# ENE 104
# Electric Circuit Theory

## Lecture 06:
## Basic RL and RC Circuits

**Week #6 :** Dejwoot KHAWPARISUTH
office: CB40906 Tel: 02-470-9065
Email: dejwoot.kha@mail.kmutt.ac.th

http://webstaff.kmutt.ac.th/~dejwoot.kha/

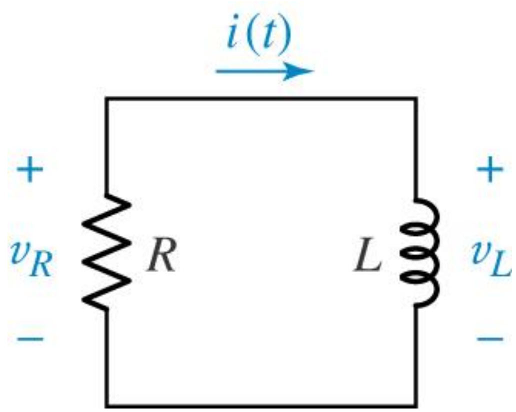
# Objectives : Ch8

- **time constants** for RL and RC circuits
- the **natural** and **forced response**
- the **total response**
- the effect of **initial conditions** on circuit response
- **RL** and **RC** circuit response

# The Source-Free RL Circuit:

- A **natural** response
- A **transient** response
- A **forced** response

# The Source-Free RL Circuit:




**A series RL circuit for which *i*(t) is to be determined, subject to the initial condition that *i*(0) = $I_0$.**

$$Ri + v_L = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

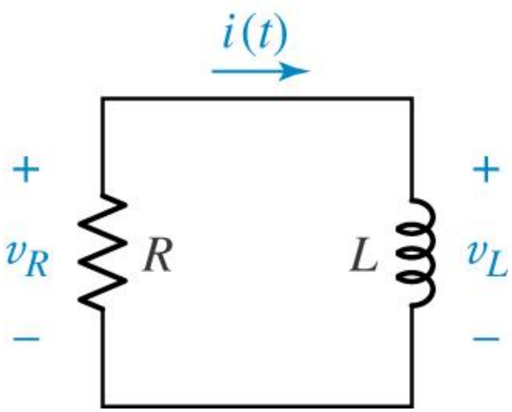
$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{t_0}^{i(t)} \frac{di'}{i'} = \int_0^t -\frac{R}{L}dt'$$

$$\ln i - \ln I_0 = -\frac{R}{L}(t-0)$$

$$so \quad \boxed{i(t) = I_0 e^{-Rt/L}}$$

# The Source-Free RL Circuit:




**A series RL circuit for which *i*(t) is to be determined, subject to the initial condition that *i*(0) = $I_0$.**

$$\boxed{i(t) = I_0 e^{-Rt/L}}$$

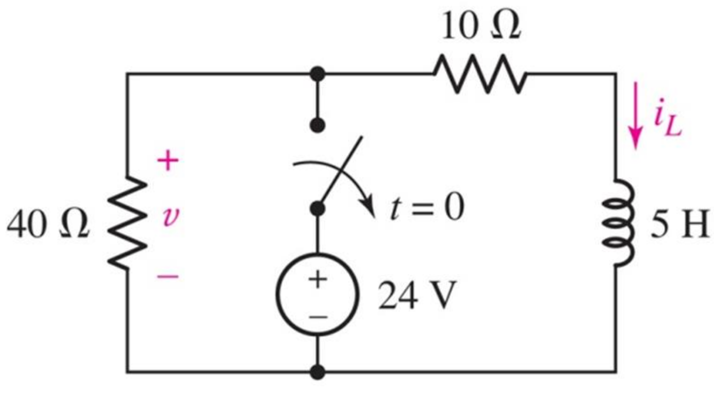
The power: $$p_R = i^2 R = I_0^2 R \cdot e^{\frac{-2Rt}{L}}$$

The total energy: $$w_R = \int_0^\infty p_R dt = I_0^2 R \int_0^\infty e^{\frac{-2Rt}{L}} dt$$

$$= I_0^2 R \left( \frac{-L}{2R} \right) e^{\frac{-2Rt}{L}} \Bigg|_0^\infty = \frac{1}{2} L I_0^2$$

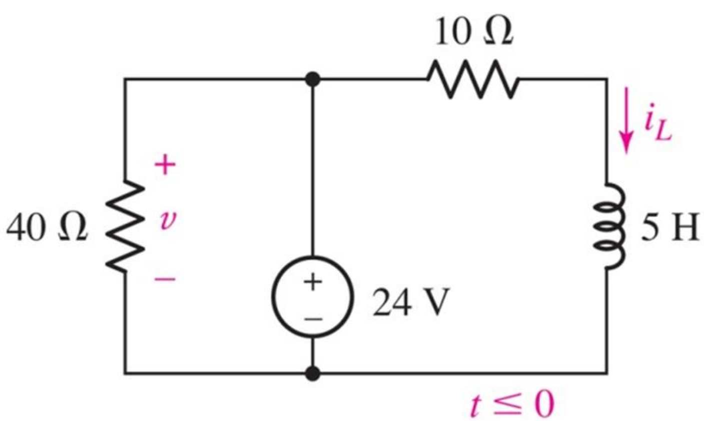
# Example 8.1:

find the current through the 5-H inductor at t = 200ms

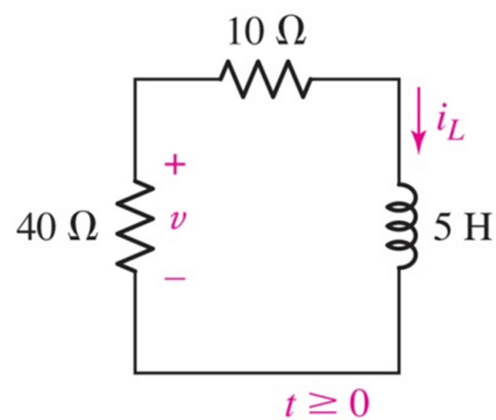



(a)

$$i_L(t) = I_0 e^{\frac{-Rt}{L}}$$



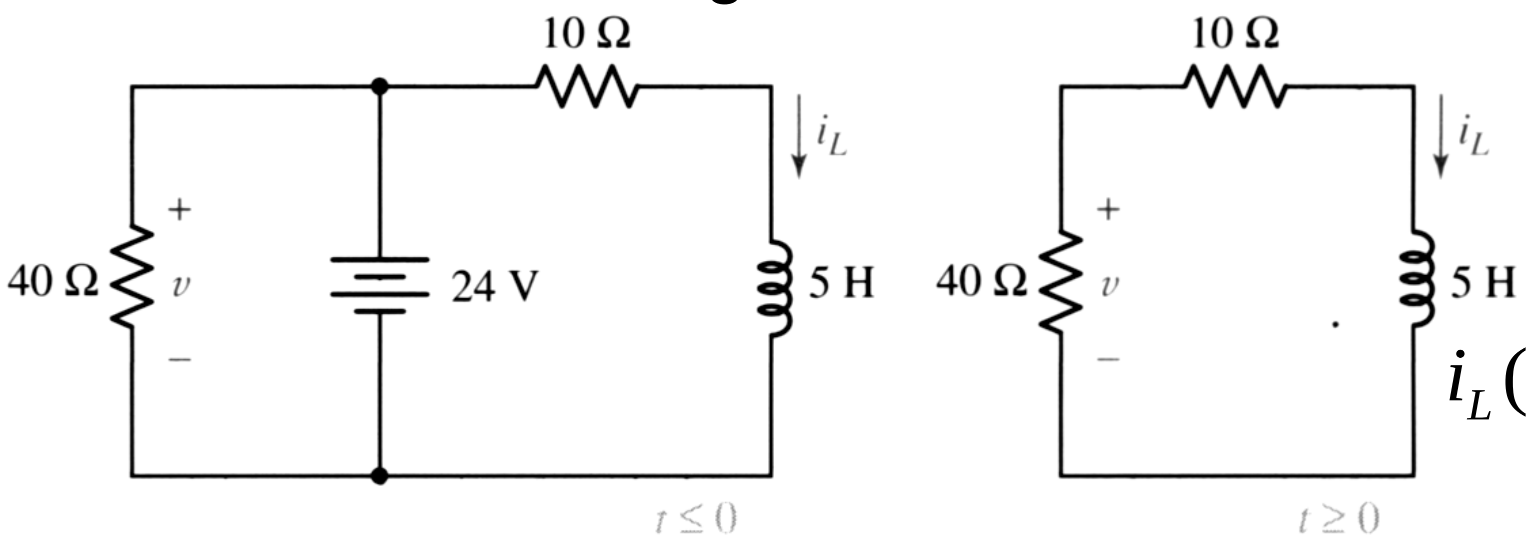

(b)




(c)

# Example :

find the current through the 5-H inductor at t = 200ms



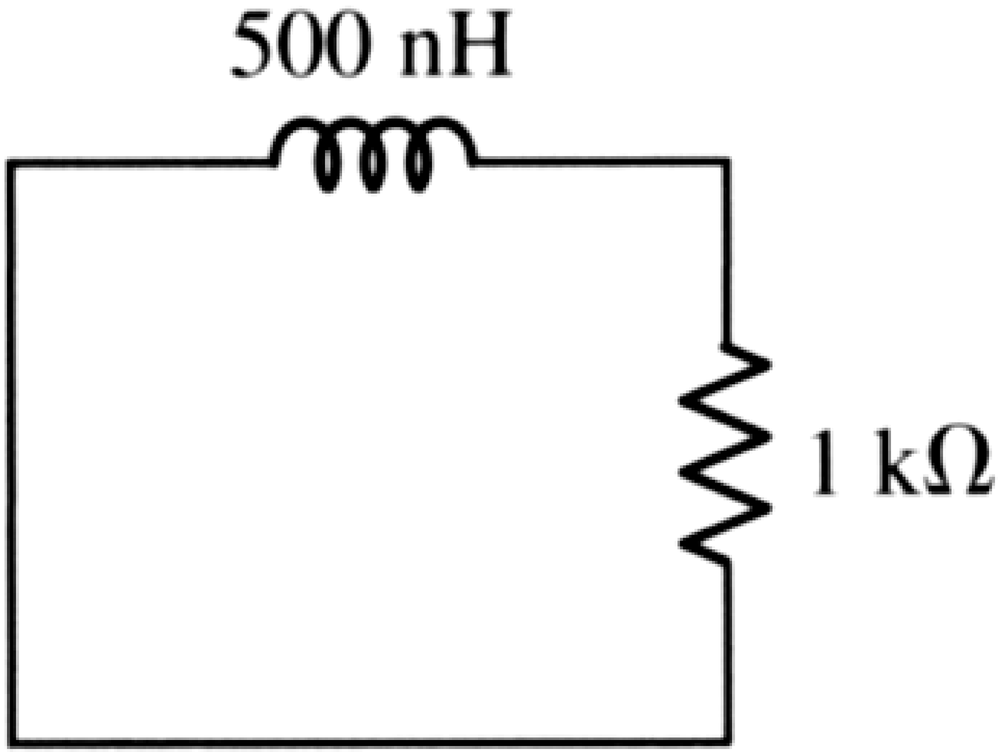

$$i_L(t) = I_0 e^{\frac{-Rt}{L}}$$

$$I_0 = \frac{24}{10} = 2.4$$

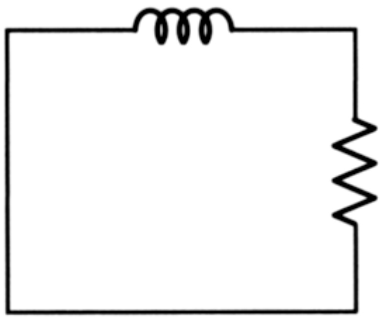
$$i_L(t = 200ms.) = 2.4e^{-10t} = 324.8mA.$$

# Practice: 8.1

Determine the energy remaining in the inductor at t = 2 ns if it is initially storing 7 μJ



# Practice: 8.1




7 µJ stored in a 500 nH inductor

a current magnitude = $\sqrt{\dfrac{2\times 7\times 10^{-6}}{500\times 10^{-9}}} = 5.292\ \text{A}$

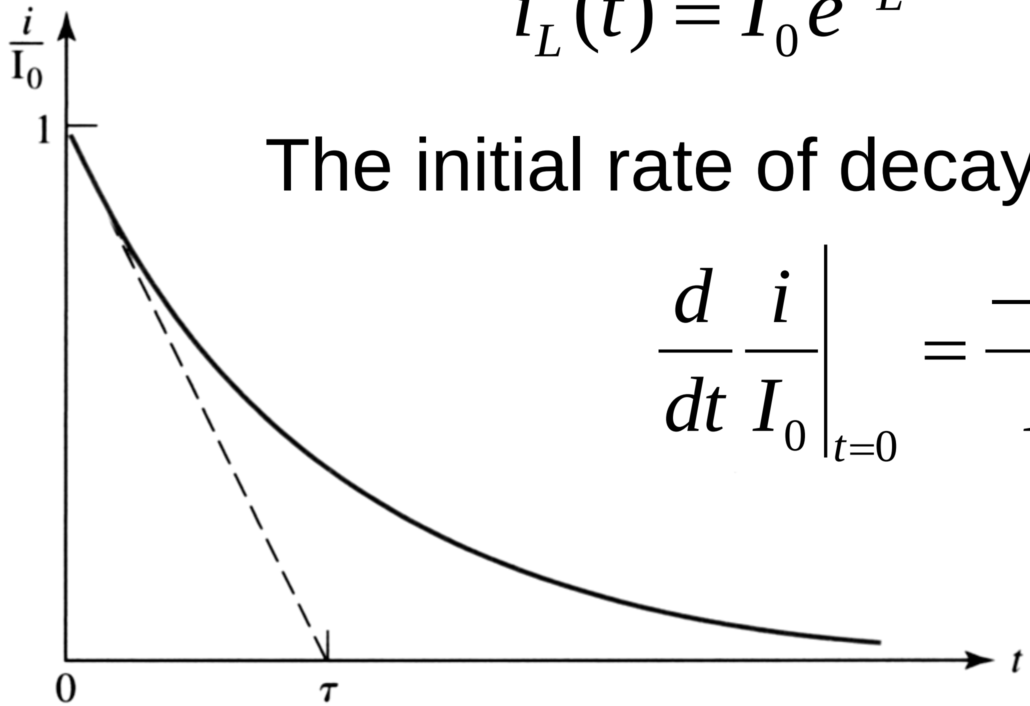
$$i(t) = I_o e^{-t/\tau} \text{ where } \tau = \frac{L}{R} = \frac{500\times 10^{-9}}{1000} = 500\ \text{ps}$$

$$i(2ns) = 5.292\exp\left(\frac{-2\times 10^{-9}}{500\times 10^{-12}}\right) = 96.93\ \text{mA}$$

$$w(2\ \text{ns}) = \frac{1}{2}Li^2 = 0.5(500\times 10^{-9})(96.93\times 10^{-3})^2$$

# Properties of the Exponential resp.:

$$i_L(t) = I_0 e^{\frac{-Rt}{L}}$$




The initial rate of decay:

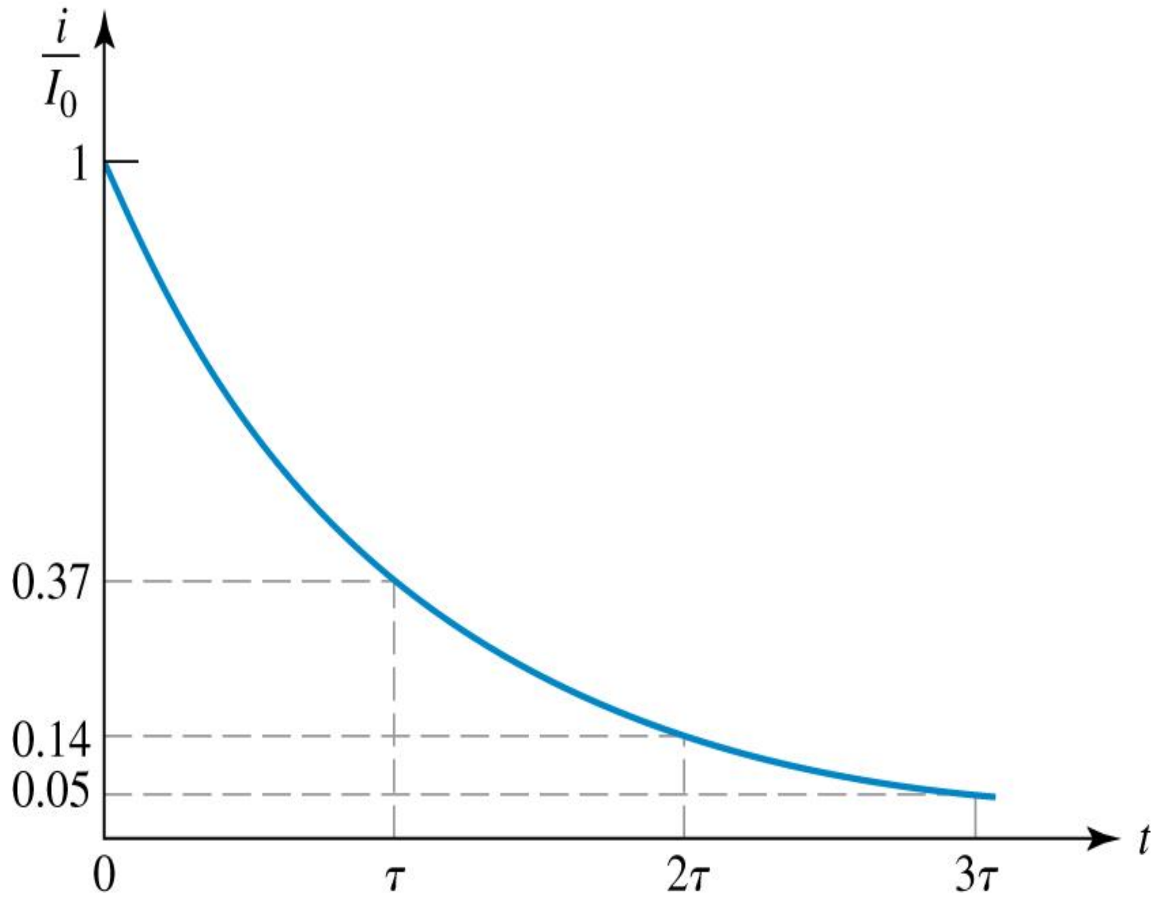
$$\left.\frac{d}{dt}\frac{i}{I_0}\right|_{t=0} = \left.\frac{-R}{L}e^{\frac{-Rt}{L}}\right|_{t=0} = \frac{-R}{L}$$

$$\tau = \frac{L}{R}$$

$$\frac{i(\tau)}{I_0} = e^{-1} = 0.3679$$ or $$i(\tau) = 0.3679 I_0$$

# Properties of the Exponential resp.






**A plot of the exponential response versus time.**

# Practice: 8.2

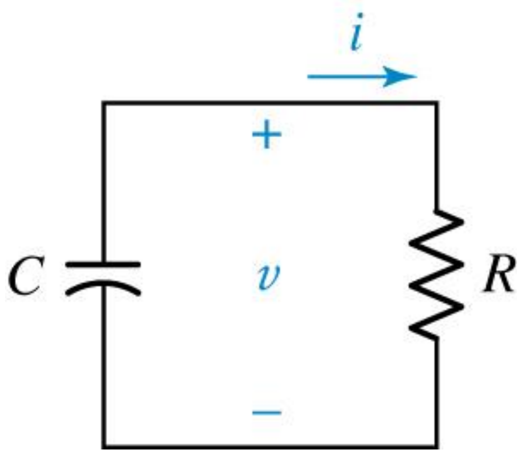
In a source-free series RL circuit, find the numerical value of the ratio: (a) $i(2\tau)/i(\tau)$, (b) $i(0.5\tau)/i(0)$, and (c) $t/\tau$ if $\frac{i(t)}{i(0)} = 0.2$; (d) $t/\tau$ if $i(0) - i(t) = i(0)ln2$ .

$$i(t) = I_o e^{-t/\tau} \qquad i(0) = I_o$$

$$\frac{i(2\tau)}{i(\tau)} = \frac{e^{-2}}{e^{-1}} = \qquad \frac{i(0.5\tau)}{i(0)} = e^{-0.5} =$$

$$\frac{i(t)}{i(0)} = e^{-t/\tau} = 0.2, \text{ so } \frac{t}{\tau} = -\ell n\, 0.2 =$$

# The Source-Free RC Circuit:



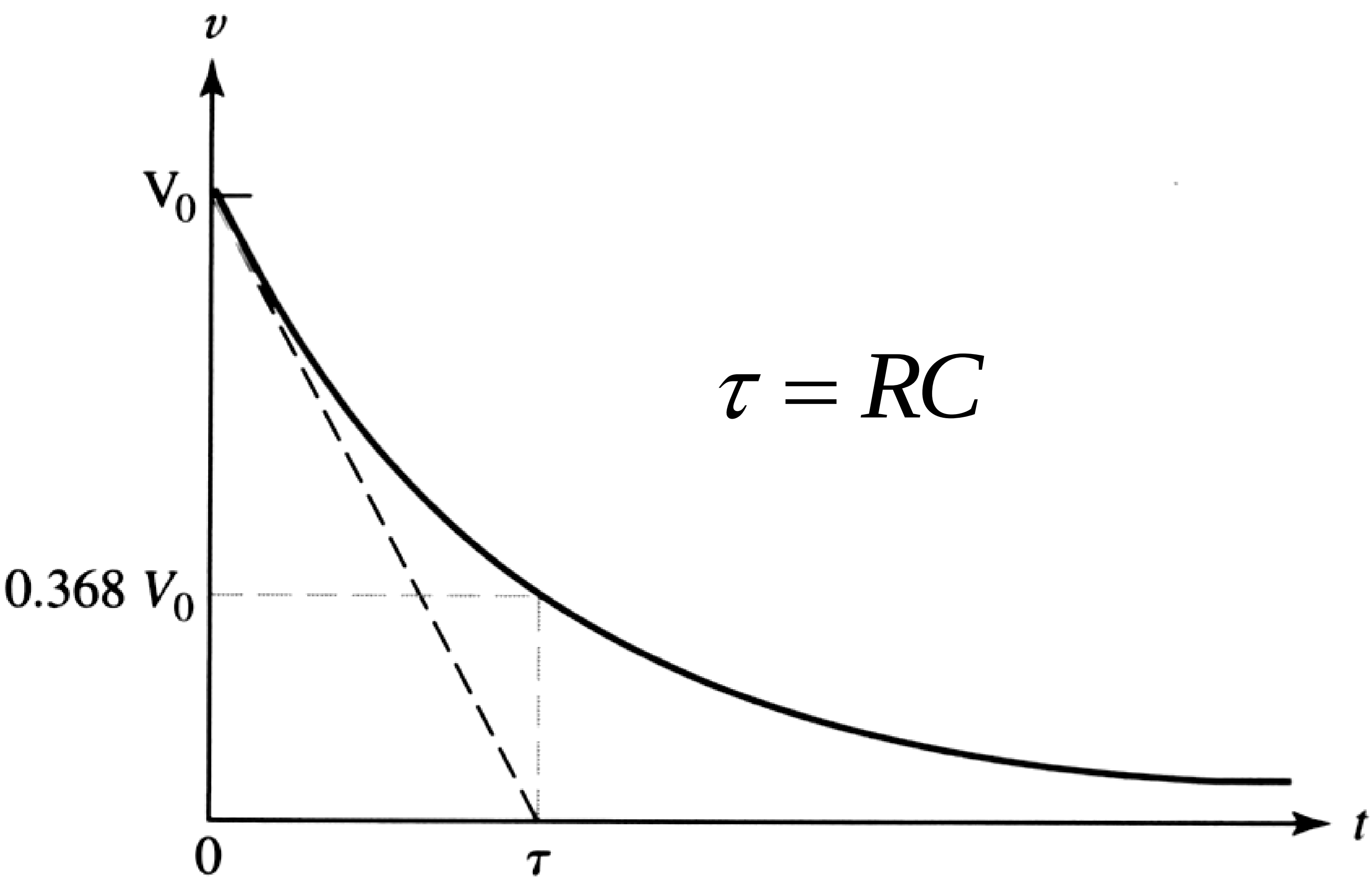

**A parallel RC circuit for which *v*(t) is to be determined, subject to the initial condition that *v*(0) = $V_0$.**

$$C\frac{dv}{dt}+\frac{v}{R}=0$$

$$\frac{dv}{dt}+\frac{v}{RC}=0$$

$$\rightarrow \quad v(t)=v(0)e^{\frac{-t}{RC}}=V_0 e^{\frac{-t}{RC}}$$

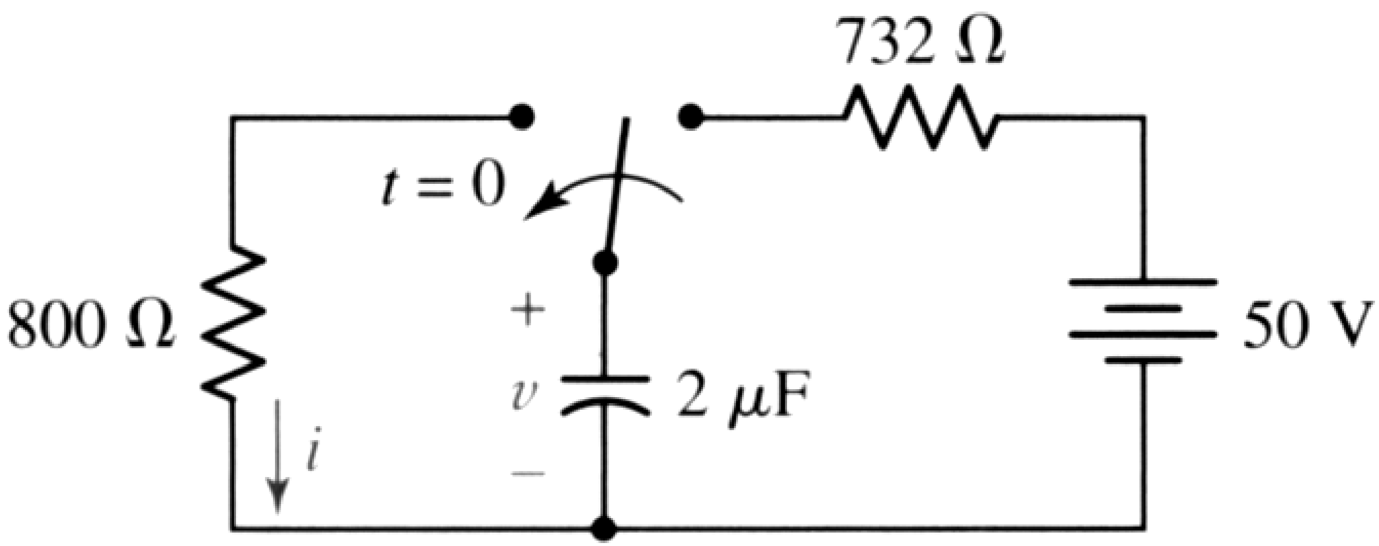
# The Source-Free RC Circuit:



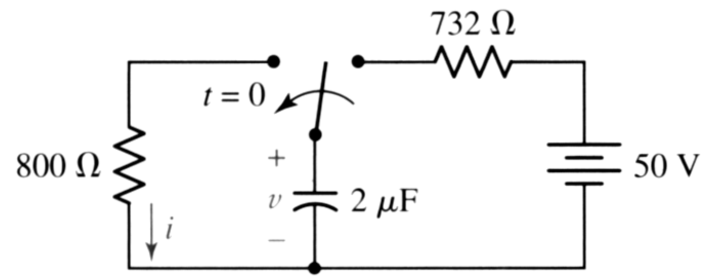

$$\tau = RC$$

# Practice: 8.3

Find $v(0)$ and $v(2ms)$



# Practice: 8.3




With no current flow permitted through the capacitor (it is assumed any transients have long since died out), $v$ = 50 V.

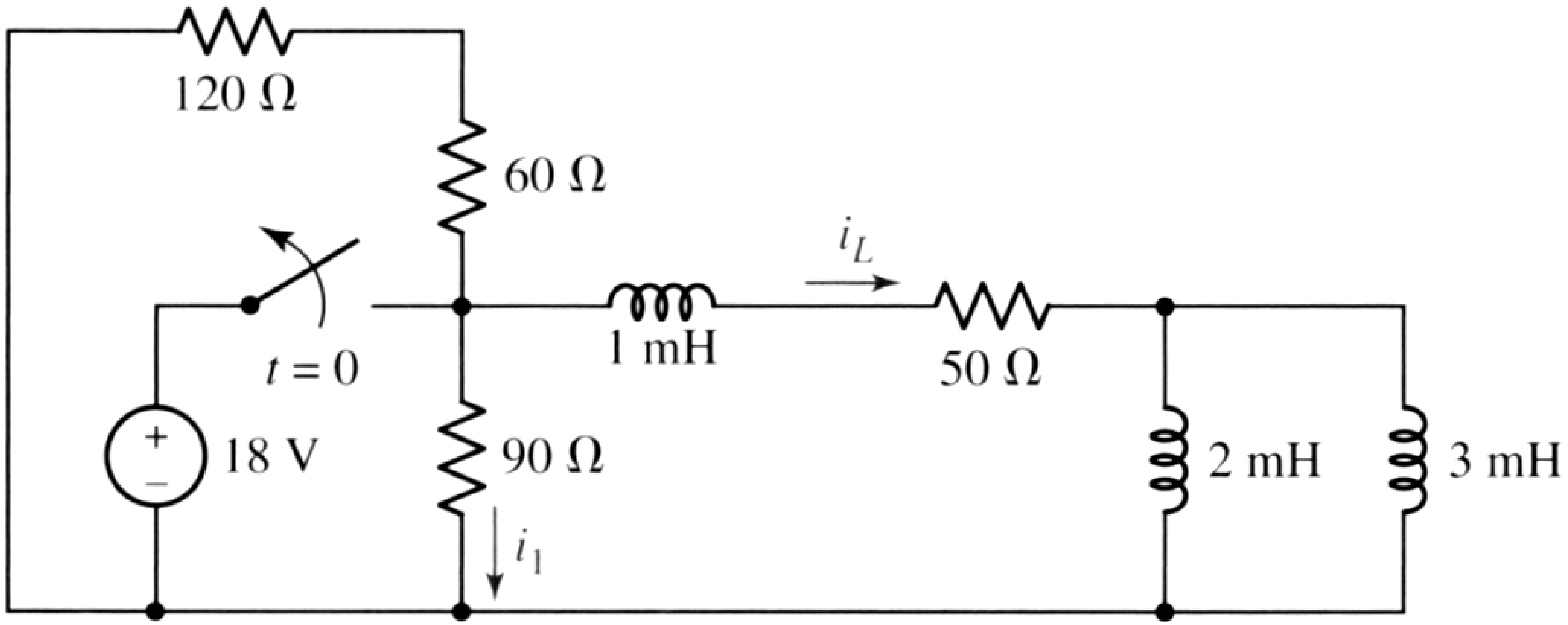
After the switch is thrown, the only remaining circuit is a simple source-free RC circuit.

$$\tau = RC = 1.6 \text{ ms}$$

$$v(t) = v(0)\,e^{-t/\tau} \quad \text{so} \quad v(2\text{ms}) = 50\exp\left(\frac{-2\times10^{-3}}{1.6\times10^{-3}}\right)$$

# Example 8.2:

determine both $i_1$ and $i_L$ in the circuit



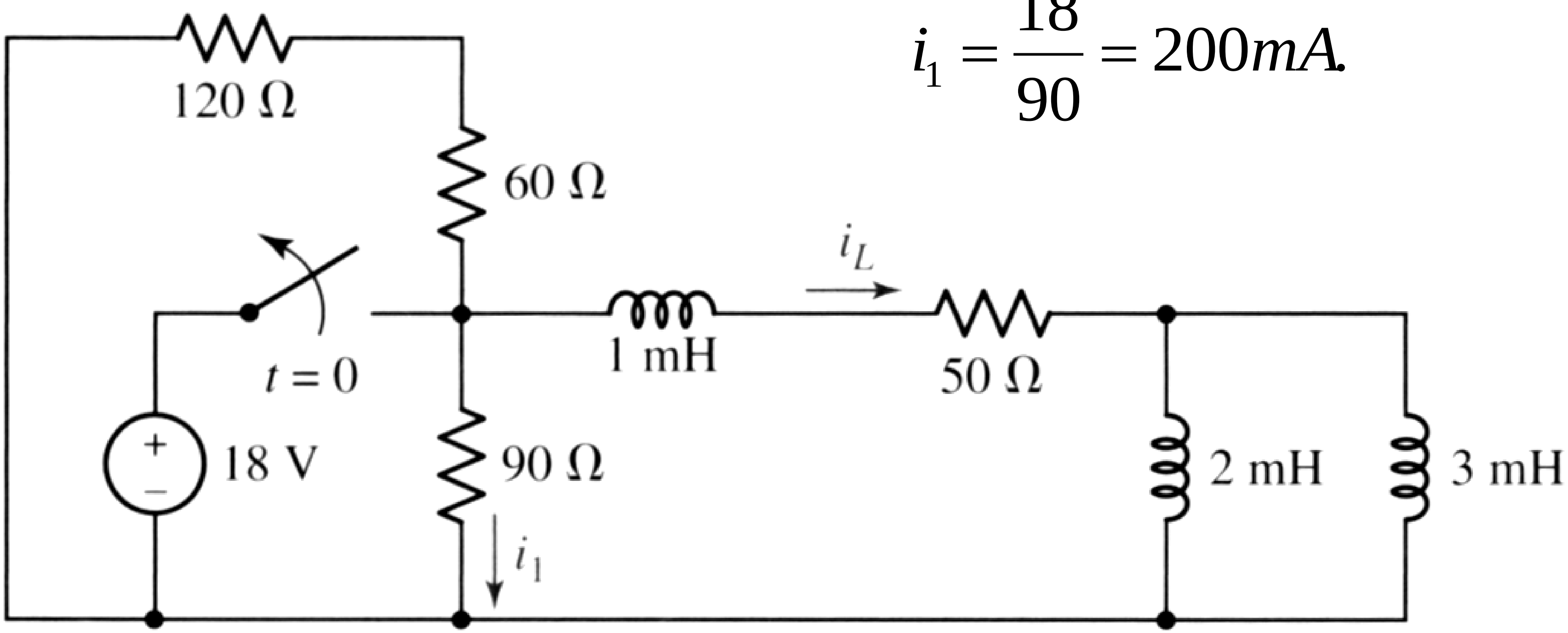

(a)

# Example :

determine both $i_1$ and $i_L$ in the circuit

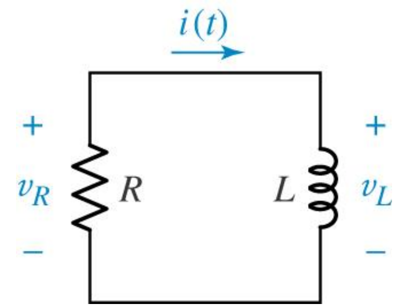
$$t < 0, i_L = \frac{18}{50} = 360mA.$$

$$i_1 = \frac{18}{90} = 200mA.$$



# Example :

determine both $i_1$ and $i_L$ in the circuit



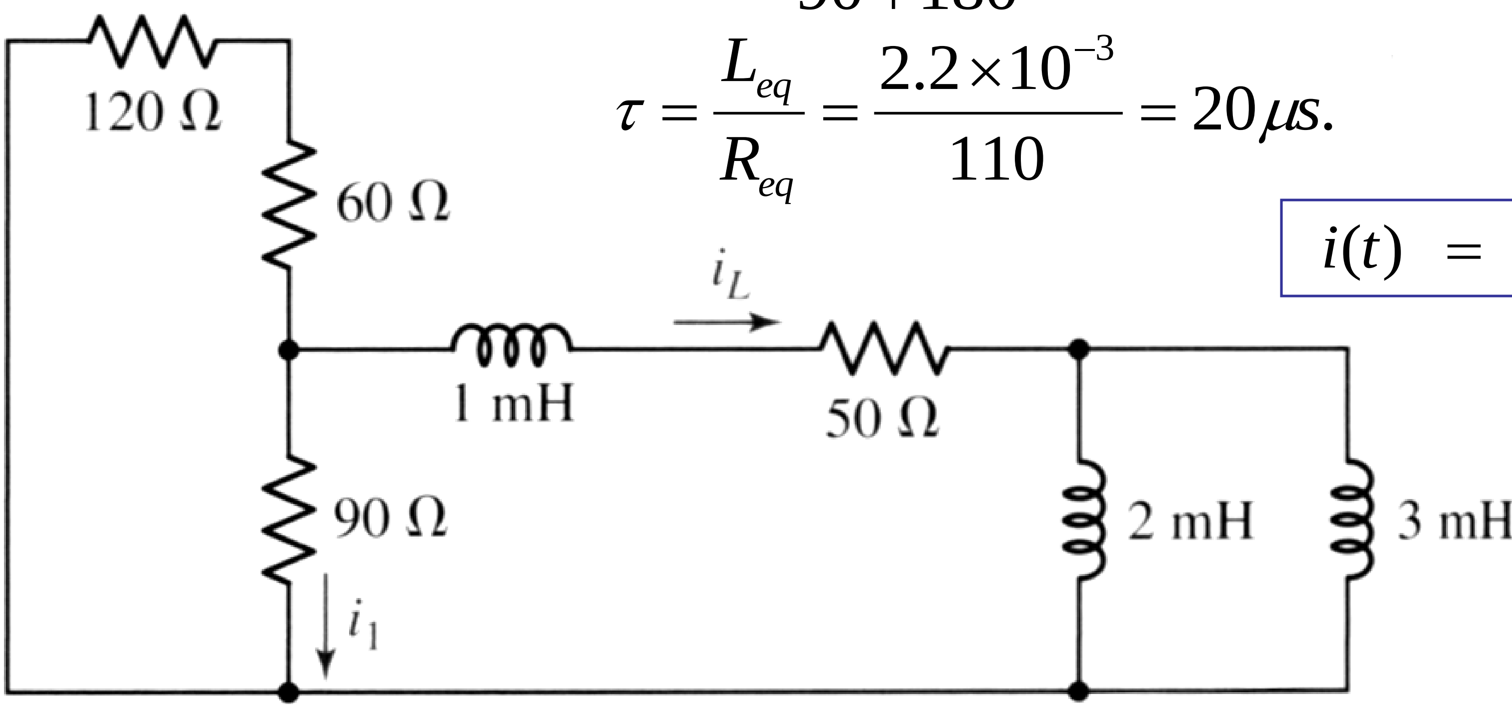

For t>0,

$$L_{eq} = \frac{2\times 3}{2+3} + 1 = 2.2mH.$$

$$R_{eq} = \frac{90\times(60+120)}{90+180} + 50 = 110\Omega$$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{2.2\times 10^{-3}}{110} = 20\mu s.$$

$$\boxed{i(t) \;=\; I_0 e^{-Rt/L}}$$

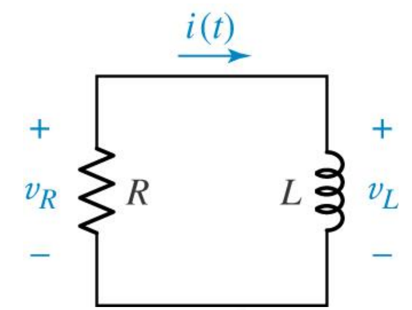


# Example :

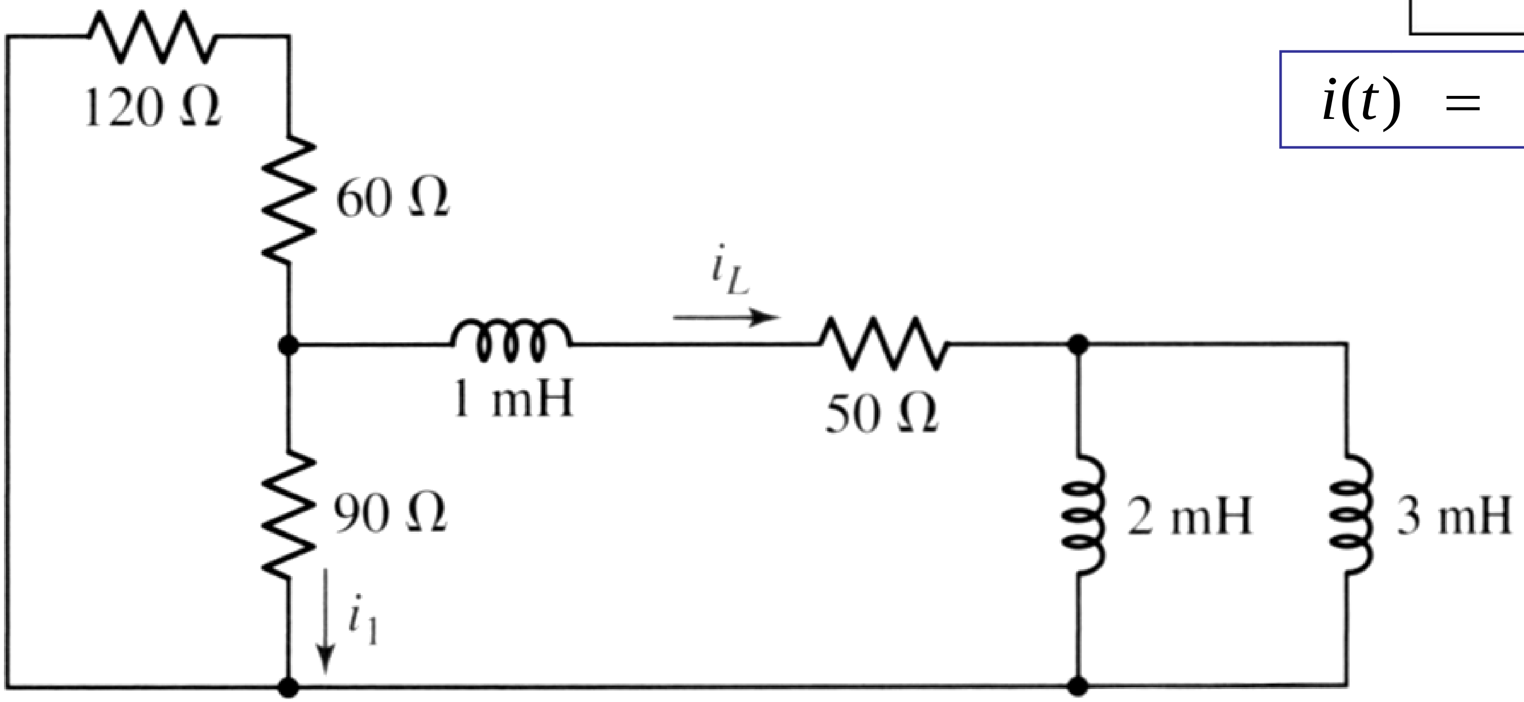
determine both $i_1$ and $i_L$ in the circuit

For t>0, $i_L(0^+) = i_L(0^-) = 360mA.$

$$i_L(t) = 360e^{\frac{-Rt}{L}} = 360e^{\frac{-t}{\tau}} = 360e^{-50000t}\,mA.$$




$$i(t) = I_0 e^{-Rt/L}$$

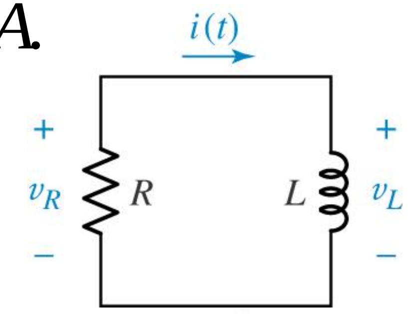


# Example :

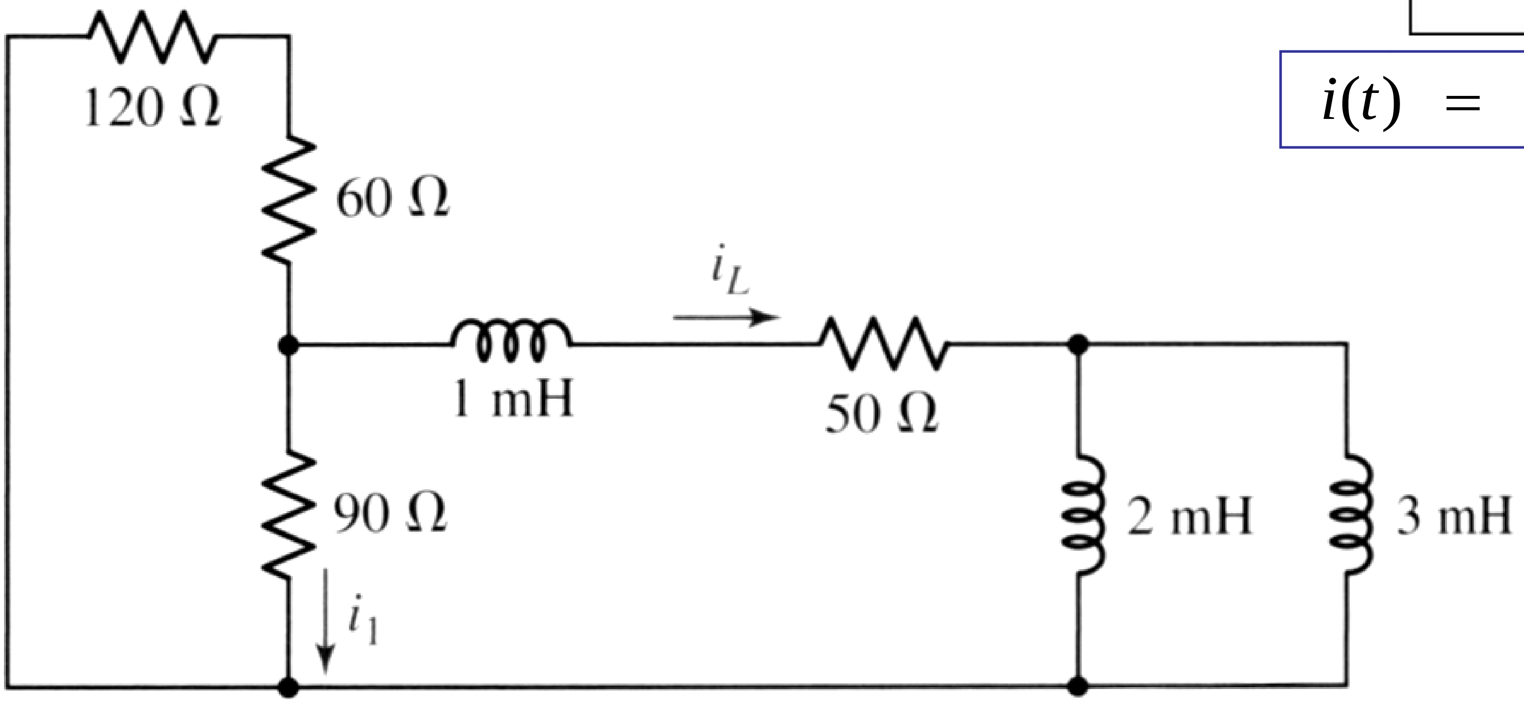
determine both $i_1$ and $i_L$ in the circuit

For t>0, $i_1(0^+) = -i_L(0^+)\dfrac{120+60}{120+60+90} = -240mA.$

$$i_1(t) = -240e^{-50000t}\,mA.$$

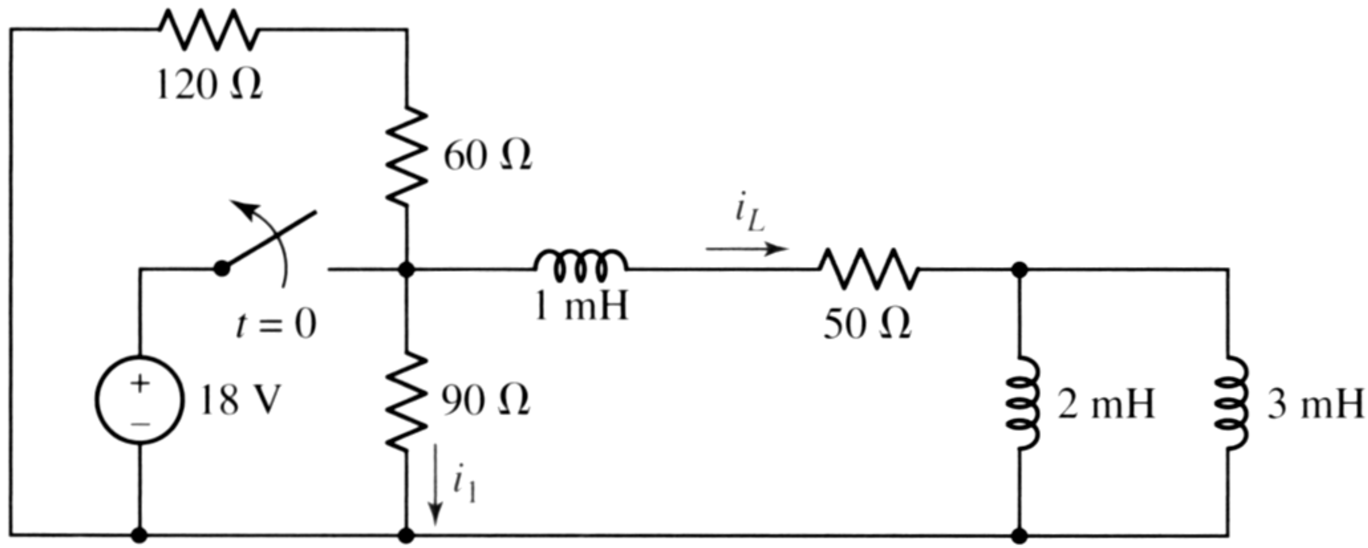



$$\boxed{i(t) \;=\; I_0 e^{-Rt/L}}$$



# Example :

determine both $i_1$ and $i_L$ in the circuit

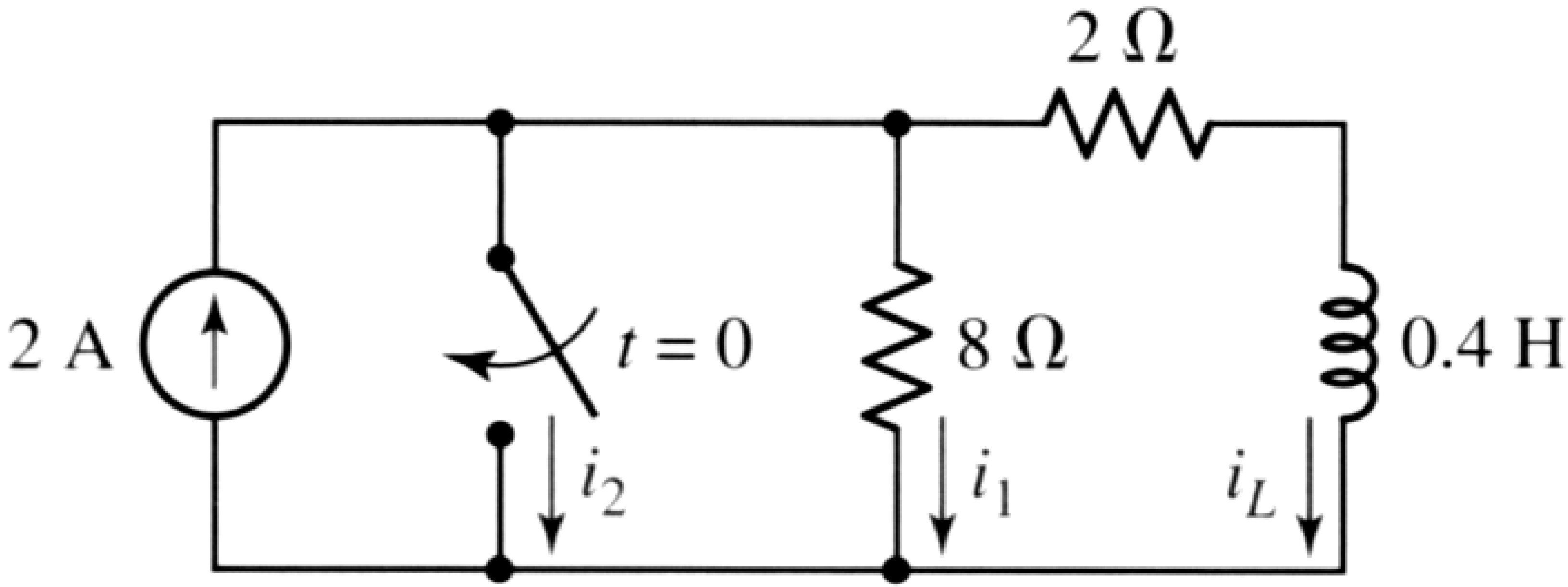



$$i_L(t) = \begin{cases} 360 mA. & (t < 0) \\ 360 e^{-50000t} mA. & (t > 0) \end{cases}$$

$$i_1(t) = \begin{cases} 200 mA. & (t < 0) \\ -240 e^{-50000t} mA. & (t > 0) \end{cases}$$

# Practice: 8.4

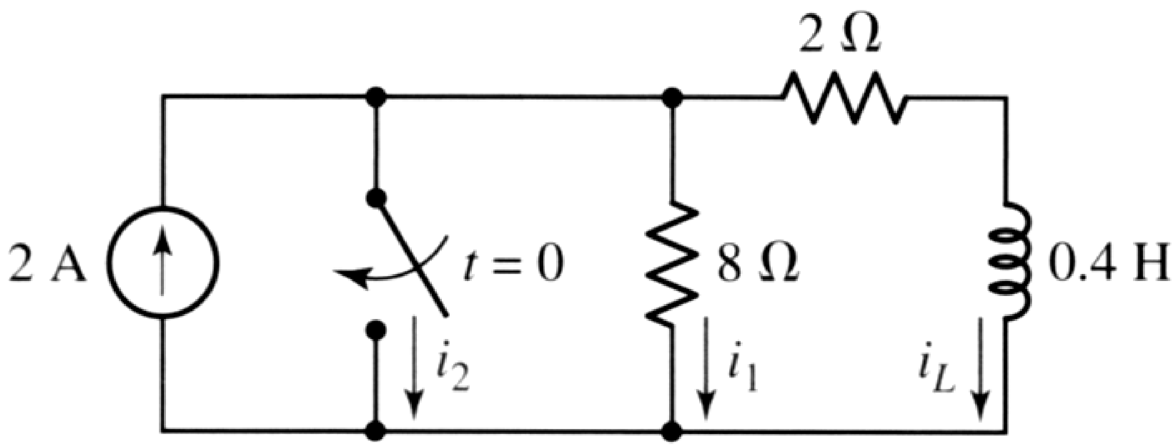
at t = 0.15s. find the value of $i_L$, $i_1$, $i_2$



# Example :

at t = 0.15 s. find the value of $i_L$, $i_1$, $i_2$

For t<0,




$$i_L = \frac{8}{8+2} \cdot 2A = 1.6A = i_L(0^-) = i_L(0^+) = I_0$$

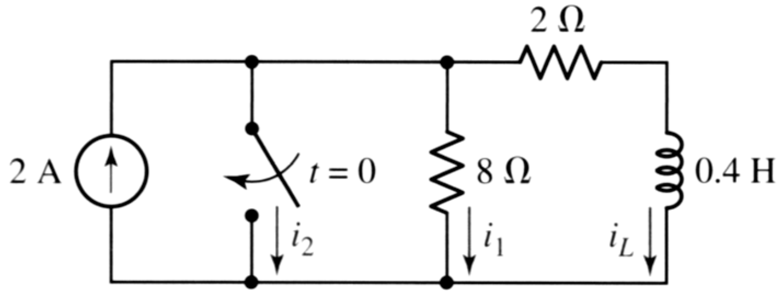
$$i_1 = \frac{2}{8+2} \cdot 2A = 0.4A$$

# Example :

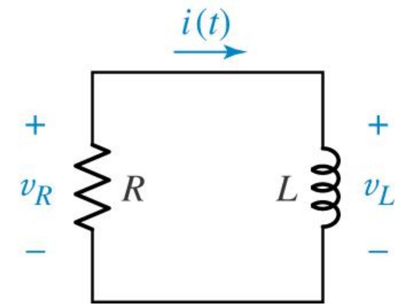


at t = 0.15s. find the value of $i_L$, $i_1$, $i_2$

For t>0,




$$R_{eq} = 2\Omega$$



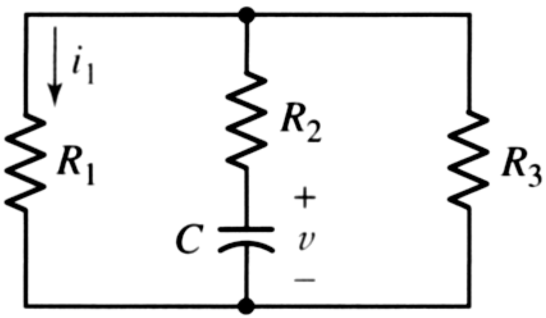

$$\boxed{i(t) = I_0 e^{-Rt/L}}$$

$$i_L(t = 0.15s.) = 1.6e^{\frac{-Rt}{L}} = 1.6e^{\frac{-2t}{0.4}} = 0.756A.$$

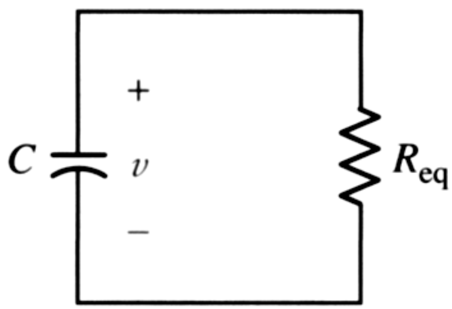
$$i_2 = 2 - i_L = 1.244A.$$

# Example 8.3:

find $v(0^+)$ and $i_1(0^+)$ if $v(0^-) = V_0$




$(a)$




$(b)$

$$R_{eq} = R_2 + R_1 // R_3$$

$$v_C(0^+) = v_C(0^-) = V_0$$

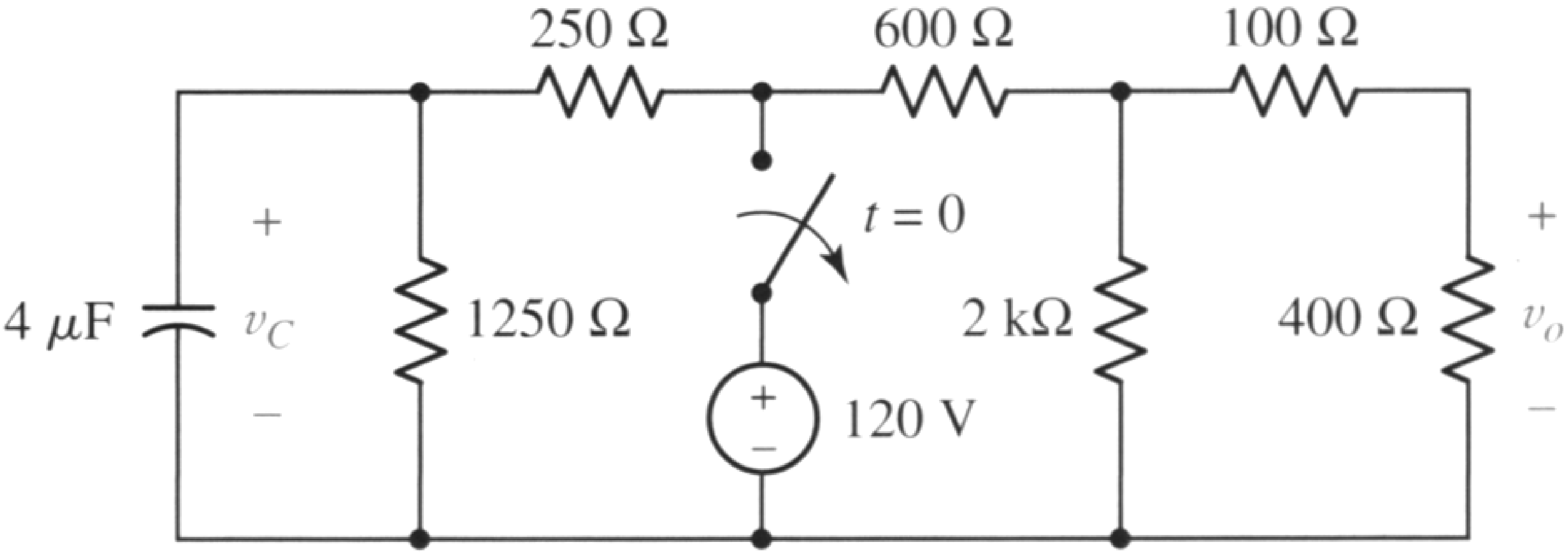
$$i_1(t) = i_1(0^+)e^{\frac{-t}{\tau}}$$

$$\tau = R_{eq}C$$

$$i_1(0^+) = \frac{V_0}{R_2 + {}^{R_1R_3}\!/\!_{R_1+R_3}} \cdot \frac{R_3}{R_1 + R_3}$$

# Practice: 8.5

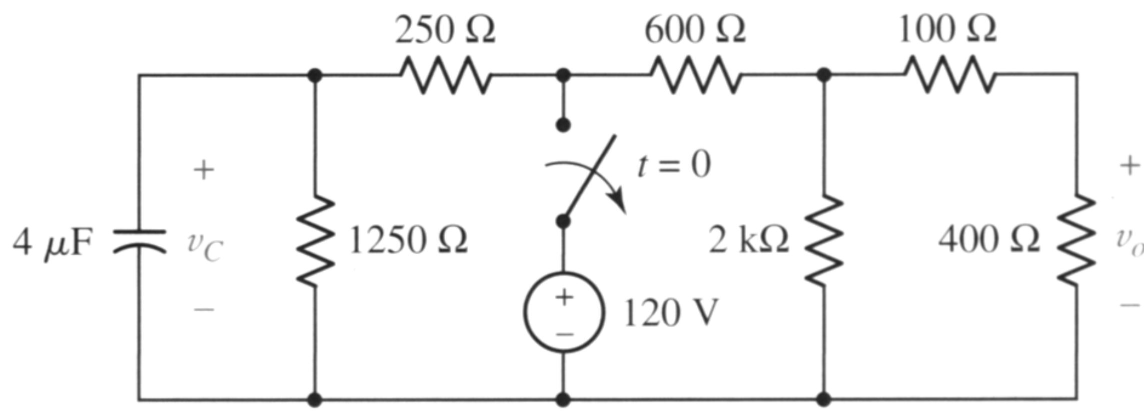
find $v_C$ and $v_0$ at $t = 0^-, 0^+$, 1.3ms.



# Example :

find $v_C$ and $v_0$ at t = $0^-$, $0^+$, 1.3ms.

For t<0,

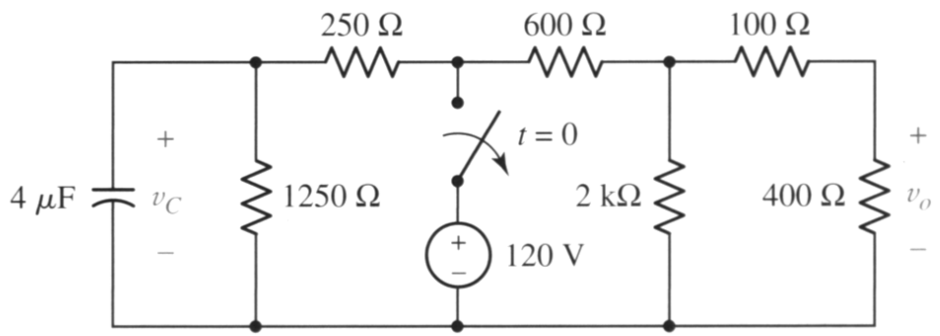



$$v_C = \frac{1250}{1250+250} \cdot 120V. = 100V. = v_C(0^-) = v_C(0^+)$$

$$v_o(0^-) = \left[\frac{(400+100)\cdot 2k}{400+100+2k} + 600\right]^{-1} \cdot 120 \cdot \frac{2k}{400+100+2k} \cdot 400 = 38.4V.$$

# Example :

find $v_C$ and $v_0$ at t = $0^-$, $0^+$, 1.3ms.




For $t=0^+$,

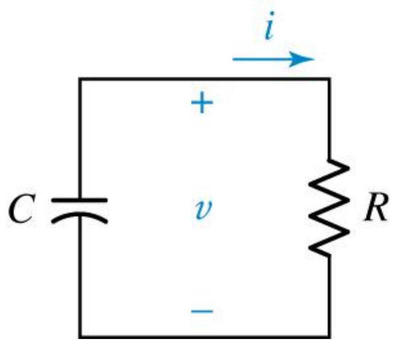
$$v_C(0^+) = v_C(0^-) = 100V.$$

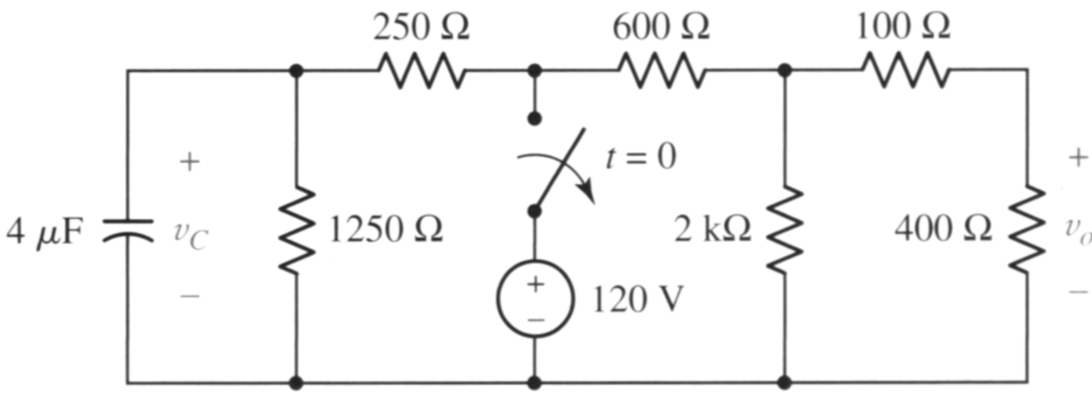
$$v_o(0^+) = \frac{100V.}{250+600+500//2k} \cdot \frac{2k}{400+100+2k} \cdot 400 = 25.6V.$$

# Example :

find $v_C$ and $v_0$ at $t = 0^-, 0^+, 1.3ms$.

For t=1.3ms.,




$$\begin{aligned} R_{eq} &= 1250 // \{250 + 600 + [2k //(100+400)]\} \\ &= 1250 // \{850 + 400\} \\ &= 625\Omega \end{aligned}$$

$$v_C(t) = v_C(0^+)e^{\frac{-t}{\tau}} = 100e^{-400t}V.$$

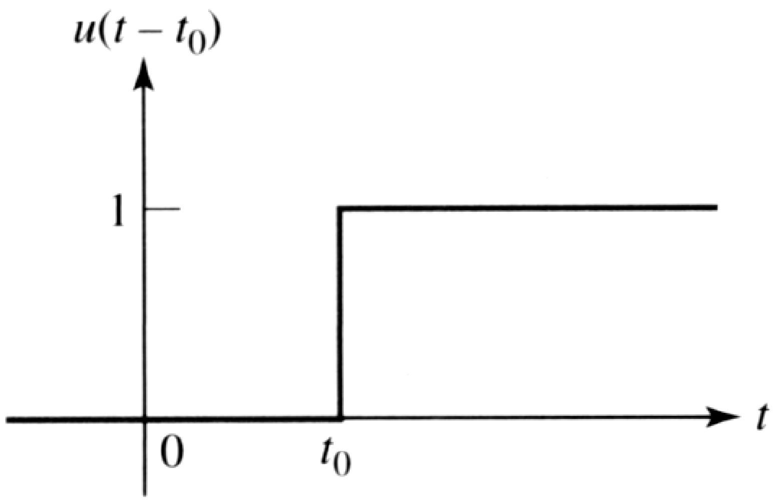
$$v_C(t = 1.3ms.) = 59.45V.$$

$$\tau = R_{eq}C = (625)\cdot(4\mu F) = 0.0025$$

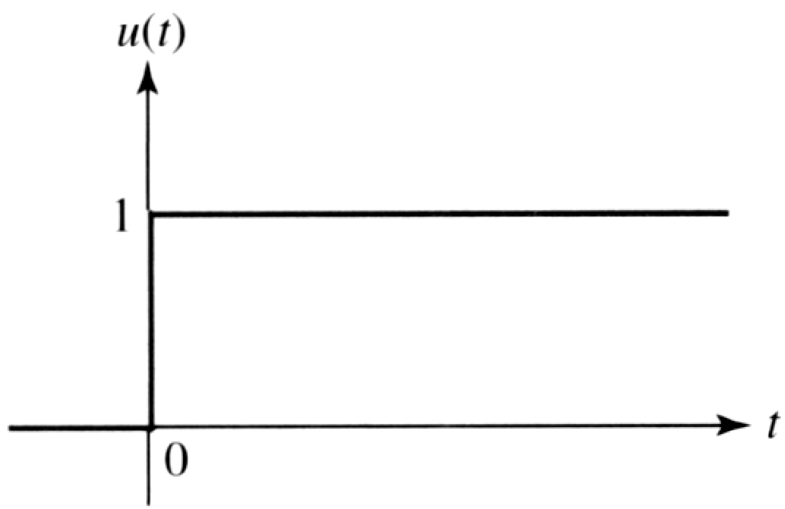
$$\frac{1}{\tau} = 400$$

$$v_0(t = 1.3ms.) = v_0(0^+)e^{-400t} = 25.6e^{-400t} = 15.22V.$$

# The Unit-Step Function:

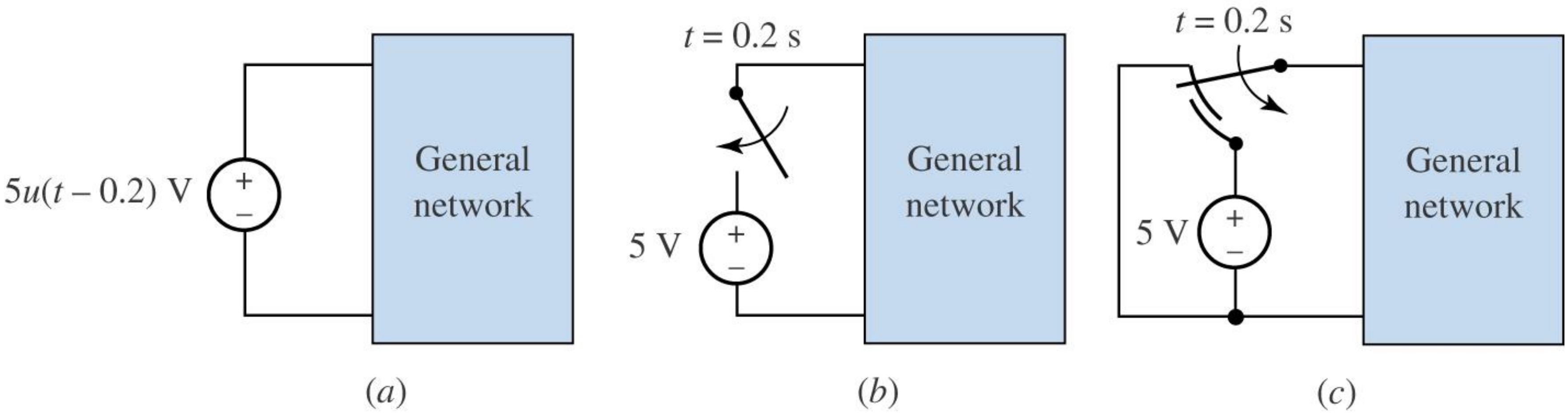



$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$



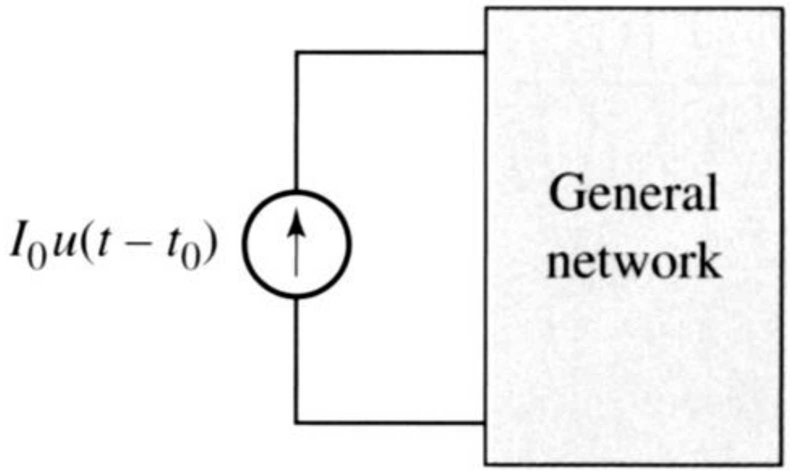

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

# The Unit-Step Function:

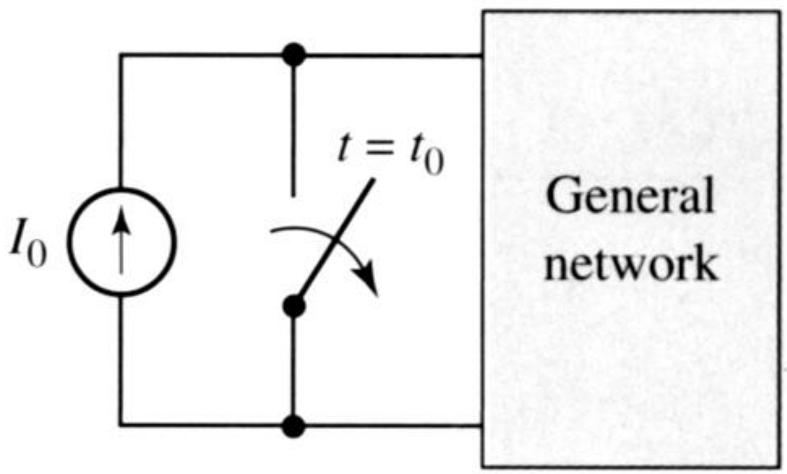



**(a) A voltage-step forcing function is shown as the source driving a general network. (b) A simple circuit which, although not the exact equivalent of part (a), may be used as its equivalent in many cases. (c) An exact equivalent of part (a).**

# The Unit-Step Function:

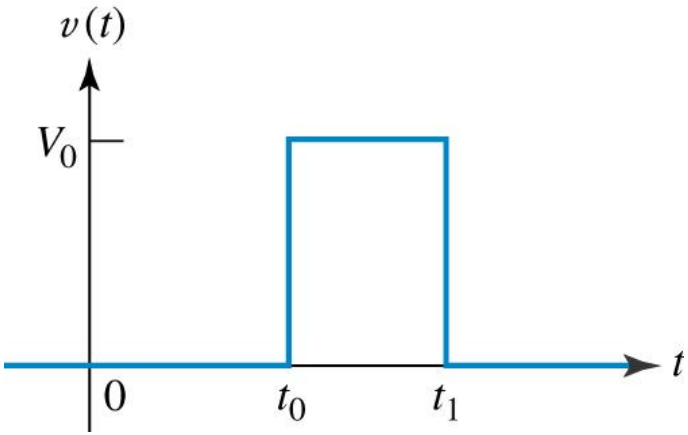



$(a)$



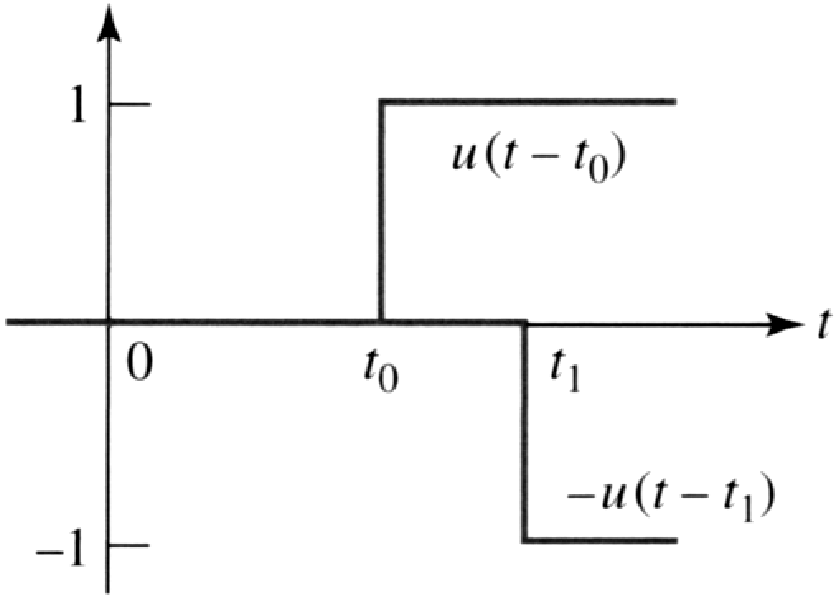

$(b)$

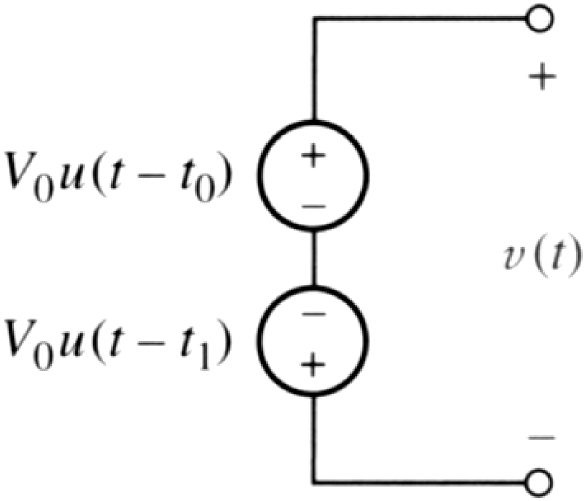
# The Rectangular Pulse Function:




**The rectangular pulse function**

$$v(t-t_0) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$



# Practice: 8.6

Evaluate each of the following at $t = 0.8$:

(a) $3u(t) - 2u(-t) + 0.8u(1-t)$; (b) $[4u(t)]u(-t)$;

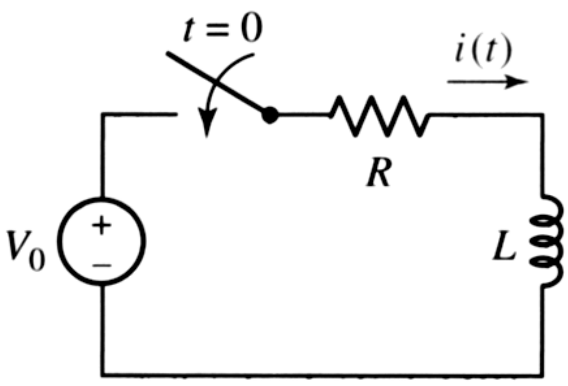
(c) $2u(t)sin\pi t$.

(a) 3 - 0 + 0.8 = <u>3.8</u>

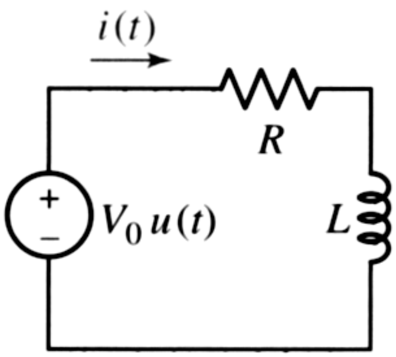
(b) [4] (0) = <u>0</u>

(c) 2 sin 0.8 ¶ = <u>1.176</u>

# Driven RL Circuits:




(a)




(b)

$$Ri(t) + L\frac{di(t)}{dt} = V_0 u(t)$$

$$i(t) = 0, t < 0$$

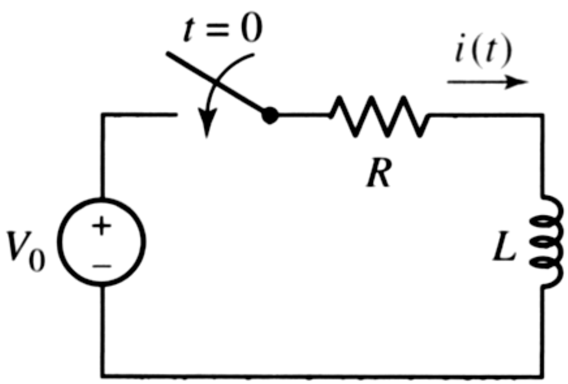
for positive time; $t > 0$

$$Ri(t) + L\frac{di(t)}{dt} = V_0$$

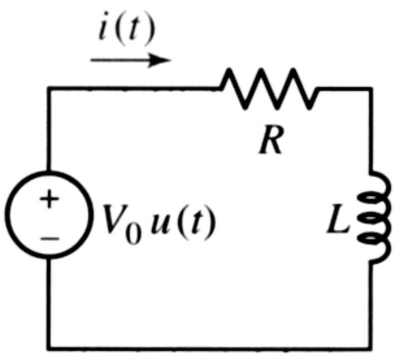
yielding; $$\frac{Ldi(t)}{V_0 - Ri(t)} = dt$$

integrated; $$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + k$$

# Driven RL Circuits:




(a)




(b)

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + k$$

setting i = 0 for t = 0, we obtain;

$$-\frac{L}{R}\ln\left(V_0\right) = k$$

And, hence,

$$-\frac{L}{R}\left\{\ln\left(V_0 - Ri\right) - \ln\left(V_0\right)\right\} = t$$

$$\frac{V_0 - Ri}{V_0} = e^{-\frac{Rt}{L}}$$

or

$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R}e^{-\frac{Rt}{L}}\right)u(t)$$

# Practice: 8.7

The voltage source $60 - 40u(t)$ V is in series with a 10-$\Omega$ resistor and a 50-mH inductor. Find the magnitudes of the inductor current and voltage at $t$ equal to (a) $0^-$; (b) $0^+$; (c) ∞; (d) $3ms$.

# Practice: 8.7

(a) $t = 0^-$, so only 60 V is across the RL circuit. Thus $|v_L(0^-)| = \underline{0}$ and $|i_L(0^-)| = \frac{60}{10} = \underline{6\ \text{A}}$

(b) At $t = 0^+$, the source voltage changes to 60 – 40 = 20 V. The inductor current cannot change, so $|i_L(0^+)| = \underline{6\ \text{A}}$. The current through the resistor is 6 A, so the voltage dropped across the inductor is 20 – 10 (6) = – 40 V. Thus, $|v_L(0^+)| = \underline{40\ \text{V}}$

(c) At $t = \infty$, the source voltage is 20 V but all transients have died out. Thus, $|i_L(\infty)| = \frac{20}{10} = \underline{2\ \text{A}}$ and $|v_L(\infty)| = \underline{0}$. The direction of $i_L$ has not changed.

(d) For $t > 0$, $|i_L(t)| = \left| i_L(\infty) + \left[ i_L(0^+) - i_L(\infty) \right] e^{-t/\tau} \right|$

where $|i_L(t)| = 2\ \text{A}$ and $\tau = L/R = \frac{50 \times 10^{-3}}{10} = 5\ \text{ms}$.

Thus, $i_L(3\ \text{ms}) = 2 + (6-2)e^{-3/5} = \underline{4.195\ \text{A}}$

We then find that $|v_L(3\ \text{ms})| = \left| 20 - 4.195(10) \right| = \underline{21.95\ \text{V}}$

# Natural and Forced Response:

The solution of any linear differential equation may be expressed as the sum of two parts:

- the complementary solution (natural response)
- the particular solution (forced response)

let consider; $\frac{di}{dt} + Pi = Q$ ;Q = a forcing function

$$di + Pi \cdot dt = Q \cdot dt$$

$$i = e^{-Pt} \int Qe^{-Pt} dt + Ae^{-Pt}$$

# Natural and Forced Response:

$$i = e^{-Pt}\int Qe^{-Pt}dt + Ae^{-Pt}$$

For a source free circuit, Q must be zero, and the solution is the natural response:

$$i_n = Ae^{-Pt}$$

The first term is also called the steady state response, the particular solution, or the particular integral.

When Q is a constant,

$$i_f = \frac{Q}{P}$$

# Natural and Forced Response:

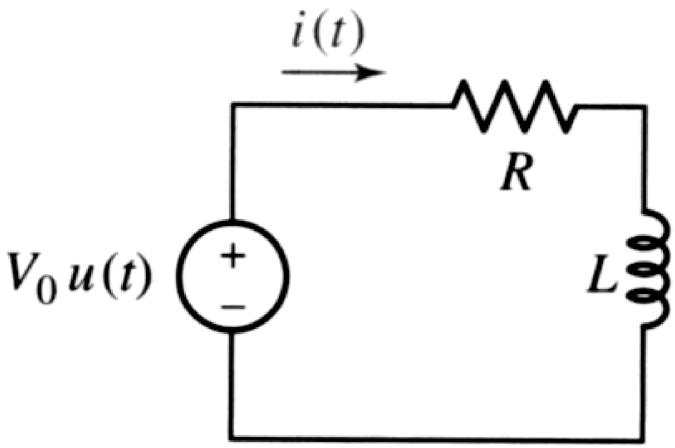
$$i = e^{-Pt}\int Qe^{-Pt}dt + Ae^{-Pt}$$

or the complete response;

$$i(t) = i_f + i_n = \frac{Q}{P} + Ae^{-Pt}$$

# Determination of the complete res.:






$$i(t) = i_f + i_n$$

$$i_n(t) = Ae^{\frac{-Rt}{L}}$$

$$i_f = \frac{V_0}{R}$$

$$\therefore i(t) = i_n(t) + i_f = Ae^{\frac{-Rt}{L}} + \frac{V_0}{R}$$

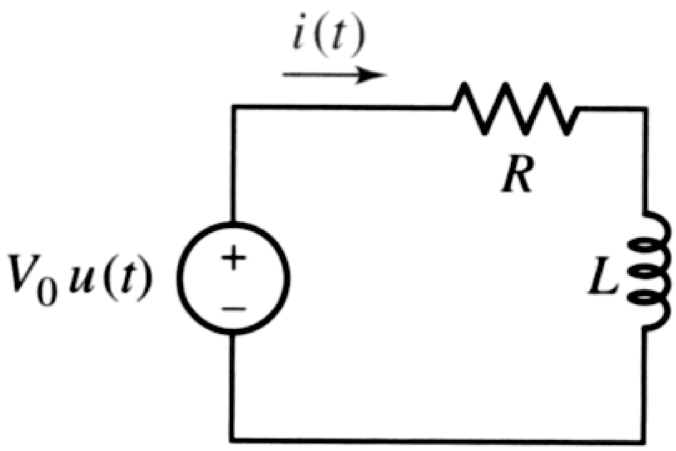
apply initial condition to evaluate A, i = 0 when t = 0;

$$0 = A + \frac{V_0}{R}$$

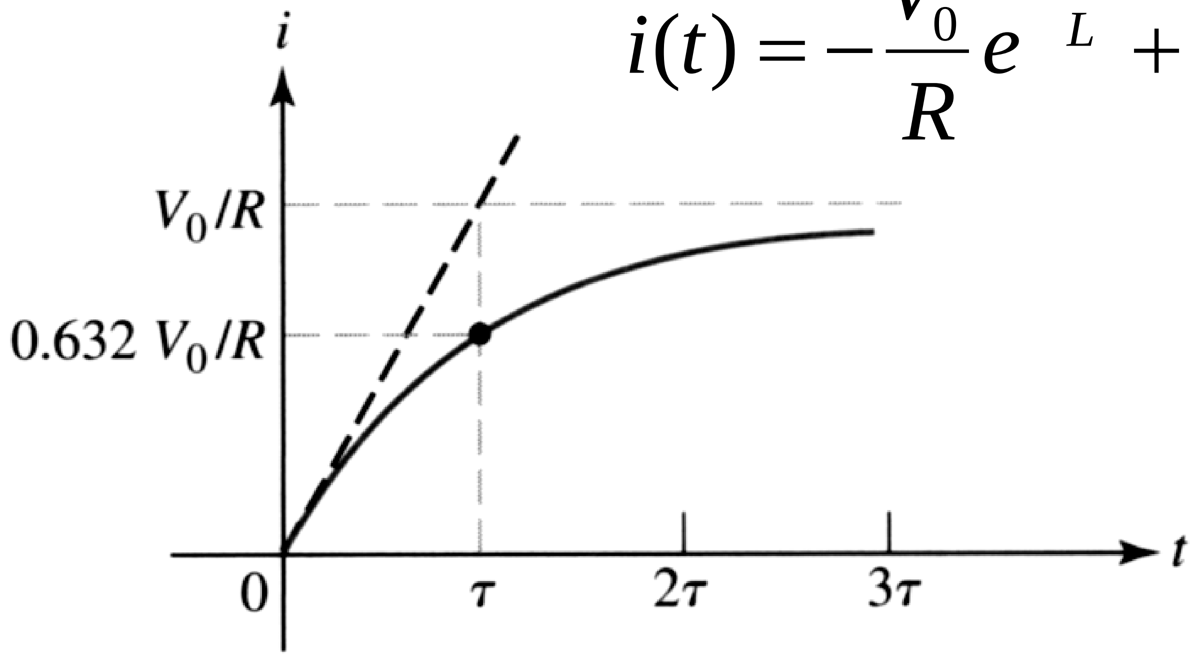
so

$$i(t) = -\frac{V_0}{R} e^{-\frac{Rt}{L}} + \frac{V_0}{R}$$

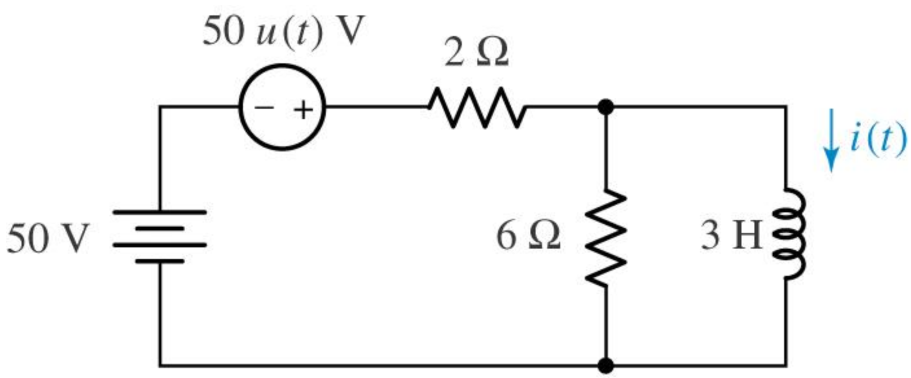
# Determination of the complete res.:



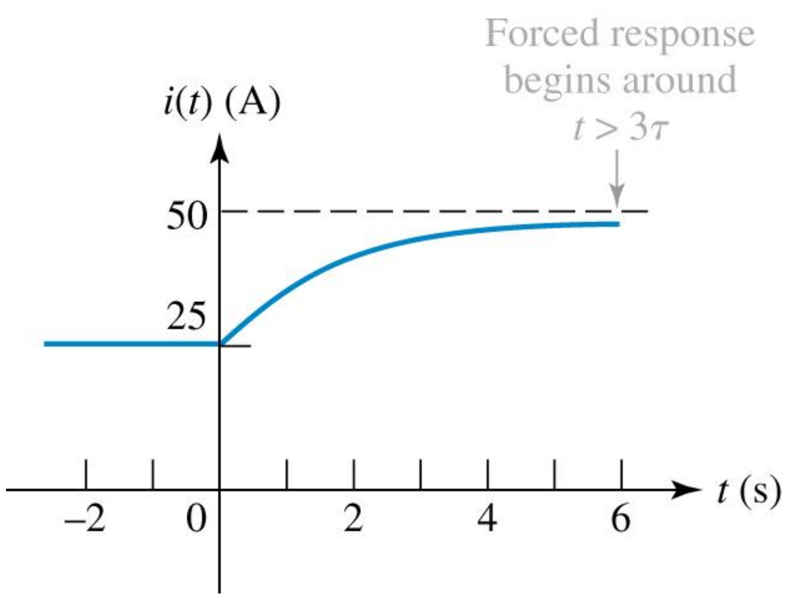

$$i(t) = -\frac{V_0}{R} e^{-\frac{Rt}{L}} + \frac{V_0}{R}$$



# Example 8.4:

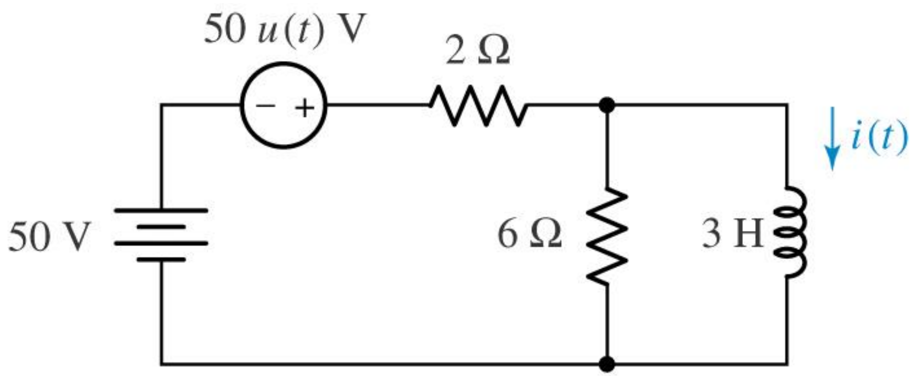



**Circuit for which a complete response *i*(t) is desired.**




**The desired current response as a function of time.**

# Example :




$$R_{eq} = 2 // 6 = \frac{2\times 6}{2+6} = 1.5\Omega$$

$$i(t) = i_f + i_n$$

$$i_n(t) = Ae^{\frac{-R_{eq}t}{L}} = Ae^{\frac{-t}{\tau}}, t > 0$$

$$i_f = \frac{100}{2} = 50A.$$

$$\therefore i(t) = 50 + Ae^{\frac{-1.5t}{3}}, t > 0$$

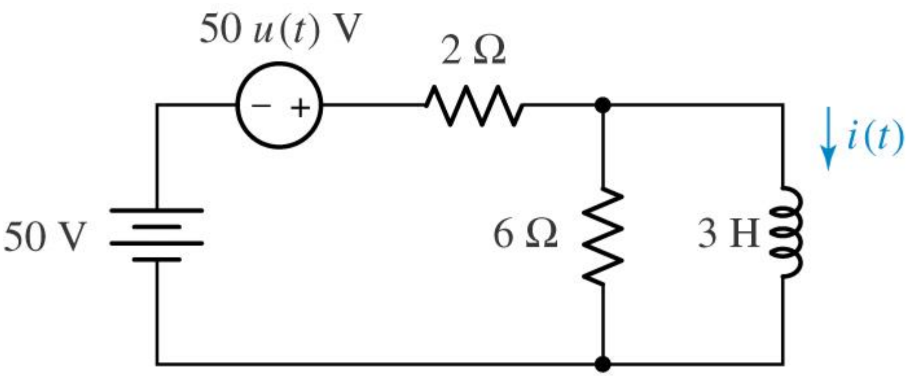
To find A, @t=0$^-$;

$$i(0^-) = \frac{50}{2} = 25A.$$

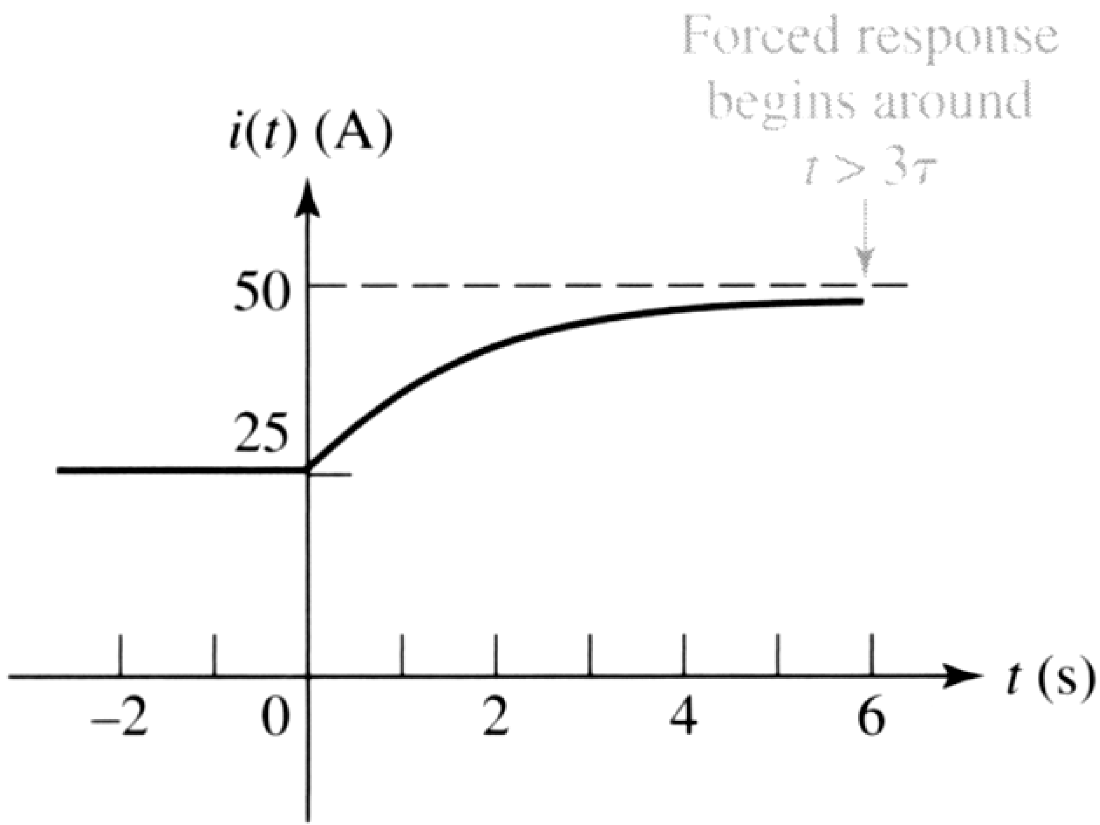
$$25 = 50 + A \qquad ; A = -25$$

$$\therefore i(t) = 50 - 25e^{-0.5t}, t > 0$$ or $$i(t) = 25 + 25\left(1 - e^{-0.5t}\right)u(t)A.$$

# Example :




$$i(t) = 25 + 25\left(1 - e^{-0.5t}\right)u(t)\,A.$$



# Practice: 8.8

A voltage source, $v_s = 20e^{-100t}u(t)$ V, is in series with a 200-Ω resistor and a 4-H inductor. Find the magnitude of the inductor current at $t$ equal to (a) $0^-$; (b) $0^+$; (c) 8 ms; (d) 15 ms.

$$v_S(0^-) = 0 \;\; \text{so} \;\; i_L(0^-) =$$

$$i_L(0^+) = i_L(0^-) \;\; \text{so} \;\; i_L(0^+) =$$

# Practice: 8.8

$$v_S(t) = 200 i_L(t) + v_L(t)$$

or

$$20e^{-100t} = 200 i_L + 4\frac{di_L}{dt} \qquad [1]$$

$$i_L(t) = i_f + i_n$$

$$i_n(t) = A\,e^{-t/\tau} \text{ where } \tau = \frac{L}{R} = \frac{4}{200} = 20 \text{ ms}$$

If we assume $i_f(t) = B\,e^{-100t}$ then $i_L(t) = A\,e^{-50t} + B\,e^{-100t}$

so $\frac{di_L}{dt} = -50A\,e^{-50t} - 100B\,e^{-100t}$

Substituting back in Eq. [1],

$$200 i_L + 4\frac{di_L}{dt} = 200A\,e^{-50t} + 200B\,e^{-100t} - 200A\,e^{-50t} - 400B\,e^{-100t} = -200B\,e^{-100t}$$

# Practice: 8.8

Thus, referring to Eq. [1], 20 = – 200 $B$ so $B = \dfrac{-20}{200} = -0.1$

Thus, $i_L(t) = Ae^{-50t} - 0.1e^{-100t}$

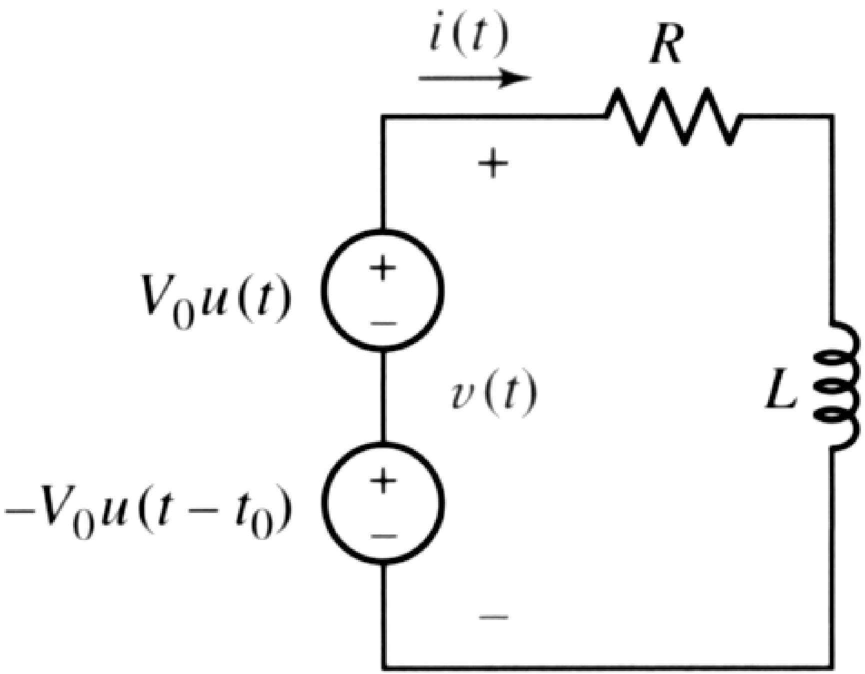
since $i_L(0^+) = A - 0.1 = 0$, $A = 0.1$ and $i_L(t) = 0.1(e^{-50t} - e^{-100t})$ amperes

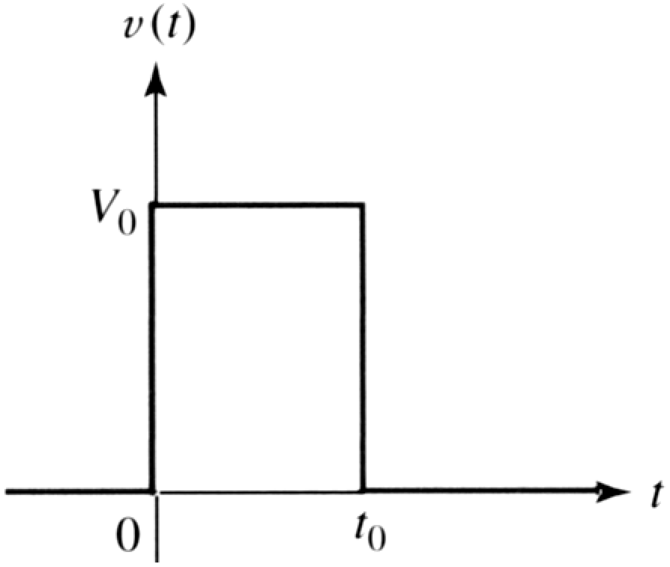
(c) $i_L(8\text{ ms}) =$ 22.10 mA

(d) $i_L(15\text{ ms}) =$ 24.92 mA

# Example 8.5:

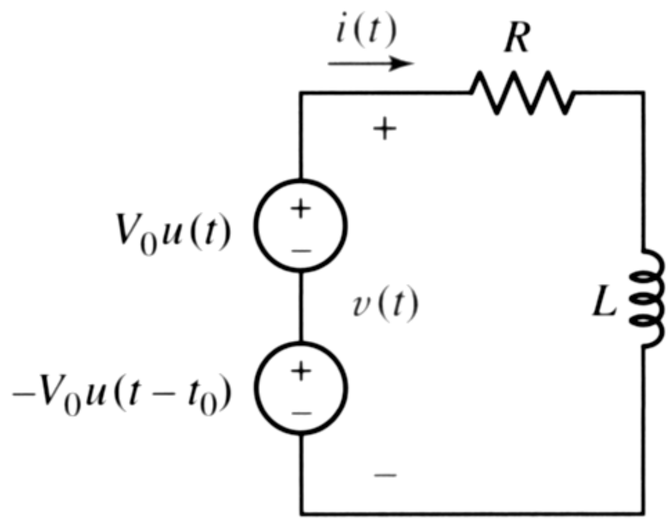
find the current response



# Example :

find the current response




$$i(t)=i_1(t)+i_2(t)$$

$$i_1(t)=\frac{V_0}{R}\left(1-e^{-\frac{Rt}{L}}\right), t>0$$

$$i_2(t)=-\frac{V_0}{R}\left(1-e^{-\frac{R(t-t_0)}{L}}\right), t>t_0$$

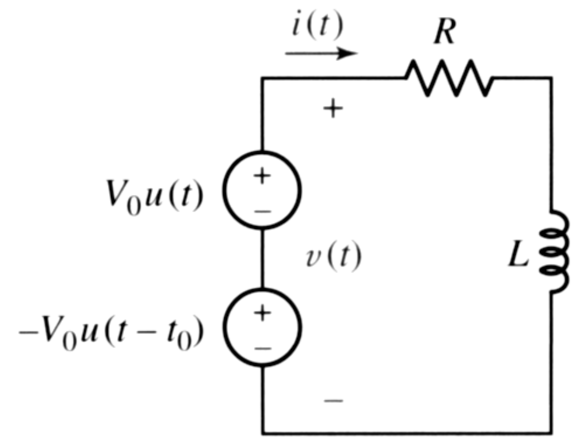
$$\therefore i(t)=\frac{V_0}{R}\left(1-e^{-\frac{Rt}{L}}\right), 0<t<t_0$$

and

$$\therefore i(t)=\frac{V_0}{R}\left(1-e^{-\frac{Rt}{L}}\right)-\frac{V_0}{R}\left(1-e^{-\frac{R(t-t_0)}{L}}\right), t>t_0$$

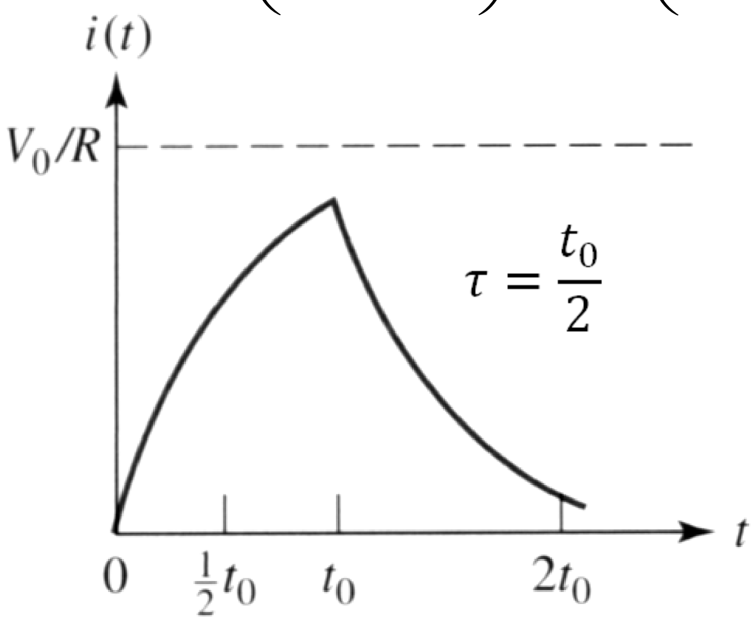
# Example :

find the current response

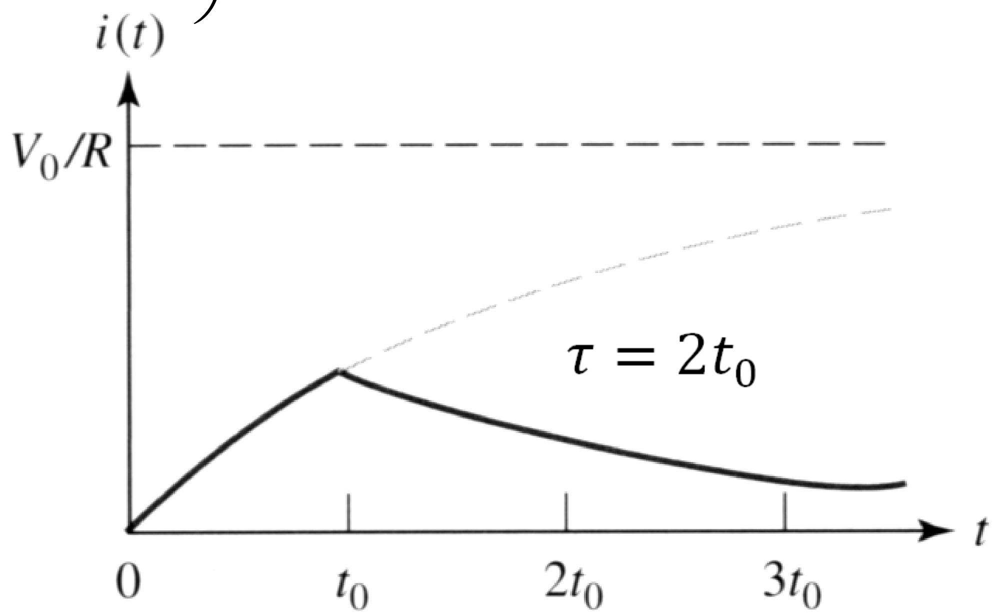



$$\therefore i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right), 0 < t < t_0$$

$$\therefore i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right) - \frac{V_0}{R}\left(1 - e^{-\frac{R(t-t_0)}{L}}\right), t > t_0$$




(a)



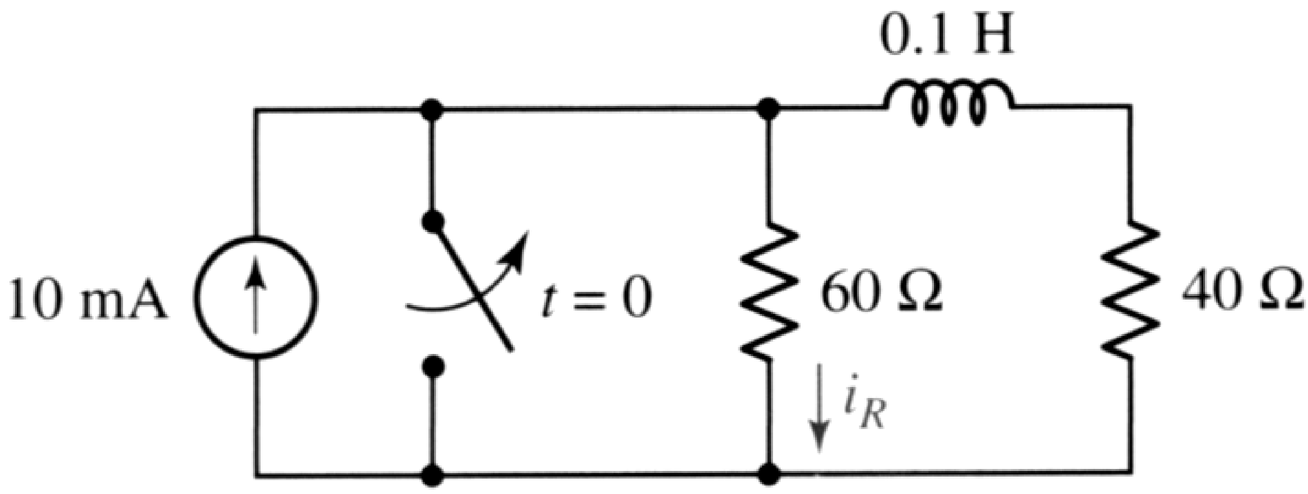

(b)

# Summary:

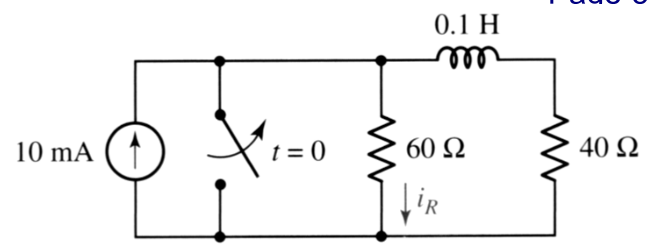
1. With all independent sources killed, simplify the circuit to determine $R_{eq}, L_{eq}$ and the time constant $\tau = L_{eq}/R_{eq}$ .
2. Viewing $L_{eq}$ as a short circuit, use dc analysis methods to find $i_L(0^-)$ the inductor current just prior to the discontinuity.
3. Again viewing $L_{eq}$ as a short circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent *it* by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$ .
5. Find $f(0^+)$ by using the condition *that* $i_L(0^+) = i_L(0^-)$. If desired, $L_{eq}$ may be replaced by a current source $i_L(0^+)$ [an open circuit *if* $i_L(0^+) = 0$ ] for this calculation. With the exception of inductor currents (and capacitor voltages), other currents and voltages in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$ , or total response = final value + (initial value — final value) $e^{-t/\tau}$ .

# Practice: 8.9

The circuit shown has been in the form shown for a very long time. The switch opens at $t = 0$. find $i_R$ at $t$ equal to (a) $0^-$; (b) $0^+$; (c) $\infty$; (d) 1.5 ms.



# Practice: 8.9




(a) $i_R(0^-) = \underline{0}$

(b) $i_L(0^-) = 0$ so $i_L(0^+) = 0$

Thus, all of the source current is shunted through the 60–Ω resistor; hence, $i_R(0^+) = \underline{10\text{ mA}}$

(c) $$i_R(\infty) = 10 \times \frac{40}{40+60} = \underline{4\text{ mA}}$$

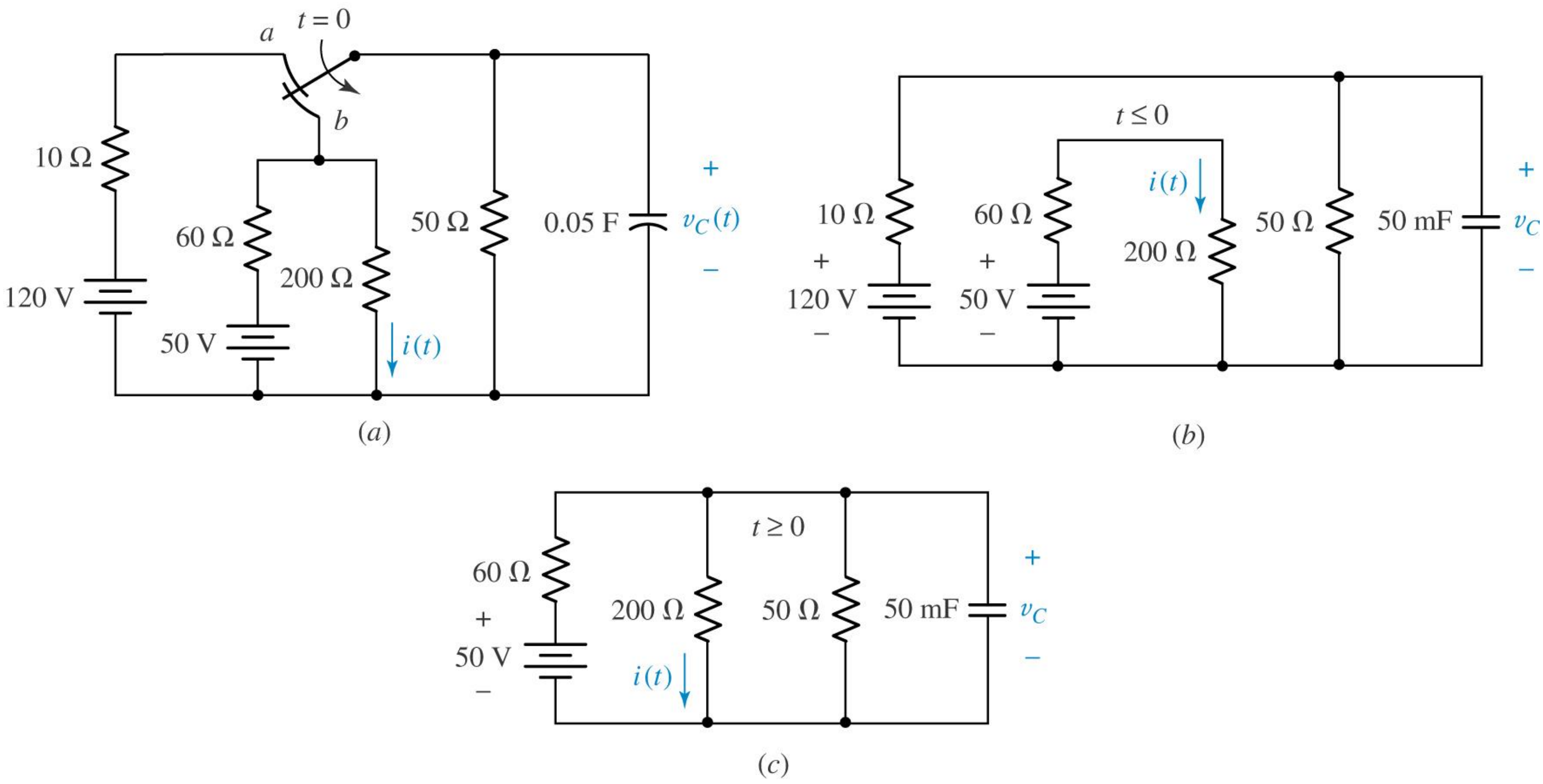
(d) $$\tau = \frac{L}{R_{eq}} = \frac{0.1}{40+60} = 1\text{ ms}$$

$$i_R(t) = i_R(\infty) + \left[i_R(0^+) - i(\infty)\right]e^{-t/\tau}$$
$$= 4 + [10-4]e^{-10^3 t}\text{ mA}$$

so $i_R(1.5\text{ ms}) = \underline{5.339\text{ mA}}$

# Driven RC Circuits: Example

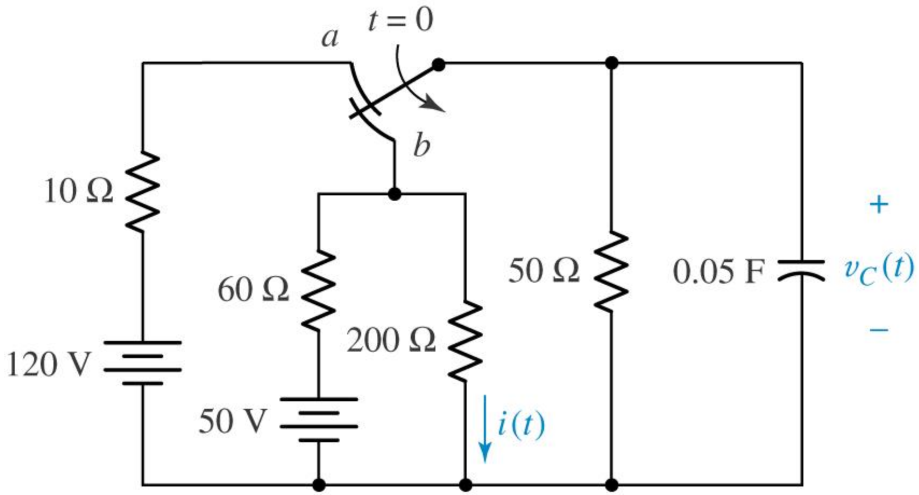
**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**

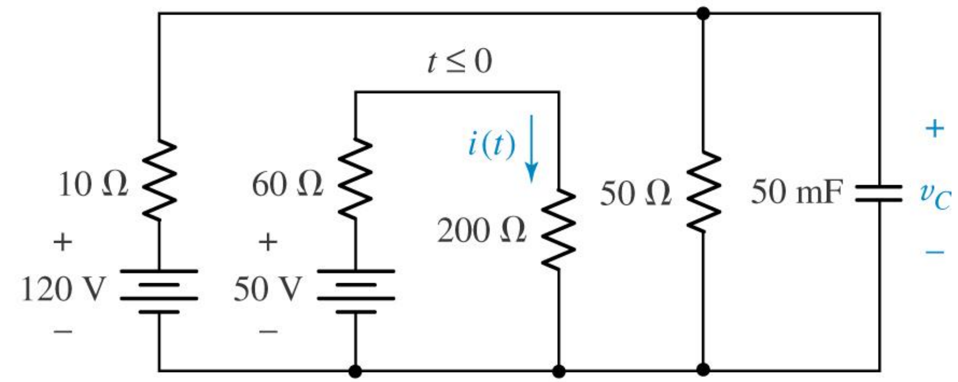



**(a) Original circuit; (b) circuit valid for $t \le 0$; (c) circuit for $t \ge 0$.**

# Driven RC Circuits: Example

**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**



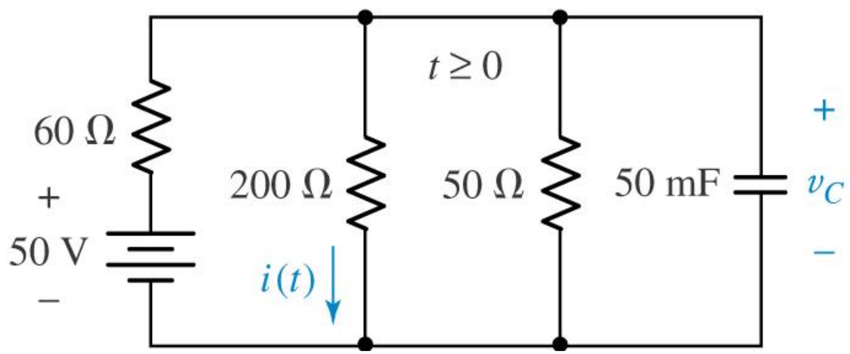

$$v_C(t) = v_{Cf} + v_{Cn}$$

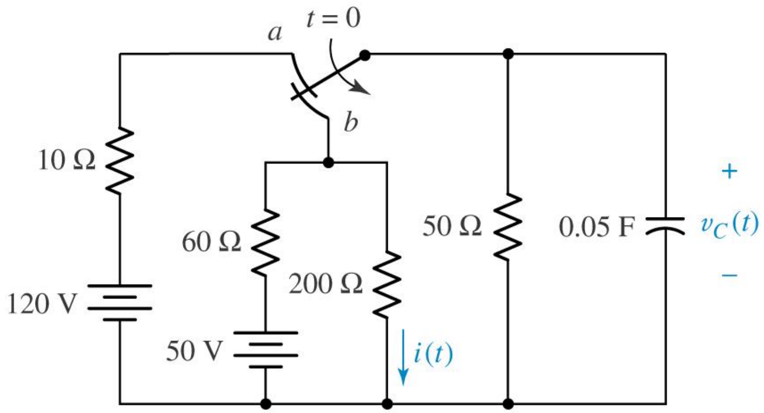
$$v_C(0^-) = \frac{50}{50+10}\cdot 120 = 100 = v_C(0^+)$$

$$i(0^-) = \frac{50}{260} = 192.3mA.$$

# Driven RC Circuits: Example

**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**




$$R_{eq} = 60 // 200 // 50$$
$$= \frac{1}{1/60 + 1/200 + 1/50} = 24\Omega$$

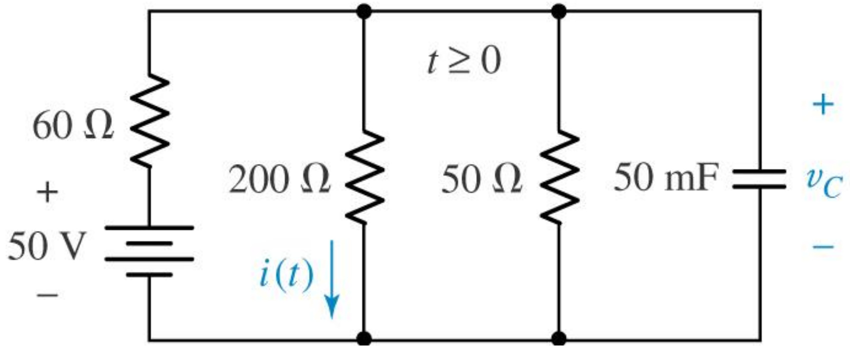
$$v_C(t) = v_{Cf} + v_{Cn}$$

$$v_{Cf} = \frac{50\times200/(50+200)}{60 + 50\times200/(50+200)} \cdot 50 = 20V.$$

$$v_{Cn}(t) = Ae^{\frac{-t}{R_{eq}C}} = Ae^{\frac{-t}{1.2}}$$

# Driven RC Circuits: Example

**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**




$$\therefore v_C(t) = 20 + Ae^{\frac{-t}{1.2}}$$

To find A, @t=0;

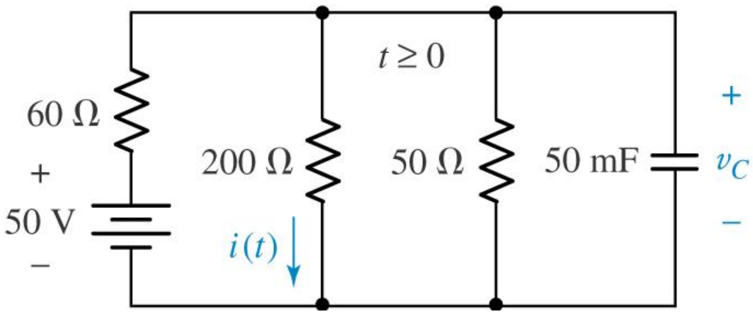
$$v_C(0^+) = v_C(0^-) = 100 = 20 + A$$

$$\therefore A = 80$$

$$; v_C(t) = 20 + 80e^{\frac{-t}{1.2}}$$

# Driven RC Circuits: Example

**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**




$$i(0^+) = \frac{v_C(0^+)}{200} = \frac{100}{200} = 0.5A.$$

$$i_f = i(t=\infty) = \frac{50}{60 + {}^{50\times 200}\!/_{(50+200)}} \cdot \frac{50}{50+200} = 0.1A.$$

$$\therefore i(t) = i_f + i_n(t) = 0.1 + Ae^{\frac{-t}{1.2}}$$

To find A, @t=0;

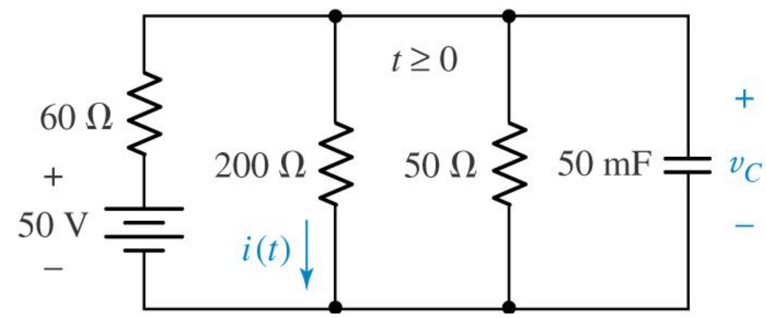
$$i(0^+) = 0.5 = 0.1 + A$$

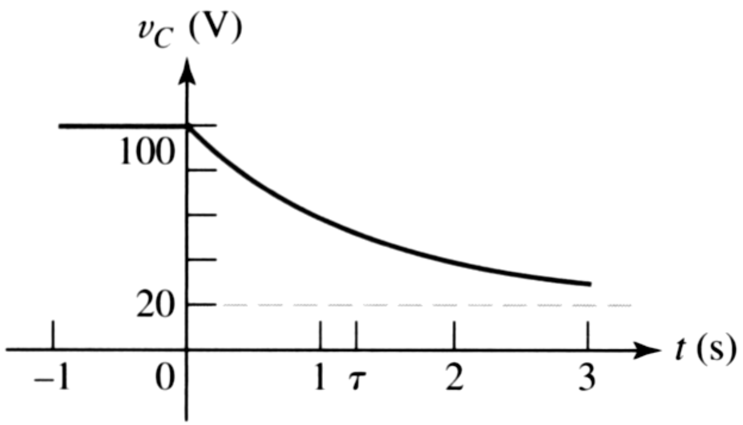
$$\therefore A = 0.4$$

$$\therefore i(t) = 0.1 + 0.4e^{\frac{-t}{1.2}}, t > 0$$

# Driven RC Circuits: Example

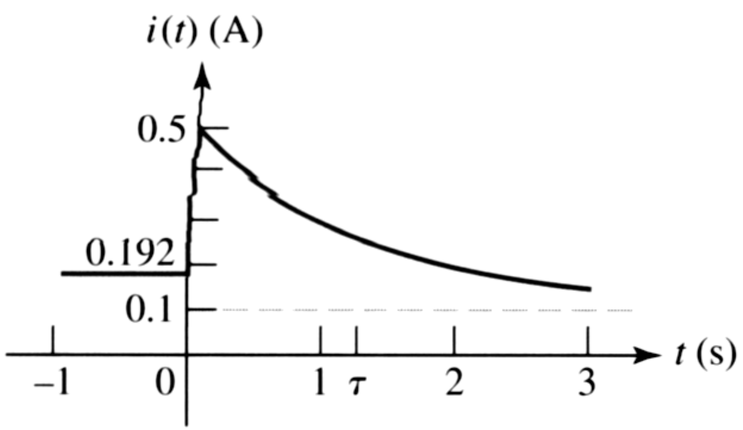
**Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200-Ω resistor of the circuit shown in (a).**




(a)

$$v_C(t) = 20 + 80e^{\frac{-t}{1.2}}$$

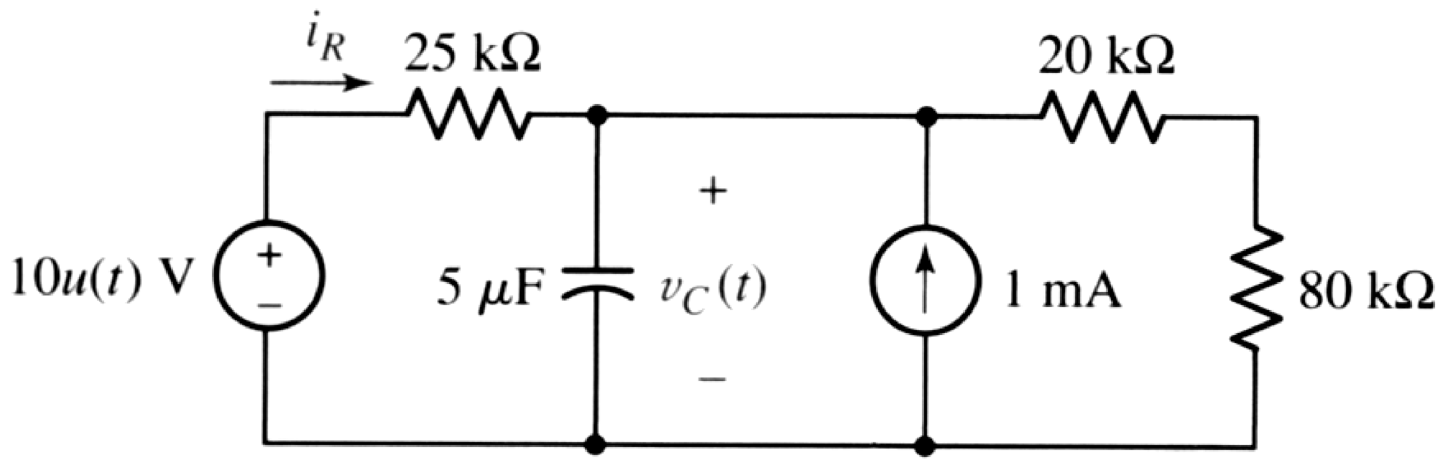



(b)

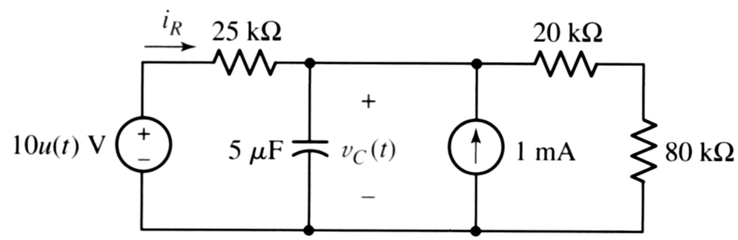
$$i(t) = 0.1 + 0.4e^{\frac{-t}{1.2}}$$

# Practice: 8.10

For the circuit, find $v_C(t)$ at $t$ equal to (a) $0^-$; (b) $0^+$; (c) $\infty$; (d) 0.08 s.



# Practice: 8.10




(a) At $t = 0^-$, only the current source is on, so

$$v_C(0^-) = 1 \times [25 // (20 + 80)] = \underline{20\ \text{V}}$$

(b) $v_C(0^+) = v_C(0^-)$, so $v_C(0^+) = \underline{20\ \text{V}}$

(c) At $t = \infty$, both sources are on, so

$$v_C(\infty) = 1 \times [25 // (20 + 80)] + 10 \times \frac{(100)}{125}$$

$$= 20 + 8 = \underline{28\ \text{V}}$$

(d) $v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right] e^{-t/\tau}$

where $\tau = R_{eq}C$

$R_{eq} = 25//100 = 20\ \text{k}\Omega$, so $\tau = 100$ ms

Thus, $v_C(80\ \text{ms}) = 28 + [20 - 28]e^{-80/100}$

$= \underline{24.41\ \text{V}}$

# Summary :

1. With all independent sources killed, simplify the circuit to determine $R_{eq}, C_{eq}$ and the time constant $\tau = R_{eq}C_{eq}$.
2. Viewing $C_{eq}$ as an open circuit, use dc analysis methods to find $v_C(0^-)$ the capacitor voltage just prior to the discontinuity.
3. Again viewing $C_{eq}$ as an open circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent *it* by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$.
5. Find $f(0^+)$ by using the condition *that* $v_C(0^+) = v_C(0^-)$. If desired, $C_{eq}$ may be replaced by a voltage source $v_C(0^+)$ [a short circuit *if* $v_C(0^+) = 0$] for this calculation. With the exception of capacitor voltages (and inductor currents), other currents and voltages in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$, or total response = final value + (initial value — final value) $e^{-t/\tau}$.

# Reference:

**W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.**