Seat Number

King Mongkut's University of Technology Thonburi
Midterm Examination
Semester 1 -- Academic Year 2013

Subject: EIE 208 Electrical and Electronic Engineering Mathematics
For: Electrical Communication and Electronic Engineering, 2^{th} Yr (Inter. Program)
Exam Date: Monday September 30, 2013 **Time:** 13.00-16.00 pm.

Instructions:-

1. This exam consists of 7 problems with a total of 7 pages, including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself (use the back pages for extra spaces).
5. A calculator compiling with the university rule is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds … No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment with university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Exam No.	1	2	3	4	5	6	7	8	TOTAL
Full Score	10	13	12	10	5	10	10		70
Graded Score									

Name ______________________________ **Student ID** ________________

This examination is designed by
Dr. Pinit Kumhom; Tel: 9075, 9070.

This examination has been approved by the committees of the ENE department.

(Assoc. Prof. Wudhichai Assawinchaichote, Ph.D.)
Head of Electronic and Telecommunication Engineering Department

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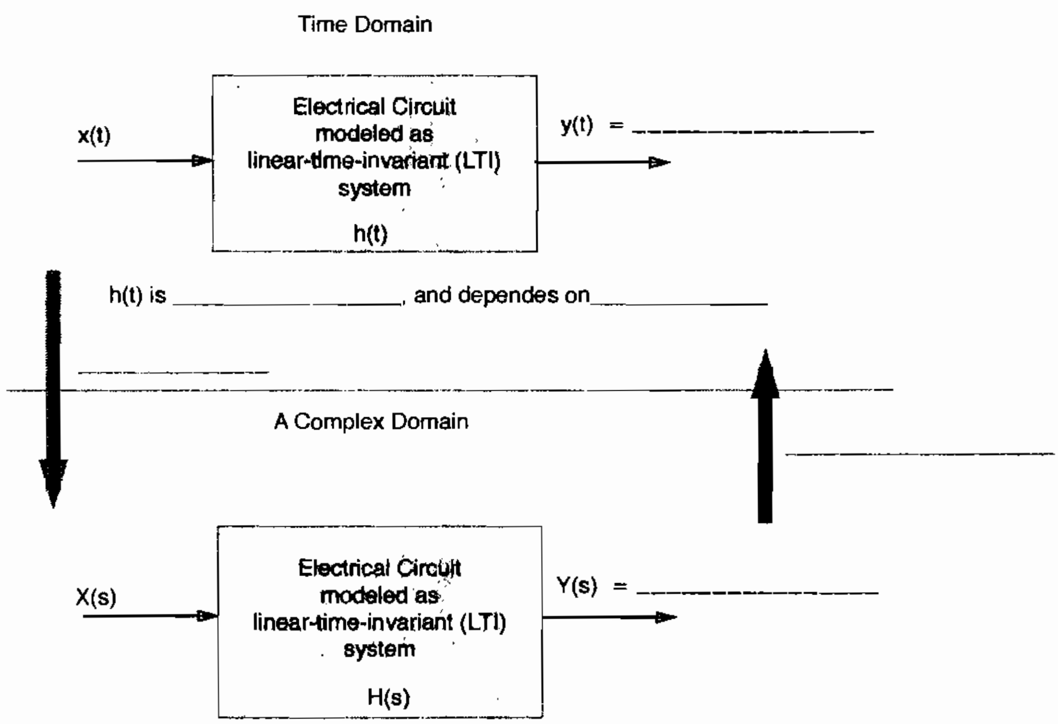
1. (10 points) Engineering and Mathematics

1.1. (5 points) Describe concisely why and how mathematics is important to engineering.

1.2. (5 points) Electrical circuits are usually modeled as linear time invariant (LTI) system. Using the provided block diagram, describe system analysis method and the applied mathematics.



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2. (13 points) Express the following complex numbers in (1) rectangular form, and (2) polar form using the exponential function. Show how the answers are obtained in details.

2.1. (6 points) $z = \left(\dfrac{1+\sqrt{3}i}{1+i}\right)^2 + \sqrt{3}e^{3\pi i} - e^{3\pi i/2}$

2.2. (5 points) $z = \dfrac{1-\sqrt{3}i}{2e^2 e^{i\pi/3}}$

2.3. (3 points) $z = (\cos\dfrac{\pi}{12} - i\sin\dfrac{\pi}{12})^3$

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3. (12 points) Complex roots

3.1. (6 points) Find all the 12^{th} roots of unity and sketch all the roots on the complex plane.

3.2. (6 point) Find the 12^{th} roots of $32\sqrt{2}(-1+i)$ and sketch all the roots on the complex plane. (Hint: use the result from 3.1)

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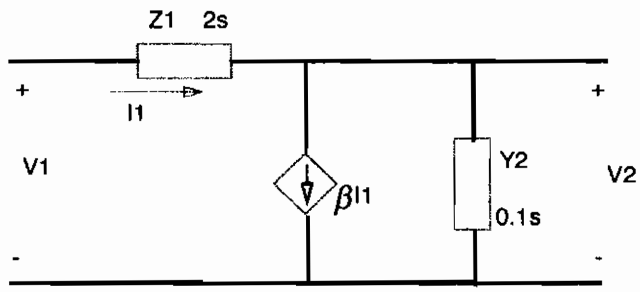
4. (10 points) Given that

- $V_1(s)$ and $V_2(s)$ are complex functions of s, representing input and output voltages in a complex domain,
- $Z_1 = 2s$ and $Y_2 = 0.1s$ are complex functions of s, representing the impedance and admittance of two components, respectively, and
- β is a positive real constant.



4.1. (7 points) Analyze the circuit to find its transfer function H(s) = V_2/V_1.

4.2. (2 points) Find all zeros and poles of H(s).

4.3. (1 points) What happen to the poles if $\beta = 0$?

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5. (5 points) Find all the poles and their multiplicities of $f(z)=\dfrac{3z^2+1}{z^4+2iz^3-z^2}$. Show how the answers are obtained in details.

6. (10 points) Given that $f(z)=\dfrac{3z^2+1}{z^2(z+1)}$,

6.1. (8 points) Write $f(z)$ in partial fraction form,

6.2. (2 points) Find the residue at each pole of $f(z)$

7. (10 points) $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega$ is called the inverse Fourier transform of $F(\omega)$. Compute

$f(t)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega$, $F(\omega)=\frac{1}{\omega^2+1}$, using the residue theorem.

<u>Hint</u>: $\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega = p.v.\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega=\lim_{\rho\to\infty}\int_{-\rho}^{\rho}F(\omega)e^{i\omega t}d\omega,\quad F(\omega)=\frac{1}{\omega^2+1}$.

Let $F(z)=\frac{1}{z^2+1}$, a complex function with no poles on the x-axis,

C_ρ^+ be the directed smooth curve along the half-circle above the x-axis with radius of ρ,

$\Gamma_\rho=(\gamma_\rho,C_\rho^+)$, where γ_ρ the directed smooth curve along the x-axis from $-\rho$ to $+\rho$, and

z_k be a pole of $F(z)e^{izt}$ above the x-axis. Then, for t > 0

$$p.v.\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega+\lim_{\rho\to\infty}\int_{C_\rho}F(z)e^{izt}\,dz=\lim_{\rho\to\infty}\int_{\Gamma_\rho}F(z)e^{izt}\,dz=2\pi i\sum_{all\,z_k}\mathrm{Res}(z_k,F(z)e^{izt})$$

and let C_ρ^- be the directed smooth curve along the half-circle below the x-axis with radius of ρ,

$-\Gamma_\rho=(\gamma_\rho,C_\rho^-)$, where γ_ρ the directed smooth curve along the x-axis from $-\rho$ to $+\rho$, and

z_l be a pole of $F(z)e^{izt}$ below the x-axis. Then, for t < 0

$$p.v.\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega+\lim_{\rho\to\infty}\int_{C_\rho^-}F(z)e^{izt}\,dz=\lim_{\rho\to\infty}\int_{-\Gamma_\rho}F(z)e^{izt}\,dz=-2\pi i\sum_{All\,z_l}\mathrm{Res}(z_l,F(z)e^{izt})$$

Based on the Jordan's Lemma if $F(z)$ is a rational function P/Q such that $\deg Q\geq \deg P+1$, then for t > 0, $\lim_{\rho\to\infty}\int_{C_\rho^+}F(z)e^{izt}\,dz=0$, and for t < 0, $\lim_{\rho\to\infty}\int_{C_\rho^-}F(z)e^{izt}\,dz=0$

Definition: For any a complex rational function $R(z)$, a residue of $R(z)$ at pole z_j denoted by

$\mathrm{Res}(z_j,R(z))$ is the coefficient of the term $\frac{1}{z-z_j}$ in the partial fraction expansion of $R(z)$.

Theorem and Formula for Partial Fraction Expansion of Complex Rational Function

Let $R_{m,n}(z)=\dfrac{a_0+a_1z+a_2z^2+\cdots+a_mz^m}{b_0+b_1z+b_2z^2+\cdots+b_nz^n}$ be a rational function whose denominator degree $n=d_1+d_2+\cdots+d_r$ exceeds its numerator degree *m*, then $R_{m,n}(z)$ has a partial fraction decomposition of the form

$$
\begin{aligned}
R_{m,n}(z) &= \frac{A_0^{(1)}}{(z-z_1)^{d_1}}+\frac{A_1^{(1)}}{(z-z_1)^{d_1-1}}+\cdots+\frac{A_s^{(1)}}{(z-z_1)^{d_1-s}}+\cdots+\frac{A_{d_1-1}^{(1)}}{(z-z_1)} \\
&+\frac{A_0^{(2)}}{(z-z_2)^{d_2}}+\frac{A_1^{(2)}}{(z-z_2)^{d_2-1}}+\cdots+\frac{A_s^{(2)}}{(z-z_2)^{d_2-s}}+\cdots+\frac{A_{d_2-1}^{(2)}}{(z-z_2)}+ \\
&\cdots \\
&+\frac{A_0^{(k)}}{(z-z_k)^{d_k}}+\frac{A_1^{(k)}}{(z-z_k)^{d_k-1}}+\cdots+\frac{A_s^{(k)}}{(z-z_k)^{d_k-s}}+\cdots+\frac{A_{d_k-1}^{(k)}}{(z-z_k)}+ \\
&\cdots \\
&+\frac{A_0^{(r)}}{(z-z_r)^{d_r}}+\frac{A_1^{(r)}}{(z-z_r)^{d_r-1}}+\cdots+\frac{A_s^{(r)}}{(z-z_r)^{d_r-s}}+\cdots+\frac{A_{d_r-1}^{(r)}}{(z-z_r)}
\end{aligned}
$$

where $\{A_s^{(k)}\}$ are constants. For each *s* and *k*, $s=0,1,\cdots,d_k-1$ and $k=1,2,\cdots,r$, $A_s^{(k)}$ is associated with z_k which is a distinct pole of $R_{m,n}(z)$ with multiplicity d_k. The constant $A_s^{(k)}$ can be computed by

$$A_s^{(k)}=\lim_{z\to z_k}\frac{1}{s!}\frac{d^{(s)}}{dz^{(s)}}\left[(z-z_k)^{d_r}R_{m,n}(z)\right]$$