Seat Number

King Mongkut's University of Technology Thonburi
Midterm Examination
Semester 1 -- Academic Year 2016

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3rd Yr (Inter. Program)
Exam Date: Tuesday September 27, 2016 **Time:** 9.00am-12.00pm

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Instructions:-

1. This exam consists of 4 problems with a total of 11 pages, including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rule is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds … No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment within university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Question No.	1	2	3	4	TOTAL
Full Score	25	25	25	25	100
Graded Score					

Name ______________________________ **Student ID** ________________

This examination is designed by
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This examination has been approved by the committees of the ENE department.

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Problem 1: Measurements from Two Sensors [25 points]

An engineer installs two sensors to measure periodically the ambient temperatures of a rainforest. The first sensor has measured and collected $n = 40$ temperatures

$$x_1, x_2, x_3, \ldots, x_{40},$$

whose sample mean is $\overline{x} = 24°\text{C}$ and sample standard deviation is $s_x = 3°\text{C}$. The second sensor has measured and collected $m = 60$ temperatures

$$y_1, y_2, y_3, \ldots, y_{60},$$

whose sample mean is $\overline{y} = 20°\text{C}$ and sample standard deviation is $s_y = 5°\text{C}$.

To analyze the overall ambient temperature, the engineer combines the measurements from both sensors to form a large sample

$$x_1, x_2, x_3, \ldots, x_{40}, y_1, y_2, y_3, \ldots, y_{60},$$

of size $n + m = 100$.

(a) [12 points] Find the sample mean of the combined temperatures

$$x_1, x_2, x_3, \ldots, x_{40}, y_1, y_2, y_3, \ldots, y_{60}.$$

(b) [13 points] Find the sample standard deviation of the combined temperatures

$$x_1, x_2, x_3, \ldots, x_{40}, y_1, y_2, y_3, \ldots, y_{60}.$$

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Problem 2: A Repair Shop [25 points]

A service center receives 15 computers to be repaired. Five of these computers are desktop (D) computers, five are laptop (L) computers, and the other five are tablet (T) computers. Suppose these computers are randomly allocated the numbers $1, 2, 3, \ldots, 15$ that establish the order in which they will be repaired: a computer that is marked "1" is repaired first, while the computer that is marked "15" is repaired last.

(a) [12 points] Find the probability that all desktop computers are among the first ten computers to be repaired.

Hint: An example of an outcome is the sequence below:

D	L	D	T	T	D	D	L	D	T	L	T	L	L	T
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(b) [13 points] Find the probability that after repairing ten of these computers, only computers of type desktop <u>and</u> type laptop remain to be repaired.

Hint: An example of an outcome is the sequence below:

D	D	D	L	L	T	T	T	T	T	D	D	L	L	L
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

First, consider the case of one desktop computer and four laptop computers at the end of the repairing sequence. Then, cover all remaining cases.

Problem 3: Airplane Flights [25 points]

An engineer who lives in Bangkok makes frequent consulting trips to Chiang mai; 40% of the time she travels on Air Asia, 50% of the time on Nok Air, and 10% of the time on Thai Airways. For Air Asia, flights are late into Chiang mai 20% of the time and late into Bangkok 30% of the time. For Nok Air, these percentages are 15% and 25%, respectively, whereas for Thai Airways the percentages are 5% and 35%, respectively.

Assume that a late arrival in Bangkok is unaffected by what happens on the flight to Chiang mai.

(a) [12 points] If we learn that on a particular trip she arrived late at Chiang mai, what is the posterior probability of having flown on Air Asia?

(b) [13 points] If we learn that on a particular trip she arrived late at Chiang mai or Bangkok, what is the posterior probability of having flown on Thai Airways?

Problem 4: Diagnosis of a Disease [25 points]

One percent of Thai people have a particular disease. To check whether a person has the disease, a doctor tests the person's blood. The result of a test is either *positive* or *negative* for the disease.

Of all Thai people who have the disease, 90% test positive for the disease. Of all Thai people who do not have the disease, 5% test positive for the disease.

A Thai person is randomly selected, and two blood samples are drawn from him or her. Suppose that the test is applied *independently* to the two blood samples.

(a) [12 points] Find the probability that both tests yield the positive results.

(b) [13 points] Find the probability that both tests yield the negative results.

Formula Sheet

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\tilde{x} = \begin{cases} x'_m & \text{if } n \text{ is odd, where } m = \frac{n+1}{2} \\ \frac{1}{2}(x'_m + x'_{m+1}) & \text{if } n \text{ is even, where } m = \frac{n}{2} \end{cases}$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \left[\left(\sum_{i=1}^{n} x_i^2 \right) - n(\bar{x})^2 \right]$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 = \frac{1}{N} \left(\sum_{i=1}^{N} x_i^2 \right) - \mu^2$$

For $y_i = ax_i + b$, where $i = 1, 2, \ldots, n$,

$$\bar{y} = a\bar{x} + b, \quad s_y^2 = a^2 s_x^2, \quad s_y = |a| s_x$$

Axioms of probability:

1. The probability of any event A is non-negative: $\mathbb{P}\{A\} \geq 0$
2. The probability of the sample space equals one: $\mathbb{P}\{\Omega\} = 1$
3. If $A_1, A_2, A_3, \ldots$ are pairwise disjoint events, then $\mathbb{P}\{\cup_{i=1}^{\infty} A_i\} = \sum_{i=1}^{\infty} \mathbb{P}\{A_i\}$

- The probability of the null event equals zero: $\mathbb{P}\{\emptyset\} = 0$
- If events $A_1, A_2, \ldots, A_n$ are pairwise disjoint, then $\mathbb{P}\{\cup_{i=1}^{n} A_i\} = \sum_{i=1}^{n} \mathbb{P}\{A_i\}$
- For any event A, $\mathbb{P}\{A'\} = 1 - \mathbb{P}\{A\}$
- The probability of any event A is at most one: $\mathbb{P}\{A\} \leq 1$
- For any two events A and B, $\mathbb{P}\{A \cup B\} = \mathbb{P}\{A\} + \mathbb{P}\{B\} - \mathbb{P}\{A \cap B\}$
- For any three events A, B, and C,

$$\begin{aligned} \mathbb{P}\{A \cup B \cup C\} = {} & \mathbb{P}\{A\} + \mathbb{P}\{B\} + \mathbb{P}\{C\} \\ & - \mathbb{P}\{A \cap B\} - \mathbb{P}\{A \cap C\} - \mathbb{P}\{B \cap C\} + \mathbb{P}\{A \cap B \cap C\} \end{aligned}$$

- When outcomes are equally likely, $\mathbb{P}\{A\} = \frac{|A|}{|\Omega|}$
- *The product rule*: the number of k-tuples is $n_1 n_2 n_3 \cdots n_k$

$$
\begin{aligned}
{}^nP_k &= n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!} \\
0! &= 1, \quad n! = n(n-1)(n-2)\cdots 1 \\
\binom{n}{k} &= \frac{{}^nP_k}{k!} = \frac{n!}{k!(n-k)!} \\
\mathbb{P}\{A \mid B\} &= \frac{\mathbb{P}\{A \cap B\}}{\mathbb{P}\{B\}} \\
\mathbb{P}\{A \cap B\} &= \mathbb{P}\{A\}\mathbb{P}\{B \mid A\} = \mathbb{P}\{B\}\mathbb{P}\{A \mid B\} \\
\mathbb{P}\{A_1 \cap A_2 \cap A_3\} &= \mathbb{P}\{A_3 \mid A_2 \cap A_1\} \cdot \mathbb{P}\{A_2 \mid A_1\} \cdot \mathbb{P}\{A_1\}
\end{aligned}
$$

The law of total probability: Suppose events $A_1, A_2, \ldots, A_n$ partition the sample space Ω. Then the probability of any other event B equals

$$\mathbb{P}\{B\} = \sum_{i=1}^{n} \mathbb{P}\{B \mid A_i\}\mathbb{P}\{A_i\}$$

Bayes' theorem: : Suppose events $A_1, A_2, \ldots, A_n$ partition the sample space Ω and have the probabilities $\mathbb{P}\{A_1\}, \mathbb{P}\{A_2\}, \ldots, \mathbb{P}\{A_n\}$. Let B denote any event that has a chance to occur, i.e., $\mathbb{P}\{B\} > 0$. Then the posterior probability of A_j given that B has occurred is

$$\mathbb{P}\{A_j \mid B\} = \frac{\mathbb{P}\{A_j \cap B\}}{\mathbb{P}\{B\}} = \frac{\mathbb{P}\{B \mid A_j\}\mathbb{P}\{A_j\}}{\sum_{i=1}^{n} \mathbb{P}\{B \mid A_i\}\mathbb{P}\{A_i\}}$$

for each index $j = 1, 2, \ldots, n$.

- Events A and B are independent $\iff \mathbb{P}\{A \mid B\} = \mathbb{P}\{A\}$ $\iff \mathbb{P}\{B \mid A\} = \mathbb{P}\{B\} \iff \mathbb{P}\{A \cap B\} = \mathbb{P}\{A\}\mathbb{P}\{B\}$
- Events $A_1, A_2, \ldots, A_n$ are mutually independent iff for every size $k \in \{2, 3, \ldots, n\}$ and for every subset of indices $i_1, i_2, \ldots, i_k$, the probability of an intersection equals the product of probabilities:

$$\mathbb{P}\{A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}\} = \mathbb{P}\{A_{i_1}\} \cdot \mathbb{P}\{A_{i_2}\} \cdots \mathbb{P}\{A_{i_k}\}$$