Electrical and Electronic Measurements:

Power and Energy Measurement

1

Work, Energy and Power

- Work is an activity of force and movement in the direction of force (Joules)
- Energy is the capacity for doing work (Joules)
- Power is the rate of using energy (Watt)

$P = W / t$, Work done in the time

$= E / t$, Energy transferred in the time

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Electrical Power

- DC: Instantaneous power at time t

$$P(t) = I(t)\,V(t)$$
$$= I^2(t)\,R$$
$$= V^2(t) / R$$

- AC: Mean power per one cycle

$$P_{avg} = 1/T \int_{t=0 \to T} P(t)\,dt$$
$$= I_{rms}^2\,R$$
$$= V_{rms}^2 / R$$

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Apparent and True Power

- True power is the power used by the load in Watt unit (W),

$$P = I^2R = V^2/R$$

- Reactive power in Volt-Amps-Reactive unit (VAR),

$$Q = I^2X = V^2/X$$

- Apparent power is the power that is applied to the load in Volt-Amps unit (VA),

$$S = I^2Z = V^2/Z$$

where impedance $Z = R + jX$

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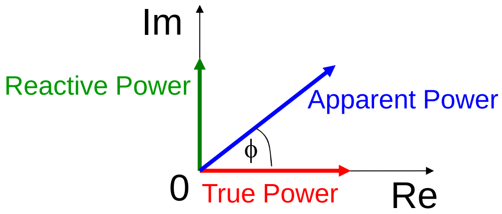
Power Factor

$\cos \phi$ = True Power / Apparent Power

- Pure resistance, $\cos(0°) = 1 \Rightarrow$ perfect
- Pure inductance, $\cos(90°) = 0$
- Pure capacitance, $\cos(–90°) = 0$



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Power Factor (Cont'd)

$P = I\ V \cos\phi$

- Up to a kind of loads being powered
- In phase or $\phi = 0$ (resistor)
- Lagging or current lags voltage, $\phi > 0$ (inductor)
- Leading or current leads voltage, $\phi < 0$ (capacitor)
- Lower power factor (higher $| \phi |$), higher losses
- Need to improve power factor

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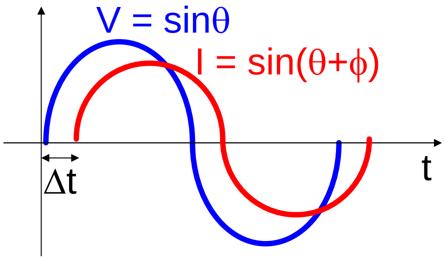
Power Factor (Cont'd)

$I = \sin\omega(t+\Delta t)$
$= \sin(\omega t+\omega\Delta t)$
$= \sin(\theta+\phi)$



$\phi = 0° \Rightarrow I = \sin(\theta)$, resistor

$\phi = 90° \Rightarrow I = \sin(\theta+90°)$
$= \cos(\theta)$, inductor

$\phi = -90° \Rightarrow I = \sin(\theta-90°)$
$= -\cos(\theta)$, capacitor

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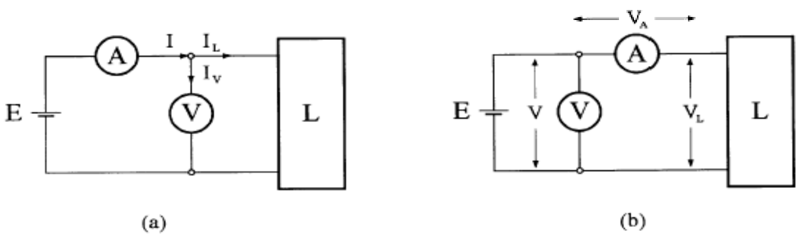
Power Measurement in DC Circuit

- Ammeter measures current which flow into the voltmeter and load, shown in (a).
- Voltmeter measures voltage drop across the ammeter in addition to that dropping across the load, shown in (b).



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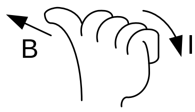
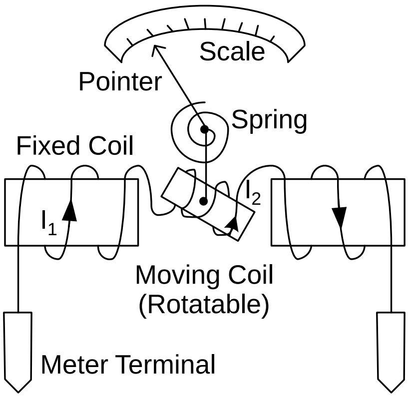
Wattmeter

- To measure the true power
- Two fixed coil is used to provide a magnetic field that is proportional to a current passing through it, instead of using a permanent magnet.



Right Hand's Rule

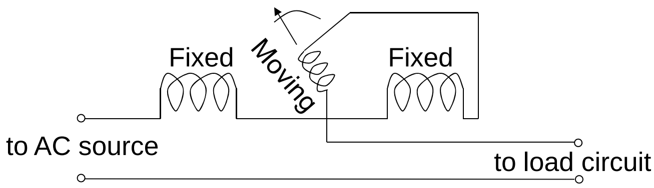


Electrodynamometer as an ammeter (I^2)

Wattmeter (Cont'd)

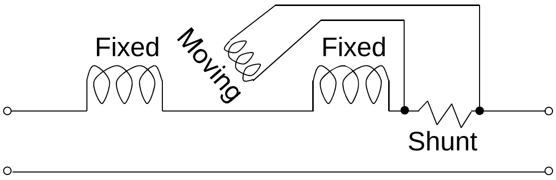
- Fixed coils, $B \propto I_1$
- Moving coil, Torgue $\propto B\, I_2$

$$\theta = K\, I_1\, I_2$$
$$= K\, I^2 \qquad , I = I_1 = I_2$$
$$\propto I^2 \quad \Rightarrow \text{ammeter (non-linear scale)}$$



Wattmeter (Cont'd)

- Full-scale deflection current 5-100 mA
- For larger FSD up to 20 A, it is shunted by a low resistance.
- Electrodynamometer can be used for both DC and AC (rms) measurement.



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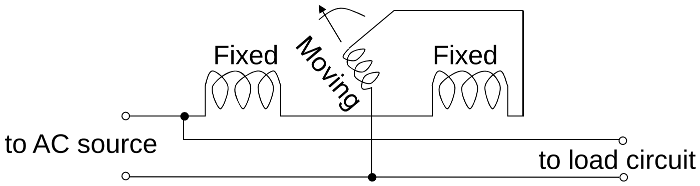
Wattmeter (Cont'd)

- Electrodynamometer as a voltmeter
  $\theta \propto V^2$



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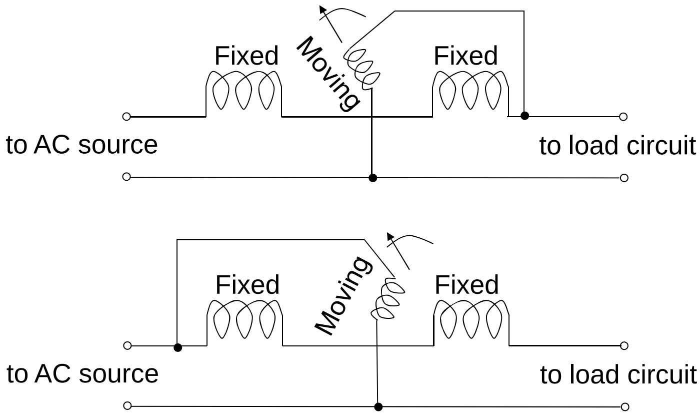
Wattmeter (Cont'd)

- Electrodynamometer as a wattmeter
  $\theta \propto IV$
- Loading error!



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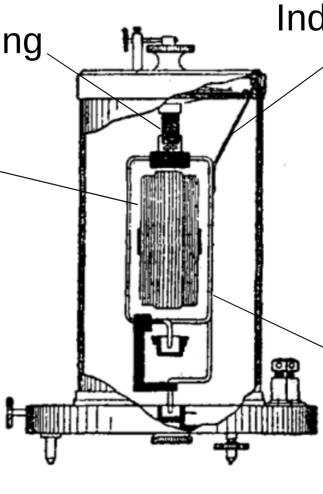
Wattmeter (Cont'd)

- Electrodynamometer



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Advantages

- Used on AC as well as DC measurements
- Free from eddy current and hysteresis error (the difference in two measurements of the same quantity when the measurement is approached from opposite directions)
- Accurate measurement of RMS values of voltages irrespective of waveforms.
- Because of precision grade accuracy and same calibration for AC and DC measurements, it is useful as transfer type and calibration instruments.

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Disadvantages

- As the instrument has square law response, the scale is non-uniform.
- These instruments have small torque/weight ratio, so the frictional error is considerable.
- More costly than PMMC instruments.
- Adequate screening of the movements against stray magnetic fields is essential.
- Power consumption is comparably high because of their construction.

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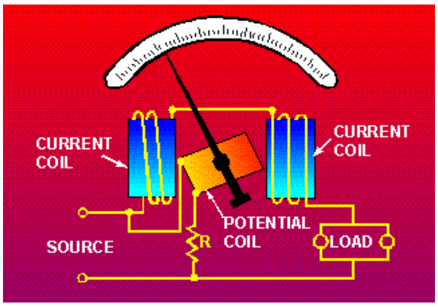
Single-Phase Wattmeter

- Based on the principles of the dynamometer
- The current passing through moving coil and load is proportional to the potential difference across the load.



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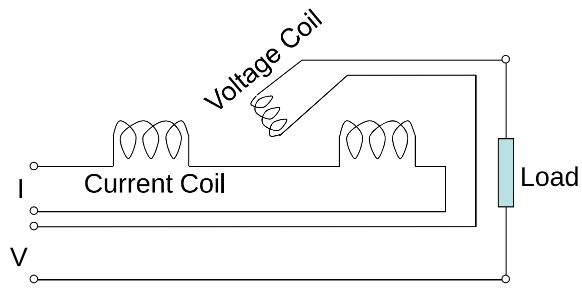
Single-Phase Wattmeter (Cont'd)



$$\theta = K\, I_1\, I_2$$
$$= K\, I\, (V/R)$$
$$= (K/R)\, (IV)$$
$$\propto P$$

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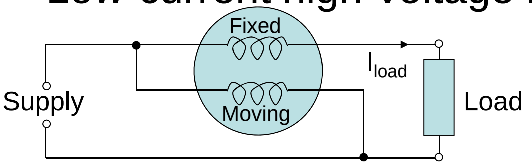
Single-Phase Wattmeter (Cont'd)

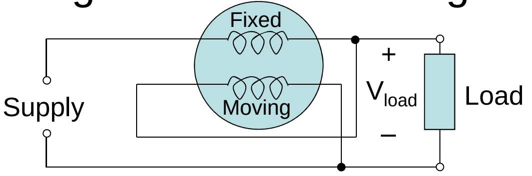
- Low-current high-voltage load



$P_{reading} = I_{load}^2(R_{fixed}+R_{load})$
$P_{load} = I_{load}^2 R_{load}$

Wattmeter reads high due to the potential drop across the fixed coil (if neglect the reactance).

- High-current low-voltage load



$$P_{reading} = V_{load}^2 / (R_{moving} \parallel R_{load}) = V_{load}^2 \frac{(R_{moving}+R_{load})}{R_{moving}R_{load}}$$

$P_{load} = V_{load}^2 / R_{load}$

Wattmeter reads high due to the current through the movable coil (if neglect the reactance).

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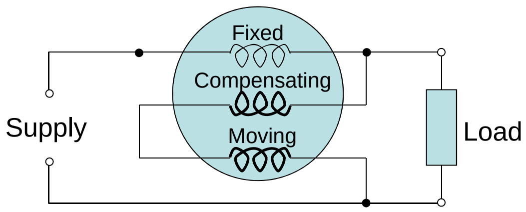
Single-Phase Wattmeter (Cont'd)

- Compensated wattmeter



To reduce the systematic error, the compensating coil current is, however, in the opposite direction to the load current and cancels out the proportion of the magnetic flux due to the moving coil current (the same number of turns).

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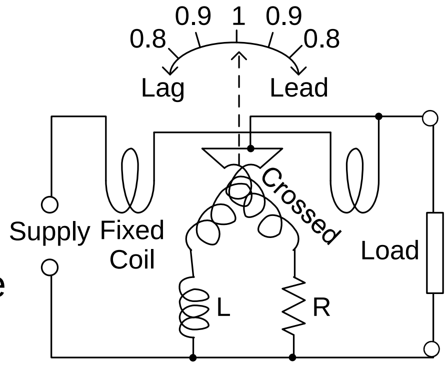
Crossed-Coil Power Factor Meter

- Two moving coils are crossed and connected in series to an inductor and resistor.
- For PF = 1, the coil with R will be in phase with the line current (moving), while the coil with L will be out of phase by 90° (no moving).
- For PF = 0, coil L moves and coil R does not move.



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Three-Phase Voltages

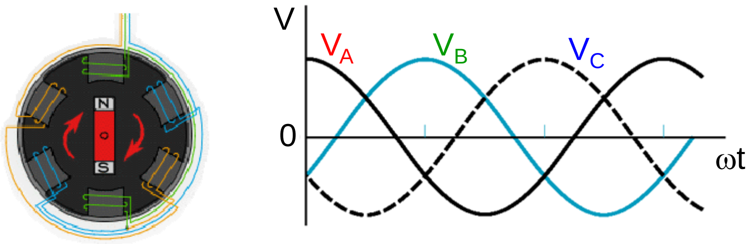
- Balanced three-phase voltage

$V_A = \sqrt{2}\ V \cos(\omega t)$

$V_B = \sqrt{2}\ V \cos(\omega t - 120°)$

$V_C = \sqrt{2}\ V \cos(\omega t - 240°)$

$= \sqrt{2}\ V \cos(\omega t + 120°)$



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Three-Phase Power Distribution

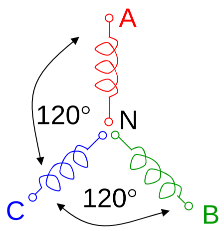
The three-phase system uses less conductor material to transmit electric power that the others.

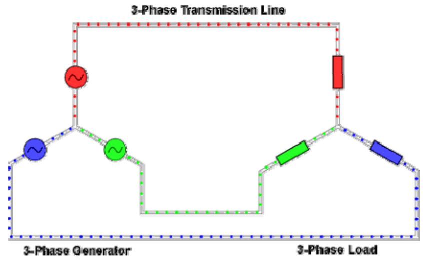
Y-Type, $\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN}$

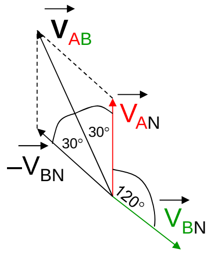
$$|\vec{V}_{AB}| = |\vec{V}_{AN}| \cos 30° + |\vec{V}_{BN}| \cos 30°$$

$$= \sqrt{3}\ |\vec{V}_{AN}| \qquad , |\vec{V}_{AN}| = |\vec{V}_{BN}|$$

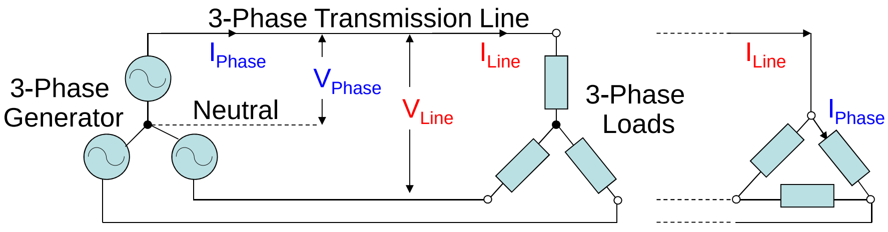
$$V_{Line} = \sqrt{3}\ V_{Phase}$$







Three-Phase Loads



Star-Type,
$V_{Line} = \sqrt{3}\ V_{Phase}$
$I_{Line} = I_{Phase}$

Delta-Type,
$V_L = V_P$
$I_L = \sqrt{3}\ I_P$

Star-type balanced-loads,

$$P_1 = P_2 = P_3 = P = I_P\, V_P \cos\phi$$

$$P_{total} = P \times 3 = 3\, I_P\, V_P \cos\phi$$

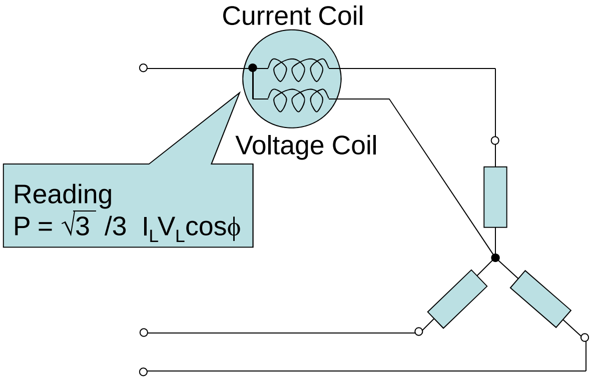
$$= 3\, (I_L)\, (V_L / \sqrt{3}\,) \cos\phi$$

$$= \sqrt{3}\ I_L\, V_L \cos\phi$$

Similarly, $Q = \sqrt{3}\ I_L\, V_L \sin\phi$

Three-Phase Wattmeter

- For three-phase system with balanced load, just using single-phase wattmeter, P×3



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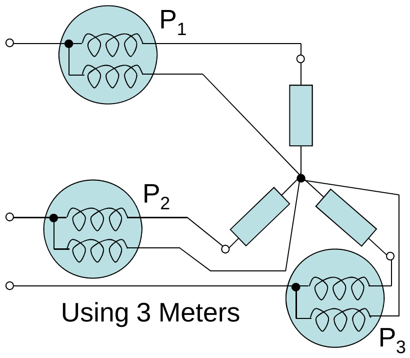
Unbalanced-Load 3-Phase Wattmeter



By using 3 wattmeters,

$P_{total} = P_1 + P_2 + P_3$

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Balanced-Load 3-Phase Wattmeter

Brondel's theory,

$P = I_1V_1 + I_2V_2 + I_3V_3$

Kirchoff's law,

$V_1 = V_3 + V_1'$

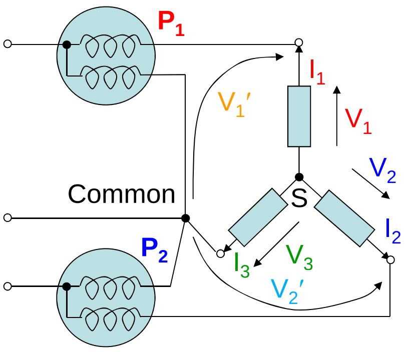
$V_2 = V_3 + V_2'$

Therefore, $\underbrace{}_{}$

$P = \underbrace{(I_1+I_2+I_3)}_{\to 0}V_3$

$\quad + I_1V_1' + I_2V_2'$

$= \mathbf{P_1} + \mathbf{P_2}$



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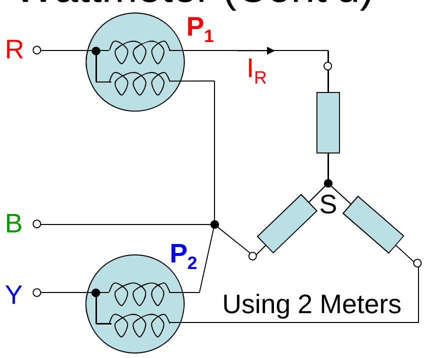
Balanced-Load 3-Phase Wattmeter (Cont'd)

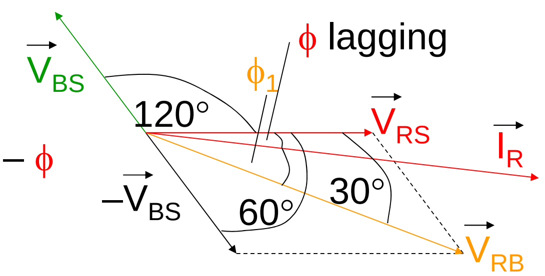
By using 2 wattmeters,

$\mathbf{P_1} = I_R V_{RB} \cos\phi_1$

$\vec{V}_{RB} = \vec{V}_{RS} - \vec{V}_{BS}$

Balanced, $|\vec{V}_{RS}| = |\vec{V}_{BS}|$





$\phi_1 = 30° - \phi$

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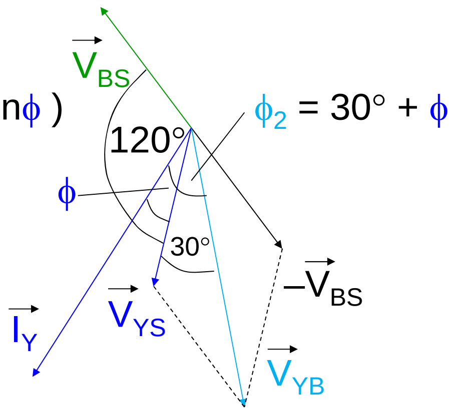
Balanced-Load 3-Phase Wattmeter (Cont'd)

$P_1 = I_L V_L \cos(30° - \phi)$
$\quad = I_L V_L \left(\sqrt{3}/2 \cos\phi + 1/2 \sin\phi \right)$

Similarly,

$P_2 = I_L V_L \left(\sqrt{3}/2 \cos\phi - 1/2 \sin\phi \right)$



Note: $\cos(\theta \mp \phi) = \cos\theta \cos\phi \pm \sin\theta \sin\phi$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

Total Power,

$$P_{total} = P_1 + P_2$$
$$= \sqrt{3}\; I_L V_L \cos\phi$$

The same as using 1 meter for balanced load system

and $P_1 - P_2 = I_L V_L \sin\phi$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

Reactive Power,

$Q = \sqrt{3}\ I_L V_L \sin\phi = \sqrt{3}\ (P_1 - P_2)$

$(P_1 – P_2)/(P_1 + P_2) = I_L V_L \sin\phi / \sqrt{3}\ I_L V_L \cos\phi$

$= \tan\phi / \sqrt{3}$

$\tan\phi = \sqrt{3}\ (P_1 - P_2) / (P_1 + P_2)$

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Balanced-Load 3-Phase Wattmeter (Cont'd)

$\sin^2\phi + \cos^2\phi = 1$

$\tan^2\phi + 1 = 1/\cos^2\phi$

$3(P_1 – P_2)^2/(P_1 + P_2)^2 + 1 = 1/\cos^2\phi$

Power factor,

$\cos\phi = \sqrt{1 / [\,1 + 3(P_1 – P_2)^2/(P_1 + P_2)^2\,]}$

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Watt-Hour-Meter

- To measure the electrical energy supplied to industrial and domestic consumers in Joules.
- Meters for smaller services, such as small residential customers, can be connected directly in-line between source and customer.
- For larger loads, more than about 200 ampere of load, current transformers are used, so that the meter can be located other than in line with the service conductors.
- The meters fall into two basic categories, electromechanical and electronic.

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Watt-Hour-Meter (Cont'd)

Electricity meters operate by continuously measuring the instantaneous voltage and current to give energy used in Unit (kW-hr).

Generated Torque by Eddy Current = k_g I V $\cos\phi$

Magnetic Breaking Torque = k_b N

where N is number of revolution per unit time

(gearing to a mechanical counter to count watt-hours)

$$k_b N = k_g I V \cos\phi$$

$$N = (k_g/k_b) IV\cos\phi$$

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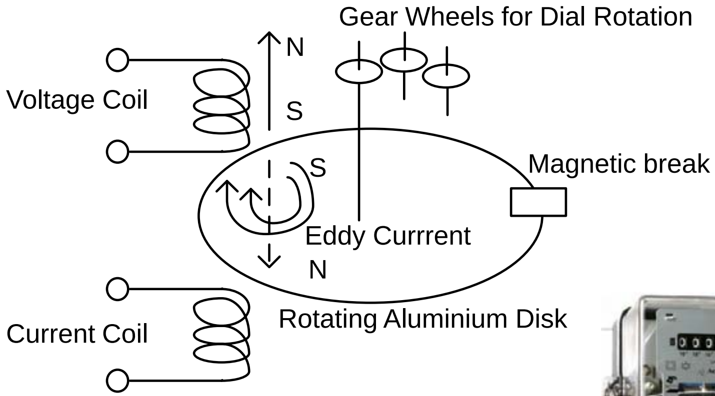
Watt-Hour-Meter (Cont'd)



Therefore, number of revolution refers to the energy used by loads.

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References

- P. Purkait, B. Biswas, S. Das and C. Koley, "Electrical and Electronics Measurements and Instrumentation"
- https://en.wikipedia.org/wiki/Electricity_meter#Types_of_meters

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