Seat No. [ ]

King Mongkut's University of Technology Thonburi

Midterm Examination, Semester 2/2011

Subject : STA 302 Statistics for Engineers   Department: Computer Engineering (English Program)

Date : 24 February 2012   Time 1.00-4.00 P.M.

Insturctions 1. There are 9 questions, 10 pages, (include this page) and 90 points

2. The books or documents are not allowed in the exam
3. The calculator is allowed in the exam
4. The statistical table is given and you have to send it back after the exam
5. Some formulae are given (2 pages)
6. Write down your ID on every sheet

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Name...ID....................Department........................

This exam paper has been considered by the board of department of mathematics.

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(Dr. Dusadee Sukawat)
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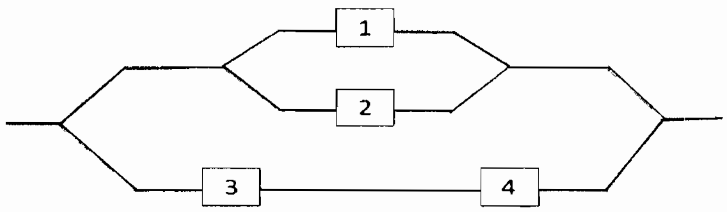
Department .... ID......................................

1. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and P(each component works)=0.9, calculate P(system works).



(10 points)

Department .... ID....................................

2. Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages.

a.) If a customer buys a film, what is the probability that its package does not show the expiration date? (8 points)

b.) If a customer complains that her package of film does not show that expiration date, what is the probability that it was inspected by John? (3 points)

Department .... ID......................................

3. Suppose the joint probability density function of (X,Y) is given by

$$f(x,y)=\begin{cases}\dfrac{6}{5}(x+y^2) & ,0\le x\le 1, 0\le y\le 1\\ 0 & ,otherwise\end{cases}$$

a.) find $P(Y\le 0.5 \mid X=0.8)$ (12 points)

Department .... ID......................................

b.) find $E(Y \mid X = 0.8)$. (6 points)

4. Given $E(X^2) = 16,\ E(X) = 3,\ E(Y^2) = 10,\ E(Y) = 2,\ E(XY) = 12$,

compute $Var\left[U(X,Y)\right]$ where $U(X,Y) = 3X - 2Y + 5$. (8 points)

Department .... ID.....................................

5. Suppose a couple decide to have children until they have two female children.

a.) What is the probability distribution of $X =$ the total number of children born to this couple? Assume that the probabilities to get male and female children are equal.

(4 points)

b.) What is the expected number of children that a couple has? (3 points)

6. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. What is the probability that the hotel get 2 defective television sets. (8 points)

Department .... ID.......................................

7. Assume that each of your calls to a popular radio station has a probability of 0.05 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent. What is the probability that your first call that connects is your sixth call? (8 points)

8. The distribution of resistance for resistors of a certain type is known to be normal with variance is 0.25. We found that 2.5% of all resistors having a resistance exceeding 10.256 ohms. What is the mean of the resistance distribution? (10 points)

Department .... ID......................................

9. Suppose only 75% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected. What is the probability that between 360 and 400 of the drivers in the sample regularly wear a seat belt? (10 points)

Department .... ID.......................................

Formula

Conditional Probability

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}\ ;\ P(B)>0$$

Law of Total Probability

$$P(A)=\sum_{i=1}^{k}P(B_i\cap A)=\sum_{i=1}^{k}P(B_i)\cdot P(A\mid B_i)$$

Bayes' Rule

$$P(B_r\mid A)=\frac{P(B_r\cap A)}{\sum_{i=1}^{k}P(B_i\cap A)}=\frac{P(B_r)P(A\mid B_r)}{\sum_{i=1}^{k}P(B_i)P(A\mid B_i)}$$

The cumulative distribution function

$$F(x)=P(X\le x)=\sum_{t\le x}f(t) \qquad \text{D.R.V}$$

$$F(x)=P(X\le x)=\int_{-\infty}^{x}f(t)dt \qquad \text{C.R.V}$$

The marginal distributions of X

$$g(x)=\sum_{for\ all\ y}f(x,y) \qquad \text{D.R.V}$$

$$g(x)=\int_{-\infty}^{\infty}f(x,y)dy \qquad \text{C.R.V}$$

The conditional distribution of X, given that Y = y

$$f(x\mid y)=\frac{f(x,y)}{h(y)},\quad h(y)>0.$$

Expectation

$$\mu=E(X)=\sum_{for\ all\ x}xf(x) \qquad \text{D.R.V}$$

$$\mu=E(X)=\int_{-\infty}^{\infty}xf(x)dx \qquad \text{C.R.V}$$

$$\mu_{U(X,Y)}=E[U(X,Y)]=\sum_{for\ all\ x}\sum_{for\ all\ y}U(x,y)f(x,y) \qquad \text{D.R.V}$$

$$\mu_{U(X,Y)}=E[U(X,Y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y)f(x,y)\,dxdy \qquad \text{C.R.V}$$

The conditional expectation of X given that Y = y

$$\mu_{X|Y}=E(X\mid Y=y)=\begin{cases}\sum_{for\ all\ x}x\,f(x\mid y) & \text{if } X \text{ and } Y \text{ are discrete}\\ \int_{-\infty}^{\infty}x\,f(x\mid y)\,dx & \text{if } X \text{ and } Y \text{ are continuous}\end{cases}$$

Variance

$$Var(X)=\sigma^2=E\left[(X-\mu)^2\right]=\sum_{for\ all\ x}(x-\mu)^2f(x) \qquad \text{D.R.V}$$

$$Var(X)=\sigma^2=E\left[(X-\mu)^2\right]=\int_{-\infty}^{\infty}(x-\mu)^2f(x)dx \qquad \text{C.R.V}$$

$$\sigma^2=E(X^2)-[E(X)]^2$$

Department .... ID.....................................

Covariance

$$\sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = \sum_{\text{for all } x}\sum_{\text{for all } y}(x-\mu_X)(y-\mu_Y)f(x,y) \quad \text{D.R.V}$$

$$\sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_X)(y-\mu_Y)f(x,y)\,dxdy \quad \text{C.R.V}$$

$$\sigma_{XY} = E(XY) - E(X)\cdot E(Y)$$

Correlation

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}$$

Discrete Uniform Distribution

$$f(x;k) = \frac{1}{k} \; ; \quad x = x_1, x_2, \ldots, x_k .$$

$$\mu = \sum_{i=1}^{k}\frac{x_i}{k}, \quad \sigma^2 = \sum_{i=1}^{k}\frac{(x_i-\mu)^2}{k}.$$

Binomial Distribution

$$b(x;n,p) = \binom{n}{x}p^x q^{n-x} \; ; \qquad x = 0,1,2,\ldots,n$$

$$\mu = np, \qquad \sigma^2 = npq$$

Hypergeometric Distribution

$$h(x;N,n,k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \; ; \qquad x = 0,1,2,\ldots,n$$

$$\mu = \frac{nk}{N}, \qquad \sigma^2 = \frac{N-n}{N-1}\cdot n\cdot\frac{k}{N}\left(1-\frac{k}{N}\right)$$

Negative Binomial Distribution

$$b^*(x;k,p) = \binom{x-1}{k-1}p^k q^{x-k} \; ; \qquad x = k, k+1, k+2, \ldots$$

$$\mu = \frac{k}{p}, \qquad \sigma^2 = \frac{kq}{p^2}$$

Geometric Distribution

$$g(x;p) = pq^{x-1} \; ; \qquad x = 1,2,3,\ldots$$

$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{q}{p^2}$$

Poisson Distribution

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \; ; \qquad x = 0,1,2,\ldots$$

$$\mu = \lambda t, \qquad \sigma^2 = \lambda t$$