Seat Number

King Mongkut's University of Technology Thonburi
Final Examination
Semester 1 -- Academic Year 2013

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3rd Yr (Inter. Program)
Exam Date: Monday December 2, 2013 **Time:** 9.00a-12.00pm

Instructions:-

1. This exam consists of 5 problems with a total of 17 pages, **not** including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rules is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds ... No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instructions might eventually result in a failure of the class or may receive the highest punishment with university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid needless calculation!**

Question No.	1	2	3	4	5	TOTAL
Full Score	20	20	20	20	20	100
Graded Score						

Name ______________________________ **Student ID** ______________

This examination is designed by
Watcharapan Suwansantisuk; Tel: 9069.

This examination has been approved by the committees of the ENE department.

A. Wudhidee

(Assoc. Prof. Wudhichai Assawinchaichote, Ph.D.)
Head of Electronic and Telecommunication Engineering Department

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Problem 1: Probability Distributions [20 points]

Consider the following probability distributions:

Poisson distribution with parameter μ:

$$\mathbb{P}\{X = x\} = \begin{cases} \frac{\mu^x e^{-\mu}}{x!}, & x = 0, 1, 2 \ldots \\ 0, & \text{otherwise} \end{cases}$$

Exponential distribution with parameter λ:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Uniform distribution on the set $\{1, 2, 3, \ldots, n\}$:

$$\mathbb{P}\{X = x\} = \begin{cases} \frac{1}{n}, & x = 1, 2, 3, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

Normal distribution with mean μ and variance σ^2:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}, \qquad -\infty < x < \infty$$

Answer the following questions. **Briefly justify your answer**.

(a) [4 points] Among the given four probability distributions, which distributions are for discrete random variables? Circle all correct answers.

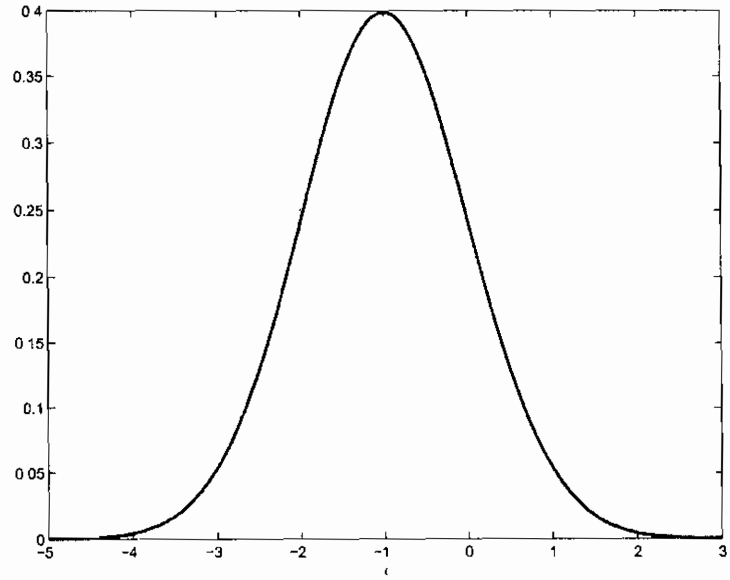
(A) Poisson distribution

(B) Exponential distribution

(C) Uniform distribution

(D) Normal distribution

(b) [4 points] What is the graph below? Circle one answer from each column.

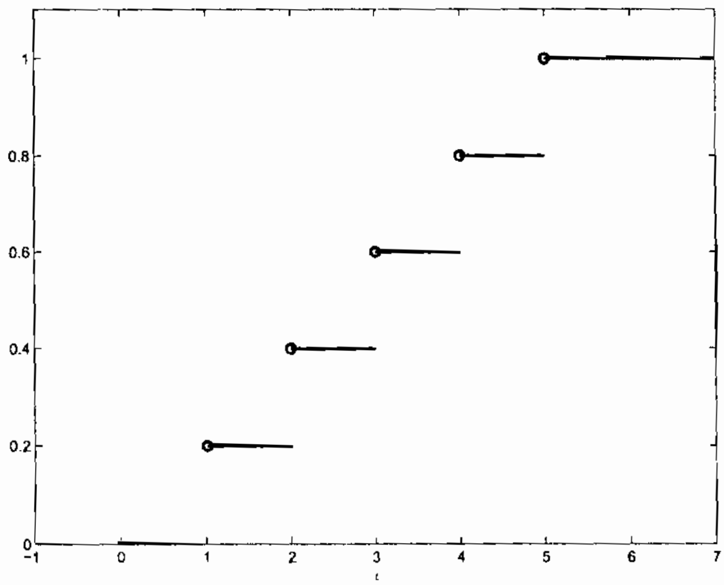


The graph is a

$$\left\{\begin{array}{c}\text{cumulative distribution function (cdf)}\\ \text{probability density function (pdf)}\\ \text{probability mass function (pmf)}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{a Poisson}\\ \text{an exponential}\\ \text{a uniform}\\ \text{a normal}\end{array}\right\}$$

random variable.

(c) [4 points] What is the graph below? Circle one answer from each column.

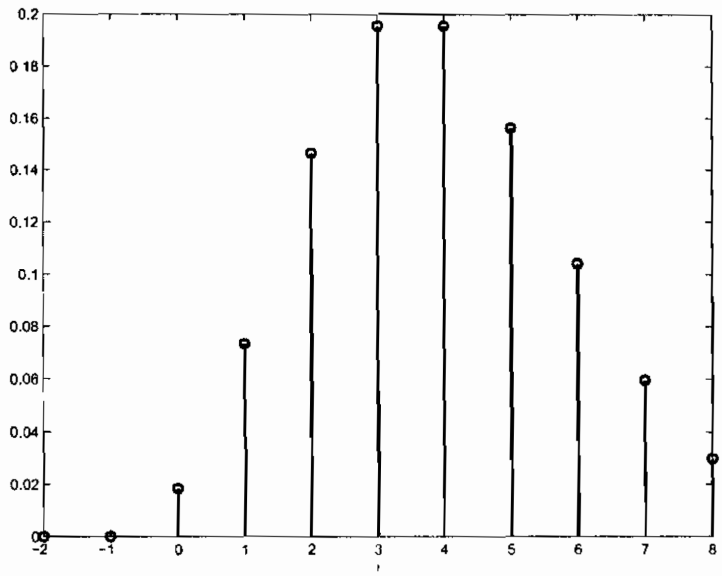


The graph is a

$$\left\{\begin{array}{c}\text{cumulative distribution function (cdf)}\\ \text{probability density function (pdf)}\\ \text{probability mass function (pmf)}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{a Poisson}\\ \text{an exponential}\\ \text{a uniform}\\ \text{a normal}\end{array}\right\}$$

random variable.

(d) [4 points] What is the graph below? Circle one answer from each column.



The graph is a

$$\left\{\begin{array}{c}\text{cumulative distribution function (cdf)}\\ \text{probability density function (pdf)}\\ \text{probability mass function (pmf)}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{a Poisson}\\ \text{an exponential}\\ \text{a uniform}\\ \text{a normal}\end{array}\right\}$$

random variable.

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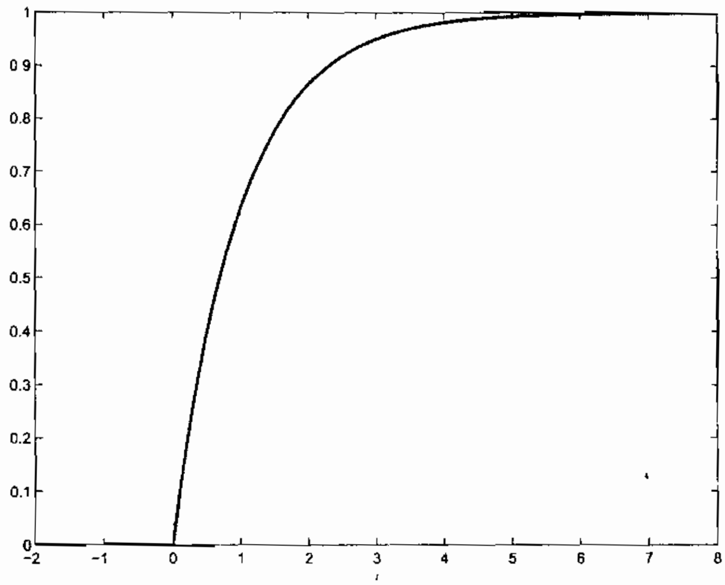
(e) [4 points] What is the graph below? Circle one answer from each column.



The graph is a

$$\left\{\begin{array}{c}\text{cumulative distribution function (cdf)}\\ \text{probability density function (pdf)}\\ \text{probability mass function (pmf)}\end{array}\right\} \text{ of } \left\{\begin{array}{c}\text{a Poisson}\\ \text{an exponential}\\ \text{a uniform}\\ \text{a normal}\end{array}\right\}$$

random variable.

Problem 2: Discrete Distributions [20 points]

TOEFL is a test of proficiency in English. To graduate, a student must score 475 points or more on a TOEFL examination. The student keeps taking the TOEFL examinations and stops only after he passes, that is, after he scores 475 points or more.

Let random variable X denote the number of times that the student takes the TOEFL, so $X = 1, 2, 3, \ldots$. The probability is $\frac{1}{3}$ that the student passes each examination. Assume that the test results ("pass" or "fail") from different examinations are **independent**.

(a) [4 points] Obtain the probability that the student takes two TOEFL examinations before he passes, that is, $\mathbb{P}\{X = 2\}$.
[*Hint*: $\mathbb{P}\{X = 2\} = \mathbb{P}\{\text{Fail the 1st exam } \textbf{and} \text{ pass the 2nd exam}\}$.]

(b) [4 points] Show that the pmf of X equals

$$p_X(x) = \begin{cases} \left(\frac{2}{3}\right)^{x-1}\frac{1}{3}, & x = 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

(c) [4 points] Fill in the blanks below for the formulas of $\mathbb{E}\{X\}$ and $\mathbb{E}\{X^2\}$.

$$\mathbb{E}\{X\} = \sum_{x=\_\_\_\_}^{\_\_\_\_} \_\_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_$$

$$\mathbb{E}\{X^2\} = \sum_{x=\_\_\_\_}^{\_\_\_\_} \_\_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_\_$$

Taking the TOEFL examinations costs money. The student pays the fees of 6,000 Bahts/examination. Suppose the student has a saving of 30,000 Bahts in a bank. After taking the TOEFL, the amount of money left is $Y = 30000 - 6000X$.

(d) [4 points] Obtain $\mathbb{E}\{Y\}$, the expected amount of money left in the bank.
[*Hint*: $\mathbb{E}\{X\} = 3$.]

(e) [4 points] Obtain the standard deviation of Y.
[*Hint*: $\mathbb{E}\{X^2\} = 15$.]

Problem 3: Continuous Distributions [20 points]

Let X denote the amount of time that a book on two-hour reserve is actually checked out. Suppose X is a continuous random variable with the cdf

$$F_X(x) = \begin{cases} 0, & x < 0 \\ Kx, & 0 \le x < 2 \\ 1, & 2 \le x. \end{cases}$$

(a) [5 points] What is K?

(b) [5 points] Obtain the pdf $f_X(x)$.

(c) [5 points] Obtain $\mathbb{P}\{X \leq 1\}$, $\mathbb{P}\{.5 \leq X \leq 1\}$, and $\mathbb{P}\{X > 1.5\}$.

(d) [5 points] Compute $\mathbb{E}\{X\}$ and $\mathbb{V}\{X\}$.

Problem 4: Joint Discrete Distributions [20 points]

A convenience store has two checkout lines: regular and express. Let X_1 denote the number of customers in the *regular* line at a particular time of day, and let X_2 denote the number of customers in the *express* line at the same time. Suppose the joint pmf of X_1 and X_2 is as given in the accompanying table.

		x_2	
		0	1
	0	.20	.07
x_1	1	.13	.35
	2	.15	.10

(a) [5 points] What is $\mathbb{P}\{X_1 + X_2 = 2\}$, that is, the probability that two customers are in the lines?
[*Hint*: Write the event as

$$\{X_1 + X_2 = 2\}$$
$$= \{\, (X_1 = 0 \text{ and } X_2 = 2) \text{ or } (X_1 = 1 \text{ and } X_2 = 1) \text{ or } (X_1 = 2 \text{ and } X_2 = 0)\,\}$$

]

(b) [5 points] Determine the cdf of X_1.
[*Hint*: Obtain the cdf from the marginal pmf of X_1.]

(c) [5 points] Are X_1 and X_2 independent random variables? Explain.

(d) [5 points] Given that $X_1 = 0$, determine the conditional pmf of X_2, that is, $p_{X_2|X_1}(0|0)$ and $p_{X_2|X_1}(1|0)$.

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Problem 5: Joint Continuous Distributions [20 points]

Andy and Bob agree to meet for dinner between 5:00pm and 6:00pm. Let X = Andy's arrival time and Y = Bob's arrival time. Suppose that X and Y are independent and have a uniform distribution on the interval $[5, 6]$. That is, the pdf's of X and Y are

$$f_X(x) = \begin{cases} 1, & 5 \leq x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 1, & 5 \leq y \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) [5 points] What is the joint pdf of X and Y?

(b) [5 points] What is the conditional pdf $f_{Y|X}(y|x)$?

(c) [5 points] What is the probability that both Andy and Bob arrive between 5:15pm and 5:45pm?

(d) [5 points] What is $\mathbb{E}\{X - Y\}$, the expected duration that Andy waits for Bob?
[*Hint*: Use linearity of the expectation.]