Measurement and Testing

EIE 240 Electrical and Electronic Measurements
Class 2: January 16, 2015

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Measurement

- **Measurement** is to determine the value or size of some quantity, e.g. a voltage or a current.
- **Analogue measurement** gives a response to a continuous quantity.
- **Digital measurement** is for the quantity at sampled times and quantized values.
- **Comparison measurement** is to compare the quantity with standards, e.g. null method.

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Testing

- **Testing** is to measure to ensure that a product conforms to its specification.
- **Manual testing** proceeded by human
- **Automatic testing** by a machine for reducing human error and increasing the performance.

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Measurement Standards

- **International standards**, for comparison with primary standards, are defined by international agreements and mentioned at *the International Bureau of Weights and Measures* in France.
- **Primary standards**, for checking the accuracy of secondary standards, are maintained at institutions in various countries around the world.
- **Secondary standards**, for verifying the accuracy of working standards, are employed in industry as references for calibrating high-accuracy equipment and component.
- **Working standards** are the principal tools of a measurement laboratory.

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Electrical Measurement Standards

- Electrical measurement standards are precise resistors, capacitors, inductors, voltage sources, and current sources, which can be used for comparison purposes when measuring electrical quantities.
- For example, the primary standard for resistance, the mercury ohm was initially defined in 1884 in as a column of mercury 106.3 cm long and 1 mm^2 in cross-section, at 0 °C.
- At the present time, Fluke 742 series working standard resistors are available in values ranging from 1 Ω - 19 MΩ used at ambient room temperatures (18-28 °C) with resistance changes ranging from ±1.5 (10 kΩ) to ± 4 (19 MΩ) ppm.

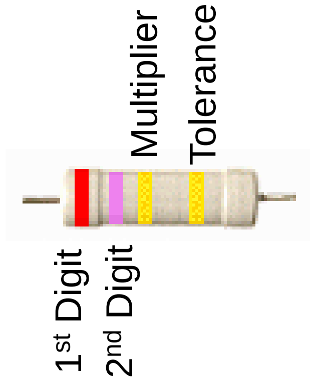
Error of a Measurement

- Errors are present in every experiment! If an experiment is well designed and carefully performed, the errors can be reduced to an acceptable level (their effects are not significant).
- Error = Measured Value – True Value
- Percentage Error = $\dfrac{\text{Error}}{\text{True Value}} \times 100\%$
- Degree of Uncertainty = ± %Error

Error of a Measurement (Cont'd)

e.g. Resistor codes

None		(±20%)
Silver	-2	(±10%)
Gold	-1	(±5%)
Black	0	
Brown	1	(±1%)
Red	2	(±2%)
Orange	3	
Yellow	4	
Green	5	(±0.5%)
Blue	6	(±0.25%)
Violet	7	(±0.1%)
Grey	8	(±0.05%)
White	9	



$27 \times 10^{-1}\ \Omega \pm 5\%$

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Random Error

- **Random error** is unpredictable for a successive reading of the same quantity.
- **Operating error** from the measurement situation leading to small variations.
- **Environmental error** such as a temperature or a humidity.
- **Stochastic error** e.g. electrical noise.

Electronic noise generated by the thermal agitation of the charge carriers inside an electrical conductor was first measured by John B. Johnson and explained by Harry Nyquist at Bell Labs in 1926.

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Systematic Error

- **Systematic error** remains constant with repeated measurements.
- **Construction error** from manufacture of an instruments
- **Calibration error** from an incorrect setting.
- **Approximation error** e.g. for a linear scales
- **Ageing error** for the old instrument.
- **Loading error** for inserting a quantity affecting its value.

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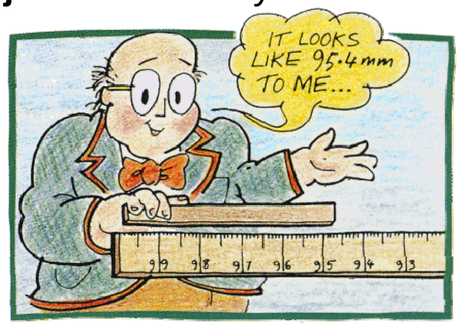
Human Error

- **Human error** or gross error is the mistake made by humans in using instruments and taking the readings.
- **Misreading** of the operator.
- **Calibration error** of the operator.
- **Incorrect instruments** chosen by the operator.
- **Incorrect adjustment** of any conditions.



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Accuracy

- Accuracy refers to how closely a measured value agrees with the correct value.

- Error = Measured Value – Expected Value

$$e = x_{measured} - x_{expected}$$

- Percent Error = (Error / Expected Value)×100

$$\%e = |\,(x_{measured} - x_{expected}) / x_{expected}\,| \times 100$$

- Accuracy = 100 – Percent Error

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Precision

- Precision refers to how closely individual measurements agree with each other.

- Deviation = Measured Value – Average Value

$$d = x_{measured} - x_{average}$$

- Percent Deviation = (Deviation / Average Value) ×100

$$\%d = |\,(x_{measured} - x_{average}) / x_{average}\,| \times 100$$

- Precision = 100 – Percent Deviation

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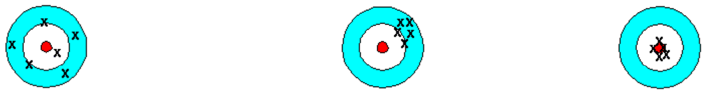
Accuracy Vs Precision

e.g. When a meter is said to be accurate to 1%, this means that a reading taken anywhere along one of its scale will not be in error by more than 1% of the full-scale value.



Accurate
(the average is accurate)
but not precise

Precise
but not accurate
(calibration needed)

Accurate
and precise

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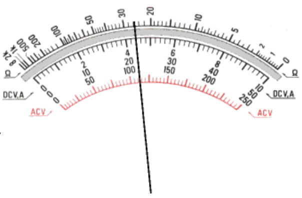
Resolution and Sensitivity

- Resolution is the significance of the least significant digits, e.g. the range of ammeter is 199 mA with a resolution of 0.1 mA. The range would be 000.0, 000.1, 000.2, …, 199.9 mA
  or 3½ meter (the most significant digits can only be either a 0 or 1.)



- Sensitivity = $\dfrac{\text{Change in the Output}}{\text{Change in the Input}}$

e.g. $\dfrac{\text{Change in instrument scale reading}}{\text{Change in the quantity being measured}}$

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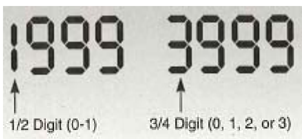
Reading Resolution of Digital Multimeter (DMM)

- For the fixed resolution of .001 V (step size)
  Range 0 - 1 V ➔ 3 digits (.999) 1,000 steps
  Range 0 - 10 V ➔ 4 digits (9.999) 10,000 steps
  Range 0 - 100 V ➔ 5 digits (99.999) 100,000 steps
  Number of digit = log(step)
- How many digit for the range 0 - 3 V ?
  It is 3½ digits (log(3000) ≈ 3.477 digits)

Reading Resolution of Digital Multimeter (DMM) (Cont'd)



- Typical for DMMs,
  3½ digit display 0000 ➔ 1999
  (e.g. full scale 2 V if enable the 1st decimal point, 0.000 ➔ 1.999)
- MSB can only be "0" or "1" (usually not visible when the reading is less than 999), whereas all the other can be "0", "1", "2", "3", …, "9"
- For 3¾ digit for the range 4 V, MSB can be "0" to "3"

Accuracy of DMM

- For example
  $\pm$ (0.5% Reading + 1 Digit LSB)
- when you read a voltage 1.8 V

$$\text{error} = \pm (0.5\% \text{ of } 1.8V \quad + \quad 0.001V)$$
$$= \pm\, 0.01V$$
$$\approx \pm\, 0.56\% \text{ of reading}$$

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Range and Bandwidth

- The range of an instrument refers to the minimum and maximum values of the input variable for which it has been designed. The range chosen should be such that the reading is large enough to give close to the required precision.
- The bandwidth of an instrument is the difference between the minimum and maximum frequencies for which it has been designed. If the signal is outside the bandwidth, it will not be able to follow changes in the quantity being measured.

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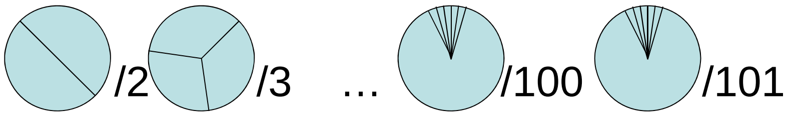
Statistical Evaluation

- Mean, $\overline{X} = \Sigma_{i=1\rightarrow N} X_i / N$ $\Rightarrow$ the best value
- Deviation = $X_i - \overline{X}$
- Mean Deviation = $\Sigma_{i=1\rightarrow N} |X_i - \overline{X}| / N$
- Standard Deviation,

  $\sigma = \sqrt{\Sigma_{i=1\rightarrow N} (X_i-\mu)^2 / N}$ for a population

  s.d. = $\sqrt{\Sigma_{i=1\rightarrow N} (X_i-\overline{X})^2 / N-1}$ for a sample (<30)

  e.g. a cake



- Variance, σ^2

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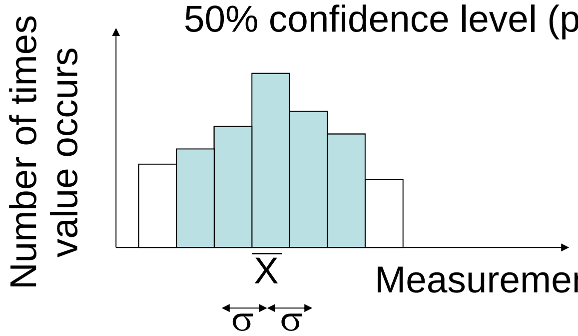
Normal Distribution (Gaussian)

Histogram

50% confidence level (probable error)



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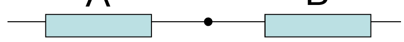
Summation of Error



Relative Error

$100\Omega \pm 10\%$ $\quad$ $200\Omega \pm 5\%$

$90\Omega \leftrightarrow 110\Omega\ (\pm 10\Omega)$ $\quad$ $190\Omega \leftrightarrow 210\Omega\ (\pm 10\Omega)$

Absolute Error

$$X = A + B$$

$$X \pm \Delta X = A \pm \Delta A + B \pm \Delta B$$

$$= (A+B) \pm (\Delta A + \Delta B)$$

$$= (100\Omega + 200\Omega) \pm (10+10)$$

$$= 300\Omega \pm 20$$

$280\Omega \leftrightarrow 320\Omega \quad \Rightarrow \quad$ Extreme!

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Summation of Error (Cont'd)

In case of summation,

$$X = A + B$$

$$X \pm \Delta X = A \pm \Delta A + B \pm \Delta B$$

$$= (A+B) \pm \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

$$= 300\Omega \pm 14.14$$

$285.86\Omega \leftrightarrow 314.14\Omega \quad \Rightarrow \quad$ Better

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Summation of Error (Cont'd)

In case of multiplying,

$$X = AB$$

$$X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \cancelto{0}{\Delta A\Delta B}$$

$$\Delta X = \pm A\Delta B \pm B\Delta A$$

$$\Delta X/X = (\pm A\Delta B \pm B\Delta A) / AB \,\% \quad \Rightarrow \quad \%\text{Error}$$

$$= \pm \Delta B/B \pm \Delta A/A \,\%$$

$$= \pm (\Delta B/B + \Delta A/A) \,\%$$

$$= \pm \sqrt{(\Delta B/B)^2 + (\Delta A/A)^2} \,\%$$

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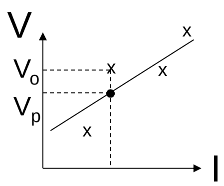
Linear Regression

$V = IR \quad \Rightarrow \quad$ Linear $y=mx+c$

Error $= V_p - V_o$

Minimum $\Sigma(V_p - V_o)^2$



Prediction Vs Observation
(Best Fitted?)

$$Y_i = \beta_0 + \beta_1 X_i + \cancelto{0}{\varepsilon_{random}} \quad , i=1,2,\ldots,N \text{ points}$$

$$\varepsilon = \Sigma_i \,[\, Y_i - (\beta_0 + \beta_1 X_i) \,]^2$$

$$\partial\varepsilon/\partial\beta_1 = -2\, \Sigma_i \,[\, Y_i - \beta_0 - \beta_1 X_i \,]\, X_i$$

$$= -2\, \Sigma_i \,[\, Y_i X_i - \beta_0 X_i - \beta_1 X_i^2 \,] = 0$$

$$\partial\varepsilon/\partial\beta_0 = -2\, \Sigma_i \,[\, Y_i - \beta_0 - \beta_1 X_i \,] = 0$$

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Linear Regression (Cont'd)

$$\Sigma_i [\, Y_iX_i - \beta_0X_i - \beta_1X_i^2 \,] = \Sigma_i [\, Y_i - \beta_0 - \beta_1X_i \,]$$

$$\Sigma_iY_iX_i - \beta_0\Sigma_iX_i - \beta_1\Sigma_iX_i^2 = \Sigma_iY_i - N\beta_0 - \beta_1\Sigma_iX_i$$

$$\Sigma_iY_iX_i - \Sigma_iY_i = \beta_0(\Sigma_iX_i - N) + \beta_1(\Sigma_iX_i^2 - \Sigma_iX_i)$$

$$(\Sigma_iY_iX_i)/N - \bar{Y} = (\bar{Y} - \beta_1\bar{X})(\bar{X} - 1) + \beta_1[\, (\Sigma_iX_i^2)/N - \bar{X} \,]$$

$$(\Sigma_iY_iX_i)/N - \cancel{\bar{Y}} = \bar{X}\bar{Y} - \cancel{\bar{Y}} + \beta_1[(\Sigma_iX_i^2)/N - \cancel{\bar{X}} - \bar{X}^2 + \cancel{\bar{X}}]$$

$$(\Sigma_iY_iX_i) = N\bar{X}\bar{Y} + \beta_1[\, \Sigma_iX_i^2 - N\bar{X}^2 \,]$$

$$\beta_1 = \frac{\Sigma_iY_iX_i - N\bar{X}\bar{Y}}{\Sigma_iX_i^2 - N\bar{X}^2} \quad \text{and } \beta_0 = \bar{Y} - \beta_1\bar{X}$$

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Linear Regression (Cont'd)

For nonlinear equation,

$Y = X^n \;\Rightarrow\; (\log Y) = n\,(\log X)$ → Logarithm

$Y = a^X \;\Rightarrow\; (\log Y) = (\log a)\,X$ → Semi-log

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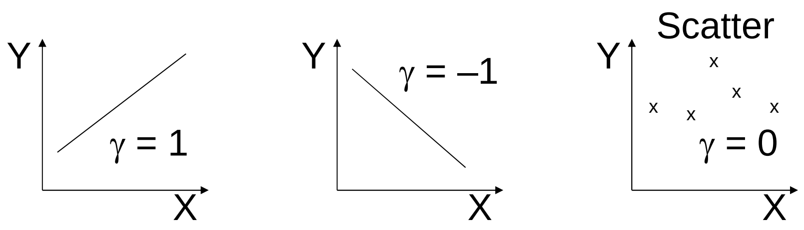
Correlation

$$\gamma = \frac{\text{Covariance}(X,Y)}{\sigma_X \sigma_Y} = \frac{1/N\ \Sigma(X - \overline{X})(Y - \overline{Y})}{\sigma_X \sigma_Y}$$



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Interesting Question

Why sample standard deviation divided by N-1, instead of N?

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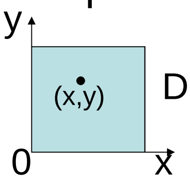
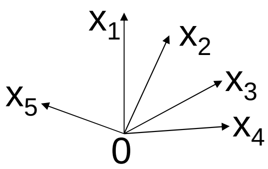
Degree of Freedom

Independent directions of movement



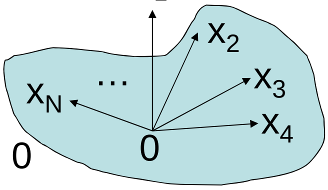
DOF = 2



DOF = 5

Fixed mean, $\bar{X}$ = sum / N $\Rightarrow$ Fixed sum

Fixed x_1 = sum



DOF = N-1

$X_2+X_3+X_4+\ldots+X_N = 0$

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