Seat Number

King Mongkut's University of Technology Thonburi
Midterm Examination
Semester 1 – Academic Year 2013

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3rd Yr (Inter. Program)
Exam Date: Monday September 30, 2013 **Time:** 9.00 am - 12.00 pm

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Instructions:-

1. This exam consists of 4 problems with a total of 12 pages, **not** including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rules is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds … No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instructions might eventually result in a failure of the class or may receive the highest punishment with university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid needless calculation!**

Question No.	1	2	3	4	TOTAL
Full Score	25	25	25	25	100
Graded Score					

Name ________________________________ **Student ID** ______________

This examination is designed by
Watcharapan Suwansantisuk; Tel: 9069.

This examination has been approved by the committees of the ENE department.

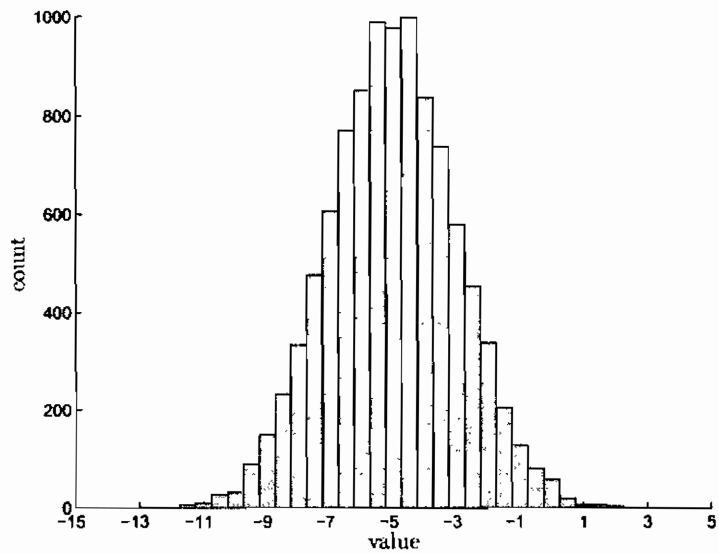
A. Wudhichai

(Assoc. Prof. Wudhichai Assawinchaichote, Ph.D.)
Head of Electronic and Telecommunication Engineering Department

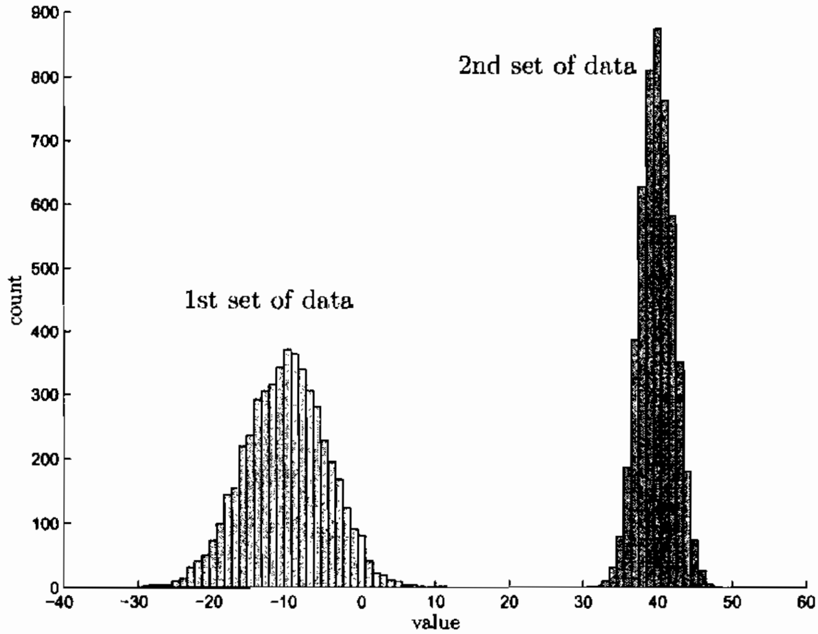
Name...Student ID.......................Seat Number......

Problem 1: Sample Mean and Sample Variance [25 points]

(a) [5 points] From the histogram below, the sample mean is equal approximately to what value? **Why?**
[*Hint*: You will receive a small or no credit if you did not answer "Why?"]



(b) [5 points] Two histograms below are for two different sets of data. The sample variance of which set of data is **larger**? **Why?**



Parts (c) and (d) below are not related to Parts (a) and (b) and can be done separately.

Let $x_1, x_2, \ldots, x_n$ be a sample. Let a and b be constants. Let $y_i = ax_i + b$ be a linear transformation of x_i $(i =, 1, 2, \ldots, n)$.

(c) [7 points] Show that the sample mean of $y_1, y_2, \ldots, y_n$ is $a\overline{x} + b$, where $\overline{x}$ is the sample mean of $x_1, x_2, \ldots, x_n$.

(d) [8 points] Show that the sample variance of $y_1, y_2, \ldots, y_n$ is $a^2 s_x^2$, where s_x^2 is the sample variance of $x_1, x_2, \ldots, x_n$.

Name .... Student ID .... Seat Number ......

Problem 2: Events and Their Probabilities [25 points]

Consider randomly selecting a student at a certain university. Let B denote the event that the selected individual owns a **black** pen, and R be the analogous event for a **red** pen. Suppose that $\mathbb{P}\{B\} = 0.5$, $\mathbb{P}\{R\} = 0.4$, and $\mathbb{P}\{B \cap R\} = 0.25$.

(a) [6 points] Compute the probability that the selected individual has at least one of the two types of pens (in other words, the probability of the event $B \cup R$).

(b) [6 points] What is the probability that the selected individual has neither type of pen?
[*Hint*: This is the probability that the selected individual does not have a black pen and does not have a red pen.]

(c) [6 points] Describe, in terms of B and R, the event that the selected student has a black pen but not a red pen, and then calculate the probability of this event.

(d) [7 points] Are the events B and R independent? **Justify your answer** to receive a credit.

Name .... Student ID .... Seat Number ......

Problem 3: Counting [25 points]

A small classroom consists of 3 students: a, b, and c.

(a) [5 points] The number of ways to select 2 students **with** the order in selection is $P_{2,3} = \frac{3!}{(3-2)!} = 6$ ways. List all the 6 ways of ordered selection.
[*Hint*: One way is (a, b).]

(b) [5 points] The number of ways to select 2 students **without** the order in selection is $\binom{3}{2} = \frac{3!}{2!(3-2)!} = 3$ ways. List all the 3 ways of unordered selection.

Parts (c) and (d) below are not related to Parts (a) and (b) and can be done separately.

An employee at a university has eight forms on his desk awaiting processing. Five of these are withdrawal petitions and the other three are course substitution requests.

(c) [8 points] If he randomly selects five of these forms to give to a subordinate, what is the probability that only one of the two types of forms remains on his desk?

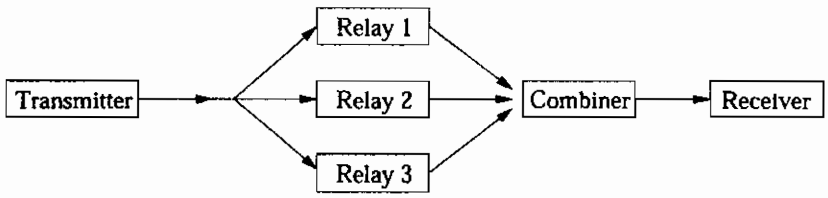
(d) [7 points] Suppose he has time to process only three of these forms before leaving for the day. If these three are randomly selected one by one, what is the probability that each succeeding form is of a different type from its predecessor?

Problem 4: Conditional Probability [25 points]

A transmitter is sending a message by using a binary code, namely a sequence of 0's and 1's. Each transmitted bit (0 or 1) must pass through three relays **in parallel** and must pass through a combiner to reach the receiver. At each relay, the probability is 0.10 that the bit sent will be different from the bit received (a reversal).

The combiner receives three bits from the three relays and sends a majority bit. In other words, the combiner sends a 1 if two or three relays send a 1. The combiner sends a 0 if two or three relays send a 0.

Assume that the three relays and the combiner operate independently of one another.



(a) [8 points] If a 1 is sent from the transmitter, what is the probability that a 1 is sent by all three relays?

(b) [8 points] If a 1 is sent from the transmitter, what is the probability that a 1 is received by the receiver?
[*Hint*: Break the desired probability into a sum of four terms:

$$\begin{aligned}&\mathbb{P}\{(\text{R1 sent 1}) \text{ and } (\text{R2 sent 1}) \text{ and } (\text{R3 sent 1}) \mid \text{transmitter sent 1}\}\\ &\quad + \mathbb{P}\{(\text{R1 sent 0}) \text{ and } (\text{R2 sent 1}) \text{ and } (\text{R3 sent 1}) \mid \text{transmitter sent 1}\}\\ &\quad + \mathbb{P}\{(\text{R1 sent 1}) \text{ and } (\text{R2 sent 0}) \text{ and } (\text{R3 sent 1}) \mid \text{transmitter sent 1}\}\\ &\quad\quad + \mathbb{P}\{(\text{R1 sent 1}) \text{ and } (\text{R2 sent 1}) \text{ and } (\text{R3 sent 0}) \mid \text{transmitter sent 1}\}.\end{aligned}$$

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(c) [9 points] Suppose 80% of all bits sent from the transmitter are 1's. If a 1 is received by the receiver, what is the probability that a 1 was sent?
[*Hint*: (Bayes' Theorem) Let $A_1, A_2, \ldots, A_k$ be a collection of mutually exclusive and exhaustive events with prior probabilities $\mathbb{P}\{A_i\}$ for $i = 1, 2, \ldots, k$. Then for any other event B for which $\mathbb{P}\{B\} > 0$, the posterior probability of A_j given that B has occurred is

$$\begin{aligned}\mathbb{P}\{A_j|B\} &= \frac{\mathbb{P}\{A_j \cap B\}}{\mathbb{P}\{B\}} \\ &= \frac{\mathbb{P}\{B|A_j\}\mathbb{P}\{A_j\}}{\sum_{i=1}^{k} \mathbb{P}\{B|A_i\}\mathbb{P}\{A_i\}}, \qquad j = 1, 2, \ldots, k.\end{aligned}$$

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