Globally Optimal Matching With Shape Priors

HOUDBERT Aurelien - OUBELLA Aymane - EL AICHI Mohammed

Graphical Models: Discrete Inference and Learning
CentraleSupélec

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Overview

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Introduction

Image segmentation and shape matching are two very similar tasks; in this work we implemented an algorithm able to perform these two tasks simultaneously based on the work of T. Schoenemann and D. Cremers. We tested our solution on different small shapes and images, testing scaling and translation invariance properties.

Optimization Problem

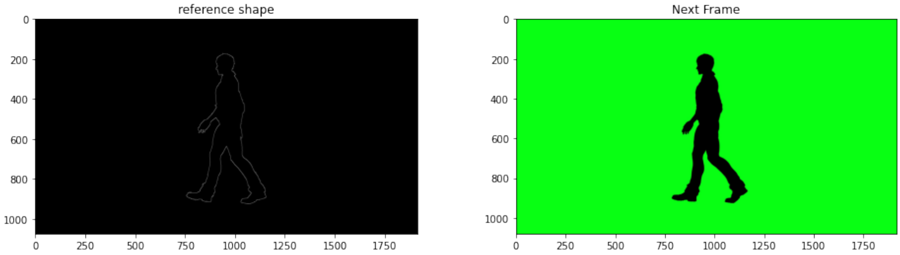


Figure: matching shape C from the reference image to next image

Given $C : [0, l(C)] \mapsto \mathrm{R}^2$ and $S : [0, l(S)] \mapsto \mathrm{R}^2$
Find : strictly increasing diffeomorphism $m : [0, l(C)] \mapsto [0, l(S)]$ which maps C into S.

Optimization Problem

The optimal matching can be found by minimizing a ratio energy :

$$\min_{C,m} \frac{E(C,m)}{l(C)}$$

Which can be discretised as :

$$\min_{C} \frac{\sum_{e \in C} n(e)}{\sum_{e \in C} d(e)}$$

The Optimal cycle of this optimization problem can be found via the Minimum Ratio Cycle Algorithm.

Optimization Problem

The numerator and the denominator are expressed by :

$$n(e) = \frac{1}{2}|q-p|(g(p)+g(q)) + \nu|q-p||\alpha_S(s) - \phi(q-p)|^2 + \lambda$$

$$d(e) = |q-p|$$

With the **edge indicator function**:

$$g(x) = \frac{1}{1+|\nabla I(x)|}$$

$\phi(.)$ denotes the angle of a vector
$\alpha_S(s)$ is the tangent angle of the shape S at s.
And ν, λ are regularization factors

Graph Structure

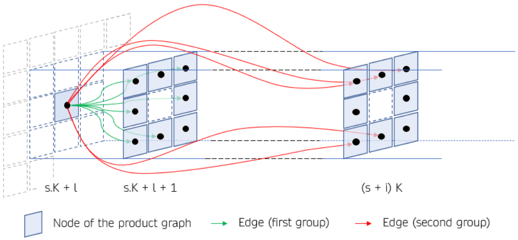
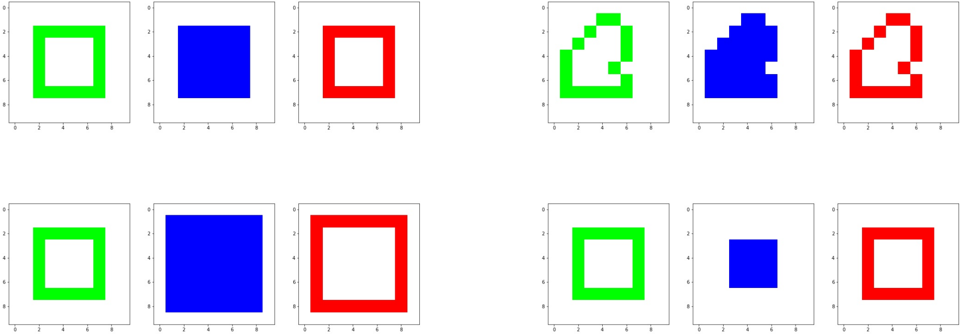


Figure: Graph structure representation. Each instance is the size of the image I. **Green** edges are of the first category and **red** edges are of the second category

The Algorithm

τ = initial_value; (upper bound)
C^* = None;
while *exist_negative_cycle(graph, τ)* **do**
    τ = negative_cycle_tau ;
    C^* = negative_cycle ;
end
return τ, C^*

Results on simple shapes



Shortcomings of our solution

- Computationally expensive resulting in very long execution time.
- Difficulty to perform down scaling (maybe because of the small size images used during the test.
- Does not handle rotations.

Conclusions

- The graph structure makes the shape matching invariant to translation or up scaling but show defaults on the down scaling task.
- It would be interesting to test the implementation on long and complex shapes to really measure its performances.
- The energy function doesn't allow to handle rotations.

References

[1] I. H.Jermynet H.Ishikawa. “GLOBALLY OPTIMAL REGIONS AND BOUNDARIES AS MINIMUMRATIO WEIGHT CYCLES.” (2001)
[2] NimrodMegiddo. “Combinatorial optimization with rational objective functions.” (1979)
[3] ThomasSchoenemannet DanielCremers. “Globally Optimal Image Segmentation with an Elastic ShapePrior.” (2007)
[4] ThomasSchoenemannet DanielCremers. “Matching Non-rigidly Deformable Shapes Across Images : A Globally Optimal Solution.” (2008)