Sources of Error

EIE 240 Electrical and Electronic Measurement

Class 3, December 22, 2011

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Errors in Electrical Measurements

- Systematic error ⇒ every times you measure e.g. loading or insertion of the measurement instrument
- Meter error ⇒ scaling (inaccurate marking), pointer bending, friction
- Random error ⇒ temperature effect, noises (unwanted signals)
- Reading error ⇒ parallax

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Background

- Electric charge, $e^{-} = 1.6\times10^{-19}$ Coulombs
- Current, $I = dq/dt$ C/sec = Amps
- Voltage, the difference in electrical potential between two points (Joules/Coulomb)
- Power, $P = IV$ Watts
- Energy, $E = Pt$ Units (kW·hr)
- Ohm's Law, $V = IR$

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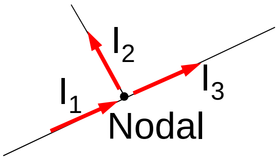
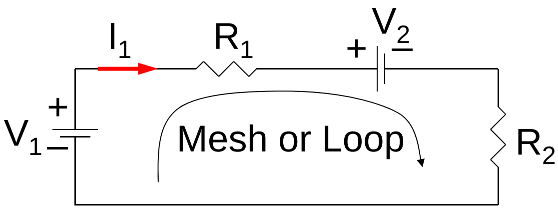
Background (Cont'd)

- Kirchhoff's Current Law (KCL)
  $I_1 - I_2 - I_3 = 0$



- Kirchhoff's Voltage Law (KVL)
  $-V_1 + I_1R_1 + V_2 + I_2R_2 = 0$



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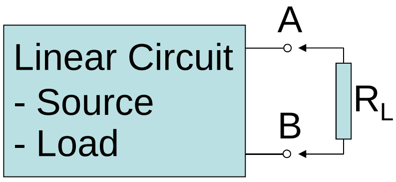
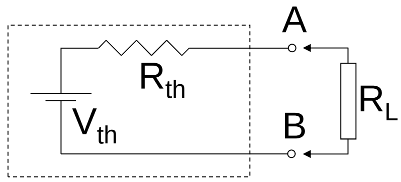
Background (Cont'd)

- Thévenin's Theorem

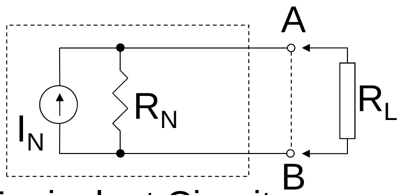




Equivalent Circuit

Open circuit to find V_{th}

- Norton's Theorem
  Short circuit to find I_N



Equivalent Circuit

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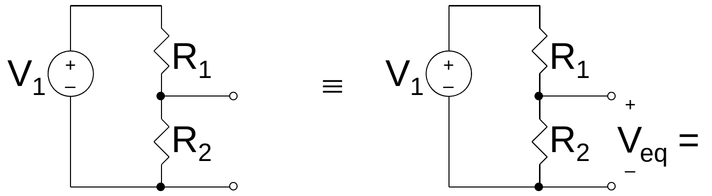
Background (Cont'd)

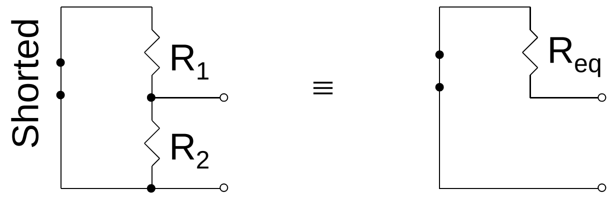
Example

No Load ⇒ Equivalent Source



$$V_{eq} = \frac{R_2}{R_1+R_2} V_1$$

No Source ⇒ Equivalent Load



$R_{eq} = R_1 \,||\, R_2$

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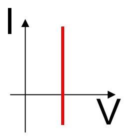
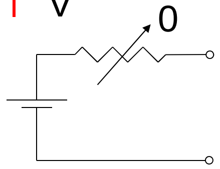
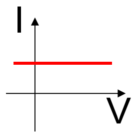
Background (Cont'd)

- Ideal Voltage Source Vs. Ideal Current Source

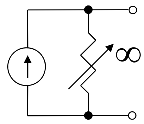


$I_\infty = V_{fixed} / R_0$

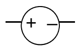
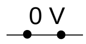


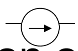
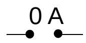


$V_\infty = I_{fixed} R_\infty$



- Superposition Theorem (for linear resistive network containing several sources)

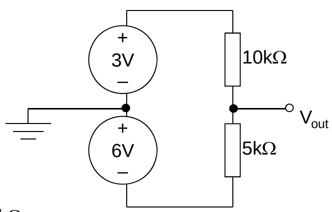
  Voltage source  ⇒ Short Circuit 

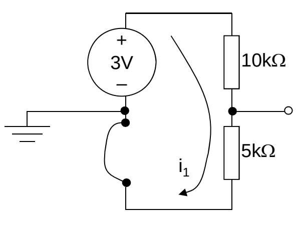
  Current source  ⇒ Open Circuit 

  Vector summation of the individual voltage or current caused by each separate source

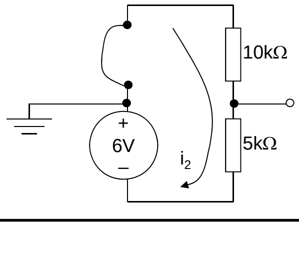
Background (Cont'd)

Example





$V_{out1} = 3V/(10k\Omega+5k\Omega) \cdot 5k\Omega = 1V$

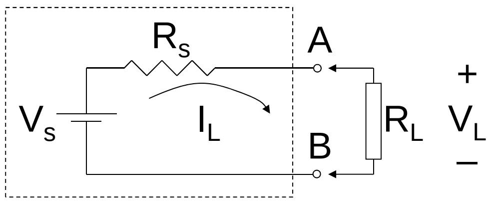


$V_{out2} = -6V/(10k\Omega+5k\Omega) \cdot 10k\Omega = -4V$

$$\left.\begin{array}{l} V_{out1} \\ V_{out2} \end{array}\right\} \quad \begin{aligned} V_{out} &= V_{out1} + V_{out2} \\ &= 1V - 4V \\ &= -3V \end{aligned}$$

Background (Cont'd)

- The Maximum Power Transfer Theorem



If V_s and R_s are fixed, when $I_L \uparrow$ then $V_L \downarrow$
Maximum power is at $R_L = R_s$
Therefore $V_L = V_s / 2$ and $I_L = V_s / 2R_L$

$$P_L = (I_L)^2 R_L = (V_s / 2R_L)^2 R_L = V_s^2 / 4R_L$$

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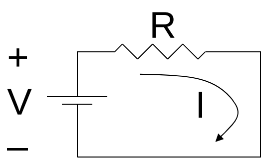
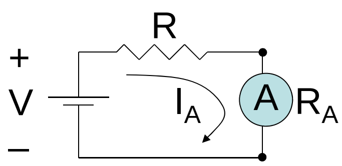
Ammeter Loading

For current measurement, open circuit to put an ammeter.

DC Circuit



$I = V/R$



$I_A = V/(R+R_A)$

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Ampmeter Loading (Cont'd)

Loading error, $I_A - I = V/(R+R_A) - V/R$

$$= \frac{VR - V(R+R_A)}{R(R+R_A)}$$

$$= \frac{-VR_A}{R(R+R_A)}$$

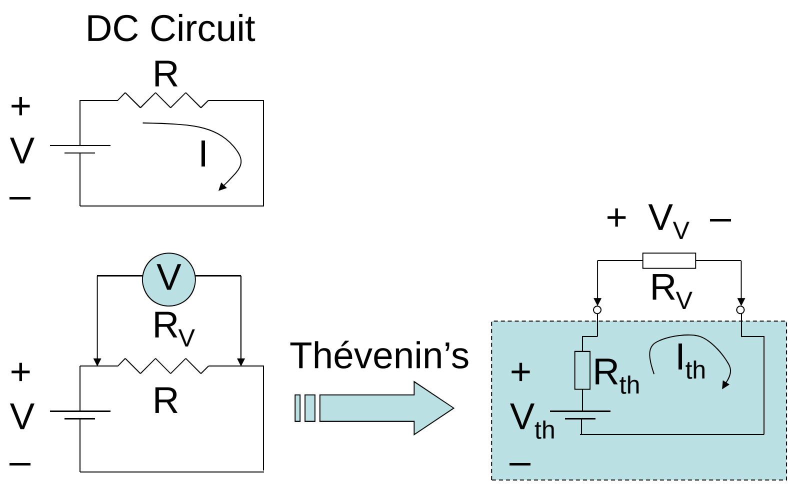
% Loading Error = $(I_A - I)/I \times 100\%$

$$= \frac{-R_A}{R+R_A} \times 100\%$$

It means if $R_A \uparrow$ then error $\uparrow$

Voltmeter Loading

For voltage measurement, voltmeter is placed in parallel with the circuit element.



Voltmeter Loading (Cont'd)

$I_{th} = V_{th} / (R_{th} + R_V)$

$V_V = I_{th}R_V = V_{th}R_V / (R_{th} + R_V)$

$\text{Error} = V_V - V_{th}$, $V_{th} = V$ (no load)

$$= \left(\frac{R_V}{R_{th} + R_V} - 1 \right) V_{th}$$

$\%\ \text{Error} = (V_V - V_{th})/V_{th} \times 100\%$

$$= \frac{-R_{th}}{R_{th}+R_V} \times 100\%$$

It means if $R_V \uparrow$ then error $\downarrow$

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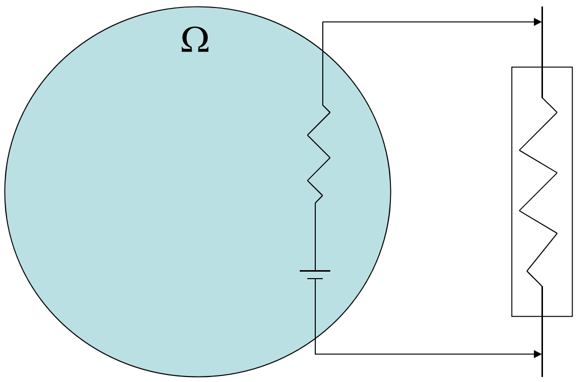
Ohmmeter Loading

Using a circuit within the instrument



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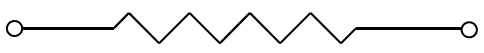
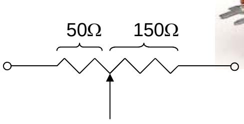
Loading for a Potentiometer

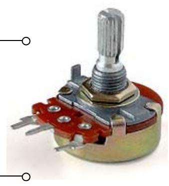
Resistor

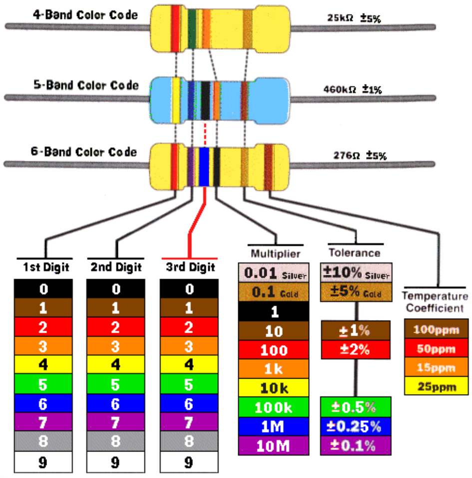
- Fixed



- Adjustable rheostat and potentiometer







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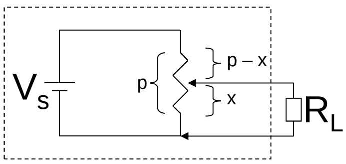
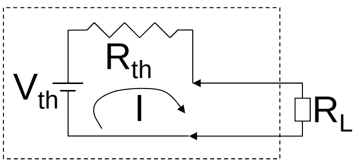
Loading of a Potentiometer (Cont'd)

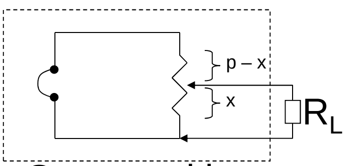




Thévenin's

Voltage Divider

$V_{th} = (x / p)\, V_s$



Superposition

$$\frac{1}{R_{th}} = \frac{1}{(x/p)R_p} + \frac{1}{((p-x)/p)R_p}$$

$$R_{th} = (x/p)R_p(1 - x/p)$$

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Loading of a Potentiometer (Cont'd)

$$V_{th} = I\,(R_{th}+R_L)$$

$$(x/p)V_s = I\,[\,(x/p)(1 - x/p)R_p + R_L\,]$$

$$I = \frac{(x/p)V_s}{(x/p)(1 - x/p)R_p + R_L}$$

$$V_L = IR_L = \frac{(x/p)V_sR_L}{(x/p)(1 - x/p)R_p + R_L}$$

$$= \frac{(x/p)V_s}{(R_p/R_L)(x/p)(1 - x/p) + 1}$$

It means the relationship between V_L and x is nonlinear!

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Loading of a Potentiometer (Cont'd)

Error,

$$V_{th} - V_L = (x/p)V_s\left(1 - \frac{1}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

$$= (x/p)V_s\left(\frac{(R_p/R_L)(x/p)(1 - x/p)}{(R_p/R_L)(x/p)(1 - x/p) + 1}\right)$$

If $R_L \gg R_p$ then $R_p/R_L \approx 0$

Error $\approx (x/p)V_s(R_p/R_L)(x/p)(1 - x/p)$

$\approx V_s(R_p/R_L)[(x/p)^2 - (x/p)^3]$, $R_L \uparrow$ error $\downarrow$

% Error $\approx (R_p/R_L)\,[(x/p) - (x/p)^2] \times 100\%$

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Loading of a Potentiometer (Cont'd)

Let d(Error)/dx = $V_s(R_p/R_L)[2x/p^2 – 3x^2/p^3] = 0$

We get $2x/p^2 = 3x^2/p^3$

$x/p = 2/3$ for the maximum

Therefore,

the max error = $V_s(R_p/R_L)[(2/3)^2 – (2/3)^3]$

$\approx 0.148\ V_s(R_p/R_L)$

and the max % error $\approx 22.2\ (R_p/R_L)$ %

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