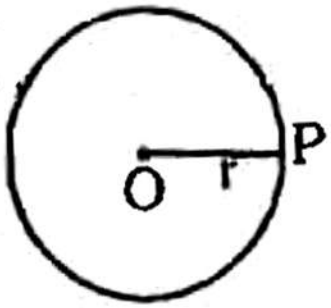
CIRCLES

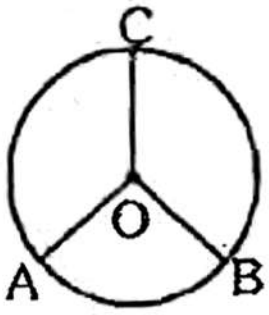
Terms and Facts Related To Circles

1. A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point remains constant. The fixed point is called the centre & the constant distance is called the radius of the circle.



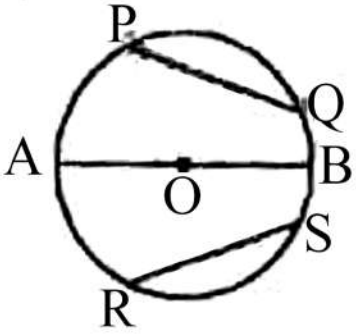
2. The perimeter of a circle is called its circumference.

Circumference $= 2\pi r$



3. Diameter = A chord of the circle passing through the centre of a circle is called its diameter.

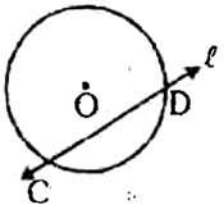
Diameter = 2 × Radius



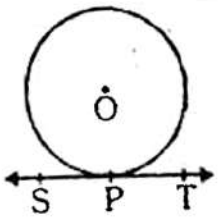
(i) Diameter is the largest chord of a circle.

(ii) All diameters of a circle are equal in length

4. **SECANT:** A Line which intersects a circle in two distinct points is called a secant of the circle.



5. **TANGENT:** A Line that touches the circle in exactly one point is called a tangent of the circle.

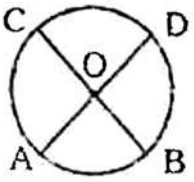


6. **Central Angle:** An Angle subtented by an arc at the centre of a circle is called its central angle.

In the given figure,

Central angle of $\overset{\frown}{PQ} = \angle POQ$

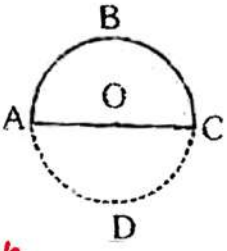
7. **Congruent Arcs:** Two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are said to be congruent, if they have same degree measure.



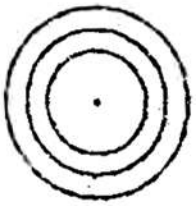
$$\overset{\frown}{AB} = \overset{\frown}{CD} \Leftrightarrow m(\overset{\frown}{AB}) = m(\overset{\frown}{CD}) \Leftrightarrow \angle AOB = \angle COD$$

8. **Semi-Circles:** A diameter divides a circle into 2 equal arcs. Each of these two arcs is called a semi-circle.

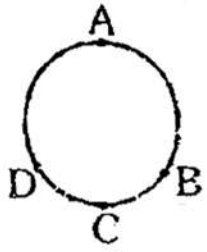
The degree measure of a semi-circle is $180°$.



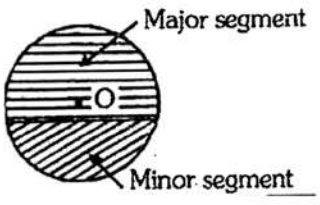
9. **Concentric Circles:** Circles having same centre but different radii are called concentric circles.



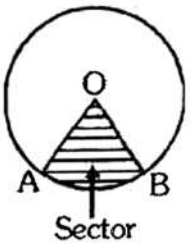
10. **Concyclic Points:** The points, which lie on the circumference of the same circle, are called concyclic points.



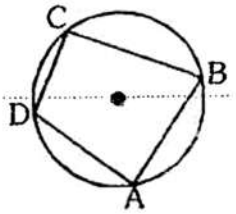
11. **Segment:** A segment is a part of a circular region bounded by an arc and a chord, including the arc and the chord.



12. **Sector of a Circle:** The part of the plane region enclosed by an arc of a circle and its two radii is called a sector of the circle.



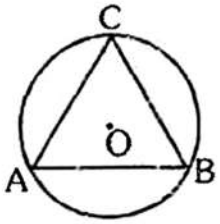
13. **Cyclic Quadrilateral:** If all the four vertices of a quadrilateral lie on a circle, such quadrilateral is called a Cyclic quadrilateral.



14. **Circum-Circle:** A circle which passes through all the three vertices of a Δ.

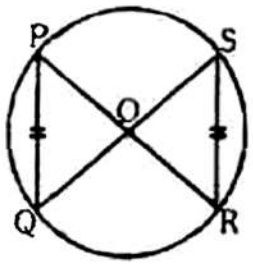
Circumcentre is always equidistant from the vertices of the Δ.

$OA = OB = OC$ (Circum radius)



Theorems related to Circles

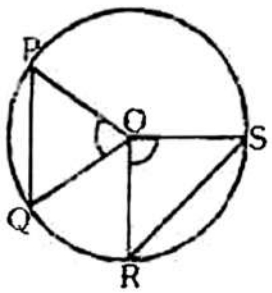
1. Equal chords of a circle subtend equal angles at the centre.



Given: A circle with centre O in which chord PQ = Chord RS.

Then: $\angle POQ = \angle ROS$

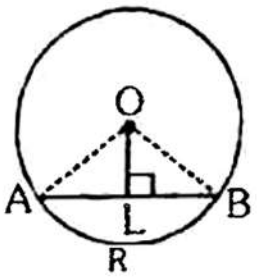
2. Converse of above theorem: If the angles subtended by the chords of a circle at the centre are equal, then chords are equal.



Given: A circle with centre O.
$\angle POQ = \angle ROS$

Then: Chord PQ = Chord RS

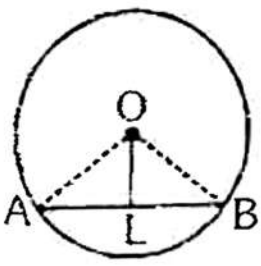
3. The perpendicular from the centre of a circle to a chord bisects the chord.



Given: AB is a chord of a circle with centre O.

Then: LA = LB

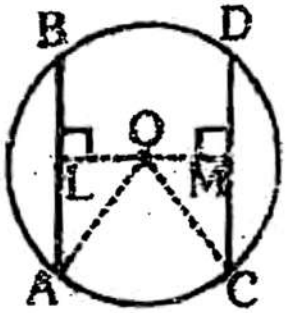
4. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Given: AB is a chord of a circle with centre O and OL bisects AB.

Then: $OL \perp AB$

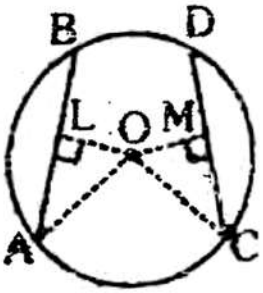
5. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Given: A circle with centre O in which chord AB = chord CD; $OL \perp AB$ and $OM \perp CD$.

Then: $OL = OM$

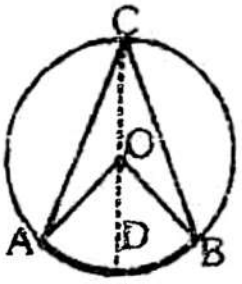
6. Chords equidistant from the centre of a circle are equal in length.



Given: AB & CD are two chords of a circle with centre O; $OL \perp AB$, $OM \perp CD$ and $OL = OM$.

Then: $AB = CD$

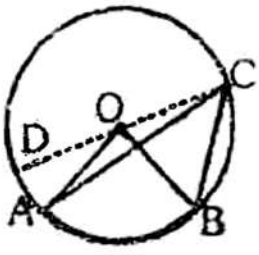
7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



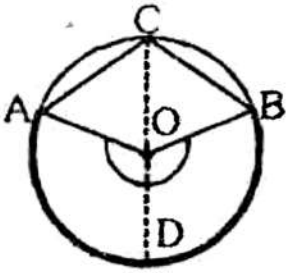
(i)

Given: A circle with centre O and an arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at any point C on the remaining part of the circle

Then: $\angle AOB = 2\angle ACB$



(ii)

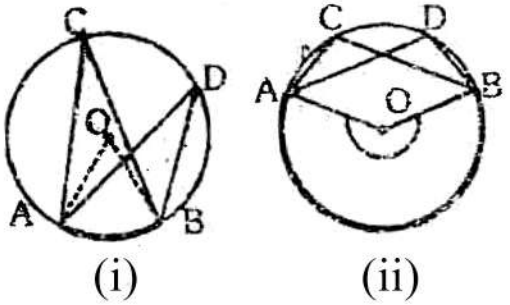


(iii)

8. Angles in the same segment of a circle are equal.

Given: A circle with centre O and two angles $\angle ACB$ and $\angle ADB$ in the same segment of the circle.

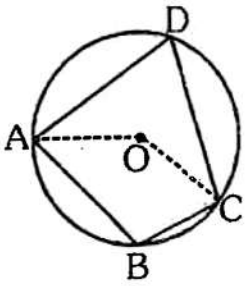
Then: $\angle ACB = \angle ADB$



9. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

$\angle ADC + \angle ABC = 180°$ and

$\angle BAD + \angle BCD = 180°$

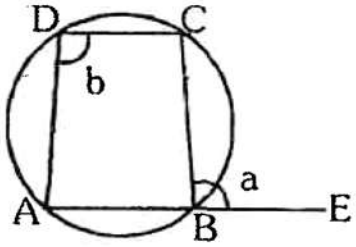


10. If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.

11. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Given: A cyclic quadrilateral whose side AB is produced to a point E.

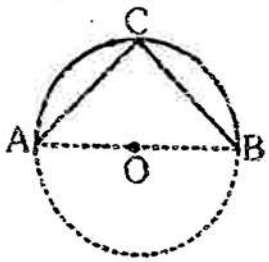
Then: $\angle CBE = \angle ADC$



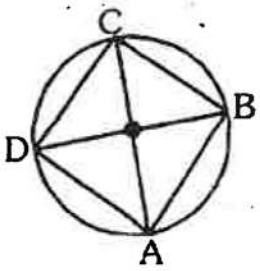
12. The angle in a semi-circle is a right angle.

Given: A semi-circle ACB of a circle with centre O.

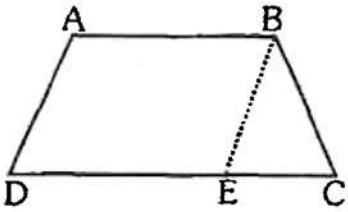
Then: $\angle ACB = 90°$



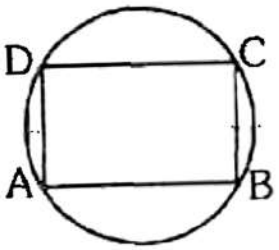
13. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, then it is a rectangle.



14. If the non-parallel sides of a trapezium are equal, then it is cyclic.



15. Every Cyclic parallelogram is a rectangle.



16. The quadrilateral formed by angle bisectors of a cyclic quadrilateral ABCD is also cyclic.

17. One and only one circle, passing through three non-collinear points.