Formal Verification of Just-in-Time Compilation

Aurèle Barrière

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Billions of people use JITs every day.

Bugs lead to vulnerabilities

May 2019: Coinbase company workers receive an email with a link.
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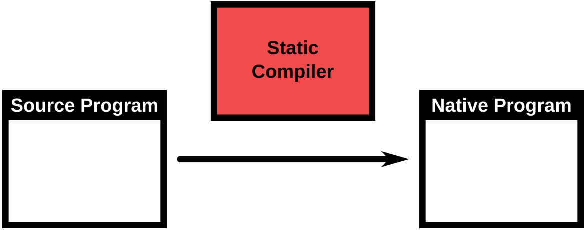
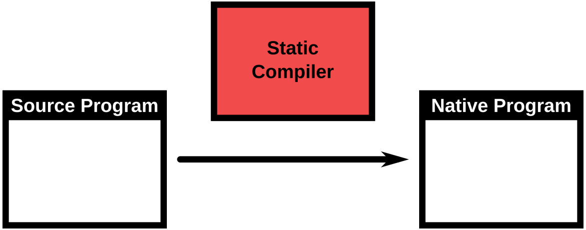
Bugs in JITs

These vulnerabilities rely on **bugs** in **Just-in-Time compilers** (JITs), and allow an attacker to execute malicious code.



Static compilation

Transforming a source program into an equivalent native program.
The resulting program can then be executed independently.
To trust this execution: trust both the source program and the compiler.



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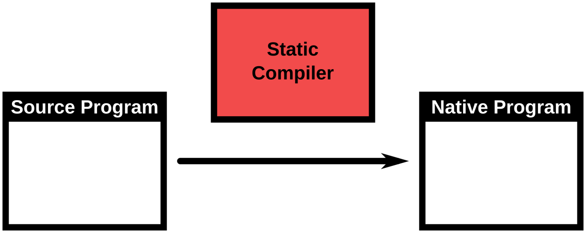
Despite testing, static compilers still contain bugs

PLDI 2011: Hundreds of bugs found in GCC and LLVM [Yang et al. 2011].

Static compilation theory is well established.



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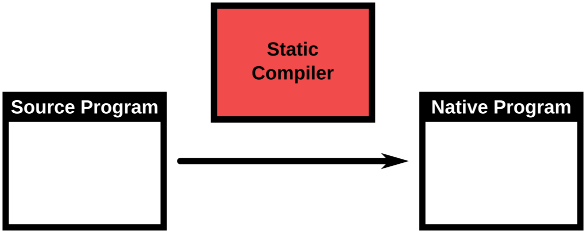

Program Semantics

We can formally define the behavior of a program, then prove the correctness of code transformations.

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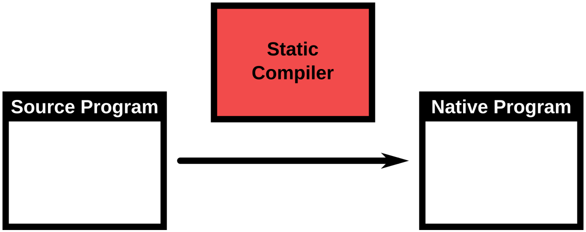

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Formally Verified Static Compilers

CompCert [Leroy 2006], CakeML [Kumar et al. 2014], VeLLVM [Zhao et al. 2012].
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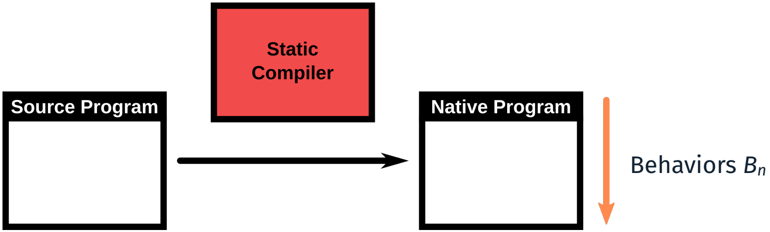

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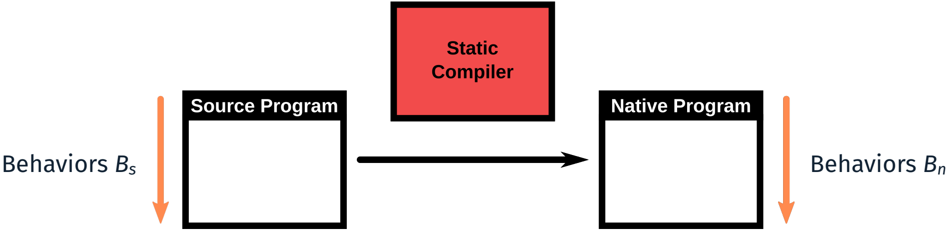

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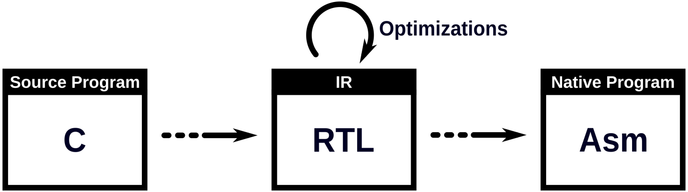
From C to assembly (x86, ARM, PowerPC, RiscV). 9 Intermediate Representations (IRs).



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Executable: can be extracted as an OCaml program.
Modular proof: each transformation proved independently.

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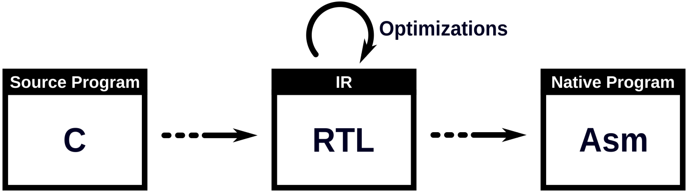

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Formal Verification brings substantial guarantees

PLDI 2011: No bugs found in the formally verified part of CompCert [Yang et al. 2011].

What about Just-in-Time Compilation ?

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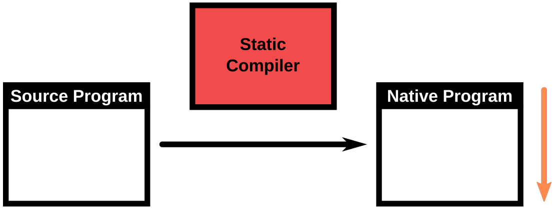

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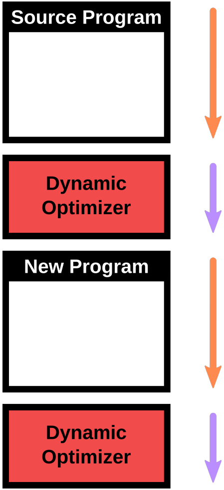

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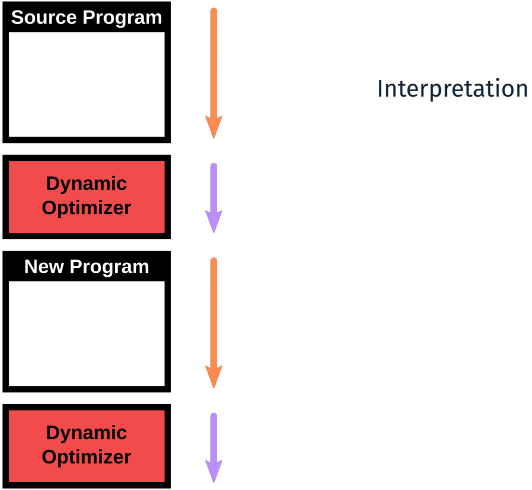

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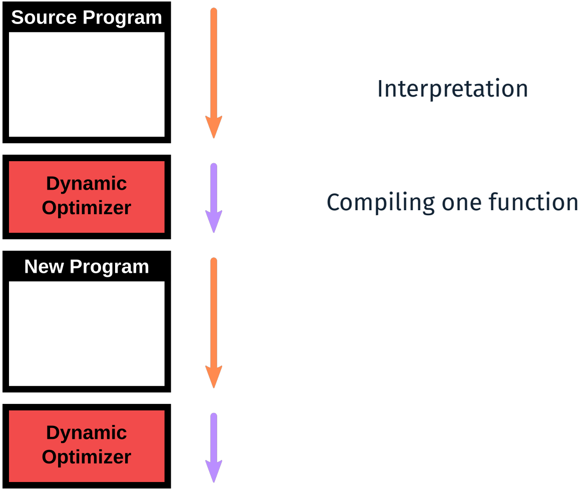

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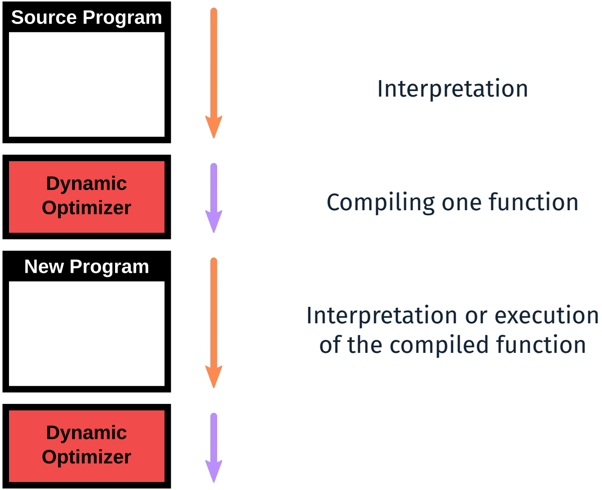

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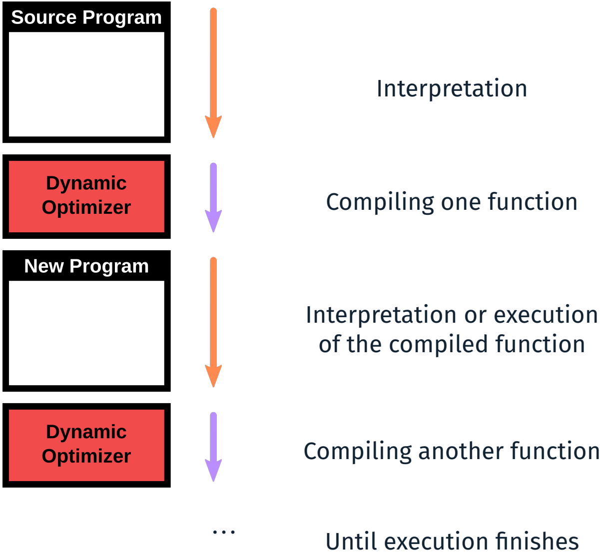

Executing operation `left + right`, an example from JavaScriptCore [WebKit 2020].

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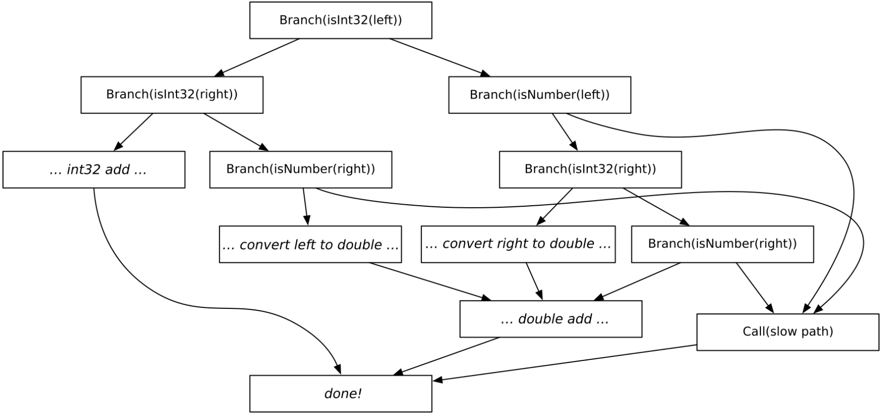

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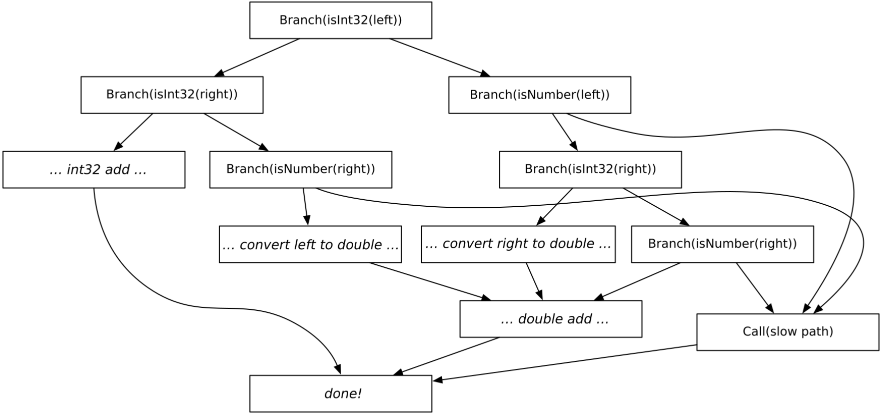

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Type Speculation

If we can speculate on the types of `left` and `right`, the graph is reduced to a single node.

Execution Timeline

Interpretation of `f`

Execution Stack

Interpreter: `f`

Program

```
Function f():
 while(...):
  g()

Function g():
 g1
 g2
```

Execution Timeline

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Execution Timeline

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Optimization of `g`

Execution Stack

Interpreter: `f`

Optimizing Compiler

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```
Function f():
 while (...):
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Function g():
 g1
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Function g_x86():
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 Speculation (x is int32)
 g2'
```

Execution Timeline

Interpretation of `f`

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Optimization of `g`

Call to `g_x86`

Execution Stack

Interpreter: `f`

Native: `g_x86`

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Execution Timeline

Interpretation of f

Interpretation of g

Optimization of g

Call to g_x86

Deoptimization to g

Execution Stack

Interpreter: f
Native: g_x86

Speculation fails

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Executing a program with a JIT with speculative optimizations

Execution Timeline

Interpretation of `f`

Interpretation of `g`

Optimization of `g`

Call to `g_x86`

Deoptimization to `g`

Execution Stack

Interpreter: `f`
~~Native: `g_x86`~~
Interpreter: `g`

Speculation fails

On-stack replacement

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Call to `g_x86`

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Execution Stack

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Deoptimization requires the JIT to synthesize interpreter stackframes in the middle of a function.

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Related Work in Formalizing JITs

- [Myreen 2010]: Verified non-optimizing x86 JIT.
- [Guo et al. 2011]: Soundness of trace optimizations in JITs.
- [Brown et al. 2020]: DSL to write verified range analyses in JITs.
- [Flückiger et al. 2018]: **Sourir**, an IR with speculation, with correctness proofs of speculative optimizations.

In this thesis, we investigate the feasability of developing formally verified JITs.

Dynamic Optimizations: JITs interleave optimizations with the program execution.

Speculative Optimizations: JITs insert and manipulate speculations in their programs.

Impure Components: Some JITs components are difficult to write in Coq (*e.g.* executing native code).

Static Compiler Reuse: Verified JITs should reuse formally verified compilers.

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Our approach: adapt the simulation proof methodology of CompCert.

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In this presentation:

- A) Our JIT Design
- B) Dynamic Optimizations
- C) Speculative Optimizations

Designing a Formally Verified JIT in Coq

CompCert Theorem

If we compile a program whose behaviors are free of errors, then any behavior of the compiled program is a behavior of the source program.

```
Theorem transf_c_program_is_refinement:
  ∀ p tp, transf_c_program p = OK tp → (* compilation of p produced tp *)
  (∀ beh,program_behaves (source_sem p) beh → not_wrong beh) → (* p has no wrong behaviors*)
  (∀ beh,program_behaves (asm_sem tp) beh → program_behaves (source_sem p) beh).
  (* every behavior of tp is a behavior of p *)
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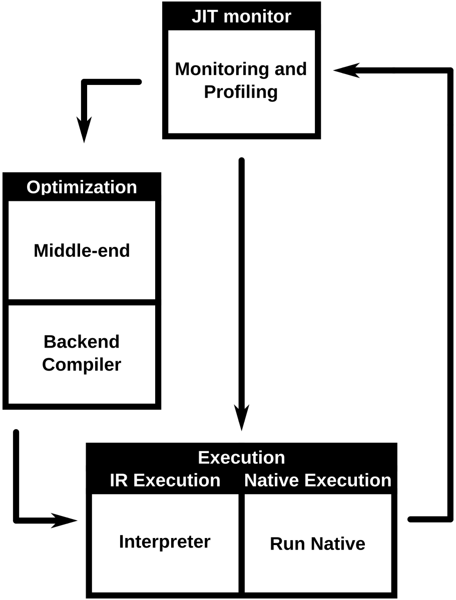
JIT Theorem

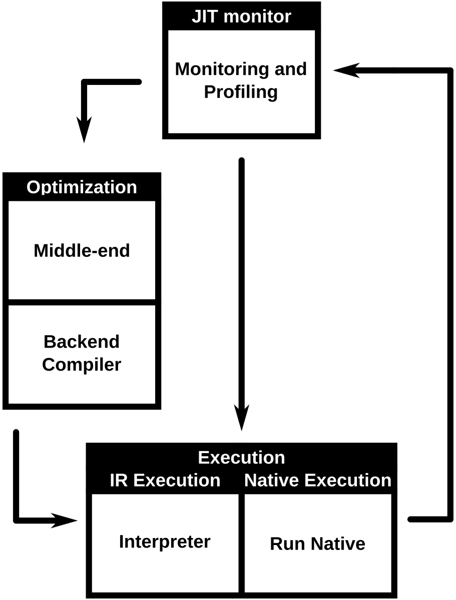
If the semantics (`source_sem`) of the program is free of errors, then any behavior of the JIT on that program (`jit_sem`) is a behavior of the program.

```
Theorem jit_behavior_refinement:
  ∀ p,
  (∀ beh,program_behaves (source_sem p) beh → not_wrong beh) → (* p has no wrong behaviors*)
  (∀ beh,program_behaves (jit_sem p) beh → program_behaves (source_sem p) beh).
  (* every behavior of the JIT executing p is a behavior of p *)
```



JIT loop

The **monitor** chooses execution or optimization. **Profiling**: inspects execution and suggests speculations and optimizations.

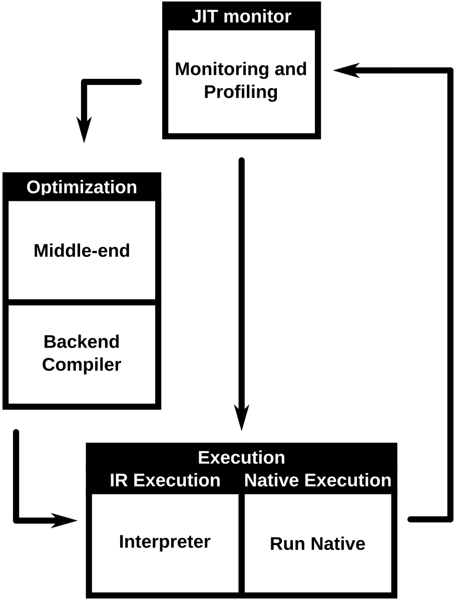


Middle-end Optimizer

From the IR to the IR.
Inserts speculation.

Backend Compiler

Generates native code.
Use the CompCert backend from RTL to x86.



Interpreter
Interpret the IR code.

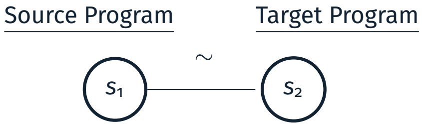
Native Code Execution
Run the generated code.

Returns to the monitor on function calls, function returns or deoptimizations.

Correctness of a JIT with dynamic optimizations

Backward Simulation in CompCert: (simplified)

- design an invariant $\sim$, a relation between semantic states.
- show that each semantic step of the target program is matched with steps of the source program preserving the invariant and the observable events e.



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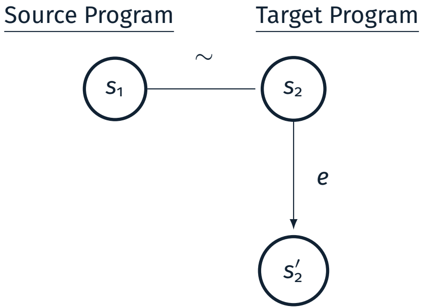

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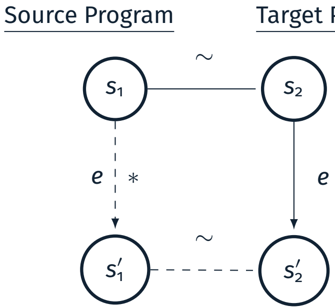

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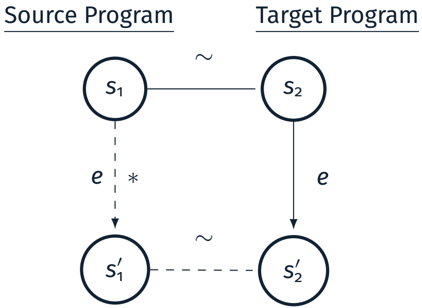

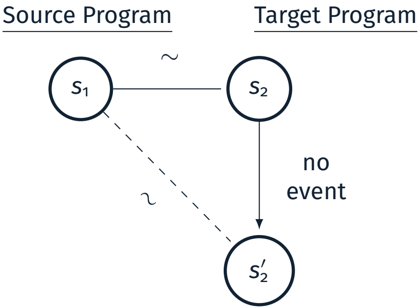

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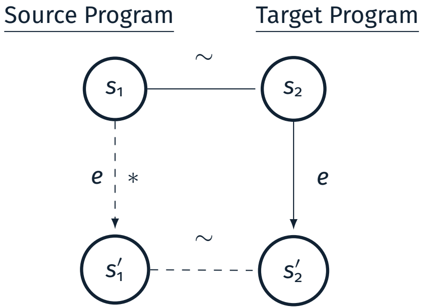
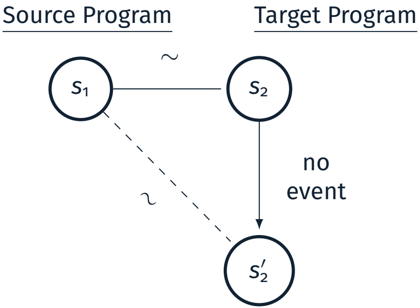
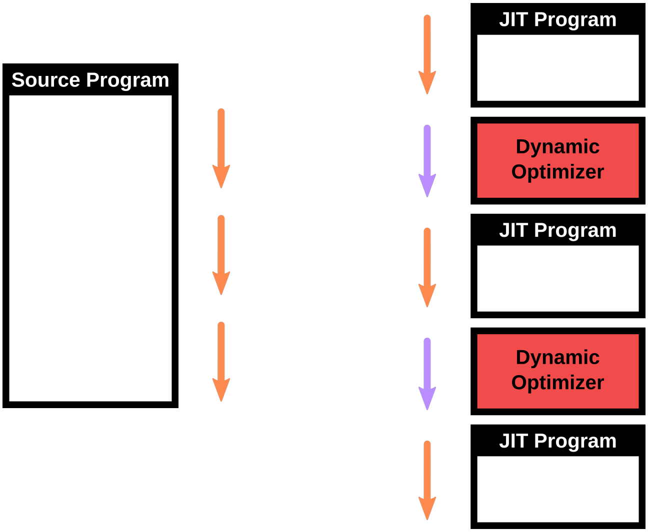




Simulations are **composable**.


Source Program
JIT Program
Dynamic Optimizer
JIT Program
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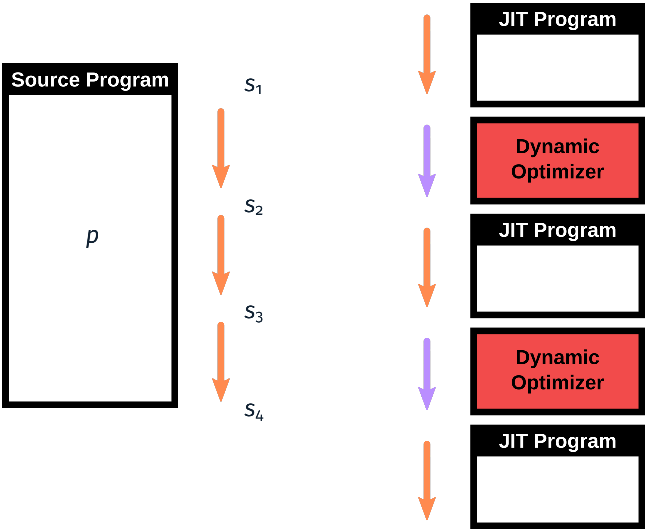

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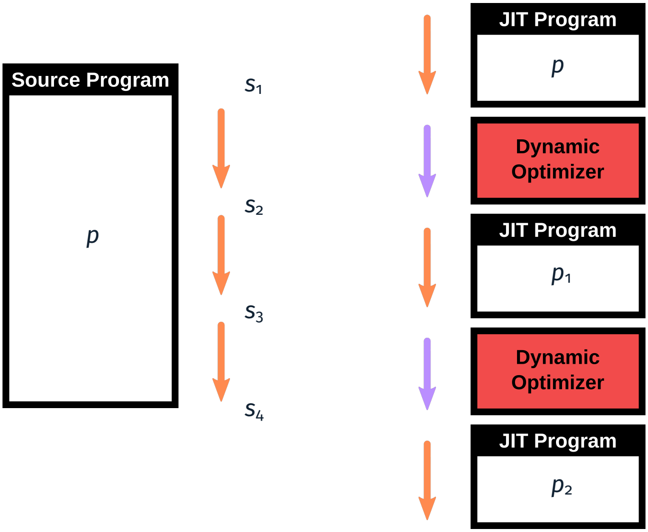
Fixed program p.
Semantic states of p: s_1, s_2, s_3, s_4.



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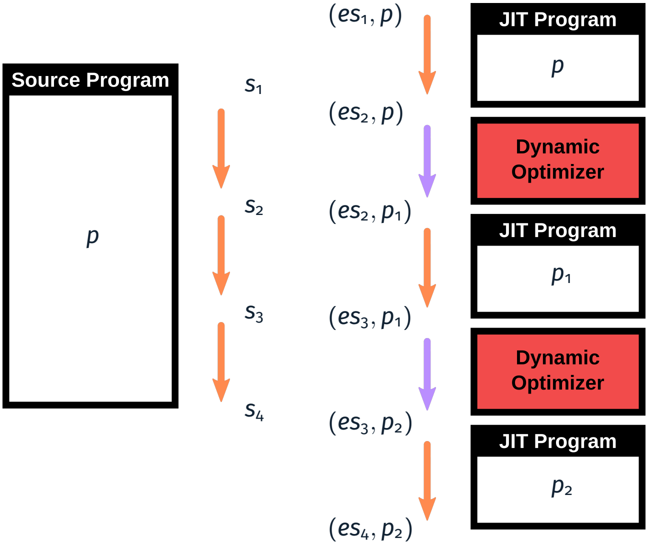

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where es is an execution state,
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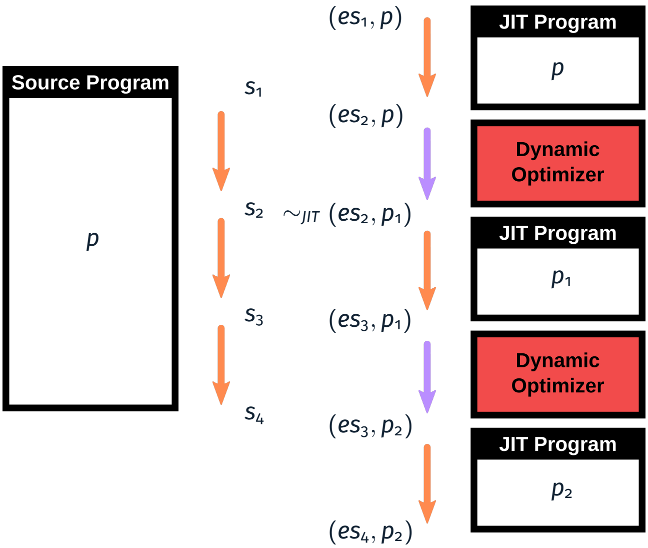

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where es is an execution state,
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To Prove a Simulation:
Find and preserve an invariant relation $\sim_{JIT}$ between source states s and JIT states (es, jp).

Invariant Intuition:
At any point in the execution,

- the current execution state corresponds to a source semantic state,
- the current JIT program is equivalent to the source program p.

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This equivalence can be expressed with a backward simulation.

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At any point in the execution,

- the current execution state corresponds to a source semantic state,
- the current JIT program is equivalent to the source program p.

This equivalence can be expressed with a backward simulation.

Defining the invariant

Nested Simulations: Our JIT simulation invariant contains another simulation (called *internal*).
$s \sim_{JIT} (es, jp)$ when

- there *exists* a simulation between p and jp, using an invariant $\sim_{in}$
- $s \sim_{in} es$.

The internal simulation changes as the JIT performs optimizations.

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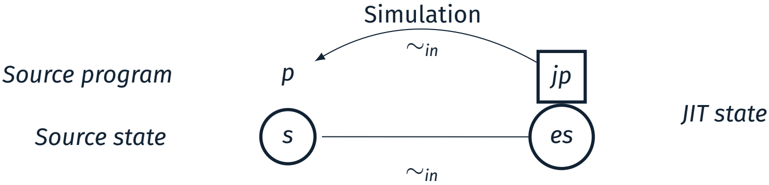
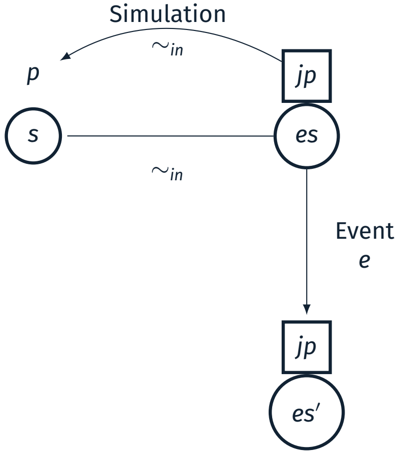


The JIT did an **execution** step (*e.g.* interpreter), and updated its execution state *es*.



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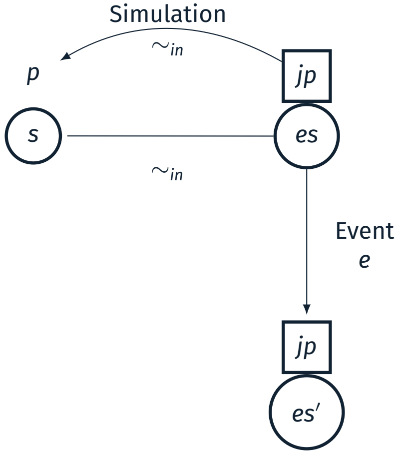

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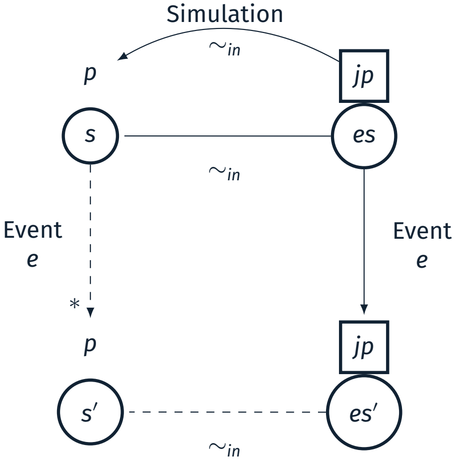

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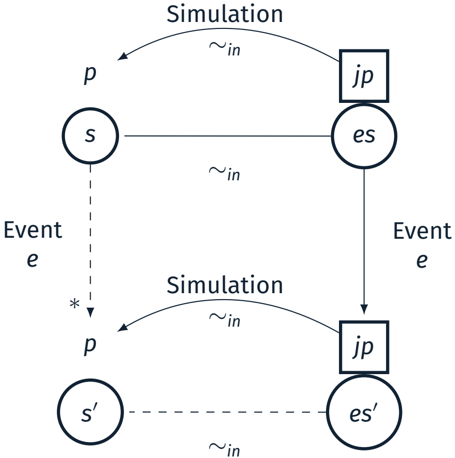
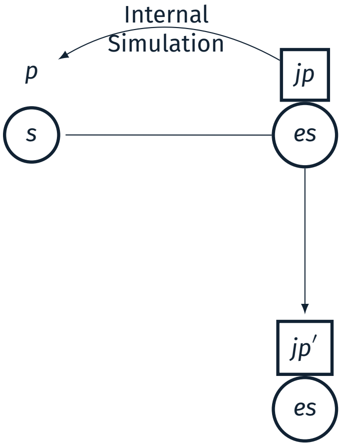


We need to prove that there exists matching steps in the source semantics preserving the invariant and the observed behavior *e*.

The JIT program *jp* did not change, and is still simulated with *p*.

The JIT did an **optimization** step and updated its program *jp*.



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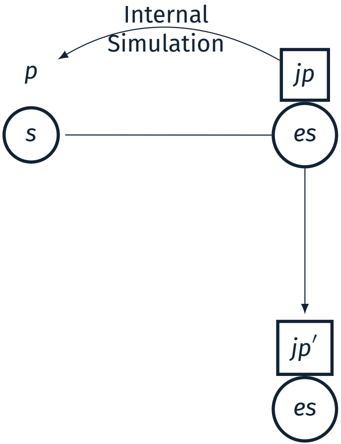

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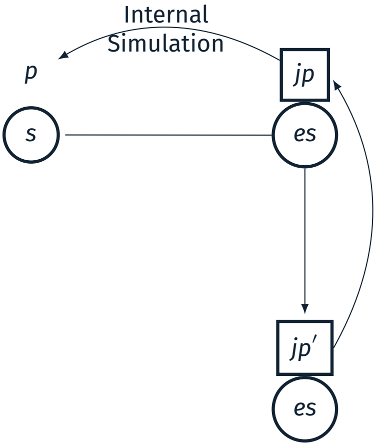
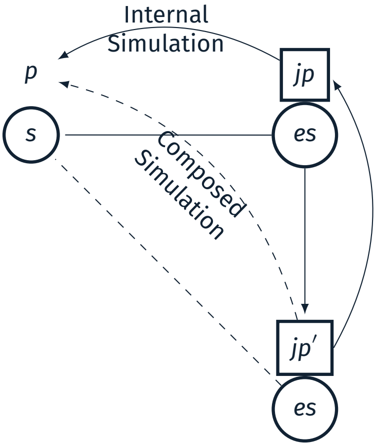


We need to prove that jp' is still simulated with p.

We prove the dynamic optimizer correct with a backward simulation.

```
Theorem optimizer_correct:
∀ jp jp’,
optimizer jp = jp’ →
backward_simulation jp jp’.
```

The JIT did an **optimization** step and updated its program *jp*.



We need to prove that *jp*′ is still simulated with *p*.

We prove the dynamic optimizer correct with a backward simulation.

```
Theorem optimizer_correct:
∀ jp jp',
optimizer jp = jp' →
backward_simulation jp jp'.
```

We compose this new simulation with the existing internal one.

To prove correct a JIT with dynamic optimizations, it suffices to prove its optimizer correct with a backward simulation.

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Like a static compiler.

Formally Verified Speculation

Speculative instructions should have precisely defined semantics.

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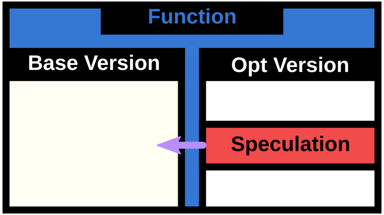
CoreIR, an Intermediate Representation for Speculation

Two speculative instructions to represent two sides of speculation.
Inspired by Sourir [Flückiger et al. 2018] and CompCert's RTL.
Each function has a *base* version and an optional *optimized* one.



```
Function F(x,y):
Version Base:
l1: a ← 1


    b ← y + x


    Return (a + b)
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```

```
Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]

    b ← y + x

    Return (a + b)
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Timeline:

- Create an `Opt` version, with synchronization points `Anchor`

```
Function F(x,y):
Version Base:
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Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]

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Timeline:

- Create an `Opt` version, with synchronization points `Anchor`

Profiler: speculate that x is 9

```
Function F(x,y):
Version Base:
l1: a ← 1



    b ← y + x



    Return (a + b)
```

```
Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]
    Assume (x=9) F.l1 [x ← x, y ← y, a ← 1]

    b ← y + x


    Return (a + b)
```

Timeline:

- Create an `Opt` version, with synchronization points `Anchor`
- Insert speculation `Assume`

Profiler: speculate that `x` is `9`

```
Function F(x,y):
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Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]
    Assume (x=9) F.l1 [x ← x, y ← y, a ← 1]

    b ← y + 9

    Return (1 + b)
```

Timeline:

- Create an `Opt` version, with synchronization points `Anchor`
- Insert speculation `Assume`
- Constant propagation

```
Function F(x,y):
Version Base:
l1: a ← 1


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Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]
    Assume (x=9) F.l1 [x ← x, y ← y, a ← 1]
    Assume (y=7) F.l1 [x ← x, y ← y, a ← 1]
    b ← y + 9


    Return (1 + b)
```

Timeline:

- Create an `Opt` version, with synchronization points `Anchor`
- Insert speculation `Assume`
- Constant propagation
- Insert speculation

Profiler: speculate that `y` is `7`

```
Function F(x,y):
Version Base:
l1: a ← 1



    b ← y + x



    Return (a + b)
```

```
Version Opt:
l1: a ← 1
    Anchor F.l1 [x ← x, y ← y, a ← 1]
    Assume (x=9) F.l1 [x ← x, y ← y, a ← 1]
    Assume (y=7) F.l1 [x ← x, y ← y, a ← 1]
    b ← 16


    Return (17)
```

Timeline:

- Create an Opt version, with synchronization points Anchor
- Insert speculation Assume
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- Constant propagation

```
Function F(x,y):
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    Return (17)
```

Timeline:

- Create an `Opt` version, with synchronization points `Anchor`
- Insert speculation `Assume`
- Constant propagation
- Insert speculation
- Constant propagation
- Dead code elimination

The Anchor instruction : How to Deoptimize

Expresses a synchronization between the original and optimized versions.

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`Assume` (y=3) `F.l` [x ← y+1] deoptimizes if y is not 3, skips to the next instruction otherwise.

Non-deterministic Semantics

The semantics of an `Anchor` are non-deterministic: either deoptimize to the original version or continue to the next instruction in the optimized version.
`Assume` is deterministic: deoptimizes when the speculation fails.

To prove each transformation, prove a backward simulation.

```
Theorem assume_insertion_correct:
  ∀ p f guard lbl newp,
    insert_assume f guard lbl p = OK newp →
    backward_simulation p newp.
```

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```
Theorem assume_insertion_correct:
  ∀ p f guard lbl newp,
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```

First, design an **invariant** relating semantic states of CoreIR.

Semantic states of CoreIR:

$(S,\ C,\ pc,\ R,\ M)$

stack

code

label

registers

memory

```
Function G():
   Return F(4)

Function F(x):
Version Base:
l1:Return (x+16)

Version Opt (Cbefore):
la:Anchor F.l1 [x ←x]

l1:Return (x+16)
```

Assume Insertion

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Simulation Invariant: $\sim$

Outside the modified version:

$$\frac{C \neq C_{before} \qquad S_1 \approx S_2}{(S_1, C, pc, R, M) \sim (S_2, C, pc, R, M)}$$

$S_1 \approx S_2$ when stackframes of C_{before} have been replaced with stackframes of C_{after}.

Assume Insertion

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   Return F(4)          ~                     Return F(4)

Function F(x):                             Function F(x):
Version Base:                              Version Base:
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Version Opt (Cbefore):                     Version Opt (Cafter):
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Inside the modified version:

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Assume Insertion

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`Version Base:`		`Version Base:`
`l1:Return (x+16)`	$\sim$	`l1:Return (x+16)`
`Version Opt (Cbefore):`		`Version Opt (Cafter):`
`la:Anchor F.l1 [x ←x]`	$\sim$	`la:Anchor F.l1 [x ←x]`
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At the new `Assume`:

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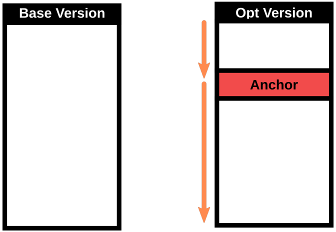
$S_1 \approx S_2$ when stackframes of C_{before} have been replaced with stackframes of C_{after}.

At the `Assume`, we reason by case analysis on the speculation:

- if it holds, we match this step to no deoptimization of the `Anchor`

Before `Assume` Insertion:



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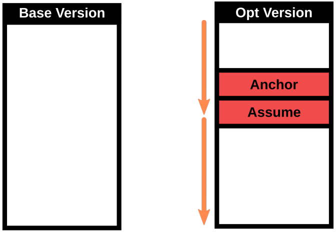

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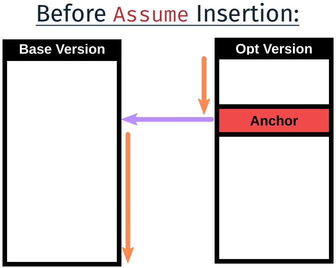

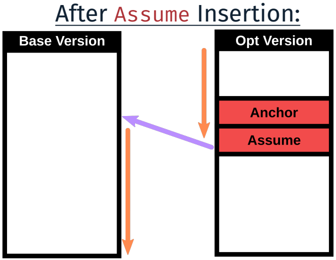

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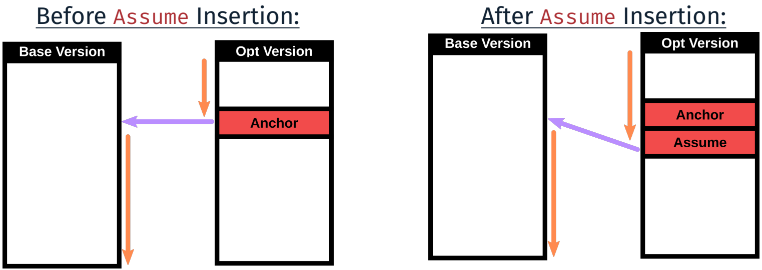

Using the non-determinism of the `Anchor`, it is correct to insert a new `Assume`.

Non-deterministic semantics are adequate to represent the possibility of a deoptimization.

Backward Simulations: Handling Speculation

- **Inserting Anchor** instructions.
- **Inserting Assume** instructions using Anchors.
- **Lowering** : Removes Anchor instructions.

A Backward Simulation for Inlining

- **Inlining** : When deoptimizing from an inlined function, we need to synthesize an additional stackframe to restore the environment.

Proving Non-speculative Optimizations

- **Constant Propagation** : reusing the *forward to backward* methodology of CompCert.

Middle-end Correctness

Correct for any speculation and optimizations done by the middle-end.

```
Theorem middle_end_correct:
  ∀ jp jp' optim_list,
  middle_end jp optim_list = jp' →
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Backend Correctness

- Transform a CoreIR function into RTL programs, using custom calling conventions to interact with the JIT.
- Then, use the CompCert backend to generate x86 code.

```
Theorem backend_correct:
  ∀ jp' jp'',
  backend jp' = jp'' → (* integrates CompCert backend *)
  backward_simulation jp' jp''.
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```

Using **nested** simulations:

```
Theorem jit_simulation:
  ∀ p, backward_simulation p (jit_sem p).
```

Which implies the JIT behavior preservation theorem.

A small set of *impure* primitives used in a JIT:

- store and load from the heap.
- push and pop from the stack.
- install, load and execute native code.

For primitives, write both C implementations and Coq specifications.

An encoding for programs using both Coq functions and Impure primitives:

```
Definition optimizer (f:function) : free unit :=
  do f2 ← ret (middle_end f);  (* written in Coq *)
  do f_x86 ← ret (backend f2); (* written in Coq, reusing CompCert *)
  Prim_Install_Code f_x86.  (* impure primitive *)
```

Solving the first two JIT challenges

POPL21: *Formally Verified Speculation and Deoptimization in a JIT compiler.* [Barrière et al. 2021]

- Dynamic Optimizations
- Speculations.

Solving the remaining two JIT challenges

POPL23: *Formally Verified Native Code Generation in an Effectful JIT*
or: Turning the CompCert Backend into a Formally Verified JIT Compiler. [Barrière et al. 2023]

- Impure Components
- Native code generation (reusing CompCert) and execution.

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- Impure Components
- Native code generation (reusing CompCert) and execution.

Executable Coq develoment: execute our JIT on CoreIR programs.
Speedups when speculating or generating native code.
`https://github.com/Aurele-Barriere/JIThm`

Designed a generic JIT architecture.

Identified JIT-specific verification challenges and presented composable correctness arguments.

A methodology to develop Formally Verified JITs, with a mechanized verified and executable Coq prototype.

Addressing our limitations

- **Impure primitive verification**: check the C implementations against their Coq specifications.
- **Generating efficient code**: the interfacing between native code and the JIT can be improved.

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A new playground to formalize/verify JIT features

- **Recompilation**: Recompiling from scratch a function that deoptimizes too often.
- **Contextual Dispatch**: Different versions of a function specialized for different call contexts. [Flückiger et al. 2020].
- **On-Stack-Replacement as en entry**: Jumping into native code from the interpreter.

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Towards Verified JITs for Realistic Languages

WebAssembly, a language for high-performance applications on web pages.
On-going work on formally verified Coq interpreter and compiler for WebAssembly [Watt et al. 2021].
We could then reuse our techniques to develop a formally verified WebAssembly JIT.

Recap - JIT-specific Verification Challenges

Dynamic Optimizations: JITs interleave optimizations with the program execution.
Our approach: adapt the simulation proof methodology of CompCert.

Speculative Optimizations: JITs insert and manipulate speculations in their programs.
Our approach: define semantics for speculative instructions, verify code transformations manipulating them.

Impure Components: Some JITs components are difficult to write in Coq (*e.g.* executing native code).
Our approach: free monadic encoding of the JIT with a pure specification of some impure primitives.

Static Compiler Reuse: Verified JITs should reuse formally verified compilers.
Our approach: integrate the CompCert backend and its proof to generate native code.

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CoreIR Syntax

Operands:

$$
\begin{array}{lll}
op & ::= r & \text{Register} \\
 & \mid v & \text{Value}
\end{array}
$$

Expressions:

$$
\begin{array}{lll}
e & ::= op + op \mid op - op \mid op * op & \text{Arithmetic} \\
 & \mid op < op \mid op = op & \text{Relational} \\
 & \mid op & \text{Register or value}
\end{array}
$$

Instructions:

$$
\begin{array}{lll}
i & ::= \mathbf{Nop}\ l & \text{Noop} \\
 & \mid (r \leftarrow e)^*\ l & \text{Operations} \\
 & \mid \mathbf{Cond}\ e\ l_t\ l_f & \text{Branch} \\
 & \mid r \leftarrow \mathbf{Call}\ f\ e^*\ l & \text{Call} \\
 & \mid \mathbf{Return}\ e & \text{Return} \\
 & \mid r \leftarrow \mathbf{Load}\ e\ l & \text{Memory load} \\
 & \mid e \leftarrow \mathbf{Store}\ e\ l & \text{Memory store} \\
 & \mid \mathbf{Print}\ e\ l & \text{Output value} \\
 & \mid \mathbf{Anchor}\ deop\ l & \text{Deoptimization anchor} \\
 & \mid \mathbf{Assume}\ e^*\ deop\ l & \text{Speculation}
\end{array}
$$

Metadata:

$$
\begin{array}{lll}
vm & ::= (r \leftarrow e)^* & \text{Varmap} \\
syn & ::= f.l\ r\ vm & \text{Stack frame} \\
deop & ::= f.l\ vm\ syn^* & \text{Deopt metadata}
\end{array}
$$

Programs:

$$
\begin{array}{lll}
V & ::= l \mapsto i & \text{Code} \\
F & ::= \{r^*, l, V, option\ V\} & \text{Function} \\
P & ::= f \mapsto F & \text{Program}
\end{array}
$$

CoreIR Speculative Instructions Semantics

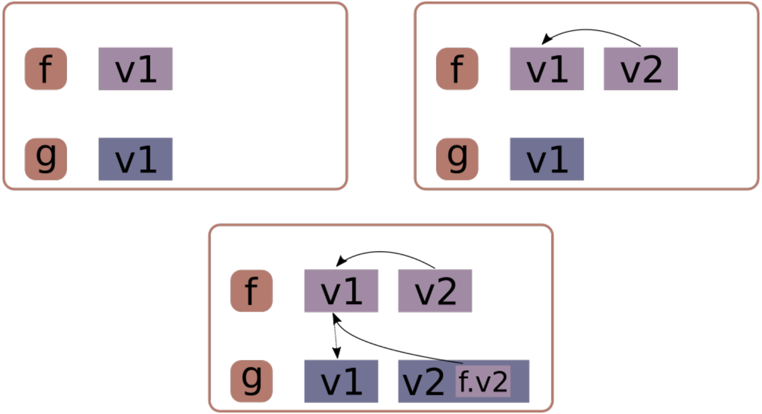
$$\textsc{Ignore}\ \frac{\begin{array}{c} V[l_{pc}] \;=\; \texttt{Anchor}\ f.l\ vm\ st^*\ l_{next} \\ \texttt{deopt\_regmap}\ vm\ R \;=\; R' \qquad \texttt{synthesize\_frame}\ R\ st^* \;=\; S' \end{array}}{S\ V\ l_{pc}\ R\ M \;\longrightarrow\; S\ V\ l_{next}\ R\ M}$$

$$\textsc{Deopt}\ \frac{\begin{array}{c} V[l_{pc}] \;=\; \texttt{Anchor}\ f.l\ vm\ st^*\ l_{next} \\ \texttt{deopt\_regmap}\ vm\ R \;=\; R' \qquad \texttt{synthesize\_frame}\ R\ st^* \;=\; S' \end{array}}{S\ V\ l_{pc}\ R\ M \;\longrightarrow\; (S' +\!\!+\, S)\ (\texttt{base\_version}_P\ f)\ l\ R'\ M}$$

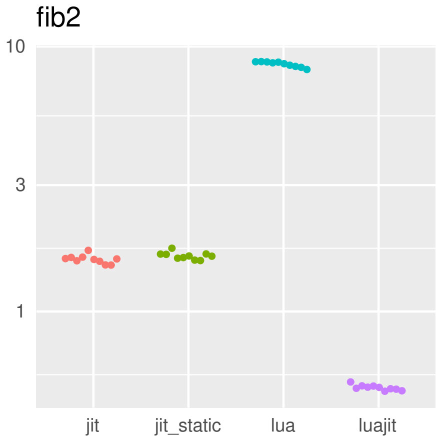
$$\textsc{AssumePass}\ \frac{V[l_{pc}] \;=\; \texttt{Assume}\ e^*\ f.l\ vm\ st^*\ l_{next} \qquad (e^*, R) \Downarrow\ true}{S\ V\ l_{pc}\ R\ M \;\longrightarrow\; S\ V\ l_{next}\ R\ M}$$

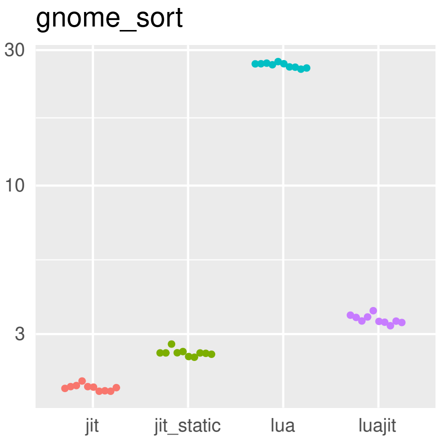
$$\textsc{AssumeFail}\ \frac{\begin{array}{c} V[l_{pc}] \;=\; \texttt{Assume}\ e^*\ f.l\ vm\ st^*\ l_{next} \qquad (e^*, R) \Downarrow\ false \\ \texttt{deopt\_regmap}\ vm\ R \;=\; R' \qquad \texttt{synthesize\_frame}\ R\ st^* \;=\; S' \end{array}}{S\ V\ l_{pc}\ R\ M \;\longrightarrow\; (S' +\!\!+\, S)\ (\texttt{base\_version}_P\ f)\ l\ R'\ M}$$

Inlining Speculation - Extra Stackframe on Deoptimization

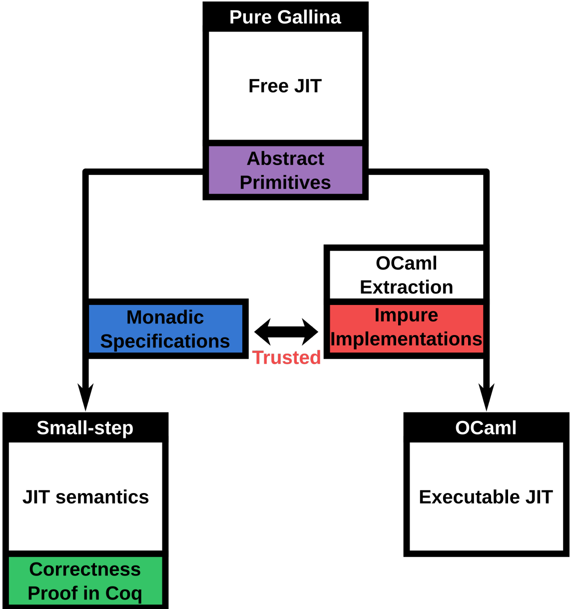


CoreIR Experiments





Free Monadic Encoding



State and Error Monad

A **pure** encoding of functions modifying and accessing a global state.
Either the function fails, or it succeeds and returns the next global state. Found in CompCert.

```
Inductive sres (state:Type) (A:Type): Type :=
| SError : errmsg → sres state A
| SOK: A → state → sres state A.
Definition state_mon {state:Type} (A:Type) : Type := state → sres state A.
```

Monadic constructors:

```
Definition state_ret {state:Type} {A:Type} (x:A) : state_mon A :=
fun (s:state) ⇒ SOK x s.

Definition state_bind {state:Type} {A B:Type} (f: state_mon A) (g:A → state_mon B) : state_mon B :=
  fun (s:state) ⇒
    match (f s) with
    | SError msg ⇒ SError msg
    | SOK a s' ⇒ g a s'
    end.
```

Executable Impure JIT

This is fine to specify the primitives, but the executable JIT should execute **impure** code.

Free Monad Definitions - An Example

In that example, we want to write programs that can access a single global variable of type nat.

A list of primitives our programs can use:

```
Inductive primitive: Type → Type :=
| Get : primitive nat
| Put (x:nat) : primitive unit.
```

We can then define the free monad:

```
Inductive free (T :Type) : Type :=
  | pure (x : T) :  free T
  | impure {R}
    (prim : primitive R) (next : R → free T).
```

Binding free monadic computations:

```
Fixpoint free_bind {X Y} (f: free X) (g: X →
free Y) : free Y :=
  match f with
  | pure x ⇒ g x
  | impure R prim next ⇒
    impure prim (fun x ⇒ free_bind (next x) g)
    end.
```

We can now encode incomplete programs.
Next, let’s see two ways to complete them.

Giving Semantics to our Free Monad - An Example

Given primitive implementations, we want to turn a free monadic function into an executable state monadic one. A **monadic specification** consists in one state monad for each primitive:

```
Record monad_spec: Type :=
  mk_mon_spec {
    prim_get: state_mon nat;
    prim_put: nat → state_mon unit; }.
```

```
Definition prim_spec {R:Type} (p:primitive R)
                  (i:monad_spec) : state_mon R :=
  match p with
  | Get ⇒ prim_get i
  | Put x ⇒ prim_put i x
  end.
```

We can now define **semantics** for our free monadic encoding, replacing each primitive with its specification:

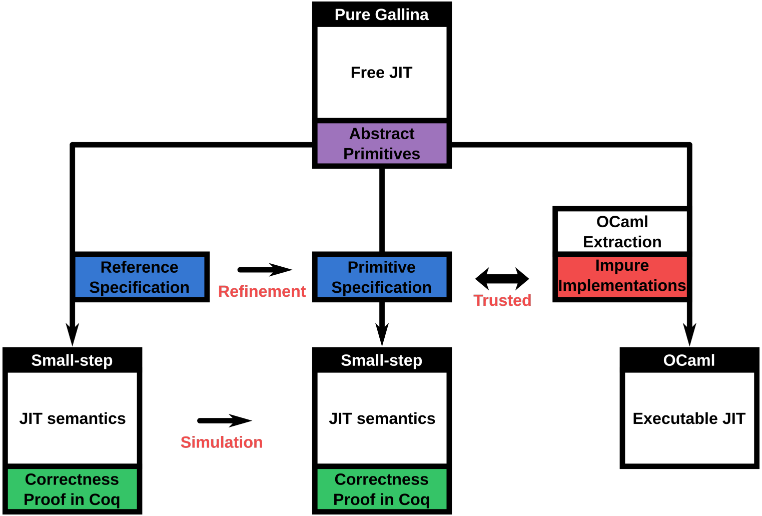
```
Fixpoint free_sem {A:Type} (f:free A) (i:monad_spec): state_mon A :=
  match f with
  | pure a ⇒ state_ret a
  | impure R prim cont ⇒
    state_bind (prim_spec prim i) (fun r:R ⇒ free_sem (cont r) i)
  end.
```

Executing the Free Monad - An Example

Finally, we extract the free monadic JIT to OCaml.
We can write a new way to **execute** a free monadic function, calling impure primitives when needed.

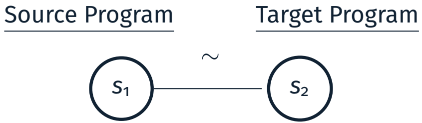
```
(* impure primitive implementations, using a global reference *)
let prim_impl (p:'x primitive) : 'x =
match p with
  | Get -> !global
  | Put (n) -> global := n
(* from OCaml, we can also call C implementations *)

(* executing free monads *)
let rec exec (f:'A free) : 'A =
  match f with
  | Coq_pure (a) -> a
  | Coq_ferror (e) -> print_error e; failwith "JIT crashed"
  | Coq_impure (prim, cont) ->
     let x = prim_impl prim in
     exec (cont x)
```

Free Monadic Refinement



Forward Simulation in CompCert:

- Design an invariant $\sim$, a relation between semantic states.
- Show that each semantic step of the source program is matched with steps of the target program preserving the invariant.



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- Design an invariant $\sim$, a relation between semantic states.
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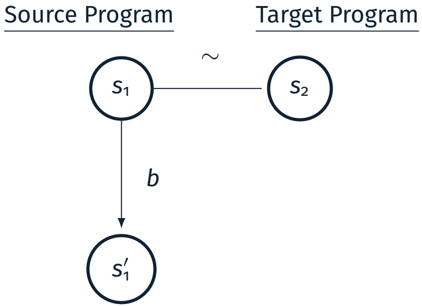

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- Design an invariant $\sim$, a relation between semantic states.
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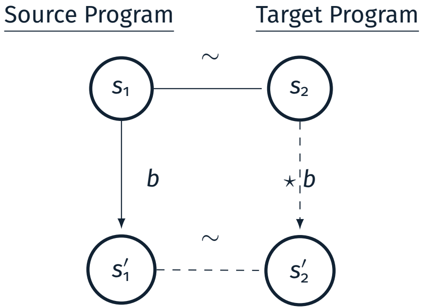
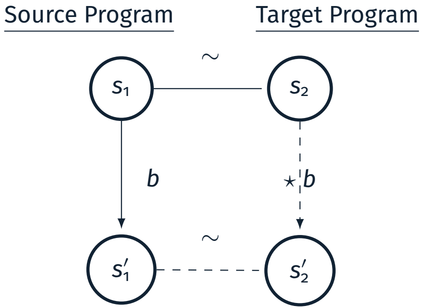


Simulations and Refinement

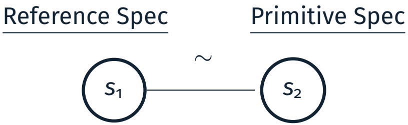
Forward Simulation in CompCert:

- Design an invariant $\sim$, a relation between semantic states.
- Show that each semantic step of the source program is matched with steps of the target program preserving the invariant.



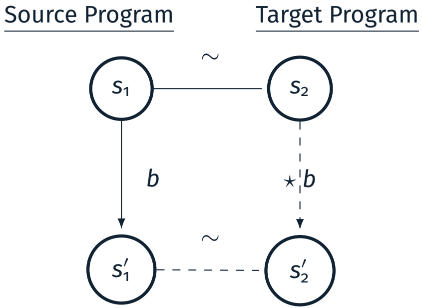
Refinement:

- Design an invariant $\sim$, a relation between monadic states.
- Show that each primitive execution of the first monadic specification is matched with a primitive execution of the second monadic specification preserving the invariant.



Forward Simulation in CompCert:

- Design an invariant $\sim$, a relation between semantic states.
- Show that each semantic step of the source program is matched with steps of the target program preserving the invariant.



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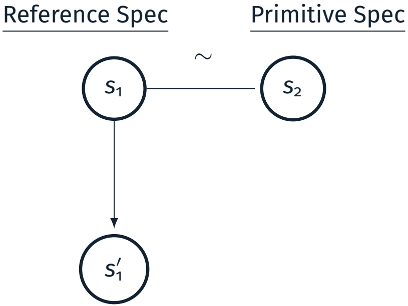

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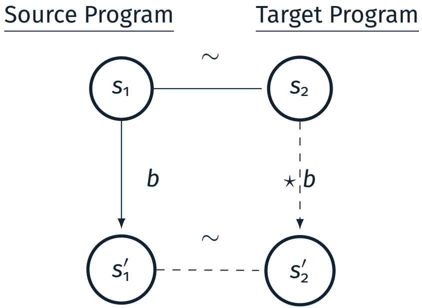

Refinement:

- Design an invariant $\sim$, a relation between monadic states.
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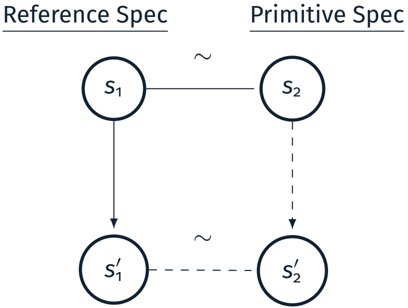
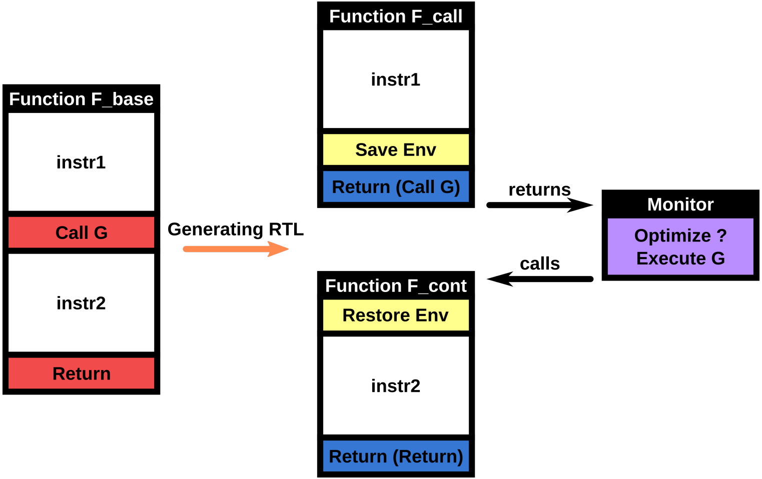


Custom Calling Conventions



Generating Natice Code using Primitives - An Example

CoreIR Function

```
Function Fun1 (reg1):
  reg2 ← 4 + reg1
  reg3 ← Call Fun7 (reg2)
  reg3 ← reg1 + reg3
  Return reg3
```

RTL Functions

```
$1() {
  x8 = "Pop"()
  x9 = x8 + 4 (int)
  // Save environment
  x1 = "Push" (x8)
  x1 = "Push"(Fun1, $2)
  x1 = "Push"(x9)
  x1 = "Push"(Fun7)
  return RETCALL }

$2() {
  x10 = "Pop"()
  // Restore environment
  x8 = "Pop"()
  x10 = x8 + x10
  x1 = "Push"(x10)
  return RETRET }
```

Assembler Continuation Function

```
# File generated by CompCert 3.8
$2:
leaq    32(%rsp), %rax
movq    %rax, 0(%rsp)
movq    %rbx, 8(%rsp)
call    _Pop
movq    %rax, %rbx
call    _Pop
leal    0(%eax,%ebx,1), %edi
call    _Push
movl    $RETRET, %eax
movq    8(%rsp), %rbx
addq    $24, %rsp
ret
```