Universidad Politécnica de Pachuca

MATEMÁTICAS PARA INGENIERÍA II
Tarea 7.

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MATEMATICAS II TAREA 07.

Parte 1. En la siguiente lista de ejercicios resolver los que son multiplo de 3.

3. $dx + e^{3x}\,dy = 0$

$$dy = \frac{-dx}{e^{3x}} \longrightarrow \int dy = -\int \frac{dx}{e^{3x}}$$

$$\longrightarrow \int dy = -\int e^{3x}\,dx$$

$$\left\{ \begin{array}{l} u = -3x \\ \frac{du}{dx} = -3 \\ du = -3dx \end{array} \right.$$

$$y = -\int e^{-3x}\,dx$$

$$y = -1/3 \int e^{-3x}(-3dx)$$

$$y = \frac{1}{3}\,\frac{1}{\ln e}\,e^{-3x} + C$$

$$y = \frac{1}{3}\,e^{-3x} + C$$

$$\frac{dy}{dx} + 2xy^2 = 0$$

$$\frac{dy}{y^2} = 2x\,dx \longrightarrow \int y^{-2} + \int 2x\,dx = 0$$

$$\rightarrow \frac{-y^{-1}}{1} + \int \cancel{2}\frac{x^2}{\cancel{2}}$$

$$= \boxed{\frac{1}{y} + x^2 + C}$$

$$y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$$

$$y \ln x \frac{dx}{dy} = \frac{(y+1)^2}{x^2}$$

$$x^2 \ln x\, dx = \frac{(y+1)^2}{y} dy$$

$$\rightarrow x^2 \ln x\, dx = \frac{y^2+2y+1}{y} dy$$

$$\rightarrow x^2 \ln x\, dx = y + 2 + \frac{1}{y} (dy)$$

$$\rightarrow \int x^2 \ln x\, dx = \int (y + 2 + 1/y)\, dy$$

$$\rightarrow \ln x \left(\frac{1}{3} x^3\right) - \int \frac{1}{3} x^3 \left(\frac{1}{x}\right) dx = \int y\, dy + 2\int dy + \frac{1}{y} dy$$

$$\frac{1}{3}\ln x\, x^3 - \frac{1}{3}\left(\frac{1}{3}x^2\right) + c = \frac{1}{2}y^2 + 2y + \ln y + c$$

$$\frac{1}{3}\ln x\, x^3 - \frac{1}{9}x^3 = \frac{1}{2}y^2 + 2y + \ln y + c$$

$$\frac{1}{3}\ln x\, x^3 - \frac{1}{9}x^3 = \frac{1}{2}y^2 + 2y + \ln y$$

$$\frac{1}{2}y^2 + 2y + \ln y = \frac{1}{3}\ln x\, x^3 - \frac{1}{9}x^3 + c$$

↳ Solución implícita.

12. $\operatorname{Sen} 3x\, dx + 2y \cos^3 3x\, dy = 0$

$$2y \cos^3 3x\, dy = -\operatorname{sen} 3x\, dx$$

$$2y\, dy = -\frac{\operatorname{Sen} 3x\, dx}{\cos^3 (3x)}$$

$$2\int y\, dy = -\int \frac{\operatorname{Sen} 3x}{\cos^3 3x}\, dx$$

$$2\int y\, dy = -\int \frac{\operatorname{Sen} 3x}{\cos 3x} \cdot \frac{1}{\cos^2 3x}\, dx \qquad \begin{cases} u = 3x \\ dx = \frac{1}{3}du \end{cases}$$

$$y^2 = -\int \tan 3x \cdot \sec^2 (3x)\, dx$$

$$y^2 = -\frac{1}{3}\int \tan(u) \cdot \sec^2(u)\, du \qquad \begin{cases} v = \sec^2 u \\ dv = \\ (\sec u)^2 \end{cases}$$

$$y^2 = -\frac{1}{3}\,\frac{1}{2} \cdot \int dv$$

$$y^2 = -\frac{1}{16} \cdot v + c$$

$$y^2 = -6 \sec^2(u) + c$$

$$y^2 = -\frac{1}{6} \sec^2(3x) + c$$

$$y = \pm \sqrt{-\frac{1}{6} \sec^2 (3x + c}$$

Solución explicita

15. $\frac{ds}{dr} = ks$

$$\frac{ds}{s} = k\,dr$$

$$\int \frac{ds}{s} = k \int dr$$

$$\ln s = kr + c$$

$$\cancel{e^{\ln s}} = e^{kr+c}$$

$$s = e^{kr+c}$$

$$s = e^{kr} \cdot \cancel{e^{c}} c_1$$

$$s = e^{kr} \cdot c_1$$

$$\underline{s = c_1 e^{kr}}$$

Solución explicita

18. $\frac{dN}{dt} + N = Nte^{t+2}$

$$\frac{dN}{dt} = Nte^{t+2} - N$$

$$\frac{dN}{dt} = N.(te^{t+2} - 1)$$

$$\frac{dN}{dt} = N(te^{t}e^{2} - 1)$$

$$\frac{1}{N}\,dN = (te^{2}e^{t}t - 1)\,dt$$

$$\int \frac{1}{N}\,dN = e^{2}\cdot\int te^{t}\,dt - \int dt$$

$$\ln N = e^{2}(te^{t} - e^{t}) - t + C$$

$$\ln N = e^{2}te^{t} - e^{2}e^{t} - t + C$$

$$\cancel{e^{\ln}}N = e^{e^{2}te^{t} - e^{2}e^{t} - t + \cancel{C}}\; C_1$$

$$\underbrace{N = C_1 e^{e^{2}e^{t}t - e^{2}e^{t} - t}}$$

solución explícita.

PARTE II. En la siguiente lista de ejercicios resolver los que son multiplos de 4.

En los problemas 1 a 24, encuentre la solución general de la ecuación diferencial dada. Proporcione el intervalo mas largo sobre el cual está definida la solución general. Determine si existe algún termino transitorio en la solución general.

4. $3\frac{dy}{dx} + 12y = 4$

$$y = e^{-\int P(x)\,dx}\int e^{\int P(x)\,dx} Q(x)\,dx$$

$$\frac{1}{3}\left[3\frac{dy}{dx} + 12y = 4\right]$$

$$\rightarrow \frac{dy}{dx} + 4y = 4/3$$

$$= e^{-\int P(x)\,dx} = e^{-\int 4 \cdot dx} = e^{-4\int dx} = e^{-4x}$$

$$e^{\int P(x)\,dx} = e^{\int 4 \cdot dx} = e^{4x}$$

$$y_p = e^{-4x}\int e^{4x}\,\frac{4}{3}\,dx \qquad \begin{cases} u = 4x \\ du = 4\,dx \end{cases}$$

$$y_p = \frac{1}{3}e^{-4x}\int e^u\,du$$

$$y_p = \frac{1}{3}e^{-4x}e^u$$

$$y_p = \frac{1}{3}e^{-4x}e^{4x} \longrightarrow y_p = 1/3$$

$$y = C_1 e^{-4x} + \frac{1}{3}$$

$$y' = 2y + x^2 + 5$$

$$y' - 2y = x^2 + 5$$

$$e^{-2x} y' - 2e^{-2x} y = x^2 e^{-2x} + 5e^{-2x}$$

$$(e^{-2x} y)' = x^2 e^{-2x} + 5e^{-2x}$$

$$e^{-2x} y = \int x^2 e^{-2x}\, dx + \int 5e^{-2x}\, dx$$

$$e^{-2x} y = -\frac{1}{4}(2x^2 + 2x + 1)e^{-2x} - \frac{5}{2} e^{-2x} + c$$

$$y = -\frac{1}{2}x^2 - \frac{1}{2}x - \frac{11}{4} + ce^{2x}$$

$$(-\infty, \infty)$$

12. $(1+x)\frac{dy}{dx} - xy = x + x^2$

$$(1+x)\frac{dy}{dx} - xy = x + x^2$$

$$u(x) = \exp \int -\frac{x}{x+1}\, dx$$

$$= e^{-x + \ln x+1} \longrightarrow e^{-x} \cancel{e^{\ln x+1}} = e^{-x}(x+1$$

$$e^{-x}(x+1)y' - xe^{-x} = x(x+1)e^{-x}$$

$$(e^{-x}(x+1)y)' = x^2 + e^{-x} + xe^{-x}$$

$$e^{-x}(x+1)y = \int (x^2 e^{-x} + xe^{-x})\, dx = e^{-x}(x^2 + 3x + 3) + c$$

$$y = \frac{-(x^2 + 3x + 3)}{x+1} + \frac{ce^x}{x+1} = \frac{ce^x - x^2 - 3x - 3}{x+1}$$

16. $y\,dx = (ye^{y} - 2x)\,dy$

$$e^{\int 2y^{-1} dy} = e^{2 \ln y} = \cancel{e^{\ln y^2}} = y^2$$

$$y^2 x' + 2y\,x = y^2 e^{y}$$

$$\int (y^2 x)' = \int y^2 e^{y}$$

$$y^2 x = \int y^2 e^{y} dy$$

$$\begin{cases} u = y^2 \\ du = 2y\,dy \\ dv = e^{y}\,dy \\ v = e^{y} \end{cases}$$

$$y^2 x = y^2 e^{y} - \int 2y e^{y}\,dy$$

$$y^2 x = y^2 e^{y} - 2y e^{y} - \int 2e^{y}\,dy$$

$$y^2 x = y^2 e^{y} - 2y e^{y} - 2y e^{y} + 2e^{y} + c$$

$$x = e^{y} - 2y^{-1} e^{y} + 2y^{-2} e^{y} + cy^{-2}$$

$$(0, \infty)$$