ENE 104
Electric Circuit Theory

Lecture 10:
AC Power Circuit Analysis

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Objectives : Ch11

- the instantaneous power
- the average power
- the rms value of a time-varying waveform
- complex power: average and reactive power
- the power factor, how to improve.

Instantaneous Power:

$$p(t) = v(t)i(t)$$

the device is a **resistor**:

$$p(t) = i^2(t)R = \frac{v^2(t)}{R}$$

the device is entirely **inductive**:

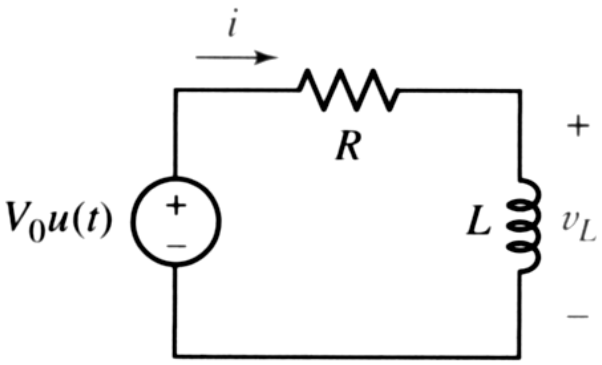
$$p(t) = Li(t)\frac{di(t)}{dt} = \frac{1}{L}v(t)\int_{-\infty}^{t} v(t')d\,t'$$

the device is entirely **capacitive**:

$$p(t) = Cv(t)\frac{dv(t)}{dt} = \frac{1}{C}i(t)\int_{-\infty}^{t} i(t')d\,t'$$

Instantaneous Power:

Consider the series RL circuit,



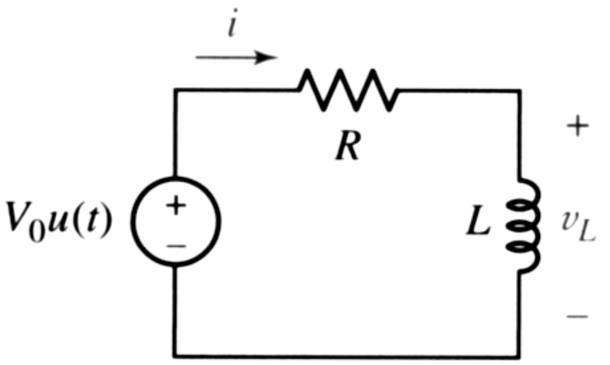
$$i(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)u(t)$$

The total power delivered by the source

$$p(t) = v(t)i(t) = \frac{V_0^2}{R}\left(1 - e^{-Rt/L}\right)u(t)$$

Instantaneous Power:

Consider the series RL circuit,



$$i(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)u(t)$$

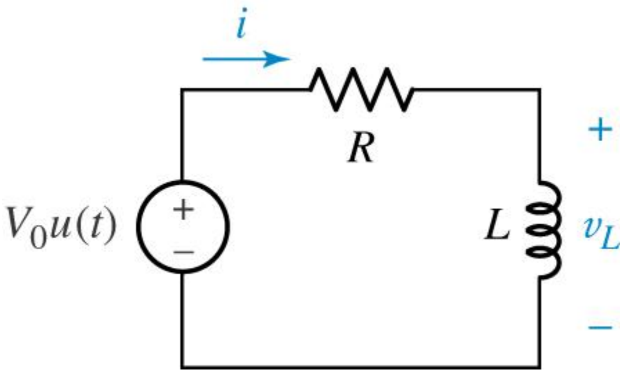
The power delivered to the resistor is

$$p_R(t) = i^2(t)R = \frac{V_0^2}{R}(1 - e^{-Rt/L})^2u(t)$$

Instantaneous Power:

Consider the series RL circuit,

$$i(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)u(t)$$



to determine the power absorbed by the inductor, first

$$\begin{aligned} v_L(t) &= L\frac{di(t)}{dt} \\ &= V_0 e^{-Rt/L}u(t) + \frac{LV_0}{R}\left(1 - e^{-Rt/L}\right)\frac{du(t)}{dt} \\ &= V_0 e^{-Rt/L}u(t) \end{aligned}$$

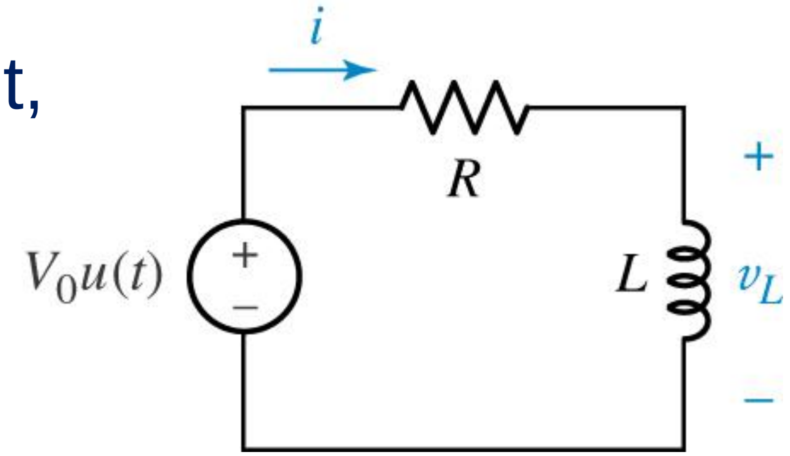
Then,

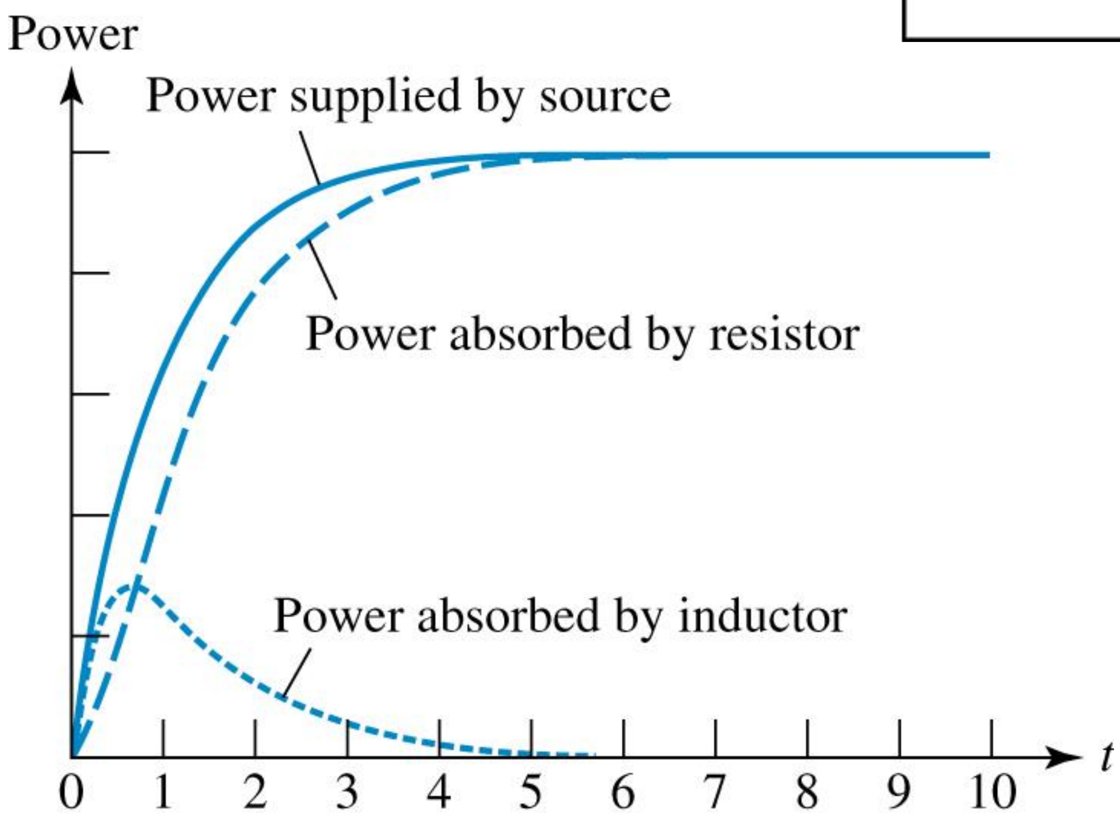
$$p_L(t) = v_L(t)i(t) = \frac{V_0^2}{R}e^{-Rt/L}\left(1 - e^{-Rt/L}\right)^2 u(t)$$

Instantaneous Power:

Consider the series RL circuit,

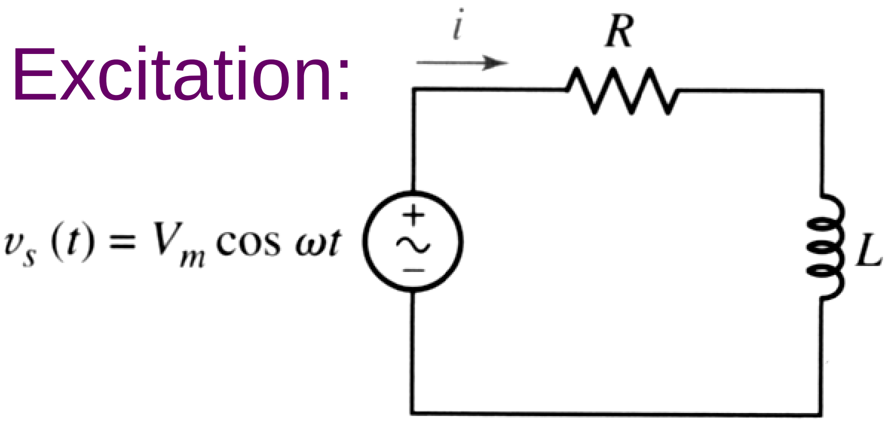
$$p(t) = p_R(t) + p_L(t)$$





Instantaneous Power:

Power Due to Sinusoidal Excitation:



the response is

$$i(t) = I_m \cos(\omega t + \phi)$$

Where

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \phi = -\tan^{-1}\frac{\omega L}{R}$$

Practice: 11.1

A current source of 12cos(2000t) A., a 200-ohm resistor, and a 0.2-H inductor are in parallel. Assume steady-state conditions exist. At t = 1ms., find the power being absorbed by the

(a) resistor

(b) inductor

(c) sinusoidal source

Practice: 11.1

- define voltage $v(t)$ across the parallel combination

$$12\angle 0^\circ = \frac{\mathbf{V}}{200} + \frac{\mathbf{V}}{j0.2(2000)}$$

where $\mathbf{V} = 2147\angle 26.57^\circ$ V so $v(t) = 2147\cos(2000t + 26.57^\circ)$ V

(a) $P_R(t) = \dfrac{v^2(t)}{200}$ so $P(0.001) = \underline{13.98\text{ kW}}$

(b) $i_L(t) = 12\cos 2000t - \dfrac{v(t)}{200}$

$v(0.001) = -1672$ V so $i_L(0.001) = 3.366$ A

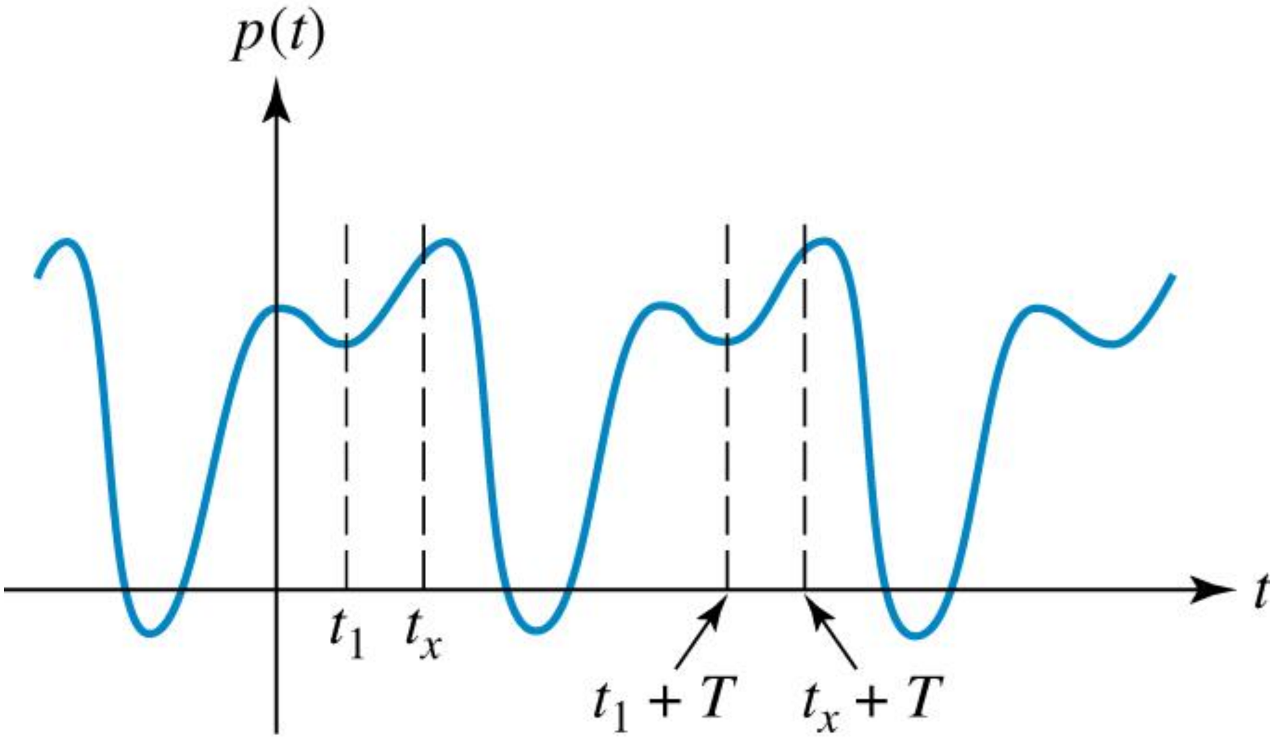
Thus, $P_L(0.001) = -1672 \times 3.366 = \underline{-5.628\text{ kW}}$

(c) $P_S(0.001) = -(13.98 - 5.628) = \underline{-8.352\text{ kW (absorbed)}}$

Average Power:

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt$$

For Periodic Waveforms: $f(t) = f(t+T)$

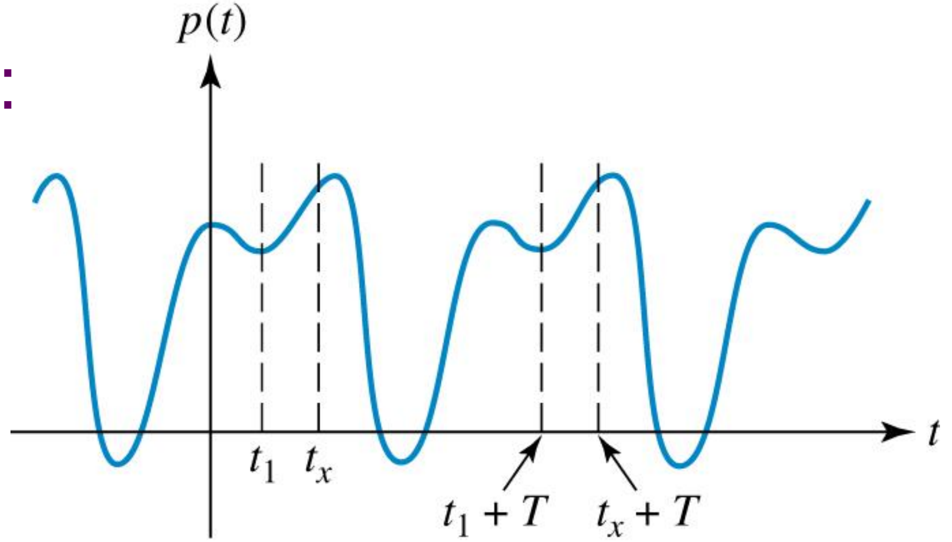


$$P_1 = \frac{1}{T}\int_{t_1}^{t_1+T} p(t)dt$$

$$P_x = \frac{1}{T}\int_{t_x}^{t_x+T} p(t)dt$$

Average Power:

For Periodic Waveforms:

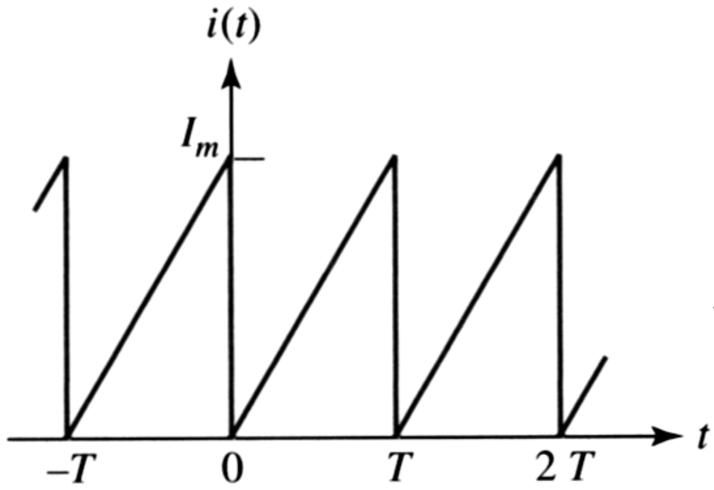


$$P = \frac{1}{nT} \int_{t_x}^{t_x + nT} p(t)\,dt \qquad n = 1, 2, 3, \ldots$$

$$P = \frac{1}{nT} \int_{-nT/2}^{nT/2} p(t)\,dt \qquad\qquad P = \lim_{n \to \infty} \frac{1}{nT} \int_{-nT/2}^{nT/2} p(t)\,dt$$

Example:

find the average power delivered to a resistor R by the periodic sawtooth current waveform

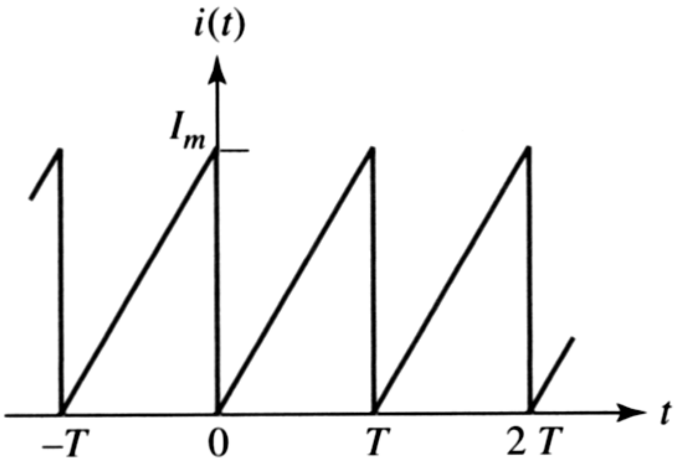


$$i(t) = \frac{I_m}{T} t, \quad 0 < t \le T$$

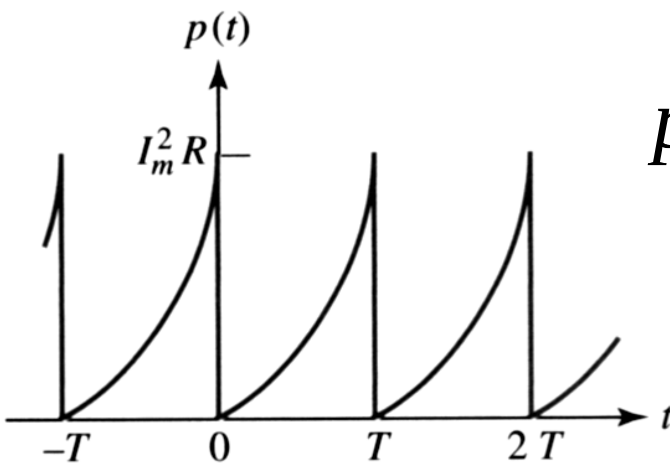
$$i(t) = \frac{I_m}{T}(t - T), \quad T < t \le 2T$$

$$p(t) = v(t)i(t) = i^2(t)R = \frac{v^2(t)}{R}$$

Example:



(a)



(b)

$$i(t) = \frac{I_m}{T} t, \quad 0 < t \le T$$

$$i(t) = \frac{I_m}{T}(t - T), \quad T < t \le 2T$$

$$p(t) = \frac{1}{T^2} I_m^2 R t^2, \quad 0 < t \le T$$

$$p(t) = \frac{1}{T^2} I_m^2 R (t - T)^2, \quad T < t \le 2T$$

$$P = \frac{1}{T} \int_0^T \frac{I_m^2 R}{T^2} t^2 dt = \frac{1}{3} I_m^2 R$$

Average Power:

In the Sinusoidal Steady State

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

the instantaneous power is

$$p(t) = V_m I_m \cos(\omega t + \theta)\cos(\omega t + \phi)$$

Or $p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$

By inspection:

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta - \phi)}$$

Example:

Given the time-domain voltage $v = 4\cos(\pi t/6)$ V., find both the average power and an expression for the instantaneous power that result when the corresponding phasor voltage $\mathbf{V} = 4\angle 0^{o}$ V. is applied across an impedance $\mathbf{Z} = 2\angle 60^{o}$ ohm.

Sol:

Example:

$v = 4\cos(\pi t/6)$ V. $\longrightarrow$ $\mathbf{V} = 4\ \angle 0^o$ V.

$\mathbf{Z} = 2\ \angle 60^o$ ohm.

Sol:

The phasor current is $\mathbf{V}/\mathbf{Z} = 2\ \angle -60^o \rightarrow i = 2\cos\left(\pi t/6 - 60^\circ\right)$

So the average power is: P

$$= \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$= \frac{1}{2}(4)(2)\cos(60^\circ) = 2W$$

Example:

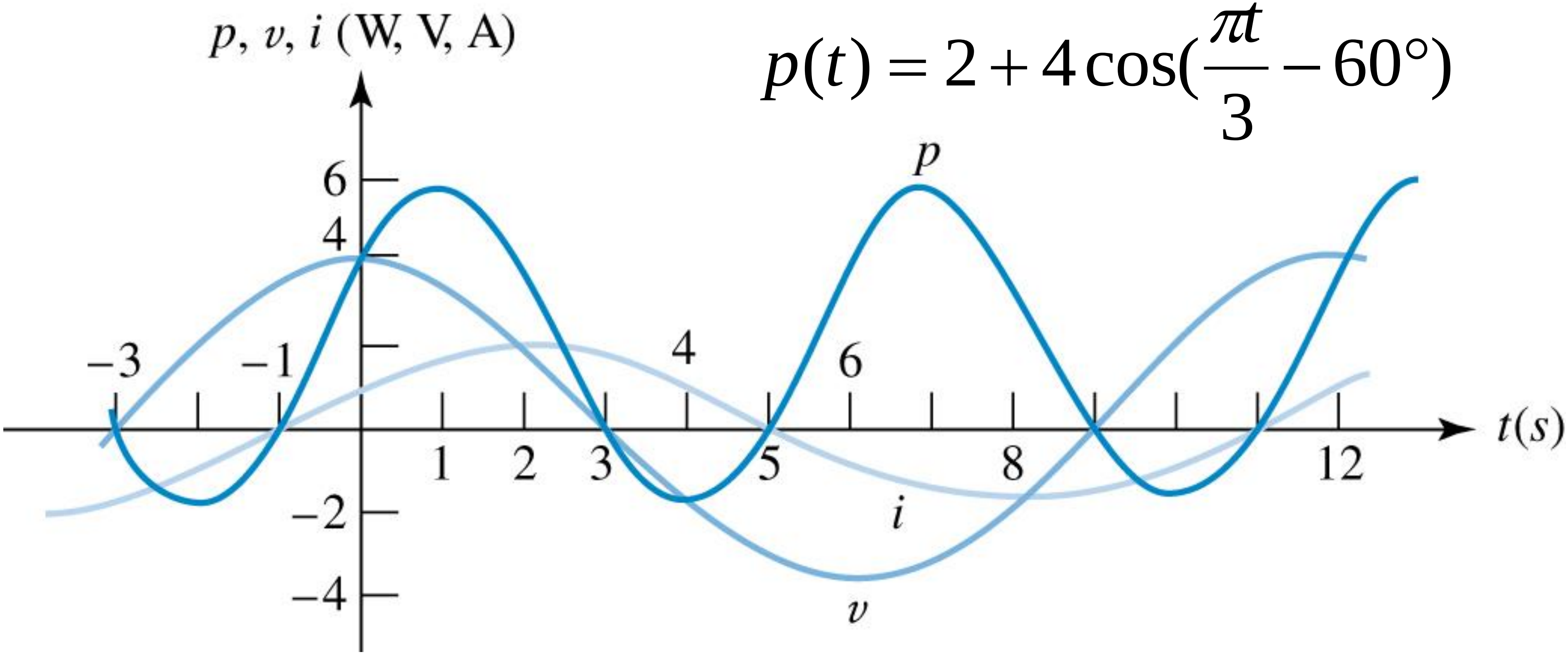
$$v = 4\cos(\pi t/6) \qquad\qquad i = 2\cos\left(\pi t/6 - 60^\circ\right)$$

the instantaneous power is:

$$\begin{aligned} p(t) &= V_m I_m \cos(\omega t + \theta)\cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi) \\ &= 8\cos(\frac{\pi t}{6})\cos(\frac{\pi t}{6} - 60^\circ) = 2 + 4\cos(\frac{\pi t}{3} - 60^\circ) \end{aligned}$$

Example:

$$p(t) = 2 + 4\cos(\frac{\pi t}{3} - 60°)$$



$$v = 4\cos(\pi t/6)$$

$$i = 2\cos(\pi t/6 - 60^{\circ})$$

Practice: 11.2

Given the phasor voltage **V** = $115\sqrt{2}\,\angle 45^\circ$ V. across an impedance **Z** = 16.26 $\angle 19.3^o$ ohm., obtain an expression for the instantaneous power, and compute the average power if $\omega = 50$ rad/s.

Sol:

$$p(t) = V_m I_m \cos(\omega t + \theta)\cos(\omega t + \phi)$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$$

Practice: 11.2

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{115\sqrt{2}\angle 45^\circ}{16.26\angle 19.3^\circ} = 10\angle 25.7^\circ \text{ A}$$

$$\begin{aligned} \mathrm{P}(t) &= 115\sqrt{2}\cos(\omega t + 45^\circ) \square 10\cos(\omega t + 25.7^\circ) \\ &= \frac{1150\sqrt{2}}{2}\left[\cos(2\omega t + 45^\circ + 25.7^\circ) + \cos(45^\circ - 25.7^\circ)\right] \\ &= \underline{767.5 + 813.2\cos(2\omega t + 70.7^\circ)\text{ W}} \quad \text{and} \quad \underline{\mathrm{P} = 767.5\text{ W}} \end{aligned}$$

Average Power:

Absorbed by an Ideal Resistor:

$$P_R = \frac{1}{2} V_m I_m \cos(0) = \frac{1}{2} V_m I_m$$

Or

$$P_R = \frac{1}{2} I_m^2 R = \frac{V_m^2}{2R}$$

Absorbed by Purely Reactive Elements:

$$P_x = 0$$

Example:

Find the average power being delivered to an impedance $\mathbf{Z_L}$ = 8-j11 ohm. by a current $\mathbf{I} = 5\angle 20^o$ A.

Sol:

$$P_R = \frac{1}{2} I_m^2 R = \frac{1}{2}(5)^2 8 = 100 \quad W.$$

Practice: 11.3

Calculate the average power delivered to the impedance $6\angle 25°\Omega$ by the current $\mathbf{I} = 2 + j5$ A.

$$\mathbf{I} = 2 + j5 = 5.385\angle 68.20° \text{ A}$$

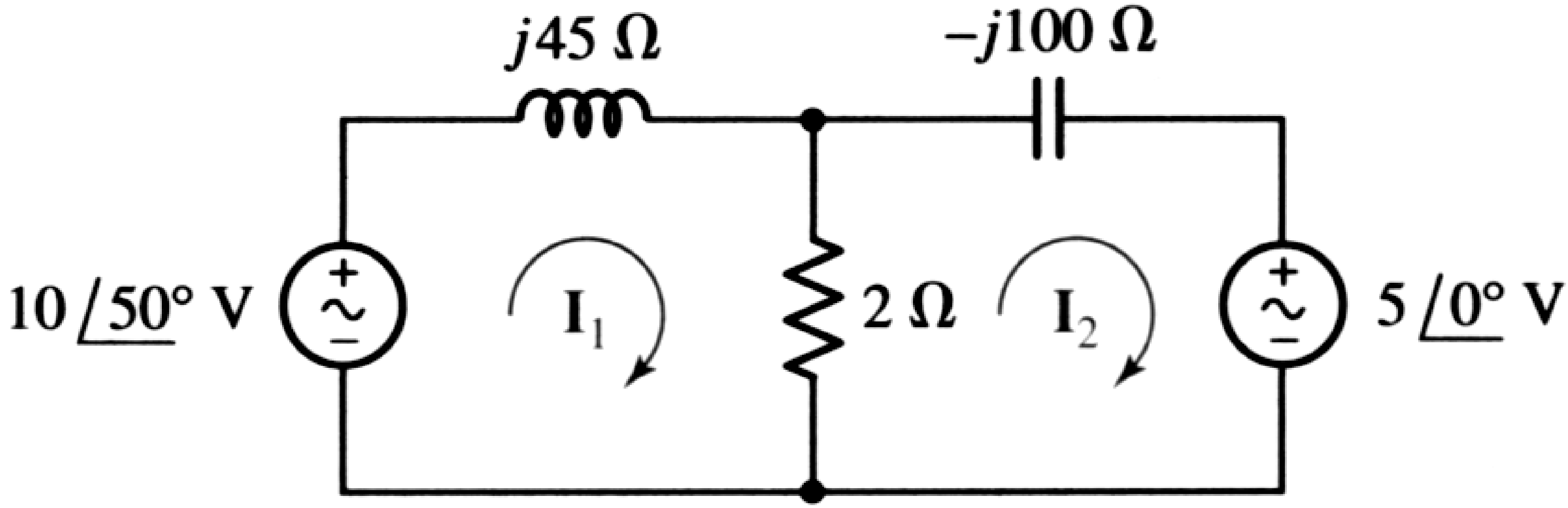
$$6\angle 25°\Omega = 5.438 + j2.536\,\Omega$$

$$\therefore \text{P} = \frac{1}{2}\left(5.385\right)^2 \square\, 5.438 = \underline{78.85 \text{ W}}$$

Practice: 11.4

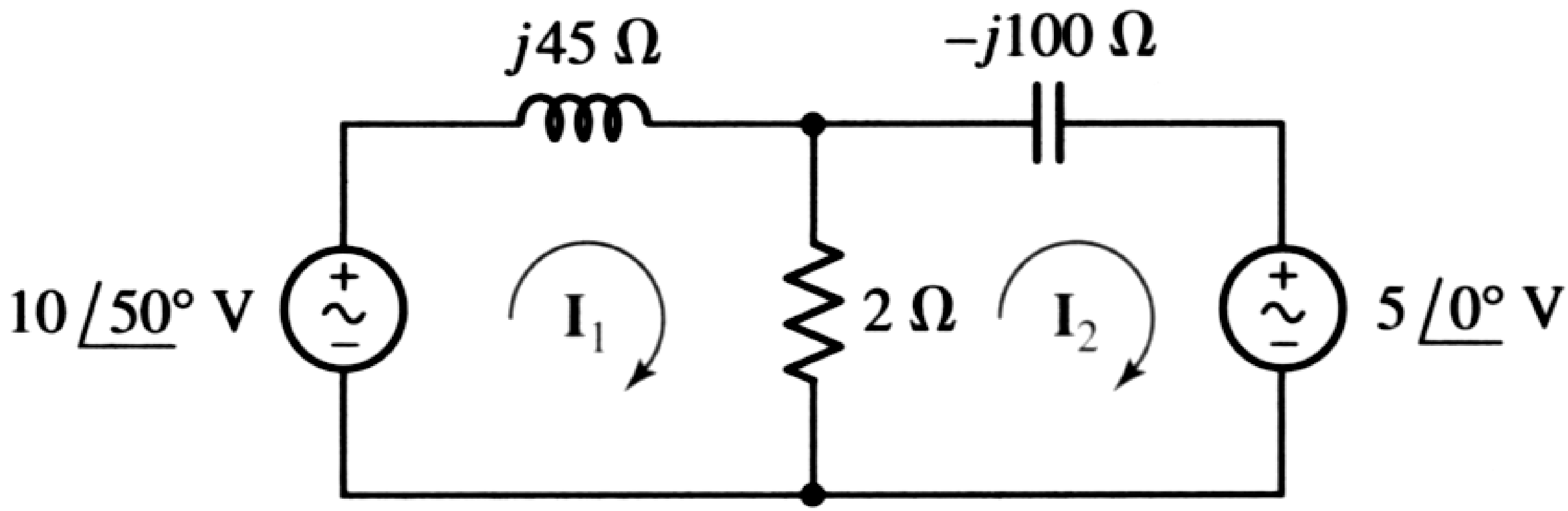
Compute the average power delivered to each of the passive elements.

Sol:

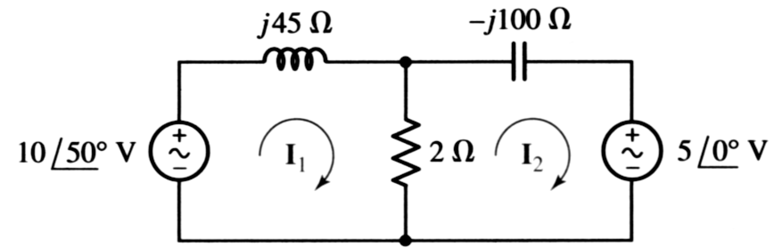


Practice: 11.4

For the circuit in the figure below, compute the average power delivered to each of the passive elements. Verify your answer by computing the power delivered by the two sources.



Practice: 11.4



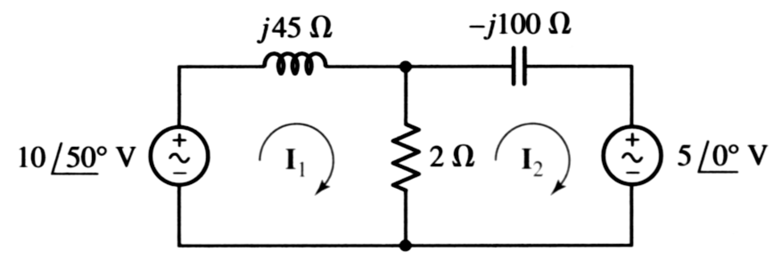
$$\text{mesh 1:}\quad 10\angle 50^\circ = (2 + j45)\mathbf{I}_1 - 2\mathbf{I}_2$$

$$\text{mesh 2:}\quad -5 = -2\mathbf{I}_1 + (2 - j100)\mathbf{I}_2$$

$$\text{Solving, } \mathbf{I}_1 = 0.1742 - j0.1352 \text{ A} = 0.2205\angle -37.82^\circ \text{ A}$$

$$\mathbf{I}_2 = 0.0018 - j0.0466 \text{ A} = 0.04664\angle -87.79^\circ \text{ A}$$

Practice: 11.4



So

$$P_{2\Omega} = \frac{1}{2}(\mathbf{I}_1 - \mathbf{I}_2)^2 \times 2 = (0.1938)^2 = \underline{37.56 \text{ mW}}$$

The left source supplies

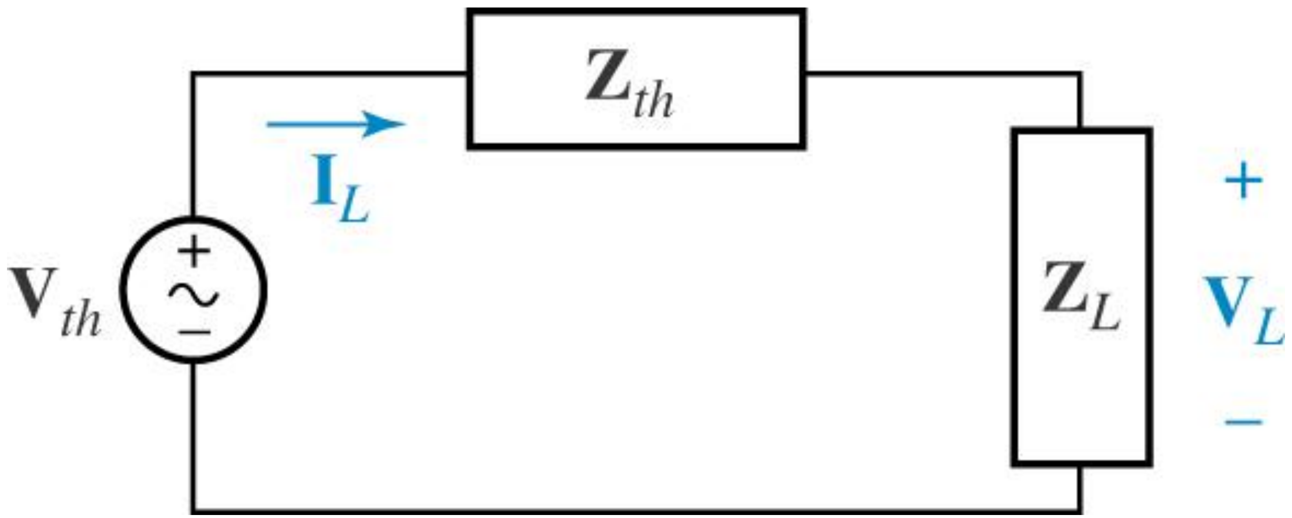
$$P = \frac{1}{2}(10)(0.2205)\cos(50^\circ + 37.82^\circ) = 41.94 \text{ mW}$$

and the right source supplies

$$P = -\frac{1}{2}(5)(0.04664)\cos(87.79^\circ) = -4.496 \text{ mW}$$

$41.94 - 4.496 = 37.44$ mW – agrees within rounding error

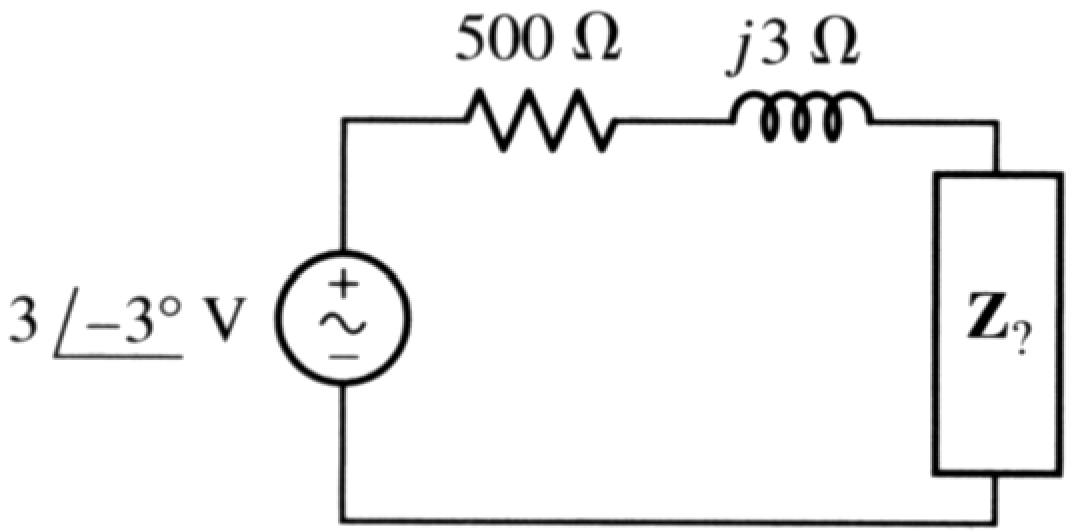
Maximum Power Transfer:



An independent voltage source in *series* with an impedance $\mathbf{Z_{th}}$ delivers a maximum average power to that load impedance $\mathbf{Z_L}$ which is the conjugate of $\mathbf{Z_{th}}$, or $\mathbf{Z_L = Z^*_{th}}$

Example 11.5:

If we are assured that the voltage source is delivering maximum average power to the unknown impedance, what is its value?

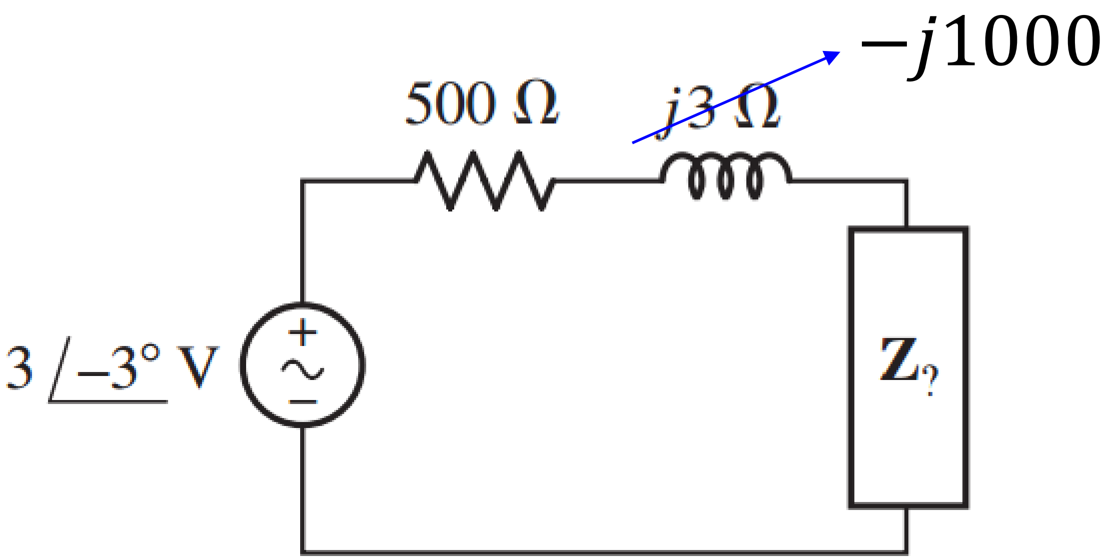


Sol:

$$Z_? = Z_{th}^* = 500 - j3 \quad \Omega$$

Practice: 11.5

If the 30-mH inductor of Example 11.5 is replaced with a 10-μF capacitor, what is the value of the inductive component of the unknown impedance $\mathbf{Z}_?$ if it is known that $\mathbf{Z}_?$ is absorbing maximum power?



Practice: 11.5

$$\mathbf{Z}_{TH} = 500 - \frac{j}{(100)(10 \times 10^{-6})} = 500 - j1000\ \Omega$$

$\mathbf{Z}_L$ must be $\mathbf{Z}^*_{TH} = 500 + j1000\ \Omega$ if it is absorbing maximum power.

Thus, the inductive component of the load is $\dfrac{1000}{100} = \underline{10\text{ H}}$

Average Power:

For Nonperiodic Function: for example

$$i(t) = \sin t + \sin \pi t$$

the average power delivered to a 1 ohm resistor:

$$P = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} (\sin^2 t + \sin^2 \pi t + 2 \sin t \sin \pi t) dt$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \quad Watt$$

Average Power:

For

$$i(t) = I_{m1}\cos\omega_1 t + I_{m2}\cos\omega_2 t + \ldots + I_{mN}\cos\omega_N t$$

the average power delivered to a resistor R:

$$P = \frac{1}{2}\left(I_{m1}^2 + I_{m2}^2 + \ldots + I_{mN}^2\right)R$$

Practice: 11.6

A voltage source v_s is connected across a 4-Ω resistor. Find the average power absorbed by the resistor if v_s equals (a) $8sin200t$ (b) $8sin200t - 6\cos(200t - 45°)$ V. (c) $8sin200t - 4sin100t$ V. (d) $8sin200t - 6\cos(200t - 45°) - 5sin100t + 4$ V.

Practice: 11.6

(a) $P = \frac{1}{2}\frac{(8)^2}{4} = \underline{8\ \text{W}}$

(b) $\mathbf{V}_S = 8\angle -90^\circ - 6\angle -45^\circ = 5.667\angle -138.5^\circ\ \text{V}$

$\therefore P = \frac{1}{2}\frac{(5.667)^2}{4} = \underline{4.014\ \text{W}}$

(c) $P = \frac{1}{2}\frac{8^2}{4} + \frac{1}{2}\frac{4^2}{4} = \underline{10\ \text{W}}$

(the sinusoids have different frequencies, so superposition applies to this power calculation).

(d) Combining the two sinusoids with $\omega = 200$ rad/s,

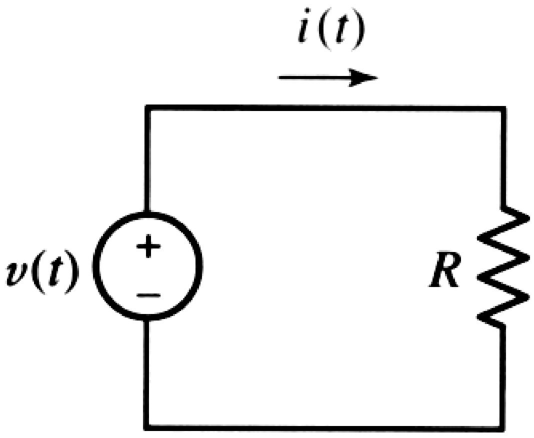
$8\angle -90^\circ - 6\angle -45^\circ = 5.667\angle -138.5^\circ\ \text{V}$

$\therefore P = \frac{1}{2}\frac{(5.667)^2}{4} + \frac{1}{2}\left(\frac{5^2}{4}\right) + \frac{4^2}{4} = \underline{11.14\ \text{W}}$

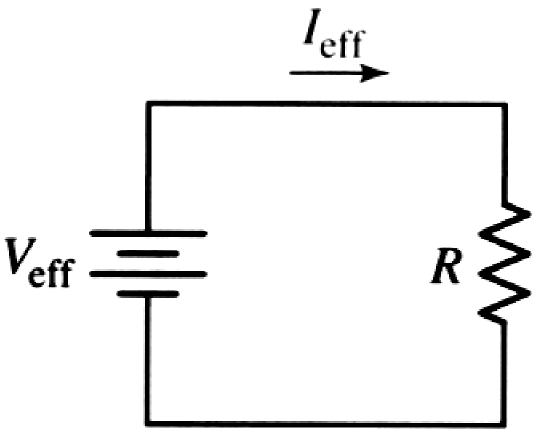
(the average power provided by the dc source is simply 4 W; it operates at a "frequency" different from the other sources).

Effective Values:

If the resistor received the same average power in part (a) and (b), then the effective value of i(t) is equal to I_{eff}, and the effective value of v(t) is equal to V_{eff}



(a)



(b)

Effective Values:

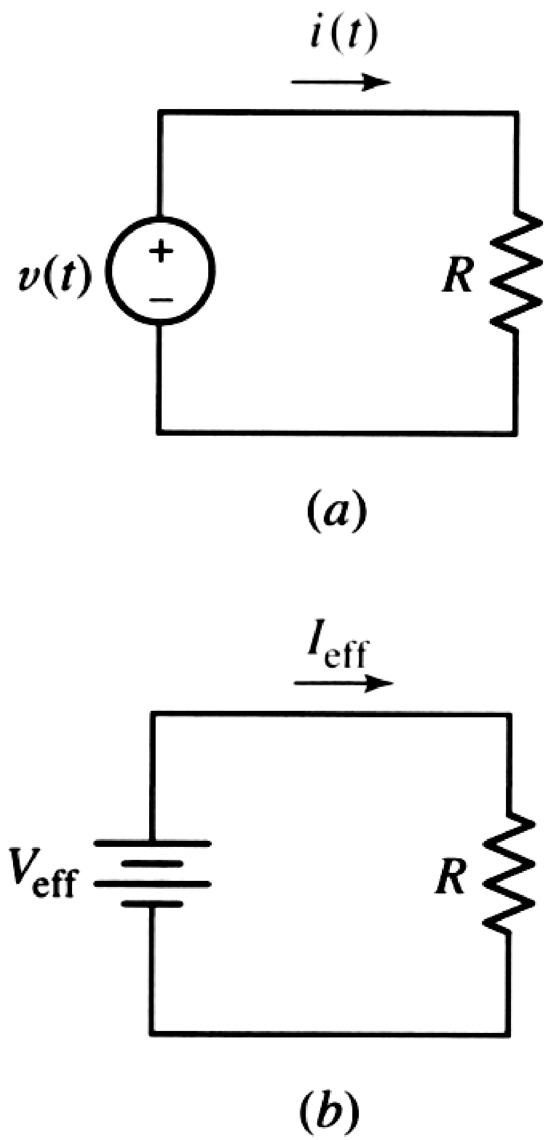
of a Periodic Waveform:

the average power delivered to the resistor,

$$P = \frac{1}{T}\int_0^T i^2 dt = \frac{R}{T}\int_0^T i^2 dt$$

the power delivered by the the direct current is:

$$P = I_{eff}^2 R$$



Effective Values:

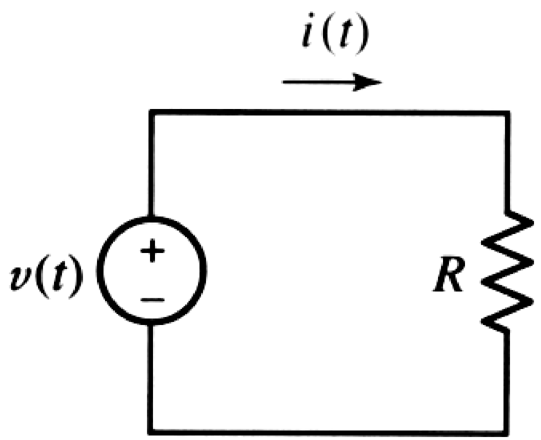
of a Periodic Waveform:

$$\frac{R}{T}\int_0^T i^2 dt = I_{eff}^2 R$$

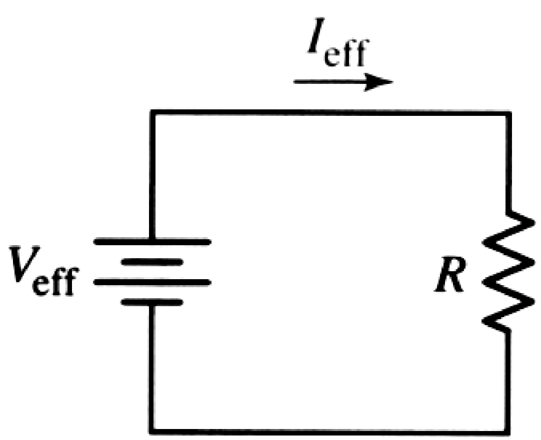
we get

$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}}$$

note: the effective value is often called the **root-mean-square** value, or simply the **rms** value.



(a)



(b)

Effective Values:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

of a sinusoidal Waveform:

$$i(t) = I_m \cos(\omega t + \phi)$$

which has a period $T = \dfrac{2\pi}{\omega}$

to obtain the effective value

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi) dt}$$

$$= I_m \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} [\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega t + 2\phi)] dt}$$

Effective Values:

of a sinusoidal Waveform

$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}}$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi) dt}$$

$$= I_m \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} [\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega t + 2\phi)] dt}$$

$$= I_m \sqrt{\frac{\omega}{4\pi}[t]_0^{2\pi/\omega}}$$

$$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$$

Effective Values:

Use of RMS Values to Compute Average Power

$$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$$

$$P = \frac{1}{2} I_m^2 R = I_{eff}^2 R = \frac{V_{eff}^2}{R}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

Effective Values:

with Multiple-Frequency Circuits

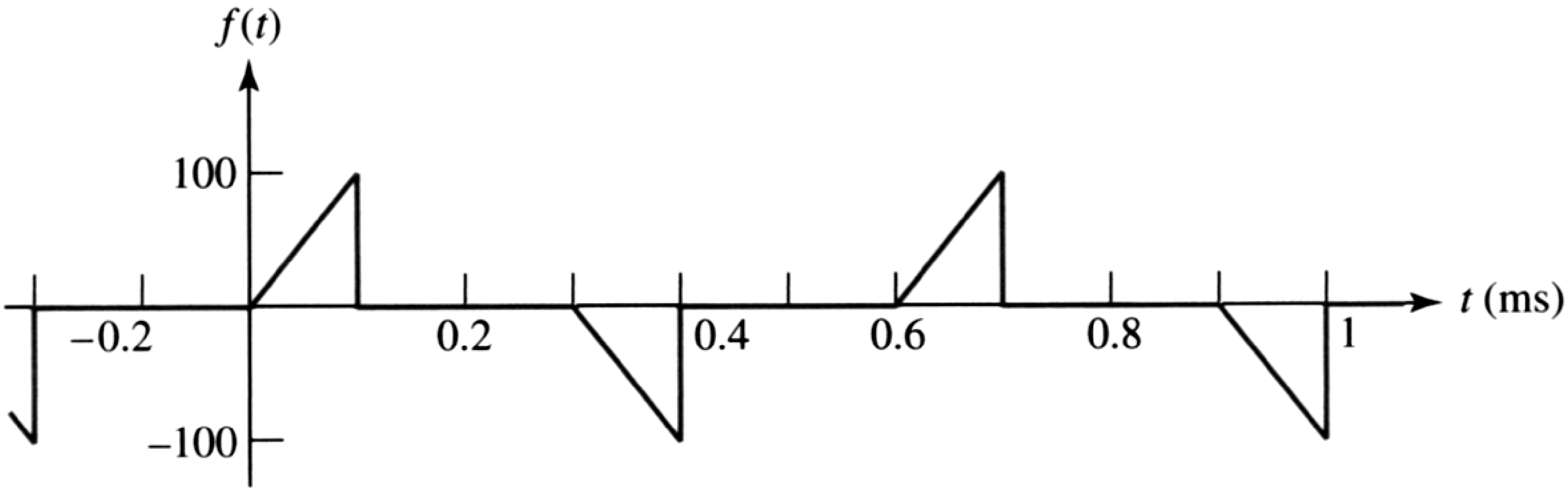
$$I_{eff} = \frac{I_m}{\sqrt{2}}$$

$$P = \left(I_{1eff}^2 + I_{2eff}^2 + \ldots + I_{Neff}^2\right)R$$

$$I_{eff} = \sqrt{I_{1eff}^2 + I_{2eff}^2 + \ldots + I_{Neff}^2}$$

Effective Values: Example

Ex22 Page 384: find the effective value of:



$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

Practice: 11.7

Calculate the effective value of each of the periodic voltages: (a) $6cos25t$; (b) $6cos25t + 4\sin(25t + 30°)$ V. (c) $6cos25t + 5cos^2(25t)$ V. (d) $6cos25t + 5sin30t + 4$ V.

Practice: 11.7

(a) $\dfrac{6}{\sqrt{2}} = \underline{4.243}$

(b) $6\cos 25t + 4\sin(25t + 30^\circ)$

may be combined: $6\angle 0^\circ + 4\angle\left(30^\circ - 90^\circ\right) = 8.718\angle -23.41^\circ$

$\therefore$ effective value is $\dfrac{8.718}{\sqrt{2}} = \underline{6.165}$

(c) $\cos^2 25t = \dfrac{1}{2}(1 + \cos 50t)$

Practice: 11.7

The effective value is

$$\left[\frac{1}{T}\int_0^T (6\cos 25t + 5\cos^2 25t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (36\cos^2 25t + 60\cos^3 25t + 25\cos^4 25t)dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T 36\cos^2 25t\, dt + \frac{1}{T}\int_0^T 60\cos^3 25t\, dt + \frac{1}{T}\int_0^T 25\cos^4 25t\, dt\right]^{1/2}$$

where $T = \dfrac{1}{25}$ seconds

Solving each integral, we obtain

Practice: 11.7

$$\left[\frac{36}{T}\int_0^T\left(\frac{1}{2}+\frac{1}{2}\cos 50t\right)dt+\frac{60}{T}\int_0^T\left(\frac{1}{2}\cos 25t+\frac{1}{4}\cos 75t+\frac{1}{4}\cos 25t\right)dt\right.$$

$$\left.+\frac{25}{T}\int_0^T\left(\frac{1}{4}+\frac{1}{2}\cos 50t+\frac{1}{8}+\frac{1}{8}\cos 100t\right)dt\right]^{1/2}$$

$$=\left[\frac{36\times 25}{2}\left(\frac{1}{25}\right)+(60)(25)(0)+(25)(25)\left(\frac{1}{4}+\frac{1}{8}\right)\left(\frac{1}{25}\right)\right]^{1/2}$$

$$=\sqrt{18+\frac{75}{8}}=\underline{5.232}$$

(d) $[36 + 25 + 16]^{1/2} = \underline{8.775}$

Apparent power and Power factor:

The sinusoidal voltage $$v = V_m \cos(\omega t + \theta)$$

is applied to the network and the resultant sinusoidal current is

$$i = I_m \cos(\omega t + \phi)$$

the average power delivered to the network

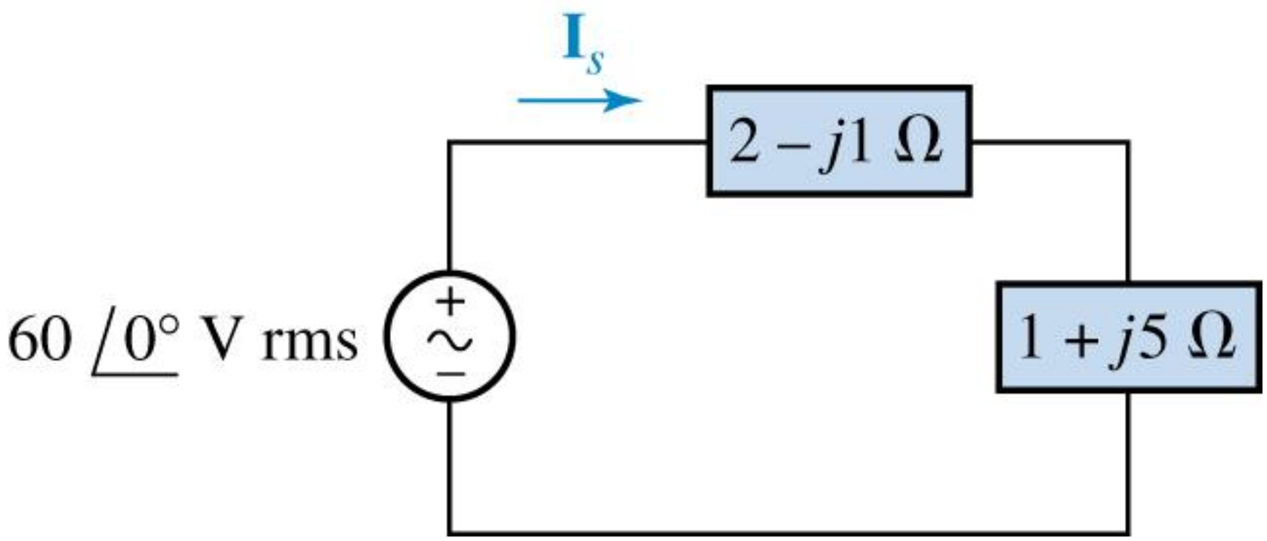
$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

the apparent power: $V_{eff} I_{eff}$

the power factor: $$PF = \frac{P}{V_{eff} I_{eff}}$$

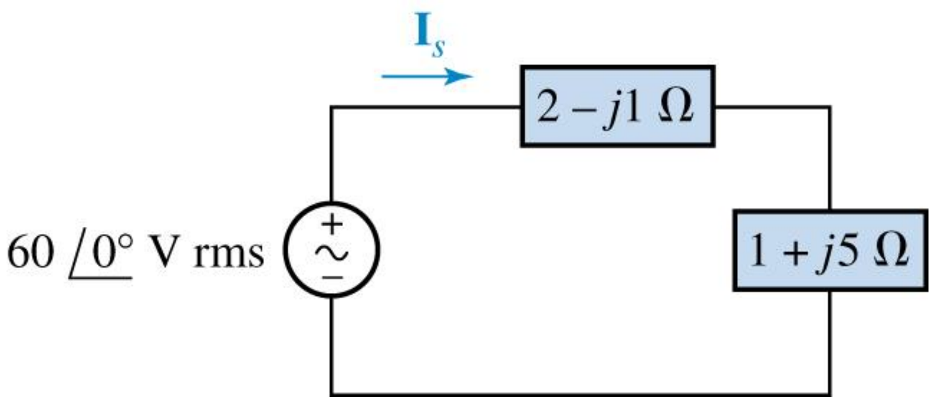
Example:

Calculate values for the average power delivered to each of the two loads, the apparent power supplied by the source, and the power factor of the combined load.



Example:

Calculate values for the average power delivered to each of the two loads, the apparent power supplied by the source, and the power factor of the combined load.

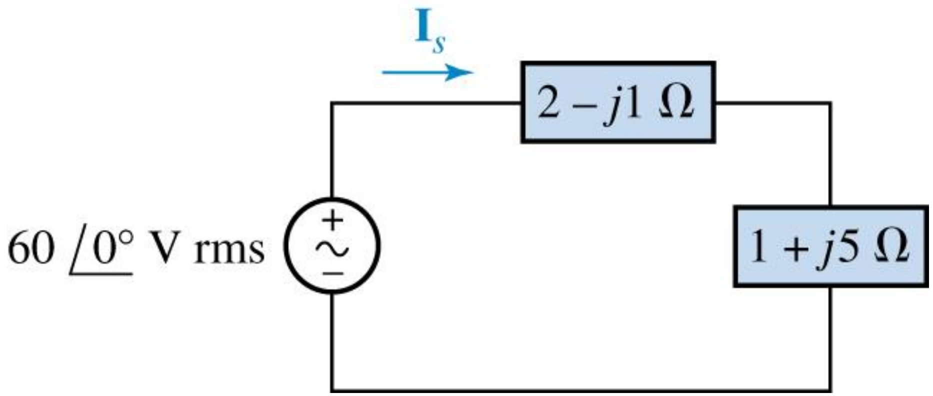


$$P = V_{eff} I_{eff} \cos(angV - angI)$$

$$V_{eff} I_{eff}$$

$$PF = \frac{P}{V_{eff} I_{eff}}$$

Example:



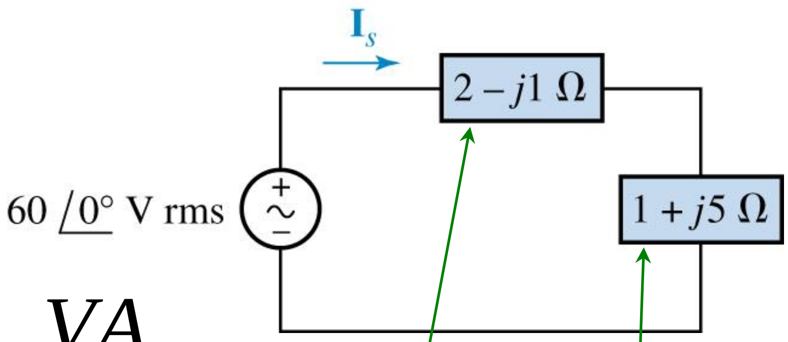
$$3 + j4 = 5\angle -53.13°$$

$$\mathrm{I}_S = \frac{60\ \angle 0^\circ}{(2 - j1) + (1 + j5)} = 12\ \angle -53.13^\circ \quad A.rms$$

$$\begin{aligned} P_S &= V_{eff} I_{eff} \cos(angV - angI) \\ &= (60)\cdot(12)\cos[0^\circ - (-53.1^\circ)] \\ &= 432 \quad W. \end{aligned}$$

Example:

the apparent power supplied by the source:



$$= V_{eff} I_{eff} = (60)\cdot(12) = 720 \quad VA.$$

$$\boxed{P = (12)^2 \cdot 2}$$

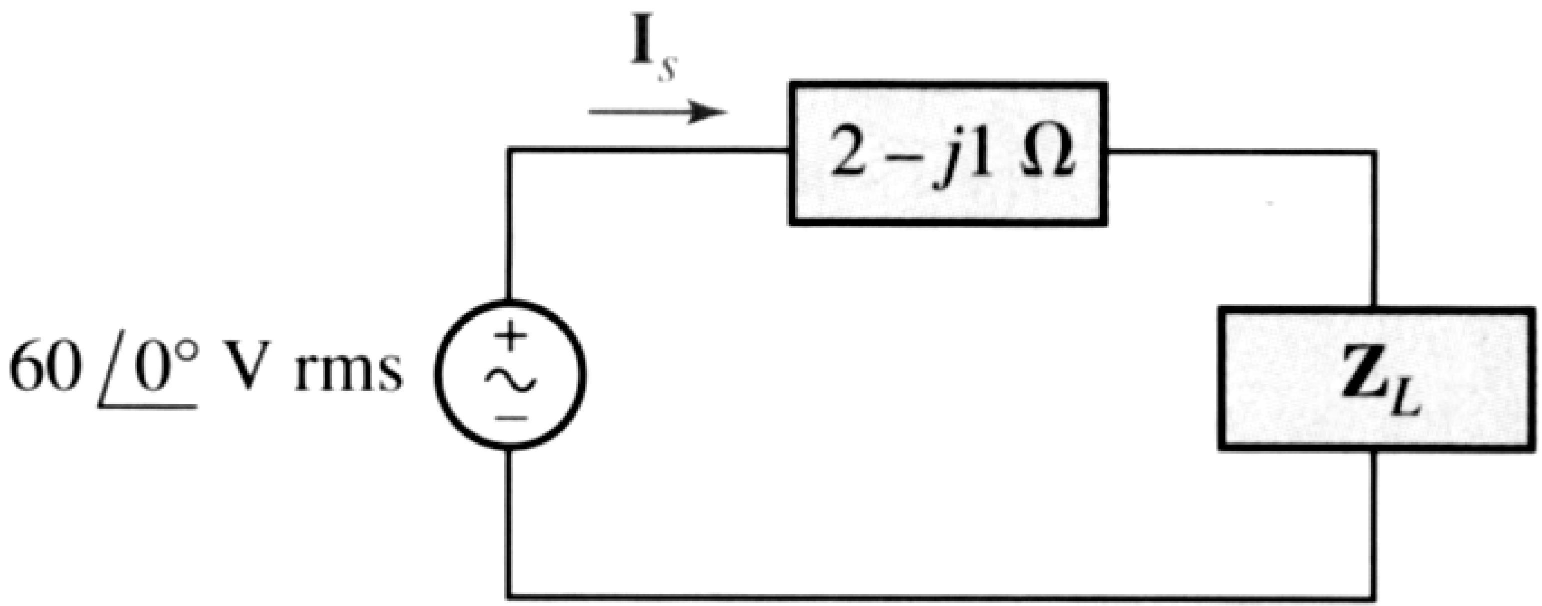
the power factor of the combined load.

$$\boxed{P = (12)^2 \cdot 1}$$

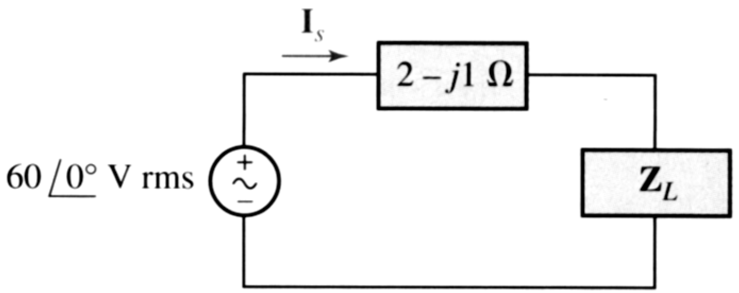
$$PF = \frac{P}{V_{eff} I_{eff}} = \frac{432}{(60)\cdot(12)} = 0.6$$

Practice: 11.8

For the circuit of the figure below, determine the power factor of the combined loads if $Z_L = 10\Omega$.



Practice: 11.8



$$\mathbf{I}_S = \frac{60}{2 - j + 10} = 4.983\angle 4.764^\circ \ \text{A rms}$$

$$PF = \cos(0^\circ - 4.764^\circ) = \underline{0.9965 \text{ leading}} \quad \text{(since the current is leading the voltage)}$$

Complex Power:

the average power, $$P = V_{eff} I_{eff} \cos(\theta - \phi)$$

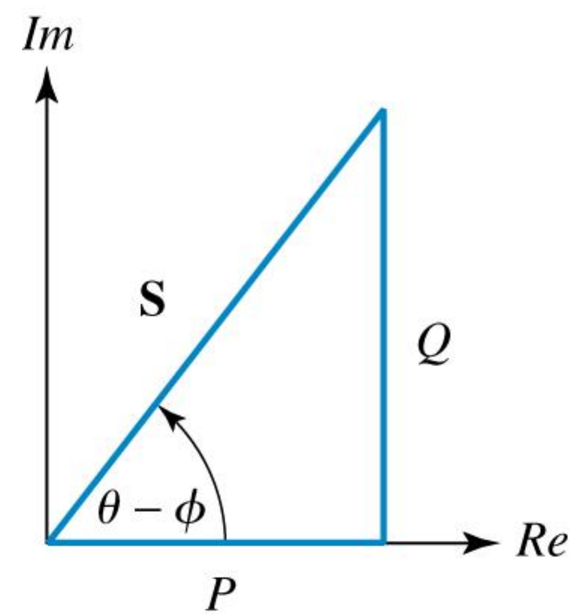
express as,

$$P = V_{eff} I_{eff} \operatorname{Re}\left\{e^{j(\theta-\phi)}\right\} = \operatorname{Re}\left\{V_{eff} e^{j\theta} I_{eff} e^{-j\phi}\right\} = \operatorname{Re}\left\{\mathrm{V}_{eff} \mathrm{I}_{eff}^{*}\right\}$$

the complex power,

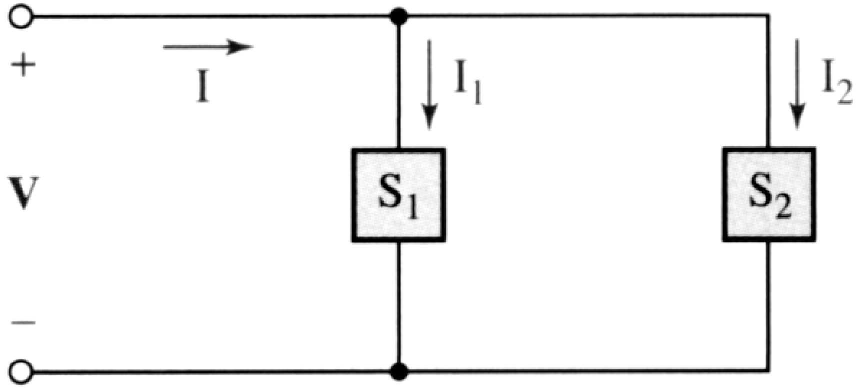
$$\mathrm{S} = \mathrm{V}_{eff} \mathrm{I}_{eff}^{*} = V_{eff} I_{eff} e^{j(\theta-\phi)} = P + jQ$$

$$Q = V_{eff} I_{eff} \sin(\theta - \phi)$$



Power Measurement:

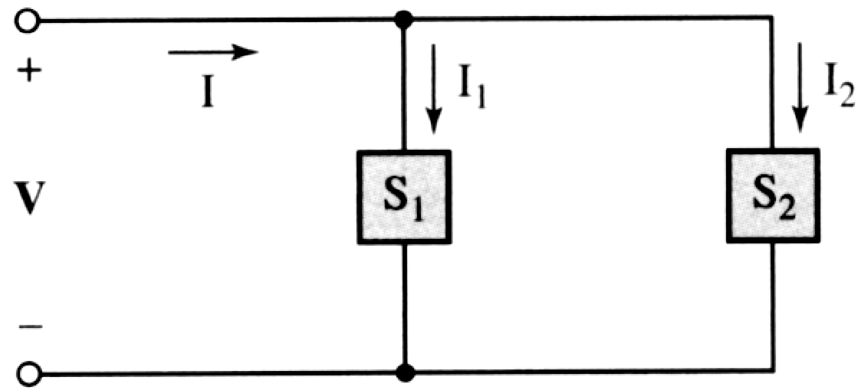
The complex power delivered to several interconnected loads is the sum of the complex power delivered to each of the individual loads.

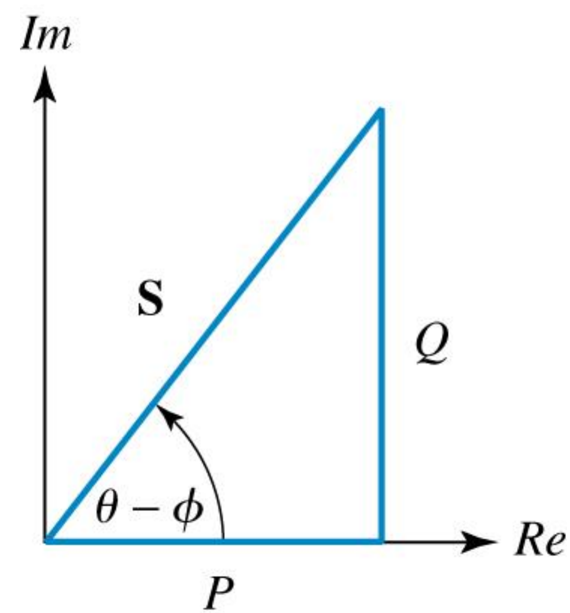


$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}(\mathbf{I}_1+\mathbf{I}_2)^* = \mathbf{V}(\mathbf{I}_1^*+\mathbf{I}_2^*) = \mathbf{V}\mathbf{I}_1^*+\mathbf{V}\mathbf{I}_2^*$$

Example:

An industrial consumer is operating a 50-kW induction motor at a lagging PF of 0.8. The source voltage is 230 Vrms. In order to obtain lower electrical rates, the customer wishes to raise the PF to 0.95 lagging. Specify a suitable solution.





Example:

A purely reactive load must be added to the system, in parallel, since the supply voltage must not change.

The complex power supplied to the motor must have a real part of 50-kW and an angle of $\cos^{-1}(0.8) = 36.9^{o}$

Hence,

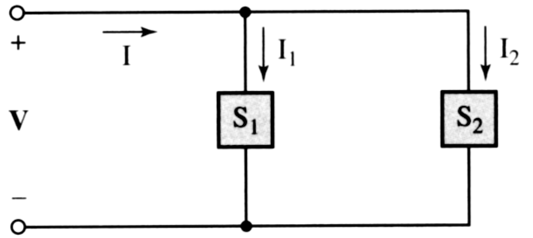
$$\mathbf{S_1} = \frac{50 \angle 36.9^\circ}{0.8} = 50 + j37.5 \quad kVA$$

Example:

$$\mathbf{S_1} = \frac{50\ \angle 36.9^\circ}{0.8} = 50 + j37.5 \quad kVA$$

To achieve a PF of 0.95, the total complex power must become:

$$\mathbf{S} = \frac{50}{0.95}\ \angle \cos^{-1}(0.95)$$
$$= 50 + j16.43 \quad kVA$$

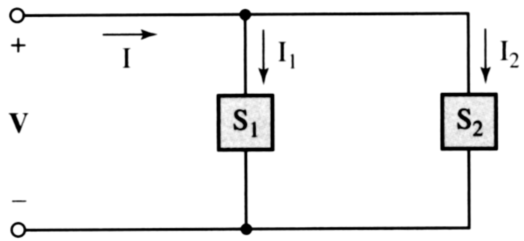


Thus the corrective load is

$$\mathbf{S_2} = \ -j21.07 \quad kVA$$

Example:

S_2 = $-j21.07 \quad kVA$



We select a phase angle of 0º for the voltage source, and the current drawn by Z_2 is

$$I_2^* = \frac{S_2}{V} = \frac{-j21070}{230} = -j91.6 \quad A.$$

Or $$I_2 = j91.6 \quad A.$$

Therefore, $$Z_2 = \frac{V}{I_2} = \frac{230}{j91.6} = -j2.51 \quad \Omega$$

Practice: 11.9

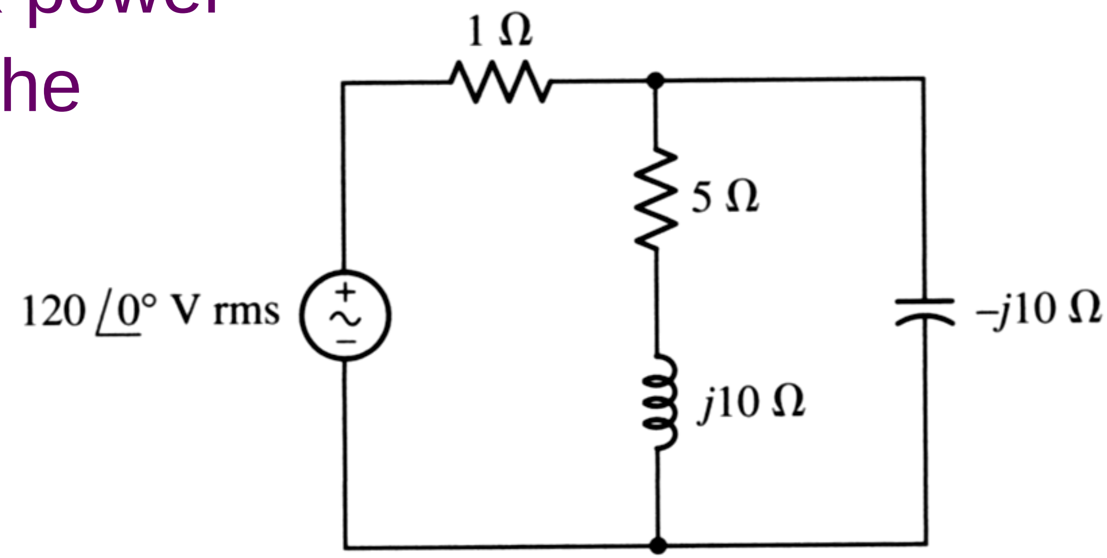
Find the complex power absorbed by the

(a) 1-ohm R

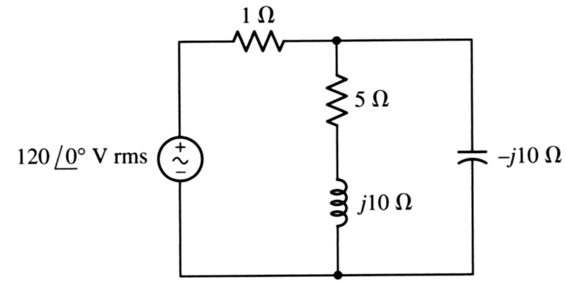
(b) -j10 C

(c) 5+j10 Z

(d) source



Practice: 11.9



$$\mathbf{Z} = 1 + (5 + j10) // - j10 = 23.26\angle -25.46°\Omega$$

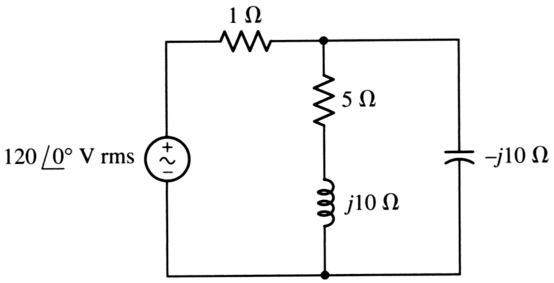
(a) The current through the 1–Ω resistor is

$$\frac{120}{23.26\angle -25.46°} = 5.159\angle 25.46° \text{ A rms}$$

so it absorbs a complex power

$$(5.159^2)(1) = \underline{26.62 + j0 \text{ VA}}$$

Practice: 11.9



(b) $(5+j10)//-j10 = 22.36\angle -26.57^\circ\ \Omega$

so the voltage across the capacitor is

$$120\angle 0^\circ\left[\frac{22.36\angle -26.57^\circ}{1+22.36\angle -26.57^\circ}\right] = 115.4\angle -1.11^\circ \text{ V rms}$$

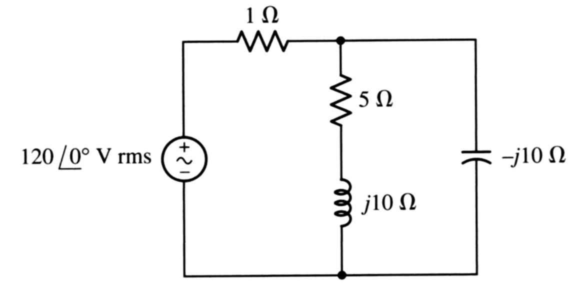
and the current through it is

$$\frac{115.4\angle -1.11^\circ}{-j10} = 11.54\angle 88.89^\circ \text{ A rms}$$

so the complex power it absorbs is

$$\mathbf{S} = (115.4\angle -1.11^\circ)(11.54\angle -88.89^\circ) = \underline{1332\angle -90^\circ \text{ VA}}$$

Practice: 11.9



(c) The current through the $5 + j10 - \Omega$ impedance is

$$\frac{115.4\angle -1.11^\circ}{5 + j10} = 10.32\angle -64.54^\circ \text{ A rms}$$

and so it absorbs complex power

$$(115.4\angle -1.11^\circ)(10.32\angle +64.54^\circ) = 1191\angle 63.43^\circ \text{ VA}$$
$$= \underline{532.7 + j1065 \text{ VA}} \text{ (either form is acceptable)}$$

(d) The complex power *absorbed* by the source is

$$\mathbf{S} = (120\angle 0^\circ)\,(-5.159\angle +25.46^\circ)^*$$
$$= (120\angle 0^\circ)\,(5.159\angle +154.54^\circ)$$
$$= 619.1\angle 154.5^\circ \text{ VA}$$
$$= \underline{-559.0 + j266.1 \text{ VA}} \text{ (either form is acceptable)}$$

Practice: 11.10

A 440-Vrms source supplies power to a load $Z_L = 10 + j2\ \Omega$ through a transmission line having a total resistance of 1.5Ω. Find (a) the average and apparent power supplied to the load; (b) the average and apparent power lost in the transmission line; (c) the average and apparent power supplied by the source; (d) the power factor at which the source operates.

Practice: 11.10

The current through the line and load is

$$\frac{440}{10+j2+1.5} = 37.70\angle -9.866^\circ \text{ A rms}$$

(a) The average power supplied to the load is simply $(37.70^2)(10)$ = 14.21 kW.
The apparent power requires the voltage across the load, which is

$$(37.70\angle -9.866^\circ)(10+j2) = 384.5\angle 1.444^\circ \text{ V rms}.$$

so the apparent power is

$$(384.5)(37.70) = 14.50 \text{ kVA}.$$

(b) The average power lost in the line is simply

$$(37.70^2)(1.5) = 2.132 \text{ kW}$$

The line impedance is purely resistive, so the apparent power is equal to the average power: 2.132 kVA

(c) The source supplies a complex power

$$(440\angle 0^\circ)(37.70\angle +9.866^\circ) = 16.59\angle 9.866^\circ \text{ kVA}$$
$$= 16.34 + j2.842 \text{ kVA}$$

Thus, it supplies an average power of 16.34 kVA and an apparent power of 16.59 kVA

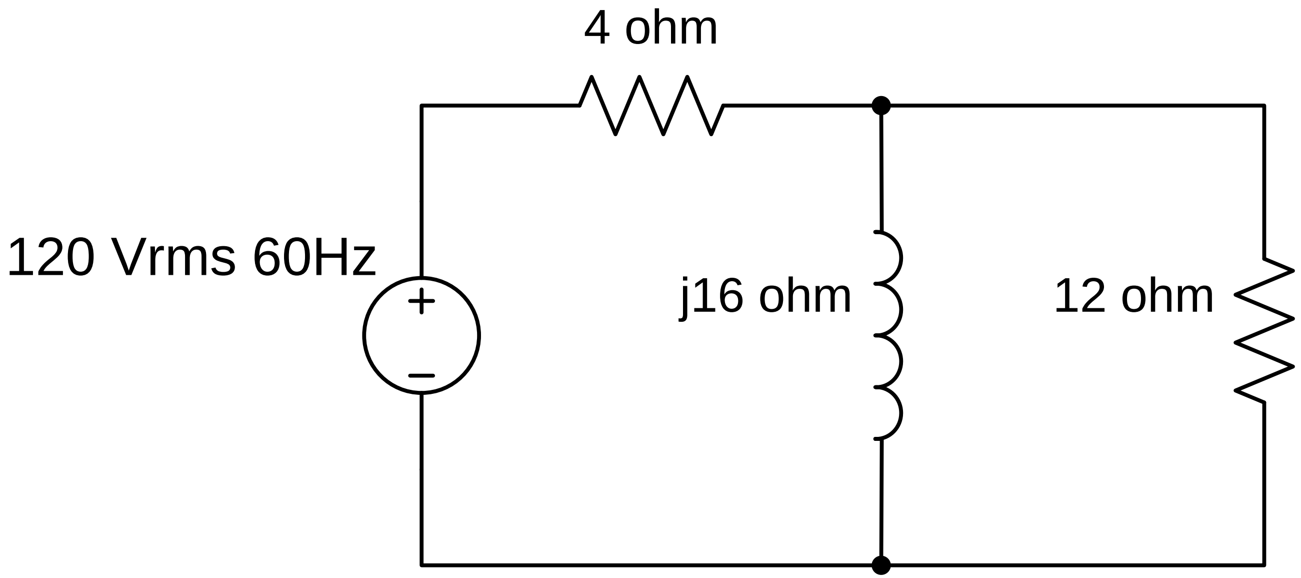
(d) The power factor of the source is

$$\cos(9.866^\circ) = 0.9852 \text{ lagging}$$

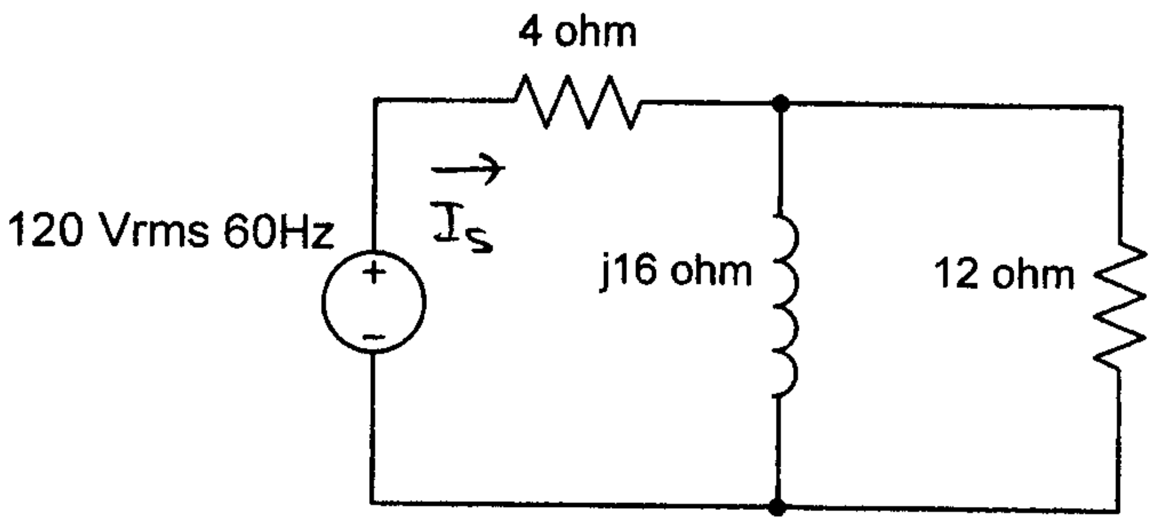
Example: Final 2/47

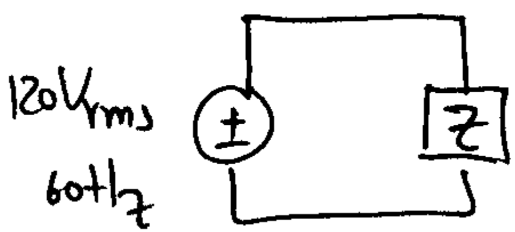
จากวงจรตามรูป ให้หา

1) the average power ที่จ่ายโดย แหล่งจ่าย (source)
2) ค่า power factor ที่แหล่งจ่าย (source)
3) ขนาดของ ตัวเก็บประจุ (capacitor) ที่เมื่อนำไปต่อขนานกับ แหล่งจ่าย ทำให้ค่า power factor = 1



Example: Final 2/47

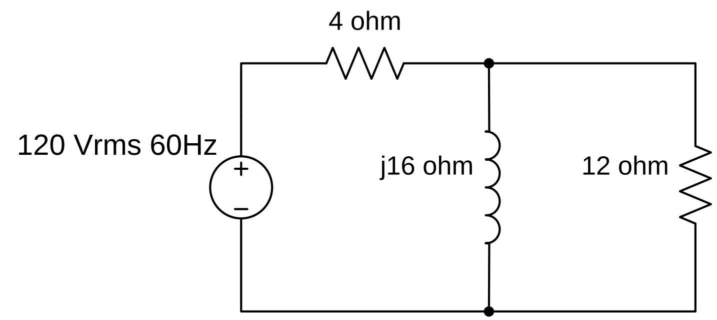




$$Z = 4 + (j16 \| 12) = 4 + \frac{j192}{12 + j16}$$

$$= 11.68 + j5.76 = 13.02\angle 26.25^\circ$$

Example: Final 2/47



[3.2]

$$I_s = \frac{120}{13.02\angle 26.25^\circ} = 9.214\angle -26.25^\circ = 8.26 - j4.08$$

$$\therefore \quad PF = \cos 26.25^\circ = 0.8969 \text{ lag} \quad \text{Ans}$$

[3.1]

$$P_s = (120)(9.214)(0.8969)$$

$$= 991.7 \text{ Watts.} \quad \text{Ans}$$

Example: Final 2/47

[3.3]

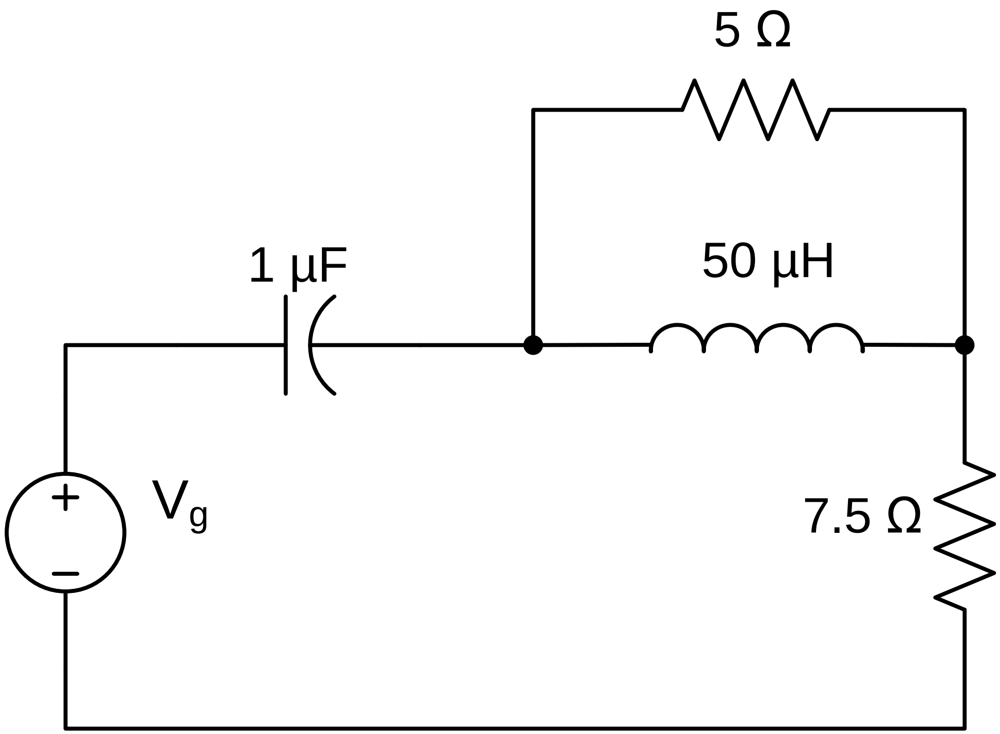
$$Z_L = 11.68 + j5.76$$

$$\frac{1}{Z_L} = Y_L = 0.068 - j0.034$$

$$\therefore \quad j120\pi C = j0.034$$

$$C = 90.1\,\mu F. \quad \underline{\text{Ans}}$$

Ex:

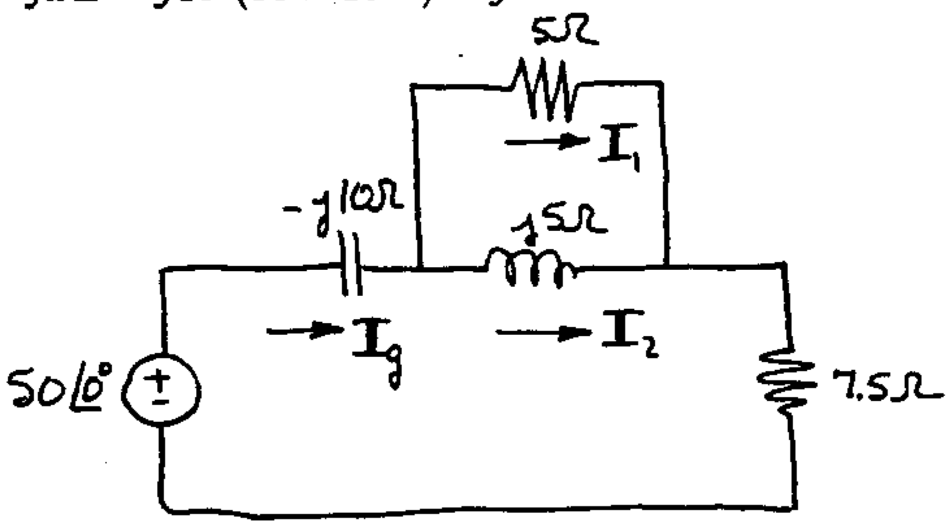


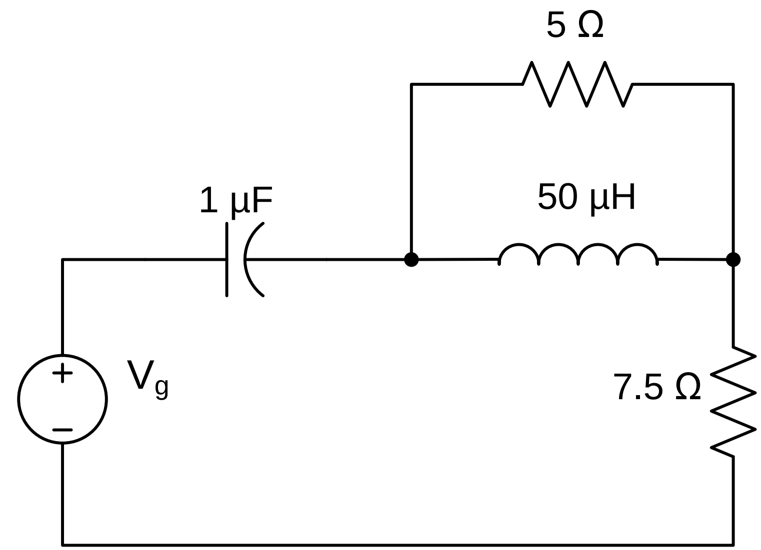
Find the average power, the reactive power, and the apparent power supplied by the voltage source in the circuit if $v_g = 50 \cos 10^5 t$ V.

Ex:

$$\frac{1}{j\omega C} = \frac{10^6}{j10^5} = -j10\,\Omega$$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5\,\Omega$$





$$Z = -j10 + \frac{(5)(j5)}{5+j5} + 7.5 = 10 - j7.5\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{10 - j7.5} = 3.2 + j2.4\text{ A}$$

$$S_g = \frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = 25(3.2 - j2.4) = 80 - j60\text{ VA}$$

$$P = 80\text{ W(del)}; \qquad Q = 60\text{ VAR(abs)}$$

$$|S| = |S_g| = 100\text{ VA}$$

Summary:

Term	Symbol	Unit	Description
Instantaneous power	$p(t)$	W	$p(t) = v(t)i(t)$. It is the value of the power at a specific instant in time. It is *not* the product of the voltage and current phasors!
Average power	P	W	In the sinusoidal steady state, $P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$, where θ is the angle of the voltage and ϕ is the angle of the current. Reactances do not contribute to P
Effective or rms value	V_{rms} or I_{rms}	V or A	Defined, e.g., as $I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$; if $i(t)$ is sinusoidal, then $I_{eff} = I_m/\sqrt{2}$
Apparent power	$\lvert\mathbf{S}\rvert$	VA	$\lvert\mathbf{S}\rvert = V_{eff} I_{eff}$, and is the maximum value the average power can be; $P = \lvert\mathbf{S}\rvert$ only for purely resistive loads
Power factor	PF	None	Ratio of the average power to the apparent power. The PF is unity for a purely resistive load, and zero for a purely reactive load
Reactive power	Q	VAR	A means of measuring the energy flow rate to and from reactive loads
Complex power	$\mathbf{S}$	VA	A convenient complex quantity that contains both the average power P and the reactive power Q: $\mathbf{S} = P + jQ$

Hw:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.