King Mongkut's University of Technology Thonburi

Midterm Examination 2/2010

CPE 222 Signals and Systems — **Computer Engineering Department**

Date: December 22, 2010 — **Time: 1:00 – 4:00 p.m.**

Instructions:

> **Violation of examination rules and regulations will not be tolerated. Serious violator could face dismissal charge.**

1. **Only one calculator and one ruler with mathematical formula are allowed** in the examination room.
2. **Books, documents, and notes are not allowed** in the examination room.
3. Carefully read the explanation in each problem and then answer each question.
4. **Do not take the examination sheets out** of the examination room.
5. Write your answers on the examination booklet(s).
6. This examination has **3 pages (7 problems, 100 points).**

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1. Determine the impulse response of these following systems:

   a) $H(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$ *(5 points)*

   b) $|H(e^{j\omega})| = \begin{cases} 1 & 0 \le |\omega| < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$ and $\measuredangle H(e^{j\omega}) = -\frac{3\omega}{2}$ *(5 points)*

2. Determine the convolution between these following signals: *(10 points)*

   $x(t) = u(t + 1) - u(t - 1)$ and $y(t) = \{u(t - 1) - u(t - 3)\} + 2\{u(t - 3) - u(t - 4)\}$.

3. a). Consider an LTI system with frequency response $H(j\omega) = \frac{1}{j\omega + 2}$. Determine the response of this system when the input is $x(t) = e^{-t}u(t)$. *(8 points)*

   b). Consider an LTI system having the response $s[n] = a^n u[n]$; $0 < a < 1$ when the input is unit step signal. Determine the impulse response of this system. *(7 points)*

4. Given an LTI system having the response: *(15 points)*

   $y(t) = [2e^{-t} - 2e^{-4t}]u(t)$ when the input: $x(t) = [e^{-t} + e^{-3t}]u(t)$.

Determine:

a) the impulse response of this system. *(8 points)*

b) the frequency response of this system. *(5 points)*

c) the differential equation described this system. *(2 points)*

5. The property of an LTI system can be represented by the following equation:

$$y(t) = 0.5(x(t) + x(t-1)).$$ *(20 points)*

Determine:

a) the magnitude and phase response of this system. *(10 points)*
b) the response of this system to the input $x(t) = 5$. *(3 points)*
c) the response of this system to the input $x(t) = \cos(0.5\pi t)$. *(3 points)*
d) the response of this system to the input $x(t) = e^{j\pi t}$. *(4 points)*

6. Consider a d-t system having the input x[n], the output y[n], and the Fourier transforms of these signals related by the following equation: *(15 points)*

$$Y(e^{j\omega}) = 2X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) - \frac{dX(e^{j\omega})}{d\omega}.$$

a) Determine the response of this system when $x[n] = \delta[n]$. *(5 points)*
b) Is this system linear? Justify your answer with rational explanations. *(5 points)*
c) Is this system time invariant? Justify your answer with rational explanations. *(5 points)*

7. Figure 7(a) illustrates the response $y_0[n]$ and the input $x_0[n]$ of a stable LTI system. Determine the impulse response and the response of this system when the input is $x_1[n]$ in Figure P7(b). *(15 points)*

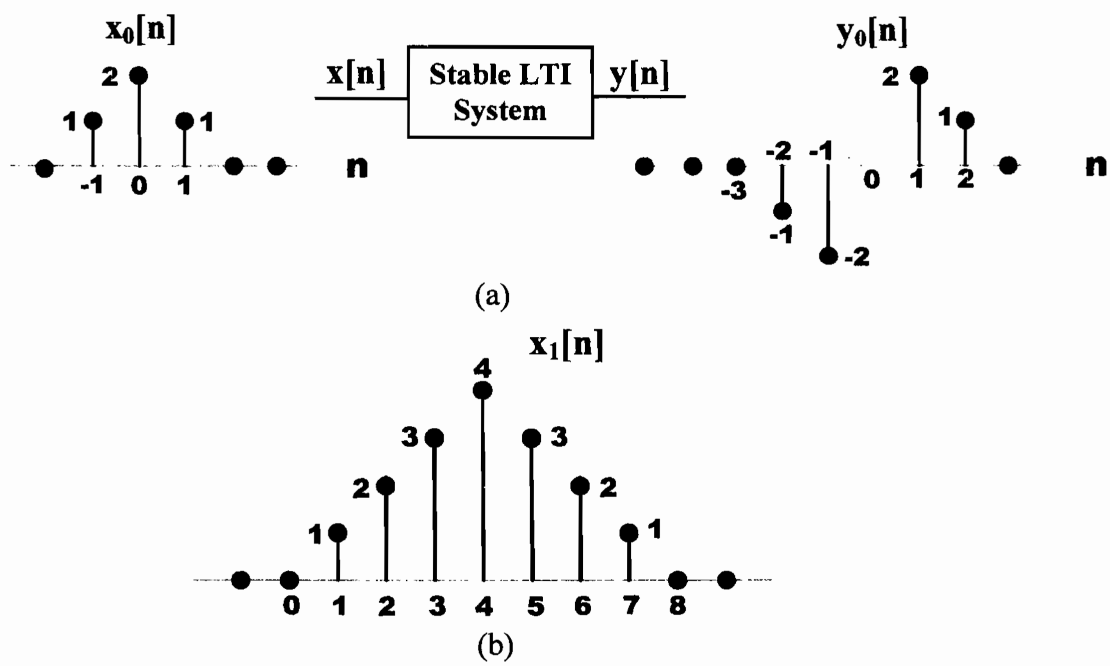

Figure P7 (a) The input – output pair for problem 7 and (b) the given $x_1[n]$

Note:

Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\Omega_0 t} \quad \text{and} \quad X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\Omega_0 t}dt$$

Discrete-Time Fourier Series:

$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\omega_0 n} \quad \text{and} \quad X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\omega_0 n}$$

Fourier Transform:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad \text{and} \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Discrete-Time Fourier Transform:

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n} \quad \text{and} \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

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TABLE 1 PROPERTIES OF THE FOURIER TRANSFORM

Property	Aperiodic Signal	Fourier Transform
	$x(t)$, $y(t)$, $h(t)$	$X(j\Omega)$, $Y(j\Omega)$, $H(j\Omega)$
Linearity	$ax(t) + by(t)$	$aX(j\Omega) + bY(j\Omega)$
Time Shifting	$x(t-t_0)$	$e^{-j\Omega t_0} X(j\Omega)$
Frequency Shifting	$e^{j\Omega_0 t} x(t)$	$X(j(\Omega-\Omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\Omega)$
Time Reversal	$x(-t)$	$X(-j\Omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{j\Omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\Omega)Y(j\Omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)Y(j(\Omega-\theta))d\theta$
Differentiation in Time	$\frac{dx(t)}{dt}$	$j\Omega X(j\Omega)$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\Omega}X(j\Omega) + \pi X(0)\delta(\Omega)$
Differentiation in Frequency	$t\,x(t)$	$j\frac{d}{d\Omega}X(j\Omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\Omega) = X^*(-j\Omega) \\ \Re e\{X(j\Omega)\} = \Re e\{X(-j\Omega)\} \\ \Im m\{X(j\Omega)\} = -\Im m\{X(-j\Omega)\} \\ \lvert X(j\Omega) \rvert = \lvert X(-j\Omega) \rvert \\ \sphericalangle X(j\Omega) = \sphericalangle X(-j\Omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\Omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\Omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = Ev\{x(t)\}$ [$x(t)$ real]	$\Re e\{X(j\Omega)\}$

$x_o(t) = Od\{x(t)\}$ [x(t) real]	$j\Im m\{X(j\Omega)\}$

Parseval's Relation for Aperiodic Signal

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\Omega)|^2 \, d\Omega$$

TABLE 2 BASIC FOURIER TRANSFORM PAIRS

Signal	**Fourier Transform**	**Fourier Series Coefficients (if periodic)**
$\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - k\Omega_0)$	a_k
$e^{j\Omega_0 t}$	$2\pi\delta(\Omega-\Omega_0)$	$a_1 = 1$ $a_k = 0$, Otherwise
$\cos(\Omega_0 t)$	$\pi[\delta(\Omega-\Omega_0) + \delta(\Omega+\Omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, Otherwise
$\sin(\Omega_0 t)$	$\frac{\pi}{j}[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, Otherwise
$x(t) = 1$	$2\pi\delta(\Omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (This is the Fourier series representation for any choice at $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1 & \lvert t\rvert \le T_1 \\ 0 & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2\sin k\Omega_0 T_1}{k} \delta(\Omega - k\Omega_0)$	$\frac{\Omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\Omega_0 T_1}{\pi}\right) =$ $\frac{\sin k\Omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k

$e^{-at}u(t), \quad Re\{a\} > 0$	$\dfrac{1}{a + j\Omega}$	
$x(t) = \begin{cases} 1 & \lvert t \rvert < T_1 \\ 0 & \lvert t \rvert > T_1 \end{cases}$	$\dfrac{2\sin \Omega T_1}{\Omega}$	
$\dfrac{\sin Wt}{\pi t}$	$X(j\Omega) = \begin{cases} 1 & \lvert\Omega\rvert < W \\ 0 & \lvert\Omega\rvert > W \end{cases}$	
$\delta(t)$	1	
$u(t)$	$\dfrac{1}{j\Omega} + \pi\delta(\Omega)$	
$t\,e^{-at}u(t), \quad Re\{a\} > 0$	$\dfrac{1}{(a + j\Omega)^2}$	
$\dfrac{t^{n-1}}{(n-1)!}e^{-at}u(t), \ Re\{a\} > 0$	$\dfrac{1}{(a + j\Omega)^n}$	

TABLE 3 BASIC DISCRETE-TIME-FOURIER TRANSFORM PAIRS

Signal	**Fourier Transform**	**Fourier Series Coefficients (if periodic)**
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \dfrac{2\pi m}{N}$ $a_k = \begin{cases} 1 & k = m, m \pm N, m \pm 2N.. \\ 0 & \text{otherwise} \end{cases}$ (b) $\dfrac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$\cos(\omega_0 n)$	$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \dfrac{2\pi m}{N}$

		$a_k = \begin{cases} \frac{1}{2} & k = m, m \pm N, m \pm 2N\ldots \\ 0 & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$\sin(\omega_0 n)$	$\frac{\pi}{j}\sum_{l=-\infty}^{\infty}\{\delta(\omega-\omega_0-2\pi l)-\delta(\omega+\omega_0-2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2j} & k = m, m \pm N, m \pm 2N \\ 0 & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\rightarrow$ The signal is aperiodic.
$x[n] = 1$	$2\pi\sum_{l=-\infty}^{\infty}\delta(\omega-2\pi l)$	$a_k = \begin{cases} 1 & k = 0, \pm N, \pm 2N\ldots \\ 0 & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1 & \lvert n\rvert \le N_1 \\ 0 & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi\sum_{k=-\infty}^{\infty} a_k\delta\left(\omega-\frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[2\pi/N)(N_1+1/2)}{N\sin[2\pi k/2N]}$, $k \ne 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1+1}{N}$, $k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{\infty}\delta[n-kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n]$, $\lvert a\rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$	
$x[n] = \begin{cases} 1 & \lvert n\rvert \le N_1 \\ 0 & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1+1/2)}{\sin(\omega/2)}$	
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1 & 0 \le \lvert\omega\rvert \le W \\ 0 & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(e^{j\omega})$ periodic with period 2π .	

$\delta[n]$	1	
$u[n]$	$\frac{1}{1-e^{j\omega}}+$ $\sum_{k=-\infty}^{\infty}\pi\delta(\omega-2\pi k)$	
$(n+1)a^{n}u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^{2}}$	
$\frac{(n+r-1)!}{n!(r-1)!}a^{n}u[n], \quad \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^{r}}$	