ENE 104
Electric Circuit Theory

Lecture 02:
Voltage and Current Laws

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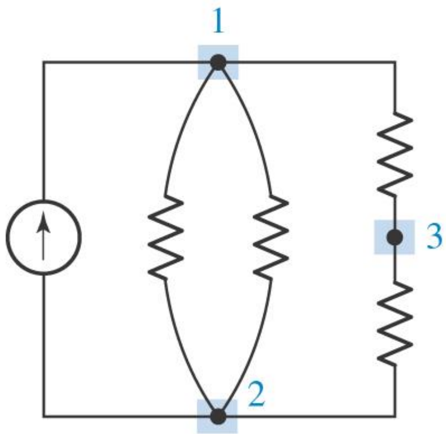
Objectives :

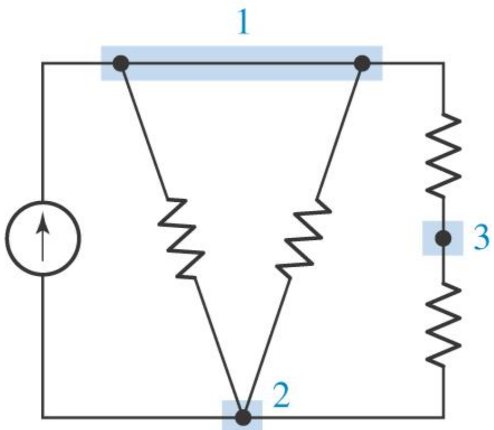
- Definition of nodes, paths, loops, and branches
- Kirchhoff's current law (KCL)
- Kirchhoff's voltage law (KVL)
- Analyzing simple series and parallel circuits
- Simplify series and parallel connected sources
- Reducing series and parallel resistor combinations
- Voltage and current division

Nodes, Paths, Loops, and Branches:



(a)

(a) A circuit containing three nodes and five branches.

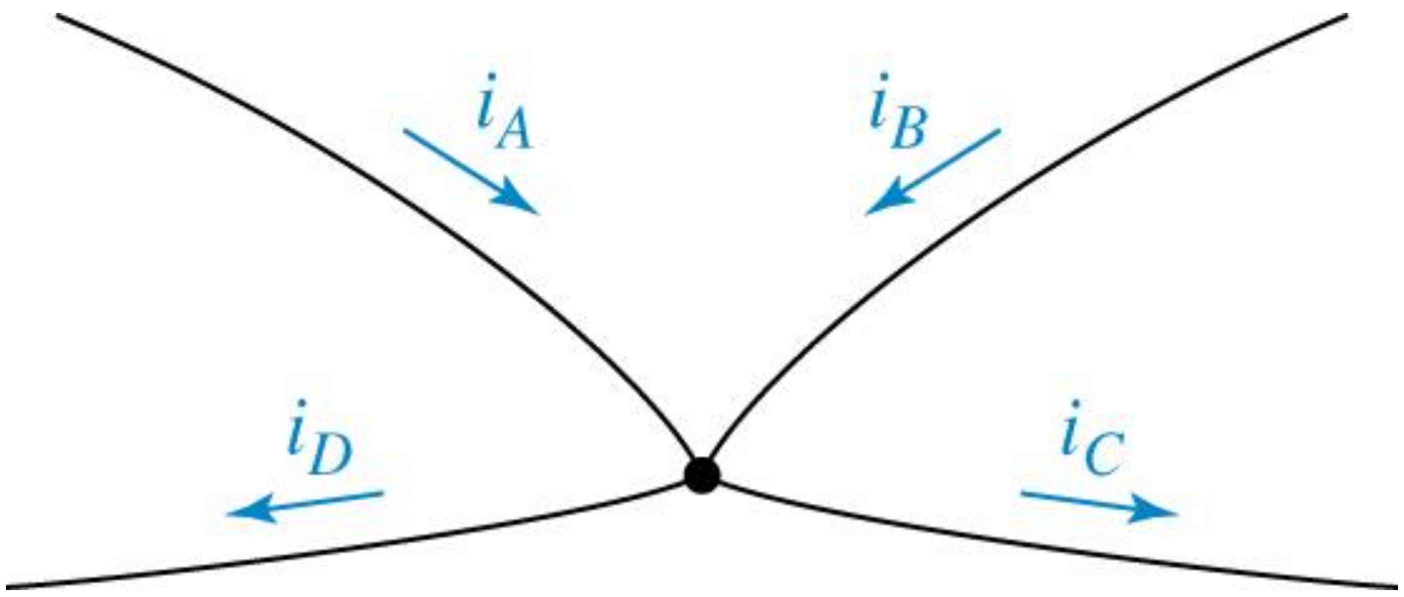


(b)

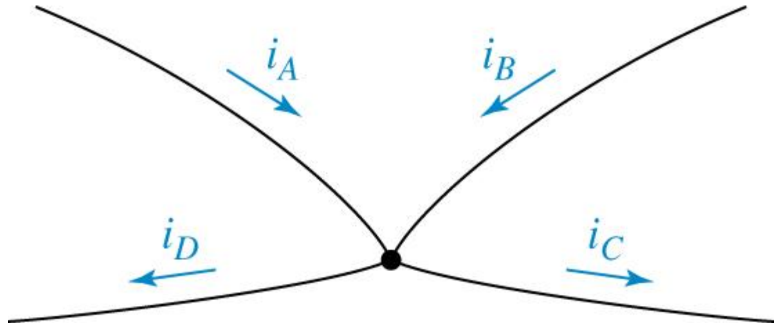
(b) Node 1 is redrawn to look like two nodes; it is still one node.

Kirchoff's Current Law (KCL):

The algebraic sum of the currents entering any node is zero



Kirchoff's Current Law (KCL):



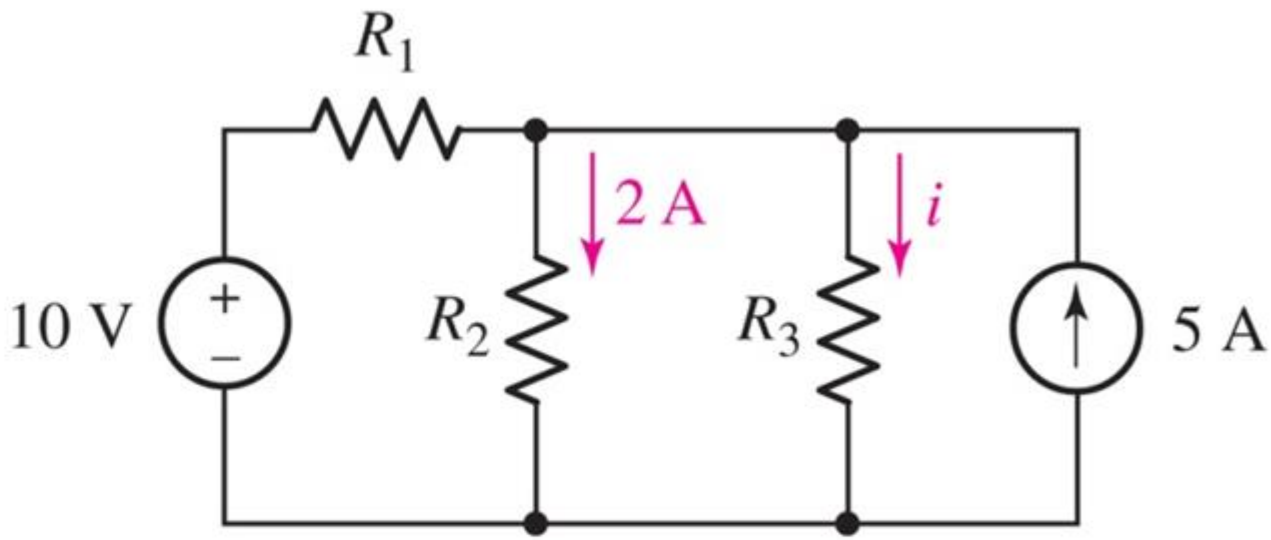
Entering the node: $i_A + i_B + (-i_C) + (-i_D) = 0$

Leaving the node: $(-i_A) + (-i_B) + i_C + i_D = 0$

Or: $i_A + i_B = i_C + i_D$

Example: 3.1

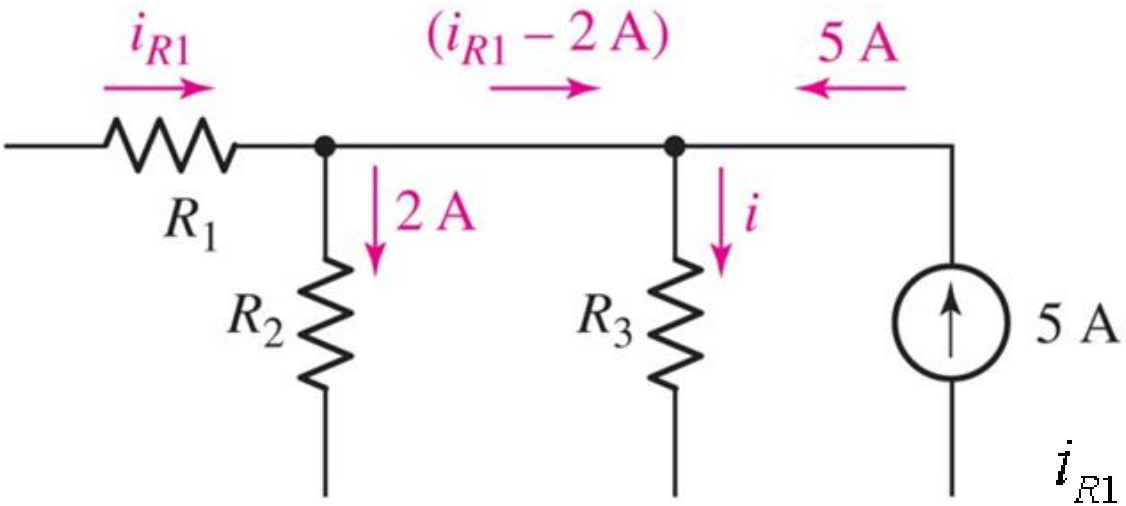
Compute the current through resister R3 if it is known that the voltage source supplies a current of 3 A.



(*a*)

Example:

Compute the current through resister R3 if it is known that the voltage source supplies a current of 3 A.

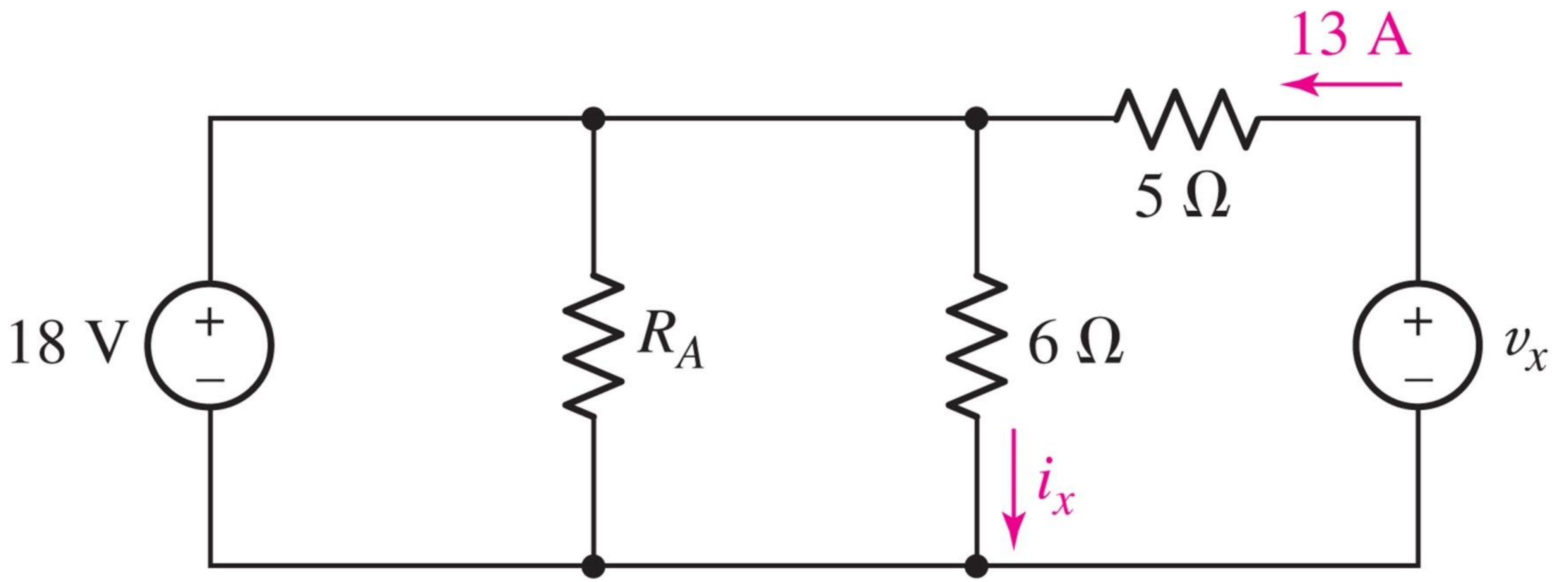


$$i_{R1} + 5A. = i + 2A.$$

$$\therefore i = i_{R1} + 5A. - 2A.$$

$$= 3A. + 5A. - 2A.$$

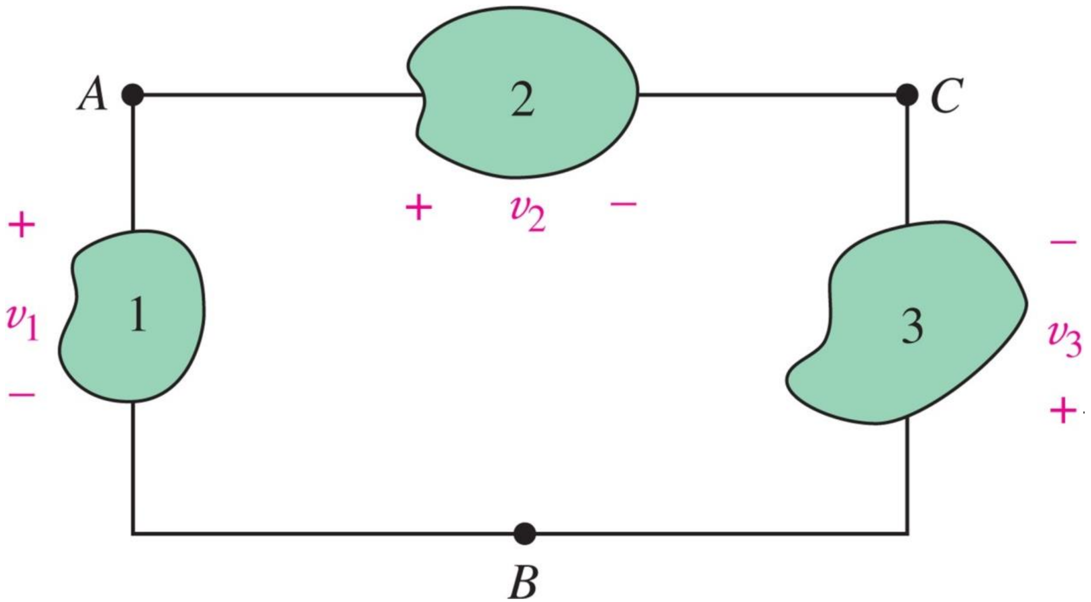
Practice: 3.1



Count the number of branches and nodes in the circuit. If i_x = 3 A. and the 18-V source delivers 8 A. of current, what is the value of R_A?

Kirchoff's Voltage Law (KVL):

The algebraic sum of the voltages around any closed path is zero

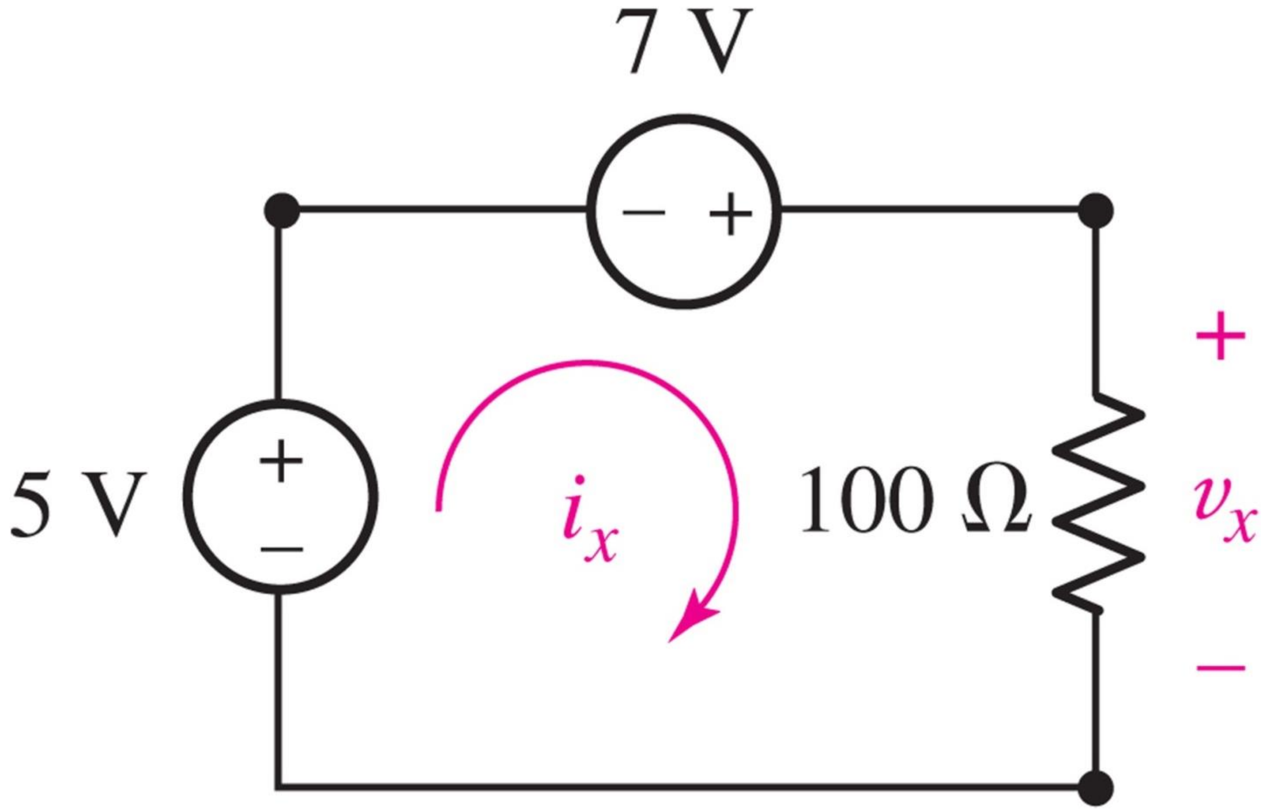


$$\sum_{n=1}^{N} v_n = 0$$

The potential difference between points A and B is independent of the path selected

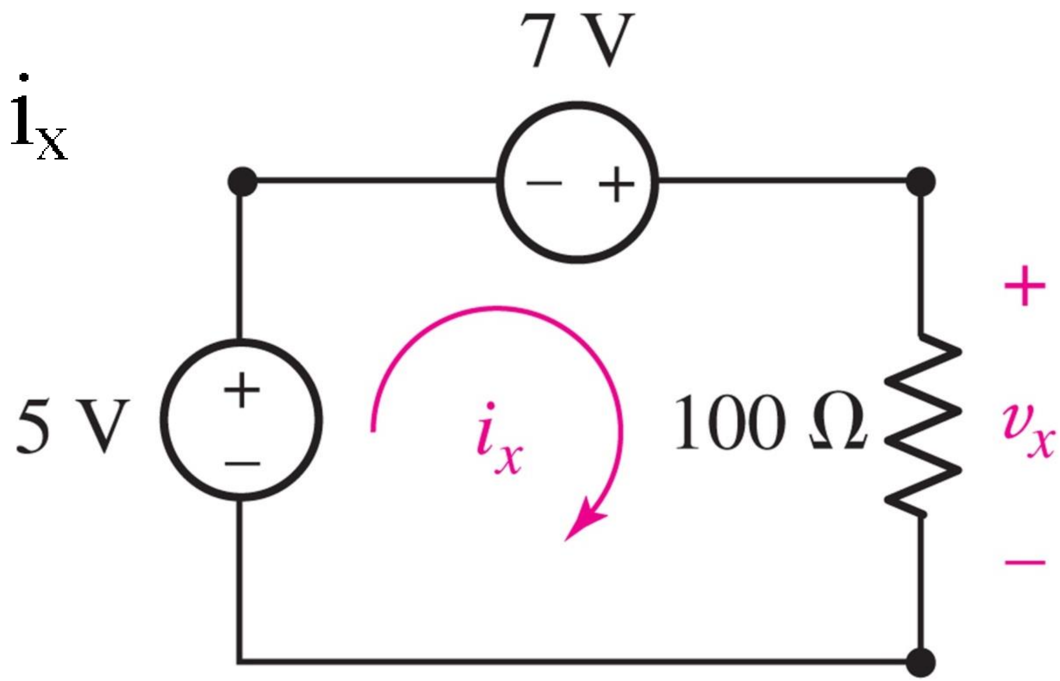
Example: 3.2

Find v_x and i_x



Example:

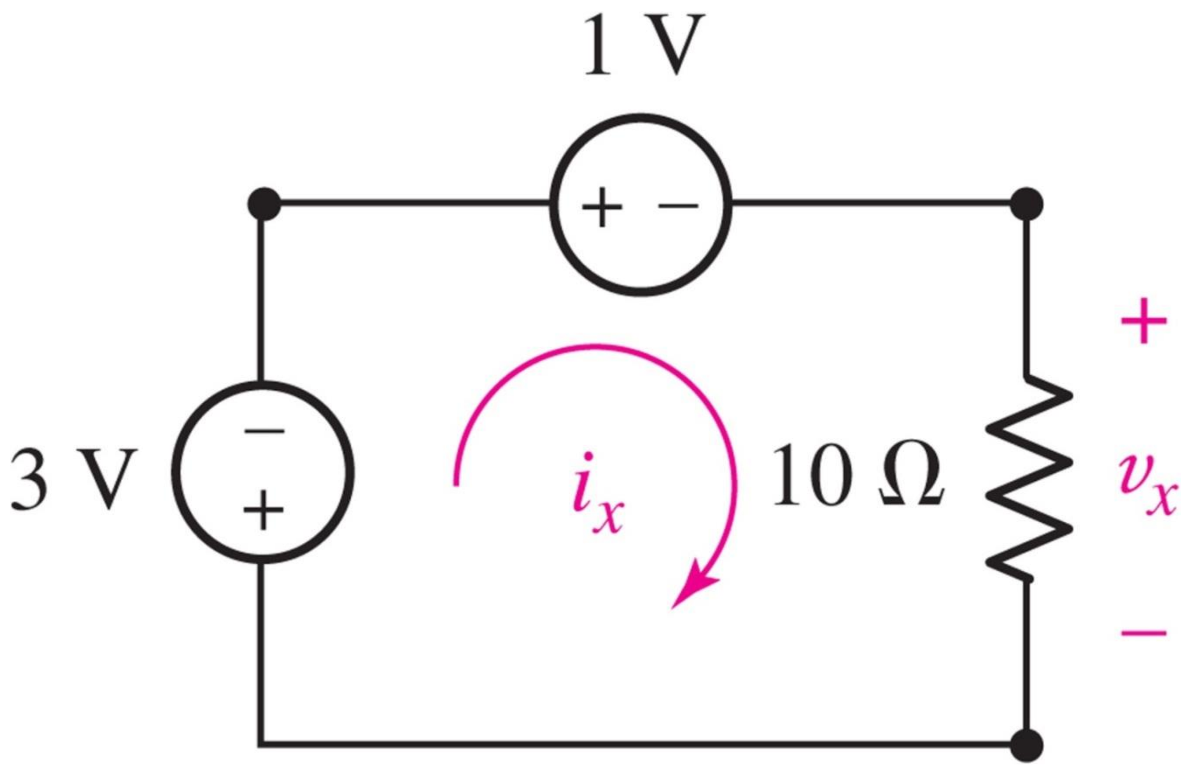
Find v_x and i_x



$$-5-7+v_x = 0$$

$$\therefore v_x = 12V.$$

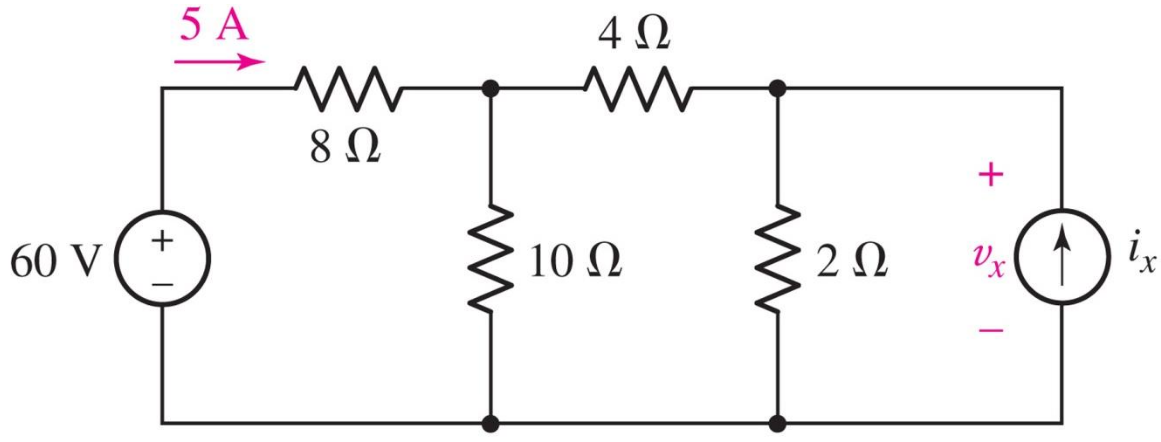
Practice: 3.2



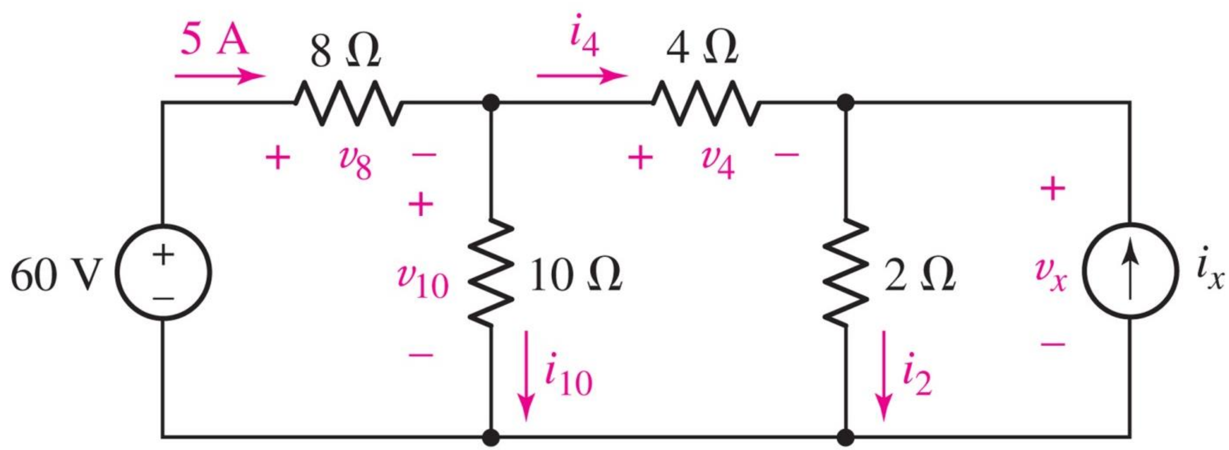
Determine i_x and v_x

Example: 3.4

Determine v_x



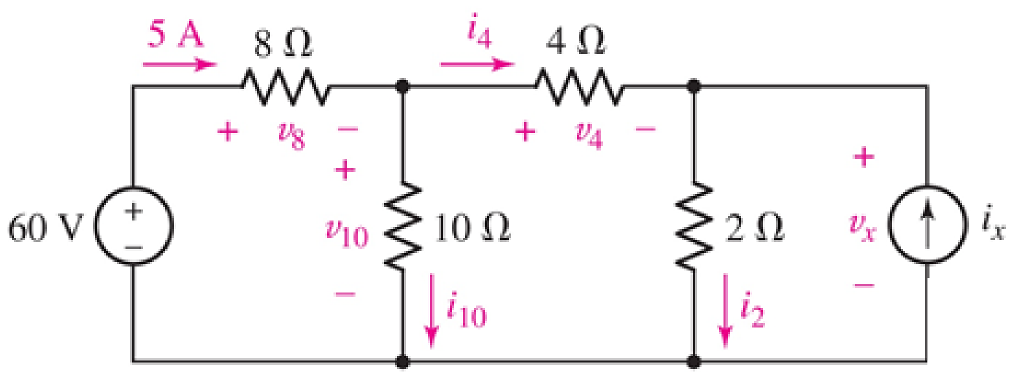
(*a*)



(*b*)

Example:

Determine v_x



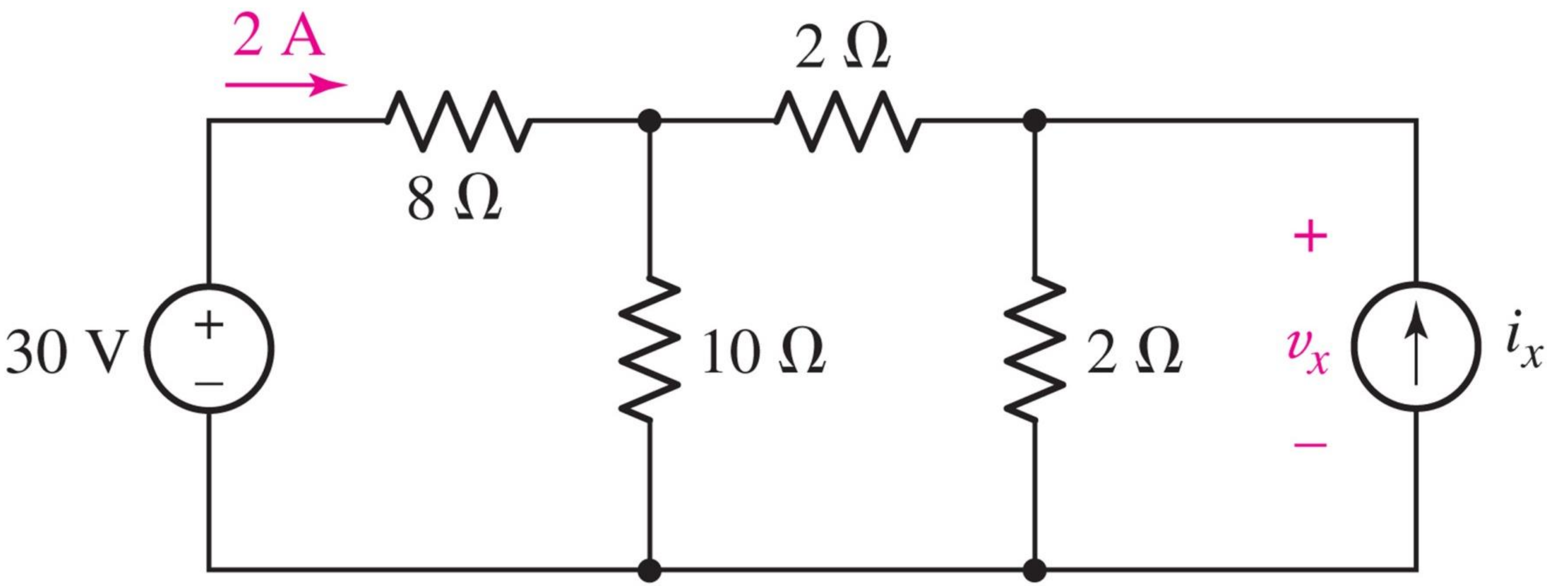
(b)

$$-60 + v_8 + v_{10} = 0$$

$$v_8 = 40 \text{ Volts}$$

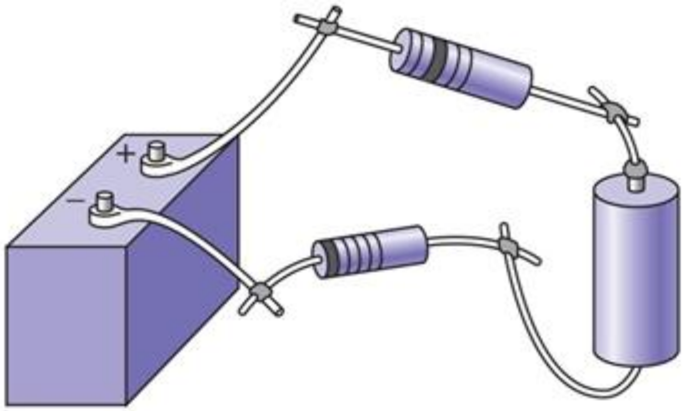
$$i_{10} = \frac{20V.}{10\Omega} = 2 \text{ Amp.}$$

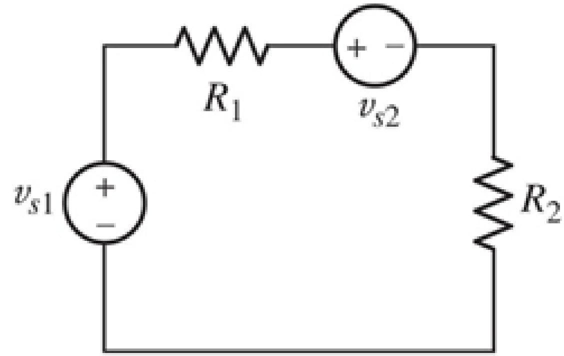
Practice: 3.3

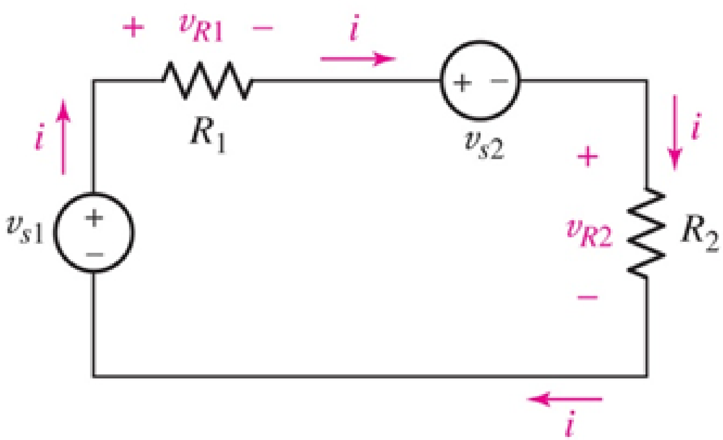


Determine v_x

The Single Loop Circuit:





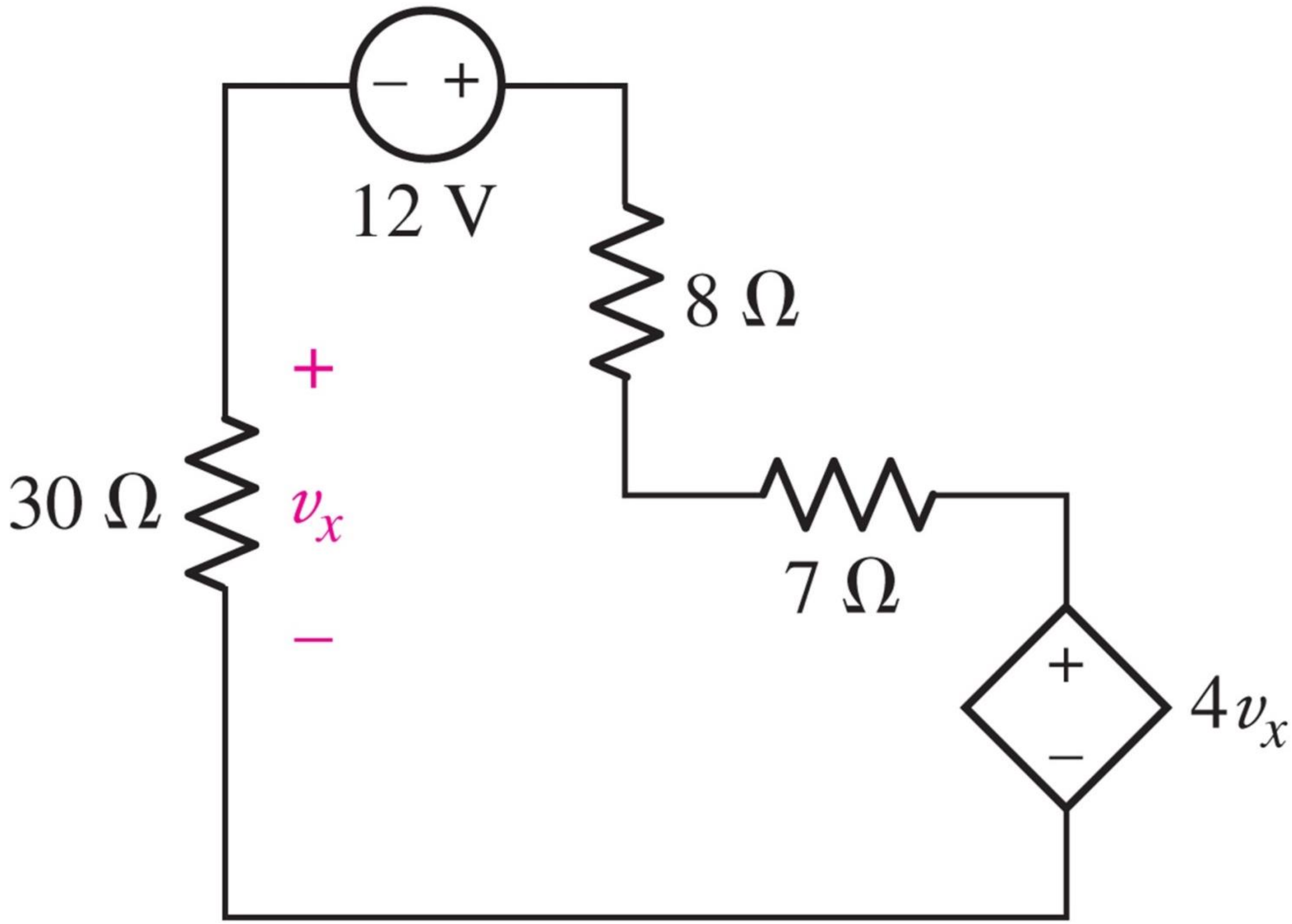


$$-v_{s1} + v_{R1} + v_{s2} + v_{R2} = 0$$

$$-v_{s1} + R_1 i + v_{s2} + R_2 i = 0$$

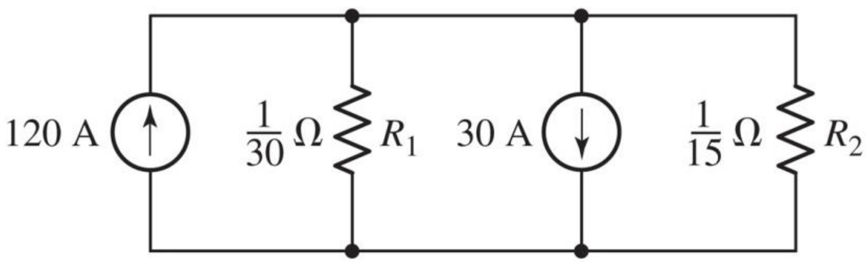
$$\therefore i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

Practice: 3.5

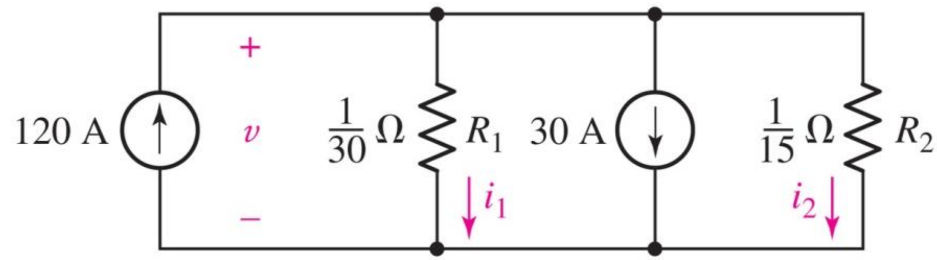


Find the power absorbed by each of the five elements in the circuit.

The Single Node-Pair Circuit:



(a)



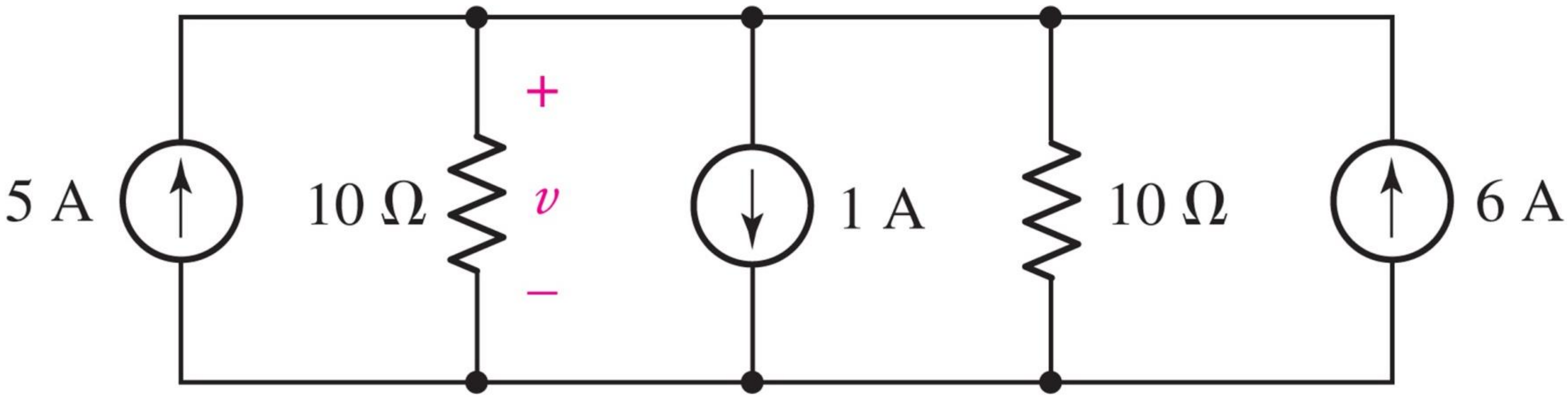
(b)

$$-120 + i_1 + 30 + i_2 = 0$$

$$-120 + 30v + 30 + 15v = 0$$

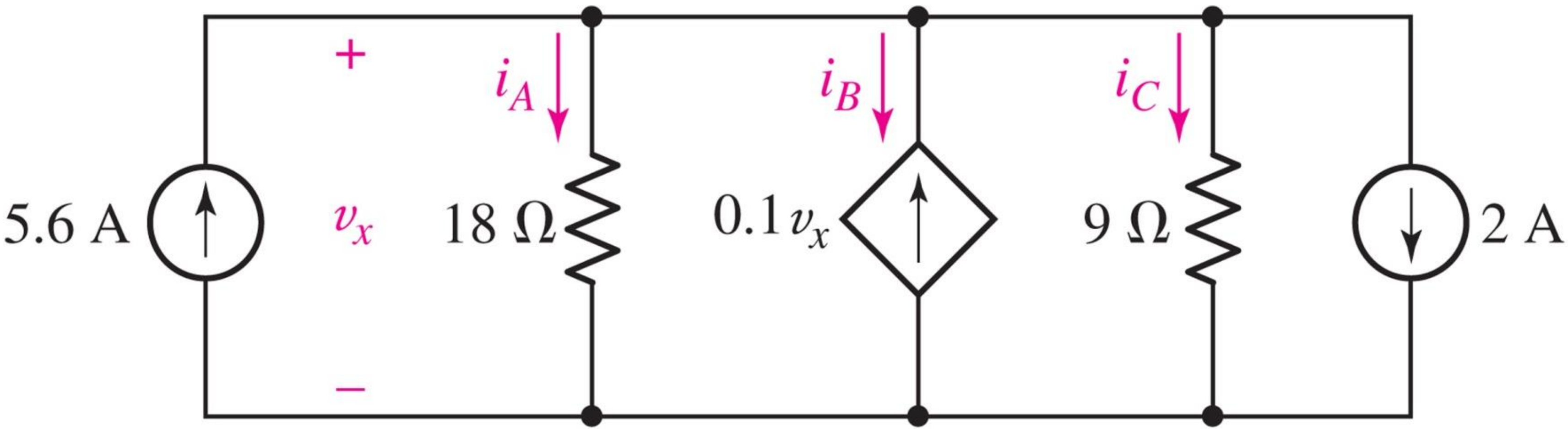
$$\therefore v = 2 \text{ Volts}$$

Practice: 3.6



Determine v

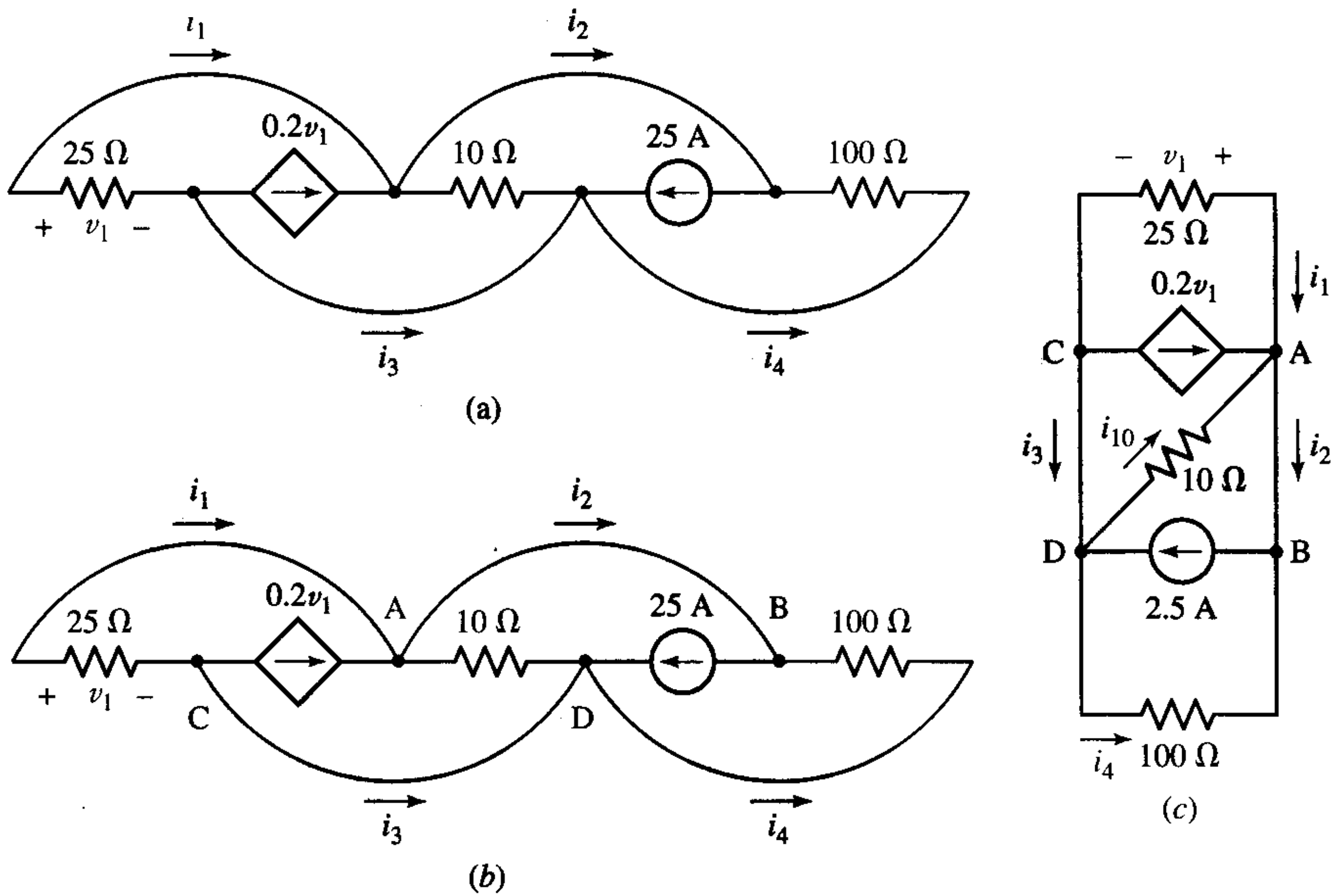
Practice: 3.7



For the single-node-pair circuit of the figure, find i_A , i_B and i_C

Example: 3.8

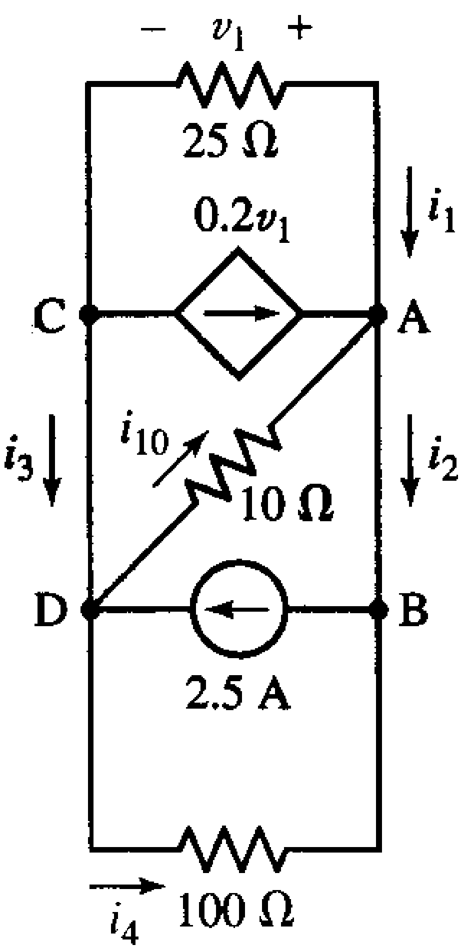
Find $i_1, i_2, i_3,$ and i_4



(*a*) A single-node-pair circuit. (*b*) Circuit with points labeled to assist in redrawing. (*c*) Redrawn circuit.

Example:

Find $i_1, i_2, i_3,$ and i_4



(c)

$$\frac{v_1}{25} - 0.2v_1 + \frac{v_1}{10} + 2.5 + \frac{v_1}{100} = 0$$

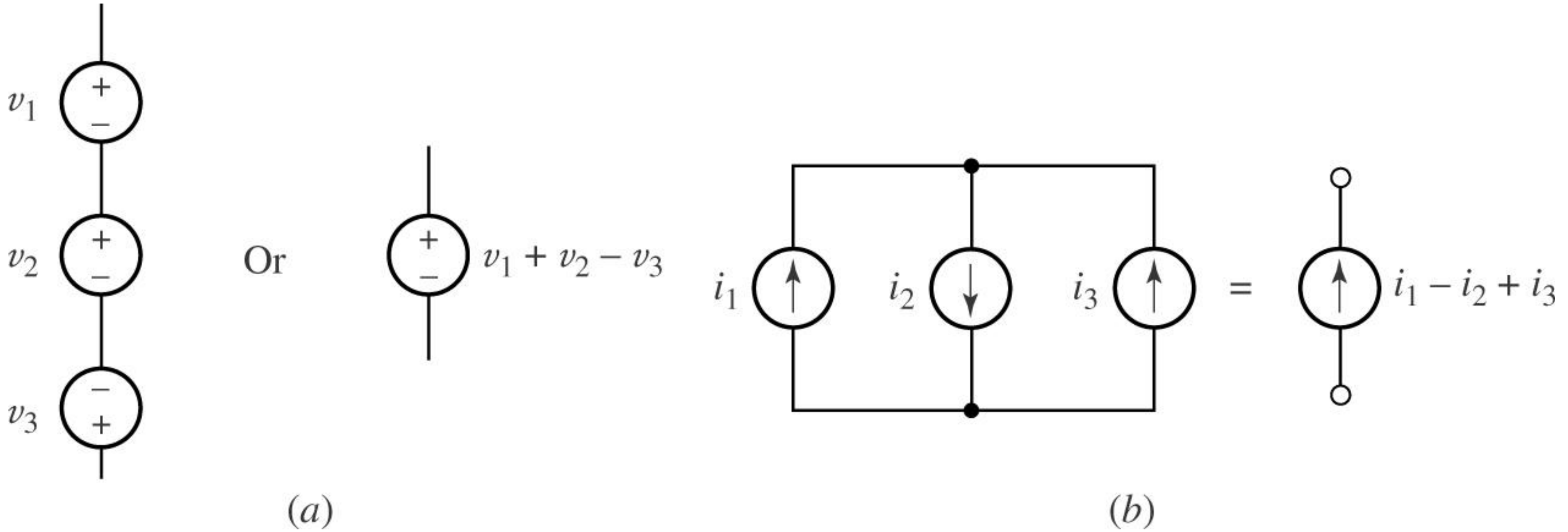
Solving, we find $v_1 = \dfrac{250}{5} = 50$ V.

$$i_1 = \frac{-v_1}{25} = -2 \text{ Amp.}$$

,...

Series and Parallel Connected:

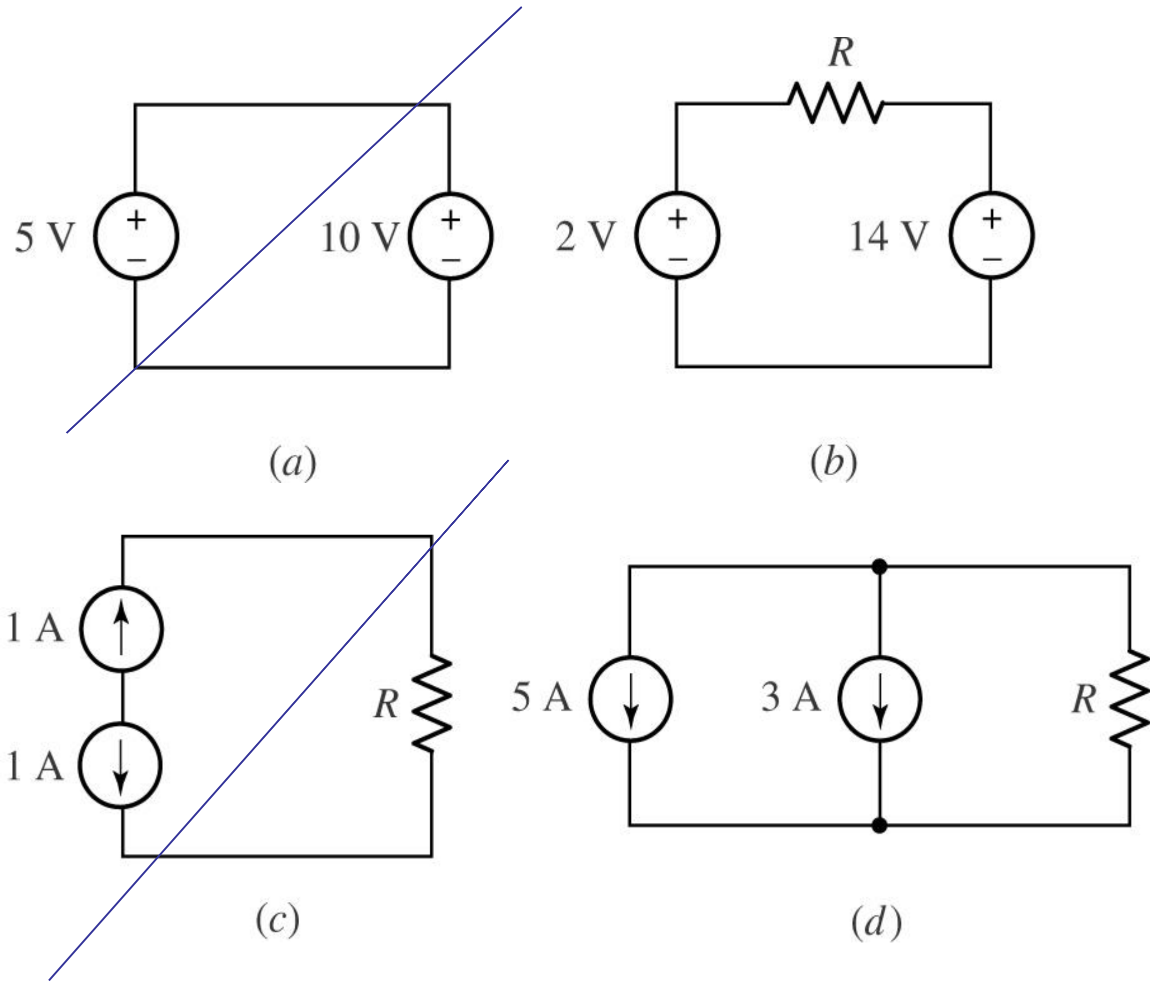
Independent Sources:



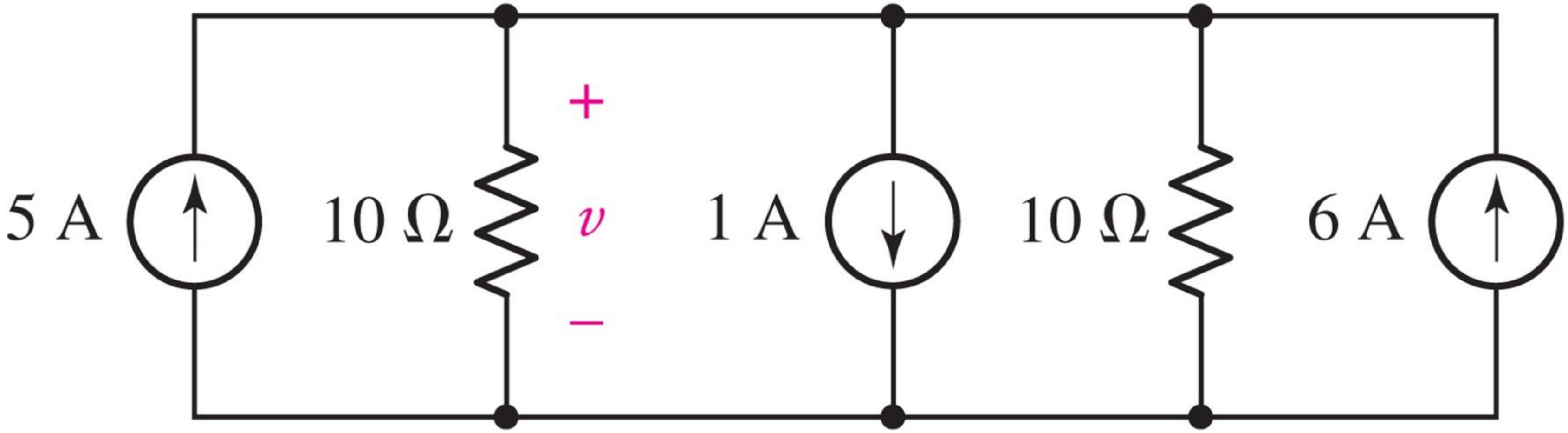
(a) Series connected voltage sources can be replaced by a single source. (b) Parallel current sources can be replaced by a single source.

Series and Parallel Connected:

Examples of circuits with multiple sources, some of which are "illegal" as they violate Kirchhoff's laws.

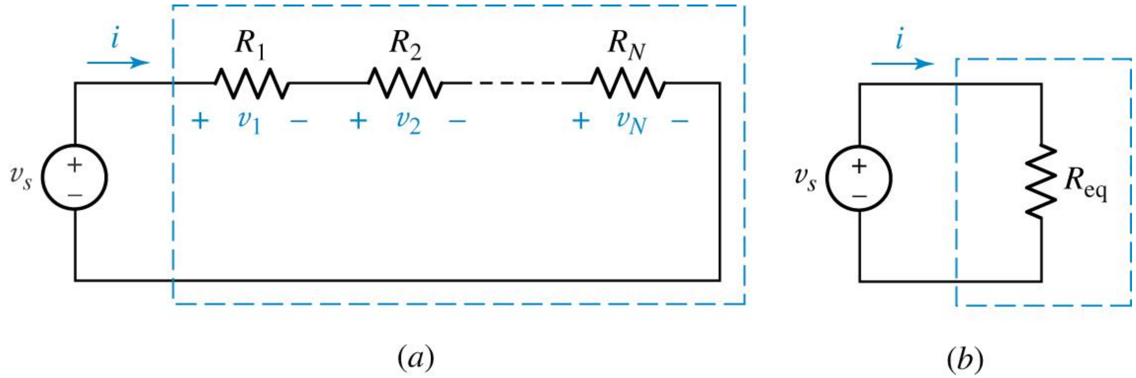


Practice: 3.8



Determine v in the circuit of the figure by first combing the three current sources.

Resistors in Series and Parallel:



(a) Series combination of *N* resistors. (b) Electrically equivalent circuit.

First, apply KVL:

$$v_s = v_1 + v_2 + \cdots + v_N$$

and then Ohm's law:

$$v_s = R_1 i + R_2 i + \cdots + R_N i = (R_1 + R_2 + \cdots + R_N) i$$

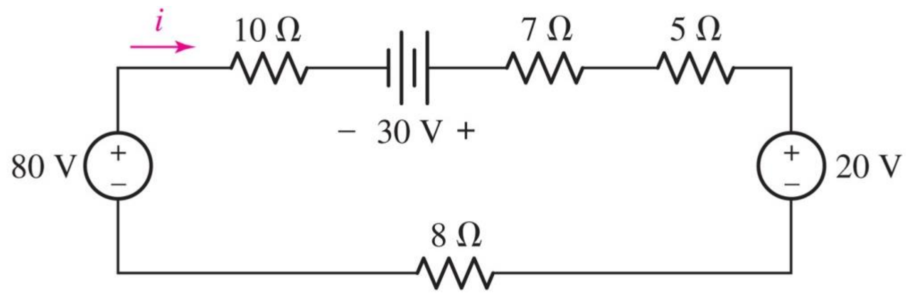
circuit shown

$$v_s = R_{eq} i$$

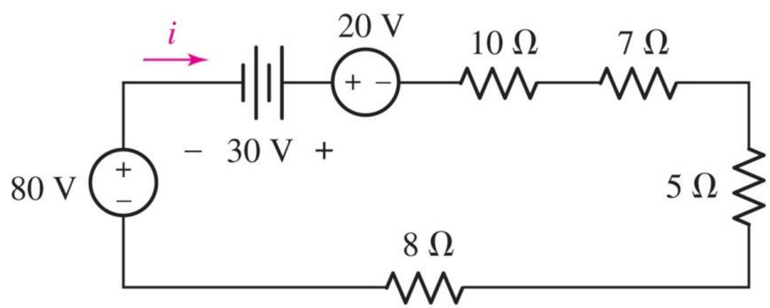
Thus, the value of the equivalent resistance for N series resistors is

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N}$$

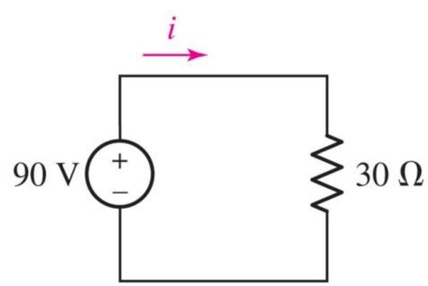
Resistors in Series and Parallel:



(a)

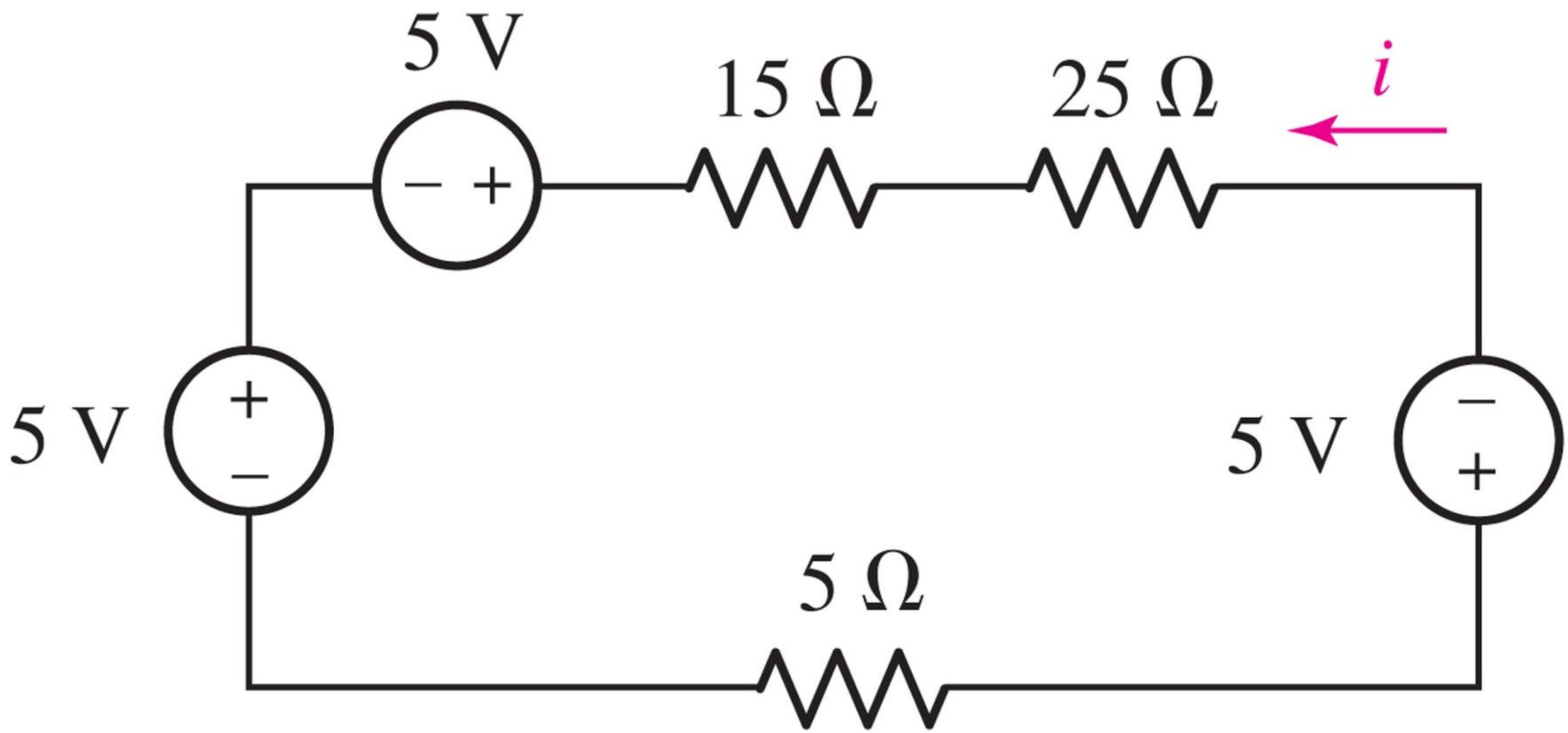


(b)



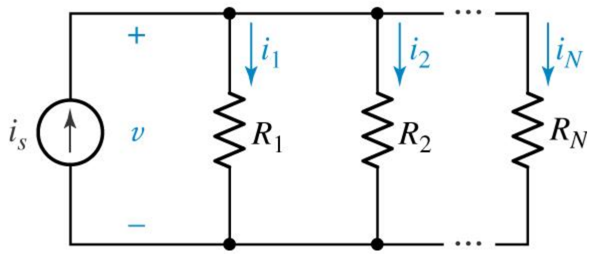
(c)

Practice: 3.9

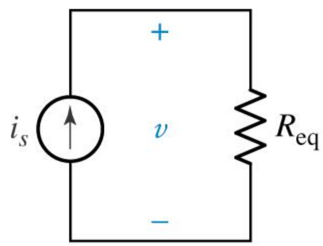


Determine i in the circuit

Resistors in Series and Parallel:



(a)



(b)

(a) A circuit with *N* resistors in parallel. (b) Equivalent circuit.

Beginning with a simple KCL equation,

$$i_s = i_1 + i_2 + \cdots + i_N$$

or $$i_S = \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N} = \frac{v}{R_{eq}}$$

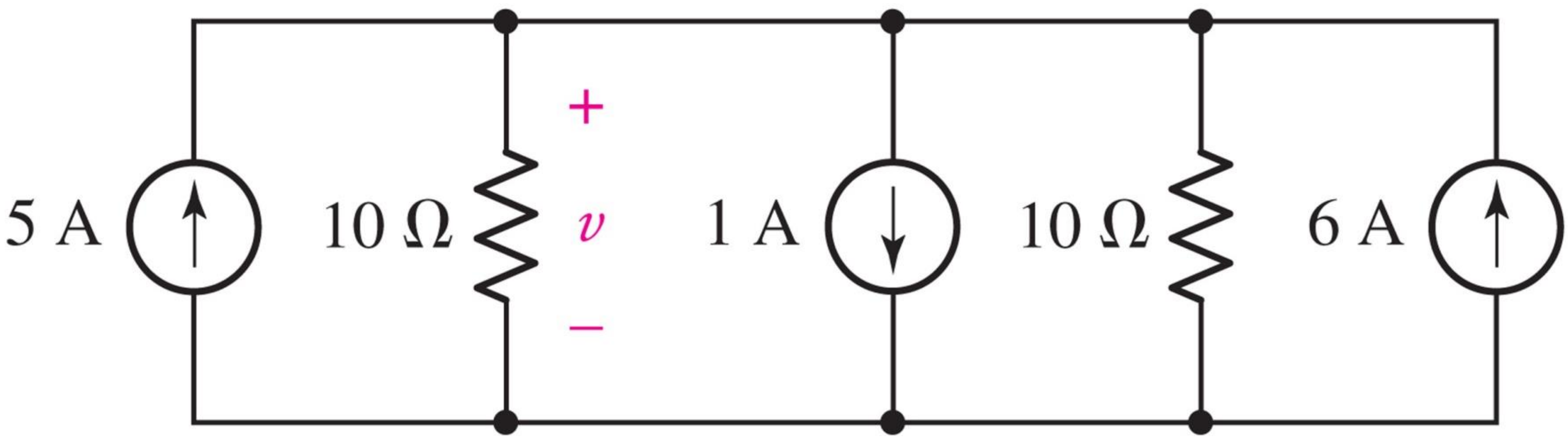
Thus, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

A special case worth remembering is

$$R_{eq} = R_1 \| R_2 = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

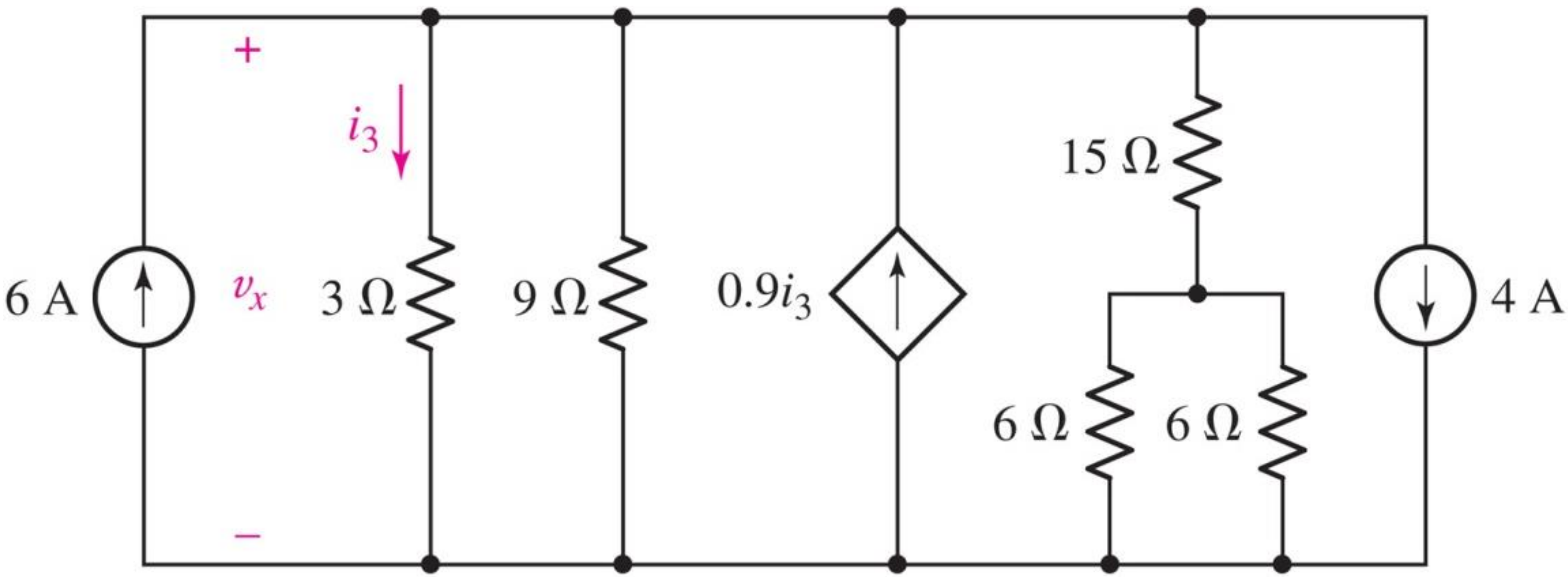
Practice: 3.10



Determine v in the circuit by first combing the three current sources, and then the two 10 Ω resistors.

Example: 3.11

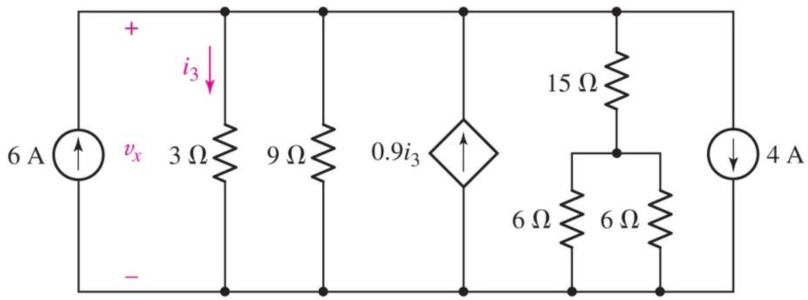
Calculate the power and voltage of the dependent source



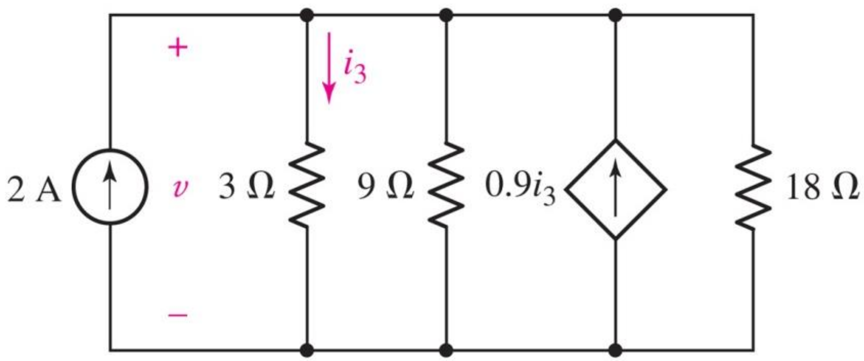
(*a*)

Example:

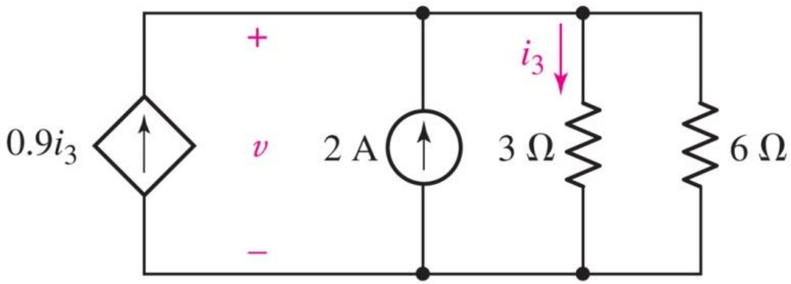
Calculate the power and voltage of the dependent source



(a)



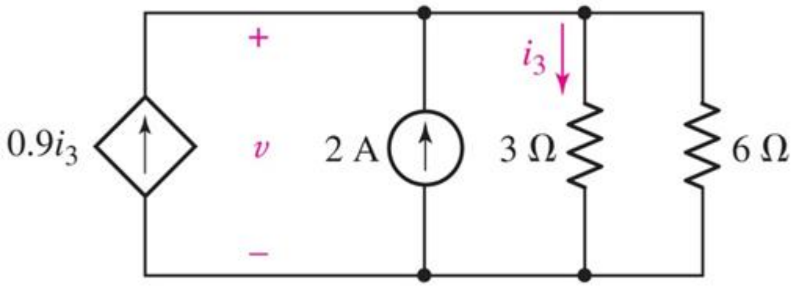
(b)



(c)

Example:

Calculate the power and voltage of the dependent source



From the top node: $-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0$

The other equation: $v = 3i_3$

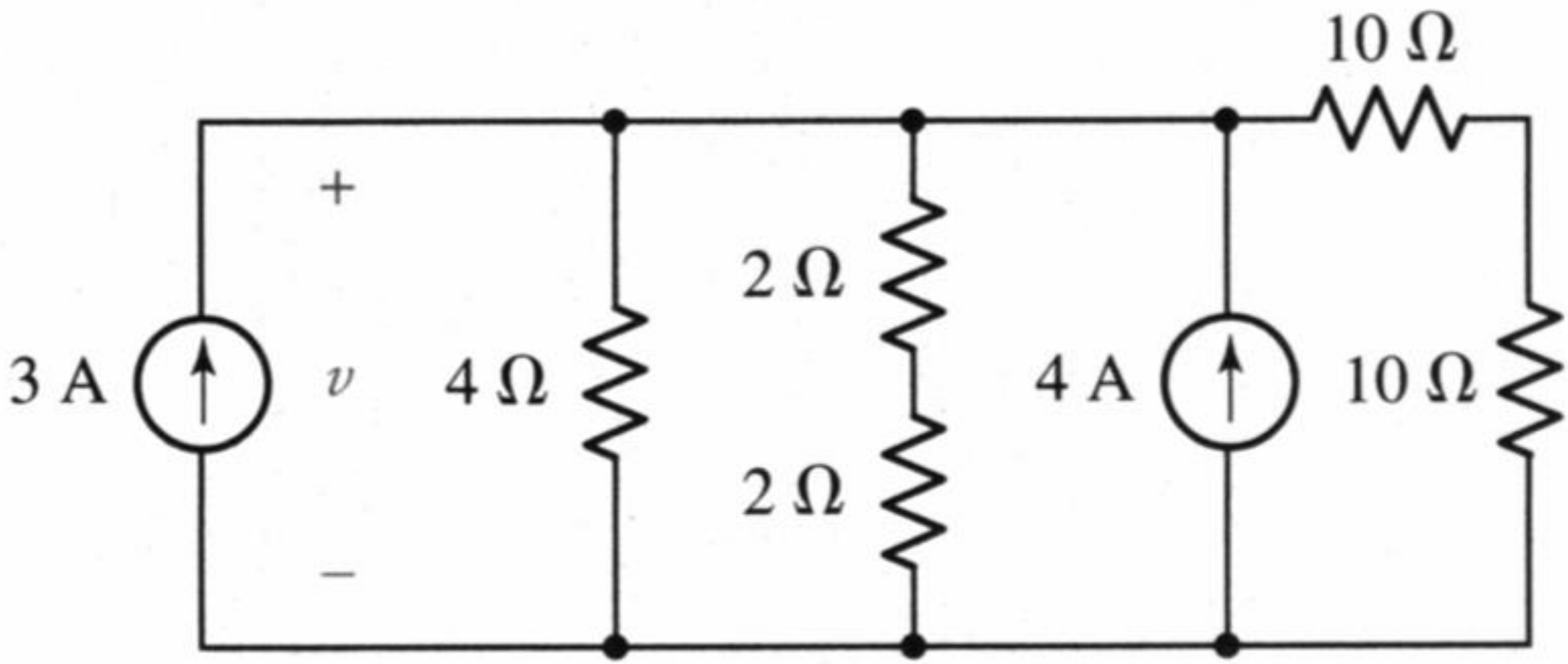
$$\therefore i_3 = \frac{10}{3} \text{ Amp.}$$

$$\therefore v = 3i_3 = 10 \text{ Volts}$$

The dependent source furnishes:

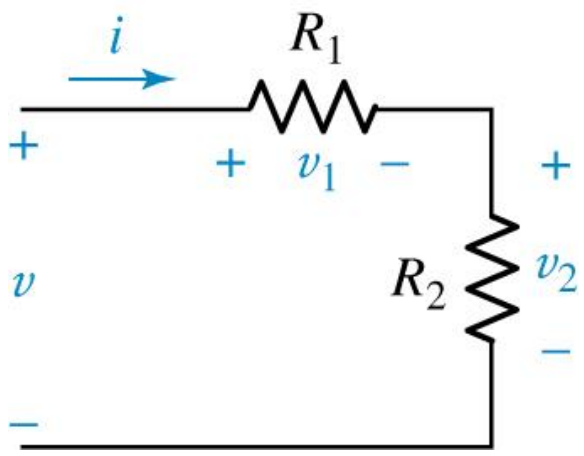
$$v \times 0.9i_3 = 10(0.9)\left(\frac{10}{3}\right) = 30 \text{ Watts.}$$

Practice: 3.11



Find the voltage v

Voltage and Current division:



An illustration of voltage division.

We may find v_2 by applying KVL and Ohm's law:

$$v = v_1 + v_2 = iR_1 + iR_2 = i(R_1 + R_2)$$

so $$i = \frac{v}{R_1 + R_2}$$

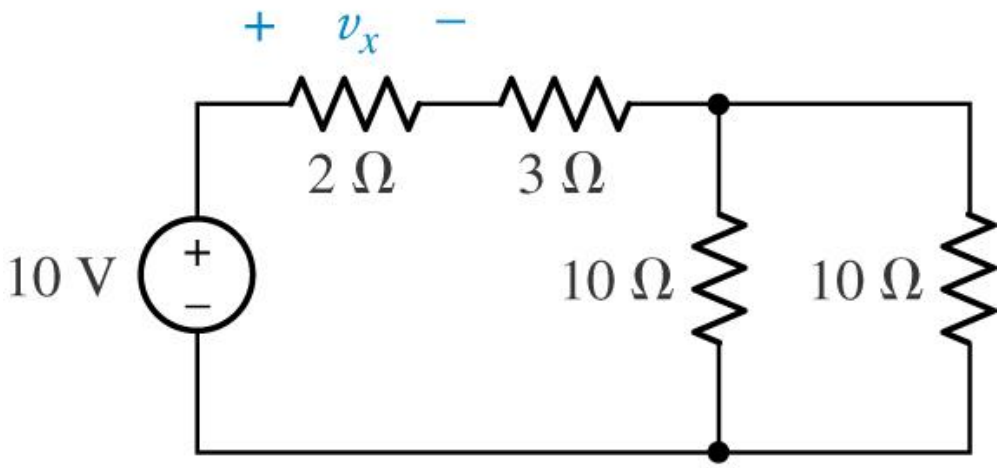
Thus, $$v_2 = iR_2 = \left(\frac{v}{R_1 + R_2}\right) R_2$$

or $$v_2 = \frac{R_2}{R_1 + R_2} v$$

For a string of *N* series resistors, we may write:

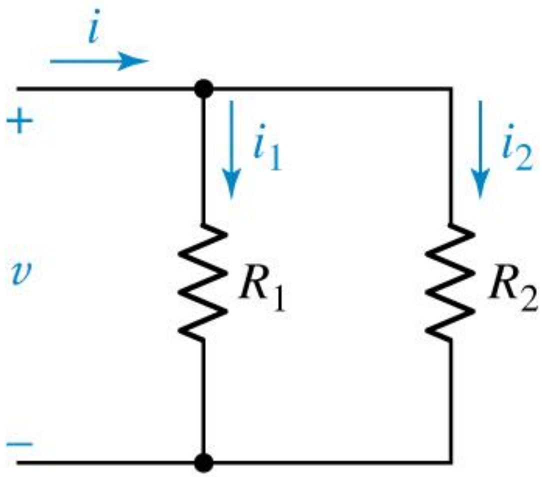
$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v}$$

Practice: 3.12



Use voltage division to determine v_x in the adjacent circuit.

Voltage and Current division:



An illustration of current division.

The current flowing through R_2 is

$$i_2 = \frac{v}{R_2} = \frac{i(R_1 \| R_2)}{R_2} = \frac{i}{R_2}\frac{R_1 R_2}{R_1 + R_2}$$

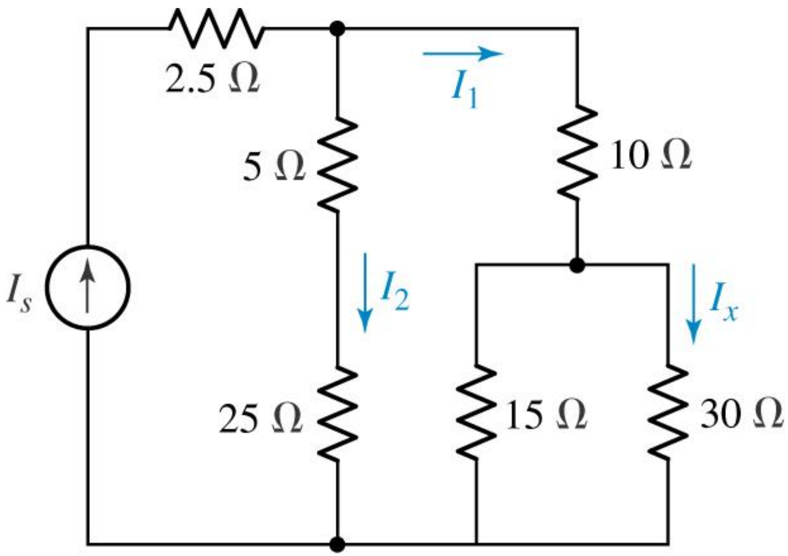
or

$$\boxed{i_2 = i\frac{R_1}{R_1 + R_2}}$$

For a parallel combination of *N* resistors, the current through R_k is

$$i_k = i\frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

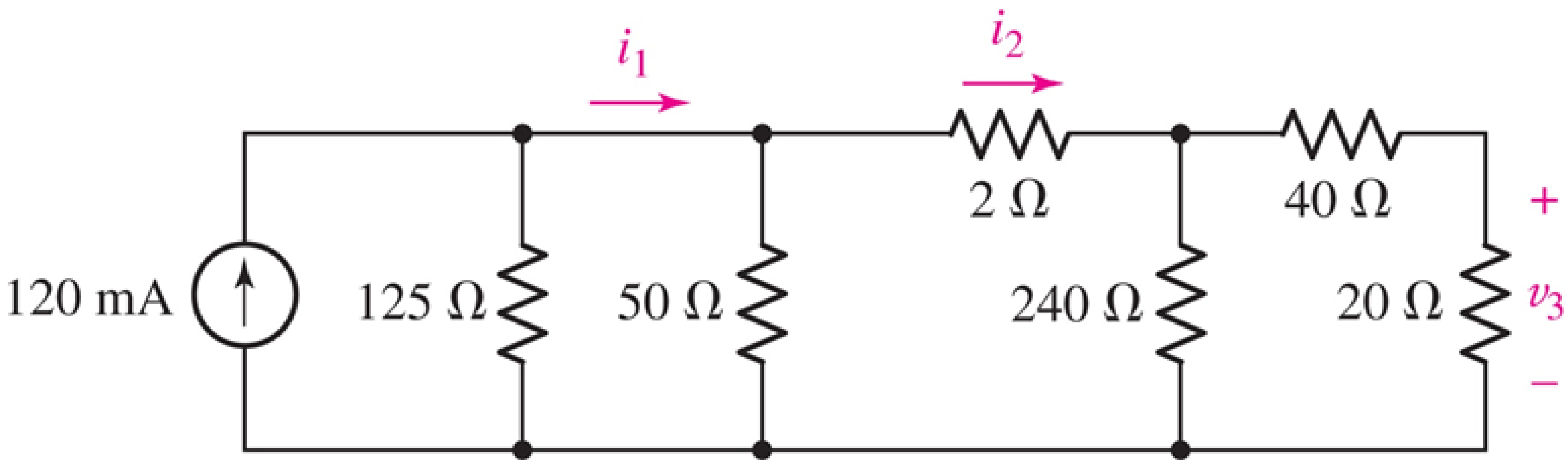
Example:



Determine the current I_x if I_1 = 100 mA.

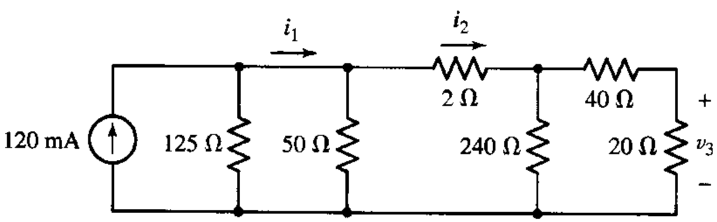
Practice: 3.13

Find i_1, i_2, and v_3



Example:

Find i_1, i_2, and v_3



$$(((40\Omega + 20\Omega) // 240\Omega) + 2\Omega) // 50\Omega$$
$$= ((60\Omega // 240\Omega) + 2\Omega) // 50\Omega$$
$$= (\frac{60 \times 240}{60 + 240} + 2\Omega) // 50\Omega$$
$$= (48\Omega + 2\Omega) // 50\Omega$$
$$= 25\Omega$$

Current division:

$$i_1 = \left(\frac{125}{125 + 25}\right) \times 120 \text{ mA.} = 100 \text{ mA.}$$

$$i_2 = \left(\frac{50}{50 + 50}\right) \times i_1 = 50 \text{ mA.}$$

$$i_{20\Omega} = \left(\frac{240}{240 + 60}\right) \times i_2 = 40 \text{ mA.}$$

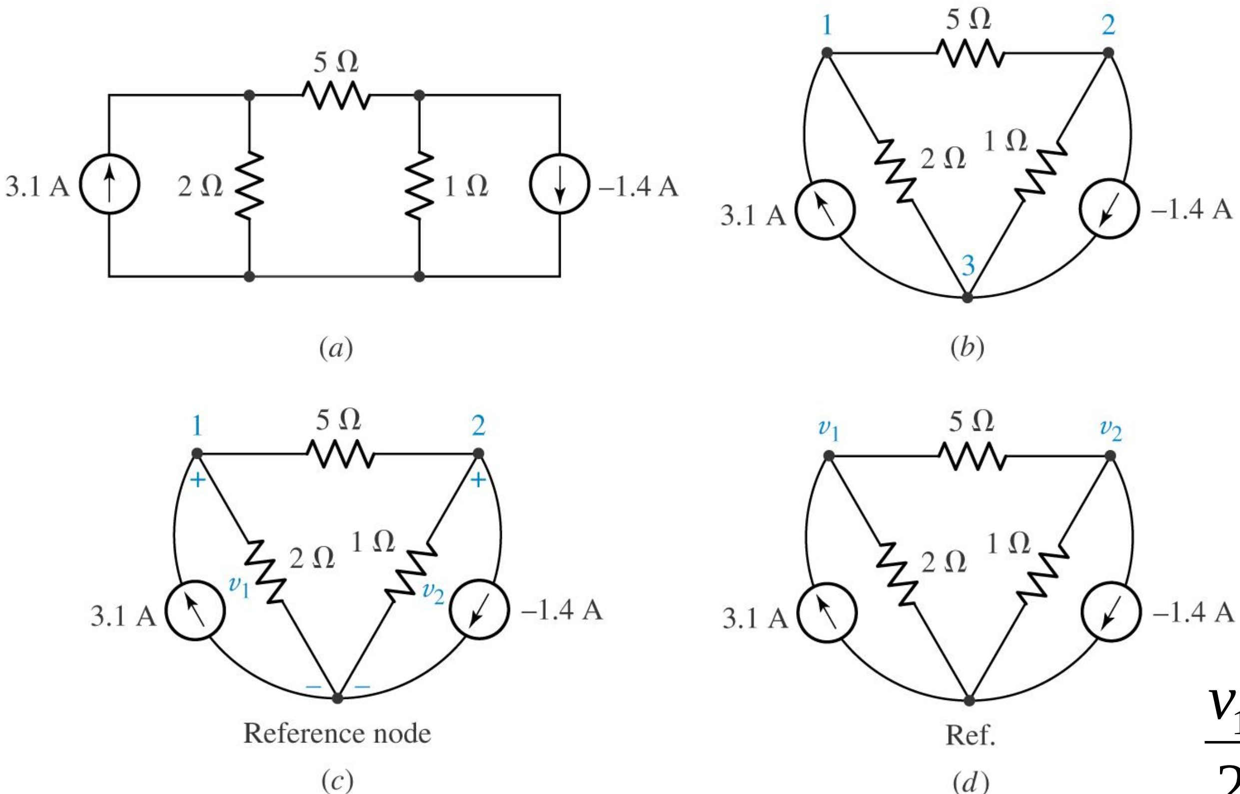
$$v_3 = i_{20\Omega} \times 20\Omega = 0.8 \text{ Volts.}$$

Homework:

Ch4-Nodal and Mesh Analysis:

Nodal Analysis: is based on KCL

Obtain values for the unknown voltages across the elements in the circuit below.



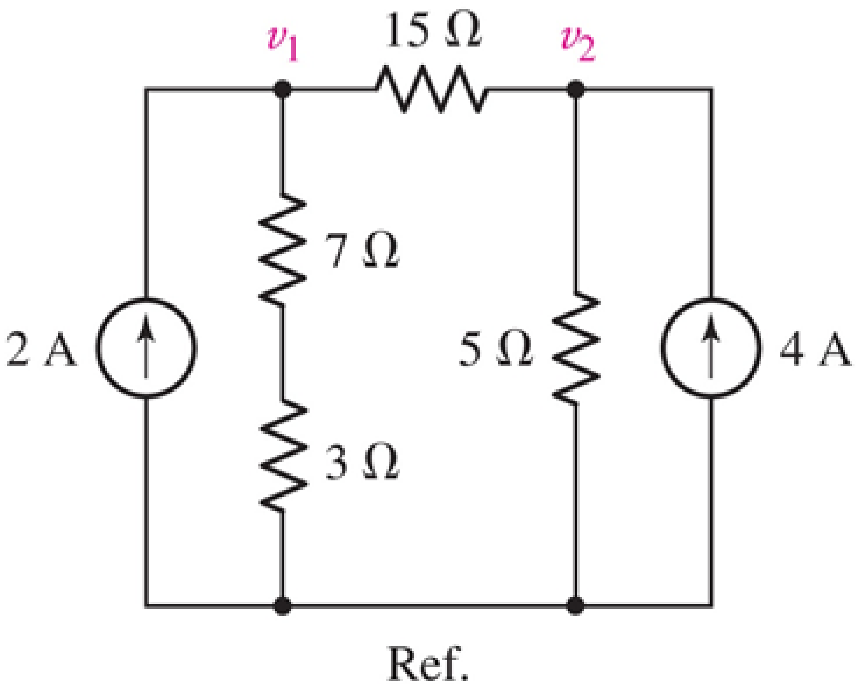
At node 1

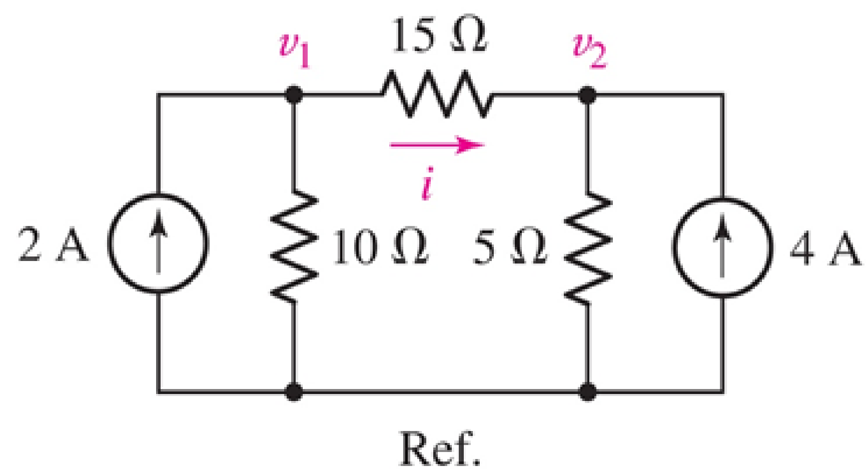
$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$

At node 2

$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4)$$

Practice: 4.1

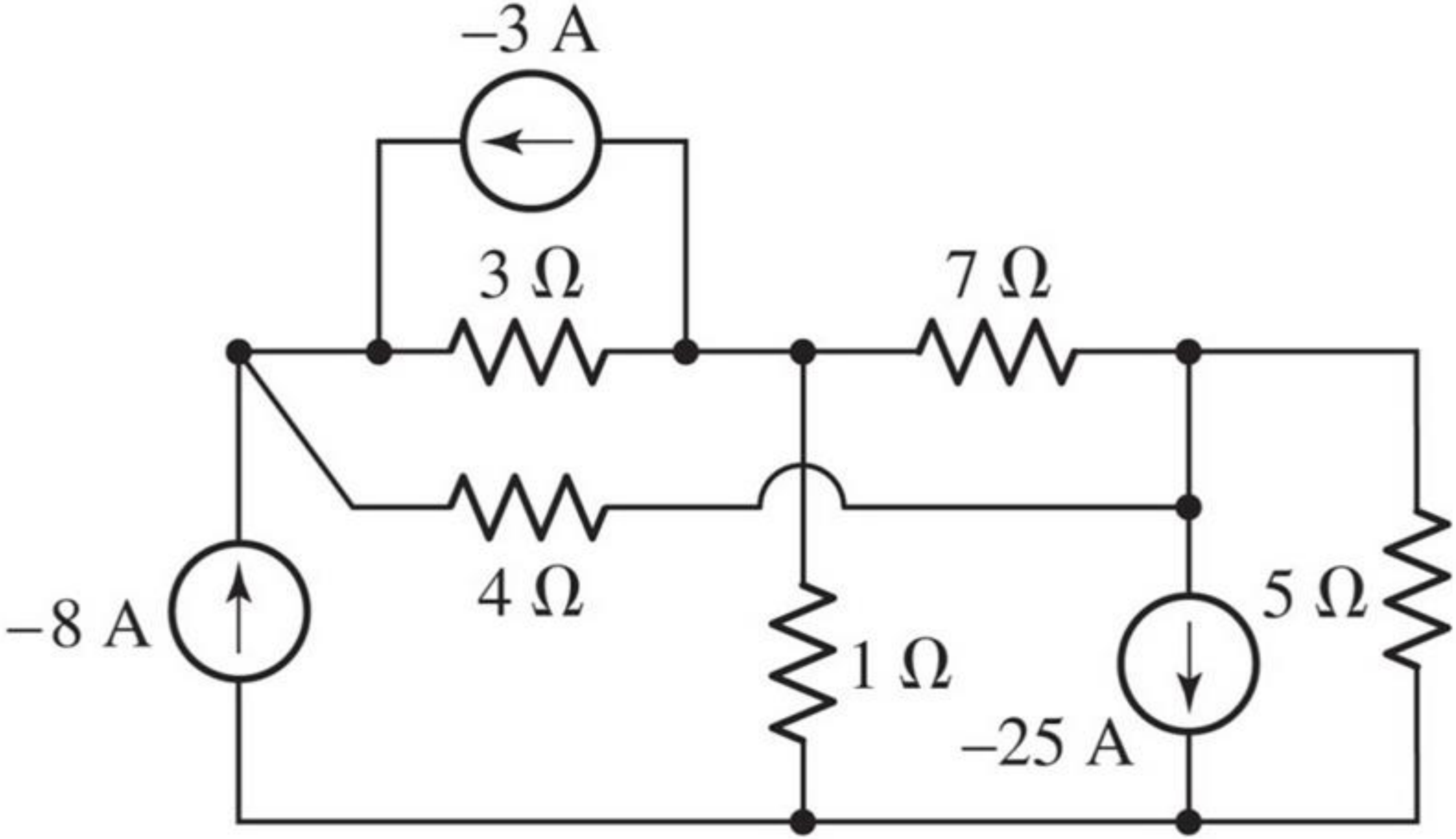




Compute the voltage across each current source.

Example: 4.2

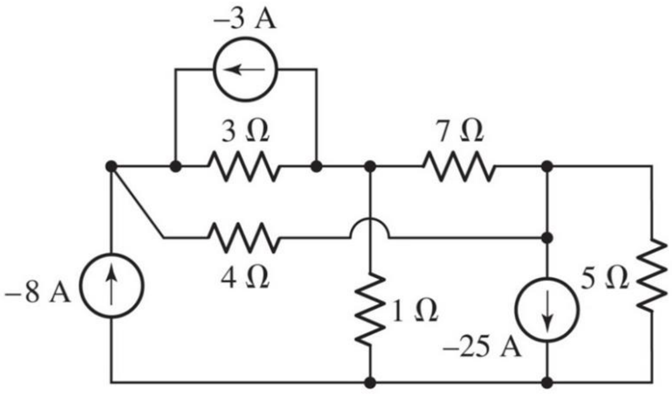
find the nodes voltages:



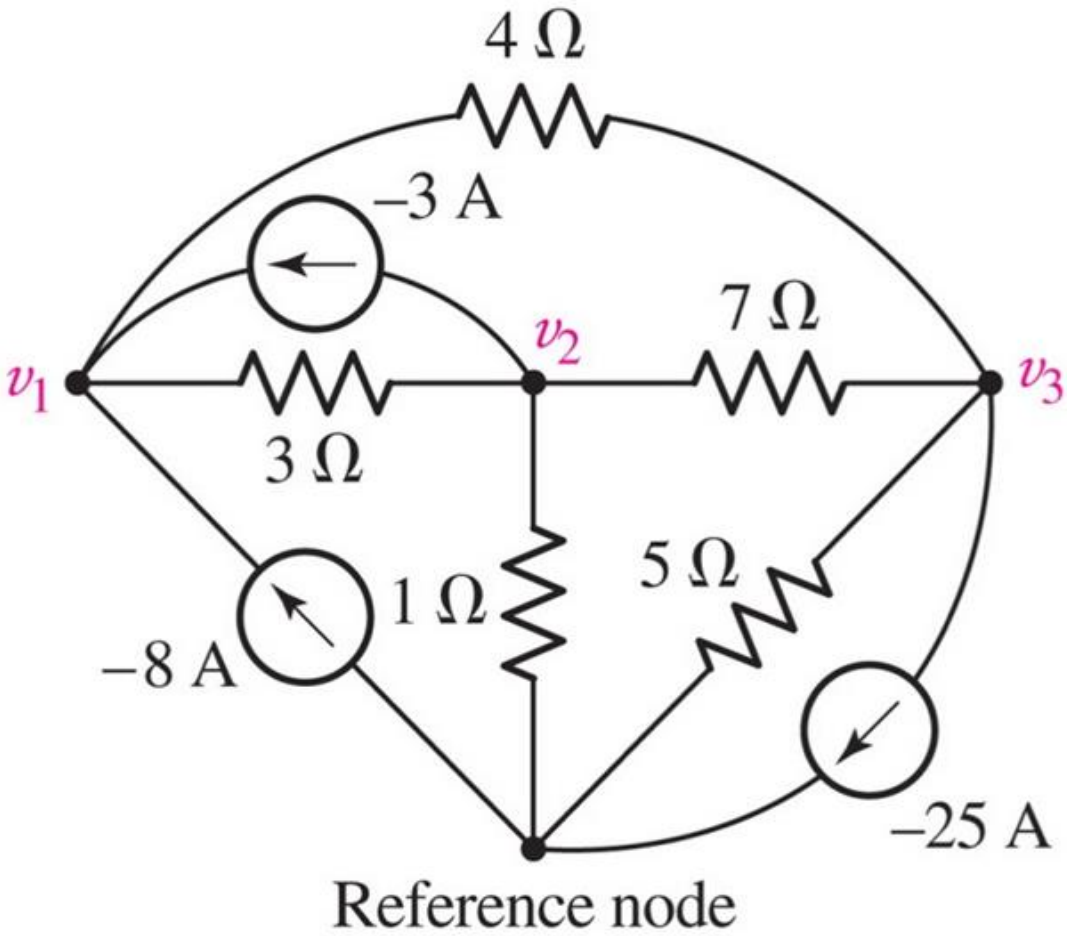
(a)

Example:

find the nodes voltages:



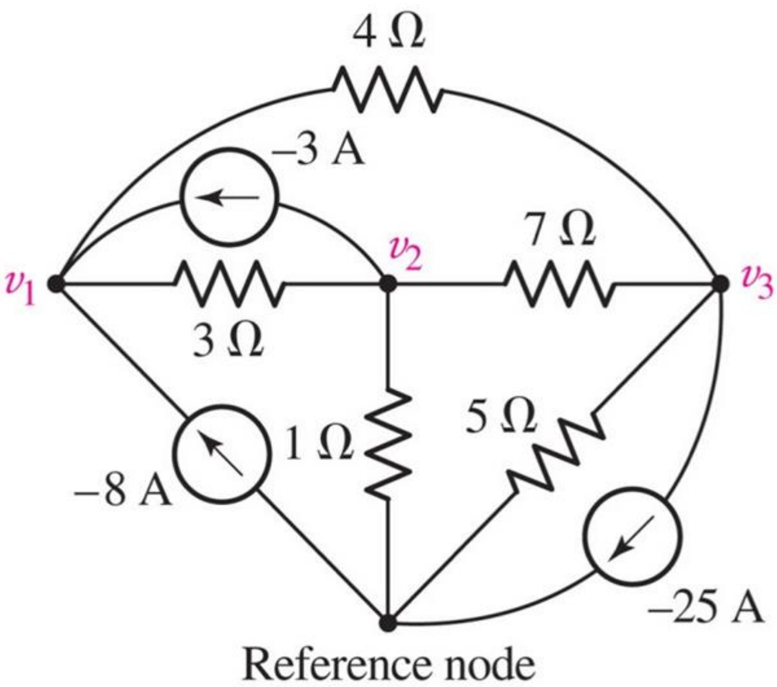
(a)



(b)

Example:

find the nodes voltages:



(b)

A KCL equation for node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

Node 2:

$$\frac{v_2 - v_3}{7} + \frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) = 0$$

Node 3:

$$\frac{v_3 - v_2}{7} + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

Example:

find the nodes voltages:

A KCL equation for node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

Node 2:

$$\frac{v_2 - v_3}{7} + \frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) = 0$$

Node 3:

$$\frac{v_3 - v_2}{7} + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

Node 1:

$$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$$

Node 2:

$$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$$

Node 3:

$$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$$

Example:

find the nodes voltages: (matrix methods)

Node 1:
$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$

Node 2:
$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$

Node 3:
$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$

$$\Rightarrow v_1 = \frac{\begin{vmatrix} -11 & -0.3333 & -0.2500 \\ 3 & 1.4762 & -0.1429 \\ 25 & -0.1429 & 0.5929 \end{vmatrix}}{\begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}}$$

$$= \frac{1.714}{0.3167} = 5.412 \text{ Volts.}$$

Example:

find the nodes voltages: (matrix methods)

Node 1:
$$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$$

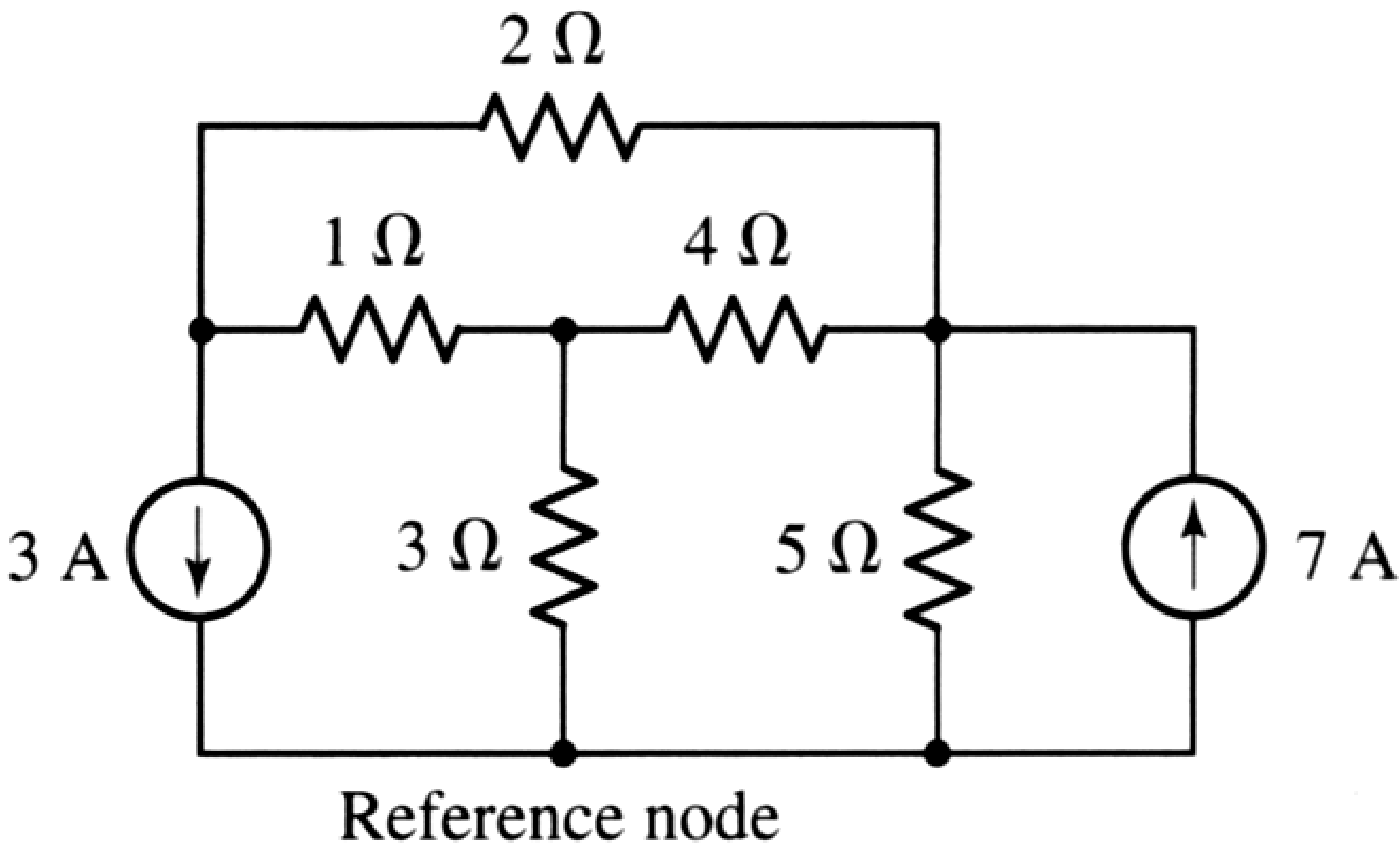
Node 2:
$$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$$

Node 3:
$$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$$

$$v_2 = \frac{\begin{vmatrix} 0.5833 & -11 & -0.2500 \\ -0.3333 & 3 & -0.1429 \\ -0.2500 & 25 & 0.5929 \end{vmatrix}}{0.3167} = \frac{2.45}{0.3167} = 7.736 \text{ Volts.}$$

$$v_3 = \frac{\begin{vmatrix} 0.5833 & -0.3333 & -11 \\ -0.3333 & 1.4762 & 3 \\ -0.2500 & -0.1429 & 25 \end{vmatrix}}{0.3167} = \frac{14.67}{0.3167} = 46.32 \text{ Volts.}$$

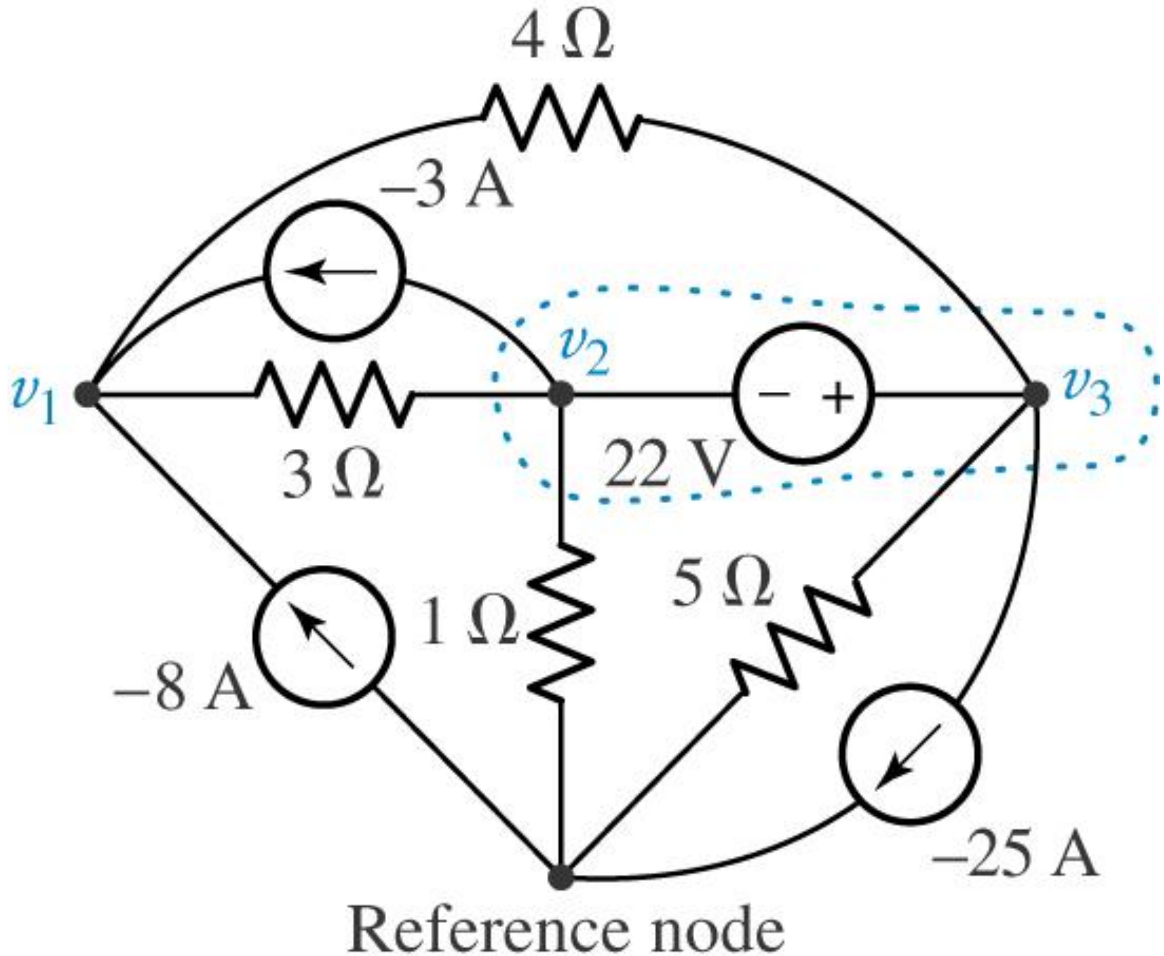
Practice: 4.2



Compute the voltage across each current source.

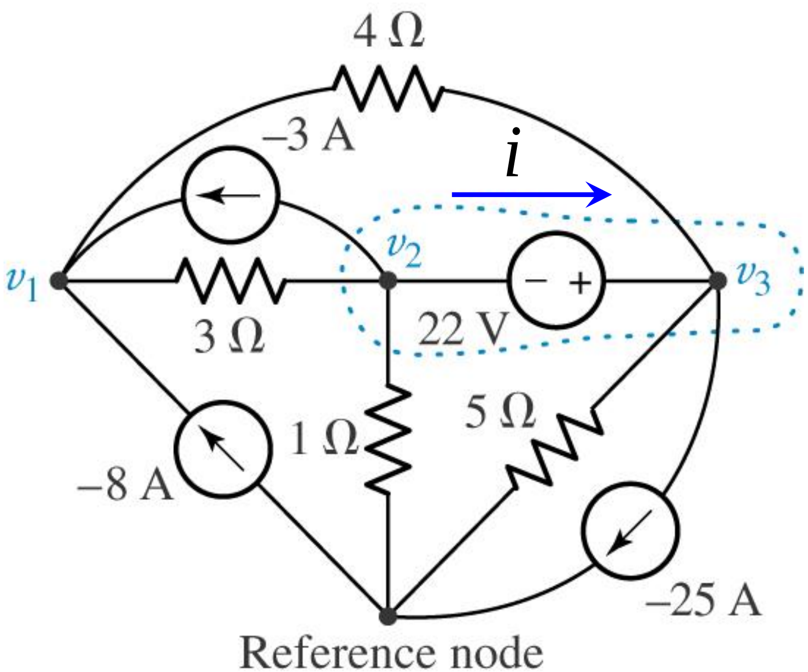
The Supernode:

find the nodes voltages:



The Supernode:

find the nodes voltages:



A KCL equation for

Node 1:

$$\frac{v_1 - v_3}{4} - (-3) + \frac{v_1 - v_2}{3} - (-8) = 0$$

Node 2:

$$i + \frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3) = 0$$

Node 3:

$$-i + \frac{v_3}{5} + (-25) + \frac{v_3 - v_1}{4} = 0$$

Reference:

W.H. Hayt, Jr., J.E. Kemmerly, S.M. Durbin, Engineering Circuit Analysis, Sixth Edition.