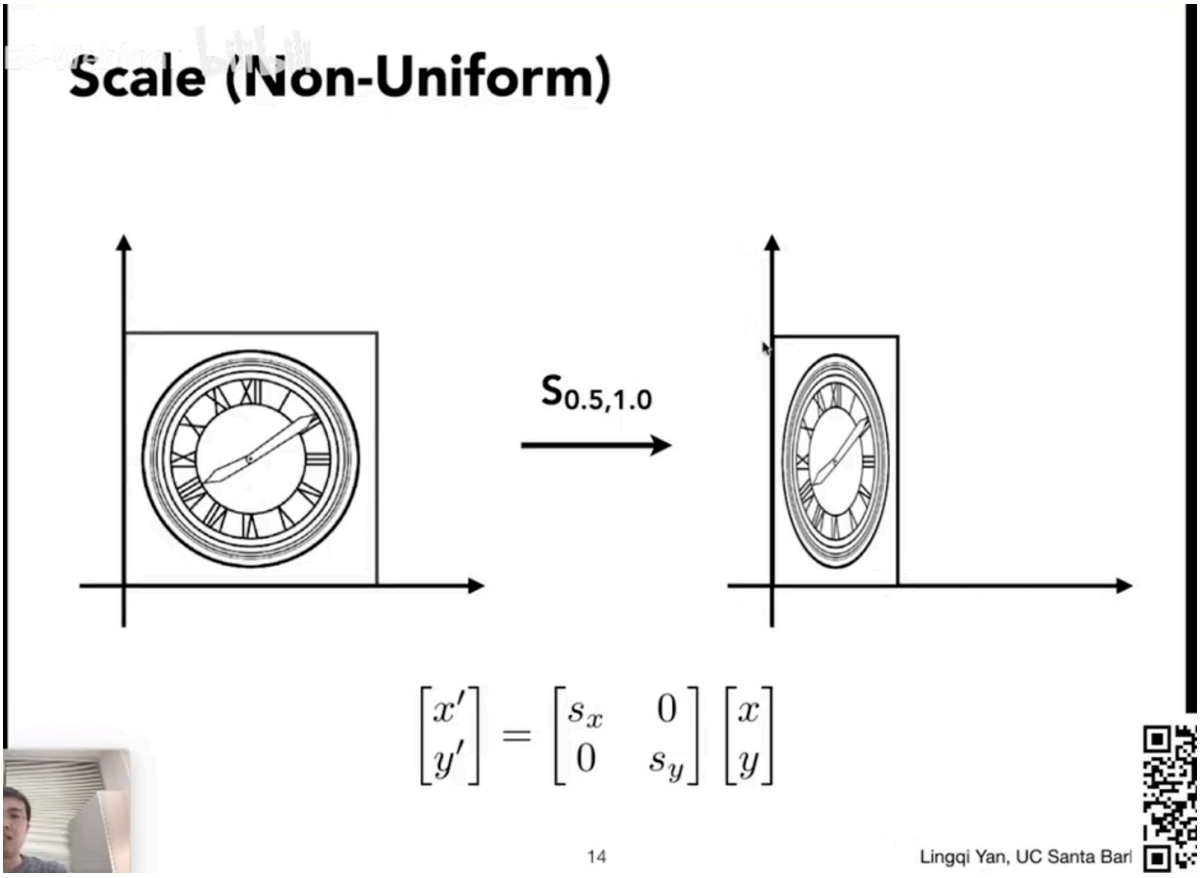
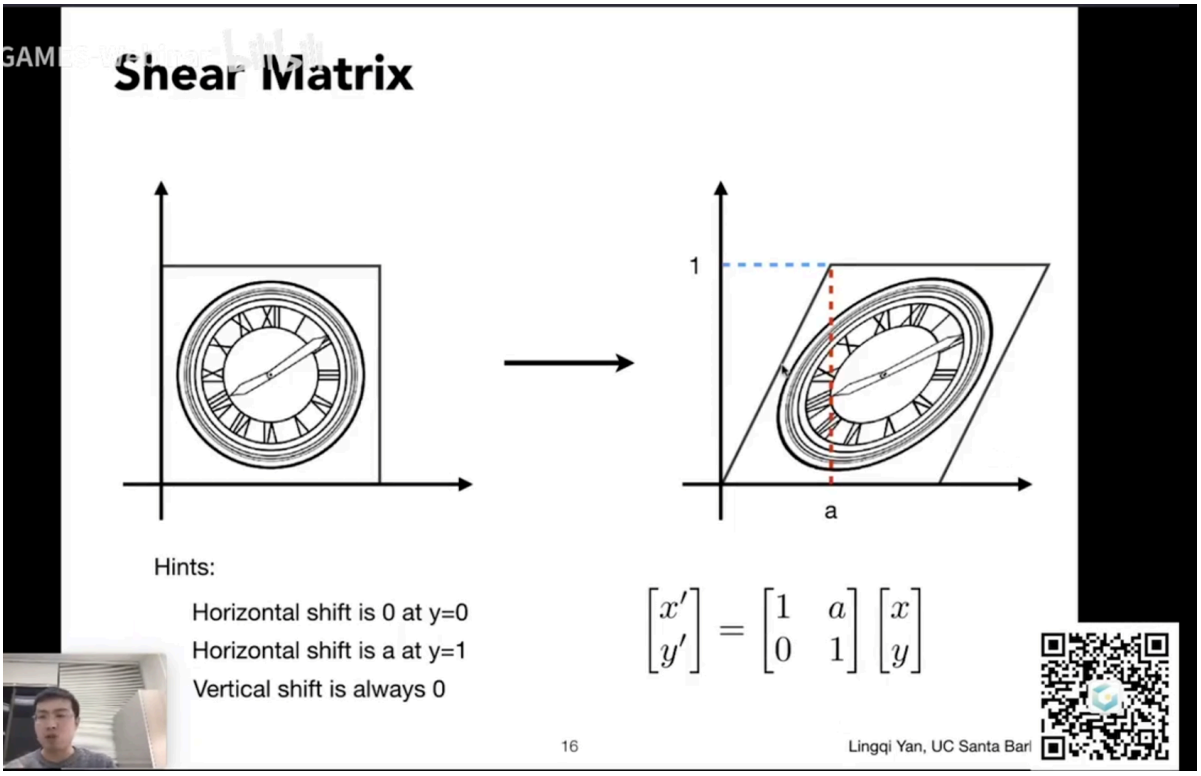
Transformation

Viewing : 3D to 2D projection

Scale

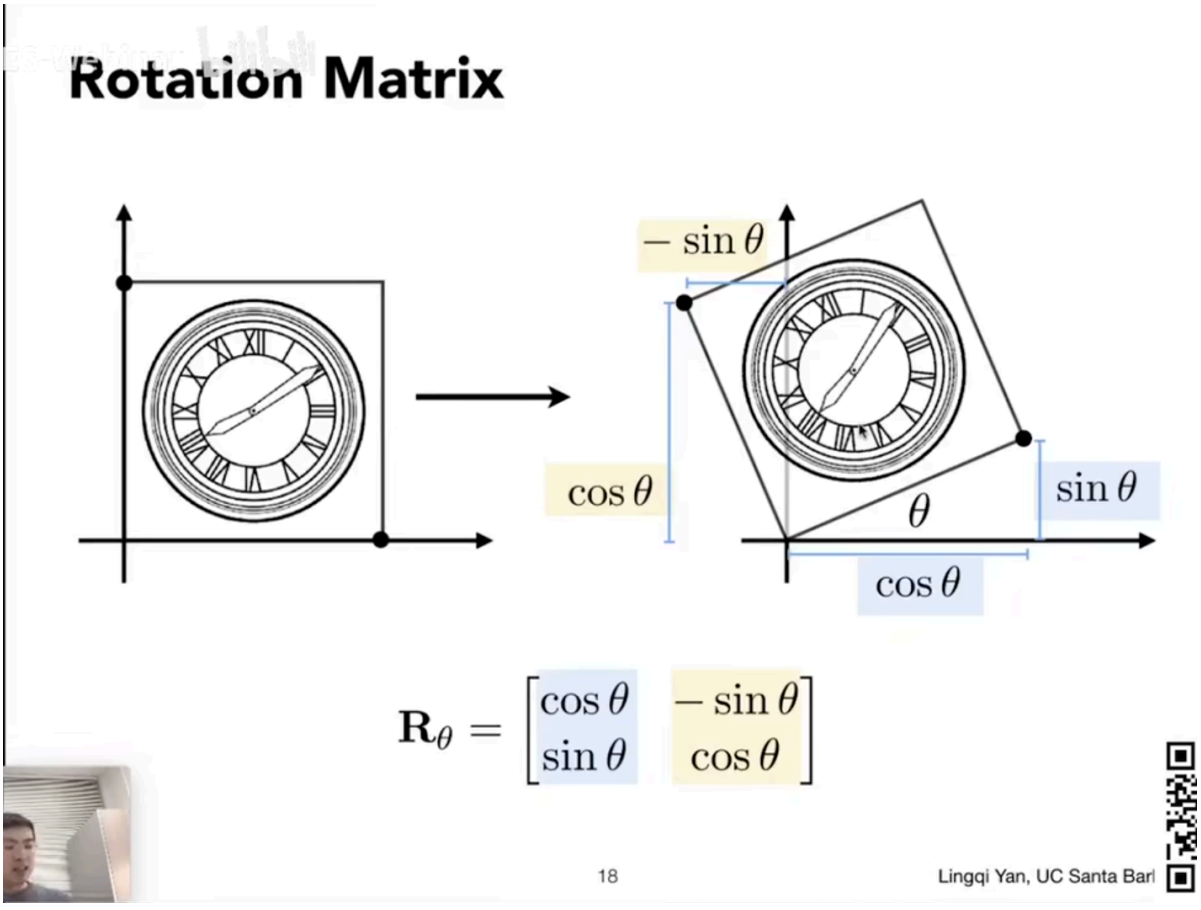


Shear

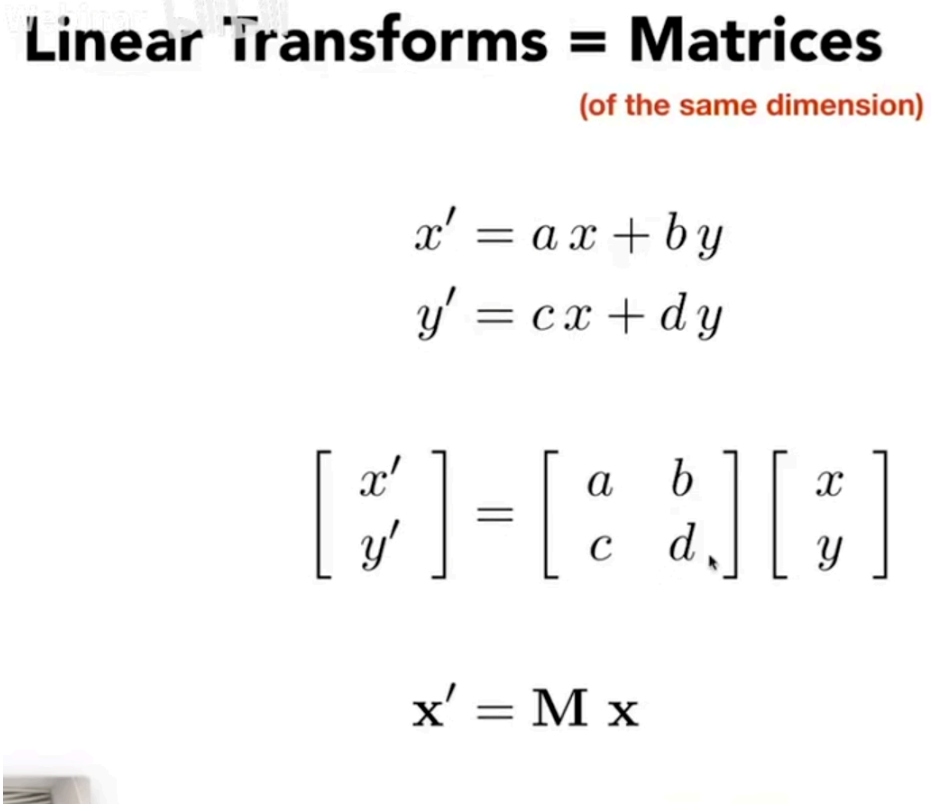


Rotate

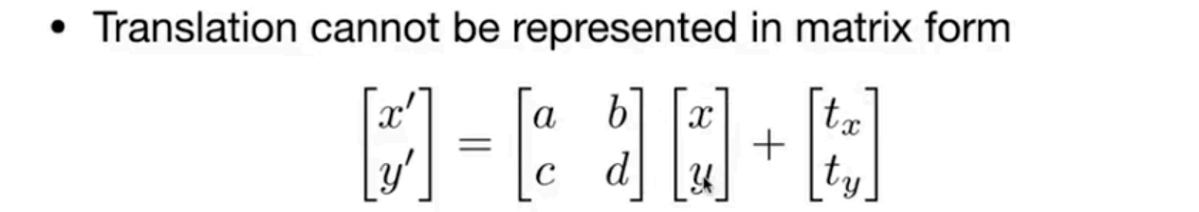
默认（0，0） CCW



变换可写成矩阵乘积数据的形式 ==> 线性变换



如果需要以统一的方法表示平移



则应该引入齐次化过程。

点的变换添加一个1，向量的变换添加一个0.

（在射线法中，计算射线的方向变换）（计算法向量时，法向量的变换）

Add a third coordinate (*w-coordinate*)

- 2D point = (x, y, 1)^T
- 2D vector = (x, y, 0)^T

Matrix representation of translations

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

Valid operation if w-coordinate of result is 1 or 0

- vector + vector = vector
- point – point = vector
- point + vector = point
- point + point = ??

仿射变换都可以写成齐次坐标的形式

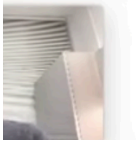
Affine Transformations

Affine map = linear map + translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Using homogenous coordinates:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



2D Transformations

Scale

$$\mathbf{S}(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation

$$\mathbf{R}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Translation

$$\mathbf{T}(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

仅在2D下的仿射变换，矩阵最后一行为001（可以在计算时作为常量引入，而不用特意存储）

矩阵的结合律：变换合成与分解

Sequence of affine transforms A_1, A_2, A_3, ...

- **Compose by matrix multiplication**
  - **Very important for performance!**

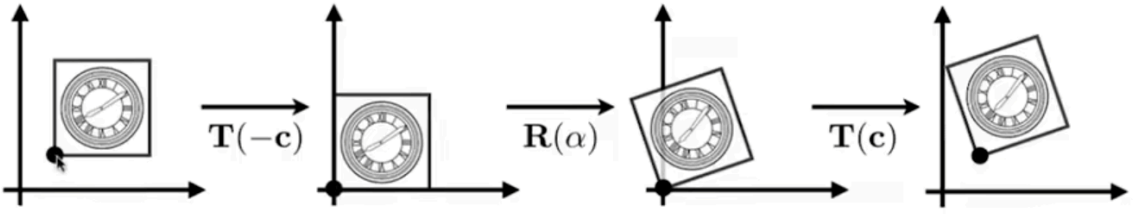
$$A_n(\dots A_2(A_1(\mathbf{x}))) = \underbrace{\mathbf{A}_n \cdots \mathbf{A}_2 \cdot \mathbf{A}_1}_{} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Pre-multiply *n* matrices to obtain a single matrix representing combined transform

对于非原点的点的变换操作：先移至原点，变换后再移回去

How to rotate around a given point c?

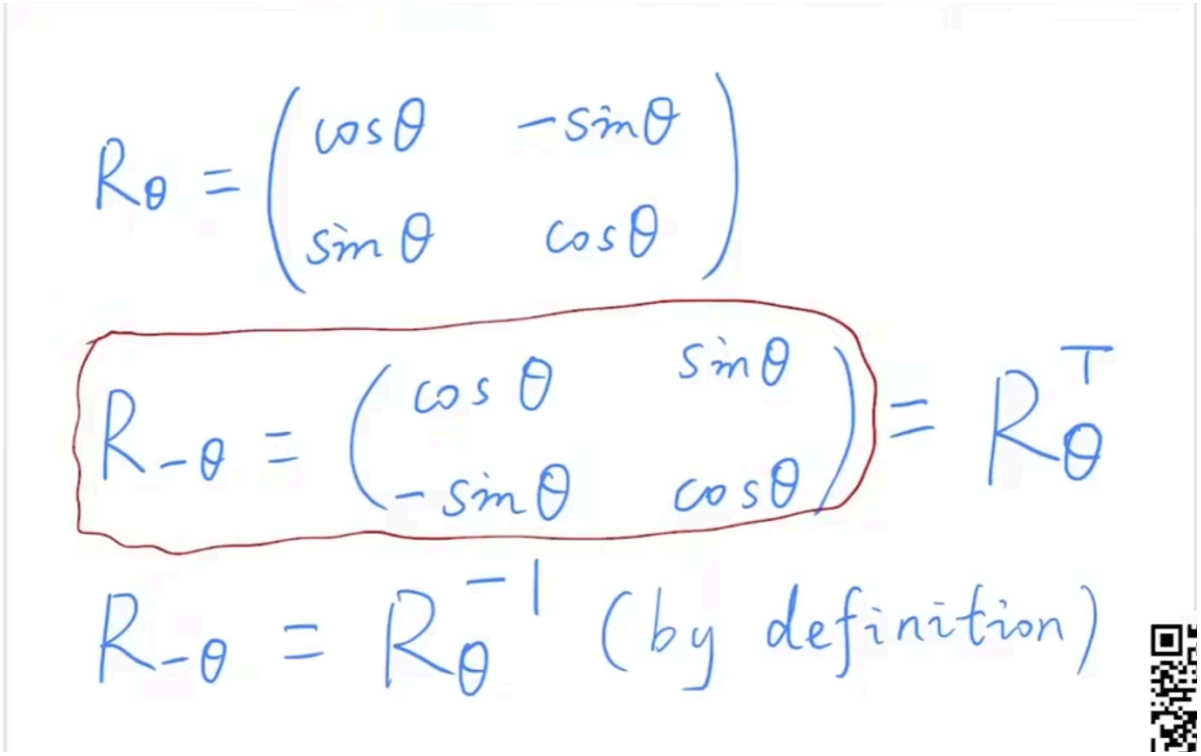
1. **Translate center to origin**
2. **Rotate**
3. **Translate back**



Matrix representation?

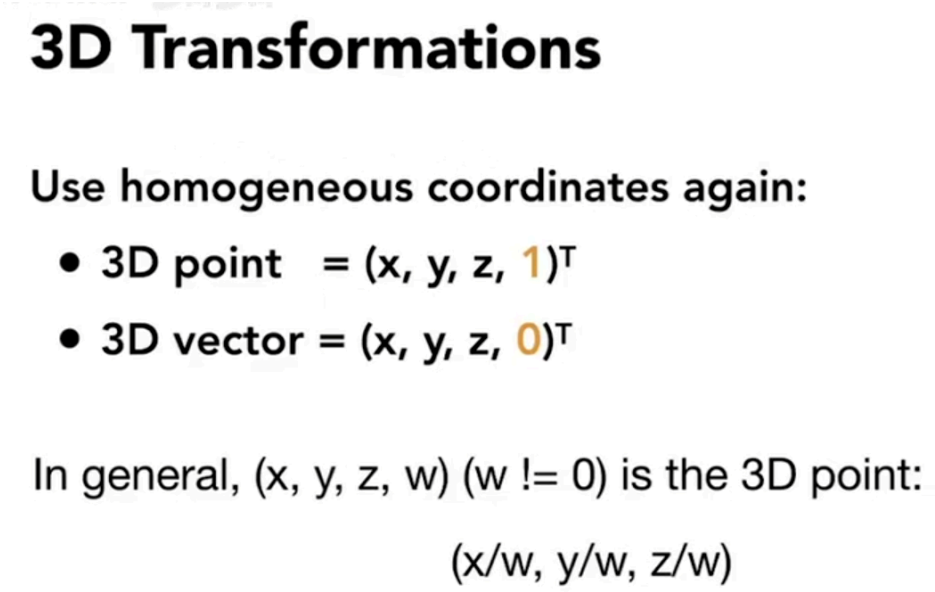
$$\mathbf{T}(\mathbf{c}) \cdot \mathbf{R}(\alpha) \cdot \mathbf{T}(-\mathbf{c})$$

旋转负角度 是旋转该角度下的旋转矩阵的逆矩阵



在旋转中，旋转矩阵的逆等于旋转矩阵的转置 ==> 正交矩阵

三维变换



3D Transformations

Use 4×4 matrices for affine transformations

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D的旋转 （右手坐标系）

（cosa 在1的右下，以此类推）

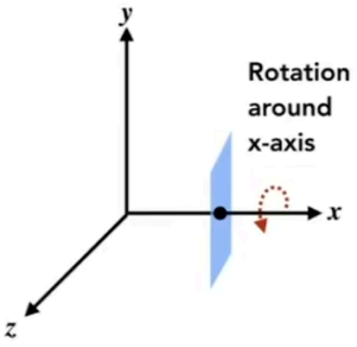
3D Transformations

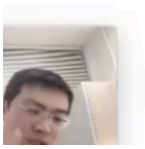
Rotation around x-, y-, or z-axis

$$\mathbf{R}_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_y(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

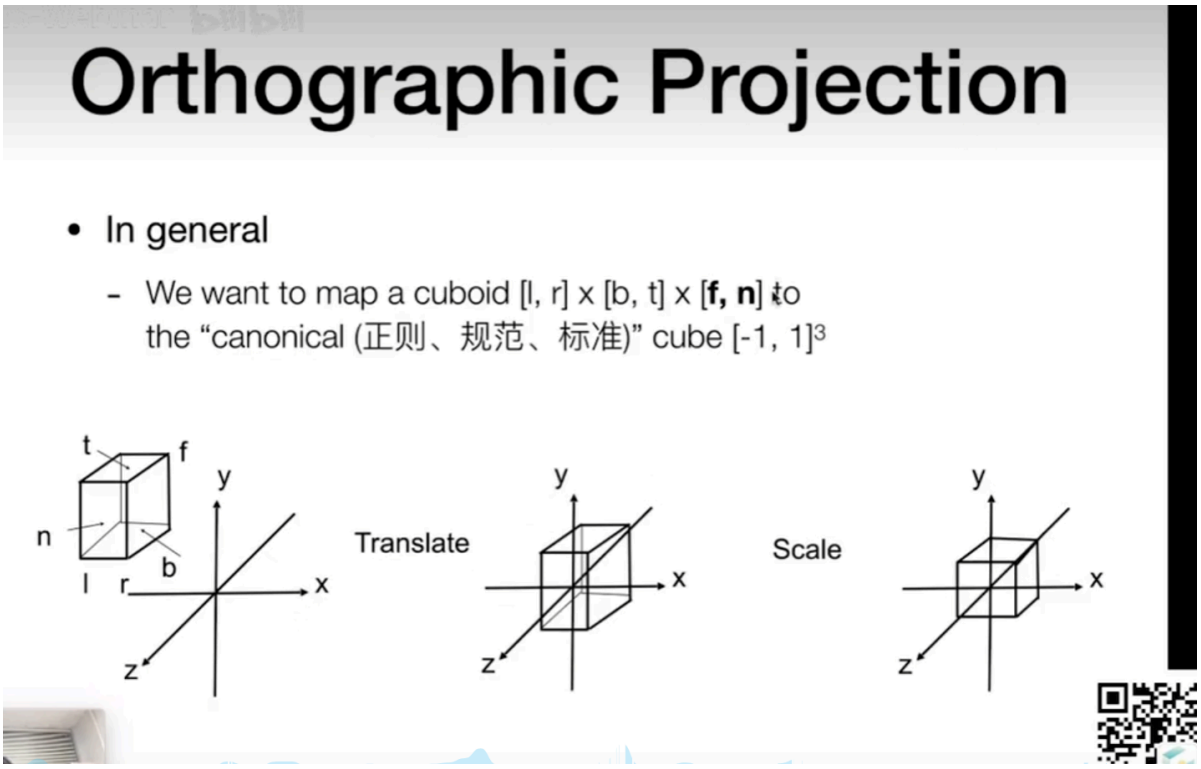
$$\mathbf{R}_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$





正交变换投影：

描述一个长方体，将其中心移动到原点，再将边长拉扯至立方体[-1,1]上。



透视投影：相似三角形原理，处于远近平面中间的点，投影至近平面所需要的xy上的变换。

Perspective Projection

- In order to find a transformation
  - Find the relationship between transformed points (x', y', z') and the original points (x, y, z)

$$y' = \frac{n}{z}y \qquad x' = \frac{n}{z}x \text{ (similar to y')}$$

- In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} nx/z \\ ny/z \\ \text{unknown} \\ 1 \end{pmatrix} \overset{\text{mult. by z}}{==} \begin{pmatrix} nx \\ ny \\ \text{still unknown} \\ z \end{pmatrix}$$

光栅化

将标准平面坐标系中的像素映射到屏幕上：视口变换

z方向不变，即屏幕平面始终朝向相机反向

Canonical Cube to Screen

- Irrelevant to z
- Transform in xy plane: $[-1, 1]^2$ to [0, width] x [0, height]
- Viewport transform matrix:

$$M_{viewport} = \begin{pmatrix} \frac{width}{2} & 0 & 0 & \frac{width}{2} \\ 0 & \frac{height}{2} & 0 & \frac{height}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

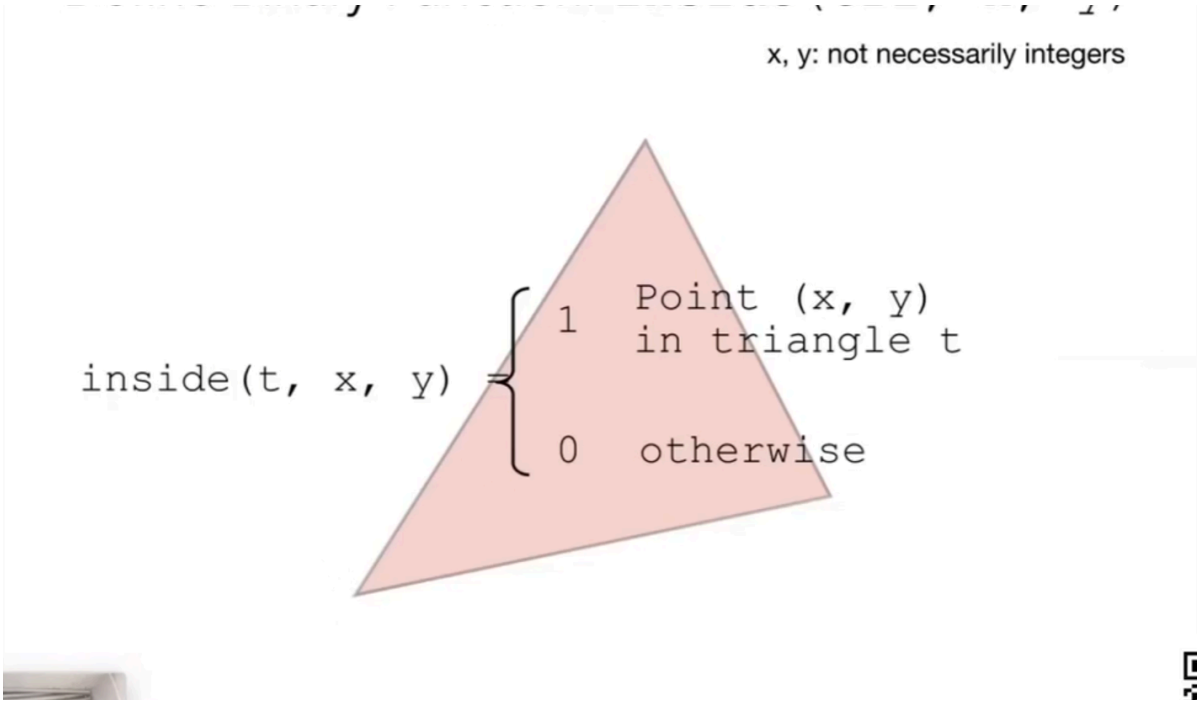
三角形是最基础的多边形。

三角形内部一定是平面的。

通过向量的叉积定义一个点是否在三角形内。

采样法：使用一个连续的函数，在一个离散的集合中询问该集合元素对应的函数值，得出的值的集合。（将函数离散化的过程）

如：屏幕空间中，不同像素的中心是否在需要渲染的三角形内



Aliasing 走样 锯齿

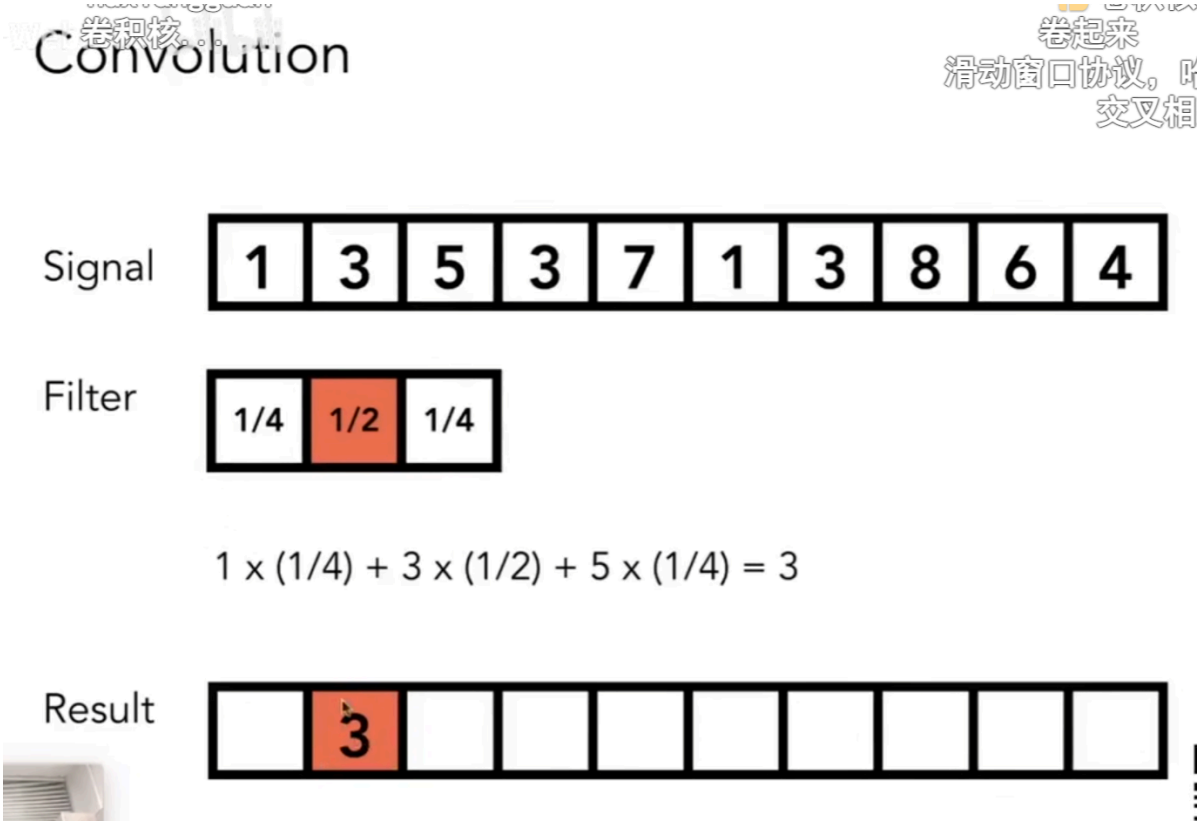
所以需要反走样，抗锯齿

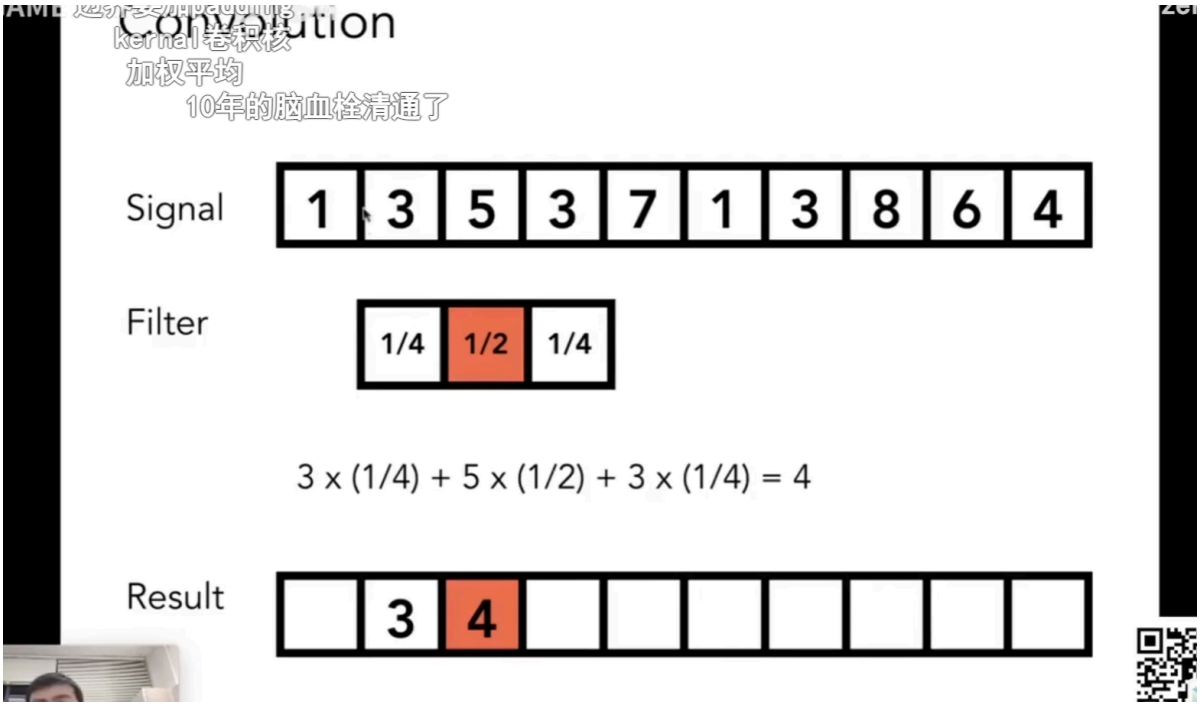
滤波：去掉特定频率的信息

对于一个特定的图像，变化大的频率高，变化小的频率低。

【高通低通带通】

卷积：对于某个窗口的特定区域的数，进行一系列的规则运算后得到的数写回该区域中（如一维卷积中，对一个数组做大小为三的卷积）



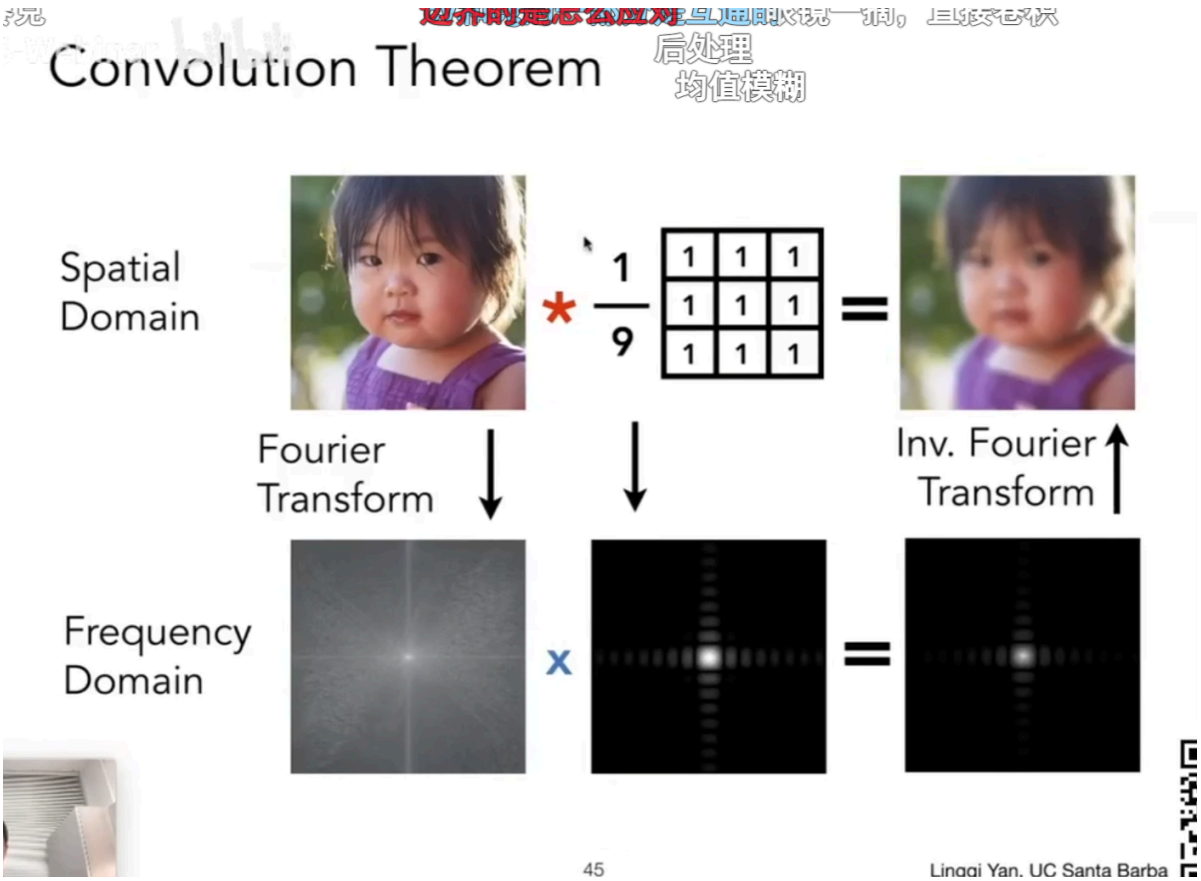


做模糊的两种方案：

对原图像取平均卷积

对原图进行傅里叶变换后，乘以卷积网络的傅里叶变换值，再逆变换回去

（时域的卷积 = 频域的乘积）



box filter 盒子滤波器 （也是低通滤波器） （模糊）

盒子越大 频域图（白色区）越小 ，可以这么理解：因为边界大（多）了。

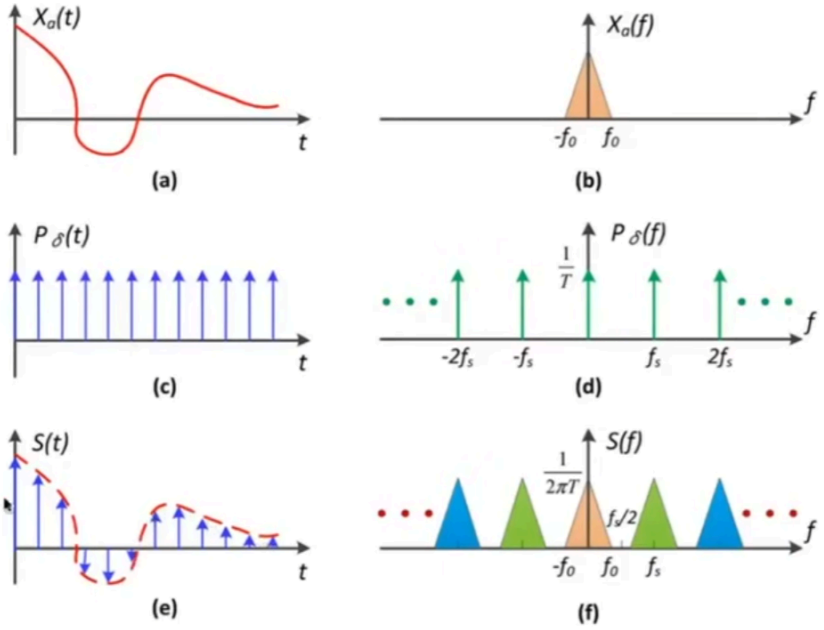
也可以这么理解：对某块区域做滤波，滤波区越大，滤后平均的范围就越大，就只能留下更低的频率，所以越模糊；而盒子越小，（如用一个像素去滤波，等于没滤）

采样就是在重复频域上的内容

用冲击函数对一块连续区域的函数进行采样，得到一系列的点。

在频域上解释，就是将一块区域复制粘贴到频域中。

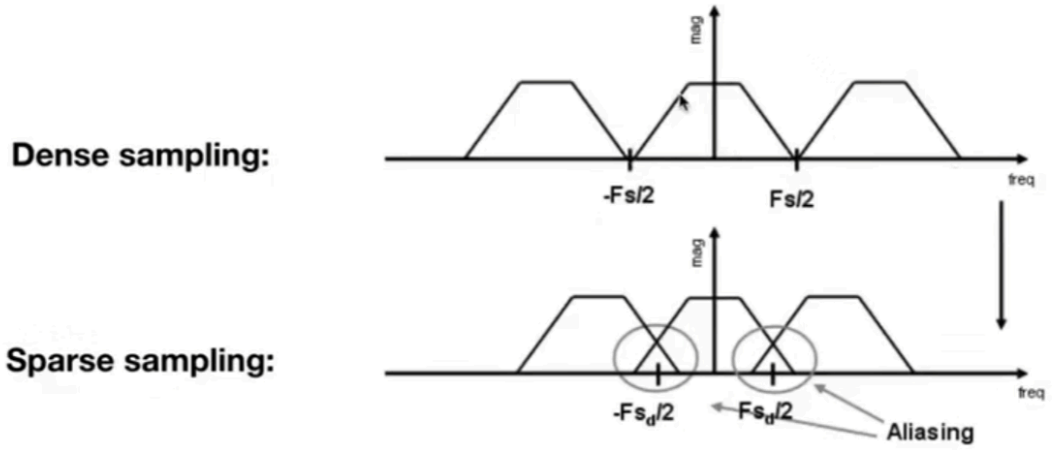
Sampling = Repeating Frequency Contents



https://www.researchgate.net/figure/The-evolution-of-sampling-theorem-a-The-time-domain-of-the-band-limited-signal-and-b_fig5_301556095

采样率不足时会造成频谱混叠

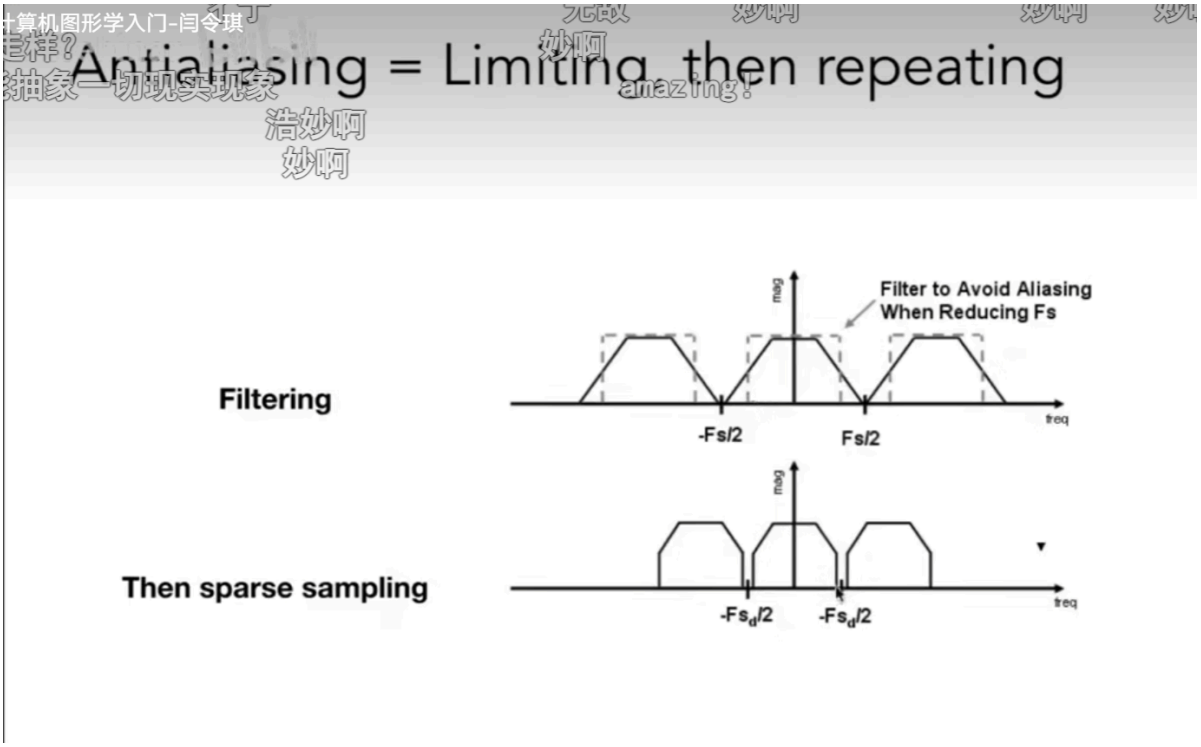
Aliasing = Mixed Frequency Contents



反走样：先做模糊，再做采样

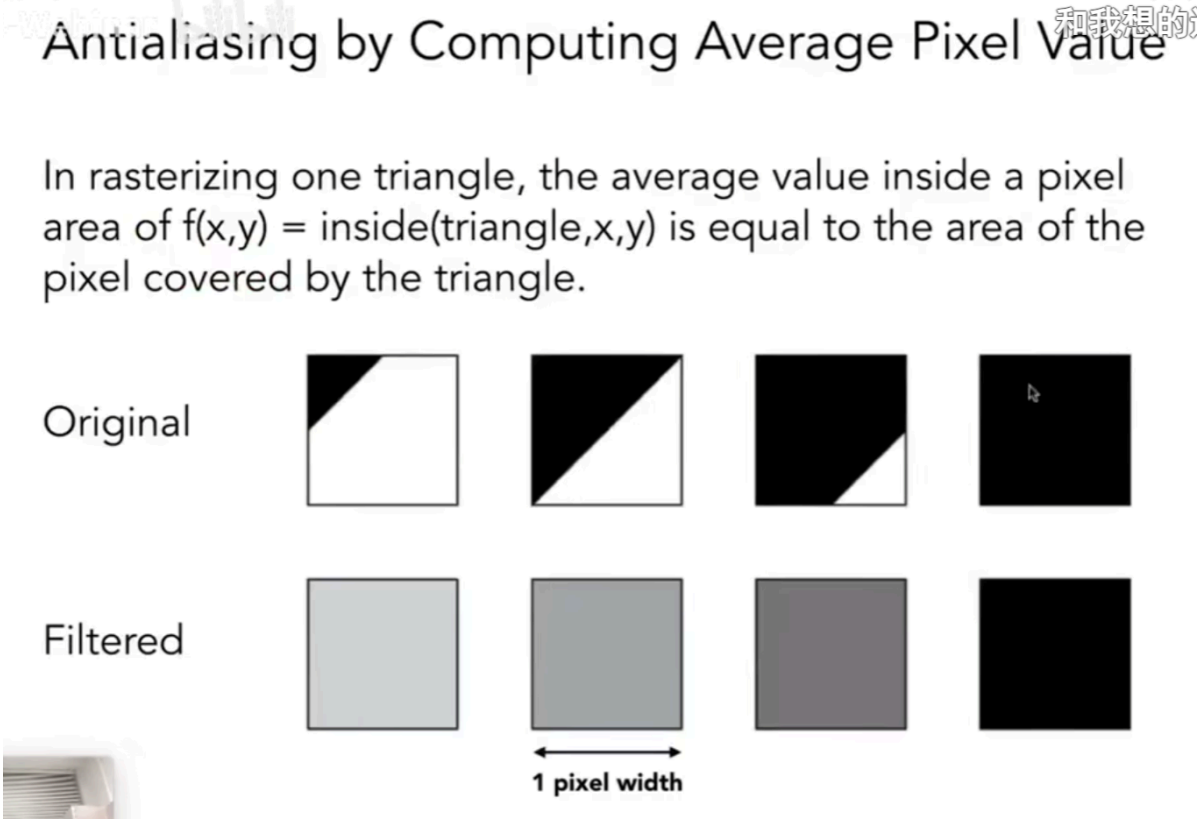
先进行低通滤波，将高频信息去掉，再进行原来的频率的采样（采样频率不变时）

另一种方案是提高分辨率，即提高采样频率



用一定大小的低通滤波器（即对图像进行卷积。）

用一个像素的box filter进行滤波，对于边界的情况可虑成灰值

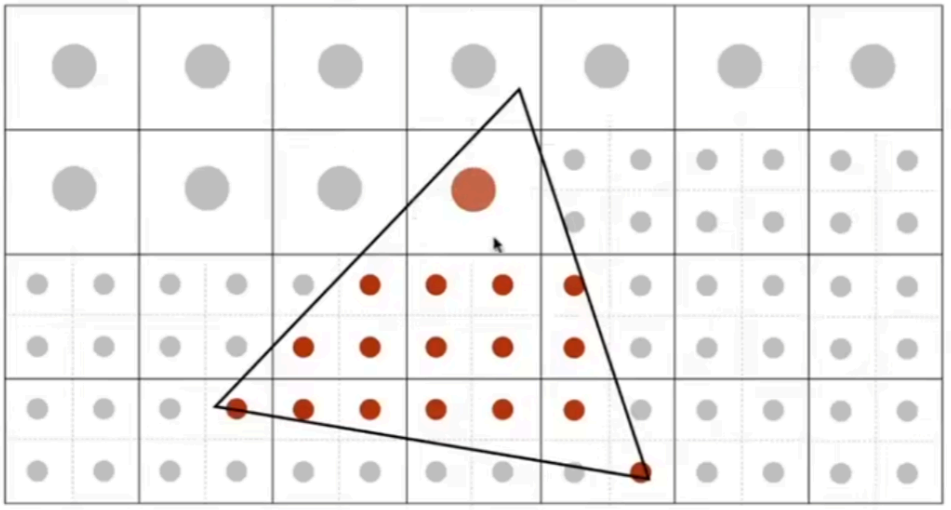


抗锯齿效果

用多像素box filter进行时，求覆盖率 MSAA

（模糊操作）

Average the NxN samples “inside” each pixel.



一个颜色用了更高的采样率

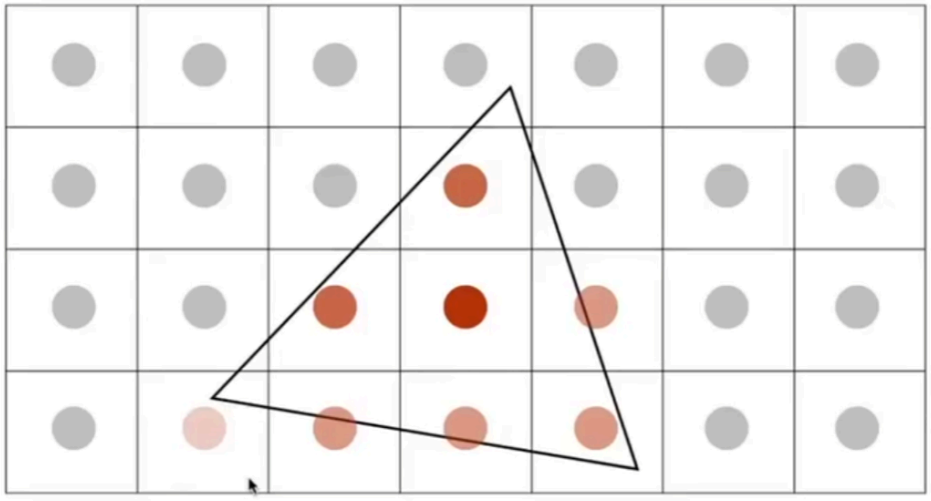
Supersampling: Step 2

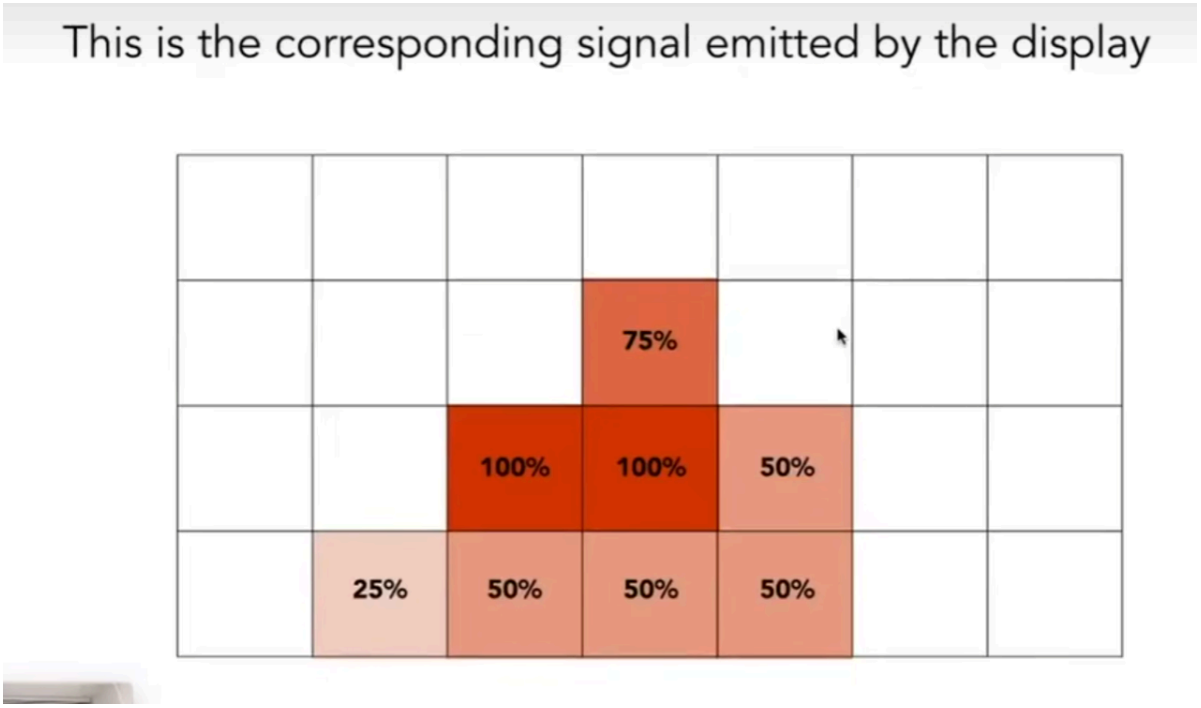
八叉树

颜色减弱吗

每个像素里本身不就应该是颜色一样的嘛？

Average the NxN samples “inside” each pixel.





再进行采样

可查 抖动分布采样 分时抗锯齿

MSAA

FXAA (图像后期处理，得到锯齿的图，找到边界，并替换边界（边缘检测））

TAA（时间相关，相邻两帧的点，复用上一帧的点进行的操作)

超分辨率

都是解决样本不足的问题

DLSS

Antialiasing Today

No free lunch!

- What's the cost of MSAA?

Milestones (personal idea)

- FXAA (Fast Approximate AA)
- TAA (Temporal AA)

Super resolution / super sampling

- From low resolution to high resolution
- Essentially still "not enough samples" problem
- DLSS (Deep Learning Super Sampling)

可见度和遮挡问题

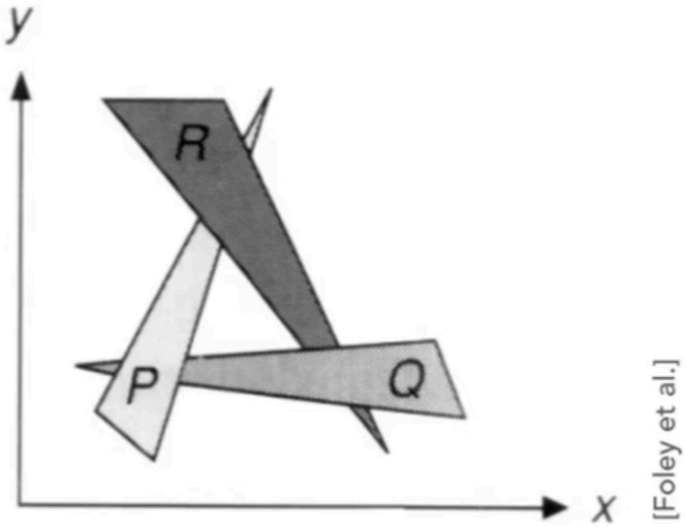
画家算法：排序，然后由远及近进行绘制。

会存在循环覆盖问题

Painter's Algorithm

Requires sorting in depth (O(n log n) for n triangles)

Can have unresolvable depth order



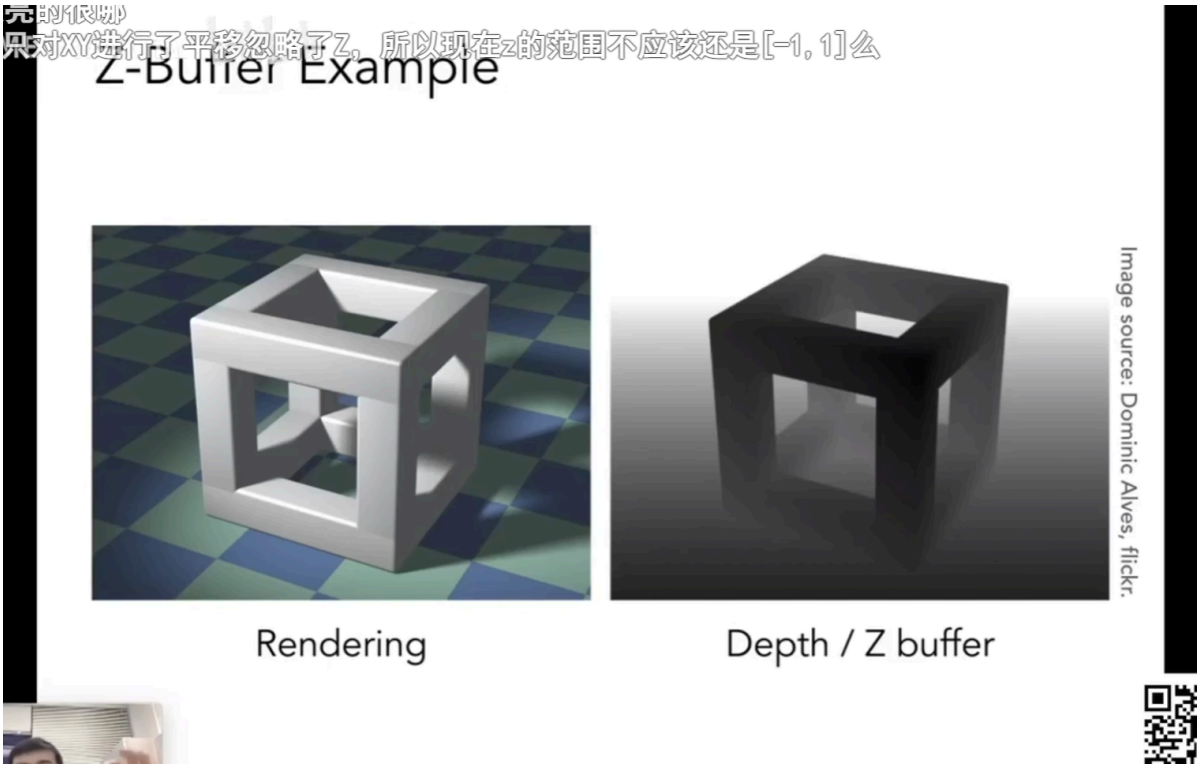
[Foley et al.]

深度缓冲

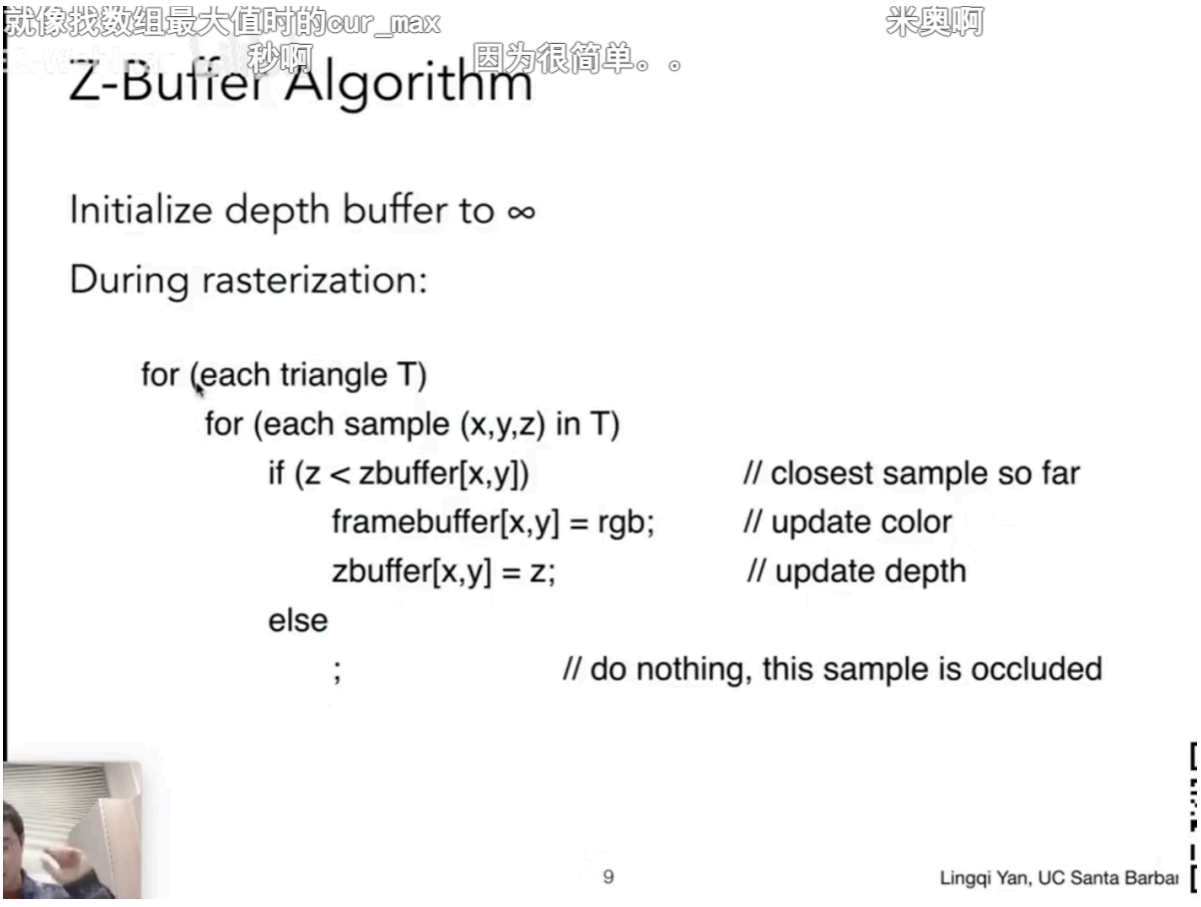
Z-Buffer

对每个像素存一个深度缓存。

在渲染时，同时生成深度缓存。同步生成。

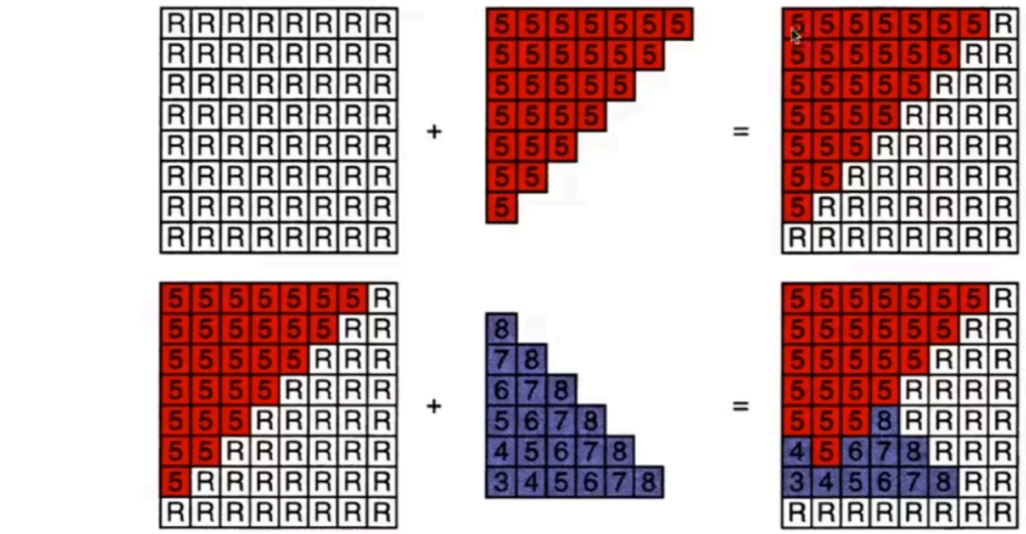


先认为所有像素的深度无限远，然后对任意三角形覆盖的所有像素与深度缓存中对应的值进行比较，取小更新，并在渲染区绘制。



同样深度 画哪张
角形中某一像素深度相同如何判断
应该是通过三角形三个顶点记录的深度信息线性插值得到的

Z-Buffer Algorithm



Z-Buffer Complexity

Complexity

- O(*n*) for *n* triangles (assuming constant coverage)
- How is it possible to sort *n* triangles in linear time?

深度缓存算法跟顺序无关

对MSAA的结果做深度缓存。

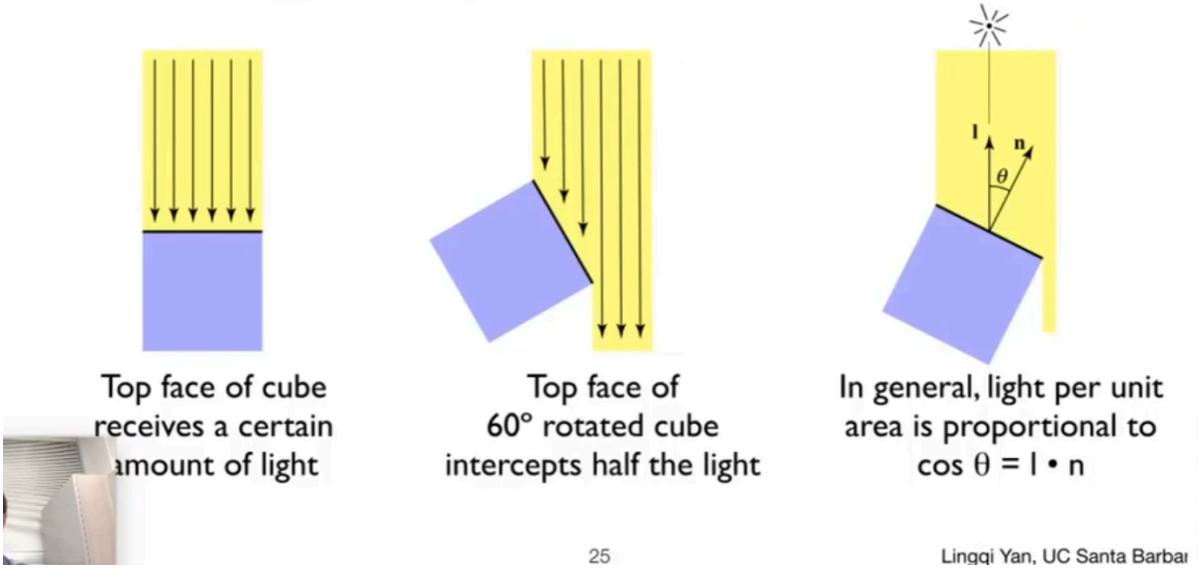
Z-Buffer对透明物体处理不了，需要进行特殊处理。

着色 Shadding

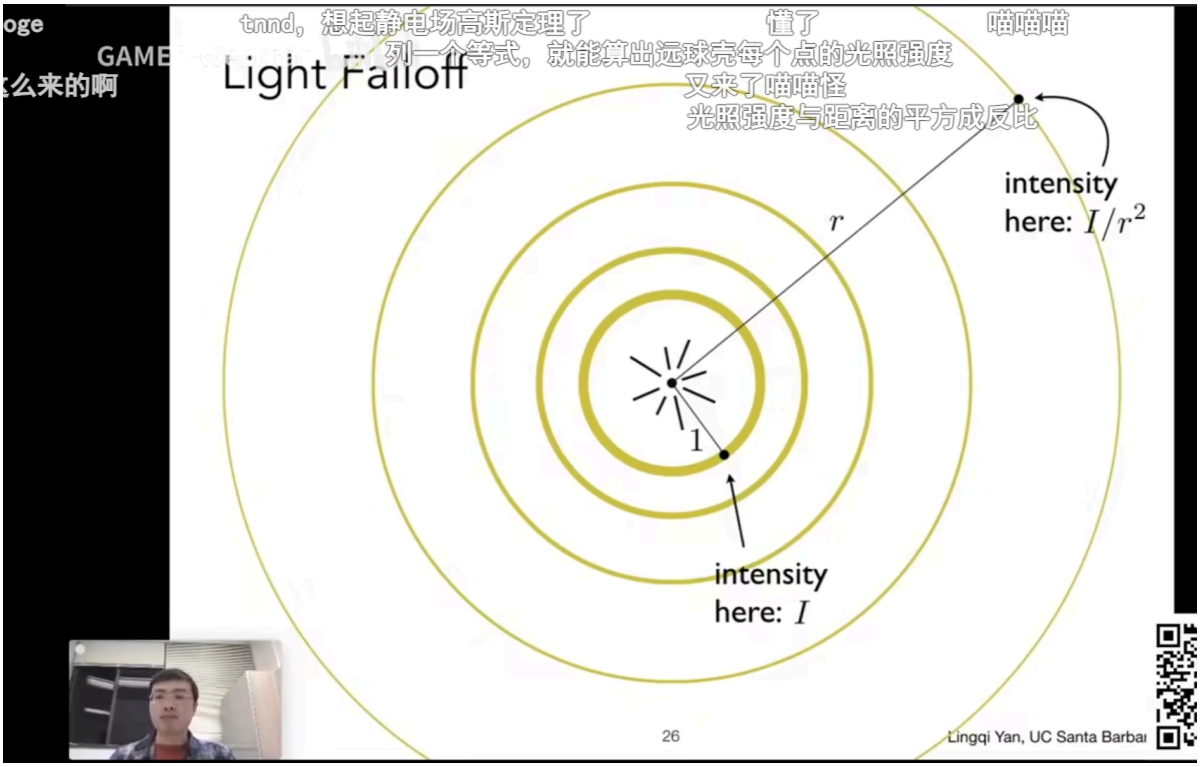
漫反射：接受到的能量 跟 法线方向与光线方向的夹角余弦有关

Diffuse Reflection

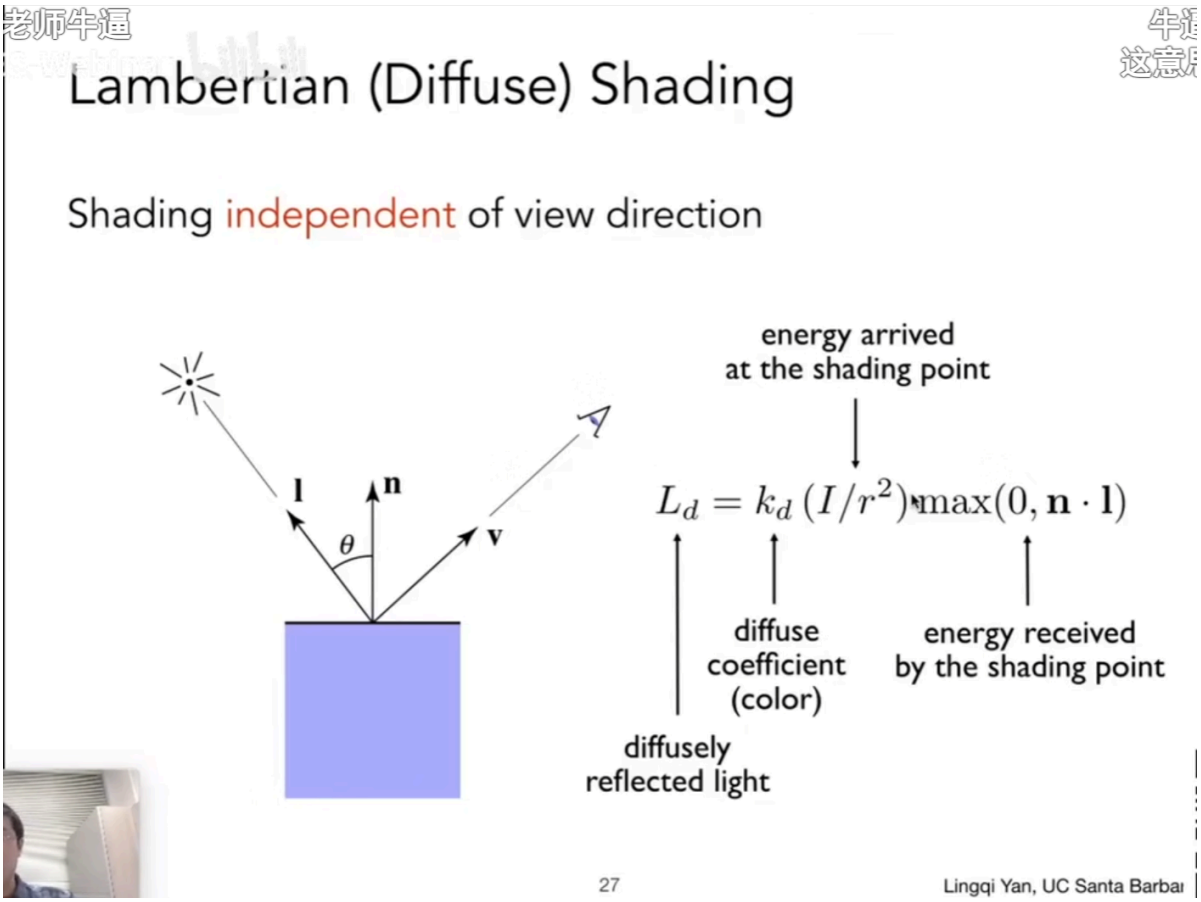
- But how much light (energy) is received?
  - Lambert's cosine law



光强与距离相关，在同距离处的光的能量相同，但由于周长原因，强度越来越小。



最终漫反射的公式如下：



kd表示为一个Color，跟物体贴图颜色相关，就可以定义漫反射的颜色。

漫反射跟观察角度无关，与物体面的方向相关。

高光

有些材料里面通常会将h为半角向量

半程向量

妙

妙啊

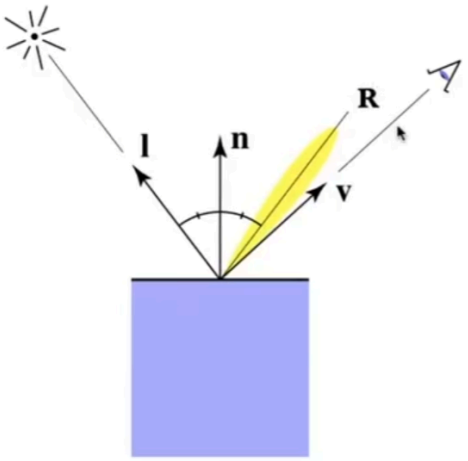
妙啊

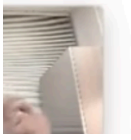
秒啊

Specular Term (Blinn-Phong)

Intensity depends on view direction

- Bright near mirror reflection direction



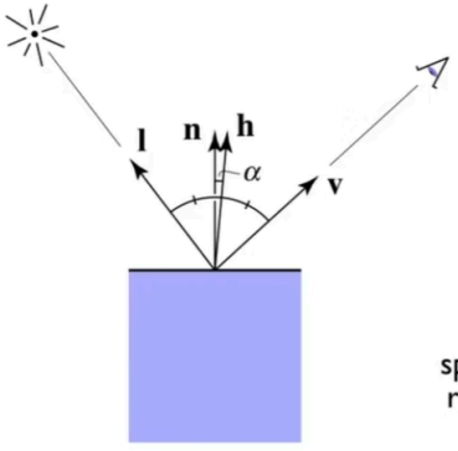


高光项

Specular Term (Blinn-Phong)

V close to mirror direction ⇔ **half vector** near normal

- Measure "near" by dot product of unit vectors



$$\mathbf{h} = \text{bisector}(\mathbf{v}, \mathbf{l})$$

(半程向量)

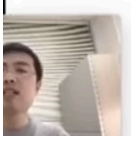
$$= \frac{\mathbf{v}+\mathbf{l}}{\|\mathbf{v}+\mathbf{l}\|}$$

$$L_s = k_s\,(I/r^2)\max(0, \cos\alpha)^p$$

$$= k_s\,(I/r^2)\max(0, \mathbf{n}\cdot\mathbf{h})^p$$

specularly reflected light

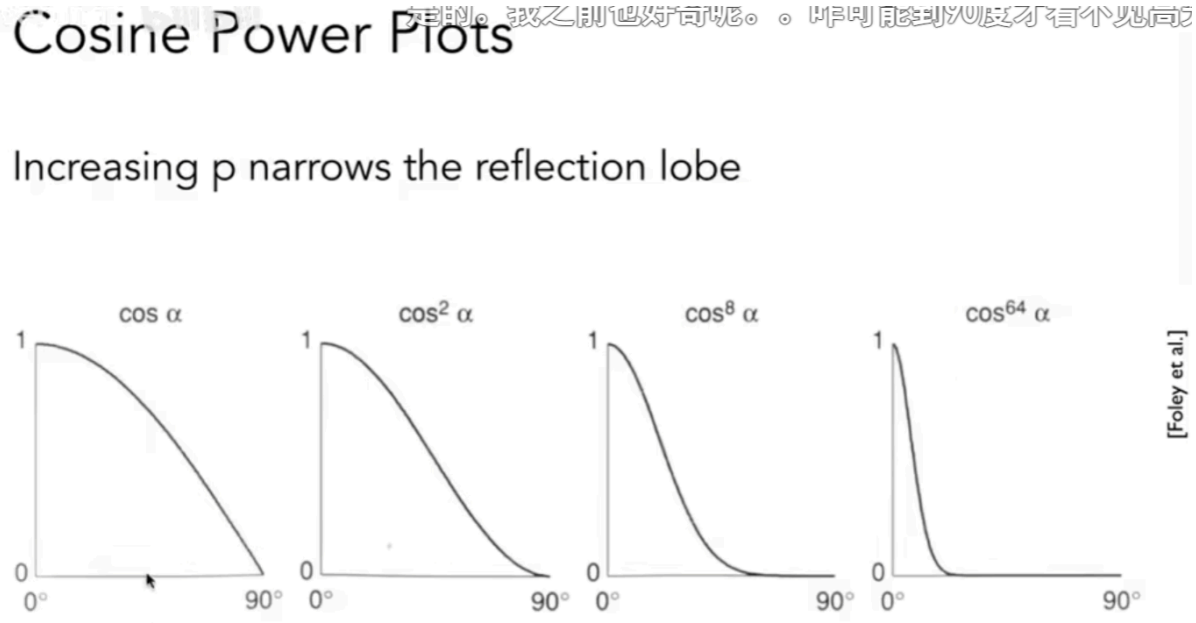
specular coefficient



当v和R接近时，n和h一定接近。

使用点乘来判断

镜面反射系数一般为白色，用夹角余弦的指数来生成高光（高光性质）。



环境光

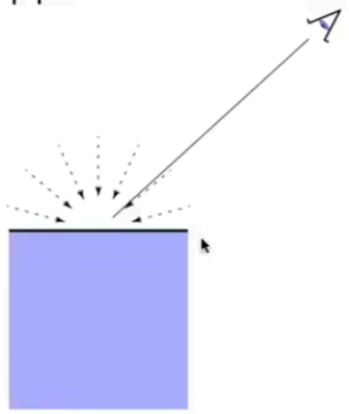
该模型下，从环境射进摄像机的值，保证所有地方都能看到，是一个常数。

更细致的，需要考虑全局光照。

Ambient Term

Shading that does not depend on anything

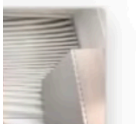
- Add constant color to account for disregarded illumination and fill in black shadows
- This is approximate / fake!



$$L_a = k_a I_a$$

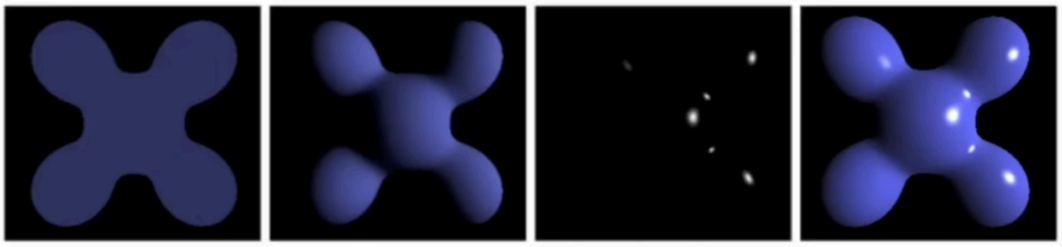
ambient coefficient

reflected ambient light



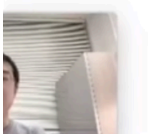
ader的ambient常数项写成零了 太明白了！信大太牛逼了！

Blinn-Phong Reflection Model



Ambient + Diffuse + Specular = Blinn-Phong Reflection

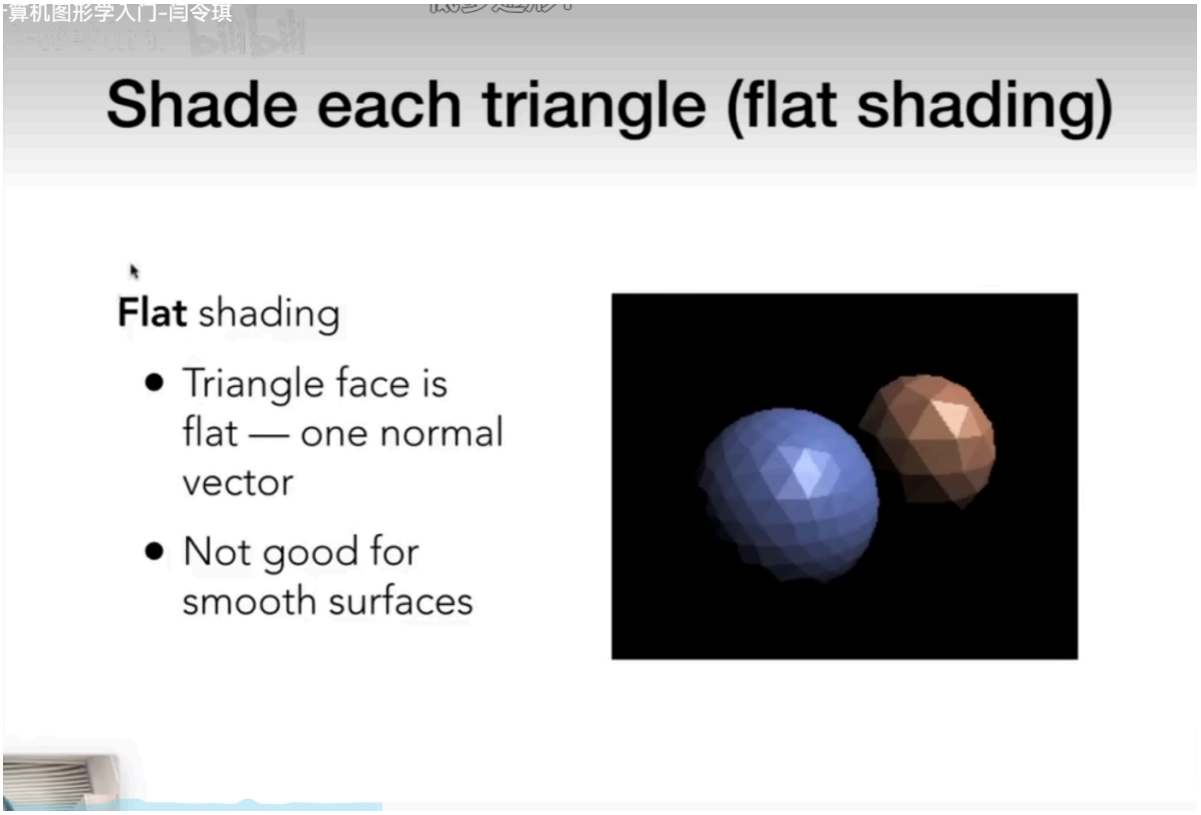
$$L = L_a + L_d + L_s$$
$$= k_a I_a + k_d (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{l}) + k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p$$



bling -feng 模型是经验模型。

着色细分

Flat shading 使用每个三角面同向法，三角形各顶点颜色一致，内部无颜色变化



Gouraud Shading

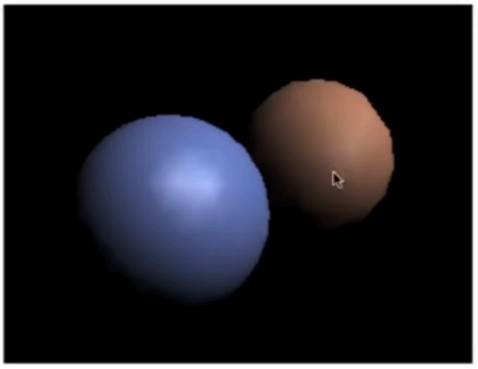
三角形每个顶点有颜色，在内部对这个三角形的颜色做插值。

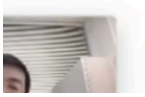
线图？ 平面着色 逐顶点着色
连接
连接的每个面求一遍法线求平均？

Shade each vertex (Gouraud shading)

Gouraud shading

- Interpolate colors from vertices across triangle
- Each vertex has a normal vector (how?)





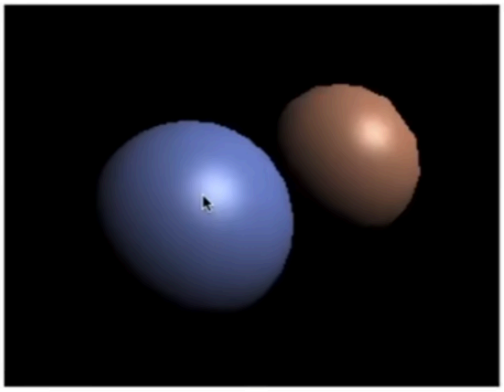
Phong shading

逐像素着色

Shade each pixel (Phong shading)

Phong shading

- Interpolate normal vectors across each triangle
- Compute full shading model at each pixel
- Not the **Blinn-Phong Reflectance Model**

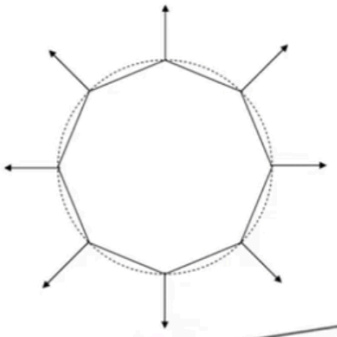


逐顶点的法线：顶点的法线为它所被共面的法线的平均（简单平均、加权平均都可）。

Defining Per-Vertex Normal Vectors

Best to get vertex normals from the underlying geometry

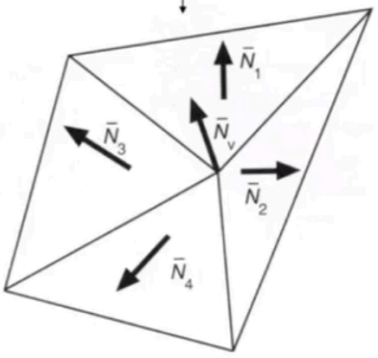
- e.g. consider a sphere

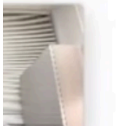


Otherwise have to infer vertex normals from triangle faces

- Simple scheme: **average surrounding face normals**

$$N_v = \frac{\sum_i N_i}{\|\sum_i N_i\|}$$



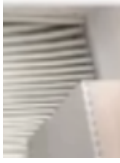


逐像素的法线：（与重心坐标相关，从一个顶点到另一个顶点的法线方向平均变化）

///====

着色器练习

Inigo Quilez



Procedurally modeled, 800 line shader.
http://shadertoy.com/view/ld3Gz2

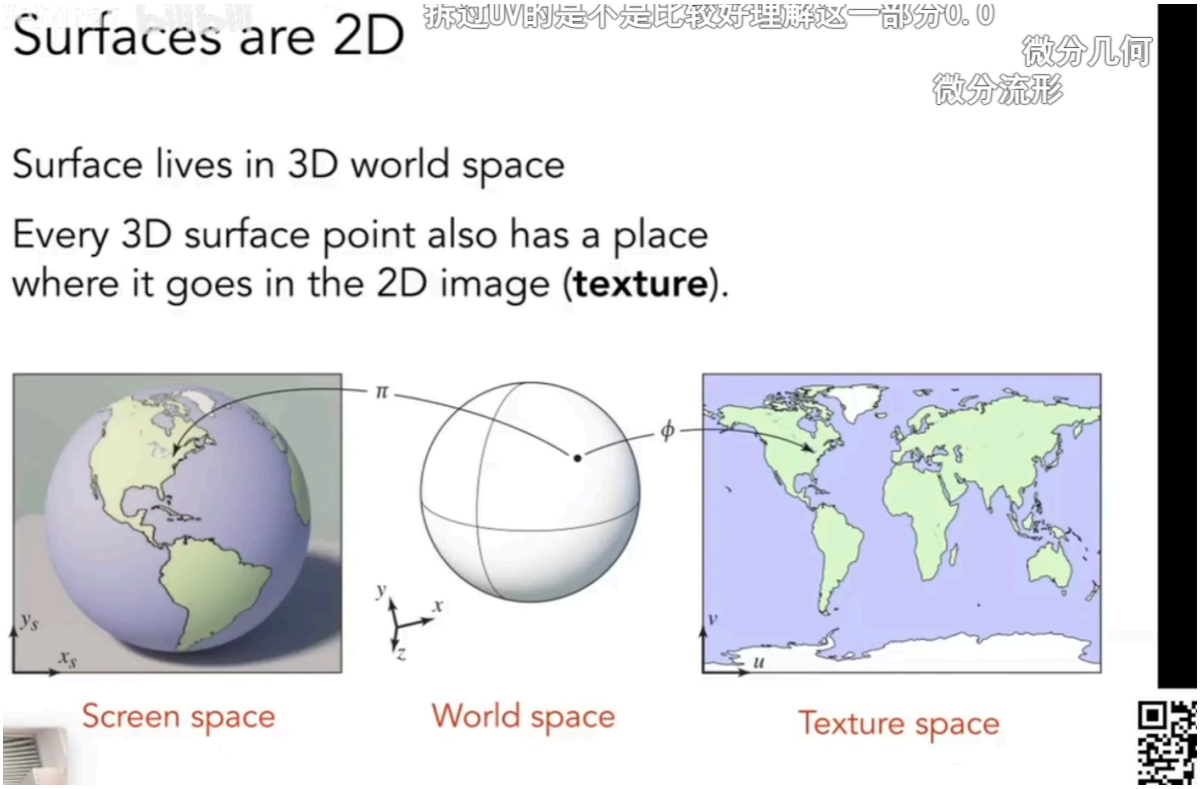
Inigo Quilez, https://youtu.be/XuSnLbB1j6E

///====

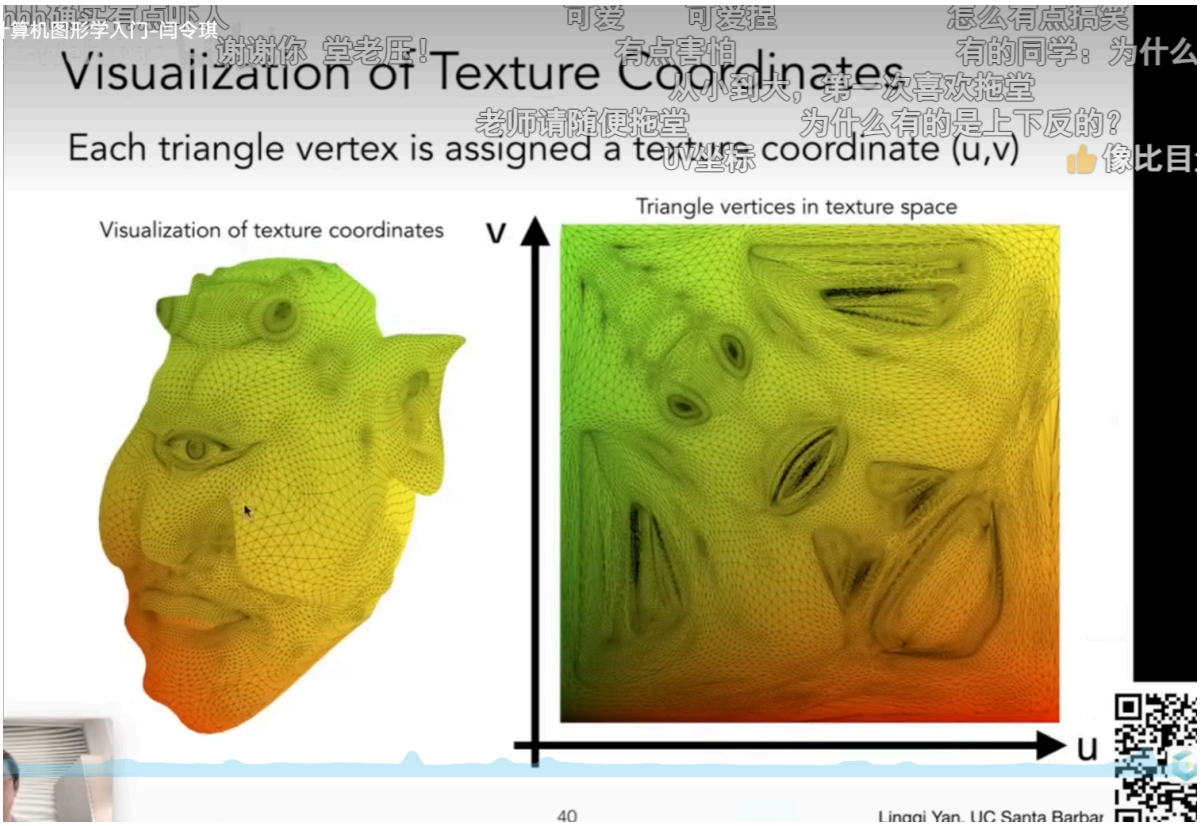
纹理映射

对于漫反射中的kd。可以通过漫反射映射出去。

三维物体的表面是二维的，可以与一个图有一一对应的关系。



纹理坐标：u-v,范围[0,1]



在三角形内部进行插值

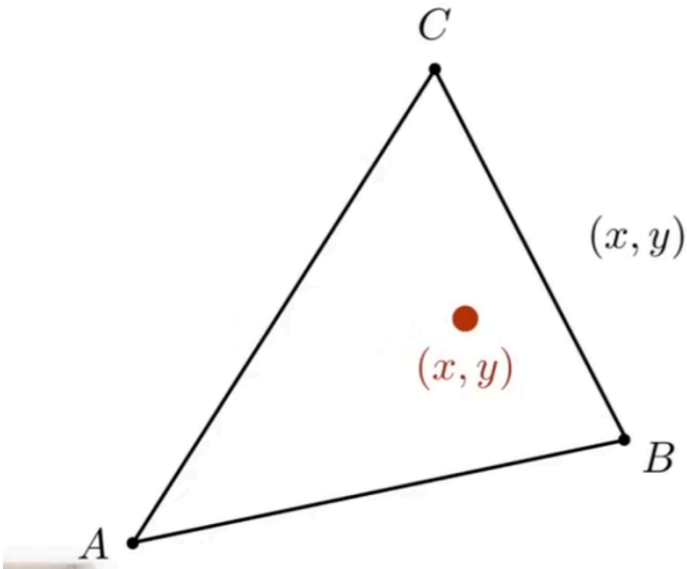
引入重心坐标

在知道顶点的情况下，知道三角形内任意一点的值。

点在三角形内，必须要α等都非负。

Barycentric Coordinates

A coordinate system for triangles (α, β, γ)



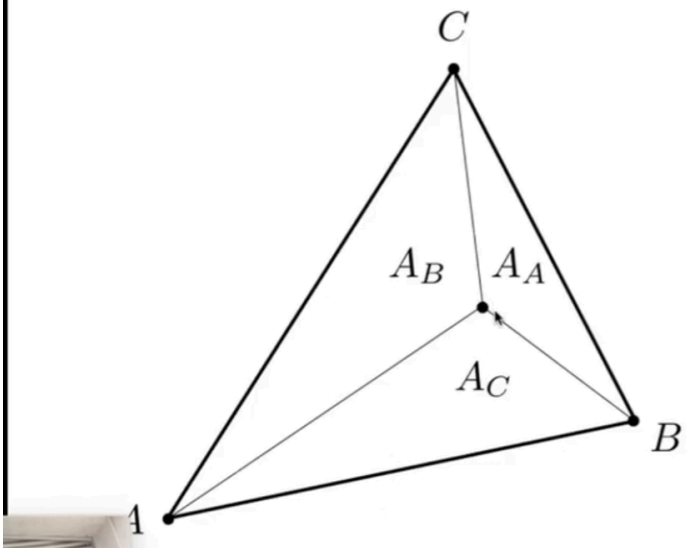
$$(x, y) = \alpha A + \beta B + \gamma C$$
$$\alpha + \beta + \gamma = 1$$

Inside the triangle if all three coordinates are non-negative

可从面积比求出任意点坐标的重心坐标系数

Barycentric Coordinates

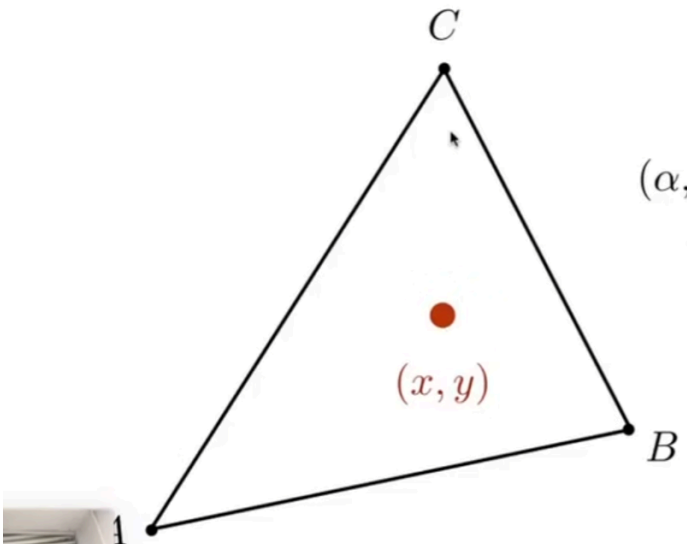
Geometric viewpoint — proportional areas



$$\alpha = \frac{A_A}{A_A + A_B + A_C}$$
$$\beta = \frac{A_B}{A_A + A_B + A_C}$$
$$\gamma = \frac{A_C}{A_A + A_B + A_C}$$

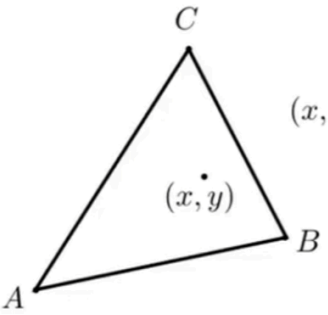
Barycentric Coordinates

What's the barycentric coordinate of the centroid?



$$(\alpha, \beta, \gamma) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$
$$(x, y) = \frac{1}{3}A + \frac{1}{3}B + \frac{1}{3}C$$

Barycentric Coordinates: Formulas



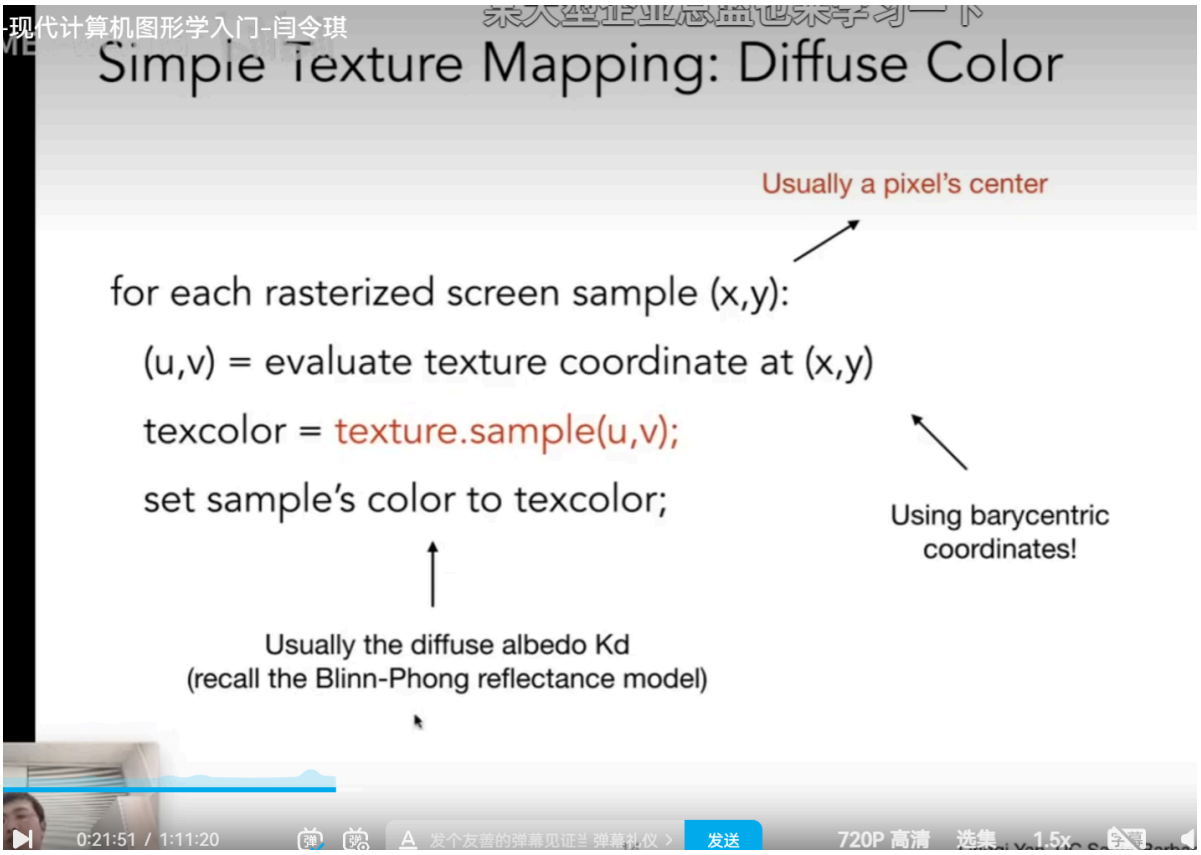
$$(x, y) = \alpha A + \beta B + \gamma C$$
$$\alpha + \beta + \gamma = 1$$

$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

重心坐标不具有投影不变性，所以应先做三维插值，再投影。



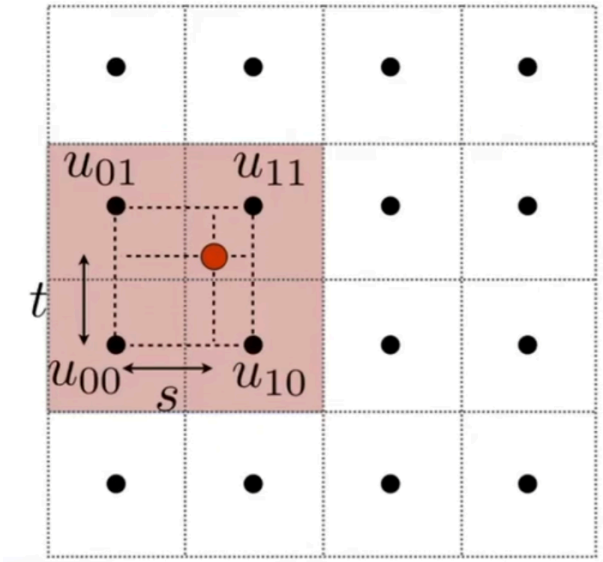
需要解决的问题

纹理放大：当纹理过小，像素映射到纹理图的一个非整数区域时，可能会出现多块物体表面像素映射同一个纹理像素的情况。

双线性插值

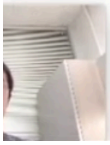
针对映射的像素位置选定需要纳入插值的纹理像素区

Bilinear interpolation



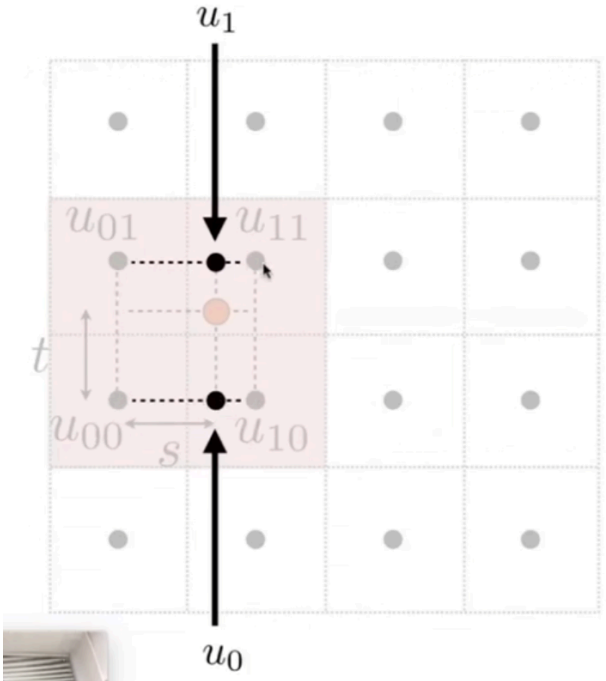
Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$



在水平上进行插值

Bilinear interpolation



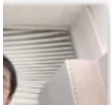
Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

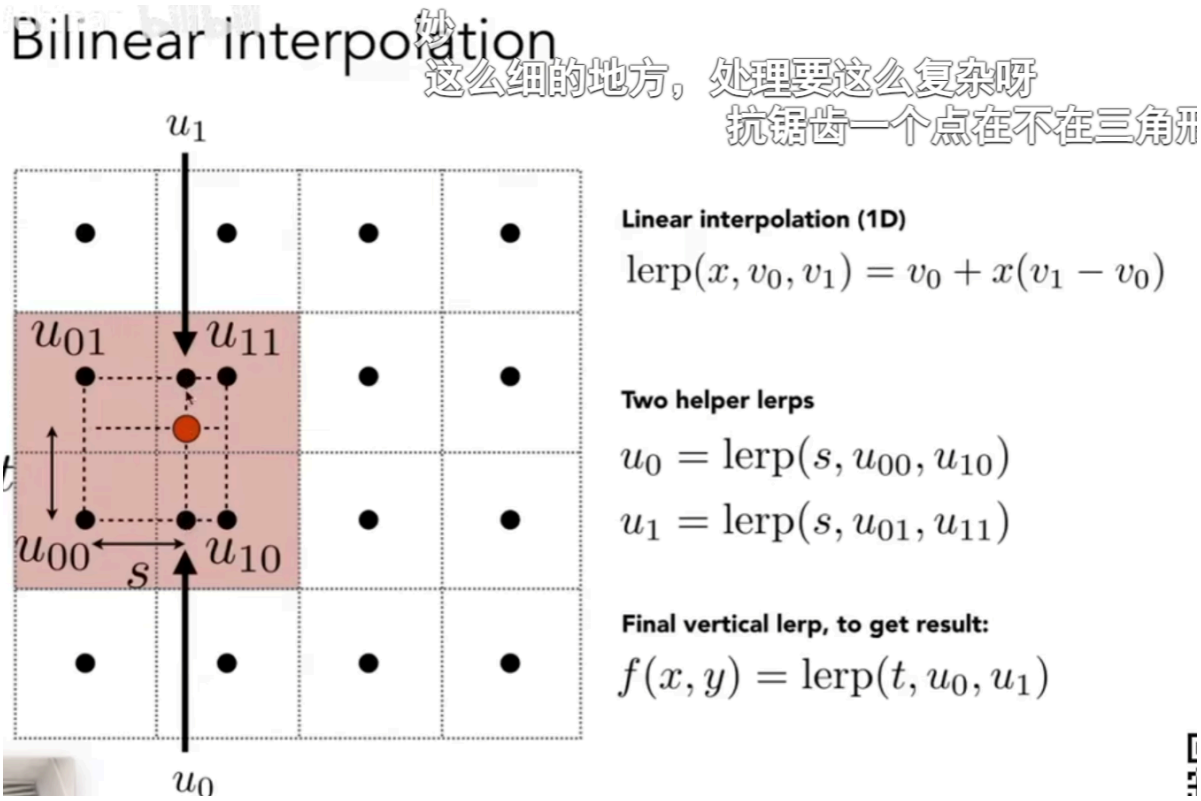
Two helper lerps (horizontal)

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

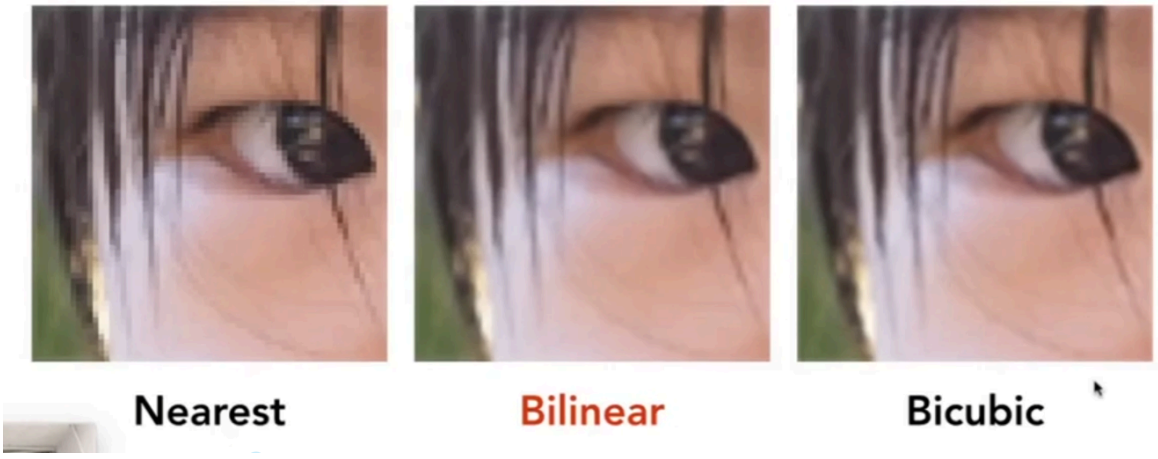
$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$



再在竖直上进行插值

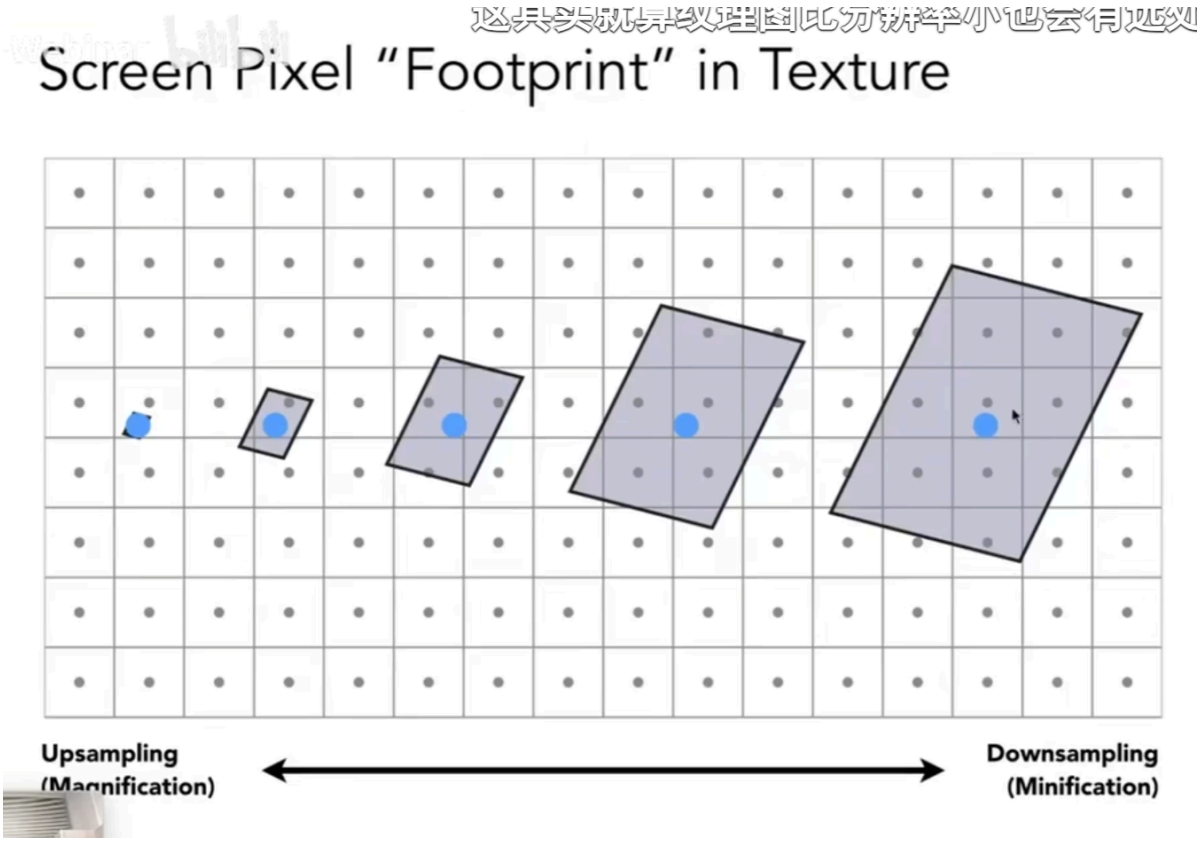


最邻近-双线性（周围4个）-立方插值（周围16个）



纹理过大时，远处出现摩尔纹，近处出现锯齿 ==> 出现了走样

像素覆盖的纹理区域：



点查询问题和范围查询问题

一种好方法是直接对该区域进行均值。

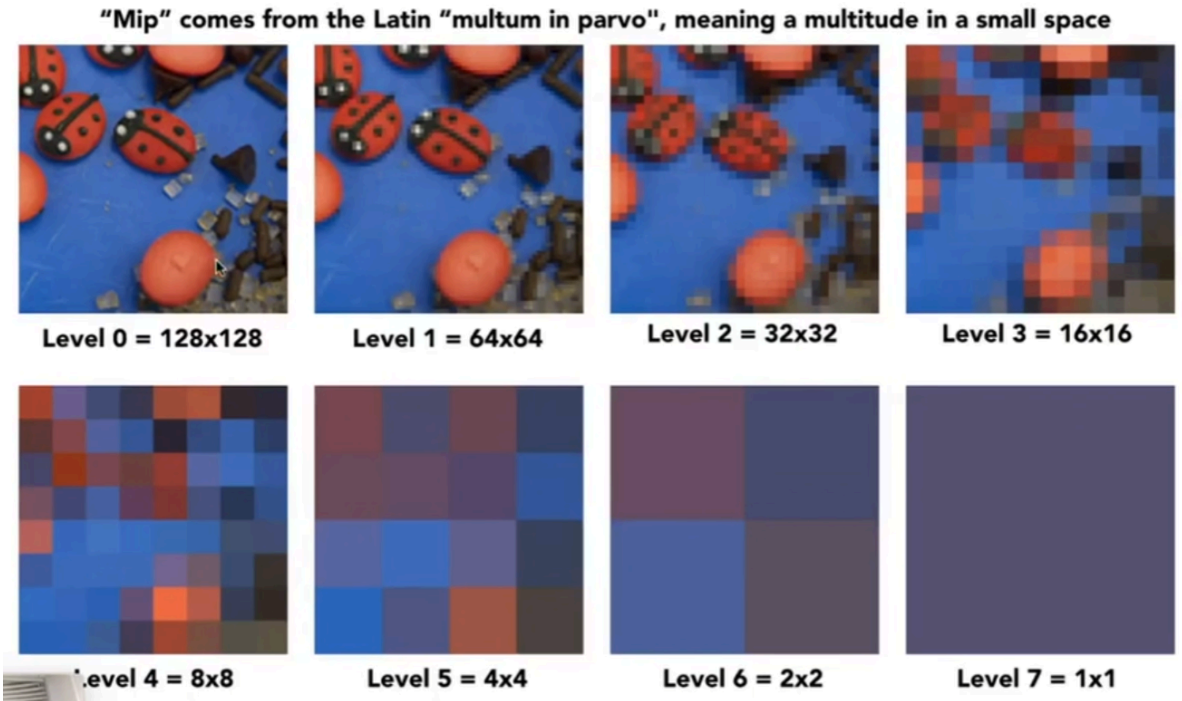
Mipmap 也称图像金字塔

近似，正方形的查询。

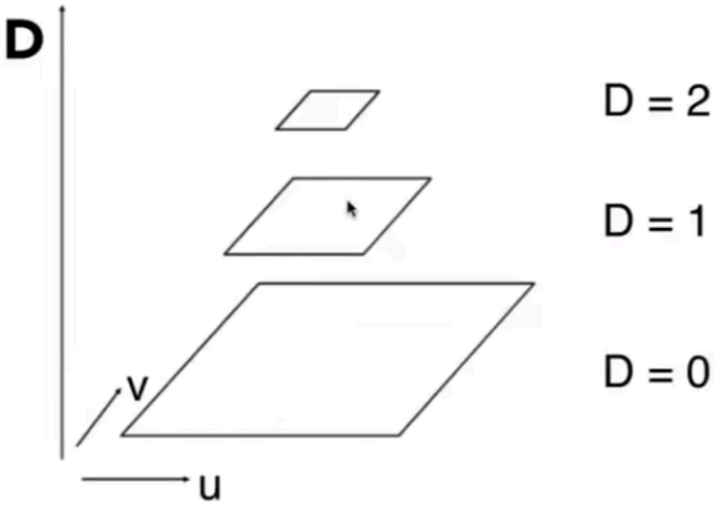
先生成一系列的纹理层组

Mipmap (L. Williams 83)

提前做好准备　图像金字塔



Mipmap (L. Williams 83)

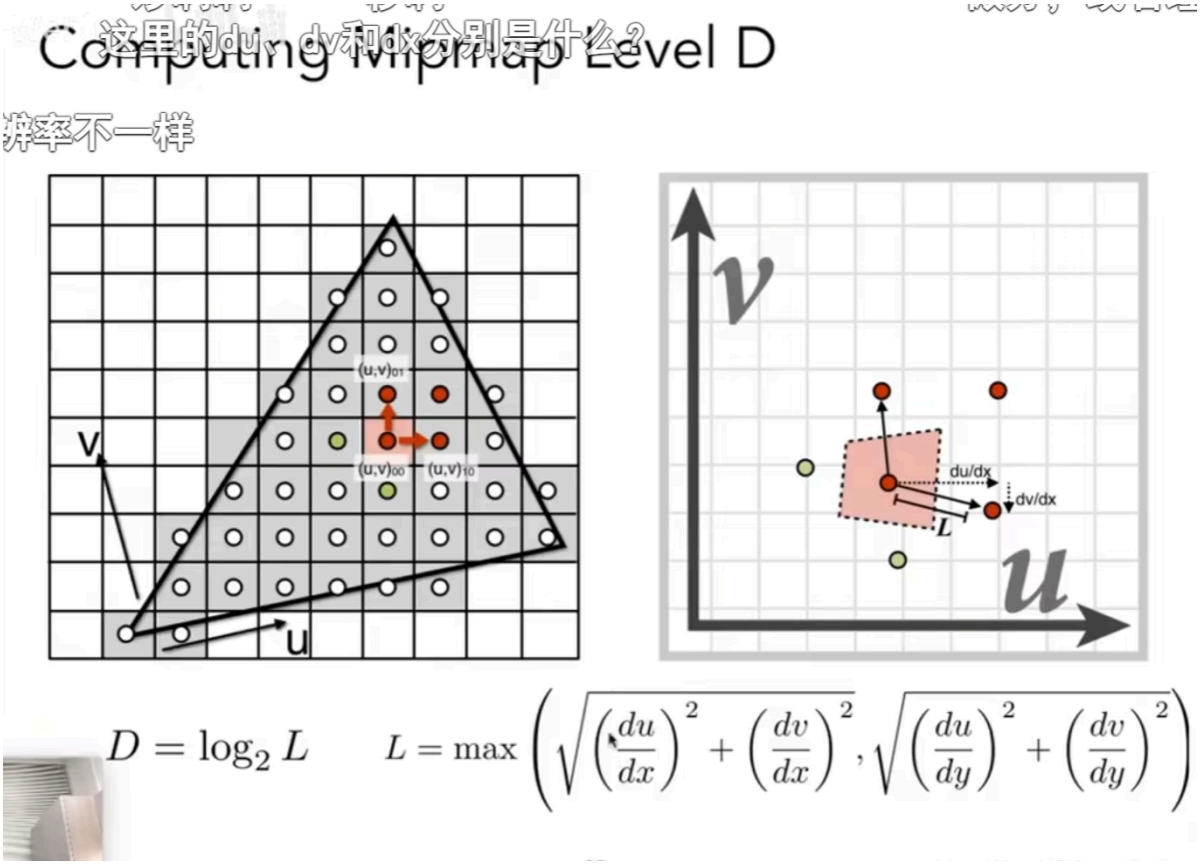


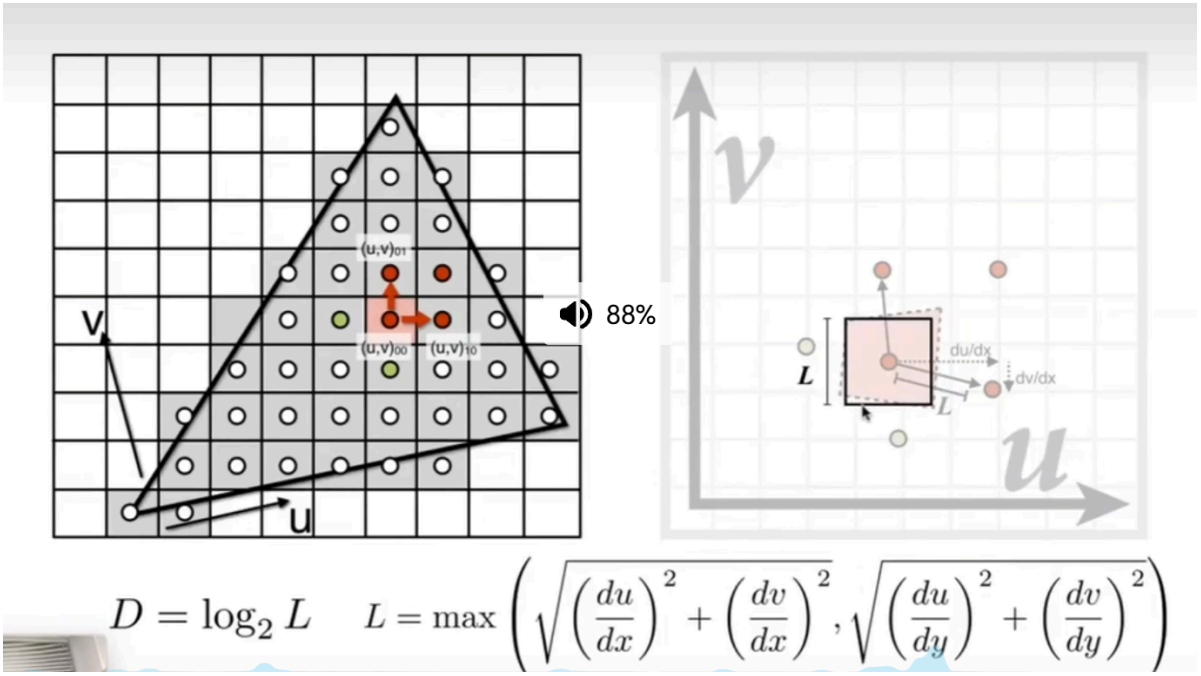
"Mip hierarchy"
level = D

What is the storage overhead of a mipmap?

引入的存储量多三分之一。（1+1/4+1/16）... =4/3

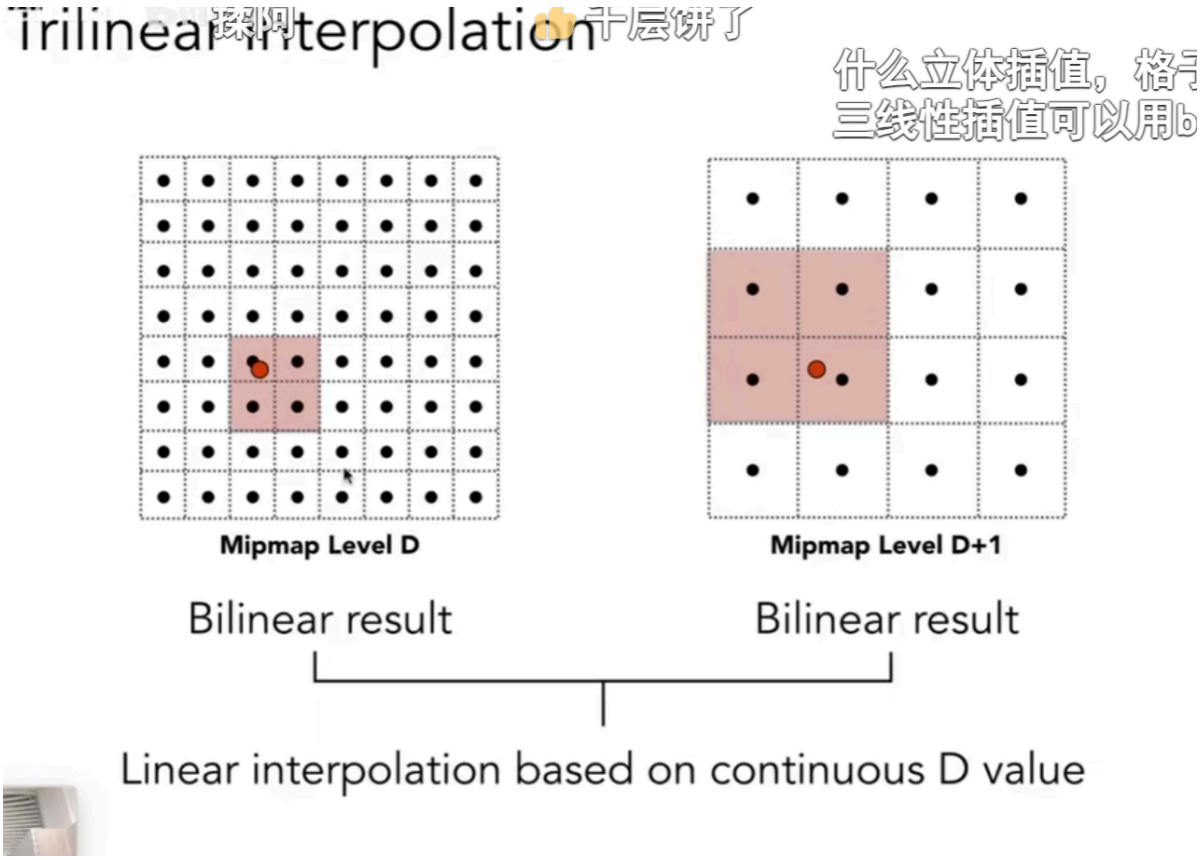
对屏幕像素所在区域映射到纹理上的对应区域：



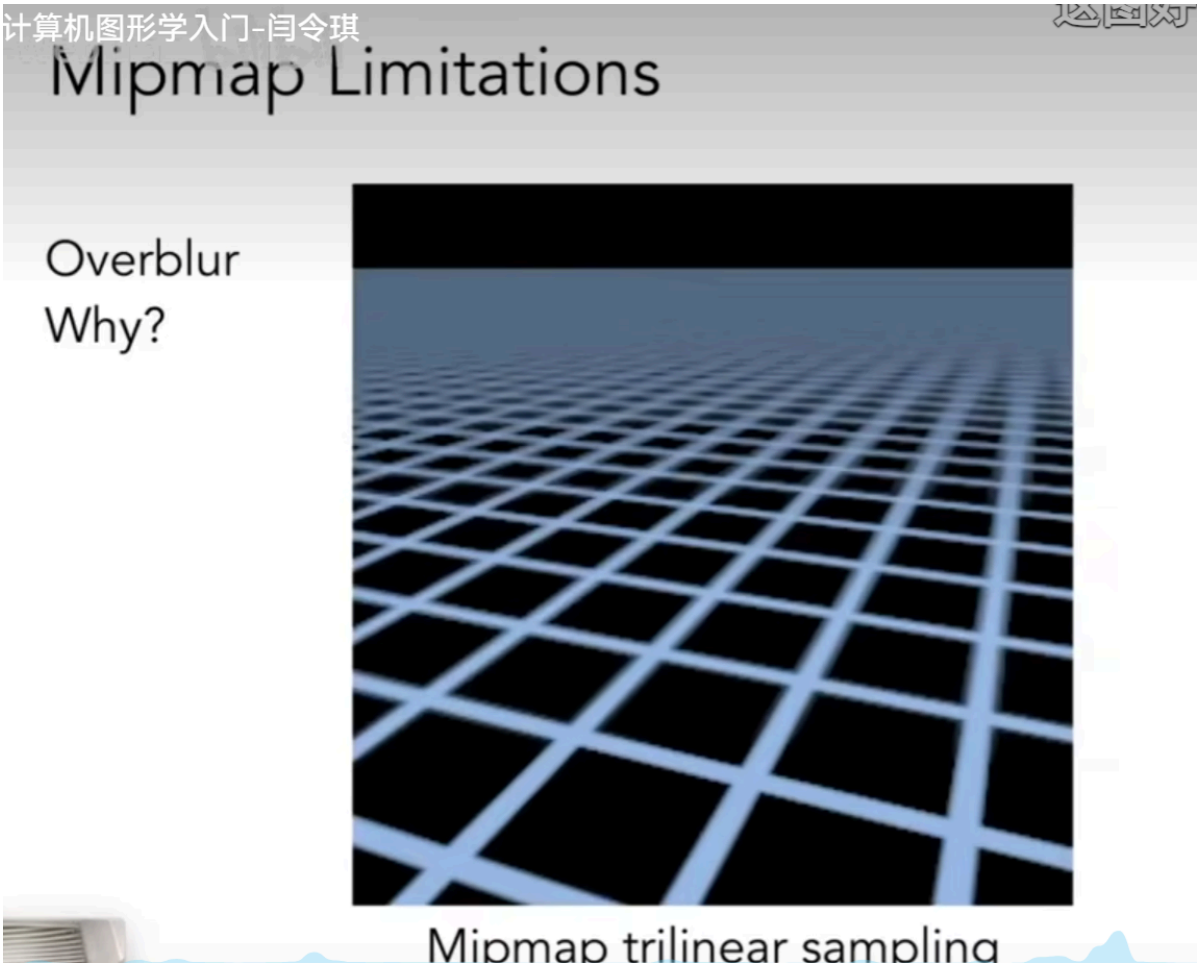


求出该区域在第log2L层能变为1个像素大小。

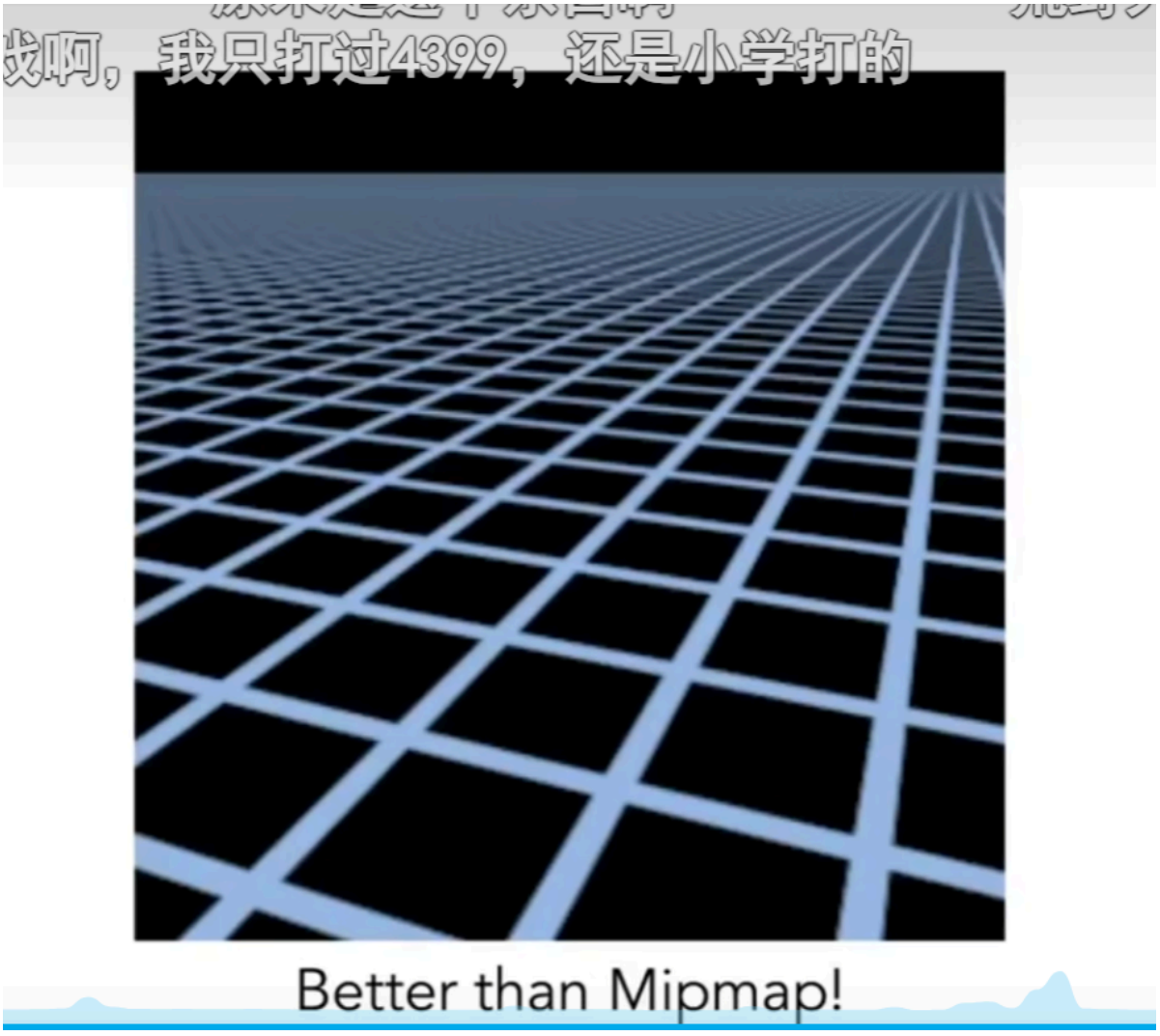
为了让各层之间连续，应该对层与层之间做一次三线性插值。（先双线性插值，然后在层与层间插值）



但是，通过三线性插值形成的mipmap会存在远处糊的情况

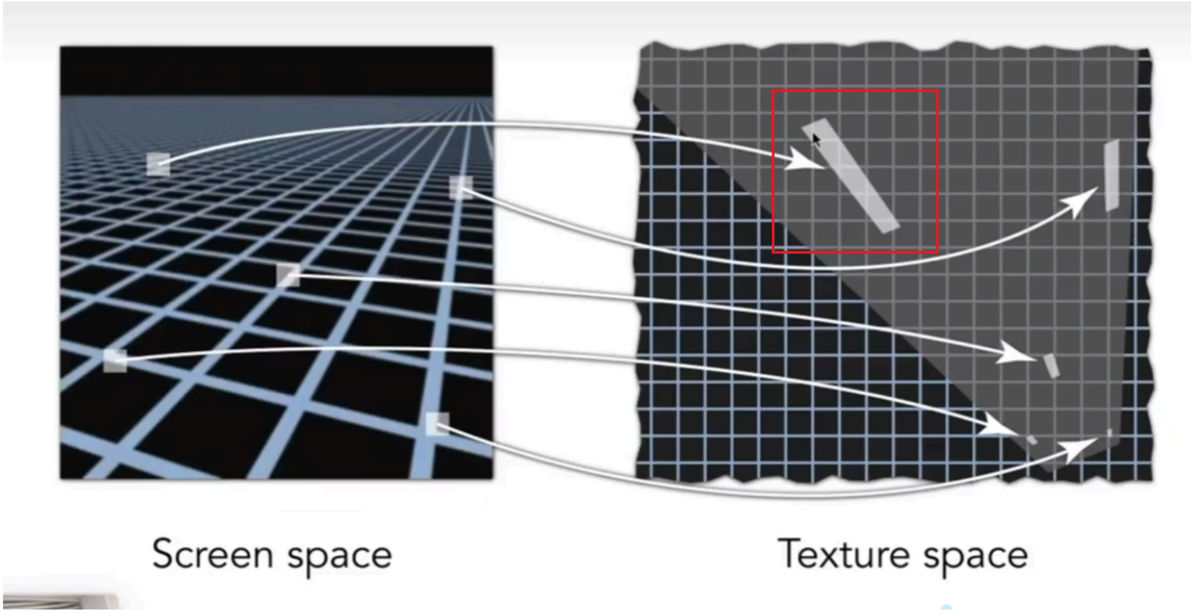


这时候，可以用各向异性过滤来解决

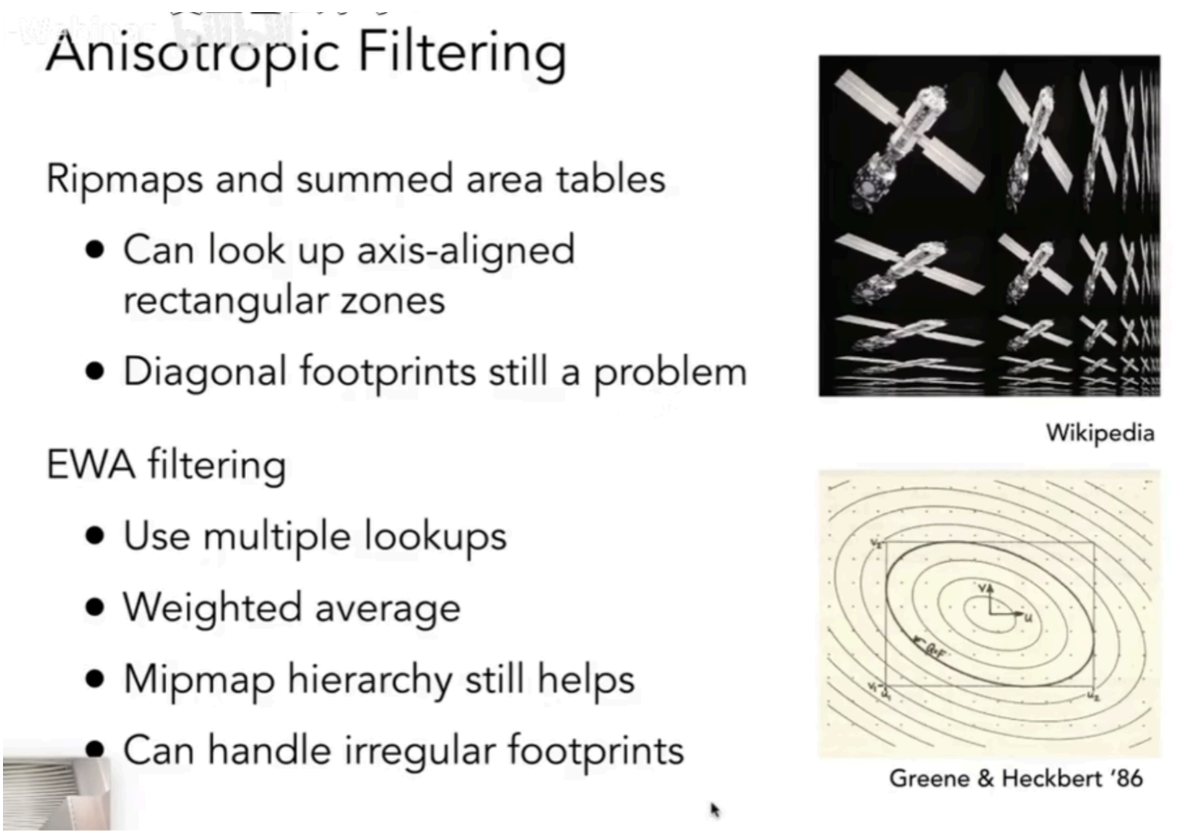


对矩形区域进行映射为正方形

但对于斜着的区域不太行


Screen space
Texture space

使用EWA


Anisotropic Filtering
Ripmaps and summed area tables
• Can look up axis-aligned rectangular zones
• Diagonal footprints still a problem
EWA filtering
• Use multiple lookups
• Weighted average
• Mipmap hierarchy still helps
• Can handle irregular footprints
Wikipedia
Greene & Heckbert '86

//= = = = = = = = = = = == == == = = = = = = = = = = = = = = =
== =

Geometry

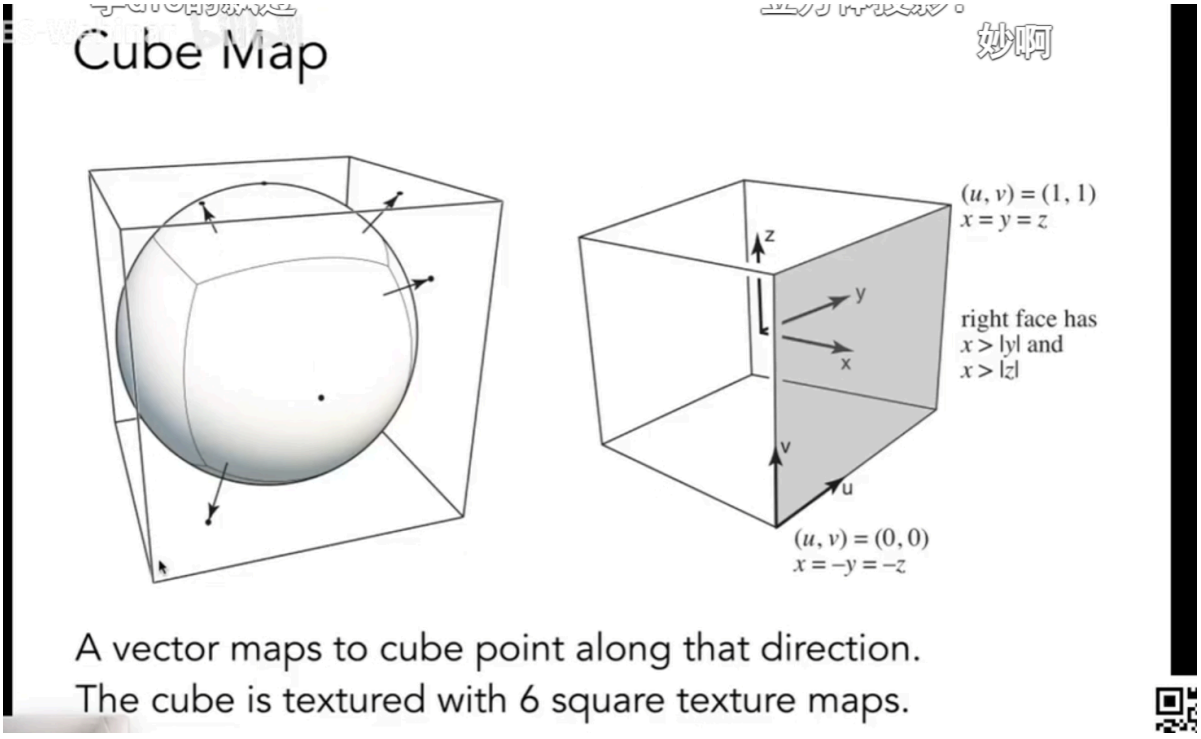
纹理可以用来表示环境光

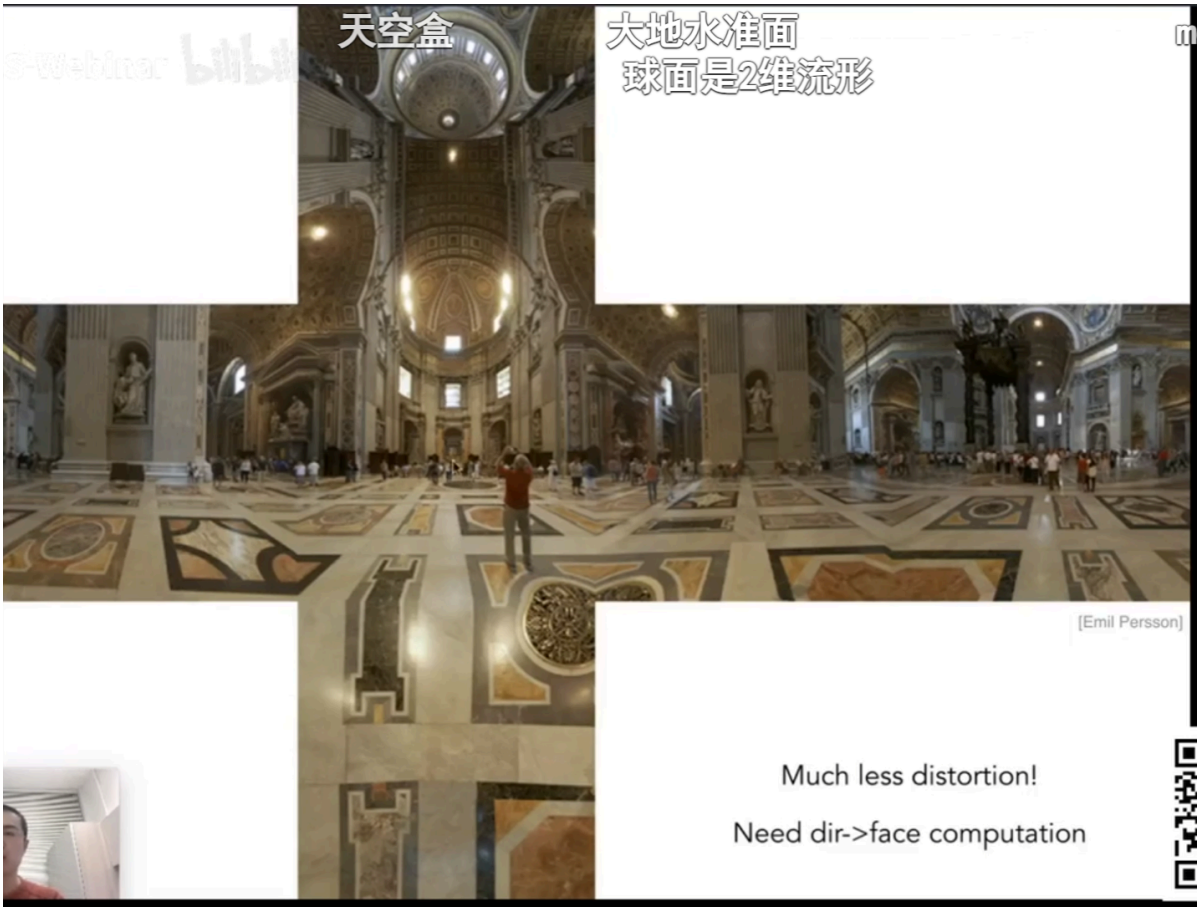
环境光都来自无穷远处，只记录方向即可。

环境光可记录在一个球面上，并进行展开

会存在一个扭曲问题

使用Cube mape 球的包围盒



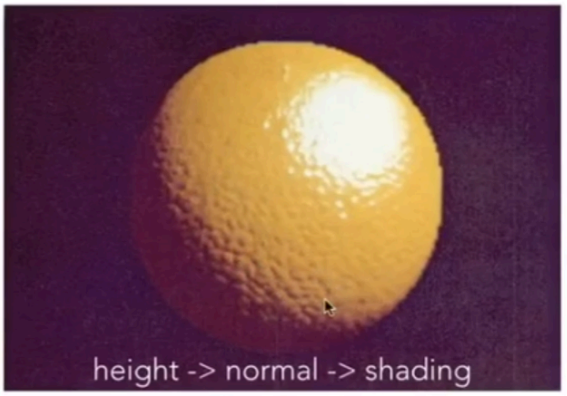


纹理还能做凹凸贴图，可以模拟出法线 法线贴图

Textures can affect shading!

- Textures doesn't have to only represent colors
  - What if it stores the height / normal?
  - Bump / normal mapping
  - **Fake** the detailed geometry

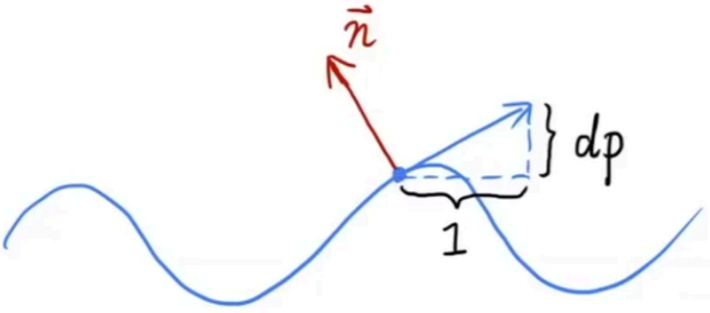
Relative height to the underlying surface



不改变几何信息，对法线进行扰动，定义任何一个像素的及其邻近位置的高度，通过高度差来重新计算法线。（相对高度移动）

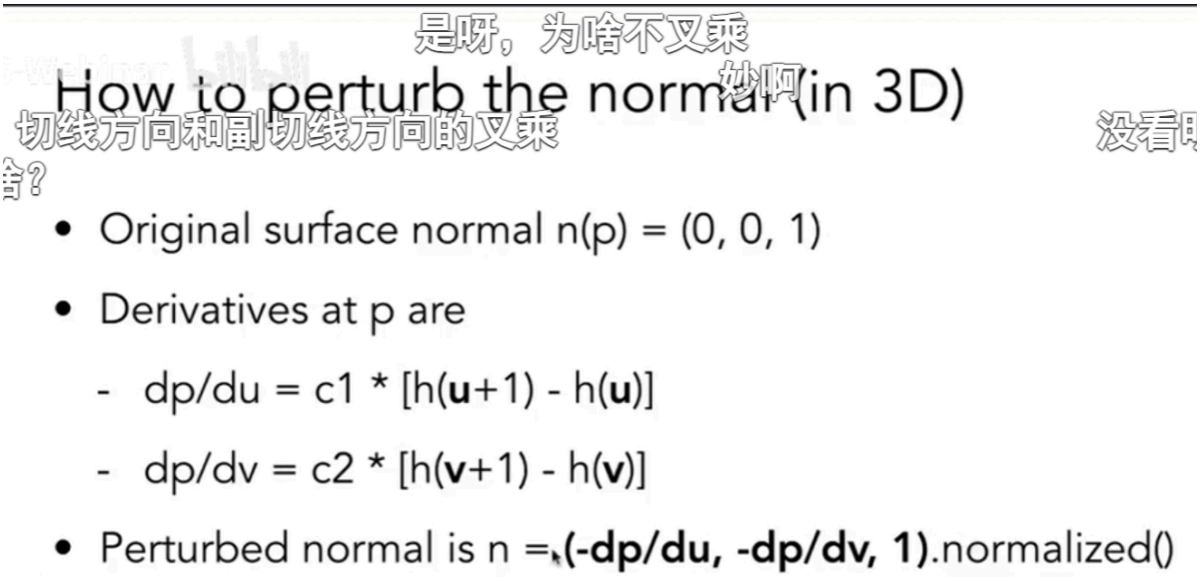
How to perturb the normal (in flatland)

- Original surface normal n(p) = (0, 1)
- Derivative at p is dp = c * [h(p+1) - h(p)]



用差分方法得切线，再逆时针旋转90°（XY对换后对Y求负）得法线。

三维情况



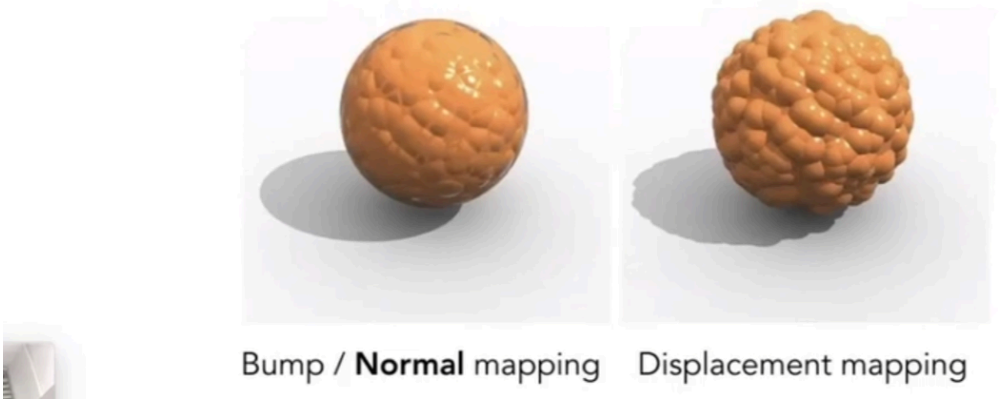
凹凸贴图在边缘和阴影处存在穿帮情况

另一种方式：位移贴图

移动了顶点的位置。但需要模型能完整反应纹理的变化。

Textures can affect shading!

- **Displacement mapping** — a more advanced approach
  - Uses the same texture as in bumping mapping
  - Actually **moves the vertices**



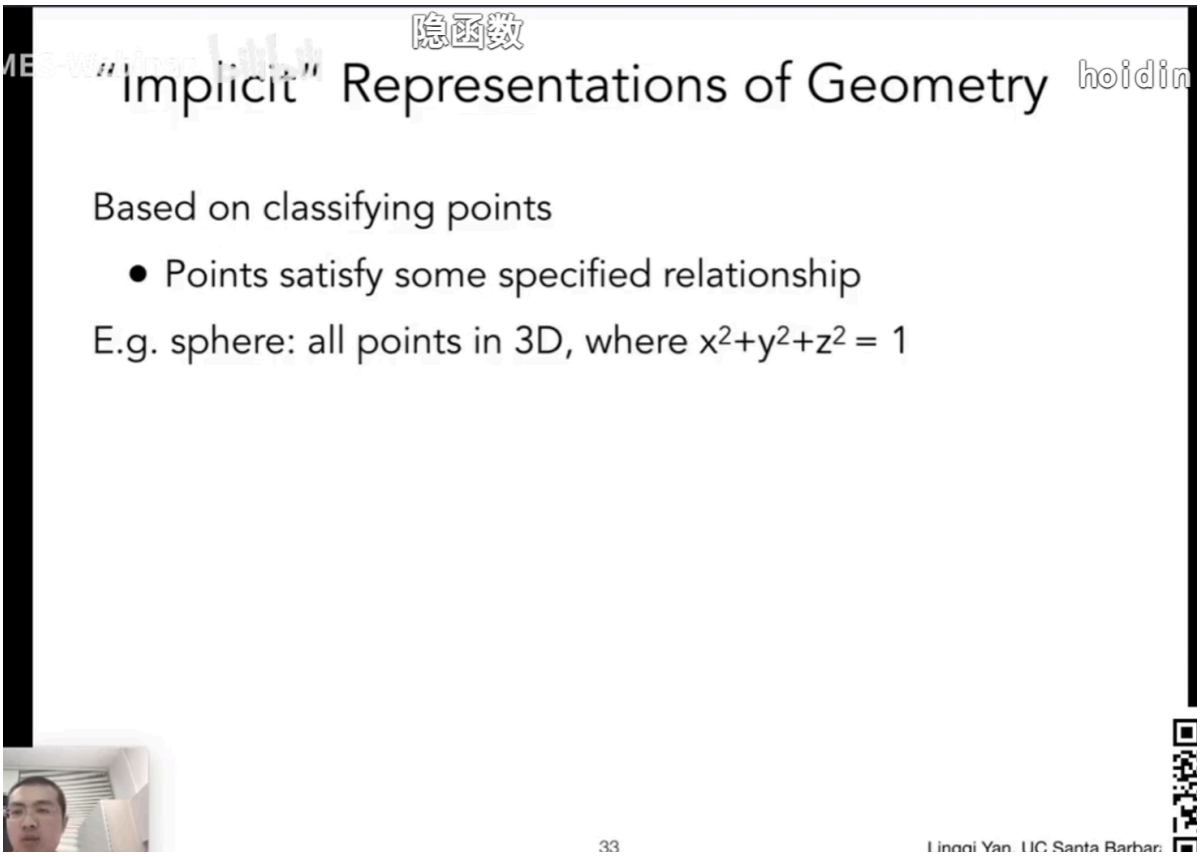
Bump / **Normal** mapping　Displacement mapping

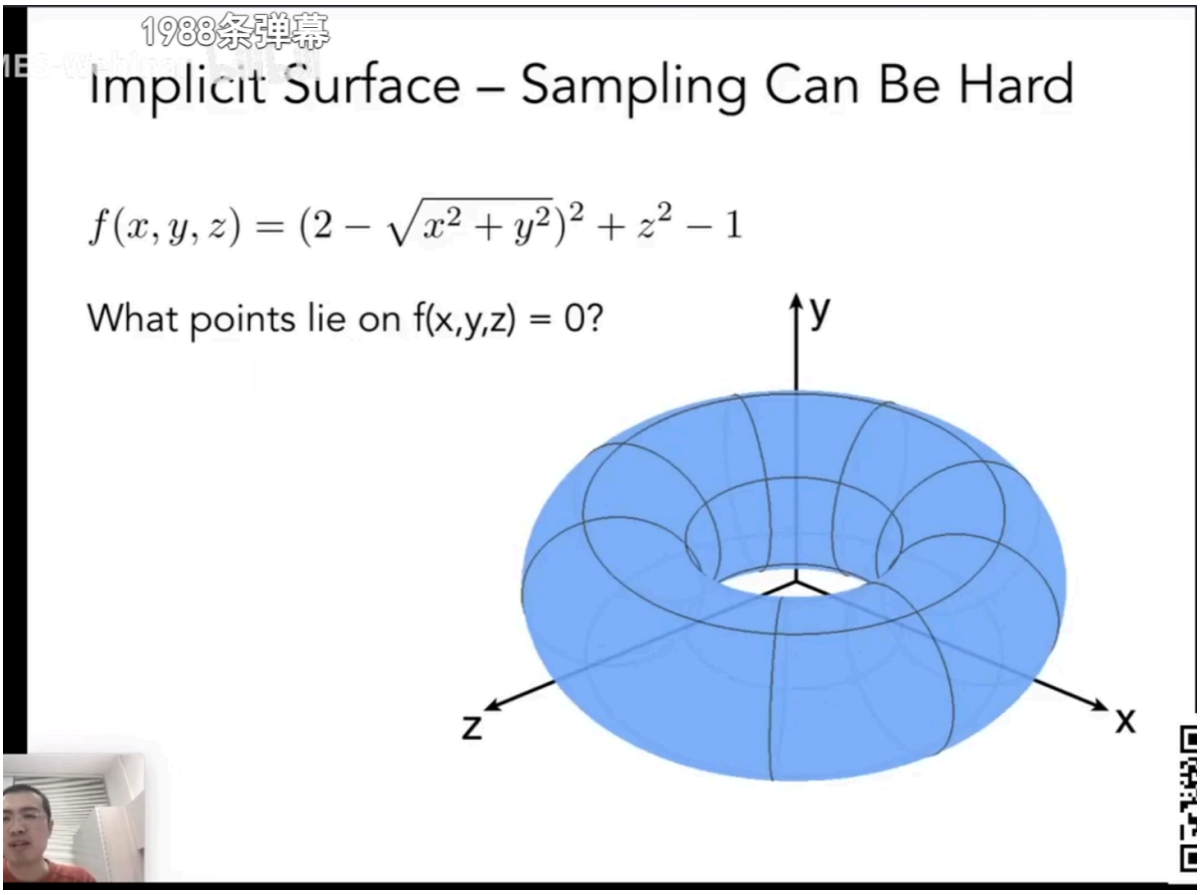
引入动态曲面细分

//====

隐式几何和显式几何

满足特定关系的点





判断问题和搜索问题。

隐式表示容易判断点是否在物体内外

Implicit Surface – Inside/Outside Tests Easy

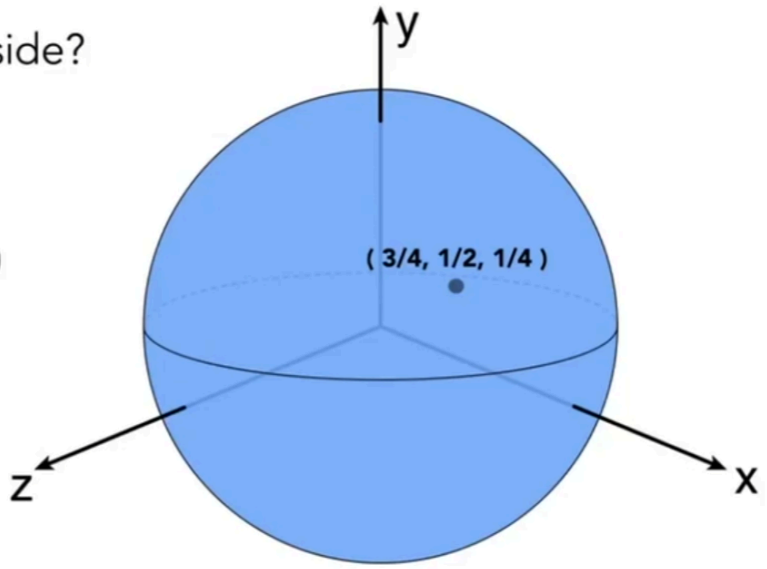
$$f(x,y,z) = x^2 + y^2 + z^2 - 1$$

Is (3/4, 1/2, 1/4) inside?

Just plug it in:

f(x,y,z) = –1/8 < 0

Yes, inside.



显式表示：

遍历像素，映射到空间中实际的点

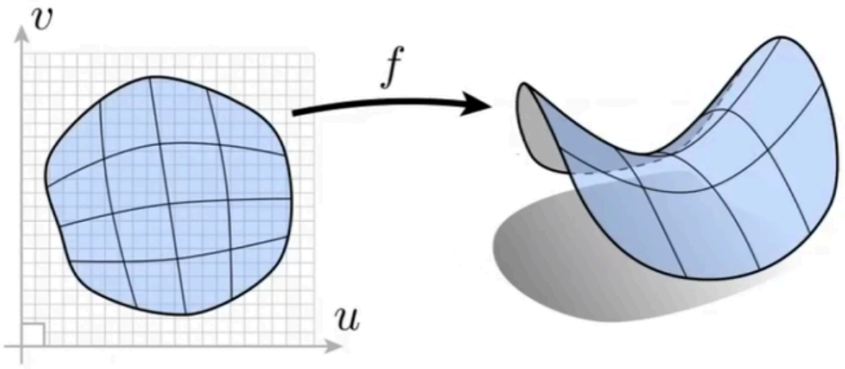
映射

只能表示面，不能表示闭合的体？

"Explicit" Representations of Geometry

All points are **given directly** or **via parameter mapping**

Generally: $f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$



参数方程？

Explicit Surface – Sampling Is Easy

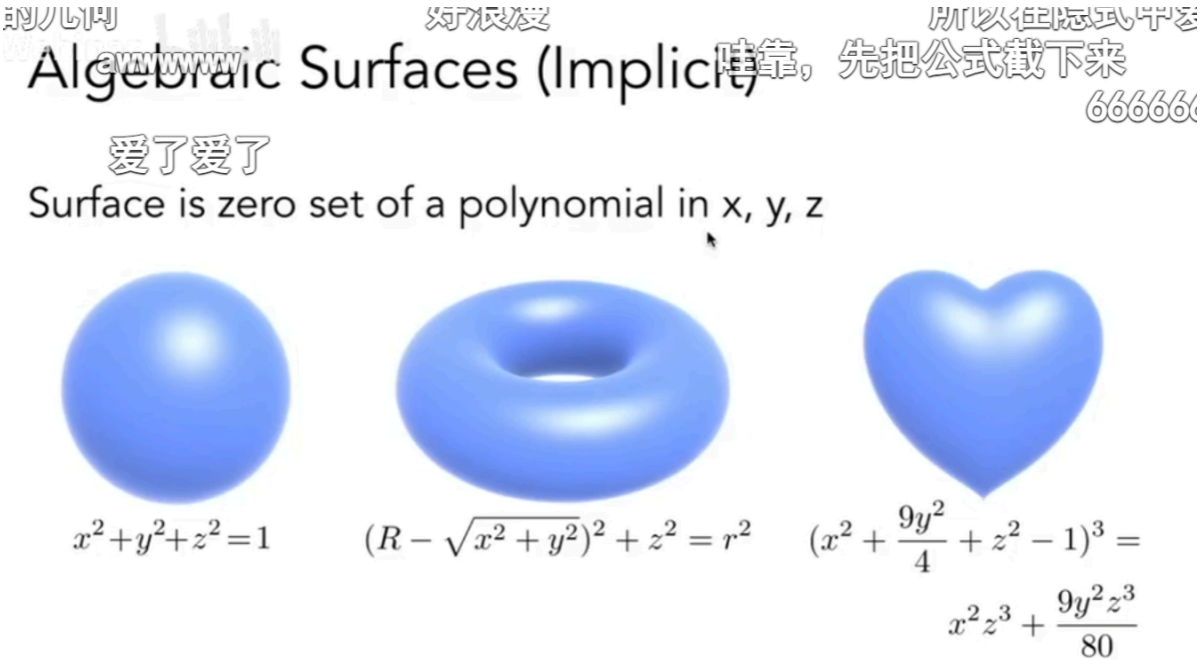
$$f(u, v) = ((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u)$$

What points lie on this surface?

Just plug in (u,v) values!

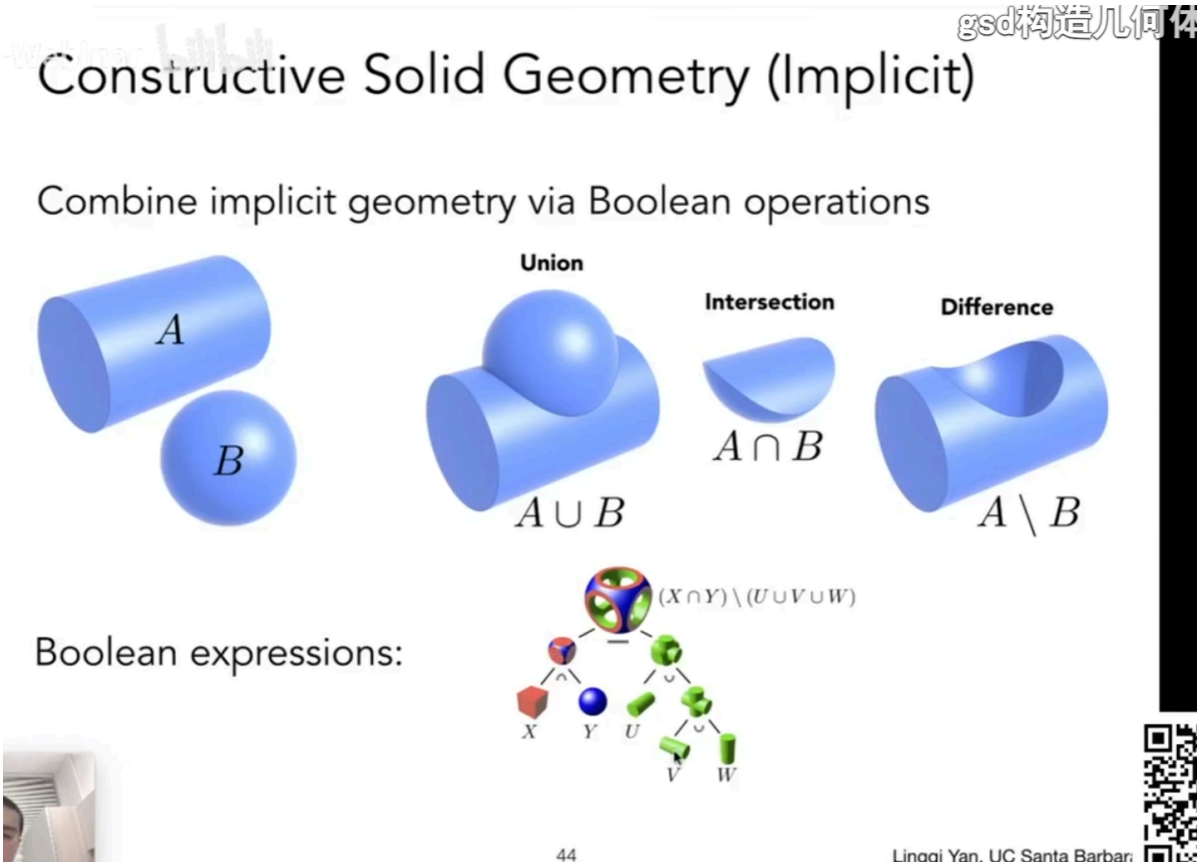
对于显式表面，判断任意点是否在物体内外 较为困难，但查询某像素的坐标较为简单。

隐式：不直观，表现出xyz的关系



基础几何的基本运算：CSG

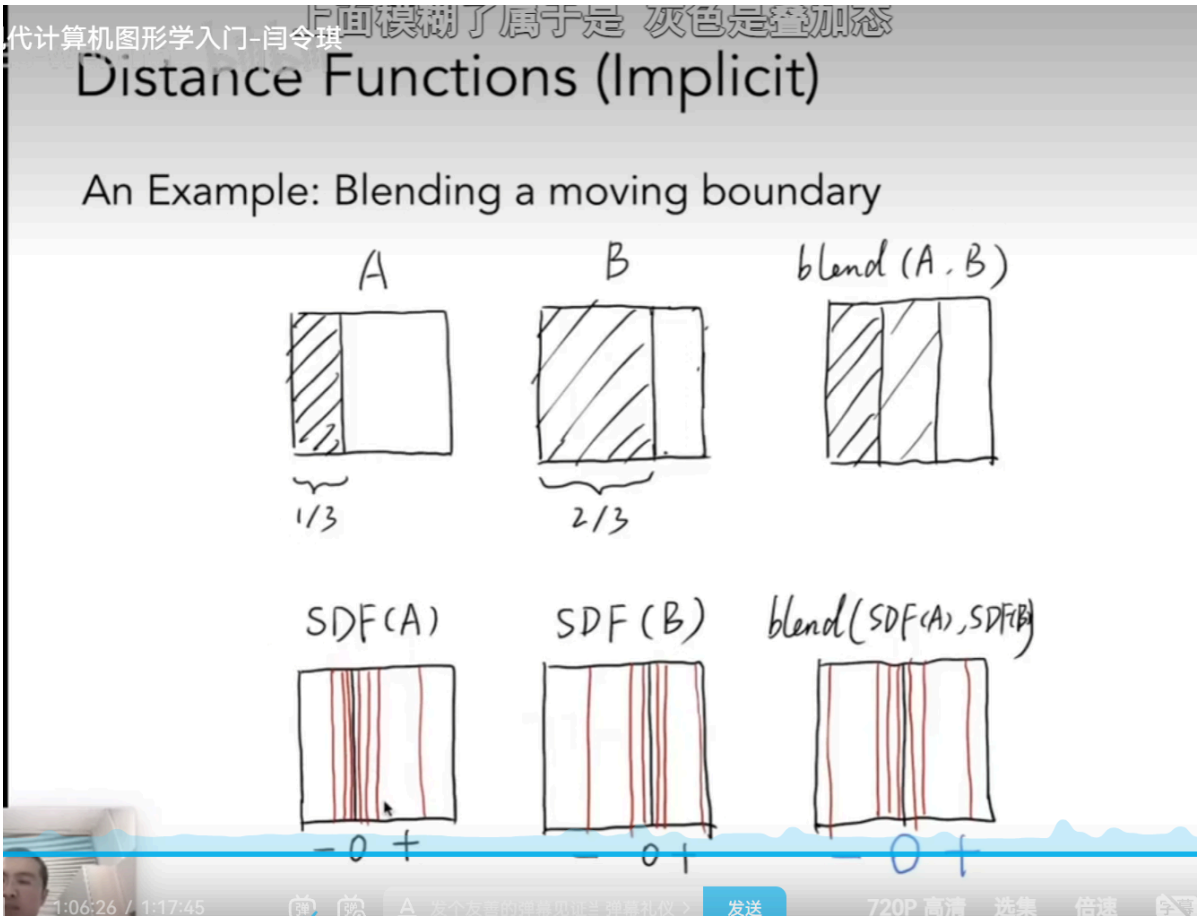
定义基础几何 和 基本运算，可形成复杂几何



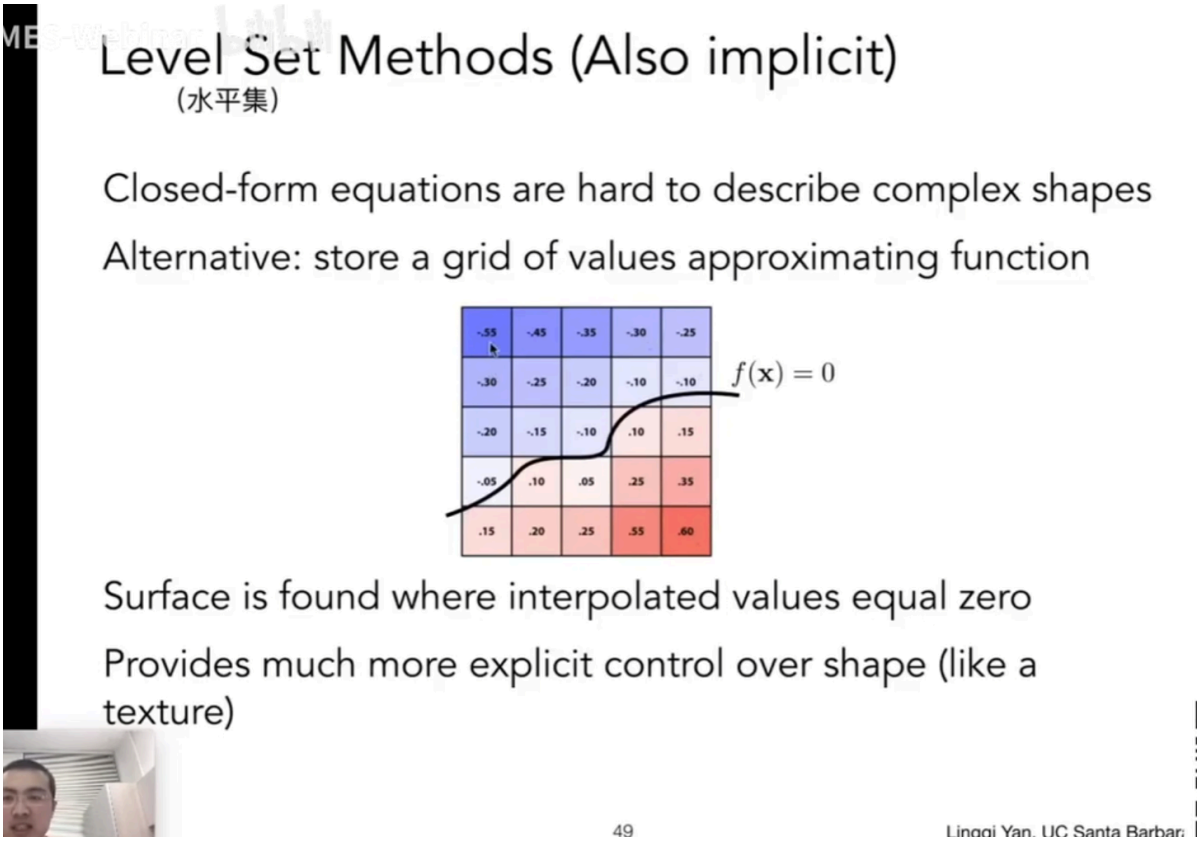
正常的叠图无法表示两个状态之间的运动关系，简单的线性blending叠加只能表示两个图各自对结果贡献了多少。

而使用距离函数，可以表示两个状态之间的运动关系。

因为距离函数是对于图形而言最近的距离，所以可以每个点形成的距离函数显示出图的边界。



类似距离函数的表示：水平集



对于三维纹理，使用等势面（水平集）可以表示层的概念

【Ray Marching】

显式几何

点云：高密度点集形成模型或面（三维扫描输出）

用三角形面表示

obj格式：

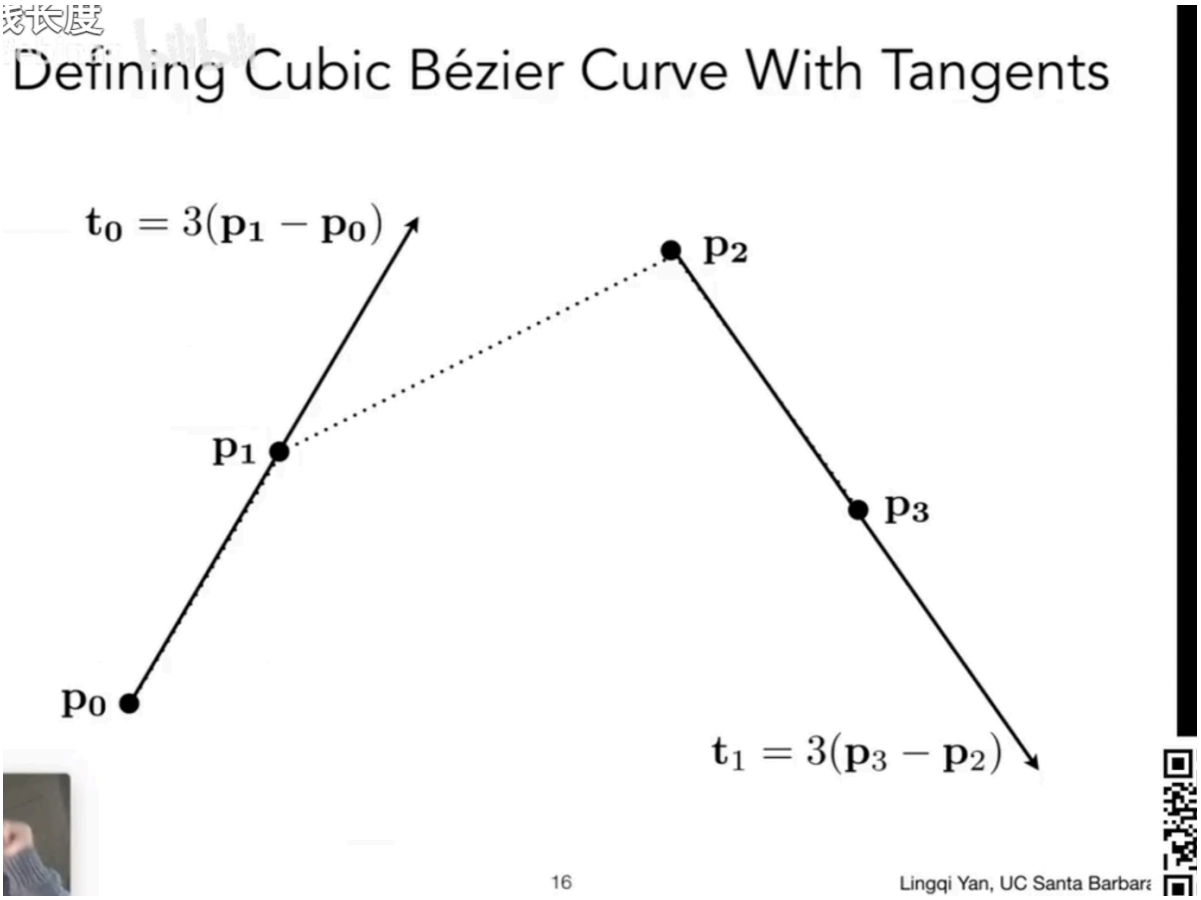
v：顶点　vn：法线　vt：纹理坐标

f：v/vt/vn（点1）v/vt/vn（点2） v/vt/vn（点3） 三个顶点构成一个面

一系列f形成一个物体（顺序）

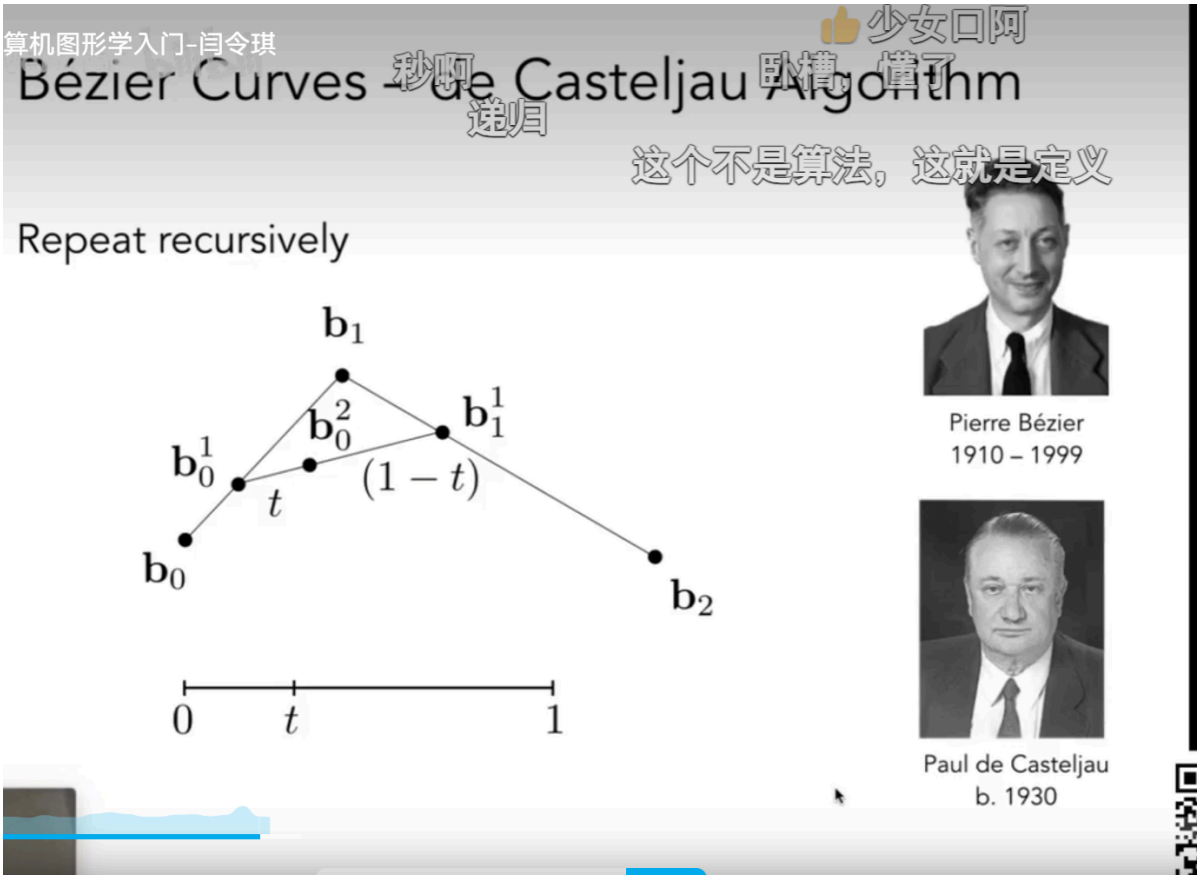
贝塞尔曲线

用一系列控制点定义某些曲线



起点p0经过p3

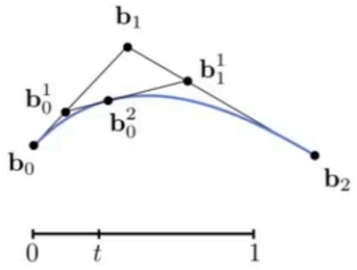
二次贝塞尔曲线



在两段中同比例距离取点b得两点，再在两点形成的连线上取同比例点c。在两段中的所有的点c形成的线就是贝塞尔曲线。

Bézier Curve – Algebraic Formula

Example: quadratic Bézier curve from three points

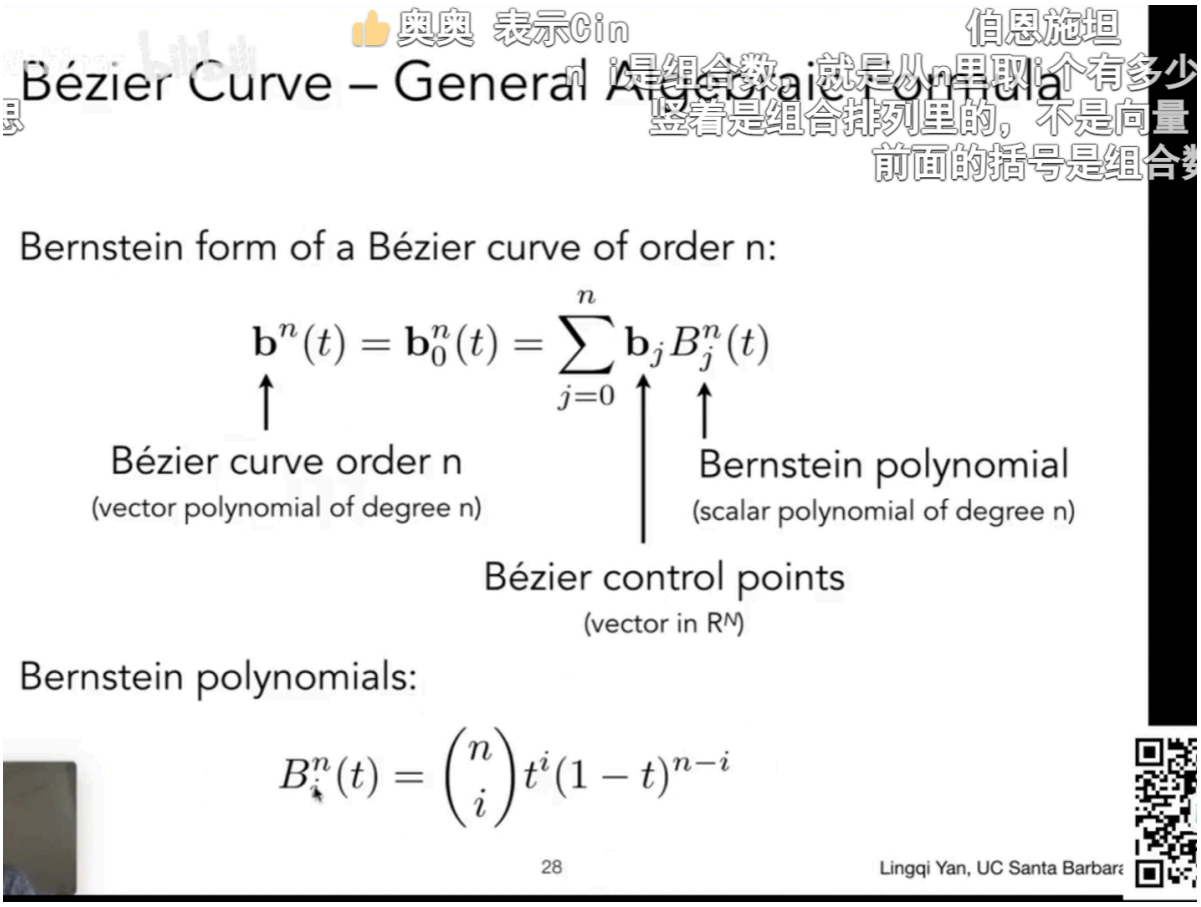


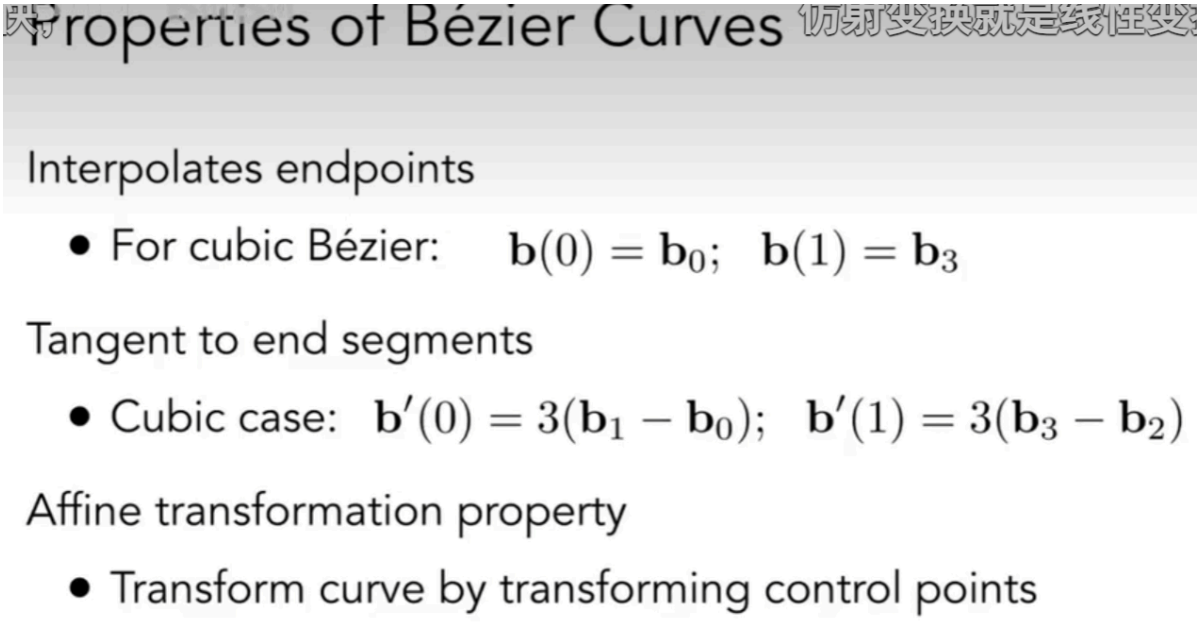
$$\mathbf{b}_0^1(t) = (1-t)\mathbf{b}_0 + t\mathbf{b}_1$$
$$\mathbf{b}_1^1(t) = (1-t)\mathbf{b}_1 + t\mathbf{b}_2$$

$$\mathbf{b}_0^2(t) = (1-t)\mathbf{b}_0^1 + t\mathbf{b}_1^1$$

$$\mathbf{b}_0^2(t) = (1-t)^2\mathbf{b}_0 + 2t(1-t)\mathbf{b}_1 + t^2\mathbf{b}_2$$

伯恩斯坦多项式

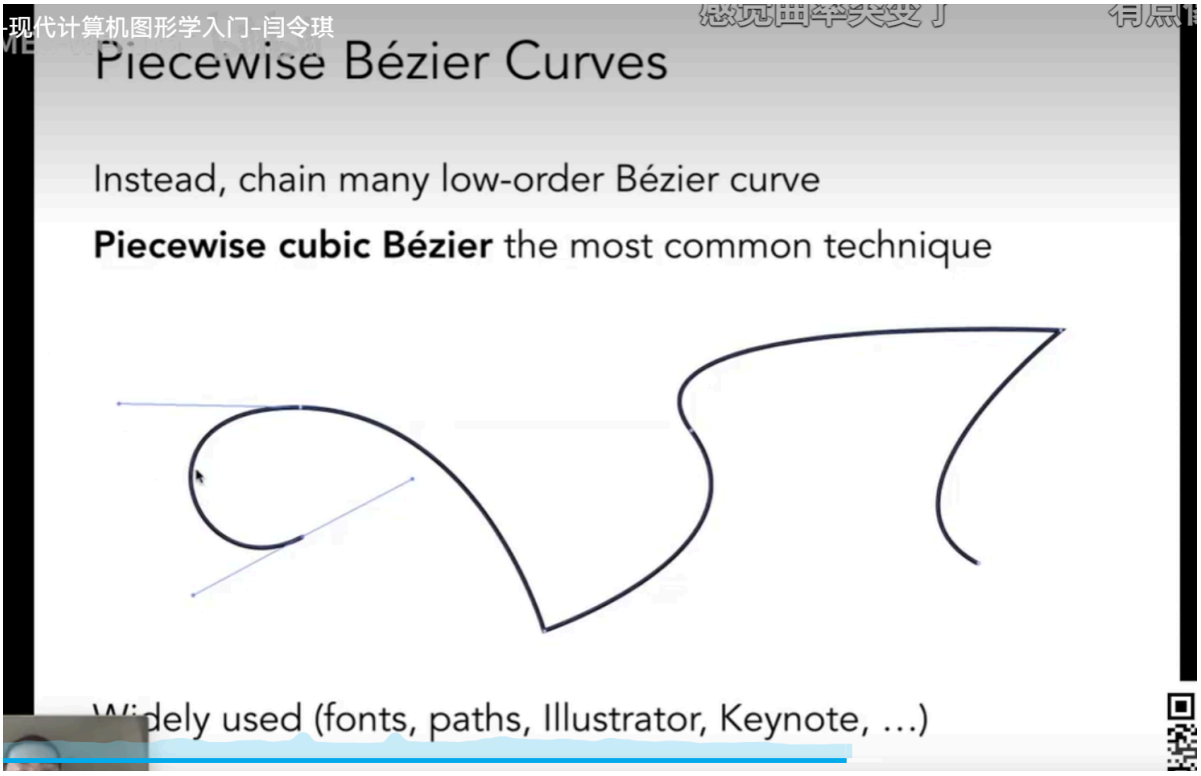


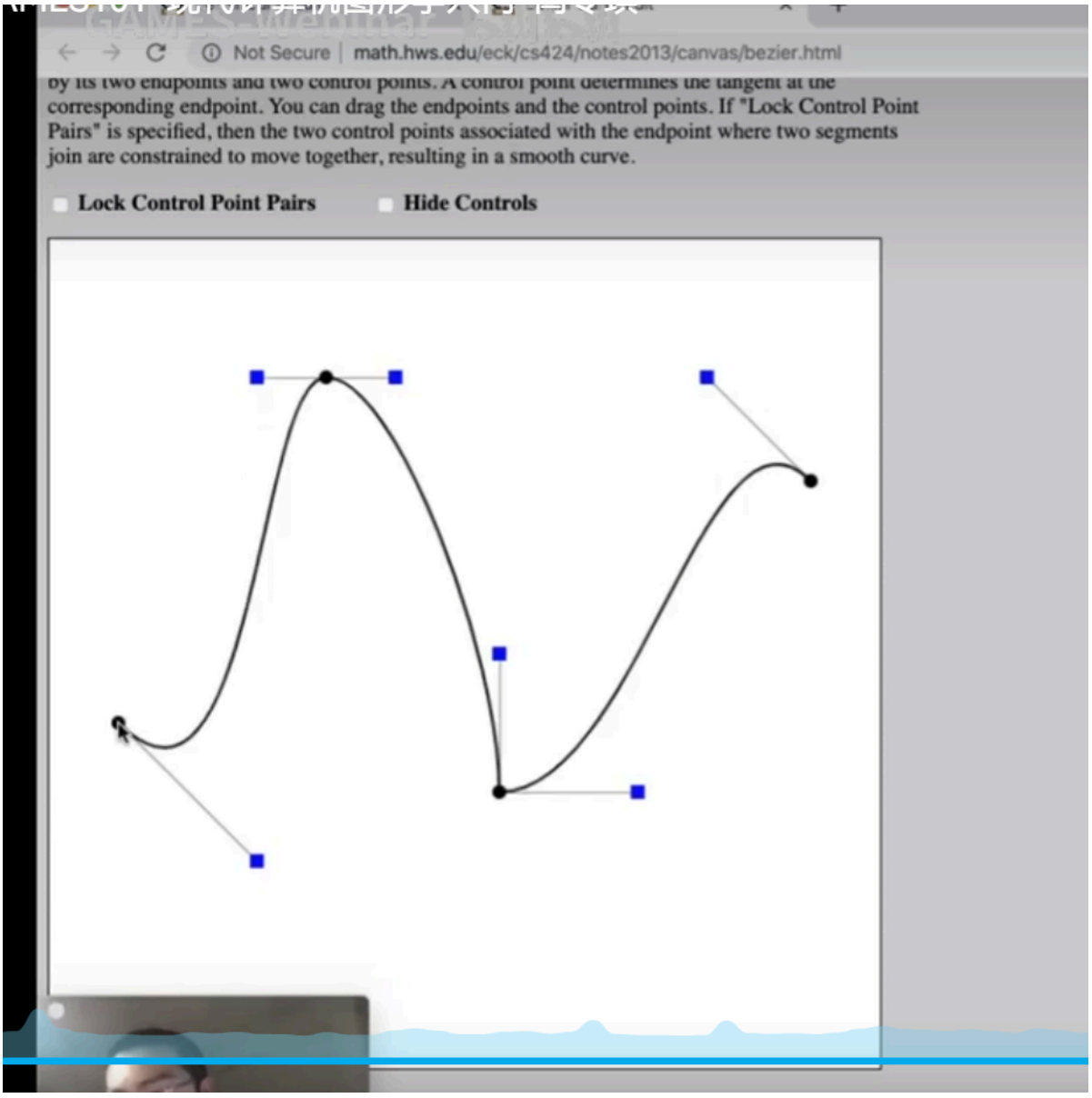


对贝塞尔曲线做仿射变换，可以只对点做仿射变换，然后做贝塞尔曲线。

贝塞尔曲线具有凸包性质，贝塞尔曲线形成一定在控制点范围内（凸包内）。

多段四控制点贝塞尔曲线：钢笔工具



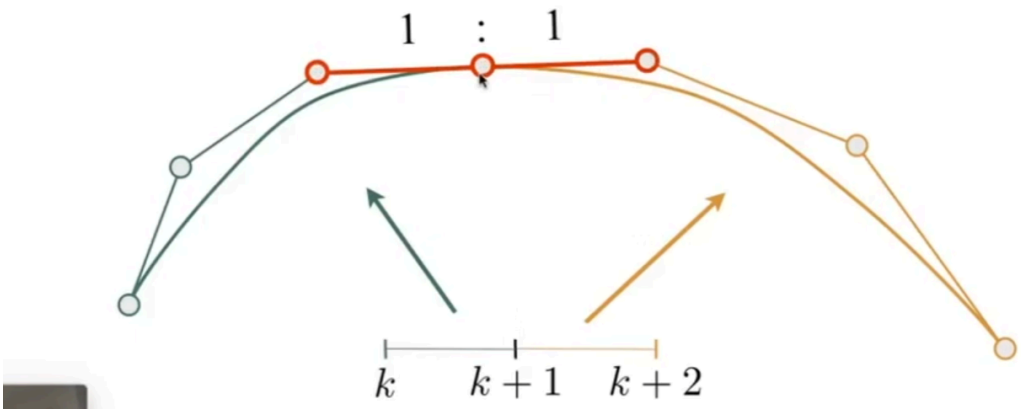


第一段终点=第二段起点:C0连续

当公共点旁的两个控制点共线且（距离）大小相等，C1连续

Piecewise Bézier Curve – Continuity

C^1 continuity: $\mathbf{a}_n = \mathbf{b}_0 = \frac{1}{2}(\mathbf{a}_{n-1} + \mathbf{b}_1)$

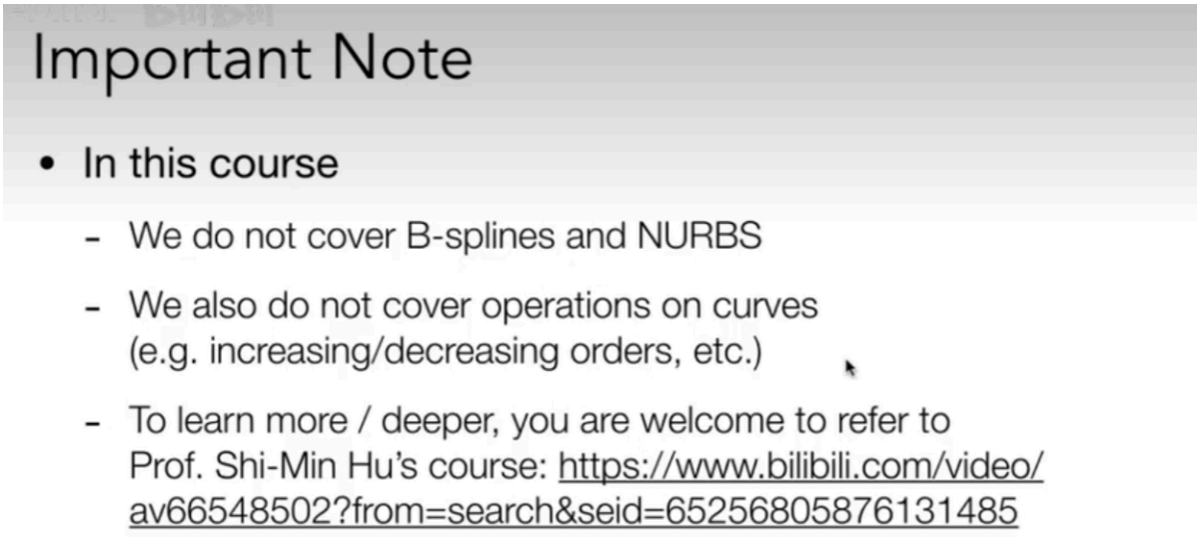


spline曲线：样条曲线：一个连续的线由一系列控制点控制。可控曲线就是样条。

伯恩斯坦多项式是基函数。

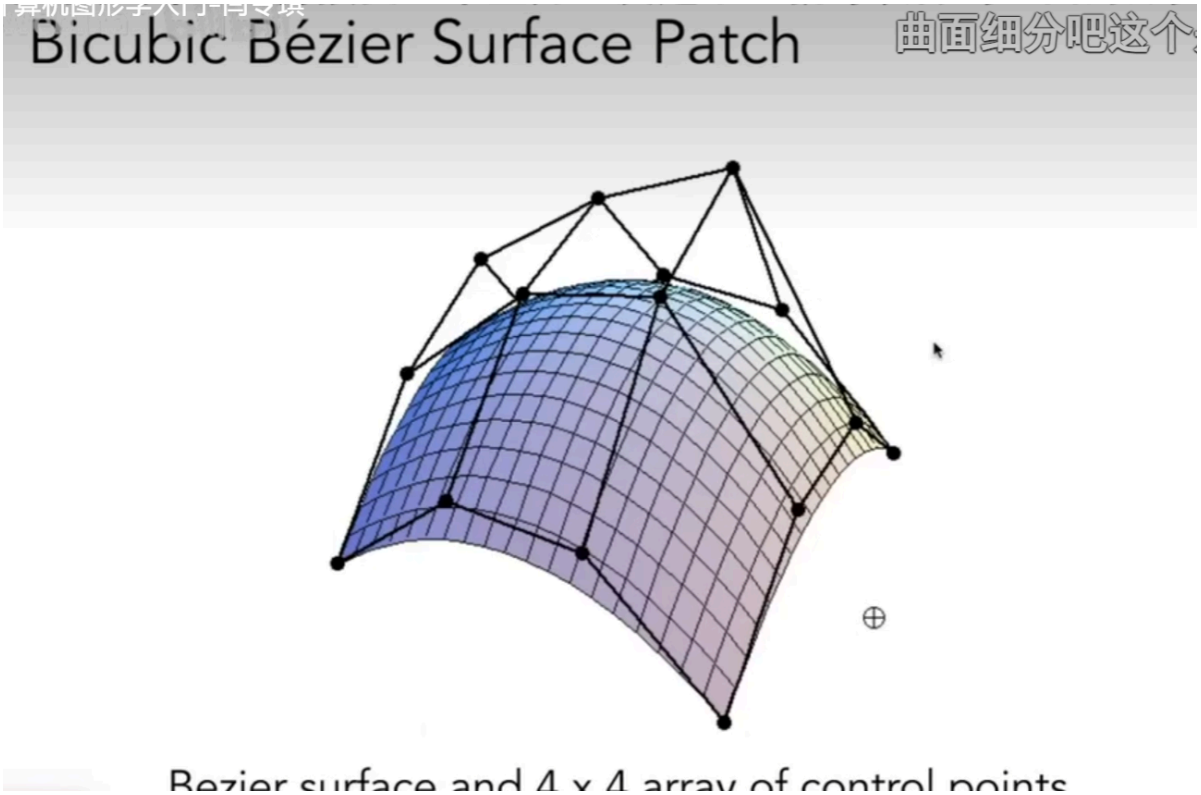
B样条：控制点影响的线条的范围可控。

（胡事民 曲线曲面）



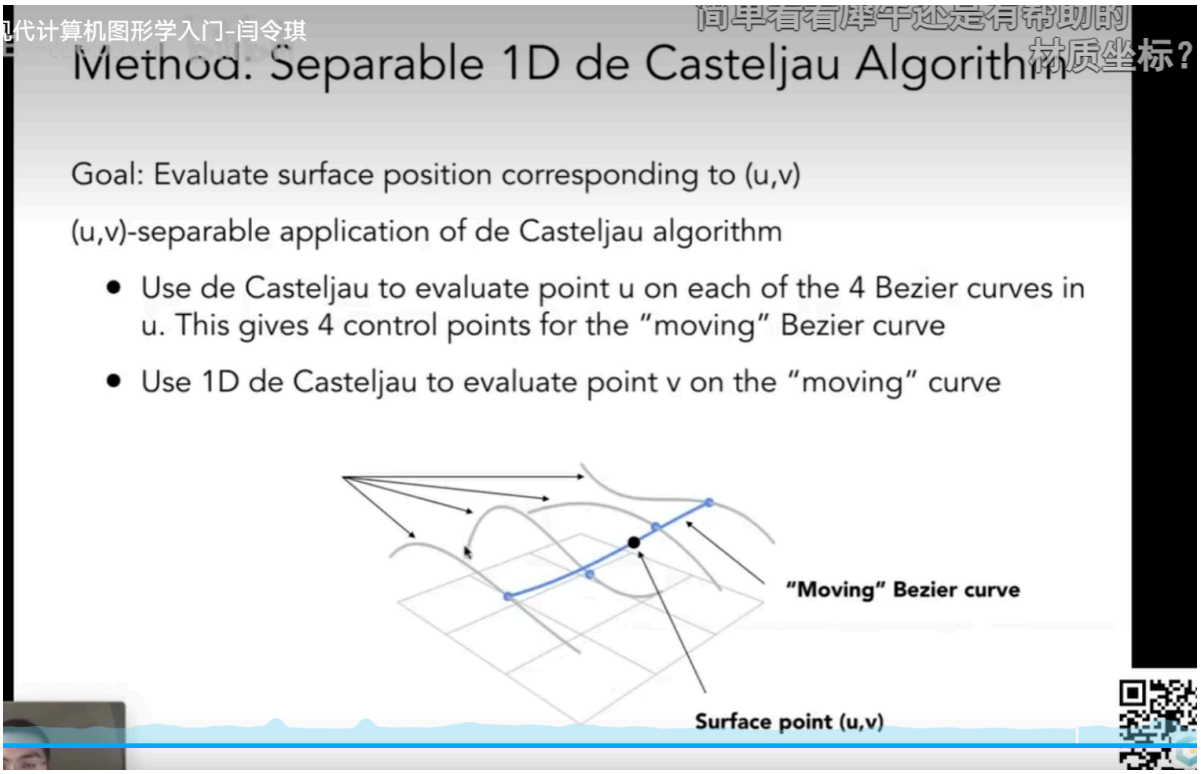
由贝塞尔曲线得贝塞尔曲面

4X4



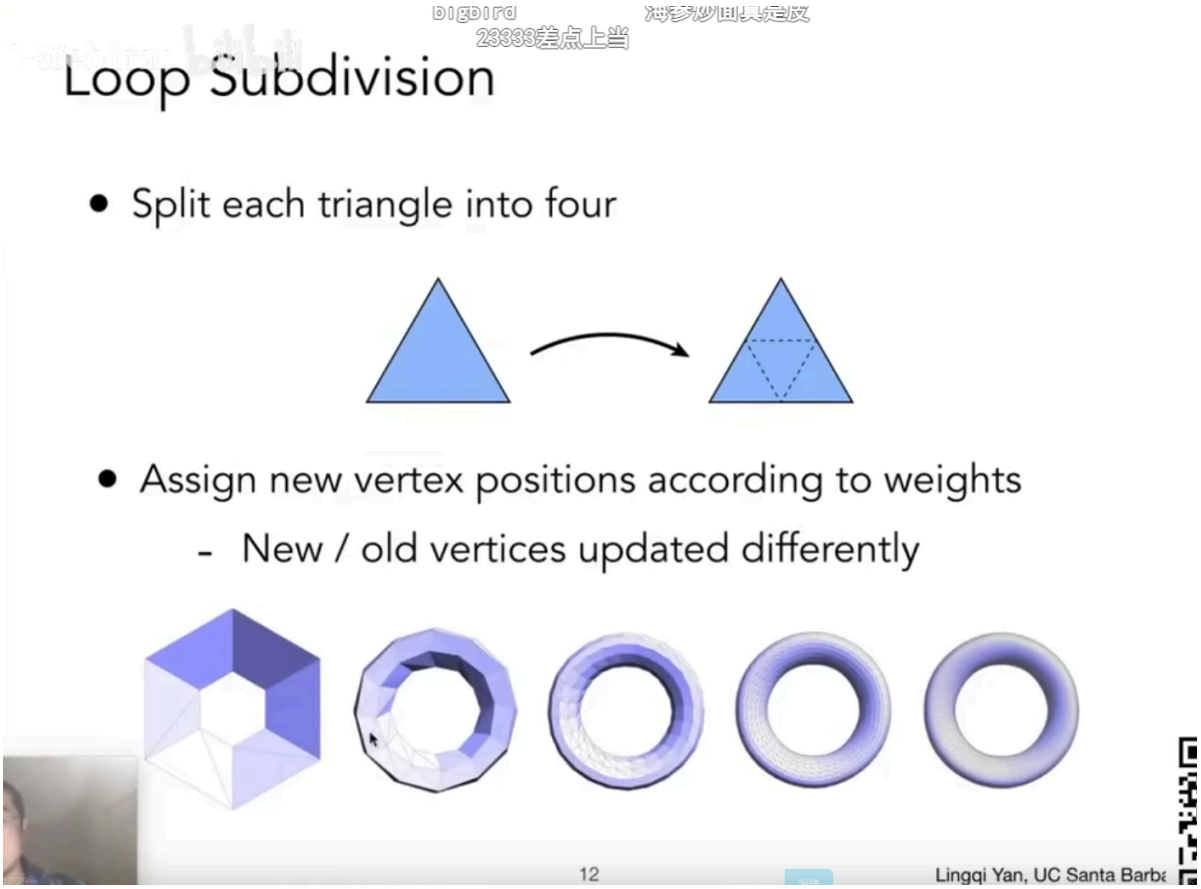
双线性插值

需要两个控制t，（u,v）u得四条贝塞尔曲线在某一个位置上的四个点，v得这四个点所形成的贝塞尔曲线的某一段位置，这个点的坐标就用（u,v）表示。

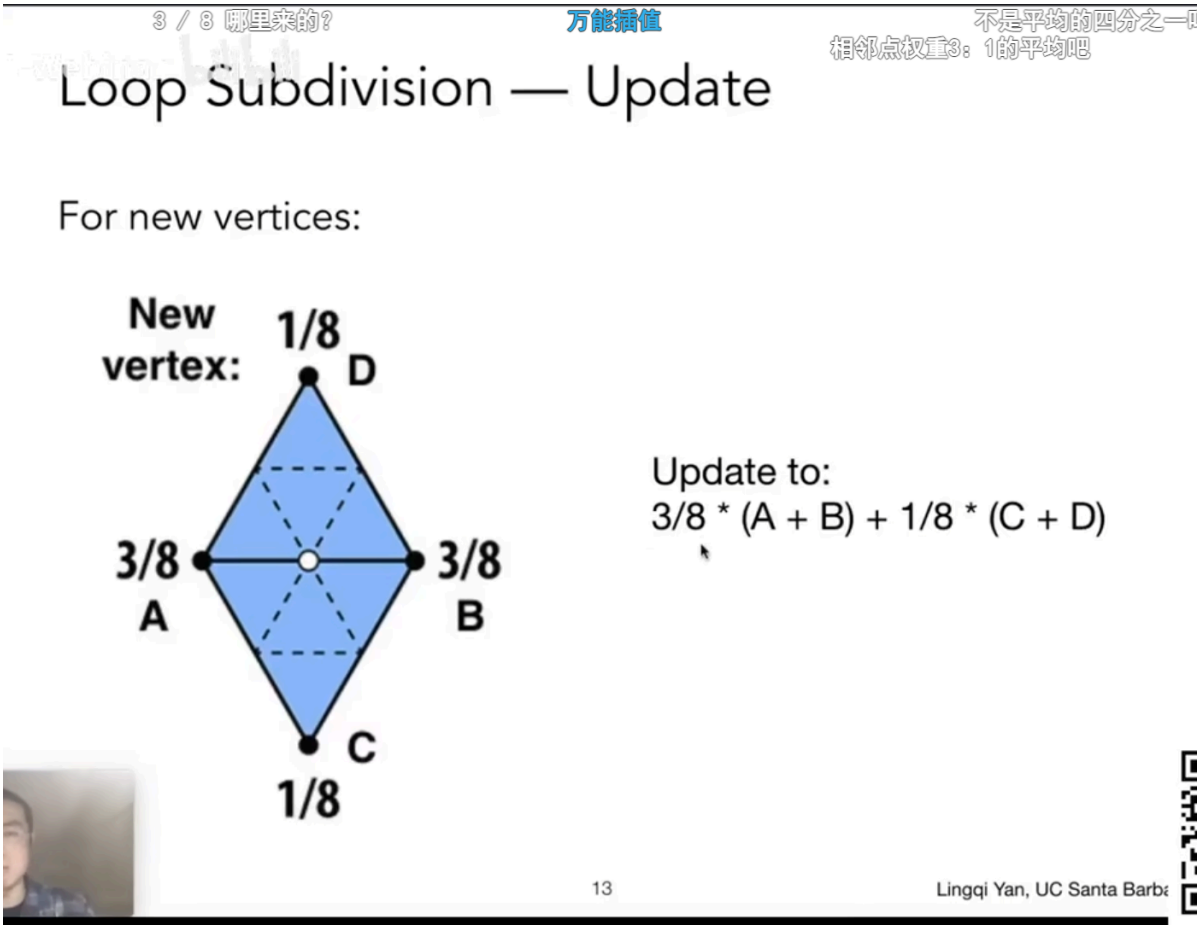


曲面细分

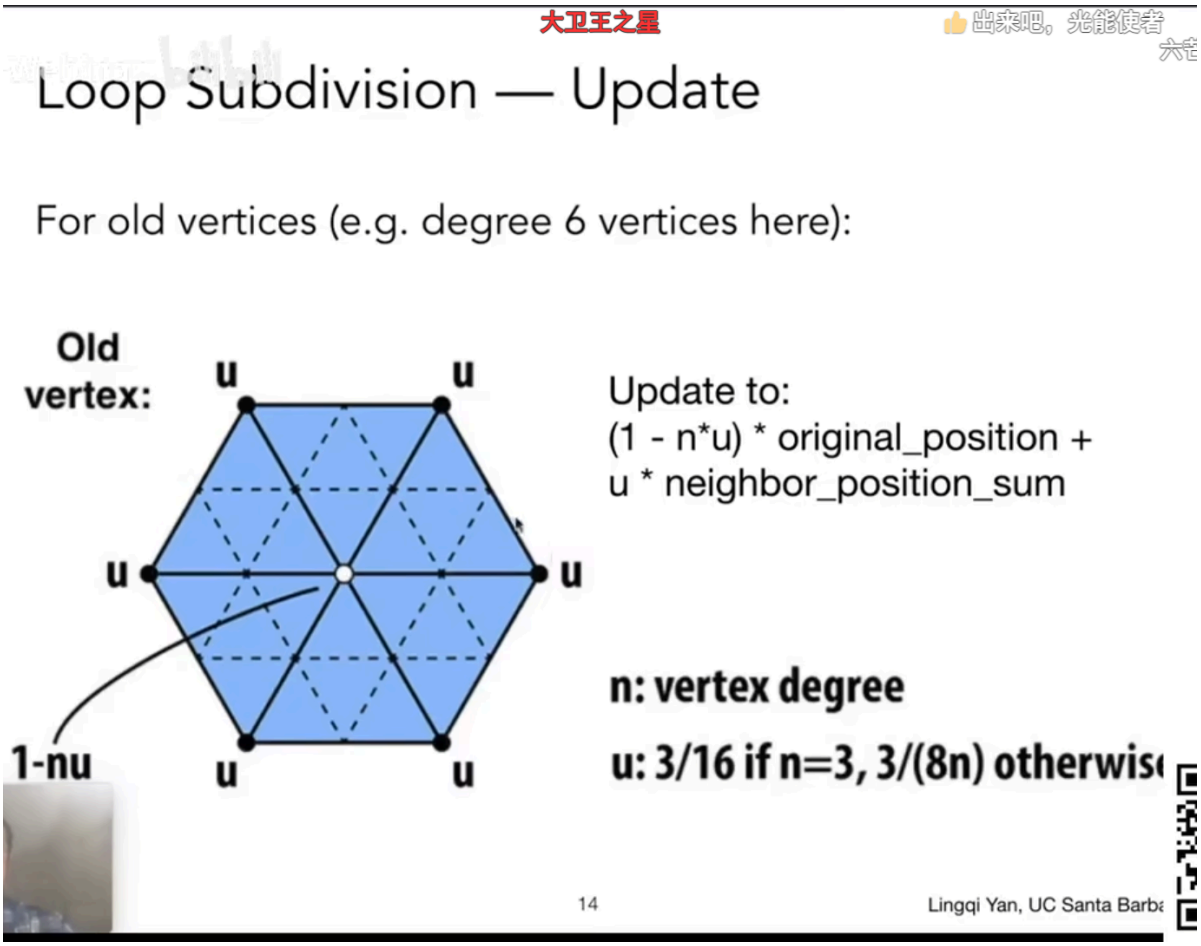
Loop细分（人叫Loop）



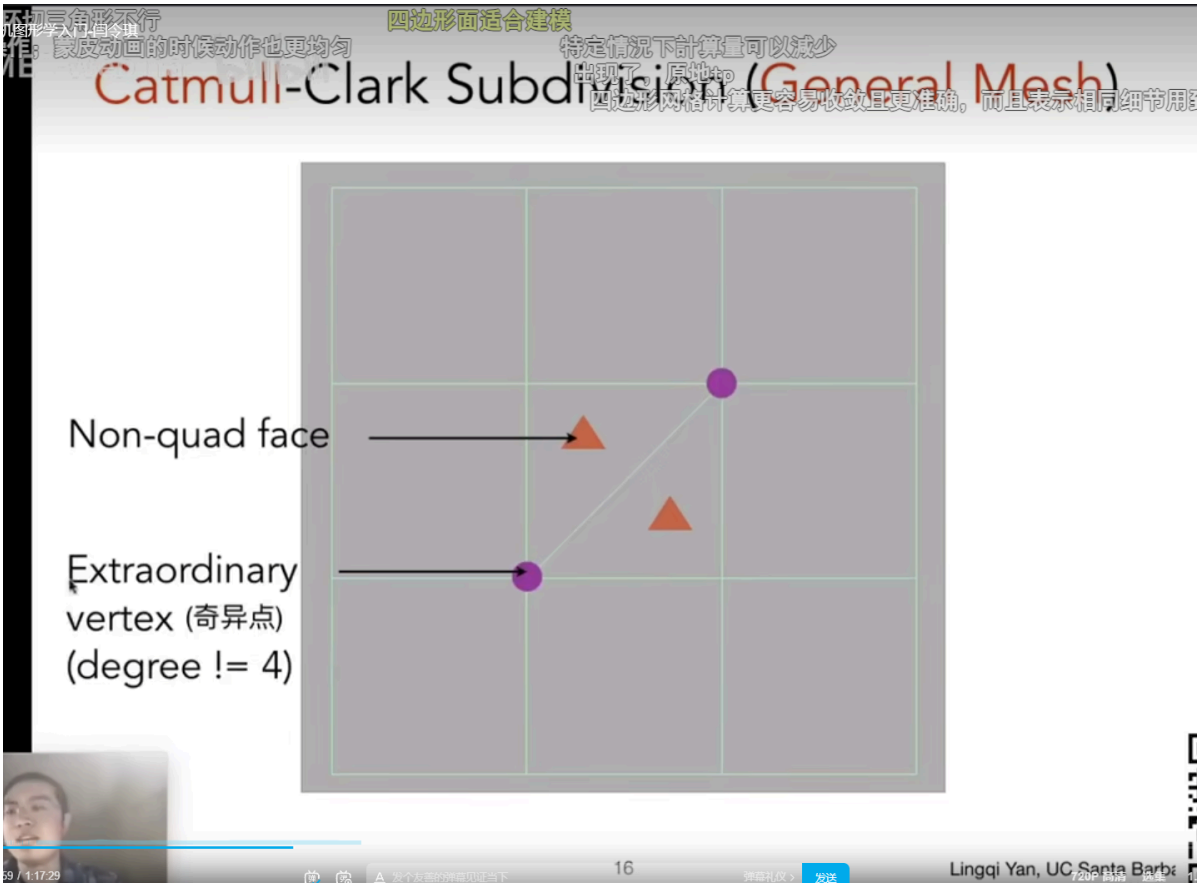
对任意两个三角形公共边的中心点，考虑这俩个三角形的四个顶点，做加权平均。



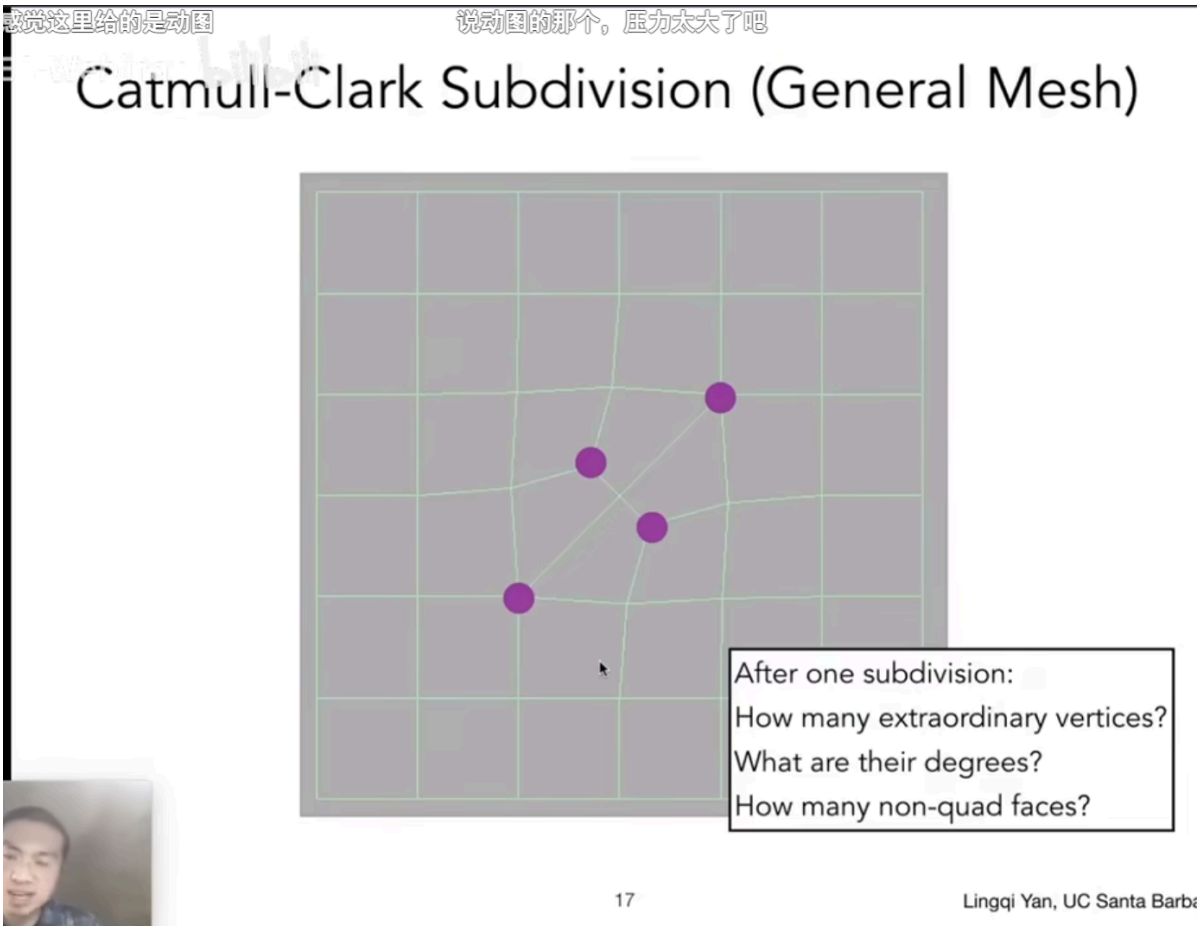
对于原有的某个顶点，定义这个顶点的度（与之相邻顶点数）与权重，其中其他顶点的权重为u，该顶点的权重为 `1 - n * u`。



catmull-clark 细分

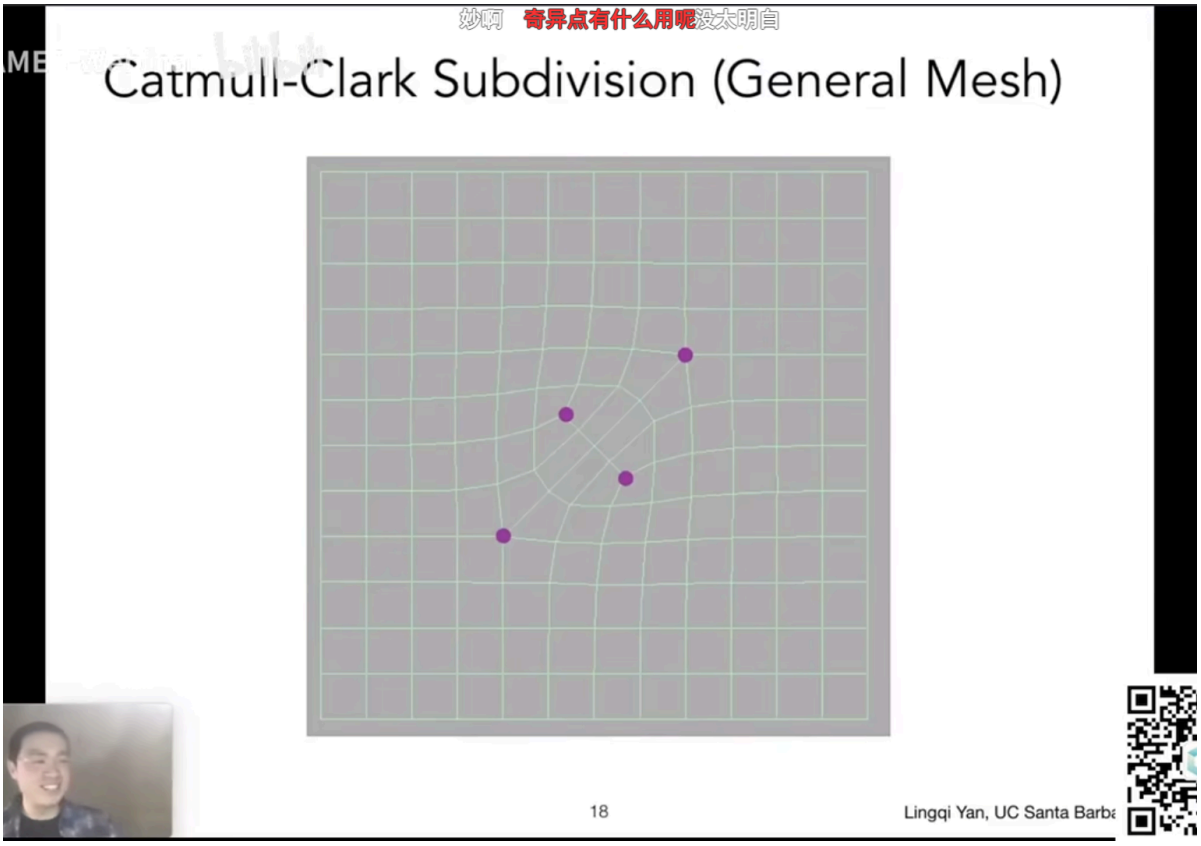


感觉这里给的是动图　　说动图的那个，压力太大了吧

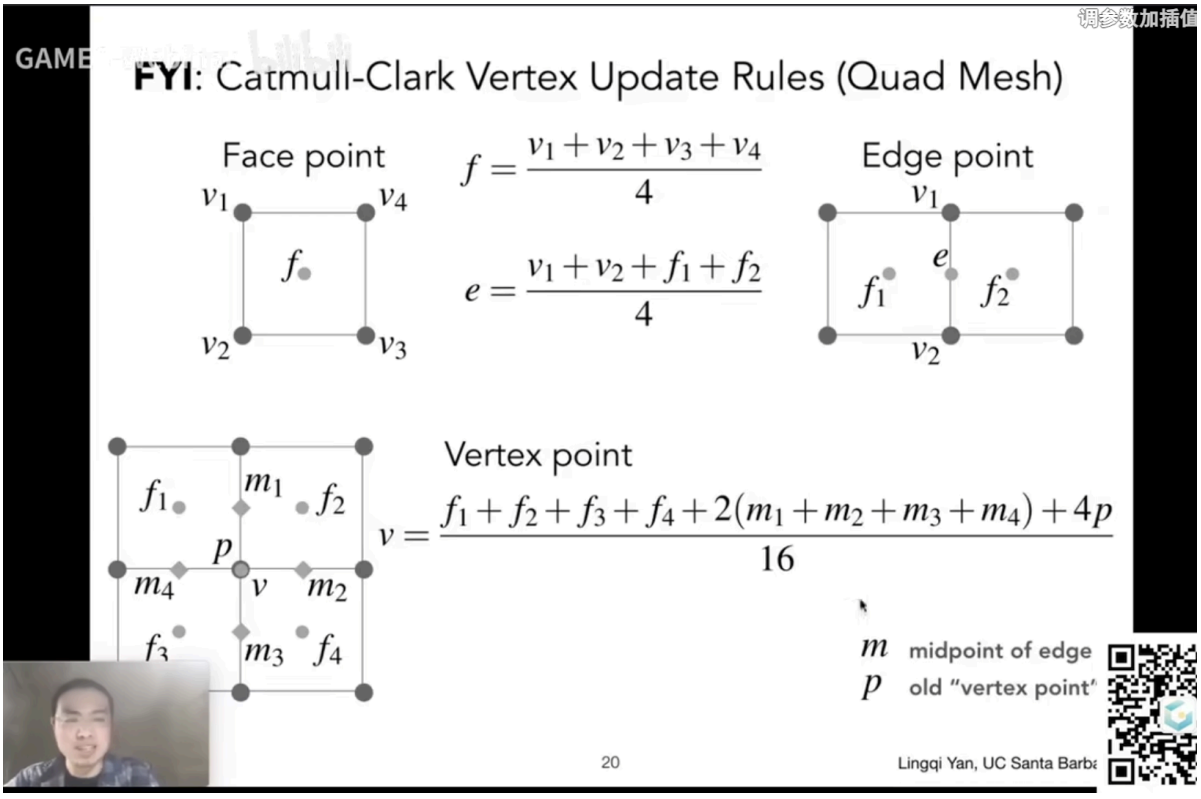


仅第一次细分时会增加奇异点，之后全部为四边形，奇异点不会增加。

妙啊　奇异点有什么用呢没太明白

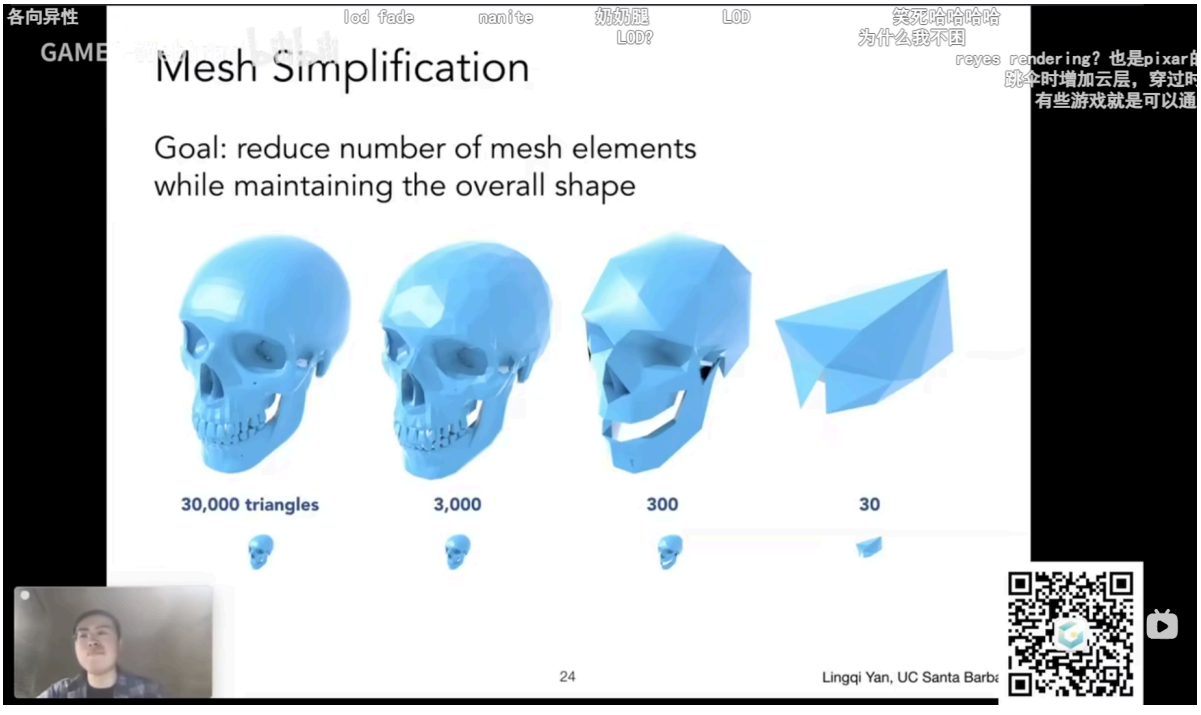


更新公式：



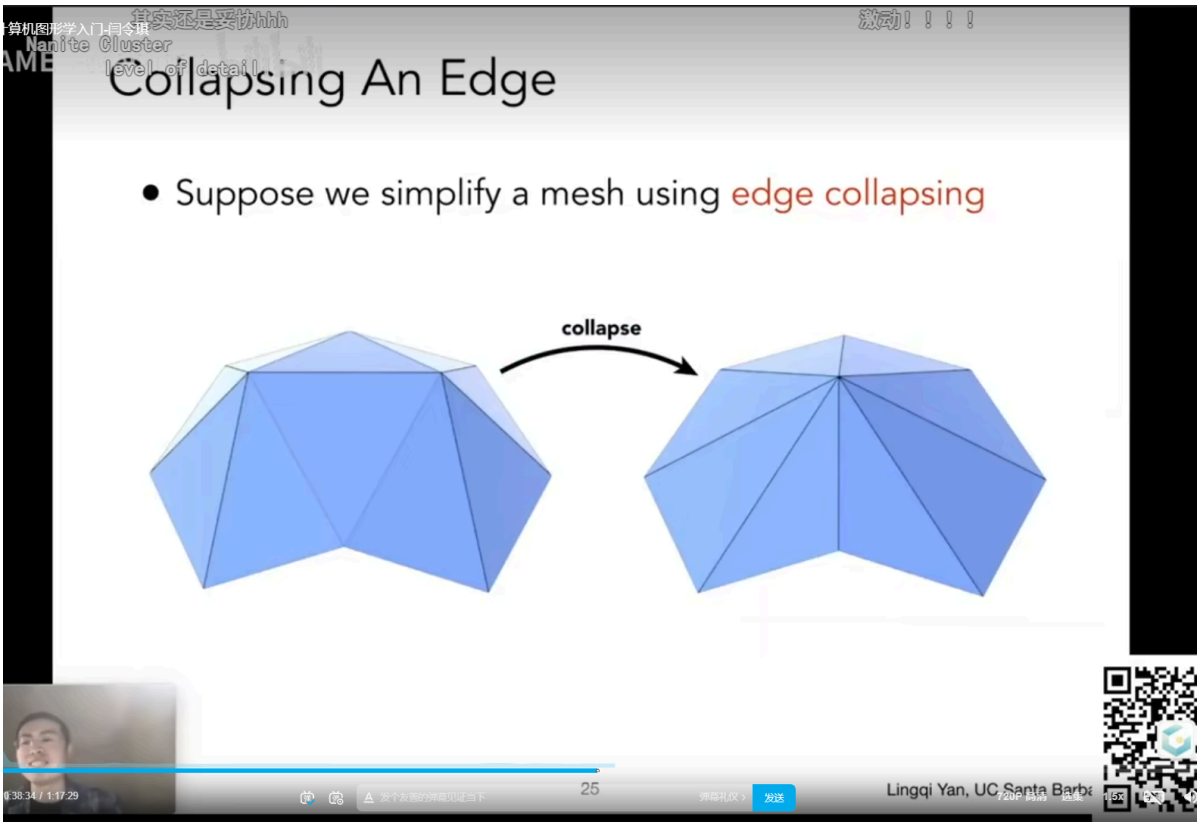
loop 细分只能用在三角形面，而cc细分可用于多边形面

曲面简化

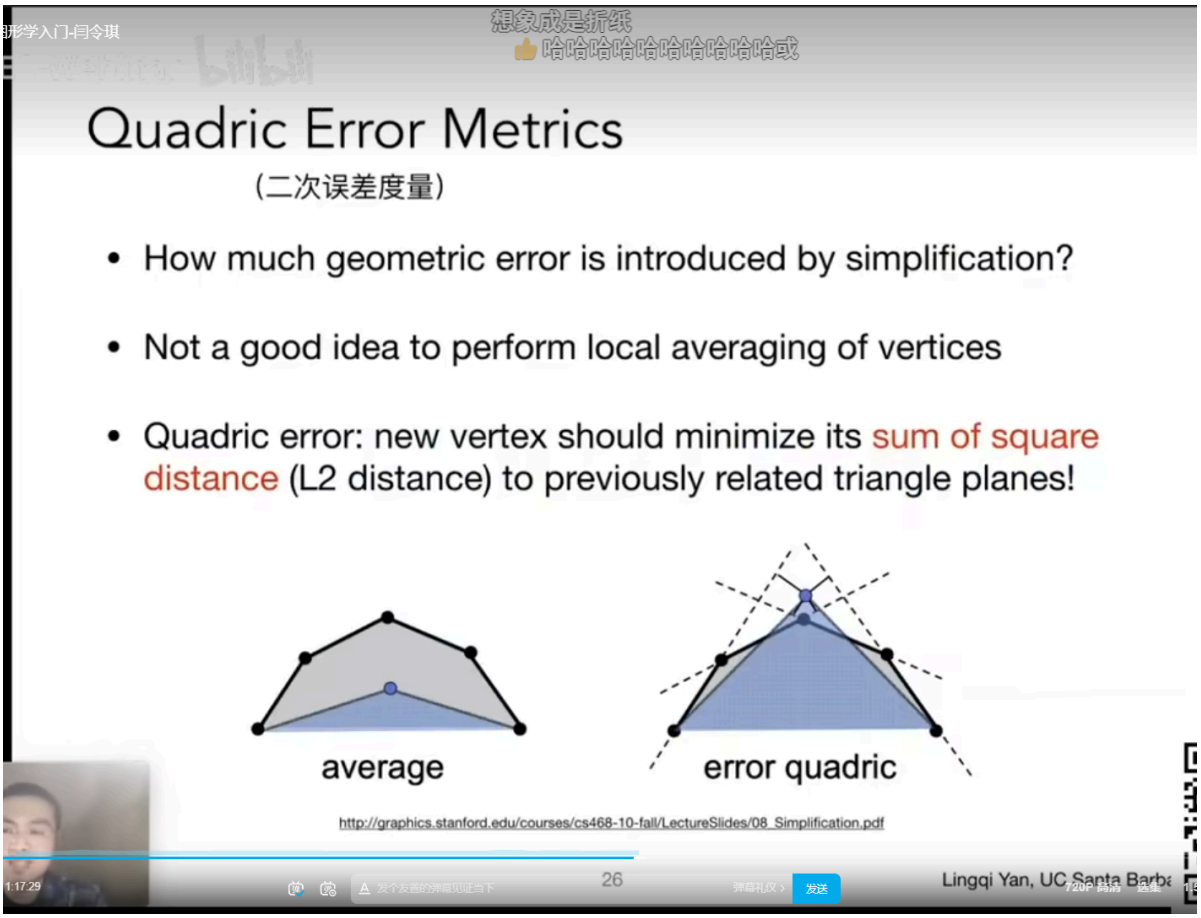


ue5 的 nanite

边坍缩

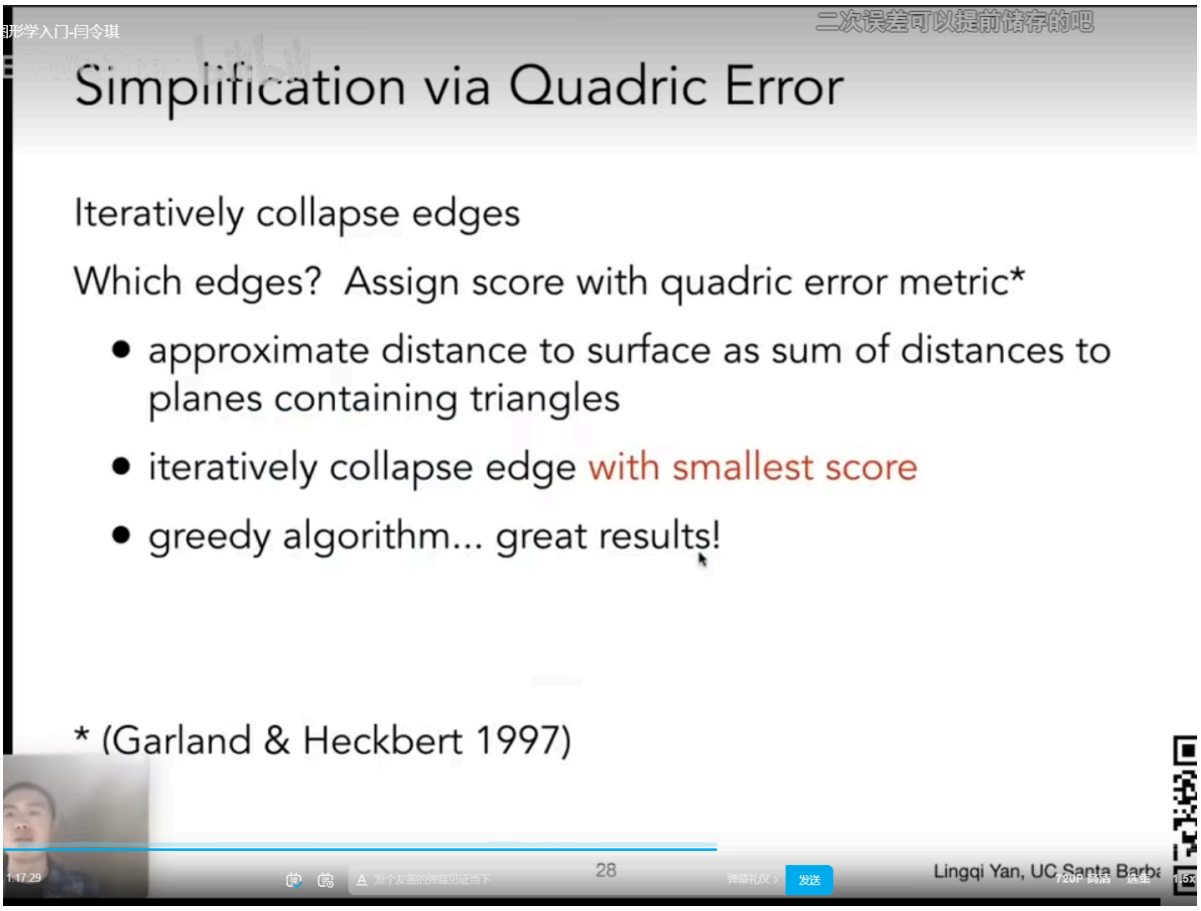


使用二次误差度量确定需要坍缩的边（重要性不高的边）



二次误差度量使用平方进行度量，寻找一个点，让它跟其余相关联面的距离的平方和最小。

对模型而言，第一次计算所有可行域（对每一条边进行计算）并存储对应面的分数，选取最优后进行坍缩，并记录更新的边（与坍缩形成的顶点相邻的边），



优先队列和堆来存储分数结果。

【可以做一个面化简的示例，使用一个滑块来定化简的面数】

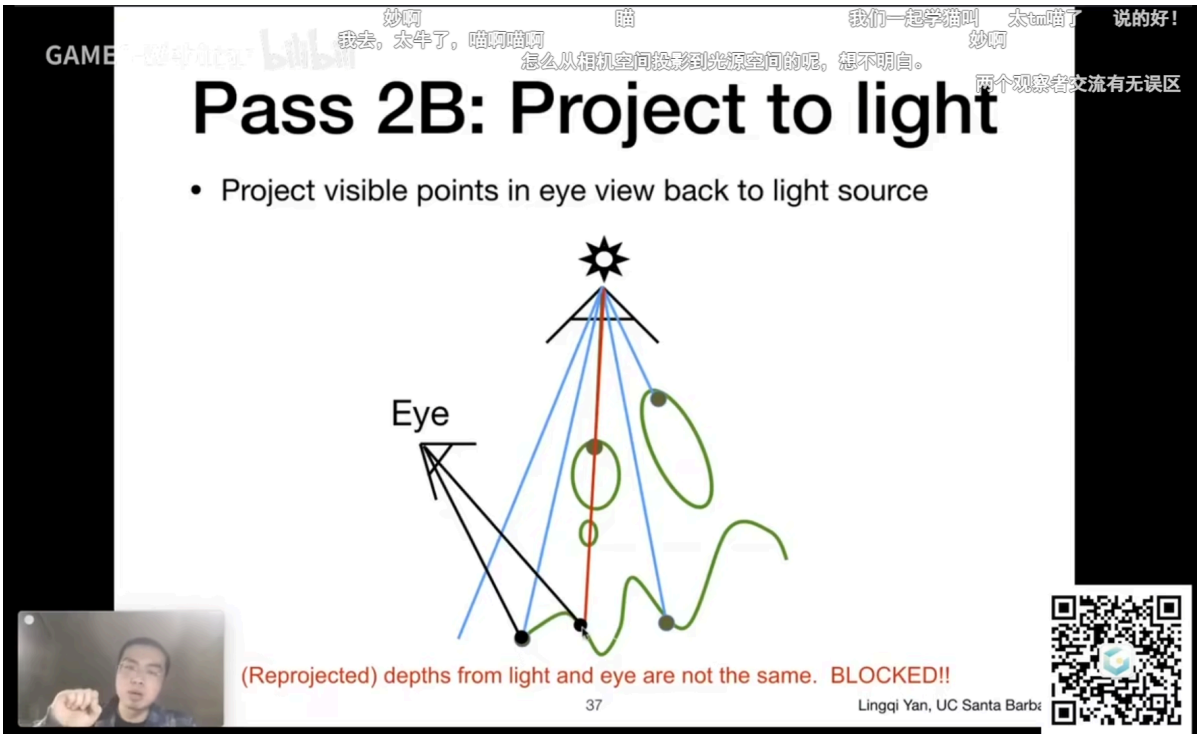
阴影生成

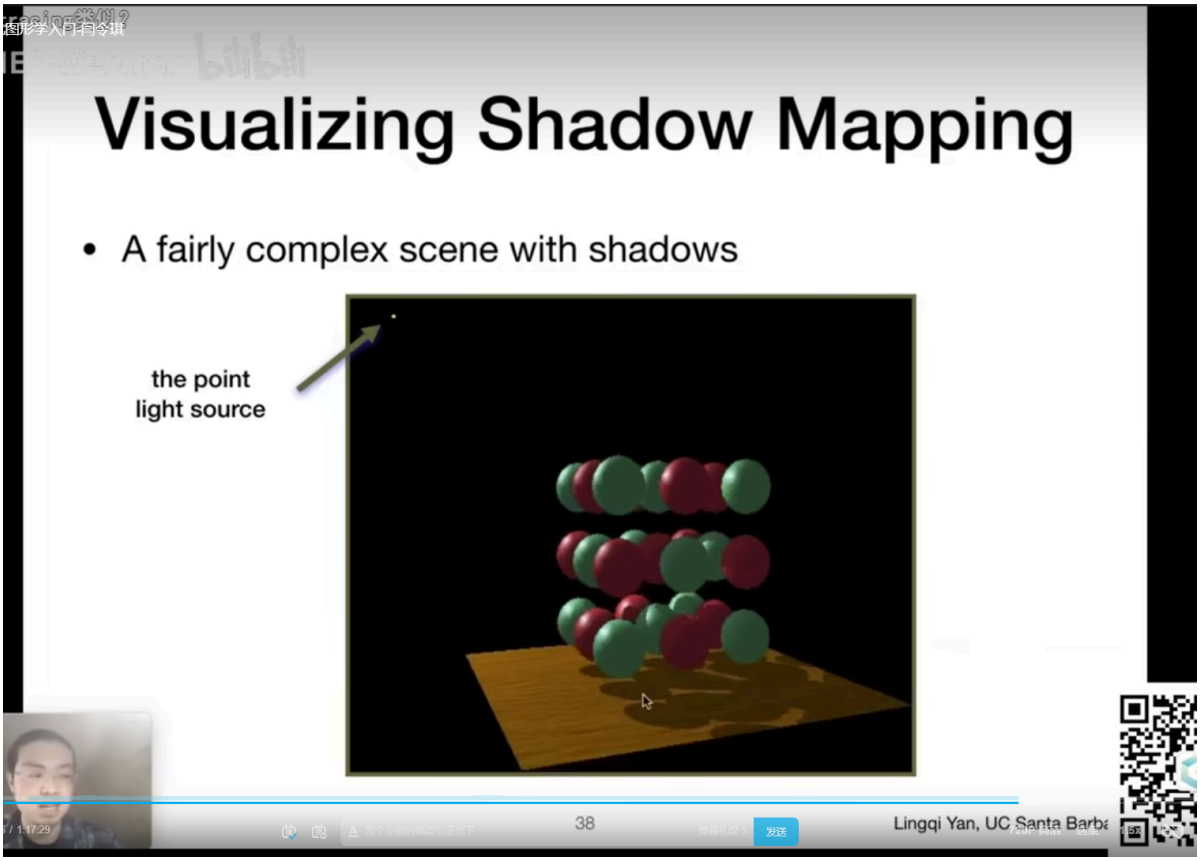
shadow mapping

1.由光源看向物体做投影，成像，计算其深度图；

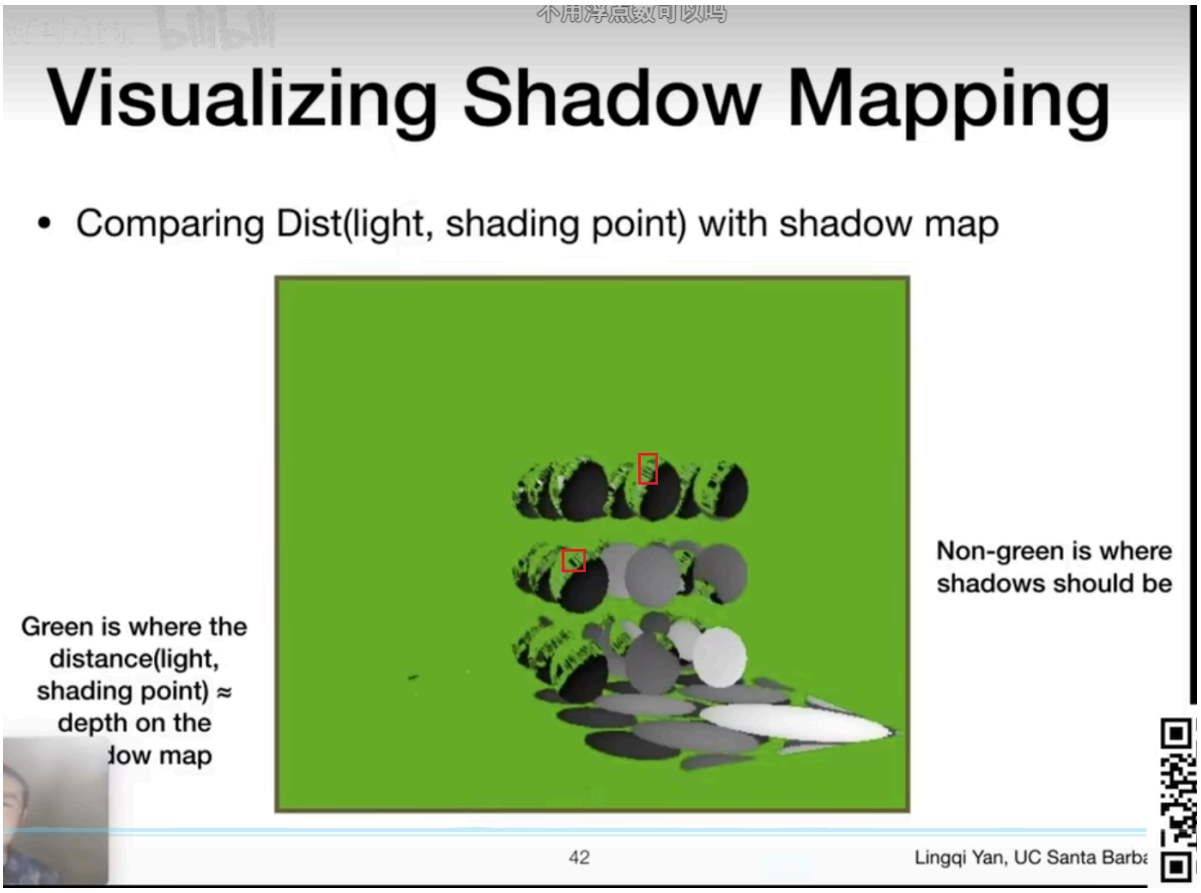
2.由眼看向物体，对可见的物体去找光源的深度图，若映射至的位置的深度值与光源先前记录的深度值一致，则该物体可见，若不是则为阴影。

（相当于问光源看得到点吗）





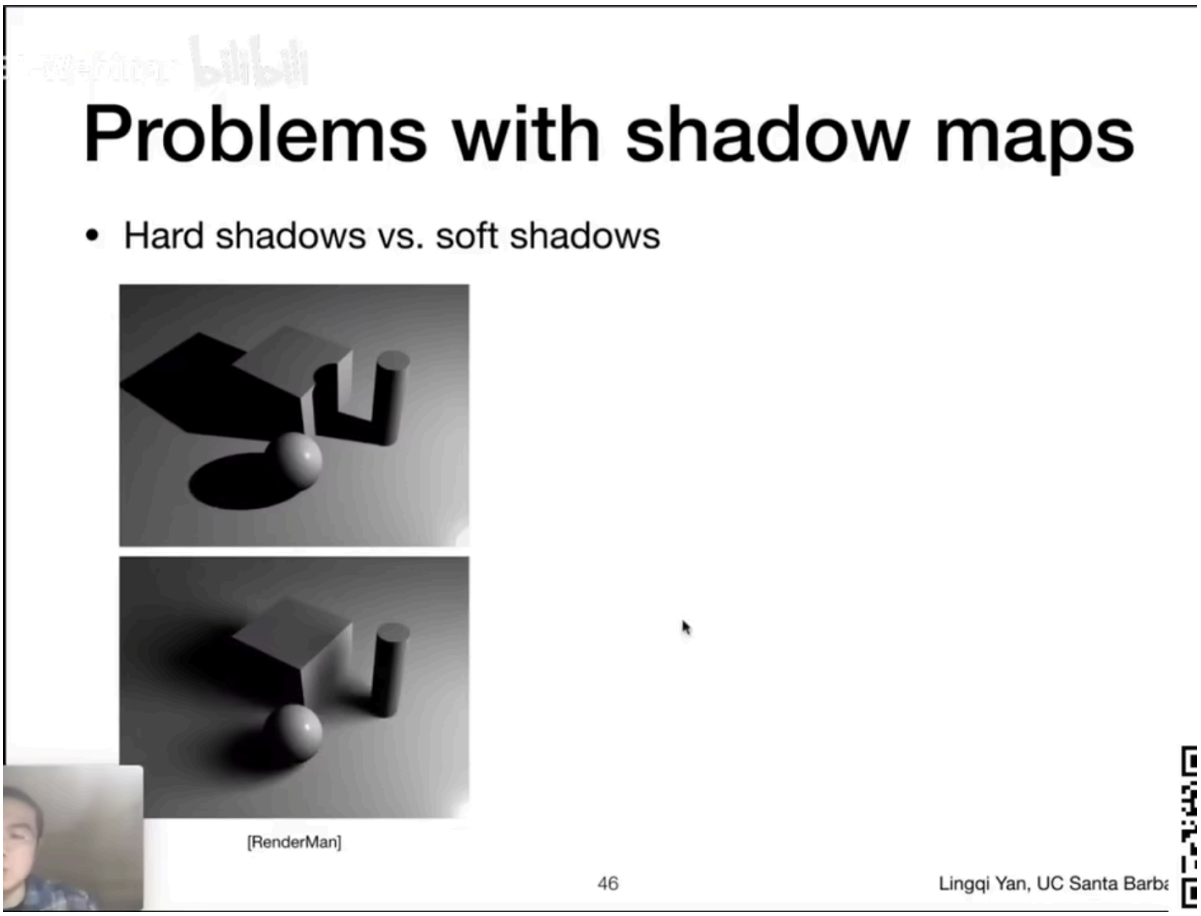
因为shadow mapping是进行相等比较，所以会存在精度问题



引入shadow bias 之类是解决方案。

还存在shadow map 本身分辨率问题，可能存在走样

硬阴影和软阴影（与物理光照相关 可见区 半可见区 不可见区）



Ray Tracing

光栅化是一种很快的近似渲染方法，它不能解决所有的光照问题，这时候需要引入光线追踪

光栅化：实时（real-time） 光线追踪：离线（offline）

光线

沿直线传播，不会发生碰撞（交叉不影响），光路可逆

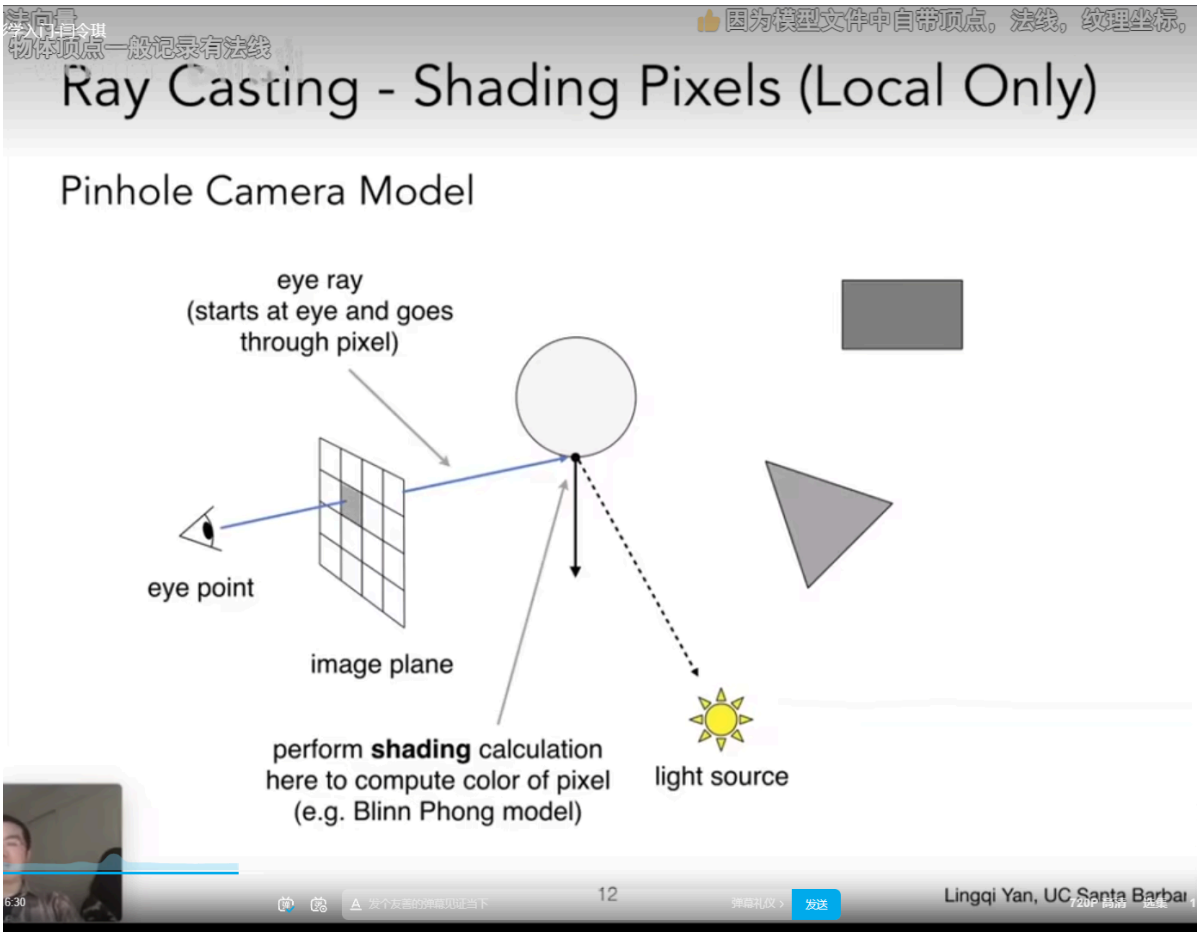
是一个异构的思想
专门的光追单元，不是单纯的gpu。
RTX是用的专用光追单元计算的，和GPU核心的计算效率没有关系
什么是RTX啊
有的天体要求光线是曲线
即使粒子，也是波
compute shader好像也可以实现光

Light Rays

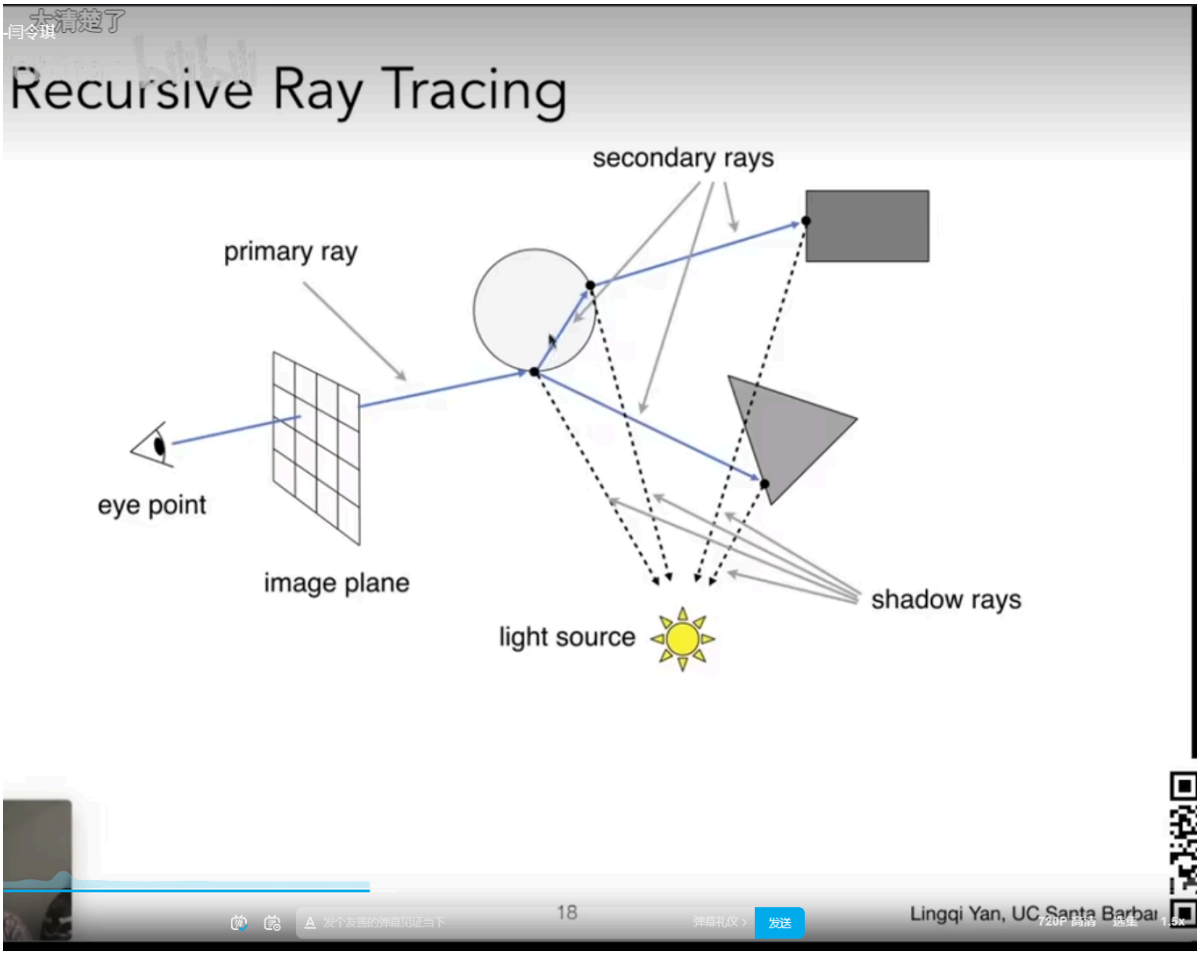
Three ideas about light rays

1. Light travels in straight lines (though this is wrong)
2. Light rays do not “collide” with each other if they cross (though this is still wrong)
3. Light rays travel from the light sources to the eye (but the physics is invariant under path reversal - reciprocity).

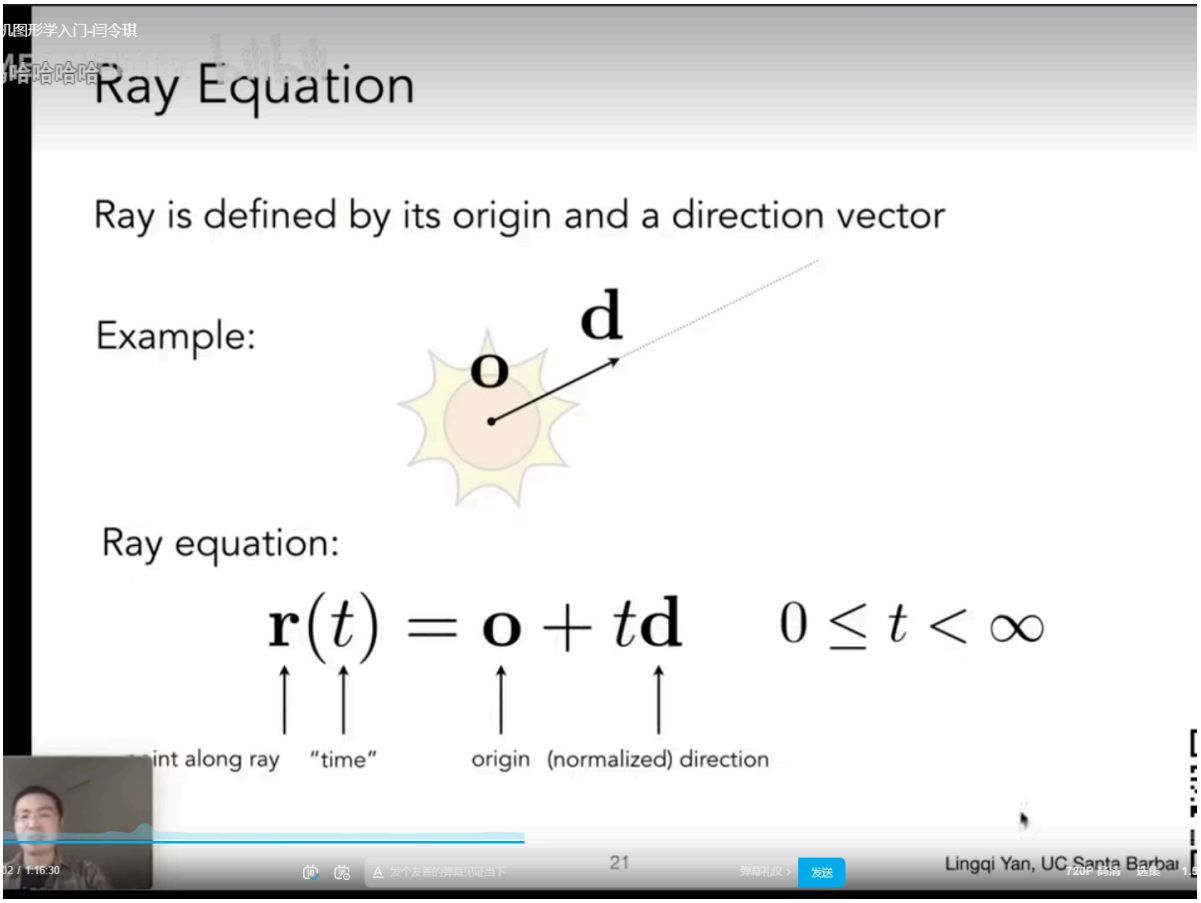
从眼睛开始，针对屏幕上的每一个点打出一条射线(eye ray)，若射线与最近的物体相交，则对物体的该点与光源相连，判断是否对光源可见，若可见则该点可见生成一条光路



多反射情况Recursive Ray Tracing



定义一条光线



光线和球的交点（t时间）

解析几何？？？

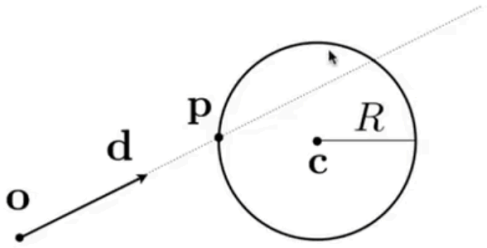
Ray Intersection With Sphere

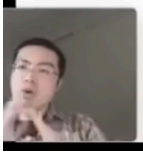
Ray: $\mathbf{r}(t) = \mathbf{o} + t\,\mathbf{d},\ 0 \le t < \infty$

Sphere: $\mathbf{p} : (\mathbf{p} - \mathbf{c})^2 - R^2 = 0$

What is an intersection?

The intersection p must satisfy both ray equation and sphere equation



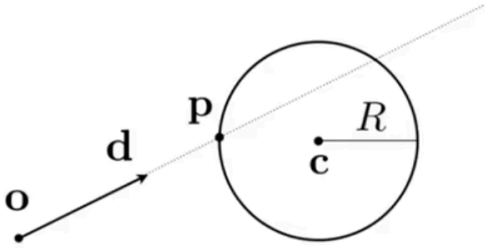


△ 得饵它～ 初中教向量？？ 死去的记忆复活了 公式不难，但不知道为什么这里用它

Ray Intersection With Sphere

Solve for intersection:

$$(\mathbf{o} + t\,\mathbf{d} - \mathbf{c})^2 - R^2 = 0$$



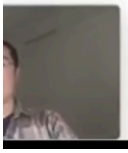
$$a\,t^2 + b\,t + c = 0, \text{ where}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2(\mathbf{o} - \mathbf{c}) \cdot \mathbf{d}$$

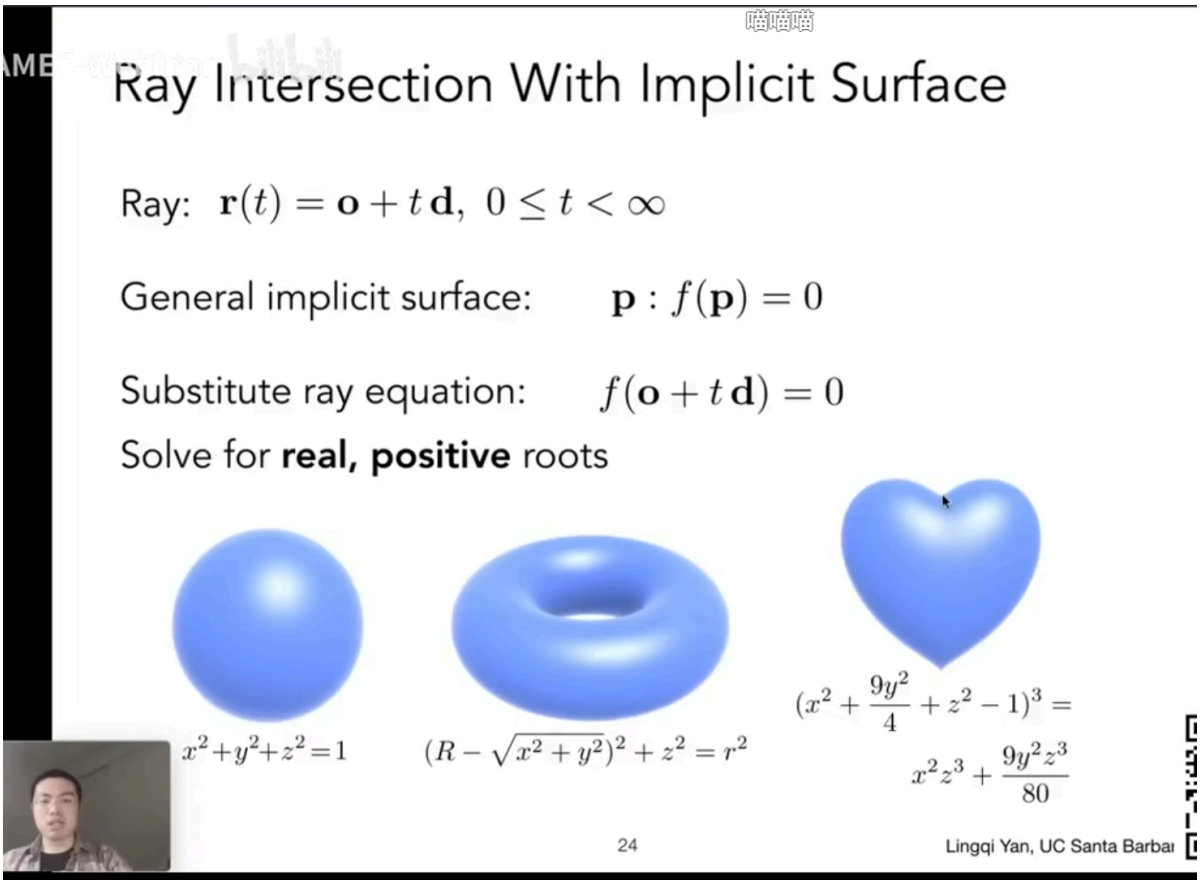
$$c = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



t>=0

与隐式表面的公式联立，用数值的方法可很快解出。



对于任意一个封闭物体，在空间上任意点一个点，若该点引出的射线与物体相交的点数为奇数，该点在物体内，若为偶数，则在物体外。

线与三角形求交

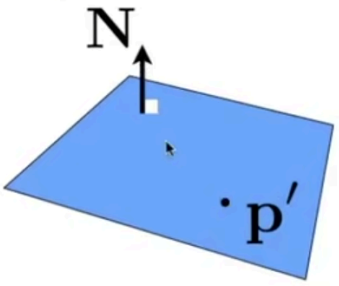
线与平面求交

平面内任意一点，与给定点的连线和这个平面的法向量点乘值为0

Plane Equation

Plane is defined by normal vector and a point on plane

Example:



Plane Equation (if p satisfies it, then p is on the plane):

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0 \qquad ax + by + cz + d = 0$$

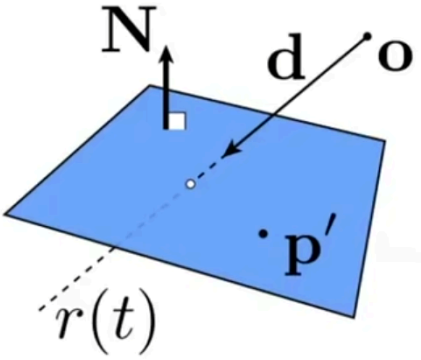
Ray Intersection With Plane

Ray equation:

$$\mathbf{r}(t) = \mathbf{o} + t\,\mathbf{d},\ 0 \le t < \infty$$

Plane equation:

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0$$



Solve for intersection

Set $\mathbf{p} = \mathbf{r}(t)$ and solve for t

$$(\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = (\mathbf{o} + t\,\mathbf{d} - \mathbf{p}') \cdot \mathbf{N} = 0$$

$$t = \frac{(\mathbf{p}' - \mathbf{o}) \cdot \mathbf{N}}{\mathbf{d} \cdot \mathbf{N}}$$

Check: $0 \le t < \infty$

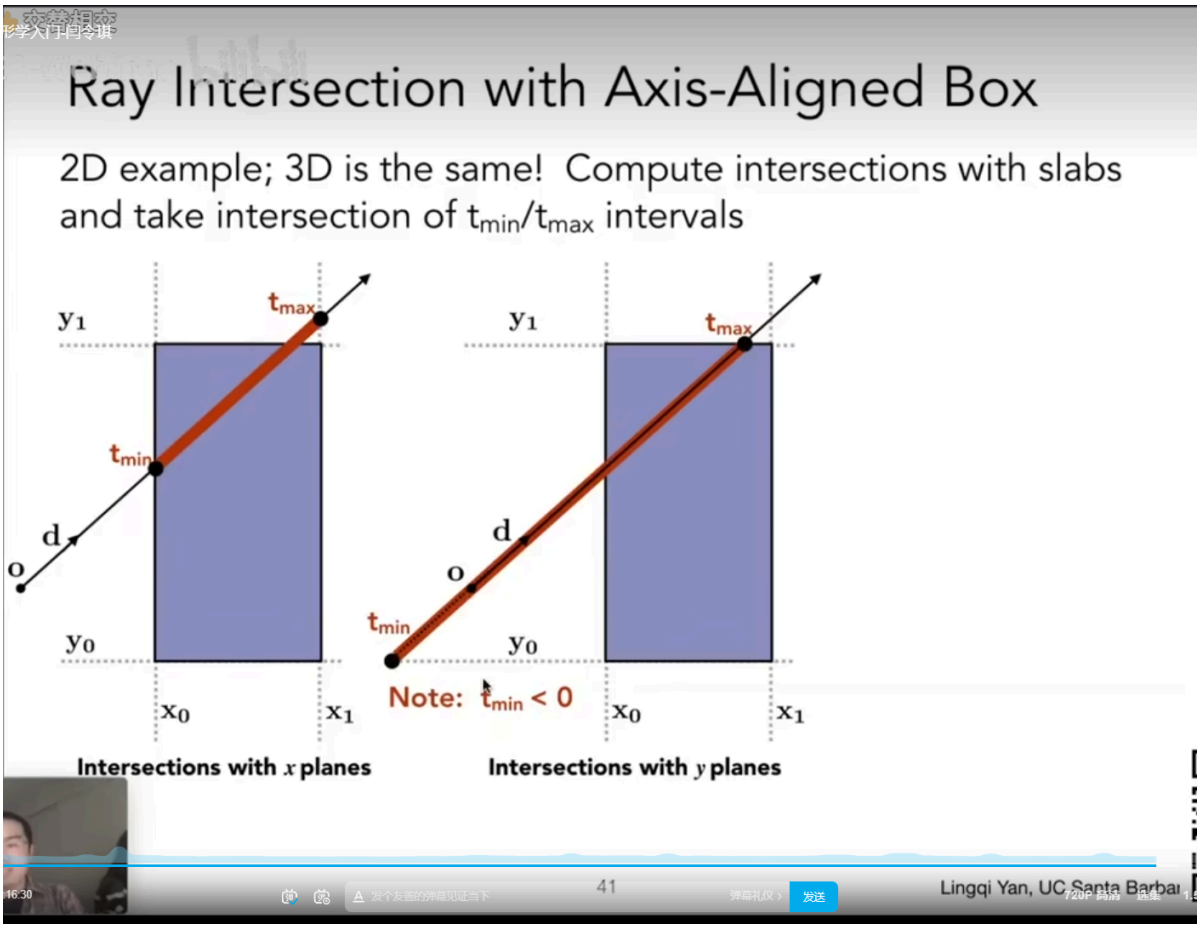
做加速

包围盒

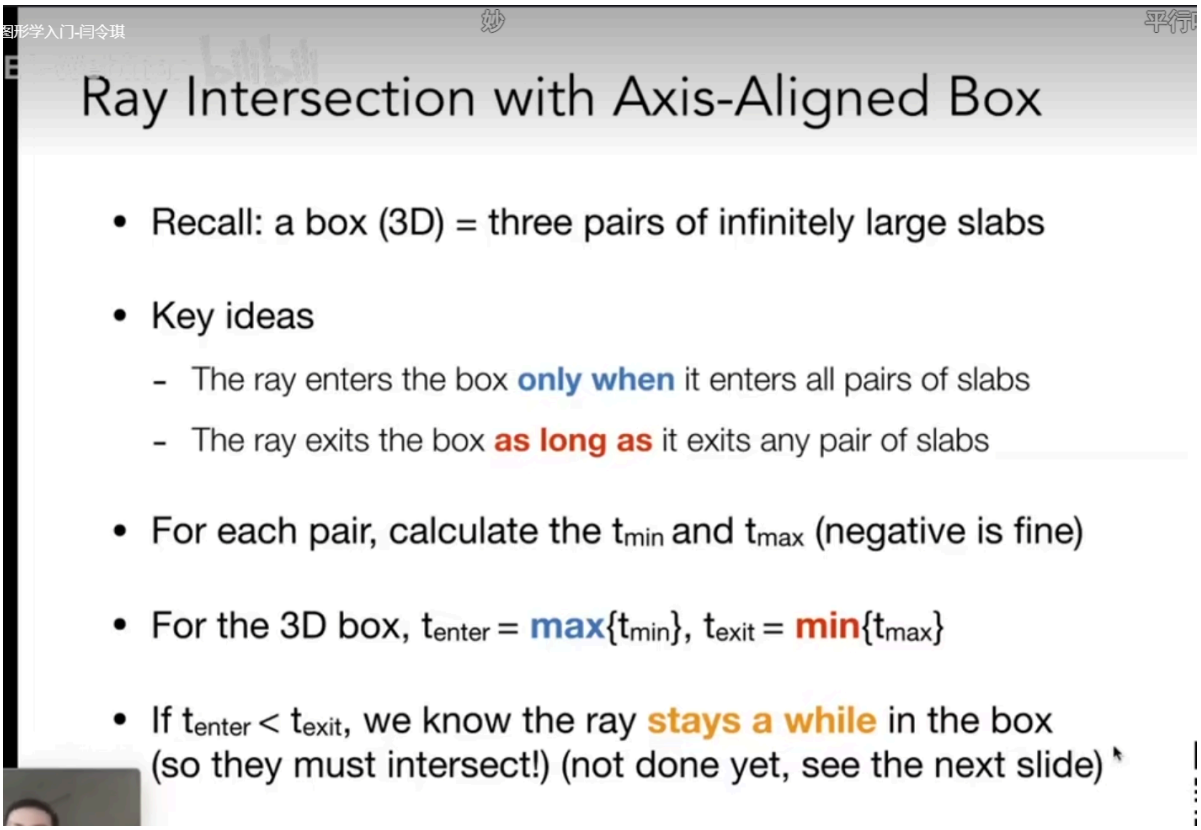
光线要首先碰到包围盒，才能碰到物体。

在2D时，对两对平行线求交点。

得到两个与时间t相关的线段，对这两个线段求交集即可求得交点对应的两个时间t

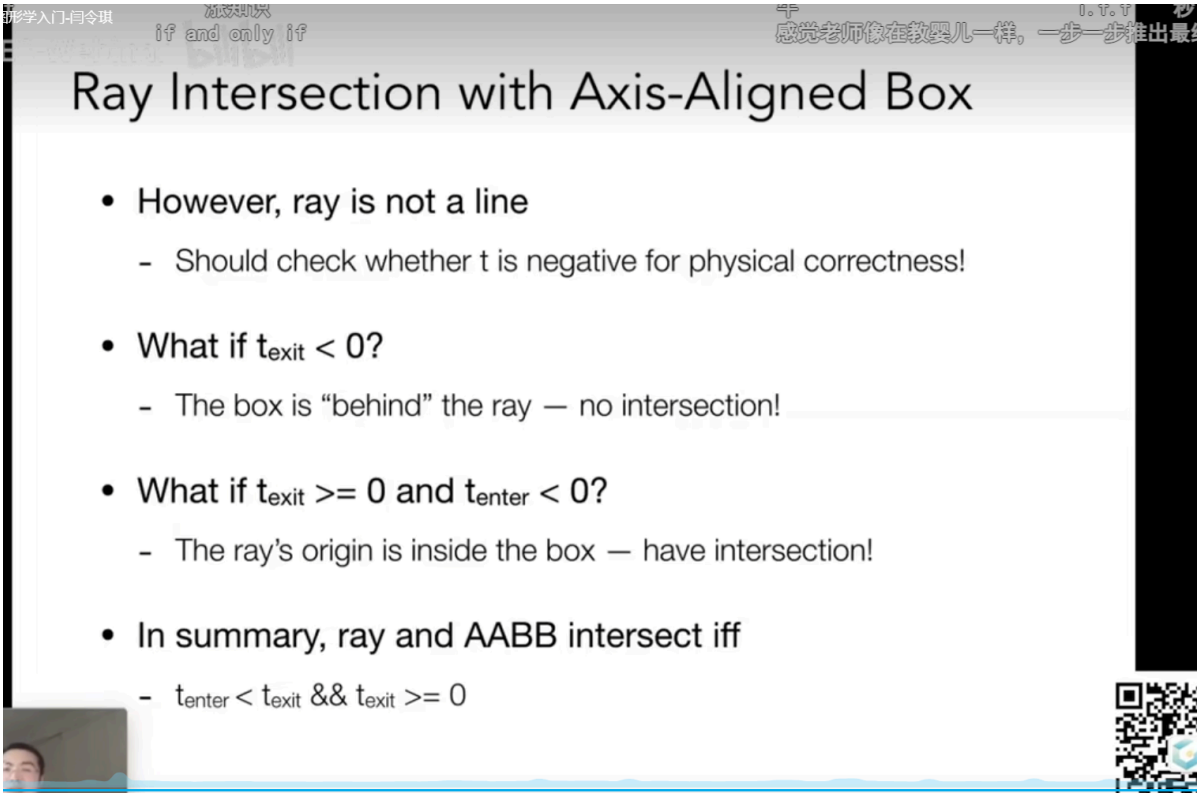


对于3D的情况



可能的各种情况

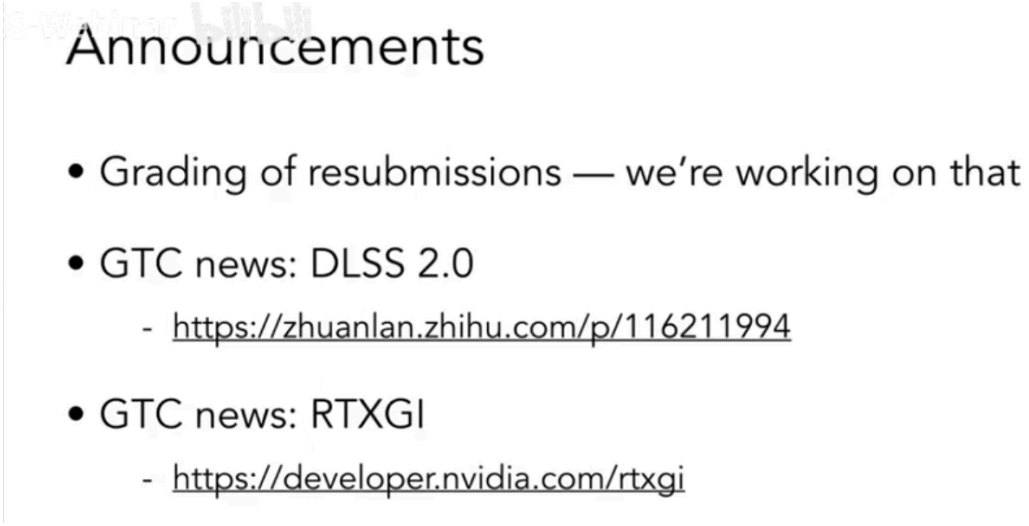
当且仅当进入时间小于出去时间，且离开时间大于等于0即可



实时渲染

DLSS2.0

RTXGI 全局光照

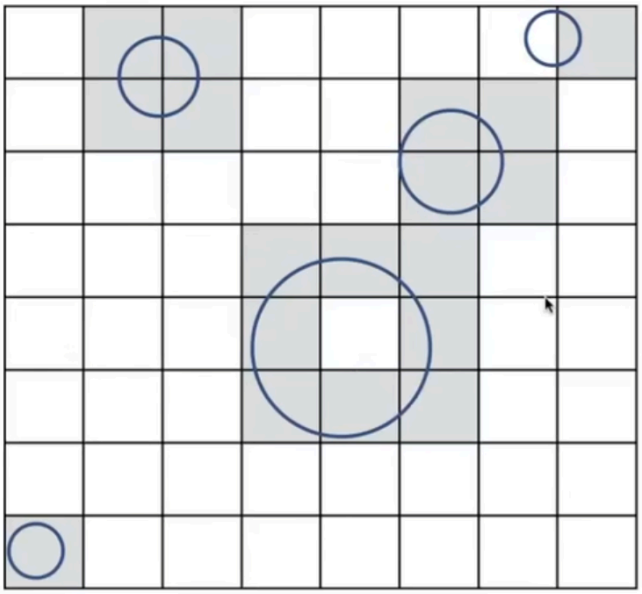


光线和AABB求交

预处理部分

找到一个场景的包围盒，划分成数个格子，找到可能存在物体表面的格子

Preprocess – Build Acceleration Grid

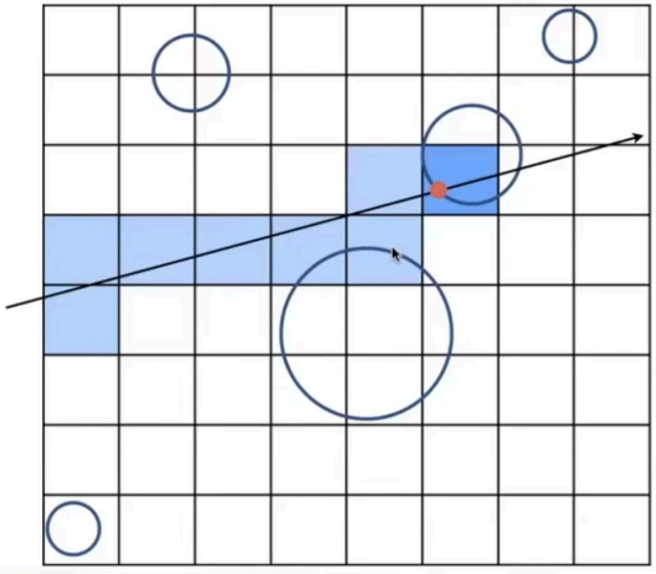


1. Find bounding box
2. Create grid
3. Store each object in overlapping cells

Lingqi Yan, UC Santa Barbar

光线和这些格子求交，当相交的格子中存在物体时，就检查光线与物体的求交

Ray-Scene Intersection



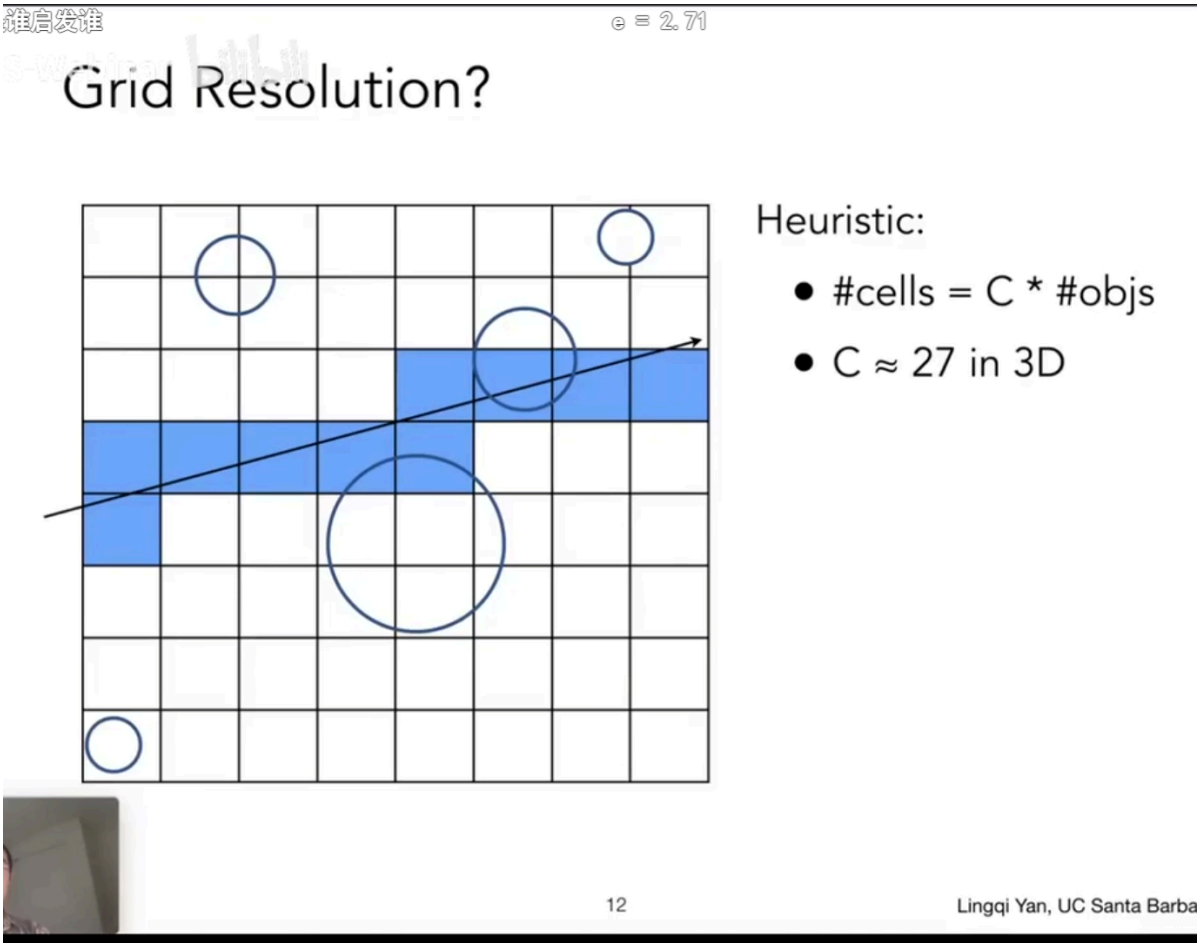
Step through grid in ray traversal order

For each grid cell
Test intersection with all objects stored at that cell

Lingqi Yan, UC Santa Barbar

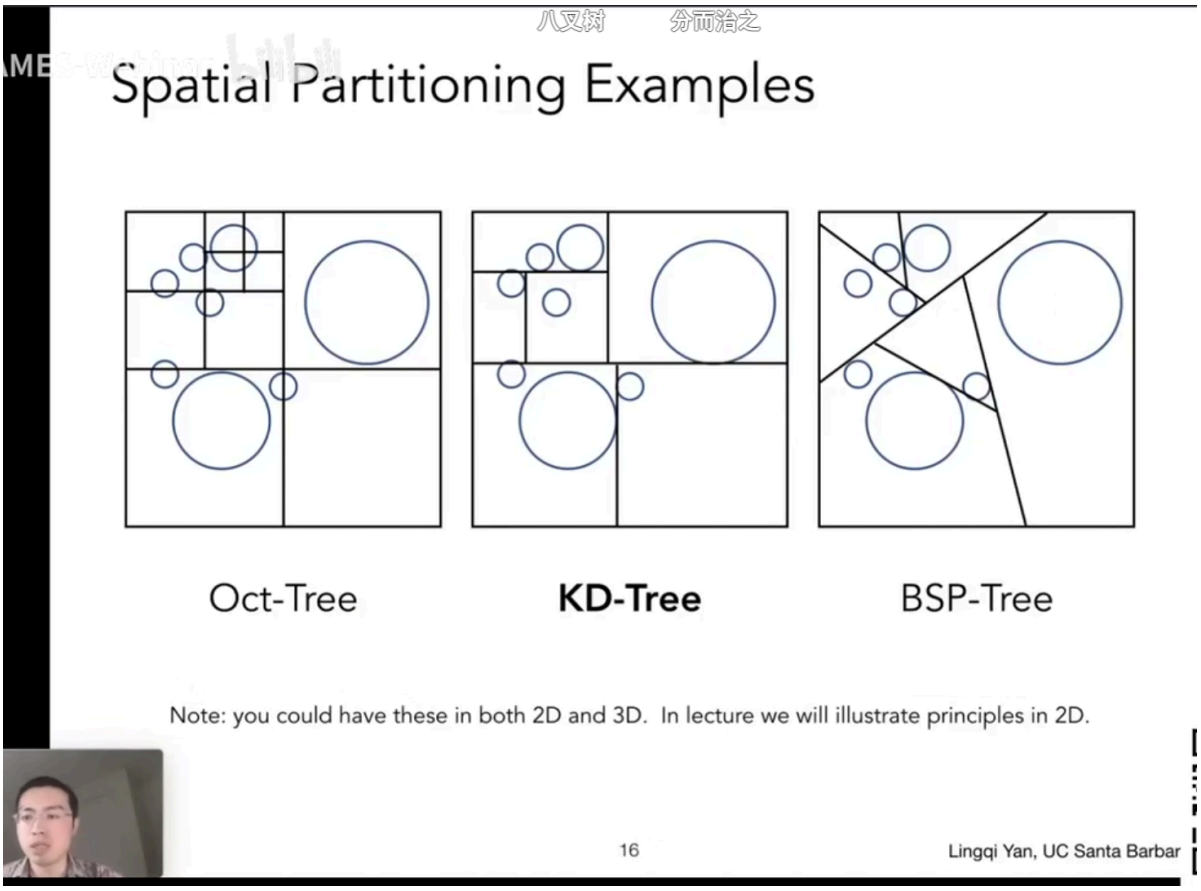
（如何光栅化一条线）

格子不能太密，也不能太稀疏



格子的不足之处（稀疏地方浪费）

使用空间划分



先建树，做预处理，再做光线追踪

kd-tree生成方法

Data Structure for KD-Trees

Internal nodes store

- split axis: x-, y-, or z-axis
- split position: coordinate of split plane along axis
- children: pointers to child nodes
- **No objects are stored in internal nodes**

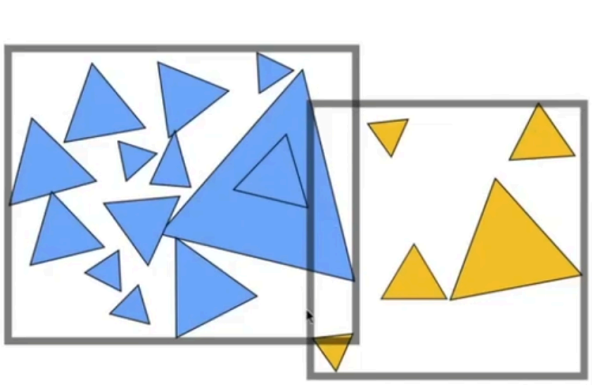
Leaf nodes store

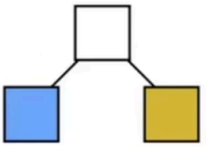
- list of objects

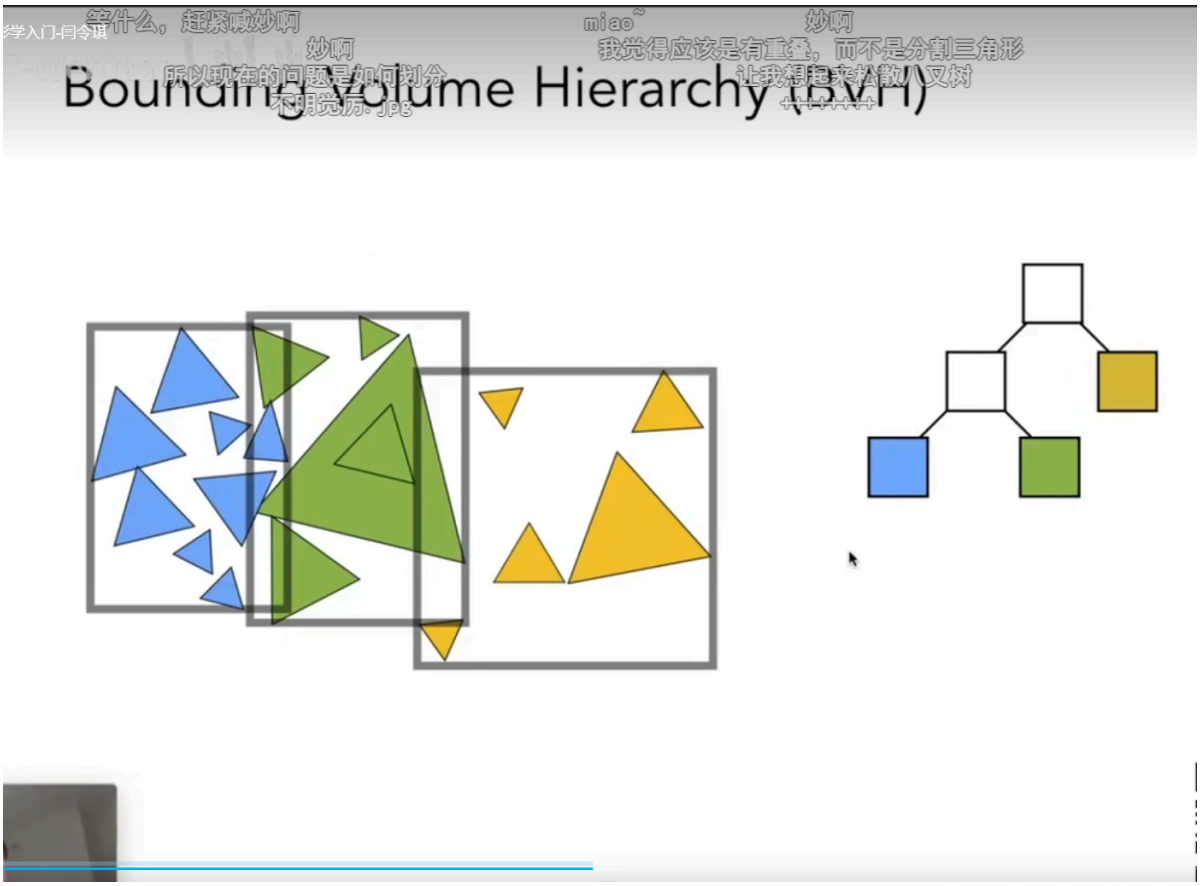
存在问题：包围盒与三角形的求交

使用BVH

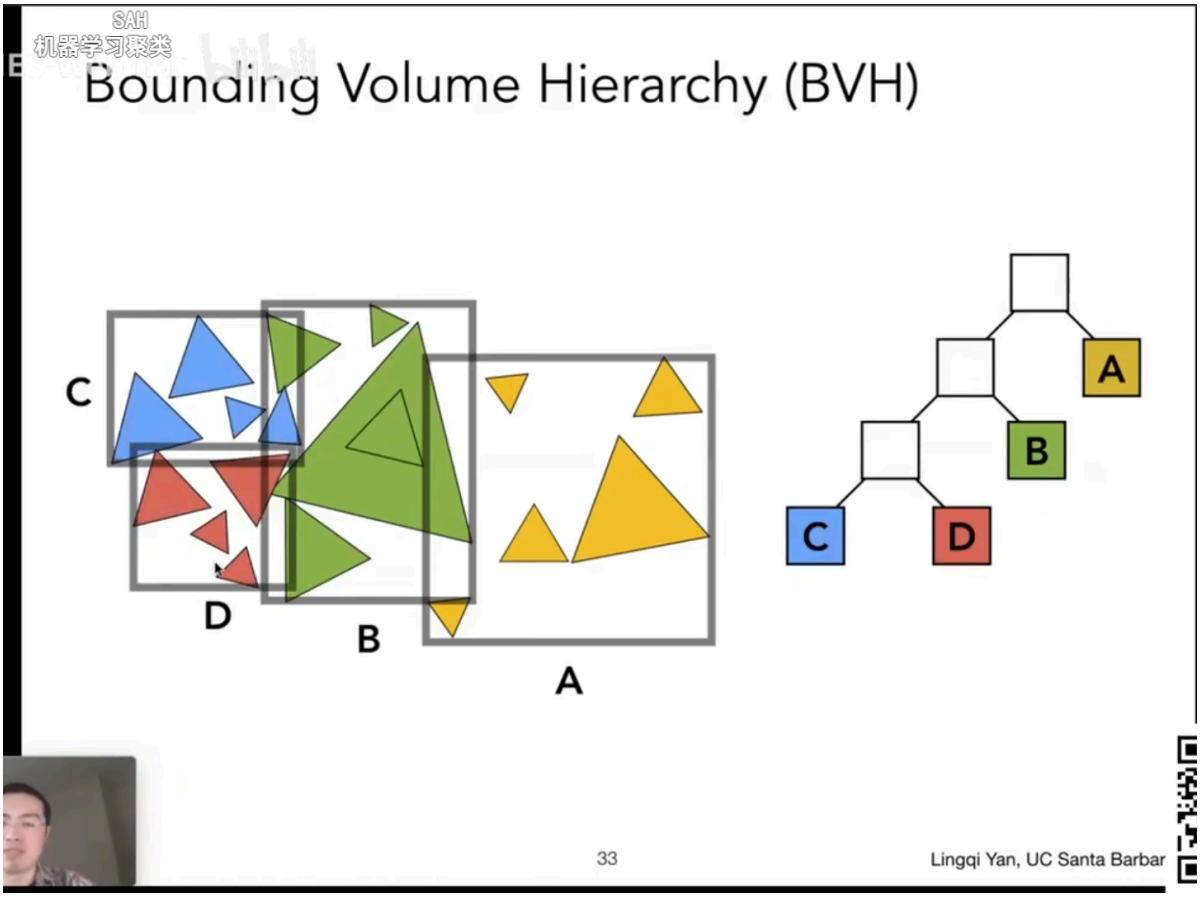
Bounding Volume Hierarchy (BVH)






Bounding Volume Hierarchy (BVH)

均匀划分，最终数量均匀


Bounding Volume Hierarchy (BVH)
C
D
B
A
A
B
C
D
33
Lingqi Yan, UC Santa Barbar

Building BVHs

How to subdivide a node?

- Choose a dimension to split
- Heuristic #1: Always choose the longest axis in node
- Heuristic #2: Split node at location of **median** object

【快速选择算法】

```
std::kth_element
```

Termination criteria?

- Heuristic: stop when node contains few elements (e.g. 5)

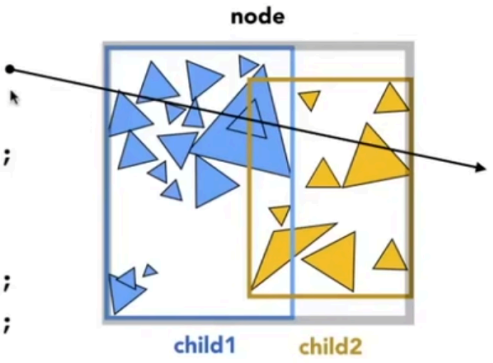
BVH Traversal

```
Intersect(Ray ray, BVH node) {
  if (ray misses node.bbox) return;

  if (node is a leaf node)
     test intersection with all objs;
     return closest intersection;

  hit1 = Intersect(ray, node.child1);
  hit2 = Intersect(ray, node.child2);

  return the closer of hit1, hit2;
}
```



kd-tree是对空间的划分，BVH是对物体的划分。（BVH在实际中更优）

辐射度量学

在物理上准确定义光照的方法