Electrical and Electronic Measurements:

Sources of Error

1

Errors in Electrical Measurements

- Systematic error ⇒ every times you measure e.g. loading or insertion of the measurement instrument
- Meter error ⇒ scaling (inaccurate marking), pointer bending, friction, no calibration
- Random error ⇒ temperature effect, noises (unwanted signals)
- Reading error ⇒ parallax, read a wrong scale
- Recording error ⇒ for many measured values

2

Background

- Electric charge, $e^{-} = 1.6 \times 10^{-19}$ Coulombs
- Current, $I = dq/dt$ Coulomb/sec = Amps
- Voltage, the difference in electrical potential between two points (Joules/Coulomb)
- Power, $P = IV$ Watts
- Energy, $E = Pt$ Units (kW·hr)
- Ohm's Law, $V = IR$

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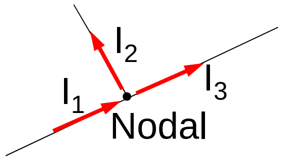
Background (Cont'd)

- Kirchhoff's Current Law (KCL)
  $I_1 - I_2 - I_3 = 0$

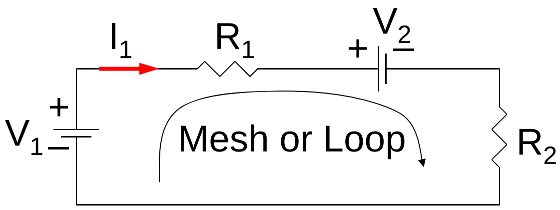


A *loop* is any closed path in a circuit, in which no node is encountered more than once.

A *mesh* is a loop that has no other loops inside of it.

A *supermesh* occurs when a current source is contained between two essential meshes.

- Kirchhoff's Voltage Law (KVL)
  $-V_1 + I_1R_1 + V_2 + I_2R_2 = 0$



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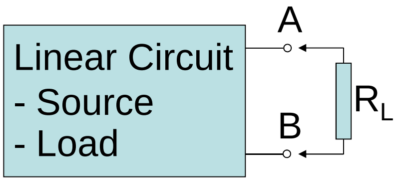
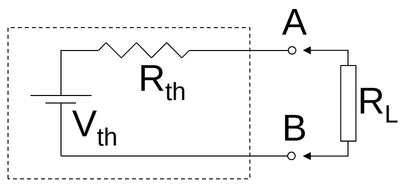
Background (Cont'd)

- Thévenin's Theorem

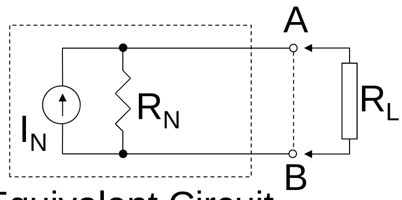




Equivalent Circuit

Open circuit to find V_{th}

- Norton's Theorem
  Short circuit to find I_N

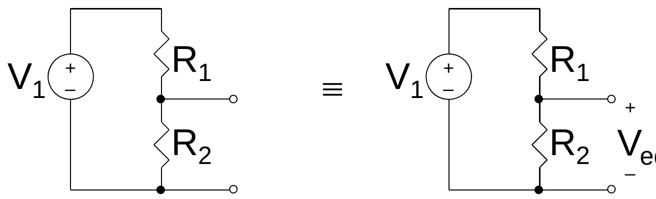


Equivalent Circuit

Background (Cont'd)

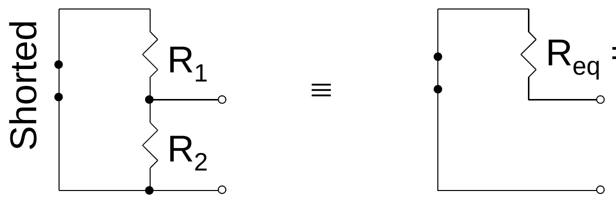
Example

No Load ⇒ Equivalent Source



$$V_{eq} = \frac{R_2}{R_1+R_2} V_1$$

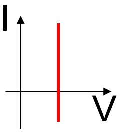
No Source ⇒ Equivalent Load



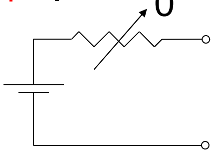
$$R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1+R_2}$$

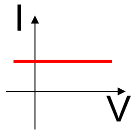
Background (Cont'd)

- Ideal Voltage Source Vs. Ideal Current Source

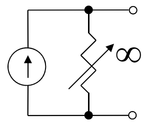


$I_{\infty} = V_{fixed} / R_0$





$V_{\infty} = I_{fixed} R_{\infty}$

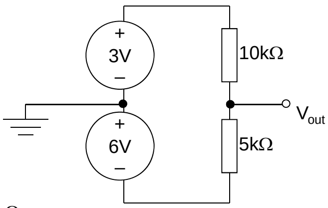


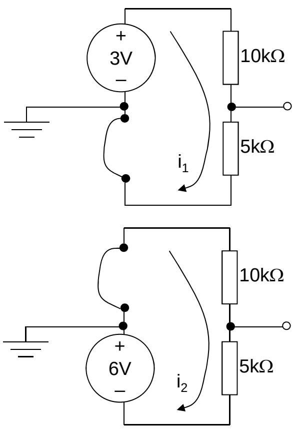
- Superposition Theorem (for linear resistive network containing several sources)

  Voltage source ⇒ Short Circuit 0 V
  Current source ⇒ Open Circuit 0 A
  Vector summation of the individual voltage or current caused by each separate source

Background (Cont'd)

Example





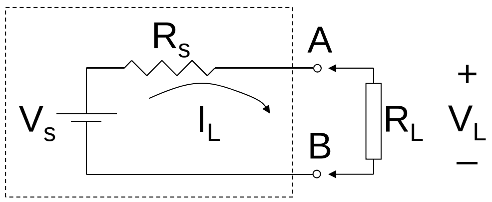
$V_{out1} = 3V/(10k\Omega+5k\Omega) \cdot 5k\Omega = 1V$

$V_{out2} = -6V/(10k\Omega+5k\Omega) \cdot 10k\Omega = -4V$

$V_{out} = V_{out1} + V_{out2}$
$= 1V - 4V$
$= -3V$

Background (Cont'd)

- The Maximum Power Transfer Theorem



If V_s and R_s are fixed, when $I_L \uparrow$ then $V_L \downarrow$

Maximum power is at $R_L = R_s$

Therefore $V_L = V_s / 2$ and $I_L = V_s / 2R_L$

$$P_L = (I_L)^2 R_L = (V_s / 2R_L)^2 R_L = V_s^2 / 4R_L$$

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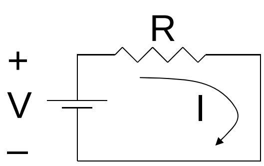
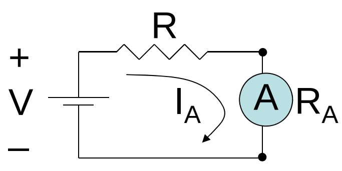
Ammeter Loading

For current measurement, open circuit to put an ammeter.



$I = V/R$

DC Circuit



$I_A = V/(R+R_A)$

While this impact is inevitable, it can be minimized through good meter design.

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Ammeter Loading (Cont'd)

Loading error, $I_A - I = V/(R+R_A) - V/R$

$$= \frac{VR - V(R+R_A)}{R(R+R_A)}$$

$$= \frac{-VR_A}{R(R+R_A)}$$

% Loading Error = $(I_A - I)/I \times 100\%$

$$= \frac{-R_A}{R+R_A} \times 100\%$$

It means if $R_A \uparrow$ then error $\uparrow$

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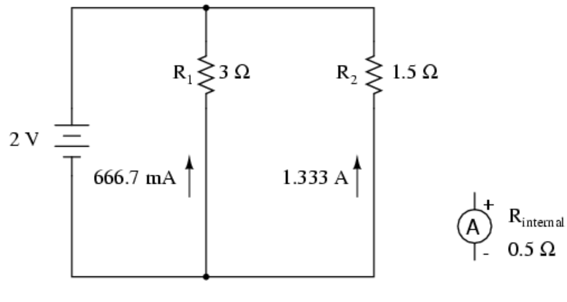
Ammeter Loading (Cont'd)

The ideal ammeter has zero internal resistance, so as to drop as little voltage as possible as electrons flow through it.

Example

without the ammeter,



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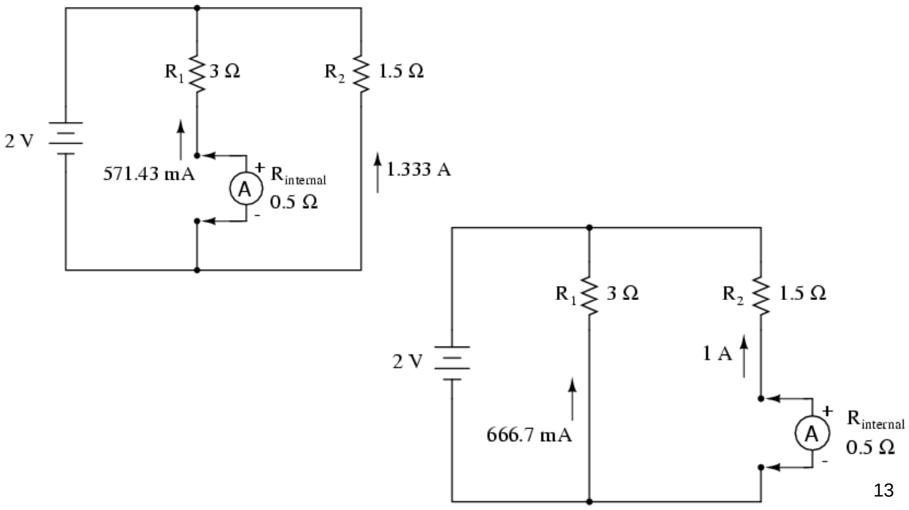
Ammeter Loading (Cont'd)

with the ammeter,



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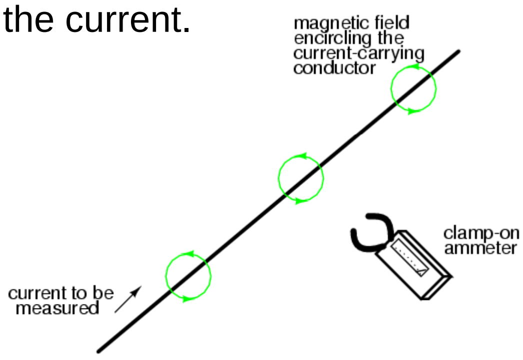
Ammeter Loading (Cont'd)

One ingenious way to reduce the impact that a current-measuring device has on a circuit is to use the circuit wire as part of the ammeter movement itself.

All current-carrying wires produce a magnetic field, the strength of which is in direct proportion to the strength of the current.



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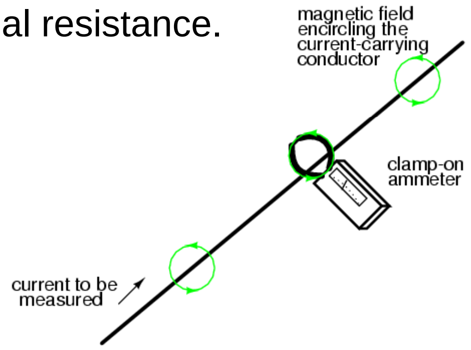
Ammeter Loading (Cont'd)

By building an instrument that measures the strength of that magnetic field, a no-contact ammeter can be produced.

Such a meter is able to measure the current through a conductor without even having to make physical contact with the circuit, much less break continuity or insert additional resistance.



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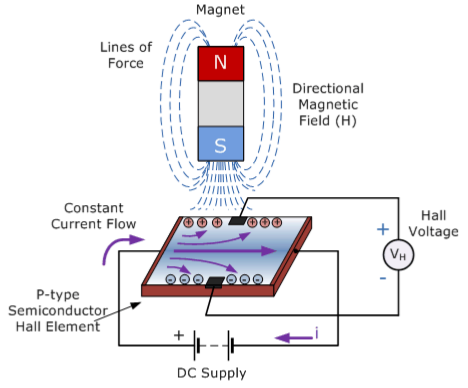
Ammeter Loading (Cont'd)

More modern designs of clamp-on ammeters utilize a small magnetic field detector device called a Hall-effect sensor to accurately determine field strength.



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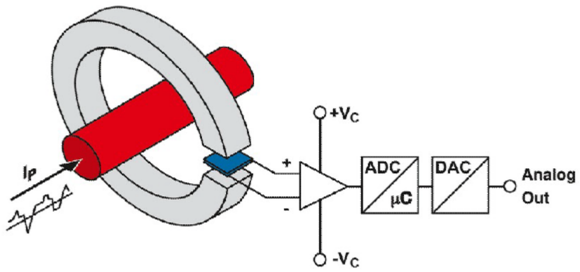
Ammeter Loading (Cont'd)

Some clamp-on meters contain electronic amplifier circuitry to generate a small voltage proportional to the current in the wire between the jaws, that small voltage connected to a voltmeter for convenient readout by a technician.



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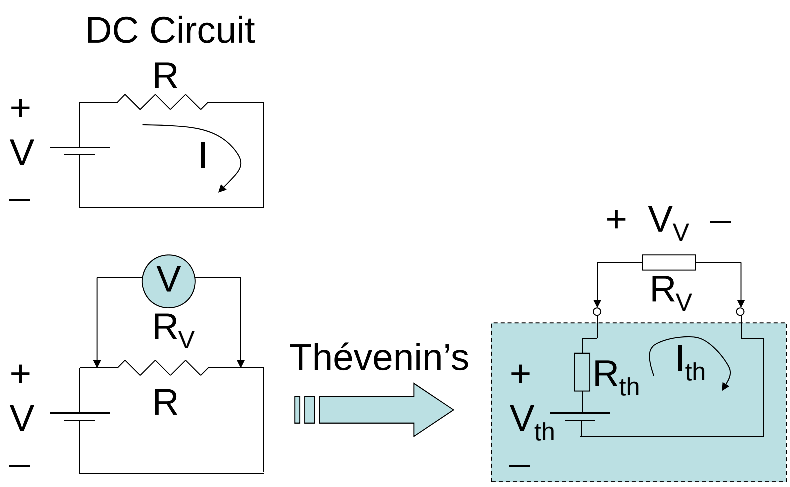
Voltmeter Loading

For voltage measurement, voltmeter is placed in parallel with the circuit element.



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Voltmeter Loading (Cont'd)

$I_{th} = V_{th} / (R_{th} + R_V)$

$V_V = I_{th}R_V = V_{th}R_V / (R_{th} + R_V)$

Error = $V_V - V_{th}$

$$= \left(\frac{R_V}{R_{th} + R_V} - 1 \right) V_{th}$$

% Error = $(V_V - V_{th})/V_{th} \times 100\%$

$$= \frac{-R_{th}}{R_{th}+R_V} \times 100\%$$

It means if $R_V \uparrow$ then error $\downarrow$

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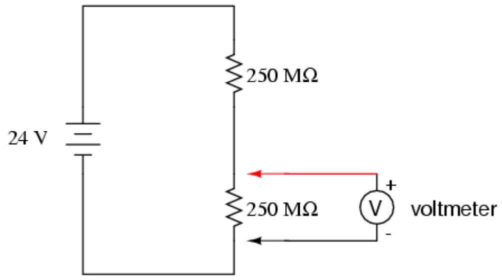
Voltmeter Loading (Cont'd)

The ideal voltmeter has infinite resistance, so that it draws no current from the circuit under test.

Example

without the voltmeter,



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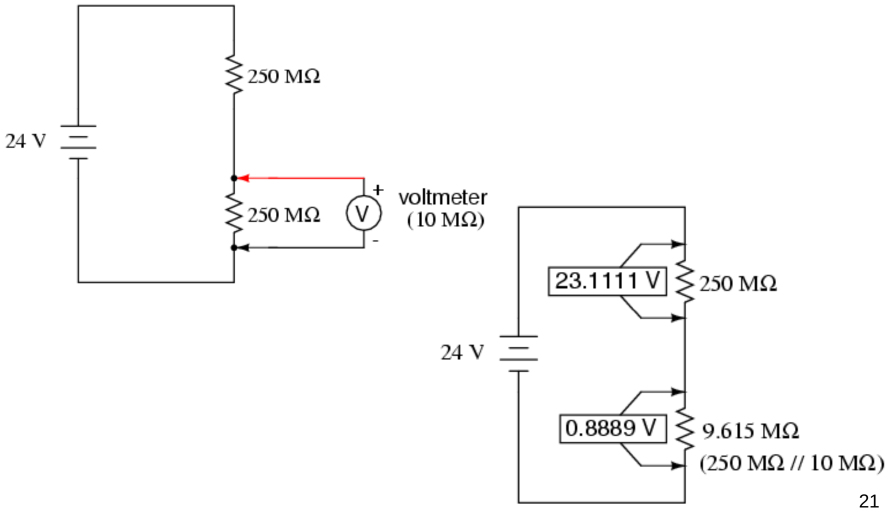
Voltmeter Loading (Cont'd)

with the voltmeter,



Voltmeter Sensitivity

- Voltmeters with electromechanical movements are typically given ratings in "ohms per volt" of range to designate the amount of circuit impact created by the current draw of the movement.
- Digital voltmeters, on the other hand, often exhibit a constant resistance across their test leads regardless of range setting, and as such are usually rated simply in ohms of input resistance, rather than "ohms per volt" sensitivity.
- Since most digital voltmeter have 50 times more impedance than analog voltmeters, digital meters are more accurate when measuring voltage.

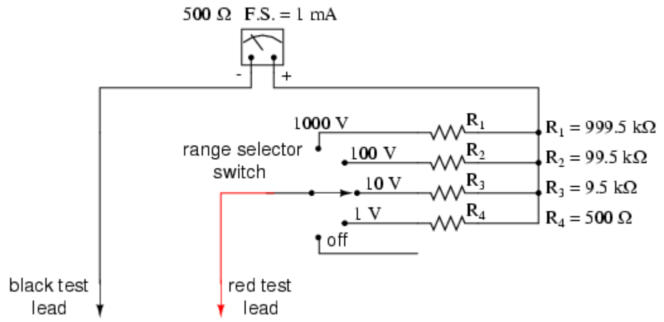
Voltmeter Sensitivity (Cont'd)

Ex On the 1000 volt scale,

the total resistance is 1 MΩ (999.5 kΩ + 500Ω),

giving 1,000,000 Ω per 1,000 volts of range,

or 1 kΩ/V (1,000 ohms per volt).



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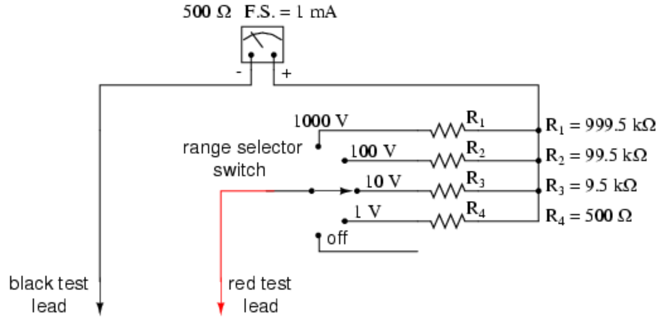
Voltmeter Sensitivity (Cont'd)

This ohms-per-volt "sensitivity" rating remains constant for any ranges of this meter:

100-V range, 100kΩ/100V = 1 kΩ/V sensitivity

10-V range, 10kΩ/10V = 1 kΩ/V sensitivity

1-V range, 1kΩ/1V = 1 kΩ/V sensitivity



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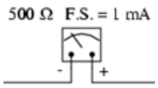
Voltmeter Sensitivity (Cont'd)

- "Ohms per volt" is the mathematical reciprocal of "volts per ohm," which is defined by Ohm's Law.
- The ohms-per-volt rating of any meter is determined by a single factor:
  the full-scale current of the movement, in this case 1 mA.



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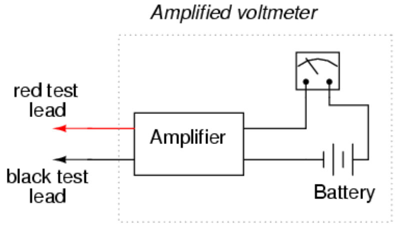
Voltmeter Sensitivity (Cont'd)

- To minimize the loading of a voltmeter on any circuit, the designer must seek to minimize the current draw of its movement.
- To electronically boost the current sent to the movement, so that very little current needs to be drawn from the circuit under test, an electronic amplifier is used.



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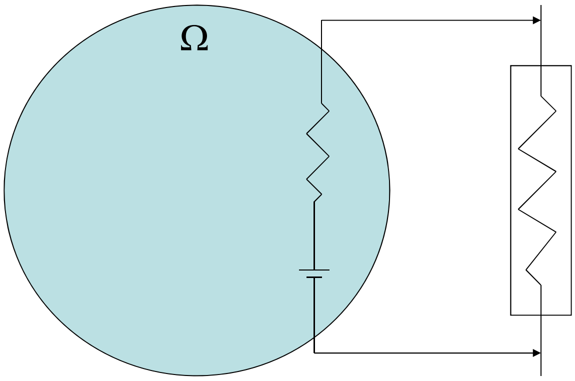
Ohmmeter Loading

Using a circuit within the instrument (self-powered)



Analog Ammeters & Voltmeters

- The users have to read the meter manual to find the basic accuracy specifications.
- Moving-coil-based analog meters are characterized by their full-scale deflection (f.s.d.) and effective resistance of the meters (few Ω - kΩ)
- Typical meters produce a f.s.d. current of 50 μA - 1 mA (this is not the ranges of the meters)

Percentage of FSD

- The accuracy of analog ammeters and voltmeters are quoted as a percentage of the full-scale deflection of pointer on the linear scale for all reading on the selected range.
- e.g. an ammeter having accuracy $\pm$2% f.s.d. within the range 0-10 A,
  Relative error $\Rightarrow \pm$ 2% of 10 A
  Absolute error $\Rightarrow \pm$ 0.2 A

- When a measured value is close to full scale, or at least above 2/3 of full scale, the published accuracy is meaningful.

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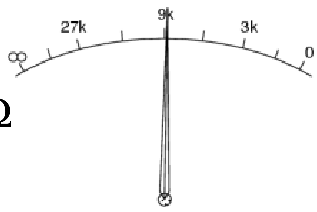
Percentage of Midscale

- Moving-coil-based ohmmeters is characterized by their midscale deflection of the pointer on nonlinear scale.
- What figure lies exactly between infinity and zero?
- e.g. $\pm$ 3% of midscale
  Relative error $\Rightarrow \pm$ 3% of 9 kΩ
  Absolute error $\Rightarrow \pm$ 0.27 kΩ



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Percentage of DMM Reading

- In the case of digital instruments, the accuracy is generally quoted as ± the percentage of the reading ± 1 digit.
- e.g. ± 0.5% Reading ± 1 Digit
  when you read a voltage 1.2 V with a digit of 0.01 V (depending on the range selected)

  error = ± 0.5% of 1.2 V ± 0.01 V
  = ± 0.016 V ⇒ absolute error
  = ± 1.3% of reading ⇒ relative error

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Potential Meter

- Before the moving coil meter, potentiometer was used in measuring the voltage in a circuit
- Calculated from a fraction of a known voltage (on resistive slide wire)
- Based on voltage divider (linearly)
- By means of a galvanometer (no deflection or null-balance)

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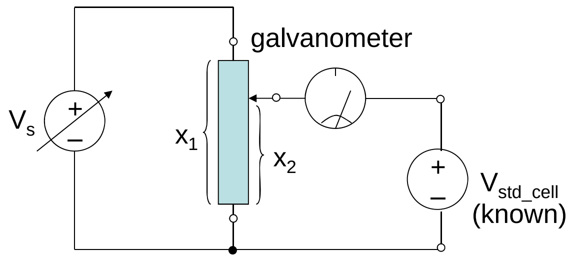
Calibration of Potentiometer



- The end of a uniform resistance wire is at x_1
- The sliding contact or wiper is then adjusted to x_2
- The standard electrochemical cell whose E.M.F. is known (e.g. 1.0183 volts for Weston cell)

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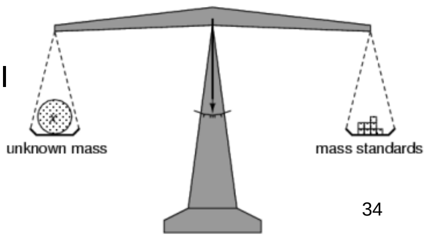
Calibration of Potentiometer (Cont'd)

- Adjust supply voltage V_s until the galvanometer shows zero or detect null (voltage on R_2 equals to the standard cell voltage or no current flows through the galvanometer)
- $x_2/x_1\ V_s = V_{std\_cell}$

  $V_s = x_1/x_2\ V_{std\_cell}$



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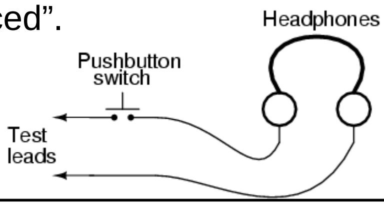
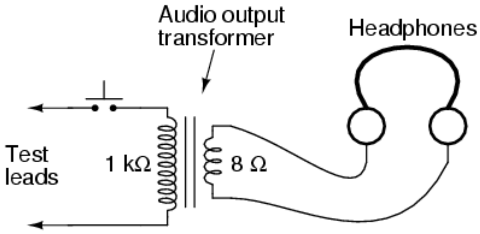
Calibration of Potentiometer (Cont'd)

- An extremely simple type of null detector, instead of the galvanometer, is a set of audio headphones, the speakers within acting as a kind of meter movement.
- When a DC voltage is initially applied to a speaker, the resulting current through it will move the speaker cone and produce an audible "click."
- Another "click" sound will be heard when the DC source is disconnected.
- The technician would repeatedly press and release the pushbutton switch, listening for silence to indicate that the circuit was "balanced".

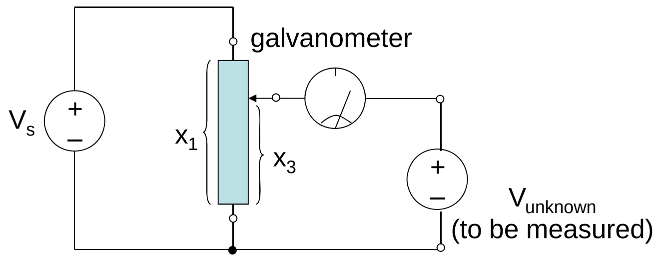


Calibration of Potentiometer (Cont'd)

- The headphone's sensitivity may be greatly increased by connecting it to a device called a transformer.
- A step-down transformer converts low-current pulses, created by closing and opening the pushbutton switch while connected to a small voltage source, into higher-current pulses to more efficiently drive the speaker cones inside the headphones ($N_p/N_s = I_s/I_p$).



Measuring of Potentiometer



- Slide the wiper (change x_3) until the galvanometer shows zero again
- $V_{unknown} = x_3/x_1\ V_s$

  $= x_3/x_2\ V_{std\_cell}$

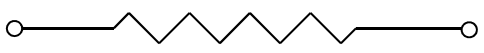
Rheostat

- Variable resistor with two terminals

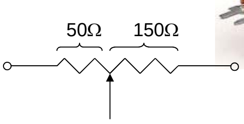
Loading for a Potentiometer

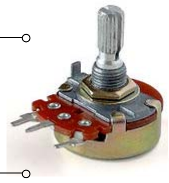
Resistor

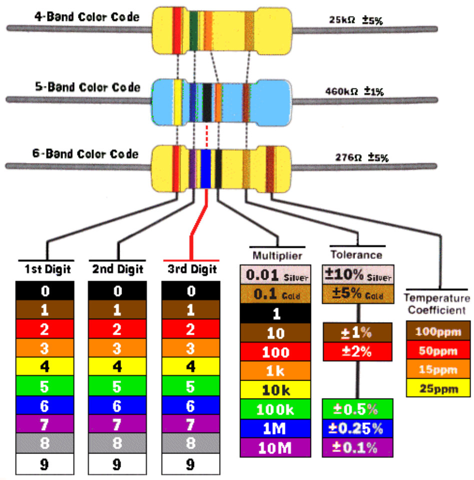
- Fixed



- Adjustable
  Potentiometer







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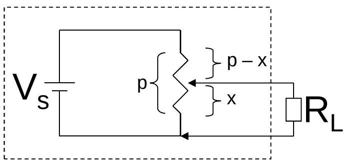
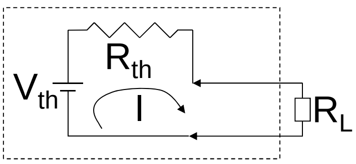
Loading of a Potentiometer (Cont'd)

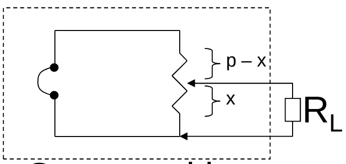




Thévenin's

Voltage Divider

$V_{th} = (x / p)\, V_s$



Superposition

$$\frac{1}{R_{th}} = \frac{1}{(x/p)R_p} + \frac{1}{((p-x)/p)R_p}$$

$$R_{th} = (x/p)R_p(1 - x/p)$$

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Loading of a Potentiometer (Cont'd)

$$V_{th} = I\,(R_{th}+R_L)$$

$$(x/p)V_s = I\,[\,(x/p)(1 - x/p)R_p + R_L\,]$$

$$I = \frac{(x/p)V_s}{(x/p)(1 - x/p)R_p + R_L}$$

$$V_L = IR_L = \frac{(x/p)V_sR_L}{(x/p)(1 - x/p)R_p + R_L}$$

$$= \frac{(x/p)V_s}{(R_p/R_L)(x/p)(1 - x/p) + 1}$$

It means the relationship between V_L and x is nonlinear!

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Loading of a Potentiometer (Cont'd)

Error,

$$V_{th} - V_L = (x/p)V_s\left(1 - \frac{1}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

$$= (x/p)V_s\left(\frac{(R_p/R_L)(x/p)(1 - x/p)}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

If $R_L \gg R_p$ then $R_p/R_L \approx 0$

Error $\approx (x/p)V_s(R_p/R_L)(x/p)(1 - x/p)$

$\approx V_s(R_p/R_L)[(x/p)^2 - (x/p)^3]$, $R_L \uparrow$ error $\downarrow$

% Error $\approx (R_p/R_L)[(x/p) - (x/p)^2] \times 100\%$

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Loading of a Potentiometer (Cont'd)

Let $d(\text{Error})/dx = V_s(R_p/R_L)[2x/p^2 - 3x^2/p^3] = 0$

We get $2x/p^2 = 3x^2/p^3$

$x/p = 2/3$ for the maximum

Therefore,

the max error $= V_s(R_p/R_L)[(2/3)^2 - (2/3)^3]$

$\approx 0.148\ V_s(R_p/R_L)$

and the max % error $\approx 22.2\ (R_p/R_L)$ %

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References

- https://www.allaboutcircuits.com/textbook/direct-current/chpt-8/voltmeter-impact-measured-circuit/

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