Electrical and Electronic Measurements:

Basic Statistical Analysis

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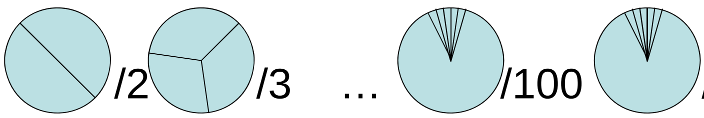
Statistical Evaluation

- Mean, $\overline{X} = \Sigma_{i=1 \rightarrow N} X_i / N$ $\Rightarrow$ the best value
- Deviation, $D = X_i - \overline{X}$
- Mean Deviation, $MD = \Sigma_{i=1 \rightarrow N} |X_i - \overline{X}| / N$
- Standard Deviation,
  $\sigma = \sqrt{\Sigma_{i=1 \rightarrow N} (X_i - \mu)^2 / N}$ for a population
  $s.d. = \sqrt{\Sigma_{i=1 \rightarrow N} (X_i - \overline{X})^2 / N-1}$ for a sample (<30)
  e.g. a cake



/2 /3 … /100 /101

- Variance, σ^2
- Probable error, $PE = \pm\, 0.6754\, \sigma$

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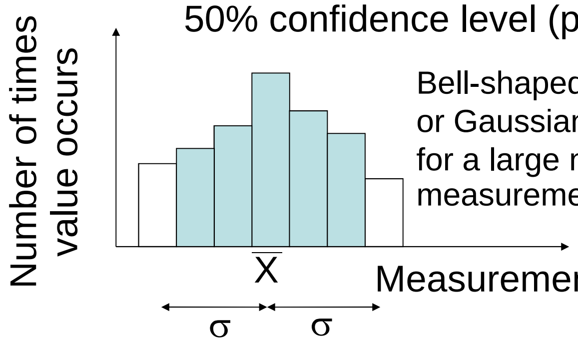
Normal Distribution (Gaussian)

Histogram is a plot showing how the number of times a measurement occurs is related to the value of the individual measurements, aka frequency distribution.

50% confidence level (probable error)



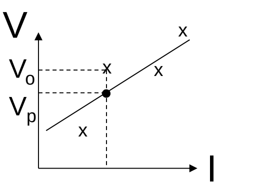
Bell-shaped normal distribution or Gaussian distribution for a large number of measurement

Linear Regression

V = IR $\Rightarrow$ Linear y=mx+c

Error = $V_p - V_o$

Minimum $\Sigma(V_p - V_o)^2$



Prediction Vs Observation (Best Fitted?)

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_{random} \rightarrow 0 \quad \text{, i=1,2,…,N points}$$

$$\varepsilon = \Sigma_i [\, Y_i - (\beta_0 + \beta_1 X_i) \,]^2$$

$$\partial\varepsilon/\partial\beta_1 = -2\, \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i \,]\, X_i$$

$$= -2\, \Sigma_i [\, Y_i X_i - \beta_0 X_i - \beta_1 X_i^2 \,] = 0$$

$$\partial\varepsilon/\partial\beta_0 = -2\, \Sigma_i [\, Y_i - \beta_0 - \beta_1 X_i \,] = 0$$

Linear Regression (Cont'd)

$$\Sigma_i [\, Y_iX_i - \beta_0X_i - \beta_1X_i^2 \,] = \Sigma_i [\, Y_i - \beta_0 - \beta_1X_i \,]$$

$$\Sigma_iY_iX_i - \beta_0\Sigma_iX_i - \beta_1\Sigma_iX_i^2 = \Sigma_iY_i - N\beta_0 - \beta_1\Sigma_iX_i$$

$$\Sigma_iY_iX_i - \Sigma_iY_i = \beta_0(\Sigma_iX_i - N) + \beta_1(\Sigma_iX_i^2 - \Sigma_iX_i)$$

$$(\Sigma_iY_iX_i)/N - \bar{Y} = (\bar{Y} - \beta_1\bar{X})(\bar{X} - 1) + \beta_1[\, (\Sigma_iX_i^2)/N - \bar{X} \,]$$

$$(\Sigma_iY_iX_i)/N \cancel{- \bar{Y}} = \bar{X}\bar{Y} \cancel{- \bar{Y}} + \beta_1[(\Sigma_iX_i^2)/N \cancel{- \bar{X}} - \bar{X}^2 \cancel{+ \bar{X}}]$$

$$(\Sigma_iY_iX_i) = N\bar{X}\bar{Y} + \beta_1[\, \Sigma_iX_i^2 - N\bar{X}^2 \,]$$

$$\beta_1 = \frac{\Sigma_iY_iX_i - N\bar{X}\bar{Y}}{\Sigma_iX_i^2 - N\,\bar{X}^2} \quad \text{and } \beta_0 = \bar{Y} - \beta_1\bar{X}$$

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Linear Regression (Cont'd)

For nonlinear equation,

$Y = X^n \;\Rightarrow\;$ (log Y) = n (log X) → Logarithm

$Y = a^X \;\Rightarrow\;$ (log Y) = (log a) X → Semi-log

or higher-degree parabolic-curve fitting

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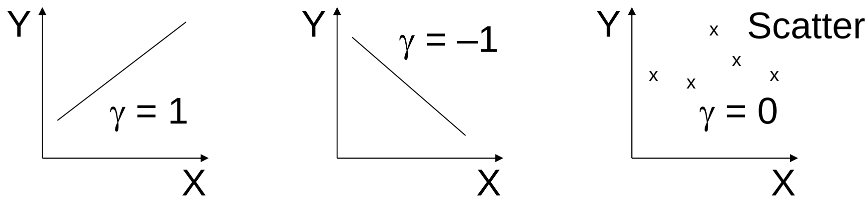
Correlation

$$\gamma_{Population} = \frac{\text{Covariance}(X,Y)}{\sigma_X \sigma_Y} = \frac{1/N\ \Sigma(X - \bar{X})(Y - \bar{Y})}{\sigma_X \sigma_Y}$$



$$\gamma_{Pearson's} = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{[\,N\Sigma X^2 - (\Sigma X)^2\,]\,[\,N\Sigma Y^2 - (\Sigma Y)^2\,]}}$$

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Estimation and Reduction of Errors

- Random errors can be estimated by taking many readings and applying statistical analysis.
- Such errors can be reduced by careful design of the measurement system to minimize interference or environmental fluctuations, e.g. by using a constant temperature enclosure, mounting the instrument on a vibration isolation mount, or shielding it against electric and magnetic fields.

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Estimation and Reduction of Errors (2)

- Systematic error can be determined and allowed for.
- We will discuss about this in the next chapter.
- Many systematic errors can be reduced by careful inspection and maintenance of instruments to ensure proper operation.

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Estimation and Reduction of Errors (3)

- Human error cannot be estimated!
- Human error can be reduced by using two or more operators to take readings or carry out operations, but the main way of reducing such error is by operators paying careful attention to what they are doing and fully understanding the techniques used and the limitations and capabilities of the instruments.

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Ex1. The following results were obtained from measurements of a resistance of 99.0Ω :

99.1Ω, 99.0Ω, 98.7Ω, 99.4Ω, 98.8Ω, 99.0Ω

- **The arithmetic mean**, $\overline{X}$

= (99.1 +99.0 +98.7 +99.4 +98.8 +99.0) / 6

= 594.0 / 6 = 99.0 Ω

- **The mean deviation, MD**

= [|99.1-99.0| +|99.0-99.0| +|98.7-99.0| +|99.4-99.0| +|98.8-99.0| +|99.0-99.0|] / 6

= [|+0.1| +|0.0| +|-0.3| +|+0.4| +|-0.2| +|0.0|]/6

= 1.0 / 6 = 0.17 Ω

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Ex1. (Cont'd)

- **The standard deviation, s.d.**

$$= \sqrt{[(+0.1)^2 +(0.0)^2 +(-0.3)^2 +(+0.4)^2 +(-0.2)^2 +(0.0)^2] / (6-1)}$$

$$= \sqrt{(0.01 +0.00 +0.09 +0.16 +0.04 +0.00) / 5}$$

$$= \sqrt{0.30 / 5}$$

$$= \sqrt{0.06}$$

= 0.24 Ω

- **Probable error, PE**

= 0.6754 × 0.24

= 0.16 Ω

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Ex1. (Cont'd)

- **Mean error, e**

= [|99.1-99.0| +|99.0-99.0| +|98.7-99.0| +|99.4-99.0| +|98.8-99.0| +|99.0-99.0|] / 6

= [|+0.1| +|0.0| +|-0.3| +|+0.4| +|-0.2| +|0.0|]/6

= 1.0 / 6 = 0.17 Ω

It is not (Mean Measured Value – True Value)!

- **Percent error, %e**

= (0.17 / 99.0) $\times$100% = 0.17%

- Accuracy

= 100 - 0.17 = 99.83%

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Ex1. (Cont'd)

- **Percent devition, %D**

= (0.17 / 99.0) $\times$100% = 0.17%

- **Precision**

= 100 - 0.17 = 99.83%

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Ex2. Consider measurements of a voltage which gave the following results, for 16 measurements :

values between 5.51 and 5.52 V come up twice
values between 5.52 and 5.53 V come up three times
values between 5.53 and 5.54 V come up six times
values between 5.54 and 5.55 V come up twice
values between 5.55 and 5.56 V come up twice
values between 5.56 and 5.57 V come up once

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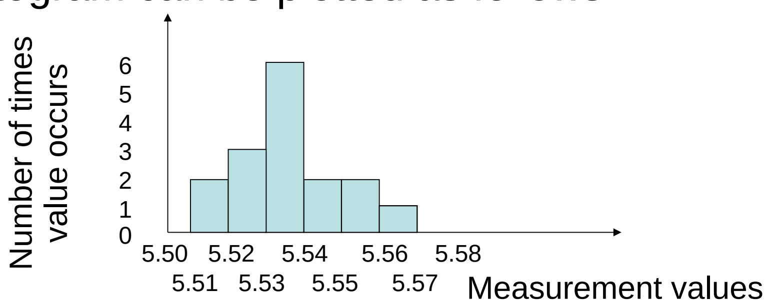
Ex2. (Cont'd)

- A histogram can be plotted as follows



- The area of a bar in histogram is related to the frequency.
- The location of the bar is related to the range of values for which that is the frequency.

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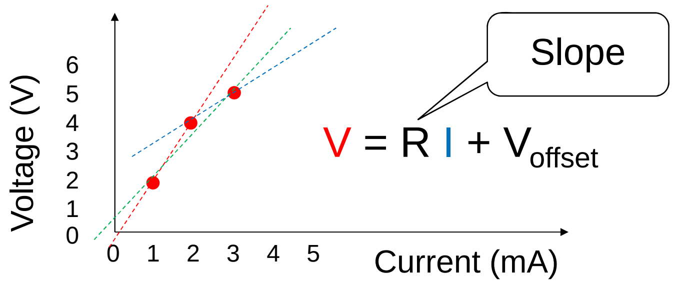
Ex3. Find the best predicted R for the following measurements :

Input Current (mA)	Measured Voltage (V)
1	2
2	4
3	5



Ex3. (Cont'd)

- Using linear regression

No.	I (mA)	V (V)	IV	I^2
1	1	2	2	1
2	2	4	8	4
3	3	5	15	9
N = 3	ΣI = 6	ΣV = 11	ΣIV = 25	$\Sigma I^2 = 14$
	$I_{avg} = 2$	$V_{avg} = 3.67$		

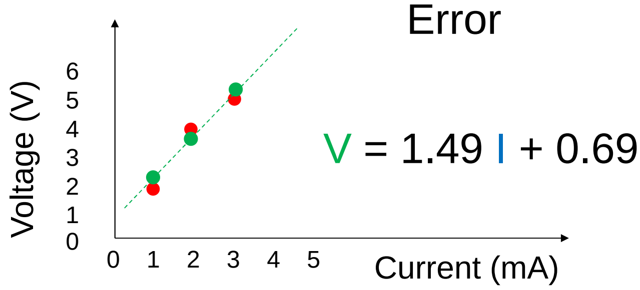
- $R = [\ 25 - (3)(2)(3.67)\ ] / [\ 14 - (3)(2)^2\ ]$

  $= 1.49\ k\Omega$
- $V_{offset} = 3.67 - (1.49)(2) = 0.69$ V

Ex3. (Cont'd)

Input Current (mA)	Measured Voltage (V)	Predicted Voltage (V)
1	2	2.18
2	4	3.67
3	5	5.16

Error



What is the average error of this measurement?

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Ex3. (Cont'd)

- The correlation coefficient for the data, γ

I (mA)	V (V)	IV	I^2	V^2
1	2	2	1	4
2	4	8	4	16
3	5	15	9	25
$\Sigma I = 6$	$\Sigma V = 11$	$\Sigma IV = 25$	$\Sigma I^2 = 14$	$\Sigma V^2 = 45$
$I_{avg} = 2$	$V_{avg} = 3.67$			

$$= [\,(3)(25)-(6)(11)\,] \,/\, \sqrt{[(3)(14)-(6)^2][(3)(45)-(11)^2]}$$

$$= 0.9820$$

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