King Mongkut's University of Technology Thonburi
Midterm Examination
Semester 1 -- Academic Year 2014

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3th Yr (Inter. Program)
Exam Date: Monday September 22, 2014   **Time:** 9.00am-12.00pm

Instructions:-
1. This exam consists of 5 problems with a total of 10 pages, not including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rule is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds ... No Cheating! GOOD LUCK!!!

Remarks:-
- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**

- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment within university rules.**

- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Question No.	1	2	3	4	5	TOTAL
Full Score	20	20	20	20	20	100
Graded Score						

Name _____Student ID_____

This examination has been approved by the committees of the ENE department.

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Problem 1: Sample Mean and Sample Variance [20 points]

The sample $x_1, x_2, \ldots, x_n$ of size $n = 11$ has the sample mean of 20 and the sample standard deviation of 2.

(a) [6 points] Find $\sum_{i=1}^{n} x_i^2$.

[*Hint*: An alternative formula of the sample variance is

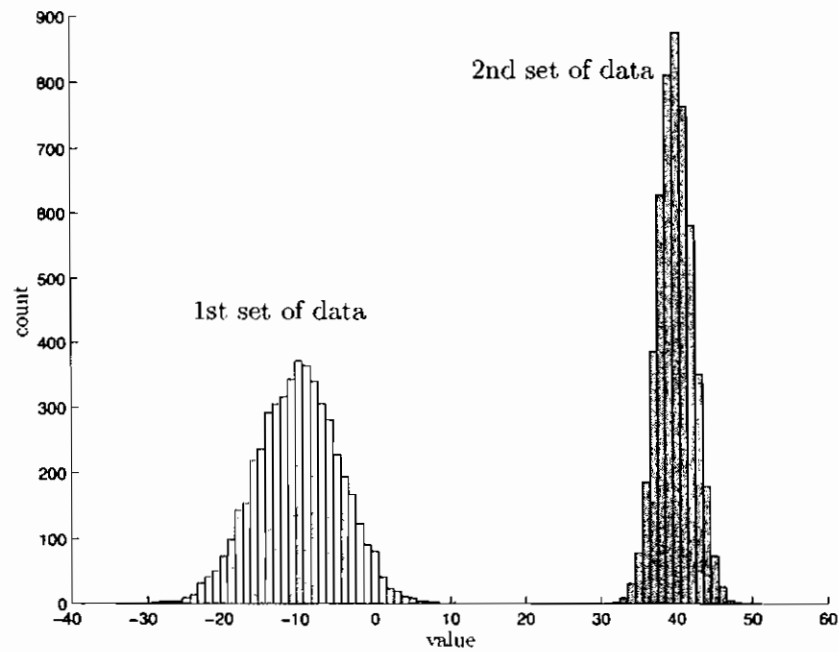$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2\right) - \frac{n}{n-1}(\overline{x})^2.$$

]

(b) [7 points] Let $y_i = -2x_i^2 + 1$ denote the transformation of x_i. Find the sample mean of $y_1, y_2, \ldots, y_n$.

[*Hint*: Use part (a)]

Part (c) below is not related to parts (a) and (b) and can be done separately.

(c) [7 points] Two histograms below are for two different sets of data. The sample variance of which set of data is **smaller**? **Why?**

Problem 2: Events and Their Probabilities [20 points]

The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is 0.3, the analogous probability for the second signal is 0.4, and the probability that he must stop at at least one of the two signals is 0.6.

(a) [6 points] What is the probability that he must stop at <u>both signals</u>?

(b) [7 points] What is the probability that he must stop at <u>the fist signal but not the second one?</u>

(c) [7 points] What is the probability that he must stop at <u>exactly one signal?</u>

Problem 3: Counting [20 points]

ATM stands for "automated teller machine." An ATM personal identification number (PIN) consists of four digits, each a 0, 1, 2, . . . , 8, or 9, in succession. Examples of PINs are 0000, 0552, and 9172.

(a) [6 points] How many different possible PINs are there?

A thief has stolen an ATM card and knows that the first and last digits of the PIN are 8 and 1, respectively:

$$8 \quad \underline{\ ?\ } \quad \underline{\ ?\ } \quad 1.$$

(b) [7 points] The thief randomly selects the 2nd and 3rd digits in the first attempt. What is the probability that he gains access to the account?

(c) [7 points] Suppose that his guess was incorrect in the first attempt. The thief then randomly selects a different pair of digits in the second attempt. What is the probability that he gains an access to the account in the second attempt?

Problem 4: Conditional Probability [20 points]

A shop sells two types of smart phones: iPhone and Samsung Galaxy. Over the past year, 40% of the phones sold are iPhone. Among the customers who buy iPhones, 30% are female. In contrast, among the customers who buy Samsung Galaxies, 50% are female.

(a) [4 points] Let I denote an event that a randomly-selected customer buys an iPhone; S denote an event that this customer buys a Samsung Galaxy; and F denote an event that this customer is female.

Based on the given information, fill in the blanks below:

$$\mathbb{P}\{I\} = \underline{\hspace{4cm}}$$

$$\mathbb{P}\{S\} = \underline{\hspace{4cm}}$$

$$\mathbb{P}\{F \mid I\} = \underline{\hspace{4cm}}$$

$$\mathbb{P}\{F \mid S\} = \underline{\hspace{4cm}}$$

(b) [8 points] Find the probability that a randomly-selected customer buys an iPhone and that the customer is female, i.e., $\mathbb{P}\{I \cap F\}$.

(c) [8 points] Given that a randomly-selected customer is female, find the probability that she buys an iPhone, i.e., $\mathbb{P}\{I \mid F\}$.

Problem 5: Random Variables [20 points]

For each random variable defined below, describe the set of possible values for the variable, and state whether the variable is discrete or continuous.

(a) [5 points] X = the number of students who are absent on the first day of a particular class.

Answer:
(Fill in the blank) The set of possible values is _____

(Circle one answer)   Discrete   Continuous

(b) [5 points] L = the length of a randomly selected snake.

Answer:
(Fill in the blank) The set of possible values is _____

(Circle one answer)   Discrete   Continuous

(c) [5 points] T = the amount of time it takes for a hard drive to fail.

Answer:
(Fill in the blank) The set of possible values is _____

(Circle one answer)   Discrete   Continuous

(d) [5 points] Y = the number of coin tosses required for three individuals to obtain a match (HHH or TTT).

Answer:
(Fill in the blank) The set of possible values is _____

(Circle one answer)   Discrete   Continuous