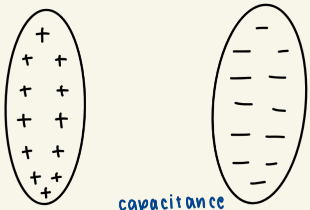
Exam #2

LECTURE (09/28/2023)



$$C = K\frac{\varepsilon_0 A}{d}$$

$Q \propto \Delta V$

charge

C does <u>NOT</u> depend on ΔV or charge

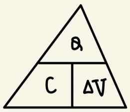
BUT depends on geometry of capacitor A, d & dielectric K

capacitance (Farads)

$$\boxed{Q = C\,\Delta V}$$

charge (coulombs)

voltage (J/C)



By changing two plates + & − we now have ΔV (potential difference)

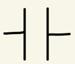
Calculating capacities

#1) Assume the capacitor has a charge Q

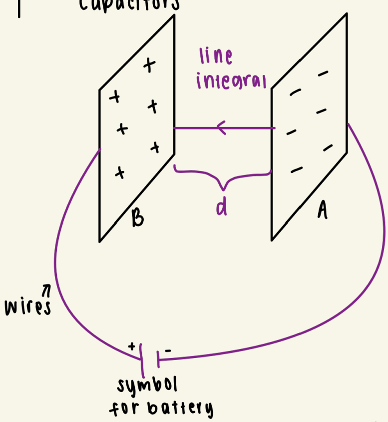
#2) Determine the $\vec{E}$ field (Electric field) between the conductors (You can use Gauss's Law for symmetry)

#3) Find the potential difference between the conductors.

#4) Then use $C = \frac{Q}{\Delta V}$



→ symbol for capacitors



#1) Q on capacitor

B ← path A

#2) $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$

$E = \frac{Q}{\varepsilon_0 A}$

#3.) $\Delta V = -\int_A^B \vec{E}\cdot\vec{dl}$

$V_B - V_A = -E\, L\Big|_A^B$

$= -E\,(\underbrace{B-A}_{-d})$

$\Delta V = Ed$

$\Delta V = \frac{Q}{A\varepsilon_0}\,d$

#4. $C = \frac{Q}{\Delta V}$

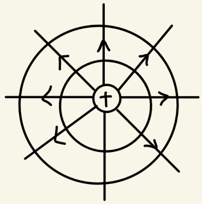
$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Connection between $\vec{E}$ & V

$\Delta V = Ed$

$\vec{E} = -\frac{\partial V}{\partial r}$

$\vec{E} = \left\langle -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right\rangle$



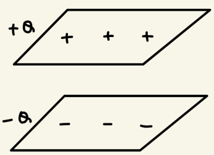
→ Direction of decreasing potential

If field is only in E_x, then:

$E_x = \frac{-\Delta V}{\Delta x}$ → The (-) gradient of the potential is the electric field

LECTURE (10/10/2023)

Capacitors



≡ (symbol)

Capacitance does NOT depend on charge & voltage

⇨ What does capacitance depend on?

Area, Distance (d) in between & Dielectric constant K

$\boxed{C = K \varepsilon_0 \frac{A}{d}}$ → distance between plates

$\boxed{C = \frac{Q}{\Delta V}}$ <=> $Q = C\Delta V$

Energy stored in a capacitor

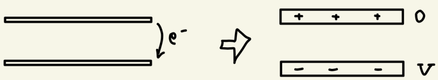
$\boxed{U_{capacitor} = \frac{1}{2} Q \Delta V}$ (Joules)

↓ C ↓ J/C

... when we started our conversation about potential (Electric)

$V = U/Q$ <=> $\boxed{U = QV}$

e^- is moving downwards



Average voltage × Q = Total Energy

$\frac{1}{2}\Delta V$ × Q = Energy or U

⇨ Where do we get this equation from?

$V = \frac{q}{C}$

$U = \int_0^Q dU$

↑ $dU = V\,dq$

$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$

$U = \frac{1}{2}\frac{Q^2}{C}$ → has only Q & ΔV $= \frac{1}{2} Q\Delta V$

→ has $C, \Delta V$ $= \frac{1}{2}\Delta V(C\cdot\Delta V) = \frac{1}{2}C\Delta V^2$

Energy $\Rightarrow \dfrac{U}{vol} = u$

energy density $= \dfrac{1}{2}\varepsilon_0 E^2$

$\rightarrow \boxed{E = -\dfrac{\Delta V}{\Delta x}}$

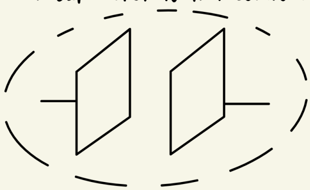
$U = \dfrac{1}{2} C \Delta V^2$

Energy density $= \dfrac{1}{2} \cdot \dfrac{\varepsilon_0 A}{d} (Ed)^2$

$= \dfrac{1}{2} \varepsilon_0 E^2 \underbrace{(A \cdot d)}_{\text{volume (v)}}$

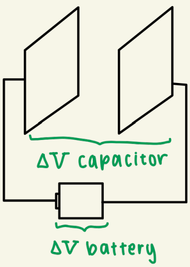
$$\boxed{u = \frac{U}{vol} = \frac{1}{2}\varepsilon_0 E^2}$$

* if capacitor is not connected to battery = isolated



→ not connected = isolated

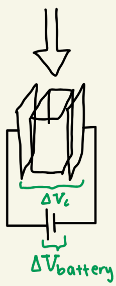
→ if you charge it & disconnect, we assume charges will still be the same.



→ connected – NOT isolated

$\Delta V_{capacitor} = \Delta V_{battery}$

→ charges are always the same

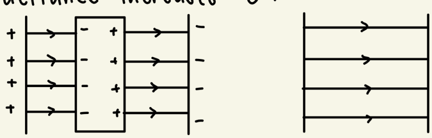


$\Delta V_{battery} = \Delta V_{capacitor}$

⇒ what happens to E? decreases / increases / stays the same

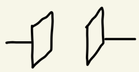
why a dielectric?

- isolate the 2 plates (insulator)
- capacitance increases $C \nearrow$

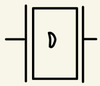


• E will stay the same as $V = E \cdot d$ or $Q = C\Delta V$. Additionally, it is hooked up to the battery. Without the battery, E will decrease.

(isolated)



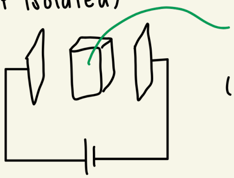
$E_{original}$



$E_{dielectric}$

$$\frac{E_{original}}{E_{dielectric}} = K$$

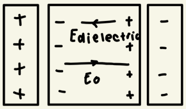
(not isolated)



E dielectric stays the same

(more charge dumped on the plates)

LECTURE (10/12/2023)



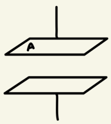
$E_{dielectric} < E_{original}$

$$\frac{E_{original}}{E_{dielectric}} = K$$ ← dielectric constant

Adding Capacitors : Series & parallel capacitors

$Q = C\Delta V$

→ Remember! $\dfrac{Q}{C \,|\, \Delta V}$

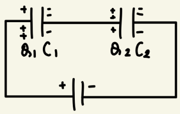


$$\boxed{C = \frac{K\varepsilon_0 A}{d}}$$

$C_{series} \Rightarrow \dfrac{1}{C_{Tot}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots + \dfrac{1}{C_n}$

$C_{parallel} \Rightarrow C_{Tot} = C_1 + C_2 + \dots + C_n$

Circuits in Series



Principle → $Q_1 = Q_2$

current flow of charge → $I_1 = I_2$

$\Delta V_{total} = \Delta V_1 + \Delta V_2$

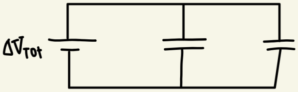
$$\frac{Q_{Total}}{C_{Total}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$Q_{Total} = Q_1 = Q_2$ → charges in series are the same

$$\frac{\cancel{Q_{Total}}}{C_{Total}} = \frac{\cancel{Q_{Total}}}{C_1} + \frac{\cancel{Q_{Total}}}{C_2}$$

$$\boxed{\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Circuits in parallel



$$\Delta V_{total} = \Delta V_1 = \Delta V_2$$

$$Q_{Total} = Q_1 + Q_2$$

$$C_{Total} \cdot \Delta V_{total} = C_1 \Delta V_1 + C_2 \Delta V_2$$

$$C_{Total} \cdot \cancel{\Delta V_{total}} = C_1 \Delta \cancel{V_{total}} + C_2 \Delta \cancel{V_{total}}$$

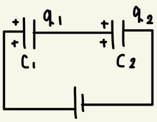
$$\boxed{C_{Total} = C_1 + C_2}$$

→ C_{eq} = C equivalent

Rule

1. Add capacitances & get C_{Total} / C_{eq}
2. use $Q = C \Delta V$

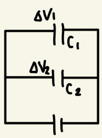
LECTURE (10/17/2023)



$q_1 = q_2$ (same charge)

series - Q same

"universal truth"

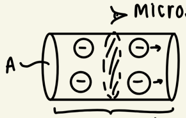


parallel - V same

$\Delta V_1 = \Delta V_2$

LECTURE (10/19/2023)

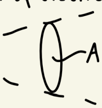
→ microscopic look



microscopic
sea of electrons

$$I = \frac{\Delta Q}{\Delta t} = \frac{C}{s} = \text{Ampere}$$

$$J = \frac{I}{A}$$

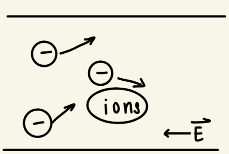


Dynamic → Electrons are moving → needs an electric field

$\vec{E} \Rightarrow \boxed{F = q\vec{E}}$

→ sea of electrons are like swamp of bees. individual bees bounce back & forth, but as a unit they move together.

wire



momentum principle

$$\boxed{\begin{array}{l} \Delta p = \vec{F}_{net} \Delta t \\ \text{or} \quad \vec{F}_{net} = m\vec{a} \end{array}}$$

⇨ Repulsion : Electron - Electron

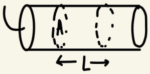
Attraction : positive ion

⇨ collide w/ atoms

⇨ sea of electron speed $\approx 1 \times 10^{-4}$ m/s

(drift speed velocity)

volume



$$I = \frac{\Delta Q}{\Delta t} \longrightarrow$$ no. of charges in a certain volume

$$I = \frac{\Delta Q}{\text{volume}} \cdot \frac{\text{volume}}{\Delta t}$$

$$I = \left(\frac{\text{no. of charges}}{\text{volume}}\right)(q) \cdot A \cdot \boxed{\frac{L}{\Delta t}}$$

→ n_e for electrons

$$I = n \cdot q \cdot A \cdot v_{drift}$$

mobility

→ $\mu E = v_{drift}$

Electric field

Examples:

1) 2.30×10^{-18} mol of Na (sodium) pass through a cell membrane in 4.0×10^{-2} seconds. Avogadro's number is 6.02×10^{23} ions/mol. Find the current.

$I = \frac{\Delta Q}{\Delta t}$ → $N_{ions} \cdot q$

$$N_{ions} = \frac{6.02 \times 10^{23} \text{ ions}}{\cancel{\text{mol}}} \cdot 2.30 \times 10^{-18} \cancel{\text{mol}} = 13.8 \times 10^{5} \text{ ions}$$

$$I = \frac{13.8 \times 10^{5} (1.6 \times 10^{-19} C)}{4.0 \times 10^{-2} s} = 5.54 \times 10^{-12} \, C/s \approx 5.54 \text{ pA}$$

2) A 5.0 Ampere current flows in a copper wire with cross-sectional area 1.0 mm^2 carried by electrons with a number density $n_e = 1.1 \times 10^{29}$ (electrons/m^3). Find the drift speed

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = nqAv_d$$

$$v_d = \frac{I}{nqA} = \frac{5 \, C/s}{(1.1 \times 10^{29} \text{ electrons}/m^3)(1.6 \times 10^{-19} C)(10^{-6} m^2)}$$

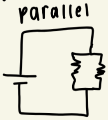
$$v_d = 2.84 \times 10^{-4} \, m/s$$

why do we use copper wires instead of any others?

because in copper, there are a lot of free electrons when compared to other wires, like ions. this will help to make the sea of electrons.

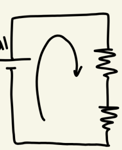
If you add resistors in series, it will be bigger. in parallel → less resistance

parallel



resistance is less (lower)

ΔV_{total}



Resistance is higher

$$\frac{\Delta V}{I \mid R}$$

$$\Delta V_{total} = |\Delta V_1 + \Delta V_2|$$
$$= I_1R_1 + I_2R_2$$

$I_1 = I_2$ series - @ (same current)

$$\frac{\Delta V_{total}}{I} = \frac{IR_1 + IR_2}{I}$$

$R_{total} = R_1 + R_2 \longrightarrow$ series

For parallel...

$$I_{bat} = I_1 + I_2$$

$$\frac{\Delta V_{total}}{R_{total}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

$$\Delta V_{total} = \Delta V_1 = \Delta V_2$$

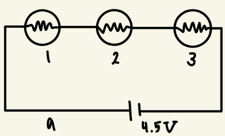
$$\frac{\Delta V}{R_{tot}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

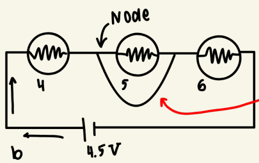
$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$R_{Tot} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \longrightarrow$ parallel

LECTURE (10/24/2023)

Chapter 23 Electric Current





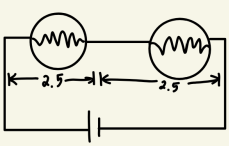
(path of least resistance)
short-circuiting (shorting)

a) In which circuit is the current greater? B

- circuit B because of less resistance

b) A

c) 4+6

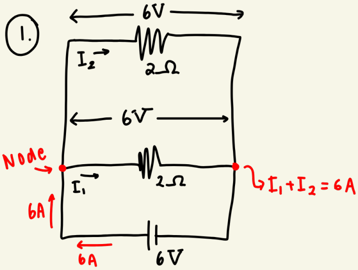


$$\frac{4.5}{2} = 2.5$$

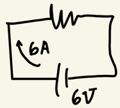
Principles

- voltages of resistors in parallel are the same.
- current in series are the same

1.



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2 \times 2}{2+2} = 1\Omega$$



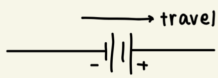
• currents into the node is equal to the node coming out.

Kirchhoff's Rule

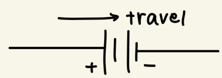
1. at the node : $\Sigma I_{in} = \Sigma I_{out}$ (conservation of charge)
2. $\Sigma \Delta V_{loop} = 0$

• using Kirchhoff's Ruler
  1) draw circuit diagram
  2) assign the direction of the current
     ↳ If you know the direction, use it
     ↳ if you don't, assign one (make an arbitrary choice)
     ↳ if it ends (-) then you chose the "wrong direction"
  3) Travel around the loop
     ↳ start at one point & go all the way + back to that point.

Ideal battery



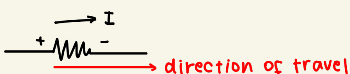
$\Delta V_{total} = +\Delta V$
(potential increases)



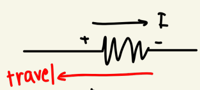
$\Delta V_{bat} = -\Delta V$
(potential decreases)

• For resistor

a)

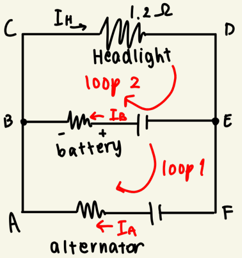


potential decreases
$\Delta V_R = -IR$



$\Delta V_R = +IR$

Q2



$\Sigma I_{in} = \Sigma I_{out}$ Equation 1

choose B (node)

$I_{in} = I_A + I_B = \overset{out}{I_H}$ —— ①

EBCDE

$\Delta V_{12} \cdot \Delta V_{0.01\Omega} - \Delta V_{1.2\Omega} = 0$

$12V - I_B(0.01) - I_H(1.2) = 0$ —— ②

FABEF

travel → ← I_B opposite

$+\Delta V_{14} - \Delta V_{0.1\Omega} + \Delta V_{0.01} - \Delta V_{12} = 0$

$14V - I_A(0.1) + I_B(0.01) - 12V = 0$ —— ③

$1 I_A + 1 I_B - 1 I_H = 0$

$0 I_A - 0.01 I_B - 1.2 I_H = -12V$

$-0.1 I_A + 0.01 I_B + 0 I_H = -2V$

matrixcalc.org

$$\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -0.01 & -1.2 & -12 \\ -0.1 & 0.01 & 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -0.01 & -1.2 \\ -0.1 & 0.01 & 0 \end{pmatrix} \begin{pmatrix} I_A \\ I_B \\ I_H \end{pmatrix} = \begin{pmatrix} 0 \\ -12 \\ -2 \end{pmatrix}$$

LECTURE (10/26/2023)

RC circuits

charging → $q(t) = q_{max}(1 - e^{t/RC})$

discharging → $q(t) = q_{max}\, e^{t/RC}$

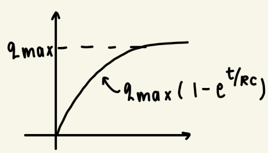


fig. charging

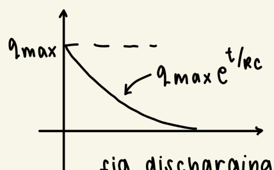
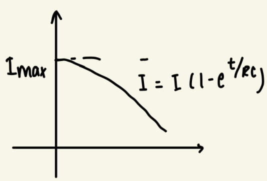
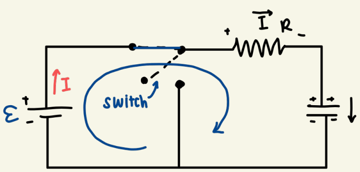


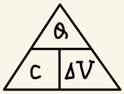
fig. discharging





Kirchhoff's Rule: $+\varepsilon + IR - \Delta V_c = 0$ $(\Sigma V = 0)$

6



$$+\varepsilon - IR - \frac{q}{C}$$

current is changing because there is capacitor

$$\frac{+\varepsilon}{R} - \frac{\frac{dq}{dt}R}{R} - \frac{\frac{q}{C}}{R} = \frac{0}{R}$$

$$\frac{\varepsilon}{R} - \frac{dq}{dt} - \frac{q}{RC} = 0$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{RC}$$

$$dq = (\varepsilon C - q)\frac{dt}{RC}$$

$$\frac{dt}{RC} = \frac{dq}{\varepsilon C - q}$$

$$\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{\varepsilon C - q}$$

let $u = \varepsilon C - q$

$du = -dq$

$$= \int_0^q \frac{1}{u} - du$$

$$= -\ln|\varepsilon C - q|\Big|_0^q$$

$$= -(\ln|\varepsilon C - q| - \ln|\varepsilon C|)$$

$$= -\ln\left(\frac{|\varepsilon C - q|}{\varepsilon C}\right)$$

$$-\frac{t}{RC} = \ln\left(\frac{|\varepsilon C - q|}{\varepsilon C}\right)$$

$$e^{-\frac{t}{RC}} = \frac{\varepsilon C - q}{\varepsilon C}$$

$$\varepsilon C e^{-t/RC} = \varepsilon C - q$$

$$\varepsilon C e^{-t/RC} - \varepsilon C = -q$$

$$\boxed{q = \underbrace{\varepsilon C}_{q_{max}}(1 - e^{-t/RC})}$$

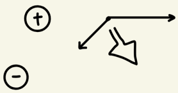
→ RC is time constant

it determines how long charging/discharging occurs.

→ in parallel, the time needed for discharge is longer than in series.

Celebration #2 (chapter 7 - 10)

- No Gauss, No Magnetism
- $\vec{E}$ (Electric field lines, equipotential lines, ...)
- $\vec{E}$ (vectors ⇨ superposition) As opposed to scalars

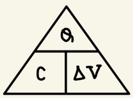


$(\Delta V_{total} = V_1 + V_2 + V_3)$
↑
No direction

- $W_{Ext} = \Delta EPE = q\Delta V$
- Conservation of Energy

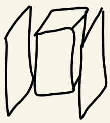
$$\boxed{K_i + U_i = K_f + U_f}$$

- Capacitors



$C = \frac{K \varepsilon_0 A}{d}$

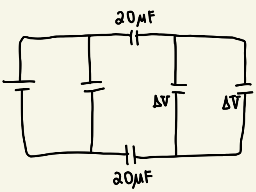
Kappa
=
dielectric



→ capacitance depends on the geometry & K

→ $\frac{\Delta V}{\Delta x} = E$ (in x-dir)

$\frac{-\partial V}{\partial r} = \vec{E}$ so, $\left\langle -\frac{dv}{dx}, -\frac{dv}{dy}, -\frac{dv}{dz} \right\rangle$ N/C



→ capacitors in series : Q (charges) same

→ " parallel : V (voltage) same

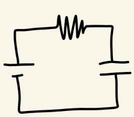
- RC Circuits

→ charging or discharging scenario ! Look @ the Question

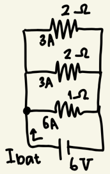
charging ⇨ $Q = Q_{max}(1 - e^{-t/RC})$

discharging ⇨ $Q = Q_{max}\, e^{-t/RC}$

} Remember, not going to be on paper / cheat sheet



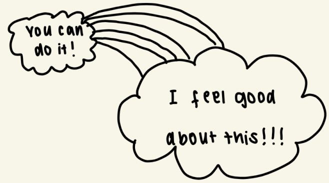
- Pink worksheet on Resistors (10/19/2023)
  - → Resistors | Bulbs
  - → Kirschoff's Rule



$I_{bat} = I_{6A} + I_{6A}$

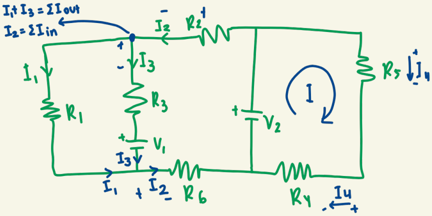
$= 12A$

if circuit is parallel, voltage is the same, so I is also the same for all $(V = I \cdot R)$



A circuit is constructed with 6 resistors and 2 batteries. The battery voltages are $V_1 = 18V$ & $V_2 = 12V$

$R_1 = R_5 = 76\Omega$ $R_2 = R_6 = 137\Omega$ $R_3 = 66\Omega$ $R_4 = 101\Omega$



1) What is V_4 the magnitude of the voltage across R_4?

loop 1 : $-I_4 R_5 - I_4 R_4 + V_2 = 0$

$-I_4(R_5 + R_4) + V_2 = 0$

$I_4 = \frac{V_2}{R_5 + R_4} = \frac{12V}{76\Omega + 101\Omega} = \frac{12V}{177\Omega}$

$= 0.0678\ A$

$V_4 = I_4 \cdot R_4$

$= (0.0678A)(101\Omega)$

$= 6.8478\ V \approx 6.8V$

2) What is I_3, the current that flows through Resistor R_3?

Ans: $-0.122666A$

(A positive value for current is defined to be in the direction of the Arrow)

3) What is I_2 the current through R_2? (A positive value is defined to be in direction of arrow)

Ans: $0.00766246\ A$

4) What is I_1 current through R_1? (+ value in direction of arrow)

Ans: $0.130316000A$