King Mongkut's University of Technology Thonburi
Midterm Exam of First Semester, Academic Year 2007

COURSE CPE 112 Discrete Mathematics for Computer Engineers **Computer Engineering Department, 2nd Yr.**
Thursday 27 Dec 2007 **9.00-12.00h.**

Instructions

1. This examination contains 6 problems, 8 pages (including this cover page).
2. The answers must be written in these examination sheets.
3. Students are allowed to use paper-based dictionaries.
4. Students are **not** allowed to use calculators.
5. No books, notes, or any other documents can be taken into the examination room.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room.
Students must not take the examination and the answers out of the examination room.

Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by

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Student Name ____________________________ Student ID ______________________

Problem 1: 20 points (+2 points for each correct answer,–1 **points** for each **incorrect answer**)
Determine the truth values (TRUE or FALSE) of the following statements:
Note: **Z** = integer numbers, **R** = real numbers

1) $0 > 1$ if and only if $2 > 1$.

2) If $1 + 1 = 3$, then $3 + 3 = 6$.

3) $[(p \to r) \wedge (q \to r)] \leftrightarrow [(p \vee q) \to r]$

4) $(p \oplus q) \to (p \oplus \neg q)$ when p is TRUE and q is FALSE.

5) $\forall x\, ((x^2 + 2) \geq 3)$ when the universe of discourse is all real numbers.

6) $\neg \exists x\, Q(x) \equiv \forall x\, Q(x)$

7) f(n) is a function from **Z** to **R** if $f(n) = 1/(n^2-4)$.

8) f(x) is a one-to-one function from **Z** to **Z** if $f(x) = x^2 + 1$.

9) $\lceil (1/2) + \lceil 5/2 \rceil - \lfloor 7/8 \rfloor \rceil = \lceil (3/2) + \lceil 5/3 \rceil \rceil$

10) x^4 is $O(g(x))$ if $g(x) = x^3 + x^4$.

Problem 2: 10 points (5 points each)

Use big-O notation to estimate the following functions:

2.1. $f(n) = 3n^3+(n^2+ 3)\log(n^2+ 1) + 2^n$

2.2. $f(n) = (x^4 + (6x \log x))\ (x^2 / (x^3+ 2))$

Problem 3: 21 points

3.1 Give a proof by contradiction that if n is an integer and $5n+7$ is odd, then n is even. (7 points)

3.2 Prove that if n is an integer and $5n+7$ is even if and only if n is odd". (7 points)

3.3 Use mathematical induction to prove that $n^2 - 7n + 12$ is nonnegative when n is an integer greater than 3. (7 points)

Problem 4: 19 points

4.1. A man has ten shirts, four pairs of pants, and three pairs of shoes. How many different outfits are possible? (4 points)

4.2. How many eight-bit strings begin with 100 or have the fourth bit 1? (5 points)

4.3. How many eight-bit strings have exactly two 1s? (5 points)

4.4. How many eight-bit strings have at least six 1s? (5 points)

Problem 5: 10 points
With pigeonhole principle, can we show that if we select 155 distinct computer engineering courses numbered between 1 and 300 inclusive, at least two are consecutively numbered. Explain your answer in detail.

Problem 6: 20 points
6.1. Discuss and show the divide-and-conquer recurrence relation for the Binary Search algorithm. What is the number of operations required to solve a problem of size n.? Define the initial condition. (6 points)

6.2. A juice vending machine accepts only 1-baht and 5-baht coins.

a) Find a recurrence relation for the number of ways to deposit *n* bahts into the vending machine, where the **order (or sequence)** in which the coins are deposited **matters**. (5 points)

b) What are the initial conditions? (5 points)

c) How many ways to deposit 8 bahts for a bottle of coffee. (4 points)