OBJECT ORIENTED PROGRAMMING

(download slides and .py files l·yŘ follow along!)

6.0001 LECTURE 8

OBJECTS

- Python supports many different kinds of data

```
1234        3.14159        "Hello"        [1, 5, 7, 11, 13]

{"CA": "California", "MA": "Massachusetts"}
```

- each is an **object**, and every object has:
  - a **type**
  - an internal **data representation** (primitive or composite)
  - a set of procedures for **interaction** with the object
- an object is an **instance** of a type
  - `1234` is an instance of an `int`
  - `"hello"` is an instance of a string

OBJECT ORIENTED PROGRAMMING (OOP)

- **EVERYTHING IN PYTHON IS AN OBJECT** (and has a type)
- can **create new objects** of some type
- can **manipulate objects**
- can **destroy objects**
  - explicitly using `del` or just "forget" about them
  - python system will reclaim destroyed or inaccessible objects – called "garbage collection"

WHAT ARE OBJECTS?

- objects are **a data abstraction** that captures…

(1) an **internal representation**

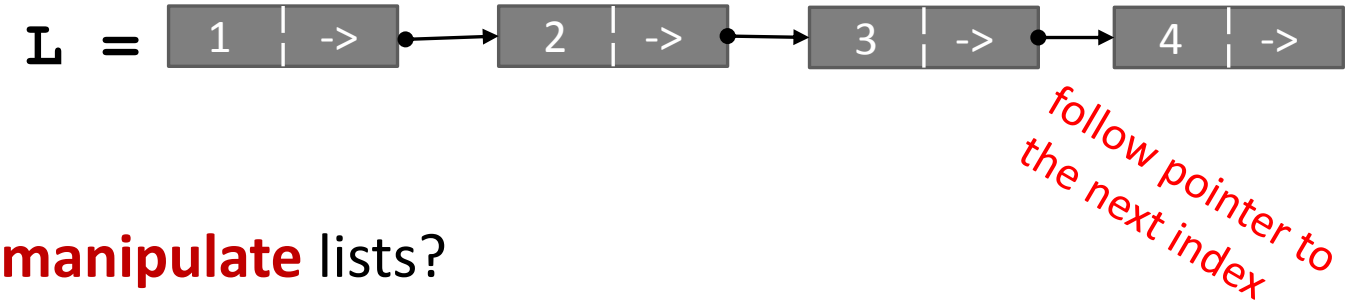
- through data attributes

(2) an **interface** for interacting with object

- through methods (aka procedures/functions)
- defines behaviors but hides implementation

EXAMPLE: [1,2,3,4] has type list

- how are lists **represented internally**? linked list of cells



- how to **manipulate** lists?
  - `L[i], L[i:j], +`
  - `len(), min(), max(), del(L[i])`
  - `L.append(),L.extend(),L.count(),L.index(), L.insert(),L.pop(),L.remove(),L.reverse(), L.sort()`
- internal representation should be private
- correct behavior may be compromised if you manipulate internal representation directly

ADVANTAGES OF OOP

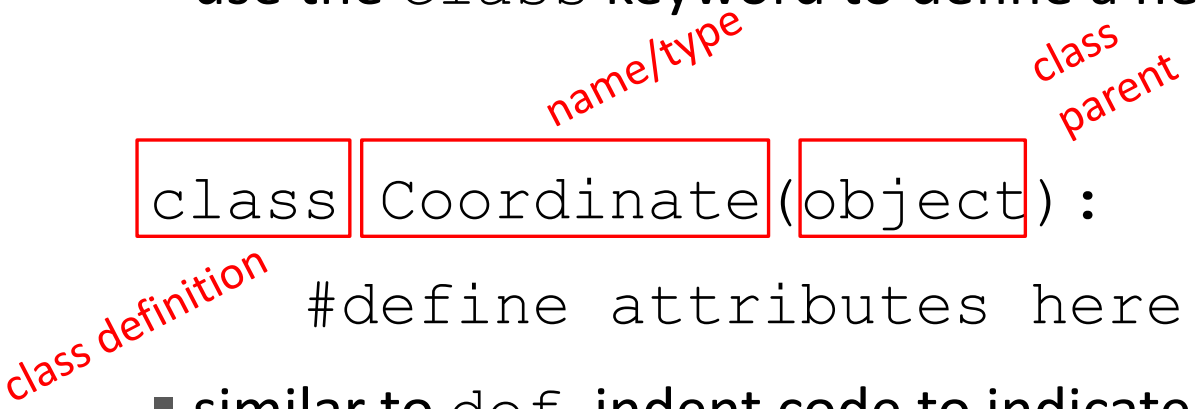
- **bundle data into packages** together with procedures that work on them through well-defined interfaces
- **divide-and-conquer** development
  - implement and test behavior of each class separately
  - increased modularity reduces complexity
- classes make it easy to **reuse** code
  - many Python modules define new classes
  - each class has a separate environment (no collision on function names)
  - inheritance allows subclasses to redefine or extend a selected subset of a superclass' behavior

CREATING AND USING YOUR OWN TYPES WITH CLASSES

- make a distinction between **creating a class** and **using an instance** of the class
- **creating** the class involves
  - defining the class name
  - defining class attributes
  - *for example, someone wrote code to implement a list class*
- **using** the class involves
  - creating new **instances** of objects
  - doing operations on the instances
  - *for example,* `L=[1,2]` *and* `len(L)`

DEFINE YOUR OWN TYPES

- use the `class` keyword to define a new type



- similar to `def`, indent code to indicate which statements are part of the **class definition**
- the word `object` means that `Coordinate` is a Python object and **inherits** all its attributes (inheritance next lecture)
  - `Coordinate` is a subclass of `object`
  - `object` is a superclass of `Coordinate`

WHAT ARE ATTRIBUTES?

- data and procedures that "**belong**" to the class
- **data attributes**
  - think of data as other objects that make up the class
  - *for example, a coordinate is made up of two numbers*
- **methods** (procedural attributes)
  - think of methods as functions that only work with this class
  - how to interact with the object
  - *for example you can define a distance between two coordinate objects but there is no meaning to a distance between two list objects*

DEFINING HOW TO CREATE AN INSTANCE OF A CLASS

- first have to define **how to create an instance** of object
- use a **special method called `__init__`** to initialize some data attributes

```
class Coordinate(object):
    def __init__(self, x, y):
        self.x = x
        self.y = y
```

special method to create an instance __ is double underscore

what data initializes a Coordinate object

parameter to refer to an instance of the class

two data attributes for every Coordinate object

ACTUALLY CREATING AN INSTANCE OF A CLASS

```
c = Coordinate(3,4)
origin = Coordinate(0,0)
print(c.x)
print(origin.x)
```

create a new object of type `Coordinate` and pass in 3 and 4 to the `__init__`

use the dot to access an attribute of instance `c`

- data attributes of an instance are called **instance variables**
- don't provide argument for `self`, Python does this automatically

WHAT IS A METHOD?

- procedural attribute, like a **function that works only with this class**
- Python always passes the object as the first argument
  - convention is to use `self` as the name of the first argument of all methods
- the "**.**" **operator** is used to access any attribute
  - a data attribute of an object
  - a method of an object

DEFINE A METHOD FOR THE `Coordinate` CLASS

```
class Coordinate(object):
    def __init__(self, x, y):
        self.x = x
        self.y = y
    def distance(self, other):
        x_diff_sq = (self.x-other.x)**2
        y_diff_sq = (self.y-other.y)**2
        return (x_diff_sq + y_diff_sq)**0.5
```

use it to refer to any instance

another parameter to method

dot notation to access data

- other than `self` and dot notation, methods behave just like functions (take params, do operations, return)

HOW TO USE A METHOD

```
def distance(self, other):
    # code here
```

method def

Using the class:

- conventional way

```
c = Coordinate(3,4)
zero = Coordinate(0,0)
print(c.distance(zero))
```

object to call method on

name of method

parameters not including `self` (`self` is implied to be `c`)

- equivalent to

```
c = Coordinate(3,4)
zero = Coordinate(0,0)
print(Coordinate.distance(c, zero))
```

name of class

name of method

parameters, including an object to call the method on, representing `self`

PRINT REPRESENTATION OF AN OBJECT

```
>>> c = Coordinate(3,4)
>>> print(c)
<__main__.Coordinate object at 0x7fa918510488>
```

- **uninformative** print representation by default
- define a **`__str__` method** for a class
- Python calls the `__str__` method when used with `print` on your class object
- you choose what it does! Say that when we print a `Coordinate` object, want to show

```
>>> print(c)
<3,4>
```

DEFINING YOUR OWN PRINT METHOD

```
class Coordinate(object):
    def __init__(self, x, y):
        self.x = x
        self.y = y
    def distance(self, other):
        x_diff_sq = (self.x-other.x)**2
        y_diff_sq = (self.y-other.y)**2
        return (x_diff_sq + y_diff_sq)**0.5
    def __str__(self):
        return "<"+str(self.x)+","+str(self.y)+">"
```

name of special method

must return a string

WRAPPING YOUR HEAD AROUND TYPES AND CLASSES

- can ask for the type of an object instance

```
>>> c = Coordinate(3,4)
>>> print(c)
<3,4>
>>> print(type(c))
<class __main__.Coordinate>
```

return of the __str__ method

the type of object c is a class Coordinate

- this makes sense since

```
>>> print(Coordinate)
<class __main__.Coordinate>
>>> print(type(Coordinate))
<type 'type'>
```

a Coordinate is a class

a Coordinate class is a type of object

- use `isinstance()` to check if an object is a `Coordinate`

```
>>> print(isinstance(c, Coordinate))
True
```

SPECIAL OPERATORS

- +, -, ==, <, >, len(), print, and many others

https://docs.python.org/3/reference/datamodel.html#basic-customization

- like `print`, can override these to work with your class
- define them with double underscores before/after

```
__add__(self, other)    →    self + other
__sub__(self, other)    →    self - other
__eq__(self, other)     →    self == other
__lt__(self, other)     →    self < other
__len__(self)           →    len(self)
__str__(self)           →    print self
```

... and others

EXAMPLE: FRACTIONS

- create a **new type** to represent a number as a fraction
- **internal representation** is two integers
  - numerator
  - denominator
- **interface** a.k.a. **methods** a.k.a **how to interact** with `Fraction` objects
  - add, subtract
  - print representation, convert to a float
  - invert the fraction
- the code for this is in the handout, check it out!

THE POWER OF OOP

- **bundle together objects** that share
  - common attributes and
  - procedures that operate on those attributes
- use **abstraction** to make a distinction between how to implement an object vs how to use the object
- build **layers** of object abstractions that inherit behaviors from other classes of objects
- create our **own classes of objects** on top of Python's basic classes

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PYTHON CLASSES and INHERITANCE

(download slides and .py files l·yŘ follow along!)

6.0001 LECTURE 9

LAST TIME

- abstract data types through classes
- `Coordinate` example
- `Fraction` example

TODAY

- more on classes
  - getters and setters
  - information hiding
  - class variables
- inheritance

IMPLEMENTING THE CLASS vs USING THE CLASS

- write code from two different perspectives

implementing a new object type with a class

- **define** the class
- define **data attributes** (WHAT IS the object)
- define **methods** (HOW TO use the object)

using the new object type in code

- create **instances** of the object type
- do **operations** with them

CLASS DEFINITION OF AN OBJECT TYPE vs INSTANCE OF A CLASS

- class name is the **type**

```
class Coordinate(object)
```

- class is defined generically
  - use `self` to refer to some instance while defining the class

```
(self.x - self.y)**2
```

  - `self` is a parameter to methods in class definition

- class defines data and methods **common across all instances**

- instance is **one specific** object

```
coord = Coordinate(1,2)
```

- data attribute values vary between instances

```
c1 = Coordinate(1,2)
c2 = Coordinate(3,4)
```

  - `c1` and `c2` have different data attribute values `c1.x` and `c2.x` because they are different objects

- instance has the **structure of the class**

WHY USE OOP AND CLASSES OF OBJECTS?

- mimic real life
- group different objects part of the same type

Jelly
1 year old
brown

5 years old
brown

Tiger
2 years old
brown

Bean
0 years old
black

2 years old
white

1 year old
b/w

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WHY USE OOP AND CLASSES OF OBJECTS?

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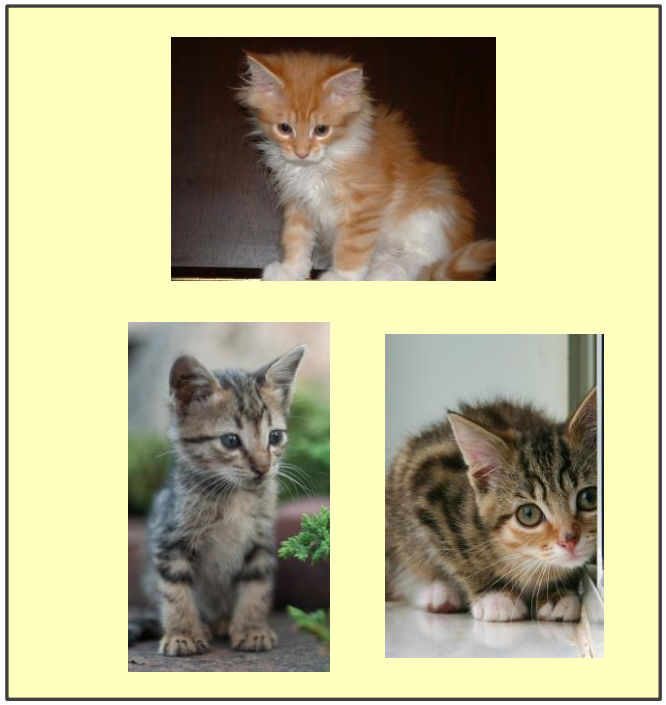

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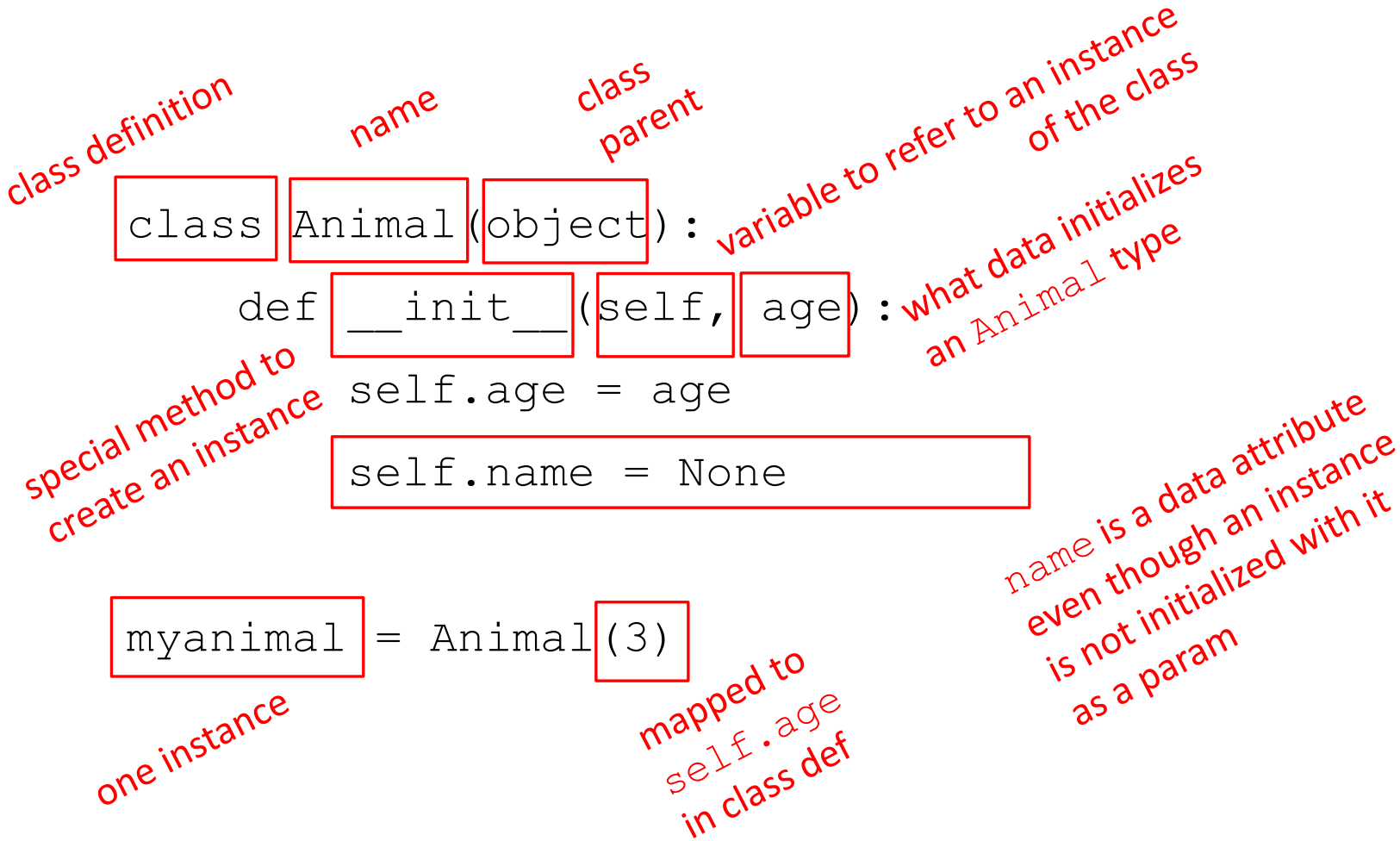
GROUPS OF OBJECTS HAVE ATTRIBUTES (RECAP)

- **data attributes**
  - how can you represent your object with data?
  - **what it is**
  - *for a coordinate: x and y values*
  - *for an animal: age, name*
- **procedural attributes** (behavior/operations/**methods**)
  - how can someone interact with the object?
  - **what it does**
  - *for a coordinate: find distance between two*
  - *for an animal: make a sound*

HOW TO DEFINE A CLASS (RECAP)



GETTER AND SETTER METHODS

```
class Animal(object):
    def __init__(self, age):
        self.age = age
        self.name = None
    def get_age(self):
        return self.age
    def get_name(self):
        return self.name
    def set_age(self, newage):
        self.age = newage
    def set_name(self, newname=""):
        self.name = newname
    def __str__(self):
        return "animal:"+str(self.name)+":"+str(self.age)
```

getter

setter

- **getters and setters** should be used outside of class to access data attributes

AN INSTANCE and DOT NOTATION (RECAP)

- instantiation creates an **instance of an object**

```
a = Animal(3)
```

- **dot notation** used to access attributes (data and methods) though it is better to use getters and setters to access data attributes

```
a.age
```

- access data attribute
- allowed, but not recommended

```
a.get_age()
```

- access method
- best to use getters and setters

INFORMATION HIDING

- author of class definition may **change data attribute** variable names

replaced `age` data attribute by `years`

```
class Animal(object):
    def __init__(self, age):
        self.years = age
    def get_age(self):
        return self.years
```

- if you are **accessing data attributes** outside the class and class **definition changes**, may get errors
- outside of class, use getters and setters instead
  use `a.get_age()` NOT `a.age`
  - good style
  - easy to maintain code
  - prevents bugs

PYTHON NOT GREAT AT INFORMATION HIDING

- allows you to **access data** from outside class definition

```
print(a.age)
```

- allows you to **write to data** from outside class definition

```
a.age = 'infinite'
```

- allows you to **create data attributes** for an instance from outside class definition

```
a.size = "tiny"
```

- it's **not good style** to do any of these!

DEFAULT ARGUMENTS

- **default arguments** for formal parameters are used if no actual argument is given

```
def set_name(self, newname=""):
    self.name = newname
```

- default argument used here

```
a = Animal(3)
a.set_name()
print(a.get_name())
```

prints ""

- argument passed in is used here

```
a = Animal(3)
a.set_name("fluffy")
print(a.get_name())
```

prints "fluffy"

HIERARCHIES

Animal

People

Student

Cat

Rabbit

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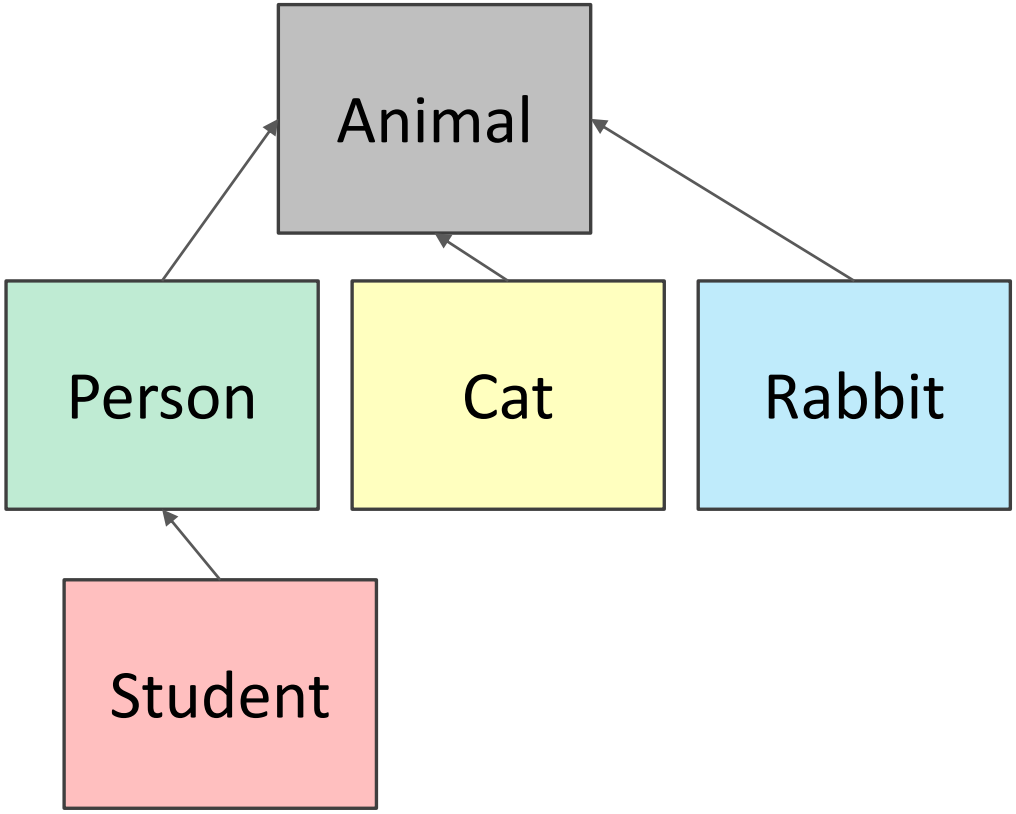
HIERARCHIES

- **parent class** (superclass)
- **child class** (subclass)
  - **inherits** all data and behaviors of parent class
  - **add** more **info**
  - **add** more **behavior**
  - **override** behavior



INHERITANCE: PARENT CLASS

```
class Animal(object):
    def __init__(self, age):
        self.age = age
        self.name = None
    def get_age(self):
        return self.age
    def get_name(self):
        return self.name
    def set_age(self, newage):
        self.age = newage
    def set_name(self, newname=""):
        self.name = newname
    def __str__(self):
        return "animal:"+str(self.name)+":"+str(self.age)
```

- everything is an object
- class `object` implements basic operations in Python, like binding variables, etc

INHERITANCE: SUBCLASS

inherits all attributes of `Animal`:
`__init__()`
`age, name`
`get_age(), get_name()`
`set_age(), set_name()`
`__str__()`

```
class Cat(Animal):
    def speak(self):
        print("meow")
    def __str__(self):
        return "cat:"+str(self.name)+":"+str(self.age)
```

add new functionality via speak method

overrides `__str__`

- add new functionality with `speak()`
  - instance of type `Cat` can be called with new methods
  - instance of type `Animal` throws error if called with `Cat`'s new method
- `__init__` is not missing, uses the `Animal` version

WHICH METHOD TO USE?

- subclass can have **methods with same name** as superclass
- for an instance of a class, look for a method name in **current class definition**
- if not found, look for method name **up the hierarchy** (in parent, then grandparent, and so on)
- use first method up the hierarchy that you found with that method name

```
class Person(Animal):
    def __init__(self, name, age):
        Animal.__init__(self, age)
        self.set_name(name)
        self.friends = []
    def get_friends(self):
        return self.friends
    def add_friend(self, fname):
        if fname not in self.friends:
            self.friends.append(fname)
    def speak(self):
        print("hello")
    def age_diff(self, other):
        diff = self.age - other.age
        print(abs(diff), "year difference")
    def __str__(self):
        return "person:"+str(self.name)+":"+str(self.age)
```

parent class is Animal

call Animal constructor

call Animal's method

add a new data attribute

new methods

override Animal's __str__ method

```
import random

class Student(Person):
    def __init__(self, name, age, major=None):
        Person.__init__(self, name, age)
        self.major = major
    def change_major(self, major):
        self.major = major
    def speak(self):
        r = random.random()
        if r < 0.25:
            print("i have homework")
        elif 0.25 <= r < 0.5:
            print("i need sleep")
        elif 0.5 <= r < 0.75:
            print("i should eat")
        else:
            print("i am watching tv")
    def __str__(self):
        return "student:"+str(self.name)+":"+str(self.age)+":"+str(self.major)
```

bring in methods from `random` class

inherits `Person` and `Animal` attributes

adds new data

- I looked up how to use the `random` class in the python docs
- `random()` method gives back float in [0, 1)

CLASS VARIABLES AND THE `Rabbit` SUBCLASS

- **class variables** and their values are shared between all instances of a class

```
class Rabbit(Animal):
    tag = 1
    def __init__(self, age, parent1=None, parent2=None):
        Animal.__init__(self, age)
        self.parent1 = parent1
        self.parent2 = parent2
        self.rid = Rabbit.tag
        Rabbit.tag += 1
```

parent class

class variable

instance variable

access class variable

incrementing class variable changes it for all instances that may reference it

- `tag` used to give **unique id** to each new rabbit instance

Rabbit GETTER METHODS

```
class Rabbit(Animal):
    tag = 1
    def __init__(self, age, parent1=None, parent2=None):
        Animal.__init__(self, age)
        self.parent1 = parent1
        self.parent2 = parent2
        self.rid = Rabbit.tag
        Rabbit.tag += 1
    def get_rid(self):
        return str(self.rid).zfill(3)
    def get_parent1(self):
        return self.parent1
    def get_parent2(self):
        return self.parent2
```

method on a string to pad the beginning with zeros for example, 001 not 1

- getter methods specific for a `Rabbit` class
- there are also getters `get_name` and `get_age` inherited from `Animal`

WORKING WITH YOUR OWN TYPES

```
def __add__(self, other):
        # returning object of same type as this class
        return Rabbit(0, self, other)
```

recall Rabbit's `__init__(self, age, parent1=None, parent2=None)`

- define **+ operator** between two `Rabbit` instances
  - define what something like this does: `r4 = r1 + r2` where `r1` and `r2` are `Rabbit` instances
  - `r4` is a new `Rabbit` instance with age 0
  - `r4` has `self` as one parent and `other` as the other parent
  - in `__init__`, **`parent1 and parent2 are of type Rabbit`**

SPECIAL METHOD TO COMPARE TWO `Rabbits`

- decide that two rabbits are equal if they have the **same two parents**

```
def __eq__(self, other):
    parents_same = self.parent1.rid == other.parent1.rid \
                   and self.parent2.rid == other.parent2.rid
    parents_opposite = self.parent2.rid == other.parent1.rid \
                       and self.parent1.rid == other.parent2.rid
    return parents_same or parents_opposite
```

booleans

- compare ids of parents since **ids are unique** (due to class var)
- note you can't compare objects directly
  - for ex. with `self.parent1 == other.parent1`
  - this calls the `__eq__` method over and over until call it on `None` and gives an `AttributeError` when it tries to do `None.parent1`

OBJECT ORIENTED PROGRAMMING

- create your own **collections of data**
- **organize** information
- **division** of work
- access information in a **consistent** manner
- add **layers** of complexity
- like functions, classes are a mechanism for **decomposition** and **abstraction** in programming

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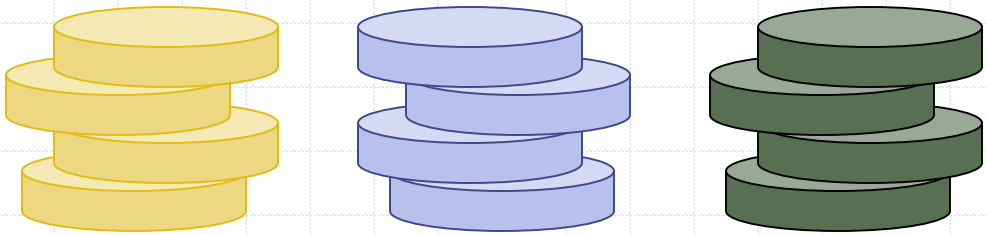
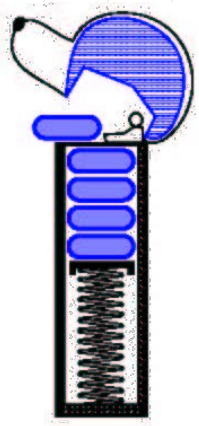
Stacks



Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored

Example

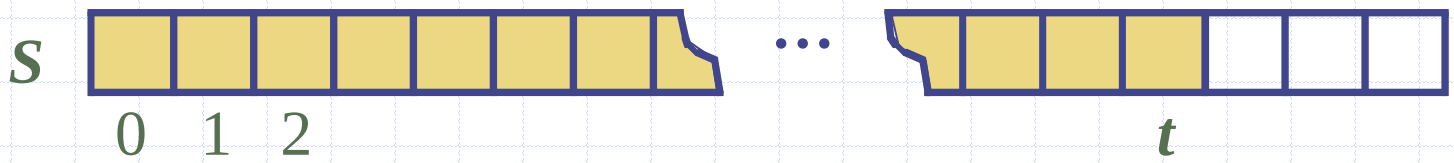
Operation	Return Value	Stack Contents
S.push(5)	–	[5]
S.push(3)	–	[5, 3]
len(S)	2	[5, 3]
S.pop( )	3	[5]
S.is_empty( )	False	[5]
S.pop( )	5	[ ]
S.is_empty( )	True	[ ]
S.pop( )	"error"	[ ]
S.push(7)	–	[7]
S.push(9)	–	[7, 9]
S.top( )	9	[7, 9]
S.push(4)	–	[7, 9, 4]
len(S)	3	[7, 9, 4]
S.pop( )	4	[7, 9]
S.push(6)	–	[7, 9, 6]
S.push(8)	–	[7, 9, 6, 8]
S.pop( )	8	[7, 9, 6]

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in a language that supports recursion
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

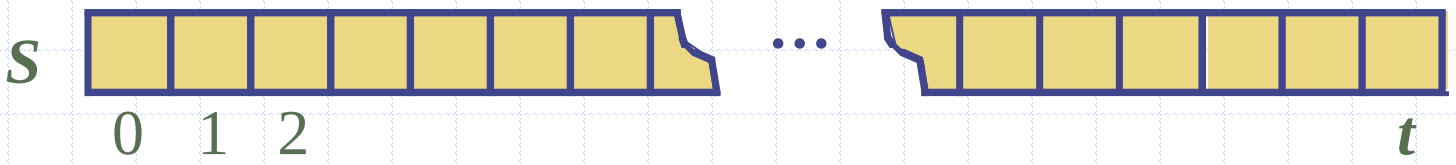
Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then need to grow the array and copy all the elements over.



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$ (amortized in the case of a push)

Array-based Stack in Python

```
1  class ArrayStack:
2    """LIFO Stack implementation using a Python list as underlying storage."""
3
4    def __init__(self):
5      """Create an empty stack."""
6      self._data = [ ]                       # nonpublic list instance
7
8    def __len__(self):
9      """Return the number of elements in the stack."""
10     return len(self._data)
11
12   def is_empty(self):
13     """Return True if the stack is empty."""
14     return len(self._data) == 0
15
16   def push(self, e):
17     """Add element e to the top of the stack."""
18     self._data.append(e)                   # new item stored at end of list
19
```

```
20   def top(self):
21     """Return (but do not remove) the element at the top of the stack.
22
23     Raise Empty exception if the stack is empty.
24     """
25     if self.is_empty( ):
26       raise Empty('Stack is empty')
27     return self._data[-1]                  # the last item in the list
28
29   def pop(self):
30     """Remove and return the element from the top of the stack (i.e., LIFO).
31
32     Raise Empty exception if the stack is empty.
33     """
34     if self.is_empty( ):
35       raise Empty('Stack is empty')
36     return self._data.pop( )               # remove last item from list
```

Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

Algorithm ParenMatch(*X*,*n*):
Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in *X* match
Let *S* be an empty stack
for *i*=0 to *n*-1 **do**
    if *X*[*i*] is an opening grouping symbol **then**
        S.push(*X*[*i*])
    else if *X*[*i*] is a closing grouping symbol **then**
        if *S*.is_empty() **then**
            return false {nothing to match with}
        if *S*.pop() does not match the type of *X*[*i*] **then**
            return false {wrong type}
if *S*.isEmpty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

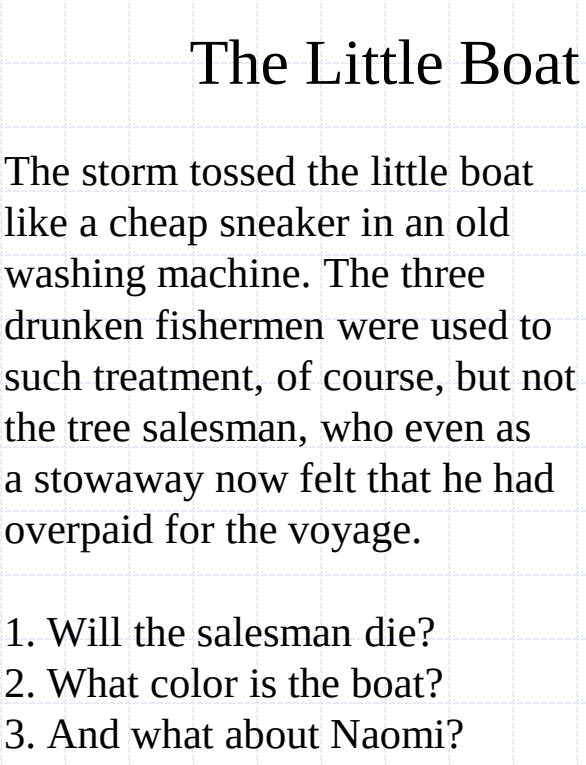
Parentheses Matching in Python

```
1  def is_matched(expr):
2    """Return True if all delimiters are properly match; False otherwise."""
3    lefty = '({['                                  # opening delimiters
4    righty = ')}]'                                 # respective closing delims
5    S = ArrayStack( )
6    for c in expr:
7      if c in lefty:
8        S.push(c)                                  # push left delimiter on stack
9      elif c in righty:
10       if S.is_empty( ):
11         return False                             # nothing to match with
12       if righty.index(c) != lefty.index(S.pop( )):
13         return False                             # mismatched
14   return S.is_empty( )                           # were all symbols matched?
```

HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```



Tag Matching Algorithm in Python

```
1  def is_matched_html(raw):
2    """Return True if all HTML tags are properly match; False otherwise."""
3    S = ArrayStack( )
4    j = raw.find('<')                    # find first '<' character (if any)
5    while j != -1:
6      k = raw.find('>', j+1)             # find next '>' character
7      if k == -1:
8        return False                     # invalid tag
9      tag = raw[j+1:k]                   # strip away < >
10     if not tag.startswith('/'):        # this is opening tag
11       S.push(tag)
12     else:                              # this is closing tag
13       if S.is_empty( ):
14         return False                   # nothing to match with
15       if tag[1:] != S.pop( ):
16         return False                   # mismatched delimiter
17     j = raw.find('<', k+1)             # find next '<' character (if any)
18   return S.is_empty( )                 # were all opening tags matched?
```

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top())
  doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

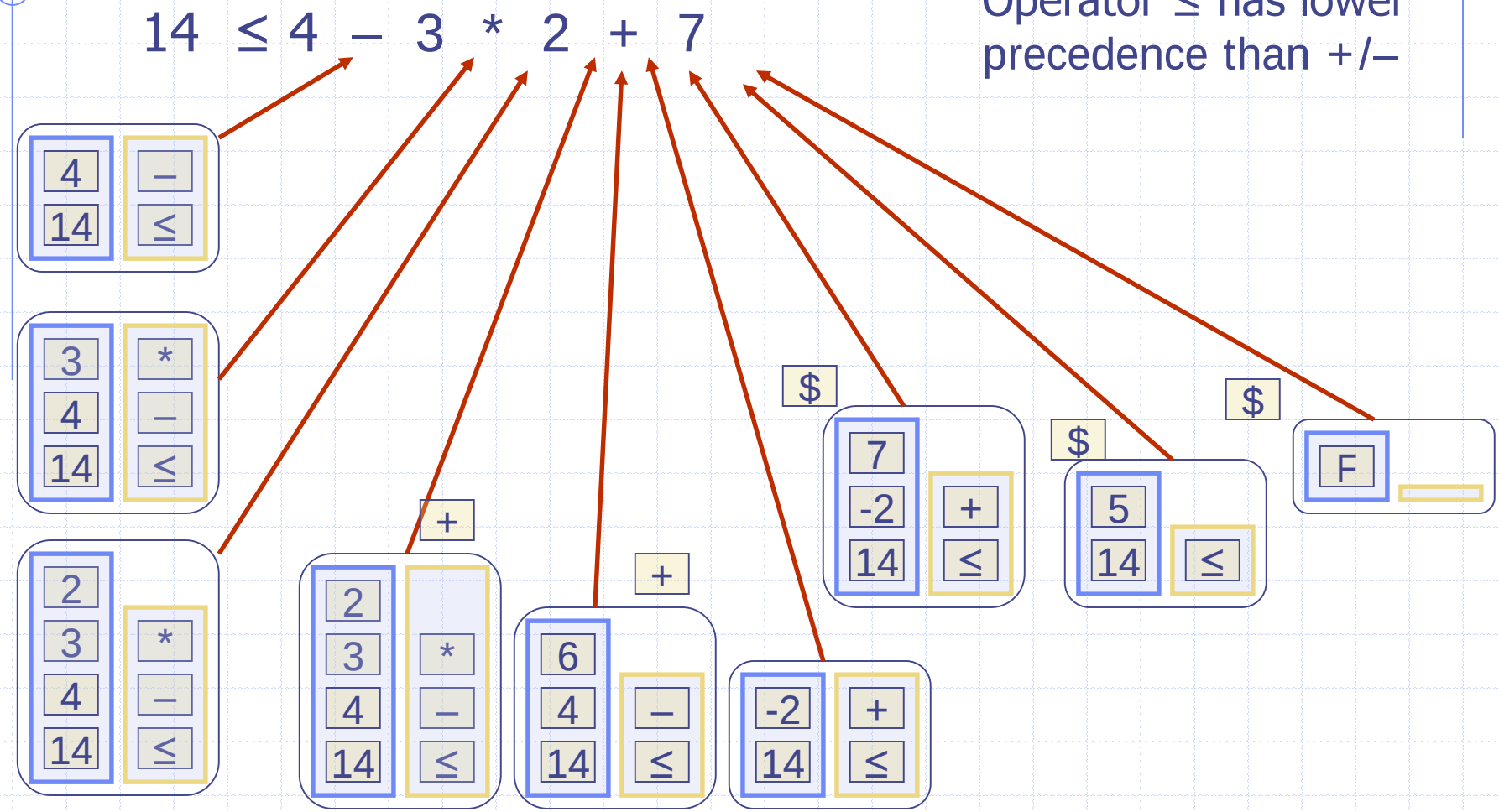
Output: the value of the expression

```
while there's another token z
  if isNumber(z) then
      valStk.push(z)
  else
      repeatOps(z);
      opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

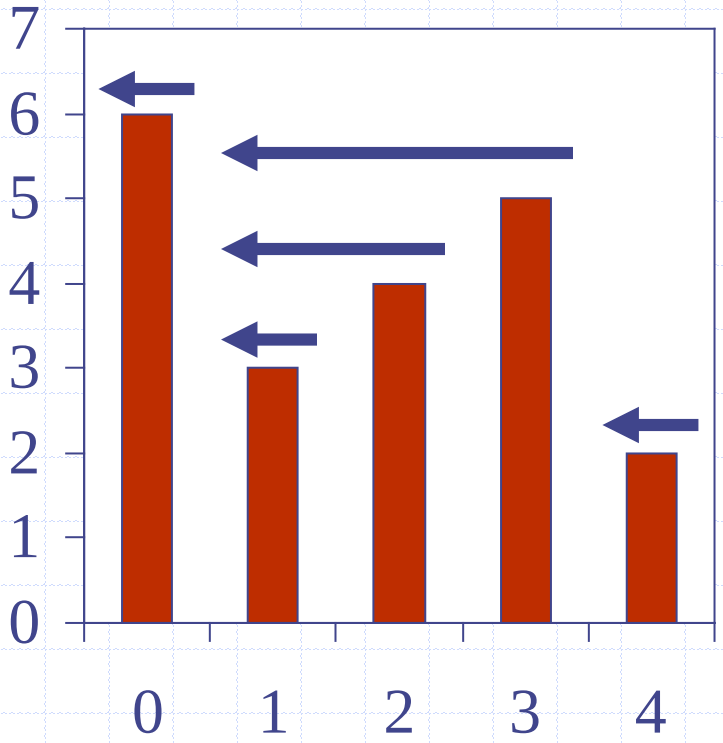
Slide by Matt Stallmann included with permission.

Operator ≤ has lower precedence than +/–



Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

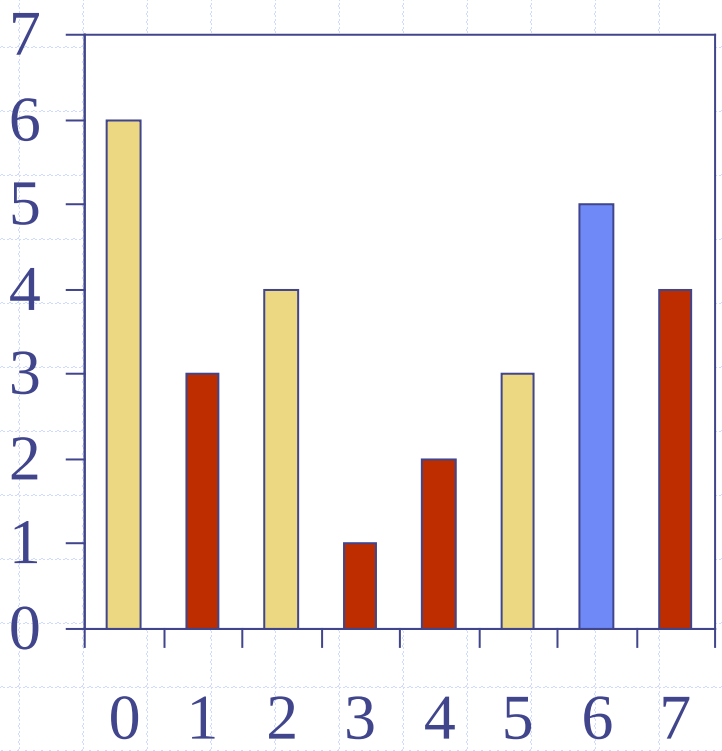
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	#
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \leq i \wedge X[i - s] \leq X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm *spans1* runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push x onto the stack

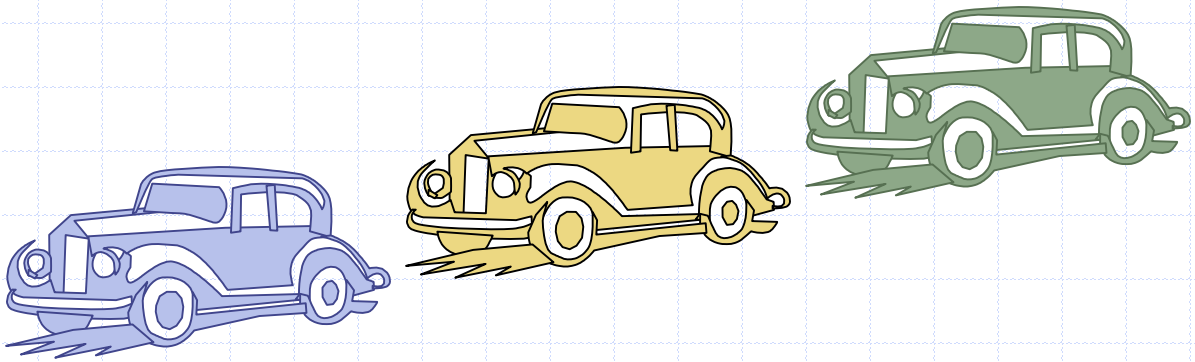


Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly one
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm *spans2* runs in $O(n)$ time

```
Algorithm spans2(X, n)                          #
    S ← new array of n integers                 n
    A ← new empty stack                         1
    for i ← 0 to n – 1 do                       n
        while (¬A.is_empty() ∧
                X[A.top()] ≤ X[i] ) do          n
            A.pop()                             n
        if A.is_empty() then                    n
            S[i] ← i + 1                        n
        else
            S[i] ← i – A.top()                  n
        A.push(i)                               n
    return S                                    1
```

Queues



The Queue ADT

- The Queue ADT stores arbitrary objects
- Insertions and deletions follow the first-in first-out scheme
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(object): inserts an element at the end of the queue
  - object dequeue(): removes and returns the element at the front of the queue
- Auxiliary queue operations:
  - object first(): returns the element at the front without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored
- Exceptions
  - Attempting the execution of dequeue or front on an empty queue throws an EmptyQueueException

Example

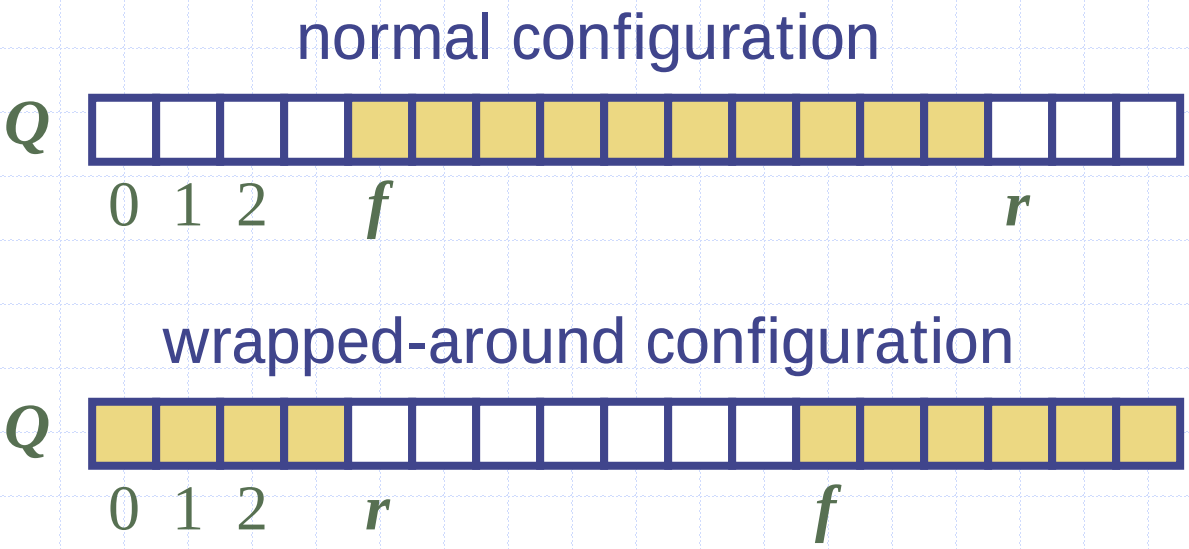
Operation	Return Value	first $\leftarrow Q \leftarrow$ last
Q.enqueue(5)	–	[5]
Q.enqueue(3)	–	[5, 3]
len(Q)	2	[5, 3]
Q.dequeue( )	5	[3]
Q.is_empty( )	False	[3]
Q.dequeue( )	3	[ ]
Q.is_empty( )	True	[ ]
Q.dequeue( )	"error"	[ ]
Q.enqueue(7)	–	[7]
Q.enqueue(9)	–	[7, 9]
Q.first( )	7	[7, 9]
Q.enqueue(4)	–	[7, 9, 4]
len(Q)	3	[7, 9, 4]
Q.dequeue( )	7	[9, 4]

Applications of Queues

- Direct applications
  - Waiting lists, bureaucracy
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Queue

- Use an array of size N in a circular fashion
- Two variables keep track of the front and rear
  - f index of the front element
  - r index immediately past the rear element
- Array location r is kept empty

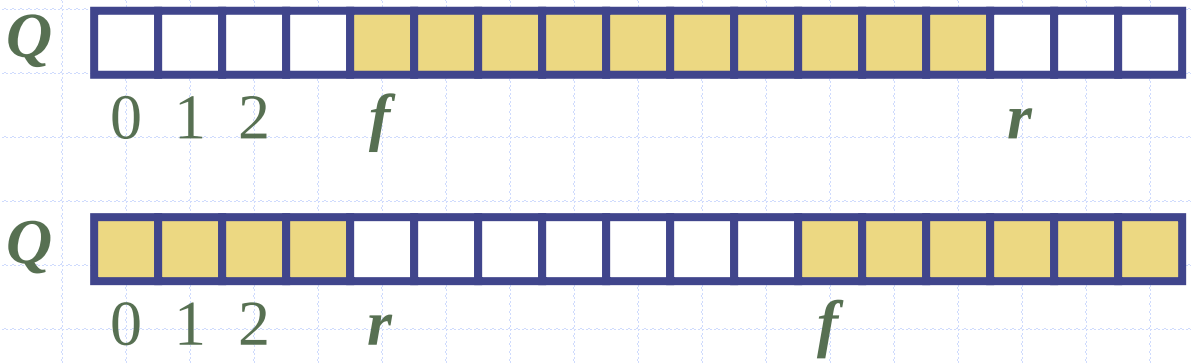


Queue Operations

- We use the modulo operator (remainder of division)

Algorithm ***size***()
  return $(N - f + r) \bmod N$

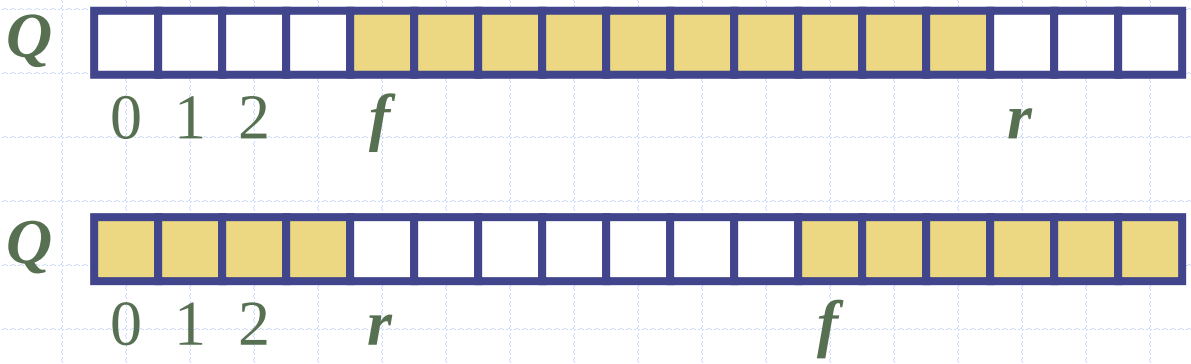
Algorithm ***isEmpty***()
  return $(f = r)$



Queue Operations (cont.)

- Operation enqueue throws an exception if the array is full
- This exception is implementation-dependent

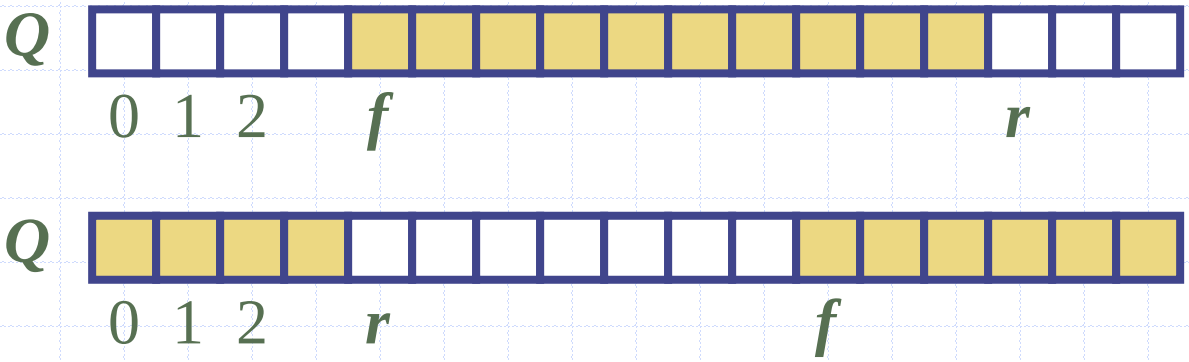
Algorithm *enqueue*(*o*)
  if *size*() = *N* – 1 **then**
    throw *FullQueueException*
  else
    Q[*r*] ← *o*
    r ← (*r* + 1) mod *N*



Queue Operations (cont.)

- Operation dequeue throws an exception if the queue is empty
- This exception is specified in the queue ADT

Algorithm *dequeue*()
  if *isEmpty*() **then**
    throw *EmptyQueueException*
  else
    $o \leftarrow Q[f]$
    $f \leftarrow (f + 1) \bmod N$
    return o



Queue in Python

- Use the following three instance variables:
  - _data: is a reference to a list instance with a fixed capacity.
  - _size: is an integer representing the current number of elements stored in the queue (as opposed to the length of the data list).
  - _front: is an integer that represents the index within data of the first element of the queue (assuming the queue is not empty).

Queue in Python, Beginning

```
1  class ArrayQueue:
2    """FIFO queue implementation using a Python list as underlying storage."""
3    DEFAULT_CAPACITY = 10          # moderate capacity for all new queues
4
5    def __init__(self):
6      """Create an empty queue."""
7      self._data = [None] * ArrayQueue.DEFAULT_CAPACITY
8      self._size = 0
9      self._front = 0
10
11   def __len__(self):
12     """Return the number of elements in the queue."""
13     return self._size
14
15   def is_empty(self):
16     """Return True if the queue is empty."""
17     return self._size == 0
18
```

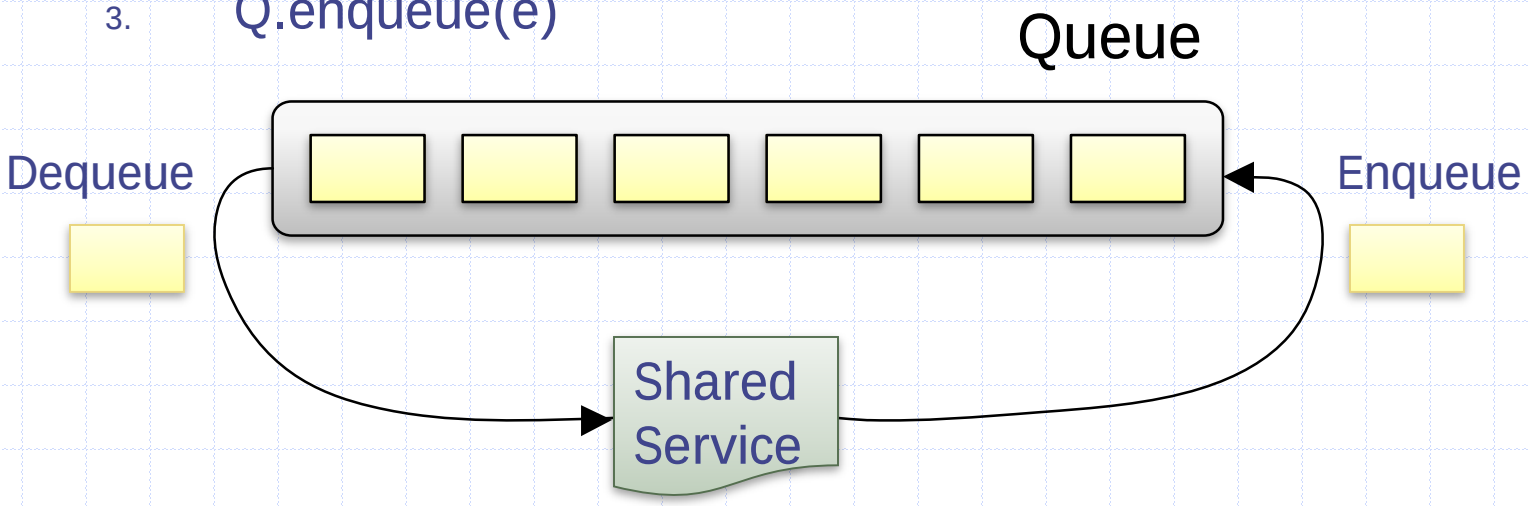
```
19   def first(self):
20     """Return (but do not remove) the element at the front of the queue.
21
22     Raise Empty exception if the queue is empty.
23     """
24     if self.is_empty():
25       raise Empty('Queue is empty')
26     return self._data[self._front]
27
28   def dequeue(self):
29     """Remove and return the first element of the queue (i.e., FIFO).
30
31     Raise Empty exception if the queue is empty.
32     """
33     if self.is_empty():
34       raise Empty('Queue is empty')
35     answer = self._data[self._front]
36     self._data[self._front] = None                # help garbage collection
37     self._front = (self._front + 1) % len(self._data)
38     self._size -= 1
39     return answer
```

Queue in Python, Continued

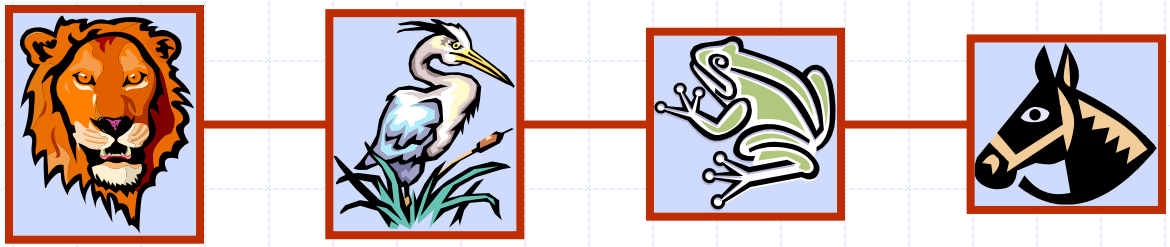
```
40    def enqueue(self, e):
41      """Add an element to the back of queue."""
42      if self._size == len(self._data):
43        self._resize(2 * len(self.data))          # double the array size
44      avail = (self._front + self._size) % len(self._data)
45      self._data[avail] = e
46      self._size += 1
47
48    def _resize(self, cap):                       # we assume cap >= len(self)
49      """Resize to a new list of capacity >= len(self)."""
50      old = self._data                            # keep track of existing list
51      self._data = [None] * cap                   # allocate list with new capacity
52      walk = self._front
53      for k in range(self._size):                 # only consider existing elements
54        self._data[k] = old[walk]                 # intentionally shift indices
55        walk = (1 + walk) % len(old)              # use old size as modulus
56      self._front = 0                             # front has been realigned
```

Application: Round Robin Schedulers

- We can implement a round robin scheduler using a queue Q by repeatedly performing the following steps:
  1. e = Q.dequeue()
  2. Service element e
  3. Q.enqueue(e)

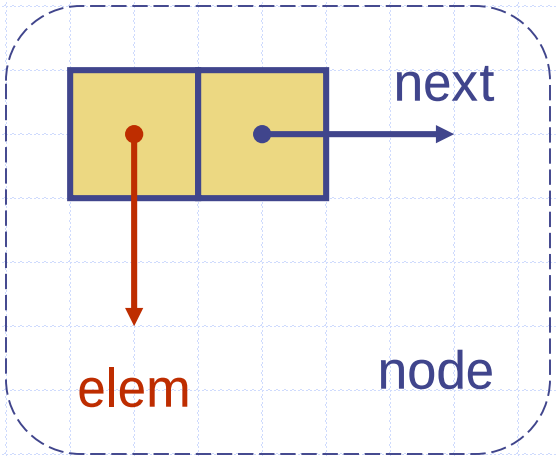


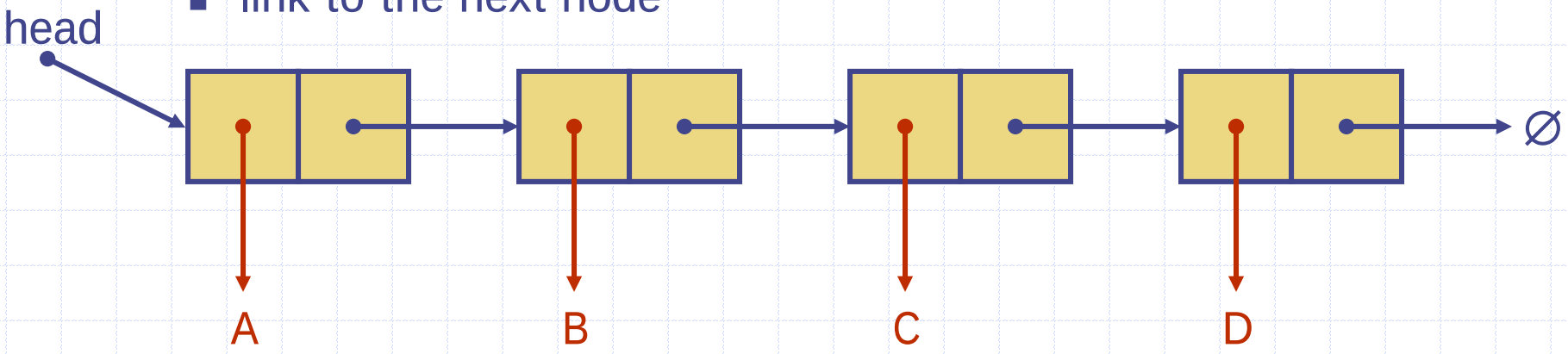
Linked Lists



Singly Linked List

- A singly linked list is a concrete data structure consisting of a sequence of nodes, starting from a head pointer
- Each node stores
  - element
  - link to the next node



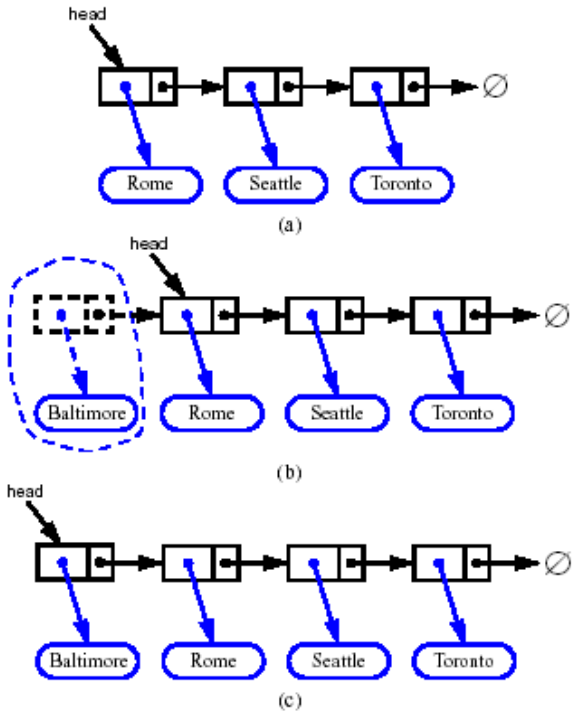


The Node Class for List Nodes

```
public class Node          {
  // Instance variables:
  private  Object element;
  private  Node next;
  /** Creates a node with null references to its element and next node. */
  public  Node()           {
    this(null,  null);
  }
  /** Creates a node with the given element and next node. */
  public  Node(Object e,  Node n)  {
      element  =  e;
      next  =  n;
  }
  // Accessor methods:
  public  Object getElement()  {
    return  element;
  }
  public  Node getNext()  {
    return  next;
  }
  // Modifier methods:
  public void  setElement(Object newElem)  {
      element  =  newElem;
  }
  public void  setNext(Node newNext)  {
      next  =  newNext;
  }
}
```

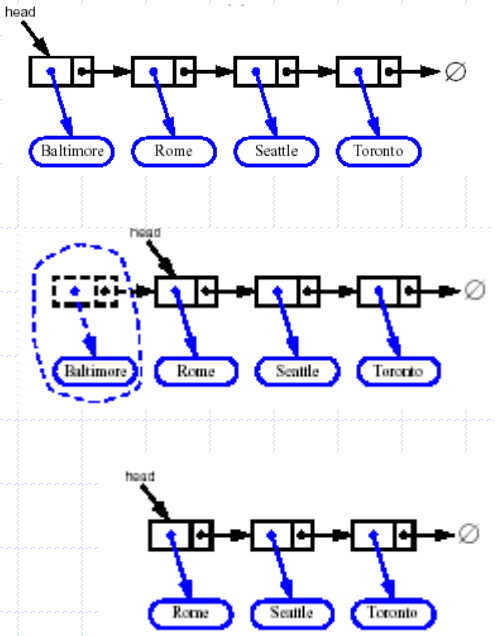
Inserting at the Head

1. Allocate a new node
2. Insert new element
3. Have new node point to old head
4. Update head to point to new node



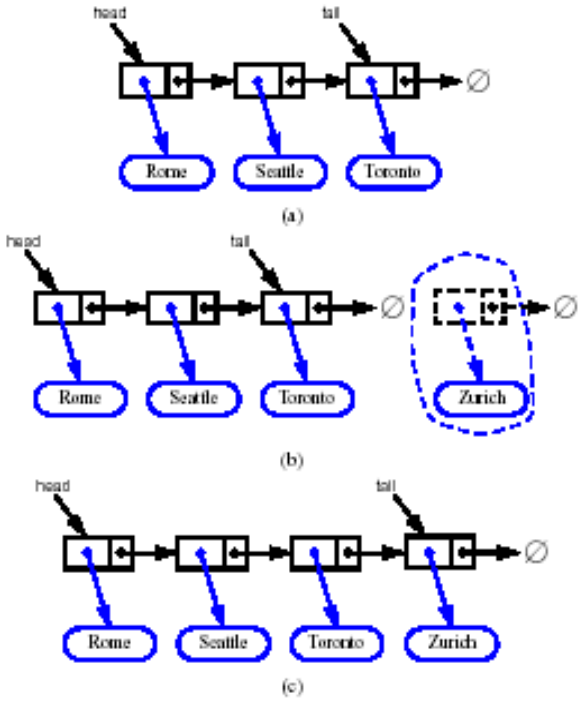
Removing at the Head

1. Update head to point to next node in the list
2. Allow garbage collector to reclaim the former first node



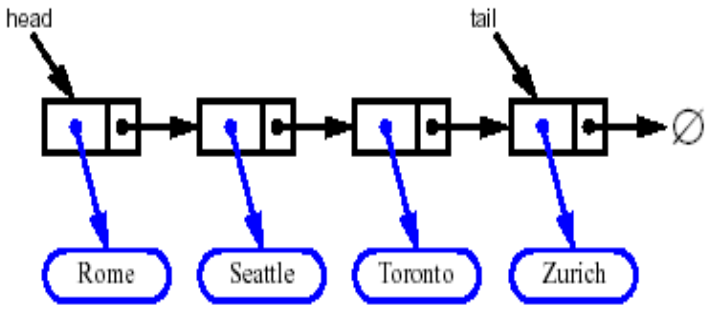
Inserting at the Tail

1. Allocate a new node
2. Insert new element
3. Have new node point to null
4. Have old last node point to new node
5. Update tail to point to new node



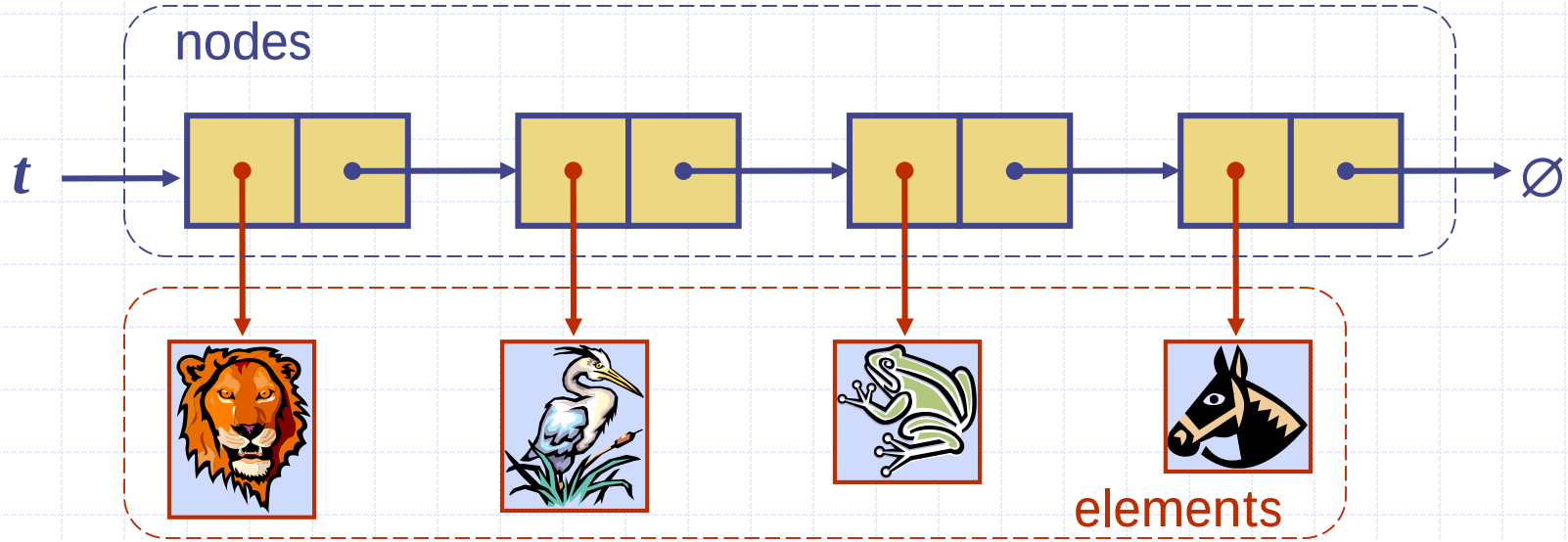
Removing at the Tail

- Removing at the tail of a singly linked list is not efficient!
- There is no constant-time way to update the tail to point to the previous node



Stack as a Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time



Linked-List Stack in Python

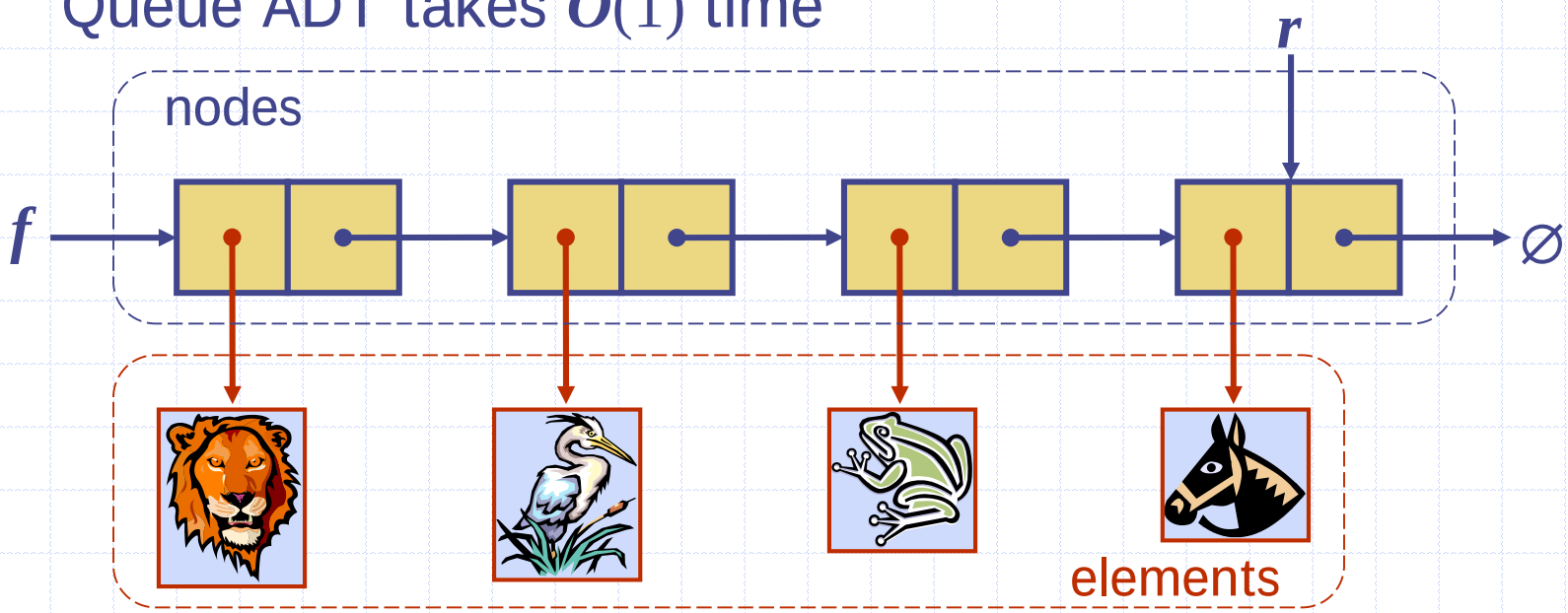
```
1  class LinkedStack:
2    """LIFO Stack implementation using a singly linked list for storage."""
3
4    #---------------------------- nested _Node class ----------------------------
5    class _Node:
6      """Lightweight, nonpublic class for storing a singly linked node."""
7      __slots__ = '_element', '_next'         # streamline memory usage
8
9      def __init__(self, element, next):      # initialize node's fields
10       self._element = element               # reference to user's element
11       self._next = next                     # reference to next node
12
13   #------------------------------ stack methods ------------------------------
14   def __init__(self):
15     """Create an empty stack."""
16     self._head = None                       # reference to the head node
17     self._size = 0                          # number of stack elements
18
19   def __len__(self):
20     """Return the number of elements in the stack."""
21     return self._size
22
```

```
23   def is_empty(self):
24     """Return True if the stack is empty."""
25     return self._size == 0
26
27   def push(self, e):
28     """Add element e to the top of the stack."""
29     self._head = self._Node(e, self._head)     # create and link a new node
30     self._size += 1
31
32   def top(self):
33     """Return (but do not remove) the element at the top of the stack.
34
35     Raise Empty exception if the stack is empty.
36     """
37     if self.is_empty():
38       raise Empty('Stack is empty')
39     return self._head._element                 # top of stack is at head of list
```

```
40   def pop(self):
41     """Remove and return the element from the top of the stack (i.e., LIFO).
42
43     Raise Empty exception if the stack is empty.
44     """
45     if self.is_empty():
46       raise Empty('Stack is empty')
47     answer = self._head._element
48     self._head = self._head._next              # bypass the former top node
49     self._size -= 1
50     return answer
```

Queue as a Linked List

- We can implement a queue with a singly linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is $O(n)$ and each operation of the Queue ADT takes $O(1)$ time

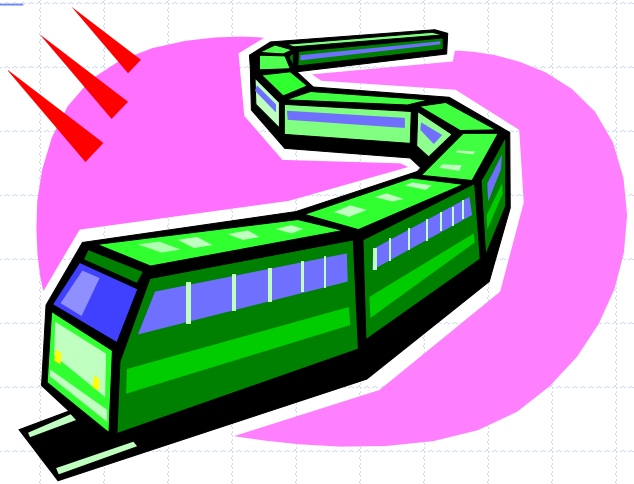


Linked-List Queue in Python

```
1  class LinkedQueue:
2    """FIFO queue implementation using a singly linked list for storage."""
3
4    class _Node:
5      """Lightweight, nonpublic class for storing a singly linked node."""
6      (omitted here; identical to that of LinkedStack._Node)
7
8    def __init__(self):
9      """Create an empty queue."""
10     self._head = None
11     self._tail = None
12     self._size = 0                          # number of queue elements
13
14   def __len__(self):
15     """Return the number of elements in the queue."""
16     return self._size
17
18   def is_empty(self):
19     """Return True if the queue is empty."""
20     return self._size == 0
21
22   def first(self):
23     """Return (but do not remove) the element at the front of the queue."""
24     if self.is_empty():
25       raise Empty('Queue is empty')
26     return self._head._element              # front aligned with head of list
```

```
27   def dequeue(self):
28     """Remove and return the first element of the queue (i.e., FIFO).
29
30     Raise Empty exception if the queue is empty.
31     """
32     if self.is_empty():
33       raise Empty('Queue is empty')
34     answer = self._head._element
35     self._head = self._head._next
36     self._size -= 1
37     if self.is_empty():                     # special case as queue is empty
38       self._tail = None                     # removed head had been the tail
39     return answer
40
41   def enqueue(self, e):
42     """Add an element to the back of queue."""
43     newest = self._Node(e, None)            # node will be new tail node
44     if self.is_empty():
45       self._head = newest                   # special case: previously empty
46     else:
47       self._tail._next = newest
48     self._tail = newest                     # update reference to tail node
49     self._size += 1
```

Doubly-Linked Lists



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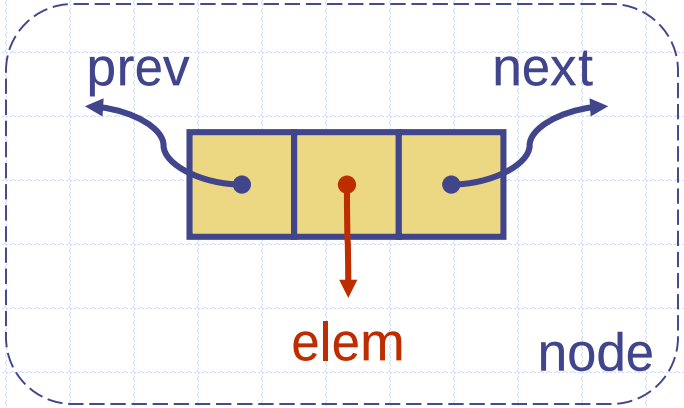
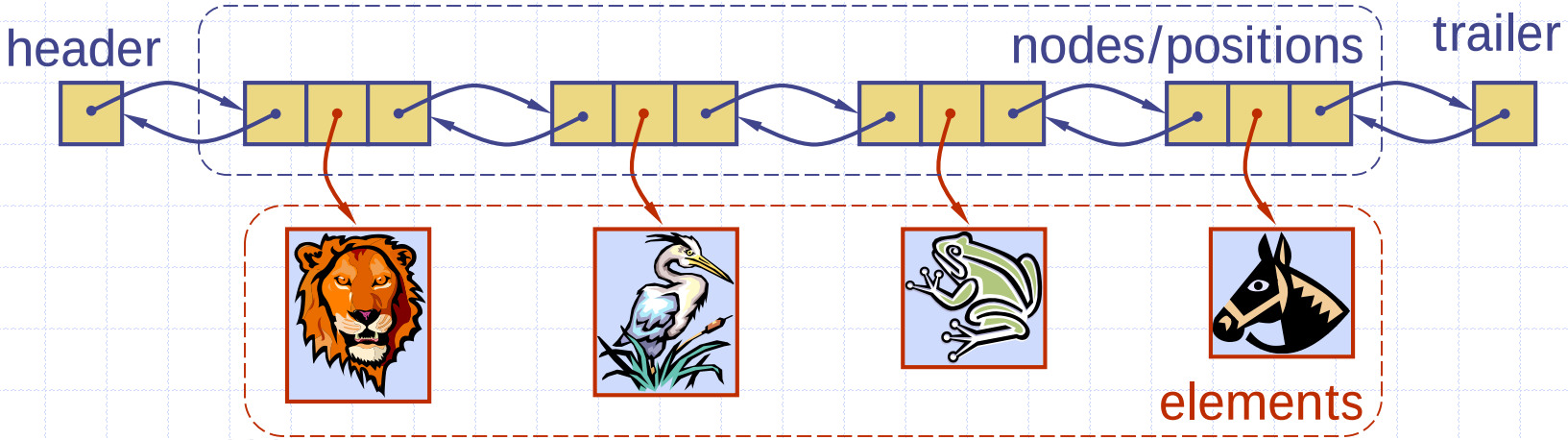
Doubly Linked List

- A doubly linked list provides a natural implementation of the Node List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes





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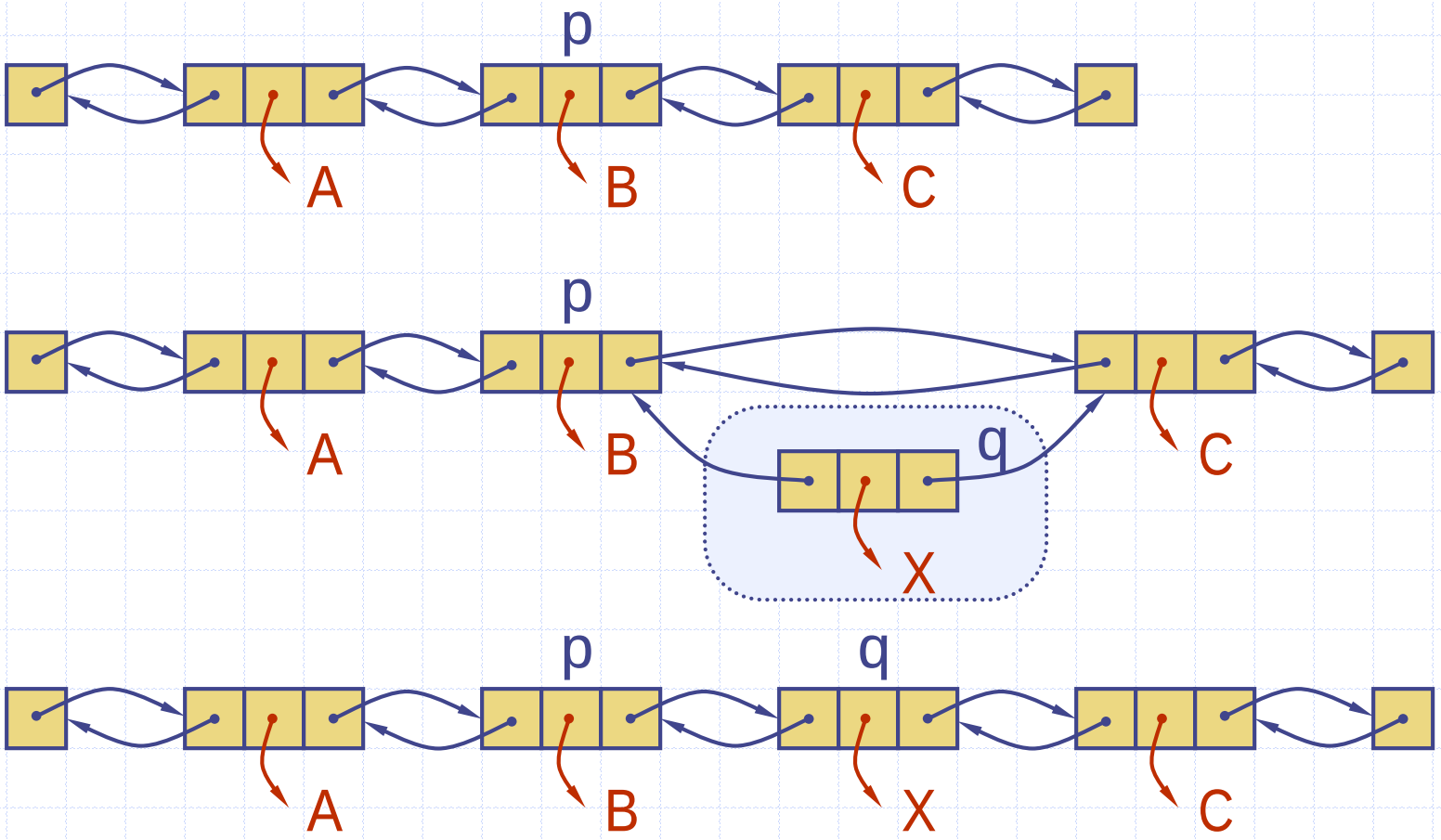
Insertion

- Insert a new node, q, between p and its successor.



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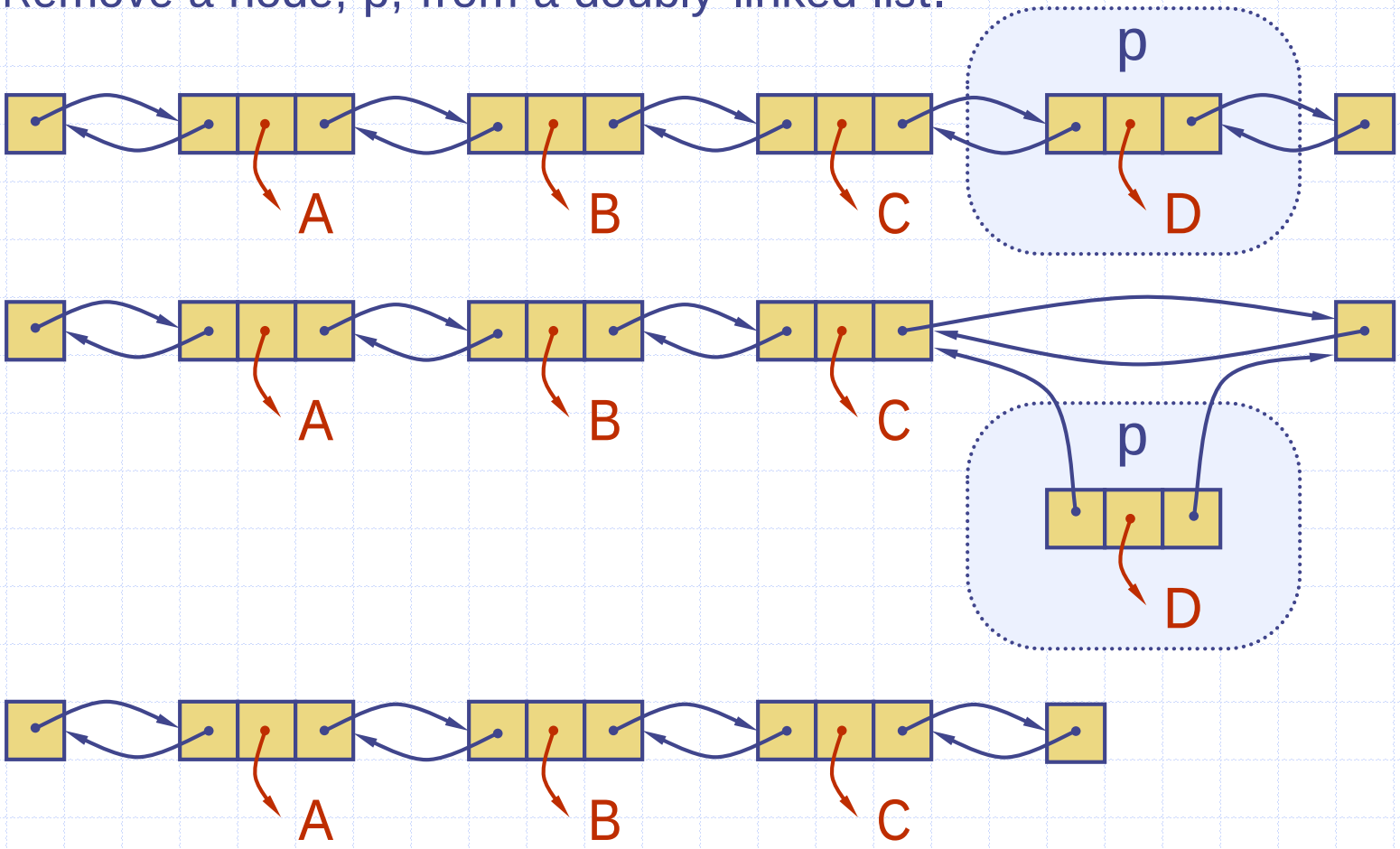
Deletion

- Remove a node, p, from a doubly-linked list.



Doubly-Linked List in Python

```
1  class _DoublyLinkedBase:
2    """A base class providing a doubly linked list representation."""
3
4    class _Node:
5      """Lightweight, nonpublic class for storing a doubly linked node."""
6      (omitted here; see previous code fragment)
7
8    def __init__(self):
9      """Create an empty list."""
10     self._header = self._Node(None, None, None)
11     self._trailer = self._Node(None, None, None)
12     self._header._next = self._trailer                # trailer is after header
13     self._trailer._prev = self._header                # header is before trailer
14     self._size = 0                                    # number of elements
15
16   def __len__(self):
17     """Return the number of elements in the list."""
18     return self._size
19
20   def is_empty(self):
21     """Return True if list is empty."""
22     return self._size == 0
23
```

```
24   def _insert_between(self, e, predecessor, successor):
25     """Add element e between two existing nodes and return new node."""
26     newest = self._Node(e, predecessor, successor)   # linked to neighbors
27     predecessor._next = newest
28     successor._prev = newest
29     self._size += 1
30     return newest
31
32   def _delete_node(self, node):
33     """Delete nonsentinel node from the list and return its element."""
34     predecessor = node._prev
35     successor = node._next
36     predecessor._next = successor
37     successor._prev = predecessor
38     self._size -= 1
39     element = node._element                          # record deleted element
40     node._prev = node._next = node._element = None   # deprecate node
41     return element                                   # return deleted element
```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Performance

- In a doubly linked list
  - The space used by a list with n elements is $O(n)$
  - The space used by each position of the list is $O(1)$
  - All the standard operations of a list run in $O(1)$ time

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Positional List

- To provide for a general abstraction of a sequence of elements with the ability to identify the location of an element, we define a **positional list** ADT.
- A position acts as a marker or token within the broader positional list.
- A position p is unaffected by changes elsewhere in a list; the only way in which a position becomes invalid is if an explicit command is issued to delete it.
- A position instance is a simple object, supporting only the following method:
  - p.element( ): Return the element stored at position p.

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Positional Accessor Operations

L.first( ):	Return the position of the first element of L, or None if L is empty.
L.last( ):	Return the position of the last element of L, or None if L is empty.
L.before(p):	Return the position of L immediately before position p, or None if p is the first position.
L.after(p):	Return the position of L immediately after position p, or None if p is the last position.
L.is_empty( ):	Return True if list L does not contain any elements.
len(L):	Return the number of elements in the list.
iter(L):	Return a forward iterator for the *elements* of the list. See Section 1.8 for discussion of iterators in Python.

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Positional Update Operations

L.add_first(e):	Insert a new element e at the front of L, returning the position of the new element.
L.add_last(e):	Insert a new element e at the back of L, returning the position of the new element.
L.add_before(p, e):	Insert a new element e just before position p in L, returning the position of the new element.
L.add_after(p, e):	Insert a new element e just after position p in L, returning the position of the new element.
L.replace(p, e):	Replace the element at position p with element e, returning the element formerly at position p.
L.delete(p):	Remove and return the element at position p in L, invalidating the position.

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Positional List in Python

```
1  class PositionalList(_DoublyLinkedBase):
2    """A sequential container of elements allowing positional access."""
3
4    #------------------------- nested Position class -------------------------
5    class Position:
6      """An abstraction representing the location of a single element."""
7
8      def __init__(self, container, node):
9        """Constructor should not be invoked by user."""
10       self._container = container
11       self._node = node
12
13     def element(self):
14       """Return the element stored at this Position."""
15       return self._node._element
16
17     def __eq__(self, other):
18       """Return True if other is a Position representing the same location."""
19       return type(other) is type(self) and other._node is self._node
20
21     def __ne__(self, other):
22       """Return True if other does not represent the same location."""
23       return not (self == other)           # opposite of __eq__
24
25   #------------------------------ utility method ------------------------------
26   def _validate(self, p):
27     """Return position's node, or raise appropriate error if invalid."""
28     if not isinstance(p, self.Position):
29       raise TypeError('p must be proper Position type')
30     if p._container is not self:
31       raise ValueError('p does not belong to this container')
32     if p._node._next is None:               # convention for deprecated nodes
33       raise ValueError('p is no longer valid')
34     return p._node
```

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Positional List in Python, Part 2

```
35    #------------------------------ utility method ------------------------------
36    def _make_position(self, node):
37      """Return Position instance for given node (or None if sentinel)."""
38      if node is self._header or node is self._trailer:
39        return None                                  # boundary violation
40      else:
41        return self.Position(self, node)             # legitimate position
42
43    #------------------------------ accessors ------------------------------
44    def first(self):
45      """Return the first Position in the list (or None if list is empty)."""
46      return self._make_position(self._header._next)
47
48    def last(self):
49      """Return the last Position in the list (or None if list is empty)."""
50      return self._make_position(self._trailer._prev)
51
52    def before(self, p):
53      """Return the Position just before Position p (or None if p is first)."""
54      node = self._validate(p)
55      return self._make_position(node._prev)
56
57    def after(self, p):
58      """Return the Position just after Position p (or None if p is last)."""
59      node = self._validate(p)
60      return self._make_position(node._next)
61
62    def __iter__(self):
63      """Generate a forward iteration of the elements of the list."""
64      cursor = self.first()
65      while cursor is not None:
66        yield cursor.element()
67        cursor = self.after(cursor)
```

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Positional List in Python, Part 3

```
68    #------------------------------ mutators ------------------------------
69    # override inherited version to return Position, rather than Node
70    def _insert_between(self, e, predecessor, successor):
71      """Add element between existing nodes and return new Position."""
72      node = super()._insert_between(e, predecessor, successor)
73      return self._make_position(node)
74
75    def add_first(self, e):
76      """Insert element e at the front of the list and return new Position."""
77      return self._insert_between(e, self._header, self._header._next)
78
79    def add_last(self, e):
80      """Insert element e at the back of the list and return new Position."""
81      return self._insert_between(e, self._trailer._prev, self._trailer)
82
83    def add_before(self, p, e):
84      """Insert element e into list before Position p and return new Position."""
85      original = self._validate(p)
86      return self._insert_between(e, original._prev, original)
87
88    def add_after(self, p, e):
89      """Insert element e into list after Position p and return new Position."""
90      original = self._validate(p)
91      return self._insert_between(e, original, original._next)
92
93    def delete(self, p):
94      """Remove and return the element at Position p."""
95      original = self._validate(p)
96      return self._delete_node(original)      # inherited method returns element
97
98    def replace(self, p, e):
99      """Replace the element at Position p with e.
100
101     Return the element formerly at Position p.
102     """
103     original = self._validate(p)
104     old_value = original._element           # temporarily store old element
105     original._element = e                   # replace with new element
106     return old_value                        # return the old element value
```

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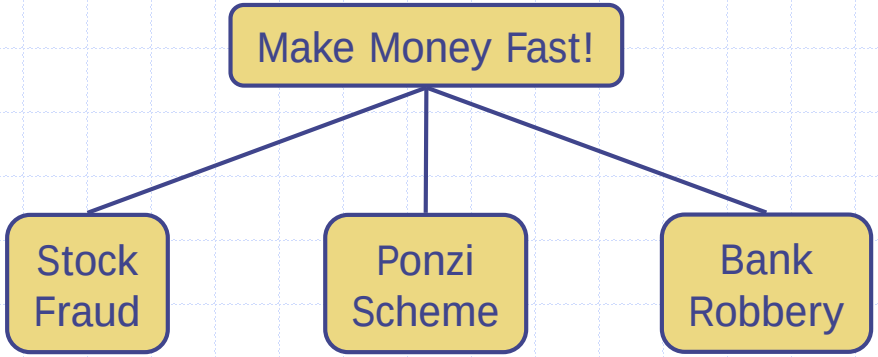
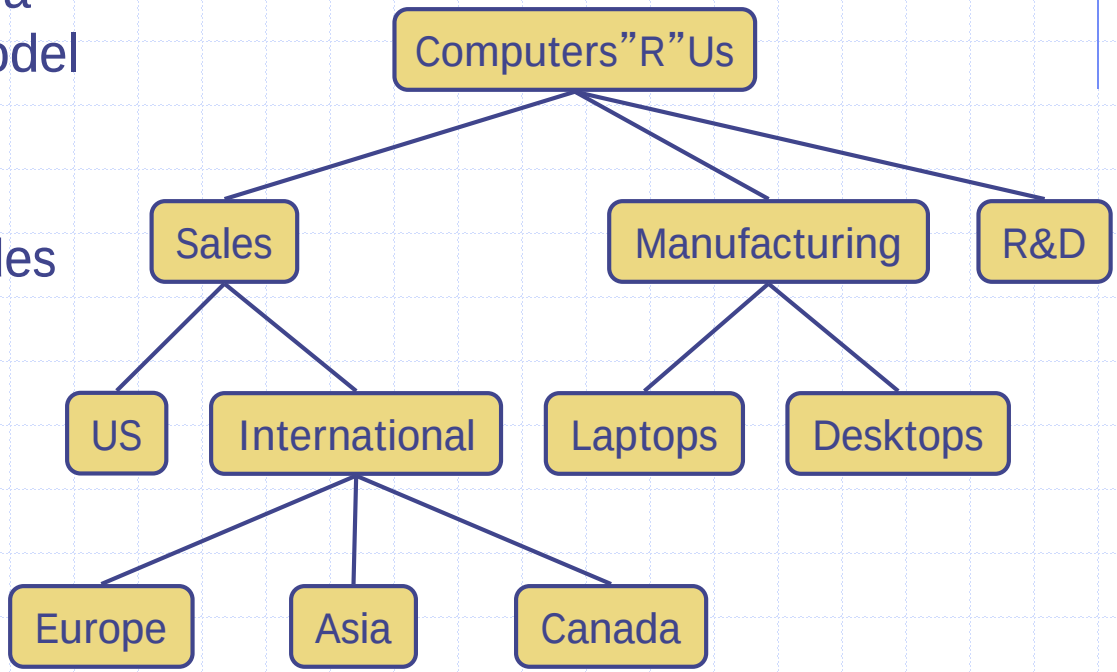
Trees



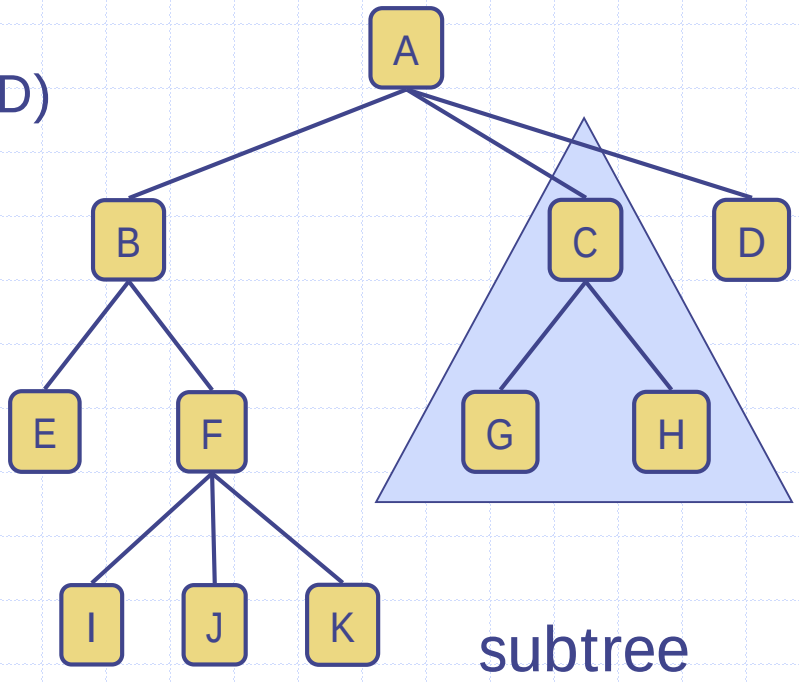
What is a Tree

- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:
  - Organization charts
  - File systems
  - Programming environments



Tree Terminology

- Root: node without parent (A)
- Internal node: node with at least one child (A, B, C, F)
- External node (a.k.a. leaf): node without children (E, I, J, K, G, H, D)
- Ancestors of a node: parent, grandparent, grand-grandparent, etc.
- Depth of a node: number of ancestors
- Height of a tree: maximum depth of any node (3)
- Descendant of a node: child, grandchild, grand-grandchild, etc.
- Subtree: tree consisting of a node and its descendants



Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - Integer len()
  - Boolean is_empty()
  - Iterator positions()
  - Iterator iter()
- Accessor methods:
  - position root()
  - position parent(p)
  - Iterator children(p)
  - Integer num_children(p)

- Query methods:
  - Boolean is_leaf(p)
  - Boolean is_root(p)
- Update method:
  - element replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Abstract Tree Class in Python

```
1  class Tree:
2    """Abstract base class representing a tree structure."""
3
4    #--------------------------- nested Position class ---------------------------
5    class Position:
6      """An abstraction representing the location of a single element."""
7
8      def element(self):
9        """Return the element stored at this Position."""
10       raise NotImplementedError('must be implemented by subclass')
11
12     def __eq__(self, other):
13       """Return True if other Position represents the same location."""
14       raise NotImplementedError('must be implemented by subclass')
15
16     def __ne__(self, other):
17       """Return True if other does not represent the same location."""
18       return not (self == other)           # opposite of __eq__
19
```

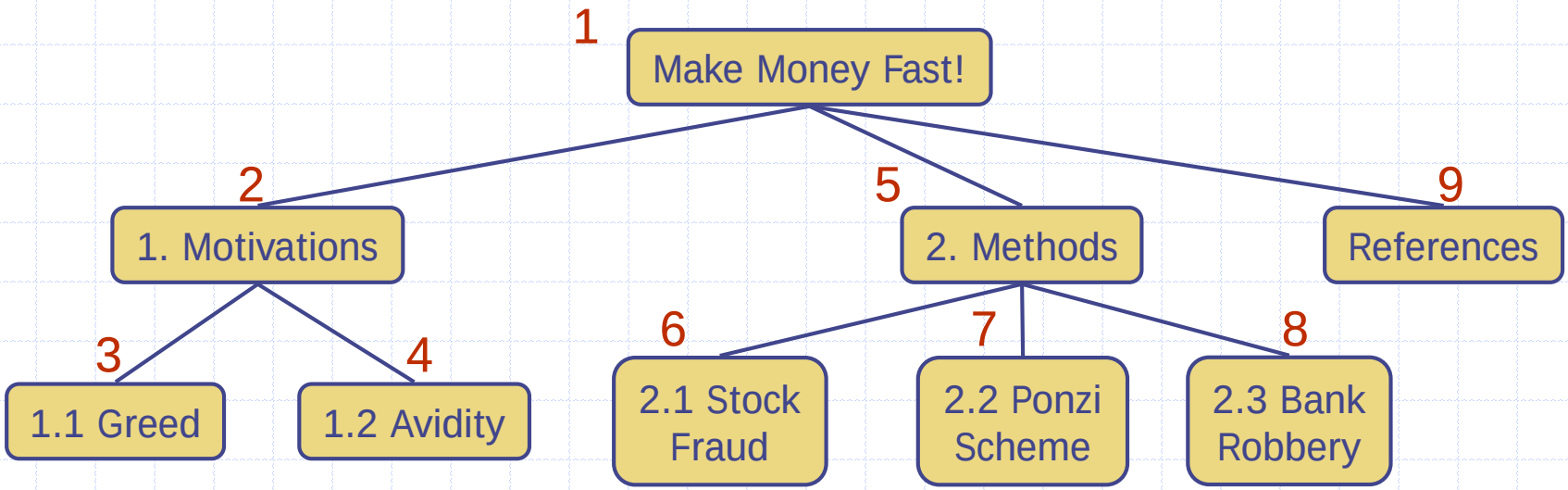
```
20   # ---------- abstract methods that concrete subclass must support ----------
21   def root(self):
22     """Return Position representing the tree's root (or None if empty)."""
23     raise NotImplementedError('must be implemented by subclass')
24
25   def parent(self, p):
26     """Return Position representing p's parent (or None if p is root)."""
27     raise NotImplementedError('must be implemented by subclass')
28
29   def num_children(self, p):
30     """Return the number of children that Position p has."""
31     raise NotImplementedError('must be implemented by subclass')
32
33   def children(self, p):
34     """Generate an iteration of Positions representing p's children."""
35     raise NotImplementedError('must be implemented by subclass')
36
37   def __len__(self):
38     """Return the total number of elements in the tree."""
39     raise NotImplementedError('must be implemented by subclass')
```

```
40   # ---------- concrete methods implemented in this class ----------
41   def is_root(self, p):
42     """Return True if Position p represents the root of the tree."""
43     return self.root( ) == p
44
45   def is_leaf(self, p):
46     """Return True if Position p does not have any children."""
47     return self.num_children(p) == 0
48
49   def is_empty(self):
50     """Return True if the tree is empty."""
51     return len(self) == 0
```

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

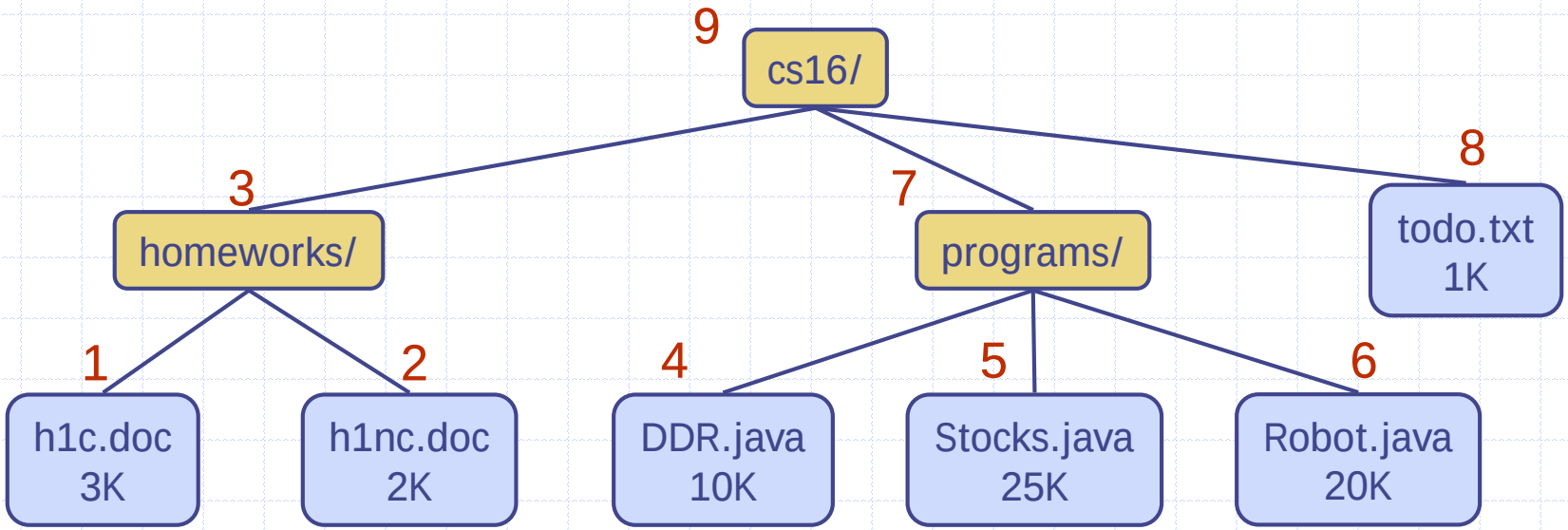
Algorithm ***preOrder(v)***
    visit(v)
    for each child ***w*** of ***v***
        preorder **(*w*)**



Postorder Traversal

- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

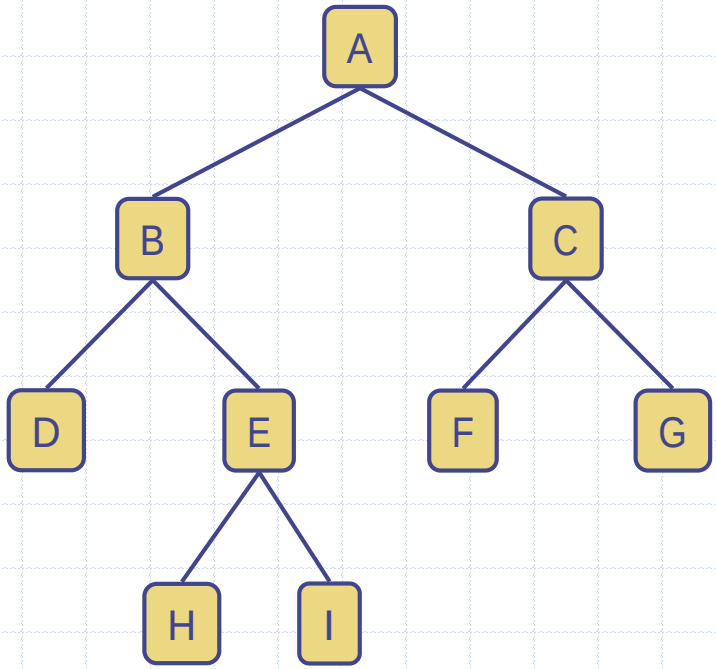
Algorithm ***postOrder(v)***
  for each child ***w*** of ***v***
    postOrder ***(w)***
  visit(v)



Binary Trees

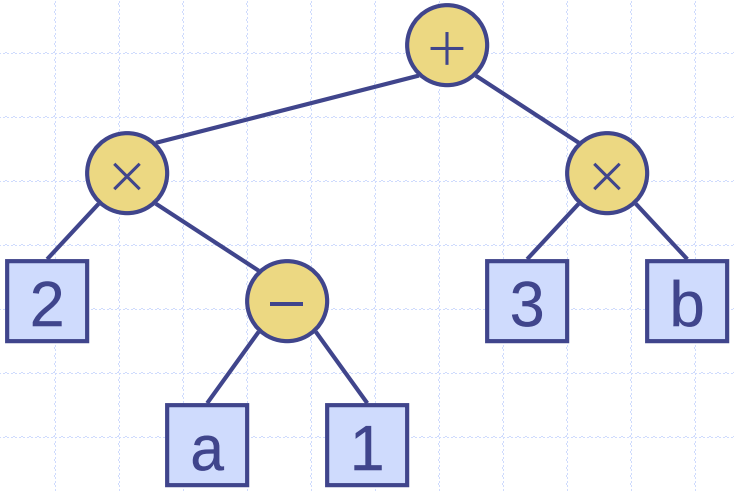
- A binary tree is a tree with the following properties:
  - Each internal node has at most two children (exactly two for proper binary trees)
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree

- Applications:
  - arithmetic expressions
  - decision processes
  - searching



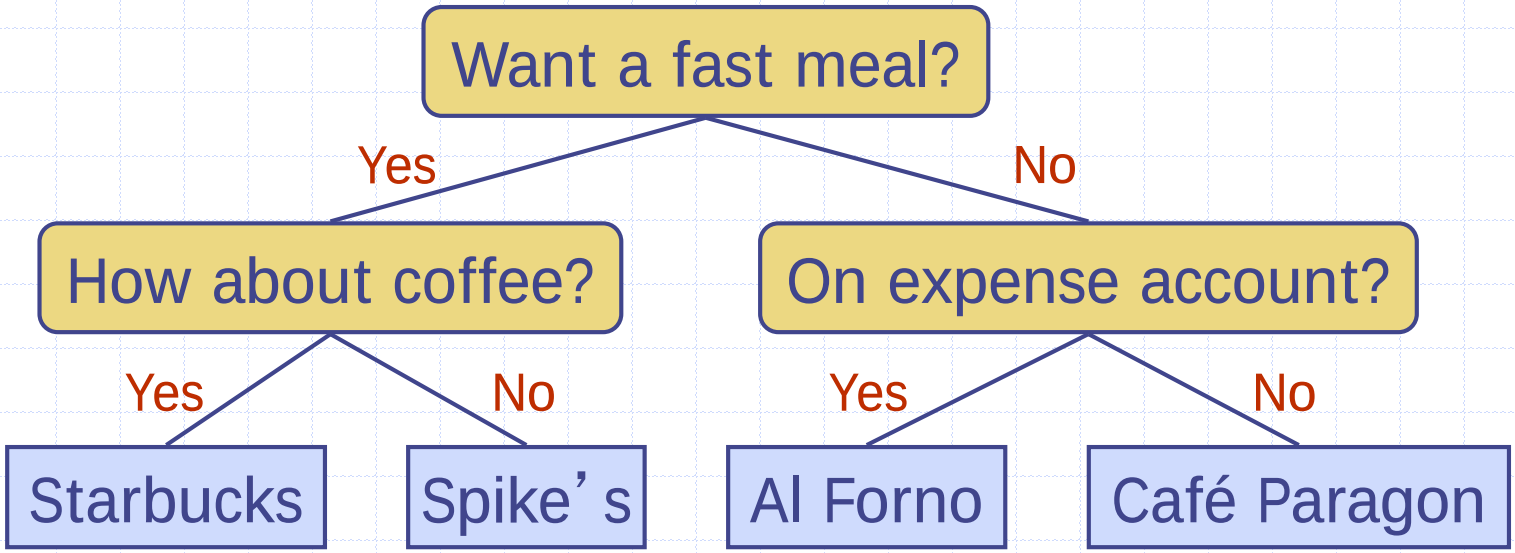
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



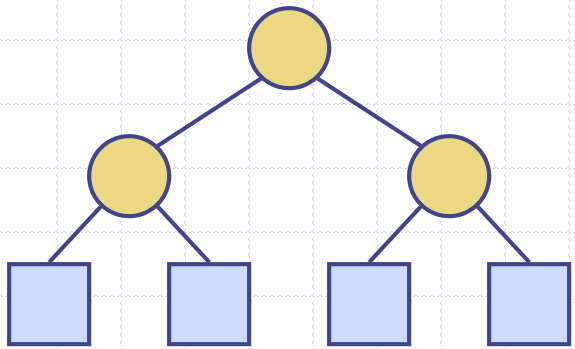
Decision Tree

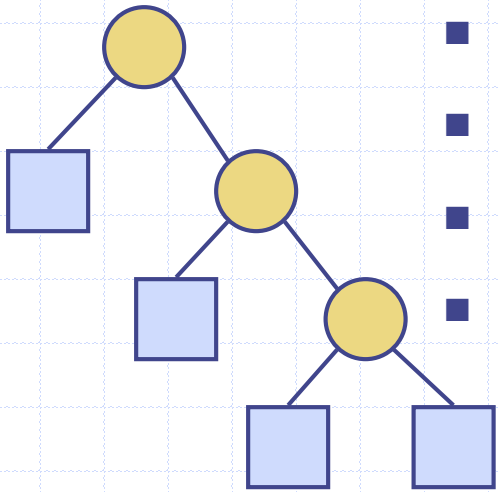
- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Properties of Proper Binary Trees

- Notation
  - n number of nodes
  - e number of external nodes
  - i number of internal nodes
  - h height





- Properties:
  - $e = i + 1$
  - $n = 2e - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $e \leq 2^h$
  - $h \geq \log_2 e$
  - $h \geq \log_2 (n + 1) - 1$

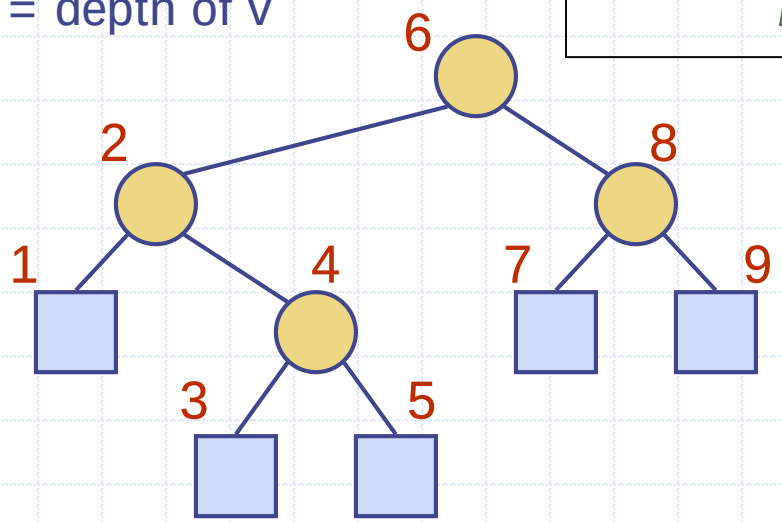
BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position left(p)
  - position right(p)
  - position sibling(p)
- Update methods may be defined by data structures implementing the BinaryTree ADT

Inorder Traversal

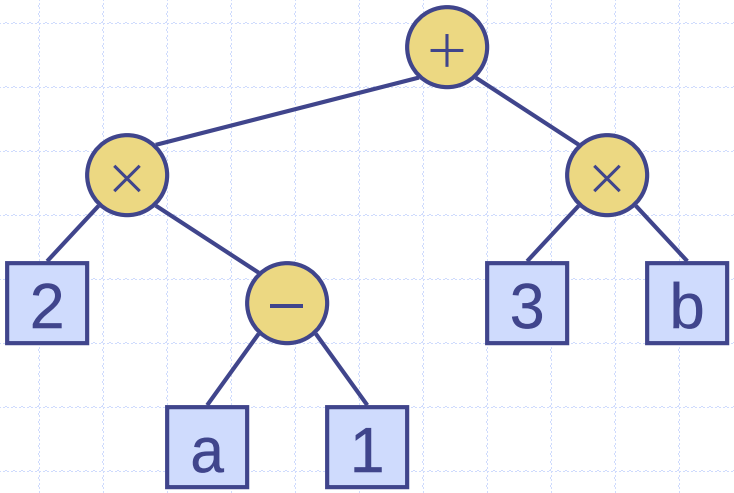
- In an inorder traversal a node is visited after its left subtree and before its right subtree
- Application: draw a binary tree
  - x(v) = inorder rank of v
  - y(v) = depth of v

Algorithm *inOrder*(*v*)
  if *v* **has a left child**
    inOrder (*left* (*v*))
  visit(*v*)
  if *v* **has a right child**
    inOrder (*right* (*v*))



Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print "(" before traversing left subtree
  - print ")" after traversing right subtree

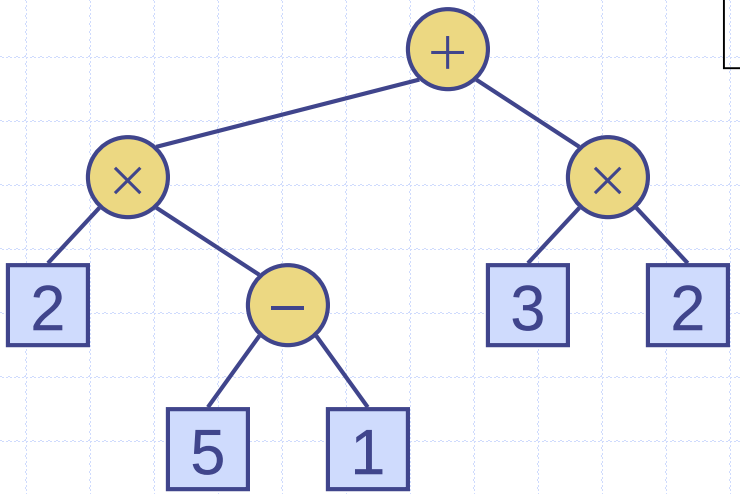


Algorithm *printExpression*(*v*)
  if *v* **has a left child**
    print("(")
    inOrder (*left*(*v*))
  print(*v.element* ())
  if *v* **has a right child**
    inOrder (*right*(*v*))
    print (")")

$((2 \times (a - 1)) + (3 \times b))$

Evaluate Arithmetic Expressions

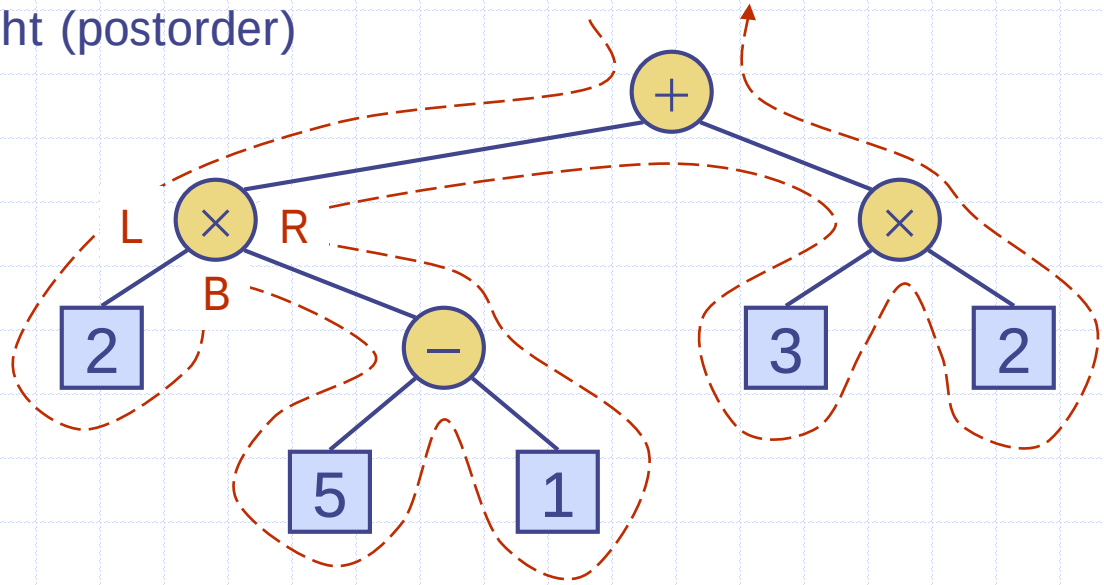
- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees



Algorithm *evalExpr*(*v*)
  if *is_leaf* (*v*)
    return *v.element* ()
  else
    $x \leftarrow$ *evalExpr*(*left* (*v*))
    $y \leftarrow$ *evalExpr*(*right* (*v*))
    $\lozenge \leftarrow$ operator stored at *v*
    return $x \lozenge y$

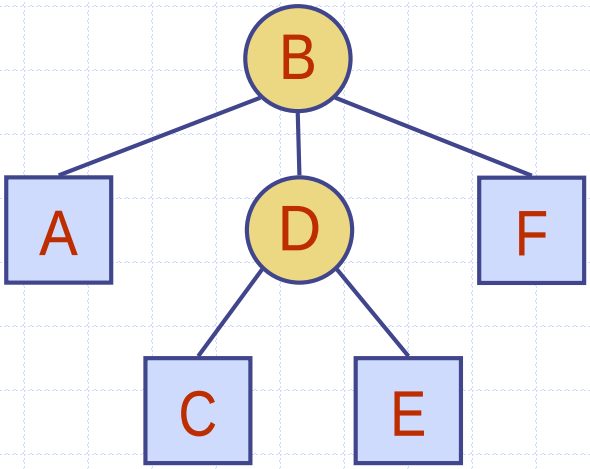
Euler Tour Traversal

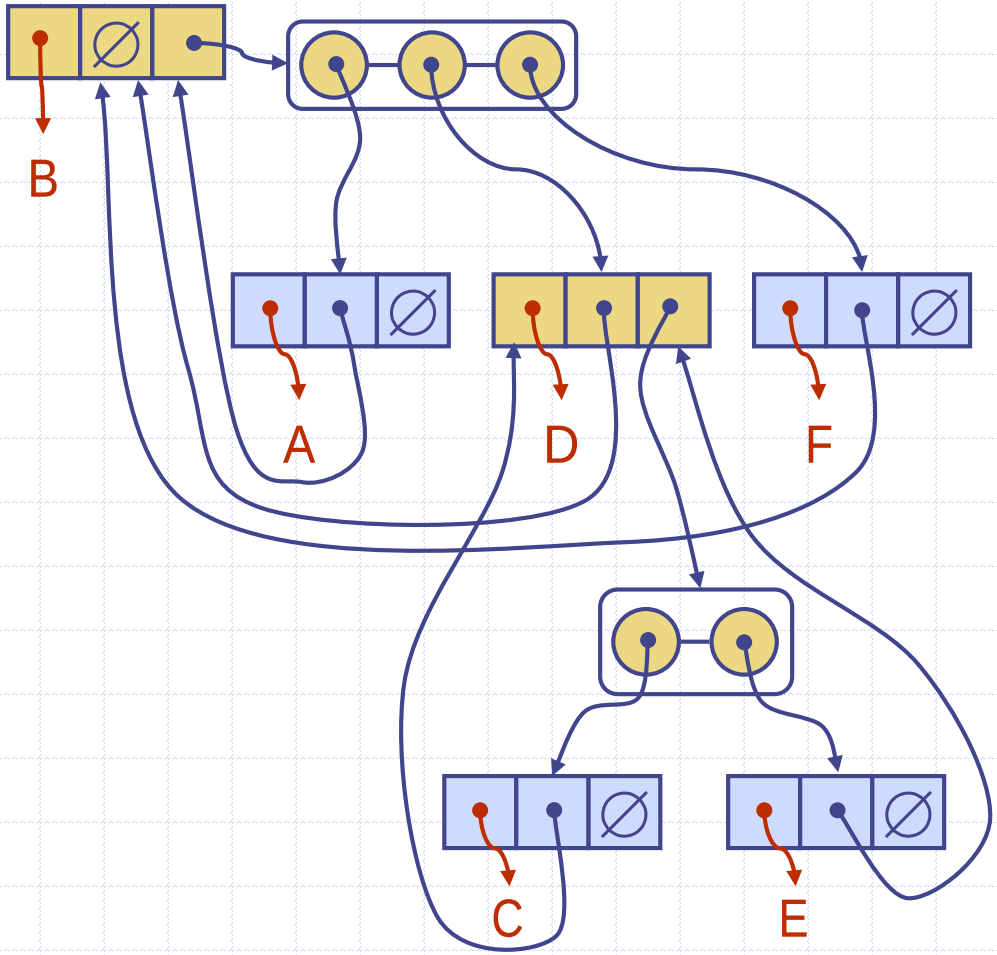
- Generic traversal of a binary tree
- Includes a special cases the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Linked Structure for Trees

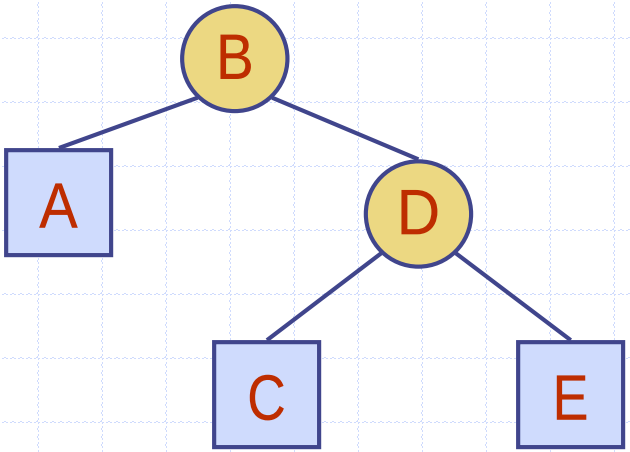
- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT

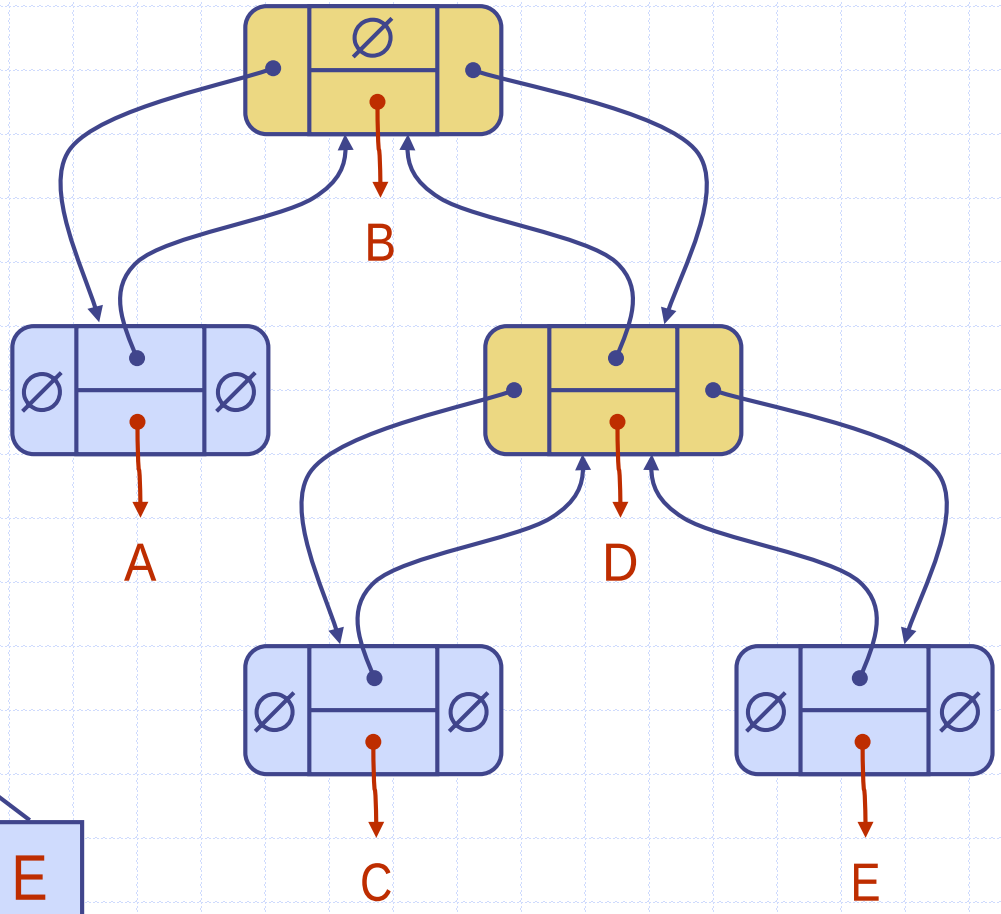




Linked Structure for Binary Trees

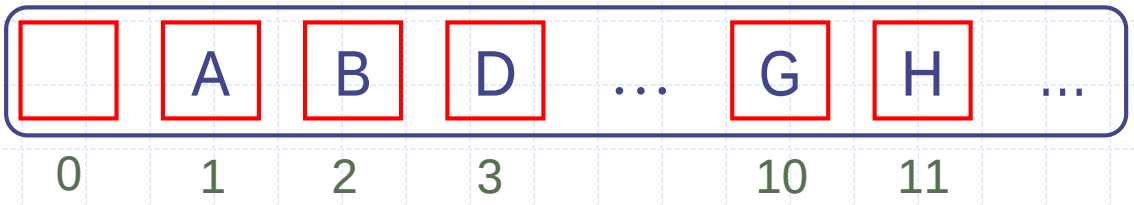
- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT

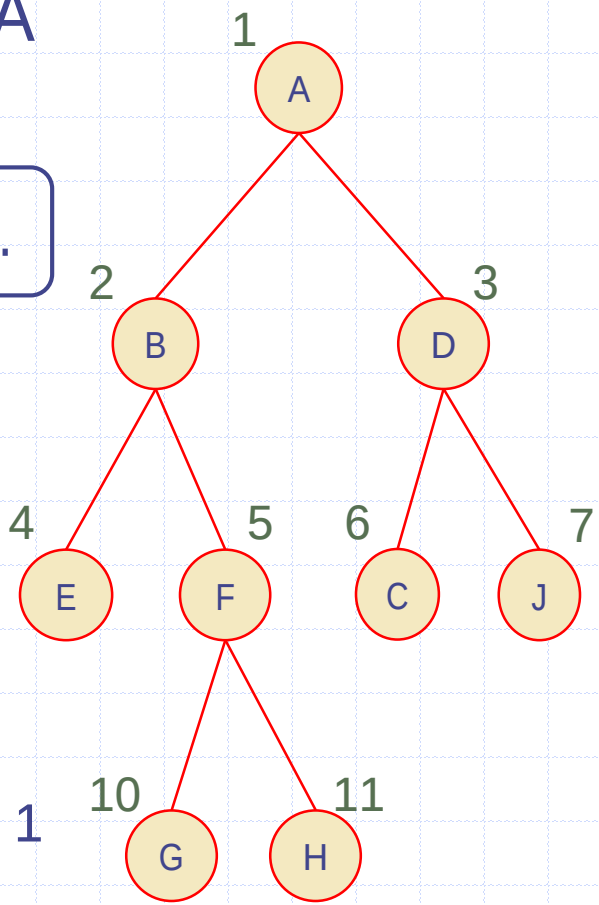




Array-Based Representation of Binary Trees

- Nodes are stored in an array A





- Node v is stored at A[rank(v)]
  - rank(root) = 1
  - if node is the left child of parent(node), rank(node) = 2 · rank(parent(node))
  - if node is the right child of parent(node), rank(node) = 2· rank(parent(node)) + 1