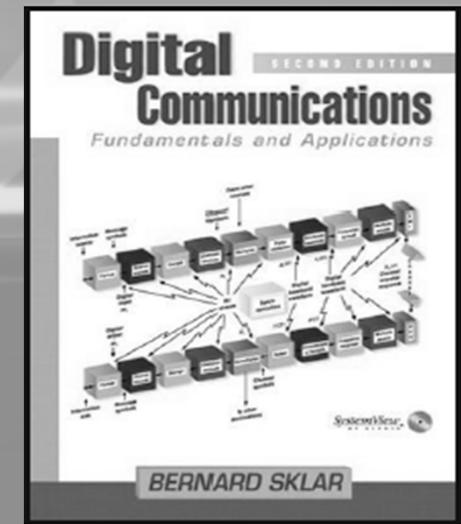


# **ENE 467**

# **Digital Communications**

**TEACHING BY**

**ASST. PROF. SUWAT PATTARAMALAI, PH.D.**



# **5. Communication and Link analysis**

- Outcome
  - Can explain the System Link Budget
  - Can explain communication channel (concept, performance, loss, and noise)
  - Can explain received signal power and noise power
  - Can analyze Link Budget (calculation)
  - Can explain Noise Figure, Noise Temperature, and System Temperature
  - Can give and explain samples or Link Budget analysis

# Communication and Link analysis

- What the system link budget tells the system engineer

What is a link analysis, and what purpose does it serve in the development of a communication system? The link analysis, and its output, the *link budget*, consist of the calculations and tabulation of the useful signal power and the interfering noise power available at the receiver. The link budget is a balance sheet of gains and losses; it outlines the detailed apportionment of transmission and reception resources, noise sources, signal attenuators, and effects of processes throughout the link. Some of the budget parameters are statistical (e.g., allowances for the fading of signals as described in Chapter 15). The budget is an *estimation* technique for evaluating communication system error performance. In Chapters 3 and 4 we examined probability of error versus  $E_b/N_0$  curves having a “waterfall-like” shape, such as the one shown in Figure 3.6. We thereby related error probability to  $E_b/N_0$  for various modulation types in Gaussian noise. Once a modulation scheme has been chosen, the requirement to meet a particular error probability dictates a particular operating point on the curve; in other words, the required error perfor-

# Communication and Link analysis

- The Channel
  - Free Space channel
  - Error performance Degradation

The concept of *free space* assumes a channel free of all hindrances to RF propagation, such as absorption, reflection, refraction, or diffraction. If there is any atmosphere in the channel, it must be perfectly uniform and meet all these conditions. Also, we assume that the earth is infinitely far away or that its reflection coefficient is negligible. The RF energy arriving at the receiver is assumed to be a function only of distance from the transmitter (following the inverse-square law as used in

For digital communications, error performance depends on the received  $E_b/N_0$ , which was defined in Equation (3.30) as

$$\frac{E_b}{N_0} = \frac{S}{N} \left( \frac{W}{R} \right)$$

In other words,  $E_b/N_0$  is a measure of normalized signal-to-noise ratio (S/N or SNR). Unless otherwise stated, SNR refers to *average* signal power and *average* noise power. The signal can be an information signal, a baseband waveform, or a modulated carrier. The SNR can degrade in two ways: (1) through the decrease of the desired signal power, and (2) through the increase of noise power, or the increase of interfering signal power. Let us refer to these degradations as *loss* and *noise* (or *interference*), respectively. Losses occur when a portion of the signal is absorbed, diverted, scattered, or reflected along its route to the intended receiver;

# Communication and Link analysis

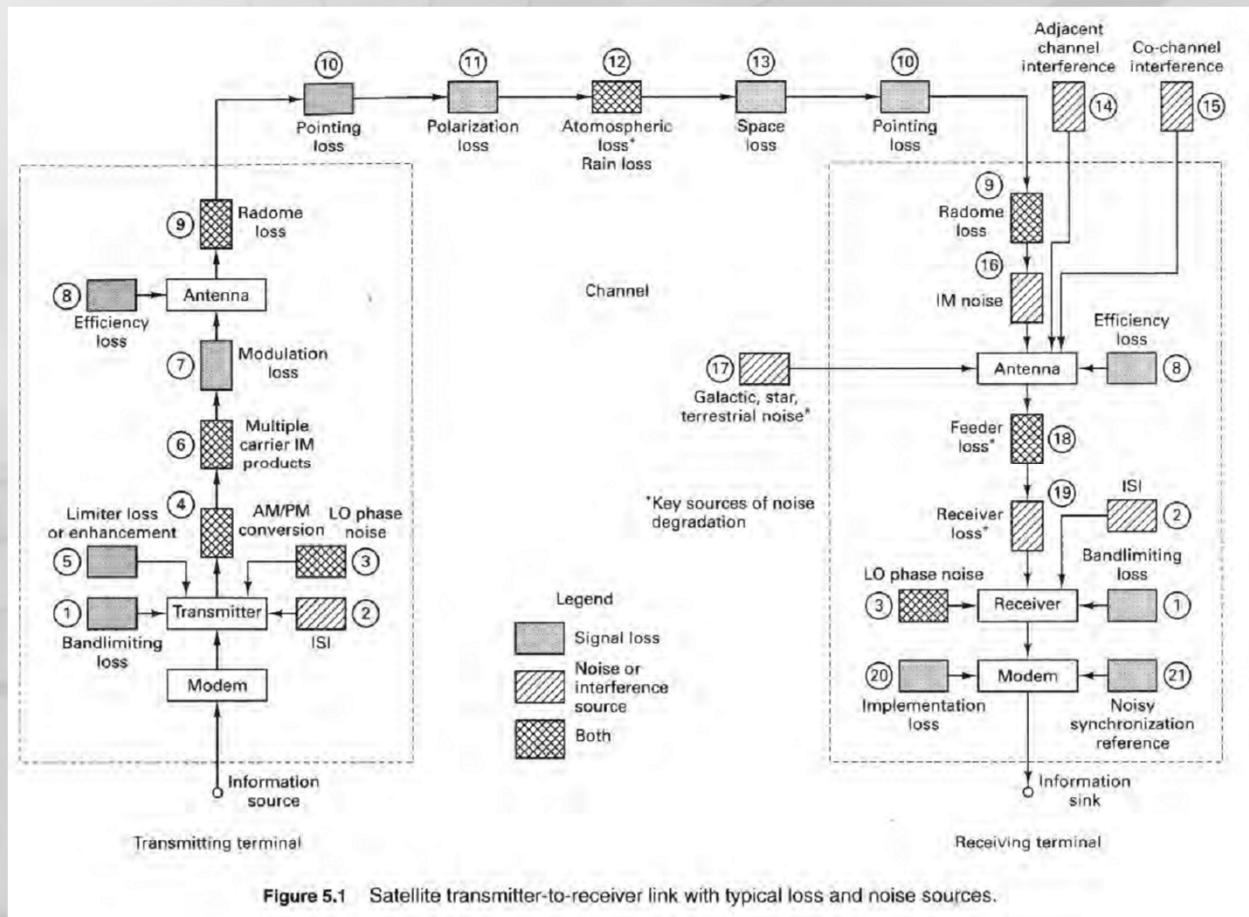


Figure 5.1 Satellite transmitter-to-receiver link with typical loss and noise sources.

# Communication and Link analysis

1. **Bandlimiting loss.** All systems use filters in the transmitter to ensure that the transmitted energy is confined to the allocated or assigned bandwidth. This is to avoid interfering with other channels or users and to meet the requirements of regulatory agencies. Such filtering reduces the total amount of energy that would otherwise have been transmitted; the result is a *loss* in signal.
2. **Intersymbol interference (ISI).** As discussed in Chapter 3, filtering throughout the system—in the transmitter, in the receiver, and in the channel—can result in ISI. The received pulses overlap one another; the tail of one pulse “smears” into adjacent symbol intervals so as to *interfere* with the detection process. Even in the absence of thermal noise, imperfect filtering, system bandwidth constraints, and fading channels lead to ISI degradation.
3. **Local oscillator (LO) phase noise.** When an LO is used in signal mixing, phase fluctuations or jitter adds phase *noise* to the signal. When used as the reference signal in a receiver correlator, phase jitter can cause detector degradation and hence signal *loss*. At the transmitter, phase jitter can cause out-of-band signal spreading, which, in turn, will be filtered out and cause a *loss* in signal.
4. **AM/PM conversion.** AM-to-PM conversion is a phase *noise* phenomenon occurring in nonlinear devices such as traveling-wave tubes (TWT). Signal amplitude fluctuations (amplitude modulation) produce phase variations that contribute phase *noise* to signals that will be coherently detected. AM-to-PM conversion can also cause extraneous sidebands, resulting in signal *loss*.

5. **Limiter loss or enhancement.** A hard limiter can enhance the stronger of two signals, and suppress the weaker; this can result in either a signal *loss* or a signal *gain* [2].
6. **Multiple-carrier intermodulation (IM) products.** When several signals having different carrier frequencies are simultaneously present in a nonlinear device, such as a TWT, the result is a multiplicative interaction between the carrier frequencies which can produce signals at all combinations of sum and difference frequencies. The energy apportioned to these spurious signals (intermodulation or IM products) represents a *loss* in signal energy. In addition, if these IM products appear within the bandwidth region of these or other signals, the effect is that of added *noise* for those signals.
7. **Modulation loss.** The link budget is a calculation of received useful power (or energy). Only the power associated with information-bearing signals is useful. Error performance is a function of energy per transmitted symbol. Any power used for transmitting the carrier rather than the modulating signal (symbols) is a modulation *loss*. (However, energy in the carrier may be useful in aiding synchronization.)
8. **Antenna efficiency.** Antennas are transducers that convert electronic signals into electromagnetic fields, and vice versa. They are also used to focus the

electromagnetic energy in a desired direction. The larger the antenna aperture (area), the larger is the resulting signal power density in the desired direction. An antenna’s efficiency is described by the ratio of its effective aperture to its physical aperture. Mechanisms contributing to a reduction in efficiency (*loss* in signal strength) are known as amplitude tapering, aperture blockage, scattering, re-radiation, spillover, edge diffraction, and dissipative loss [3]. Typical efficiencies due to the combined effects of these mechanisms range between 50 and 80%.

# Communication and Link analysis

9. **Radome loss and noise.** A radome is a protective cover, used with some antennas, for shielding against weather effects. The radome, being in the path of the signal, will scatter and absorb some of the signal energy, thus resulting in a signal *loss*. A basic law of physics holds that a body capable of absorbing energy also radiates energy (at temperatures above 0 K). Some of this energy falls in the bandwidth of the receiver and constitutes injected *noise*.
10. **Pointing loss.** There is a *loss* of signal when either the transmitting antenna or the receiving antenna is imperfectly pointed.
11. **Polarization loss.** The polarization of an electromagnetic (EM) field is defined as the direction in space along which the field lines point, and the polarization of an antenna is described by the polarization of its radiated field. There is a *loss* of signal due to any polarization mismatch between the transmitting and receiving antennas.
12. **Atmospheric loss and noise.** The atmosphere is responsible for signal loss and is also a contributor of unwanted noise. The bulk of the atmosphere extends to an altitude of approximately 20 km; yet within that relatively short path, important loss and noise mechanisms are at work. Figure 5.2 is a plot of the theoretical one-way attenuation from a specified height to the top of the atmosphere. The calculations were made for several heights (0 km is sea level) and for a water vapor content of 7.5 g/m<sup>3</sup> at the earth's surface. The magnitude of signal *loss* due to oxygen (O<sub>2</sub>) and water vapor absorption is plotted as a function of carrier frequency. Local maxima of attenuation occur in the vicinities of 22 GHz (water vapor), and 60 and 120 GHz (O<sub>2</sub>). The atmosphere also contributes *noise* energy into the link. As in the case of the radome, molecules that absorb energy also radiate energy. The oxygen and water vapor molecules radiate noise throughout the RF spectrum. The portion of this noise that falls within the bandwidth of a given communication system will degrade its SNR. A primary atmospheric cause of signal *loss* and contributor of *noise* is rainfall. The more intense the rainfall, the more signal energy it will absorb. Also, on a day when rain passes through the antenna beam, there is a larger amount of atmospheric noise radiated into the system receiver than there is on a clear day. More will be said about atmospheric noise in later sections.
13. **Space loss.** There is a decrease in the electric field strength, and thus in signal strength (power density or flux density), as a function of distance. For a satellite communications link, the space loss is the largest single *loss* in the system. It is a loss in the sense that all the radiated energy is not focused on the intended receiving antenna.
14. **Adjacent channel interference.** This *interference* is characterized by unwanted signals from other frequency channels "spilling over" or injecting energy into the channel of interest. The proximity with which channels can be located in frequency is determined by the modulation spectral roll-off and the width and shape of the main spectral lobe.
15. **Co-channel interference.** This *interference* refers to the degradation caused by an interfering waveform appearing within the signal bandwidth. It can be introduced by a variety of ways, such as accidental transmissions, insufficient vertical and horizontal polarization discrimination, or by radiation spillover from an antenna sidelobe (low-energy beam surrounding the main antenna beam). It can be brought about by other authorized users of the same spectrum.
16. **Intermodulation (IM) noise.** The IM products described in item 6 result from multiple-carrier signals interacting in a nonlinear device. Such IM products are sometimes called *active intermods*; as described in item 6, they can either cause a loss in signal energy or be responsible for noise injected into a link. Here we consider *passive intermods*; these are caused by multiple-carrier transmission signals interacting with nonlinear components at the transmitter output. These nonlinearities generally occur at the junction of waveguide coupling joints, at corroded surfaces, and at surfaces having poor electrical contact. When large EM fields impinge on surfaces that have a diode-like transfer function (work potential), they cause multiplicative products, and hence *noise*. If such noise radiates into a closely located receiving antenna, it can seriously degrade the receiver performance.

# Link analysis

17. *Galactic or cosmic, star, and terrestrial noise.* All the celestial bodies, such as the stars and the planets, radiate energy. Such *noise* energy in the field of view of the antenna will degrade the SNR.
18. *Feeder line loss.* The level of the received signal might be very small (e.g.,  $10^{-12}$  W), and thus will be particularly susceptible to noise degradation. The receiver front end, therefore, is a region where great care is taken to keep the noise as small as possible until the signal has been suitably amplified. The waveguide or cable (feeder line) between the receiving antenna and the receiver front end contributes both signal *attenuation* and thermal *noise*; the details are treated in Section 5.5.3.
19. *Receiver noise.* This is the thermal *noise* generated within the receiver; the details are treated in Sections 5.5.1 to 5.5.4.
20. *Implementation loss.* This *loss* in performance is the difference between theoretical detection performance and the actual performance due to imperfections such as timing errors, frequency offsets, finite rise and fall times of waveforms, and finite-value arithmetic.
21. *Imperfect synchronization reference.* When the carrier phase, the subcarrier phase, and the symbol timing references are all derived perfectly, the error probability is a well-defined function of  $E_b/N_0$  discussed in Chapters 3 and 4. In general, they are not derived perfectly, resulting in a system *loss*.

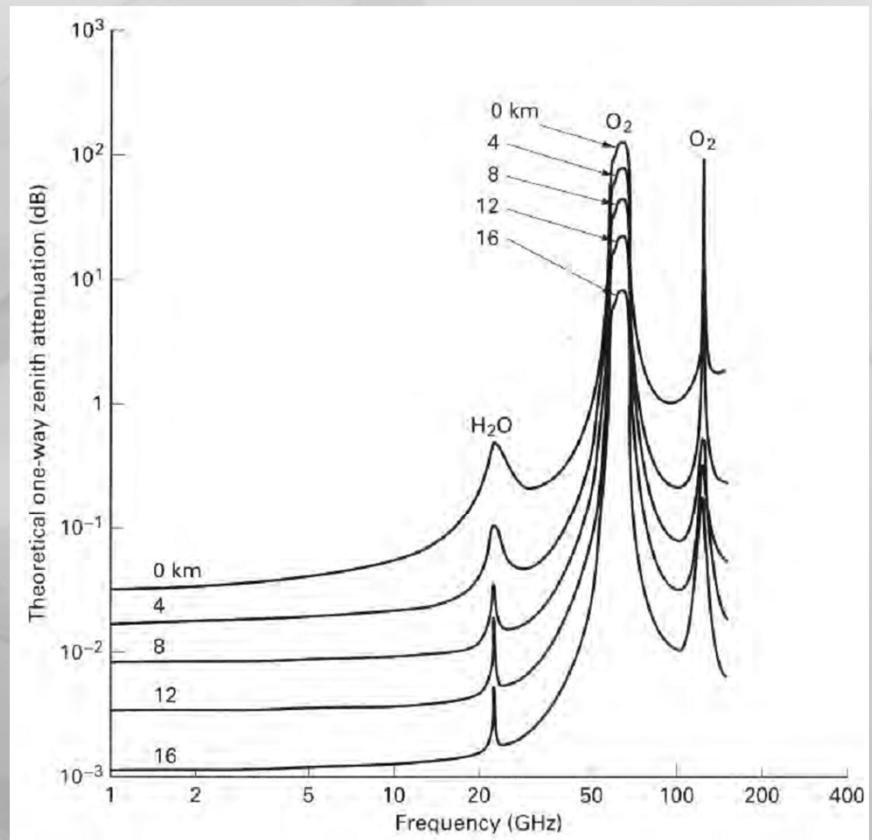


Figure 5.2 Theoretical vertical one-way attenuation from specified height to top of atmosphere for  $7.5 \text{ g/m}^3$  of water vapor at the surface. (Does not include effect of rain or cloud attenuation.) (Reprinted from NASA Reference Publication 1082(03), "Propagation Effects Handbook for Satellite Systems Design," June 1983, Fig. 6.2-1, p. 218, courtesy of the National Aeronautics and Space Administration.)

# Received signal power and noise power

- The Range Equation

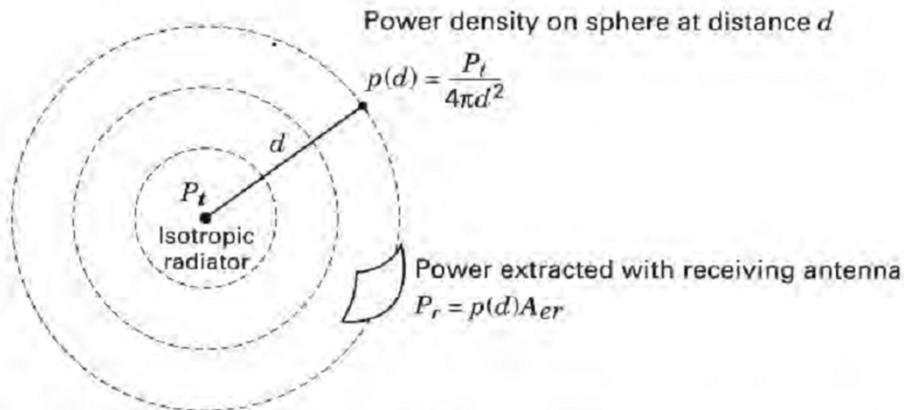


Figure 5.3 Range equation. Expresses received power in terms of distance.

An antenna's effective area  $A_e$  and physical area  $A_p$  are related by an efficiency parameter  $\eta$  as

$$A_e = \eta A_p \quad (5.4)$$

which accounts for the fact that the total incident power is not extracted; it is lost through various mechanisms [3]. Nominal values for  $\eta$  are 0.55 for a dish (parabolic-shaped reflector) and 0.75 for a horn-shaped antenna.

In radio communication systems, the carrier wave is propagated from the transmitter by the use of a transmitting antenna. The transmitting antenna is a transducer that converts electronic signals into electromagnetic (EM) fields. At the receiver, a receiving antenna performs the reverse function; it converts EM fields into electronic signals. The development of the fundamental power relationship between the receiver and transmitter usually begins with the assumption of an omnidirectional RF source, transmitting uniformly over  $4\pi$  steradians. Such an ideal source, called an *isotropic radiator*, is illustrated in Figure 5.3. The power density  $p(d)$  on a hypothetical sphere at a distance  $d$  from the source is related to the transmitted power  $P_t$  by

$$p(d) = \frac{P_t}{4\pi d^2} \quad \text{watts/m}^2 \quad (5.1)$$

since  $4\pi d^2$  is the area of the sphere. The power extracted with the receiving antenna can be written as

$$P_r = p(d)A_{er} = \frac{P_t A_{er}}{4\pi d^2} \quad (5.2)$$

where the parameter  $A_{er}$  is the absorption cross section (effective area) of the receiving antenna, defined by

$$A_{er} = \frac{\text{total power extracted}}{\text{incident power flux density}} \quad (5.3)$$

# Received signal power and noise power

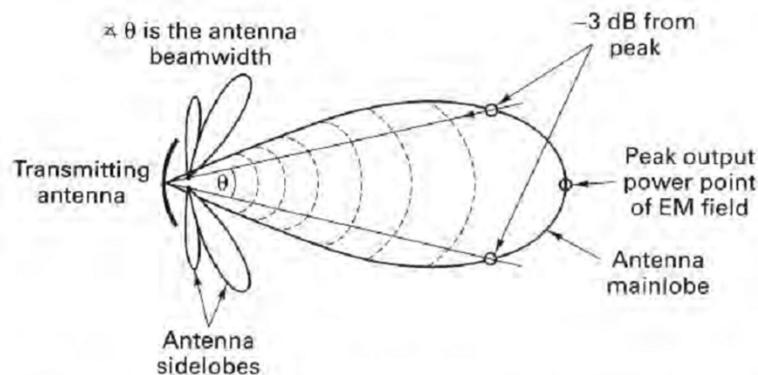
- The Range Equation

The antenna parameter that relates the power output (or input) to that of an isotropic radiator as a purely geometric ratio is the antenna directivity or *directive gain*

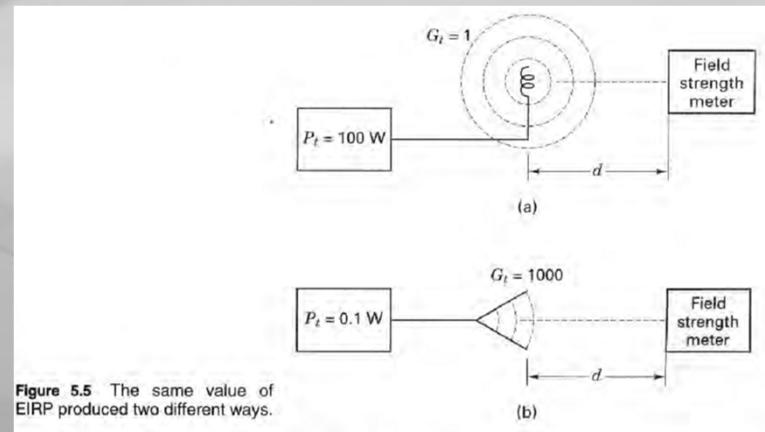
$$G = \frac{\text{maximum power intensity}}{\text{average power intensity over } 4\pi \text{ steradians}} \quad (5.5)$$

ans, as shown in Figure 5.4. Now we can define an *effective radiated power*, with respect to an isotropic radiator (EIRP), as the product of the transmitted power  $P_t$  and the gain of the transmitting antenna  $G_t$ , as follows:

$$\text{EIRP} = P_t G_t \quad (5.6)$$



**Figure 5.4** Antenna gain is the result of concentrating the isotropic RF flux.



**Figure 5.5** The same value of EIRP produced two different ways.

*Solution*

Figure 5.5a depicts a 100-W transmitter coupled to an isotropic antenna; the  $\text{EIRP} = P_t G_t = 100 \times 1 = 100 \text{ W}$ . Figure 5.5b depicts a 0.1-W transmitter coupled to an antenna with gain  $G_t = 1000$ ; the  $\text{EIRP} = P_t G_t = 0.1 \times 1000 = 100 \text{ W}$ . If field-strength meters were positioned, as shown, to measure the effective power, the measurements could not distinguish between the two cases.

# Received signal power and noise power

- The Range Equation

For the more general case in which the transmitter has some antenna gain relative to an isotropic antenna, we replace  $P_t$  with EIRP in Equation (5.2) to yield

$$P_r = \text{EIRP} \frac{A_{er}}{4\pi d^2} \quad (5.7)$$

The relationship between antenna gain  $G$  and antenna effective area  $A_e$  is [4]

$$G = \frac{4\pi A_e}{\lambda^2} \quad (\text{for } A_e \gg \lambda^2) \quad (5.8)$$

where  $\lambda$  is the wavelength of the carrier. Wavelength  $\lambda$  and frequency  $f$  are reciprocally related by  $\lambda = c/f$ , where  $c$  is the speed of light ( $= 3 \times 10^8$  m/s). Similar expressions apply for both the transmitting and receiving antennas. The *reciprocity theorem* states that for a given antenna and carrier wavelength, the transmitting and receiving gains are identical [4].

We can calculate the effective area of an isotropic antenna by setting  $G = 1$  in Equation (5.8) and solving for  $A_e$  as follows:

$$A_e = \frac{\lambda^2}{4\pi} \quad (5.9)$$

Then to find the power received,  $P_r$ , when the receiving antenna is isotropic, we substitute Equation (5.9) into Equation (5.7) to get

$$P_r = \frac{\text{EIRP}}{(4\pi d/\lambda)^2} = \frac{\text{EIRP}}{L_s} \quad (5.10)$$

where the collection of terms  $(4\pi d/\lambda)^2$ , called the *path loss* or *free-space loss*, is designated by  $L_s$ . Notice that Equation (5.10) states that the power received by an isotropic antenna is equal to the effective transmitted power, reduced only by the path loss. When the receiving antenna is not isotropic, replacing  $A_{er}$  in Equation (5.7) with  $G_r \lambda^2 / 4\pi$  from Equation (5.8) yields the more general expression

$$P_r = \frac{\text{EIRP} G_r \lambda^2}{(4\pi d)^2} = \frac{\text{EIRP} G_r}{L_s} \quad (5.11)$$

where  $G_r$  is the receiving antenna gain. Equation (5.11) can be termed the *range equation*.

# Received signal power and noise power

- Received signal power

Since the transmitting antenna and the receiving antenna can each be expressed as a gain or an area,  $P_r$ , can be expressed four different ways:

$$P_r = \frac{P_t G_t A_{er}}{4\pi d^2} \quad (5.12)$$

$$P_r = \frac{P_t A_{et} A_{er}}{\lambda^2 d^2} \quad (5.13)$$

$$P_r = \frac{P_t A_{et} G_r}{4\pi d^2} \quad (5.14)$$

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} \quad (5.15)$$

In these equations,  $A_{er}$  and  $A_{et}$  are the effective areas of the receiving and transmitting antennas, respectively.

Figure 5.6 illustrates a satellite application where the downlink antenna beam is required to provide earth coverage (a beamwidth of approximately 17° from synchronous altitude). Since the satellite antenna gain  $G_t$  must be fixed, the resulting  $P_r$  is independent of wavelength, as shown in Equation (5.12). If the transmission at some frequency  $f_1$  ( $= c/\lambda_1$ ) provides earth coverage, then a frequency change to  $f_2$ , where  $f_2 > f_1$ , will result in reduced coverage (since for a given antenna,  $G_t$  will increase); hence the antenna size must be reduced to maintain the required earth coverage or beamwidth. Thus earth coverage antennas become smaller as the carrier frequency is increased.

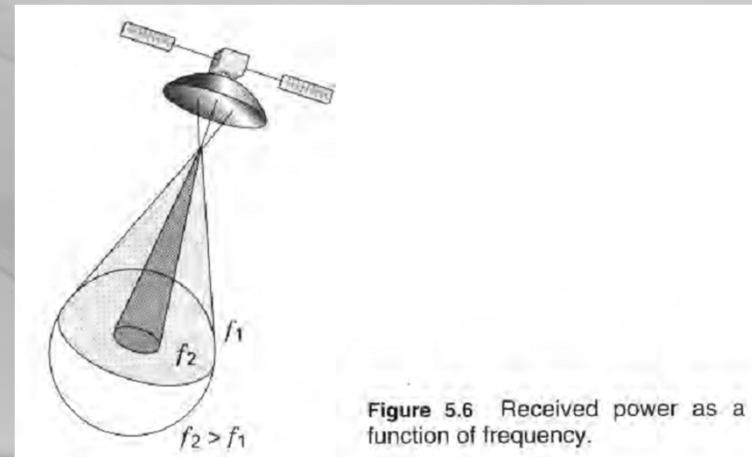


Figure 5.6 Received power as a function of frequency.

# Received signal power and noise power

- Path loss is frequency dependent

## Example 5.2 Antenna Design for Measuring Path Loss

Design a hypothetical experiment to measure path loss  $L_s$ , at frequencies  $f_1 = 30$  MHz and  $f_2 = 60$  MHz, when the distance between the transmitter and receiver is 100 km. Find the effective area of the receiving antenna, and calculate the path loss in decibels for each case.

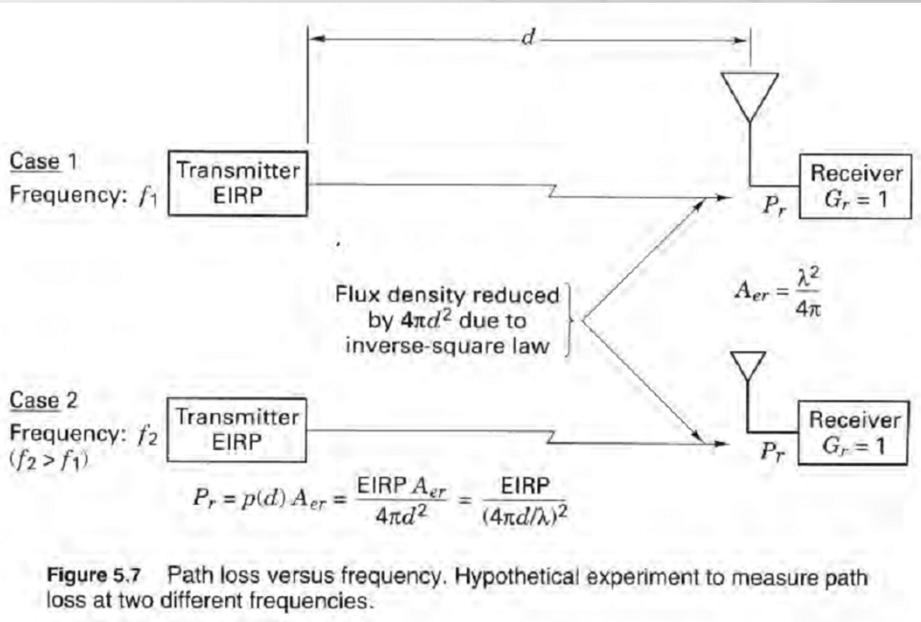


Figure 5.7 Path loss versus frequency. Hypothetical experiment to measure path loss at two different frequencies.

## Solution

Figure 5.7 illustrates the two links for measuring  $L_s$  at frequencies  $f_1$  and  $f_2$ , respectively. The power density,  $p(d)$ , at each receiver is identical and equal to

$$p(d) = \frac{\text{EIRP}}{4\pi d^2}$$

This reduction in power density is due *only* to the inverse-square law. The actual power received at each receiver is found by multiplying the power density  $p(d)$  at the receiver by the effective area,  $A_{er}$ , of the collecting antenna, as shown in Equation (5.7). Since path loss is predicated on  $G_r = 1$ , we compute the effective area  $A_{er1}$  at frequency  $f_1$ , and  $A_{er2}$  at frequency  $f_2$ , using Equation (5.9):

$$\begin{aligned} A_{er} &= \frac{\lambda^2}{4\pi} = \frac{(c/f)^2}{4\pi} \\ A_{er1} &= \frac{(3 \times 10^8/30 \times 10^6)^2}{4\pi} \approx 8 \text{ m}^2 \\ A_{er2} &= \frac{(3 \times 10^8/60 \times 10^6)^2}{4\pi} \approx 2 \text{ m}^2 \end{aligned}$$

The path loss for each case in decibels is

$$L_{s1} = 10 \times \log_{10} \left( \frac{4\pi d}{\lambda_1} \right)^2 = 10 \times \log_{10} \left( \frac{4\pi \times 10^5}{3 \times 10^8/30 \times 10^6} \right)^2 = 102 \text{ dB}$$

$$L_{s2} = 10 \times \log_{10} \left( \frac{4\pi d}{\lambda_2} \right)^2 = 10 \times \log_{10} \left( \frac{4\pi \times 10^5}{3 \times 10^8/60 \times 10^6} \right)^2 = 108 \text{ dB}$$

# Received signal power and noise power

## • Thermal noise power

Thermal noise is caused by the thermal motion of electrons in all conductors. It is generated in the lossy coupling between an antenna and receiver and in the first stages of the receiver. The noise power spectral density is constant at all frequencies up to about  $10^{12}$  Hz, giving rise to the name *white noise*. The thermal noise process in communication receivers is modeled as an additive white Gaussian noise (AWGN) process, as described in Section 1.5.5. The physical model [5, 6] for thermal or Johnson noise is a noise generator with an open-circuit mean-square voltage of  $4\kappa T^\circ W \mathcal{R}$ , where

$$\begin{aligned}\kappa &= \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K or W/K-Hz} \\ &= -228.6 \text{ dBW/K-Hz},\end{aligned}$$

$T^\circ$  = temperature, kelvin,

$W$  = bandwidth, hertz,

and

$\mathcal{R}$  = resistance, ohms.

The maximum thermal noise power  $N$  that could be coupled from the noise generator into the front end of an amplifier is

$$N = \kappa T^\circ W \quad \text{watts} \quad (5.16)$$

Thus, the maximum single-sided noise power spectral density  $N_0$  (noise power in a 1-Hz bandwidth), available at the amplifier input is

$$N_0 = \frac{N}{W} = \kappa T^\circ \quad \text{watts/hertz} \quad (5.17)$$

### Example 5.3 Maximum Available Noise Power

Using a noise generator with mean-square voltage equal to  $4\kappa T^\circ W \mathcal{R}$ , demonstrate that the maximum amount of noise power that can be coupled from this source into an amplifier is  $N_i = \kappa T^\circ W$ .

*Solution*

A theorem from network theory states that maximum power is delivered to a load when the value of the load impedance is made equal to the complex conjugate of the generator impedance [7]. In this case the generator impedance is a pure resistance,  $\mathcal{R}$ ; therefore, the condition for maximum power transfer is fulfilled when the input resistance of the amplifier equals  $\mathcal{R}$ . Figure 5.8 illustrates such a network. The input thermal noise source is represented by an electrically equivalent model consisting of a noiseless source resistor in series with an ideal voltage generator whose rms noise voltage is  $\sqrt{4\kappa T^\circ W \mathcal{R}}$ . The input resistance of the amplifier is made equal to  $\mathcal{R}$ . The noise voltage delivered to the amplifier input is just one-half the generator voltage, following basic circuit principles. The noise power delivered to the amplifier input can accordingly be expressed as

$$\begin{aligned}N_i &= \frac{(\sqrt{4\kappa T^\circ W \mathcal{R}}/2)^2}{\mathcal{R}} = \frac{4\kappa T^\circ W \mathcal{R}}{4\mathcal{R}} \\ &= \kappa T^\circ W\end{aligned}$$

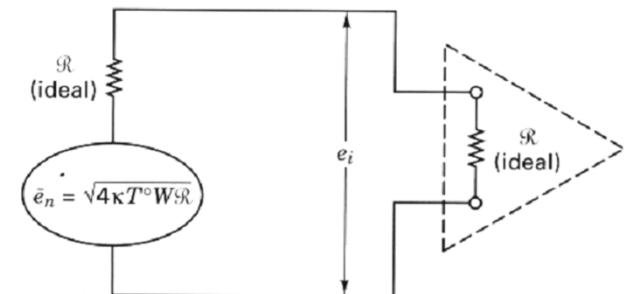


Figure 5.8 Electrical model of maximum available thermal noise power at amplifier input.

# Link Budget Analysis

In evaluating system performance, the quantity of greatest interest is the signal-to-noise ratio (SNR) or  $E_b/N_0$ , since a major concern is the ability to detect signals in the presence of noise with an acceptable error probability. Since in the case of satellite communication systems, the most usual signal structure is a modulated carrier with constant envelope, we can use average *carrier power-to-noise power* ( $C/N$ )

ratio as the predetection SNR of interest. In fact, for constant-envelope signaling, the predetection SNR is often expressed by using any of the equivalent ratio terms

$$\frac{P_r}{N} = \frac{S}{N} = \frac{C}{N} = \frac{C}{\kappa T^\circ W}$$

where  $P_r$ ,  $S$ ,  $C$ , and  $N$  are received power, signal power, carrier power, and noise power, respectively. And,  $k$ ,  $T^\circ$ , and  $W$  are Boltzmann's constant, temperature in Kelvin, and bandwidth, respectively. Is  $P_r/N$  or  $S/N$  actually the same as carrier-to-noise ( $C/N$ ) at all times? No, the signal power and the carrier power are only the same for the case of constant envelope signaling (angle modulation). For example, consider a frequency modulated (FM) carrier wave expressed in terms of the modulating message waveform  $m(t)$  as

$$s(t) = A \cos(\omega_0 t + K \int m(t) dt)$$

where  $K$  is a constant of the system. The average power in the modulating signal is  $\overline{m^2(t)}$ . Increasing this modulating power only serves to increase the frequency deviation of  $s(t)$ , which means that the carrier is spread over a wider spectrum, but its average power  $\overline{s^2(t)}$  remains equal to  $A^2/2$ , regardless of the power in the modulating signal. Thus, we can see that FM, which is an example of constant-envelope signaling, is characterized by the fact that the received signal power is the same as the carrier power.

For linear modulation, such as amplitude modulation (AM), the power in the carrier is quite different than the power in the modulating signal. For example, consider an AM carrier wave in terms of the modulating signal  $m(t)$ :

$$\begin{aligned} s(t) &= [1 + m(t)] A \cos \omega_0 t \\ \overline{s^2(t)} &= [1 + m(t)]^2 \frac{A^2}{2} \\ &= \frac{A^2}{2} [1 + \overline{m^2(t)} + 2 \overline{m(t)}] \end{aligned}$$

If we assume that  $m(t)$  has a zero mean, then the average carrier power can be written as

$$\overline{s^2(t)} = \frac{A^2}{2} + \frac{A^2}{2} \overline{m^2(t)}$$

From the above expression, it should be clear that in this case the power in the carrier wave is not the same as the signal power. In summary, the parameters  $C/N$  and  $P_r/N$  are the same for constant-envelope signaling (e.g., PSK or FSK) but are different for nonconstant-envelope signaling (e.g., ASK, QAM).

We obtain  $P_r/N$  by dividing Equation (5.11) by noise power  $N$ , as follows:

$$\frac{P_r}{N} = \frac{\text{EIRP } G_r/N}{L_s} \quad (5.18)$$

Equation (5.18) applies to any one-way RF link. With *analog receivers*, the noise bandwidth (generally referred to as the effective or equivalent noise bandwidth) seen by the demodulator is usually greater than the signal bandwidth, and  $P_r/N$  is the main parameter for measuring signal detectability and performance quality. With *digital receivers*, however, correlators or matched filters are usually implemented, and signal bandwidth is taken to be equal to noise bandwidth. Rather than consider input noise power, a common formulation for digital links is to replace noise power with *noise power spectral density*. We can use Equation (5.17) to rewrite Equation (5.18) as

$$\frac{P_r}{N_0} = \frac{\text{EIRP } G_r/T^\circ}{\kappa L_s L_n} \quad (5.19)$$

where the system effective temperature  $T^\circ$  is a function of the noise radiated into the antenna and the thermal noise generated within the first stages of the receiver. Note that the receiving antenna gain  $G_r$  and system temperature  $T^\circ$  are grouped together. The grouping  $G_r/T^\circ$  is sometimes called the *receiver figure-of-merit*. The reason for treating these terms in this way is explained in Section 5.6.2.

Assuming that all the received power  $P_r$  is in the modulating (information-bearing) signal, we now relate  $E_b/N_0$  and SNR from Equation (3.30) and write

$$\frac{E_b}{N_0} = \frac{P_r}{N} \left( \frac{W}{R} \right) \quad (5.20a)$$

$$\frac{E_b}{N_0} = \frac{P_r}{N_0} \left( \frac{1}{R} \right) \quad (5.20b)$$

and

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R \quad (5.20c)$$

# Link Budget Analysis

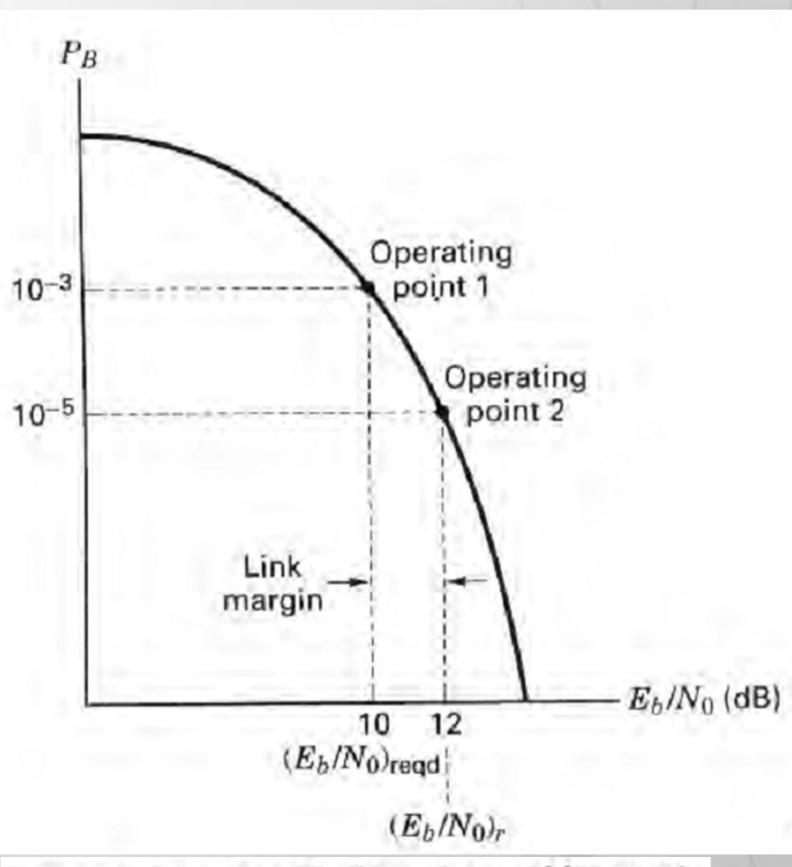


Figure 5.9 Two  $E_b/N_0$  values of interest.

$$\frac{P_r}{N_0} = \left( \frac{E_b}{N_0} \right)_r \quad R = M \left( \frac{E_b}{N_0} \right)_{\text{reqd}} \quad R$$

$$M(\text{dB}) = \left( \frac{E_b}{N_0} \right)_r (\text{dB}) - \left( \frac{E_b}{N_0} \right)_{\text{reqd}} (\text{dB})$$

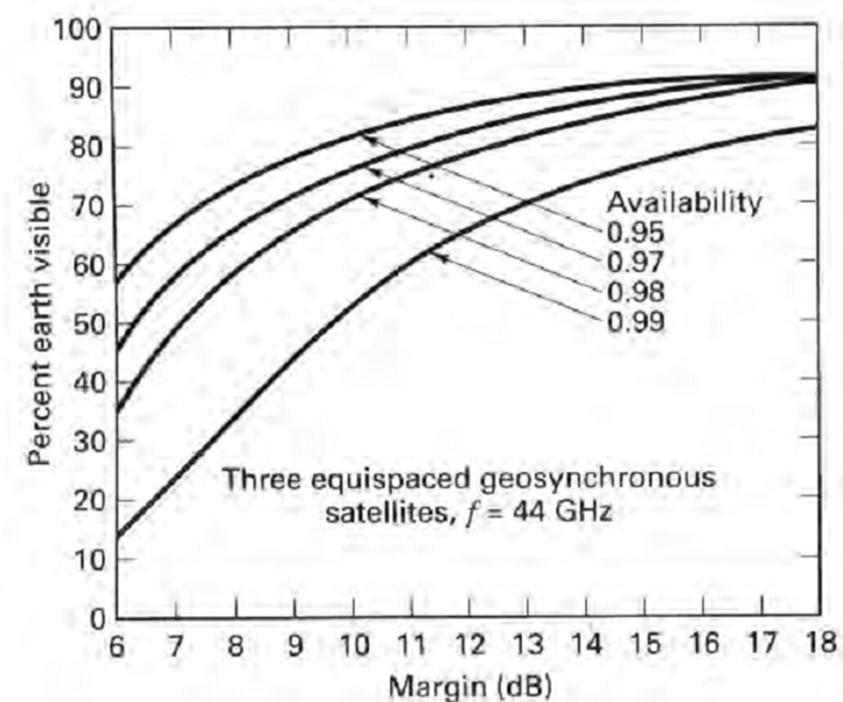
$$M = \frac{\text{EIRP } G_t / T^o}{(E_b/N_0)_{\text{reqd}} R \kappa L_s L_o}$$

$$M(\text{dB}) = \text{EIRP} (\text{dBW}) + G_t (\text{dBi}) - \left( \frac{E_b}{N_0} \right)_{\text{reqd}} (\text{dB}) - R (\text{dB-bit/s}) \\ - \kappa T^o (\text{dBW/Hz}) - L_s (\text{dB}) - L_o (\text{dB}) \quad (5.24)$$

# Link Budget Analysis

**TABLE 5.1** Proposed Direct Broadcast Satellite (DBS) from Satellite Television Corp.

Uplink		
Earth station EIRP	86.6 dBW	
Free-space loss (17.6 GHz, 48° elevation)	208.9 dB	
Assumed rain attenuation	12.0 dB	
Satellite $G/T^\circ$	7.7 dB/K	
Uplink $C/\kappa T^\circ$	102.0 dB-Hz	
Atmospheric Condition		
Downlink	Clear	5-dB Rain Attenuation
Satellite EIRP	57.0 dBW	57.0 dBW
Free-space loss (12.5 GHz, 30° elevation)	206.1 dB	206.1 dB
Atmospheric attenuation	0.14 dB	5.0 dB
Home receiver $G/T^\circ$ (0.75 m dish)	9.4 dB/K	8.1 dB/K
Receiver pointing loss (0.5" error)	0.6 dB	0.6 dB
Polarization mismatch loss (average)	0.04 dB	0.04 dB
Downlink $C/\kappa T^\circ$	88.1 dB-Hz	82.0 dB-Hz
Overall $C/\kappa T^\circ$	87.9 dB-Hz	82.0 dB-Hz
Overall $C/N$ (in 16 MHz)	15.9 dB	10.0 dB
Reference threshold $C/N$	10.0 dB	10.0 dB
Margin over threshold	5.9 dB	0.0 dB



**Figure 5.10** Earth coverage versus link margin for various values of link availability. (Reprinted from L. M. Schwab, "World-Wide Link Availability for Geostationary and Critically Inclined Orbits Including Rain Effects," *Lincoln Laboratory Rep. DCA-9*, Jan. 27, 1981, Fig. 14, p. 38, courtesy of Lincoln Laboratory.)

# Link Budget Analysis

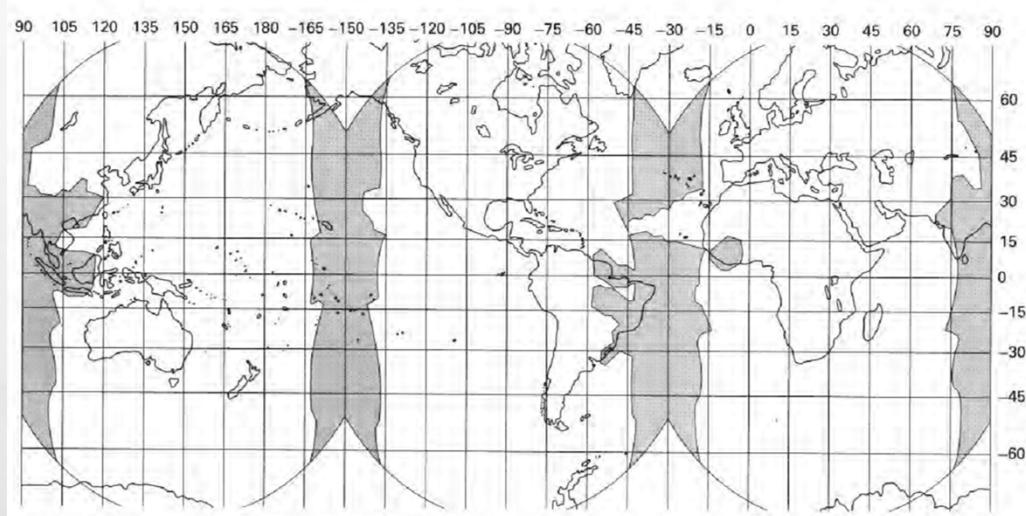


Figure 5.11 Earth coverage (unshaded) for 0.99 link availability for three equi-spaced geostationary satellites,  $f = 44$  GHz, link margin = 14 dB. (Reprinted from L. M. Schwab, "World-Wide Link Availability for Geostationary and Critically Inclined Orbits Including Rain Effects," *Lincoln Laboratory, Rep. DCA-9*, Jan. 27, 1981, Fig. 17, p. 42, courtesy of Lincoln Laboratory.)

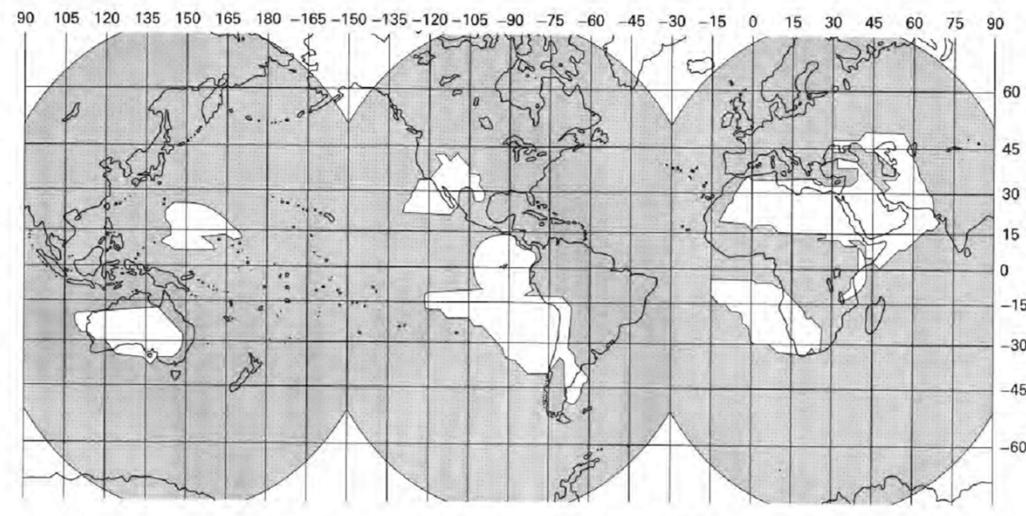
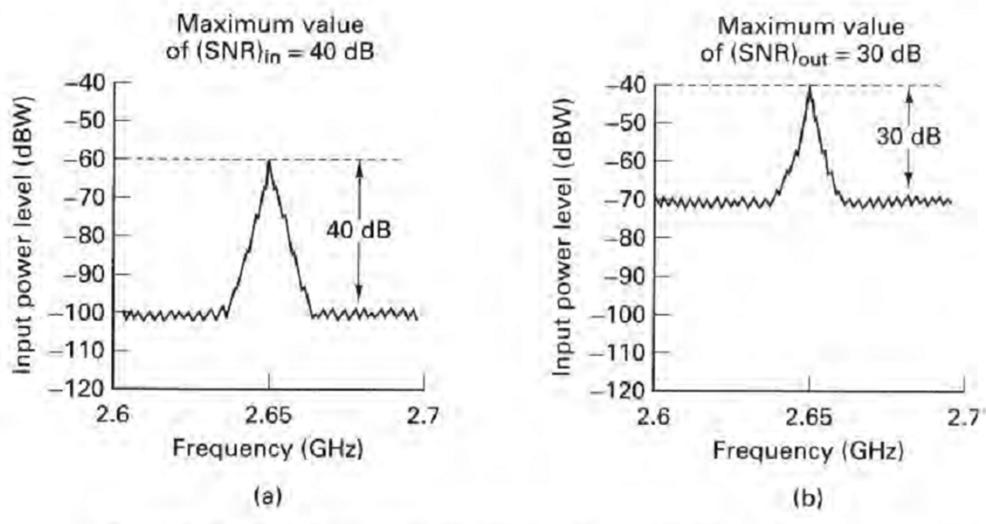


Figure 5.13 Earth coverage (unshaded) for 0.99 link availability for three equi-spaced geostationary satellites,  $f = 44$  GHz, link margin = 6 dB. (Reprinted from L. M. Schwab, "World-Wide Link Availability for Geostationary and Critically Inclined Orbits Including Rain Effects," *Lincoln Laboratory, Rep. DCA-9*, Jan. 27, 1981, Fig. 19, p. 44, courtesy of Lincoln Laboratory.)

# Noise Figure



**Figure 5.14** Amplifier signal and noise levels as a function of frequency.  
(a) Amplifier input. (b) Amplifier output.

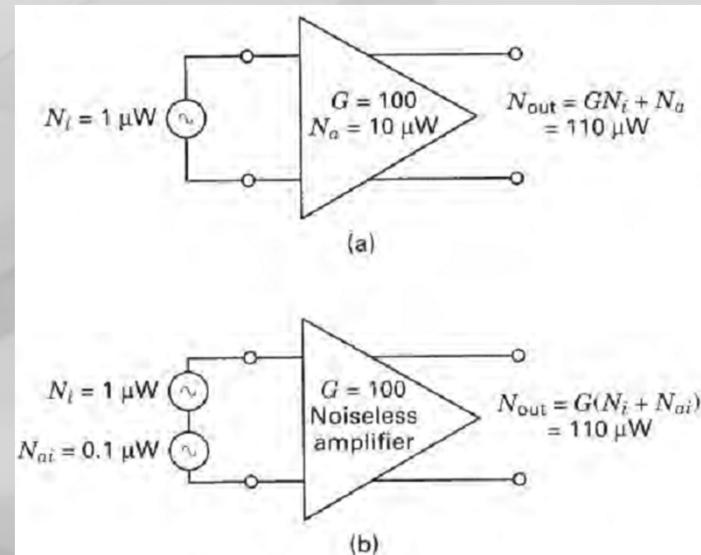
$$F = \frac{(\text{SNR})_{\text{in}}}{(\text{SNR})_{\text{out}}} = \frac{S_i/N_i}{GS_i/G(N_i + N_{ai})}$$

$S_i$  = signal power at the amplifier input port,

$N_i$  = noise power at the amplifier input port,

$N_{ai}$  = amplifier noise referred to the input port,

$G$  = amplifier gain.



**Figure 5.15** Example of noise treatment in amplifiers.

$$F = \frac{N_i + N_{ai}}{N_i} = 1 + \frac{N_{ai}}{N_i} \quad T_0^\circ = 290 \text{ K}$$

$$N_0 = \kappa T_0^\circ = 1.38 \times 10^{-23} \times 290 = 4.00 \times 10^{-21} \text{ W/Hz}$$

$$N_0 = -204 \text{ dBW/Hz}$$

# Noise Temperature

Rearranging Equation (5.26), we can write

$$N_{ai} = (F - 1)N_i \quad . \quad (5.27)$$

From Equation (5.16) we can replace  $N_i$  with  $\kappa T_0^{\circ}W$  and  $N_{ai}$  with  $\kappa T_R^{\circ}W$ , where  $T_0^{\circ}$  is the reference temperature of the source and  $T_R^{\circ}$  is called the *effective noise temperature* of the receiver (or network). We can then write

$$\kappa T_R^{\circ}W = (F - 1)\kappa T_0^{\circ}W$$

or

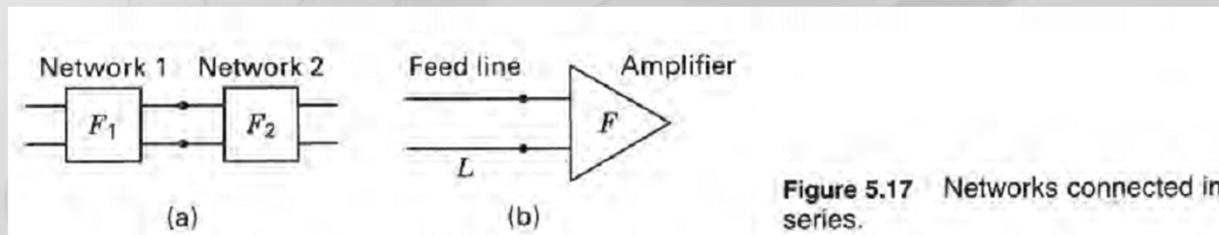
$$T_R^{\circ} = (F - 1) T_0^{\circ}$$

Or, since  $T_0^{\circ}$  has been chosen to be 290 K,

$$T_R^{\circ} = (F - 1)290 \text{ K} \quad (5.28)$$

Equation (5.26) uses the concept of noise figure to characterize the noisiness of an amplifier. Equation (5.28) represents an alternative but equivalent characterization known as *effective noise temperature*. Note that the noise figure is a measurement relative to a reference. However, noise temperature has no such constraint.

# Composite Noise Figure and Noise Temperature



**Figure 5.17** Networks connected in series.

When two networks are connected in series, as shown in Figure 5.17a, their composite noise figure can be written as

$$F_{\text{comp}} = F_1 + \frac{F_2 - 1}{G_1} \quad (5.39)$$

where  $G_1$  is the gain associated with network 1. When  $n$  networks are connected in series the relationship between stages expressed in Equation (5.39) continues, so that the *composite noise figure* for a sequence of  $n$  stages is written as

$$F_{\text{comp}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}} \quad (5.40)$$

Equations (5.40) and (5.28) can be combined to express the composite effective noise temperature of a sequence of  $n$  stages:

$$T_{\text{comp}}^{\circ} = T_1^{\circ} + \frac{T_2^{\circ}}{G_1} + \frac{T_3^{\circ}}{G_1 G_2} + \dots + \frac{T_n^{\circ}}{G_1 G_2 \dots G_{n-1}} \quad (5.41)$$

# Sky Noise Temperature

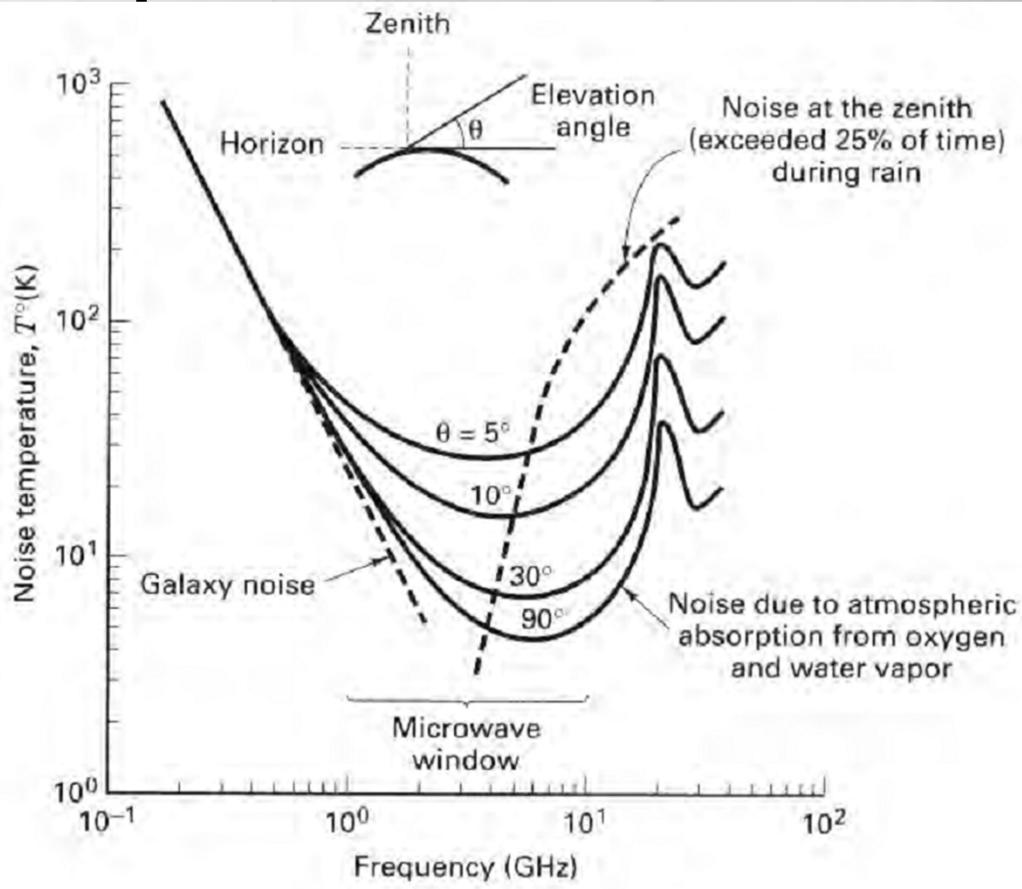
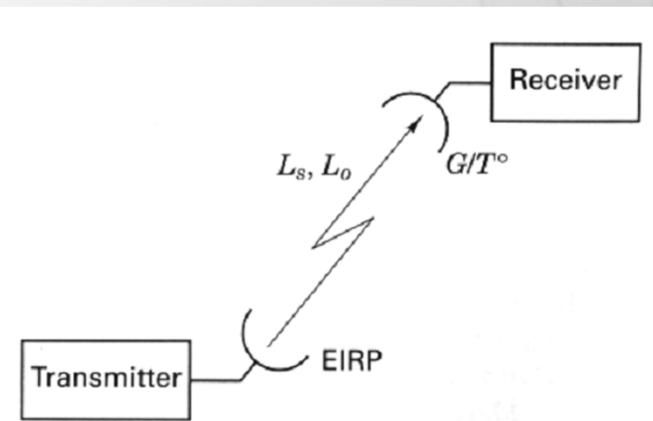


Figure 5.20 Sky noise temperature.

# Link Budget Sample



**Figure 5.23** Key parameters of a link analysis.

$$M(\text{dB}) = \text{EIRP}(\text{dBW}) + \frac{G_r}{T^o}(\text{dB/K}) - \left( \frac{E_b}{N_0} \right)_{\text{reqd}}(\text{dB}) - R(\text{dB-bits/s}) \\ - \kappa(\text{dBW/K-Hz}) - L_s(\text{dB}) - L_o(\text{dB})$$

**TABLE 5.2** Earth Terminal to Satellite Link Budget Example: Frequency = 8 GHz, Range = 21,915 Nautical Miles.

1. Transmitter power (dBW)	(100.00W)	20.0	P <sub>t</sub>
2. Transmitter circuit loss (dB)		⟨2.0⟩	L <sub>a</sub>
3. Transmitter antenna gain (peak dBi)		51.6	G <sub>t</sub>
Dish diameter (ft)	20.00		
Half-power beamwidth (degrees)	0.45		
4. Terminal EIRP (dBW)		69.6	EIRP
5. Path loss (dB)	(10° elev.)	⟨202.7⟩	L <sub>s</sub>
6. Fade allowance (dB)		⟨4.0⟩	L <sub>a</sub>
7. Other losses (dB)		⟨6.0⟩	L <sub>o</sub>
8. Received isotropic power (dBW)		-143.1	
9. Receiver antenna gain (peak dBi)		35.1	G <sub>r</sub>
Dish diameter (ft)	3.00		
Half-power beamwidth (degrees)	2.99		
10. Edge-of-coverage loss (dB)		⟨2.0⟩	L <sub>o</sub>
11. Received signal power (dBW)		-110.0	P <sub>r</sub>
Receiver noise figure at antenna port (dB)			11.5
Receiver temperature (dB-K)			35.8 (3806 K)
Receiver antenna temperature (dB-K)			24.8 (300 K)
12. System temperature (dB-K)			36.1 (4106 K)
13. System G/T° (dB/K)	-1.0		G/T°
14. Boltzmann's constant (dBW/K-Hz)			-228.60
15. Noise spectral density (dBW/Hz)	⟨-192.5⟩	N <sub>0</sub> = kT°	
16. Received P <sub>r</sub> /N <sub>0</sub> (dB-Hz)	82.5	(P <sub>r</sub> /N <sub>0</sub> ) <sub>r</sub>	
17. Data rate (dB-bit/s)	(2 Mbits/s)	⟨63.0⟩	R
18. Received E <sub>b</sub> /N <sub>0</sub> (dB)	19.5	(E <sub>b</sub> /N <sub>0</sub> ) <sub>r</sub>	
19. Implementation loss (dB)	⟨1.5⟩	L <sub>o</sub>	
20. Required E <sub>b</sub> /N <sub>0</sub> (dB)	⟨10.0⟩	(E <sub>b</sub> /N <sub>0</sub> ) <sub>reqd</sub>	
21. Margin (dB)	8.0	M	

# Link Budget Sample

1. Transmitter power is 100 W (20 dBW).
2. Circuit loss between the transmitter and antenna is 2 dB.
3. Transmitting antenna gain is 51.6 dBi.
4. The net tally of items 1 to 3 yields the EIRP = 69.6 dBW.
5. The path loss has been calculated for the range shown in the table title, corresponding to a 10° elevation angle at the earth terminal.
- 6 and 7. Here are allowances made for weather fades and a variety of other, unspecified losses.
8. Received isotropic power refers to the power that would be received, -143.1 dBW, if the receiving antenna were isotropic.
9. The peak gain of the receiving antenna is 35.1 dBi.
10. Edge-of-coverage loss is due to the off-axis antenna gain (compared to peak gain) and to the increased range for users at the extreme edge of communication coverage (a nominal 2-dB loss is shown here.)
11. The input power to the receiver, tallied from items 8, 9, and 10, is -110 dBW.
12. System temperature is found using Equation (5.46). However, in this example we are assuming a lossless line from the receiver antenna to the front end, so that the line loss factor  $L$  is equal to 1, and system temperature is computed in column 3, as  $T_S^o = T_A + T_R$ .

**TABLE 5.2** Earth Terminal to Satellite Link Budget Example: Frequency = 8 GHz, Range = 21,915 Nautical Miles.

1. Transmitter power (dBW)	(100.00W)	20.0	$P_t$
2. Transmitter circuit loss (dB)		$\langle 2.0 \rangle$	$L_o$
3. Transmitter antenna gain (peak dBi) Dish diameter (ft) Half-power beamwidth (degrees)	20.00 0.45	51.6	$G_t$
4. Terminal EIRP (dBW)		69.6	EIRP
5. Path loss (dB) (10° elev.)		202.7	$L_s$
6. Fade allowance (dB)		4.0	$L_o$
7. Other losses (dB)		6.0	$L_o$
8. Received isotropic power (dBW)		-143.1	
9. Receiver antenna gain (peak dBi) Dish diameter (ft) Half-power beamwidth (degrees)	3.00 2.99	35.1	$G_r$
10. Edge-of-coverage loss (dB)		2.0	$L_o$
11. Received signal power (dBW) Receiver noise figure at antenna port (dB) Receiver temperature (dB-K) Receiver antenna temperature (dB-K)	-110.0	$P_r$	11.5 35.8 (3806 K) 24.8 (300 K) 36.1 (4106 K)
12. System temperature (dB-K)			
13. System $G/T^o$ (dB/K)	-1.0		$G/T^o$
14. Boltzmann's constant (dBW/K-Hz)			-228.60
15. Noise spectral density (dBW/Hz)		$\langle -192.5 \rangle$	$N_0 = kT^o$
16. Received $P_r/N_0$ (dB-Hz)	82.5		$(P_r/N_0)_r$
17. Data rate (dB-bit/s)		$\langle 63.0 \rangle$	$R$
18. Received $E_b/N_0$ (dB)	19.5		$(E_b/N_0)_r$
19. Implementation loss (dB)		1.5	$L_o$
20. Required $E_b/N_0$ (dB)		10.0	$(E_b/N_0)_{\text{reqd}}$
21. Margin (dB)	8.0		$M$

# Link Budget Sample

13. We form the receiver figure-of-merit ratio  $G/T^o$  by combining the gain of the receiver antenna  $G_r$  (see item 9) with system temperature  $T_s$ . This ratio is placed in the left column as a parameter of interest, rather than in the middle column. This is because  $G_r$  is accounted for in link budget item 9, and  $T_s$  is accounted for in item 15. If  $G/T^o$  were to be placed in the center column, it would represent a double tabulation.
14. Boltzmann's constant is  $-228.6 \text{ dBW/K-Hz}$ .
15. Boltzmann's constant in decibels (item 14), plus system temperature in decibels (item 12), yields noise power spectral density.
16. Finally, we can form the received signal-to-noise spectral density,  $82.5 \text{ dB-Hz}$ , by subtracting noise spectral density in decibels (item 15), from received signal power in decibels (item 11).
17. The data rate is listed in dB-bit/s.
18. Since  $E_b/N_0 = (1/R)(P_r/N_0)$ , we need to subtract  $R$  in decibels (item 17), from  $P_r/N_0$  in decibels (item 16), yielding  $(E_b/N_0)_r = 19.5 \text{ dB}$ .
19. An implementation loss, here taken to be  $1.5 \text{ dB}$ , accounts for the difference between theoretically predicted detection performance, and the performance of the actual detector.
20. This is our required  $E_b/N_0$ , a result of the modulation and coding chosen, and the probability of error specified.
21. The difference between the received and the required  $E_b/N_0$  in decibels (taking implementation loss into account), yields the final margin.

**TABLE 5.2** Earth Terminal to Satellite Link Budget Example: Frequency = 8 GHz, Range = 21,915 Nautical Miles.

1. Transmitter power (dBW)	(100.00W)	20.0	$P_t$
2. Transmitter circuit loss (dB)		$\langle 2.0 \rangle$	$L_o$
3. Transmitter antenna gain (peak dBi) Dish diameter (ft) Half-power beamwidth (degrees)	20.00 0.45	51.6	$G_t$
4. Terminal EIRP (dBW)		69.6	EIRP
5. Path loss (dB)	(10° elev.)	$\langle 202.7 \rangle$	$L_s$
6. Fade allowance (dB)		$\langle 4.0 \rangle$	$L_o$
7. Other losses (dB)		$\langle 6.0 \rangle$	$L_o$
8. Received isotropic power (dBW)		-143.1	
9. Receiver antenna gain (peak dBi) Dish diameter (ft) Half-power beamwidth (degrees)	3.00 2.99	35.1	$G_r$
10. Edge-of-coverage loss (dB)		$\langle 2.0 \rangle$	$L_o$
11. Received signal power (dBW)		-110.0	$P_r$
Receiver noise figure at antenna port (dB)			11.5
Receiver temperature (dB-K)			35.8 (3806 K)
Receiver antenna temperature (dB-K)			24.8 (300 K)
12. System temperature (dB-K)			36.1 (4106 K)
13. System $G/T^o$ (dB/K)	-1.0		$G/T^o$
14. Boltzmann's constant (dBW/K-Hz)			-228.60
15. Noise spectral density (dBW/Hz)		$\langle -192.5 \rangle$	$N_0 = kT^o$
16. Received $P_r/N_0$ (dB-Hz)	82.5		$(P_r/N_0)_r$
17. Data rate (dB-bit/s)		$\langle 63.0 \rangle$	$R$
18. Received $E_b/N_0$ (dB)	19.5		$(E_b/N_0)_r$
19. Implementation loss (dB)		$\langle 1.5 \rangle$	$L_o$
20. Required $E_b/N_0$ (dB)		$\langle 10.0 \rangle$	$(E_b/N_0)_{\text{reqd}}$
21. Margin (dB)	8.0		$M$

# Conclusion

Of the many analyses that support a developing communication system, the link budget stands out in its ability to provide overall system insight. By examining the link budget, one can learn many things about the overall system design and performance. For example, from the link margin, one learns whether the system will meet

## Problems

its requirements comfortably, marginally, or not at all. It will be evident if there are any hardware constraints, and whether such constraints can be compensated for in other parts of the link. The link budget is often used for considering system trade-offs and configuration changes, and in understanding subsystem nuances and inter-dependencies. Together with other modeling techniques, the link budget can help predict weight, size, and cost. We have considered how to formulate this budget and how it might be used for system trade-offs. The link budget is one of the system manager's most useful documents; it represents a "bottom-line" tally in the search for optimum error performance of the system.

# Problems and Questions

- Problems

- 5.1.** (a) What is the value in decibels of the free-space loss for a carrier frequency of 100 MHz and a range of 3 miles?  
(b) The transmitter output power is 10 W. Assume that both the transmitting and receiving antennas are isotropic and that there are no other losses. Calculate the received power in dBW.  
(c) If in part (b) the EIRP is equal to 20 W, calculate the received power in dBW.  
(d) If the diameter of a dish antenna is doubled, calculate the antenna gain increase in decibels.  
(e) For the system of part (a), what must the diameter of a dish antenna be in order for the antenna gain to be 10 dB? Assume an antenna efficiency of 0.55.
- 5.7.** A receiver preamplifier has a noise figure of 13 dB, a gain of 60 dB, and a bandwidth of 2 MHz. The antenna temperature is 490 K, and the input signal power is  $10^{-12}$  W.  
(a) Find the effective temperature, in kelvin, of the preamplifier.  
(b) Find the system temperature in kelvin.  
(c) Find the output SNR in decibels.
- 5.16.** A receiver with 80-dB gain and an effective noise temperature of 3000 K is connected to an antenna that has a noise temperature of 600 K.  
(a) Find the noise power that is available from the source over a 40-MHz band.  
(b) Find the receiver noise power referenced to the receiver input.  
(c) Find the receiver output noise power over a 40-MHz band.

# Problems and Questions

- Questions

- 5.1.** Why is *free-space loss* a function of wavelength? (See Section 5.3.3.)
- 5.2.** What is the relationship between received signal-to-noise (*S/N*) ratio and carrier-to-noise (*C/N*) ratio? (See Section 5.4.)
- 5.3.** How much *link margin* is enough? (See Section 5.4.3.)
- 5.4.** There are two primary sources of noise and interference degradation at the input of a receiver. What are they? (See Section 5.5.5.)
- 5.5.** In order to achieve equitable sharing of a *nonregenerative* satellite repeater, what important relationship must exist among multiple users? (See Section 5.7.1.)