King Mongkut's University of Technology Thonburi
Midterm Examination of First Semester, Academic Year 2016
International Program

COURSE CPE 121 Discrete Mathematics for Computer Engineers **Computer Engineering Department, 1st Yr.**
Thursday September 22, 2016 **13.00-16.00hr.**

Instructions

1. This examination contains 11 problems, 7 pages (including this cover page).
2. The answers must be written in these examination sheets.
3. Each Student is **allowed** to use a calculator.
4. No books, notes, or any other documents can be taken into the examination room.

Students must raise their hand to inform to the proctor upon their completion of the examination, to ask for permission to leave the examination room.
Students must not take the examination and the answers out of the examination room.

Students will be punished if they violate any examination rules. The highest punishment is dismissal.

This examination is designed by

(Assoc.Prof. Dr. Naruemon Wattanapongsakorn), Tel. 0-2470-9083

This Examination is approved by the department of Computer Engineering

(Asst.Prof. Sanan Srakaew), Department Chair

Student Name ________________________ Student ID ____________ Seat Number_____

	Score	obtained		Score	obtained
Problem 1	6		Problem 7	15	
Problem 2	4		Problem 8	15	
Problem 3	4		Problem 9	5	
Problem 4	3		Problem 10	4	
Problem 5	10		Problem 11	6	
Problem 6	6				
				Total = 80	

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Problem 1: Let p and q be the propositions (6 points)

p: It is sunny.

q: you wear a hat.

Write these propositions using p and q and logic connectives. (2 points each)

1.1) You wear a hat if it is sunny.

1.2) If it is not sunny, then you do not wear a hat.

1.3) Whenever you wear a hat, it is sunny.

Problem 2: Construct a truth table of the compound statement (4 points)

$$(q \rightarrow \sim p) \leftrightarrow (p \leftrightarrow q)$$

Problem 3: Build a logic circuit for $(p \vee \sim r) \wedge (\sim p \vee q \wedge r)$ (4 points)

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Problem 4: Determine if each of these functions is a bijective (one-to-one and onto) from **R** to **R**. (3 points)

4.1) $f(x) = 4x + 3$

4.2) $f(x) = x^2 - 1$

4.3) $f(x) = x^3 + 2$

Problem 5: (10 points)
Using Binary Search Algorithm to determine the number of comparisons to search for 19 in the list {5, 6, 8, 13, 18, 19, 22, 33, 49}. Determine the complexity of this algorithm. Show your work.

Procedure binary search(x: integer, $a_1, a_2, \ldots, a_n$: increasing integers)
i := 1 {i is left endpoint of search interval}
j := n {j is right endpoint of search interval}
while i < j
begin m := $\lfloor (i+j)/2 \rfloor$
    if x > a_m **then** i := m+1
    else j := m
end
if x = a_i **then** location := i
else location := 0
{location is the subscript of the term equal to x, or 0 if x is not found}

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Problem 6: Use big-O notation to closely estimate the following functions. Show your work. (6 points)

6.1) $f(n) = (\log n)^4 + (5n^3) + n/3$ (3 points)

6.2) $f(n) = (n! + 2^n)(n^3 + \log(n^3 + 7))$ (3 points)

Problem 7: Mathematical Proof. (15 points)

7.1) Prove that "if n is a positive integer, then n is odd if and only if 5n +6 is odd". (5 points)

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7.2) Give a proof by contradiction that if n is an integer and $n^2 + 5$ is even, then n is odd. (5 points)

7.3) Use mathematical induction to show if $n^2 - 7n + 12$ is nonnegative when n is an integer greater than 3. (5 points)

Problem 8: 15 points

8.1) How many non-negative integers that have value no greater than 500 that are divisible by 5 or by 7 ? (5 points)

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8.2) How many strings of four decimal digits (5 points)

8.2.1) do not contain the same digit twice (each digit is unique)? (2 points)

8.2.3) have exactly three digits that are 5s? (3 points)

8.3) How many bit strings of length 8 contain (5 points)

8.3.1) At least four 1s? (3 points)

8.3.2) Exactly two zeros? (2 points)

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Problem 9: Consider that a department contains 9 men and 12 women. How many ways to form a committee with eight members if it must have the number of women equal to five? (5 points)

Problem 10: What is the coefficient of x^9y^6 in the expansion of $(2x - 3y)^{15}$? (4 points)

Given $(x + y)^n = \sum_{j=0}^{n} (C(n, j) (x^{n-j})(y^j))$

Problem 11: Recurrence Relation. (6 points)

Find a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

11.1) $a_n = 4n - 2$

11.2) $a_n = n^2$

11.3) $a_n = 10^n$