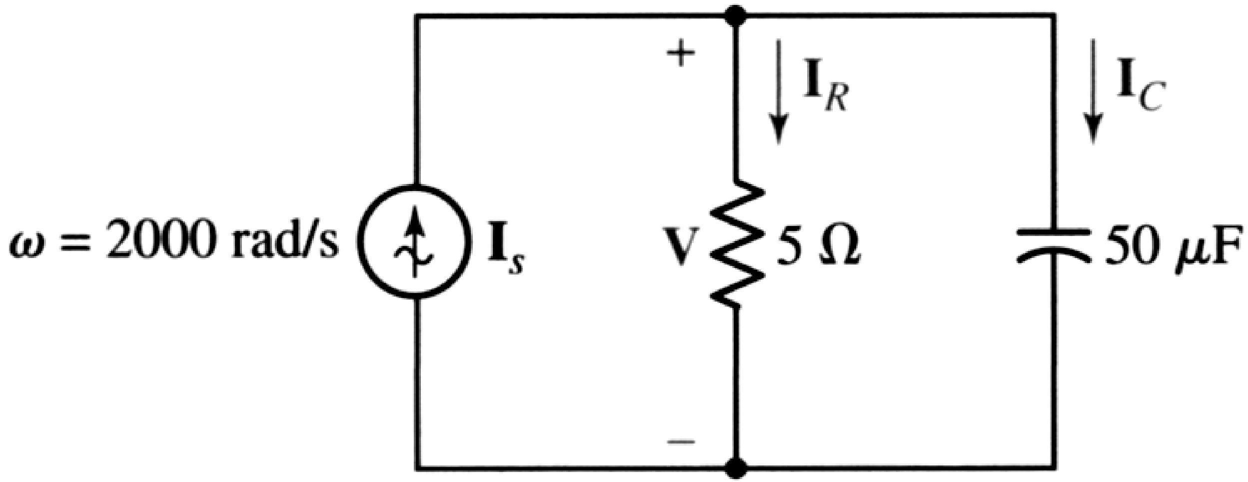
(*a*) (*b*)

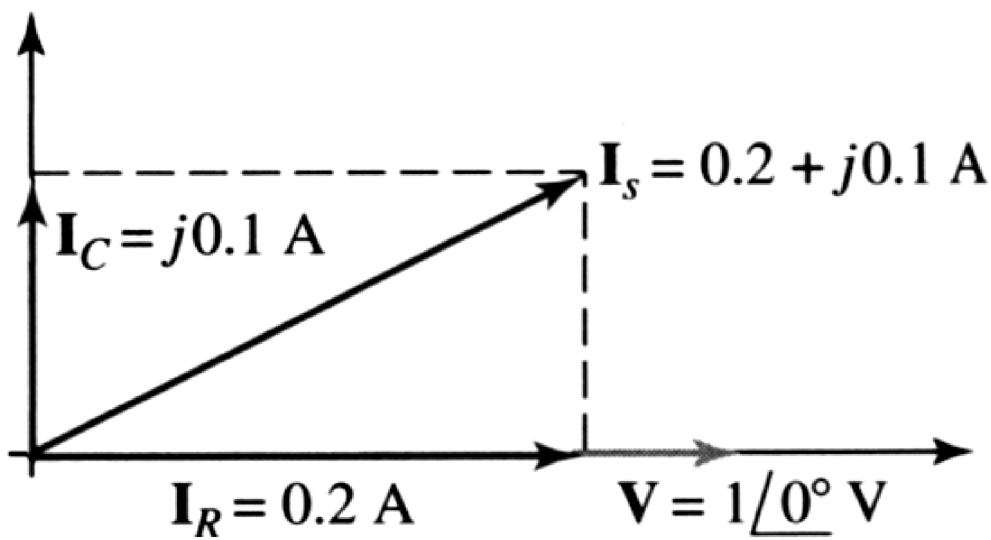
Phasor Diagram:



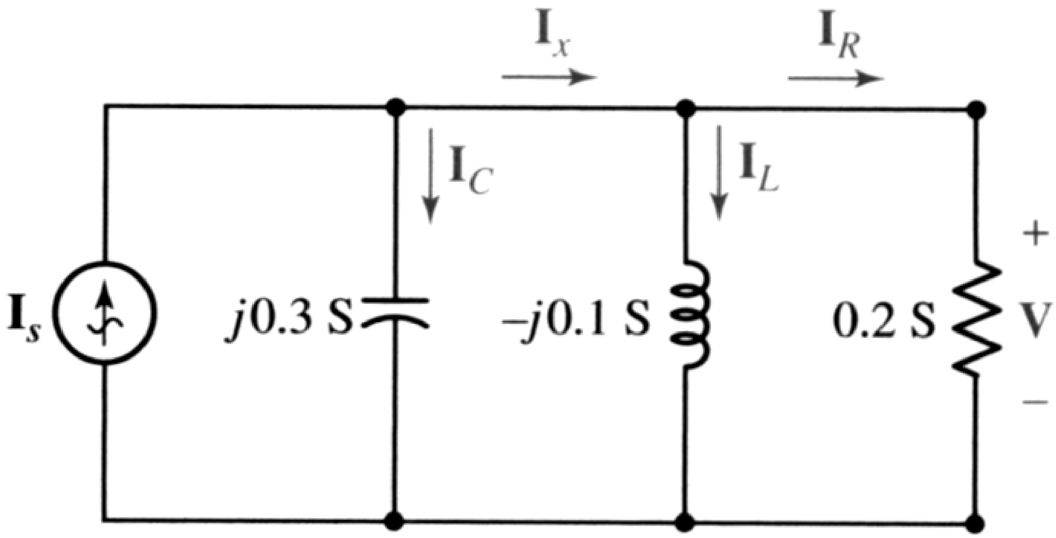


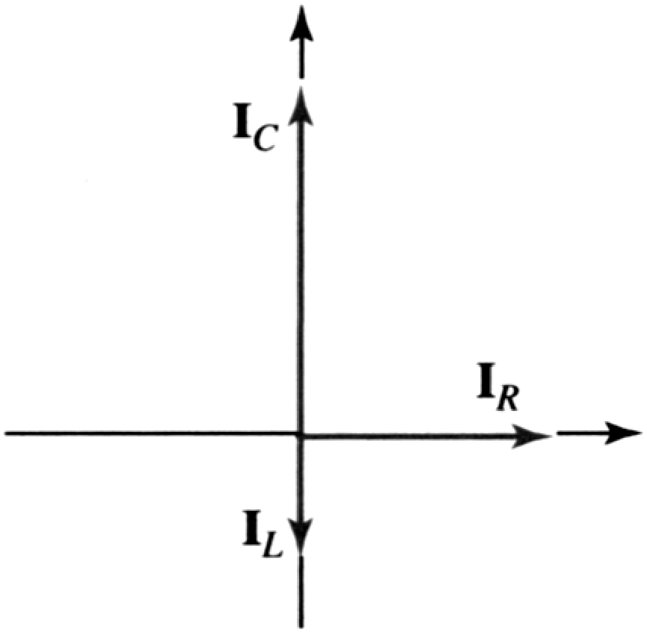
Phasor Diagram:





Example:



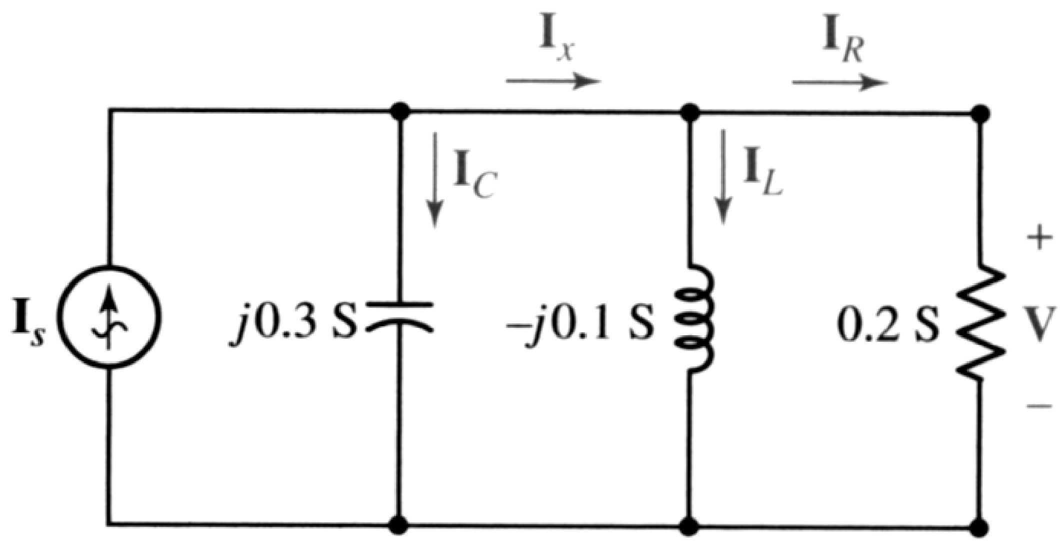


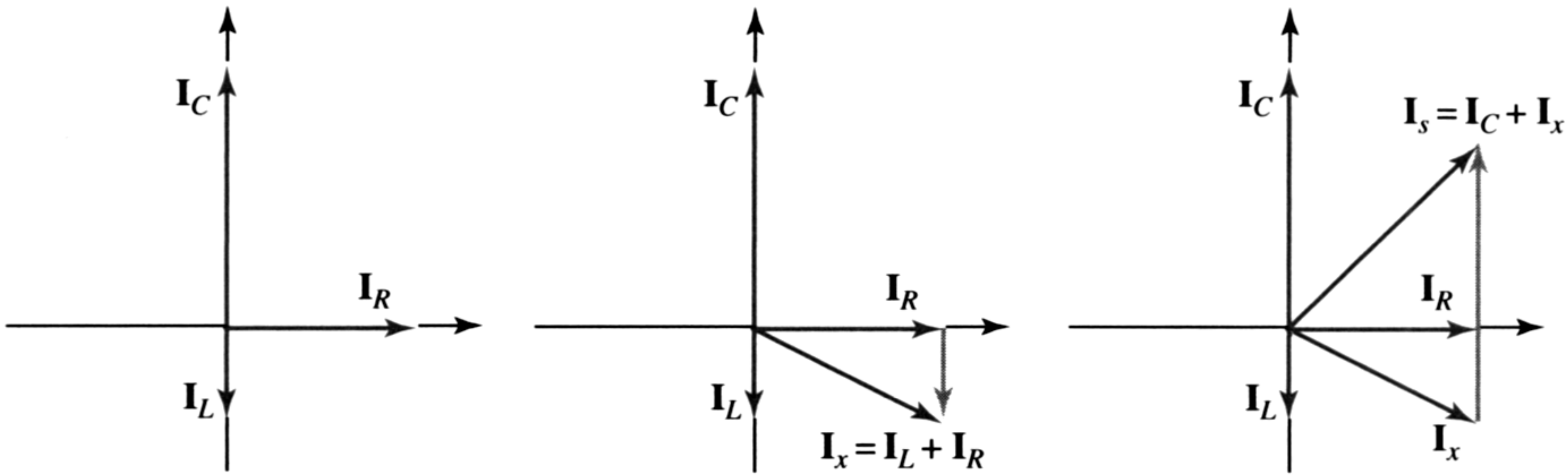
Select $\mathbf{V} = 1\angle 0^\circ$

;

$$\mathbf{I}_\mathrm{R} = (0.2)\cdot 1\angle 0^\circ$$
$$\mathbf{I}_\mathrm{L} = (-j0.1)\cdot 1\angle 0^\circ = 0.1\angle -90^\circ$$
$$\mathbf{I}_\mathrm{C} = (j0.3)\cdot 1\angle 0^\circ = 0.3\angle 90^\circ$$

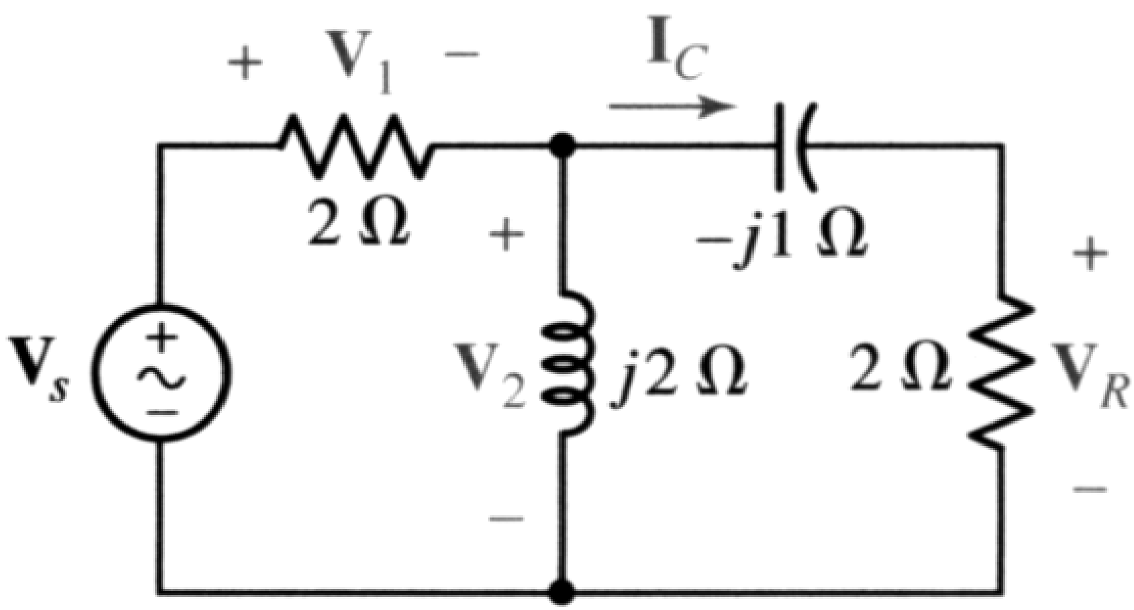
Example:


$\mathbf{I}_x$
$\mathbf{I}_R$
$\mathbf{I}_C$
$\mathbf{I}_L$
$\mathbf{I}_s$
$j0.3$ S
$-j0.1$ S
0.2 S
+
$\mathbf{V}$
–

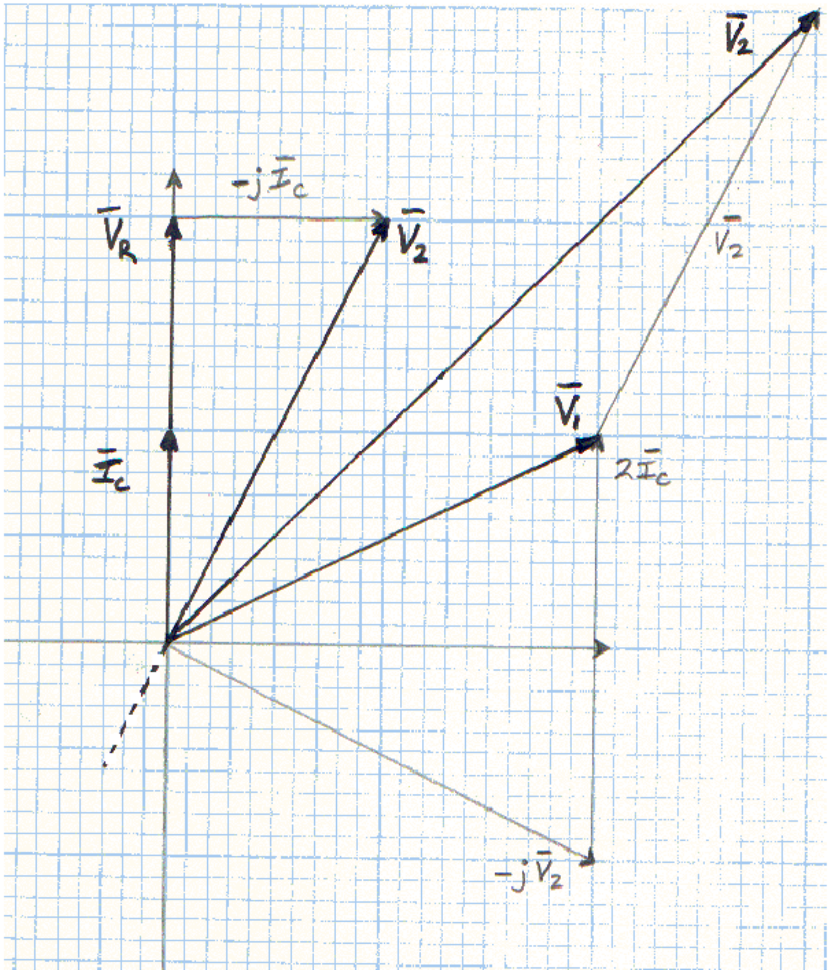

$\mathbf{I}_C$
$\mathbf{I}_R$
$\mathbf{I}_L$
$\mathbf{I}_C$
$\mathbf{I}_R$
$\mathbf{I}_L$
$\mathbf{I}_x = \mathbf{I}_L + \mathbf{I}_R$
$\mathbf{I}_C$
$\mathbf{I}_s = \mathbf{I}_C + \mathbf{I}_x$
$\mathbf{I}_R$
$\mathbf{I}_L$
$\mathbf{I}_x$

Practice: 10.17

Select some convenient reference value for $\mathbf{I}_C$ in the circuit, draw a phasor diagram showing $\mathbf{V}_R$, $\mathbf{V}_2$, $\mathbf{V}_1$, and $\mathbf{V}_s$, and measure the ratio of the lengths of (a) $\mathbf{V}_s$ to $\mathbf{V}_1$; (b) $\mathbf{V}_1$ to $\mathbf{V}_2$; (c) $\mathbf{V}_s$ to $\mathbf{V}_\mathrm{R}$



Practice: 10.17



$\mathbf{V}_R = 2\mathbf{I}_C$

$\mathbf{V}_2 = -j\mathbf{I}_C + \mathbf{V}_R = \mathbf{I}_C\angle\text{-}90^\circ + \mathbf{V}_R$

$$\mathbf{V}_1 = 2\left(\frac{\mathbf{V}_2}{j2} + \mathbf{I}_C\right) = \mathbf{V}_2\angle\text{-}90^\circ + 2\mathbf{I}_C$$

$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2$

Using a ruler on the actual size graph (10 small squares = 25.5 mm): $\mathbf{V}_s$ = 10.8 mm, $\mathbf{V}_1$ = 5.7 mm, $\mathbf{V}_2$ = 5.7 mm, $\mathbf{V}_R$ = 5.1 mm.

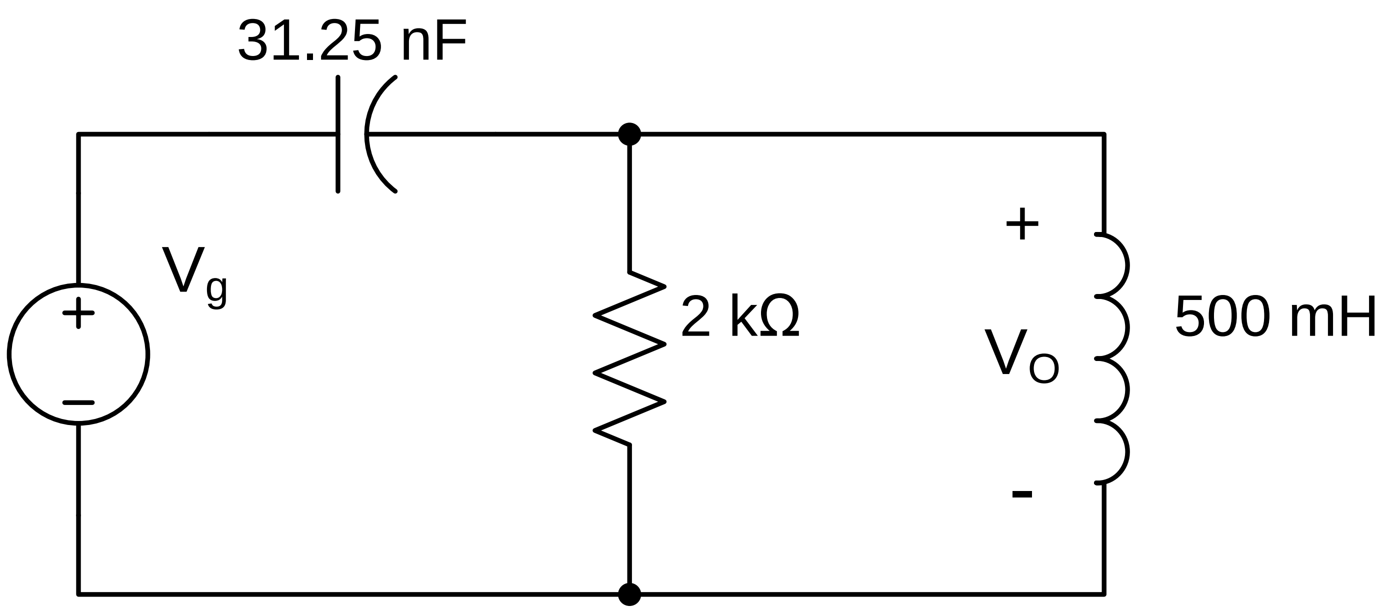
(a) $\mathbf{V}_s / \mathbf{V}_1$ = 1.90

(b) $\mathbf{V}_1 / \mathbf{V}_2$ = 1.00

(c) $\mathbf{V}_s / \mathbf{V}_R$ = 2.12

Ex:

The circuit is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g(t) = 64 \cos 8000t$ V.

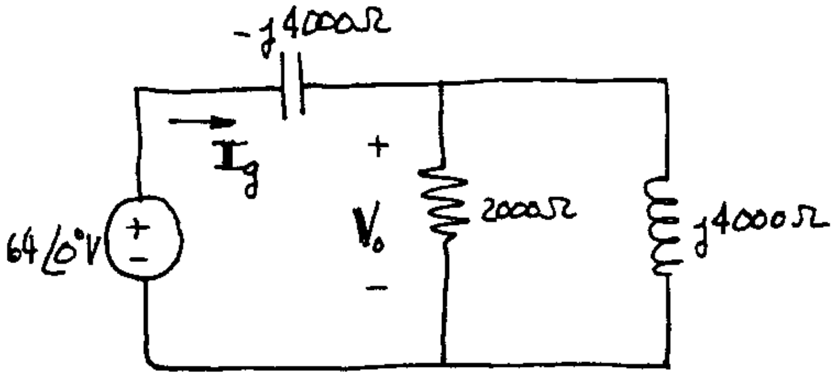


Ex:

$$\frac{1}{j\omega C} = \frac{10^9}{(31.25)(8000)} = -j4000\,\Omega$$

$$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$$

$$\mathbf{V}_g = 64\underline{/0^\circ}\text{ V}$$



$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\text{ mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}$$

$$v_o = 32\cos(8000t + 90^\circ)\text{ V}$$

Hw:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.