

ENE/EIE 211 : Electronic Devices
and Circuit Design II
Lecture 11: Stability and Oscillator
Circuits

4. Amplifier with a two-pole response

Consider an amplifier whose open-loop transfer function is characterized by 2 real-axis poles:

$$A(s) = \frac{A_0}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})} \quad \dots (1)$$

In this case, the closed-loop poles are obtained from $1 + A(s)\beta = 0$, which lead to

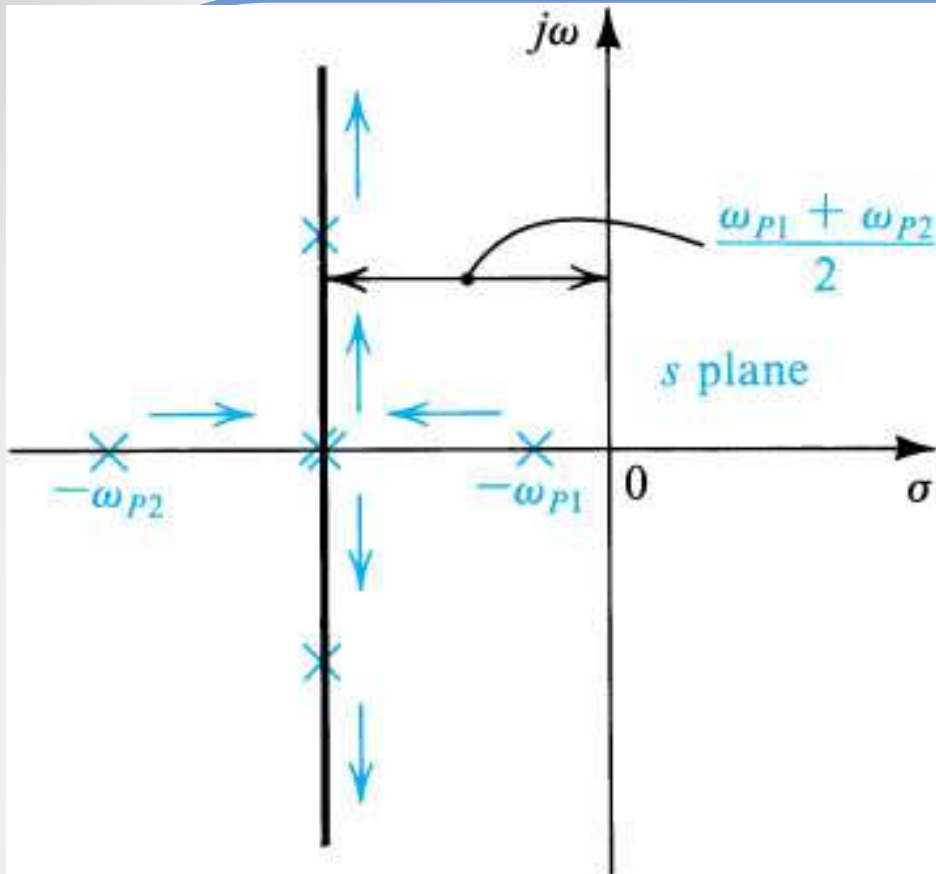
$$s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_0\beta)\omega_{P1}\omega_{P2} = 0 \quad \dots (2)$$

Thus, the closed-loop poles are given by

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}} \quad \dots (3)$$

From eq (3):

- When the loop gain $A_0\beta = 0$, we'll get $s_1, s_2 = -\omega_{P1}, -\omega_{P2}$
- When the loop gain $A_0\beta$ increases to the point where $(\omega_{P1} + \omega_{P2})^2 = 4(1 + A_0\beta)\omega_{P1}\omega_{P2}$, then $s_1, s_2 = -0.5(\omega_{P1} + \omega_{P2})$. This means the two poles are at the same location.
- When the loop gain $A_0\beta$ increases further, the term $(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}$ would become negative, making the poles be complex conjugate pair.
- Therefore, as the loop gain $A_0\beta$ increases from 0, the poles are brought together, then coincide and become complex conjugate and move along a vertical line.



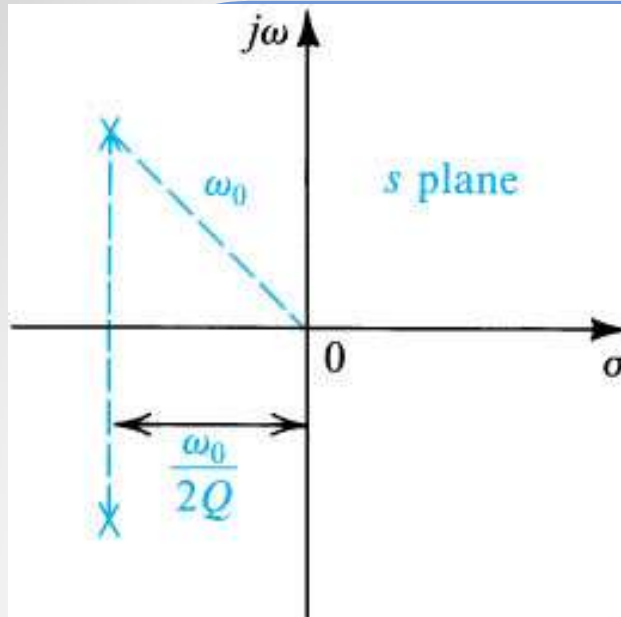
The plot that shows locus of the poles for increasing loop gain is called a root-locus diagram. From this diagram, we also see that the feedback amp is unconditionally stable.

As is the case with 2nd order responses generally, the closed-loop response can show a peak. The characteristic eqn of 2nd order network can be written in standard form as

$$s^2 + s \frac{\omega_o}{Q} + \omega_o^2 = 0$$

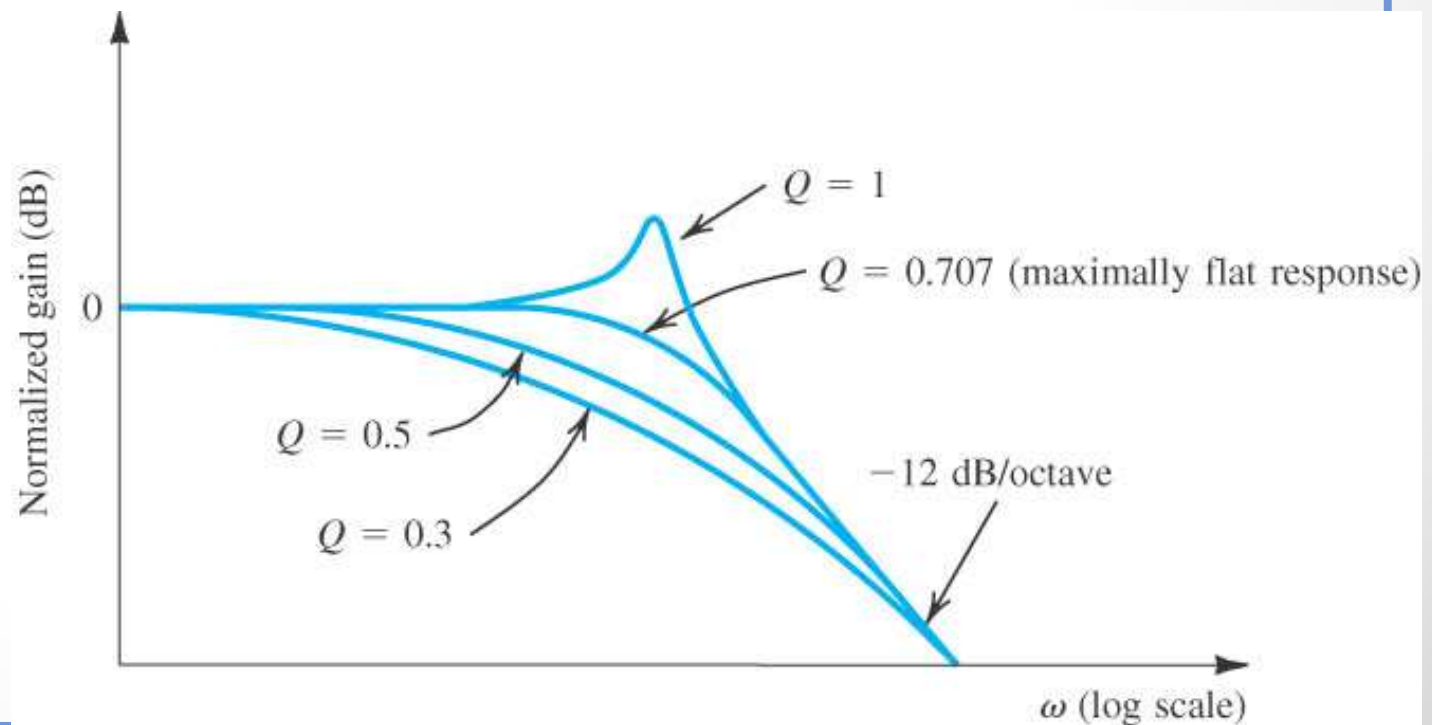
where ω_o is called the **pole frequency** and Q is called **pole Q factor**. The poles are complex if Q is greater than 0.5.

$$Q = \frac{\sqrt{(1 + A_0 \beta) \omega_{P1} \omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

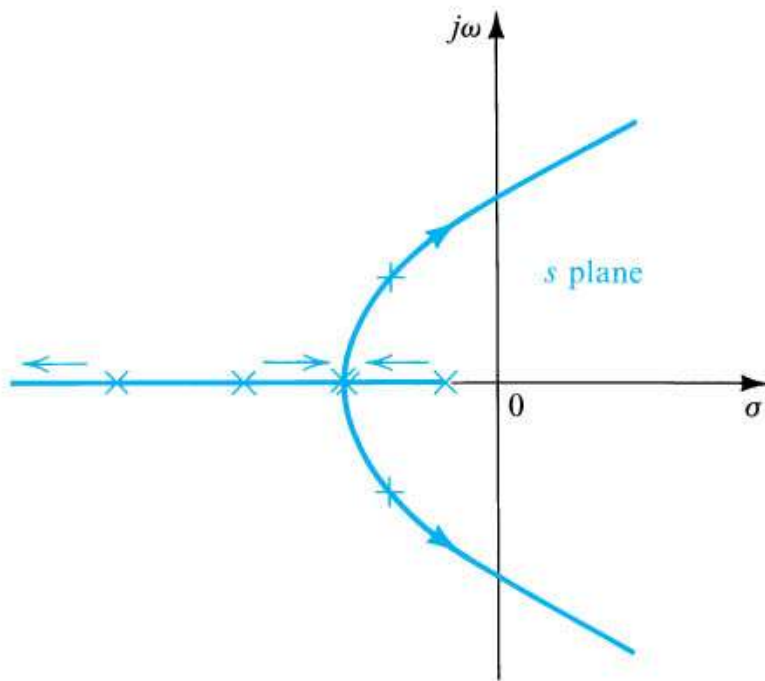


ω_0 is the radial distance of the poles from the origin and Q indicates the distance of the poles from the $j\omega$ axis. Poles on the $j\omega$ axis have $Q = \infty$.

The response of the feedback amplifier shows no peaking for $Q \leq 0.707$ or $Q \leq 1/\sqrt{2}$. The boundary case ($Q = 0.707$) (poles at 45° angles) results in **the maximally flat response**.



5. Amplifier with 3 or more poles

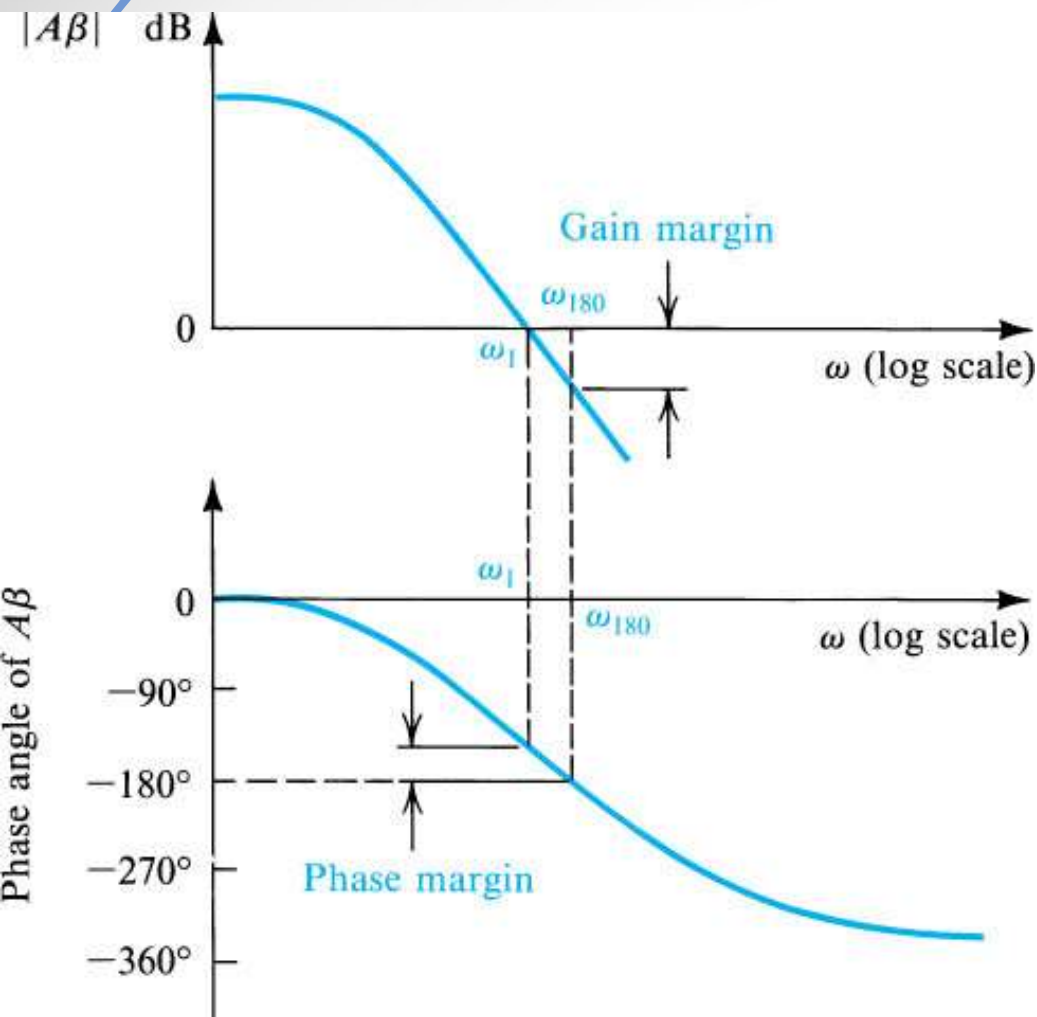


From the root-locus diagram, as the loop gain increases from zero, the highest-freq pole moves outward while the 2 other poles are brought closer together. As $A_o\beta$ increases further, the two poles become coincident and then become complex and conjugate.

A value of $A_o\beta$ exists at which this pair of complex conjugate poles enters the right half of the s plane, thus causing the amp to become unstable.

Thus, one can always maintain amp stability by keeping the loop gain $A_o\beta$ smaller than the value corresponding to the poles entering the right half plane.

Gain and Phase Margins



The difference between the value of $|A\beta|$ at ω_{180} (the freq of 180° phase shift) and unity, called the gain margin, is usually expressed in decibels.

The gain margin represents the amount by which the loop gain can be increased while stability is maintained.

The difference between the phase angle at $|A\beta| = 1$ and 180° is termed the phase margin. If at the freq where $|A\beta| = 1$, the phase angle is less than 180° , then the amplifier is stable.

An alternative approach for investigating stability

We can investigate the network stability by constructing a Bode plot for the open-loop gain $20\log|A(j\omega)|$ and $20\log|1/\beta|$ which is a straight line, assuming that β is independent of frequency. The difference between the two curves will be

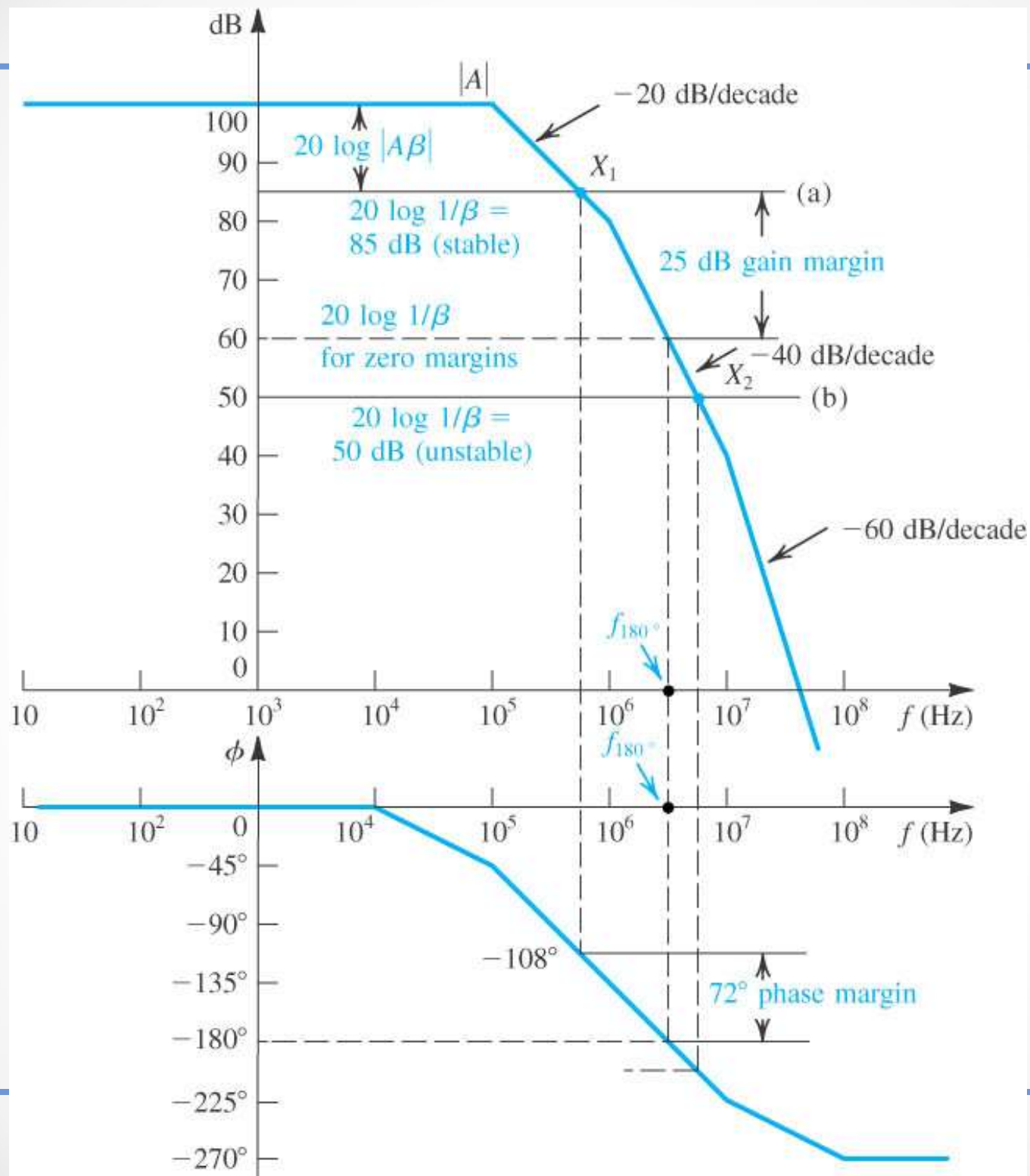
$$20\log|A(j\omega)| - 20\log\frac{1}{\beta} = 20\log|A\beta|$$

which is the loop gain (in dB). We may therefore study stability by examining the difference between the two plots.

For example, consider an amplifier whose open-loop transfer function is characterized by three poles at 0.1 MHz, 1 MHz and 10MHz, as shown on the next page. Note that because the poles are widely spaced, the phase is approximately -45° at the first pole freq, -135° at the 2nd pole freq and -225° at the 3rd. The freq at which the phase of $A(j\omega)$ is -180° lies on the -40-dB/decade segment. The amplitude and phase of the open-loop gain of this amp can be expressed as

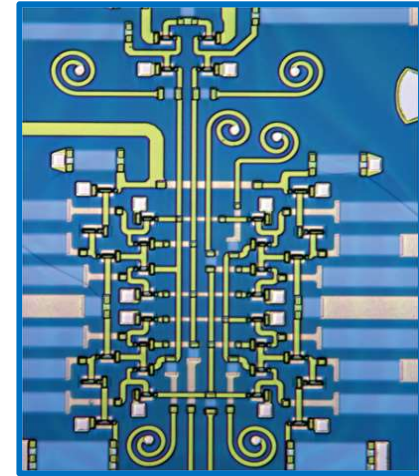
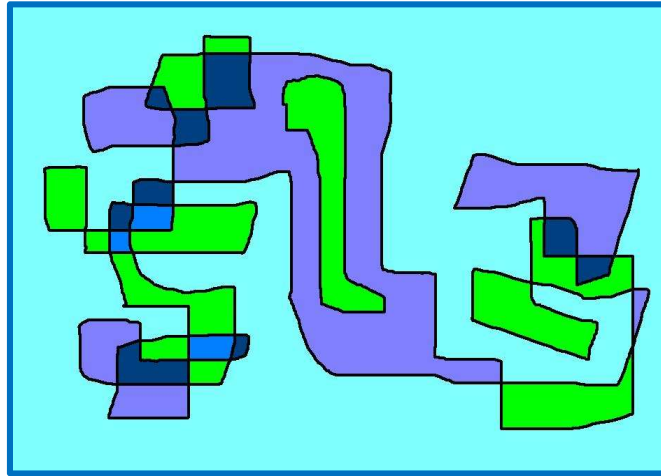
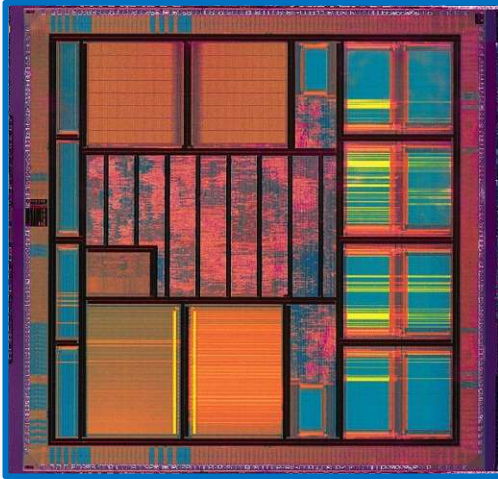
$$A(f) = \frac{10^5}{(1 + jf/10^5)(1 + jf/10^6)(1 + jf/10^7)}$$

$$\phi(f) = -[\tan^{-1}(f/10^5) + \tan^{-1}(f/10^6) + \tan^{-1}(f/10^7)]$$



We can either use the plot or the equations to help estimate where the frequency f_{180} at which the phase angle is 180° . The f_{180} is found to be at 3.34×10^6 Hz and at this freq, the magnitude is 58.2 dB.

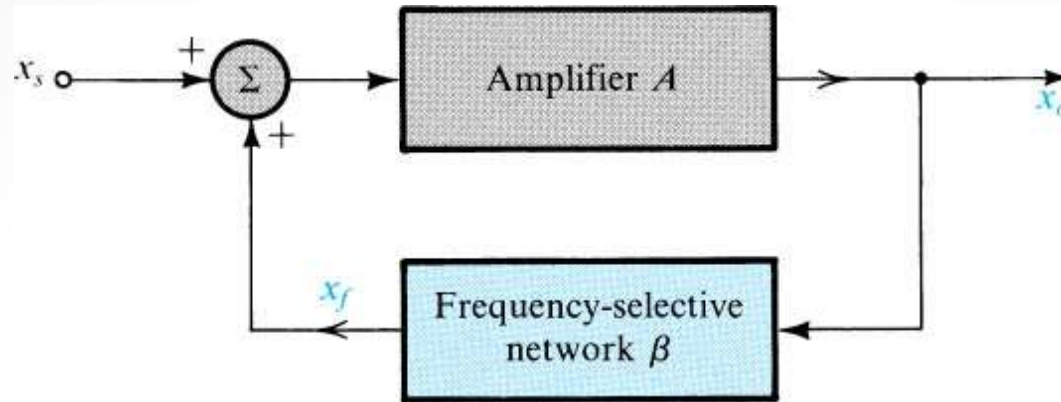
Consider next the straight line labeled (a) which represents a feedback factor having $20\log(1/\beta) = 85$ dB, which corresponds to $\beta = 5.623 \times 10^{-5}$. Since the loop gain is the difference between the $|A|$ curve and the $1/\beta$ line, the point of intersection X_1 corresponds to the freq at which $|A\beta| = 1$. From the inspection of the plot, this freq is 5.6×10^5 Hz. At this freq, the phase angle is approximately -108° . Thus, the closed-loop amplifier for which $20\log(1/\beta) = 85$ dB will be stable with a phase margin of 72° . The gain margin can be obtained from the plot; it is 25 dB.



ENE/EIE 211 : Electronic Devices and Circuit Design II

Oscillator Circuits

Oscillators



The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network. In an actual oscillator circuit, no input signal will be present; here an input signal x_s is employed to help explain the principle of operation.

- * **Feedback amplifier but frequency dependent feedback**

$$A_f(s) = \frac{A(s)}{1 + \beta_f(s)A(s)}$$

- * **Positive feedback, i.e. $\beta_f(s)A(s) < 0$**

- * **Oscillator gain defined by** $A_f(s) = \frac{A(s)}{1 - \beta_f(s)A(s)}$

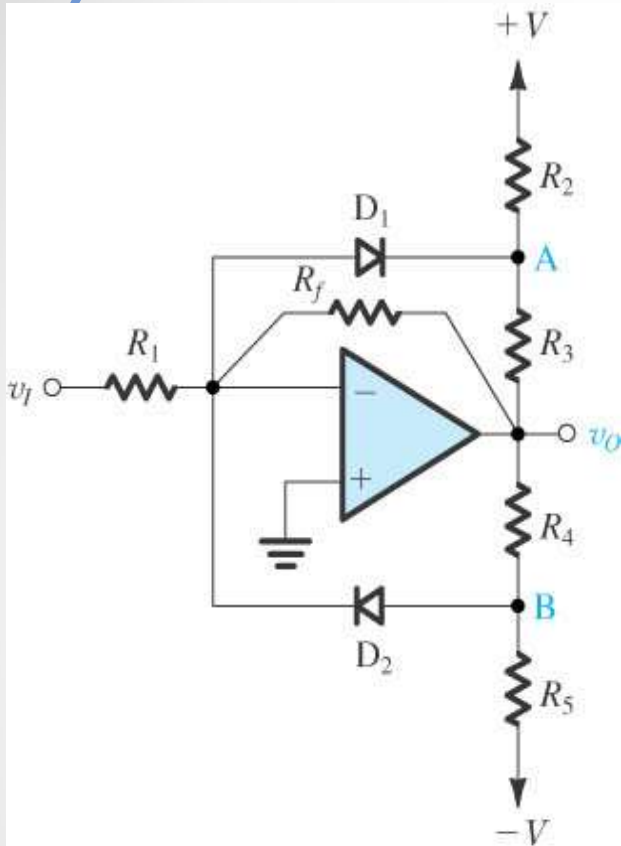
- * **Oscillation condition at $\omega = \omega_o$ (Barkhausen's criterion) $A_f(\omega_o) = \infty$**

$$L(\omega_o) = \beta_f(\omega_o)A(\omega_o) = \left| \beta_f(\omega_o)A(\omega_o) \right| e^{j\phi(\omega_o)} = 1$$

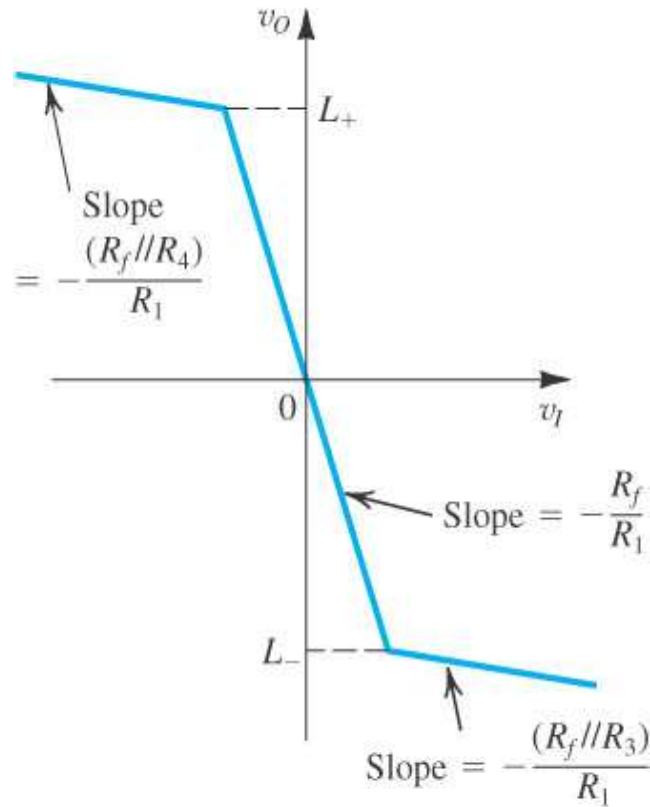
$$\phi(\omega_o) = \text{phase of } \beta_f(\omega_o)A(\omega_o)$$

Nonlinear Amplitude Control

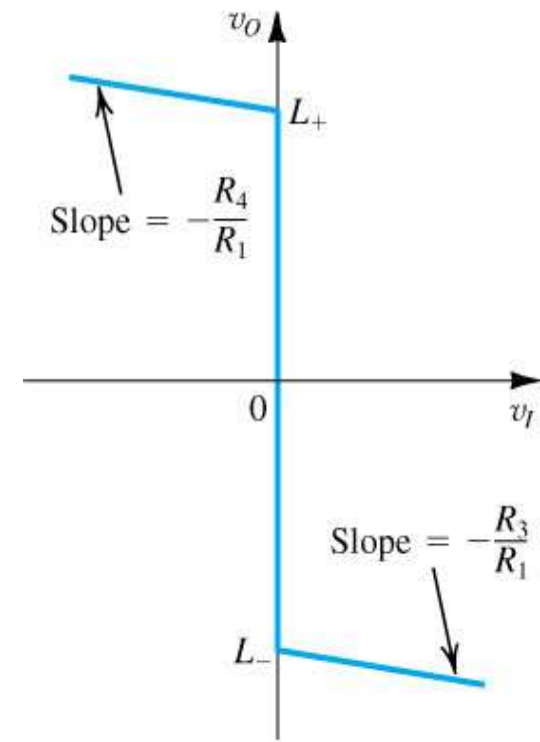
The oscillation condition, the Barkhausen criterion, guarantees sustained oscillations in a mathematical sense. However, the parameters of any physical system cannot be maintained constant for any length of time. In other words, suppose $A\beta = 1$ at $\omega = \omega_0$ and then the temperature changes and $A\beta$ becomes slightly less than unity. Oscillations will cease in this case. Conversely, if $A\beta$ exceeds unity, oscillations will grow in amplitude. We therefore need a mechanism for forcing $A\beta$ to remain equal to unity at the desired value of output amplitude. This task is accomplished by providing a nonlinear circuit for gain control. A popular limiter circuit is shown on the next page.



(a)

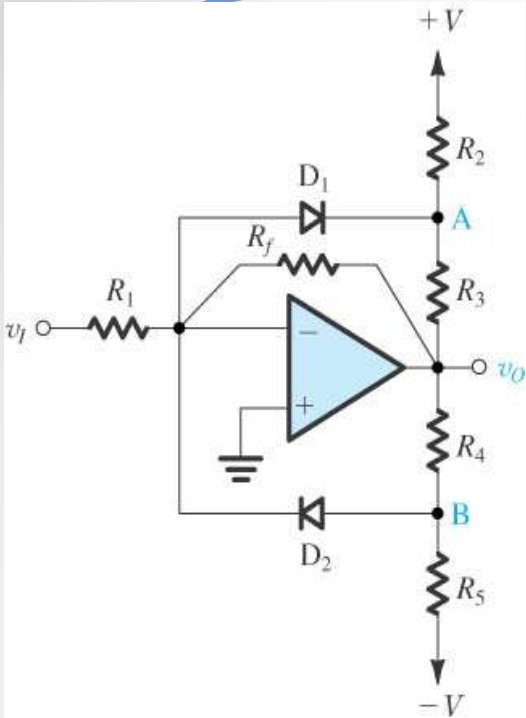


(b)



(c)

(a) A popular limiter circuit. **(b)** Transfer characteristic of the limiter circuit. **(c)** When R_f is removed, the limiter turns into a comparator with the characteristic shown.



(a)

Consider in the case small $v_i (\approx 0)$ and a small v_o so that v_A is positive and v_B is negative. Both diodes D_1 and D_2 will be off.

Thus,
$$v_o = -(R_f / R_1)v_i \quad \dots (1)$$

This is the linear portion of the limiter transfer characteristic. We now can use superposition to find the voltages at nodes A and B in terms of $\pm V$ and v_o as

$$v_A = V \frac{R_3}{R_3 + R_2} + v_o \frac{R_2}{R_3 + R_2} \quad \dots (2)$$

$$v_B = -V \frac{R_4}{R_4 + R_5} + v_o \frac{R_5}{R_4 + R_5} \quad \dots (3)$$

As v_i goes positive, v_o goes negative (from (1)) and v_B becomes more negative (from (3)), thus keeping D_2 off. From (2), v_A also becomes less positive and as v_i increases to the point where v_A becomes -0.7 V (called V_D), diode D_1 conducts. To find the value of v_o at which D_1 conducts (called this v_o as " L_- "), we'll get the negative limiting level given by

$$L_- = -V \frac{R_3}{R_2} - V_D \left(1 + \frac{R_3}{R_2} \right)$$

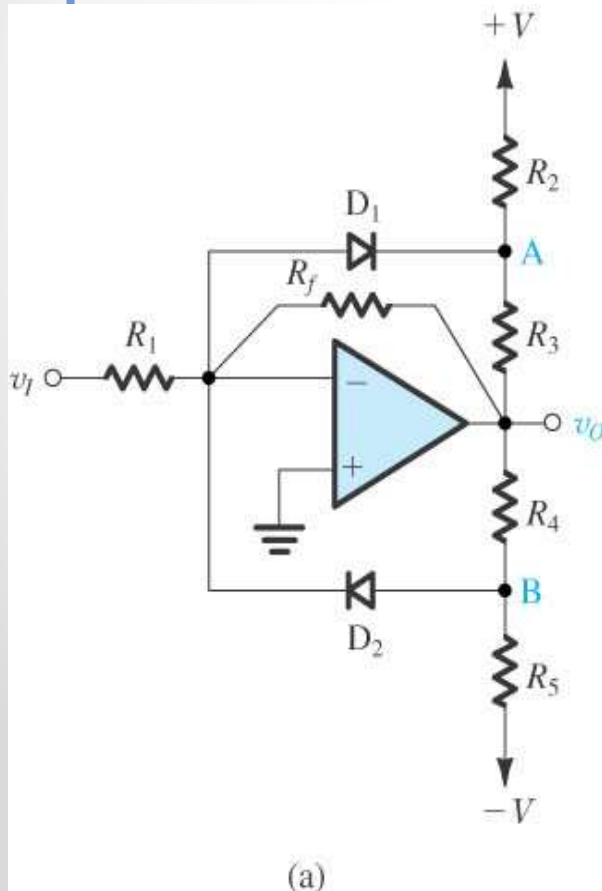
The corresponding value of v_i can be found by dividing L_- by the limiter gain $-R_f/R_1$. As v_i is increased, more current flows through the diode D_1 and v_A remains at approximately constant. R_3 appears in effect in parallel with R_f and the incremental gain is $-(R_f || R_3)/R_1$.

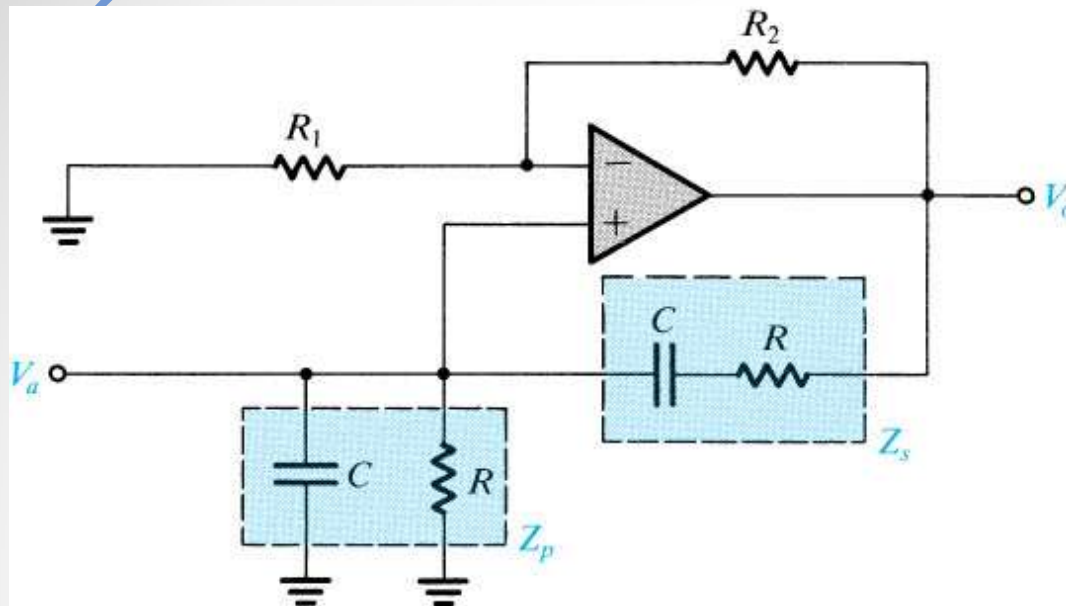
The transfer characteristic for the negative v_i can be found in a similar manner. The positive limiting level L_+ can be found to be

$$L_+ = V \frac{R_4}{R_5} + V_D \left(1 + \frac{R_4}{R_5} \right)$$

and the slope of the transfer characteristic in the positive limiting region is $-(R_f \parallel R_4)/R_1$.

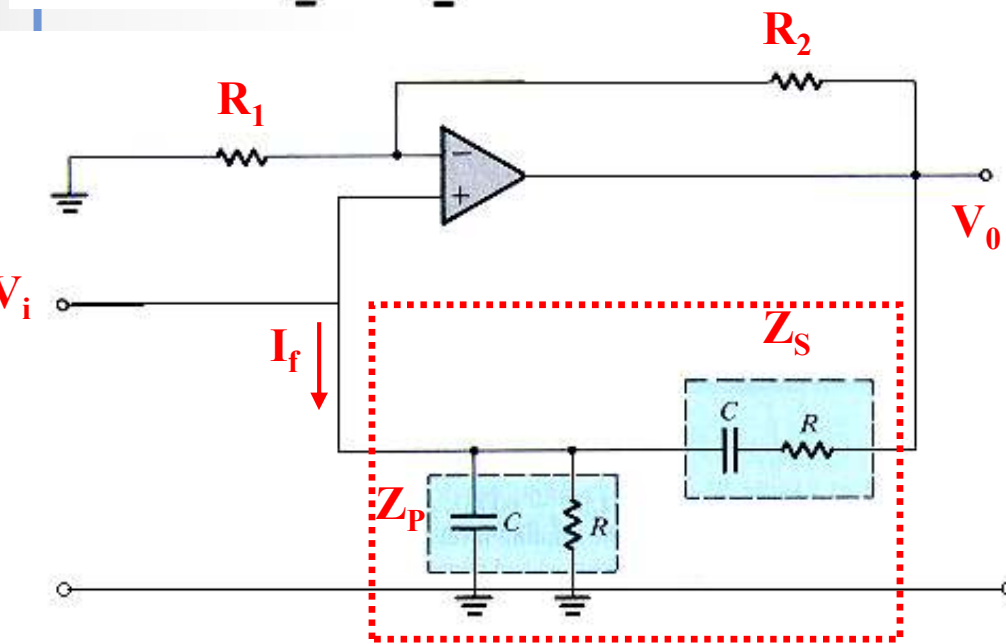
Finally, increasing R_f results in higher gain the linear region. Removing R_f altogether results in transfer characteristic of a comparator. That is, the ckt compares v_i with the comparator reference value of 0 V: $v_i > 0$ results in $v_o = L_-$, $v_i < 0$ results in $v_o = L_+$.

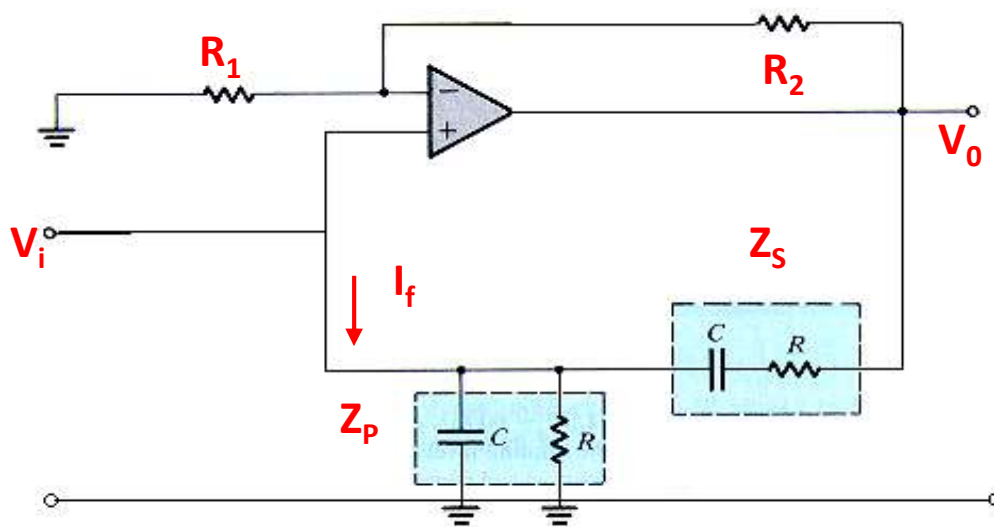




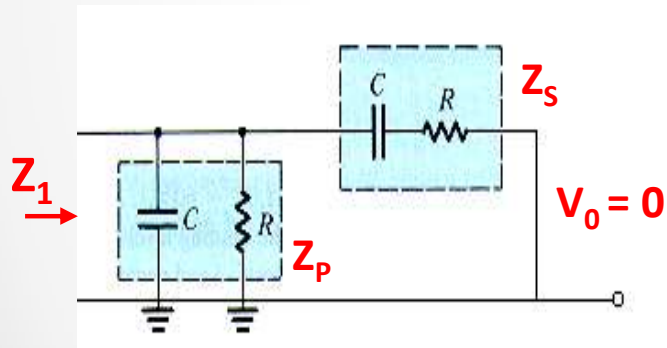
Wien Bridge Oscillator

- * Based on op amp
- * Combination of R's and C's in feedback loop so feedback factor β_f has a frequency dependence.
- * Analysis assumes op amp is ideal.
 - ★ Gain A is very large
 - ★ Input currents are negligibly small ($I_+ \approx I_- \approx 0$).
 - ★ Input terminals are virtually shorted ($V_+ \approx V_-$).
- * Analyze like a normal feedback amplifier.
 - ★ Determine input and output loading.
 - ★ Determine feedback factor.
 - ★ Determine gain with feedback.
- * Shunt-shunt configuration.





Input Loading



$$Z_1 = Z_p \parallel Z_i = \left[\frac{1}{Z_p} + \frac{1}{Z_i} \right]^{-1} =$$

$$\left[\frac{1+sCR}{R} + \frac{sC}{1+sCR} \right]^{-1} = \frac{R(1+sCR)}{sCR + (1+sCR)^2}$$

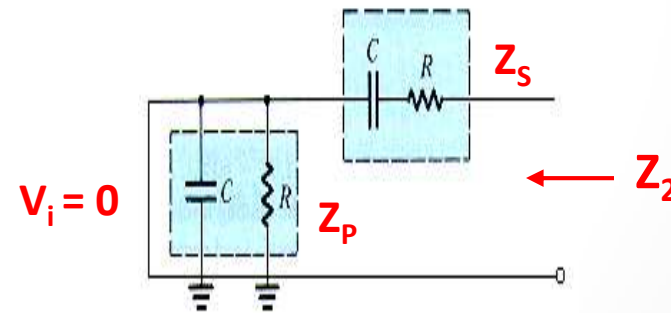
Define

$$Z_s = R + Z_C = R + \frac{1}{sC} = \frac{1+sRC}{sC}$$

$$Z_p = R \parallel Z_C = \left(\frac{1}{R} + \frac{1}{Z_C} \right)^{-1} = \left(\frac{1}{R} + sC \right)^{-1}$$

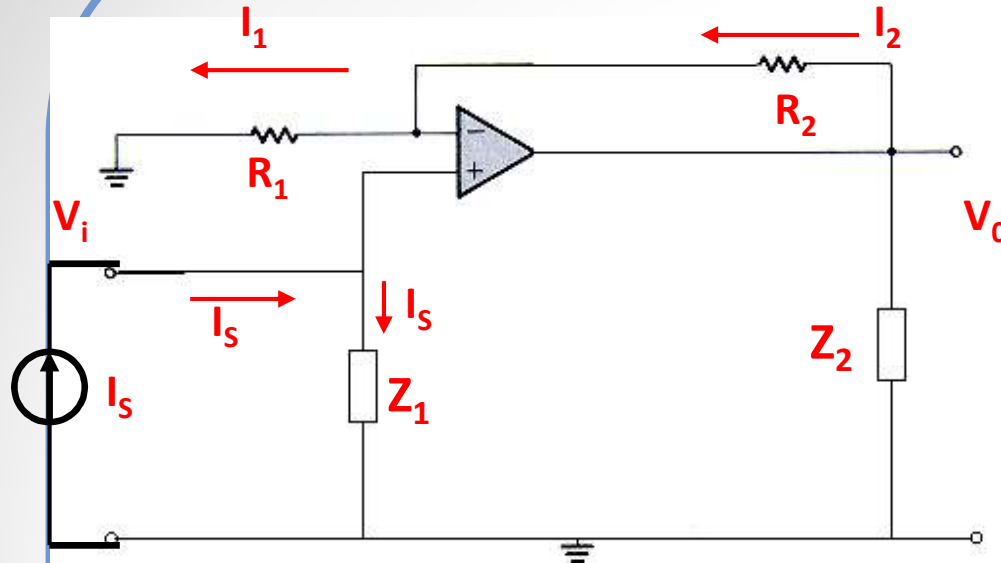
$$= \frac{R}{1+sCR}$$

Output Loading



$$Z_2 = Z_s = R + Z_C = \frac{1+sRC}{sC}$$

Wien Bridge Oscillator



Amplifier gain including loading effects

$$A_r = \frac{V_0}{I_s} = \frac{V_0}{V_i} \frac{V_i}{I_s}$$

To get $\frac{V_0}{V_i}$, we use $I_1 = I_2 = \frac{V_0}{R_1 + R_2}$ and

$$V_i = V_+ = V_- = I_1 R_1 = \frac{V_0}{R_1 + R_2} R_1 \quad \text{so}$$

$$\frac{V_0}{V_i} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

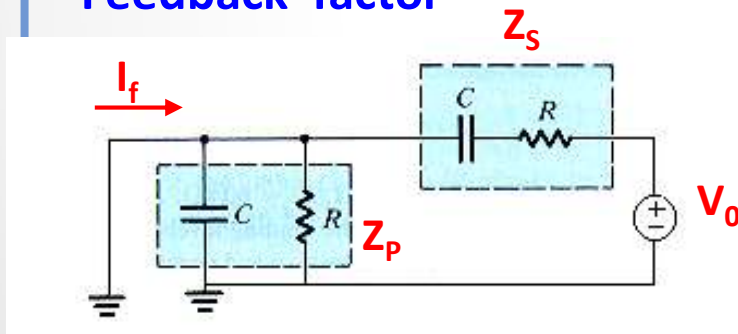
Since $I_+ = 0$, $\frac{V_i}{I_s} = Z_1$ and

$$A_r = \frac{V_0}{V_i} \frac{V_i}{I_s} = Z_1 \left(1 + \frac{R_2}{R_1} \right)$$

where $Z_1 = \frac{R(1 + sCR)}{sCR + (1 + sCR)^2}$ so

$$A_r = \left(1 + \frac{R_2}{R_1} \right) \frac{R(1 + sCR)}{sCR + (1 + sCR)^2}$$

Feedback factor



$$\begin{aligned} \beta_f &= \frac{X_f}{X_o} = \frac{I_f}{V_o} = -\frac{1}{Z_s} \\ &= -\frac{sC}{1 + sRC} \end{aligned}$$

Wien Bridge Oscillator

Oscillation condition

Phase of $\beta_f A_r$ equal to 180° . It already is since $\beta_f A_r < 0$.

Then need only $|\beta_f A_r| = \left(1 + \frac{R_2}{R_1}\right) \frac{sCR}{sCR + (1 + sCR)^2} = 1$

Rewriting

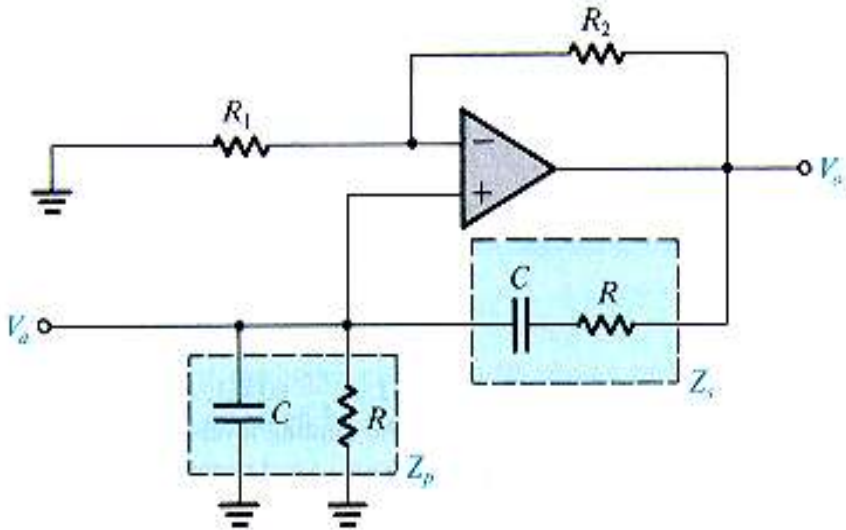
$$\begin{aligned} |\beta_f A_r| &= \left(1 + \frac{R_2}{R_1}\right) \frac{sCR}{sCR + (1 + sCR)^2} \\ &= \left(1 + \frac{R_2}{R_1}\right) \frac{sCR}{sCR + (1 + 2sCR + s^2 C^2 R^2)} \\ &= \left(1 + \frac{R_2}{R_1}\right) \frac{sCR}{1 + 3sCR + s^2 C^2 R^2} = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + \frac{1}{sCR} + sCR} \\ &= \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)} \end{aligned}$$

Then imaginary term = 0 at the oscillation frequency

$$\omega = \omega_o = \frac{1}{RC}$$

Then, we can get $|\beta_f A_r| = 1$ by selecting the resistors R_1 and R_2 appropriately using

$$\left(1 + \frac{R_2}{R_1}\right) \frac{1}{3} = 1 \quad \text{or} \quad \frac{R_2}{R_1} = 2$$



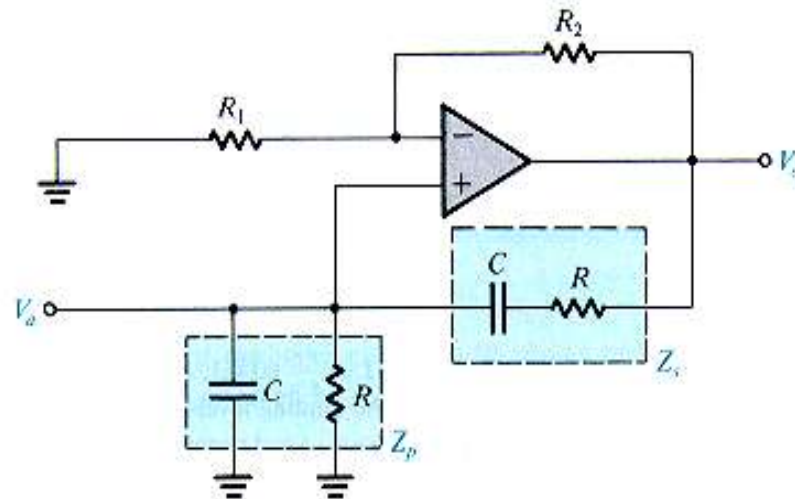
Loop Gain

$$\begin{aligned} \beta_f A_r &= \left(-\frac{sC}{1 + sCR}\right) A_r \\ &= \left(-\frac{sC}{1 + sCR}\right) \left(1 + \frac{R_2}{R_1}\right) \frac{R(1 + sCR)}{sCR + (1 + sCR)^2} \\ &= -\left(1 + \frac{R_2}{R_1}\right) \frac{sCR}{sCR + (1 + sCR)^2} \end{aligned}$$

Gain with feedback is

$$A_{rf} = \frac{A_r}{1 + \beta_f A_r}$$

Wien Bridge Oscillator - Example



Oscillator specifications: $\omega_o = 1 \times 10^6 \text{ rad/s}$

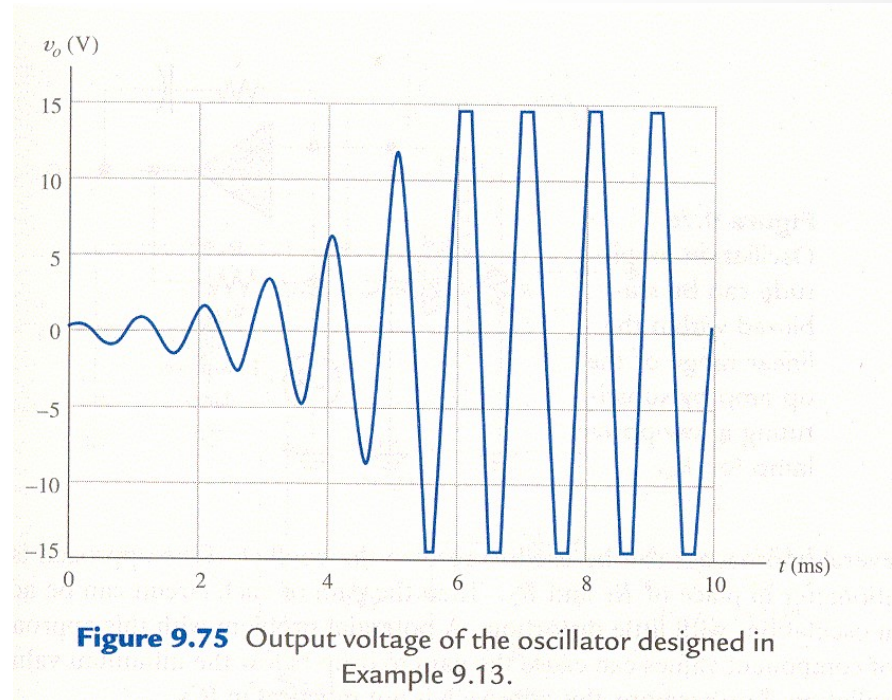
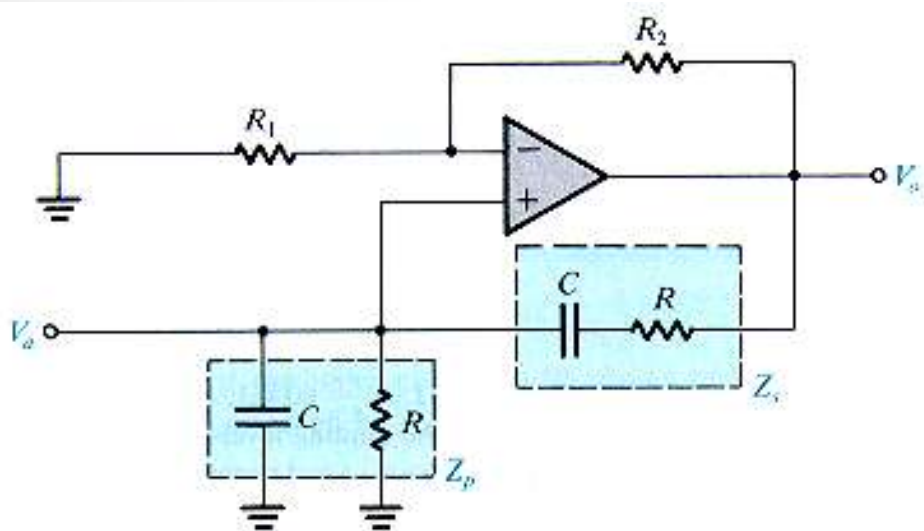
Selecting for convenience $C = 10 \text{ nF}$, then from $\omega_o = \frac{1}{RC}$

$$R = \frac{1}{\omega_o C} = \frac{1}{10 \text{ nF} (1 \times 10^6 \text{ rad/s})} = 100 \text{ } \Omega$$

Choosing $R_1 = 10 \text{ K}$, then since $R_2 = 2 R_1$ we get

$$R_2 = 2(10 \text{ K}) = 20 \text{ K}$$

Wien Bridge Oscillator



Final note: *No input signal is needed.* Noise at the desired oscillation frequency will likely be present at the input and when picked up by the oscillator when the DC power is turned on, it will start the oscillator and the output will quickly buildup to an acceptable level.

- * Once oscillations start, a **limiting circuit** is needed to prevent them from growing too large in amplitude

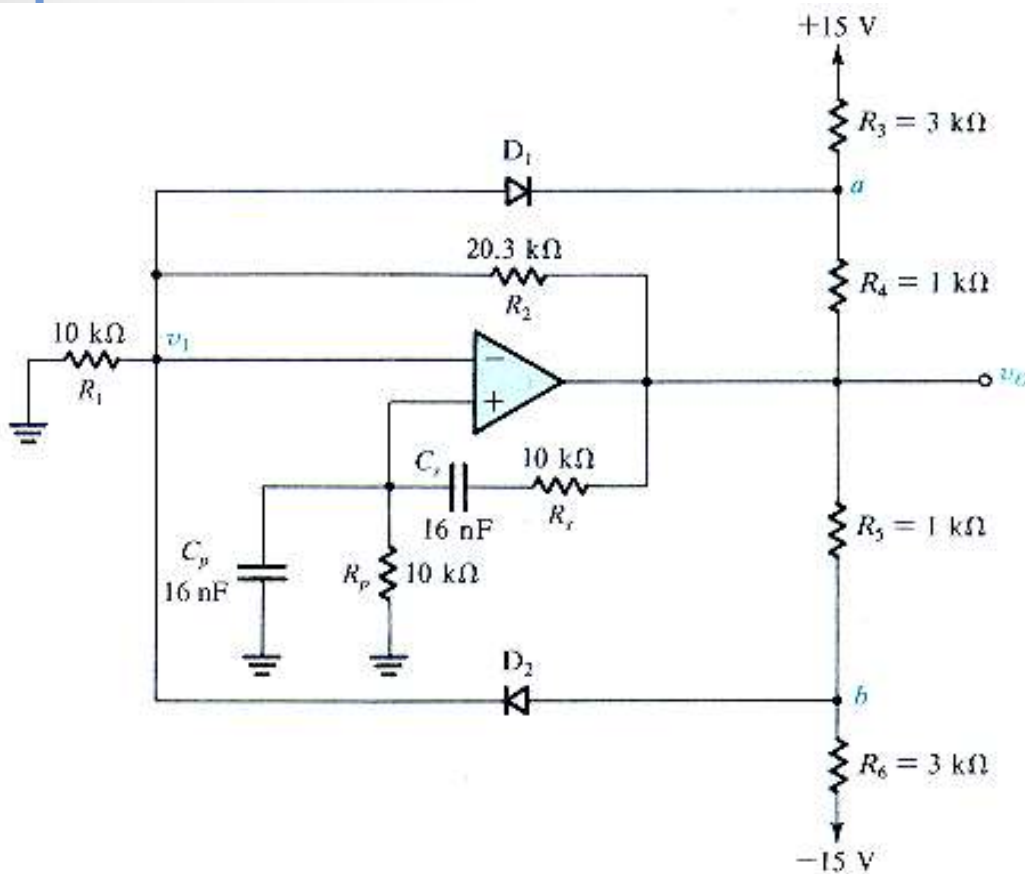


Fig. 12.5 A Wien-bridge oscillator with a limiter used for amplitude control.

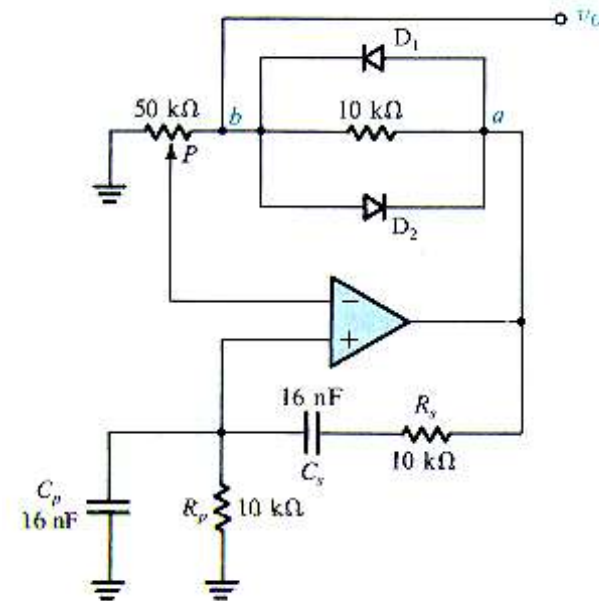


Fig. 12.6 A Wien-bridge oscillator with an alternative method for amplitude stabilization.

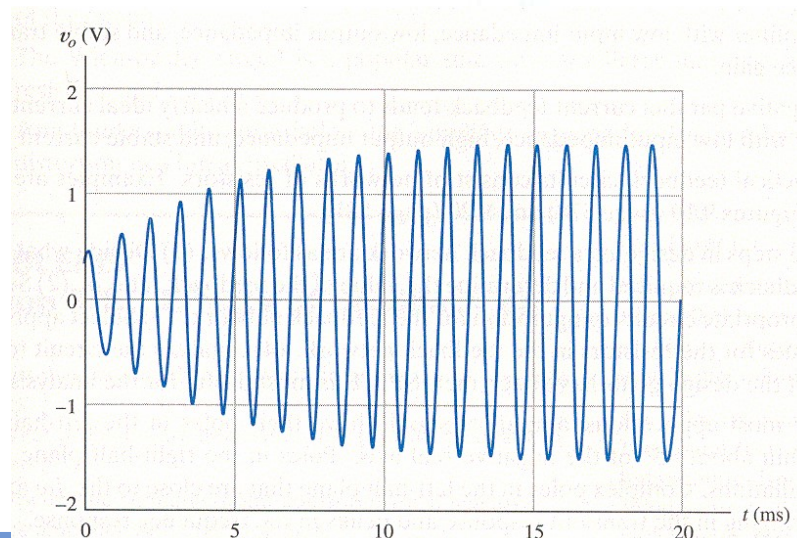
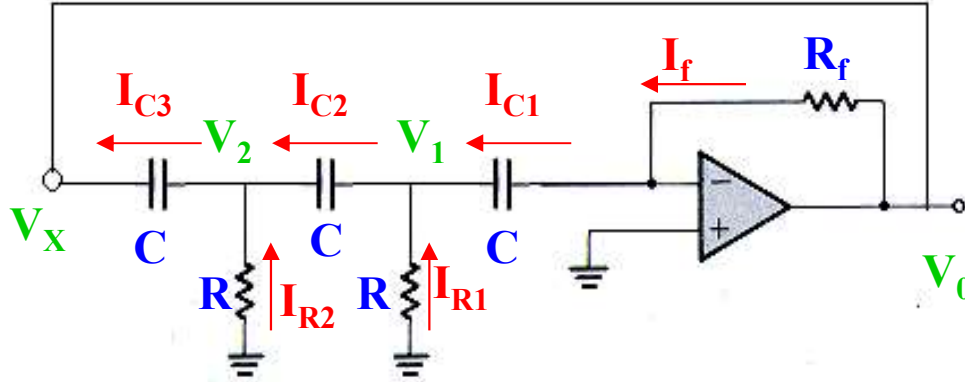


Figure 9.78 Output voltage of the oscillator of Figure 9.77.

Phase Shift Oscillator



$$I_{C2} = I_{R1} + I_{C1} = \frac{V_o}{sCRR_f} + \frac{V_o}{R_f} = \frac{V_o}{R_f} \left(1 + \frac{1}{sCR} \right)$$

$$V_2 = V_1 - I_{C2}Z_C = -\frac{V_o}{sCR_f} - \frac{V_o}{R_f} \left(1 + \frac{1}{sCR} \right) \frac{1}{sC}$$

$$= -\frac{V_o}{sCR_f} \left(2 + \frac{1}{sCR} \right)$$

$$I_{R2} = \frac{-V_2}{R} = \frac{V_o}{sCRR_f} \left(2 + \frac{1}{sCR} \right)$$

$$I_{C3} = I_{R2} + I_{C2} = \frac{V_o}{sCRR_f} \left(2 + \frac{1}{sCR} \right) + \frac{V_o}{R_f} \left(1 + \frac{1}{sCR} \right)$$

$$= \frac{V_o}{R_f} \left[\left(1 + \frac{1}{sCR} \right) + \frac{1}{sCR} \left(2 + \frac{1}{sCR} \right) \right] = \frac{V_o}{R_f} \left[1 + \frac{3}{sCR} + \frac{1}{(sCR)^2} \right]$$

Finally

$$V_X = V_2 - \frac{I_{C3}}{sC} = -\frac{V_o}{sCR_f} \left(2 + \frac{1}{sCR} \right) - \frac{V_o}{sCR_f} \left[1 + \frac{3}{sCR} + \frac{1}{(sCR)^2} \right]$$

$$= -\frac{V_o}{sCR_f} \left[3 + \frac{4}{sCR} + \frac{1}{(sCR)^2} \right]$$

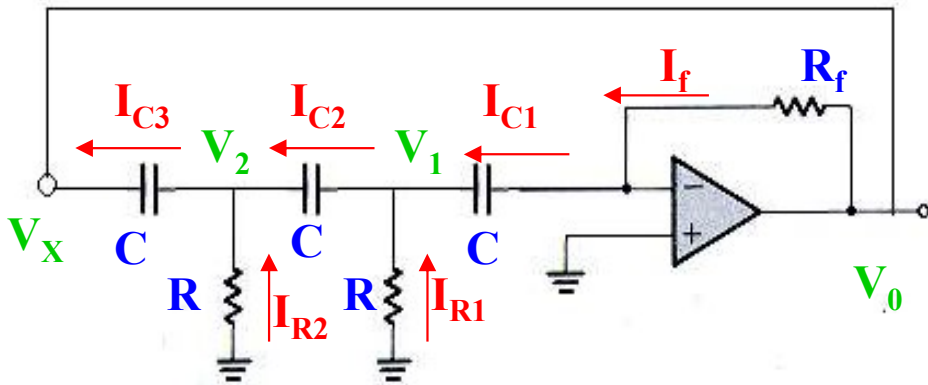
- * Based on op amp using inverting input
- * Combination of R's and C's in feedback loop so get additional phase shift. Target 180° to get oscillation.
- * Analysis assumes op amp is ideal.

$$V_- \approx V_+ = 0 \quad \text{so} \quad I_f = \frac{V_o}{R_f} = I_{C1}$$

$$V_1 = V_- - I_{C1}Z_C = -\frac{V_o}{sCR_f}$$

$$I_{R1} = \frac{-V_1}{R} = \frac{-1}{R} \left(-\frac{V_o}{sCR_f} \right) = \frac{V_o}{sCRR_f}$$

Phase Shift Oscillator



Rearranging

$$V_x = -\frac{V_o}{sCR_f} \left[3 + \frac{4}{sCR} + \frac{1}{(sCR)^2} \right]$$

we get for the loop gain

$$L(\omega) = \beta(\omega)A(\omega) = \frac{V_o}{V_x} = \frac{-sCR_f}{\left[3 + \frac{4}{sCR} + \frac{1}{(sCR)^2} \right]} = 1$$

$$= \frac{-j\omega CR_f}{\left[3 - j\frac{4}{\omega CR} - \frac{1}{(\omega CR)^2} \right]} = \frac{\omega^2 C^2 R R_f}{\left[4 + j\left(3\omega CR - \frac{1}{\omega CR} \right) \right]}$$

To get oscillations, we need the imaginary term to go to zero.

We can achieve this at one frequency ω_o so

$$3\omega CR = \frac{1}{\omega CR} \quad \text{so} \quad \omega = \omega_o = \frac{1}{\sqrt{3}RC}$$

To get oscillations, we also need $L(\omega_o) = 1$ so

$$L(\omega_o) = \frac{\omega_o^2 C^2 R R_f}{4} = 1 \quad \text{and substituting for } \omega_o \text{ we get}$$

$$\frac{\omega_o^2 C^2 R R_f}{4} = \frac{C^2 R R_f}{4} \frac{1}{3R^2 C^2} = \frac{R_f}{12R} = 1 \quad \text{so}$$

$$R_f = 12R$$

Example**Oscillator specifications:** $\omega_o = 1 \times 10^6 \text{ rad/s}$ Selecting for convenience $C = 10 \text{ nF}$,

$$\text{then from } \omega_o = \frac{1}{\sqrt{3}RC}$$

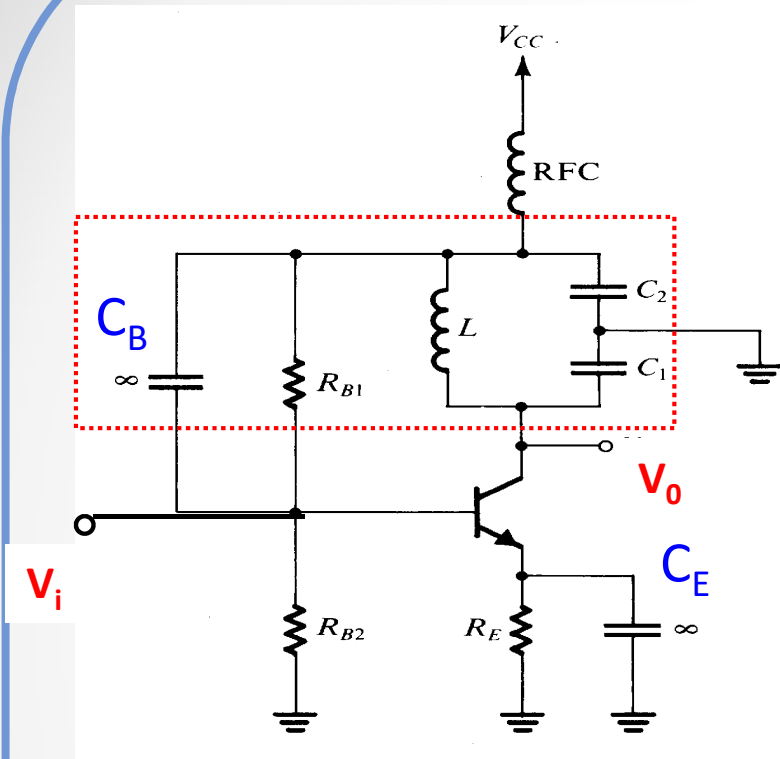
$$R = \frac{1}{\sqrt{3}\omega_o C} = \frac{1}{\sqrt{3} \cdot 10 \text{ nF} \cdot (1 \times 10^6 \text{ rad/s})} = 58 \, \Omega$$

Then

$$R_f = 12(58 \, \Omega) = 0.67 \text{ K}$$

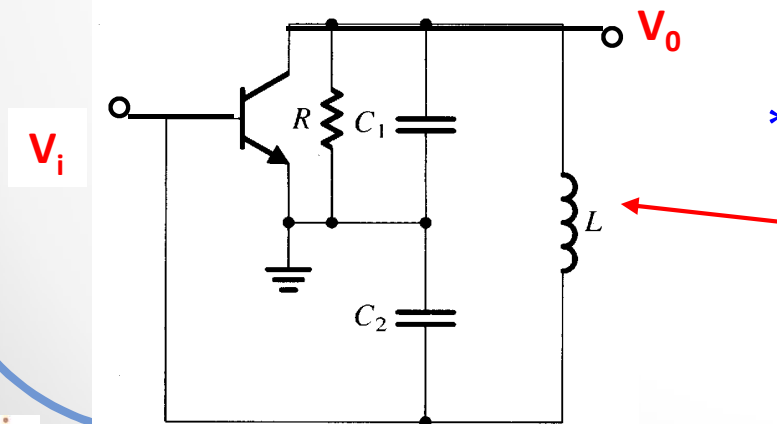
Note: We get 180° phase shift from op amp since input is to inverting terminal and another 180° from the RC ladder.

Colpitts LC-Tuned Oscillator

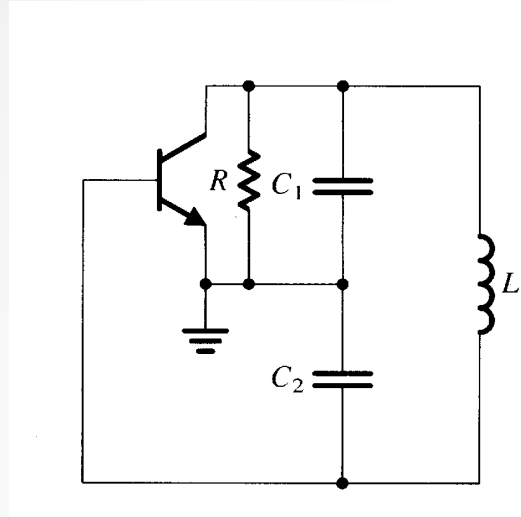


- * Feedback amplifier with inductor L and capacitors C_1 and C_2 in feedback network.
 - ★ Feedback is frequency dependent.
 - ★ Aim to adjust components to get positive feedback and oscillation.
 - ★ Output taken at collector V_o .
 - ★ No input needed, noise at oscillation frequency ω_o is picked up and amplified.

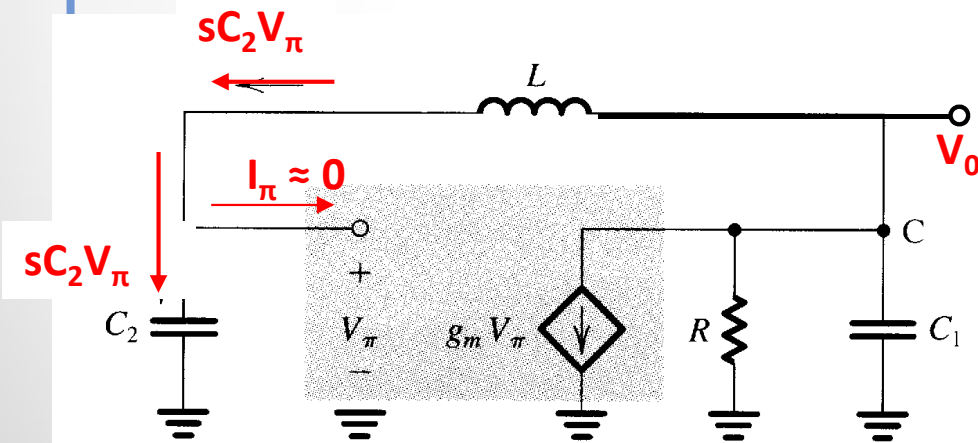
- * R_{B1} and R_{B2} are biasing resistors.
- * RFC is RF Choke (inductor) to allow dc current flow for transistor biasing, but to block ac current flow to ac ground.



- * Simplified circuit shown at **midband frequencies** where large emitter bypass capacitor C_E and base capacitor C_B are shorts and transistor capacitances (C_π and C_μ) are opens.



AC equivalent circuit



- * Voltage across C_2 is just V_π

$$I_{C2} = \frac{V_\pi}{Z_{C2}} = sC_2V_\pi$$

- * Neglecting input current to transistor ($I_\pi \approx 0$),

$$I_L = I_{C2} = \frac{V_\pi}{Z_{C2}} = sC_2V_\pi$$

- * Then, output voltage V_o is

$$V_o = V_\pi + I_L Z_L = V_\pi + (sC_2V_\pi)(sL) = V_\pi(1 + s^2LC_2)$$

- * KCL at output node (C)

Assuming oscillations have started, then $V_\pi \neq 0$ and $V_o \neq 0$, so

$$sC_2V_\pi + g_mV_\pi + \left(\frac{1}{R} + sC_1\right)V_o = 0$$

$$sC_2V_\pi + g_mV_\pi + \left(\frac{1}{R} + sC_1\right)V_\pi(1 + s^2LC_2) = 0$$

$$s^3LC_1C_2 + s^2\left(\frac{LC_2}{R}\right) + s(C_1 + C_2) + \left(g_m + \frac{1}{R}\right) = 0$$

Setting $s = j\omega$

$$\left(g_m + \frac{1}{R} - \frac{\omega^2LC_2}{R}\right) + j[\omega(C_1 + C_2) - \omega^3LC_1C_2] = 0$$

- * To get oscillations, both the real and imaginary parts of this equation must be set equal to zero.

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R} \right) + j[\omega(C_1 + C_2) - \omega^3 LC_1 C_2] = 0$$

- * From the imaginary part we get the expression for the **oscillation frequency**

$$\omega_o(C_1 + C_2) - \omega_o^3 LC_1 C_2 = 0$$

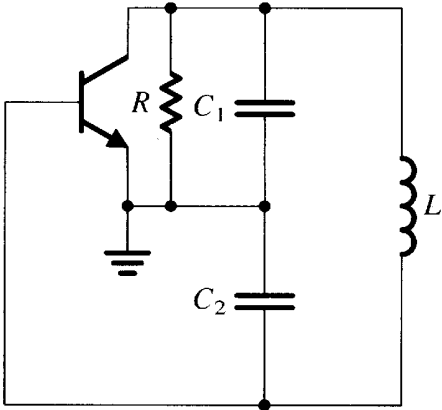
$$\omega_o = \sqrt{\frac{C_1 + C_2}{LC_1 C_2}} = \frac{1}{\sqrt{L \left(\frac{C_1 C_2}{C_1 + C_2} \right)}}$$

- * From the real part, we get the condition on the ratio of C_2/C_1

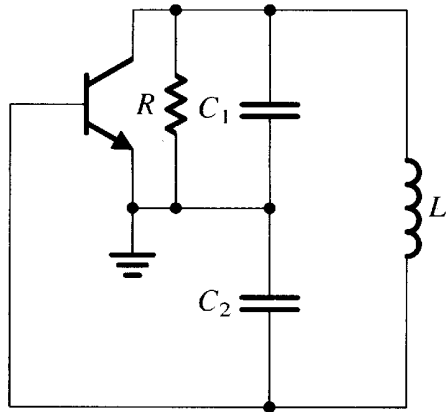
$$g_m + \frac{1}{R} - \frac{\omega_o^2 LC_2}{R} = 0$$

$$1 + g_m R = \omega_o^2 LC_2 = LC_2 \left[\frac{C_1 + C_2}{LC_1 C_2} \right] = 1 + \frac{C_2}{C_1}$$

$$\frac{C_2}{C_1} = g_m R$$



Example



* Given:

★ Design oscillator at **150 MHz**

$$\omega_o = 2\pi f = 2\pi(150 \times 10^6) = 9.4 \times 10^8 \text{ rad/s}$$

★ Transistor $g_m = 100 \text{ mA/V}$, $R = 0.5 \text{ K}$

* Design:

$$\frac{C_2}{C_1} = g_m R = (100 \text{ mA/V})(0.5 \text{ K}) = 50$$

★ Select $L = 50 \text{ nH}$, then calculate C_2 , and then C_1

$$\omega_o = \sqrt{\frac{C_1 + C_2}{LC_1 C_2}} = \sqrt{\frac{1}{LC_2} \left(1 + \frac{C_2}{C_1}\right)}$$

$$C_2 = \frac{1}{L\omega_o^2} \left(1 + \frac{C_2}{C_1}\right) = \frac{1}{50 \text{ nH} (9.4 \times 10^8)^2} (1 + 50) = 1.13 \times 10^{-9} \text{ F} = \boxed{1,130 \text{ pF}}$$

$$C_1 = \frac{C_2}{50} = \frac{1,130 \text{ pF}}{50} = \boxed{23 \text{ pF}}$$

References

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Microelectronic Circuit Design by Richard C. Jaeger. The McGraw-Hill Companies, Inc. 2011

Microelectronics Circuit Analysis and Design by Donald Neamen, The McGraw-Hill Companies, Inc. 2010



HOPE

Once you choose hope, anything's possible!

~Christopher Reeve



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