Capacitance Measurement

EIE 240 Electrical and Electronic Measurement
Class 10, March 5, 2012

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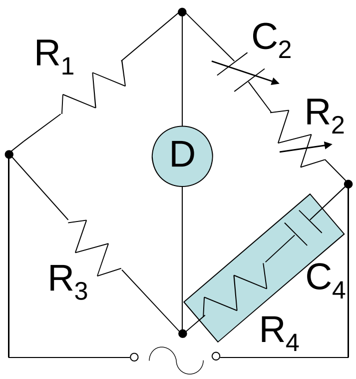
Series Capacitance Bridge

Capacitor can be considered to be a pure capacitance in series with, or sometimes in parallel with, a pure resistance.



Impedances,

$Z_1 = R_1$
$Z_2 = R_2 - j/\omega C_2$
$Z_3 = R_3$
$Z_4 = R_4 - j/\omega C_4$

Series Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 - j/\omega C_4 = (R_2 - j/\omega C_2)\, R_3 / R_1$$

$$= R_2 R_3/R_1 - j(R_3/\omega C_2 R_1)$$

Real part: $R_4 = R_2 R_3/R_1$

Imagination part: $C_4 = C_2 R_1/R_3$

Used for low D = 0.001-0.1

$D = 1/Q = \omega R_4 C_4 = \omega R_2 C_2$

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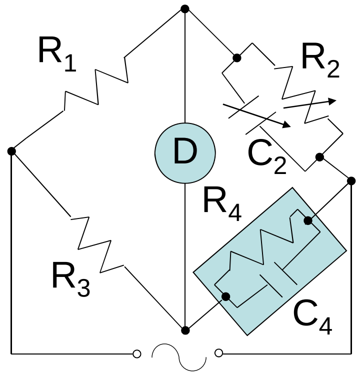
Parallel Capacitance Bridge

Used for D = 0.05-50



Impedances,

$Z_1 = R_1$

$Z_2 = 1 / (1/R_2 + j\omega C_2)$

$= R_2/(1+j\omega C_2 R_2)$

$Z_3 = R_3$

$Z_4 = R_4/(1+j\omega C_4 R_4)$

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Parallel Capacitance Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$

$$R_4 / (1+j\omega C_4R_4) = R_2R_3 / (1+j\omega C_2R_2)R_1$$

$$R_1R_4 + j\omega C_2R_1R_2R_4 = R_2R_3 + j\omega C_4R_2R_3R_4$$

Real part: $R_1R_4 = R_2R_3$

$$R_4 = R_2R_3/R_1$$

Imagination part: $C_2R_1R_2R_4 = C_4\,R_2R_3R_4$

$$C_4 = C_2R_1/R_3$$

$D = \omega R_4C_4 = \omega R_2C_2$

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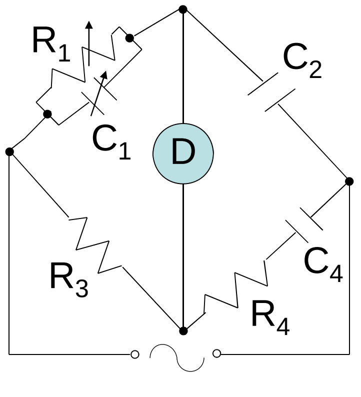
Schering Bridge

Used for very low D



Impedances,

$Z_1 = R_1/(1+j\omega C_1R_1)$

$Z_2 = 1/j\omega C_2$

$Z_3 = R_3$

$Z_4 = R_4 - j/\omega C_4$

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Schering Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2Z_3 / Z_1$$
$$R_4 - j/\omega C_4 = R_3(1+j\omega C_1R_1) / j\omega C_2R_1$$
$$= (\omega C_1R_1R_3 - jR_3) / \omega C_2R_1$$
$$= R_3C_1/C_2 - j(R_3/\omega R_1C_2)$$

Real part: $R_4 = R_3C_1/C_2$
Imagination part: $1/C_4 = R_3/R_1C_2$
$$C_4 = C_2R_1/R_3$$

$D = \omega R_4C_4 = \omega R_1C_1$

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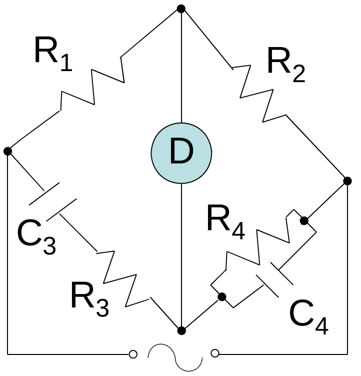
Wien Bridge

Used as frequency-dependent circuit



Impedances,
$Z_1 = R_1$
$Z_2 = R_2$
$Z_3 = R_3 - j/\omega C_3$
$Z_4 = R_4/(1+j\omega C_4R_4)$

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Wien Bridge (Cont'd)

Balanced bridge,

$$Z_4 = Z_2 Z_3 / Z_1$$

$$R_4 / (1+j\omega C_4 R_4) = R_2(R_3 - j/\omega C_3) / R_1$$

$$R_1 R_4 / R_2 = R_3 + R_4 C_4 / C_3 + j(\omega C_4 R_3 R_4 - 1/\omega C_3)$$

Imagination part: $\omega C_4 R_3 R_4 = 1/\omega C_3$

$$C_4 R_4 = 1/\omega^2 C_3 R_3$$

Real part: $R_1 R_4 / R_2 = R_3 + R_4 C_4 / C_3$

$$R_4 = (R_3 R_2 C_3 + R_2 R_4 C_4) / R_1 C_3$$

$$= (R_3 R_2 C_3 + R_2 / \omega^2 C_3 R_3) / R_1 C_3$$

$$= R_2(\omega^2 C_3^2 R_3^2 + 1) / (\omega^2 C_3^2 R_1 R_3)$$

and $C_4 = 1/\omega^2 C_3 R_3 R_4$

$$= C_3 R_1 / R_2(\omega^2 C_3^2 R_3^2 + 1)$$

$D = \omega R_4 C_4 = 1/\omega R_3 C_3$

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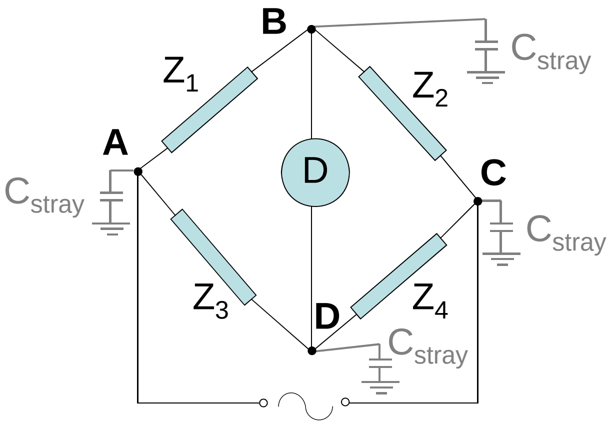
Stray Impedance

There are stray capacitances between the various element and the ground and it mat affect bridge balance.



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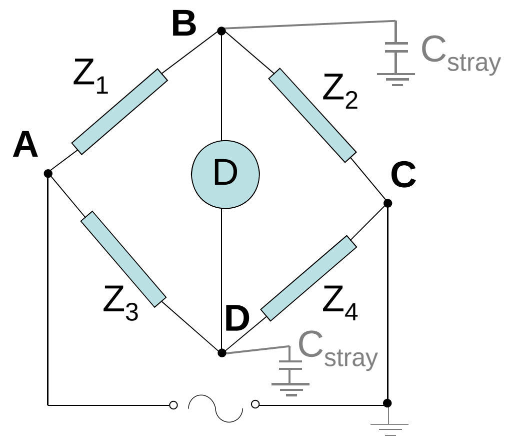
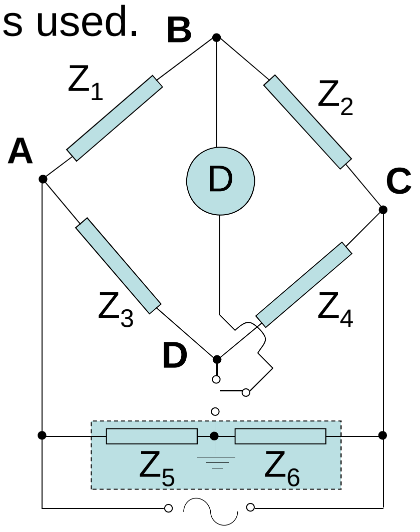
Stray Impedance (Cont'd)

The stray capacitances can be reduced by earthing one side of AC supply.



Stray Impedance (Cont'd)

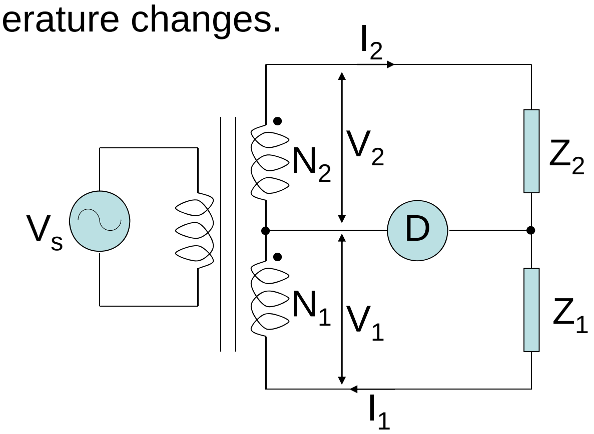
The minimize stray capacitances between the detector terminals and earth, Wagner earth is used.



To ensuring that the points B and D of a balanced bridge are at ground potential

Transformer Ratio Bridges

Not only varying the impedances, but bridge can be also balanced by varying the turns ratio of a transformer. There is a small number of standard resistors and capacitors and no effect of temperature changes.



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Single Ratio Transformer Bridge

Tap a transformer $\Rightarrow$ voltage divider of V_s

$V_1 = kN_1 = I_1Z_1 \Rightarrow I_1 = kN_1 / Z_1$

$V_2 = kN_2 = I_2Z_2 \Rightarrow I_2 = kN_2 / Z_2$

To balance the bridge or no current through the detector, D = Null

$I_1 = I_2 \quad \Rightarrow \quad Z_1 / Z_2 = N_1 / N_2$

Impedance Ratio   Turn Ratio

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Single Ratio Transformer Bridge (Cont'd)

- Resistance Measurement

  Z_1 = Unknown resistor R_x

  Z_2 = Standard resistor R_s

$$R_x = R_s \frac{N_1}{N_2}$$

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Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement

  Z_1 = Unknown $C_x \parallel R_x$ (leakage resistance)

  $= 1/(1/R_x + j\omega C_x)$

  $= R_x / (1+j\omega R_x C_x)$

  Z_2 = Standard $C_s \parallel R_s$

  $= R_s / (1+j\omega R_s C_s)$

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Single Ratio Transformer Bridge (Cont'd)

- Capacitance Measurement (Cont'd)

  Balanced, $Z_1 / Z_2 = N_1 / N_2$

$$1/Z_1 = (N_2/N_1)\ 1/Z_2$$

$$(1+j\omega R_xC_x)/R_x = (N_2/N_1)\ (1+j\omega R_sC_s)/R_s$$

$$1/R_x + j\omega C_x = (N_2 / N_1R_s) + j\omega C_sN_2/N_1$$

Real part: $R_x = R_sN_1/N_2$

Imagination part: $C_x = C_sN_2/N_1$

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Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement

  Z_1 = Unknown $L_x \parallel R_x$

  $= 1/(1/R_x + 1/j\omega L_x)$

  $= 1/(1/R_x - j/\omega L_x)$

  Z_2 = Standard $C_s \parallel R_s$

  $= 1/(1/R_s + j\omega C_s)$

  $= 1/(1/R_s - j\omega C_s)$, Reversed Current

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Single Ratio Transformer Bridge (Cont'd)

- Inductance Measurement (Cont'd)

  Balanced, $1/Z_1 = (N_2/N_1)\ 1/Z_2$

  $$1/R_x - j/\omega L_x = (N_2/N_1)\ (1/R_s - j\omega C_s)$$

Real part: $R_x = R_s N_1/N_2$

Imagination part: $1/\omega L_x = (N_2/N_1)\omega C_s$

$$L_x = N_1 / N_2\omega^2 C_s$$

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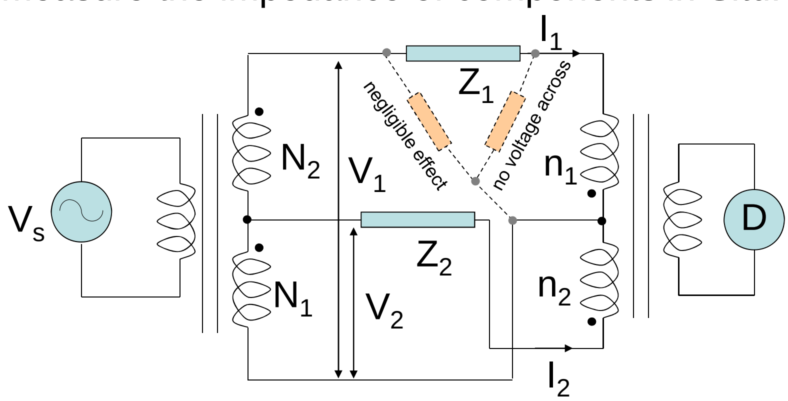
Double Ratio Transformer Bridge

To measure the impedance of components *in Situ.*



$I_1 = V_1 / Z_1 = k(N_1+N_2)/Z_1$

$I_2 = V_2 / Z_2 = kN_2/Z_2$

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Double Ratio Transformer Bridge (Cont'd)

Balanced, null current or zero magnetic flux,

$$n_1 I_1 = n_2 I_2$$

$$n_1(N_1+N_2)/Z_1 = n_2 N_2/Z_2$$

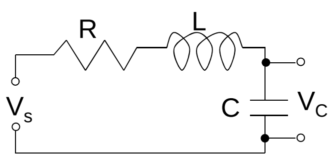
$$Z_1 = Z_2 \frac{n_1(N_1+N_2)}{n_2 N_2}$$

Q-Meter

RLC Series Resonance

$Z = R + j\omega L + 1/j\omega C$

$\quad = R + j(\omega L - 1/\omega C)$

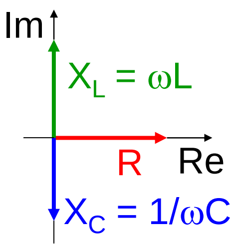


Resonant frequency

$\omega_0 L = 1/\omega_0 C$

$\omega_0^2 = 1/LC$

$\omega_0 = 1/\sqrt{LC}$



Q-Meter (Cont'd)

$$I_0 = V_s/R$$

$$\begin{aligned} V_C &= I_0 X_C \\ &= (V_s/R)(1/\omega_0 C) \\ &= (1/\omega_0 RC)\, V_s \\ &= Q\, V_s \\ &\propto Q \end{aligned}$$

where Q = Reactance/Resistance

$$\begin{aligned} &= \omega_0 L / R \\ &= \omega_0 (1/\omega_0^2 C) / R \\ &= 1 / \omega_0 RC \end{aligned}$$

(Unloaded)

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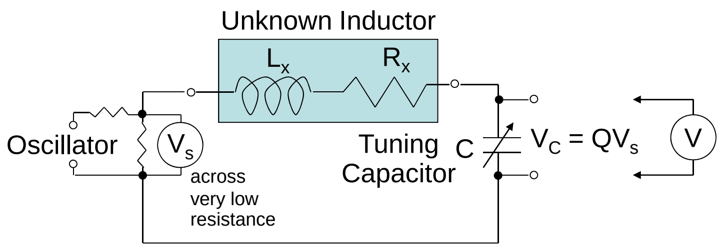
Q-Meter (Cont'd)



Tuning to resonance, $\omega_0 = 1/\sqrt{LC}$

$$L_x = 1 / \omega_0^2 C$$

$$R_x = 1 / Q\omega_0 C$$

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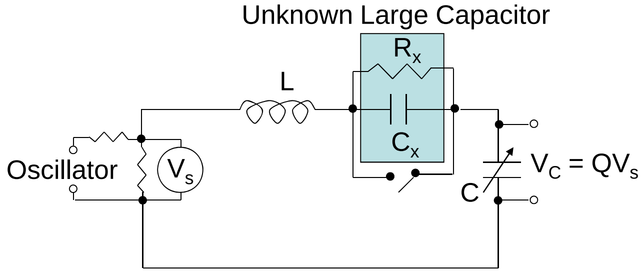
Q-Meter: Low Impedance Measurement



Short circuit ⇒ tuning $C = C_1$, $L_1 = L$, R_1

$Q_1 = 1/\omega_0 R_1 C_1$

$R_1 = 1/\omega_0 C_1 Q_1$

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Q-Meter: Low Impedance Measurement (Cont'd)

Remove short circuit ⇒ then tuning $C = C_2$

C_2 & $C_x = C_1$, $L_2 = L$, $R_2 = R_1$ & R_x

Series capacitors, $C_1 = 1 / (1/C_x + 1/C_2)$

$= C_2C_x / (C_x+C_2)$

$C_1C_x + C_1C_2 = C_2C_x$

$C_x = C_1C_2 / (C_2 – C_1)$

$Q_2 = 1/\omega_0 R_2 C_2$

$R_2 = 1/\omega_0 C_2 Q_2$

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Q-Meter: Low Impedance Measurement (Cont'd)

$R_1 = R_2 - R_x$

$R_x = R_2 - R_1$, Leakage resistance

$= 1/\omega_0 C_2 Q_2 - 1/\omega_0 C_1 Q_1$

$= (C_1 Q_1 - C_2 Q_2) / (\omega_0 C_1 C_2 Q_1 Q_2)$

$Q_x = 1/\omega_0 R_x C_x$

$= (\omega_0 C_1 C_2 Q_1 Q_2)(C_2 - C_1) /$

$\omega_0 (C_1 Q_1 - C_2 Q_2) C_1 C_2$

$= Q_1 Q_2 (C_2 - C_1) / (C_1 Q_1 - C_2 Q_2)$

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Q-Meter: Low Impedance Measurement (Cont'd)

If the unknown component is an inductor,

$L_x = 1/\omega_0^2 C_x$

$= (C_2 - C_1) / \omega_0^2 C_1 C_2$

If the unknown component is a pure resistor (no reactance),

$R_x = (C_1 Q_1 - C_2 Q_2) / (\omega_0 C_1 C_2 Q_1 Q_2)$

$= (Q_1 - Q_2) / \omega_0 C_1 Q_1 Q_2$, $C_1 = C_2$

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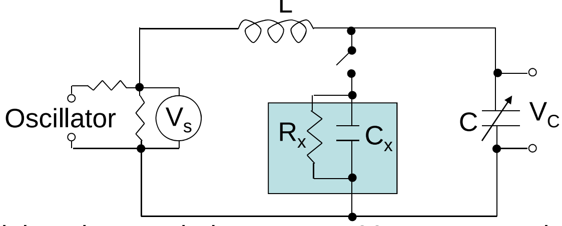
Q-Meter: High Impedance Measurement



For high resistance, inductance > 100mH, or capacitance < 400 pF

Open circuit and tune $C = C_1$, $L_1 = L$, R_1

Then short circuit and tune $C = C_2$

$$C_2 \,//\, C_x = C_1,\ L_2 = L,\ R_2 = R_x \,//\, R_1$$

$C_x + C_2 = C_1 \Rightarrow C_x = C_1 - C_2$

$R_2 = R_x || R_1 = R_x R_1/(R_x+R_1) \Rightarrow R_x = R_1 R_2/(R_1 - R_2)$

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Q-Meter: High Impedance Measurement (Cont'd)

Parallel RLC $\Rightarrow Q = \omega_0 RC$ (loaded)

$R_1 = Q_1/\omega_0 C_1$ and $R_2 = Q_2/\omega_0 C_2$

Therefore

$$R_x = Q_1 Q_2 \omega_0 C_1 C_2 / \omega_0^2 C_1 C_2 (Q_1 C_2 - Q_2 C_1)$$
$$= Q_1 Q_2 / \omega_0 (Q_1 C_2 - Q_2 C_1)$$
$$Q_x = \omega_0 R_x C_x$$
$$= \omega_0 Q_1 Q_2 (C_1 - C_2) / \omega_0 (Q_1 C_2 - Q_2 C_1)$$
$$= Q_1 Q_2 (C_1 - C_2) / (Q_1 C_2 - Q_2 C_1)$$

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Q-Meter: High Impedance Measurement (Cont'd)

For unknown inductance,

$$L_x = 1/\omega_0^2 C_x$$
$$= 1/\omega_0^2(C_1 – C_2)$$

For pure resistance,

$$R_x = Q_1Q_2 / \omega_0(Q_1C_2 – Q_2C_1)$$
$$= Q_1Q_2 / \omega_0C_1(Q_1 – Q_2) \qquad , C_1 = C_2$$

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