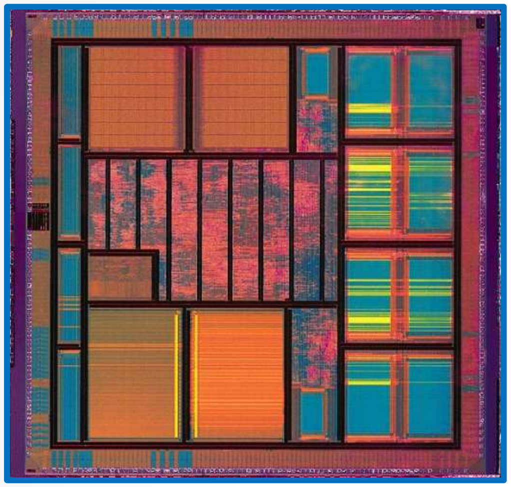
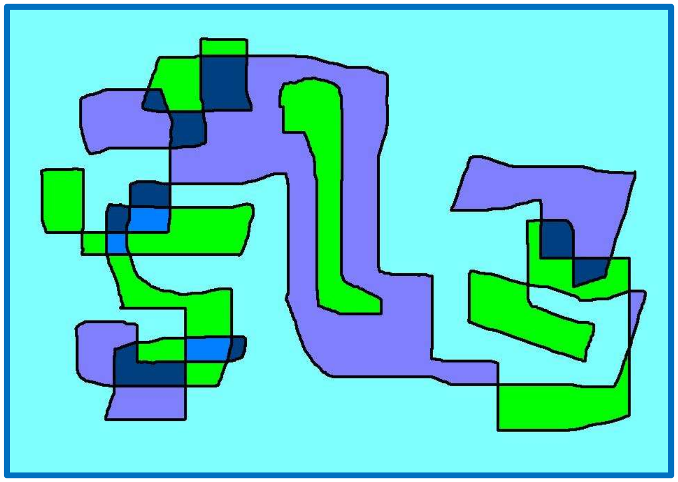
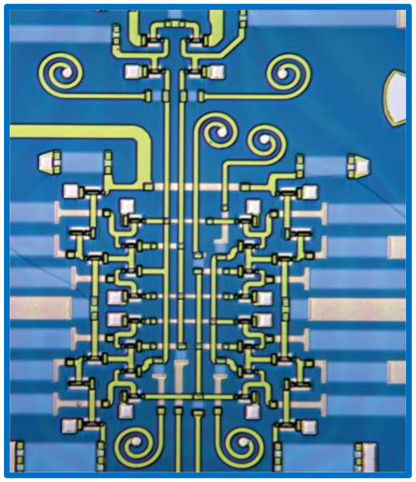






ENE/EIE 211: Electronic Devices and Circuit Design II

Lecture 7: Filters

Outline of topics

- Introduction
- Filter Transmission, Types and Specification
- Filter Transfer Function
- Bessel, Butterworth and Chebyshev Filters
- First-Order and Second-Order Filter Functions
  - Passive Realization
  - Active Realization (Op-amp RC Realization)
  - LCR Resonators
  - Second-Order Active Filters Based on Inductor Replacement
    - Antoniou Induction Simulation Circuits

Introduction

Filter is an important building block of communications and instrumentation systems. Filter design is one of the very few areas of engineering for which a complete design theory exists, starting from specification and ending with a circuit realization.

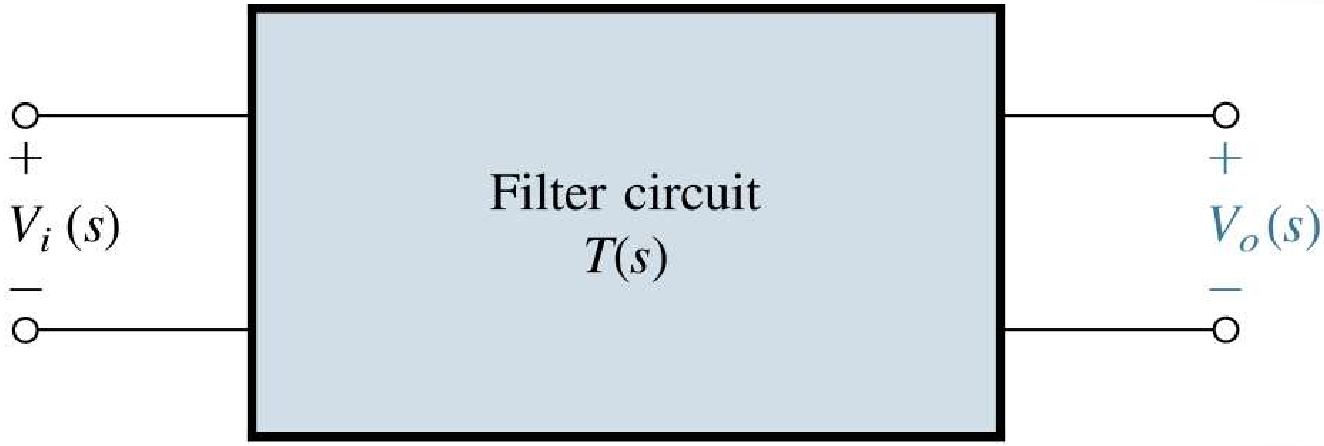
Passive LC filters make use of inductors and capacitors. They work well at high freq. However, in low freq, the required inductors are large and physically bulky and their characteristics are nonideal. So, people resort to inductorless filters such as the active RC filters and switched capacitor filters.

Active RC filters utilize op amps together with resistors and capacitors and are fabricated using discrete, hybrid thick-film, or hybrid thin-film technology. However, for large-volume production, such technologies do not yield the economies achieved by monolithic (IC) fabrication.

At present time, the most viable approach for realizing fully integrated monolithic filters is the switched-capacitor technique.

Filter Transmission & Types

Filter, a two port device



Transfer function T(s)

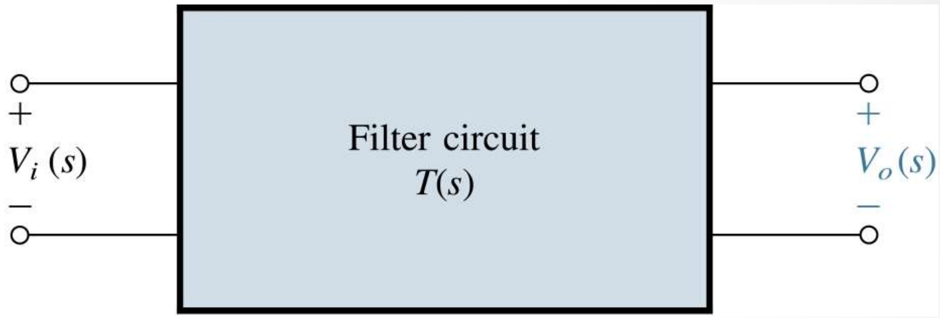
$$T(s) = V_0(s) / V_i(s)$$

Filter Transmission, Types and Specification

Linear Filters

Transfer Function

$$T(s) = \frac{V_o(s)}{V_i(s)}$$



The Filter Transmisson found by evaluating T(s) for physical frequencie:

$$s = j\cdot\omega \qquad T(j\omega) = |T(j\omega)|\cdot e^{j\phi(\omega)}$$

Gain Function

$$G(\omega) = 20\cdot\log(|T(j\omega)|) \qquad \text{dB}$$

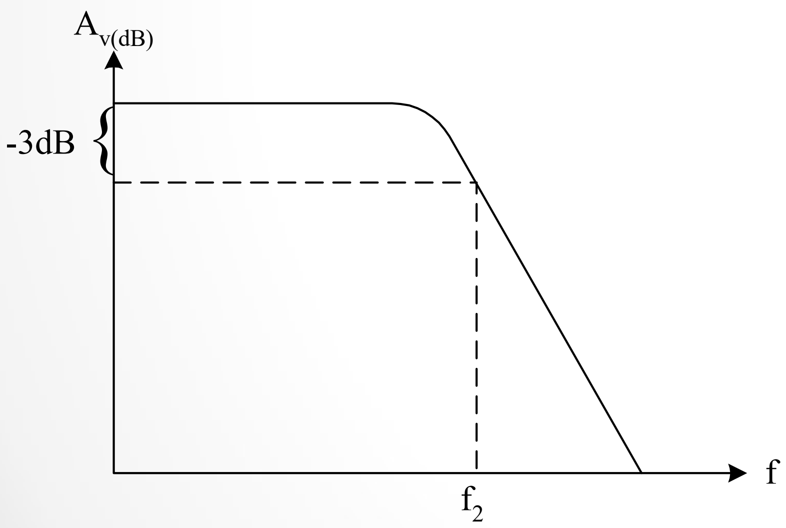
Attenuation Function

$$A(\omega) = -20\cdot\log(|T(j\omega)|) \qquad \text{dB}$$

Categories of Filters

***Low Pass Filters*:**

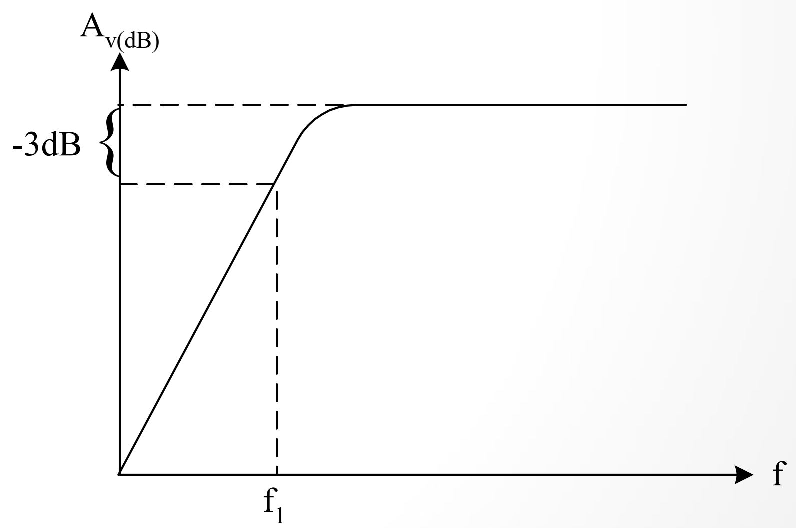
pass all frequencies from dc up to the upper cutoff frequency.



Low-pass response

***High Pass Filters*:**

pass all frequencies that are above its lower cutoff frequency

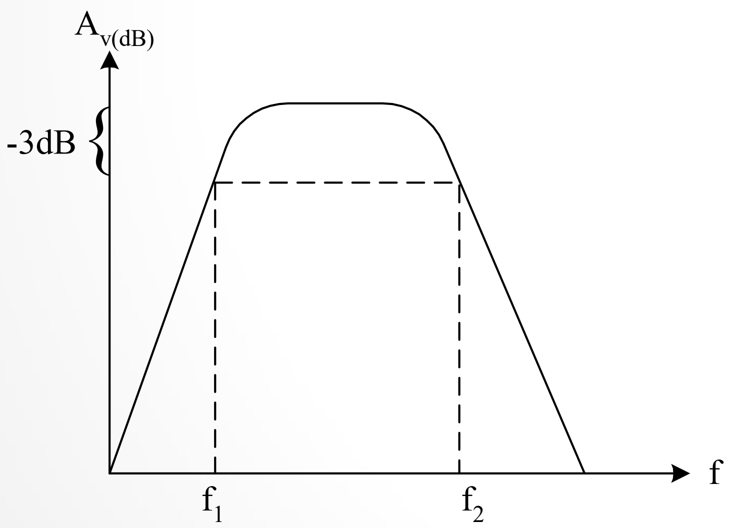


High-pass response

Categories of Filters

Band Pass Filters:

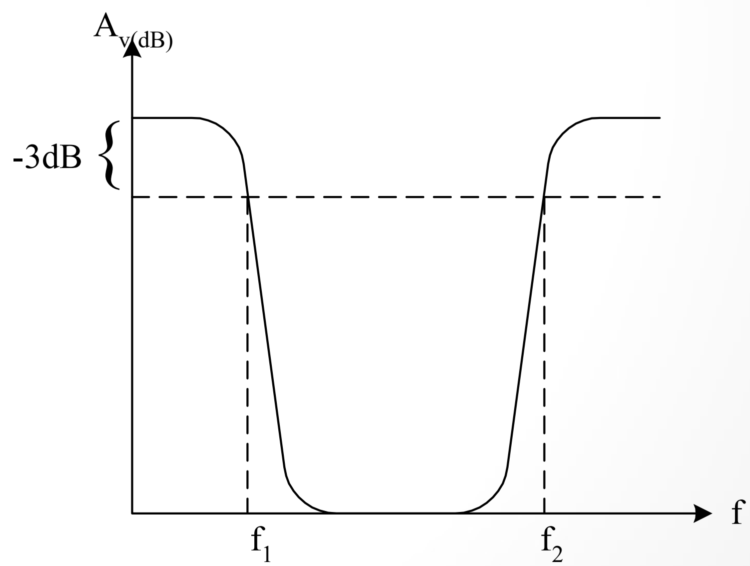
pass only the frequencies that fall between its values of the lower and upper cutoff frequencies.



Band Pass Response

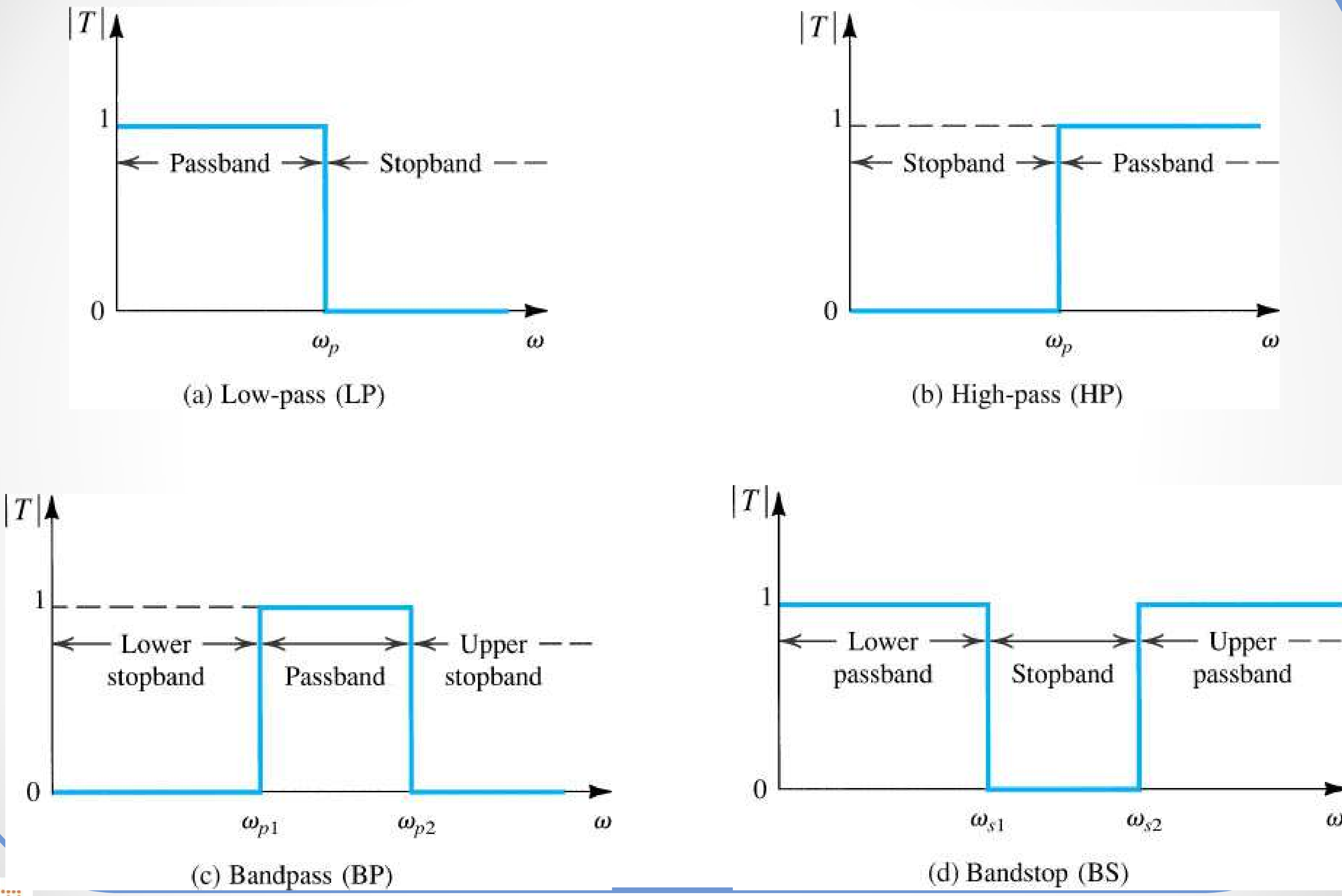
Band Stop (Notch) Filters:

eliminate all signals within the stop band while passing all frequencies outside this band.



Band Stop Response

Filter Specification

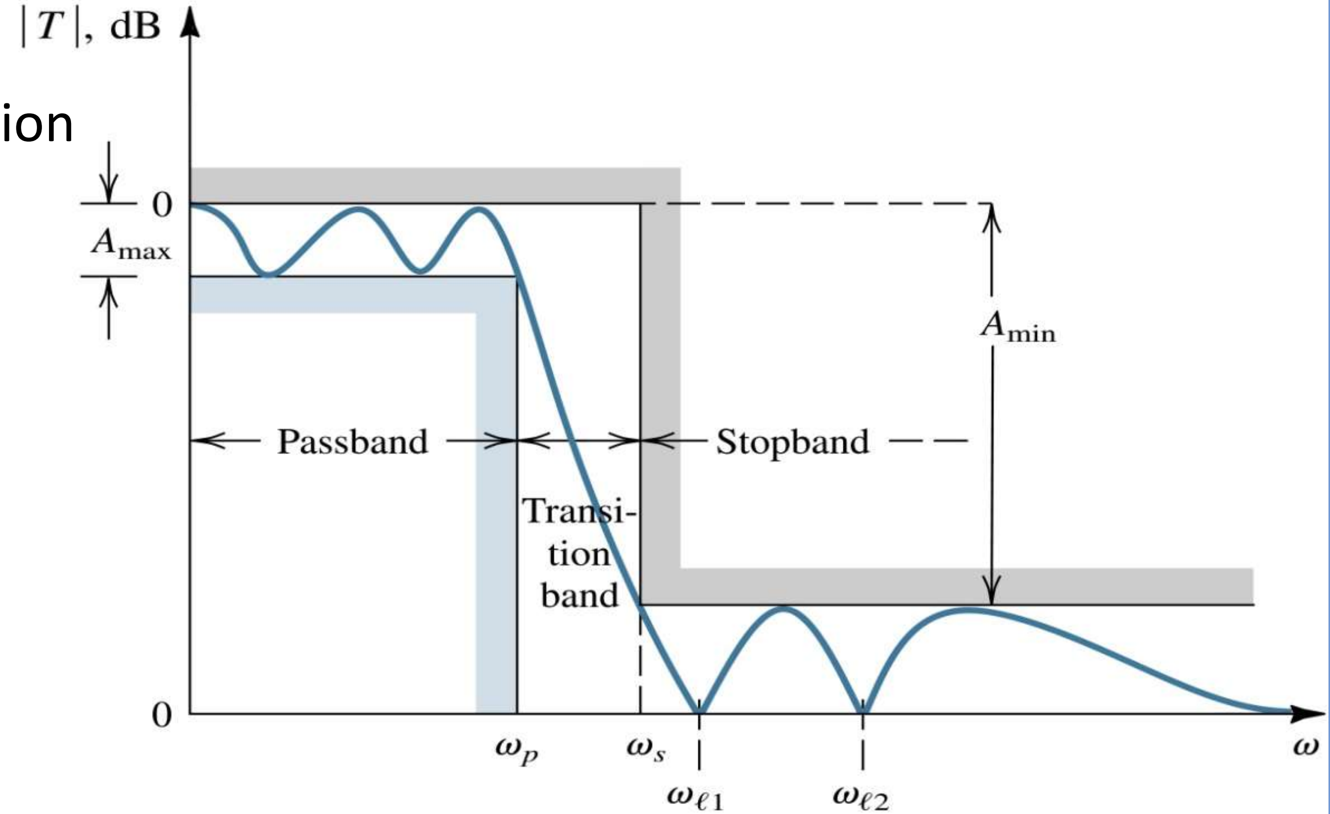


Filter Specification

Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

Frequency-Selection function

- Passing
- Stopping
- Pass-Band
- Low-Pass
- High-Pass
- Band-Pass
- Band-Stop
- Band-Reject



Passband ripple (A_{max})
Ripple bandwidth (ω_p)

Summary – Low-pass specs

-the passband edge, ω_p
-the maximum allowed variation in passband, Amax
-the stopband edge, ω_s
-the minimum required stopband attenuation, Amin

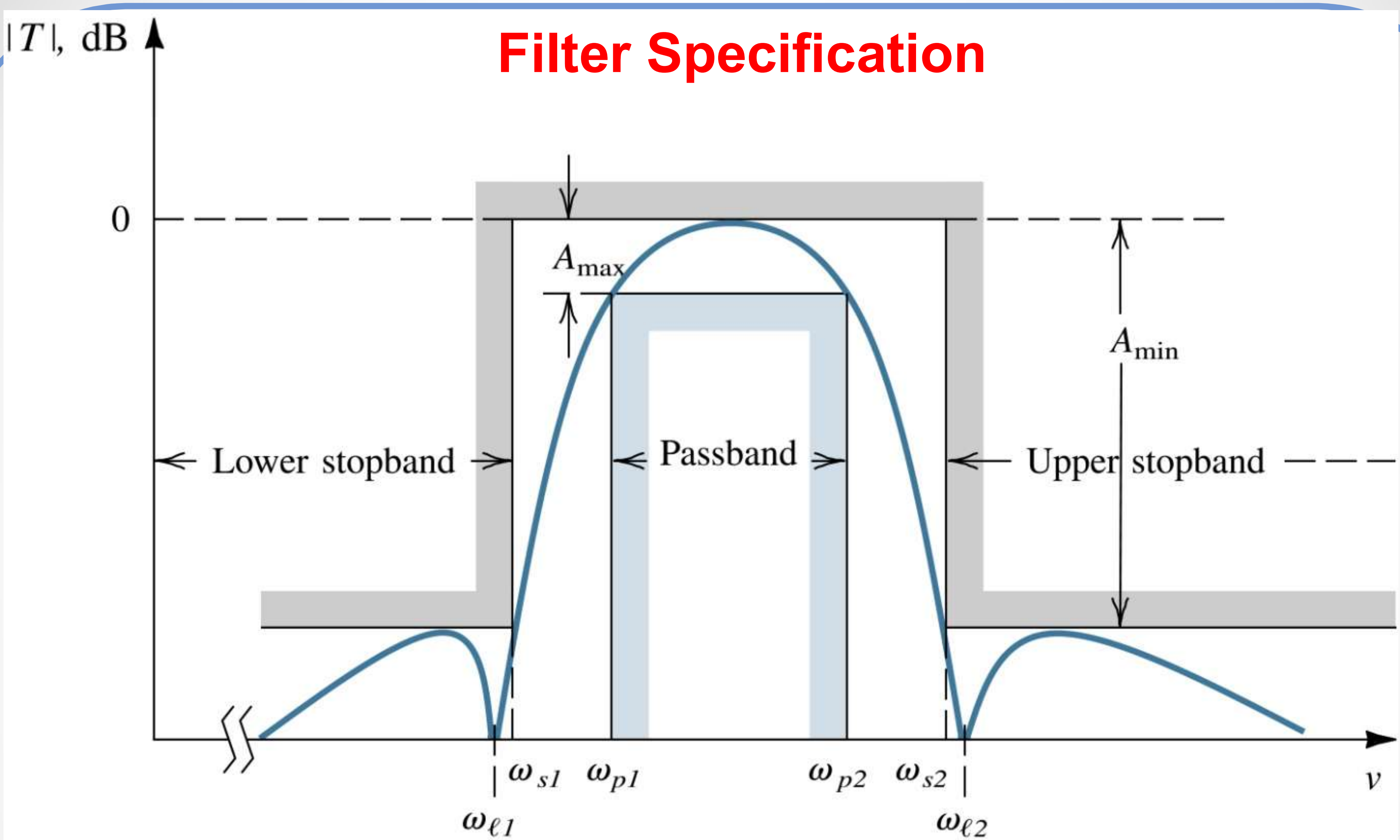
Filter Specification

The **selectivity factor** is a measure of sharpness of the low-pass filter response and equals to ω_s / ω_p . The **transition band** extends from the passband edge (**ωp)** to the stopband edge (**ωs)** .

Also, the more tightly one specifies a filter---that is, lower Amax, higher Amin, and/or selectivity ratio closer to unity---the closer the response of the resulting filter will be to the ideal. However, the resulting filter circuit must be of higher order and thus more complex and expensive.

Next step is to find a **transfer function** whose magnitude meets the specification. To meet specification, the magnitude-response curve must lie in the unshaded area. In the example, the filter response shows ripples in the passband and the stopband with ripple peaks all equal. This response is said to be **equiripple**.

Filter Specification

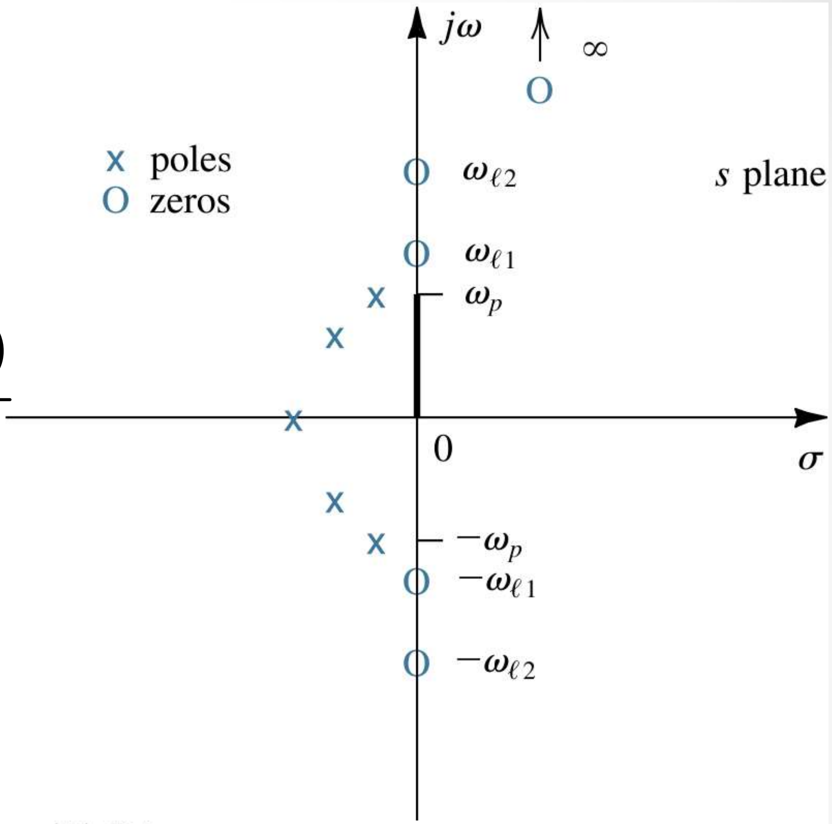


Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

The Filter Transfer Function

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0}$$

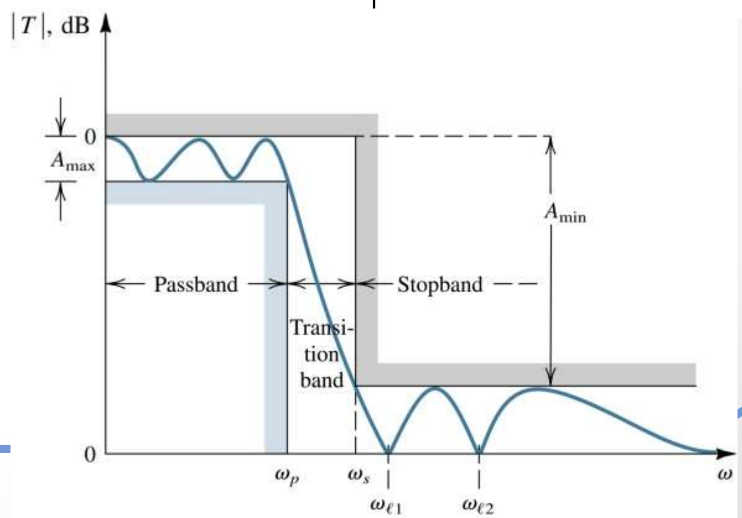
$$T(s) = \frac{a_M (s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$



The degree of the denominator (N) is the filter order

For filter to be stable, $\boxed{M \leq N}$

The numerator roots, z_1, z_2, … z_m are the transfer function zeros or transmission zeros; and the denominator roots, p_1, p_2, p_3, ….p_n are the transfer-function poles or the natural modes.

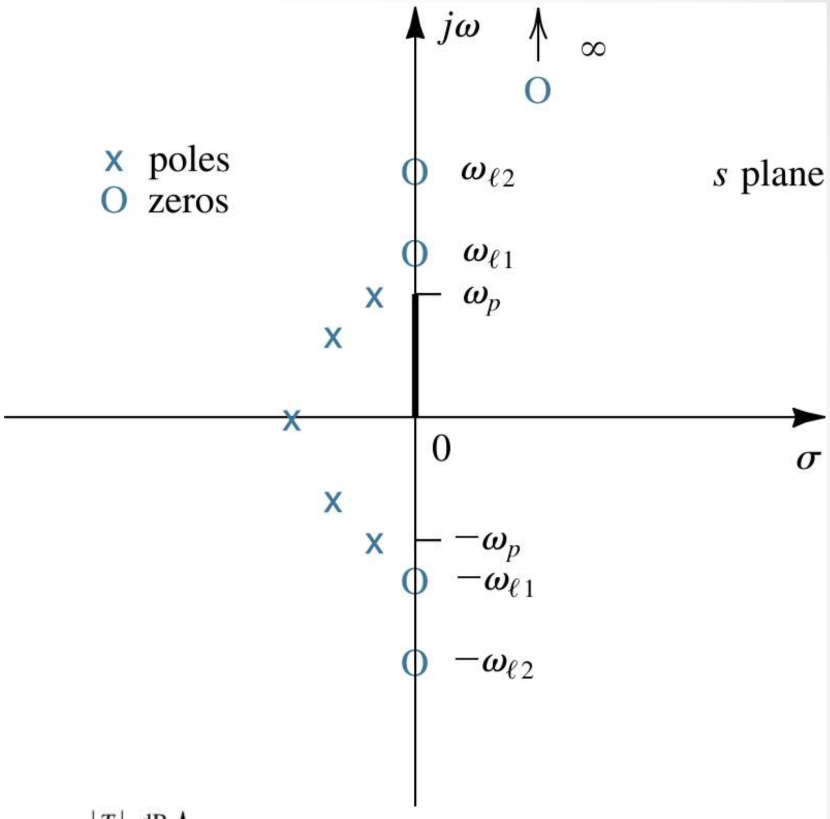


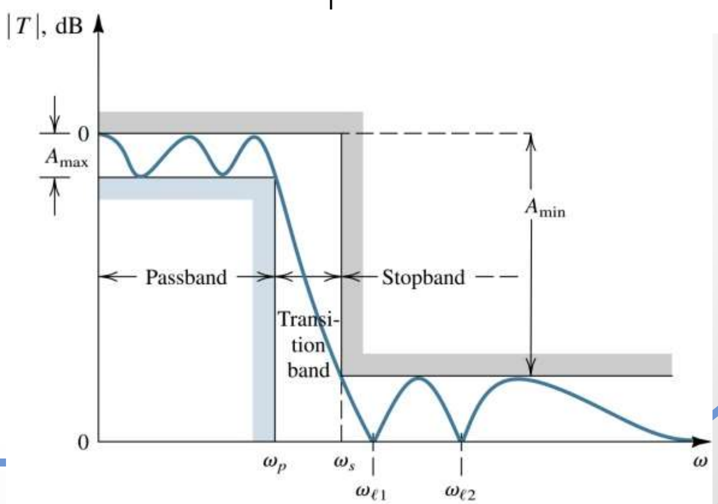
Each zero or pole an be either real or complex number. Complex zeros and poles must occur in conjugate pairs.

Since in the filter stopband, the transmission is required to be zero or small, the filter transmission zeros are usually placed on the jω axis at stopband frequencies.

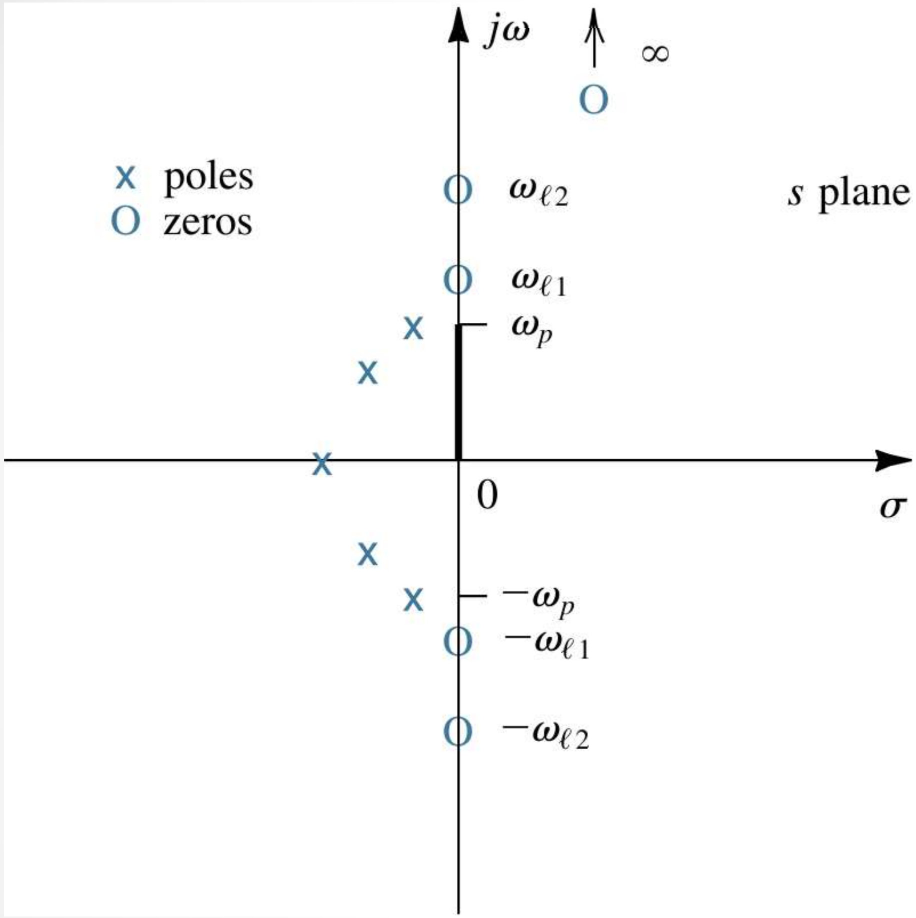
The number of transmission zeros at s = ∞ is the difference between N and M. This is because as s approaches ∞ , T(s) approaches $a_M/s^{N-M}$and thus it said to have N-M zeros at s = ∞.

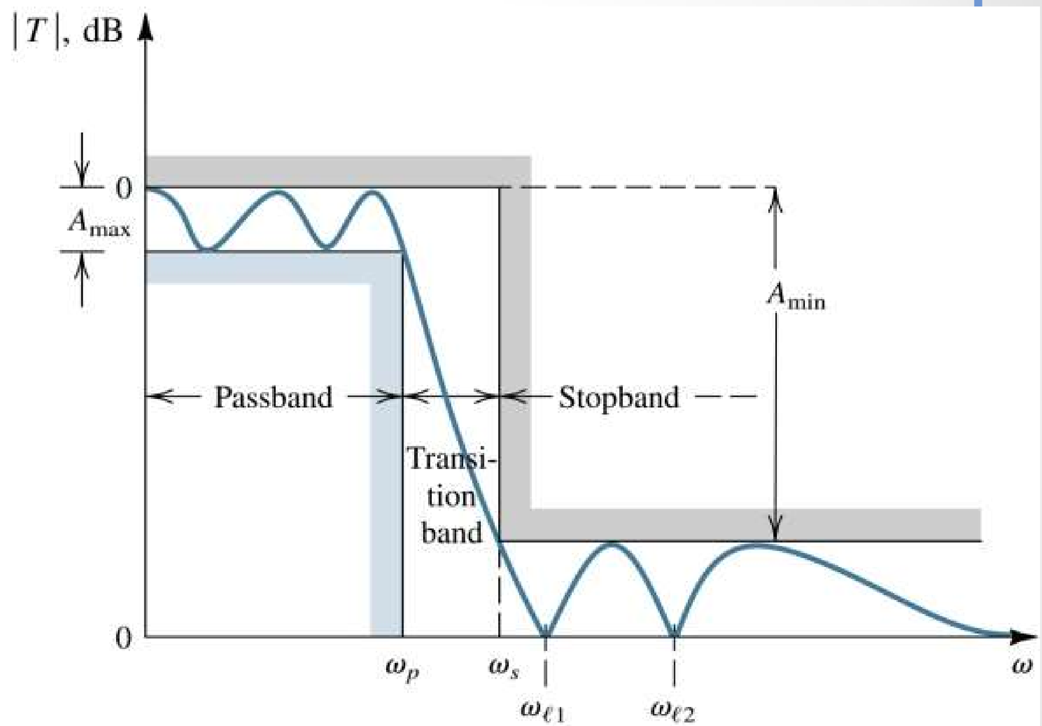
For a filter to be stable, all its poles must lie in the left half of the s plane, and thus p_1, p_2, …, p_N must all have negative real parts. All poles lie in the vicinity of the passband.





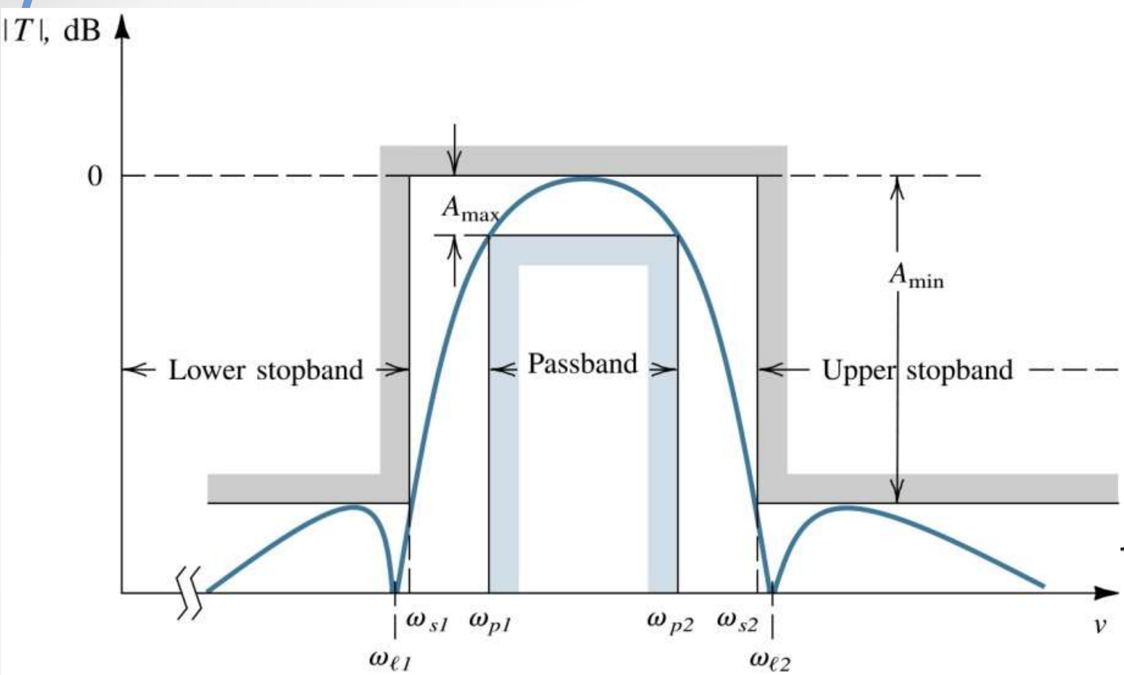
Example: lowpass Filter

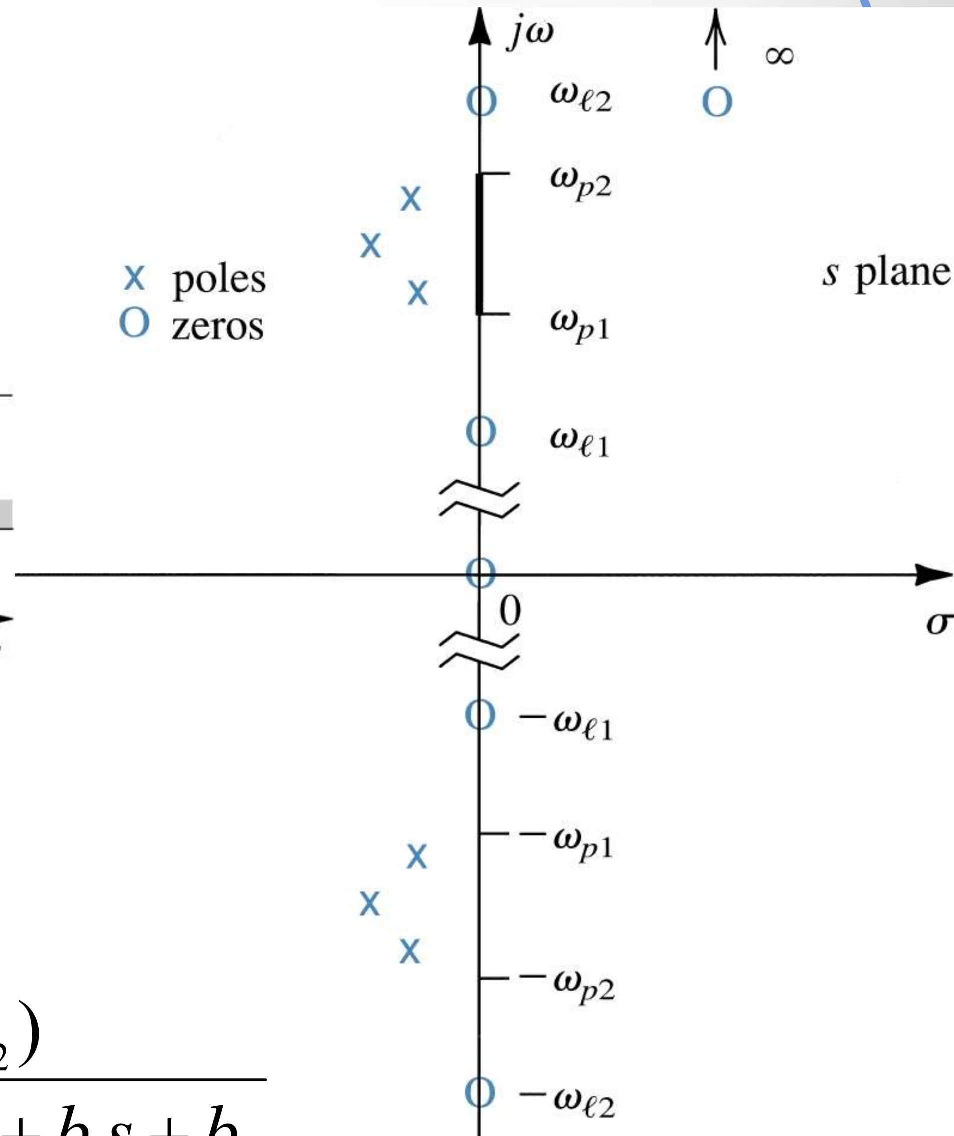




$$T(s)=\frac{a_4(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)}{s^5+b_4s^4+b_3s^3+b_2s^2+b_1s+b_0}$$

Example: Bandpass Filter



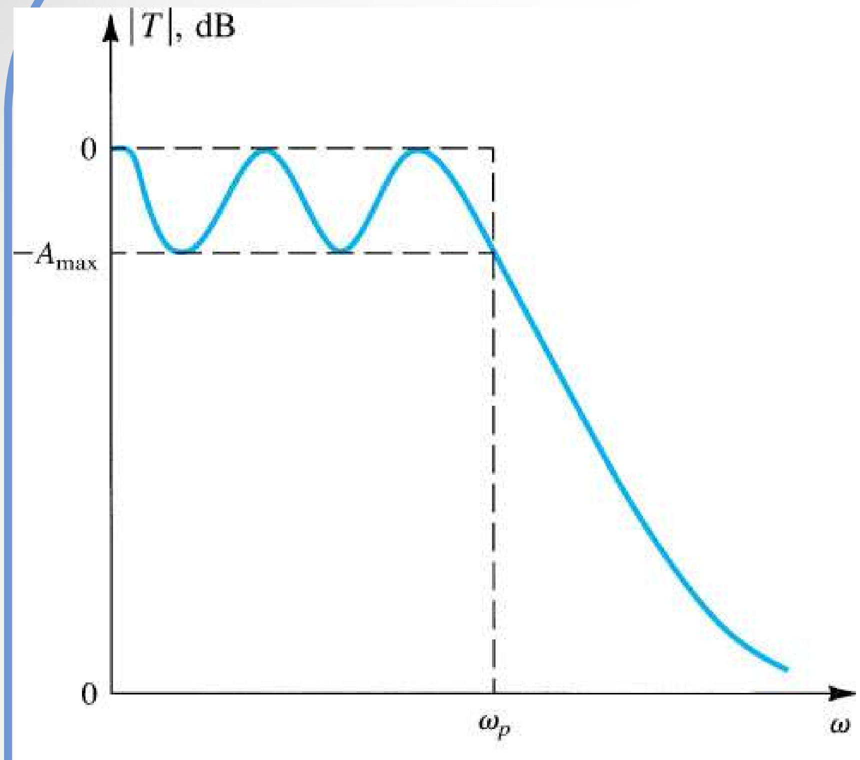


Transmission zeros at

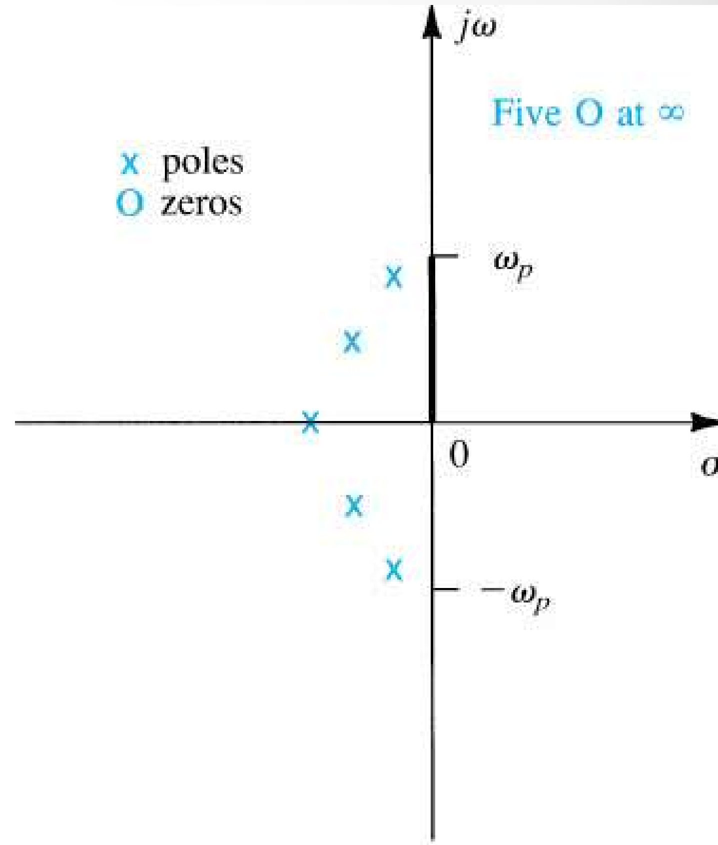
$$s = \pm j\omega_{l1}, s = \pm j\omega_{l2}, s = 0, s = \infty$$

$$T(s) = \frac{a_5 s(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^6 + b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

Example: lowpass Filter



(a)



(b)

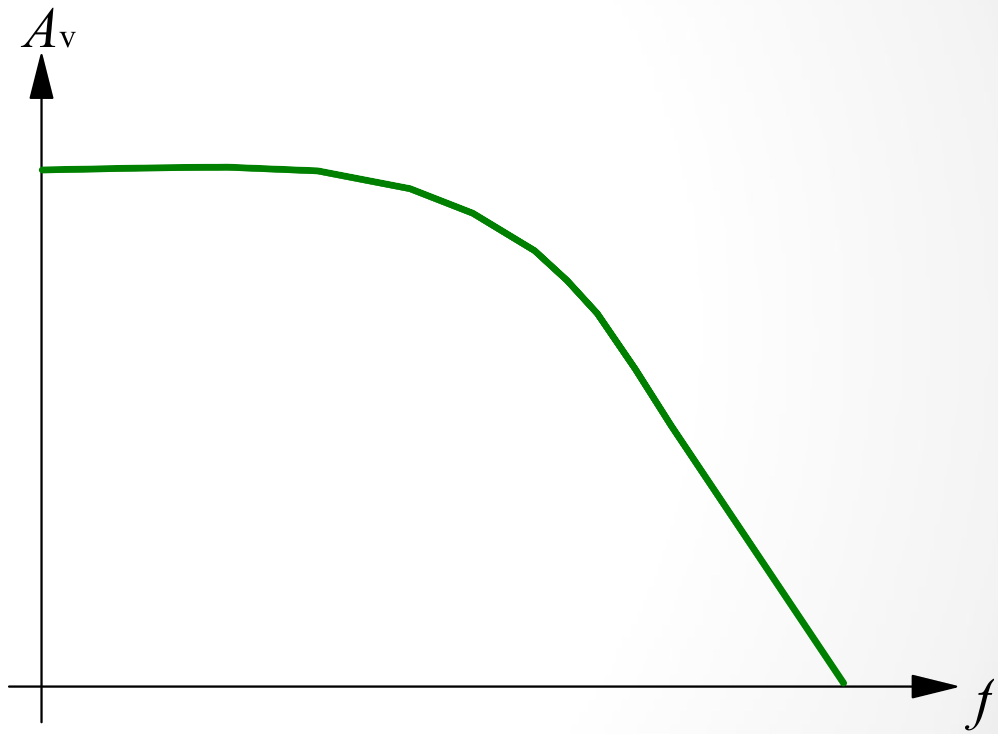
Since there are no finite values of ω at which the attenuation is infinite (zero transmission). All the transmission zeros are at s = ∞. This is a fifth-order all-pole filter.

$$T(s) = \frac{a_0}{s^N + b_{N-1}s^{N-1} + \cdots + b_0}$$

Bessel, Butterworth and Chebyshev Filters

1. Bessel Filter:

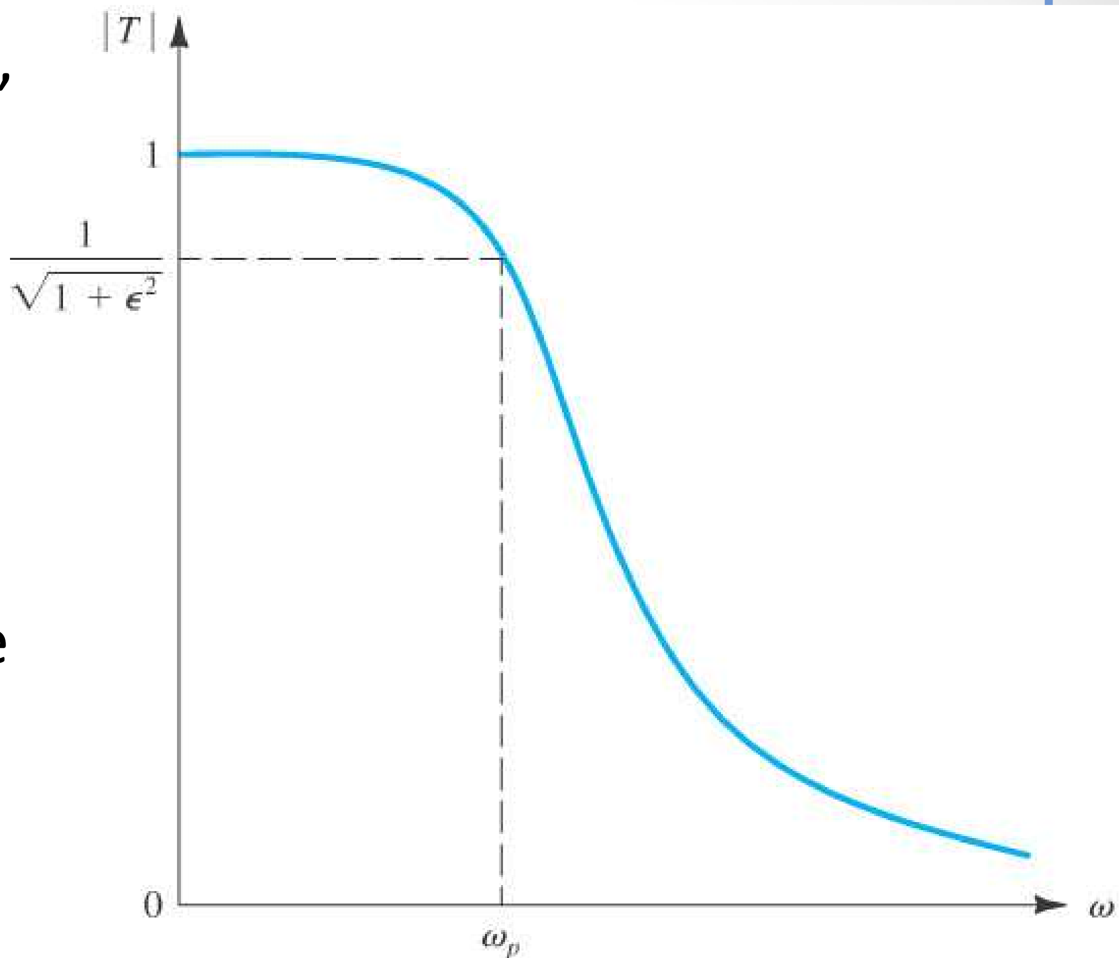
- Flat response in the passband.
- Roll-off rate is less than *20dB*/decade/pole.
- Phase response is linear.
- Used for filtering pulse waveforms without distorting the shape of the waveform.



Bessel, Butterworth and Chebyshev Filters

2. Butterworth Filter: an all-pole filter

- Very flat amplitude, $A_{v(dB)}$, response in the passband.
- Roll-off rate is *20dB*/decade/pole.
- Phase response is not linear.
- Used when all frequencies in the passband must have the same gain.
- Often referred to as a *maximally flat response*.



$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_P}\right)^{2N}}} \quad \text{.... (1)}$$

At $\omega = \omega_p$,

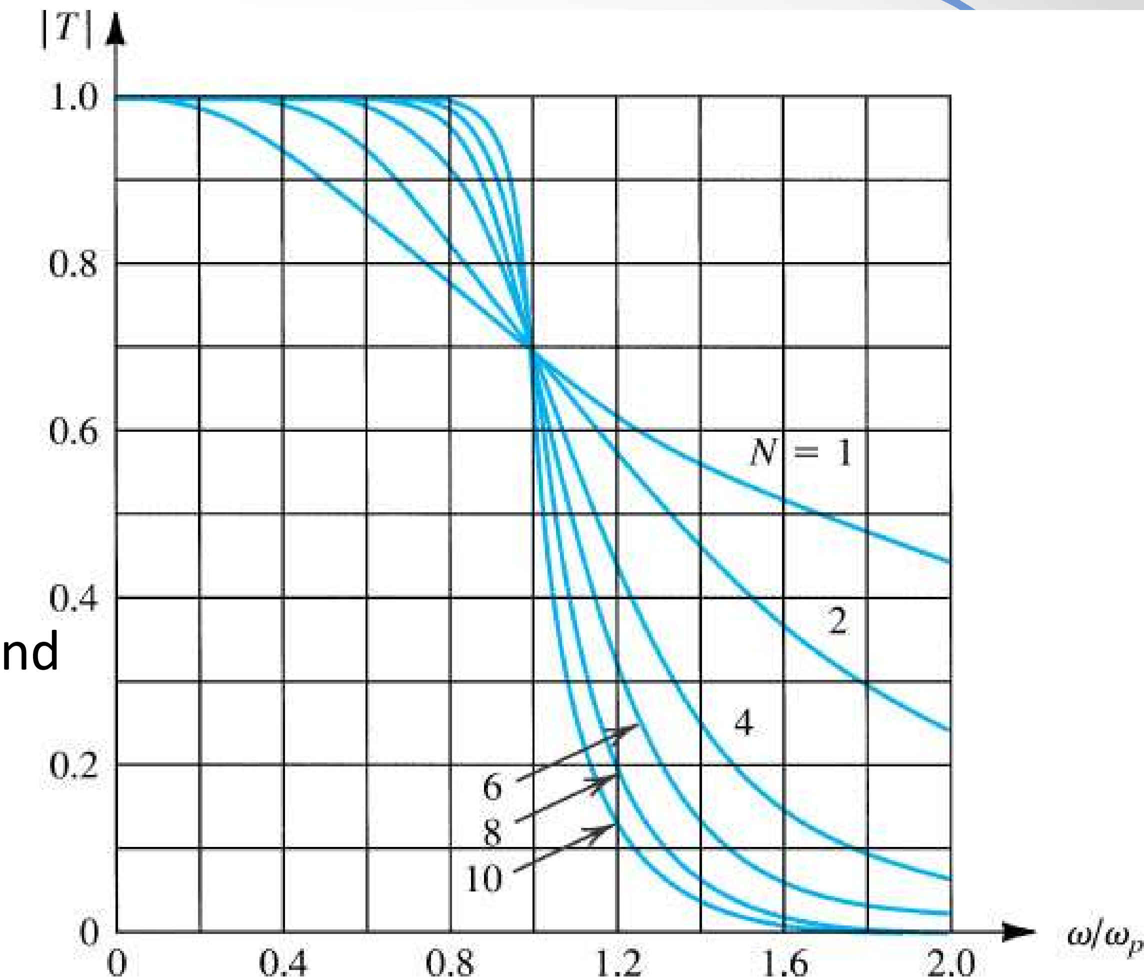
$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}} \quad \text{.... (2)}$$

ε determines max variation in passband

$$A_{max} = 20\log\sqrt{1+\varepsilon^2} \quad \text{.... (3)}$$

Or we can write

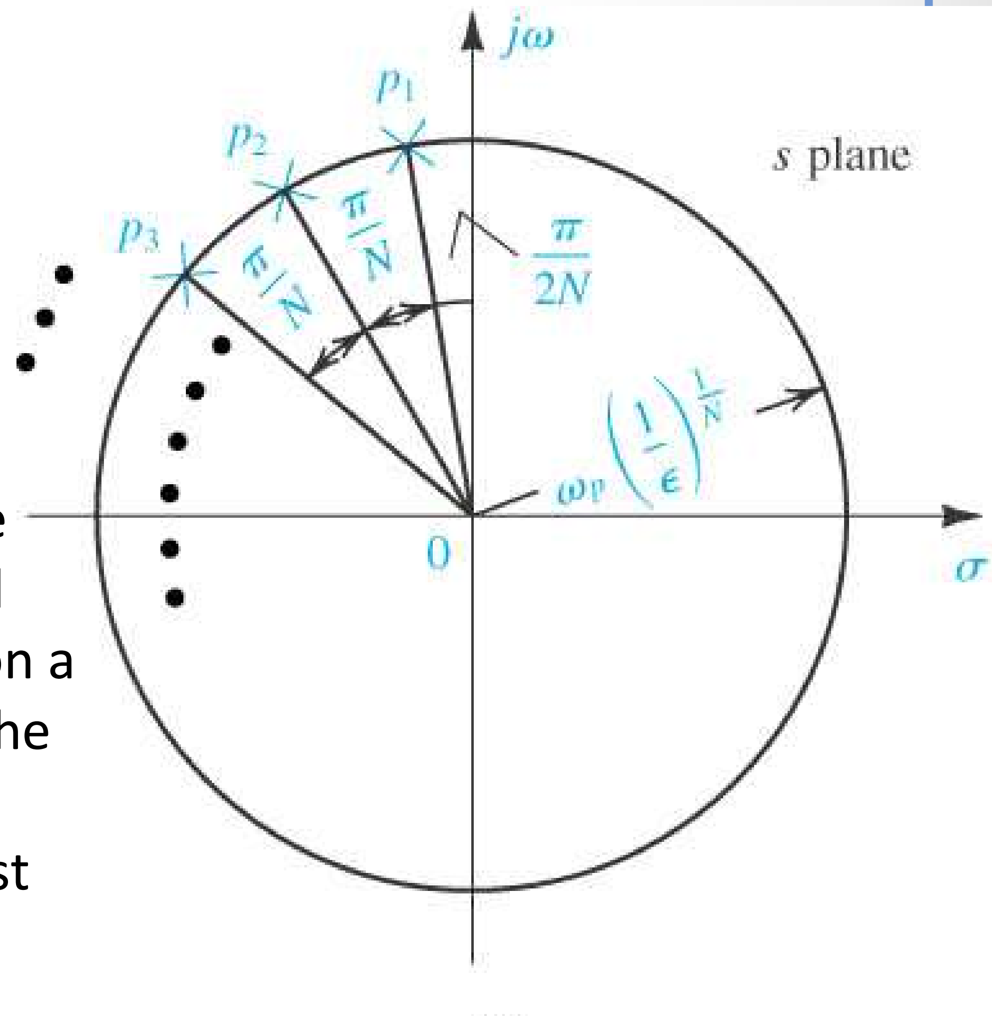
$$\varepsilon = \sqrt{10^{A_{max}/10} - 1} \quad \text{.... (4)}$$



At the edge of stopband, $\omega = \omega_s$, the attenuation is given by

$$A(\omega_s) = -20\log[1/\sqrt{1+\varepsilon^2(\omega_s/\omega_P)^{2N}}] \qquad \text{.... (5)}$$
$$= 10\log[1+\varepsilon^2(\omega_s/\omega_P)^{2N}]$$

This equation can be used to determine the filter order required, which is the lowest integer value of N that yields $A(\omega_s) \geq A_{\min}$

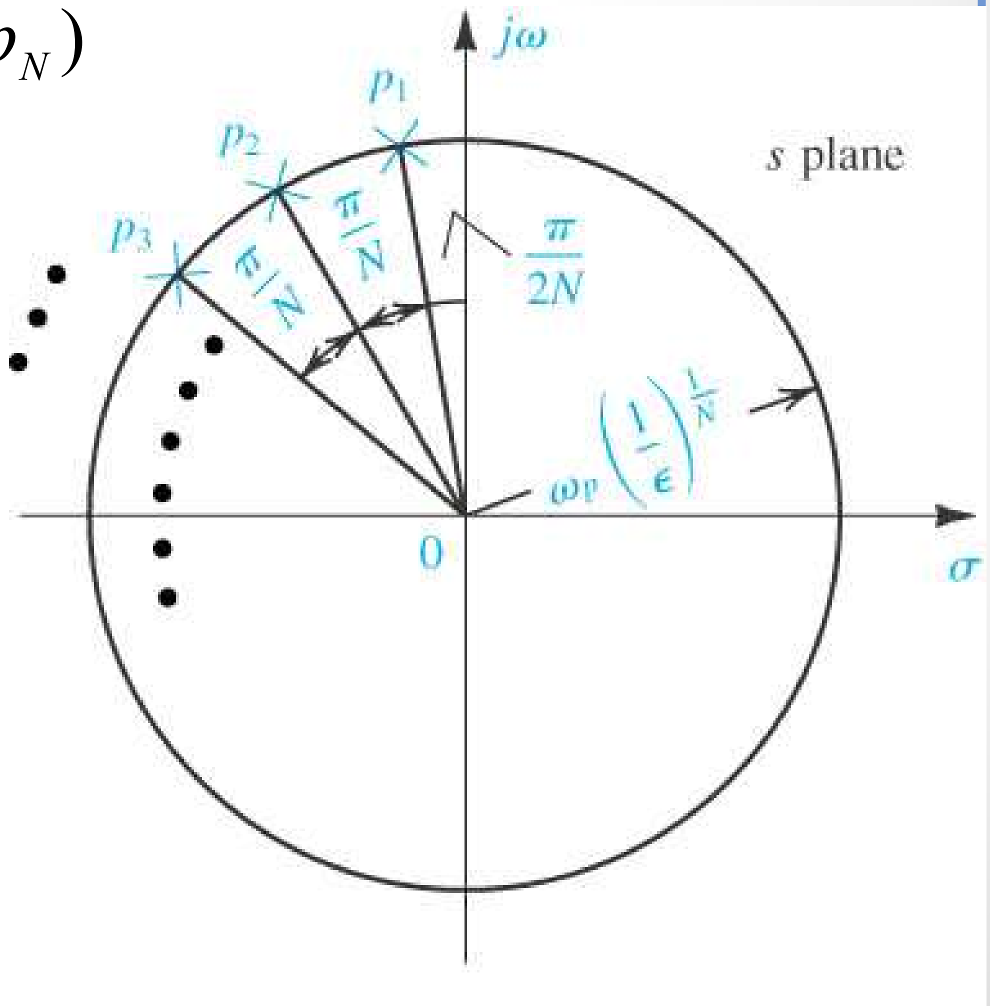


(a)

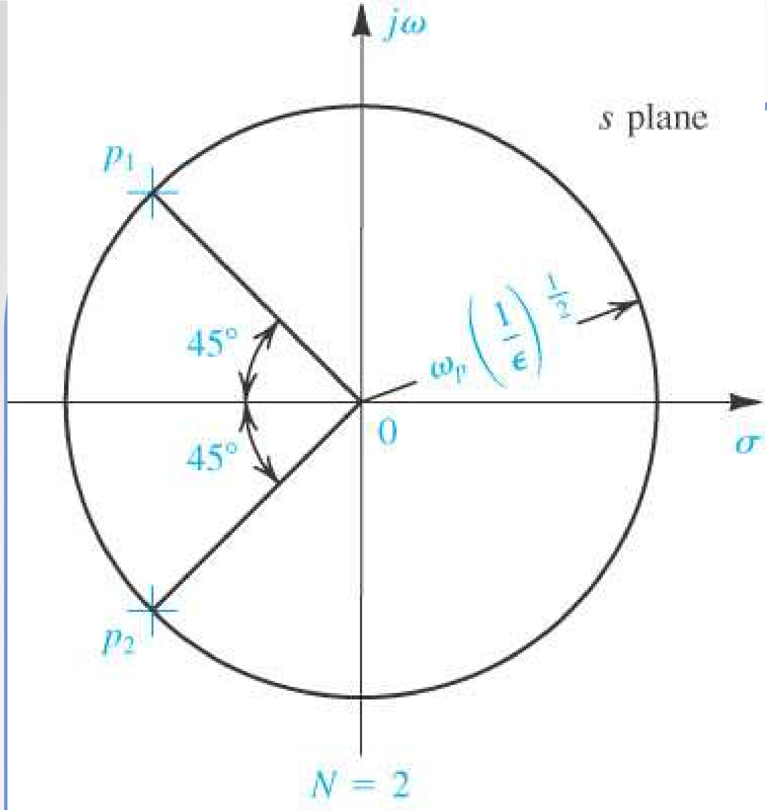
Graphical construction for determining the poles of a Butterworth filter of order *N*. All the poles lie in the left half of the *s*-plane on a circle of radius $\omega_0 = \omega_p(1/\varepsilon)^{1/N}$, where ε is the passband deviation parameter, and are spaced by equal angles of π/N, with the first mode at angle $\pi/2N$ from the $+j\omega$ axis.

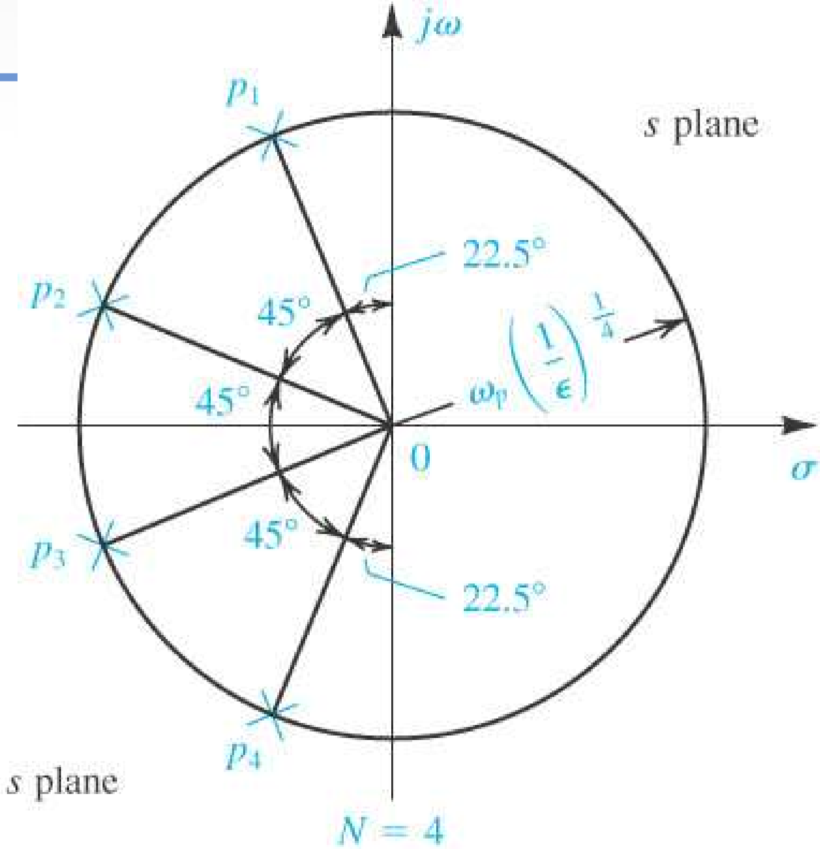
The transfer function can be written as

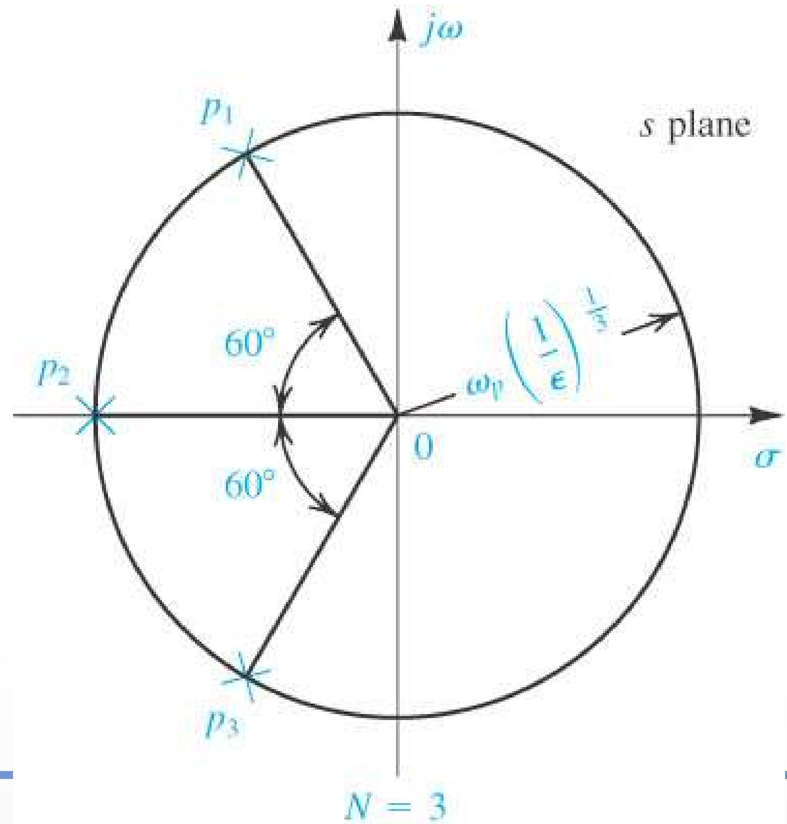
$$T(s) = \frac{K\omega_o^N}{(s-p_1)(s-p_2)\cdots(s-p_N)} \quad \text{.... (6)}$$



(a)


jω
s plane
p_1
p_2
45°
45°
$\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{2}}$
0
σ
N = 2
(b)

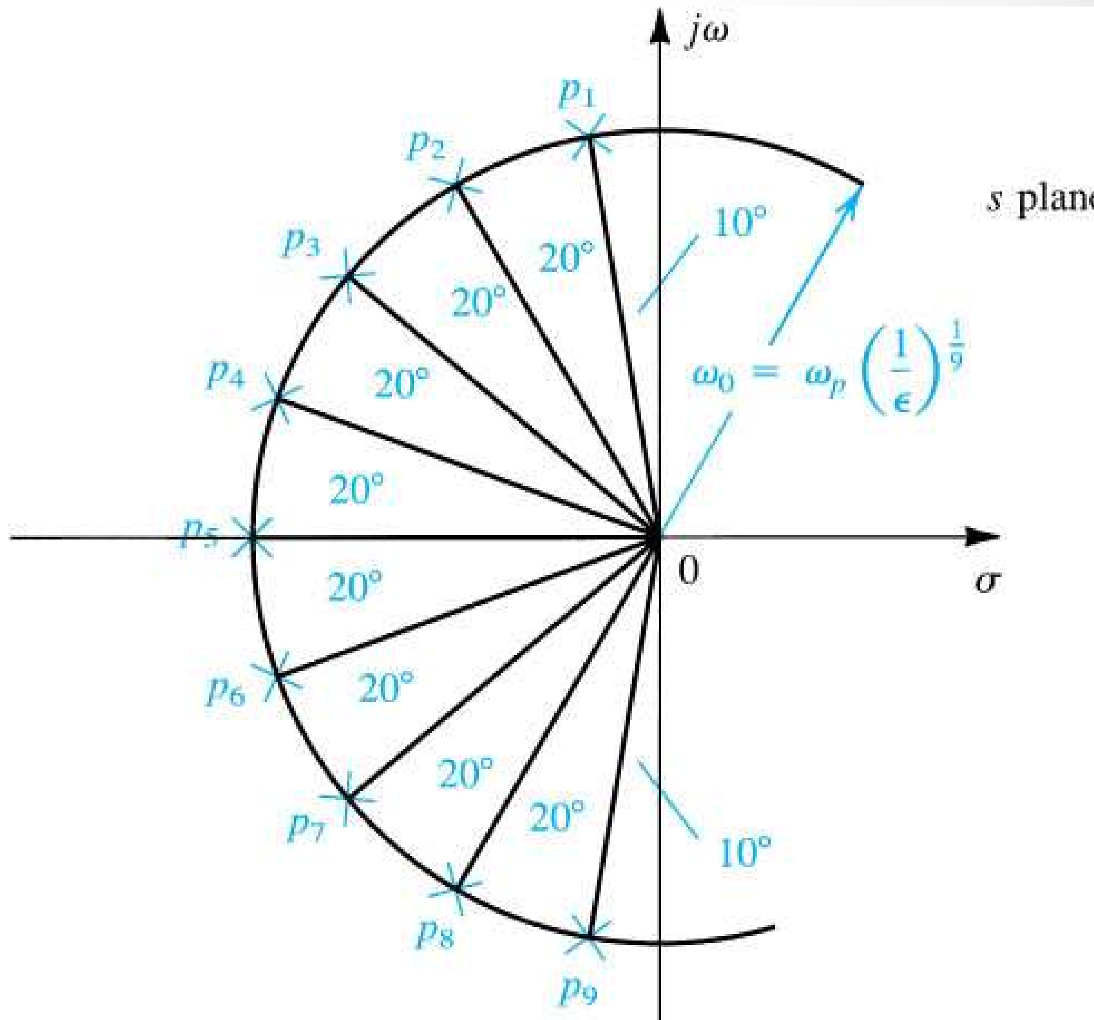

jω
s plane
p_1
p_2
p_3
p_4
22.5°
45°
45°
45°
22.5°
$\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{4}}$
0
σ
N = 4
(d)


jω
s plane
p_1
p_2
p_3
60°
60°
$\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{3}}$
0
σ
N = 3
(c)

To summarize, to find a Butterworth transfer function, we do the followings:

1. Determine ε from equation (4)
2. Use eq. (5) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{\min}$
3. Use graphical construction to determine the N natural modes.
4. Use eq. (6) to determine T(s).

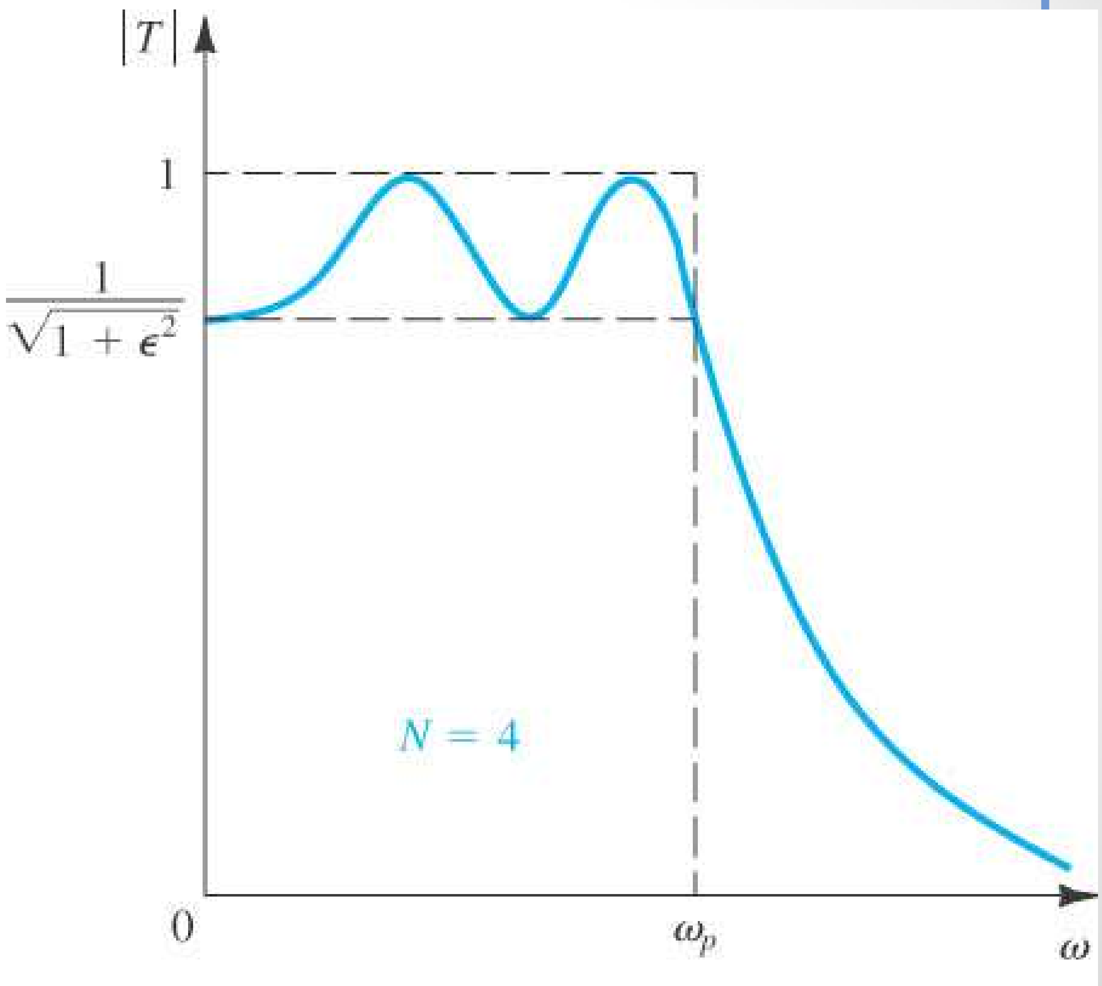
Example: Find the Butterworth transfer function that meets the following low-pass specification: fp = 10 kHz, fs = 15 kHz, Amax = 1 dB, Amin = 25 dB, dc gain = 1



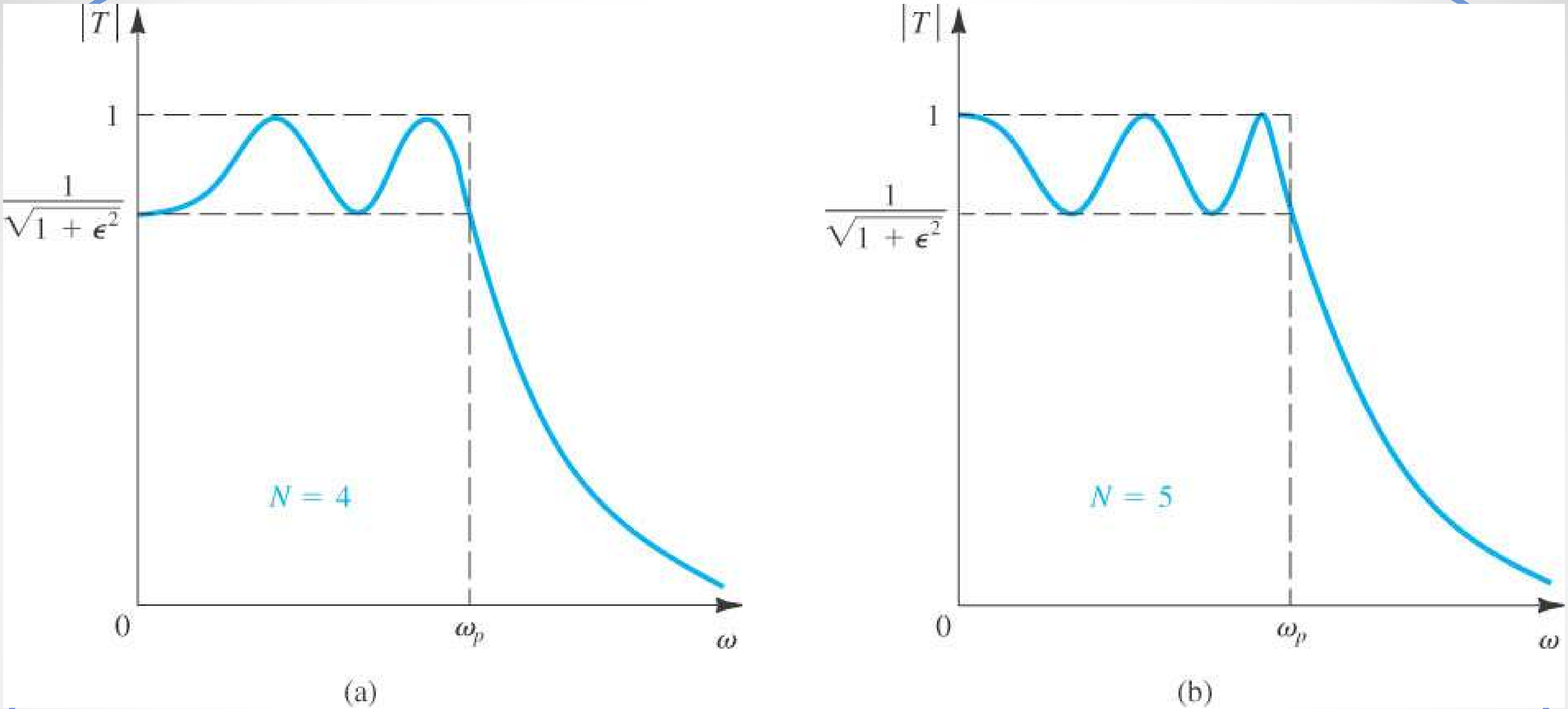
Bessel, Butterworth and Chebyshev Filters

3. Chebyshev Filter: an all-pole filter (again)

- Overshoot or ripples in the pass band.
- Roll-off rate greater than *20dB*/decade/pole.
- Phase response is not linear - worse than Butterworth.
- Used when a rapid roll-off is required.



(a)



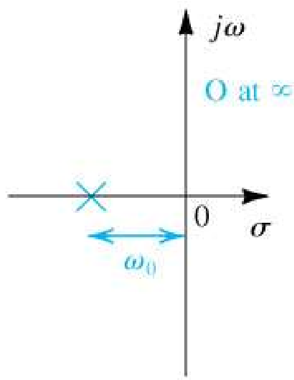
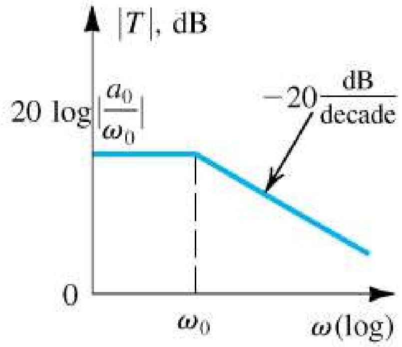
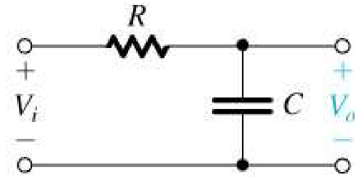
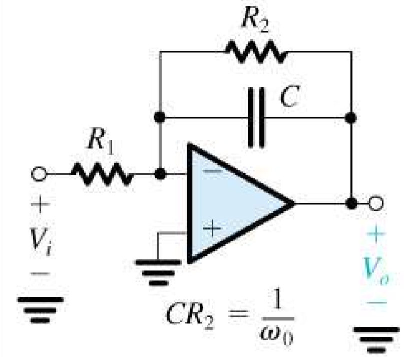
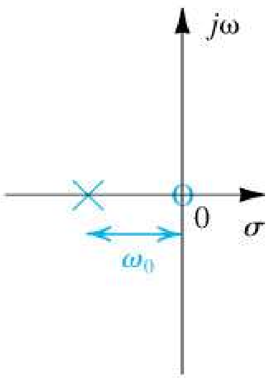
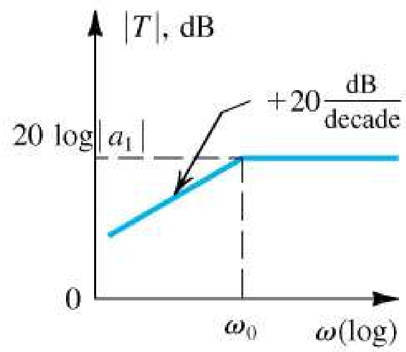
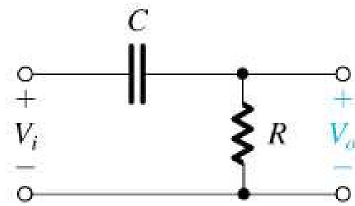
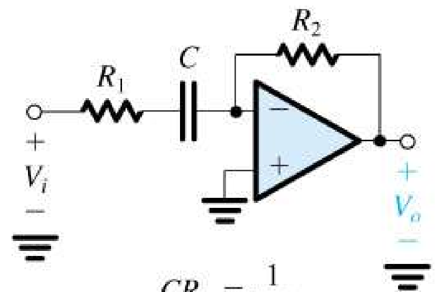
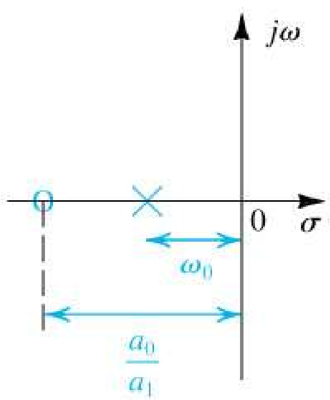
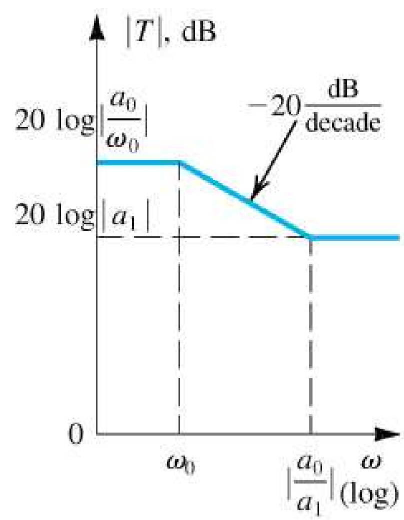
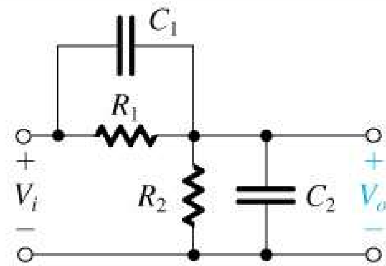
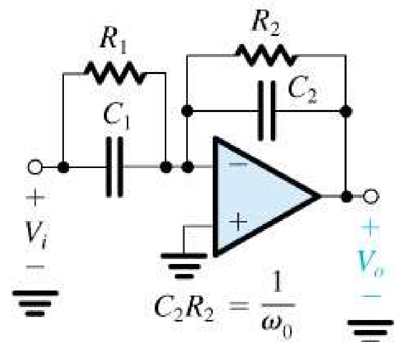
- While the odd number filter has |T(0)| = 1, the even number filter exhibits its maximum magnitude deviation at ω = 0.
- In both cases, the total number of passband maxima and minima equal to the order of the filter, N.
- All transmission zeros are at ω = ∞

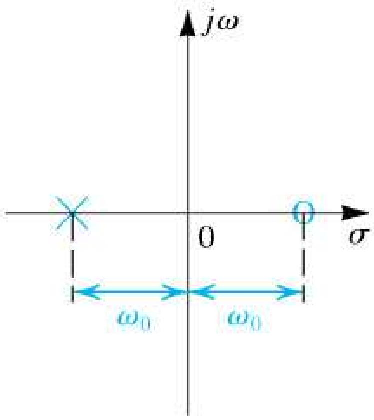
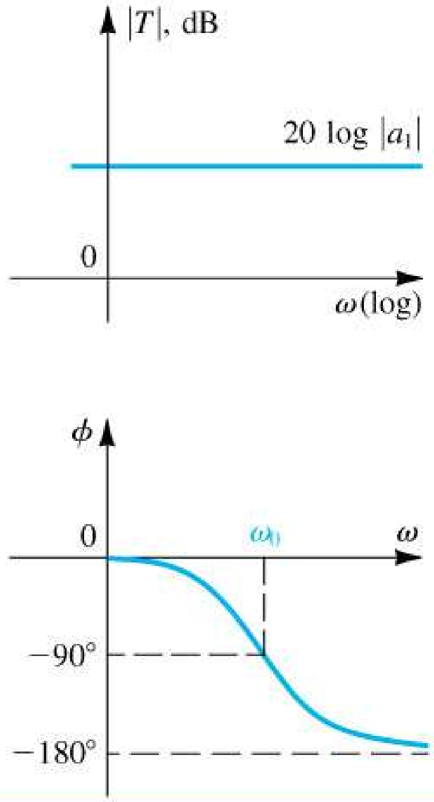
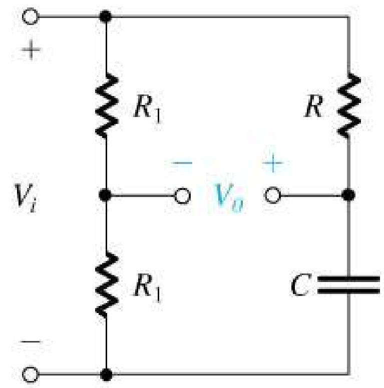
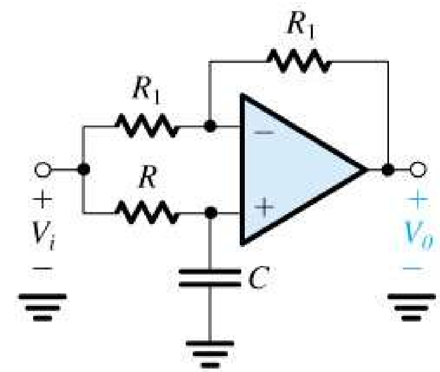
First-Order and Second-Order Filter Functions

1. First-Order Filters: the general first-order filter transfer function is

$$\boxed{T(s) = \frac{a_1 s + a_0}{s + \omega_0}}$$

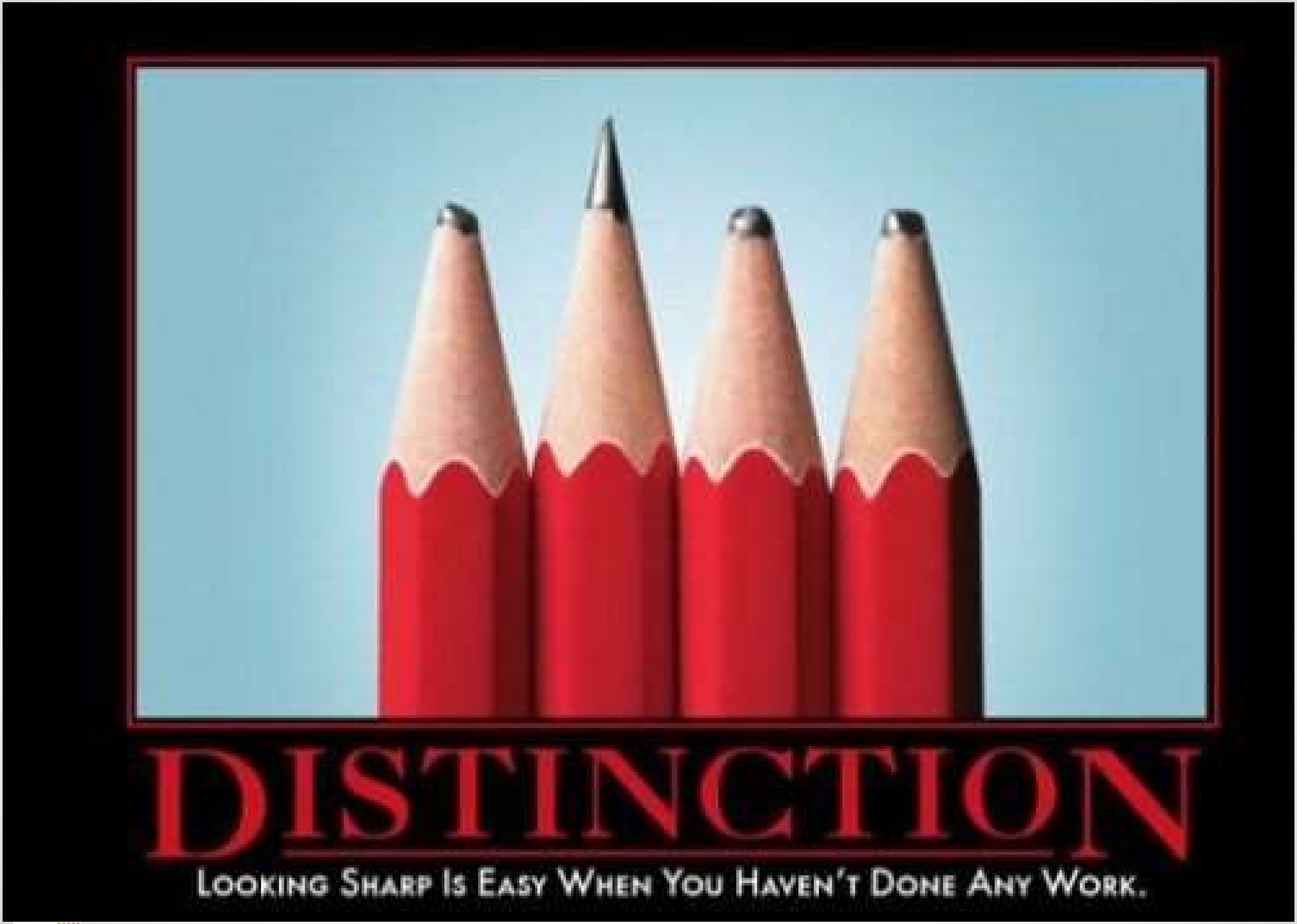
- This *bilinear transfer function* characterizes a 1st order filter with a natural mode at s = $-\omega_0$, a transmission zero at s = $-a_0/a_1$, and a high-freq gain that approaches a_1.
- The numerator coefficients, a_0 and a_1, determine the type of filter (e.g. low pass, high pass, etc.).
- Some special cases together with passive (RC) and active (op amp-RC) realizations are shown on the next two pages.

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\|T\|$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \dfrac{1}{\omega_0}$ DC gain $= -\dfrac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

$T(s)$	Singularities	$\lvert T \rvert$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

Reference

Microelectronic Circuits by Adel S. Sedra & Kenneth C. Smith. Saunders College Publishing


DISTINCTION
LOOKING SHARP IS EASY WHEN YOU HAVEN'T DONE ANY WORK.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