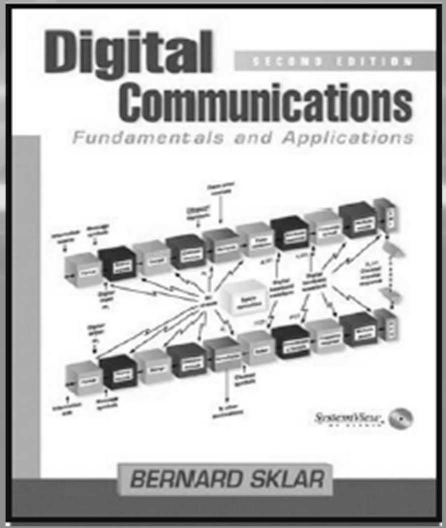


ENE 467
Digital Communications

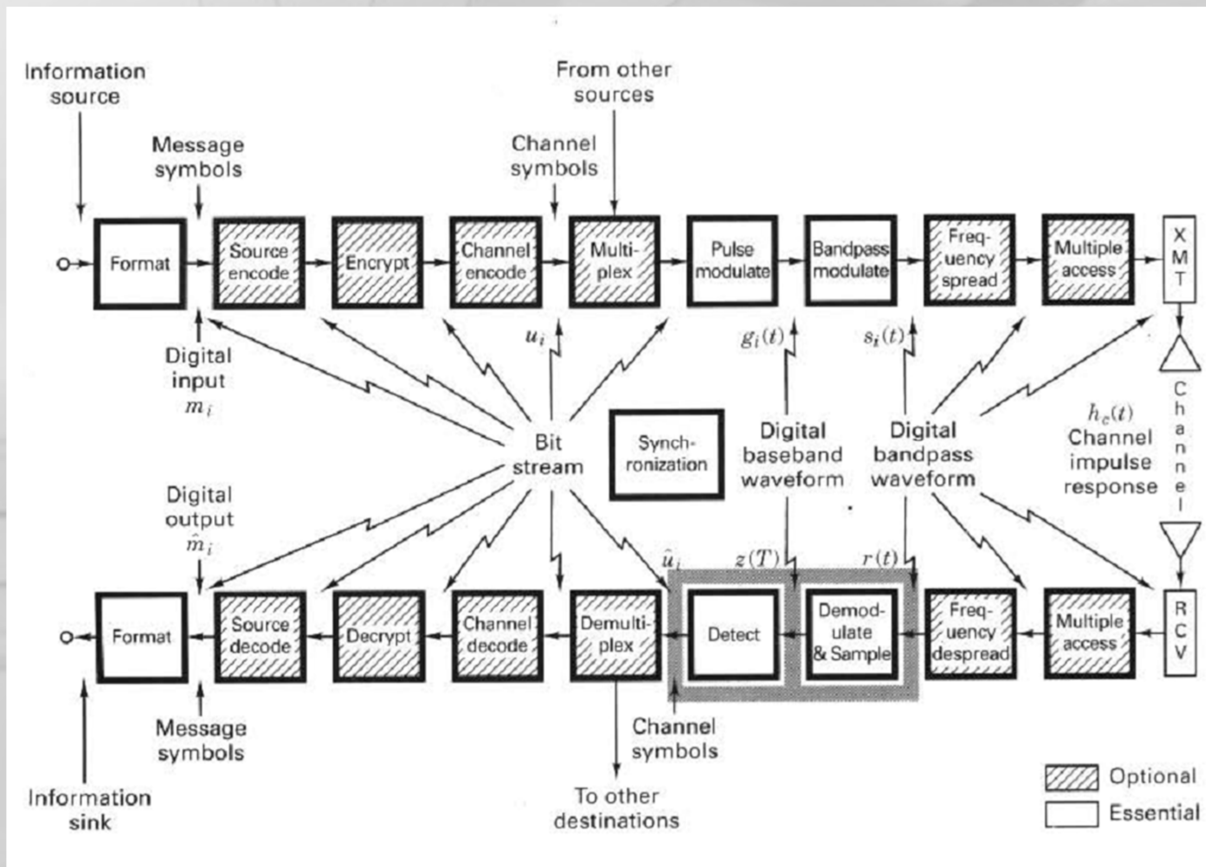
TEACHING BY

ASST. PROF. SUWAT PATTARAMALAI, PH.D.

3. Baseband Demodulation/Detection

- Outcome
  - Can explain Signals and Noise (Error, detection, SNR, and Figure of Merit)
  - Can design detection of binary signals in Gaussian Noise (ML receiver, Matched filter, Correlator, Error performance and probability)
  - Can explain Inter-symbol Interference (ISI): Pulse shaping
  - Can design Equalization (channel characteristic)
  - Can explain Eye-pattern

3. Baseband Demodulation/Detection



3. Baseband Demodulation/Detection

- Signals and Noise
  - Error Performance Degradation in Communication Systems
    - Inter-symbol Interference (ISI)
    - Thermal noise (white noise: AWGN) $G_n(f) = N_0/2$
  - Demodulation and Detection
    - Demodulation
      - Recovery of waveform
    - Detection
      - Decision making process

$$s_i(t) = \begin{cases} s_1(t) & 0 \le t \le T \quad \text{for a binary 1} \\ s_2(t) & 0 \le t \le T \quad \text{for a binary 0} \end{cases}$$

$$r(t) = s_i(t) * h_c(t) + n(t) \qquad i = 1, \ldots, M$$

$h_c(t)$ is an impulse function

$$r(t) = s_i(t) + n(t) \qquad i = 1, 2, \qquad 0 \le t \le T$$

3. Baseband Demodulation/Detection

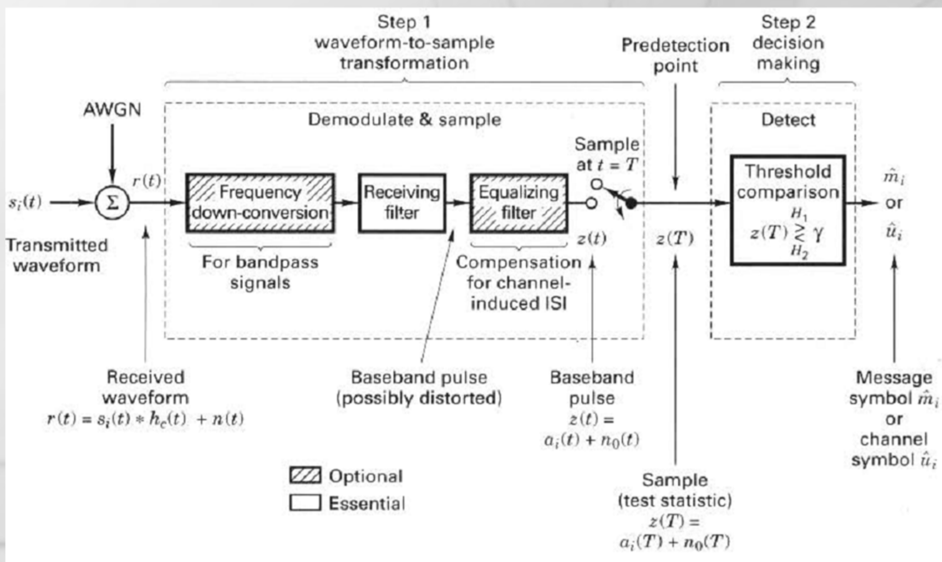


Figure 3.1 Two basic steps in the demodulation/detection of digital signals.

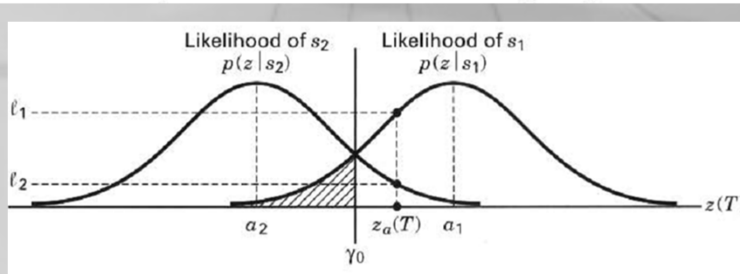


Figure 3.2 Conditional probability density functions: $p(z|s_1)$ and $p(z|s_2)$.

$$z(T) = a_i(T) + n_0(T) \qquad i = 1, 2$$

$$p(n_0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right]$$

$$p(z|s_1) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_1}{\sigma_0}\right)^2\right]$$

and

$$p(z|s_2) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_2}{\sigma_0}\right)^2\right]$$

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

3. Baseband Demodulation/Detection

- A Vector View of Signals and Noise
  - Orthogonal space: a space characterized by a set of N linearly independent functions called basis functions

$$\int_0^T \psi_j(t)\psi_k(t)\,dt = K_j\delta_{jk} \qquad 0 \le t \le T \qquad j, k = 1, \ldots, N$$

Kronecker delta function

$$\delta_{jk} = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{otherwise} \end{cases}$$

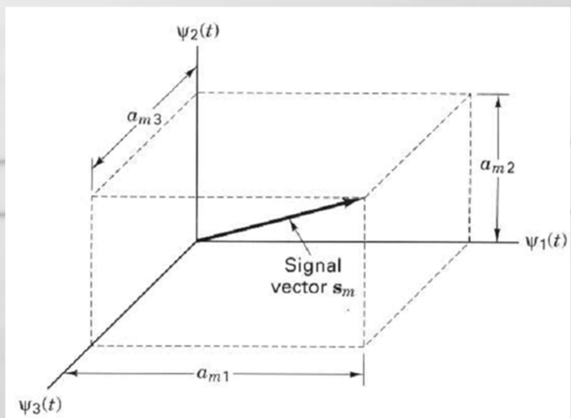


Figure 3.3 Vectorial representation of the signal waveform $s_m(t)$.

$$E_j = \int_0^T \psi_j^2(t)\,dt = K_j$$

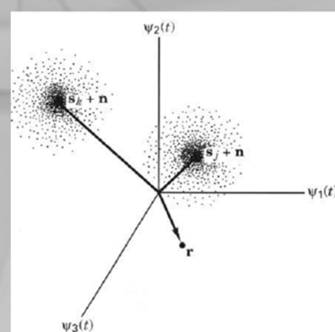


Figure 3.4 Signals and noise in a three-dimensional vector space.

$$\begin{aligned} s_1(t) &= a_{11}\psi_1(t) + a_{12}\psi_2(t) + \cdots + a_{1N}\psi_N(t) \\ s_2(t) &= a_{21}\psi_1(t) + a_{22}\psi_2(t) + \cdots + a_{2N}\psi_N(t) \\ &\vdots \\ s_M(t) &= a_{M1}\psi_1(t) + a_{M2}\psi_2(t) + \cdots + a_{MN}\psi_N(t) \end{aligned}$$

$$s_i(t) = \sum_{j=1}^{N} a_{ij}\,\psi_j(t) \qquad i = 1, \ldots, M \qquad N \le M$$

$$a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\psi_j(t)\,dt \qquad i = 1, \ldots, M \quad 0 \le t \le T \quad j = 1, \ldots, N$$

$$\mathbf{s}_i = (a_{i1}, a_{i2}, \ldots, a_{iN}) \qquad i = 1, \ldots, M$$

3. Baseband Demodulation/Detection

- Waveform Energy

$$
\begin{aligned}
E_i &= \int_0^T s_i^2(t)\,dt = \int_0^T \Big[\sum_j a_{ij}\,\psi_j(t) \Big]^2 dt \\
&= \int_0^T \sum_j a_{ij}\,\psi_j(t) \sum_k a_{ik}\psi_k(t)\,dt \\
&= \sum_j \sum_k a_{ij}\,a_{ik} \int_0^T \psi_j(t)\psi_k(t)\,dt \\
&= \sum_j \sum_k a_{ij}\,a_{ik} K_j \delta_{jk} \\
&= \sum_{j=1}^N a_{ij}^2 K_j \qquad i = 1, \dots, M
\end{aligned}
$$

$$E_i = \sum_{j=1}^N a_{ij}^2$$

- Ordinary Fourier Transforms
  - Basis functions are sine and cosine harmonic function

3. Baseband Demodulation/Detection

Example 3.1 Orthogonal Representation of Waveforms

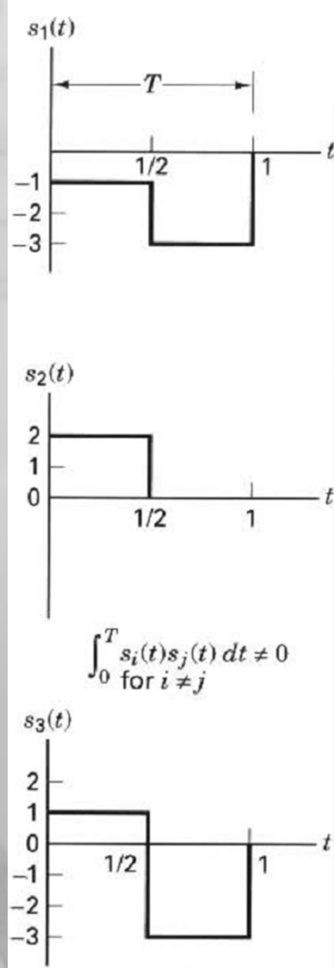
Figure 3.5 illustrates the statement that any arbitrary integrable waveform set can be represented as a linear combination of orthogonal waveforms. Figure 3.5a shows a set of three waveforms, $s_1(t)$, $s_2(t)$, and $s_3(t)$.

(a) Demonstrate that these waveforms *do not* form an orthogonal set.

$$\int_0^T s_1(t)s_2(t)\,dt = \int_0^{T/2} s_1(t)s_2(t)\,dt + \int_{T/2}^T s_1(t)s_2(t)\,dt$$

$$= \int_0^{T/2} (-1)(2)\,dt + \int_{T/2}^T (-3)(0)\,dt = -T$$

Figure 3.5a is not an orthogonal set.



(a)

3. Baseband Demodulation/Detection

Example 3.1 Orthogonal Representation of Waveforms

(b) Figure 3.5b shows a set of two waveforms, $\psi_1(t)$ and $\psi_2(t)$. Verify that these waveforms form an orthogonal set.

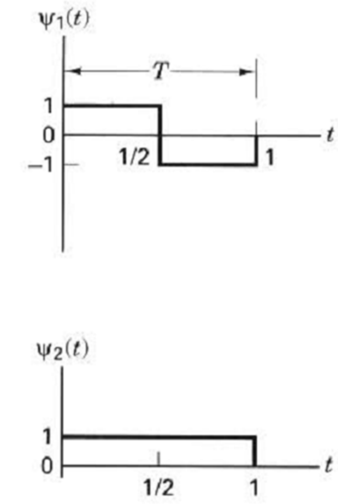
$$\int_0^T \psi_1(t)\psi_2(t)\,dt = \int_0^{T/2}(1)(1)\,dt + \int_{T/2}^{T}(-1)(1)\,dt = 0$$

(c) Show how the nonorthogonal waveform set in part (a) can be expressed as a linear combination of the orthogonal set in part (b).

$$a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\psi_j(t)\,dt$$

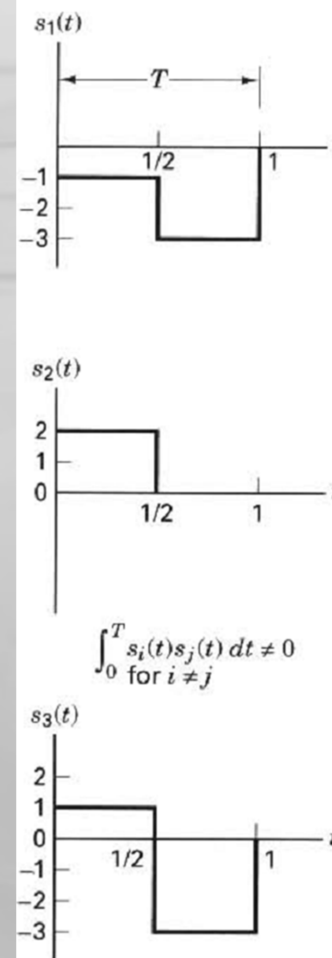
$$E_j = \int_0^T \psi_j^2(t)\,dt = K_j$$

$$s_1(t) = \psi_1(t) - 2\psi_2(t)$$
$$s_2(t) = \psi_1(t) + \psi_2(t)$$
$$s_3(t) = 2\psi_1(t) - \psi_2(t)$$



$$\int_0^T \psi_j(t)\psi_k(t)\,dt = \begin{cases} T \text{ for } j = k \\ 0 \text{ otherwise} \end{cases}$$

(b)



(a)

3. Baseband Demodulation/Detection

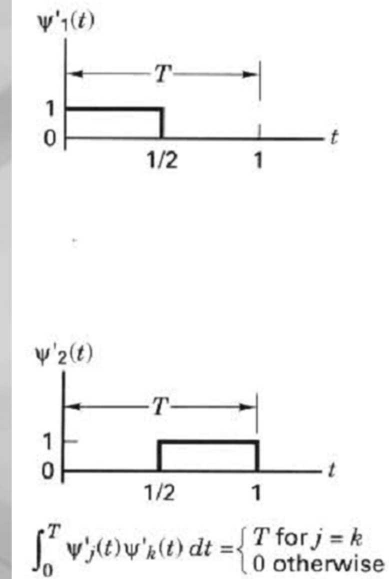
Example 3.1 Orthogonal Representation of Waveforms

(d) Figure 3.5c illustrates another orthogonal set of two waveforms, $\psi_1'(t)$ and $\psi_2'(t)$. Show how the nonorthogonal set in Figure 3.5a can be expressed as a linear combination of the set in Figure 3.5c.

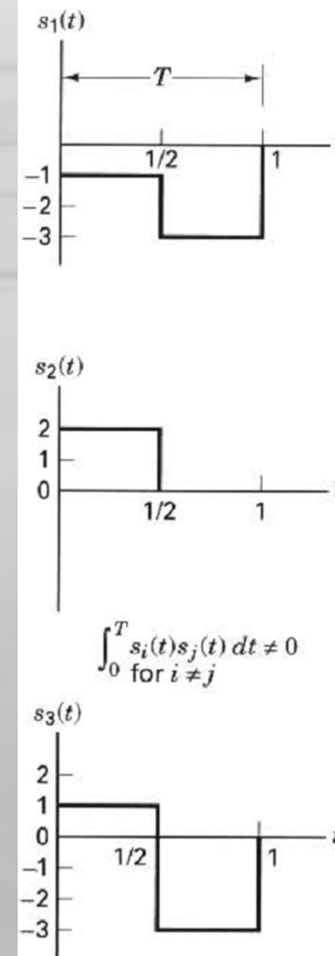
$$a_{ij} = \frac{1}{K_j} \int_0^T s_i(t)\psi_j(t)\,dt$$

$$E_j = \int_0^T \psi_j^2(t)\,dt = K_j$$

$$s_1(t) = -\psi_1'(t) - 3\psi_2'(t)$$
$$s_2(t) = 2\psi_1'(t)$$
$$s_3(t) = \psi_1'(t) - 3\psi_2'(t)$$



(c)



(a)

3. Baseband Demodulation/Detection

- Gram-Schmidt Procedure

  – Construct a set of orthonormal waveforms

$$\phi_1(t)=\frac{s_1(t)}{\sqrt{\xi_1}}$$

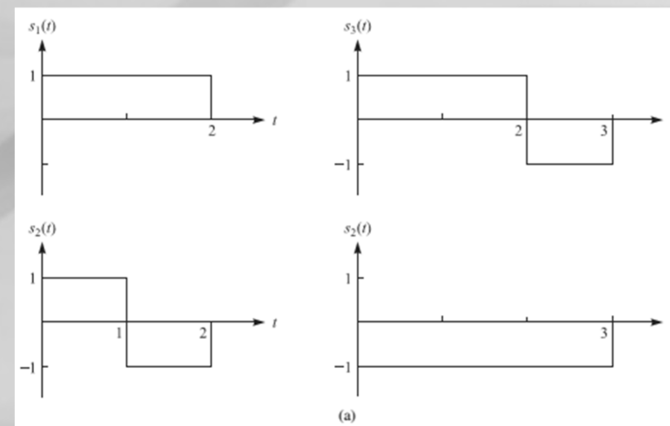
$$c_{21}=\langle s_2(t),\phi_1(t)\rangle=\int_{-\infty}^{\infty}s_2(t)\phi_1^*(t)\,dt$$

$$\gamma_2(t)=s_2(t)-c_{21}\phi_1(t),\qquad \xi_2=\int_{-\infty}^{\infty}\gamma_2^2(t)\,dt$$

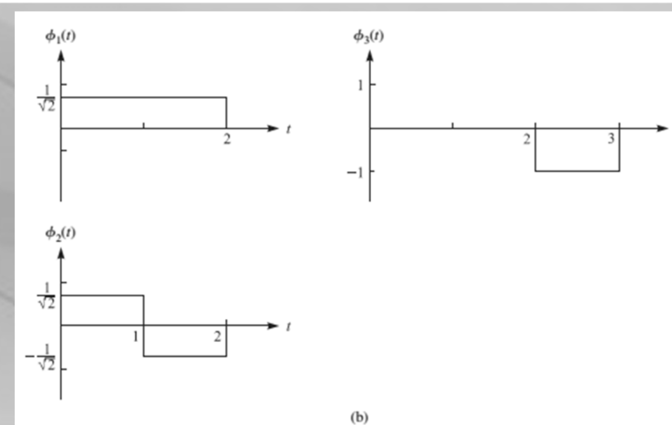
$$\phi_2(t)=\frac{\gamma_2(t)}{\sqrt{\xi_2}}$$

$$\gamma_k(t)=s_k(t)-\sum_{i=1}^{k-1}c_{ki}\phi_i(t),\ \ c_{ki}=\langle s_k(t),\phi_i(t)\rangle=\int_{-\infty}^{\infty}s_k(t)\phi_i^*(t)\,dt$$

$$\xi_k=\int_{-\infty}^{\infty}\gamma_k^2(t)\,dt,\qquad \phi_k(t)=\frac{\gamma_k(t)}{\sqrt{\xi_k}}$$



Gram-Schmidt orthogonalization of the signal $\{s_m(t), m = 1, 2, 3, 4\}$ and the corresponding orthonormal basis.



3. Baseband Demodulation/Detection

- Representing White Noise with Orthogonal Waveform

$$n(t) = \hat{n}(t) + \tilde{n}(t)$$

$$\hat{n}(t) = \sum_{j=1}^{N} n_j \psi_j(t)$$

$$n_j = \frac{1}{K_j} \int_0^T n(t)\psi_j(t)\, dt \qquad \text{for all } j$$

$$n(t) = \sum_{j=1}^{N} n_j \psi_j(t) + \tilde{n}(t)$$

$$\int_0^T \tilde{n}(t)\psi_j(t)\, dt = 0 \qquad \text{for all } j$$

- Variance of White Noise

$$\sigma^2 = \text{var}\,[n(t)] = \int_{-\infty}^{\infty} \left(\frac{N_0}{2}\right) df = \infty$$

the variance for *filtered* AWGN

$$\sigma^2 = \text{var}\,(n_j) = \mathbf{E}\left\{\left[\int_0^T n(t)\psi_j(t)\, dt\right]^2\right\} = \frac{N_0}{2}$$

3. Baseband Demodulation/Detection

- The Basic SNR Parameter for Digital Communication Systems

average signal power to average noise power ratio (S/N or SNR)

E_b/N_0, a normalized version of SNR

$$\frac{E_b}{N_0} = \frac{S\,T_b}{N/W} = \frac{S/R_b}{N/W}$$

$$\frac{E_b}{N_0} = \frac{S}{N}\left(\frac{W}{R}\right)$$

$$\frac{E_b}{N_0} = \frac{\text{Joule}}{\text{Watt per Hz}} = \frac{\text{Watt-s}}{\text{Watt-s}}$$

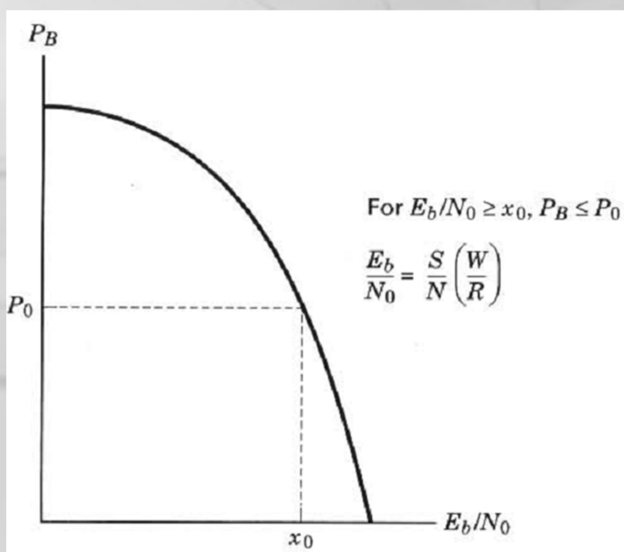


Figure 3.6 General shape of the P_B versus E_b/N_0 curve.

3. Baseband Demodulation/Detection

- Detection of Binary Signals in Gaussian Noise

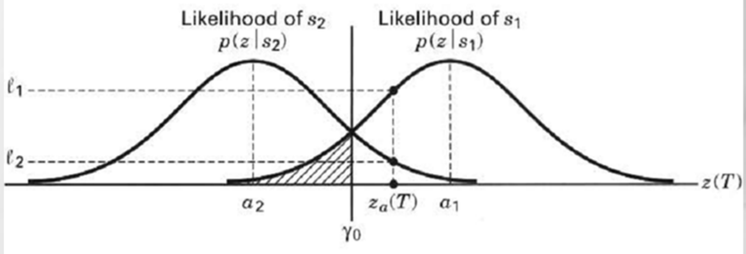


Figure 3.2 Conditional probability density functions: $p(z|s_1)$ and $p(z|s_2)$.

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

likelihood ratio test.

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

minimum error criterion

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1 + a_2}{2} = \gamma_0$$

For equally likely signals

$$p(z_a|s_1) > p(z_a|s_2)$$

maximum likelihood detector

3. Baseband Demodulation/Detection

- Error Probability

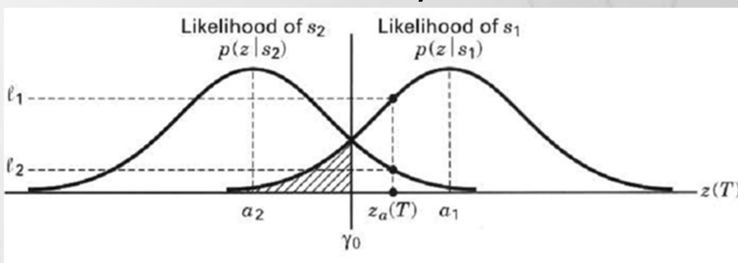


Figure 3.2 Conditional probability density functions: $p(z|s_1)$ and $p(z|s_2)$.

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

$$P_B = \sum_{i=1}^{2} P(e, s_i) = \sum_{i=1}^{2} P(e|s_i)\, P(s_i)$$

$$P(e|s_1) = P(H_2|s_1) = \int_{-\infty}^{\gamma_0} p(z|s_1)\, dz$$

$$P_B = P(e|s_1)P(s_1) + P(e|s_2)P(s_2)$$

$$P(e|s_2) = P(H_1|s_2) = \int_{\gamma_0}^{\infty} p(z|s_2)\, dz$$

$$P_B = \tfrac{1}{2} P(H_2|s_1) + \tfrac{1}{2} P(H_1|s_2) = P(H_2|s_1) = P(H_1|s_2)$$

$$P_B = \int_{\gamma_0=(a_1+a_2)/2}^{\infty} p(z|s_2)\, dz = \int_{\gamma_0=(a_1+a_2)/2}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right] dz = \int_{u=(a_1-a_2)/2\sigma_0}^{u=\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du = Q\left(\frac{a_1-a_2}{2\sigma_0}\right)$$

$$Q(x) \approx \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) du$$

$x > 3$. $$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

3. Baseband Demodulation/Detection

- Matched Filter

$$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2}$$

$$a_i(t) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}\,df$$

$$\sigma_0^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

$$\left(\frac{S}{N}\right)_T = \frac{\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi fT}\,df\right|^2}{N_0/2\int_{-\infty}^{\infty} |H(f)|^2\,df}$$

Schwarz's inequality if $f_1(x) = kf_2^*(x)$

$$\left|\int_{-\infty}^{\infty} f_1(x)f_2(x)\,dx\right|^2 \le \int_{-\infty}^{\infty} |f_1(x)|^2\,dx \int_{-\infty}^{\infty} |f_2(x)|^2\,dx$$

$$\left(\frac{S}{N}\right)_T \le \frac{2}{N_0}\int_{-\infty}^{\infty} |S(f)|^2\,df$$

$$\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi fT}df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\,df \int_{-\infty}^{\infty} |S(f)|^2\,df$$

$$\max\left(\frac{S}{N}\right)_T = \frac{2E}{N_0}$$

$$E = \int_{-\infty}^{\infty} |S(f)|^2\,df$$

$$H(f) = H_0(f) = kS^*(f)e^{-j2\pi fT}$$

$$h(t) = \mathcal{F}^{-1}\{kS^*(f)e^{-j2\pi fT}\}$$

$$h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

3. Baseband Demodulation/Detection

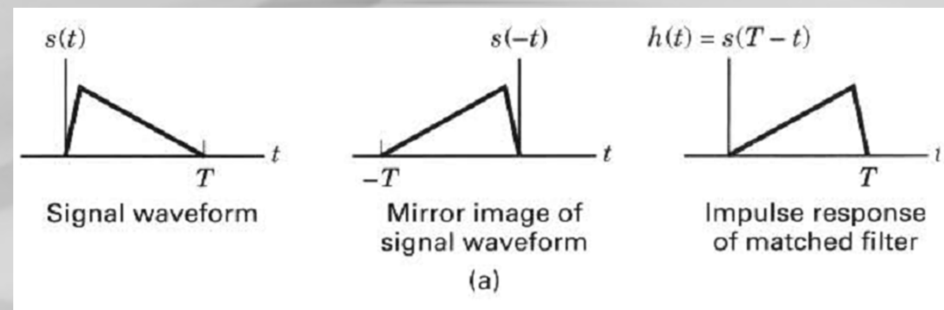
- Correlation Realization of the Matched Filter

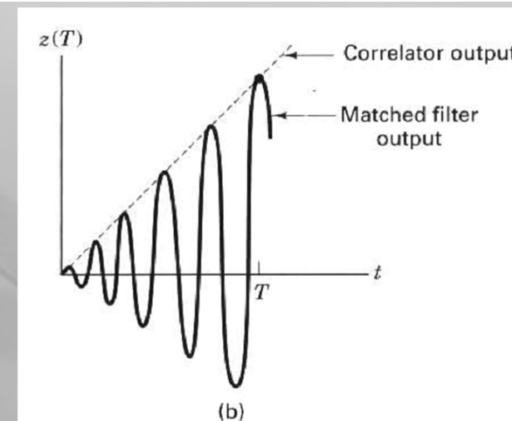
$$z(t) = r(t) * h(t) = \int_0^t r(\tau)h(t-\tau)\,d\tau$$

$$z(t) = \int_0^t r(\tau)s[T-(t-\tau)]\,d\tau$$
$$= \int_0^t r(\tau)s(T-t+\tau)\,d\tau$$

When $t = T$, we can write Equation (3.58) as

$$z(T) = \int_0^T r(\tau)s(\tau)\,d\tau$$





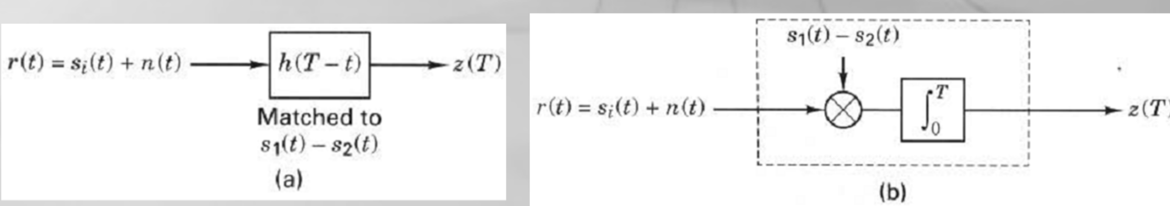


Figure 3.8 Equivalence of matched filter and correlator. (a) Matched filter. (b) Correlator.

Figure 3.7 Correlator and matched filter. (a) Matched filter characteristic. (b) Comparison of correlator and matched filter outputs.

3. Baseband Demodulation/Detection

- Optimizing Error Performance

$$\left(\frac{S}{N}\right)_T = \frac{(a_1 - a_2)^2}{\sigma_0^2} = \frac{2E_d}{N_0}$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2\, dt$$

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

time cross-correlation coefficient ρ

$$\rho = \frac{1}{E_b}\int_0^T s_1(t)\, s_2(t)\, dt$$

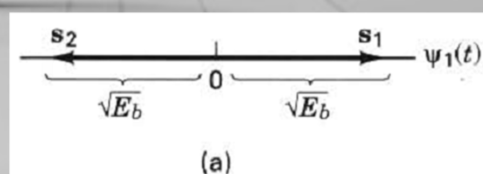
$$\rho = \cos\theta$$

$$-1 \leq \rho \leq 1$$

$$E_d = \int_0^T s_1^2(t)\, dt + \int_0^T s_2^2(t)\, dt - 2\int_0^T s_1(t)\, s_2(t)\, dt = E_b + E_b - 2\rho E_b = 2E_b(1-\rho)$$

$$E_b = \int_0^T s_1^2(t)\, dt = \int_0^T s_2^2(t)\, dt$$

$$P_B = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right)$$



$$\rho = -1$$

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

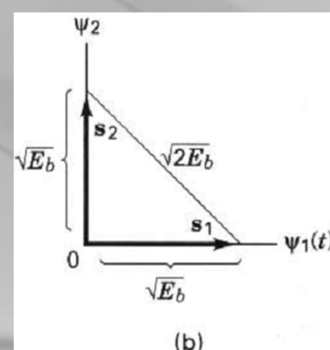


Figure 3.10 Binary signal vectors. (a) Antipodal. (b) Orthogonal.

$$\rho = 0$$

$$P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

3. Baseband Demodulation/Detection

Example 3.2 Matched Filter Detection of Antipodal Signals

Consider a binary communications system that receives equally likely signals $s_1(t)$ and $s_2(t)$ plus AWGN. See Figure 3.11. Assume that the receiving filter is a matched filter (MF), and that the noise-power spectral density N_0 is equal to 10^{-12} Watt/Hz. Use the values of received signal voltage and time shown on Figure 3.11 to compute the bit-error probability.

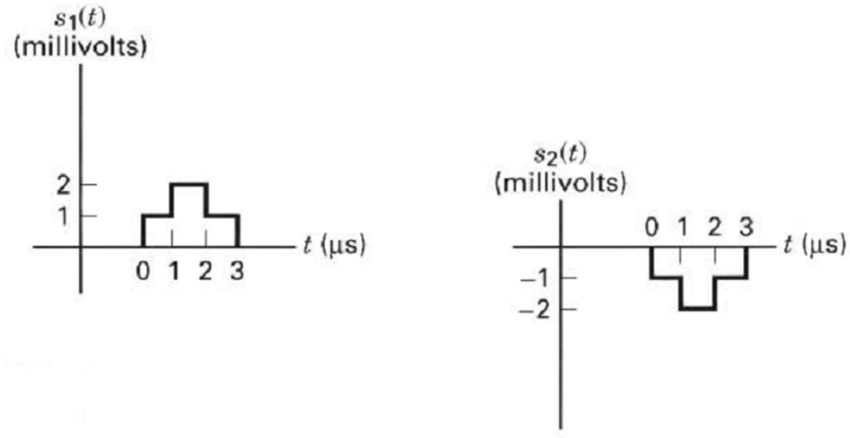


Figure 3.11 Baseband antipodal waveforms.

Solution

We can graphically determine the received energy per bit from the plot of either $s_1(t)$ or $s_2(t)$ shown in Figure 3.11 by integrating to find the energy (area under the voltage-squared pulse). Doing this in piecewise fashion, we get

$$
\begin{aligned}
E_b &= \int_0^3 v^2(t)\, dt \\
&= (10^{-3}\text{ V})^2 \times (10^{-6}\text{ s}) + (2 \times 10^{-3}\text{ V})^2 \times (10^{-6}\text{ s}) + (10^{-3}\text{ V})^2 \times (10^{-6}\text{ s}) \\
&= 6 \times 10^{-12}\text{ joule}
\end{aligned}
$$

Since the waveforms depicted in Figure 3.11 are antipodal and are detected with a matched filter, we use Equation (3.70) to find the bit-error probability, as

$$Q\left(\sqrt{\frac{12 \times 10^{-12}}{10^{-12}}}\right) = Q(\sqrt{12}) = Q(3.46)$$

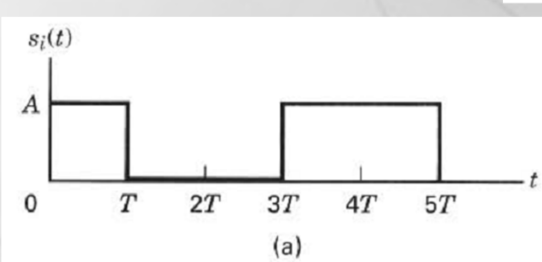
$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 3 \times 10^{-4}. \quad \text{Or} \quad \approx 2.9 \times 10^{-4}$$

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

3. Baseband Demodulation/Detection

- Error Probability Performance of Binary Signaling
  - Unipolar Signaling

$$s_1(t) = A \qquad 0 \le t \le T \qquad \text{for binary 1}$$
$$s_2(t) = 0 \qquad 0 \le t \le T \qquad \text{for binary 0}$$



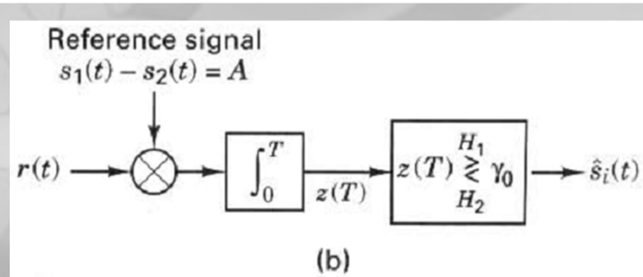


Figure 3.12 Detection of unipolar baseband signaling. (a) Unipolar signaling example. (b) Correlator detector.

$$a_1(T) = \mathrm{E}\{z(T)|s_1(t)\} = \mathrm{E}\left\{\int_0^T A^2 + An(t)\,dt\right\} = A^2T$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2\,dt = A^2T.$$

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

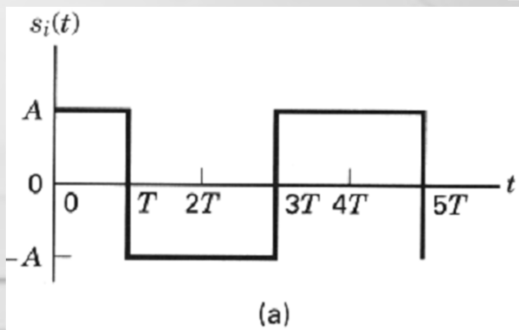
3. Baseband Demodulation/Detection

- Error Probability Performance of Binary Signaling
  - Bipolar Signaling

$$s_1(t) = +A \qquad 0 \le t \le T \qquad \text{for binary 1}$$

$$s_2(t) = -A \qquad 0 \le t \le T \qquad \text{for binary 0}$$

$$a_1 = -a_2$$



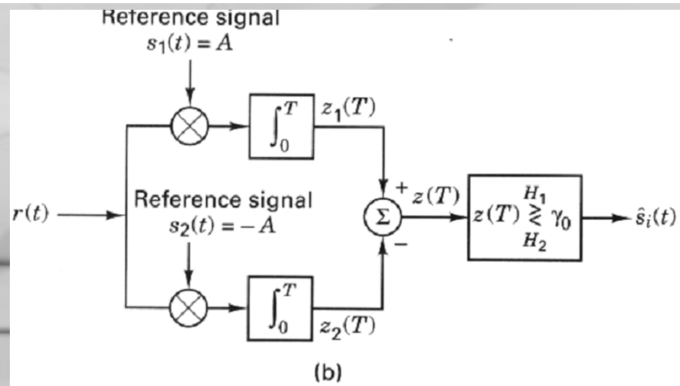


Figure 3.13 Detection of bipolar baseband signaling. (a) Bipolar signaling example. (b) Correlator detector.

$$z(T) = z_1(T) - z_2(T) \qquad \gamma_0 = 0$$

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Error Probability Performance of Binary Signaling

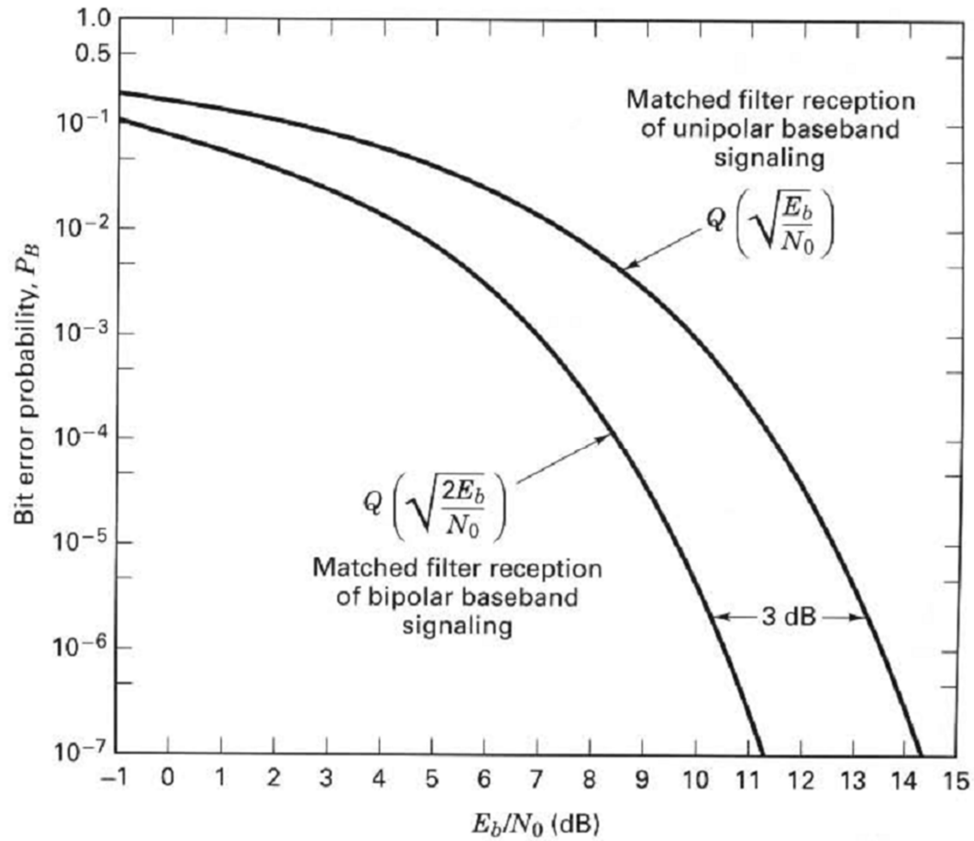


Figure 3.14 Bit error performance of unipolar and bipolar signaling.

Inter-Symbol Interference (ISI)

Nyquist filter

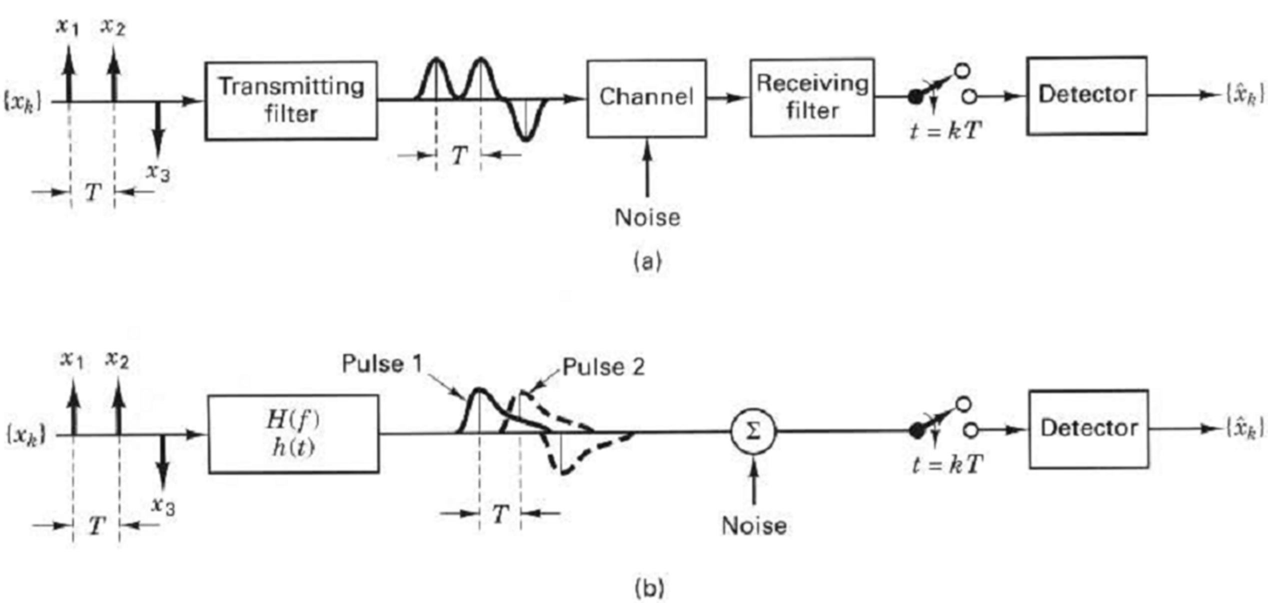


Figure 3.15 Intersymbol interference in the detection process. (a) Typical baseband digital system. (b) Equivalent model.

$$H(f) = H_t(f)\, H_c(f)\, H_r(f)$$

Nyquist pulse

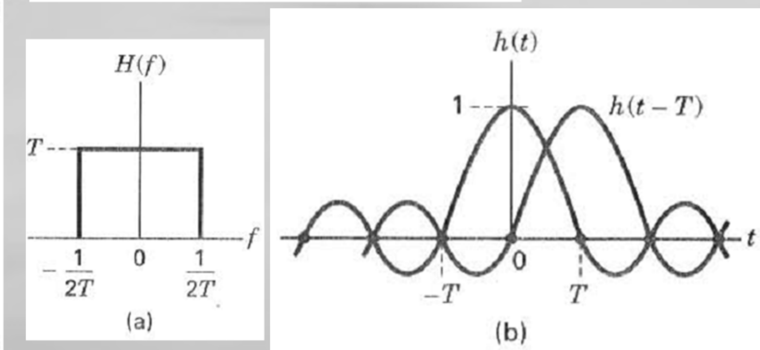


Figure 3.16 Nyquist channels for zero ISI. (a) Rectangular system transfer function $H(f)$. (b) Received pulse shape $h(t) = \text{sinc}\,(t/T)$.

$W = 1/2T = R_s/2$ hertz

$h(t) = \text{sinc}\,(t/T)$

$2W = 1/T = R_s$ symbols/s

$W \geq \frac{R_s}{2}$ hertz

symbol-rate packing,

2 symbols/s/Hz

64-ary PAM $M = 2^k = 64$ amplitudes $k = 6$ bits per symbol maximum bandwidth efficiency without ISI is 12 bits/s/Hz

Inter-Symbol Interference (ISI)

Raised-Cosine Filter

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left(\dfrac{\pi}{4}\dfrac{|f| + W - 2W_0}{W - W_0}\right) & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases} \quad (3.78)$$

$$h(t) = 2W_0(\operatorname{sinc} 2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

$$W = \frac{1}{2}(1 + r)R_s \qquad W_{DSB} = (1 + r)R_s$$

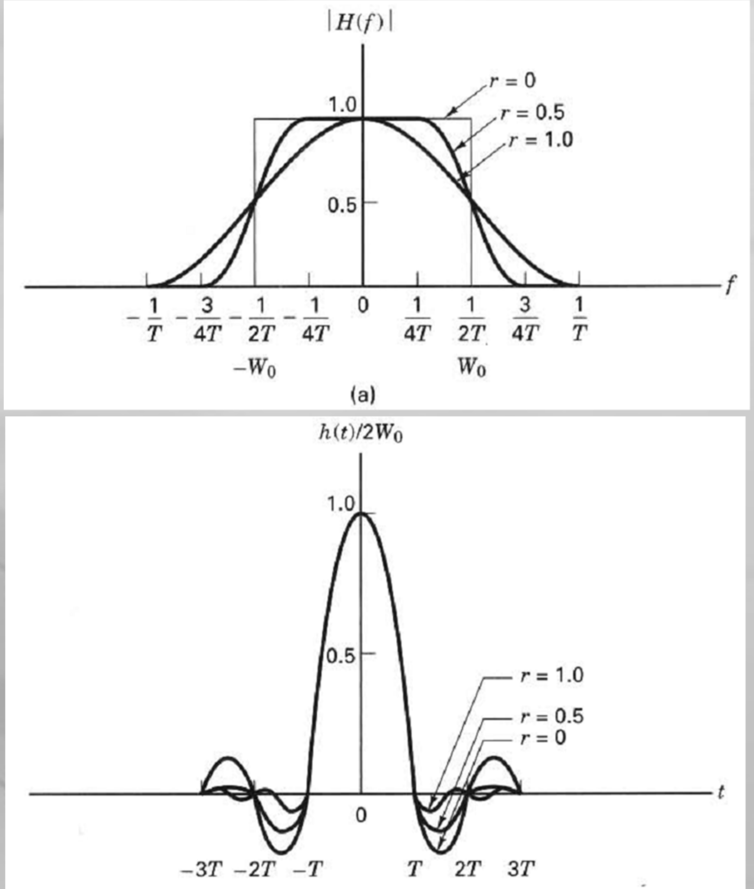


Figure 3.17 Raised-cosine filter characteristics. (a) System transfer function. (b) System impulse response.

Two Types of Error Performance Degradation

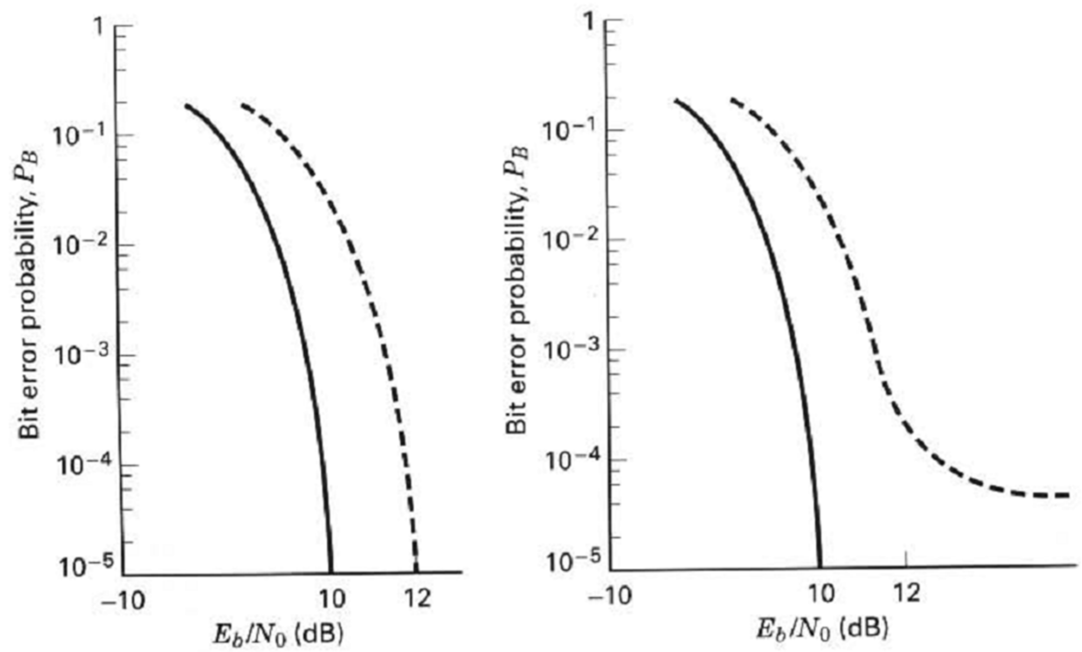


Figure 3.18 (a) Loss in E_b/N_0. (b) Irreducible P_B caused by distortion.

(a) Loss in E_b/N_0

A loss in E_b/N_0 has come about because of some signal losses or an increased level of noise or interference

(b) Irreducible P_B caused by distortion.

there is a degradation effect brought about by ISI

Bandwidth requirements

Example 3.3 Bandwidth Requirements

(a) Find the minimum required bandwidth for the baseband transmission of a four-level PAM pulse sequence having a data rate of $R = 2400$ bits/s if the system transfer characteristic consists of a raised-cosine spectrum with 100% excess bandwidth ($r = 1$).

(b) The same 4-ary PAM sequence is modulated onto a carrier wave, so that the baseband spectrum is shifted and centered at frequency f_0. Find the minimum required DSB bandwidth for transmitting the modulated PAM sequence. Assume that the system transfer characteristic is the same as in part (a).

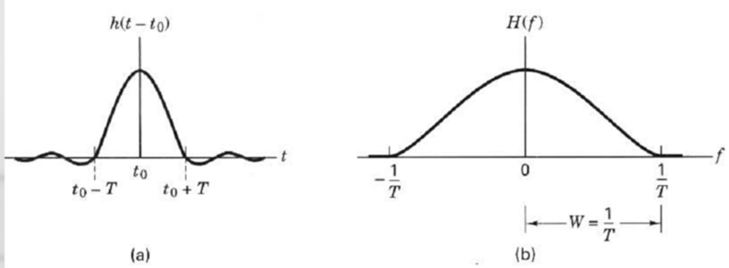


Figure 3.19 (a) Shaped pulse. (b) Baseband raised cosine spectrum.

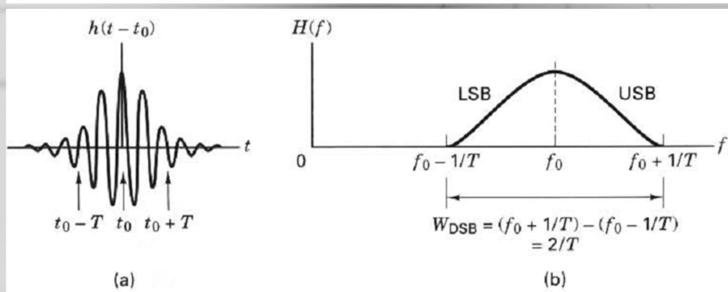


Figure 3.20 (a) Modulated shaped pulse. (b) DSB-modulated raised cosine spectrum.

Solution

(a) $M = 2^k$; since $M = 4$ levels, $k = 2$.

$$\text{Symbol or pulse rate } R_s = \frac{R}{k} = \frac{2400}{2} = 1200 \text{ symbols/s;}$$

$$\text{Minimum bandwidth } W = \tfrac{1}{2}(1 + r)R_s = \tfrac{1}{2}(2)(1200) = 1200 \text{ Hz.}$$

Figure 3.19a illustrates the baseband PAM received pulse in the time domain—an approximation to the $h(t)$ in Equation (3.79). Figure 3.19b illustrates the Fourier transform of $h(t)$—the raised cosine spectrum. Notice that the required bandwidth, W, starts at zero frequency and extends to $f = 1/T$; it is twice the size of the Nyquist theoretical minimum bandwidth.

(b) As in part (a),

$$R_s = 1200 \text{ symbols/s;}$$

$$W_{DSB} = (1 + r)R_s = 2(1200) = 2400 \text{ Hz.}$$

Figure 3.20a illustrates the modulated PAM received pulse. This waveform can be viewed as the product of a high-frequency sinusoidal carrier wave and a waveform with the pulse shape of Figure 3.19a. The single-sided spectral plot in Figure 3.20b illustrates that the modulated bandwidth is

$$W_{DSB} = \left(f_0 + \frac{1}{T}\right) - \left(f_0 - \frac{1}{T}\right) = \frac{2}{T}.$$

When the spectrum of Figure 3.19b is shifted up in frequency, the negative and positive halves of the baseband spectrum are shifted up in frequency, thereby doubling the required transmission bandwidth. As the name implies, the DSB signal has two sidebands: the upper sideband (USB), derived from the baseband positive half, and the lower sideband (LSB), derived from the baseband negative half.

Demodulation/Detection of Shaped Pulse

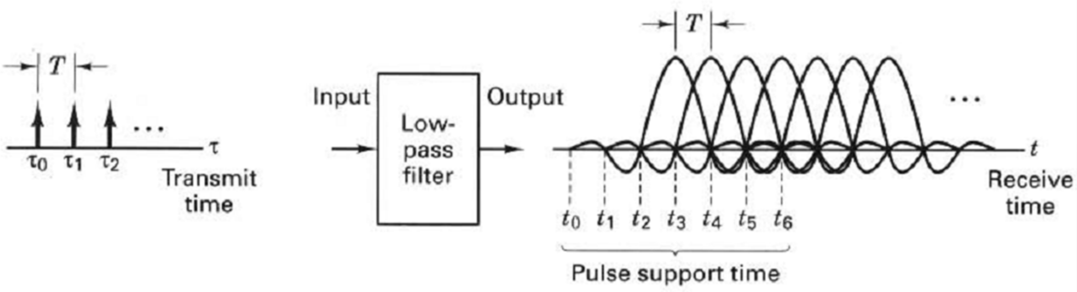


Figure 3.21 Filtered impulse sequence: output versus input.

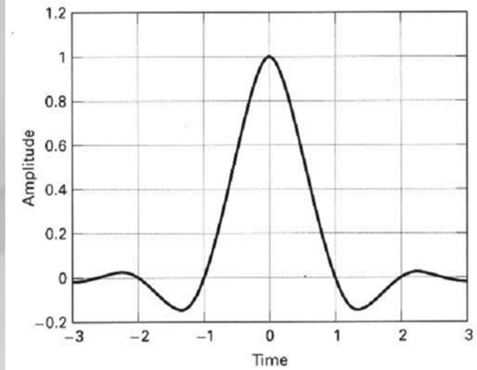


Figure 3.22b Nyquist pulse.

Nyquist Pulse

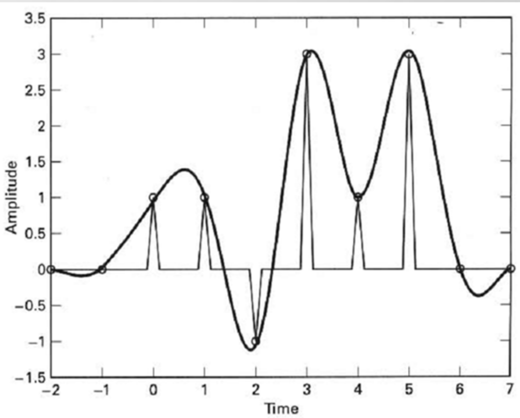


Figure 3.23b Output of raised-cosine matched filter and delayed-input sample values.

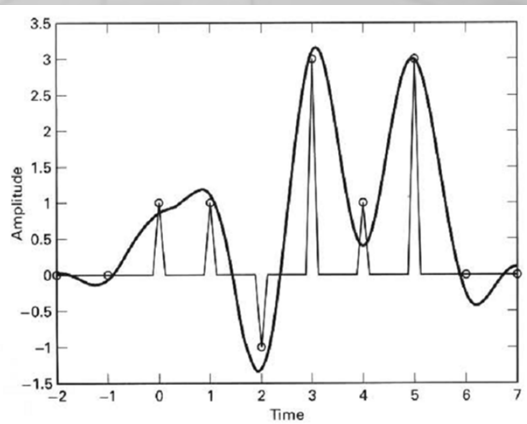


Figure 3.23a Square-root Nyquist-shaped *M*-ary waveform and delayed-input sample values.

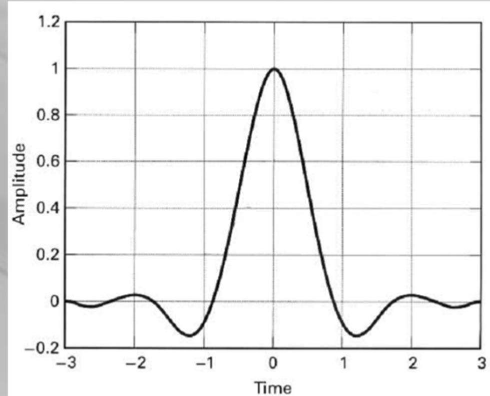


Figure 3.22a Square-root Nyquist pulse.

Square-Root Nyquist Pulse

Demodulation/Detection of Shaped Pulse

- Equalization

$$H_{RC}(f) = H_t(f)\, H_c(f)\, H_r(f)\, H_e(f)$$

$$H_{RC}(f) = H_t(f)\, H_r(f)$$

$$H_c(f) = |H_c(f)|\, e^{j\theta_c(f)}$$

$$H_e(f) = \frac{1}{H_c(f)} = \frac{1}{|H_c(f)|}\, e^{-j\theta_c(f)}$$

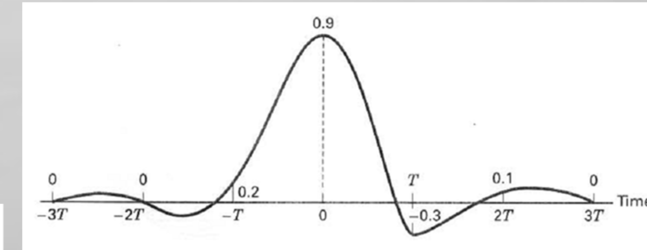


Figure 3.25 Received pulse exhibiting distortion.

- Eye Pattern

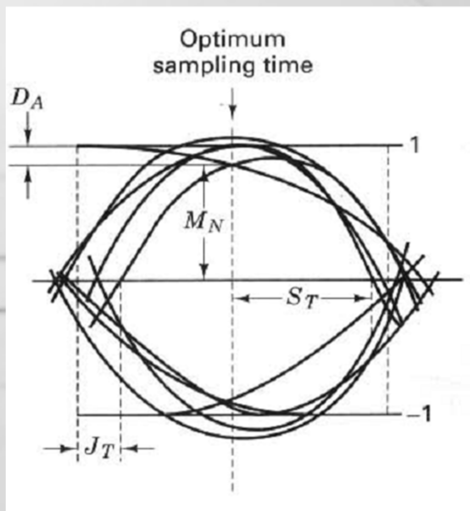


Figure 3.24 Eye Pattern.

- Transversal Equalizer

$$z(k) = \sum_{n=-N}^{N} x(k-n)\, c_n \quad k = -2N, \ldots, 2N \quad n = -N, \ldots, N$$

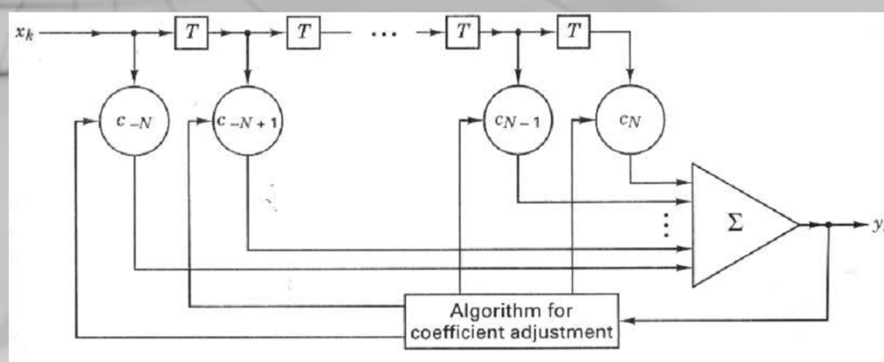


Figure 3.26 Transversal filter.

$$\mathbf{z} = \mathbf{x}\,\mathbf{c}$$

$$\mathbf{c} = \mathbf{x}^{-1}\mathbf{z}$$

Demodulation/Detection of Shaped Pulse

- Equalizer $\mathbf{z} = \mathbf{x}\,\mathbf{c}$ $\quad \mathbf{c} = \mathbf{x}^{-1}\mathbf{z}$
  - Zero-Forcing Solution: minimizes the peak ISI so that $z(k) = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k = \pm 1, \pm 2, \ldots, \pm N \end{cases}$

Example 3.5 A Zero-Forcing Three-Tap Equalizer

Consider that the tap weights of an equalizing transversal filter are to be determined by transmitting a single impulse as a training signal. Let the equalizer circuit in Figure 3.26 be made up of just three taps. Given a received distorted set of pulse samples $\{x(k)\}$, with voltage values 0.0, 0.2, 0.9, –0.3, 0.1, as shown in Figure 3.25, use a zero-forcing solution to find the weights $\{c_{-1}, c_0, c_1\}$ that reduce the ISI so that the equalized pulse samples $\{z(k)\}$ have the values, $\{z(-1) = 0,\ z(0) = 1,\ z(1) = 0\}$. Using these weights, calculate the ISI values of the equalized pulse at the sample times $k = \pm 2, \pm 3$. What is the largest magnitude sample contributing to ISI, and what is the sum of all the ISI magnitudes?

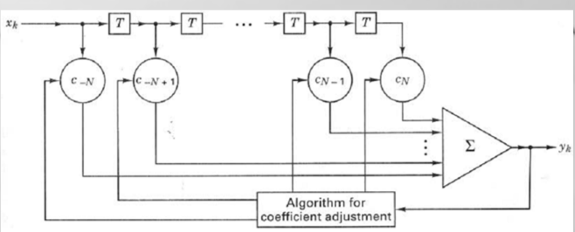


Figure 3.26 Transversal filter.

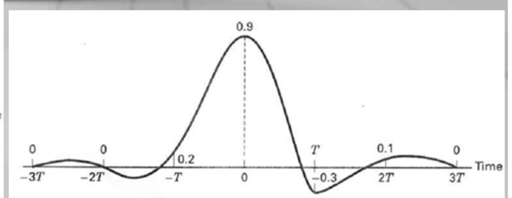


Figure 3.25 Received pulse exhibiting distortion.

Solution

For the channel impulse response specified, Equation (3.89) yields

$$\mathbf{z} = \mathbf{x}\,\mathbf{c}$$

or

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

Solving these three simultaneous equations results in the following weights:

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.2140 \\ 0.9631 \\ 0.3448 \end{bmatrix}$$

The values of the equalized pulse samples $\{z(k)\}$ corresponding to sample times $k = -3, -2, -1, 0, 1, 2, 3$ are computed by using the preceding weights in Equation (3.89a), yielding

Demodulation/Detection of Shaped Pulse

- Equalizer
  - Minimum MSE Solution: minimize the mean-square error (MSE) of all the ISI term plus the noise power at the output of the equalizer

$$\mathbf{z} = \mathbf{x}\,\mathbf{c} \qquad \mathbf{x}^T\mathbf{z} = \mathbf{x}^T\mathbf{x}\mathbf{c} \qquad \mathbf{R}_{xz} = \mathbf{R}_{xx}\mathbf{c} \qquad \mathbf{c} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xz}$$

$\mathbf{R}_{xz} = \mathbf{x}^T\mathbf{z}$ is called the *cross-correlation* vector

$\mathbf{R}_{xx} = \mathbf{x}^T\mathbf{x}$ is called the *autocorrelation* matrix of the input noisy signal

- Decision Feedback Equalizer
  - For mobile radio application
  - Nonlinear equalizer
  - Uses previous decision
  - To eliminate ISI on current pulse

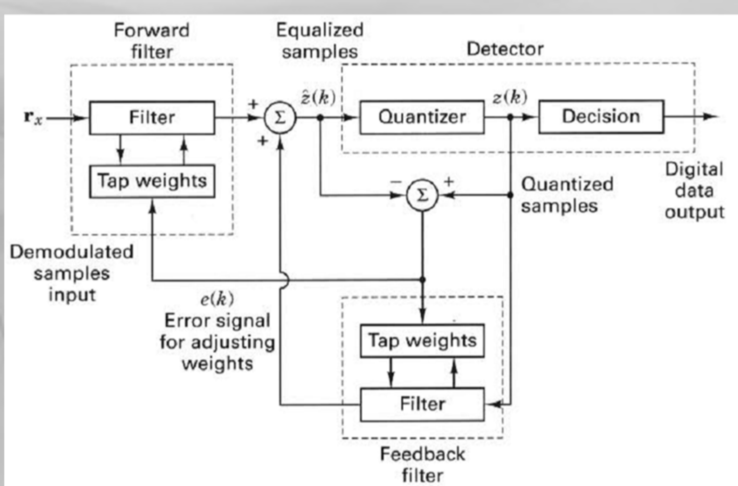


Figure 3.27 Decision Feedback Equalizer.

Problems and Questions

- Conclusion

In this chapter, we described the detection of binary signals plus Gaussian noise in terms of two basic steps. In the first step the received waveform is reduced to a single number $z(T)$, and in the second step a decision is made as to which signal was transmitted, on the basis of comparing $z(T)$ to a threshold. We discussed how to best choose this threshold. We also showed that a linear filter known as a matched filter or correlator is the optimum choice for maximizing the output signal-to-noise ratio and thus minimizing the probability of error.

We defined intersymbol interference (ISI) and explained the importance of Nyquist's work in establishing a theoretical minimum bandwidth for symbol detection without ISI. We partitioned error-performance degradation into two main types. The first is a simple loss in signal-to-noise ratio. The second, resulting from distortion, is a bottoming-out of the error probability versus the E_b/N_0 curve.

Finally, we described equalization techniques that can be used to mitigate the effects of ISI.

Problems and Questions

3.2. **(a)** Show that the three functions illustrated in Figure P3.1 are pairwise orthogonal over the interval (–2, 2).

(b) Determine the value of the constant, A, that makes the set of functions in part (a) an orthonormal set.

(c) Express the following waveform, $x(t)$, in terms of the orthonormal set of part (b).

$$x(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

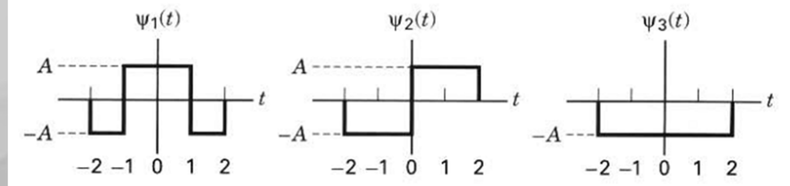


Figure P3.2

3.8. **(a)** What is the theoretical minimum system bandwidth needed for a 10-Mbits/s signal using 16-level PAM without ISI?

(b) How large can the filter roll-off factor be if the allowable system bandwidth is 1.375 MHz?

3.11. A voice signal in the range 300 to 3300 Hz is sampled at 8000 samples/s. We may transmit these samples directly as PAM pulses or we may first convert each sample to a PCM format and use binary (PCM) waveforms for transmission.

(a) What is the minimum system bandwidth required for the detection of PAM with no ISI and with a filter roll-off characteristic of $r = 1$?

(b) Using the same filter roll-off characteristic, what is the minimum bandwidth required for the detection of binary (PCM) waveforms if the samples are quantized to eight levels?

(c) Repeat part (b) using 128 quantization levels.

3.14. Consider that NRZ binary pulses are transmitted along a cable that attenuates the signal power by 3 dB (from transmitter to receiver). The pulses are coherently detected at the receiver, and the data rate is 56 kbit/s. Assume Gaussian noise with $N_0 = 10^{-6}$ Watt/Hz. What is the minimum amount of power needed at the transmitter in order to maintain a bit-error probability of $P_B = 10^{-3}$?

Problems and Questions

- Questions

3.1. In the case of *baseband* signaling, the received waveforms are already in a pulse-like form. Why then, is a demodulator needed to recover the pulse waveform? (See Chapter 3, introduction.)

3.2. Why is E_b/N_0 a natural figure-of-merit for digital communication systems? (See Section 3.1.5.)

3.3. When representing timed events, what dilemma can easily result in confusing the most-significant bit (MSB) and the least-significant bit (LSB)? (See Section 3.2.3.1.)

3.4. The term *matched-filter* is often used synonymously with *correlator.* How is that possible when their mathematical operations are different? (See Section 3.2.3.1.)

3.5. Describe the two fair ways of comparing different curves that depict bit-error probability versus E_b/N_0. (See Section 3.2.5.3.)

3.6. Are there other pulse-shaping filter functions, besides the *raised-cosine,* that exhibit zero ISI? (See Section 3.3.)

3.7. Describe a reasonable goal in endeavoring to *compress bandwidth* to the minimum possible, without incurring ISI. (See Section 3.3.1.1.)

3.8. The error performance of digital signaling suffers primarily from two degradation types: *loss* in signal-to-noise ratio, and *distortion* resulting in an irreducible bit-error probability. How do they differ? (See Section 3.3.2.)

3.9. Often times, providing more E_b/N_0 will not mitigate the degradation due to *intersymbol interference* (ISI). Explain why this is the case. (See Section 3.3.2.)

3.10. Describe the difference between equalizers that use a *zero-forcing* solution, and those that use a *minimum mean-square error* solution? (See Section 3.4.3.1.)