Measurement and Testing

ENE 240 Electrical and Electronic Measurements (2/2008)
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Measurement

- **Measurement** is to determine the value or size of some quantity, e.g. a voltage or a current.
- **Analogue measurement** gives a response to a continuous quantity.
- **Digital measurement** is for the quantity at sampled times and quantized values.
- **Comparison measurement** is to compare the quantity with standards, e.g. null method.

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Testing

- **Testing** is to measure to ensure that a product conforms to its specification.
- **Manual testing** proceeded by human
- **Automatic testing** for reducing human error and increasing the performance.

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Error of a Measurement

- Error = Measured Value – True Value
- Percentage Error = $\frac{\text{Error}}{\text{True Value}} \times 100\%$
- Degree of Uncertainty = $\pm$ %Error

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Random Error

- **Random error** is unpredictable for a successive reading of the same quantity.
- **Operating error** from the measurement situation leading to small variations.
- **Environmental error** such as a temperature or a humidity.
- **Stochastic error** e.g. electrical noise.

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Systematic Error

- **Systematic error** remains constant with repeated measurements.
- **Construction error** from manufacture of an instruments
- **Calibration error** from an incorrect setting.
- **Approximation error** e.g. for a linear scales
- **Ageing error** for the old instrument.
- **Loading error** for inserting a quantity affecting its value.

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Human Error

- **Human error** is the mistake made by humans in using instruments and taking the readings.
- **Misreading** of the operator.
- **Calculation error** of the operator.
- **Incorrect instruments** chosen by the operator.
- **Incorrect adjustment** of any conditions.

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Accuracy

- Error = Measured Value – Expected Value

$$e = x_{measured} - x_{expected}$$

- Percent Error = (Error / Expected Value)$\times$100

$$\%e = \left| (x_{measured} - x_{expected}) / x_{expected} \right| \times 100$$

- Accuracy = 100 – Percent Error

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Precision

- Deviation = Measured Value – Average Value

$$d = x_{measured} - x_{average}$$

- Percent Deviation = (Deviation / Average Value) $\times$100

$$\%d = |\,(x_{measured} - x_{average}) / x_{average}\,| \times 100$$

- Precision = 100 – Percent Deviation

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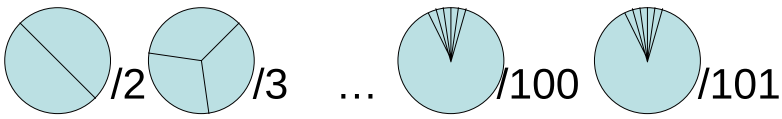
Statistics of Error

- Mean, $\overline{X} = \Sigma_{i=1\rightarrow N} X_i / N$ $\Rightarrow$ the best value
- Deviation = $X_i - \overline{X}$
- Mean Deviation = $\Sigma_{i=1\rightarrow N} |X_i - \overline{X}| / N$
- Standard Deviation,

  $\sigma = \sqrt{\Sigma_{i=1\rightarrow N} (X_i - \mu)^2 / N}$ for a population

  s.d. = $\sqrt{\Sigma_{i=1\rightarrow N} (X_i - \overline{X})^2 / N-1}$ for a sample

  e.g. a cake



- Variance, σ^2

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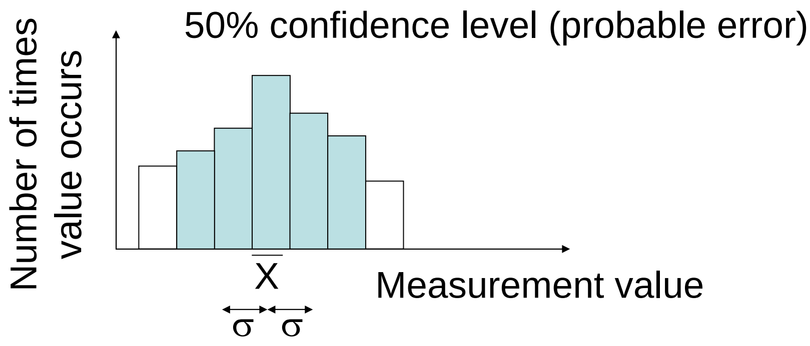
Normal Distribution (Gaussian)

Histogram



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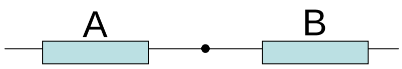
Summation of Error



$100\Omega \pm 10\%$ $\quad$ $200\Omega \pm 5\%$

$90\Omega \leftrightarrow 110\Omega$ $\quad$ $190\Omega \leftrightarrow 210\Omega$

$$X = A + B$$

$$\begin{aligned} X \pm \Delta X &= A \pm \Delta A + B \pm \Delta B \\ &= (A+B) \pm (\Delta A + \Delta B) \\ &= (100\Omega + 200\Omega) \pm (10+10) \\ &= 300\Omega \pm 20 \end{aligned}$$

$280\Omega \leftrightarrow 320\Omega \quad \Rightarrow \quad$ Extreme!

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Summation of Error (Cont'd)

$$X = A + B$$

$$X \pm \Delta X = A \pm \Delta A + B \pm \Delta B$$

$$= (A+B) \pm \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

$$= 300\Omega \pm 14.14$$

$$285.86\Omega \leftrightarrow 314.14\Omega \quad \Rightarrow \quad \text{Better}$$

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Summation of Error (Cont'd)

$$X = AB$$

$$X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \underbrace{\cancel{\Delta A \Delta B}}_{0}$$

$$\Delta X = \pm A\Delta B \pm B\Delta A$$

$$\Delta X/X = (\pm A\Delta B \pm B\Delta A) / AB \ \% \quad \Rightarrow \quad \%\text{Error}$$

$$= \pm \Delta B/B \pm \Delta A/A \ \%$$

$$= \pm (\Delta B/B + \Delta A/A) \ \%$$

$$= \pm \sqrt{(\Delta B/B)^2 + (\Delta A/A)^2} \ \%$$

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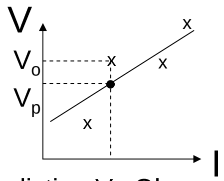
Linear Regression

$V = IR \Rightarrow$ Linear $y=mx+c$

Error $= V_p - V_o$

Minimum $\Sigma(V_p - V_o)^2$



Prediction Vs Observation
(Best Fitted?)

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_{random} \to 0$$

$$\varepsilon = \Sigma_i [\, Y_i - (\beta_0 + \beta_1 X_i)\,]^2$$

$$\partial\varepsilon/\partial\beta_1 = -2\, \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i\,]\, X_i$$

$$= -2\, \Sigma_i [\, Y_i X_i - \beta_0 X_i - \beta_1 X_i^2\,] = 0$$

$$\partial\varepsilon/\partial\beta_0 = -2\, \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i\,] = 0$$

Linear Regression (Cont'd)

$$\Sigma_i [\, Y_i X_i - \beta_0 X_i - \beta_1 X_i^2\,] = \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i\,]$$

$$\Sigma_i Y_i X_i - \beta_0 \Sigma_i X_i - \beta_1 \Sigma_i X_i^2 = \Sigma_i Y_i - N\beta_0 - \beta_1 \Sigma_i X_i$$

$$\beta_1 = \frac{\Sigma_i Y_i X_i - N\overline{X}\,\overline{Y}}{\Sigma_i X_i^2 - N\overline{X}^2}$$

$$\beta_0 = \overline{Y} - \beta_1 \overline{X}$$

Linear Regression (Cont'd)

For nonlinear equation,

$Y = X^n \Rightarrow (\log Y) = n (\log X)$ Logarithm

$Y = a^X \Rightarrow (\log Y) = (\log a) X$ Semi-log

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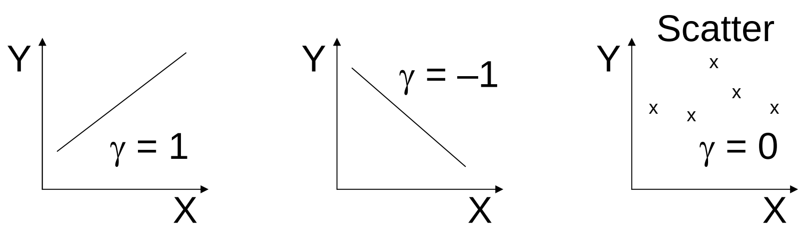
Correlation

$$\gamma = 1/N \frac{\Sigma(X - X)(Y - Y)}{\sigma_X \sigma_Y}$$



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