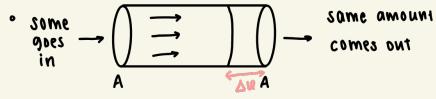


EXAM #2



Fluid Dynamics

- energy involved
- sloshes, fun  (turbulent flow)
- vector field  (laminar flow)



$V, \rho, \text{Volume}, P, \Delta t$

! mass is conserved

$$M_{\text{in}} = M_{\text{out}}$$

$$\frac{\rho V_m}{\Delta t} = \frac{\rho A \Delta u}{\Delta t}$$

divide by Δt to turn into flow rate

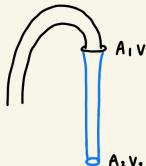
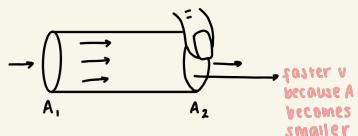
$$\frac{\rho V}{\Delta t} = \rho A v$$

$$\rho A_1 v_1 = \rho A_2 v_2$$

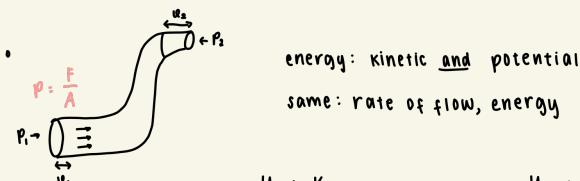
$$\boxed{A_1 v_1 = A_2 v_2}$$

velocity

continuity of flow



$v_2 > v_1$ so $A_2 < A_1$
for $A_1 v_1 = A_2 v_2$
to stay true



$$U_1 + K_1 = U_2 + K_2$$

$$mgh_1 + \frac{1}{2}mv_1^2 + \underbrace{F_1u_1 - F_2u_2}_{\text{work done}} = mgh_2 + \frac{1}{2}mv_2^2$$

\downarrow
depends on time
 $m = \rho V$

$$\frac{\rho Y g h_1}{Y} + \frac{\frac{1}{2} \rho V_1^2}{X} + \frac{F_1 u_1}{V} - \frac{F_2 u_2}{V} = \frac{\rho Y g h_2}{Y} + \frac{\frac{1}{2} \rho V_2^2}{X}$$

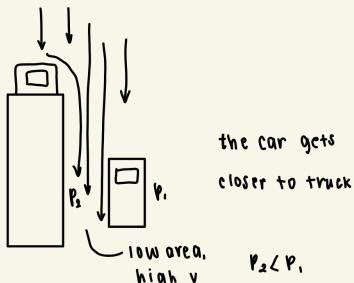
$$\rho g h_1 + \frac{1}{2} \rho v_1^2 + \frac{F_1 u_1}{A_1 Y_1} - \frac{F_2 u_2}{A_2 Y_2} = \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\boxed{P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2}$$

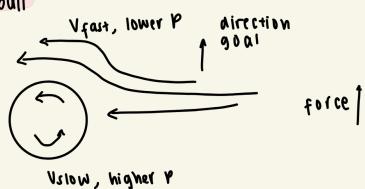
Bernoulli's Equation

ex: syringe (pressure on both sides; force from hand and when fluid goes into human body; are different)

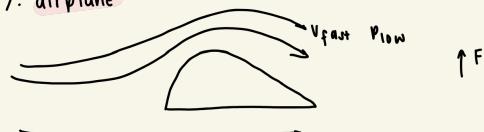
- pressure wants to go from high pressure - low pressure
- height(h) increase, lower pressure (P)
- velocity (v) increase, lower pressure (P)



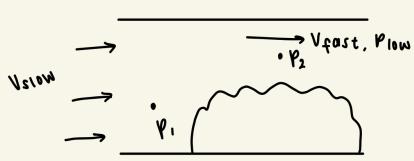
- ex: curveball



- ex: airplane



- ex: artery paradox
blood pressure increases w/ blockage



blood has viscosity. the thicker the fluid

the more R is gonna matter. so, blood pressure ↑

resistance to flow (R) - pressure is highest @ center
 "eta"
 - viscosity (how elastic fluid is)
 - radius

$$R = \frac{8\eta L}{\pi r^4}$$

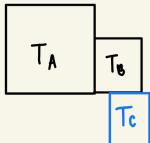
$$\frac{\Delta V}{\Delta t} = Q = \frac{\text{flow}}{\text{rate}} = \frac{\Delta P}{R}$$

* flow rate of blood has to be the SAME or you'll die ☹

$$\text{so } \frac{\Delta P}{R} \rightarrow \text{low}$$

TEMPERATURE

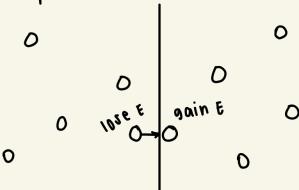
- hotter - more motion of atoms
- [definition] a measure of average kinetic energy of a particle in a system.
- heat - transmission of thermal energy due to a difference in temperature.
- 0th Law of Thermodynamics



if $T_A > T_B$, heat will flow from A \rightarrow B
until they reach thermal equilibrium

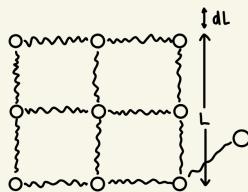
- If $T_A = T_B$ and $T_c = T_B$, $T_c = T_A$

(fast particle) A B (slower particles)



statistically more likely for A particles to transfer energy to B particles.

- $dL = \alpha L dT$ ex: solid
linear expansion constant



$$\begin{aligned} V &= L^3 \\ dV &= 3L^2 dL \\ &= 3L^2 \propto L dT \\ &= 3L^3 \propto dT \end{aligned}$$

$$dV = 3\alpha V dT$$

$$dV = \beta V dT$$

\downarrow
volume
expansion
coefficient

ex: fluids

- When $T \uparrow$
 1. hole will grow
 2. hole will shrink

because all atoms in the material are moving further apart

ans:

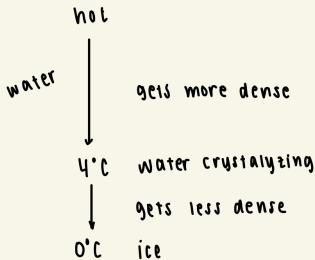


- 

object becomes less dense → metal expands first than glass
- 

bimetallic strip (diff color) — one will expand faster than the other, and so it will curl

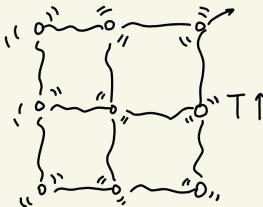
- water crystals forming 4°C
ice 0°C



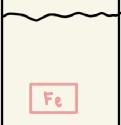
- densest water — 4°C

PHASES OF MATTER

- solid, liquid, gas, plasma + weird others ??
- low energy - medium - high - super high energy



during phase change
molecules do not gain KE,
so T remains constant
ex: same T for boiling & simmering
bcs already at phase change. T
would not increase

- 

room temp water + hot piece iron
water heats up

$$\Delta T = \frac{Q_{\text{heat}}}{mc} \rightarrow Q_{\text{heat}} = mc \Delta T$$

$$c_{\text{aluminum}} = 900 \frac{\text{Joules}}{\text{kg K}}$$

$$c_{\text{water}} = 4,186 \frac{\text{Joules}}{\text{kg K}}$$

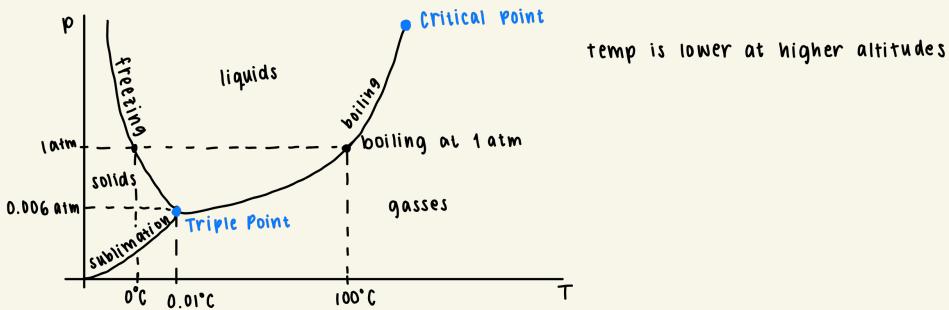
→ $Q_{\text{heat}} = mc \Delta T$
only applies if there
is no phase change



- during a phase change, ΔT plateaus
- $Q_f = m L_f$ → latent heat of fusion

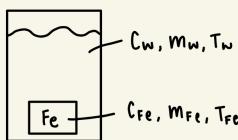
$$Q_v = m L_v \rightarrow \text{Latent heat of vaporization}$$

↑ applies if there is a phase change



temp is lower at higher altitudes

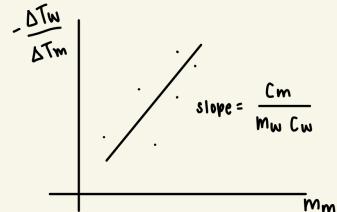
HEAT TRANSFER, CALORIMETRY



$$\text{Heat} \rightarrow Q = mc\Delta T$$

$$\theta_{\text{Tw}} = -\theta_{\text{Fe}}$$

$$\begin{aligned} \text{any metal } m & \quad m_w C_w \Delta T_w = -m_{\text{Fe}} C_{\text{Fe}} \Delta T_{\text{Fe}} \\ & \quad -\frac{\Delta T_w}{\Delta T_m} = \frac{C_m}{m_w C_w} M_m \end{aligned}$$



* the object has to touch

METHODS OF HEAT TRANSFER (1.6)

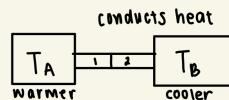
- conduction, convection, radiation
- convection → needs a medium (gas, liquids, ...)

→ hot material expands → lower density → becomes more buoyant (floats)



cools → sinks

- conduction →



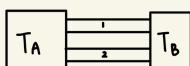
$$\frac{d\theta}{dt} = \frac{T_A - T_B}{R}$$

⇒ same as
ohm's law

$$V = IR$$

$$I = \frac{V}{R}$$

$$R_{\text{series}} = R_1 + R_2$$



$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{d}{kA}$$

length/thickness
of the material

k conductivity
a property of the
material that allows
the flow of energy



$$R_S = R_1 + R_2 + R_3$$

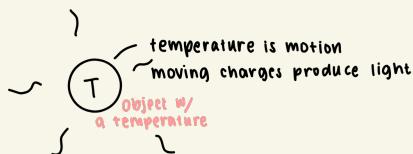
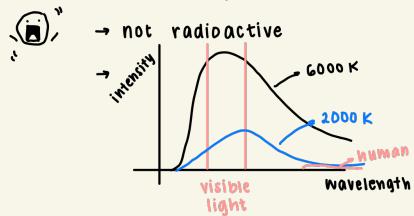
large k : metals, water

low k : wood, plastics, rubber,
(insulator)
asbestos, vacuum, air

* If u open oven, the hot air is still safe,

rather than pouring boiling water, so air is best insulator

- radiation : light, does not need medium



* we feel ice is cold because the human body is warmer than ice, so ice actually has heat.

Black body - absorbs all light, reflects nothing

- all light from a black body is associated with its temperature

$$\frac{d\Theta}{dt} = \sigma A e T^4$$

stephan-Boltzman constant

emissivity

- heat flow depends very heavily on temperature T.



$$\frac{d\Theta}{dt} = \sigma A e (T_0^4 - T_s^4)$$

object

surrounding

* $\frac{E}{t} = \text{Power}$

* the object doesn't have to be in contact with radiation

$$\Theta = m L$$

Joules

kg

$$\Theta = m c \Delta T$$

Joules

kg

$\frac{J}{kg \cdot ^\circ C}$

$^\circ C$

IDEAL GAS (2.1)

(in physics)

$$PV = NkT$$

Boltzman Constant

number of gas particles

(in chemistry)

$$PV = nRT$$

ideal gas constant

number of molecules

$$n = \frac{N}{N_A} (\text{Avogadro constant}) = \frac{N}{6.02 \times 10^{23}}$$

$$R = \frac{k}{6.02 \times 10^{23}}$$

• What makes gas ideal:

- low P, low T
- atoms small relative to volume, lots of space between molecules.
- no intermolecular forces
- collisions are elastic $O \rightarrow \leftarrow O \Rightarrow \leftarrow O O \rightarrow$ inelastic (stick) $O \rightarrow \leftarrow O \Rightarrow \infty$
- Uniform everywhere = homogenous (same everywhere)
- isotropic (same in every direction, not flowing, directions random)

• not ideal gas (Van Der Waal's Potential)

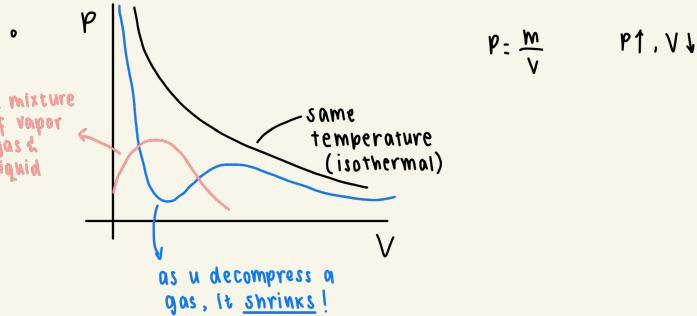
$$(P + a(\frac{N}{V})^2)(V - Nb) = NkT$$

constant depend
on material

density \downarrow Volume of container \downarrow Total volume of atoms \downarrow

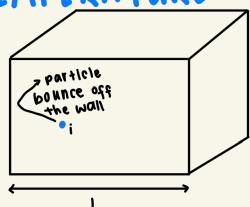
$\overline{KE}_{CO_2} = \overline{KE}_{H_2}$ $V_{CO_2} < V_H$

$$\frac{1}{2} m_{CO_2} V_{CO_2}^2 = \frac{1}{2} m_{H_2} V_{H_2}^2$$

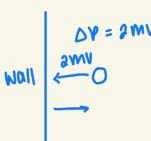


PRESSURE, TEMPERATURE

• $P = \frac{F}{A}$



$$F_i = \frac{\Delta P}{\Delta t} = \frac{2mV_i}{\Delta t}$$



as particle bounce in any direction before returning back to its point:

$$\Delta t = \frac{\Delta R}{V_{iu}} = \frac{2L}{V_{iu}}$$

$$F_i = \frac{2mV_i}{\Delta t / V_{iu}} = \frac{m}{L} V_{iu}^2$$

$$F = \sum_{i=1}^N F_i = \sum_{i=1}^N \frac{m}{L} v_i u^2 = \frac{m}{L} \sum_{i=1}^N v_i u^2 \quad (\text{multiply by } \frac{N}{N})$$

the particles inherently have $KE = \frac{1}{2} mv^2$ as they move

$$F = \frac{Nm}{L} \overline{v_u^2}$$

average

as there are 3 directions
(u, v, z)

$$\overline{v^2} = \overline{v_u^2} + \overline{v_v^2} + \overline{v_z^2}$$

$$\overline{v^2} = 3\overline{v_u^2} \quad \text{or} \quad \overline{v_u^2} = \frac{1}{3}\overline{v^2}$$

$$F = \frac{Nm}{3L} \overline{v^2}$$

$$p = \frac{F}{A} \Rightarrow p = \frac{Nm}{3LA} \overline{v^2}$$

VOLUME!

$$p = \frac{Nm}{3V} \overline{v^2}$$

→ related pressure, volume,
& velocity

$$(PV = NkT)$$

$$p = \frac{NkT}{V} = \frac{Nm}{3V} \overline{v^2}$$

$$kT = \frac{1}{3} m \overline{v^2}$$

$$3kT = m \overline{v^2}$$

$$\frac{3}{2} kT = \frac{1}{2} m \overline{v^2}$$

→ average KE

$$\overline{KE} = \frac{3}{2} kT$$

→ related Temperature & Kinetic Energy

$$\overline{v^2} = \frac{3kT}{m}$$

$$\bar{v} = \sqrt{\frac{3kT}{m}}$$

→ root mean square

- Temperature in gas comes from translational motion of particles

→ rotation

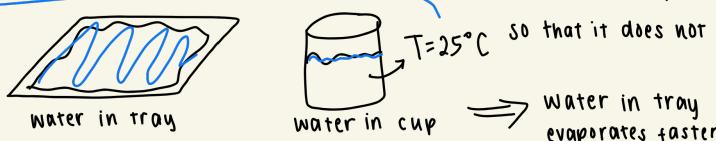
→ vibrating



- other ways to add E w/o increasing translational motion
- the more ways a molecule can move, the more E you can add w/o increasing ↑ Temperature


Specific Heat

EVAPORATION

- Temperature ∝ average velocity
 - It HAS something to do w/ surface area → only the top particles are able to escape if it's going fast enough & are directed so that it does not bump into other particles
- 

- When the higher velocity particle leaves, the average temperature will go down.
but the surroundings makes the system equilibrium that keeps the water temp at 25°C.

HUMIDITY

- Water vapor
 - sweat → condensation → skin
 - there is some water molecule from the water vapor that brings heat in (why sweating works well in humidity)
 - some molecules travel slower than others
 - condensation deposits E
 - evaporation carries E away
 - relative % humidity = $\frac{\text{partial } P_{\text{H}_2\text{O}}}{\text{vapor } P_{\text{H}_2\text{O}}} \times 100\%$
 - dew point: temperature at which spontaneous condensation occurs
- 

PARTIAL PRESSURE

→ change w/ T

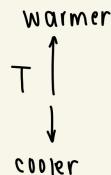
- pressure from 1 type of molecule
- high partial pressure of H₂O more likely for condensation to occur

VAPOR PRESSURE

- maximum partial pressure for a given Temp.

T	vapor pressure for H ₂ O
0°C	610 Pa
10°C	1228 Pa

- As the T rises, you need higher pressure
- Air can hold more moisture as it gets warmer



EQUIPARTITION, SPECIFIC HEAT

$$P = \frac{F}{A}$$



motor heat capacity of
a gas constant V
 $Q = n C_v \Delta T$

* if a gas can change (volume)V, it does work.

Energy in does not always contribute to ΔT .

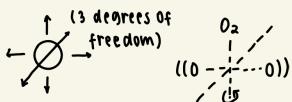
$$\Delta E_{int} = \frac{3}{2} n k \Delta T = \text{average KE}$$

$$\Delta E_{int} = \frac{3}{2} n R \Delta T = \gamma n C_v \Delta T$$

$$C_v = \frac{3}{2} R$$

comes from 3 directions
for translational KE

→ true for a monoatomic gas; He, Ne, K_e



vibration - 2 degrees of freedom

spin - 2 degrees of freedom

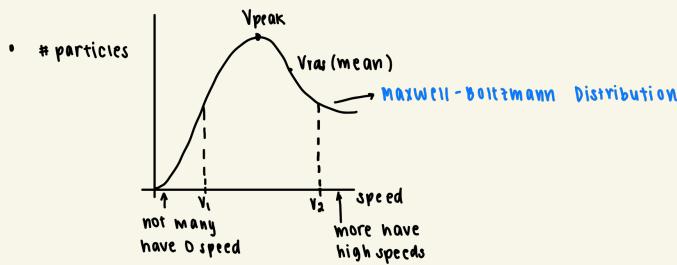
$$C_v = \frac{5}{2} \text{ at room temp.}$$

* quantum mechanics

normal air is diatomic : O₂, N₂, Cl₂

triatomic gasses : NO₂, CO₂, H₂O

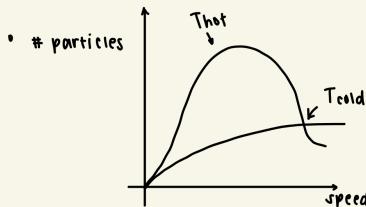
# atm	gas	C_v / R
1	Ar	1.5
1	He	1.5
1	Neon	1.5
2	CO	2.5
2	H ₂	2.47
3	CO ₂	3.48
3	H ₂ S	3.13
3	N ₂ O	3.68



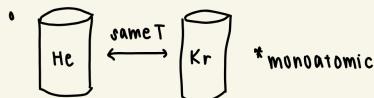
- $N(v_1, v_2) = N \int_{v_1}^{v_2} f(v) dv$

$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

normalization constant



speed of particles \leftrightarrow Temp \leftrightarrow Wave length of light emitted



T/F/more info: RMS speeds of both gasses are the same.

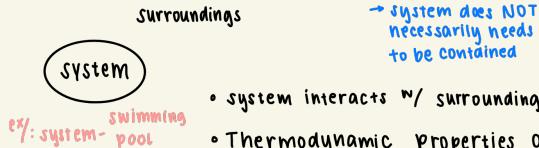
$$\frac{3}{2} kT = \frac{1}{2} mv^2$$

$$V_{rms} = \sqrt{\frac{3kT}{m}} \quad \text{At same } T,$$

V_{rms} depends on mass

internal energy of one mol of gas in each container is the same.

3.1 THERMODYNAMIC SYSTEMS

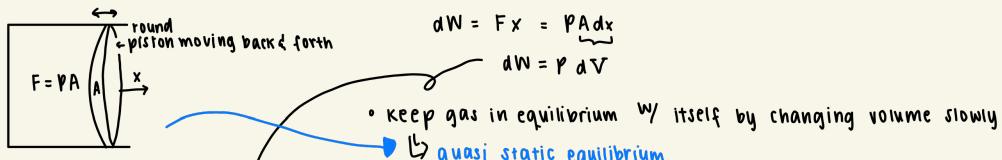


- system interacts w/ surroundings
- Thermodynamic Properties of the system:
 - Eint T (temperature), P (pressure), V (volume)
- assume energy is conserved
- assume system is in thermodynamic equilibrium w/ itself
- assume system can exchange energy w/ surroundings
but no matter is exchanged → "closed system"
- goal: build Equation of State

closed system: $f(P, V, T) = 0$

$$PV = NKT$$

$$\text{ex: } PV - NKT = 0$$



$$dW = P dV$$

$$P = \frac{NRT}{V}$$

$$W = \int \frac{NRT}{V} dV$$

ISOHERMAL PROCESS if T is constant,

constant or same

$$W = NRT \int_{V_i}^{V_f} \frac{1}{V} dV$$

$$W = nRT \ln \left(\frac{V_f}{V_i} \right)$$

work done for isothermal expansion

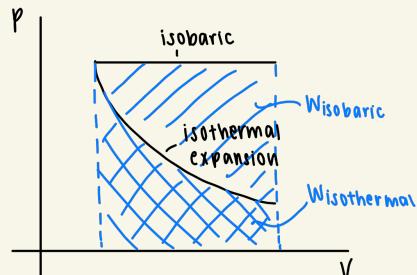
ISOBARIC PROCESS keep P constant

$$dW = P dV$$

$$W = \int_{V_i}^{V_f} P dV$$

$$W = P \Delta V$$

* WORK IS area under the curve



LAB: thermal conduction of a PVC pipe

constant	How to achieve	process	internal	W	ΔU
T	change volume slowly so thermal equilibrium is maintained	isothermal	○	$nRT \ln\left(\frac{V_f}{V_i}\right)$	$nRT \ln\left(\frac{V_f}{V_i}\right)$
P	allow piston to move while gas is in thermal equilibrium	isobaric	$\frac{3}{2} nR\Delta T$	$P\Delta V$	$ \begin{aligned} &= \frac{3}{2} nR\Delta T + P\Delta V \rightarrow PV = nRT \\ &= \frac{3}{2} nR\Delta T + nR\Delta T \\ &= n\Delta T \left(\frac{3}{2} R + R \right) \\ &= n\Delta T \left(\underbrace{C_V + R}_{C_P = C_V + R} \right) \\ \boxed{\Delta U = nC_P\Delta T} \end{aligned} $
V	lock the piston	isochoric	$\frac{3}{2} nR\Delta T$	○	$\frac{3}{2} nR\Delta T$
ΔU	insulate well (vacuum), quickly change V	adiabatic	$\frac{3}{2} nR\Delta T$	$\frac{3}{2} nR\Delta T$	○

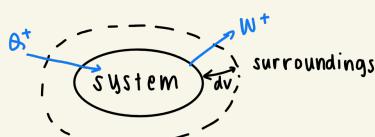
- 1st Law of Thermodynamics →

Remember!

$E_{int} = \frac{3}{2} nRT$ for monoatomic ideal gas

$$\Delta E_{int} = \Delta U = \Delta Q - \Delta W$$

heat going into system + work done by the system
 + + +

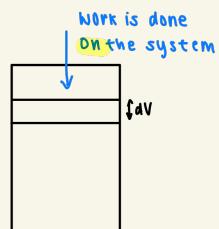
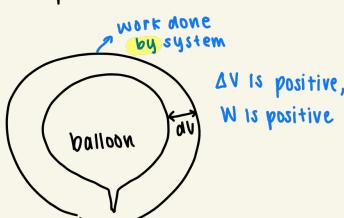


- $W = P\Delta V$ Work is positive when ΔV is positive (expansion)

other convention...

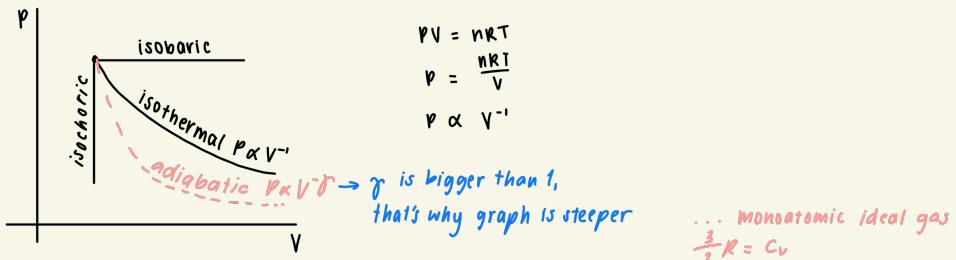
$$\begin{aligned}
 \Delta U &= \Delta Q + \Delta W \\
 \downarrow \\
 W &= -P\Delta V
 \end{aligned}$$

work done **on** the system



Sign conventions

process	convention	
Heat added to the system	+ Q	* all assuming 1st LAW $\Delta U = Q - W$
Heat removed from system	- Q	* switch work signs if you write $\Delta U = Q + W$
Work done by the system	+ W	
Work done on the system	- W	



What about adiabatic?

$$\Delta U = Q^0 - W$$

$$\Delta U = -W$$

for adiabatic process

$$W = PV = \frac{3}{2}nRT$$

$$dW = d(PV) = \frac{3}{2}nRdT$$

$$\frac{3}{2}nRdT = -d(PV)$$

$$C_v n dT = -P dV$$

$$dT = \frac{-P dV}{C_v N}$$

$$PV = nRT$$

$$d(PV) = nRdT$$

$$PdV + VdP = nRdT$$

$$dT = \frac{PdV + VdP}{nR}$$

Set equal to each other

$$\frac{PdV + VdP}{nR} = \frac{-PdV}{C_v N}$$

$$C_v (PdV + VdP) = -R PdV$$

$$C_v PdV + C_v VdP + R PdV = 0$$

$$(C_v + R) PdV + C_v VdP = 0$$

$$\frac{C_p}{P} PdV + \frac{C_v}{P} VdP = 0$$

$$C_p \frac{dV}{V} + C_v \frac{dP}{P} = 0$$

divide by C_v

$$\gamma \int_{V_1}^{V_2} \frac{dV}{V} + \int_{P_1}^{P_2} \frac{dP}{P} = 0$$

$$\gamma \ln(V_2) - \gamma \ln(V_1) + \ln(P_2) - \ln(P_1) = 0$$

$$\gamma \ln(V_2) + \ln(P_2) = \gamma \ln(V_1) + \ln(P_1)$$

$$P_2 V_2^\gamma = P_1 V_1^\gamma$$

* $PV^\gamma = \text{constant}$



- We can rewrite

$$PV^\gamma = \text{constant}$$

$$\longrightarrow P = \alpha V^{-\gamma}$$

\downarrow
constant
 $P \propto V^{-\gamma}$
 \uparrow
proportional to

note!: isothermal
 $P \propto V^{-1}$

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} \text{ (monoatomic)} > 1 \text{ (see graph)}$$

$\frac{7}{5}$ (diatomic)

- 1st Law

$$\Delta U = \cancel{Q} - W$$

$$\frac{3}{2}nR\Delta T = -P\Delta V$$

⋮

can also be

$$\begin{cases} dT = \dots \\ dP = \dots \\ dV = \dots \end{cases}$$

ideal gas

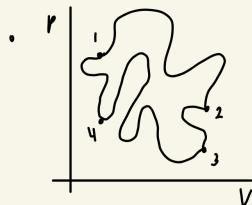
$$PV = nRT$$

⋮

$$dT = \dots$$

if you get rid of $dP \rightarrow TV^{\gamma-1} = \text{constant}$

if you get rid of $dV \rightarrow P^{1-\gamma}T^\gamma = \text{constant}$

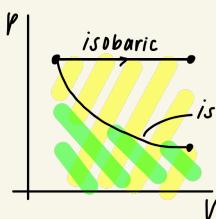


all changes/process can do this
(doesn't necessarily need to be adiabatic, etc...)

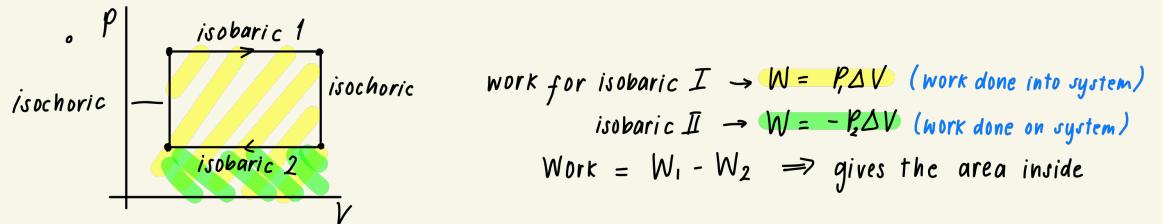
← CYCLIC PROCESS (E_{int} will change while you're going through the process of the cycle)

→ but once you go back to original state $\rightarrow \Delta U = 0$

* however you will still do work during the cycle



isobaric does more W than isothermal



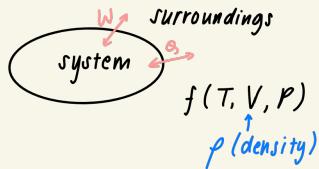
- Work done by cyclic process = area
- If internal energy (ΔU) = 0, the amount of heat (Q) is equal to the amount of work (W)

$$\Delta U = Q - W$$

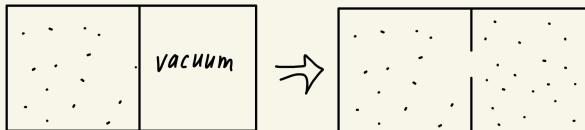
$$Q = W$$

Reversible Process

- change our system back to the original state



- return system & surroundings to original state



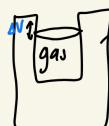
NOT REVERSIBLE

Why? because surroundings must do W to return system to original state

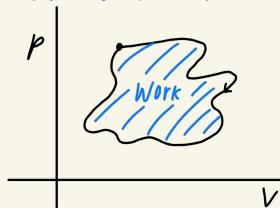
- o What does a process need to be considered reversible?

- no effect on environment
 - heat is not dissipated (no heat loss)
 - done in small increments,
so system remains in equilibrium w/ environment
- why? ↗

Examples of reversible processes

process	constant	example
phase change at equilibrium temp.		melting ice at 0°C , give a little $\Theta \rightarrow$ melt take a little $\Theta \rightarrow$ freeze
isothermal (change in ΔV)	T	gas is expanded by small ΔV , loses small ΔT , replaced by small $\Delta \Theta$
adiabatic ΔV	Θ	 <p>perfect insulation</p> <p>gas compressed in insulated chamber, exchanges small W w/ surroundings</p> <p>diff w/ isothermal: T doesn't have to stay the same</p> <p>piston</p> <p>add a little Θ to make piston expand lose a little $\Theta \rightarrow$ piston moves back</p>
isobaric ΔV	P	

HEAT ENGINE



: extract Θ from a source, do W , returns system to original state

some type of working fluid

e.g.: air in piston,

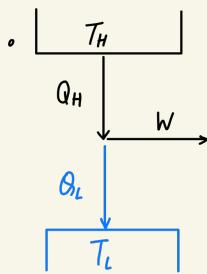
steam in steam engine

→ takes Θ turns it into W

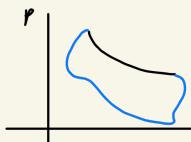
$$\Delta E_{\text{int}} = \Delta U = \Theta - W$$

← 1st Law of Thermodynamics

returning to initial state
means $\Delta U = 0$

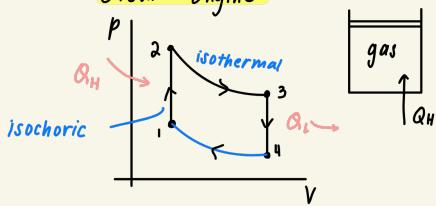


What you did



but it has to return
engine to its OG state

Steam Engine



$$\text{Total Heat added : } Q_{\text{total}} = Q_H - Q_L$$

$$\text{Total Work done : } W_{\text{net}} = W_{23} - W_{14}$$

$$= nRT_{23} \ln \frac{V_3}{V_2} + nRT_{14} \ln \frac{V_1}{V_4}$$

iso thermal
doesn't matter if it's T_2 or T_3

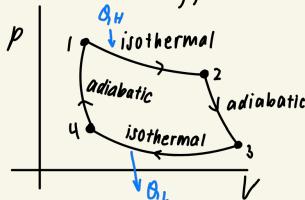
$$\text{efficiency } (\epsilon) = \frac{\text{what you get out}}{\text{what you get in}} = \frac{W_{\text{net}}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

$$\begin{aligned} \text{1}^{\text{st}} \text{ Law} \rightarrow \Delta U &= Q - W \\ \text{if } 0 & \quad W_{\text{net}} = Q_{\text{total}} \\ & \quad W_{\text{net}} = Q_H - Q_L \end{aligned}$$

H - high / hot SAME
L - low / cold THING

Carnot Cycle

What is the most efficient engine?



pressure (P) is zero when absolute zero!

can you make reversible cycle w/ only adiabatic?
No, we need other processes

Why not use isobaric instead of isothermal?

\downarrow
It changes T of gas
rather than doing W

- Isobaric has most W , so why use isothermal?
 - T is not constant
 - T is constant
- heat is absorbed, so putting more heat (our goal is to keep change of heat as little as possible)
 We are using adiabatic & isothermal because sure isobaric is putting more work, but in return it's going to need more heat = less efficient \rightarrow Recall: $e = 1 - \frac{Q_L}{Q_H}$

- referring to graph

$$W_{\text{total}} = W_{12} + W_{23} + W_{34} + W_{41}$$

$$Q_H = W_{12} = nRT_{12} \ln \frac{V_2}{V_1}$$

$$Q_L = W_{34} = -nRT_{34} \ln \frac{V_4}{V_3} = nRT_{34} \ln \frac{V_3}{V_4}$$

(-) since we are going to the other direction

$$e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{nRT_3 \ln \frac{V_3}{V_4}}{nRT_2 \ln \frac{V_2}{V_1}}$$

$$e = 1 - \frac{T_3}{T_2}$$

$$e = 1 - \frac{T_L}{T_H}$$

! use Kelvin (K)

efficiency of carnot engine

you can cancel
the $\ln(\frac{V_n}{V_x})$

Recall: $PV^\gamma = \text{constant}$

$$P_2 V_2^\gamma = P_3 V_3^\gamma \quad (\text{Recall: } PV = nRT)$$

$$\frac{nRT_2}{V_2} V_2^\gamma = \frac{nRT_3}{V_3} V_3^\gamma \quad P = \frac{nRT}{V}$$

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$$

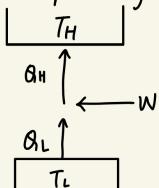
$$T_1 V_1^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$\frac{T_2 V_2^{\gamma-1}}{T_1 V_1^{\gamma-1}} = \frac{T_3 V_3^{\gamma-1}}{T_4 V_4^{\gamma-1}}$$

$$T_1 = T_2 \quad T_3 = T_4$$

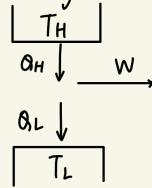
$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

- Heat Pump / Fridge

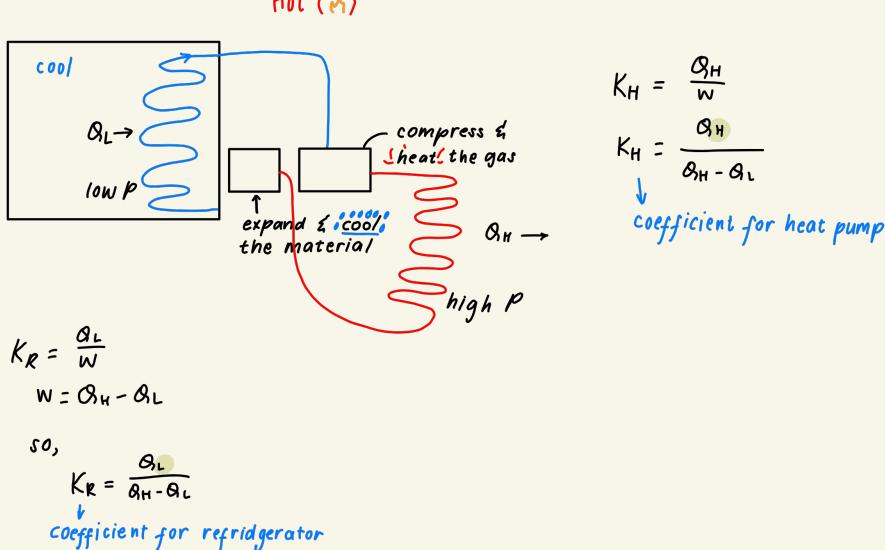


removes heat from the
low temperature

- Heat Engine



add heat



- Would there still be heat when system is isothermal?
- Isothermal means constant temperature regardless of addition of heat. Work would be in exchange of heat**
- When a gas expands isothermally, it does work. Where does the energy for this work come from? **Surroundings**
 - If you leave your refrigerator door open, does the temperature of the room increase or decrease? **increase**

ENTROPY (S)

- disorder — measure of how likely a system is to return to a previous state
- chaos — system that depends strongly on initial conditions (*nothing to do w/ entropy!*)

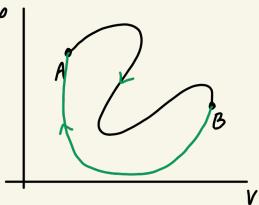
MICROSTATES

- maximum # of microstates gives you the most likely macrostate
- [def] # of ways to accomplish macrostate
- so what are microstates: Temperature, Volume, Pressure
- ex/:

increase volume isothermally, increase entropy (because there are more possible places for molecules/microstates)

- Entropy (S) could depend on...

$T, Q, n, V,$



entropy is a state function

- ΔS is independent of path

$$\boxed{\Delta S = \frac{\Theta}{T}} \quad (\text{for isothermal process})$$

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right)$$

- for isochoric (constant V)

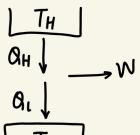
$$\Delta S = \int_A^B dS = \int_A^B \frac{d\Theta}{T} = \frac{3}{2} nR \int_A^B \frac{dT}{T} = \frac{3}{2} nR \ln\left(\frac{T_B}{T_A}\right)$$

- for adiabatic

since $\Delta Q = 0$ so, $\Delta S = 0$

- if you return to the same place, ΔS for system = 0

but there is work being done to the surroundings



so, ΔS for surroundings

$$\Delta S = \frac{\Theta}{T}$$

$$= -\frac{\Theta_H}{T_H} + \frac{\Theta_L}{T_L}$$

$$\begin{aligned} \Delta S &= -\frac{(\Theta_H + W)}{T_H} + \frac{\Theta_L}{T_L} \\ &= -\frac{\Theta_L}{T_H} - \frac{W}{T_H} + \frac{\Theta_L}{T_L} \\ &= -\frac{\Theta_L}{T_H} - \frac{\Theta_H}{T_H} + \frac{T_L \Theta_H}{T_H^2} + \frac{\Theta_L}{T_L} \\ &= -\frac{T_L \Theta_H}{T_H^2} - \frac{\Theta_H}{T_H} + \frac{T_L \Theta_H}{T_H^2} + \frac{\Theta_L}{T_L} \\ &= -\frac{\Theta_H}{T_H} + \frac{\Theta_H}{T_H} = 0 \end{aligned}$$

recall: $\Theta_H = \Theta_L + W$

$$\Theta_H = \Theta_L + \cancel{W} - \frac{T_L}{T_H} \Theta_H$$

$$\Theta_L = \frac{T_L}{T_H} \Theta_H$$

recall: $e = 1 - \frac{\Theta_L}{\Theta_H}$

$$= \frac{W}{\Theta_H} = 1 - \frac{T_L}{T_H}$$

↑
for most efficient (Carnot) engine

$$W = \Theta_H - \frac{T_L}{T_H} \Theta_H$$

(most efficient)
∴ Assuming your engine is ideal, w/ all reversible processes ΔS for surroundings = 0

Not real! IRL, ΔS for surroundings always increases!