EXAM #1

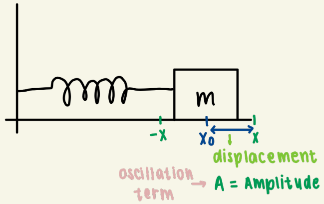
OSCILLATIONS - anything that repeats in a pattern

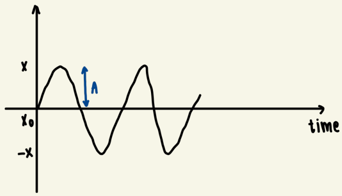
→ period : · time to repeat an oscillation

· from one peak to another (peak-to-peak)

→ $x(t)$ equation of motion

→





★ $F = ma$

$F = -kx$ (spring)

spring constant

$$F = ma = -kx$$

↓

$$m\frac{d^2x}{dt^2} = -kx \longrightarrow (1)\ \frac{d^2x}{dt^2} = -\frac{k}{m}x(t)$$

what if $x(t) = A\sin(\omega t + \phi)$

★ $V = \frac{dx}{dt}$ $\quad \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$$(2)\ \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

plug (2) into (1)

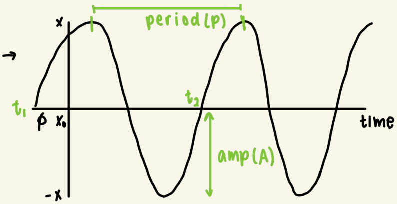
$$-A\omega^2\sin(\omega t + \phi) = -\frac{k}{m}A\sin(\omega t + \phi)$$

$$\omega^2 = \frac{k}{m}$$

angular frequency $\boxed{\omega = \sqrt{\frac{k}{m}}}$

$$\boxed{\begin{aligned} x(t) &= A\sin(\omega t + \phi) \\ v(t) &= A\omega\cos(\omega t + \phi) \\ a(t) &= -A\omega^2\sin(\omega t + \phi) \end{aligned}}$$

→



frequency $\boxed{f = \frac{1}{p}}$ period

strong spring = less oscillation

strong mass = more oscillation

$$x(t_2) - x(t_1) = 0$$

$$A\sin(\omega t_2 + \phi) - A\sin(\omega t_1 + \phi) = 0$$

$$\omega t_2 + \phi - (\omega t_1 + \phi) = 2\pi$$ after one oscillation

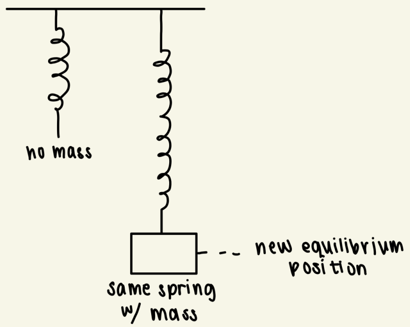
$$\omega t_2 - \omega t_1 = 2\pi$$

$$\omega(t_2 - t_1) = 2\pi$$

$$\omega P = 2\pi$$

$\boxed{P = \frac{2\pi}{\omega}}$ or $\boxed{\omega = \frac{2\pi}{P}}$

$\boxed{P = 2\pi f}$ $\quad \boxed{P = 2\pi\sqrt{\frac{m}{k}}}$



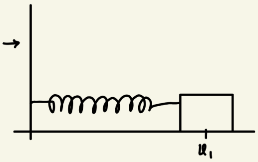
weight of mass

spring force

$$F = mg - kx_0 = 0$$

$$k\underbrace{\left(\frac{m}{k}g - x_0\right)}_{y \text{ (displacement from equilibrium)}} = 0$$

* adding mass to hanging spring DOES NOT change oscillation; ONLY equillibrium position



energy in the system:

spring counteracts the work

- Potential energy (U) $= \int_0^x kx\,dx = \boxed{\frac{1}{2}kx^2}$
- Kinetic energy (W) $= \int_0^x F\,dx = \boxed{\frac{1}{2}mv^2}$

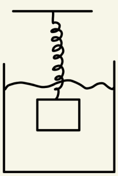
$$E = U + K$$
$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

- $x(t) = A\sin(\omega t + \phi)$
- $v(t) = A\omega\cos(\omega t + \phi)$ $\omega = \sqrt{\frac{k}{m}}$ (WACK-AM)

$$= \frac{1}{2}kA^2\sin^2(\omega t+\phi) + \frac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$
$$= \frac{1}{2}kA^2\sin^2(\omega t+\phi) + \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$
$$= \frac{1}{2}kA^2\left[\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right]$$
$$= \frac{1}{2}kA^2$$

$$\therefore \boxed{E = \frac{1}{2}kA^2}$$

→ Damping



coefficient of drag

$$F_d = bv$$

drag force

$$F_{net} = -kx - bv = ma$$
$$ma + bv + kx = 0$$
$$mx'' + bx' + kx = 0$$

? what if $b \to 0$

$$x(t) = A\sin(\omega t + \phi)$$

? what if $k \to 0$

$$mx'' = -bx'$$
$$x(t) = Ae^{-\alpha t}$$

$u(t) = A\ e^{-\alpha t} \sin(\omega t + \phi)$

sub to $mu'' + bu' + ku = 0$

$$u(t) = \underbrace{Ae^{-\alpha t}}_{u}\ \underbrace{\sin(\omega t+\phi)}_{v} \qquad u'v + uv'$$

$$u'(t) = \underbrace{-A\alpha e^{-\alpha t}}_{u}\ \underbrace{\sin(\omega t+\phi)}_{v} + \underbrace{Ae^{-\alpha t}}_{u}\ \underbrace{\omega\cos(\omega t+\phi)}_{v}$$

$$u''(t) = A\alpha^2 e^{-\alpha t}\sin(\omega t+\phi) + (-A\alpha e^{-\alpha t})(\omega\cos(\omega t+\phi)) + A(-\alpha)e^{-\alpha t}\omega\cos(\omega t+\phi) - Ae^{-\alpha t}\omega^2\sin(\omega t+\phi)$$

$$= A\alpha^2 e^{-\alpha t}\sin(\omega t+\phi) - A\omega\alpha e^{-\alpha t}\cos(\omega t+\phi) - A\omega\alpha e^{-\alpha t}\cos(\omega t+\phi) - Ae^{-\alpha t}\omega^2\sin(\omega t+\phi)$$

$$= A\alpha^2 e^{-\alpha t}\sin(\omega t+\phi) - 2A\omega\alpha e^{-\alpha t}\cos(\omega t+\phi) - A\omega^2 e^{-\alpha t}\sin(\omega t+\phi)$$

$$= A\left[\alpha^2 e^{-\alpha t}\sin(\omega t+\phi) - 2\alpha\omega e^{-\alpha t}\cos(\omega t+\phi) - e^{\alpha t}\omega^2\sin(\omega t+\phi)\right]$$

plug back into $mu'' + bu' + ku = 0$

$$Ae^{-\alpha t}\left[m\alpha^2\sin(\omega t+\phi) - 2m\omega\alpha\cos(\omega t+\phi) - m\omega^2\sin(\omega t+\phi) + b\omega\cos(\omega t+\phi) - b\alpha\sin(\omega t+\phi) + k\sin(\omega t+\phi)\right] = 0$$

$$Ae^{-\alpha t}\left[(m\alpha^2 - b\alpha - m\omega^2 + k)\sin(\omega t+\phi) + (b\omega - 2m\omega\alpha)\cos(\omega t+\phi)\right] = 0$$

$$m\alpha^2 - b\alpha - m\omega^2 + k = 0 \qquad b\omega - 2m\omega\alpha = 0$$

$$\frac{b^2}{4m} - \frac{b^2}{2m} - m\omega^2 + k = 0 \qquad \alpha = \frac{b}{2m}$$

$$m\omega^2 = k - \frac{b^2}{4m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

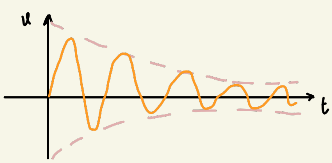
solving ω w/ drag force

→ frequency of damped oscillator

* NOT time dependent

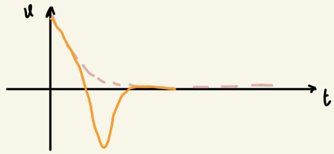
→ equation of motion for a damped oscillator

$x(t) = Ae^{-\alpha t}\sin(\omega t + \phi)$



under damped → when the damping term is weak, causing many oscillations and then coming to a stop.

ex/: pendulum, grandfather clock



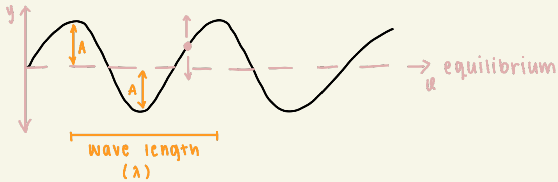
Overdamped → When the damping is too strong; It doesn't complete a cycle of oscillation; goes directly back to point of equillibrium

ex/: car going over a speed bump

* amplitude depends on time
* ω does not depend on time

WAVES : oscillation that travels through space

→ transverse wave



Amplitude (A)	m
Period (T)	s
frequency (f)	$\frac{1}{s}$ or Hz
wave length (λ)	m
wave speed (c)	m/s

$$c = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}$$

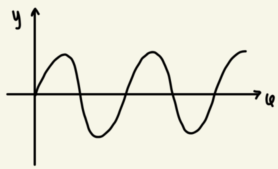
$$\boxed{c = \frac{\lambda}{T}} \qquad f = \frac{1}{T}$$

$$\boxed{c = \lambda f}$$

→ Period : time to complete a cycle [s]

frequency : how often $[\frac{1}{s}]$ aka Hz

→



$y(u,t) = A\sin(ku)$ ← NOT spring constant

let $t=0$

$y(u,0) = y(u+\lambda, 0)$ aka wave repeats

$A\sin(ku) = A\sin(ku + k\lambda)$

↓

same as right side when we add 2π

$ku + 2\pi = ku + k\pi$

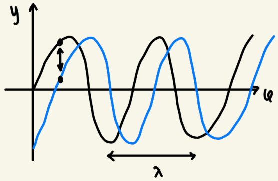
$\boxed{k = \frac{2\pi}{\lambda}}$ → **wave number** (NOT spring constant) $[\frac{1}{m}$ or $\frac{rad}{m}]$

→ $y(x,0) = A\sin(kx)$

↓ let t go

$$\boxed{y(x,t) = A\sin(kx - \omega t + \phi)}$$

formula for oscillating waves



$$\frac{\partial y}{\partial t} = A\cos(kx - \omega t + \phi)\,[-\omega]$$

$$\frac{\partial y}{\partial x} = Ak\cos(kx - \omega t + \phi)$$ slope of the wave

velocity

$$v = \frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t + \phi)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx - \omega t + \phi)$$ curvature (concavity)

acceleration

$$a = \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx - \omega t + \phi)$$

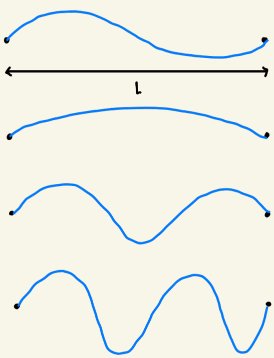
$$\frac{\frac{\partial^2 y}{\partial t^2}}{\frac{\partial^2 y}{\partial x^2}} = \frac{\cancel{-A}\omega^2\cancel{\sin(\ldots)}}{\cancel{-A}k^2\cancel{\sin(\ldots)}} = \frac{\omega^2}{k^2} = \frac{\left(\frac{2\pi}{T}\right)^2}{\left(\frac{2\pi}{\lambda}\right)^2} = \frac{4\pi^2}{T^2}\cdot\frac{\lambda^2}{4\pi^2} = \frac{\lambda^2}{T^2} = c^2$$

→ $\boxed{c = \frac{\lambda}{T}}$

→ $\boxed{c = \frac{\omega}{k}}$

linear wave equation $\boxed{\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}}$

→



$L = \lambda = \frac{2\lambda}{2}$

$L = \frac{\lambda}{2}$

$L = \frac{3\lambda}{2}$

$L = 2\lambda = \frac{4\lambda}{2}$

$$\boxed{L = \frac{n\lambda}{2}}$$

L = length

n = harmonic #

→ $y_1(x,t) = A\sin(kx - \omega t)$

$y_2(x,t) = A\sin(kx + \omega t)$

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$$

$$y_{Tot} = y_1 + y_2 = A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$y_T = A\left[\sin(kx)\cos(\omega t) - \cancel{\cos(kx)\sin(\omega t)} + \sin(kx)\cos(\omega t) + \cancel{\cos(kx)\sin(\omega t)}\right]$$

$$y_T = A\left(2\sin(kx)\cos(\omega t)\right)$$

if $\sin(kx) = 0 \implies y_{TOT} = 0$

$$\sin(kx) = \sin(\pi)$$

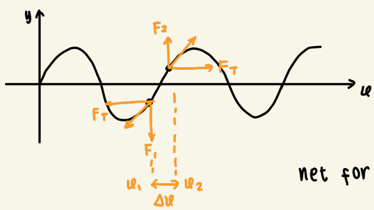
if $x = L \implies kL = \pi$ or 2π or 3π or $n\pi$

$$kL = n\pi \quad \left(k = \frac{2\pi}{\lambda}\right)$$

$$\frac{2\pi}{\lambda} L = n\pi$$

$$\boxed{L = \frac{n\lambda}{2}}$$

SPEED OF WAVE IN A STRING



net force (x - dir) = 0

$$\frac{F_1}{F_T} = -\left.\frac{\partial y}{\partial x}\right|_{x_1} \qquad \frac{F_2}{F_T} = \left.\frac{\partial y}{\partial x}\right|_{x_2}$$

ΣF_y (string is just going up & down)

$$F_{net} = F_1 + F_2$$

$$= F_T\left[\left.\frac{\partial y}{\partial x}\right|_{x_2} - \left.\frac{\partial y}{\partial x}\right|_{x_1}\right] = ma$$

density

$$\mu = \frac{M_{string}}{L_{string}}$$

$m = \mu \Delta x \rightarrow$ for part of string

$$a = \frac{F_T}{\mu} \underbrace{\frac{\left[\left.\frac{\partial y}{\partial x}\right|_{x_2} - \left.\frac{\partial y}{\partial x}\right|_{x_1}\right]}{\Delta x}}_{\text{definition of derivative}}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{F_T}{\mu} \frac{\partial^2 y}{\partial x^2}$$

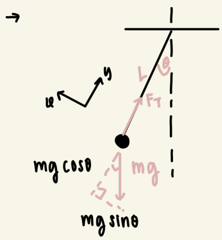
<from prev lesson> $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$

$$c^2 = \frac{F_T}{\mu} \implies c = \sqrt{\frac{F_T}{\mu}}$$

$$c^2 = \frac{1}{\mu} F_T$$

PENDULUM

→



$$F_{net\,y} = F_T - mg\cos\theta = 0$$

$$F_{net\,x} = -\cancel{m}g\sin\theta = \cancel{m}a \qquad \text{(mass does not matter)}$$

$$a = -g\sin\theta$$

$$\frac{d^2x}{dt^2} = -g\sin\theta \qquad x \text{ measures arc length}$$

$$x = L\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

series expansion of sinθ

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \qquad \theta \text{ is small}$$

for small θ, $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

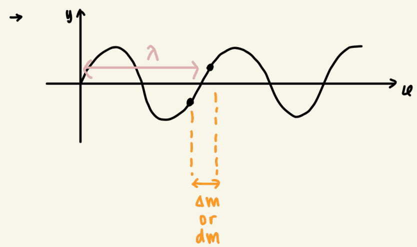
no more than 10°

$$\theta = \theta_{max}\sin(\omega t + \phi)$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}} \qquad \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

ENERGY IN WAVES (E)

→



$$dE = dK + dU$$

$$dE = \frac{1}{2}dm\,v^2 + \frac{1}{2}k_s y^2 \qquad \text{spring constant}$$

$$\left[\ \omega = \sqrt{\frac{k}{m}} \Rightarrow k_s = \omega^2 m\ \right]$$

$$dE = \frac{1}{2}dm\,v^2 + \frac{1}{2}y^2\omega^2\,dm$$

$$dE = \frac{1}{2}v^2\mu\,dx + \frac{1}{2}y^2\omega^2\mu\,dx \qquad [m = \mu\,\Delta x]$$

remember $y(x,t) = A\sin(kx - \omega t)$ $\qquad \left[k = \frac{2\pi}{\lambda}\right]$

not k_s … wave # !

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$$

$$v(x,t) = -A\omega\cos\left(\frac{2\pi}{\lambda}x - \omega t\right)$$

plug these in

$$dE = \frac{1}{2}\mu\left[A^2\omega^2\cos^2\left(\frac{2\pi}{\lambda}x - \omega t\right) + A^2\omega^2\sin^2\left(\frac{2\pi}{\lambda}x - \omega t\right)\right]dx$$

$$dE = \frac{1}{2}\mu\left(A^2\omega^2\right)dx$$

$$dE = \frac{1}{2}\mu A^2\omega^2\,dx$$

$$E = \int_0^\lambda \frac{1}{2}\mu A^2\omega^2\,dx$$

$$\boxed{E_\lambda = \frac{1}{2}A^2\omega^2\mu\lambda}$$

POWER IN WAVES (P)

power

$$P = \frac{E_\lambda}{T} = \frac{1}{2}A^2\omega^2\mu\frac{\lambda}{T}$$

$$\boxed{P = \frac{E_\lambda}{\Delta t}} \qquad \boxed{P = \frac{1}{2}A^2\omega^2\mu C}$$

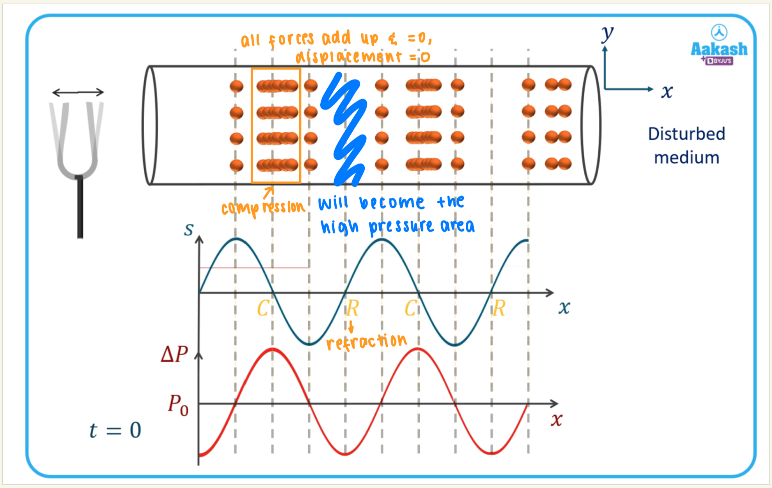
INTENSITY IN WAVES (I)

$$\boxed{I = \frac{P}{A}}$$

area of sphere → $I = \dfrac{P}{4\pi r^2}$

area of circle → $I = \dfrac{P}{\pi r^2}$

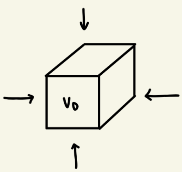
SOUND WAVES



$$P(x,t) = P_{max}\sin(kx - \omega t + \phi) \qquad \phi \to \frac{\pi}{2}$$

$$s(x,t) = S_{max}\sin(kx - \omega t)$$

imagine...



$$\frac{\Delta V}{V_0} = \frac{\Delta P}{B}$$

bulk modulus : property of material that resists changes to volume

$$P = \frac{F}{A} \quad \left[\frac{N}{m^2} = Pa\right]$$ Pascal

pressure

1 atm = 101, 325 Pa

(wave speed for spring)

→ $c = \sqrt{\frac{F_T}{\mu} \cdot \frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)}} = \sqrt{\frac{P}{\rho}}$ pressure, density

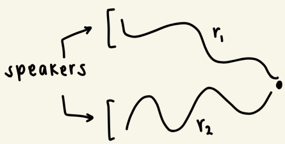
$\mu = \frac{M}{L} \cdot \frac{1}{A} = \frac{M}{V} = \rho$

(sound wave speed) $\boxed{c = \sqrt{\frac{B}{\rho}}}$

→

W/m^2	what we hear	"Bel" A. G. Bell (Alexander Graham Bell)	deciBel (dB)
10^{-12}	ant footprint	0	0
10^{-11}	breathing	1	10
10^{-10}	leaves rustling	2	20
10^{-9}	whisper	3	30
10^{-8}	quiet room	4	40
10^{-7}	normal conversation	5	50
10^{-1}	power tools	11	110
10^{0}	concert - - - - - pain	12	120

→

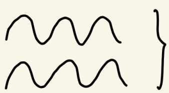


$r_1 = r_2$ (they are in phase)

$\Delta\phi = \overset{k}{\frac{2\pi}{\lambda}} r_1 - \frac{2\pi}{\lambda} r_2 = 0$ or 2π

$= \frac{2\pi}{\lambda}(r_1 - r_2) = 2\pi n$

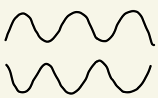
$\phi = 2\pi n$



} in phase → constructive

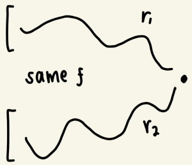
$\phi = 2\pi(2n+1)$

(2 sounds make no sound at all or cancels out)

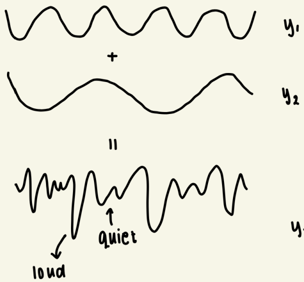


} out of phase → destructive ______

BEATS



- Beats occur when you have different frequency



$y_1(x,t) = A\cos(k_1 x - \omega_1 t)$

$y_2(x,t) = A\cos(k_2 x - \omega_2 t)$

$$y_T = y_1 + y_2 = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$y_T = 2A\cos\left(\frac{k_1 x - \omega_1 t + k_2 x - \omega_2 t}{2}\right)\cos\left(\frac{k_1 x - \omega_1 t - k_2 x + \omega_2 t}{2}\right)$$

set $x = 0$

$$y_T = 2A\cos\left(\frac{-\omega_1 t - \omega_2 t}{2}\right)\cos\left(\frac{-\omega_1 t + \omega_2 t}{2}\right)$$

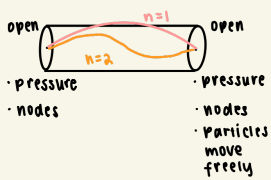
$\omega = 2\pi f$, multiply the inside by $\ominus$

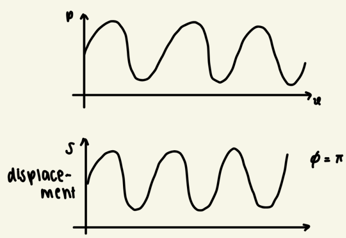
$$= 2A\cos\left(\frac{(2\pi f_1 + 2\pi f_2)}{2}t\right)\cos\left(\frac{(2\pi f_1 - 2\pi f_2)}{2}t\right)$$

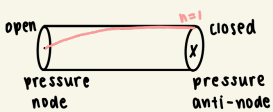
$\frac{f_1 + f_2}{2}$ = faverage = fav

$$y_T = 2A\cos(2\pi \underbrace{f_{av}}_{\text{frequency that you hear}} t)\cos(\pi \underbrace{(f_1 - f_2)}_{\text{beat frequency}} t)$$

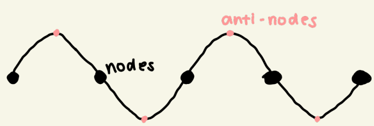
WAVES IN TUBES





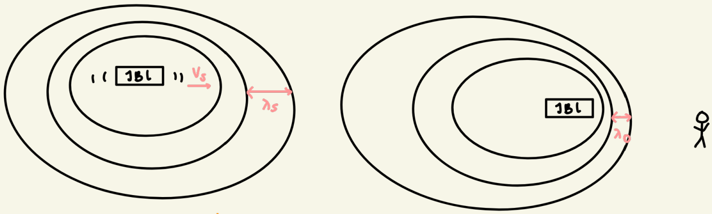


* odd harmonics only , if it's closed on one end



DOPPLER EFFECT

- motion between observer and source causing change in frequency.



how far the source moved in 1 T_s

$$\lambda_o = \lambda_s + V_s \underbrace{\Delta t}_{\text{period (T) of the source } T_s}$$

$c = \lambda f = \frac{\lambda}{T} \Rightarrow \lambda = cT$

$$\underbrace{cT_o}_{\text{period observed}} = cT_s + V_s T_s$$

* V_s is ⊕ if source is moving AWAY from observer

$$\frac{c}{f_o} = \frac{c}{f_s} + \frac{V_s}{f_s}$$

$$\frac{c}{f_o} = \frac{1}{f_s}(c + V_s)$$

$$\frac{f_o}{f_s} = \frac{c}{c + V_s}$$

$$\boxed{f_o = \left(\frac{c}{c + V_s}\right) f_s}$$

frequency observed of a moving source, stationary observer

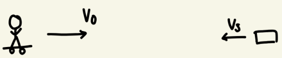
$$T_o = T_s - \Delta t$$

$$T_o = \frac{\lambda_s}{c + V_o}$$

$c = \lambda f$

$$\frac{1}{f_o} = \frac{\frac{c}{f_s}}{c + V_o} \rightarrow \boxed{f_o = \left(\frac{c + V_o}{c}\right) f_s}$$

frequency observed by a moving observer



$$f' = \left(\frac{c}{c + V_s}\right) f_s$$

$$f_o = \left(\frac{c + V_o}{c}\right) f'$$

$$= \left(\frac{c + V_o}{\cancel{c}}\right)\left(\frac{\cancel{c}}{c + V_s}\right) f_s \rightarrow \boxed{f_o = \left(\frac{c + V_o}{c + V_s}\right) f_s}$$

⊕ if observer is moving TOWARDS the source

⊕ if source is moving AWAY from observer

both source & observer moving