ENE 104
Electric Circuit Theory

Lecture 11:
Polyphase Circuits

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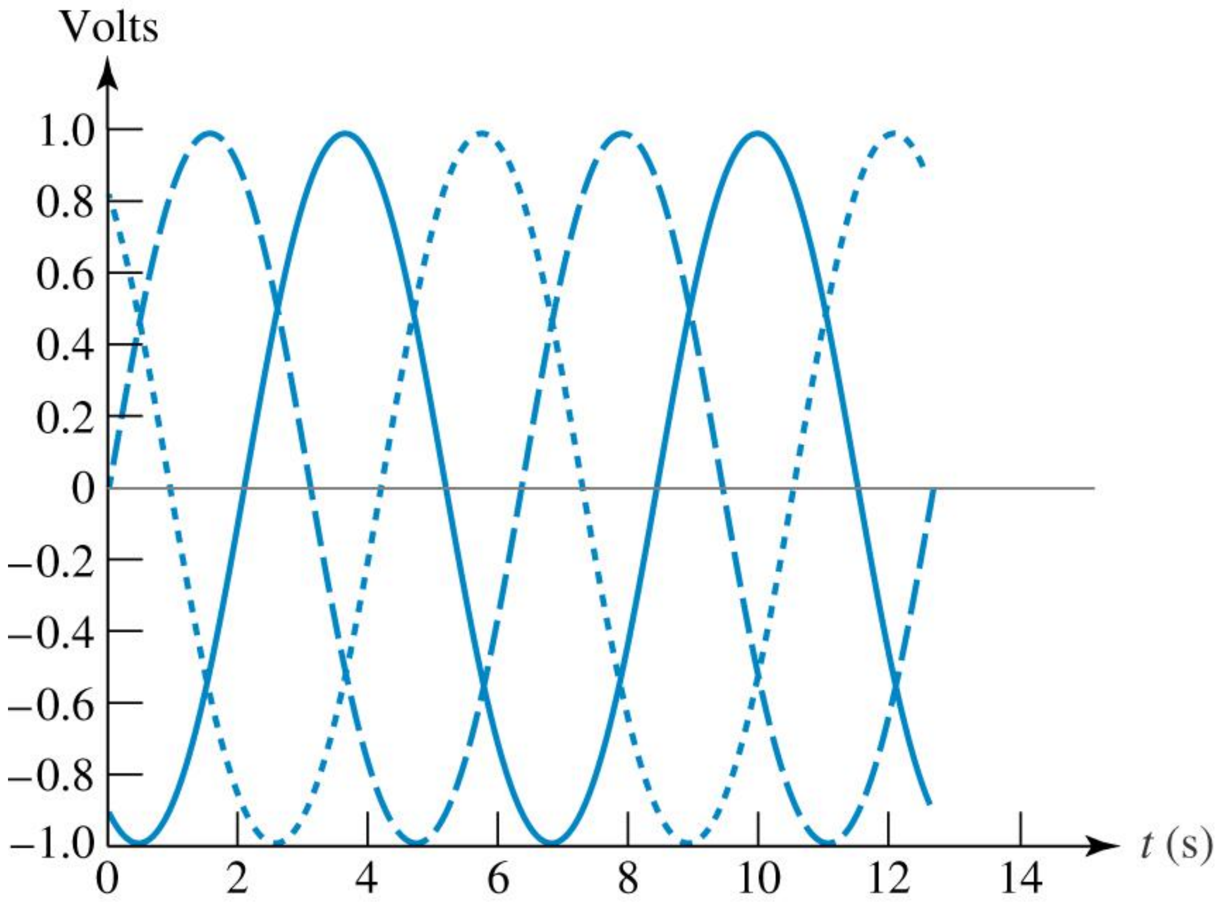
Objectives : Ch12

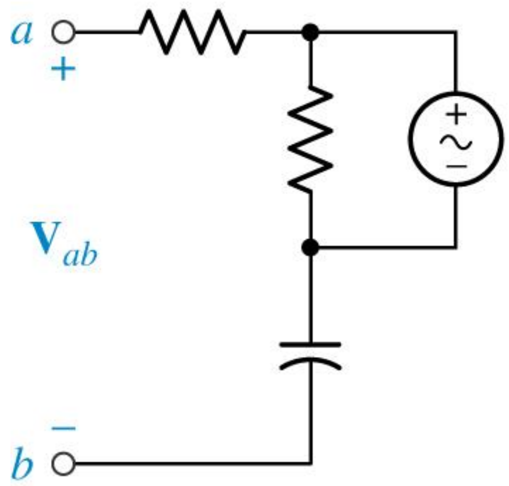
- single-phase and polyphase systems
- Y- and Δ- connected three-phase system
- per-phase analysis of three-phase systems

Introduction:

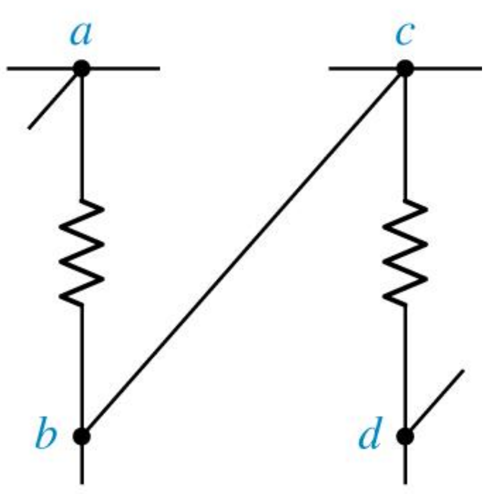


An example set of three voltages, each of which is 120º out of phase with the other two. As can be seen, only one of the voltages will be zero at a particular instant.

Double-Subscript Notation:



(a)

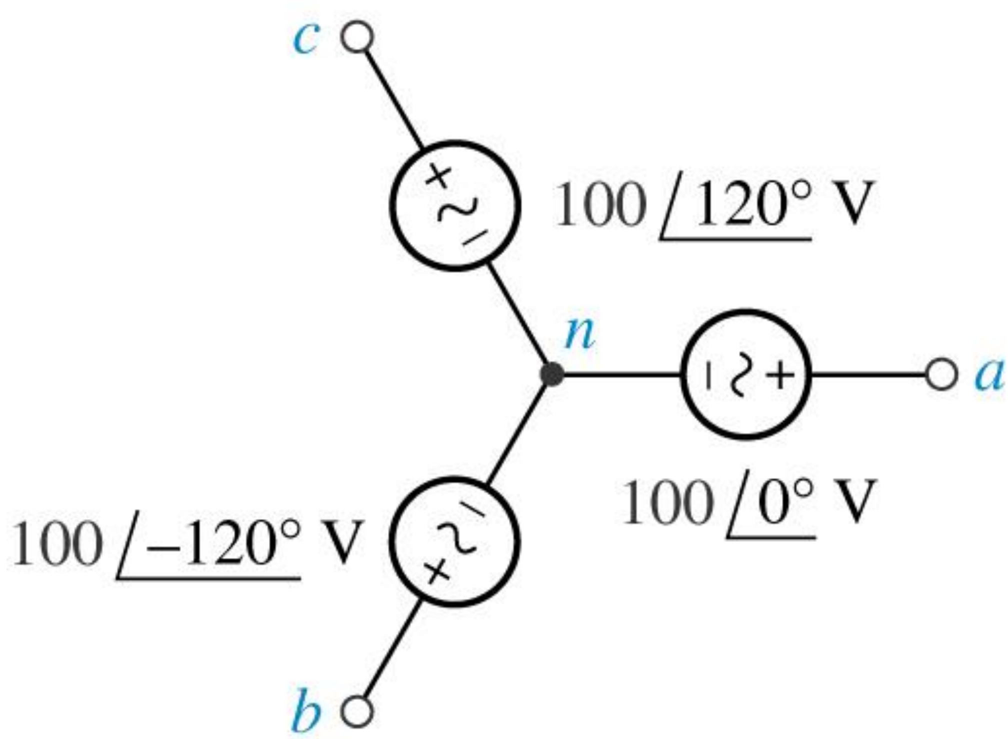


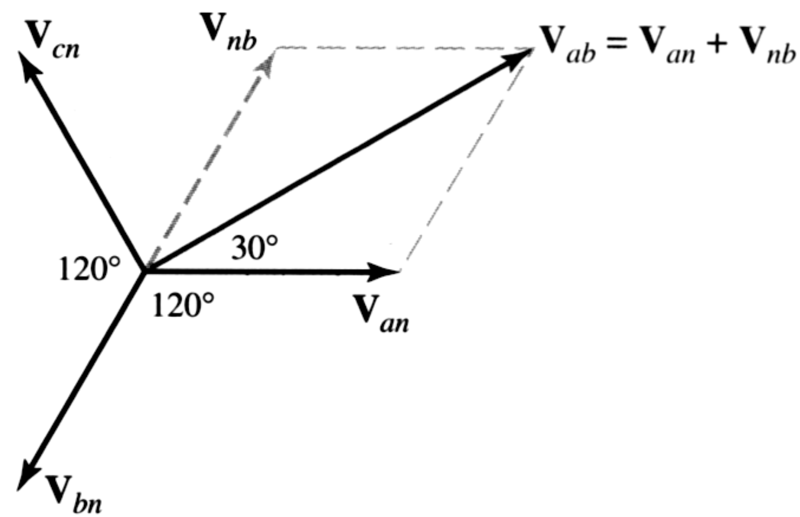
(b)

(a) The definition of the voltage $\mathbf{V}_{ab}$.

(b) $$\mathbf{V}_{ad} = \mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{cd}$$
$$= \mathbf{V}_{ab} + \mathbf{V}_{cd}.$$

Double-Subscript Notation:



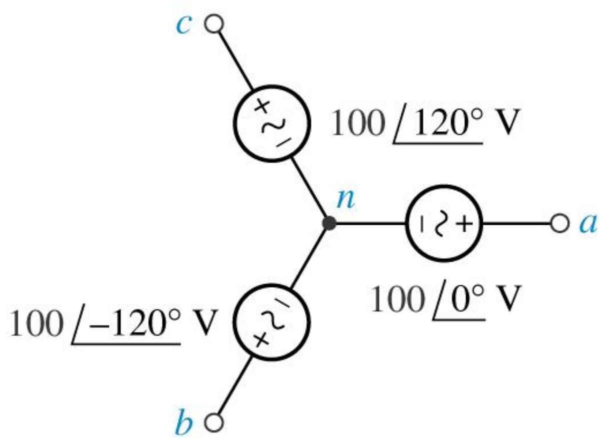


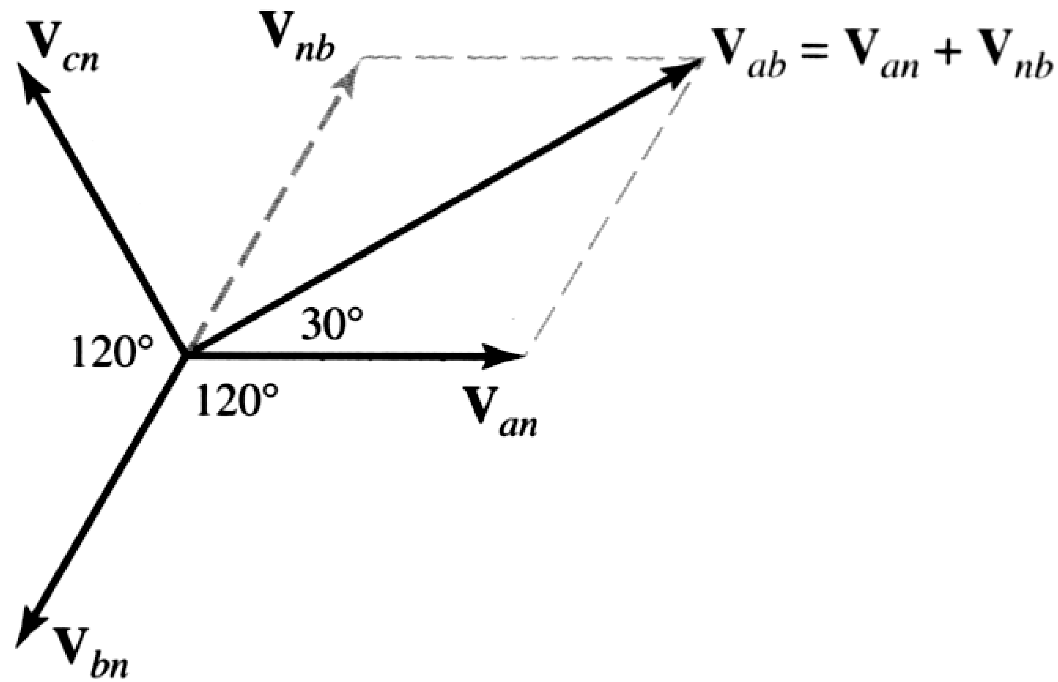
$$\mathbf{V}_{an} = 100 \angle 0^\circ \quad \text{V.}$$

$$\mathbf{V}_{bn} = 100 \angle -120^\circ \quad \text{V.}$$

$$\mathbf{V}_{cn} = 100 \angle -240^\circ \quad \text{V.}$$

Double-Subscript Notation:





$$
\begin{aligned}
\mathbf{V}_{ab} &= \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} \\
&= 100 \underline{/ 0^\circ} \quad -100 \underline{/ -120^\circ} \quad \text{V.} \\
&= 100 - (-50 - j86.6) = 173.2 \underline{/ 30^\circ} \quad \text{V.}
\end{aligned}
$$

Practice: 12.1

Let $\mathbf{V}_{ab} = 100\angle 0°$ V, $\mathbf{V}_{bd} = 40\angle 80°$ V, and $\mathbf{V}_{ca} = 70\angle 200°$ V. Find (a) $\mathbf{V}_{ad}$; (b) $\mathbf{V}_{bc}$; (c) $\mathbf{V}_{cd}$

Practice: 12.1

$$\mathbf{V}_{ab} = 100\angle 0^\circ \text{ V} = 100 + j0 \text{ V}$$

$$\mathbf{V}_{bd} = 40\angle 80^\circ \text{ V} = 6.946 + j39.39 \text{ V}$$

$$\mathbf{V}_{ca} = 70\angle 200^\circ \text{ V} = -65.78 - j23.94 \text{ V}$$

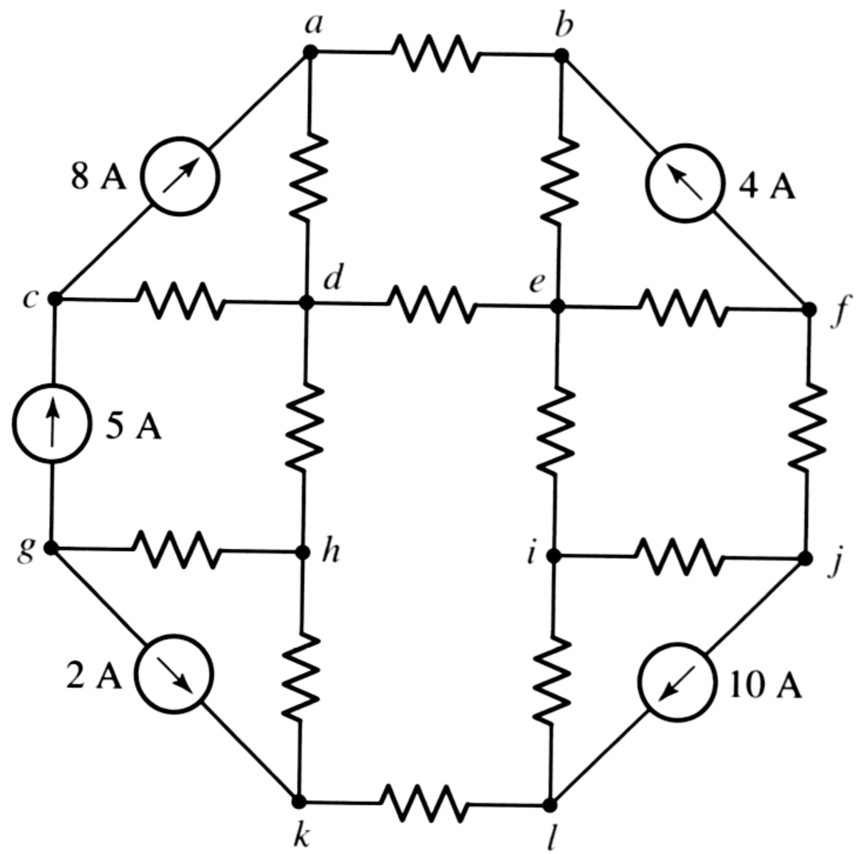
(a) $\mathbf{V}_{ad} = \mathbf{V}_{ab} + \mathbf{V}_{bd} = 106.946 + j39.39 \text{ V} = \underline{114.0\angle 20.22^\circ \text{ V}}$

(b) $\mathbf{V}_{bc} = -\mathbf{V}_{ab} - \mathbf{V}_{ca} = -34.22 + j23.94 \text{ V} = \underline{41.76\angle 145.0^\circ \text{ V}}$

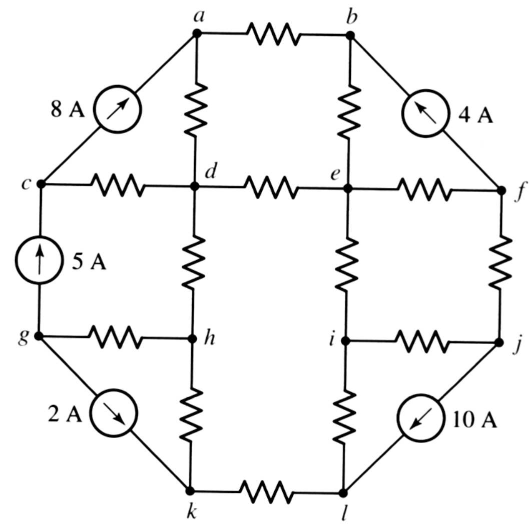
(c) $\mathbf{V}_{cd} = \mathbf{V}_{ca} + \mathbf{V}_{ab} + \mathbf{V}_{bd} = 41.17 + j15.45 \text{ V} = \underline{43.97\angle 20.57^\circ \text{ V}}$

Practice: 12.2

Refer to the circuit of figure below and let $\mathbf{I}_{fj} = 3$ A, $\mathbf{I}_{de} = 2$ A, and $\mathbf{I}_{hd} = -6$ A. Find (a), $\mathbf{I}_{cd}$; (b) $\mathbf{I}_{ef}$; (c) $\mathbf{I}_{ij}$



Practice: 12.2



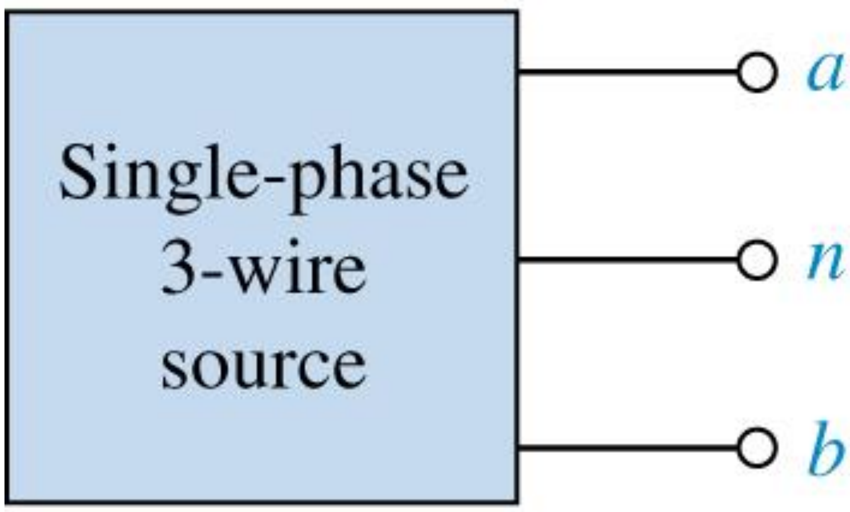
- Being a purely resistive circuit, no phases will change.

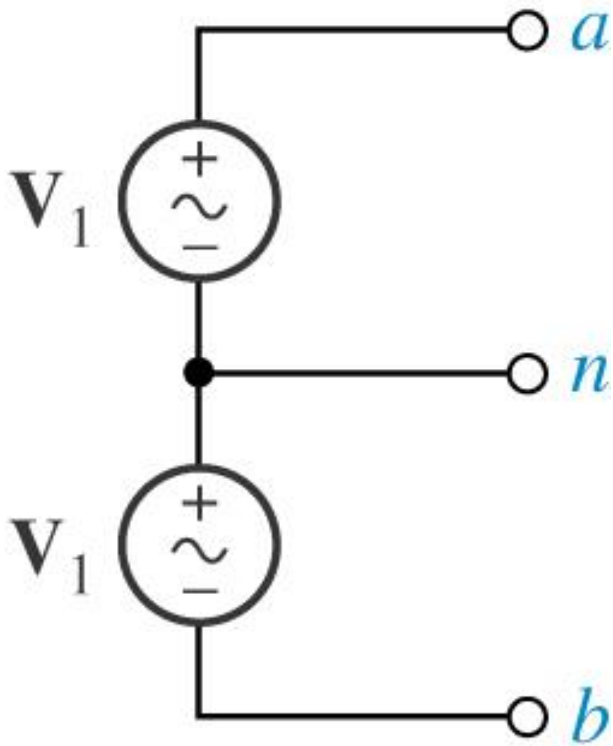
(a) $I_{cd} = 5 - 8 = \underline{-\ 3A}$

(b) $I_{fe} = -4 - 3 = -7A \qquad \therefore I_{ef} = -I_{fe} = \underline{+\ 7A}$

(c) $I_{ij} + 3 = 10 \qquad \therefore I_{ij} = \underline{7A}$

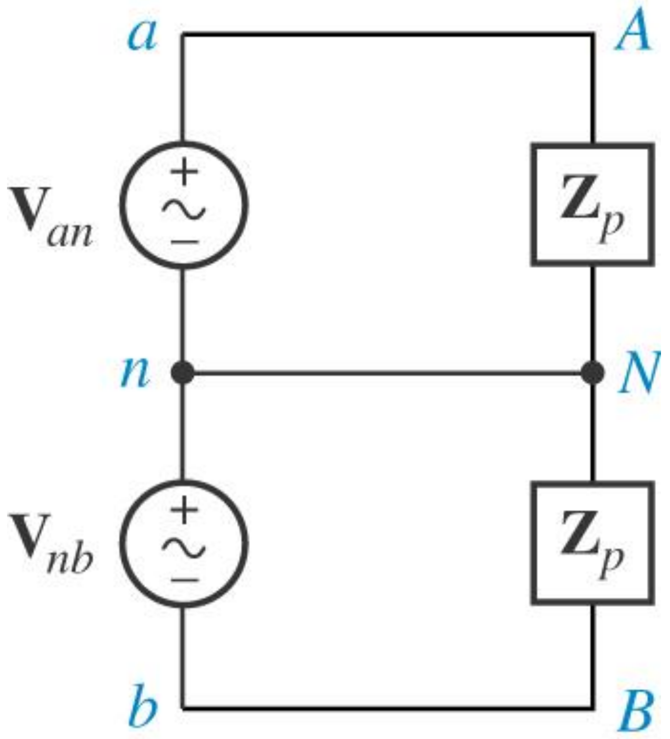
Single-Phase Three-Wire System:





Single-Phase Three-Wire System:

Consider:



Since $\mathbf{V}_{\mathrm{an}} = \mathbf{V}_{\mathrm{nb}}$

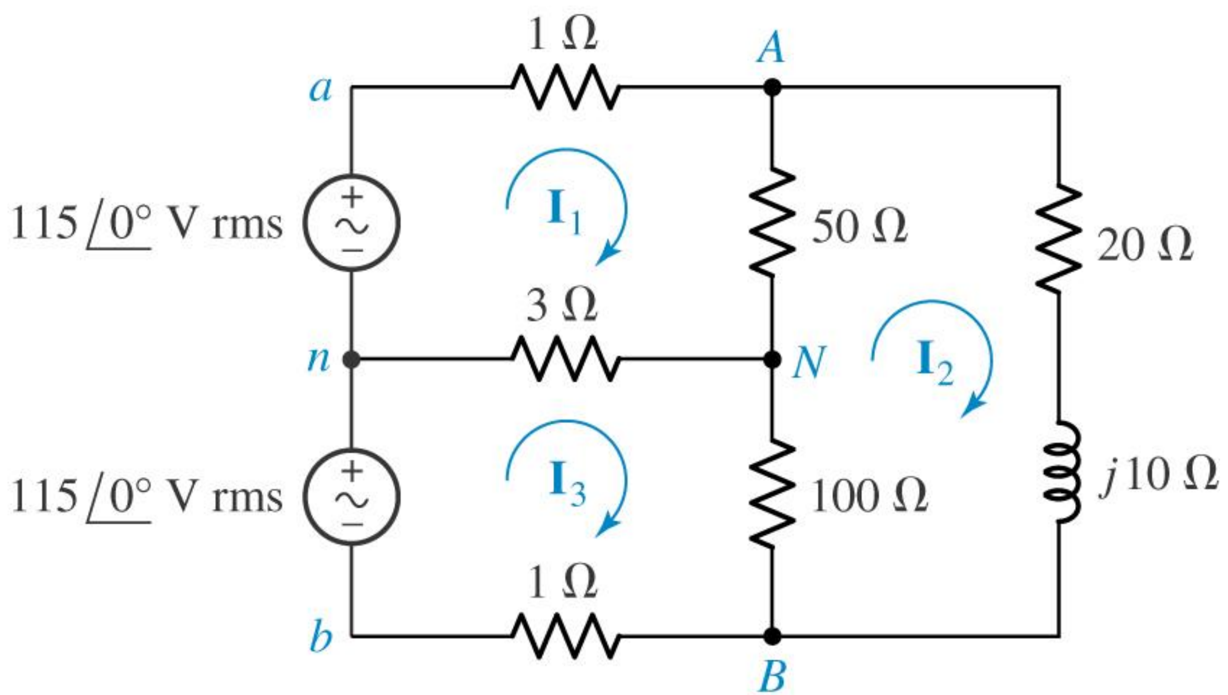
Then $$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p} = \mathbf{I}_{Bb} = \frac{\mathbf{V}_{nb}}{\mathbf{Z}_p}$$

And $\mathbf{I}_{\mathrm{nN}} = \mathbf{I}_{\mathrm{Bb}} + \mathbf{I}_{\mathrm{Aa}} = \mathbf{I}_{\mathrm{Bb}} - \mathbf{I}_{\mathrm{aA}} = 0$

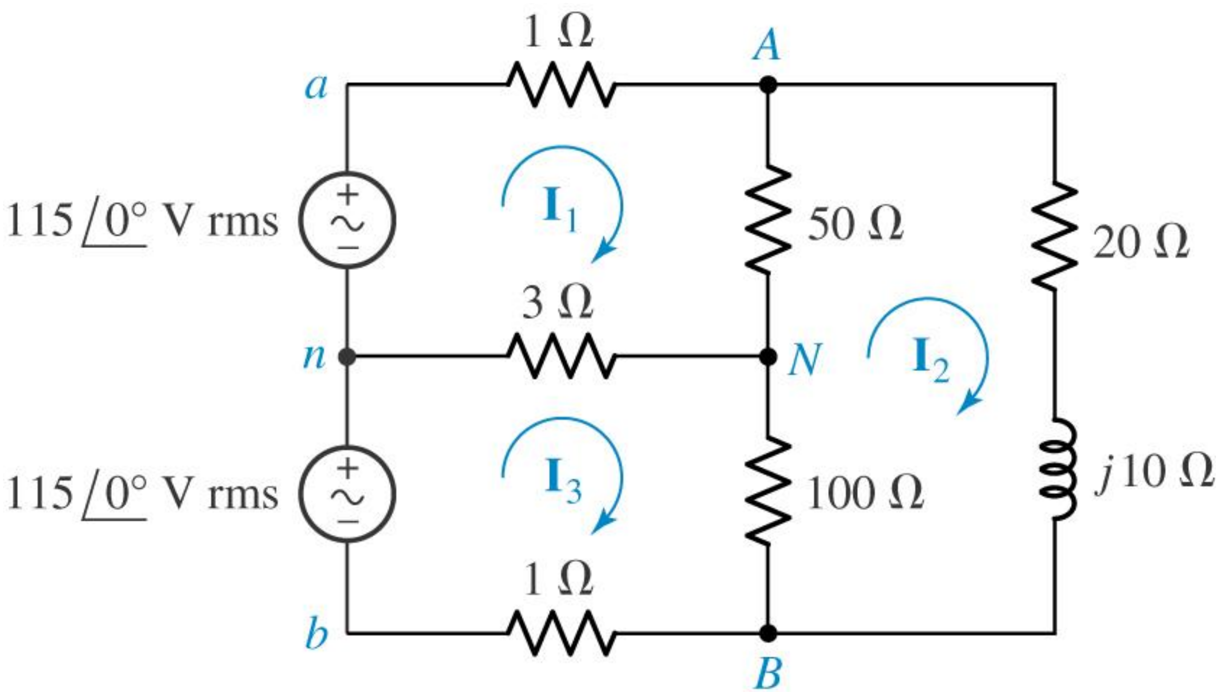
Example:

Effect of Finite Wire Impedance:

Analyze the system below and determine the power delivered to each of the three loads as well as the power lost in the neutral wire and each of the two lines.



Example:

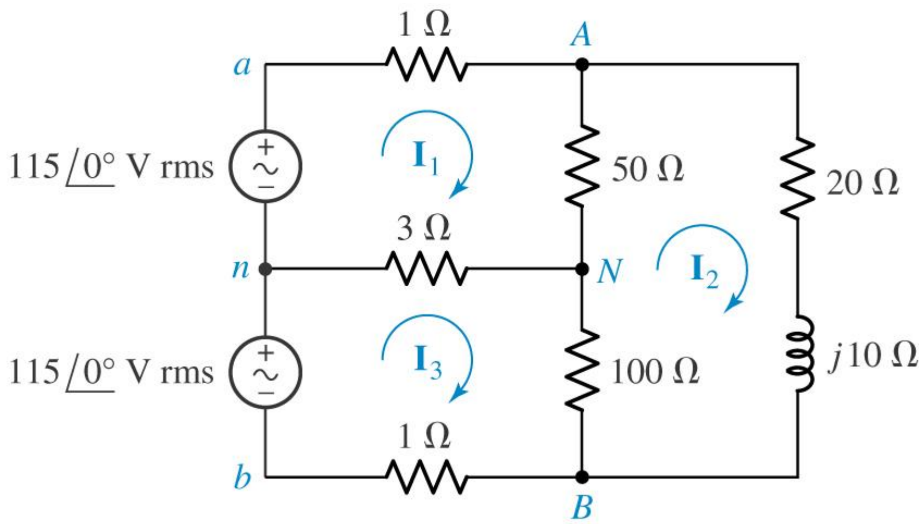


$$-115\angle 0^\circ + \mathbf{I}_1 + 50(\mathbf{I}_1 - \mathbf{I}_2) + 3(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$(20 + j10)\mathbf{I}_2 + 100(\mathbf{I}_2 - \mathbf{I}_3) + 50(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$-115\angle 0^\circ + 3(\mathbf{I}_3 - \mathbf{I}_1) + 100(\mathbf{I}_3 - \mathbf{I}_2) + \mathbf{I}_3 = 0$$

Example:



$$\mathbf{I}_1 = 11.24 \angle -19.83° \quad \text{A.}$$

$$\mathbf{I}_2 = 9.389 \angle -24.47° \quad \text{A.}$$

$$\mathbf{I}_3 = 10.37 \angle -21.80° \quad \text{A.}$$

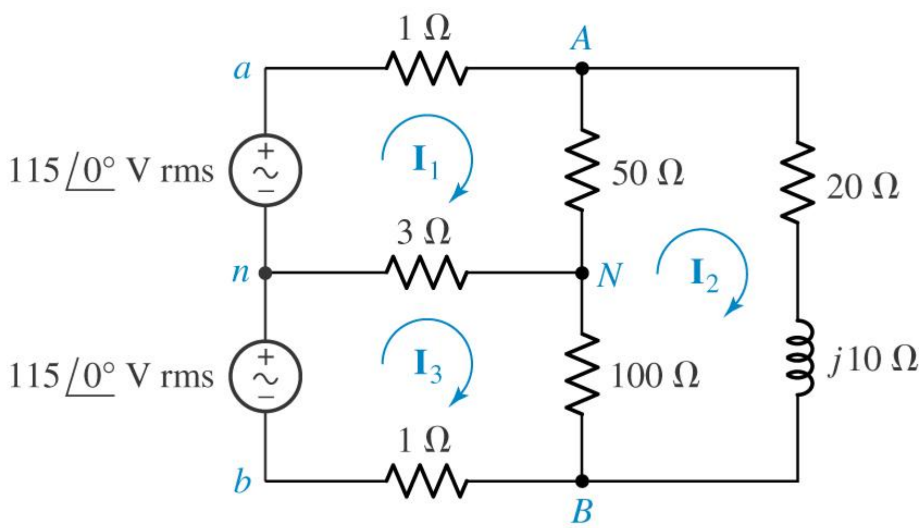
The average power drawn by each load:

$$P_{50} = \left|\mathbf{I}_1 - \mathbf{I}_2\right|^2 \cdot 50 = 206 \quad \text{W.}$$

$$P_{100} = \left|\mathbf{I}_3 - \mathbf{I}_2\right|^2 \cdot 100 = 117 \quad \text{W.}$$

$$P_{20+j10} = \left|\mathbf{I}_2\right|^2 \cdot 20 = 1763 \quad \text{W.}$$

Example:



$$\mathbf{I}_1 = 11.24 \angle -19.83^\circ \quad \text{A.}$$

$$\mathbf{I}_2 = 9.389 \angle -24.47^\circ \quad \text{A.}$$

$$\mathbf{I}_3 = 10.37 \angle -21.80^\circ \quad \text{A.}$$

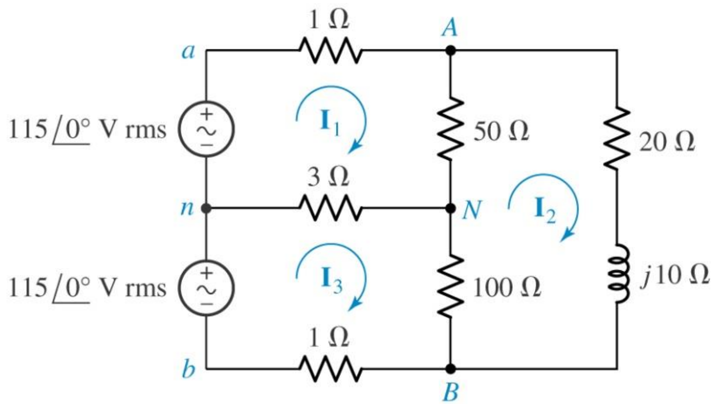
The loss in each of the wires:

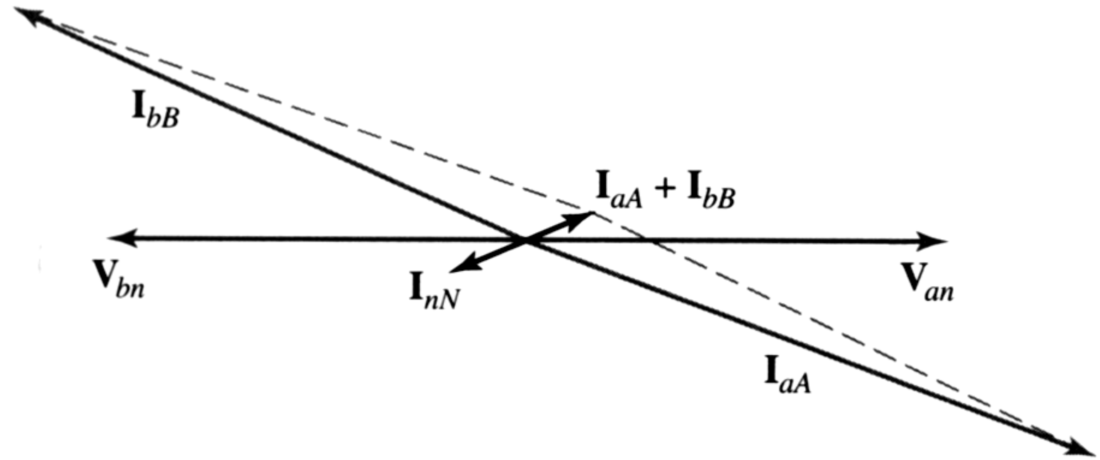
$$P_{aA} = |\mathbf{I}_1|^2 \cdot 1 = 126 \quad \text{W.}$$

$$P_{bB} = |\mathbf{I}_3|^2 \cdot 1 = 108 \quad \text{W.}$$

$$P_{nN} = |\mathbf{I}_3 - \mathbf{I}_1|^2 \cdot 3 = 3 \quad \text{W.}$$

Example:





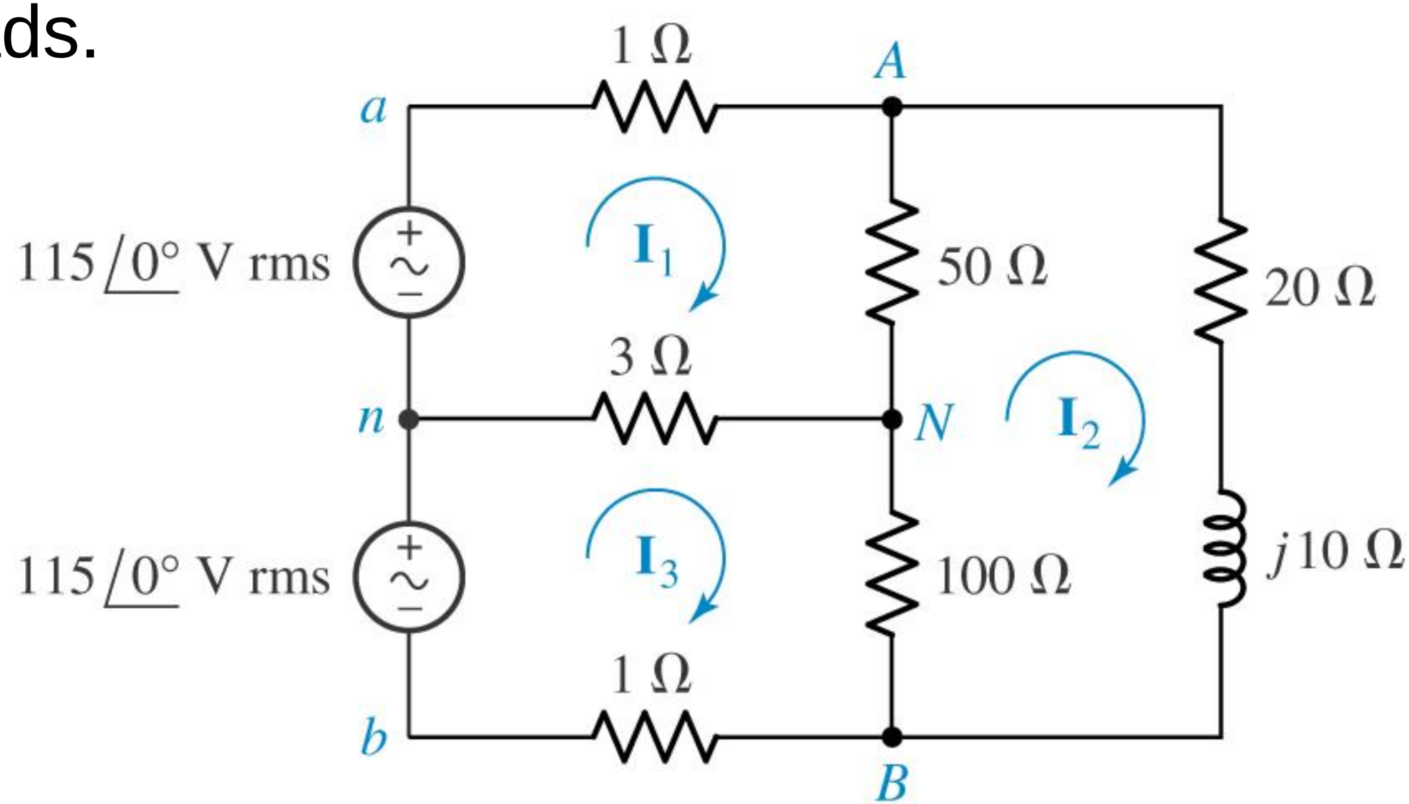
The transmission efficiency, η:

$$\eta = \frac{total \quad power \quad delivered \quad to \quad load}{total \quad power \quad generated}$$

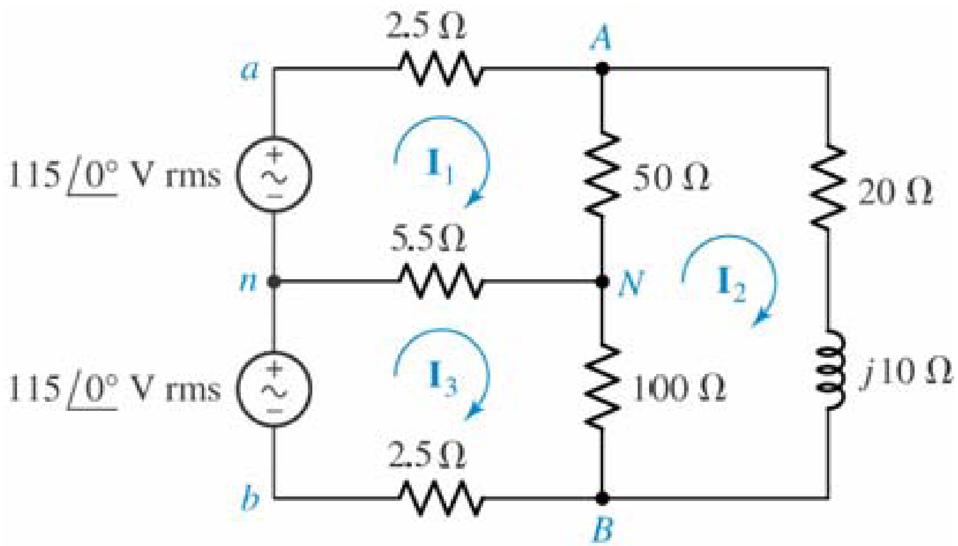
$$= \frac{2086}{2086 + 237} = 89.8\%$$

Practice: 12.3

Modified Figure below by adding a 1.5 Ω resistance to each of the two outer lines, and a 2.5 Ω resistance to the neutral wire. Find the average power delivered to each of the three loads.



Practice: 12.3



$$58\mathbf{I}_1 - 5.5\mathbf{I}_2 - 50\mathbf{I}_3 = 115 \qquad [1]$$

$$-5.5\mathbf{I}_1 + 108\mathbf{I}_2 - 100\mathbf{I}_3 = 115 \qquad [2]$$

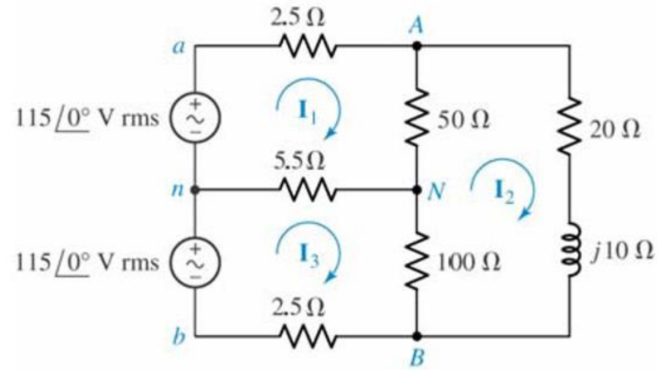
$$-50\mathbf{I}_1 - 100\mathbf{I}_2 + (170 + j10)\mathbf{I}_3 = 0 \qquad [3]$$

Solving, $\mathbf{I}_1 = 9.885\angle -17.40^\circ \text{ A rms} = 9.433 - j2.957 \text{ A rms}$

$\mathbf{I}_2 = 9.175\angle -19.21^\circ \text{ A rms} = 8.665 - j3.019 \text{ A rms}$

$\mathbf{I}_3 = 8.299\angle -21.94^\circ \text{ A rms} = 7.689 - j3.098 \text{ A rms}$

Practice: 12.3



$$\mathbf{I}_1 - \mathbf{I}_3 = 1.750\angle 4.622^\circ \text{ A rms}$$

$$\mathbf{I}_2 - \mathbf{I}_3 = 0.9792\angle 4.628^\circ \text{ A rms}$$

$$P_{50} = (1.750)^2 \times 50 = \underline{153.1 \text{ W}}$$

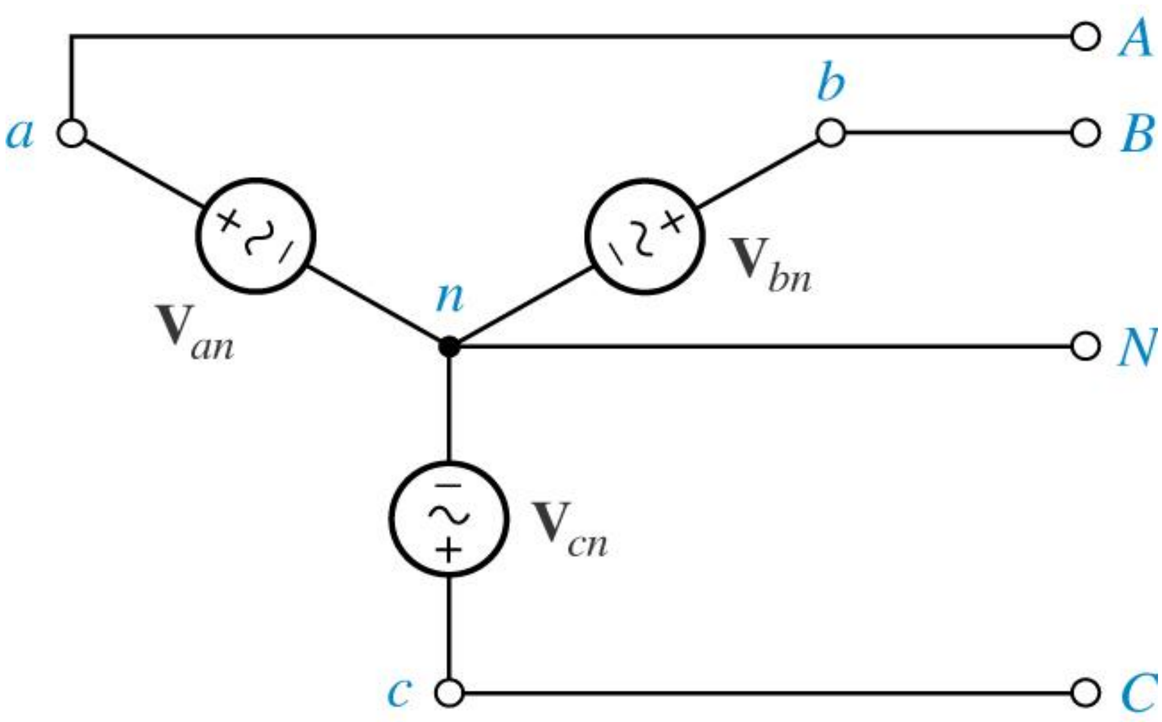
$$P_{100} = (0.9792)^2 \times 100 = \underline{95.88 \text{ W}}$$

$$P_{20+j10} = (8.299)^2 \times 20 = \underline{1377 \text{ W}}$$

Three-Phase Y-Y Connection:

A balanced three-phase system:

may be defined as having



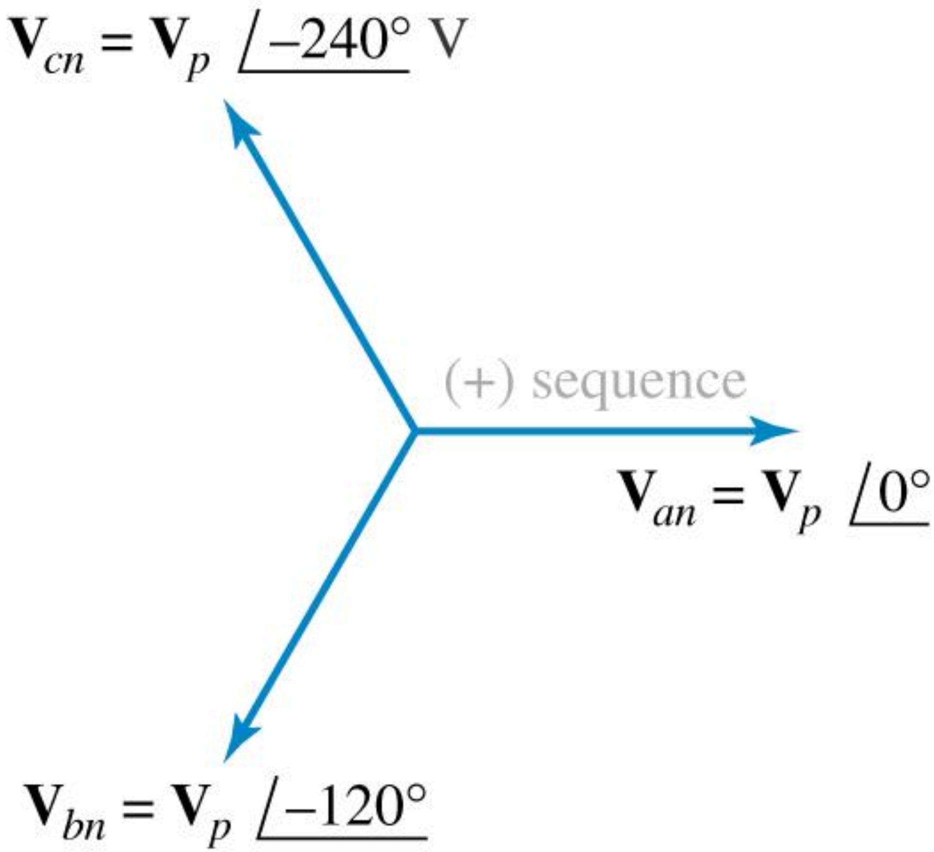
$$\left|\mathbf{V}_{an}\right| = \left|\mathbf{V}_{bn}\right| = \left|\mathbf{V}_{cn}\right|$$

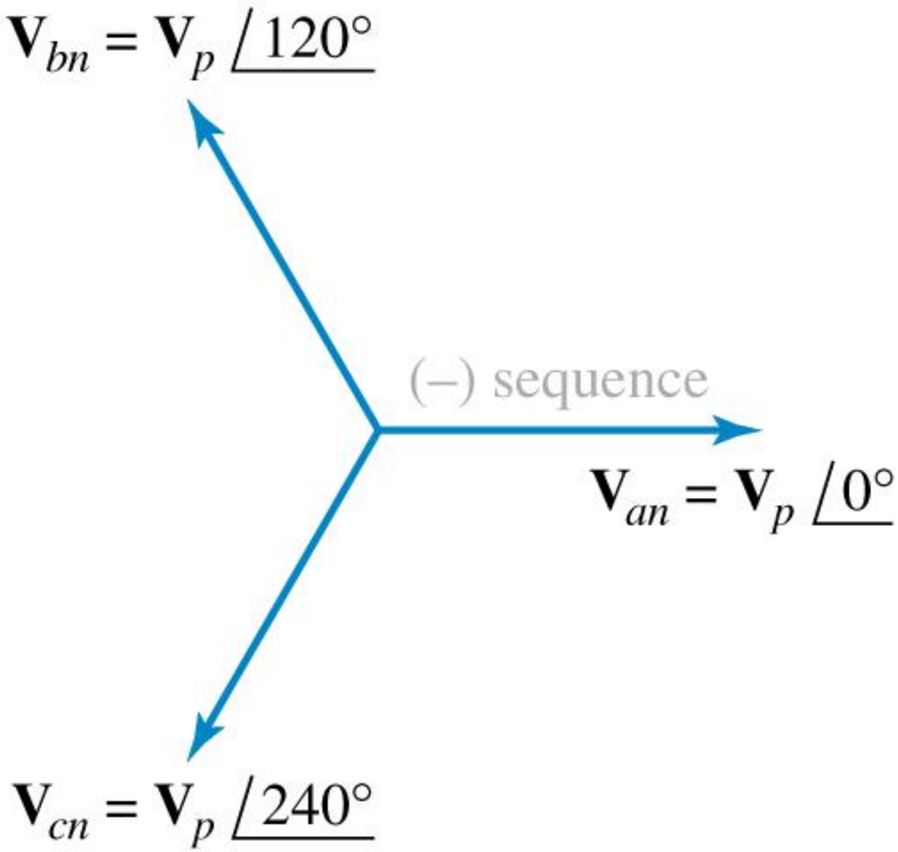
and

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

A Y-connected three-phase four-wire source.

Three-Phase Y-Y Connection:



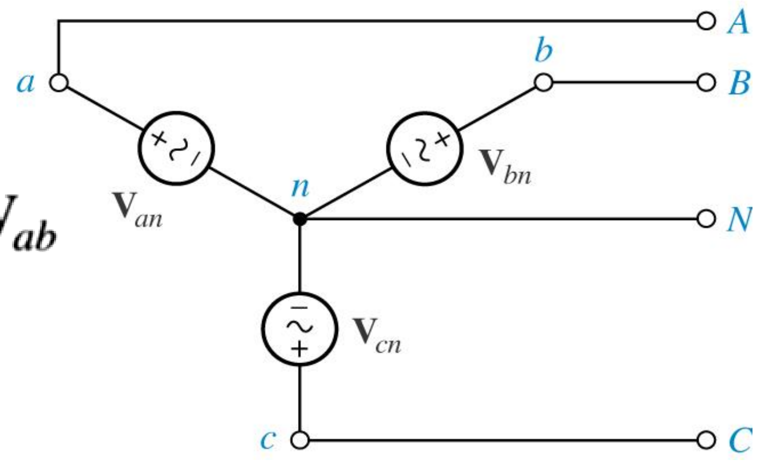


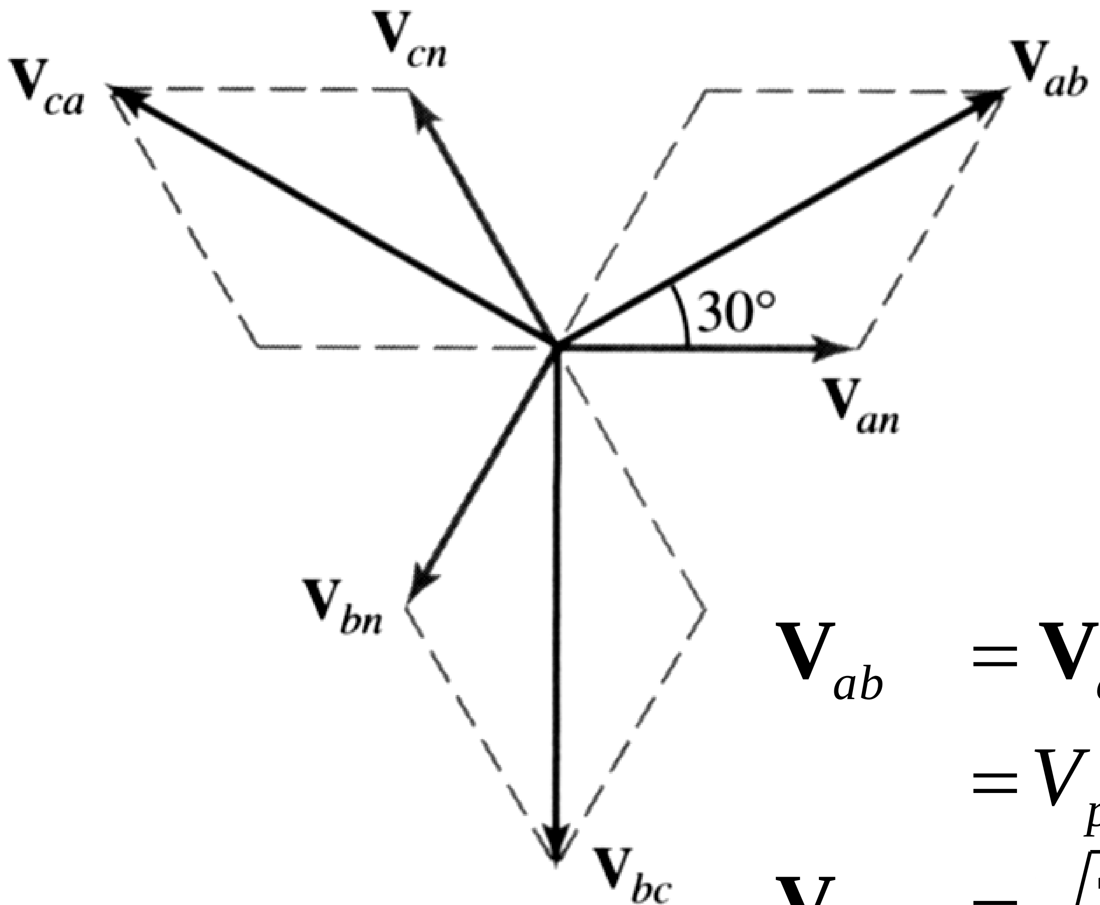
Positive, or abc, phase sequence.

Negative, or cba, phase sequence.

Three-Phase Y-Y Connection:

Line-to-Line Voltage:





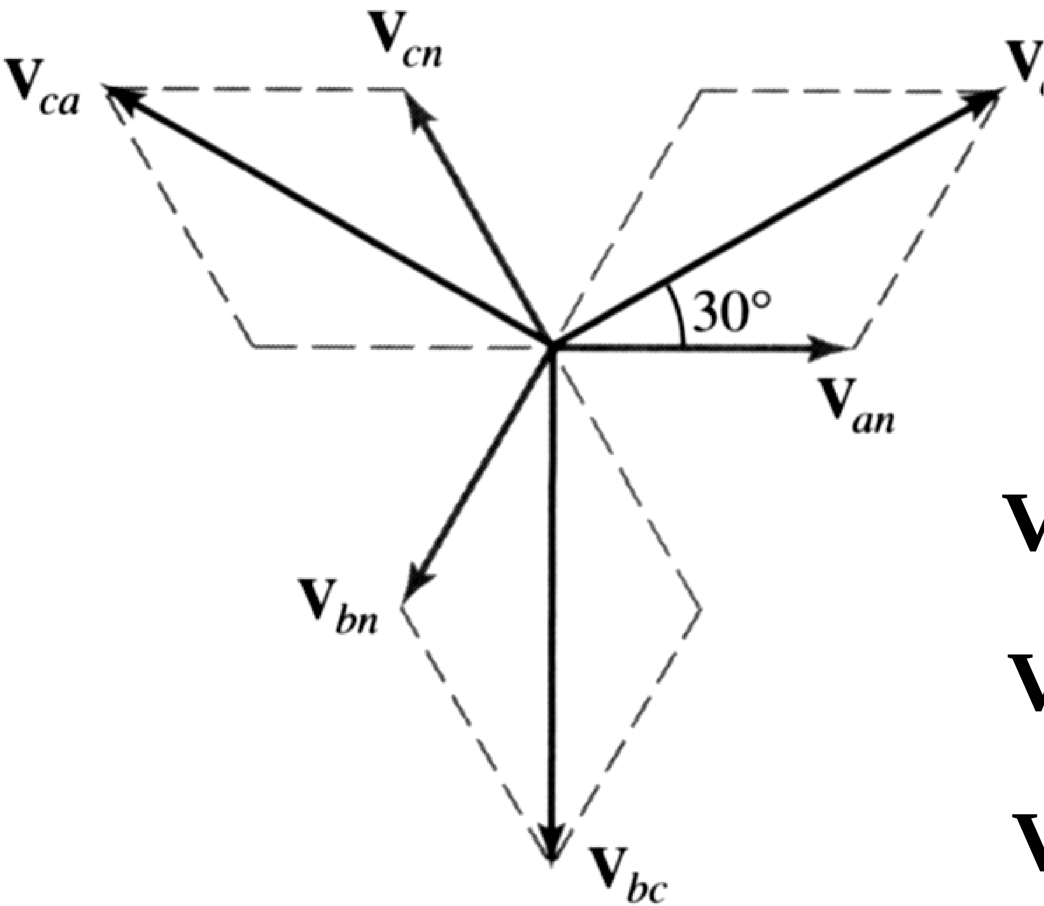
$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}$$

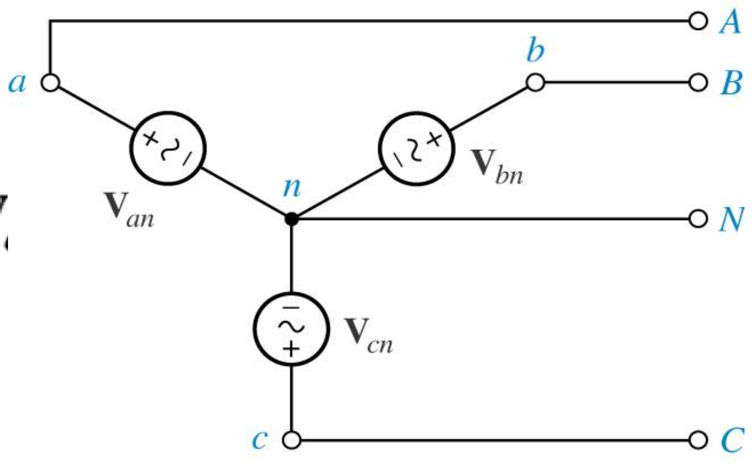
$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$\mathbf{V}_{ab} = \sqrt{3} V_p \angle 30^\circ$$

Three-Phase Y-Y Connection:

Line-to-Line Voltage:





$$\mathbf{V}_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$\mathbf{V}_{bc} = \sqrt{3} V_p \angle -90^\circ$$

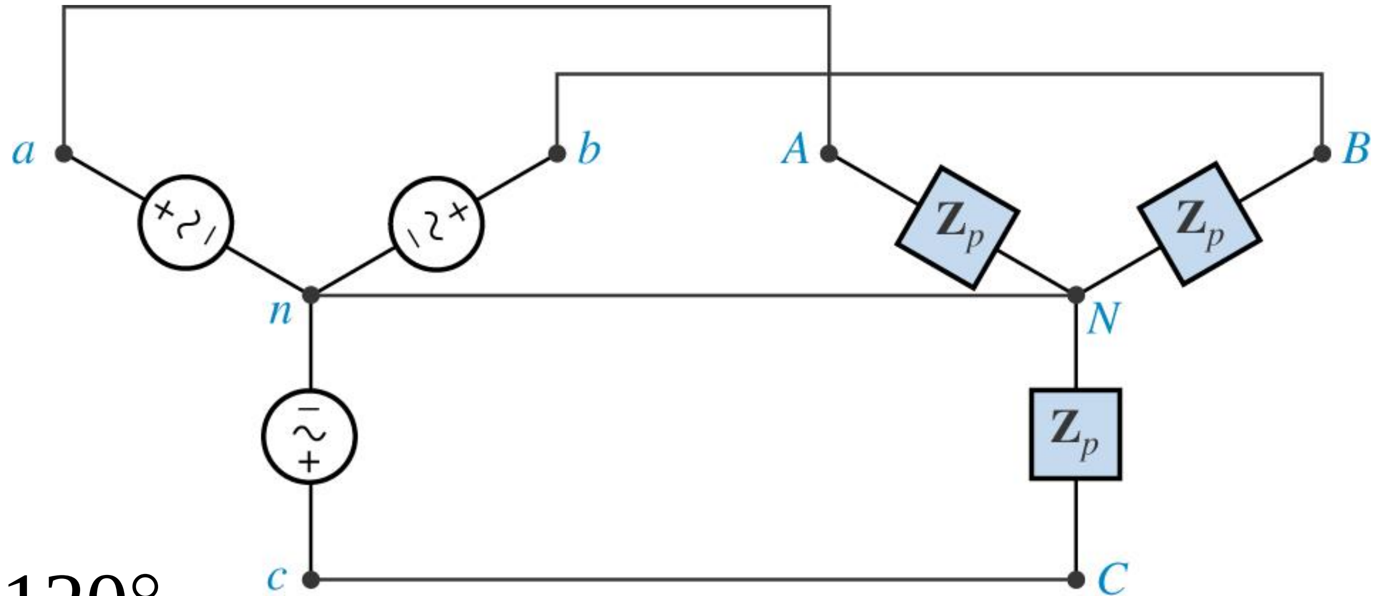
$$\mathbf{V}_{ca} = \sqrt{3} V_p \angle -210^\circ$$

Three-Phase Y-Y Connection:

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p}$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_p} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_p} = \mathbf{I}_{aA}\angle -120^\circ$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle -240^\circ$$



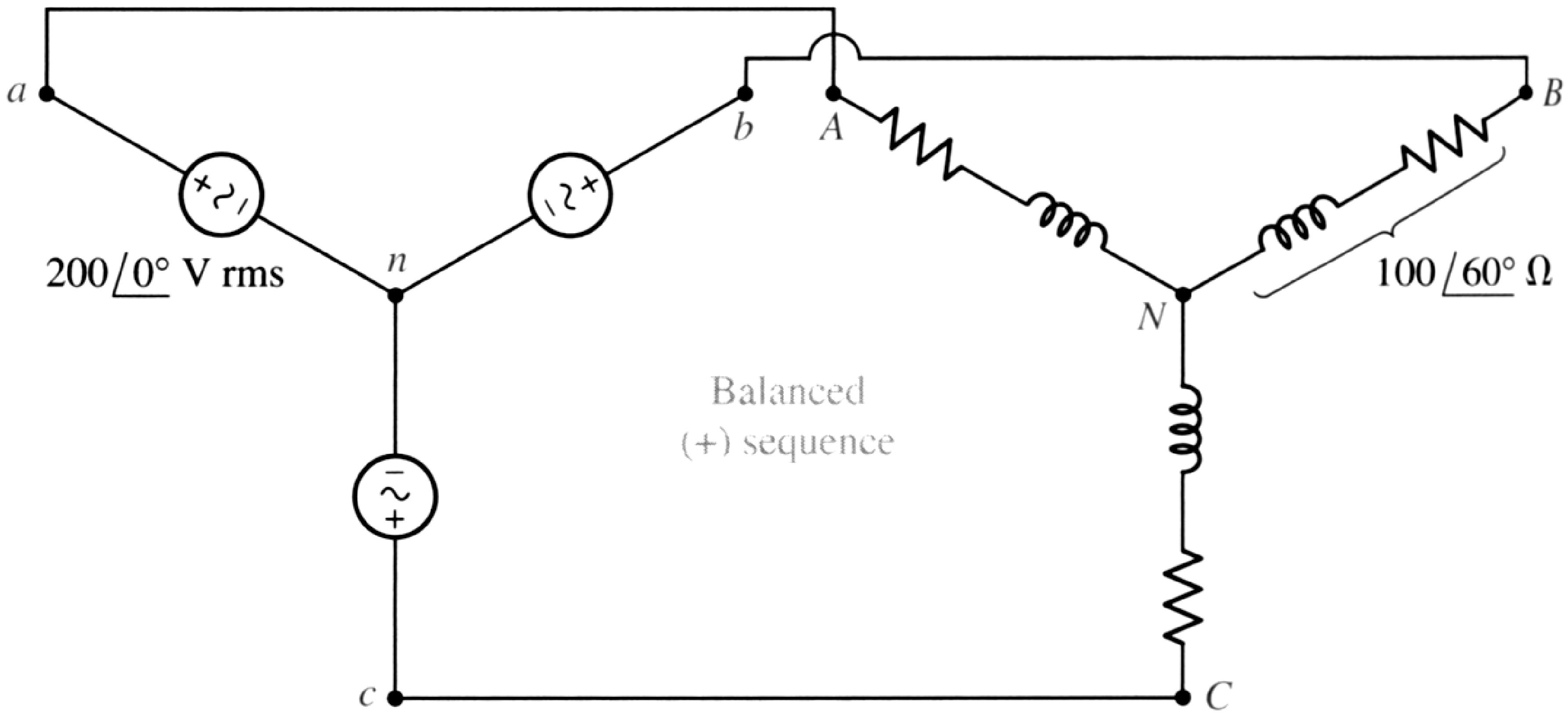
A balanced three-phase system, connected Y-Y and including a neutral.

Therefore:

$$\mathbf{I}_{Nn} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

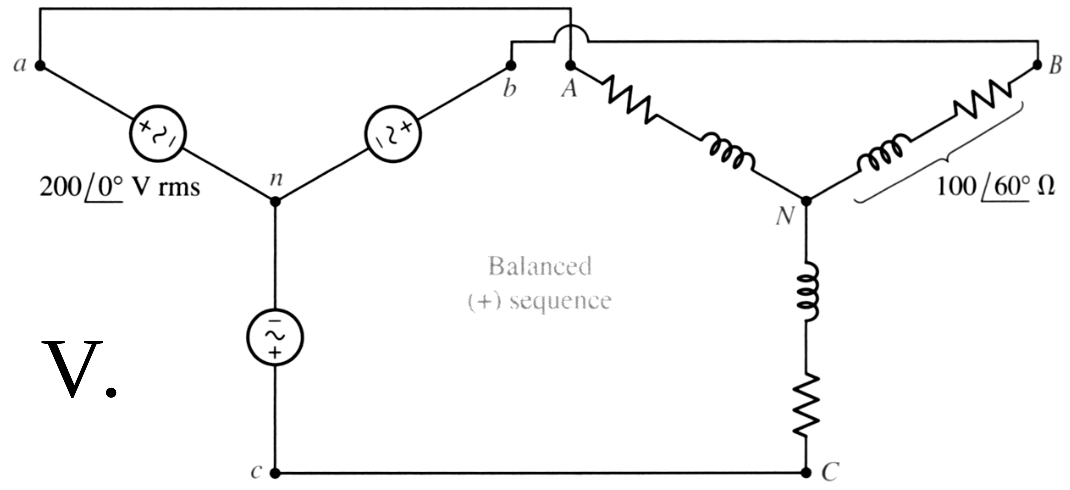
Example:

Find the currents, voltages and the total power dissipated in the load



Example:

"Per-phase":



$$\mathbf{V}_{an} = 200 \angle 0^\circ \quad \text{V.}$$

$$\mathbf{V}_{bn} = 200 \angle -120^\circ \quad \text{V.}$$

$$\mathbf{V}_{cn} = 200 \angle -240^\circ \quad \text{V.}$$

$$\mathbf{V}_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$\mathbf{V}_{bc} = \sqrt{3} V_p \angle -90^\circ$$

$$\mathbf{V}_{ca} = \sqrt{3} V_p \angle -210^\circ$$

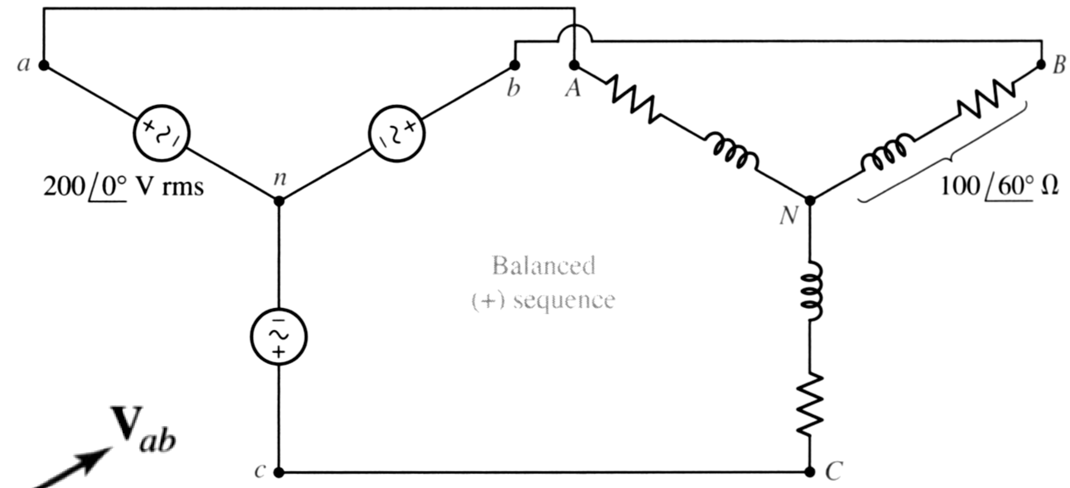
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p} = 2 \angle -60^\circ$$

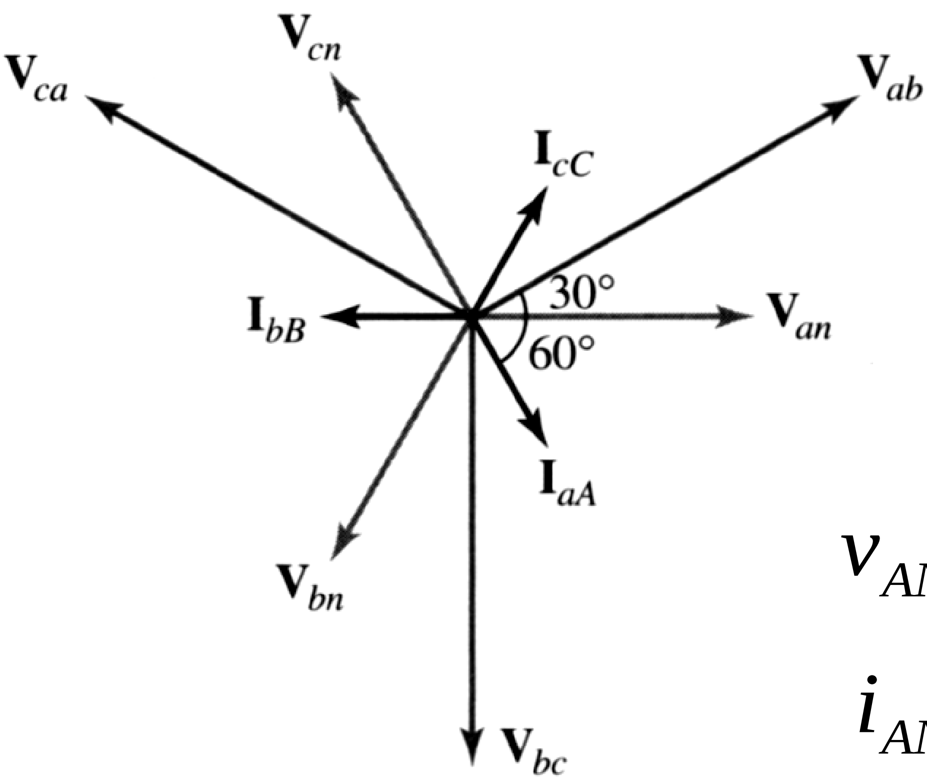
$$\mathbf{I}_{bB} = 2 \angle -180^\circ$$

$$\mathbf{I}_{cC} = 2 \angle -300^\circ$$

Example:

“Per-phase”:





$$v_{AN} = 200\sqrt{2}\cos(120\pi t + 0^\circ)$$

$$i_{AN} = 2\sqrt{2}\cos(120\pi t - 60^\circ)$$

Example:

even though phase voltages and currents have zero value at specific instants in time, the instantaneous power delivered to the total load is never zero.

$$p_A(t) = v_{AN} i_{AN} \quad = 800\cos(120\pi t)\cos(120\pi t - 60°)$$

$$= 400[\cos(-60°) + \cos(240\pi t - 60°)]$$

$$= 200 + 400\cos(240\pi t - 60°)]$$

$$p_B(t) = 200 + 400\cos(240\pi t - 300°)]$$

$$p_C(t) = 200 + 400\cos(240\pi t - 180°)]$$

Example:

the instantaneous power absorbed by the total load is therefore

$$p(t) = p_A(t) + p_B(t) + p_C(t) = 600W$$

Practice: 12.4

A balanced three-phase three-wire system has a Y-connected load. Each phase contains three loads in parallel: $-\mathrm{j}100\ \Omega$, $100\ \Omega$, and $50+\mathrm{j}50\ \Omega$. Assume positive phase sequence with $\mathbf{V}_{ab} = 400\angle 0°$ V. Find (a) $\mathbf{V}_{an}$; (b) $\mathbf{I}_{aA}$; (c) the total power drawn by the load

Practice: 12.4

(a) $\mathbf{V}_{ab} = 400\angle 0^\circ \text{ V} = \sqrt{3}\,\mathbf{V}_{an}\angle 30^\circ \text{ V}$

$$\therefore \mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle -30^\circ = \underline{230.9\angle -30^\circ \text{ V}}$$

(b) $\mathbf{I}_{aA} = \dfrac{\mathbf{V}_{an}}{\mathbf{Z}}$

where $\mathbf{Z} = -j100 // 100 // (50 + j50)$

$= 50\ \Omega$

$$\therefore \mathbf{I}_{aA} = \frac{230.9}{50}\angle -30^\circ = \underline{4.618\angle -30^\circ \text{ A}}$$

(c) We assume the above to be in rms units.

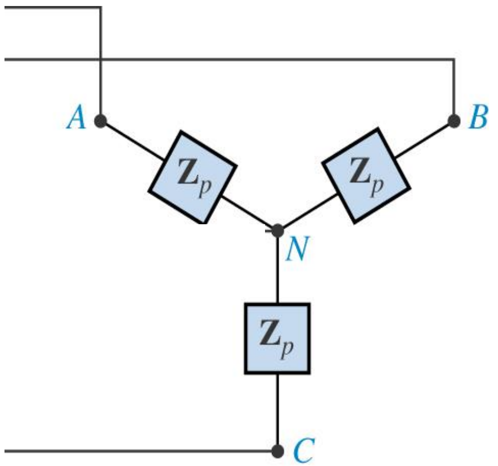
The total power absorbed by each phase is

$$P_p = 230.9 \times 4.618\cos(-30^\circ + 30^\circ)$$
$$= 1.066 \text{ kW}$$

$\therefore$ total power absorbed by the load is $3P_p = \underline{3.199 \text{ kW}}$

Example:

A balanced three-phase system with a line voltage of 300V is supplying a balanced Y-connected load with 1200W at a leading PF of 0.8. Find the line current and the per-phase load impedance.



The phase voltage is $V_p = \frac{300}{\sqrt{3}} \quad V.$

The per-phase power is $\frac{1200}{3} = 400 \quad W.$

Example:

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

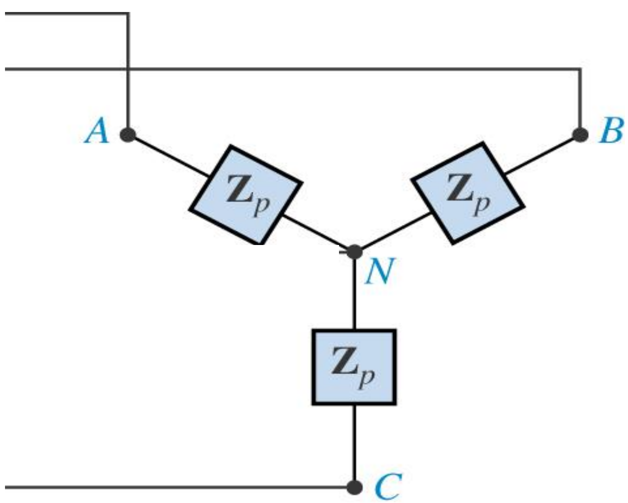
$$V_p = \frac{300}{\sqrt{3}} \quad V. \qquad\qquad P_p = \frac{1200}{3} = 400 \quad W.$$

The line current is

$$I_L = \frac{P_p}{V_p \cos(\theta - \phi)} = \frac{400}{\frac{300}{\sqrt{3}} \cdot 0.8} = 2.89 \quad A.$$

The phase impedance is

$$\left|\mathbf{Z}_p\right| = \frac{V_p}{I_L} = \frac{\frac{300}{\sqrt{3}}}{2.89}$$



Example:

The PF is 0.8 leading, so the impedance phase angle is $\cos^{-1}(0.8) = -36.9°$

thus $\mathbf{Z}_p = 60 \angle -36.9° \quad \Omega$

Practice: 12.5

A balanced three-phase three-wire system has a line voltage of 500 V. Two balanced Y-connected loads are present. One is a capacitive load with $7 - \mathrm{j}2\ \Omega$ per phase, and the other is an inductive load with $4 + \mathrm{j}2\ \Omega$ per phase. Find (a) the phase voltage; (b) the line current; (c) the total power drawn by the load; (d) the power factor at which the source is operating.

Practice: 12.5

(a) Phase voltage $= \dfrac{500}{\sqrt{3}} = \underline{288.7\text{ V}}$

(b) Line current $= \dfrac{288.7}{\mathbf{Z}}$ where $\mathbf{Z} = (7 - j2)//(4+j2)$

$$= 2.960\ \angle 10.62^\circ\ \Omega$$

$$= 2.909 + j0.5455\ \Omega$$

$\therefore$ Line current magnitude $= \dfrac{288.7}{2.960} = \underline{97.53\text{ A}}$

(c) Power drawn per phase $= (97.53)^2 \times 2.909$

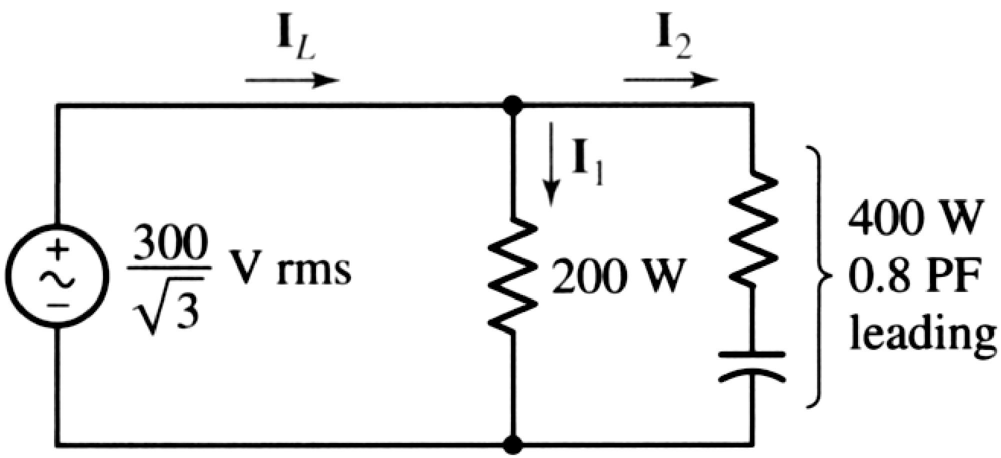
$$= 27.67\text{ kW}$$

$\therefore$ Total power drawn by load $= 3\ (27.67) = \underline{83.01\text{ kW}}$

(d) The source operates at a power factor of $\cos(-10.62^\circ) = \underline{0.9829\text{ lagging}}$ (as the current lags the voltage by 10.62°).

Example:

A balanced 600-W lighting load is added (in parallel) to the previous example. Determine the new line current.



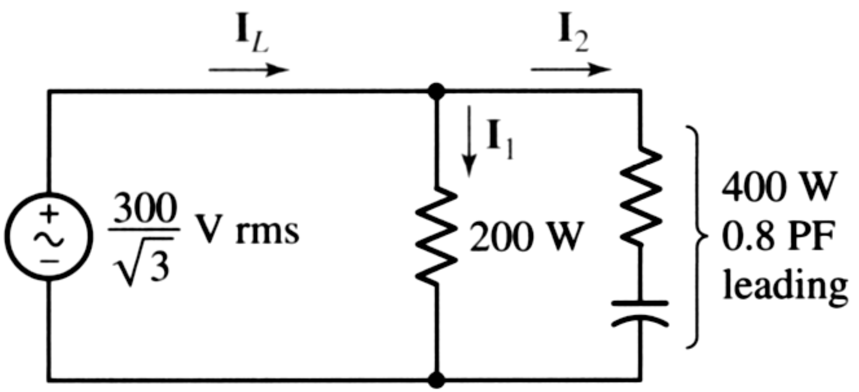
$$|\mathbf{I}_2| = 2.89$$

per-phase circuit

$$\mathbf{Z}_p = 60 \angle -36.9^\circ \quad \Omega$$

Example:

The amplitude of the lighting current is determined by



$$P = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$200 = \frac{300}{\sqrt{3}} |\mathbf{I}_1| \cos 0°$$

So $|\mathbf{I}_1| = 1.155$ assume $\mathbf{I}_1 = 1.155 \underline{/\ 0°}$ A.

$\longrightarrow$ $\mathbf{I}_2 = 2.89 \underline{/+36.9°}$ A.

And the line current is

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 = 3.87 \underline{/+26.6°} \text{ A.}$$

Practice: 12.6

Three balanced Y-connected loads are installed on a balanced three-phase four-wire system. Load 1 draws a total power of 6kW at unity PF, load 2 requires 10kVA at PF=0.96 lagging, and load 3 needs 7kW at 0.85 lagging. If the phase voltage at the load is 135 V, if each line has a resistance of $0.1\ \Omega$, and if the neutral has a resistance of $1\ \Omega$, find (a) the total power drawn by the load; (b) the combined PF of the load; (c) the total power lost in the four lines; (d) the phase voltage at the source; (e) the power factor at which the source is operating.

Practice: 12.6

(a) Load 1 draws 6 kW

Load 2 draws 10(0.96) = 9.6 kW

Load 3 draws 7 kW

$\therefore$ The total average power consumption is 6 + 9.6 + 7 = 22.6 kW

(we need to watch units carefully).

(b) The complex power drawn by the load is $6\angle 0^\circ + 10\angle\cos^{-1}0.96 + \dfrac{7}{0.85}\angle\cos^{-1}0.85$ kVA

$= 6 + j0 + 9.6 + j2.8 + 7 + j4.338 = 22.6 + j7.138 = 23.7\angle +17.53^\circ$ kVA

$\therefore$ combined load power factor = cos (+17.53°) = 0.9536 lagging

Practice: 12.6

(c) At the loads, $V_p = 135$ V and the per phase load creates a complex power demand of

$$\frac{23.7}{3}\angle +17.53^\circ \text{ kVA} = 7.9\angle +17.53^\circ \text{ kVA}$$

Thus, the line current is $\left(\dfrac{7.9\angle +17.53^\circ}{135\angle 0^\circ}\right)^* = 58.52\angle -17.53^\circ$ A

The loss in each line is $(58.52)^2\,(0.1) = 342.5$ W for a total line loss of $\underline{1027.4 \text{ W}}$

(d) The phase voltage at the source is

$$135 + 0.1\,(58.52) = \underline{140.9 \text{ V}}$$

(e) The source is connected to an impedance (line + load) of

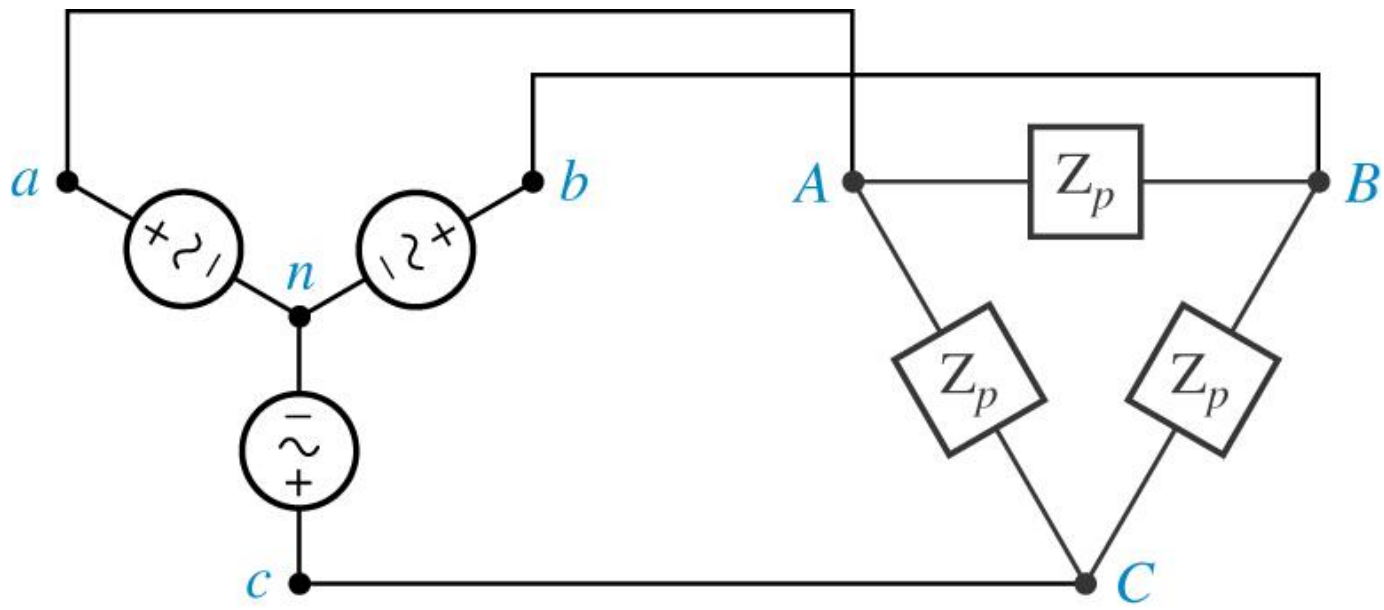
$0.1 + \dfrac{135}{58.52\angle -17.53^\circ} = 2.402\angle +16.81^\circ\,\Omega$, so the source operates at a power factor of

$\cos(-16.81^\circ) = \underline{0.9573 \text{ lagging}}$

The Delta (Δ) Connection:

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

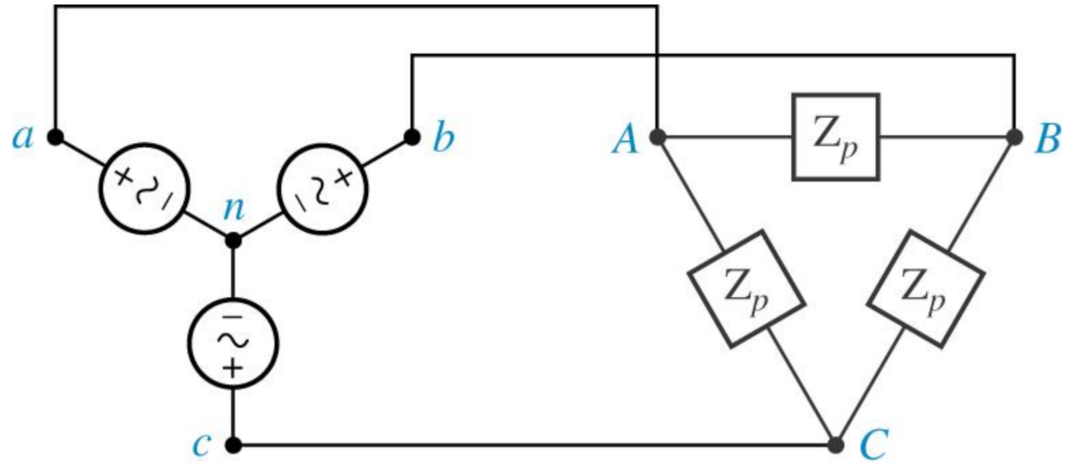
$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$



A balanced Δ-connected load is present on a three-wire three-phase system. The source happens to be Y-connected.

Where $V_L = \sqrt{3}V_p$ and $\mathbf{V}_{ab} = \sqrt{3}\,V_p \angle 30^\circ$

The Delta (Δ) Connection:



The phase current

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_p}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_p}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_p}$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

the line current:

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

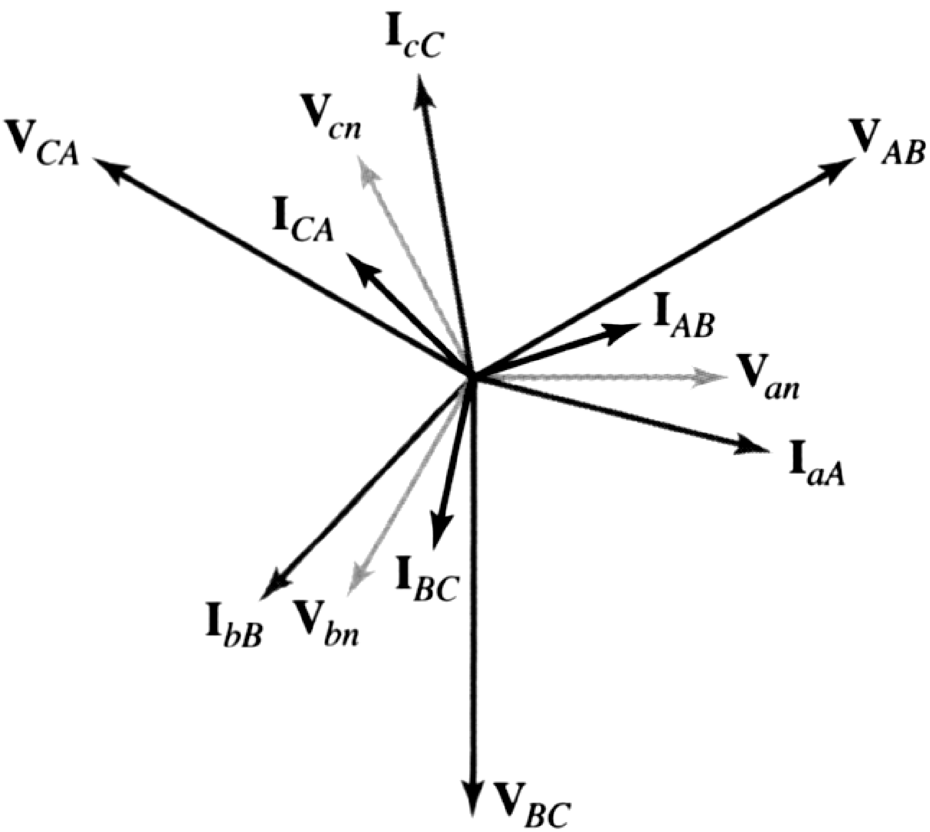
$$I_L = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|$$

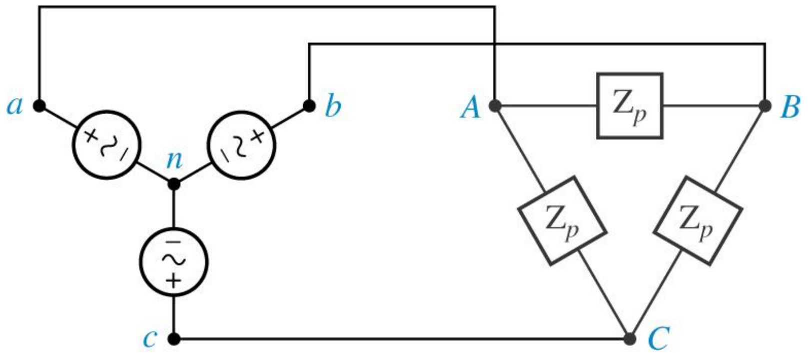
and

$$I_L = \sqrt{3} I_p$$

The Delta (Δ) Connection:

An example with an inductive load





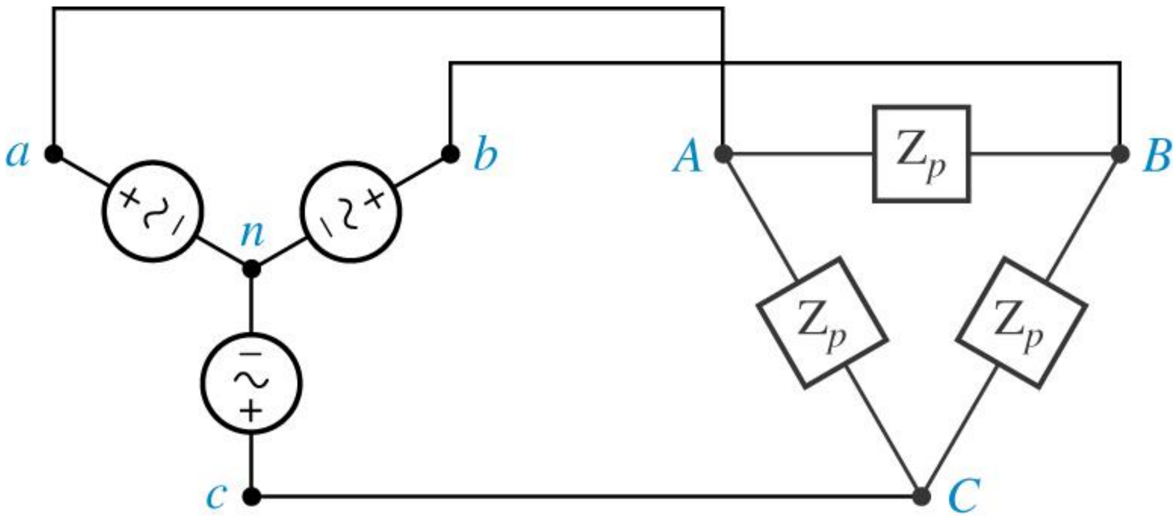
$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

$$I_L = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

$$I_L = \sqrt{3} I_p$$

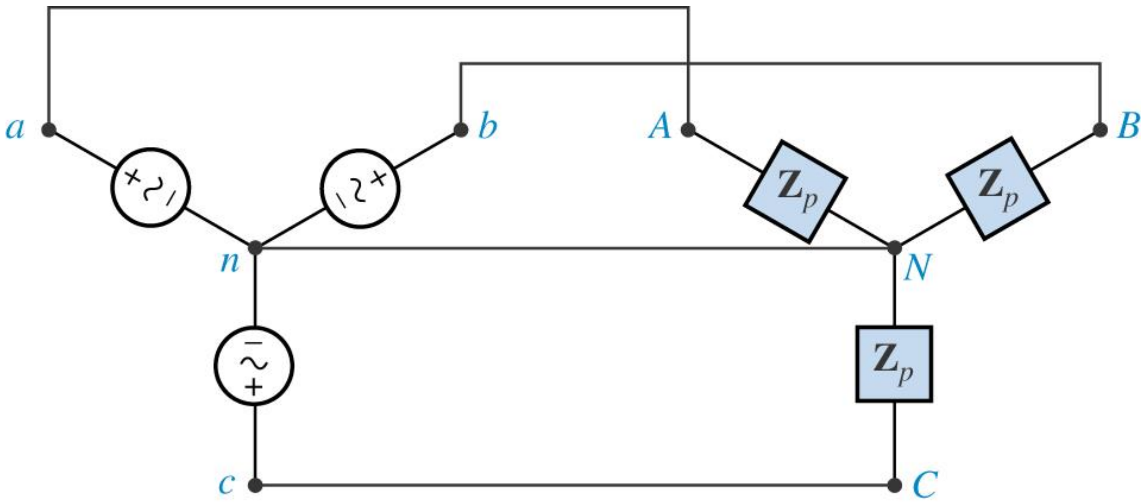
Δ-Connected:



$$P_p = V_p I_p \cos\theta$$
$$= V_L I_p \cos\theta$$
$$= V_L \frac{I_L}{\sqrt{3}} \cos\theta$$

total power, $$P = 3P_p$$
$$= \sqrt{3} V_L I_L \cos\theta$$

Y-connected:



$$
\begin{aligned}
P_p &= V_p I_p \cos\theta \\
&= V_p I_L \cos\theta \\
&= \frac{V_L}{\sqrt{3}} I_L \cos\theta
\end{aligned}
$$

total power, $$\begin{aligned} P &= 3P_p \\ &= \sqrt{3} V_L I_L \cos\theta \end{aligned}$$

Practice: 12.7

Each phase of a balanced three-phase Δ-connected load consists of a 0.2 H inductor in series with the parallel combination of a $5\ \mu$F capacitor and a $200\ \Omega$ resistance. Assume zero line resistance and a phase voltage of 200 V at $\omega = 400$ rad/s. Find (a) the phase current; (b) the line current; (c) the total power absorbed by the load

Practice: 12.7

$$\mathbf{Z} = j0.2(400) + 200 \,//\, \left[\frac{-j}{(400)(5\times10^{-6})} \right] = 172.8\angle 3.662^\circ\ \Omega$$

$$= 172.4 + j11.04\ \Omega$$

(a) $\left|\mathbf{I}_p\right| = \dfrac{200}{172.8} = \underline{1.157\ \mathrm{A}}$

(b) $\mathrm{I_L} = \sqrt{3}\,\mathrm{I_p} = \underline{2.005\ \mathrm{A}}$

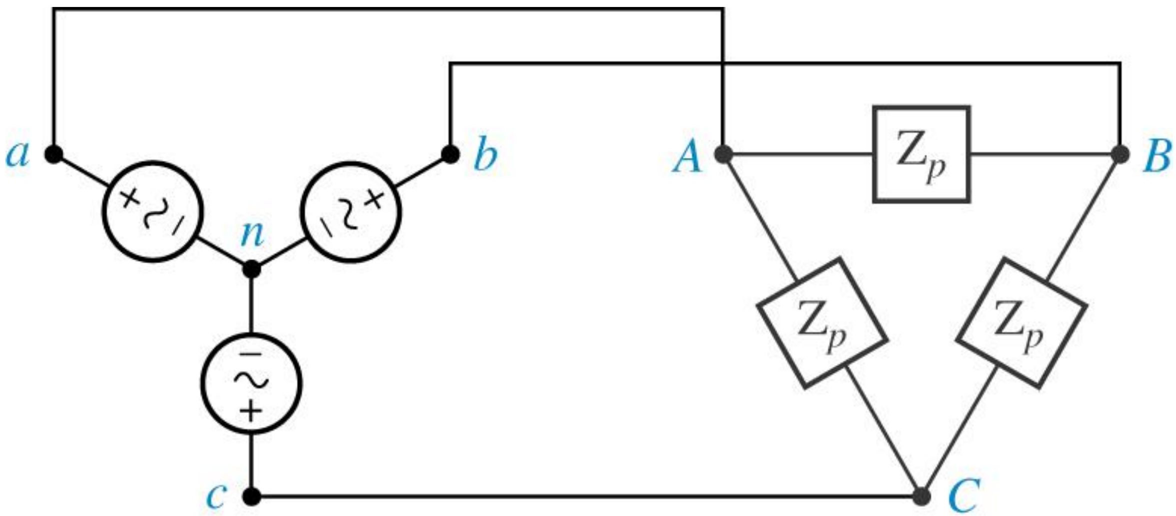
(c) per phase, the load draws average power

$$P = \frac{(200)^2}{172.4} = 232.0\ \mathrm{W}$$

Thus, the total power absorbed by the load is $3 \times 232 = \underline{696\ \mathrm{W}}$

Example 12.5:

Determine the amplitude of the line current in a three-phase system with a line voltage of 300 V that supplies 1200 W to a Δ-connected load at a lagging PF of 0.8

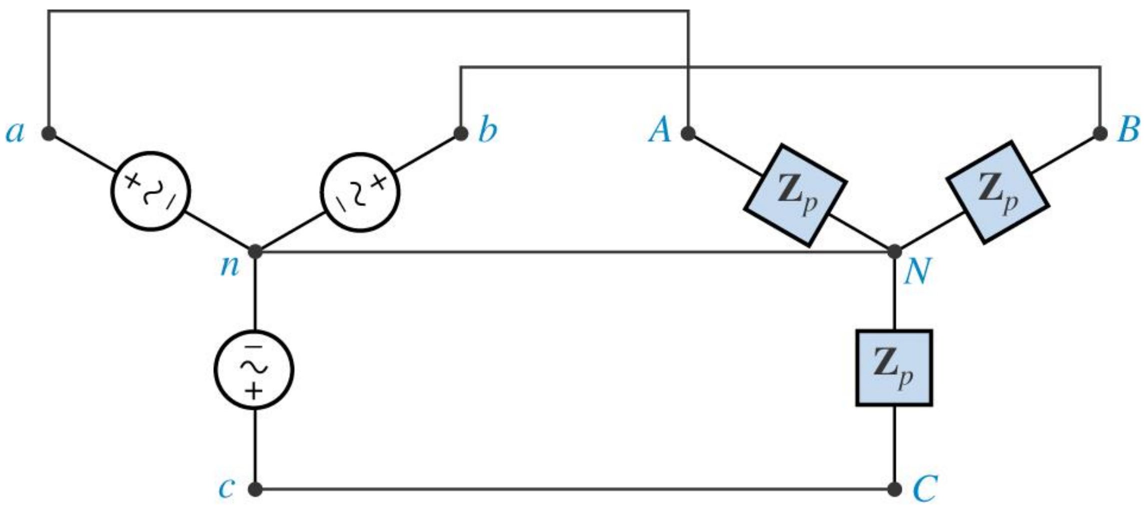


$$P_p = V_p I_p \cos\theta = V_L I_p \cos\theta = V_L \frac{I_L}{\sqrt{3}} \cos\theta$$

$$I_L = 2.89 \quad A., \quad \mathbf{Z}_p = 180 \angle 36.9^\circ$$

Example 12.6:

Determine the amplitude of the line current in a three-phase system with a line voltage of 300 V that supplies 1200 W to a Y-connected load at a lagging PF of 0.8



$$\begin{aligned} P_p &= V_p I_p \cos\theta \\ &= V_p I_L \cos\theta \\ &= \frac{V_L}{\sqrt{3}} I_L \cos\theta \end{aligned}$$

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$

$$I_L = 2.89 \quad A., \quad \mathbf{Z}_p = 60 \angle 36.9^\circ$$

Practice: 12.8

A balanced three-phase three-wire system is terminated with two Δ-connected load in parallel. Load 1 draws 40kVA at lagging PF of 0.8, while load 2 absorbs 24 kW at a leading PF of 0.9. Assume no line resistance, and let $\mathbf{V}_{ab} = 440\angle 30°$ V. Find (a) the total power drawn by the load; (b) the phase current $\mathbf{I}_{AB1}$ for the lagging load; (c) $\mathbf{I}_{AB2}$; (d) $\mathbf{I}_{aA}$.

Practice: 12.8

(a) Paying close attention to the units quoted, we see that
load 1 draws $40 \times 0.8 = 32$ kW and
load 2 draws 24 kW
$\therefore$ the total power demand is $32 + 24 = \underline{56\text{ kW}}$

(b) Working on a per-phase basis, load 1 draws an apparent power of $\dfrac{40}{3} = 13.33\,\text{kVA}$ at a power factor of 0.8 lagging.

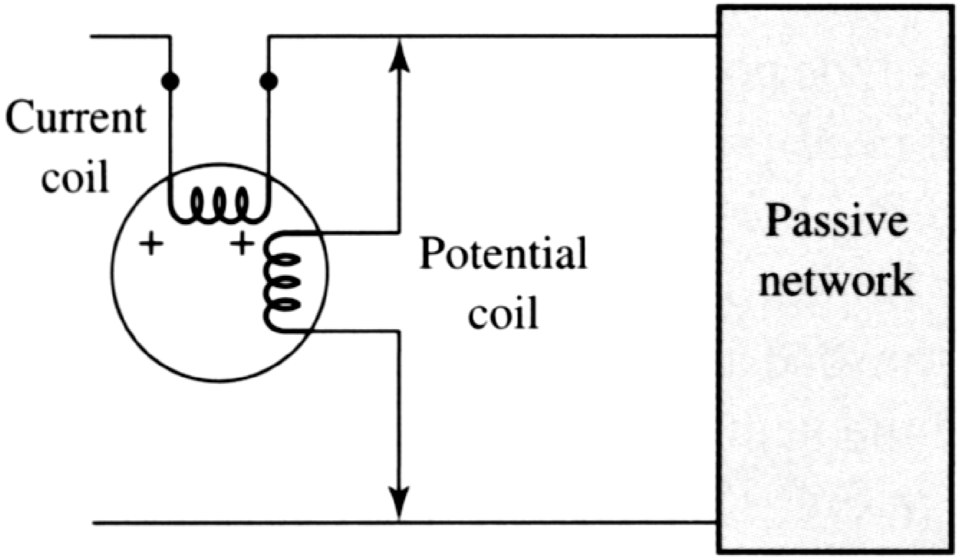
$$\text{Thus, } \mathbf{I}_{AB_1} = \left(\frac{13330\angle\cos^{-1}0.8}{440\angle 30^\circ}\right)^* = \underline{30.30\angle{-6.870^\circ}\text{ A}}$$

(c) Load 2 draws $\dfrac{1}{3}\,\dfrac{24}{0.9} = 8.889$ kVA per phase at a leading PF of 0.9.

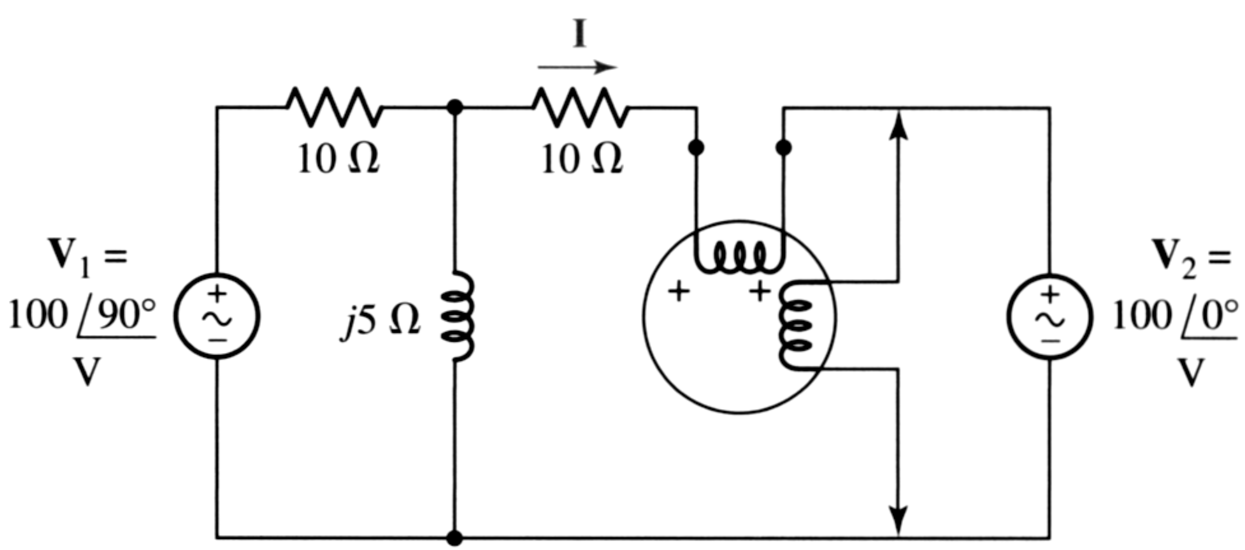
$$\text{Thus, } \mathbf{I}_{AB_2} = \left(\frac{8889\angle-\cos^{-1}0.9}{440\angle 30^\circ}\right)^* = \underline{20.20\angle{+\,55.84^\circ}\text{ A}}$$

(d) $\mathbf{I}_{aA} = \sqrt{3}\ \mathbf{I}_{AB}\angle-30^\circ = \left(\mathbf{I}_{AB_1} + \mathbf{I}_{AB_2}\right)\angle-30^\circ = \left(75.23\angle 17.54^\circ\right)\angle-30^\circ = \underline{75.23\angle{-12.46^\circ}\text{ A}}$

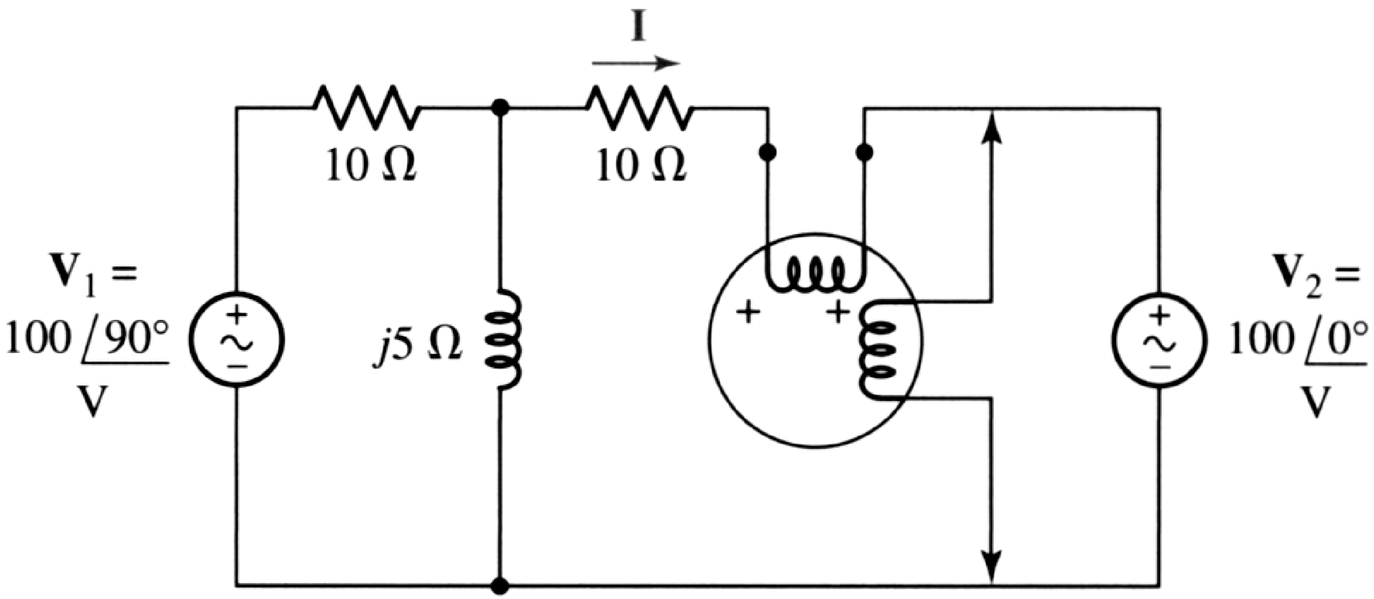
Power Measurement:



wattmeter



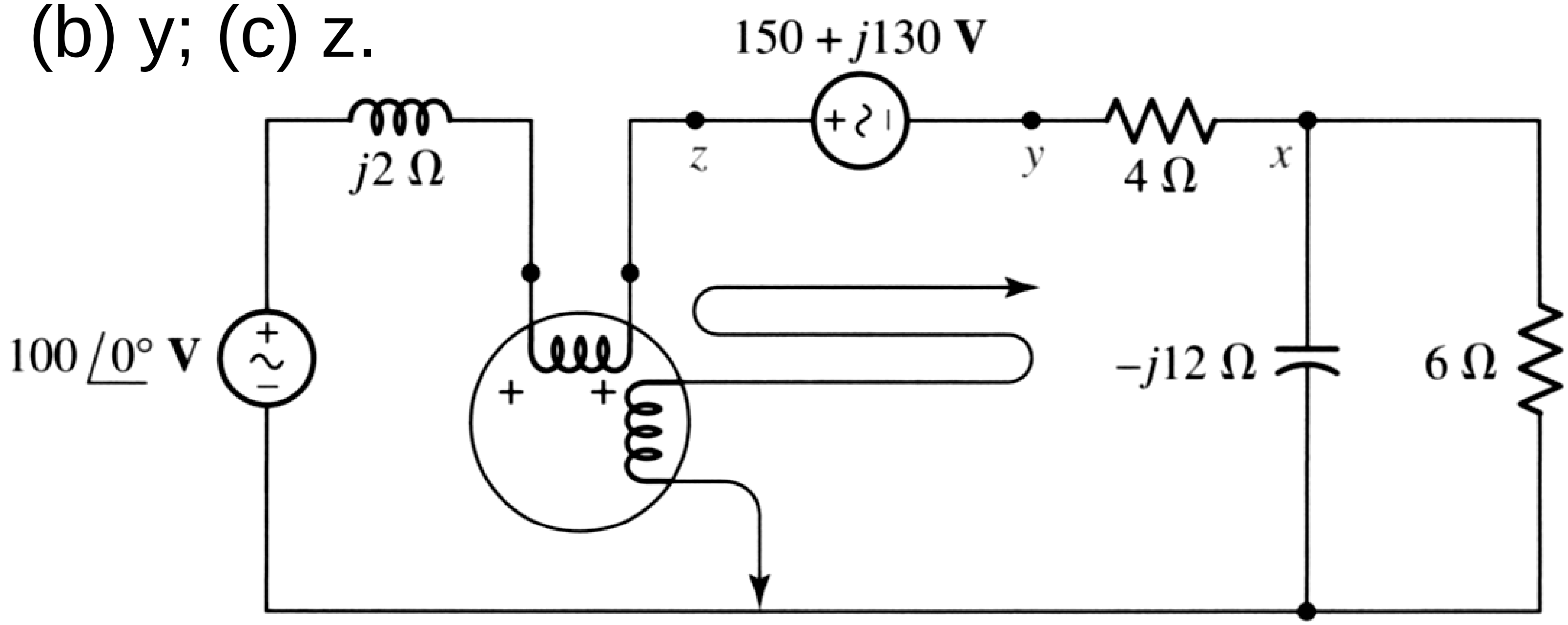
Power Measurement:



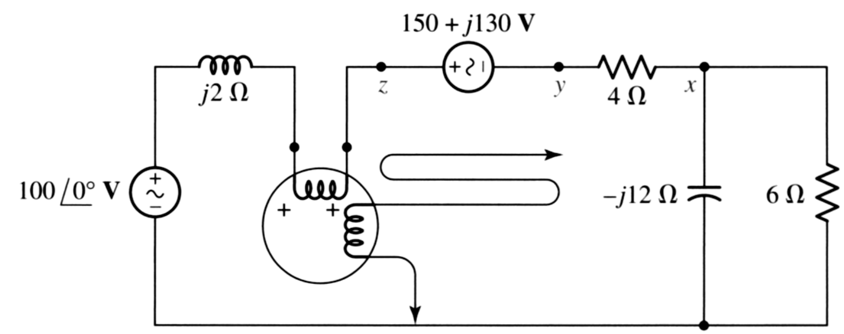
$$P = \left|\mathbf{V}_2\right| \cdot \left|\mathbf{I}\right| \cos\left(ang\mathbf{V}_2 - ang\mathbf{I}\right)$$

Practice: 12.9

Determine the wattmeter reading in Figure below, state weather or not the potential coil had to be reversed in order to obtain an upscale reading, and identify the device or devices absorbing or generating this power. The (+) terminal of the wattmeter is connected to: (a) x; (b) y; (c) z.



Practice: 12.9



- define current **I** flowing into the "+" current coil terminal using mesh analysis,

$$-100 + j2\mathbf{I} + (150 + j130) + 4\mathbf{I} + (6 // -j12)\mathbf{I} = 0$$

or

$$8.809\angle -2.603^\circ\ \mathbf{I} = -50 - j130 \quad \text{so} \quad \mathbf{I} = 15.81\angle -108.4^\circ\ \text{A}$$

(a) With the probe at point x, we measure the potential

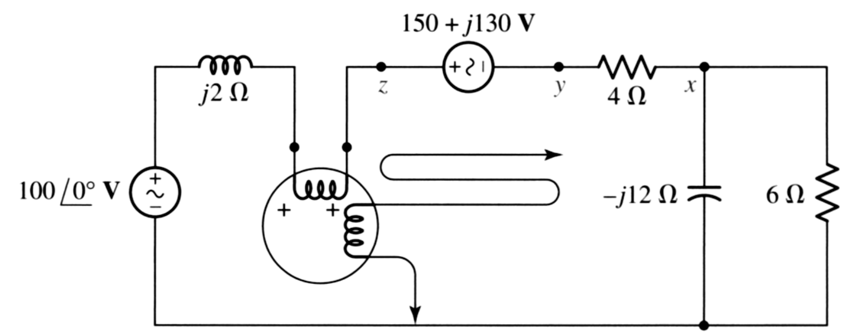
$$(6 // -j12) \times \mathbf{I} = (5.367\angle -26.57^\circ)(15.81\angle -108.4^\circ) = 84.85\angle -135.0^\circ\ \text{V}$$

The power reading is therefore $P = (84.85)(15.81)\cos(-135^\circ + 108.4^\circ)$

$= 1199.5$ W, as is.

This power is absorbed by the 6–Ω resistor.

Practice: 12.9



(b) With the probe at point y, a potential

$$-(150+j130)-j2\mathbf{I}+100=-50-j130-j2\left[15.81\angle-108.4^\circ\right]$$
$$=144.2\angle-123.7^\circ \text{ V is measured}$$

The power reading is therefore

$$P = (144.2)\,(15.81)\cos(-123.7^\circ + 108.4^\circ)$$
$$= 2199 \text{ W}$$, with no need to flip probes.

This power is absorbed by the 4–Ω and 6–Ω resistors.

(c) With the probe at point z, we measure a potential

$$-j2\mathbf{I}+100=100-j2(15.81\angle-108.4^\circ)=70.70\angle 8.115^\circ \text{ V}$$

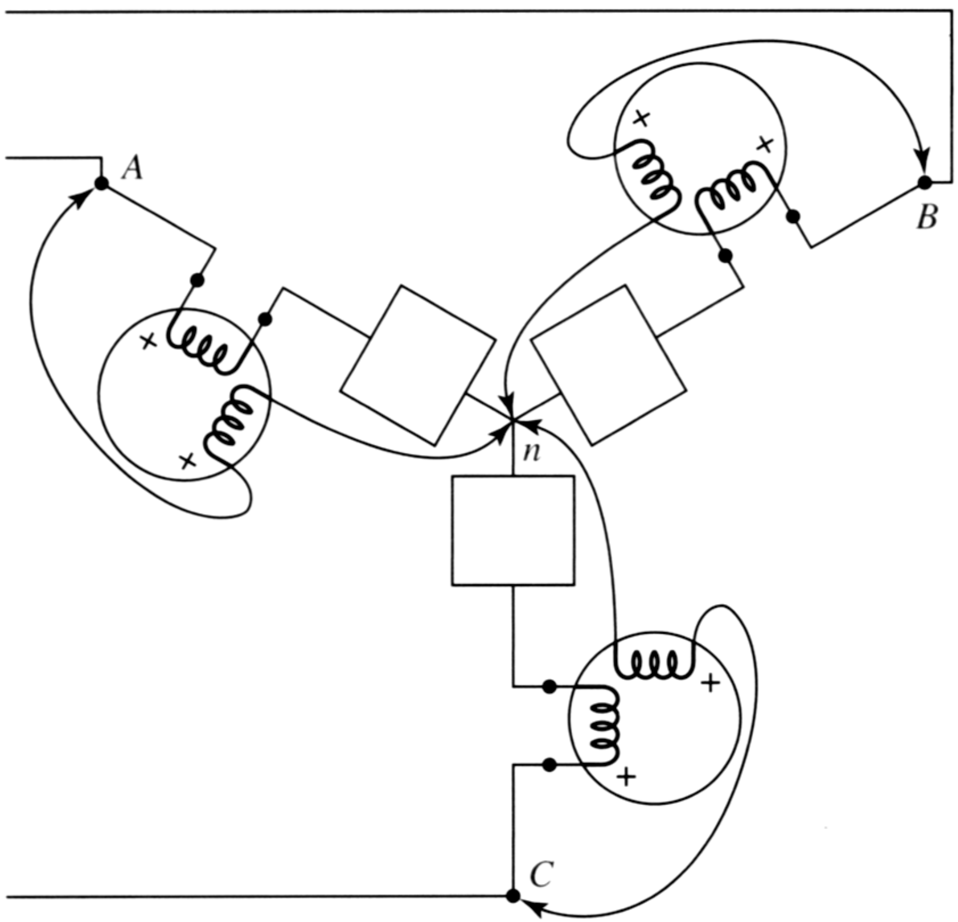
The power reading is therefore

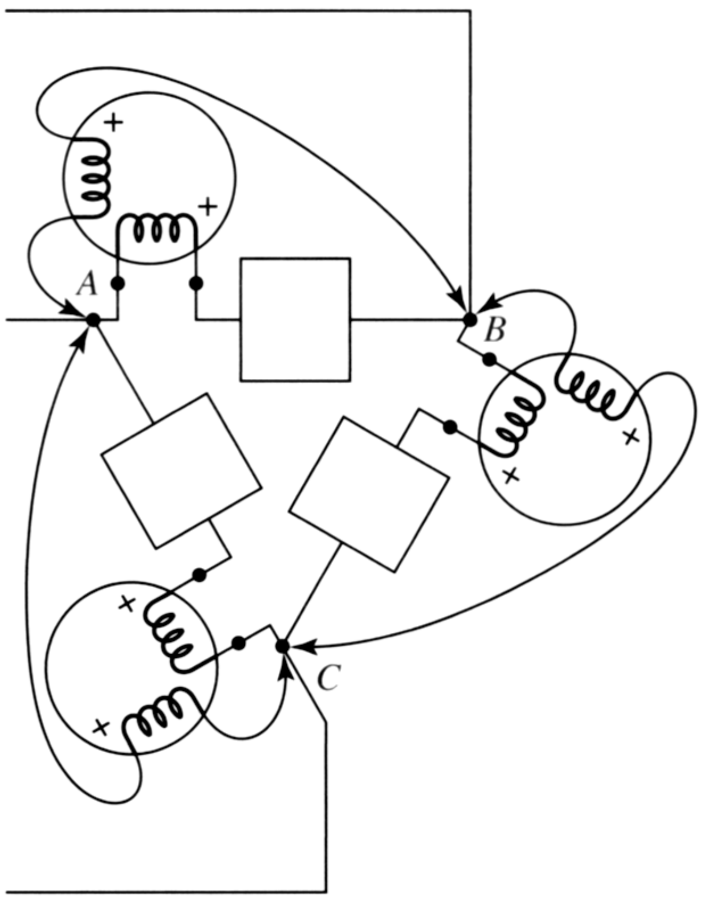
$$P=(70.70)(15.81)\cos(8.115^\circ+108.4^\circ)=-499.0 \text{ W}$$

This would peg an analog meter at downscale, so we read + 499.0 W with leads reversed.
This power is absorbed by the 100–V source.

Power Measurement:

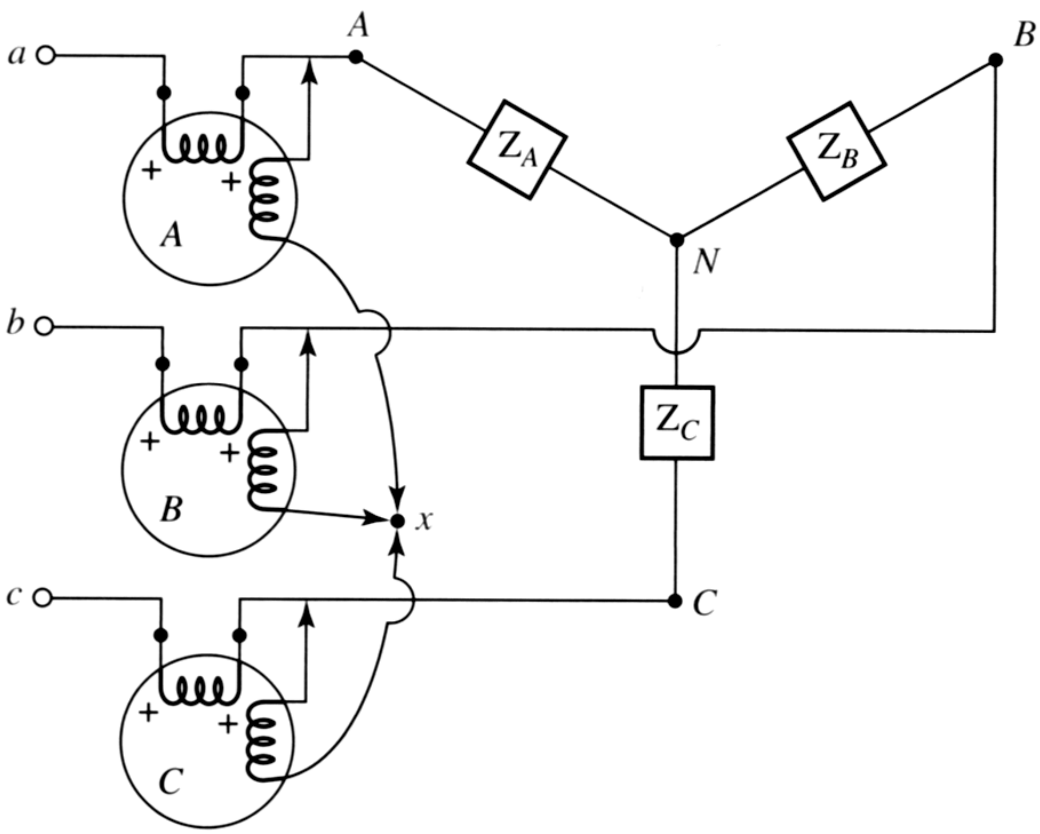
The wattmeter in a Three-Phase System:





Power Measurement:

The wattmeter in a Three-Phase System:



$$P_A = \frac{1}{T}\int_0^T v_{Ax} i_{aA} dt$$

$$v_{Ax} = v_{AN} + v_{Nx}$$

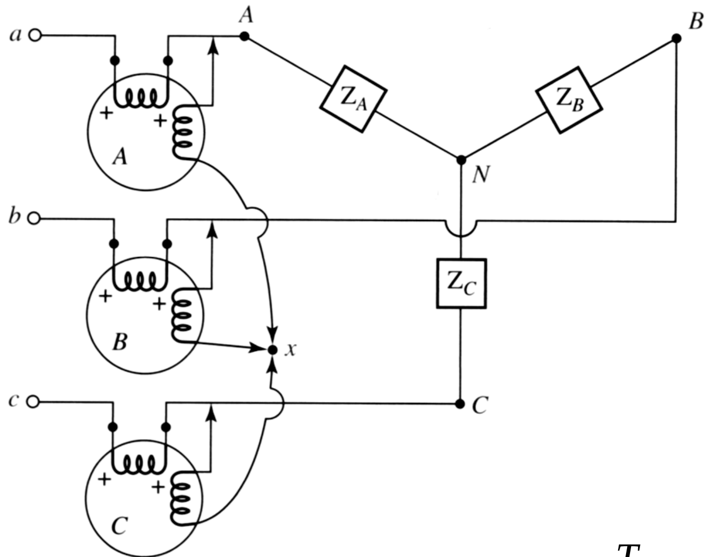
$$v_{Bx} = v_{BN} + v_{Nx}$$

$$v_{Cx} = v_{CN} + v_{Nx}$$

$$P = P_A + P_B + P_C = \frac{1}{T}\int_0^T (v_{Ax} i_{aA} + v_{Bx} i_{bB} + v_{Cx} i_{cC}) dt$$

Power Measurement:

The wattmeter in a Three-Phase System:



$$v_{Ax} = v_{AN} + v_{Nx}$$

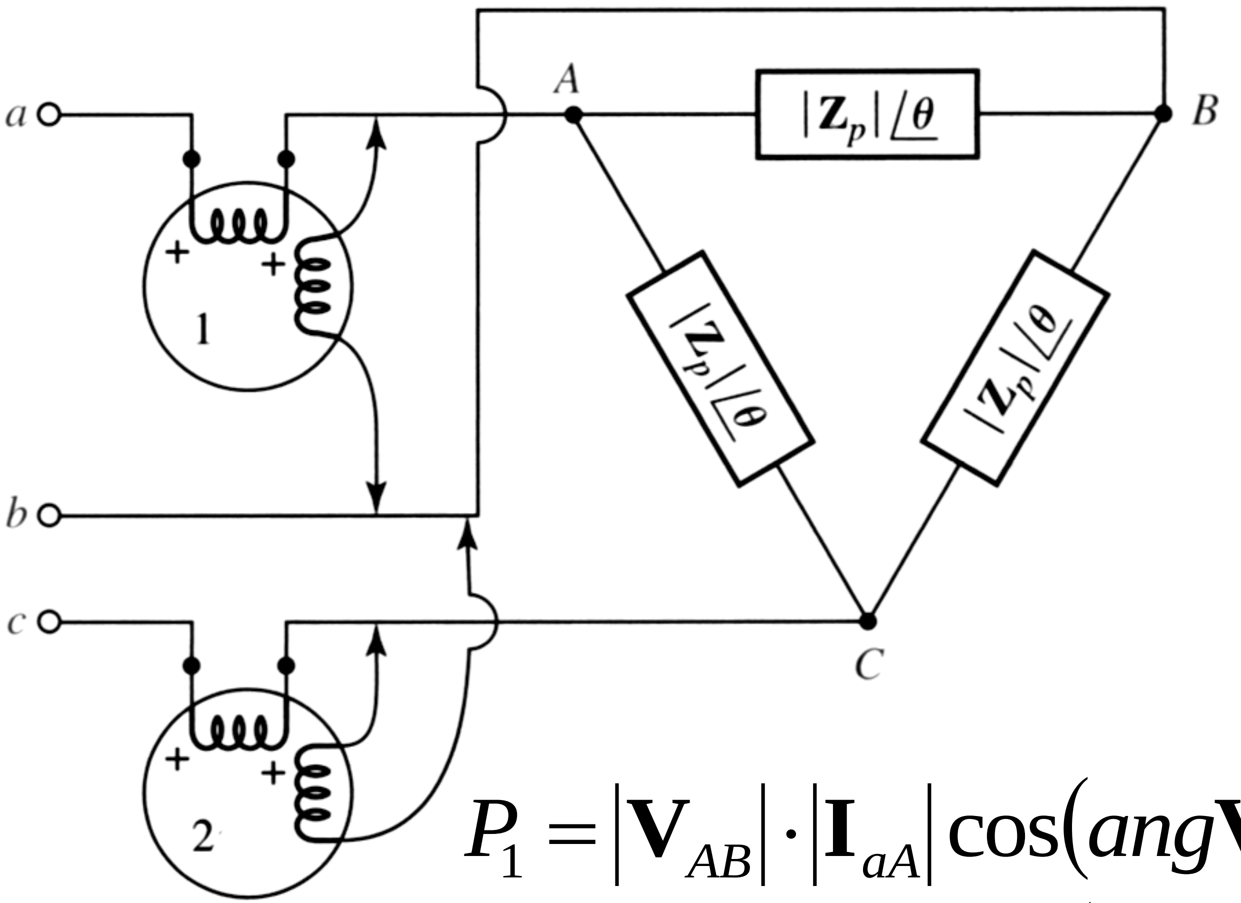
$$v_{Bx} = v_{BN} + v_{Nx}$$

$$v_{Cx} = v_{CN} + v_{Nx}$$

$$\begin{aligned} P &= P_A + P_B + P_C = \frac{1}{T}\int_0^T (v_{Ax}i_{aA} + v_{Bx}i_{bB} + v_{Cx}i_{cC})dt \\ &= \frac{1}{T}\int_0^T (v_{AN}i_{aA} + v_{BN}i_{bB} + v_{CN}i_{cC})dt + \frac{1}{T}\int_0^T v_{Nx}(i_{aA} + i_{bB} + i_{cC})dt \\ &= \frac{1}{T}\int_0^T (v_{AN}i_{aA} + v_{BN}i_{bB} + v_{CN}i_{cC})dt \end{aligned}$$

Power Measurement:

The Two-Wattmeter Method:

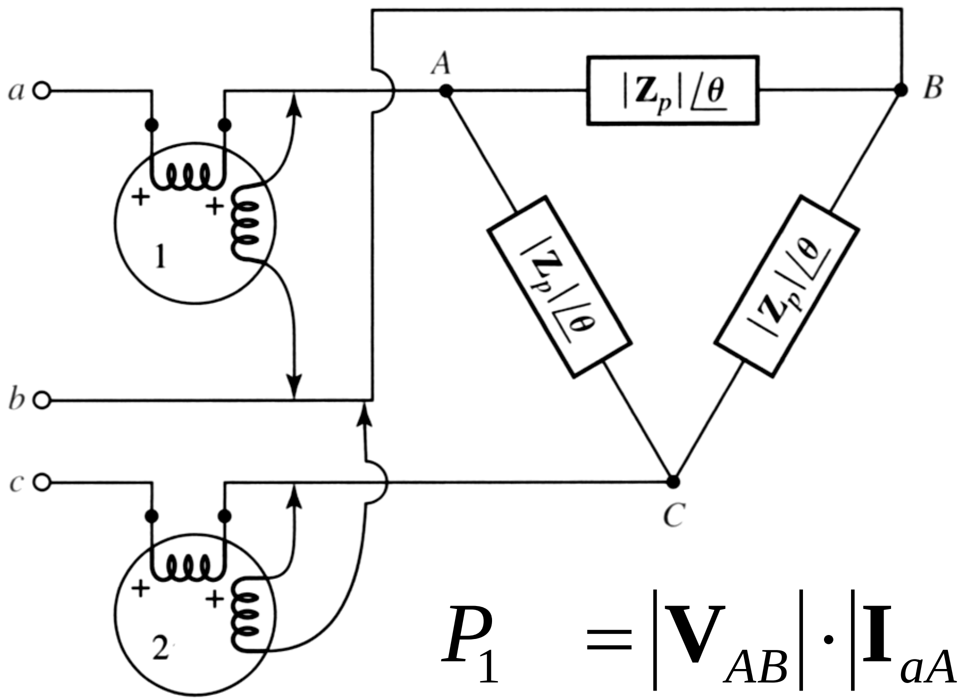


$$P_1 = |\mathbf{V}_{AB}| \cdot |\mathbf{I}_{aA}| \cos(ang\mathbf{V}_{AB} - ang\mathbf{I}_{aA})$$

$$P_2 = |\mathbf{V}_{CB}| \cdot |\mathbf{I}_{cC}| \cos(ang\mathbf{V}_{CB} - ang\mathbf{I}_{cC})$$

Power Measurement:

The Two-Wattmeter Method:



in the case of a balance load:

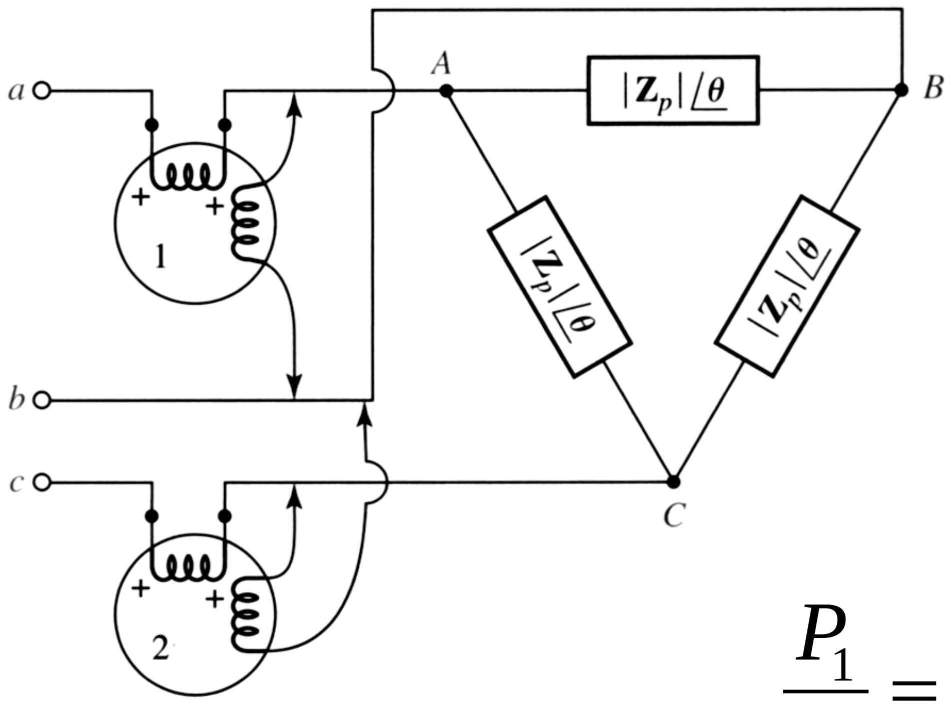
we can find PF

$$P_1 = |\mathbf{V}_{AB}| \cdot |\mathbf{I}_{aA}| \cos\left(ang\mathbf{V}_{AB} - ang\mathbf{I}_{aA}\right)$$
$$= V_L I_L \cos\left(30^\circ + \theta\right)$$

$$P_2 = |\mathbf{V}_{CB}| \cdot |\mathbf{I}_{cC}| \cos\left(ang\mathbf{V}_{CB} - ang\mathbf{I}_{cC}\right)$$
$$= V_L I_L \cos\left(30^\circ - \theta\right)$$

Power Measurement:

The Two-Wattmeter Method:



in the case of a balance load:

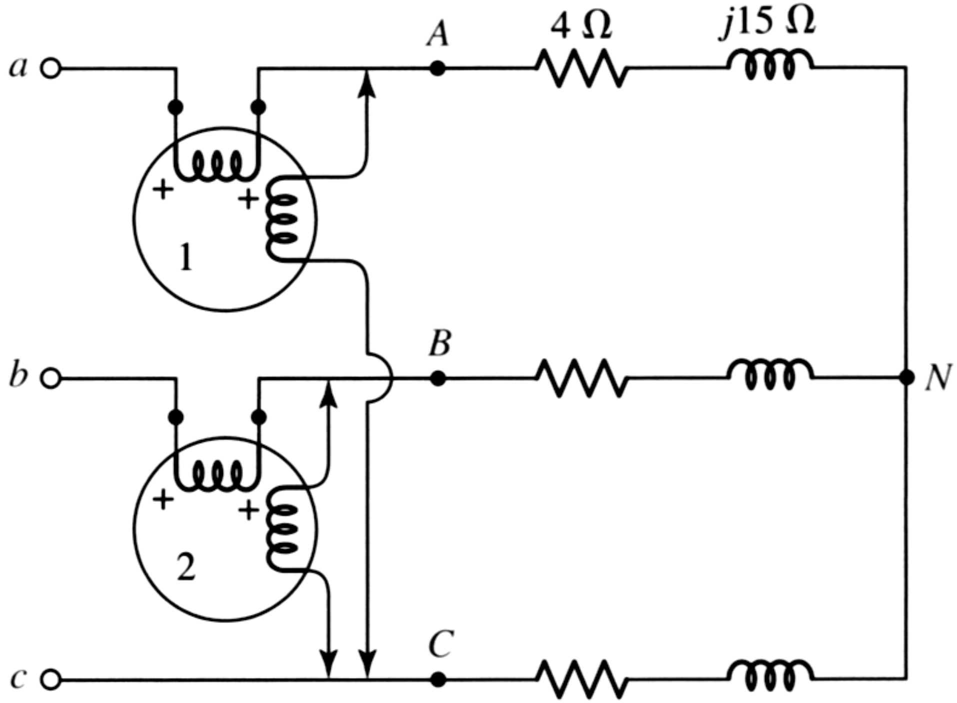
we can find PF

$$\frac{P_1}{P_2} = \frac{\cos(30^\circ + \theta)}{\cos(30^\circ - \theta)}$$

$$\tan\theta = \sqrt{3}\,\frac{P_2 - P_1}{P_2 + P_1}$$

Example 12.7:

The balanced load is fed by a balanced three-phase system having $V_{ab} = 230\angle 0°$ and positive phase sequence. Find the reading of each wattmeter and the total power drawn by the load.

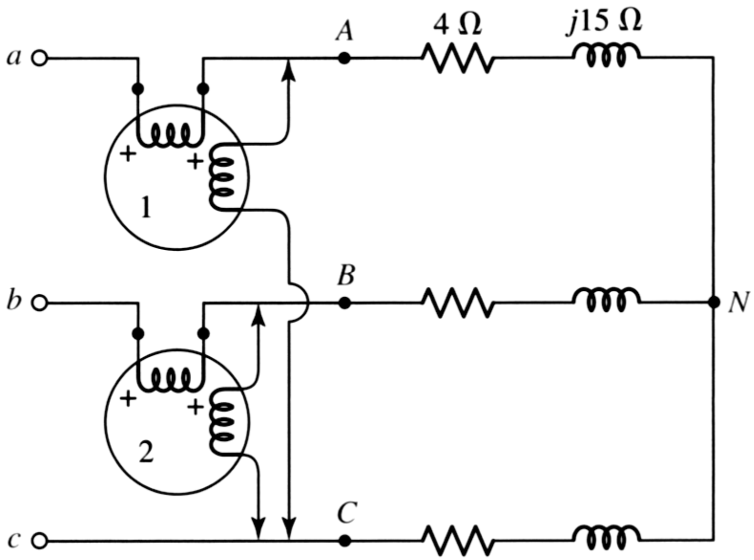


$$\mathbf{V}_{ab} = 230\angle 0°$$

$$\mathbf{V}_{bc} = 230\angle -120°$$

$$\mathbf{V}_{ca} = 230\angle 120°$$

Example:



$$\mathbf{V}_{ab} = 230\angle 0^\circ$$

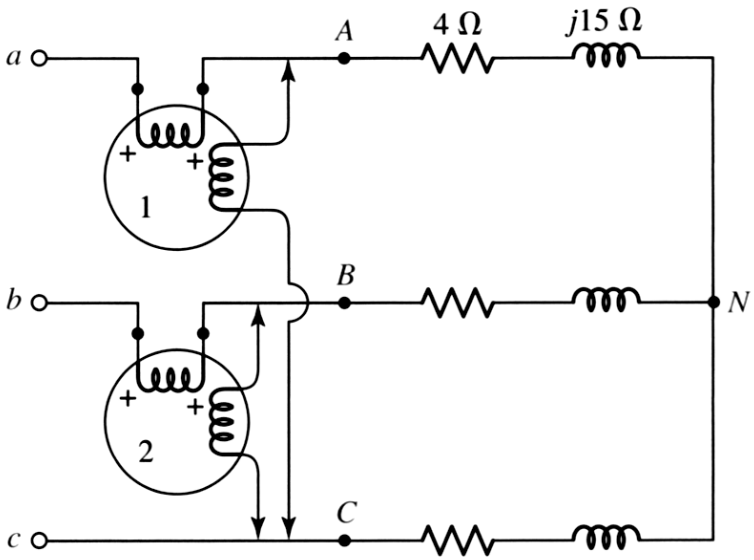
$$\mathbf{V}_{ca} = 230\angle 120^\circ$$

$$\mathbf{V}_{ac} = 230\angle -60^\circ$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{4+j15} = \frac{{}^{230}\!/\!_{\sqrt{3}}\angle -30^\circ}{4+j15} = 8.554\angle -105.1^\circ$$

$$\begin{aligned} P_1 &= |\mathbf{V}_{ac}|\cdot|\mathbf{I}_{aA}|\cos\left(ang\mathbf{V}_{ac} - ang\mathbf{I}_{aA}\right) \\ &= 230\cdot 8.554\cos\left(-60^\circ + 105.1^\circ\right) = 1389 \quad \mathrm{W} \end{aligned}$$

Example:



$$\mathbf{V}_{bc} = 230 \angle -120^\circ$$

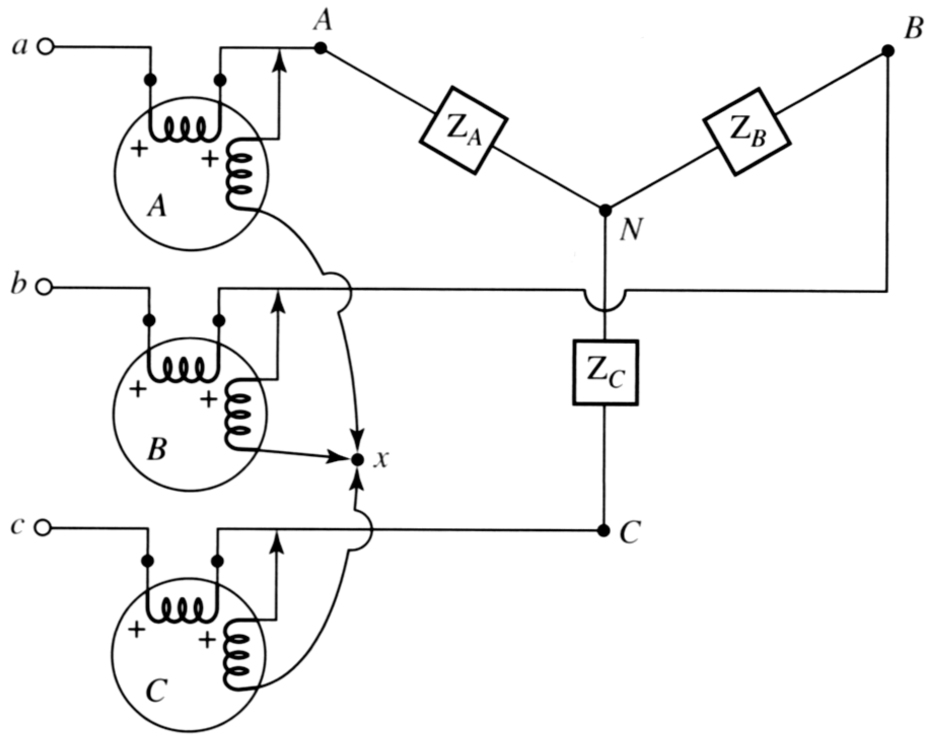
$$\mathbf{I}_{aA} = 8.554 \angle -105.1^\circ$$

$$\mathbf{I}_{bB} = 8.554 \angle +134.9^\circ$$

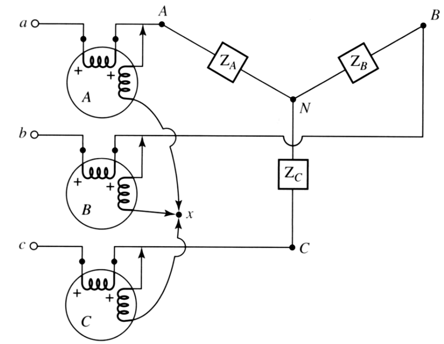
$$\begin{aligned} P_2 &= \left|\mathbf{V}_{bc}\right| \cdot \left|\mathbf{I}_{bB}\right| \cos\left(ang\mathbf{V}_{bc} - ang\mathbf{I}_{bB}\right) \\ &= 230 \cdot 8.554 \cos\left(-120^\circ - 134.9^\circ\right) = -512.5 \quad \text{W} \end{aligned}$$

Practice: 12.10

For the circuit of Figure below, let the loads be $\mathbf{Z}_A = 25\angle 60°\Omega$, $\mathbf{Z}_B = 50\angle -60°\Omega$, $\mathbf{Z}_C = 50\angle 60°\Omega$, $\mathbf{V}_{AB} = 600\angle 0°$ V rms with (+) phase sequence, and locate point x at C. Find (a) P_A; (b) P_B; (c) P_C



Practice: 12.10



$$\mathbf{V}_{AB} = 600\angle 0^\circ \text{ V} \;\text{ so }\; \mathbf{V}_{BC} = 600\angle -120^\circ \text{ V and } \mathbf{V}_{CA} = 600\angle +120^\circ \text{ V}$$

By mesh analysis, (we have an unbalanced load and no neutral wire)

$$\mathbf{V}_{AB} + (\mathbf{Z}_A + \mathbf{Z}_B)\mathbf{I}_1 \quad - \mathbf{Z}_B\mathbf{I}_2 \quad - \mathbf{Z}_A\mathbf{I}_3 = 0 \qquad [1]$$

$$-\mathbf{V}_{BC} \quad - \mathbf{Z}_B\mathbf{I}_1 + (\mathbf{Z}_B + \mathbf{Z}_C)\mathbf{I}_2 \quad - \mathbf{Z}_C\mathbf{I}_3 = 0 \qquad [2]$$

$$-\mathbf{V}_{AC} \quad - \mathbf{Z}_A\mathbf{I}_1 \quad - \mathbf{Z}_c\mathbf{I}_2 + (\mathbf{Z}_A + \mathbf{Z}_C)\mathbf{I}_3 = 0 \qquad [3]$$

Substituting values,

$$(25\angle 60^\circ + 50\angle -60^\circ)\mathbf{I}_1 \quad - 50\angle -60^\circ\,\mathbf{I}_2 \quad - 25\angle 60^\circ\,\mathbf{I}_3 = -600$$

$$-50\angle -60^\circ\,\mathbf{I}_1 + (50\angle -60^\circ + 50\angle 60^\circ)\mathbf{I}_2 \quad - 50\angle 60^\circ\,\mathbf{I}_3 = 600\angle -120^\circ$$

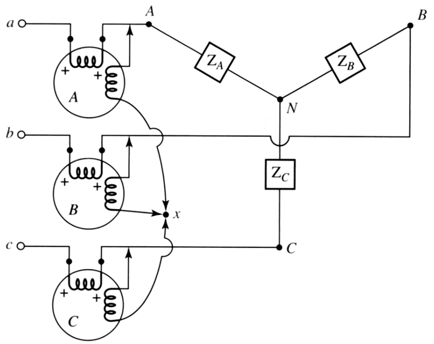
$$-25\angle 60^\circ\,\mathbf{I}_1 \quad - 50\angle 60^\circ\,\mathbf{I}_2 + (25\angle 60^\circ + 50\angle 60^\circ)\mathbf{I}_3 = 600\angle 120^\circ$$

Solving, $\mathbf{I}_1 = -j300$ A

$\mathbf{I}_2 = -6 - j310.4$ A

$\mathbf{I}_3 = -j300$ A

Practice: 12.10



$$\mathbf{I}_{aA} = \mathbf{I}_3 - \mathbf{I}_1 = 0$$

$$\mathbf{I}_{bB} = \mathbf{I}_1 - \mathbf{I}_2 = 12.00\angle 60.02^\circ$$

(a) $P_A = \mathbf{V}_{Ac}\,\mathbf{I}_{aA}\cos(ang\,\mathbf{V}_{Ac} - 0) = \underline{0}$

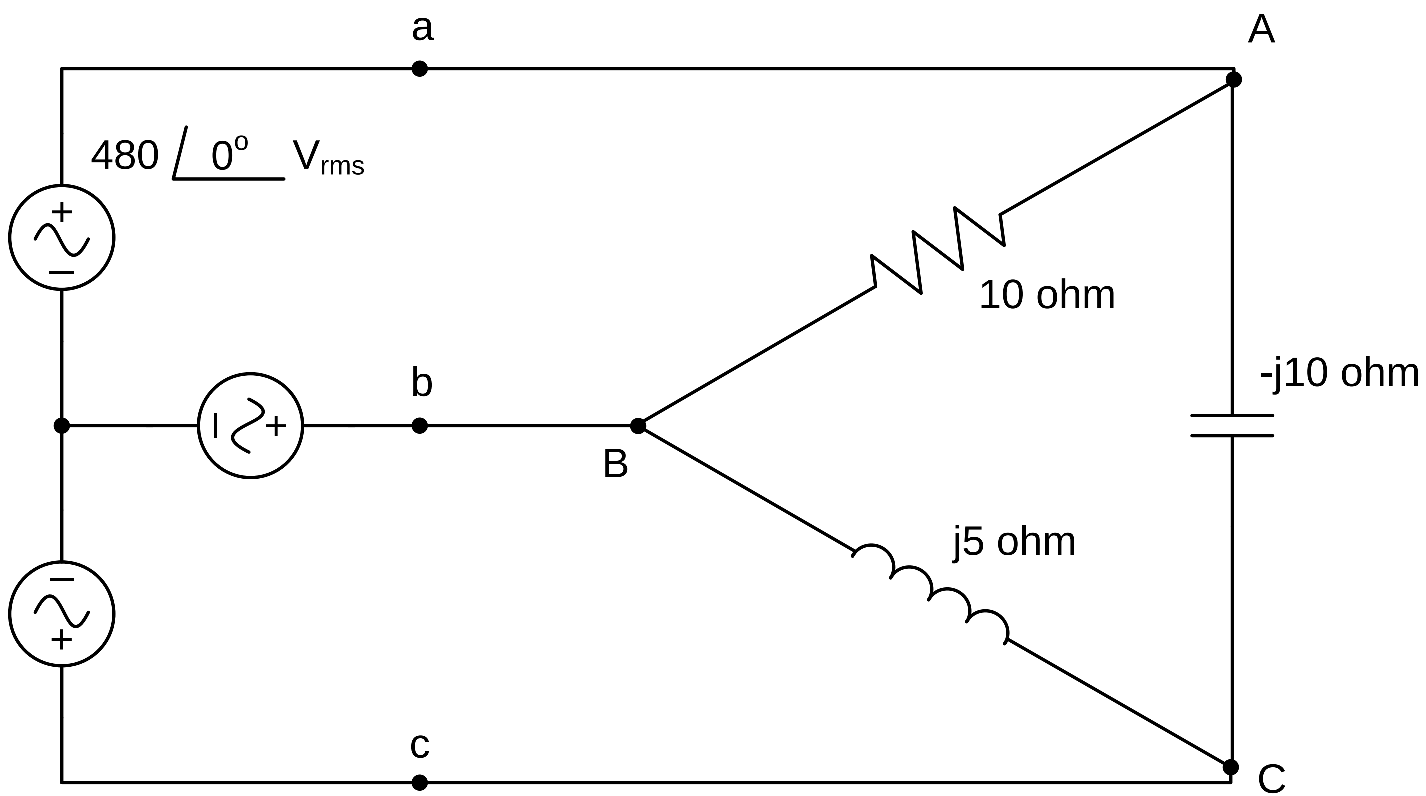
(b) $P_B = \mathbf{V}_{Bc}\,\mathbf{I}_{bB}\cos(-120^\circ - 60^\circ) = (600)(12)\cos(-180^\circ)$

$= \underline{-7200\text{ W, so the leads need to be reversed (to read }+7200\text{ W).}}$

(c) $P_C = \underline{0}$

Example: Final 2/47

เมื่อ แหล่งจ่ายในวงจร เป็นแบบ balanced และ positive phase sequence จงหา $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, $\mathbf{I}_{cC}$ และ the total complex power supplied by the source



Example: Final 2/47

$$\mathbf{V}_{an} = 480\angle 0^\circ \quad \text{V.}$$

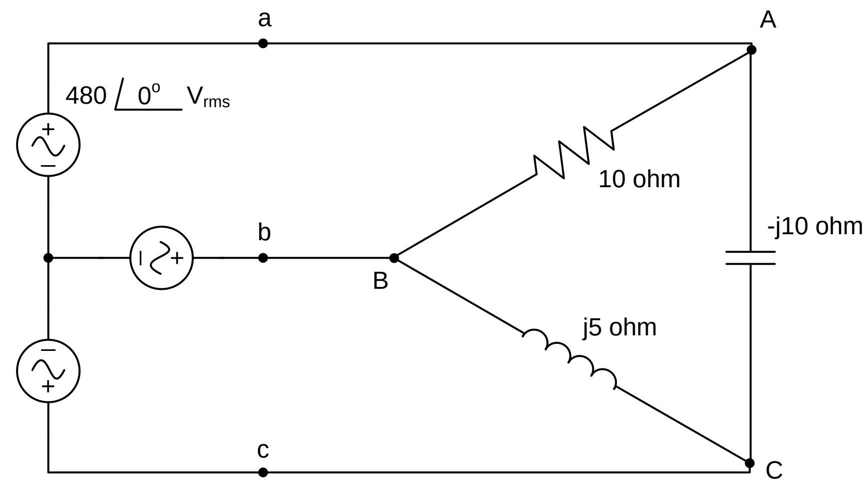
$$\mathbf{V}_{bn} = 480\angle -120^\circ \quad \text{V}$$

$$\mathbf{V}_{cn} = 480\angle -240^\circ \quad \text{V.}$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p \angle 30^\circ$$

$$\mathbf{V}_{bc} = \sqrt{3}\cdot 480\angle -90^\circ$$

$$\mathbf{V}_{ca} = 831.38\angle -210^\circ$$



$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{10} = 83.1\angle 30^\circ$$

$$\mathbf{I}_{BC} = \frac{831.38\angle -90^\circ}{j5} = -166.3$$

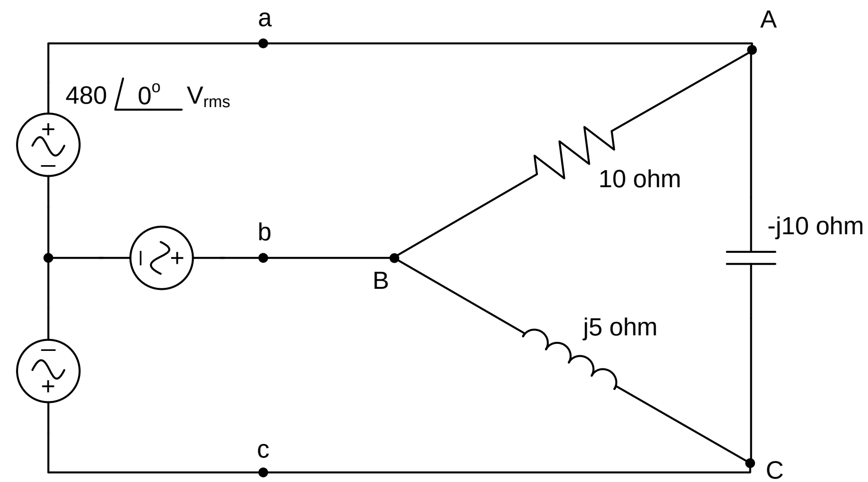
$$\mathbf{I}_{CA} = \frac{831.38\angle 150^\circ}{-j10} = 83.1\angle -120^\circ$$

Example: Final 2/47

$$\mathbf{I}_{AB} = 83.1 \angle 30^\circ$$

$$\mathbf{I}_{BC} = -166.3$$

$$\mathbf{I}_{CA} = 83.1 \angle -120^\circ$$



$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

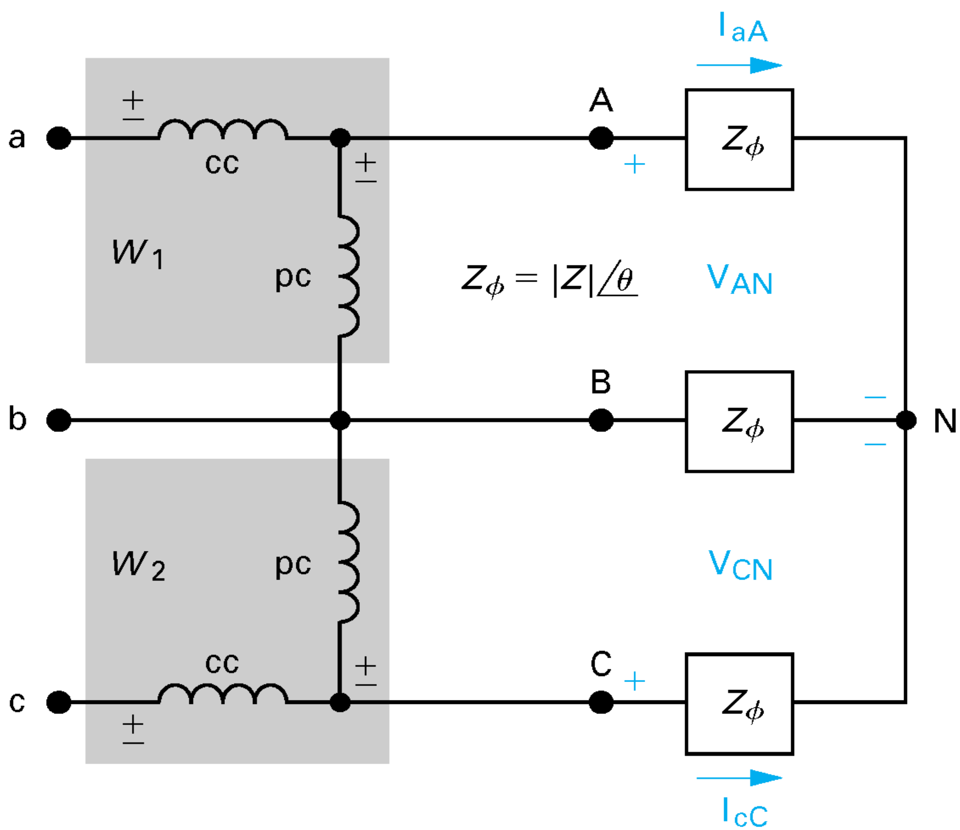
$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$\mathbf{S}_{total} = \mathbf{V}_{AB}\mathbf{I}^*_{AB} + \mathbf{V}_{BC}\mathbf{I}^*_{BC} + \mathbf{V}_{CA}\mathbf{I}^*_{CA}$$

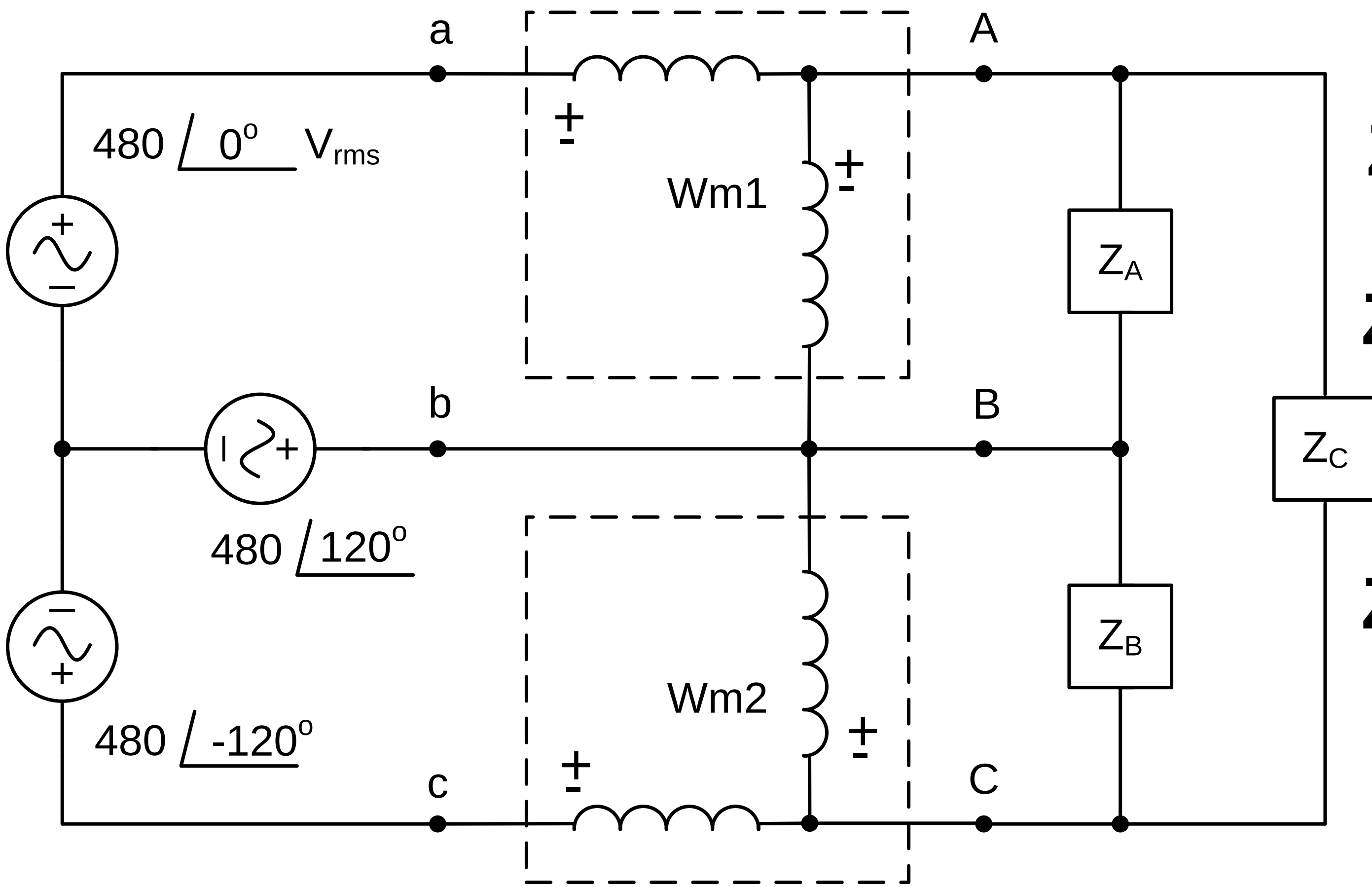
Example: Final 2/46

เมื่อ Wattmeters ในวงจร อ่านค่าได้ $W_1 = 37{,}297.54$ W. และ $W_2 = 139{,}196.31$ W. เมื่อค่า the magnitude ของ the line voltage มีค่า 4160 V. จงหา Z_ϕ



$$\tan\theta = \sqrt{3}\,\frac{P_2 - P_1}{P_2 + P_1}$$

Ex:



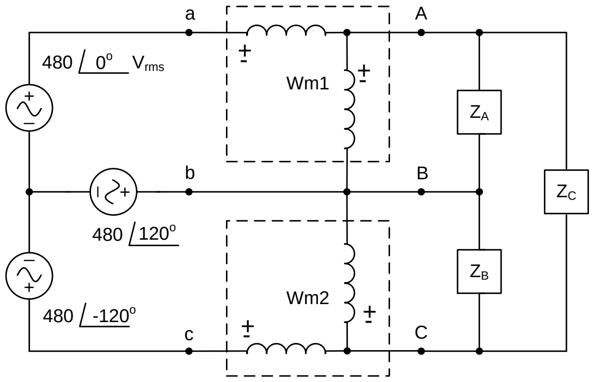
$Z_A = 60\angle{-30^\circ}$

$Z_B = 24\angle{30^\circ}$

$Z_C = 80\angle{0^\circ}$

ให้หา V_{AB}, V_{BC}, I_{AB}, I_{BC}, I_{aA}, I_{cC} และค่าที่อ่านได้ที่ wattmeter ทั้งสอง

Ex:



Negative phase sequence:

$$\mathbf{V}_{AB} = 480\sqrt{3}\underline{/-30^\circ}\ \mathrm{V}$$

$$\mathbf{V}_{BC} = 480\sqrt{3}\underline{/90^\circ}\ \mathrm{V}$$

$$\mathbf{V}_{CA} = 480\sqrt{3}\underline{/-150^\circ}\ \mathrm{V}$$

$$\mathbf{I}_{AB} = \frac{480\sqrt{3}\underline{/-30^\circ}}{60\underline{/-30^\circ}} = 8\sqrt{3}\underline{/0^\circ}\ \mathrm{A}$$

$$\mathbf{I}_{BC} = \frac{480\sqrt{3}\underline{/90^\circ}}{24\underline{/30^\circ}} = 20\sqrt{3}\underline{/60^\circ}\ \mathrm{A}$$

$$\mathbf{I}_{CA} = \frac{480\sqrt{3}\underline{/-150^\circ}}{80\underline{/0^\circ}} = 6\sqrt{3}\underline{/-150^\circ}\ \mathrm{A}$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC}$$

$$= 8\sqrt{3}\underline{/0^\circ} + 6\sqrt{3}\underline{/30^\circ} = 23.44\underline{/12.81^\circ}\ \mathrm{A}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CB} + \mathbf{I}_{CA}$$

$$= 20\sqrt{3}\underline{/-120^\circ} + 6\sqrt{3}\underline{/-150^\circ} = 43.95\underline{/-126.79^\circ}\ \mathrm{A}$$

$$W_{m1} = 480\sqrt{3}(23.44)\cos(-30 - 12.81^\circ) = 14{,}296.61\ \mathrm{W}$$

$$W_{m2} = 480\sqrt{3}(43.95)\cos(-90 + 126.79^\circ) = 29{,}261.53\ \mathrm{W}$$

Hw:

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.