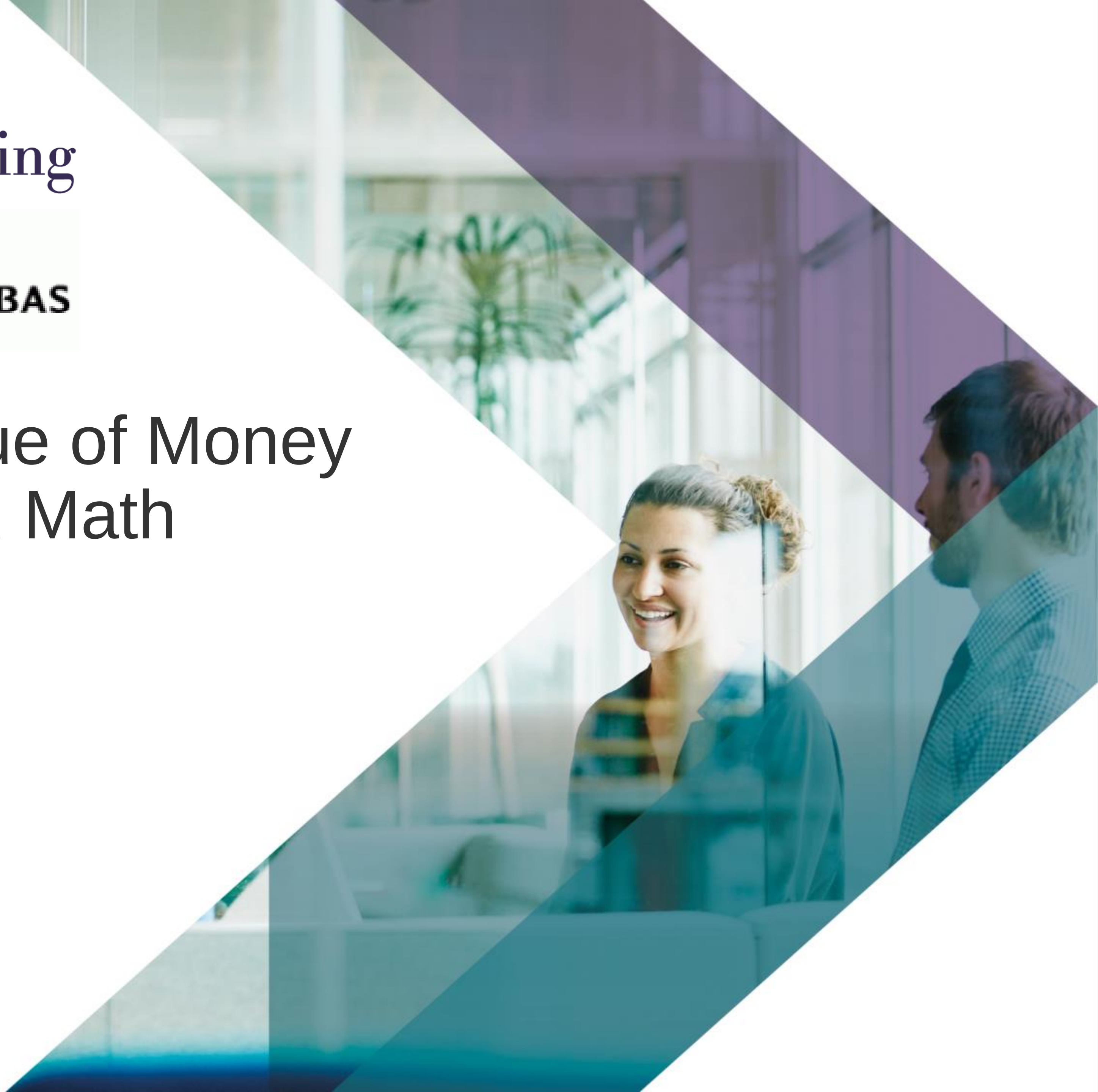
Time Value of Money and Bond Math



Agenda

1. Time Value of Money Review
2. Case Study
   - Case Study 1: Time Value of Money using Excel
3. Core Bond Math
4. Case Study
   - Case Study 2: Building a Bond Pricing Model in Excel
5. Fixed Income Risk Management
6. Case Study
   - Case Study 3: Bond Hedging

1

Time Value of Money Review

FitchLearning

Compounding

We can formally express this process as:

$$FV = PV\,(1+r)^n$$

Where FV is the future value of an amount, PV, invested today for n periods at an interest rate of r per period.

Example:

6,000 placed on deposit for five years at an annual interest rate of 6 ½% pa. What is the final sum assuming compound interest?

$$6{,}000 \times (1 + 0.065)^5 = 8{,}220.52$$

Compounding where interest is paid more than once a year (nominal vs. effective rates)

More often than not, interest is quoted as an annual rate but paid in 2 semi-annual instalments, or 4 quarterly instalments, or 12 monthly instalments…etc. Here we divide the (nominal) rate by the frequency and multiply the periods by the frequency. Using the 100,000 at 2% for 1 year above:

- Semi-annual interest payment (SA):

$$100{,}000 \times \left(1 + \frac{0.02}{2}\right)^{1\times 2} = 102{,}010.00$$

- Nominal rate is 2% pa, effective rate is 2.01% pa
- Quarterly interest payment (Q):

$$100{,}000 \times \left(1 + \frac{0.02}{4}\right)^{1\times 4} = 102{,}015.05$$

- Nominal rate is 2% pa, effective rate is 2.01505% pa

Compounding Money Market Rates

EUR1,000,000 on deposit for 1 week at 1% pa:

$$1,000,000 \times \left(1 + \left(0.01 * \frac{7}{360}\right)\right) = 1,000,194.44$$

The term (0.01*7/360) is the 1-week periodic rate. Convention dictates that with nominal rates we obtain the periodic rate by using the appropriate day-count fraction. In Excel we can use the FV function, or not.

Present Value

We can formally express this process as:

$$PV = \frac{FV}{(1+r)^n}$$

Where FV is the future value of an amount, PV, invested today for n periods at an interest rate of r per period.

Discount Factors

We could also write the discounting formula as:

$$PV = FV \times \frac{1}{(1+r)^n}$$

Where the $\frac{1}{(1+r)^n}$ element is known as the discount factor.

The discount factor is the reducing factor for that period at that interest rate.

Example

The 2-year rate is 2%, the two-year discount factor would therefore be:

$$\frac{1}{(1.02)^2} = 0.96116878$$

Using the discount factor to determine the present value of 104,040:

104,040 x 0.96116878 = 100,000

Discounting for Money Market Periods

What is the PV of EUR1,000,194.44 paid in 1 week. The 1-week rate is currently at 1% pa:

$$\frac{1,000,194.44}{\left(1+\left(0.01*\frac{7}{360}\right)\right)}=1,000,000$$

2

Case Study

FitchLearning

Case Study 1

Case Study1: Time Value of Money Using Excel

- Excel template
- Prepare solutions within Excel sheet – there are many ways to answer the questions so do not feel that you have to use the template
- Be ready to discuss your solutions

3

Core Bond Math

FitchLearning

Yield to Maturity Defined

Yield to maturity (called yield in bond markets) is used to discount the cash flow from a bond to give the current price of the bond.

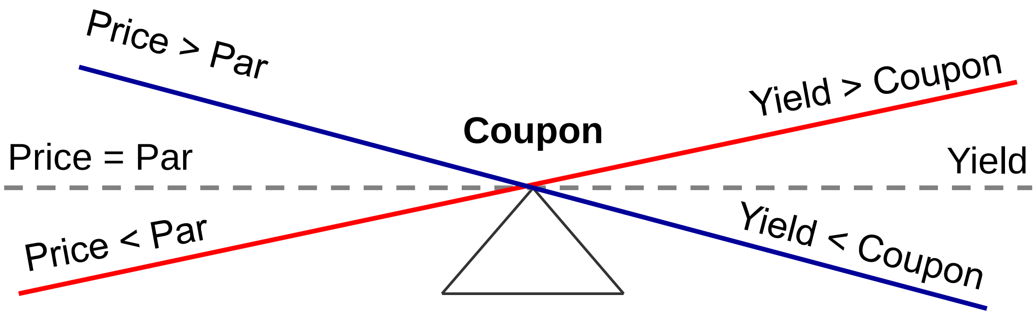
Thus, bond price and yield have an inverse relationship.

Price vs. Yields

If the prevailing required yield from a bond rises, its price will fall.

If the required yield from a bond falls, its price will rise.

Bond prices have an inverse relationship with their yields:



If, as a new investor, you can receive the same fixed cash flow but only pay a lower price, then your yield (return) must be higher.

Bond Pricing

Bonds are quoted on a clean/quoted price basis as if a full coupon period always existed. However, they are settled on a dirty/full price basis (inclusive of accrued interest) which is the PV of the bond's cash flows to the settlement date.

Consider a generic 3-year annual 5% coupon paying bond with a constant YTM of 5% and 2.5 years remaining...

Drivers of Bond Interest Rate Sensitivity

A bond's price is derived from three factors

- Maturity
- Coupons
- Required return (yield)

Risk metrics measure the bond price vs. yield inverse relationship.

Macaulay Duration

The effect of both coupons and redemption date may be expressed using Macaulay duration:

$$\text{Macaulay duration} = \frac{\sum PVCF \text{ x Time to cash flow}}{\sum PVCF}$$

Where PVCFt is the present value of a cash-flow from the bond

Bond E			
YTM year	5 cash-flow	PV cash-flow	PV cash-flow x Time
1	7.5	7.14	7.14
2	7.5	6.80	13.61
3	7.5	6.48	19.44
4	7.5	6.17	24.68
5	7.5	5.88	29.38
6	7.5	5.60	33.58
7	7.5	5.33	37.31
8	7.5	5.08	40.61
9	107.5	69.30	623.66
		117.77	829.41
Macaulay duration		7.04	

Bond F			
YTM year	5.5 cash-flow	PV cash-flow	PV cash-flow x Time
1	8	7.58	7.58
2	8	7.19	14.38
3	8	6.81	20.44
4	8	6.46	25.83
5	8	6.12	30.61
6	8	5.80	34.81
7	8	5.50	38.50
8	8	5.21	41.70
9	8	4.94	44.47
10	108	63.23	632.27
		118.84	890.58
Macaulay duration		7.49	

Modified Duration

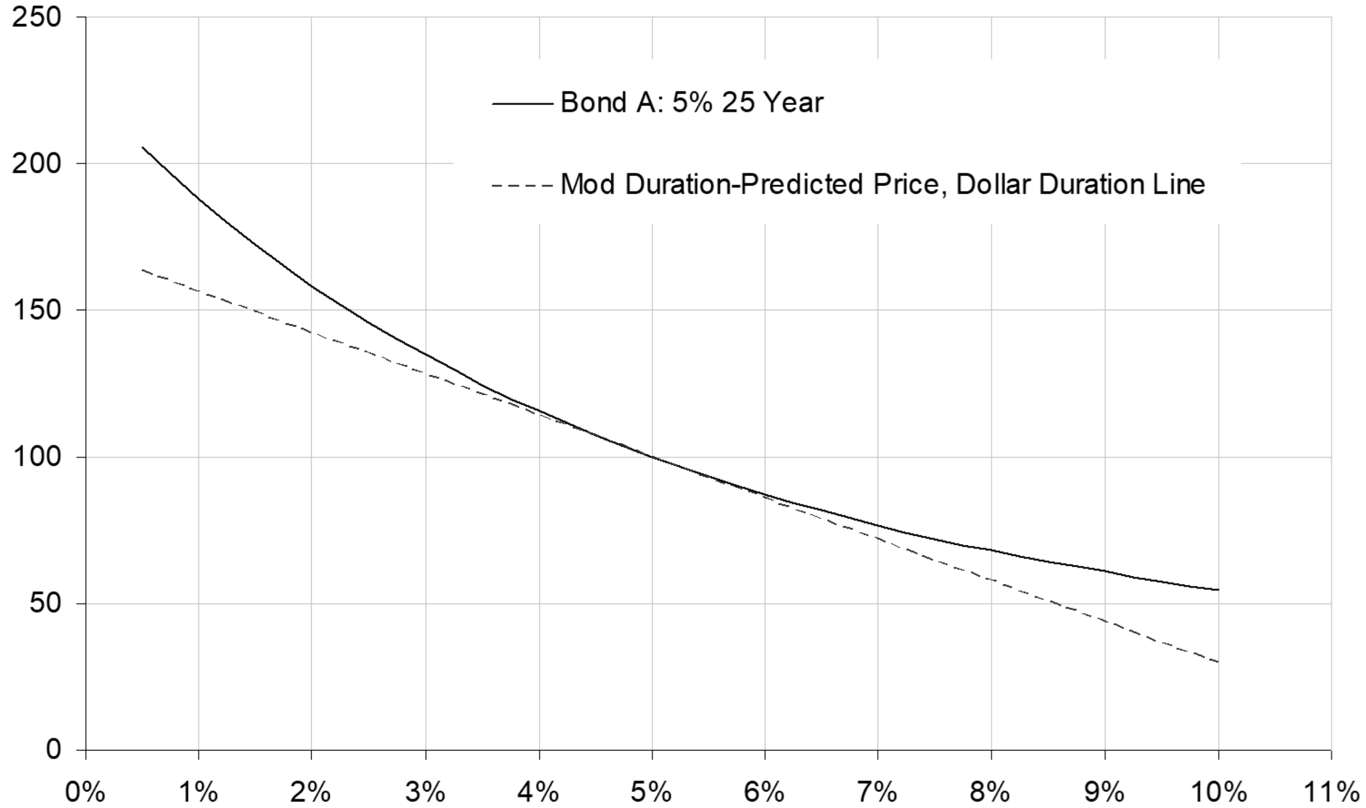
Macaulay duration will not predict the bond’s reaction to interest rates. For this we need to calculate the modified duration:

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{\left(1 + \frac{r}{k}\right)}$$

Where r is the **yield to maturity** of the bond, and k is the frequency of the coupons (in our calculations we are operating on an annual basis and Macaulay duration is expressed in number of years. If we operated on a six-monthly basis Macaulay duration would appear as the number of six-month periods).

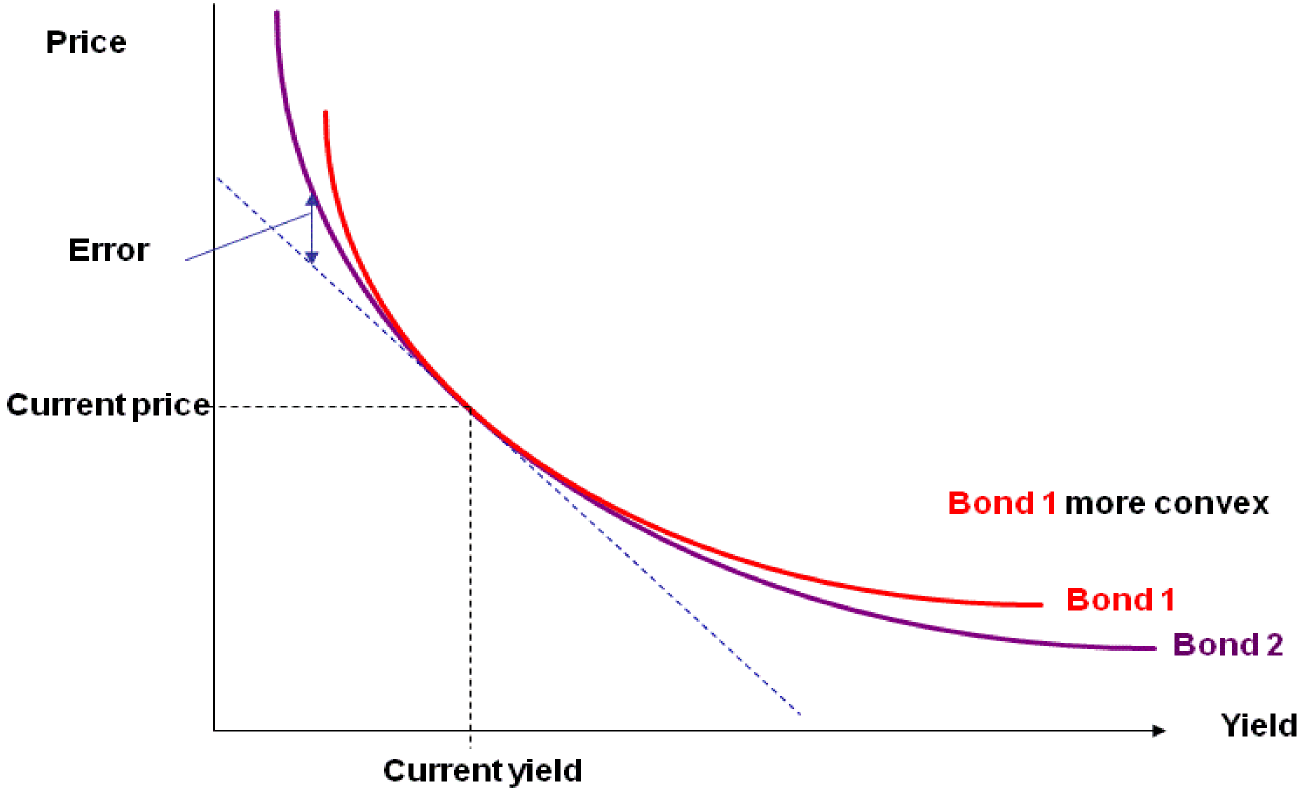
Modified duration may be described as the approximate change in a bond’s price for a 1% change in its yield to maturity

But, modified duration does not predict very accurately for large changes in yield. This is due to convexity.



Convexity

Modified duration will underestimate price changes leading to a pessimistic view of the bond's behavior. This is due to the price-yield curve's 'curviness' or convexity:



Price/Dollar Value Basis Point (PV01, PVBP, DV01)

Modified duration and PV01

Modified duration gives us the percentage price change for a 1% shift in yields. Interest rates and bond yields do not tend to move by whole percentage points on a day-to-day basis.

More useful:

If rates change by 1bp what is the absolute change in price in cents?

- Called PVBP (price/dollar value of a basis point)

PVBP = Dirty price of bond/100 x Mod dur/100

- If rates change by 1bp what is the P/L on a bond position?
- Market risk exposure per 1m, called position DV01 (P/L for a 1bp move in yield)

= 1,000,000 x PVBP/100

4

Case Study

Case Study 2

Case Study 2: Building a Bond Pricing Model in Excel

- Bloomberg screenshots
- Excel template
- Trainer will lead

5

Fixed Income Risk Management

FitchLearning

Risk Management of Bond Positions

- Buyers of new bonds will sell other bonds to generate the cash for the new purchase, and to hedge their new long position
- Relationship between bond nominal of these two trades is called the ‘hedge ratio’
- Nominal of new bonds purchased or existing bonds sold may be calculated in two ways:

  - Invested cash-weighted

  - Market risk-weighted

Investing in a New Bond

A pension fund manager wants to invest in **USD100m** nominal of a corporate bond and will sell a US Treasury from her existing portfolio to hedge the purchase:

Details of the corporate bond:

Dirty price: 105.250

PVBP: 0.0455

Details of US Treasury:

Dirty price: 98.750

PVBP: 0.0525

Investing Using a Cash-Weighted Calculation

The pension fund manager generates the cash needed to buy USD100m nominal of the corporate bond from the sale of the Treasury. But what nominal of the US Treasury needs to be sold?

Step 1: Calculate the cash needed to buy the corporate bond

Cash = Corp nominal x Corp dirty price / 100

= 100m x 105.250 / 100

= 105,250,000

Step 2: Calculate the nominal of the US Treasury needed to be sold

Treasury nominal = Cash x 100 / Treasury dirty price

= 105,250,000 x 100 / 98.750

= 106,582,278

In order to have the cash to buy USD100m nominal of the corporate bond, the pension fund manager needs to sell USD106.6m nominal of the US Treasury.

Hedge Ratio

Invested cash-weighted calculation

$$\text{Nominal of hedge bond} = \text{Nominal new bond} \times \frac{\text{Dirty price new bond}}{\text{Dirty price hedge bond}}$$

Where ‘hedge bond’ is the US Treasury and ‘new bond’ is the corporate bond

Investing Using a Market Risk-Weighted Calculation

The pension fund manager wishes to sell a nominal amount of the US Treasury that has the same market risk as the USD100m nominal of the corporate bond that they wish to buy. But how much of the US Treasury needs to be sold?

Step 1: Calculate the market risk on USD100m nominal of the corporate bond

Market risk = Corp nominal x Corp PVBP / 100

= 100m x 0.0455 / 100

= 45,500 per bp

Step 2: Calculate the nominal of the US Treasury needed to be sold

Treasury nominal = Market risk x 100 / US Treasury PVBP

= 45,500 x 100 / 0.0525

= 86,666,667

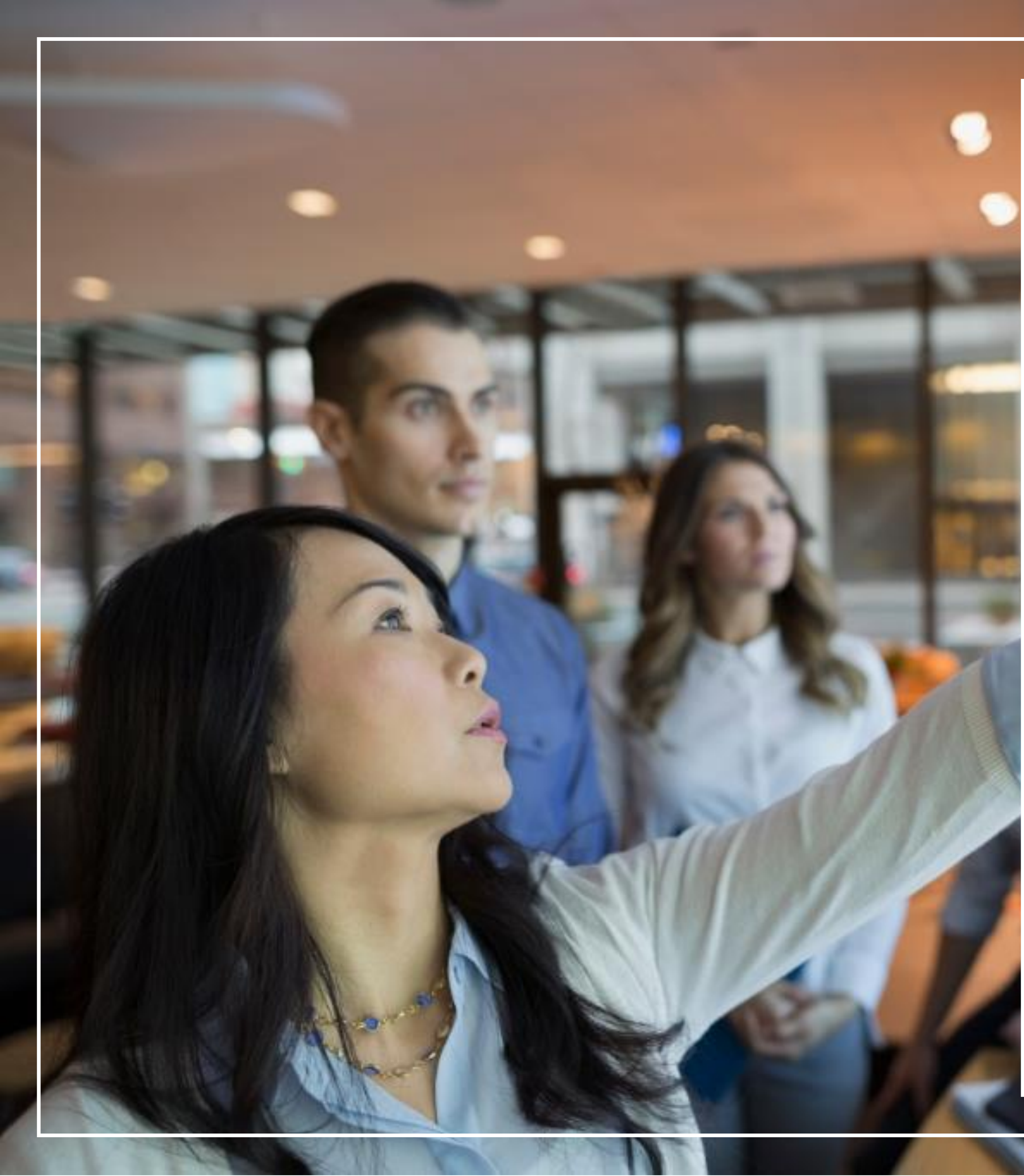
In order to have the same market risk exposure on the portfolio, the pension fund manager needs to sell **USD86.7m nominal** of the **US Treasury** in order to buy **USD100m nominal** of the **corporate bond**.

Hedge Ratio

Market risk-weighted calculation

$$\text{Nominal of hedge bond} = \text{Nominal new bond} \times \frac{\text{PVBP new bond}}{\text{PVBP hedge bond}}$$

Where ‘hedge bond’ is the US Treasury and ‘new bond’ is the corporate bond.



6

Case Study

Case Study 3

Case Study 3: Bond Hedging

- Prepare solutions
- Be ready to discuss in class

FitchLearning

BNP PARIBAS

Hedging Using Derivatives

Agenda

1. Equity Index Futures
2. Forward and Future Pricing Principles
3. Using Equity Index Futures
4. Short Term Interest Rate Futures
5. Case Studies
   - Case Study 1: Using Equity Index Futures
   - Case Study 2: STIR Futures

1

Equity Index Futures

FitchLearning

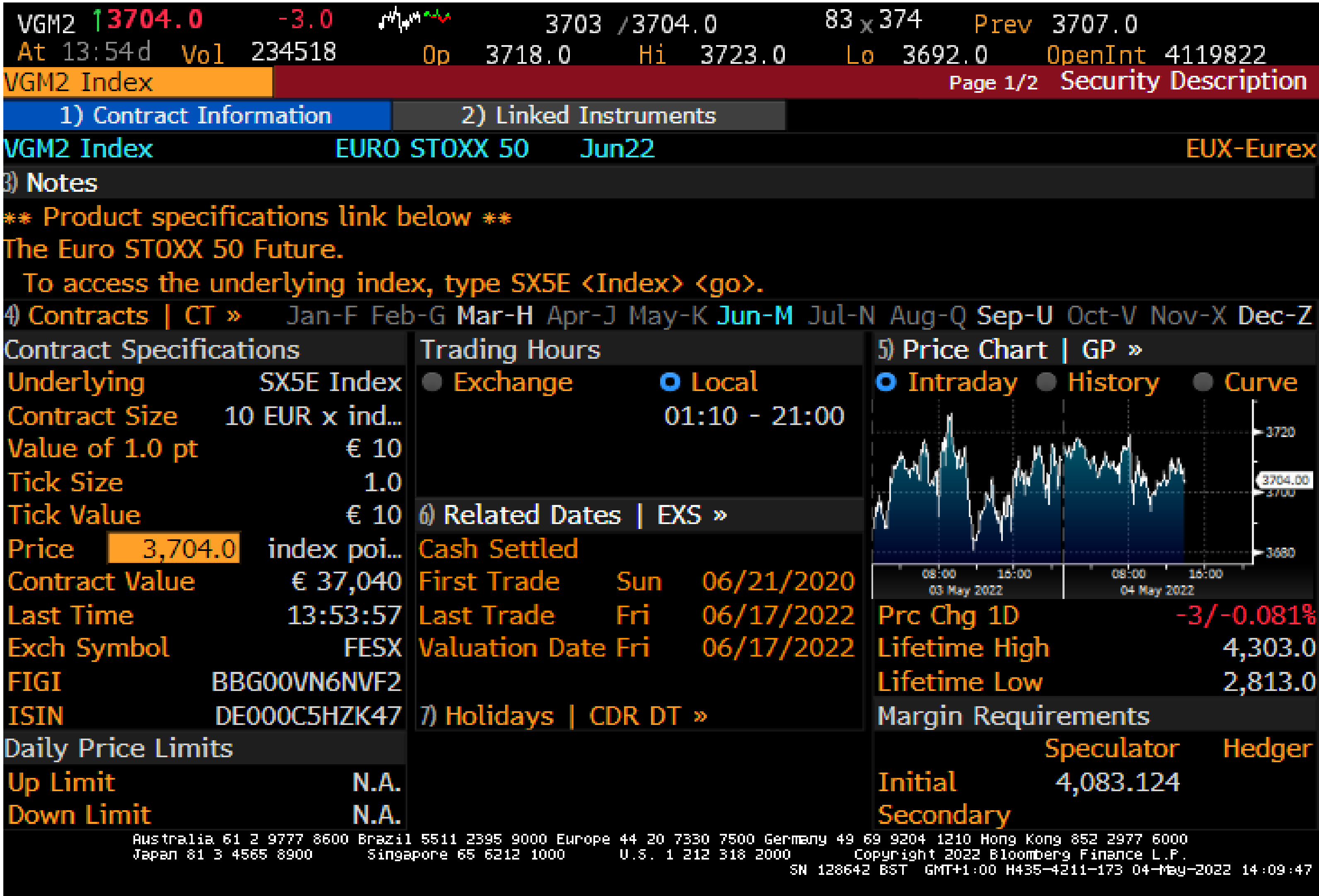
Key Features of Equity Index Futures

- Underlying: equity index
- Contract size: fixed multiplier
- Value of 1 point
- Tick size: minimum price movement
- Tick value: P/L
- Contract value: Index level * contract size
- Cash settled at maturity

Contract Specifications		
Underlying		Z M2 Index
Contract Size		10 £ x index
Value of 1.0 pt		£ 10
Tick Size		0.5
Tick Value		£ 5
Price	7,465.5	index points
Contract Value		£ 74,655
Last Time		11:30:34
Roll Method		BBG Default
Generics		4
Exch Symbol		Z
FIGI		BBG000GXXSN6

Contract Specification S&P500 Future

Contract Specification EuroStoxx50 Future



2

Forward and Future Pricing Principles

FitchLearning

Fair Value and the Cost of Carry

Fair value illustration

Imagine you are asked to quote someone a price for delivering equity in a year's time.

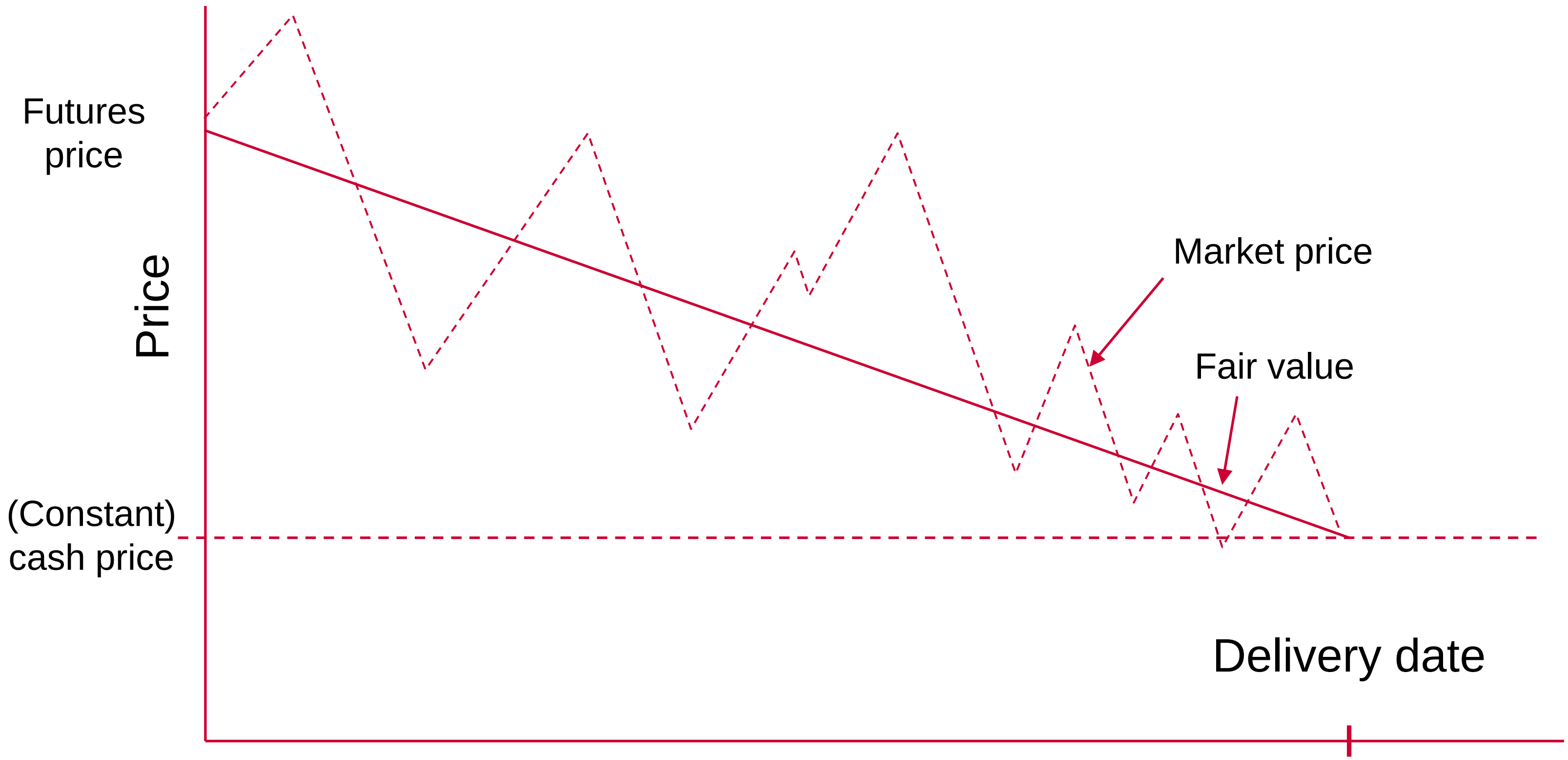
- How would you determine the 12-month delivery price? You could:
- Make an educated guess
- Buy the asset now and hold it until the delivery date (the less risky choice)
- In the second instance you would need to know:
- The price of the asset now (cash price)
- Your income and costs of holding the asset until delivery

In principle a future's theoretical, or fair value is:

Fair value = Cash price – Carry income + Carry costs

Convergence

The future's fair value and the cash price of the underlying converge as the cost of carry reduces:



However, a more accurate model would be:

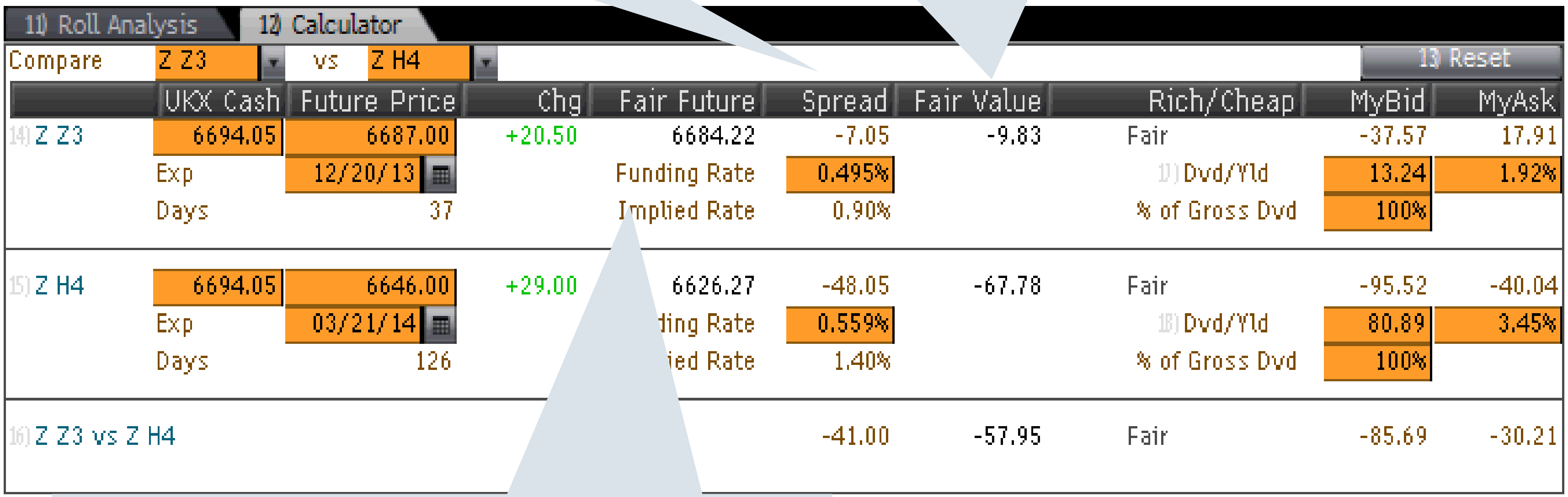
$$F_{0,T} = [S_0 - PVCF] \times (1 + r^*t)$$

- $F_{0,T}$ is the forward price agreed to at time 0 for a forward contract expiring at time
- t is the time fraction from contract initiation to expiration
- S_0 is the spot price of the underlying asset at contract initiation
- PVCF is the present value of any cash flows to the index during life of contract
- r is the borrowing and lending rate

Below is a screenshot from Bloomberg illustrating the Dec FTSE 100 (Z Z3) fair value details:

6,687.00 (futures price) - 6,694.05 (cash price)

6,684.22 (fair future) - 6,694.05 (cash price)



11) Roll Analysis | 12) Calculator

Compare Z Z3 vs Z H4 | 13) Reset

	UKX Cash	Future Price	Chg	Fair Future	Spread	Fair Value	Rich/Cheap	MyBid	MyAsk
14) Z Z3	6694.05	6687.00	+20.50	6684.22	-7.05	-9.83	Fair	-37.57	17.91
	Exp	12/20/13		Funding Rate	0.495%		11) Dvd/Yld	13.24	1.92%
	Days	37		Implied Rate	0.90%		% of Gross Dvd	100%	
15) Z H4	6694.05	6646.00	+29.00	6626.27	-48.05	-67.78	Fair	-95.52	-40.04
	Exp	03/21/14		ding Rate	0.559%		18) Dvd/Yld	80.89	3.45%
	Days	126		ied Rate	1.40%		% of Gross Dvd	100%	
16) Z Z3 vs Z H4					-41.00	-57.95	Fair	-85.69	-30.21

$$\left[6{,}694.05 - \left[\frac{13.24}{\left(1 + 0.00495 * \frac{37}{360}\right)}\right]\right] * \left(1 + 0.00495 * \frac{37}{360}\right)$$

3

Using Equity Index Futures

Profit and Loss

Futures P / L

= number contracts * number ticks * tick value

Tick size and tick value are defined in futures description

S&P futures example:

- Tick size 0.1, tick value $25
- Sell 200 futures at 2,670.1, buy them back at 2660.1
- P/L = 200 * (1,670.1 – 1,660.1) / 0.1 * 25 = 500,000

Pricing Issues

- Cash and carry pricing principles
- Not perfect arbitrage pricing
- Futures can move away from fair value
- Dividends may vary in size and timing
- Trading entire index very inefficient

  - Too many securities to trade the basket

  - Settlement costs
- Arbitragers use benchmark stocks versus futures

Hedging Example - Parameters

Investor holds EUR100m in stock

Profile very similar to Eurostoxx 50

It is June and investor wishes to hedge portfolio fearing that the market will fall

Eurostoxx trading at 2,920

- Dividend yield 3%
- 6m Libor 1%

December futures trading at 2,891

- Positive carry so forward lower than spot

Client holding futures to expiry, so using the **index** is an appropriate method to calculate the number of future to use to hedge.

Practitioners use the futures price to calculate the number of futures. But if the hedge is held until expiry, there will be daily re-hedging due to the futures converging to the index.

Hedging Example - Decision

Investor will sell December futures

- Portfolio will then generate cash returns
- Futures hedge against falling prices

Calculating number of futures

- Investment to hedge / contract value of futures
- €100m / (2,920 * 10)
- 3,425 futures sold at 2,891

Hedging Example – Analysing the Hedge I

Market rallies

As at December futures maturity market rises by 3% - Eurostoxx 50 index at 3008

Analyzing investments

- Value of stock portfolio EUR103m
- Dividends received EUR1.5m
- Futures P / L = (EUR4m)

  3,425 * (2,891 – 3,008) * 10
- Final value of investments EUR100.5m

Investments have performed as if invested in cash, so hedge has been successful, as EUR100m * 1% * 180/360 = EUR0.5m interest on cash investment

Hedging Example – Analysing the Hedge II

Market sell-off

As at December futures maturity market falls by 2% - Eurostoxx 50 index at 2862

Analyzing investments

- Value of stock portfolio EUR98m
- Dividends received EUR1.5m
- Futures P / L = EUR1m

  3,425 * (2,891 – 2,862) * 10
- Final value of investments EUR100.5m

Investments have performed as if invested in cash, so hedge has been successful, as EUR100m * 1% * 180/360 = EUR0.5m interest on cash investment

Hedging Example – Performance of the Hedge

Investor has generated a perfect hedge

Returns changed between cash from equity

Why was hedge perfect ?

- Dividends received match dividend yield priced into futures
- Re-investment of dividends ignored
- Portfolio behaved just like index

Possible to generate a good macro hedge

Perfect hedge is unlikely

Synthetic Investment Using Equity Index Futures

Buying futures creates a long exposure to the index

- Returns similar to holding investment in cash index
- No voting rights

Fund managers use futures to invest whilst also placing orders to buy cash stocks

4

Short Term Interest Rate Futures

FitchLearning

Short Term Interest Rate Futures (STIRs)

Short-term interest rate futures are agreements (obligations) on future interest rates. They are priced as 100 - Implied (or expected) interest rate.

Essentially these are bets on what 100 - Reference rate will be on the delivery date of the future. The reference rate is either an overnight rate (e.g. SOFR) or a longer term rate (e.g. EURIBOR).

Selling STIRs is like borrowing; generating profits if rates rise

Buying STIRs is like lending; generating profits if rates fall

STIR Maturities

Standard maturities are defined by the exchange

Each contract refers to the period which ends at the maturity date of the future.
Four major maturities:

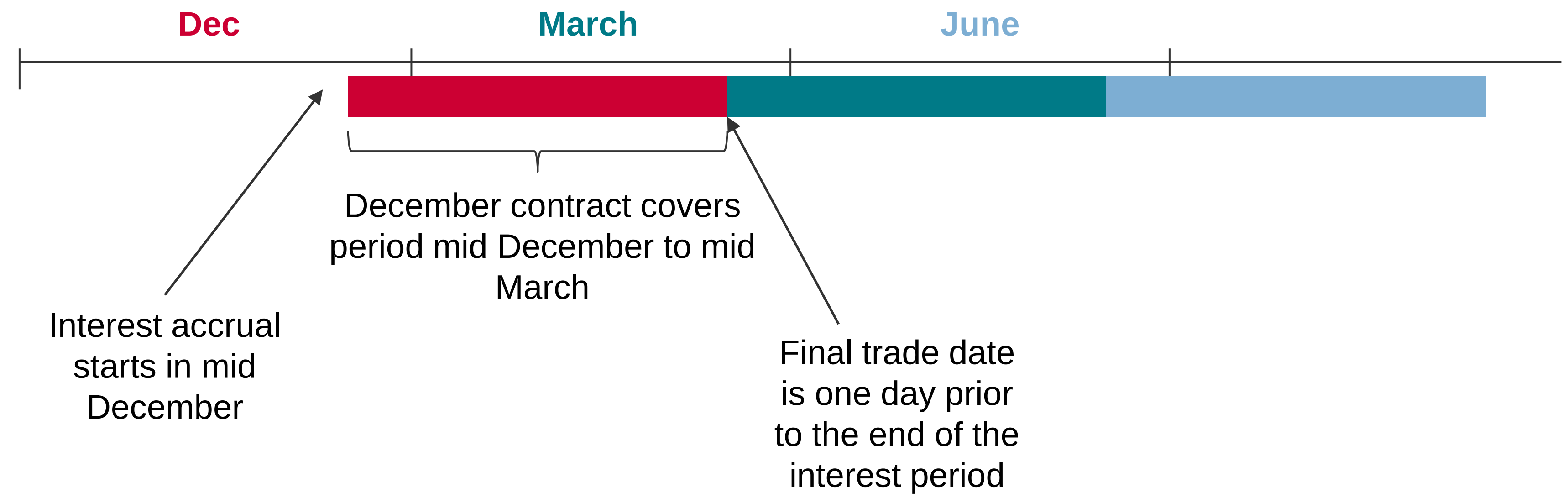
- March (H)
- June (M)
- September (U)
- December (Z)

STIRs Using Overnight Reference Rates

The maturity/delivery/settlement date and the period covered by the future

Using the December contract as an example:

The future contract will cover an interest rate period starting from mid December and ending in mid March. Interest rate is known at the ***end*** of the accrual period



SOFR Future Description

SOFR Future Contract Table

SFRZ2 97.255 +.020 i97.255 / 97.260 ic 360 x 1447 Prev 97.235

At 11:14d Vol 30506 Op 97.235 Hi 97.275 Lo 97.195 OpenInt 560318

SFRZ2 COMB Comdty | Export | Settings | Futures Contract Table

3 Month SOFR Future — As of 04/26/22

Source: Chicago Mercantile Exchange — Session: COMB — Display: Quoted Value — Type: Specific

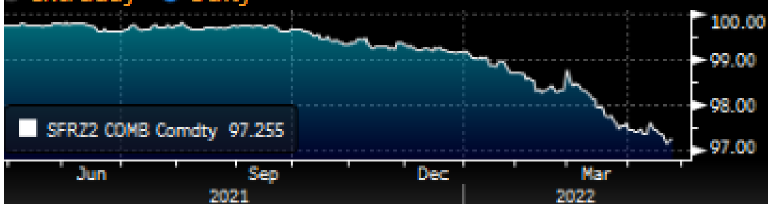
Exchange Symbol: SR3 — Currency: USD

Aggr Vol: 237,739 — Aggr Open Int: 4,610,583

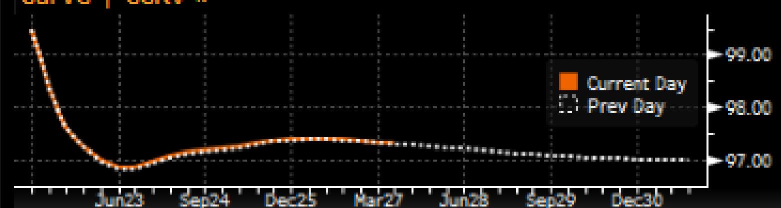
Futures | Spreads | Strategies

Show: Weekly, Monthly, Quarterly, Seasonal, Yearly — Color Legend

Intraday, Daily



Curve | CCRV »



Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
1) Spot			4/22	.270	.270			
2) Mar22	99.4850	--	9:15	99.4825	99.4850	530311	849	99.4850
3) Jun22	98.380	--	11:15	98.375	98.380	562108	23386	98.380
4) Sep22	97.615	--	11:14	97.615	97.620	519045	32485	97.615
5) Dec22	97.255	+.020	11:14	97.255	97.260	560318	30506	97.235
6) Mar23	96.985	+.035	11:14	96.985	96.990	533920	29232	96.950
7) Jun23	96.855	+.045	11:14	96.855	96.860	619930	33986	96.810
8) Sep23	96.860	+.040	11:15	96.855	96.865	312979	21983	96.820
9) Dec23	96.960	+.040	11:15	96.955	96.965	246400	19312	96.920
10) Mar24	97.060	+.035	11:14	97.060	97.065	122597	12814	97.025
11) Jun24	97.120	+.035	11:15	97.115	97.125	133189	7682	97.085

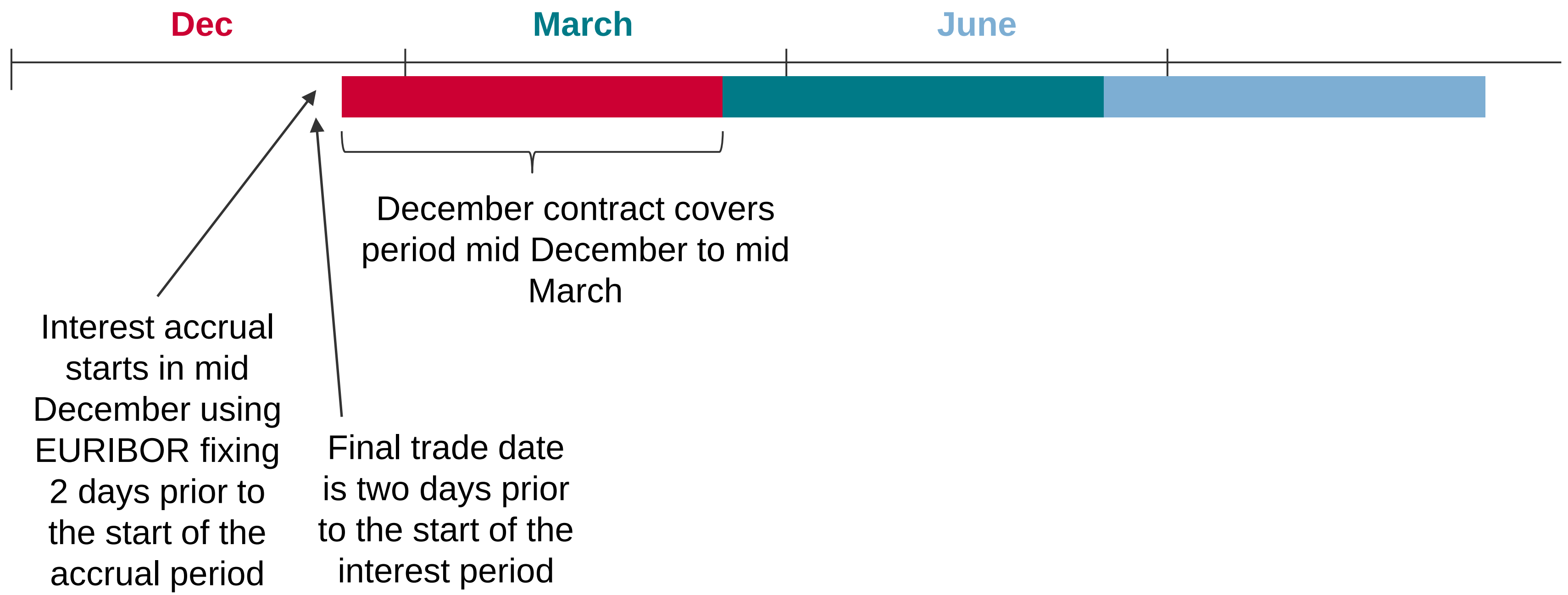
Delayed Pricing

STIRs Using EURIBOR Rates

The maturity/delivery/settlement date and the period covered by the future

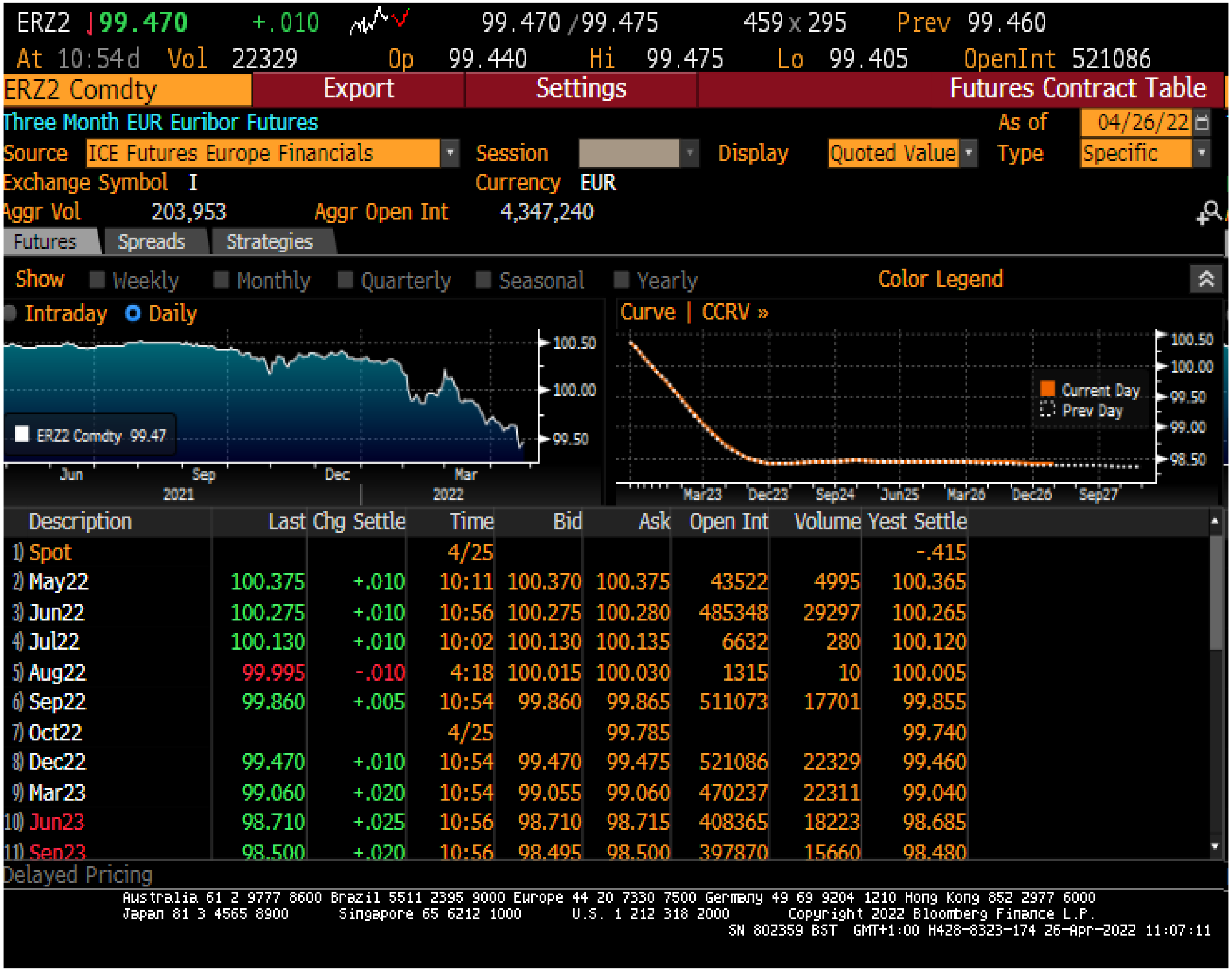
Using the December contract as an example:

The future contract will cover an interest rate period starting from mid December and ending in mid March. Interest rate is known at the ***start*** of the accrual period



EURIBOR Future Description

EURIBOR Future Contract Table



Description	Last	Chg	Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
1) Spot			4/25						-.415
2) May22	100.375	+.010		10:11	100.370	100.375	43522	4995	100.365
3) Jun22	100.275	+.010		10:56	100.275	100.280	485348	29297	100.265
4) Jul22	100.130	+.010		10:02	100.130	100.135	6632	280	100.120
5) Aug22	99.995	-.010		4:18	100.015	100.030	1315	10	100.005
6) Sep22	99.860	+.005		10:54	99.860	99.865	511073	17701	99.855
7) Oct22				4/25		99.785			99.740
8) Dec22	99.470	+.010		10:54	99.470	99.475	521086	22329	99.460
9) Mar23	99.060	+.020		10:54	99.055	99.060	470237	22311	99.040
10) Jun23	98.710	+.025		10:56	98.710	98.715	408365	18223	98.685
11) Sep23	98.500	+.020		10:56	98.495	98.500	397870	15660	98.480

Eurodollar Future Description (Until June 2023)

EDZ2 96.715 -.045 ic96.715 / 96.720 ic 289 x 478 Prev 96.760

At 9:10d Vol 16431 Op 96.755 Hi 96.770 Lo 96.705 OpenInt 1352967

EDZ2 COMB Comdty — Page 1/2 Security Description

1) Contract Information | 2) Linked Instruments

EDZ2 Comdty — 90DAY EURO$ FUTR Dec22 — CME-Chicago Mercantile Exchange

3) Notes

** Product specifications link below **

Eurodollar Futures. **Effective November 17, 2018, CME amended Eurodollar Futures product rules to clarify that contract value is $2,500 per price point. This amendment made no change to

4) Contracts | CT » Jan-F Feb-G Mar-H Apr-J May-K Jun-M Jul-N Aug-Q Sep-U Oct-V Nov-X Dec-Z

Contract Specifications

Underlying	Euro$ 3Mo TD
Contract Size	1,000,000 US...
Value of 1.0 pt	$ 2,500
Tick Size	0.005
Tick Value	$ 12.5
Price	96.715 100 - yield
Pt. Val x Price	$ 241,787.5
Last Time	09:10:43
Exch Symbol	GE
FIGI	BBG003QGMD60

Daily Price Limits

Up Limit	N.A.
Down Limit	0.003

Trading Hours

Exchange / Local

Electronic	23:00 - 22:00

6) Related Dates | EXS »

Cash Settled		
First Trade	Tue	12/18/2012
Last Trade	Mon	12/19/2022
Valuation Date	Mon	12/19/2022

7) Holidays | CDR CE »

5) Price Chart | GP »

Intraday / History / Curve

Prc Chg 1D	-0.045/-0.047%
Lifetime High	99.820
Lifetime Low	94.745

Margin Requirements

	Speculator	Hedger
Initial	852	775
Secondary	775	775

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000
SN 459137 BST GMT+1:00 H435-6406-173 03-May-2022 09:24:42

Eurodollar Future Contract Table

EDZ2 ↓96.925 +.030 ic96.920 / 96.925 ic 1070 x 6 Prev 96.895

At 10:52d Vol 22904 Op 96.895 Hi 96.940 Lo 96.860 OpenInt 1343988

EDZ2 COMB Comdty | Export | Settings | Futures Contract Table

3 Month Eurodollar Future — As of 04/26/22

Source: Chicago Mercantile Exchange — Session: COMB — Display: Quoted Value — Type: Specific

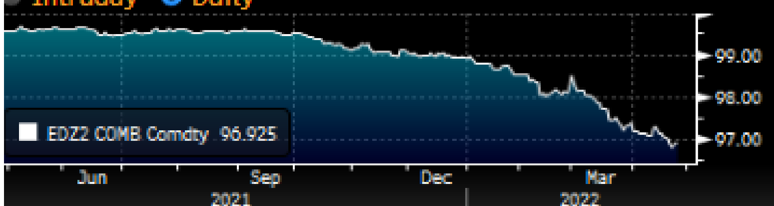
Exchange Symbol: GE — Currency: USD

Aggr Vol 251,948 — Aggr Open Int 10,617,641

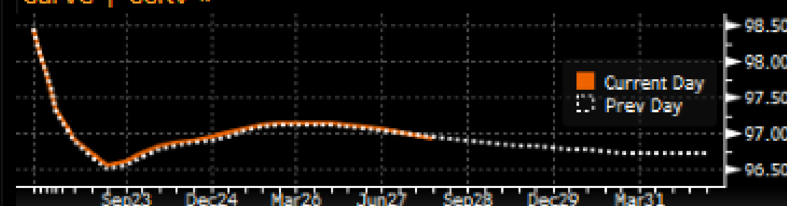
Futures | Spreads | Strategies

Show: Weekly, Monthly, Quarterly, Seasonal, Yearly — Color Legend

Intraday — Daily



Curve | CCRV »



Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
1) May22	98.4425	-.0100	10:48	98.4400	98.4450	124165	2867	98.4525
2) Jun22	98.120	+.005	10:50	98.115	98.120	1036337	18508	98.115
3) Jul22	97.850	+.005	10:44	97.845	97.855	66797	1382	97.845
4) Aug22			4/25	97.625	97.635	11768		97.620
5) Sep22	97.340	+.015	10:51	97.335	97.340	1020094	22964	97.325
6) Oct22			4/25	97.160	97.245			97.165
7) Dec22	96.925	+.030	10:52	96.920	96.925	1343988	22904	96.895
8) Mar23	96.705	+.045	10:52	96.700	96.705	935192	22384	96.660
9) Jun23	96.560	+.055	10:51	96.555	96.560	962717	35038	96.505
10) Sep23	96.610	+.050	10:52	96.605	96.615	842740	16527	96.560
11) Dec23	96.715	+.055	10:51	96.710	96.715	1251905	25397	96.660

Delayed Pricing

Using STIRs in Reality

Very heavy volume

- Hedging tools for IRS traders
- Popular risk instrument, especially spreads between consecutive contracts

Benchmarks for short maturity interest rates

- Used to construct IRS curves

Forward rates are very volatile around the turn in the rate cycle

- 2nd year (called reds) and 3rd year of rates (called greens)

5

Case Studies

FitchLearning

Case study 1: Using Equity Index Futures

- Answer questions including preparing calculations in Excel and chart for final question
- Be prepared to share calculations and chart and present verbally in class

Case study 2: STIR Futures

- Answer questions including preparing calculations in Excel
- Be prepared to share calculations and present verbally in class

Interest Rate Derivatives

Agenda

1. Interest Rates Review
2. Reference Interest Rates
3. Forwards and Futures
4. Swaps on Interest Rates
5. Short Term Swaps on Interest Rates
6. Long Term Swaps on Interest Rates
7. Interest Rate Options
8. Case studies
   - Case Study 1: Using Swaps on Interest Rates
   - Case Study 2: Analyzing Rates Derivatives Solutions

1

Interest Rates Review

Interest Rate Landscape

Reference Interest Rates

- Overnight rates – SOFR, ESTR, SONIA
- Short term rates – ‘IBORs
- Long term rates – OIS, IRS

Interest Rate Derivatives

- Forward rate agreements
- STIR futures
- Swaps on interest rates
- Options on interest rates
- Swaptions

2

Reference Interest Rates

FitchLearning

Overnight Index Rates

An **overnight interest rate** is a rate paid on a deposit / loan for a period between two consecutive business days.

Overnight periods can therefore be:

- 1 day: say borrow money on Thursday and repay Friday
- 3 days: borrow money Friday and repay Monday
- 4 days: borrow money Friday before a public holiday and repay on Tuesday.

An **overnight index rate** is a weighted average interest rate calculated from eligible trades in the market by the Central Bank and published (usually) the following day.

Volume (000s)	Market rate	Weight	Market rate x Weight
20,000	0.06%	0.128205	0.008%
32,000	0.07%	0.205128	0.014%
60,000	0.08%	0.384615	0.031%
29,000	0.09%	0.185897	0.017%
15,000	0.10%	0.096154	0.010%
		Overnight index rate	0.079%

Overnight Index Rates

Here are some important overnight index rates.

These rates are used as benchmark reference rates (RFRs) for these currencies (in the case of the USD its SOFR).

Overnight index rate	Calculating Agent	Source Market	Currency	Secured or unsecured
Effective Fed Funds Rate (EFFR)	Federal Reserve	Fed Funds Market	USD	Unsecured
Secured Overnight Financing Rate (SOFR)	Federal Reserve	A number of USD GC repo markets	USD	Secured
Euro Short Term Rate (ESTR)	ECB	Euro inter-bank market. Euro area banks	EUR	Unsecured
Swiss Average Overnight Rate (SARON)	SIX Swiss Exchange	SIX Repo AG electronic trading platform	CHF	Secured
Tokyo Overnight Average Rate (TONA)	Bank of Japan	Japanese yen unsecured overnight money market	JPY	Unsecured
Sterling Overnight Index Average (SONIA)	Bank of England	Sterling overnight inter-bank money market	GBP	Unsecured

‘IBORs

Widely used within specific markets, e.g. EURIBOR within the Eurozone

EURIBOR is set each working day by European Money Markets Institute.

- Fixing based upon submission by panel banks
- Using ‘waterfall method’ calculation from eligible trades and related instruments
- Periods: O/N, 1 week, 1, 2, 3, 6,12 months.
- Used as a reference rate for a range of interest related instruments

Other markets where ‘IBORs are used include: USD until June 2023, SEK, DKK, NOK, many emerging markets.

3

Forwards and Futures

FitchLearning

Forward Rate Agreements (FRAs)

Basic Details

- A cash-settled **OTC contract** between two counterparties where:
  - the **buyer is borrowing**
  - the **seller is lending**

  a notional principal amount at a **fixed interest rate** (the FRA rate) for a specified period of time starting at an agreed date in the future
- The difference between the agreed FRA rate and the reference interest rate on the agreed future date is cash settled on the settlement date
- Underlying - Interest payment made in a specified currency e.g. EURIBOR
- Settlement amount is calculated based on a notional principal amount
- No exchange of principal

Buy FRA

- Pay fixed rate, **receive floating rate**
- Makes a profit if EURIBOR fixes higher than FRA rate

Sell FRA

- Receive fixed rate, **pay floating rate**
- Makes a profit if EURIBOR fixes below FRA rate

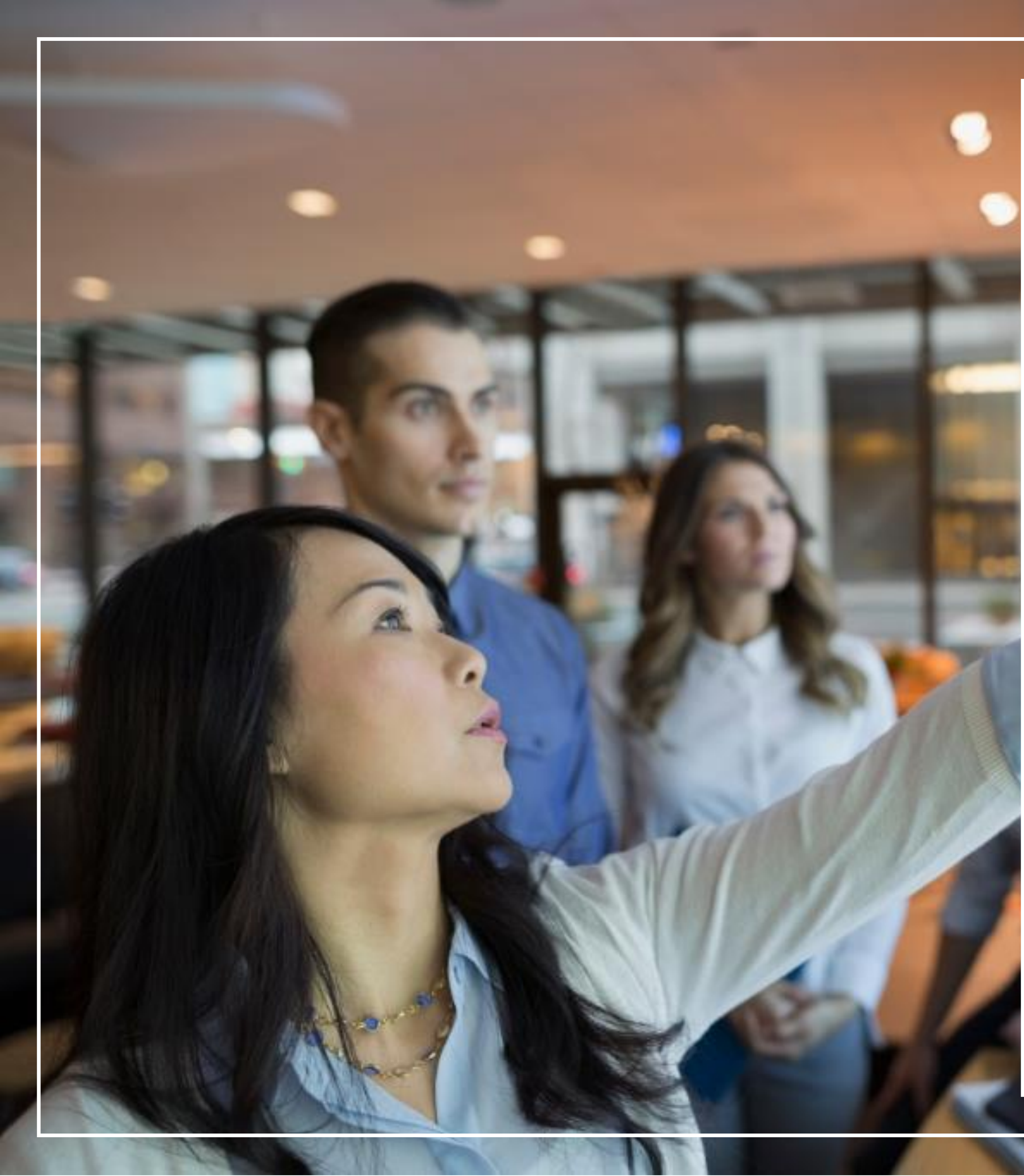
Short Term Interest Rate Futures (STIRs)

Short-term interest rate futures are agreements (obligations) on future interest rates. They are priced as 100 - Implied (or expected) interest rate.

Essentially these are bets on what 100 - Reference rate will be on the delivery date of the future. The reference rate is either an overnight rate (e.g. SOFR) or a longer term rate (e.g. EURIBOR).

Selling STIRs is like borrowing; generating profits if rates rise

Buying STIRs is like lending; generating profits if rates fall



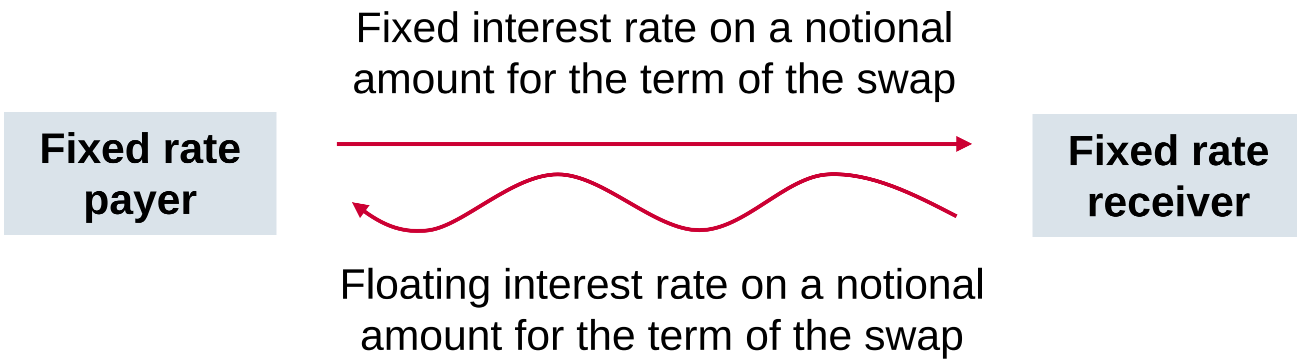
4

Swaps on Interest Rates

FitchLearning

What is a Swap on Interest Rates?

A swap on interest rates is an agreement in which **floating** interest payments are paid in exchange for receiving **fixed** interest payments on a reference or **notional**, principal amount.



It is important to remember the main features of an interest rate swap:

- The term: This is the length of time the swap lasts for.
- The notional amount: This is the size of the swap. The notional is a reference amount and not paid by anyone to anyone else.
- The frequency: This is how often each year the payments are swapped.
- A swap rate: This is the fixed rate component of the swap.
- Floating rate: Differing conventions in different markets, overnight or longer term rates.

Calculating Interest

Any swap on interest rates is ultimately a series of interest payments

- Fixed leg is known cash flow at trade date
- Floating leg is implied cash flow at trade date

Interest payment = notional x interest rate x proportion of a year

Please note:

- Interest rates are quoted as annual rates
- Proportion of a year is calculated on appropriate **day count basis**, e.g. Act/360, 30/360, Act/365

Valuation of a swap on interest rates is the PV of this future cash flow. **Current market swaps on interest rates have a PV of zero.**

5

Short Term Swaps on Interest Rates

FitchLearning

Overnight Index Swaps (OISs)

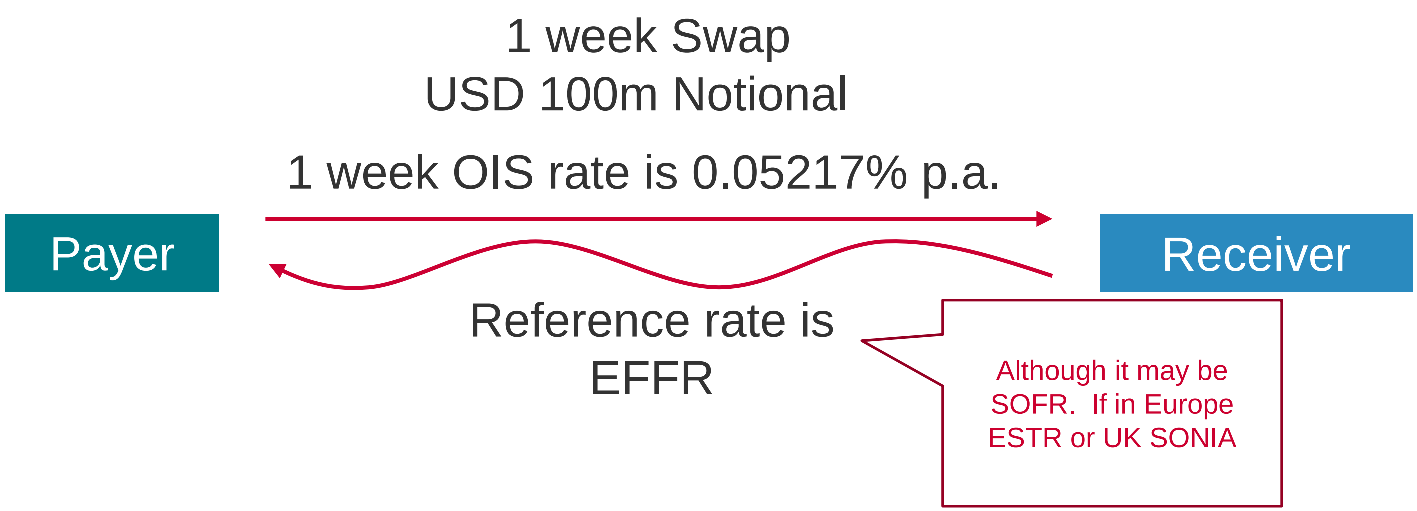
Overnight Index Swaps are agreements in which one party agrees to pay a fixed rate (the **OIS rate**) on a **notional** amount at the end of a period (the **term**). The counterparty agrees to pay an amount calculated by compounding the notional at the overnight-indexed rates for the period:

EFFR, the effective fed funds rate is the floating reference rate.

Overnight Index Swaps (OISs)

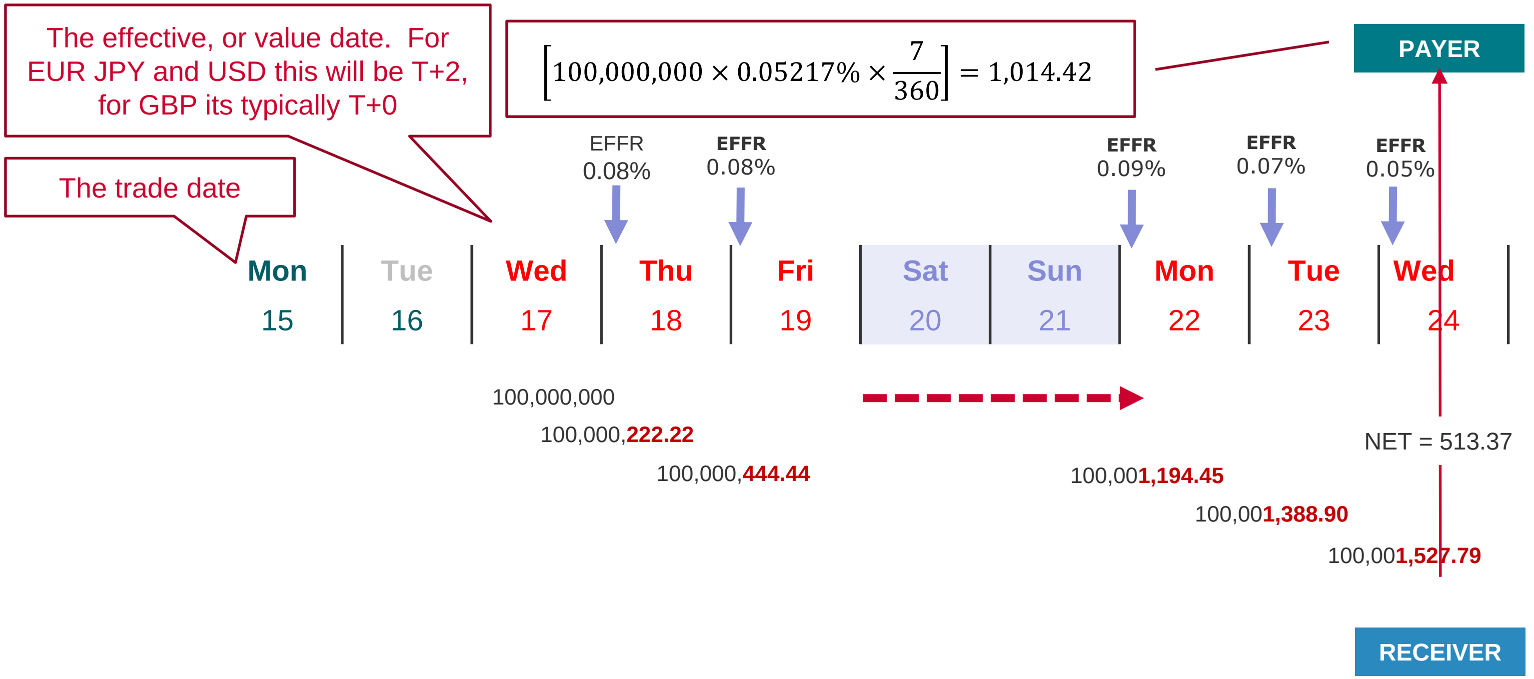
An OIS is an agreement between two parties: a **payer** and a **receiver**.

- They agree a **term**: how long the swap will last, this could be as short as a week, or as long as 30 years.
- They agree a **notional amount**. This is a reference amount used to determine the swap's cash flows. It is not an amount paid or received and is therefore not subject to any credit risk.
- They agree a **swap rate**. This is the rate, on the notional, the payer has to pay to the receiver.
- They agree a **reference overnight index rate**. The receiver will pay the payer the compounded interest on the notional at all published (printed) overnight index rates throughout the swap's term.



1 Week OIS Illustration

Here's an illustration of how the cash flows might transpire for a 1 week 100m USD OIS agreed at 0.05217% p.a.:



For all sub 12-month OISs the cash flows are paid at the end of the swap's term. They are **Zero-coupon** swaps.

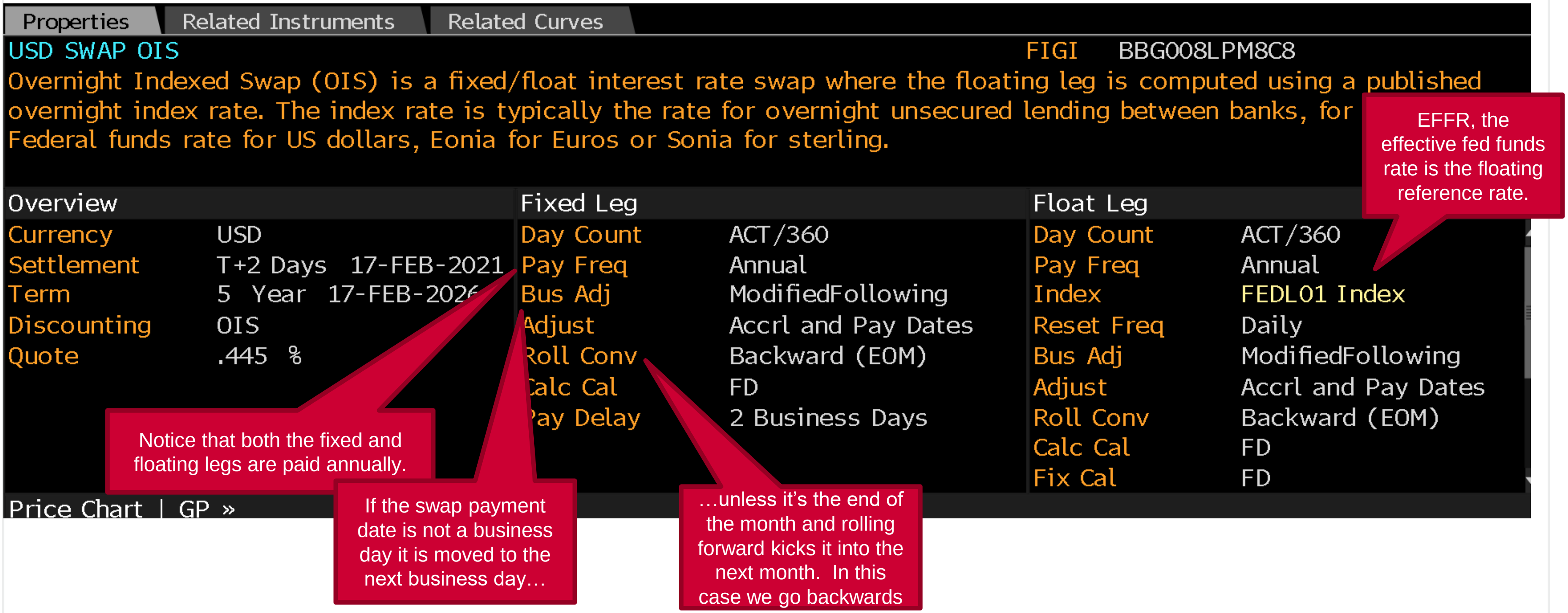
6

Long Term Swaps on Interest Rates

FitchLearning

5-year OIS illustration

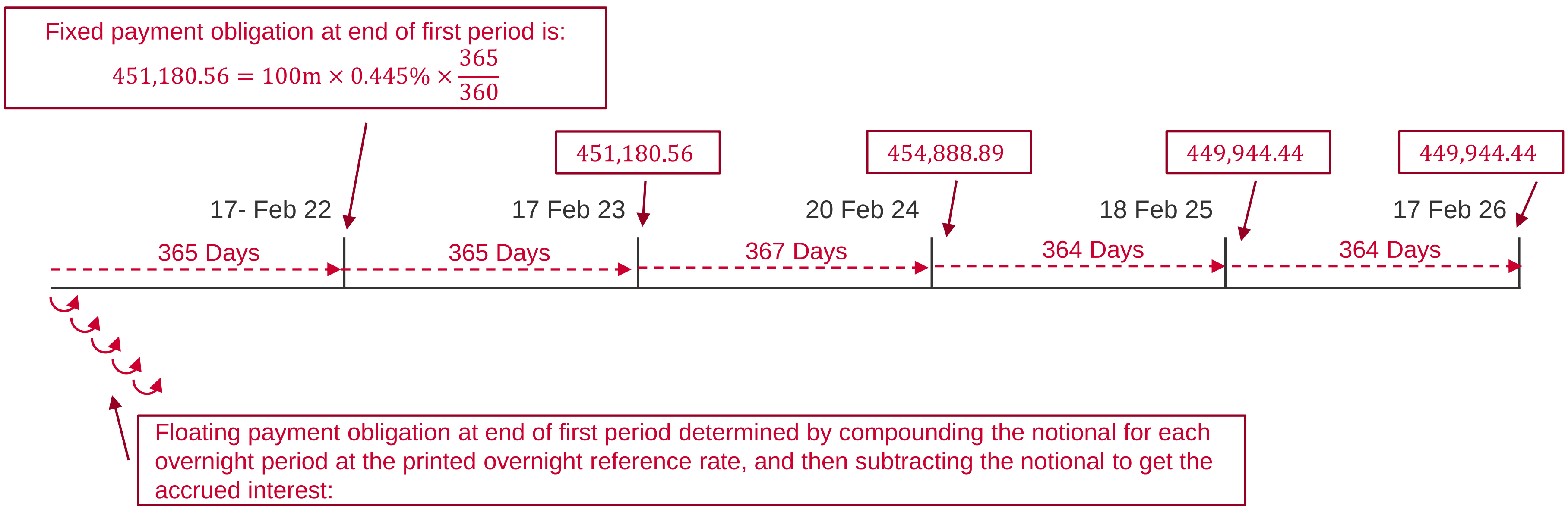
OISs are traded in tenors of up to 30 years. Here's an example of a 5-year USD OIS:



5-year OIS Illustration

For OISs with a term longer than a year the coupon and floating legs are paid annually.

(17 Feb in ‘24 and ‘25 moved due to weekends and public holidays).



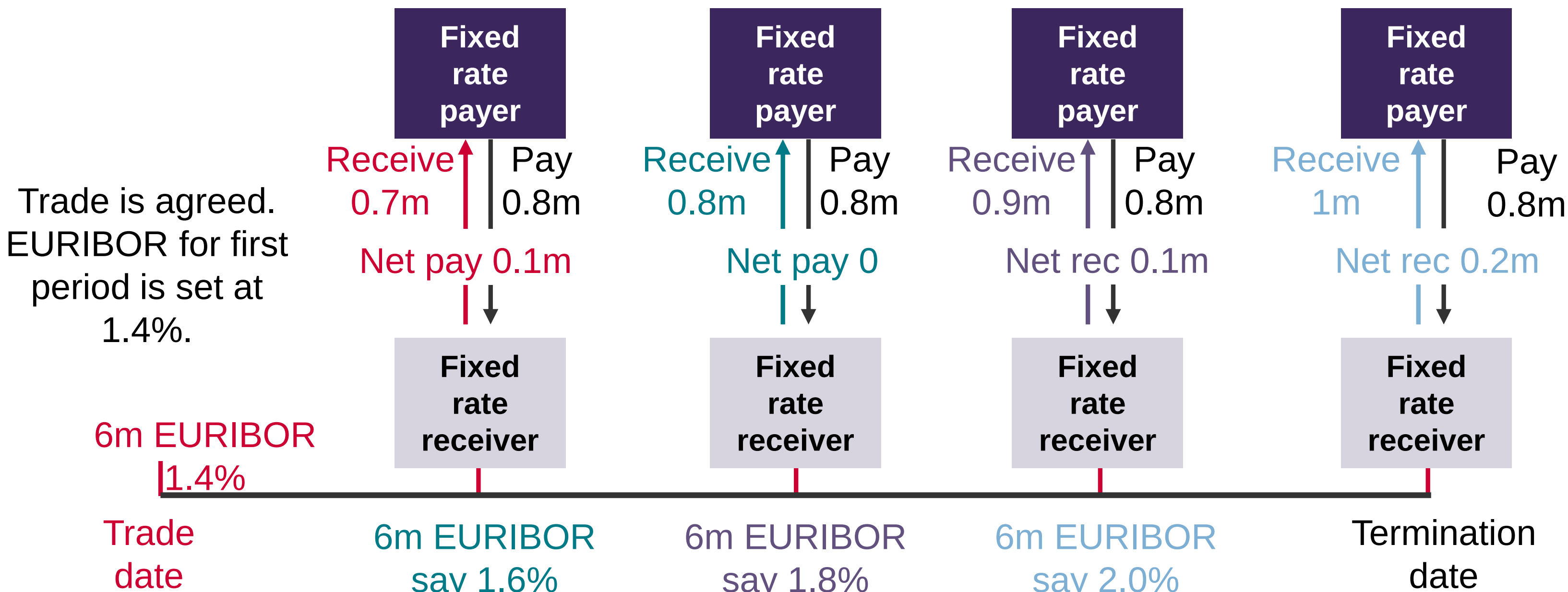
2-Year EUR Interest Rate Swap Example

Example: A bank is quoting the two-year semi-semi swap as 1.58-1.60%. Suppose we choose to pay fixed and receive floating on €100m:

Swap rate quote	1.58%-1.60%
Tenor	Two years
Notional	€100m
Fixed pay frequency	Semi
Floating reset frequency	Semi
Floating pay frequency	Semi
Floating rate index	Six-month EURIBOR
Reset	Advance

The two-way quote relates to bid ask for the fixed rate:

Below are the cash flows we would pay and receive over the swap's term:



Notice that:

- The payment amounts are calculated on a six-monthly basis
- The payments are netted
- The EURIBOR for each period must be set in advance

Example Cash Flows

- You entered into a five-year annual fixed, versus 3 month EURIBOR swap in EUR20m nine months ago
- You are receiving fixed at 2.4% pa and three-month EURIBOR for the fourth quarterly period has just reset at 1.5%
- Interest on this swap accrues on the fixed leg on a 30/360 basis and on the floating leg on an ACT/360 basis
- There are 365 days in the one-year period and 91 days in the three-month period
- What is the net settlement on the swap in three months' time?

- Fixed =

- Floating =

- Net =

Cross-Currency Swaps (CCSs)

Definition: A cross-currency swap (aka currency swap) is an exchange of cash flows in one currency for cash flows in another currency, at agreed intervals, over an agreed period and based on agreed interest rates.

Interest payment variations

- Fixed/fixed
- Fixed/floating
- Floating/floating (cross-currency basis swap)

Unlike an IRS, the currency swap involves a notional exchange at:

- Inception and maturity
- Maturity only

The notional exchange FX rate is agreed at the beginning at the current spot rate. There is no netting of payments as flows are in different currencies.

Uses: Comparative Advantage

A US company wishes to expand into Europe and needs EUR10m. Banks in Europe have quoted an 8.5% coupon to raise this money since the US company is unknown in Europe.

A US investment bank advising the company suggests issuing a USD-denominated bond at 8% and swapping the proceeds into EUR, using the following transaction:

Term = Five years

Notional exchange: Start and end

EUR notional = 10m

Spot EUR/USD = 1.20

USD notional = 12m

Company pays EUR fixed rate = 5.5%, Frequency = S/A

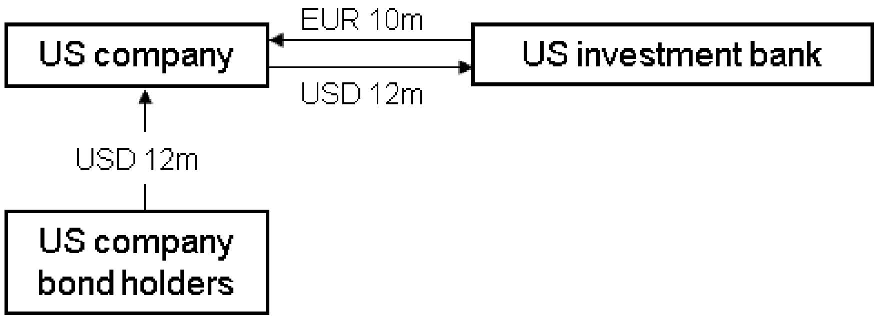
Company receives USD fixed rate = 6.0%, Frequency = S/A

Question: What is the effective borrowing cost of obtaining the EUR via the currency swap, and how much can the company save by doing so?

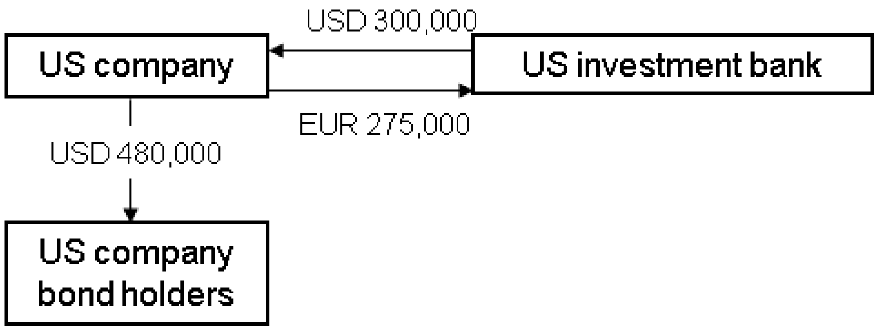
Uses: Comparative Advantage (cont.)

Cash flows

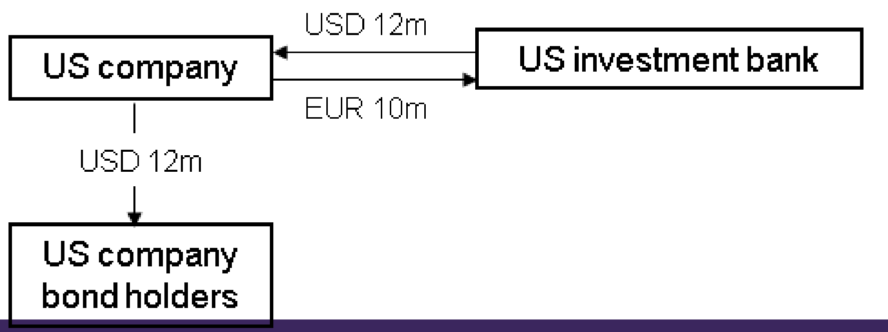
At the inception of the swap:



On every bond coupon date:



At maturity of the swap:



Other Swap Variations

Variable Nominal Swaps

- Amortizing
- Accreting
- Rollercoaster

Off Market Swap

- PV not zero, so upfront payment is made

Asset Swap

- Off market swap with spread on floating leg

7

Interest Rate Options

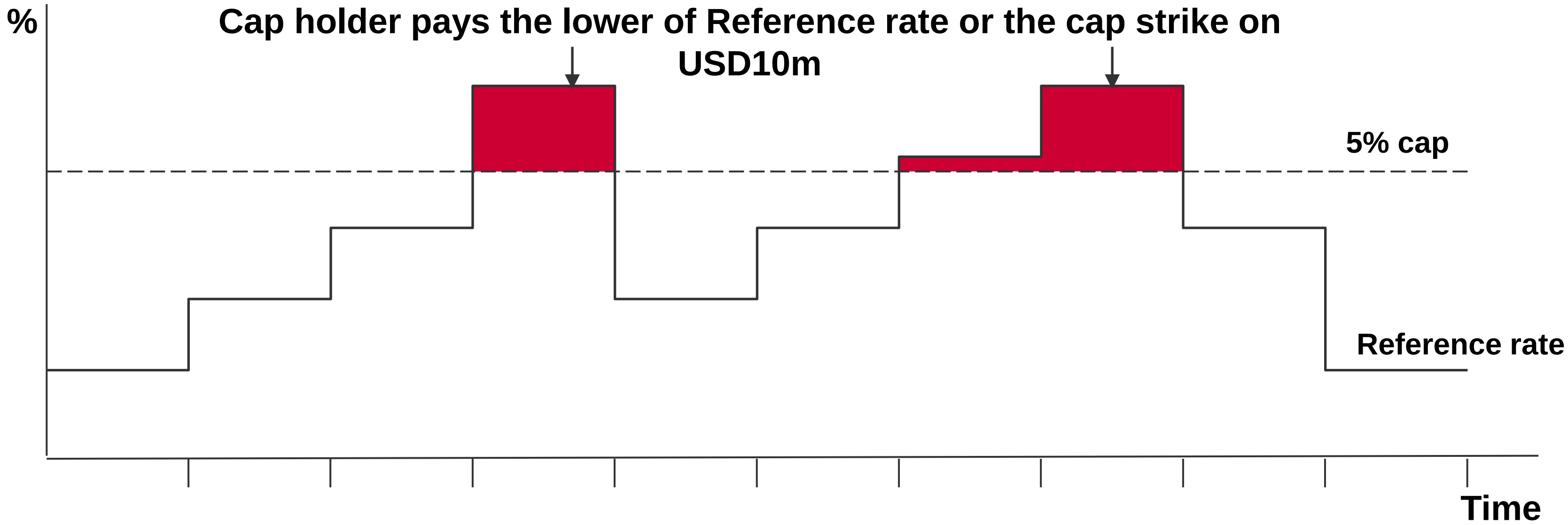
FitchLearning

Caps, Floors and Collars

Interest rate caps are used by corporates to manage variable rate loan risk.

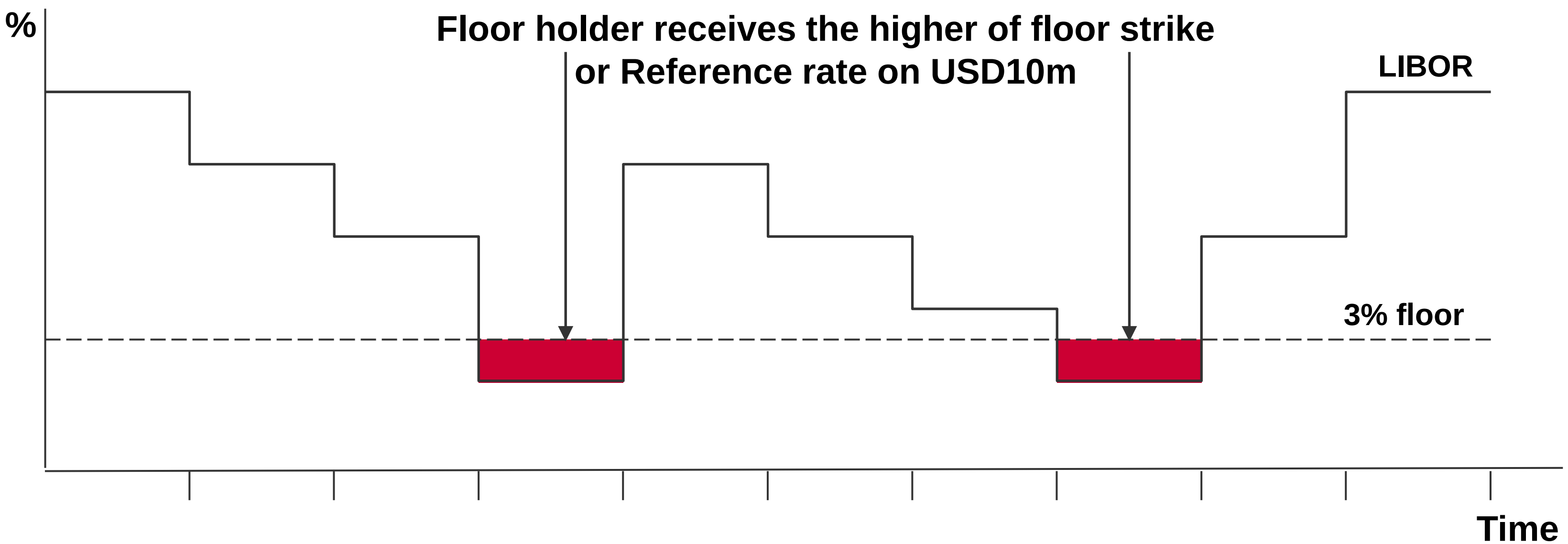
They are instruments that may be purchased (for a premium paid upfront) to ensure that the interest rate on a floating rate loan does not go above a certain level (the strike price).

Example: A borrower has a USD10m loan, and buys a 5% interest rate cap on a notional value of USD10m. This cap pays out any extra interest above 5% on the notional value of USD10m:



Interest Rate Floors

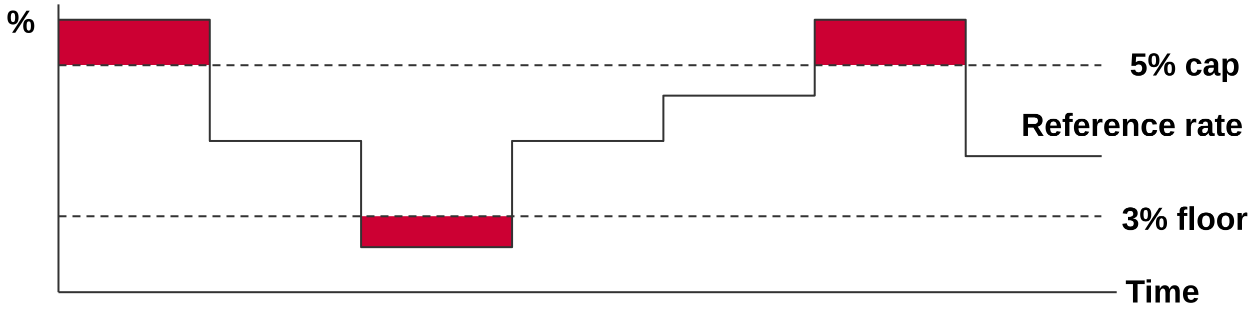
Floors pay the holder if interest rates fall below a certain level (the strike price). They are useful to **deposit holders** wanting to insure against interest rates falling.



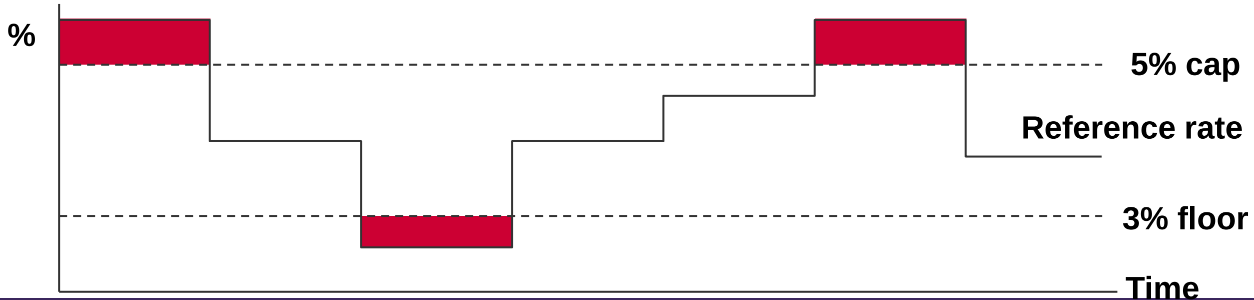
Interest Rate Collars

A **collar** is a cap and a floor combined, often in such a way as to net out to zero cost:

- **Borrowers:** Buy caps and sell floors to create a collar



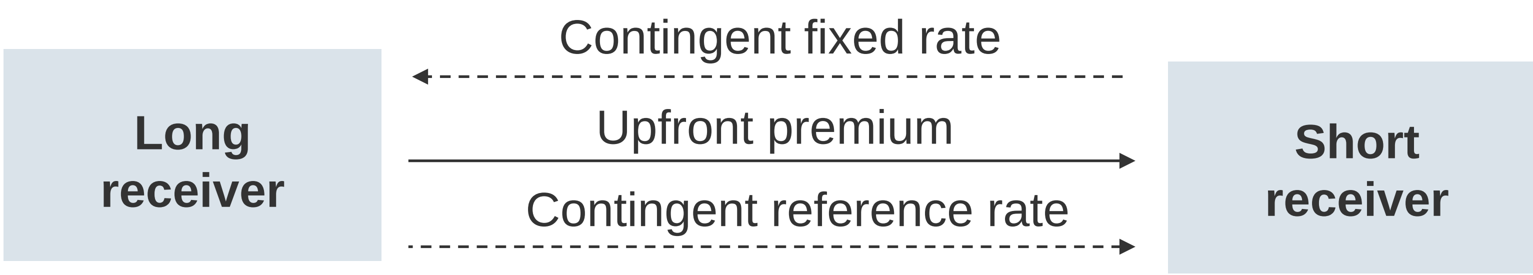
- **Lenders:** Buy floors and sell caps to create a collar



Swaptions

Swaptions are simply options on (forward) swaps where the holder has the right (but not the obligation) to enter into a fixed payer swap (a payer's swaption) or a fixed receiver swap (a receiver's swaption) at a pre-determined strike in the future in exchange for an upfront premium.

Here's what a receiver's swaption looks like:



An option which lasts 2 years and is exercisable into a 5-year swap is called a '2-year-into-5-year' swaption or just '2y5y'. Note this contract effectively runs from today for 7 years.

Cancelable vs. Extendable Swaps

Cancelable swaps:

- Swaps can be offset with swaptions in order to effectively cancel them in the future if rates move adversely:
  - Fixed rate **payers**/floating rate receivers should buy **receivers** swaptions in case rates fall
  - Fixed rate **receivers**/floating rate receivers should buy **payers** swaptions in case rates rise

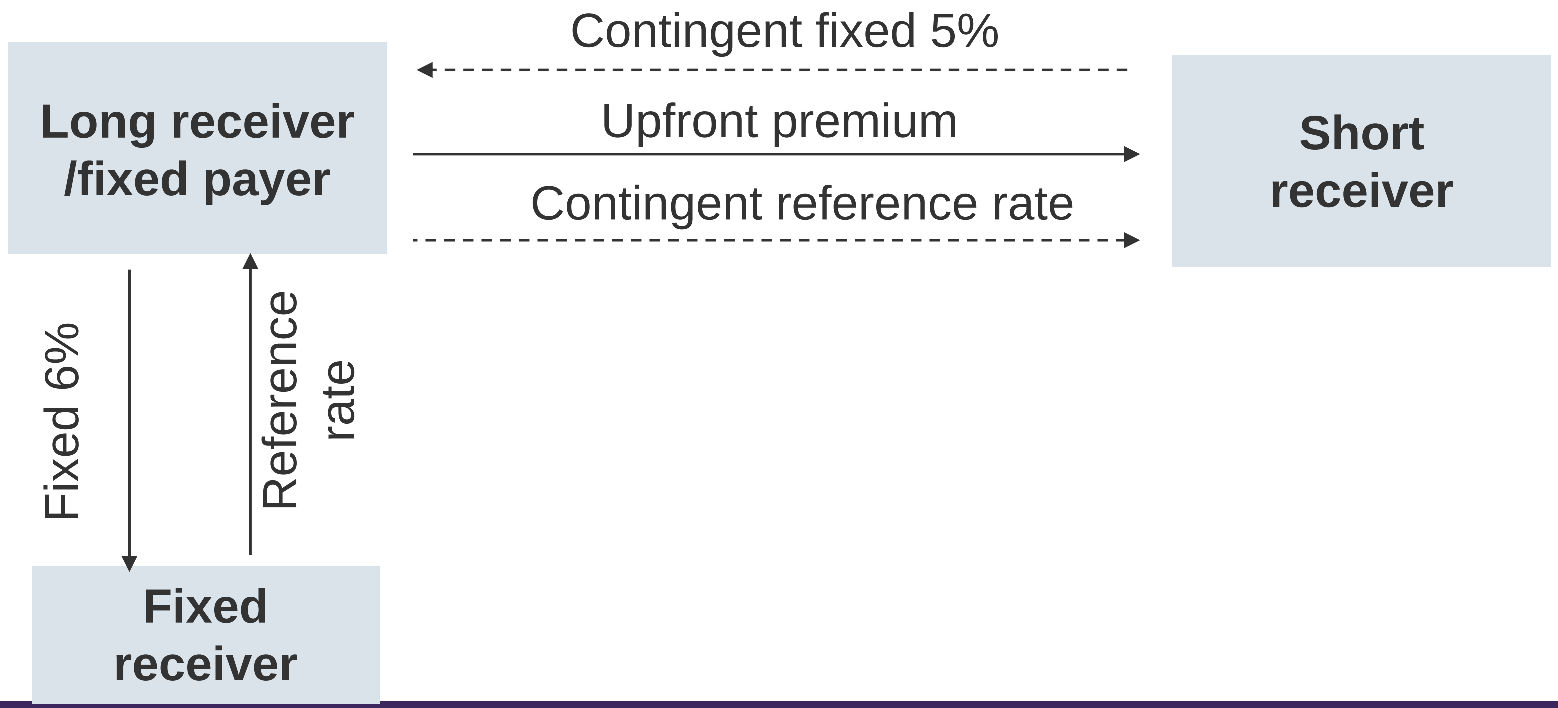
Extendible swaps:

- Swaps can be extended with swaptions in order to take advantage of favorable movements in the rates:
  - Fixed rate **payers**/floating rate receivers should buy **payers** swaptions in case rates rise
  - Fixed rate **receivers**/floating rate receivers should buy **receivers** swaptions in case rates fall

Example: Cancelable Swap

A client is currently paying 6% fixed for 7 years and wants the right to cancel this swap if rates fall below 5% in the next 2 years.

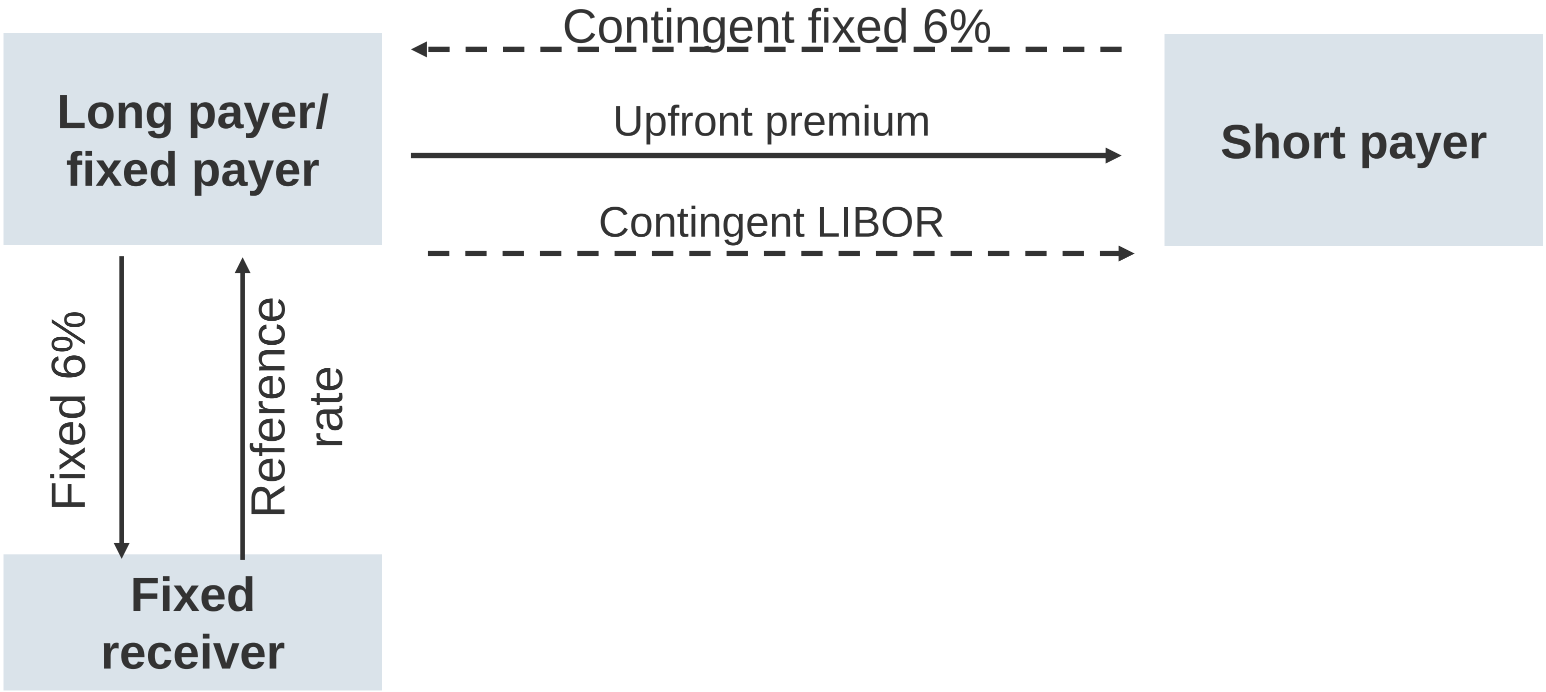
Therefore, she buys an OTM 2y5y receivers swaption struck at 5% as a hedge (physical exercise):



Example: Extendable Swap

A client is currently paying 6% fixed for 3 years, and wants the right to extend this swap for a further 1y, if rates rise above their current levels.

Therefore, she buys an ATM 3y1y payers swaption struck at 6% (physical exercise):



Equivalence of Interest Rate Products

Many of the interest rate products available have an equivalence in terms of their exposure to interest rates. This is useful to keep in mind when considering how to hedge corporate risk.

View/hedge concern	STIRs	Interest rate swaps	Swaptions	Caps/ floors
Bullish = IR down	Long	Fixed receiver	Long receiver	Buy floor
Bearish = IR up	Short	Fixed payer	Long payer	Buy cap
Bullish/neutral = IR same/ down	Long	Fixed receiver	Short payer	Sell cap
Bearish/neutral = IR same/up	Short	Fixed payer	Short receiver	Sell floor

8

Case Studies

Case Study 1: Using Swaps on Interest Rates

- Answer questions including preparing diagrams of cash flow (straight line is fixed rate, wavy line is floating rate)
- Be prepared to share diagrams and present verbally in class

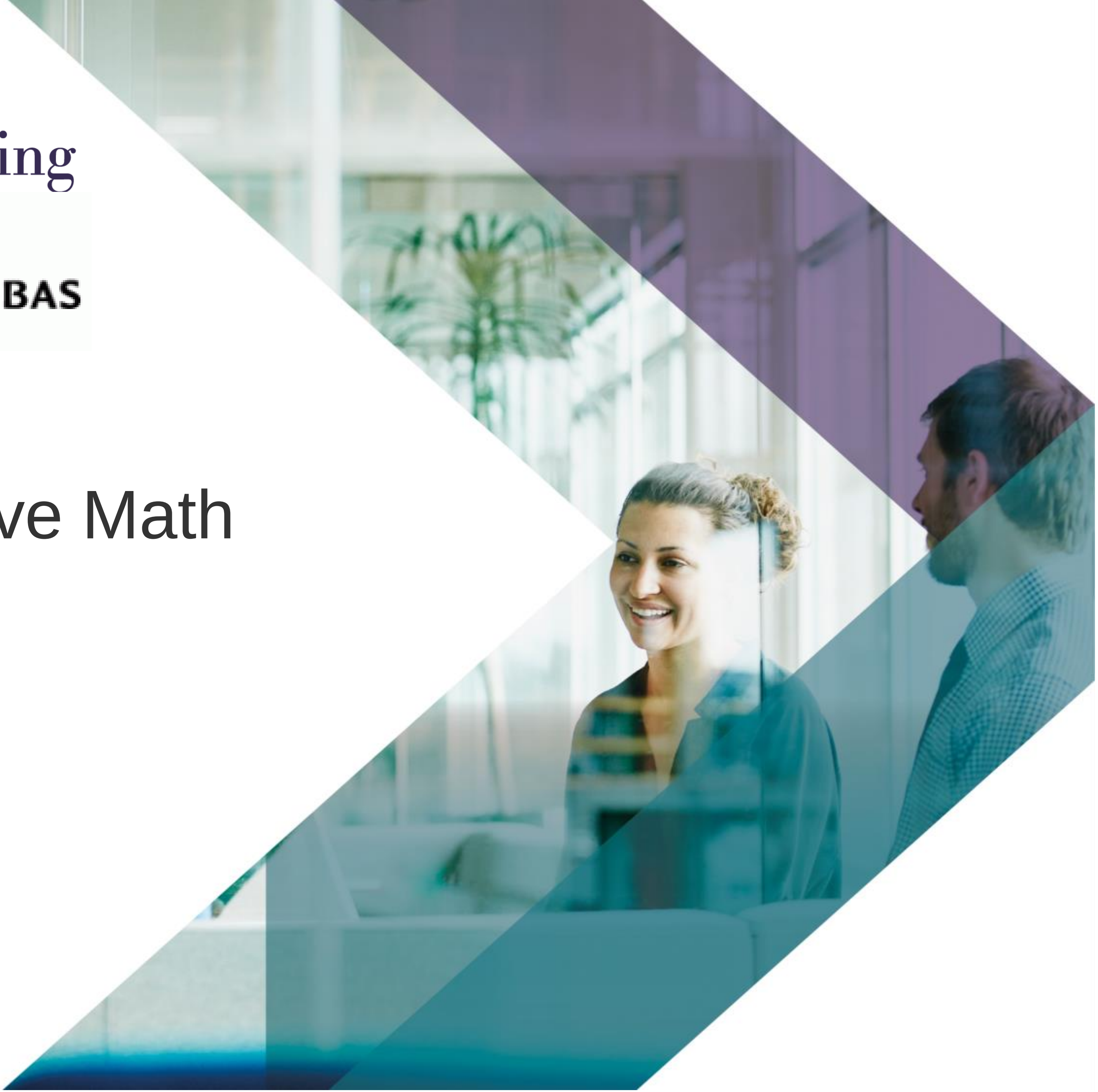
Case Study 2: Analyzing Rates Derivatives Solutions

- Prepare written answers to the questions
- One or more solutions will be shown on screen to whole class
- Peer review of documents

FitchLearning

BNP PARIBAS

Yield Curve Math



Agenda

1. Yield Curve Construction
2. Swap Valuation
3. Building a 3-Year IRS Pricing Model
4. Case Study
   – Case Study: IRS Valuation

1

Yield Curve Construction

Case Study: Yield Curve Construction

Which traded instruments could be used to construct the market rates in a yield curve?

2

Swap Valuation

FitchLearning

Basic Pricing of Interest Rate Swaps

Swap pricing illustration

We are going to look at the price swap for a 3-year annual pay swap. The main principle behind swap pricing is that the swap has **no value** when it is traded, i.e. the present value of the floating payments will equal the present value of the fixed. We call this **zero NPV**.

Yield Curve Mathematics

- Strict rules for construction of this yield curve
- Mathematical relationship between
  – Swap (spot) rates
  – Discount factors
  – Forward rates

Yield Curve Mathematics

Swap rates from discount factors

- 1-year swap = (1 – 1yr DF) / 1yr DF
- 2-year swap = (1 – 2yr DF) / sum(1yr to 2yr DF)
- 3-year swap = (1 – 3yr DF) / sum(1yr to 3yr DF)

Discount factors from swap rates

- 1-year DF = 1 / (1+ 1yr swap)
- 2-year DF = [1 - (2yr swap * 1yr DF)] / (1 + 2yr swap)
- 3-year DF = [1 - (3yr swap * sum (1yr to 2yr DF))] / (1 + 3yr swap)

Yield Curve Mathematics

Forward rates from discount factors

- Forward rate (time X - 1 to time X) = [DF (time X – 1) / DF (time X)] – 1

Swap rates from forward rates

- 2yr swap =
  [(1yr DF * 0 – 1yr fwd) + (2yr DF * 1 – 2yr fwd)] / sum (1yr to 2yr DF)
- 3yr swap =
  [(1yr DF * 0 – 1yr fwd) + (2yr DF * 1 – 2yr fwd) + (3yr DF * 2 – 3yr fwd)] / sum (1yr to 3yr DF)

Swap rate is weighted average of forward rates, where time and discount factors are the weights

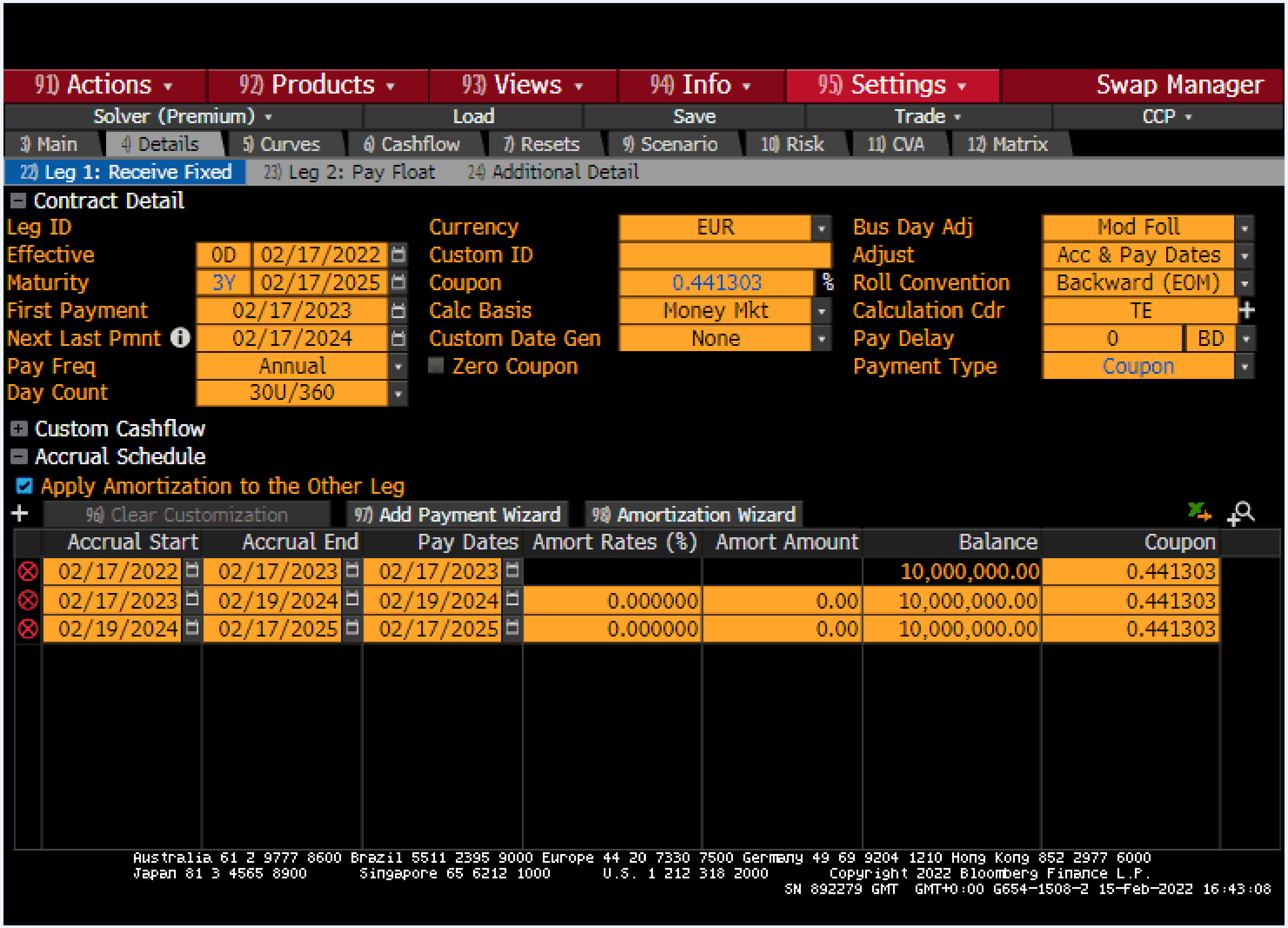
3

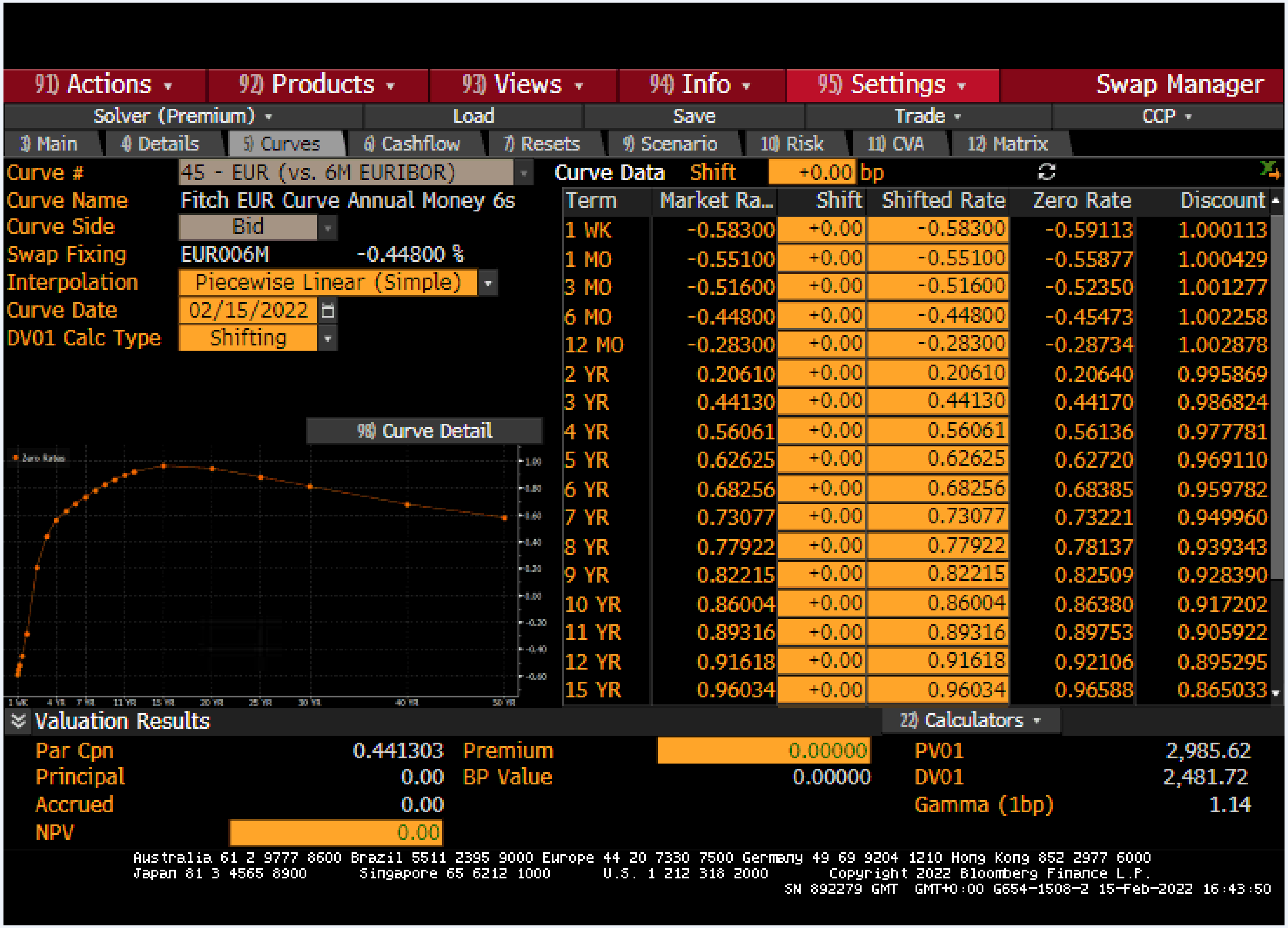
Building a 3-Year IRS Model

Bloomberg Data

FitchLearning

Cancel: Screen not saved
91) Actions
92) Products
93) Views
94) Info
95) Settings
Swap Manager
RFR curves in Excel Curves Toolkit (CTK) will require license Feb 15. See more »
Solver (Premium)
Load
Save
Trade
CCP
3) Main
4) Details
5) Curves
6) Cashflow
7) Resets
9) Scenario
10) Risk
11) CVA
12) Matrix
Deal
Fixed Float Swap
Counterparty
SWAP CNTRPARTY
Ticker
SWAP
20) Properties
CCP
OTC
Trade Date
02/15/2022
Swap
Valuation Settings
Leg 1:Fixed
Receive
Leg 2:Float
Pay
Curve Date
02/15/2022
Notional
10MM
Notional
10MM
Valuation
02/17/2022
Currency
EUR
Currency
EUR
CSA Coll Ccy
N/A
Effective
0D
02/17/2022
Effective
0D
02/17/2022
OIS DC Stripping
Maturity
3Y
02/17/2025
Maturity
3Y
02/17/2025
Coupon
0.441303
%
Index
6M
EUR006M
Pay Freq
Annual
Spread
0.000
bp
Day Count
30U/360
Leverage
1.00000
Calc Basis
Money Mkt
Latest Index
-0.44800
Reset Freq
SemiAnnual
Pay Freq
SemiAnnual
Day Count
ACT/360
Market
Dscnt
45
B
Fitch EUR Curve Ar
Dscnt
45
B
Fitch EUR Curve Ar
Fwd
45
B
Fitch EUR Curve Ar
Valuation Results
22) Calculators
Par Cpn
0.441303
Premium
0.00000
PV01
2,985.62
Principal
0.00
BP Value
0.00000
DV01
2,481.72
Accrued
0.00
Gamma (1bp)
1.14
NPV
0.00
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2022 Bloomberg Finance L.P.
SN 892279 GMT GMT+0:00 G654-1508-2 15-Feb-2022 16:42:09


91) Actions
92) Products
93) Views
94) Info
95) Settings
Swap Manager
Solver (Premium)
Load
Save
Trade
CCP
3) Main
4) Details
5) Curves
6) Cashflow
7) Resets
9) Scenario
10) Risk
11) CVA
12) Matrix
22) Leg 1: Receive Fixed
23) Leg 2: Pay Float
24) Additional Detail
Contract Detail
Leg ID
Effective 0D 02/17/2022
Maturity 3Y 02/17/2025
First Payment 02/17/2023
Next Last Pmnt 02/17/2024
Pay Freq Annual
Day Count 30U/360
Currency EUR
Custom ID
Coupon 0.441303 %
Calc Basis Money Mkt
Custom Date Gen None
Zero Coupon
Bus Day Adj Mod Foll
Adjust Acc & Pay Dates
Roll Convention Backward (EOM)
Calculation Cdr TE
Pay Delay 0 BD
Payment Type Coupon
Custom Cashflow
Accrual Schedule
Apply Amortization to the Other Leg
96) Clear Customization
97) Add Payment Wizard
98) Amortization Wizard
Accrual Start | Accrual End | Pay Dates | Amort Rates (%) | Amort Amount | Balance | Coupon
02/17/2022 | 02/17/2023 | 02/17/2023 | | | 10,000,000.00 | 0.441303
02/17/2023 | 02/19/2024 | 02/19/2024 | 0.000000 | 0.00 | 10,000,000.00 | 0.441303
02/19/2024 | 02/17/2025 | 02/17/2025 | 0.000000 | 0.00 | 10,000,000.00 | 0.441303
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2022 Bloomberg Finance L.P.
SN 892279 GMT GMT+0:00 G654-1508-2 15-Feb-2022 16:43:08



91) Actions | 92) Products | 93) Views | 94) Info | 95) Settings | Swap Manager

Solver (Premium) | Load | Save | Trade | CCP

3) Main | 4) Details | 5) Curves | 6) Cashflow | 7) Resets | 9) Scenario | 10) Risk | 11) CVA | 12) Matrix

Curve # 45 - EUR (vs. 6M EURIBOR)
Curve Name Fitch EUR Curve Annual Money 6s
Curve Side Bid
Swap Fixing EUR006M -0.44800 %
Interpolation Piecewise Linear (Simple)
Curve Date 02/15/2022
DV01 Calc Type Shifting

98) Curve Detail

Curve Data Shift +0.00 bp

Term	Market Ra...	Shift	Shifted Rate	Zero Rate	Discount
1 WK	-0.58300	+0.00	-0.58300	-0.59113	1.000113
1 MO	-0.55100	+0.00	-0.55100	-0.55877	1.000429
3 MO	-0.51600	+0.00	-0.51600	-0.52350	1.001277
6 MO	-0.44800	+0.00	-0.44800	-0.45473	1.002258
12 MO	-0.28300	+0.00	-0.28300	-0.28734	1.002878
2 YR	0.20610	+0.00	0.20610	0.20640	0.995869
3 YR	0.44130	+0.00	0.44130	0.44170	0.986824
4 YR	0.56061	+0.00	0.56061	0.56136	0.977781
5 YR	0.62625	+0.00	0.62625	0.62720	0.969110
6 YR	0.68256	+0.00	0.68256	0.68385	0.959782
7 YR	0.73077	+0.00	0.73077	0.73221	0.949960
8 YR	0.77922	+0.00	0.77922	0.78137	0.939343
9 YR	0.82215	+0.00	0.82215	0.82509	0.928390
10 YR	0.86004	+0.00	0.86004	0.86380	0.917202
11 YR	0.89316	+0.00	0.89316	0.89753	0.905922
12 YR	0.91618	+0.00	0.91618	0.92106	0.895295
15 YR	0.96034	+0.00	0.96034	0.96588	0.865033

Valuation Results | 22) Calculators

Par Cpn	0.441303	Premium	0.00000	PV01	2,985.62
Principal	0.00	BP Value	0.00000	DV01	2,481.72
Accrued	0.00			Gamma (1bp)	1.14
NPV	0.00				

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2022 Bloomberg Finance L.P.
SN 892279 GMT GMT+0:00 G654-1508-2 15-Feb-2022 16:43:50

Pay Date	Payments(Rcv)	Payments(Pay)	Net Payments	Discount	PV
08/17/2022	0.00	22,524.44	22,524.44	1.002258	22,575.29
02/17/2023	44,130.30	6,182.54	50,312.84	1.002878	50,457.62
08/17/2023	0.00	-22,275.88	-22,275.88	1.000649	-22,290.33
02/19/2024	44,375.47	-47,991.94	-3,616.47	0.995869	-3,601.53
08/19/2024	0.00	-39,916.10	-39,916.10	0.991910	-39,593.17
02/17/2025	10,043,885.13	-10,051,533.79	-7,648.66	0.986824	-7,547.89

4

Case Study

FitchLearning

Case Study: IRS Valuation

- Use Excel template to answer questions
- Be prepared to explain methodology and share solutions and Excel sheet in class

FitchLearning

BNP PARIBAS

Option Risk Management

Agenda

1. Option Pricing Review
2. Volatility Primer
3. Option Risk Management
4. Case Studies
   – Case Study 1: Option Greeks
   – Case Study 2: Risk Management
5. Appendix

1

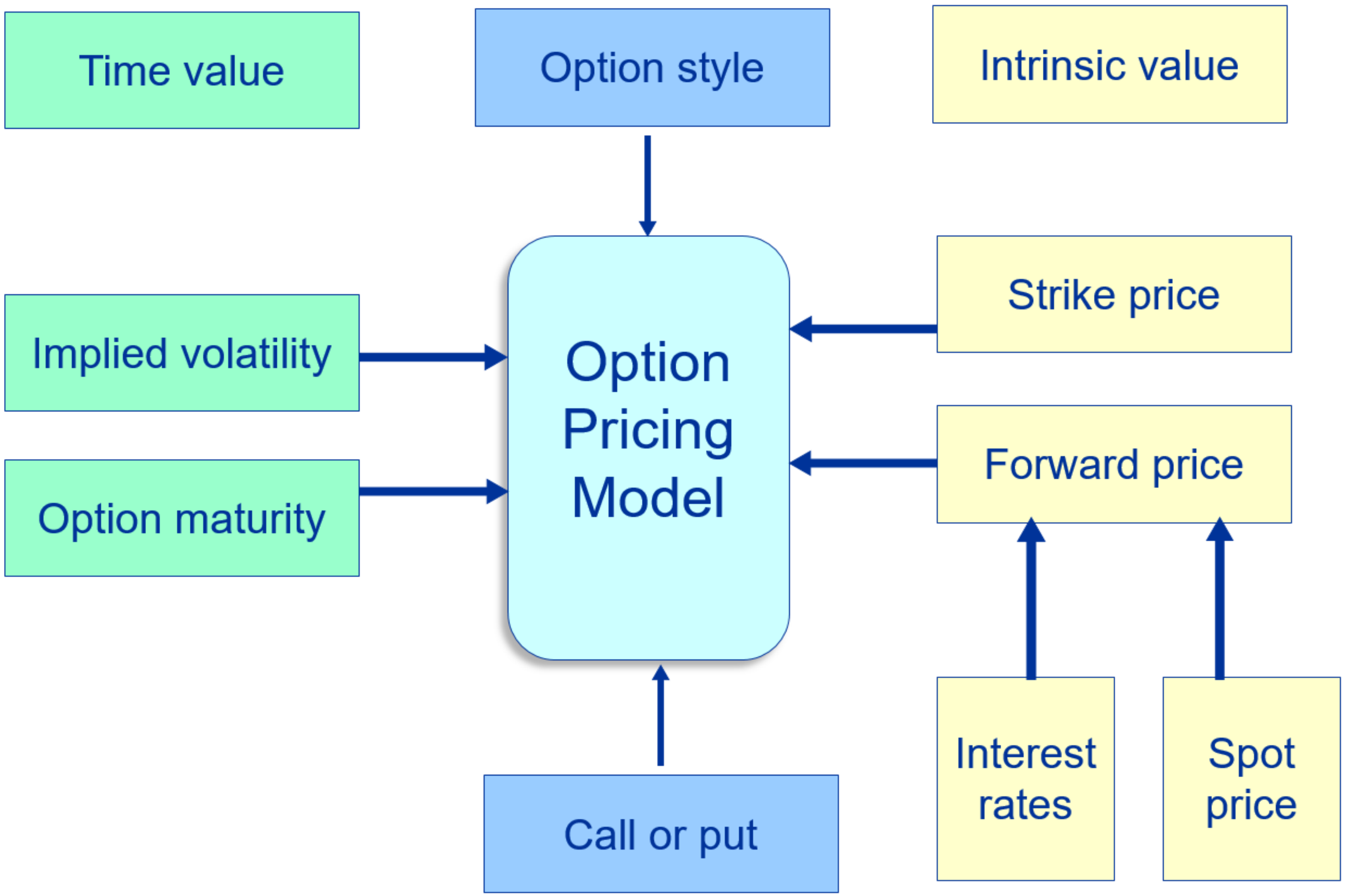
Option Pricing Review

Option Pricing Models

All option pricing models are constructed to solve the same questions:

- What is the probability of an option being exercised?
- IF exercised what will the value of the option be? How far will the option be in-the-money?
- The PV of the answer to these questions is the option premium, which represents the current expected pay-out of the option.

What Does the Model Need?



Summary: Effects of Changes in Model Inputs

	An increase in…	Calls	Puts
Intrinsic value	Underlying	↑	↓
	Strike	↓	↑
Time value	Volatility	↑	↑
	Time to expiry	↑	↑
	Interest rates	↑	↓
	Dividends	↓	↑

Option Pricing: Black-Scholes Model

Drivers of the premium

An option's premium is driven by the following five factors:

- The current price of the underlying asset (S_0)
- The option's strike price (*X* or sometimes *K*)
- Interest rates (*r*) offset by dividends or foreign interest rates
- Time to expiry (the option's remaining life) (*t*)
- The [expected] volatility (variability) of the underlying asset's returns (σ)

In terms of intrinsic and time value we could say:

- Intrinsic value is a function of S_0 and *K*; and
- Time value is a function of *r*, *t*, and σ

At the moment we price the option, all of these factors will be known with the exception of volatility. Volatility is at the centre of option pricing.

Applying the Black-Scholes Formula

The formula for the Black-Scholes equation:

Call premium = $S_0 N(d_1) - Xe^{-rt} N(d_2)$

Where $$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$ and $$d_2 = d_1 - \sigma\sqrt{t}$$

Option Pricing: Monte Carlo Model

Premium is calculated by running simulations which move the market price many thousands of separate times and calculating the PV of the expected pay-out of the option.

Monte Carlo simulations use the following formula to move markets prices

$$dS = (S \times \mu \times dt) + (S \times \sigma \times dX)$$

S = spot price

μ = market drift

t = time

σ = implied volatility

dX = random number

2

Volatility Primer

Quantifying Volatility

What is volatility?

Volatility is the **annualised** standard deviation (SD) of the distribution (natural) log of stock returns

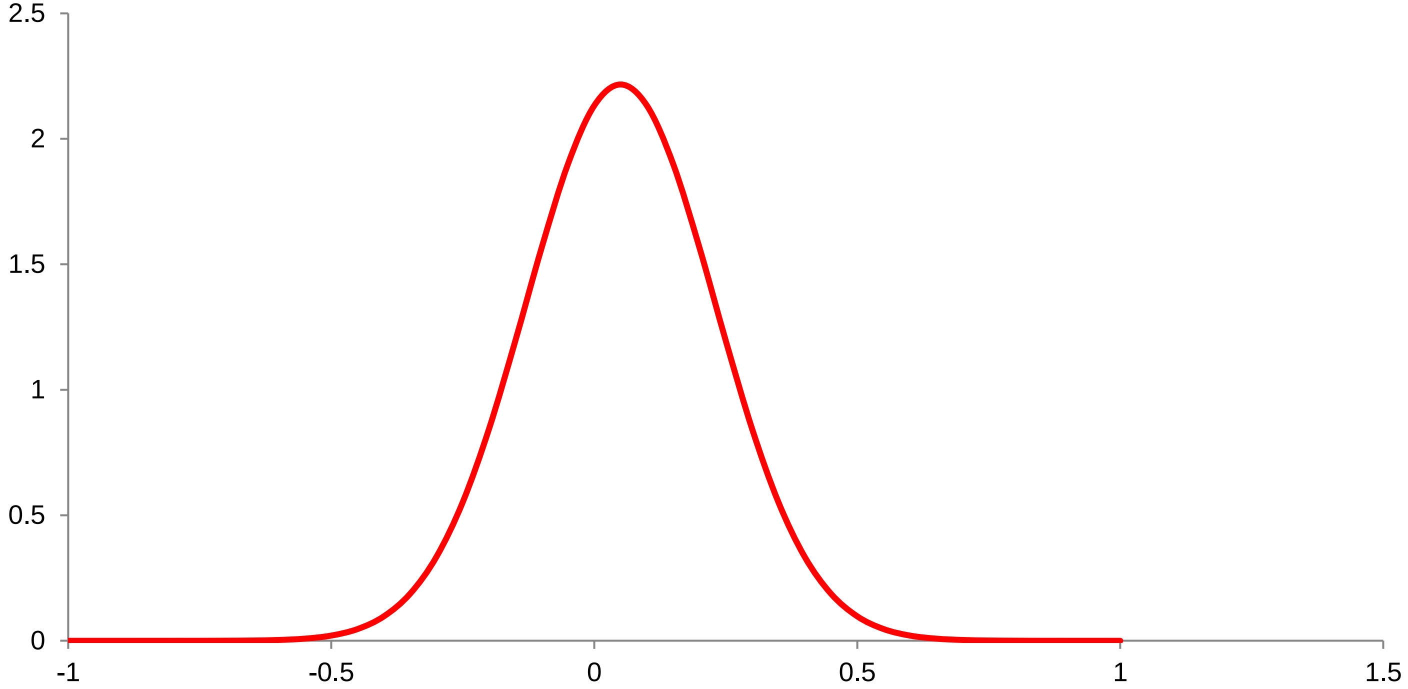
The sigma (σ) in the option pricing models

If x is the continuously compounded periodic return, the standard deviation would be calculated using:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}}$$

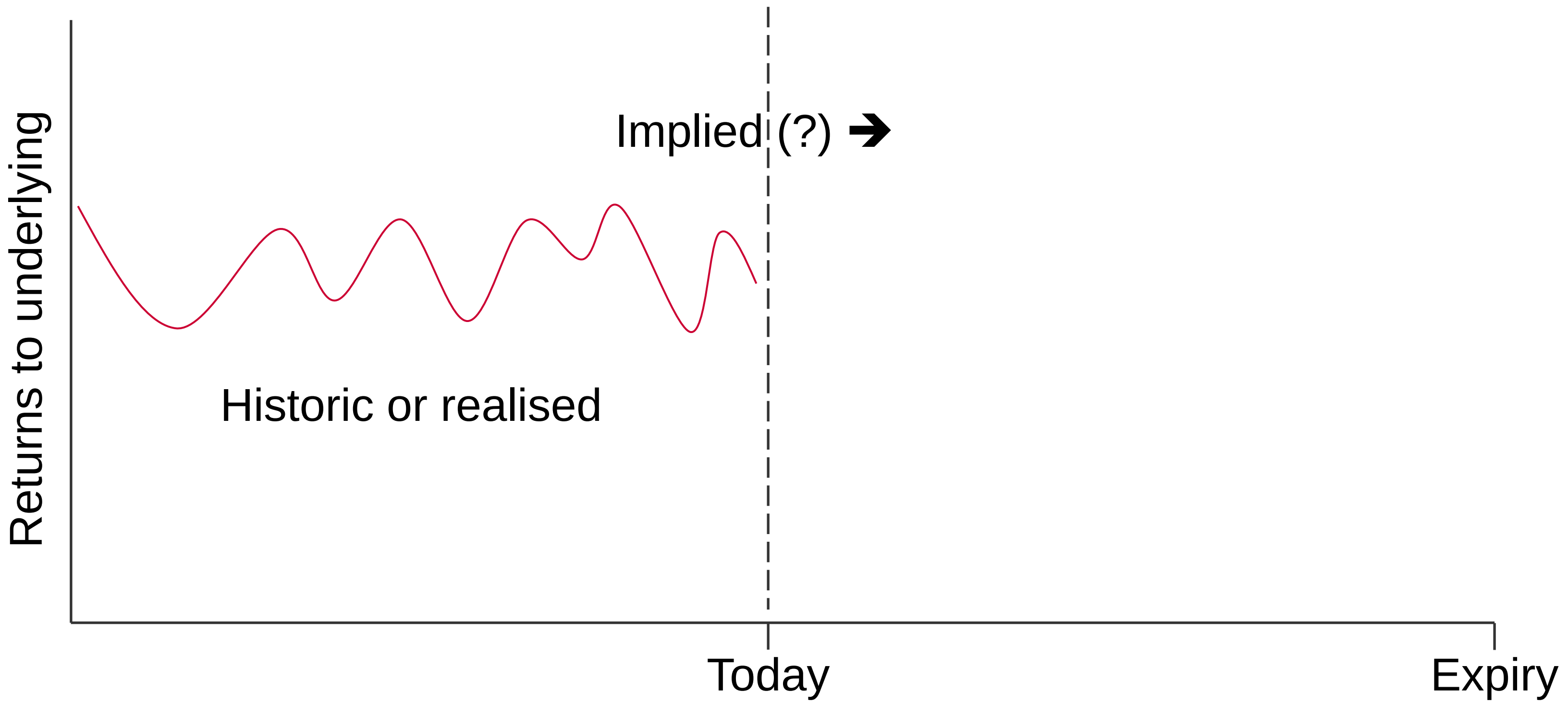
Or using the standard deviation function in Excel (=STDEV()).

Remember we typically assume that the log of the stock returns are normally distributed. This assumption would mean that the distribution of an asset with a mean return of 5% pa and a standard deviation of 18% would look like this:



Implied and Realised (Historic) Volatility

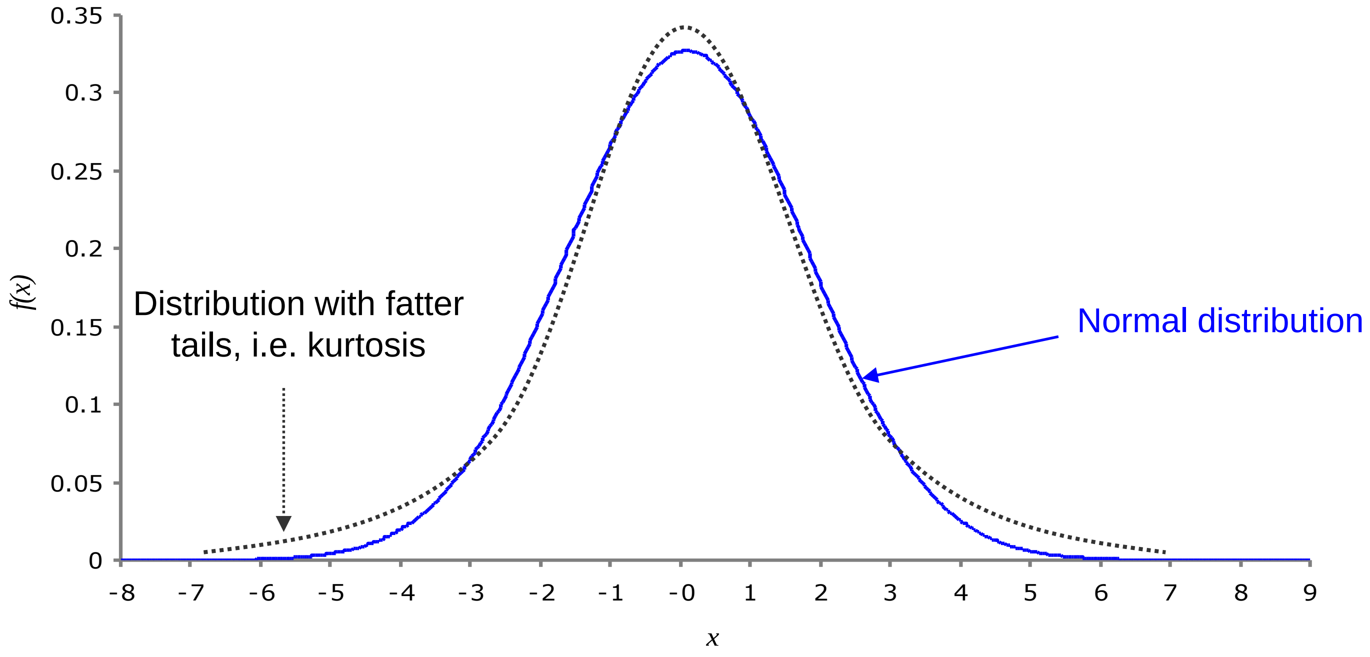
Our volatility calculations give us values for realised vol. The market's aggregate opinion of what vol **will** be for a given period and a given strike is called **implied volatility:**



Real Market Return Distributions

Kurtosis – fat tails

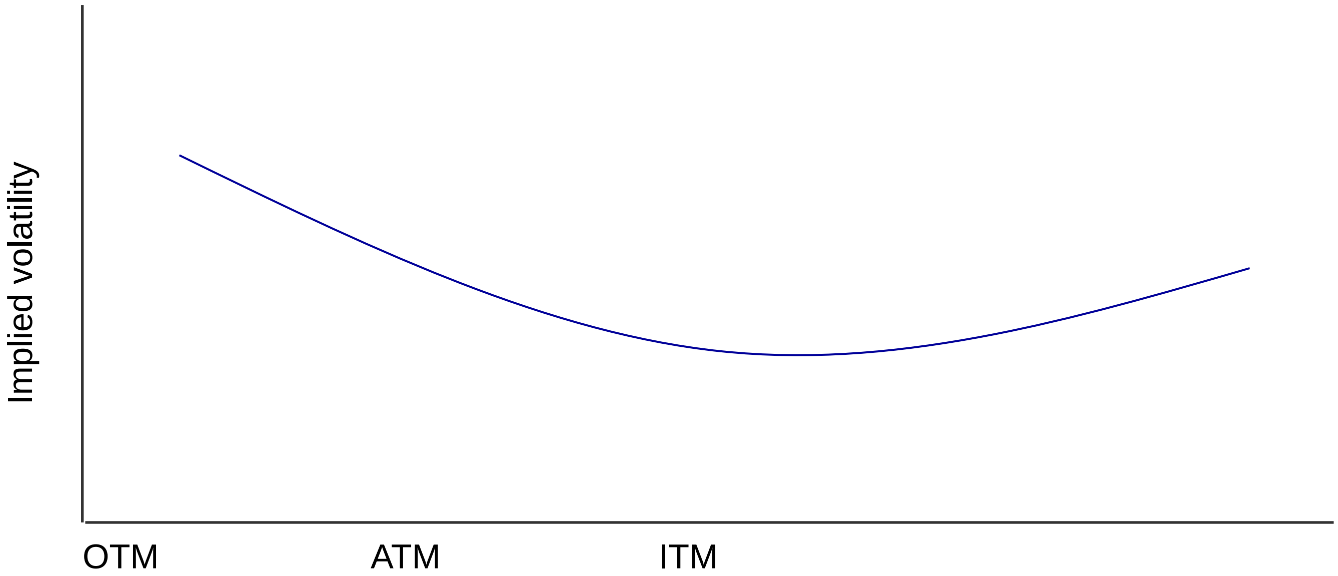
Reality, however, would seem to indicate that asset returns are not normally distributed but instead have fatter tails. This phenomenon is known as **kurtosis**.



Volatility Smiles, Skews and Smirks

Volatility smile

The implied volatility in all options on the same underlying should all be equal; but it is not. For different strike prices, implied volatility forms a **smile** or **skew:**



There are many factors that cause this, not least of which is the fat tails problem mentioned earlier.

Implied Volatility: Reasons for Skew

Distribution of asset prices appears to exhibit fat tails, more likely to be extreme values than normal distribution would suggest.

Smile may reflect directional expectations of market participants.

Appears to be a clientele effect in market for different types of option.

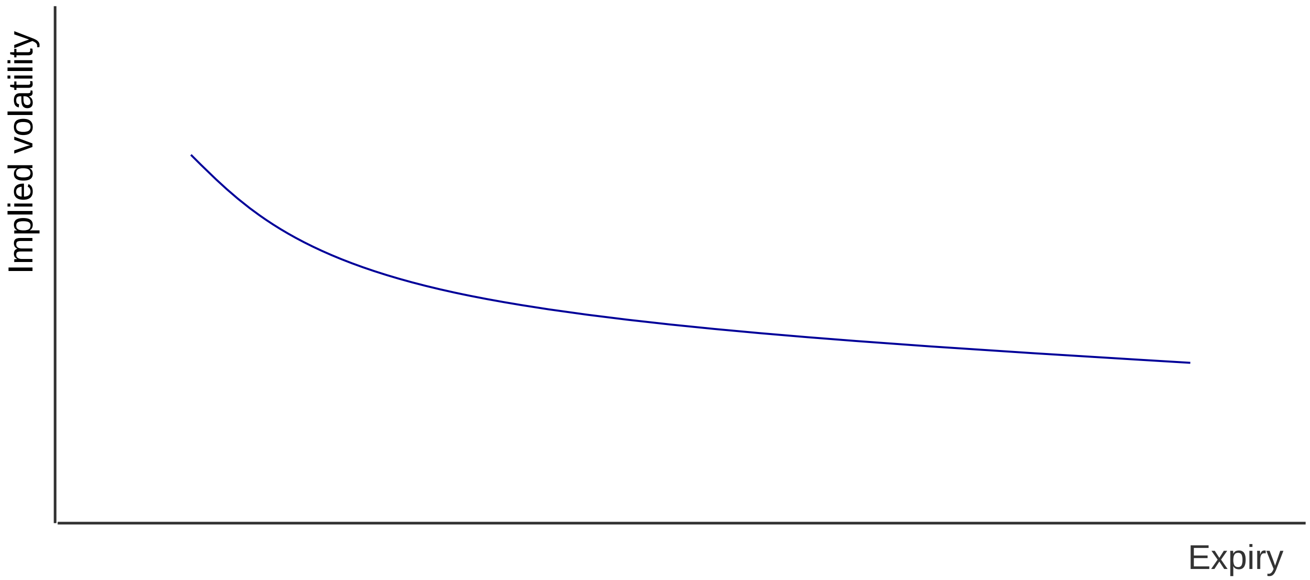
OTM options are attractive for speculators due to higher gearing

Option traders may be reluctant to sell OTM options – liquidity problems

Option traders may be reluctant to sell deeply ITM options – will need to either buy or sell the underlying

Term Structure of Volatility

For options with different expiry dates, we would also observe different values of implied volatility. This is known as the **term structure of volatility:**



Amongst the causes for this are the expectations of larger price movements in the near future.

3

Option Risk Management

FitchLearning

Option Sensitivities

An option's premium, with a fixed strike and expiry, will be sensitive to changes in the following factors:

- Movement in the underlying, the option's sensitivity to this factor is called **Delta**
- Changes in implied volatility – **Vega** (sometimes **Kappa**)
- The passing of time – **Theta**
- Changes in interest rates – **Rho**

We also look at how sensitive **Delta** is to changes in the underlying, this is called **Gamma**.

Measuring the Greeks

The Greeks below are the main **first order** derivatives (Gamma is second order):

Greek	Change in:	For a:	Notation
Delta	Option price	1-unit increase in the underlying	dPrice/dSpot
Gamma	Delta	1-unit increase in the underlying Gamma % is change in Delta for a 1% change in the underlying	dDelta/dSpot
Vega	Option price	1% point increase in implied volatility (1bp or 10bp change in certain models)	dPrice/dVol
Theta	Option price	One day lapse in time to maturity (could be calendar or trading days, depending on model)	dPrice/dTime
Rho	Option price	1% point increase in interest rates (1bp or 10bp change in certain models)	dPrice/dInterest
Phi/Rho-2	Option price	1% point increase in dividend yield (1bp or 10bp change in certain models)	dPrice/ dDividends

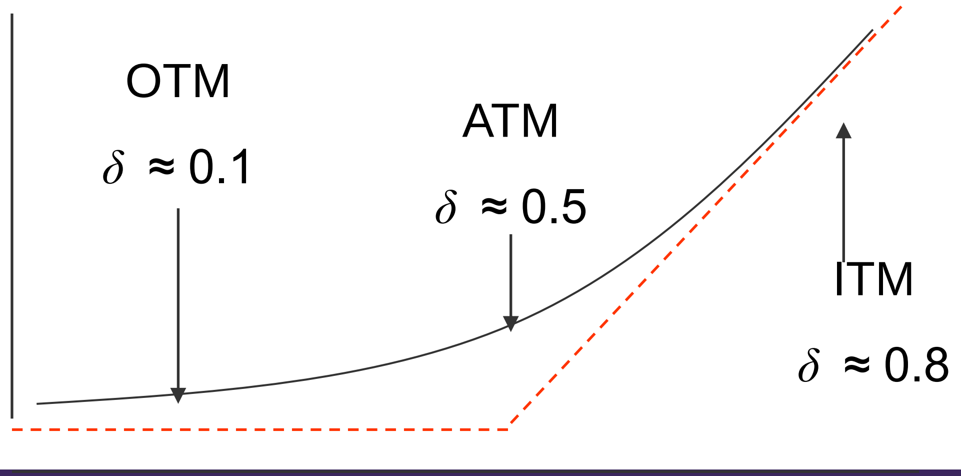
Sensitivity to the Underlying (Delta, δ)

For call options, an increase in the underlying will lead to an increase in the premium and for a put option, it will lead to a decrease in the premium.

Because of the curved nature of the premium value vs. underlying price line, the option's sensitivity (Delta, δ) to changes in the underlying varies. Delta is calculated as:

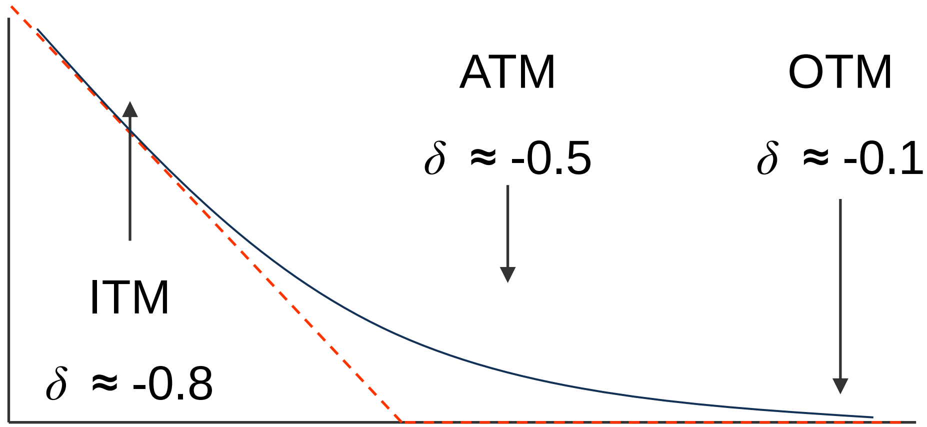
$$\delta = \frac{\text{Change in the option's premium}}{\text{Change in the underlying}}$$

It is the slope of the line. For a call option, Deltas are between 0 and +1:



Sensitivity to the Underlying (Delta, δ)

For a put, Deltas are negative – they range from 0 to -1.

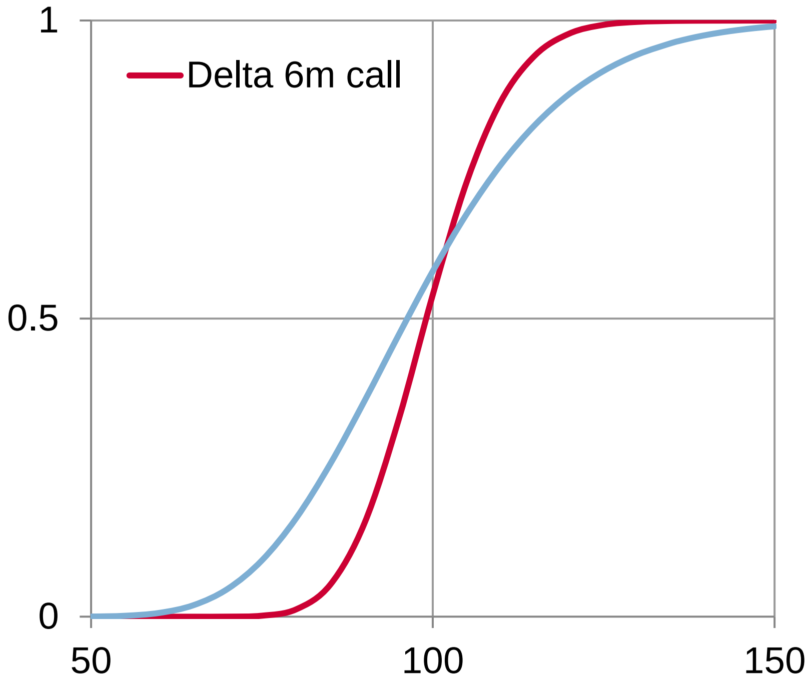


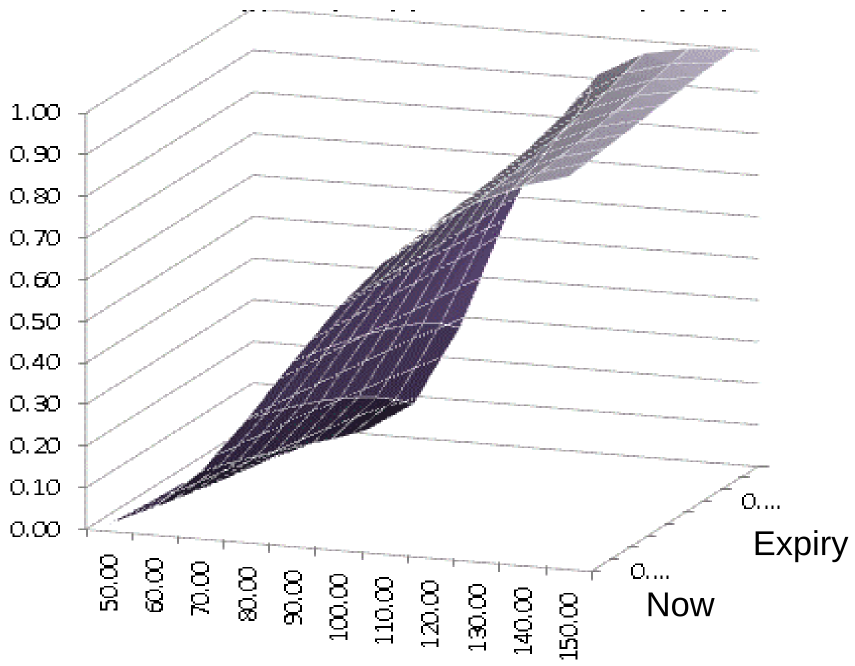
Also: The **absolute** Deltas for a put and a call with the same underlying, strike and expiry **will sum to one**.

Delta

Call Delta at different strikes and different times to expiry

This 3D graph shows how a call (100 strike, vol. 15%) Delta changes for different values of the underlying (x-axis) as the option approaches expiry:





Delta Position

Although calls have positive Delta and puts have negative Delta, our **Delta position** is governed by whether we are long of short of the option, **in addition** to whether the option is a call or put.

Long/short	Call/put/ shares	Quantity	Delta	Delta position in shares
Long	Shares	5m	+1.00	+5m
Short	Put	3m	-0.46	+1.38m
Short	Call	1.5m	+0.80	-1.2m
Long	Call	7m	+0.28	+1.96m

Portfolio Delta position	+7.14m

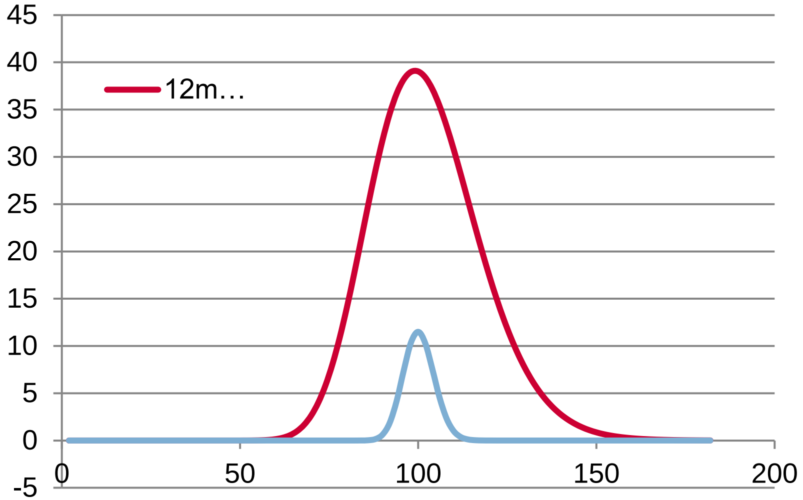
Sensitivity to Implied Volatility (Vega)

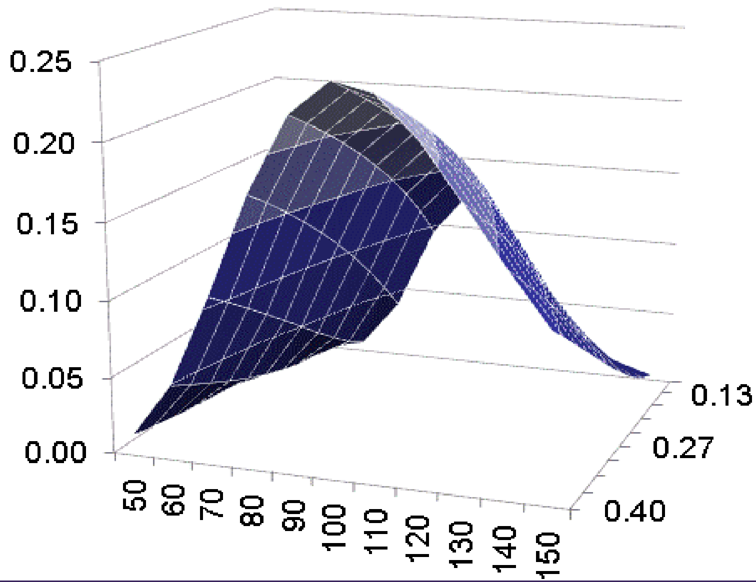
An option premium's sensitivity to changes in implied vol. is called **Vega**.

Example: If an option has a Vega of 93.95, the change in an option's premium for a 1% increase in implied volatility would be:

93.95 x 0.01 = 0.9395

Call Vega for different values of underlying and time to expiry



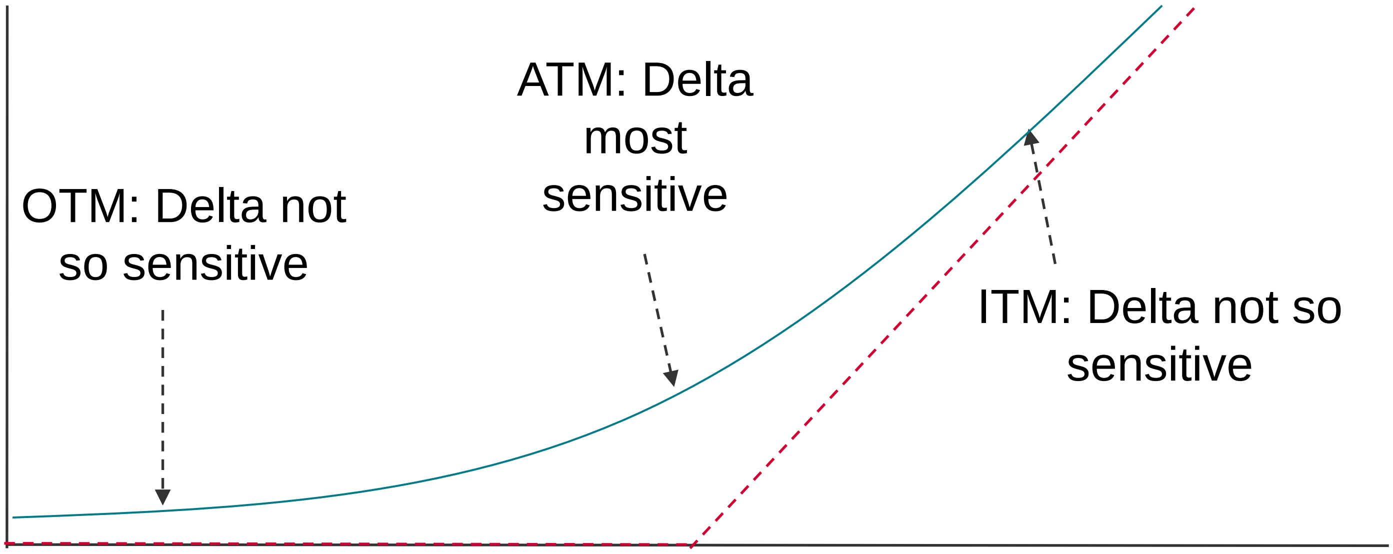


Delta's Sensitivity to Changes in the Underlying

Gamma, Γ, is the change in Delta for a change in the underlying:

$$\Gamma = \frac{\text{Change in the option's Delta}}{\text{Change in the underlying}}$$

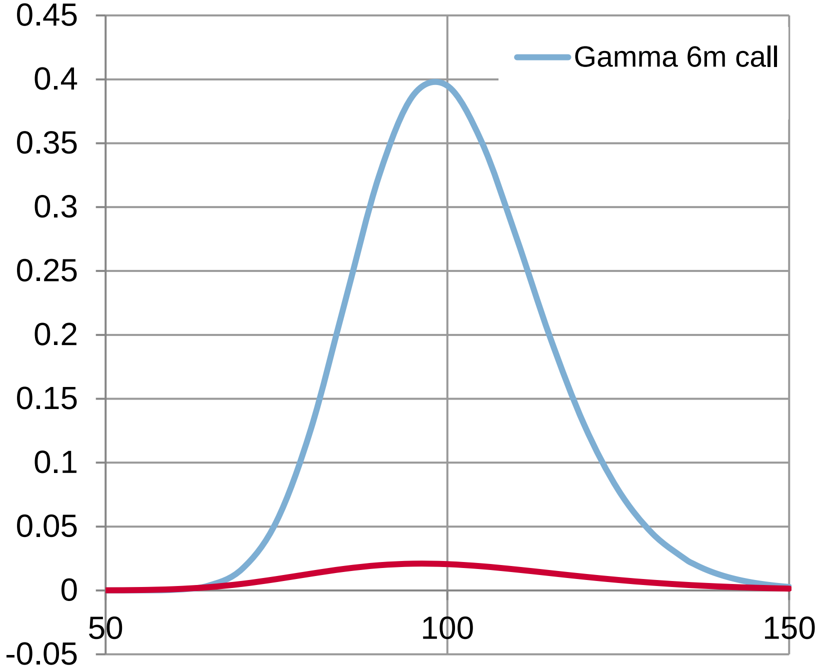
Gamma is highest when the option is at-the-money:

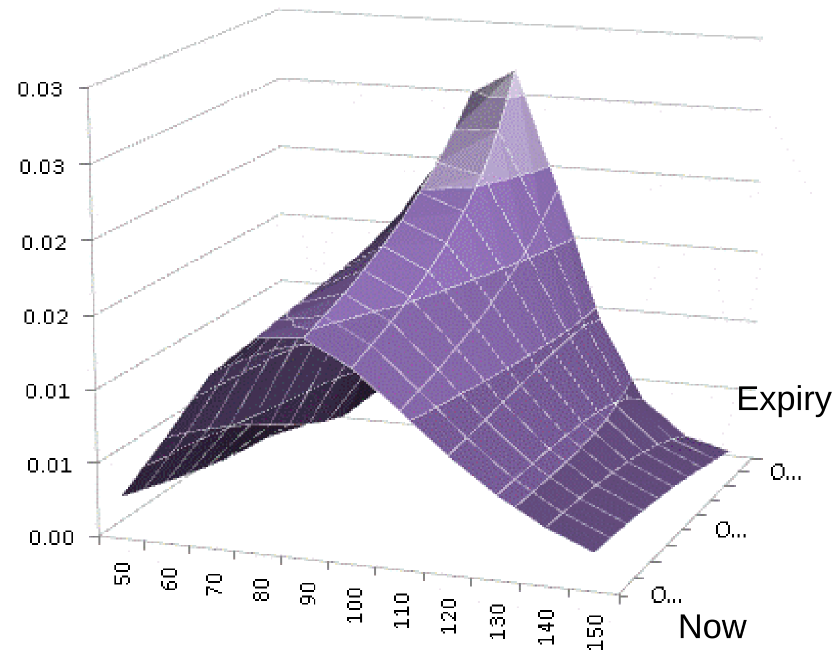


Gamma

Call Gamma at different strikes and different times to expiry

This 3D graph sows how gamma changes for different values of the underlying (x-axis) as the option approaches expiry:



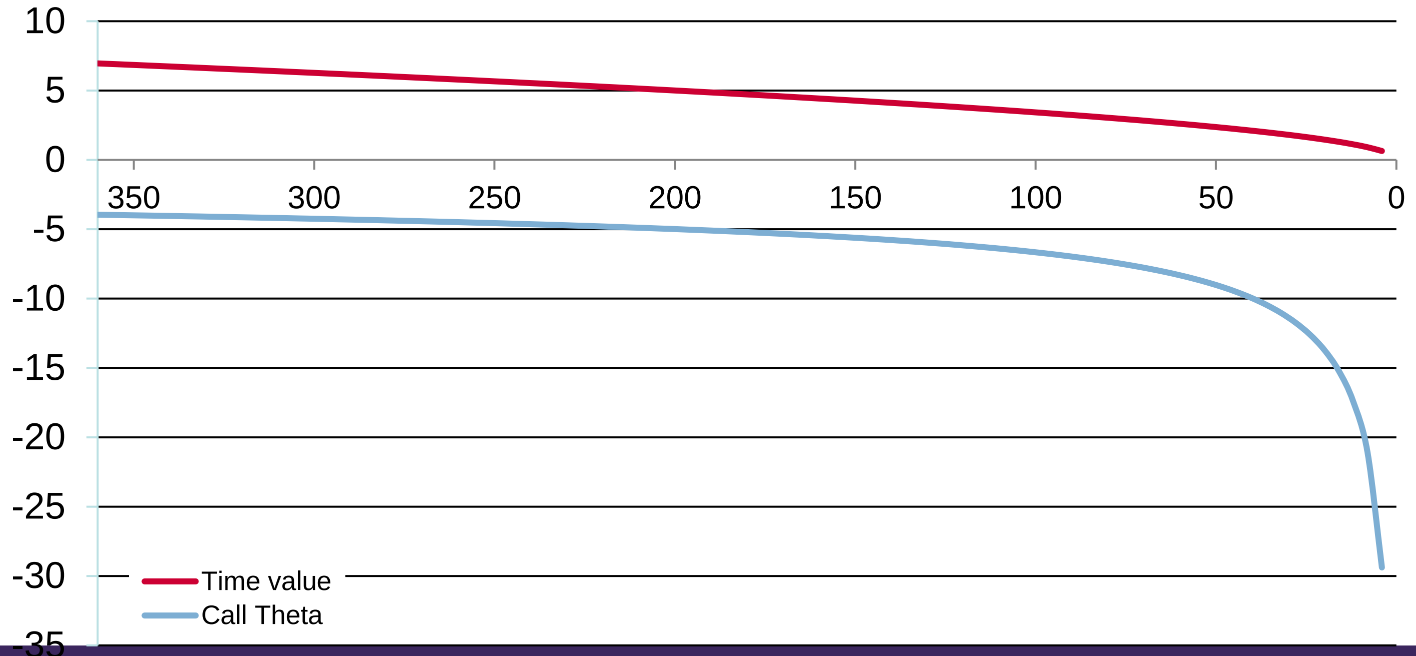


- Where is Delta most sensitive: In, at or out-of-the-money?
- What do you think this would look like for a put?

Sensitivity to Time to Expiry (Theta θ)

The longer the remaining life of an option, the more expensive it will be, or conversely – as time passes, the option's premium will fall.

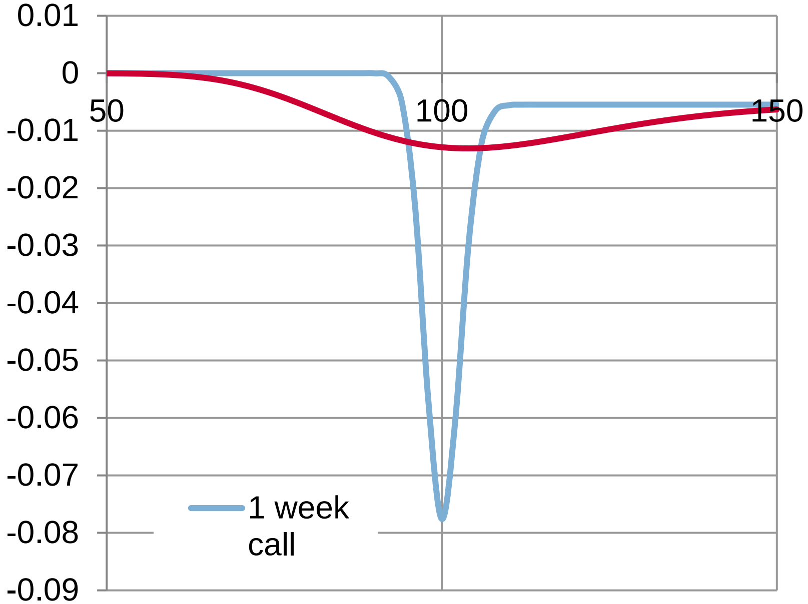
- The premium's sensitivity to the passing of time is measured by Theta. Long option positions 'leak Theta'. Theta is often quoted as a $ amount.
- The chart below plots the Theta and time value (premium) of a 12m ATM call as it moves towards expiry.

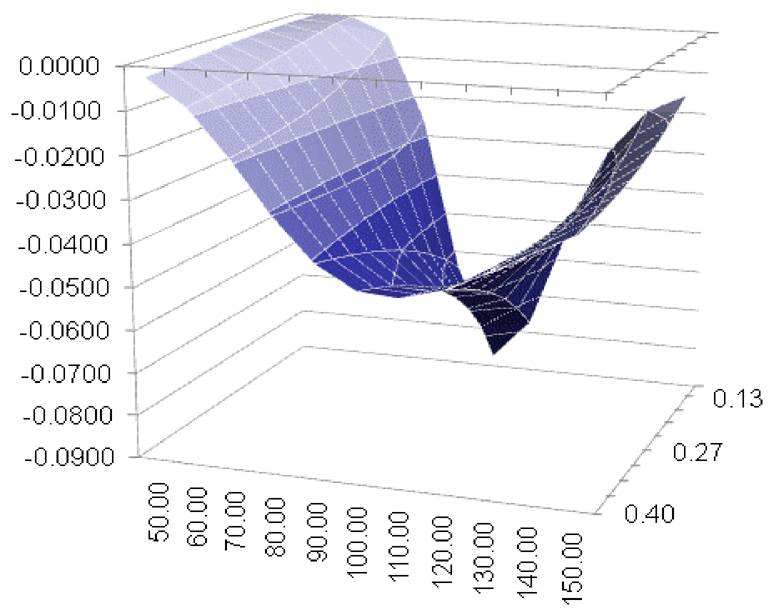


Sensitivity to Time to Expiry (Theta θ)

Example: If an option has a Theta of -38.197, its daily Theta is -38.197/365 per calendar day = -0.1046, i.e. one day of the option's remaining life passing means the premium will fall by 0.1046.

Call Theta for different values of underlying and time to expiry





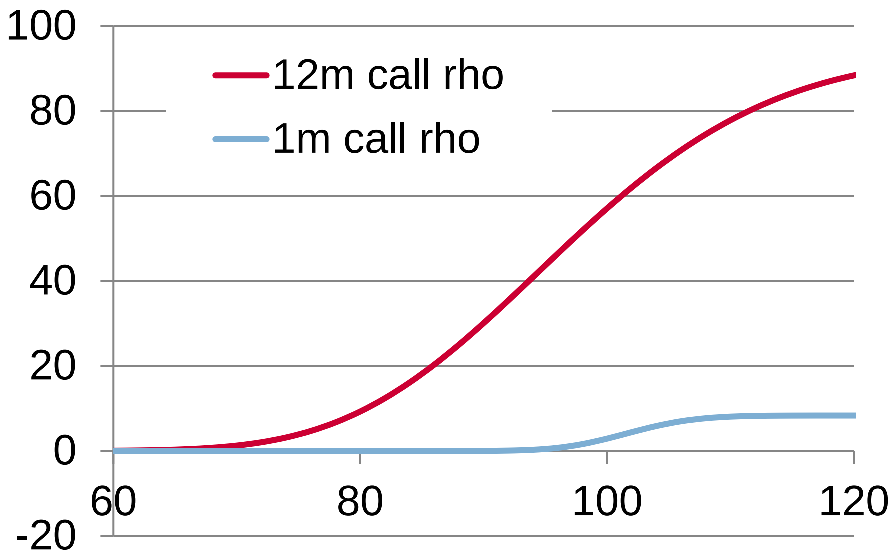
How would this look for a put?

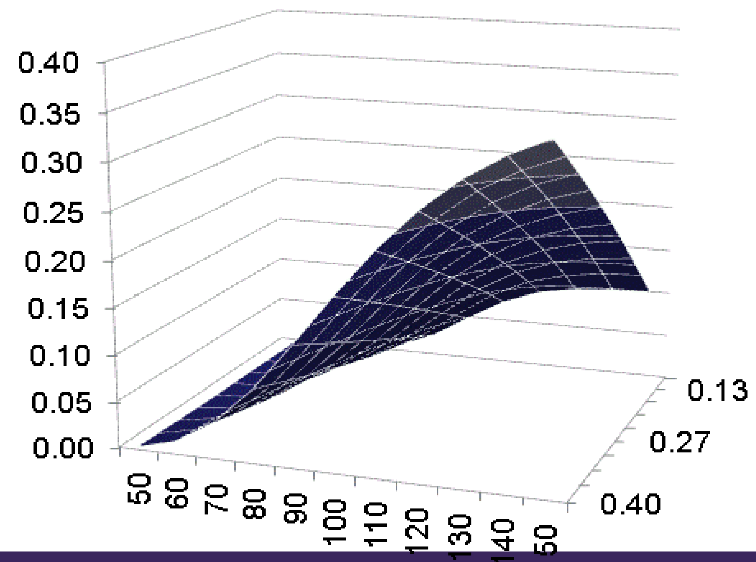
Sensitivity to Interest Rates (Rho, ρ)

The present value of the strike is what the option holder will have to pay (call), or receive:

- Higher interest rates reduce the present value of the amount the call holder pays: Calls are therefore more attractive if interest rates rise
- Higher interest rates reduce the present value of the amount the put holder receives: Puts are therefore less attractive if interest rates rise

Call Rho for different values of underlying and time to expiry





Summary of Greek Risk

	Delta exposure	Gamma exposure (Curvature)	Theta exposure (Time decay)	Vega exposure (Implied volatility)
Long underlying	Long	N/A	N/A	N/A
Short underlying	Short	N/A	N/A	N/A
Long puts	Short	Long	Short	Long
Short puts	Long	Short	Long	Short
Long calls	Long	Long	Short	Long
Short calls	Short	Short	Long	Short

4

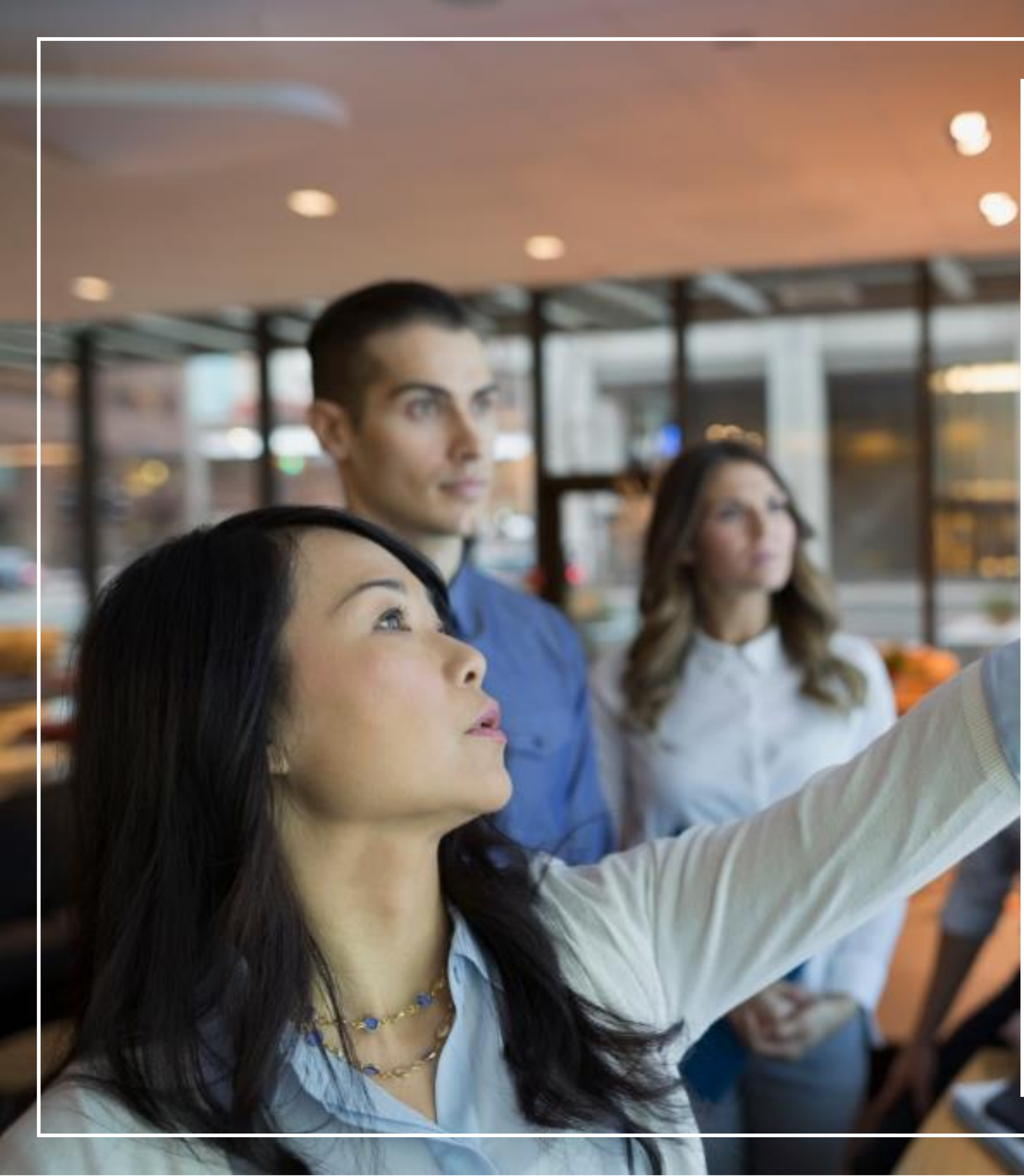
Case Studies

Case Study 1: Option Greeks

- Use Excel spreadsheet
- Answer questions and be prepared to present verbally in class

Case Study 2: Risk Management

- Answer questions and be prepared to present verbally in class



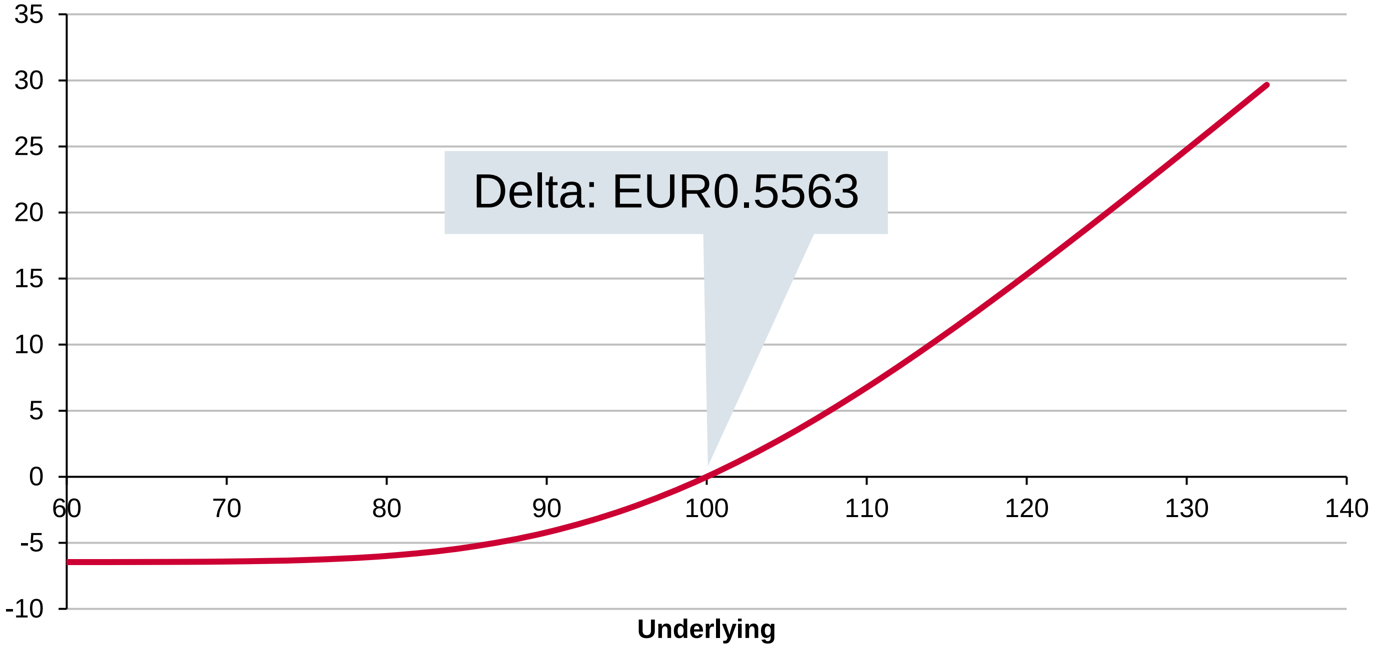
5

Appendix

Position Greeks

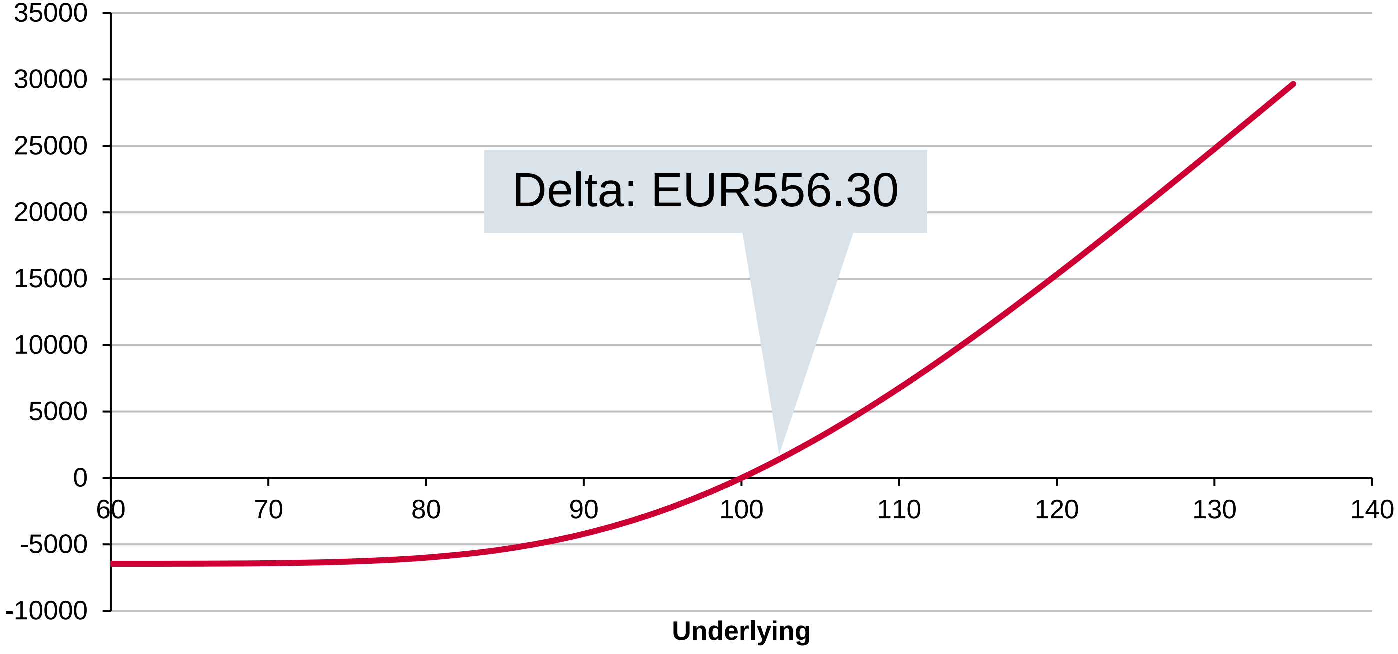
Position Delta

Position Delta is how sensitive P/L is to changes in the underlying. Suppose we buy a 1-year EUR100 strike ATM call option for EUR6.46 (vol. 15%, interest rates 1%). Here's our P/L and Delta **per share:**



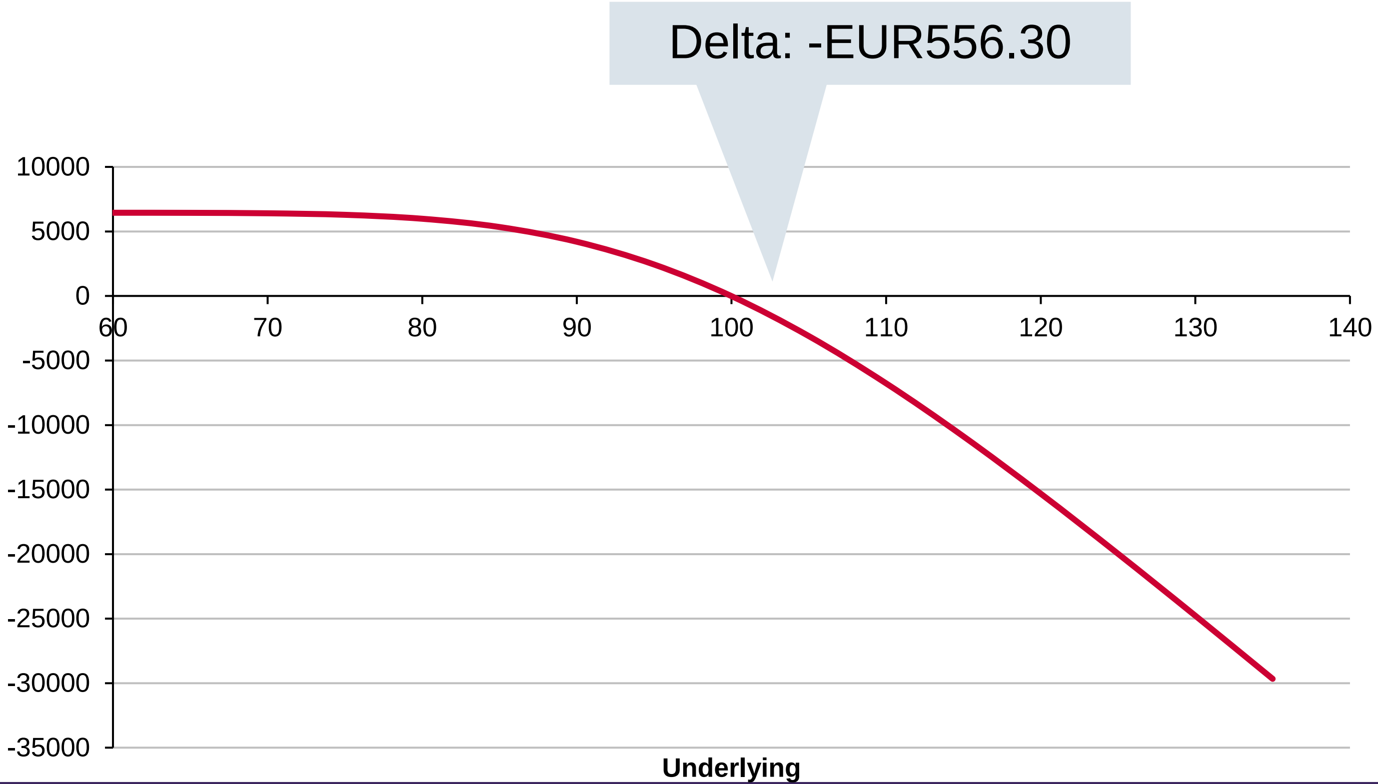
And here's the P/L and Delta for a position for options on 1,000 shares:

If the underlying moves up by EUR our position will make a profit of:



If we had sold the options instead, our position P/L and Delta would now look like this:

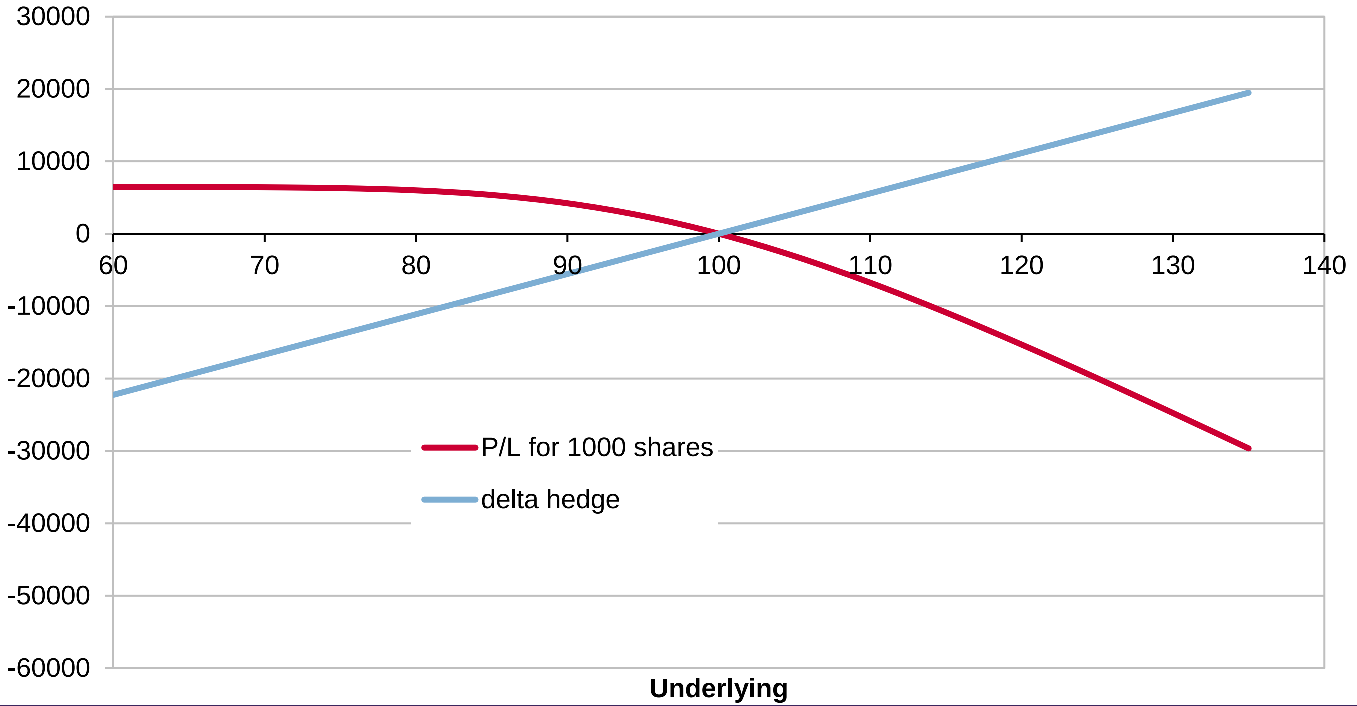
Notice how, even though this is a call option, the position Delta is now negative:



Delta Hedging

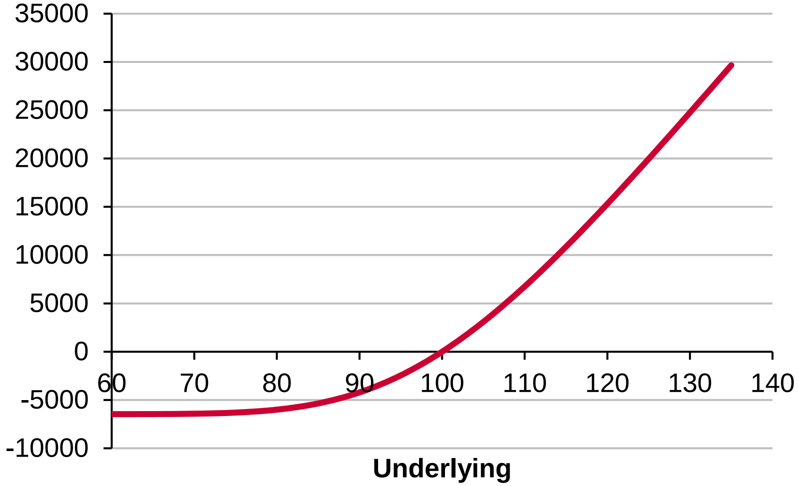
If we were an options market maker and had sold the options to a customer we might want to hedge the Delta exposure with the underlying. The amount of shares we would need to buy would be:

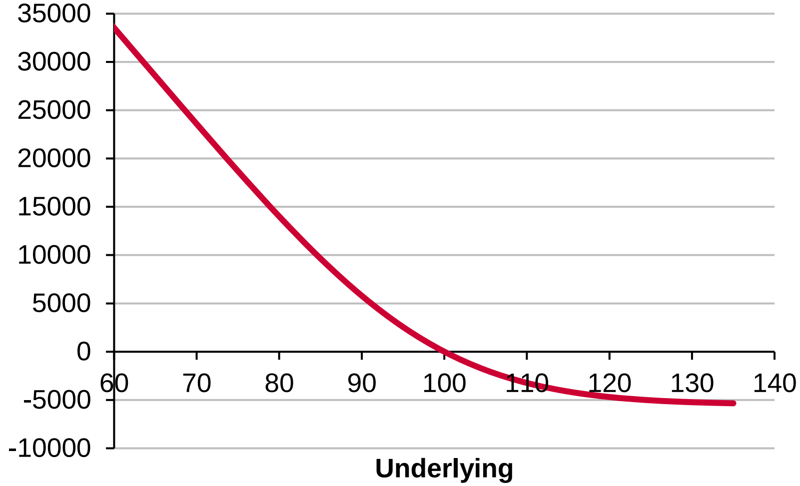
Position's per share Delta × No. of shares in the position



Position Gamma

A long option position will have long Gamma; put or call



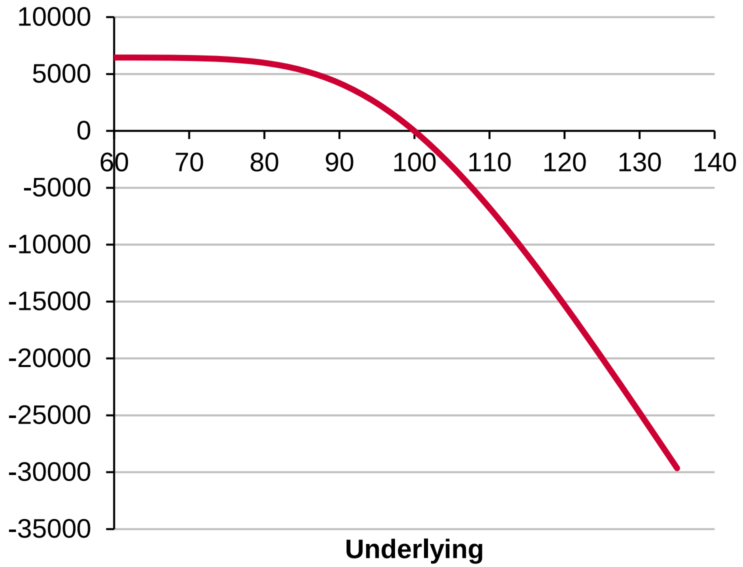


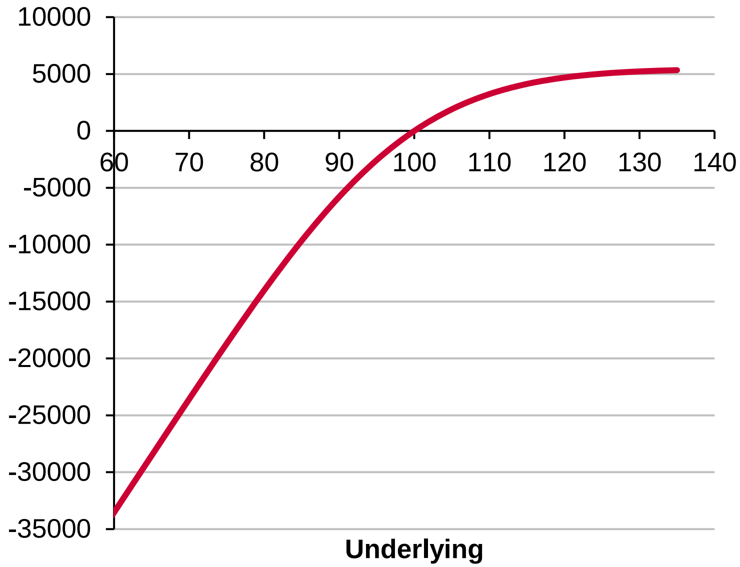
For the long call
If the underlying rises, the position's Delta becomes more positive.

For the long put
If the underlying rises, the position's Delta becomes less negative.

Position Gamma

A short option position will have short Gamma; put or call





For the short call
If the underlying rises, the position's Delta becomes more negative.

For the short put
If the underlying rises, the position's Delta becomes less positive.

FitchLearning

BNP PARIBAS

Structured Products and Exotic Options

Agenda

1. Introduction to Exotic Options
2. Structured Note Construction
3. Case Study
   – Case Study: Structured Product Term Sheets

1

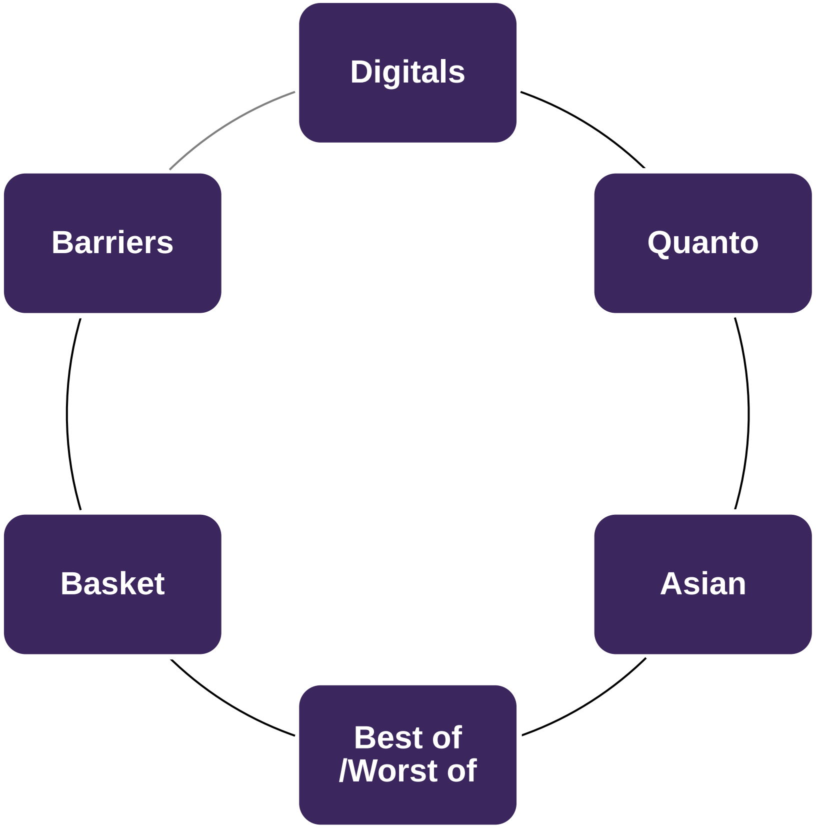
Introduction to Exotic Options

Exotic Options

Exotic options are options that differ from vanilla options by nature of their non-standard payoffs.

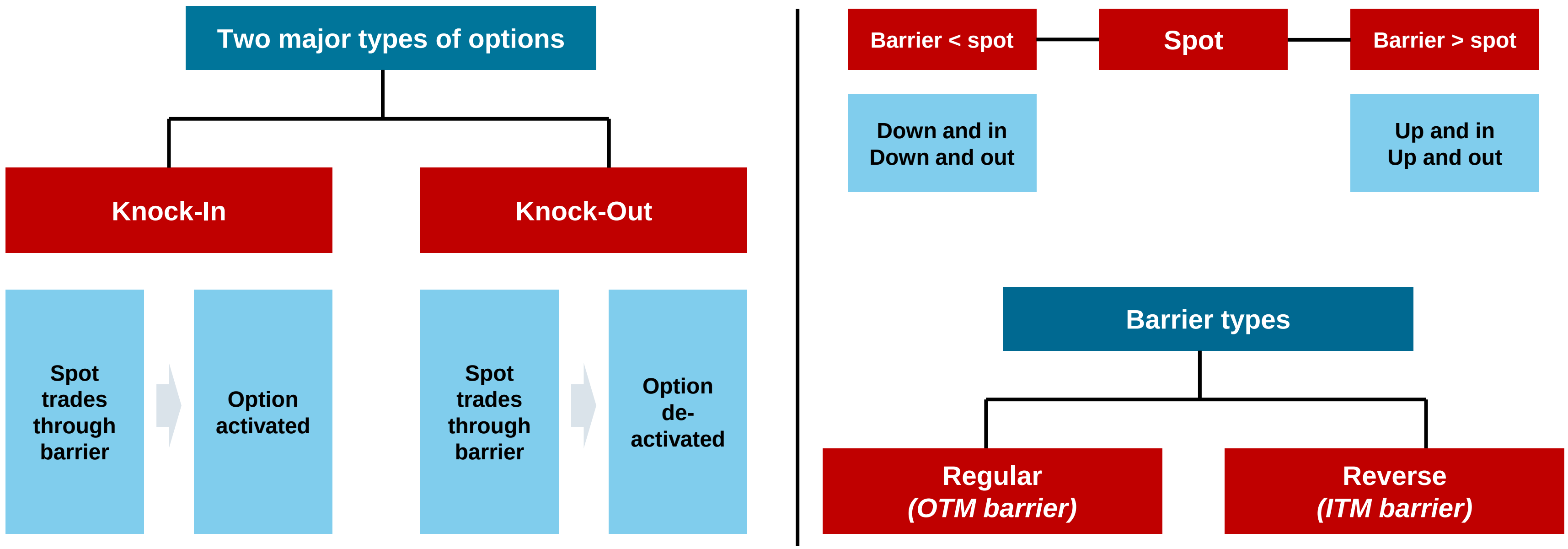
These payoffs can be dependent on

- Asset price paths
- Limits that activate or extinguish the option
  - Barriers
- Time constraints
- Modified payoffs
  - Digitals
- Where the underlying asset involves combined or multiple assets
  - Baskets
  - Worst -Of/Best-Of
    - a.k.a Rainbow
  - Quanto



Barrier Options

Barriers options are activated or extinguished depending on whether spot trades at a predetermined trigger

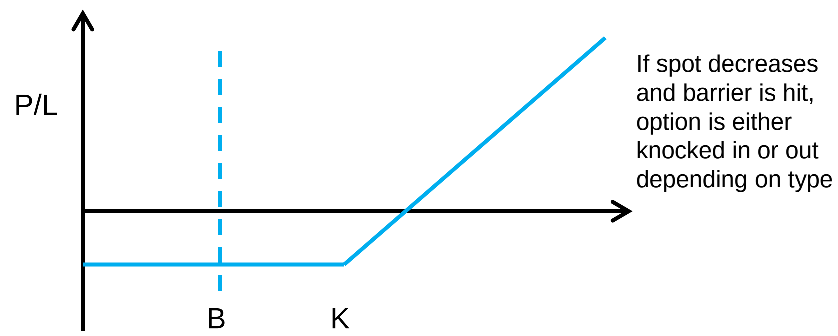


Key advantages

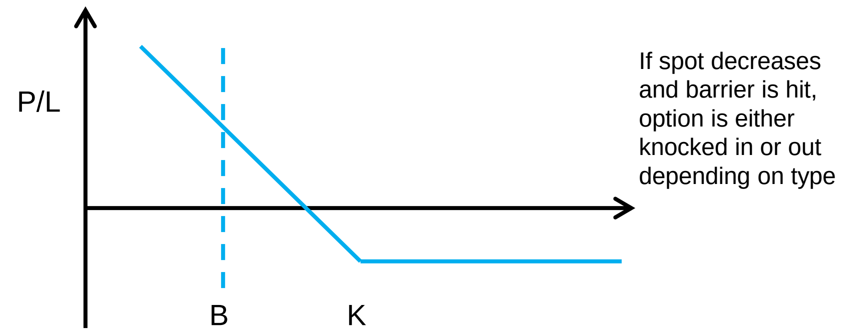
- Cheaper than standard options
- Flexibility in setting the barrier level and thus the cost of the option
- customizable

Barrier Options: Example Payoffs

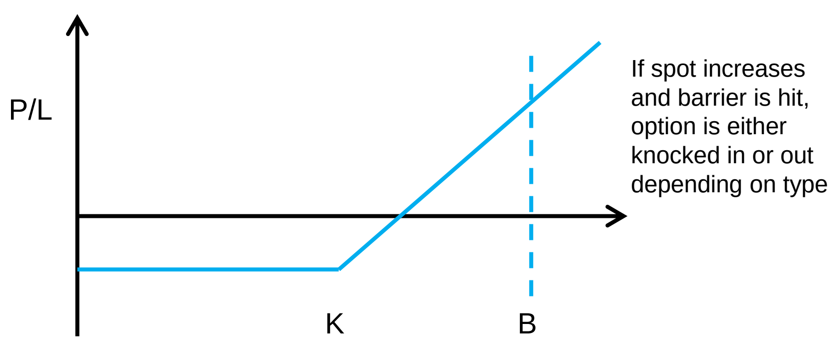
Knock-in/ Knock-out (DI/DO) call



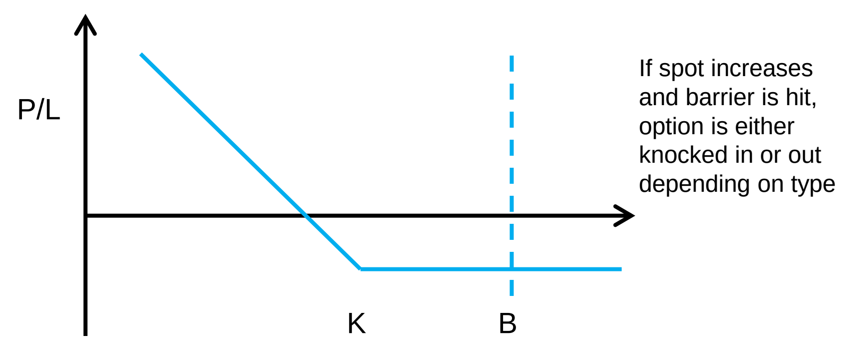
Reverse knock in/Knock out (DI/DO) put



Reverse knock-in/ Knock-out (UI/UO) call



Knock-in/ Knock-out (UI/UO) put

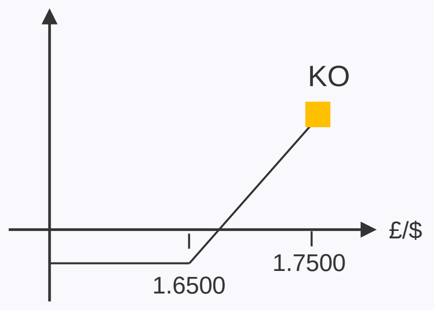


Key: DI= down and in, DO= down and out, UI = up and in, UO= up and out

Why Barrier Options?

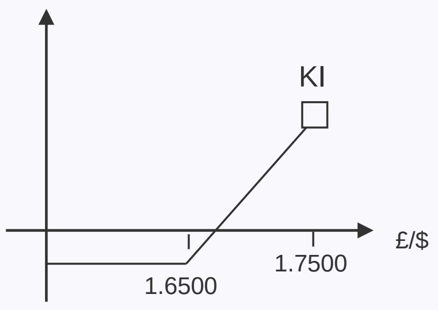
Price = 0.16%
Spot = 1.6400
Strike = 1. 6500
Maturity = 6m

Knock-out Call



\+

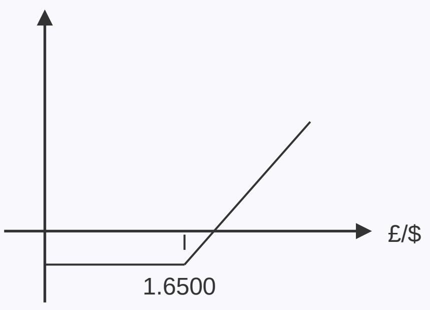
Knock-in Call



Price = 3.23%
Spot = 1.6400
Strike = 1.6500
Maturity = 6m

Price = 3.39%
Spot = 1.6400
Strike = 1.6500
Maturity = 6m

Plain Vanilla Call



If at any point, spot breaches the 1.75 barrier, the KO call becomes obsolete however the KI call comes alive. The net effect is the same as owning a vanilla call option to start with.

Barrier options are individually cheaper than the vanilla option

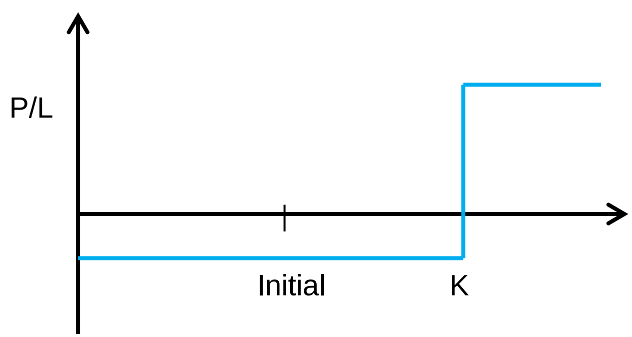
Binary/Digital Options: European

A European digital is a binary option where the trigger (barrier) is observed only at expiry

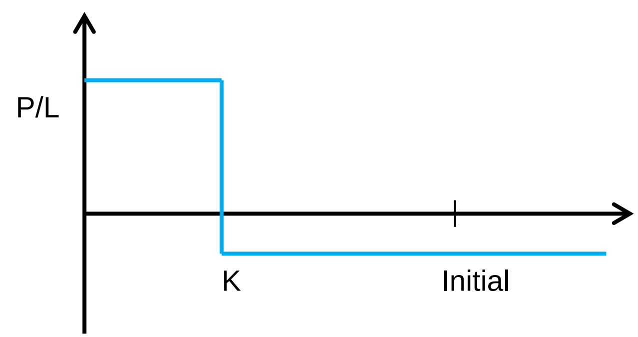
Types

- **Digital Call**: Spot > barrier at expiry → Payout = full notional
- **Digital Put**: Spot < barrier at expiry → Payout = full notional

Digital Call/ Up and in



Digital Put/ Down and in



European digitals are initially struck OTM

Binary/Digital Options: American

An American digital is a binary option where the trigger (barrier) is observed over the life of the trade

American

One touch

- Starts inactive → Payout = full notional (else nothing)
- If spot EVER touches barrier

No touch

- Starts inactive → Payout = full notional
- If spot NEVER touches barrier

Different American Payouts

	One touch	No touch	Double touch	Double no touch
Up	ALL if spot>barrier at any time prior to expiry	NOTHING if spot>barrier at any time prior to expiry	ALL if Spot>up barrier at any time prior to expiry OR spot < down barrier at any time prior to expiry	NOTHING if Spot>up barrier at any time prior to expiry OR spot < down barrier at any time prior to expiry
Down	ALL if spot<barrier at any time prior to expiry	NOTHING if spot<barrier at any time prior to expiry		

Multifactor Options

Where the underlying asset involves combined or multiple assets

Payoffs are based on the relationship between multiple assets rather than the single asset of a vanilla option.

	Description	Payout
Basket Options	Payout dependent on the cumulative performance of a Basket of assets	Max [0, Σ (W^i x $Stock_T^i$ / $Stock_0^i$) - Strike] W^i = weight $Stock_T^i$ = closing level of stock i on final observation date T $Stock_0^i$ = closing level of stock i on the strike date
Worst Of (WO)/Best Of (BO) (aka Rainbow options)	Payout based on the relationship between multiple assets, usually in a best-of or worst-of scenario (Not to be confused with Basket options which offer an exposure to the cumulative performance of a group of assets)	Best-of call option =Max [Max (S_1 , S_2) - X, 0] Worst-of put option =Max [X- Min (S_1 , S_2), 0] S_1 = final price of asset A S_2 = final price of asset B X = exercise price
Quanto	Quantity adjusting option with a payout denominated in a currency other than that of the underlying asset Option payout is converted to the domestic currency at expiry, at a predetermined exchange rate but the amount of money to be converted will not be known until expiry.	Call = E* Max (S – X, 0) Put = E* Max (X – S, 0) S = final price of the asset E = predetermined exchange rate X = exercise price

Sample Structure: Basket Call Option

Hypothetical terms

Structure:	Call option
Underlying Basket:	Apple Inc (AAPL) Caterpillar Inc (CAT) ConocoPhillips (COP) Ford Motor Co (F) Intel Corp (INTC) Coca-Cola Co (KO) Pfizer Inc (PFE) QUALCOMM Inc (QCOM) Target Corp (TGT) Verizon Communications Inc (VZ)
At Maturity	The investor receives Max $[0, \Sigma (W^i \times Stock_T^i / Stock_0^i) - Strike]$ With W^i = weight $Stock_T^i$ = closing level of the stock i on the final observation date T $Stock_0^i$ = closing level of the stock i on the strike date

Indicative Option Premiums

Maturity	Strike	Basket Call	Average Call*	SPX
3 Months	100%	2.70%	3.60%	2.45%
6 Months	100%	3.65%	5.05%	3.75%
12 Months	100%	5.10%	7.25%	5.90%

* Average premium of calls on each of the underlying

Basket Options

Correlation impacts the implied volatility of the basket

For Basket options, the correlation impacts the implied volatility of the basket

Example

- Take a basket of SX5E and SPX, equally weighted
- Asset price = 0.5 * SX5E + 0.5 * SPX with
- Volatility of SX5E is 15%, volatility of SPX is 10% and they are 70% correlated.
- What is the volatility of the basket?

$$\sigma_{\text{Portfolio}} = \sqrt{0.5^2_{\text{weight SX5E}} 15\%^2_{\text{Vol SX5E}} + 0.5^2_{\text{weight SPX}} 10\%^2_{\text{Vol SPX}} + 2 * 0.5_{\text{weight SX5E}} 0.5_{\text{weight SPX}} 0.70_{\text{correl}} 15\%_{\text{Vol SX5E}} 10\%_{\text{Vol SPX}}} = 11.56\%$$

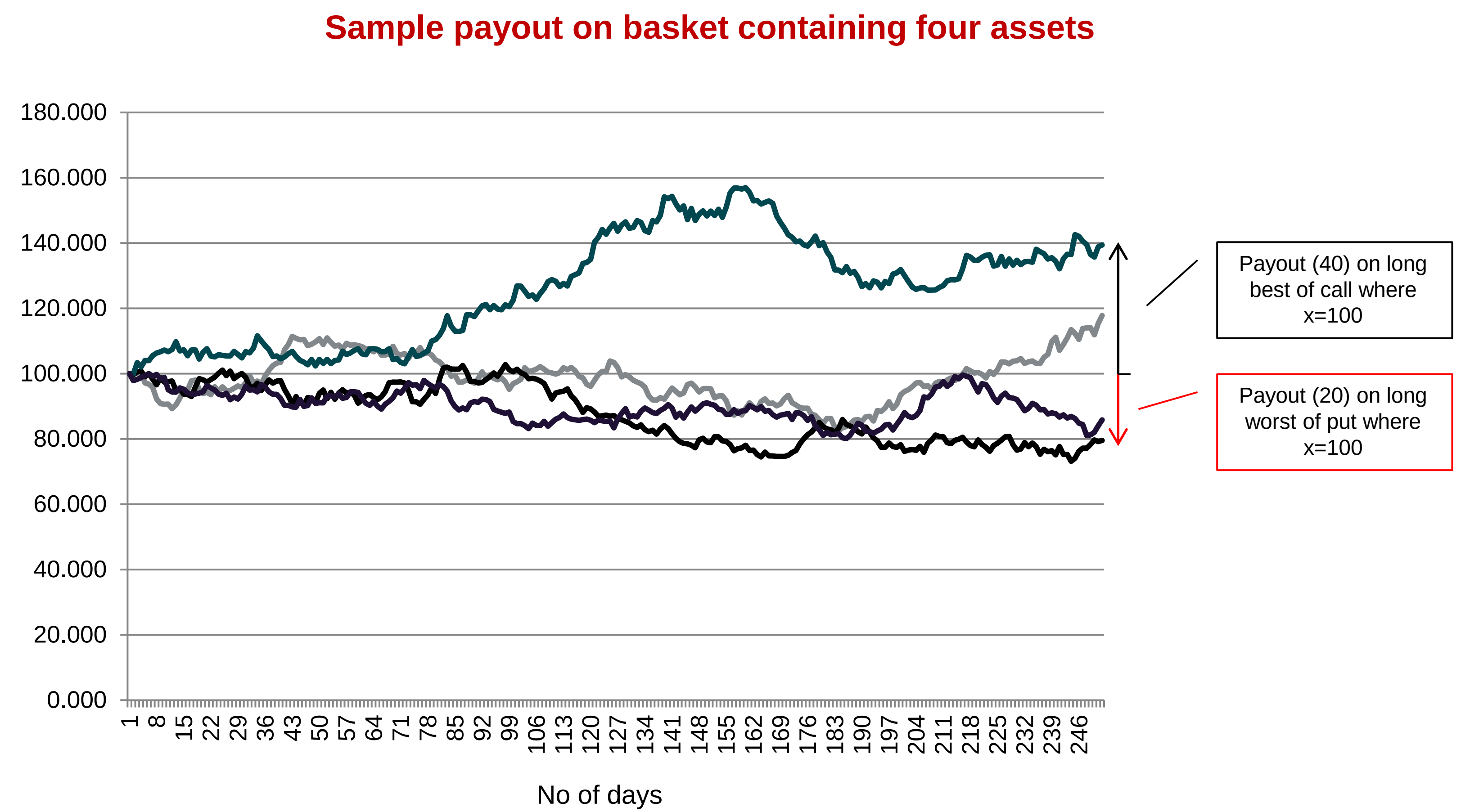
Lower correlation = Cheaper option price

Correlation risk is illiquid -> Typical dealers warehouse it

Correlation risk is hand-in-hand with X-Gamma risk

Worst-Of / Best-Of Options

Payout based on the worst or best performing asset in a multivariate basket.



Asian (or average rate) Options

Asian payoffs are dependent on asset price paths

Path dependent options where the expiry payoff is determined by comparing the strike price with the average of the spot price over the option life

Payoff is given by:

- Call = Max($S_{average}$ – X,0)
- Put = Max(X - $S_{average}$,0)
  - The most common averaging process of the asset price is arithmetic, given by:
  - $\sum (S_1+S_2+S_3.....+S_n)/n$ where S_n is the price at time n and n is the number of price observations

The averaging process is the key driver of the Asian option price

- Average price at expiry > strike price for the (call) option to have any intrinsic value

Volatility of average prices < volatility of the individual asset prices

- As an increasing proportion of asset prices are known, a greater part of the average price is known.
- Therefore individual price moves have a proportionally lower impact on the average price.

Delta and gamma become more stable towards expiry as the price path becomes increasingly averaged, making dynamic hedging easier.

These factors make Asian options cheaper than vanilla options

Example: Base Metals Consumer

Commodity Price Hedging Using Average Rate Option

Gencorp are a manufacturer of generators which use copper wiring within their motors

Problem

- Currently the price they sell generators at is set 6 months before they buy their copper
- They expect to consume around 7,000mt copper per annum and would like to hedge 3000mt of this
- Gencorp want to fix their copper price early in order to alleviate the 6 month risk they currently have

Solution

- Purchased average rate call options
  - Protection at a known price (Cap) whilst allowing
  - Full participation in favourable moves in market prices
  - Requires payment of an up-front premium
- Cash settlement for Gencorp is calculated using an average price over Reference Period.

2

Structured Notes

FitchLearning

Structured Notes

Structured products are debt securities or bank CDs that provide returns based on the performance of one or more reference underlyings

- e.g. stocks, ETFs, indices, commodities, interest rates, FX, etc.

Bond/Deposit + Exposure to Reference Underlying

The bond is usually a zero coupon bond or deposit

Exposure to reference underlying usually achieved via embedding vanilla or exotic options

The combination of the bond and embedded optionality creates a funded product with a unique payoff

- Aimed at High Net Worth Individuals (HNWI) and certain Institutions

Major Payoff Types

Capital protection

Investors receive low or no coupon and forgone interest is used to finance the purchase of options to gain exposure to a particular market, e.g. call spread note

Yield

Investors receive an above market coupon, achieved through the sale of options on a particular market, e.g. reverse convertibles, auto-callables

Participation

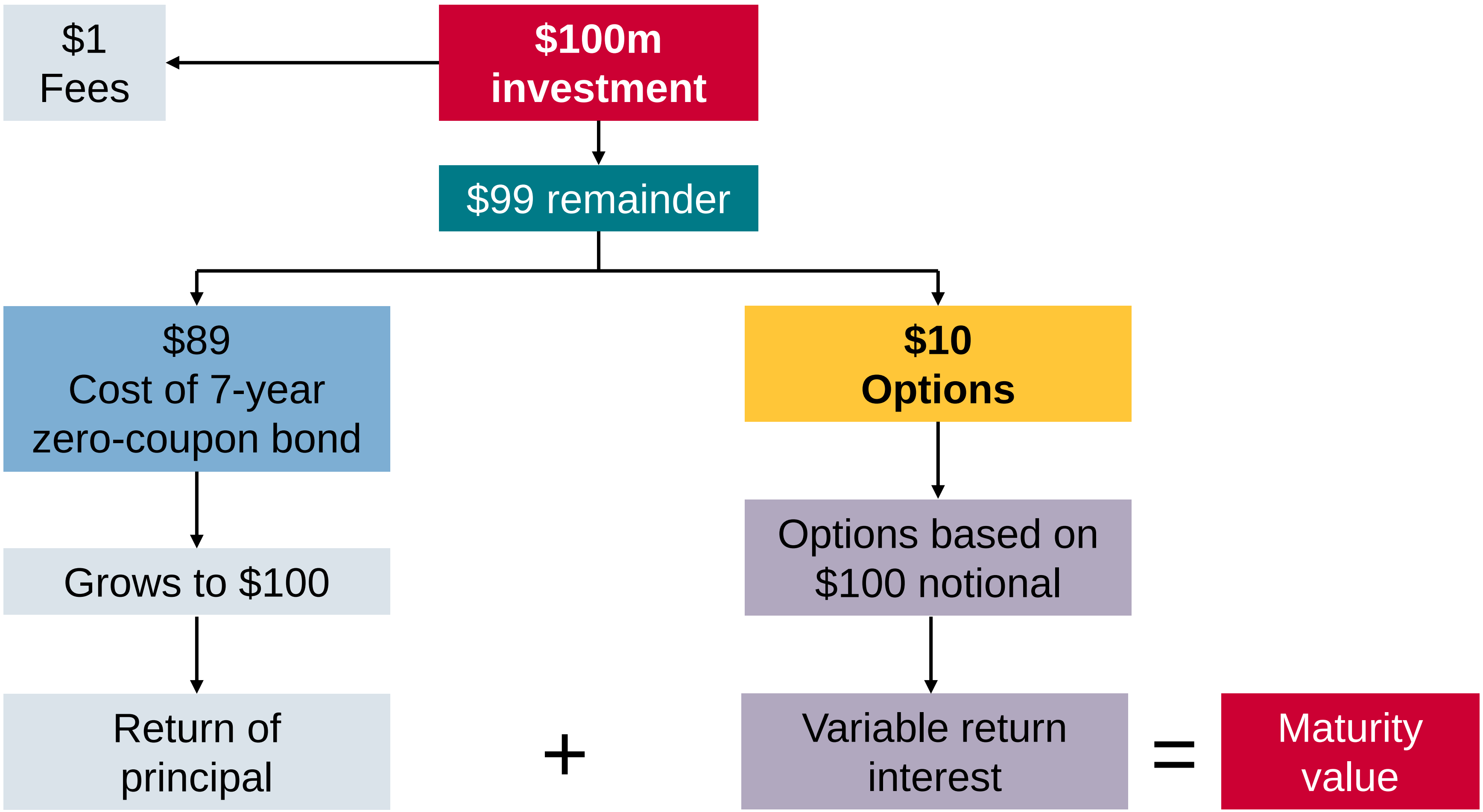
Investors participate in the returns (positive/**long or** negative/**short**) of a market

Leverage

Investors participate in the **magnified** returns (positive/**long or** negative/**short**) of a market

Higher risk/ potential return

Capital Protection Note Example



Structured Note Construction

An investor pays $100 for a 5 year ELN, 100% capital protection and equity participation in a defined index

Part of the upfront payment of $100 is used to purchase a zero coupon bond/deposit

- Assume $90 upfront guarantees capital protection
- If index returns are negative over the five years, the bond matures at $100 enabling the return of the initial investment.
- The remaining $10 is used to purchase 5yr optionality providing the equity participation.

Levels of participation can be increased depending on the price of the zero coupon bond and the optionality, and also if the level of capital protection is lowered below 100%

Example 1

The zero coupon bond is purchased for $90 but the optionality costs $20.

- The note could be issued with 50% participation in the equity index performance
- Maintain 100% capital protection

Example 2

If the optionality costs $20, the note could be created by:

- Offer 100% participation in the equity index performance
- But only 88% capital protection (if retrurn = 2%):

$$\%\ of\ \ protection = \frac{(100-20)\times(1+2\%)^5}{100} = 88.3\%$$

Structured Notes Pricing Considerations

Note Pricing is impacted by a variety of factors

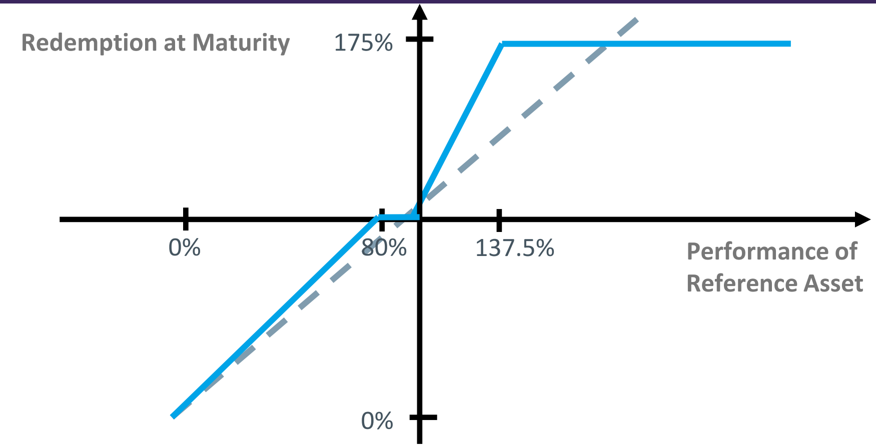
	Effect on Note Pricing
Volatility & Correlation	High Volatility = expensive options: structures that sell options will be more prevalent (such as reverse convertibles). Low volatility = cheaper options: structures that buy options favoured (such as index participation) Low/negative correlation between assets in a basket will lower basket option volatility and vice-versa
Interest rates	Impacts cost of zero coupon product and hence structured products
Dividends	Higher dividend yields on assets reduce the cost of derivatives
Instruments	The use of exotic options, like barriers or basket options can reduce the price of structured products but the use of capped payoffs created by option spreads like call spreads can do the same.
Term	Generally, the longer the term of the product, the more money will be available to purchase derivatives

Structured Notes Example – 2 Year Supertrack

Description

- Offers 200% upside participation in the index subject to a cap of 137.5% of the starting value
- Capital protection is subject to an 80% downside barrier. If the index breaches the 80% downside barrier, then clients are exposed 1.25% for every 1% in the downside movement of the index.

Payoff



Hypothetical Example

2Y Zero Coupon Bond

\+

Long 2x ATM Call

−

Short 2x 137.5% Call

−

Short 1.25x 80% Put

Pricing Considerations

Factor	Pricing Impact
Issuer's Credit	Higher funding spread = cheaper ZCB ► better economic terms
Interest Rates	Higher rates = cheaper ZCB ► better economic terms
Volatility	Short Vega risk. Lower volatility ►better economic terms
Dividend	Higher dividend = lower forward price = cheaper call spread & more valuable puts ►better economic terms
Upside Skew	Flatter skew = OTM call vol > ATM = cheaper call spread ► better economic terms

Structured Notes Example – Reverse Convertible

Description

A reverse convertible places the principal at risk but it offers an above market return.

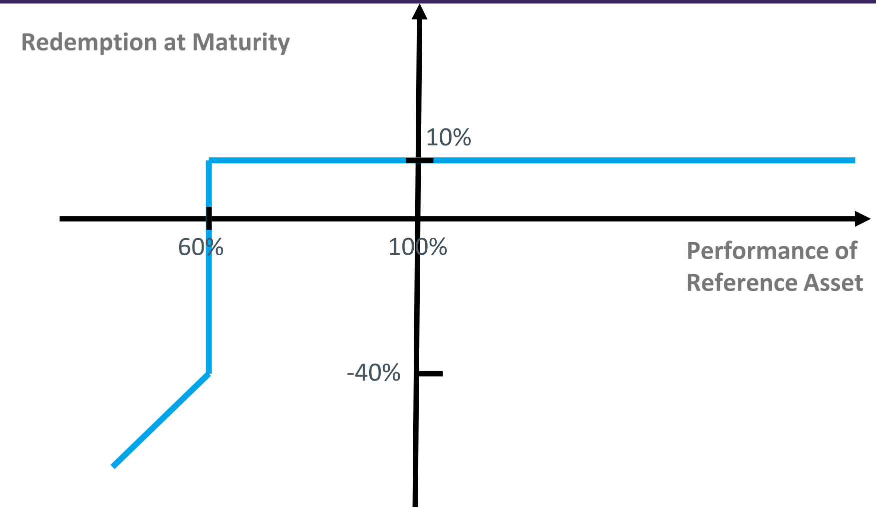
Example

Term	1 Year
Price	$100
Issue price	100% of index
Interest	10.00% ($\text{Index}_{1year} \geq 60\%$)

Capital Payout if $\text{Index}_{1year} \geq 60\%$: 100%

Capital Payout if $\text{Index}_{1year} < 60\%$:
100% x ($\text{Index}_{1year}/\text{Index}_{today}$)

Payoff



Construction

1Y Zero Coupon Bond

Short RKI Put x =100%, b =60%

Pricing Considerations

The potential return of 10% comes from the proceeds of the RKI put option added to the price paid ($100) and invested at *r* for one year

Payout at maturity where index is lower than barrier

If Index_{1year} was 50%, the payout would be: $100 x (50/100) = $50

Structured Notes Example – Range Accrual

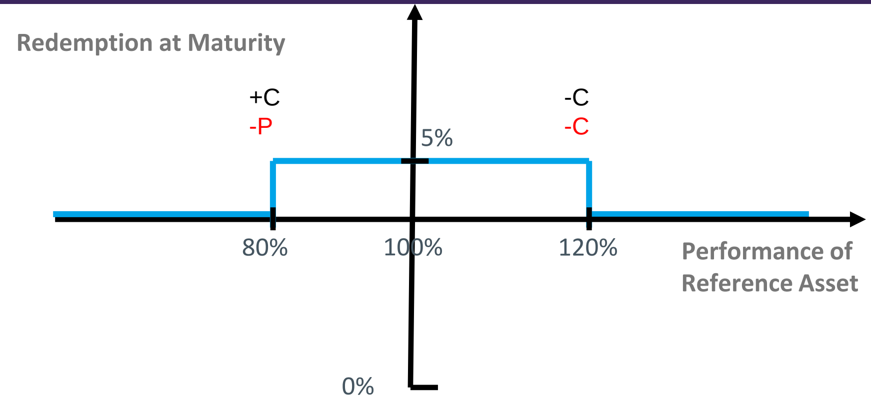
Description

Usually principal protected products that offer above market returns provided the underlying asset remains in a *defined trading range*.

<u>For example</u>

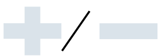
An asset that is within the range of 80%/120% more often than it is not might offer an above market return.

Payoff



Construction

Zero coupon bond



Digital calls + Puts

Pricing Considerations

Can be constructed via a ZCB plus either a digital call spread ($+C_{80\%}$, $-C_{120\%}$) or a combination of $-P_{80\%}$ and $-C_{120\%}$

The payout is made on assessment dates, which can be daily, weekly or monthly for the term of the RAN. If within range accruing interest equals r%/n x nominal, otherwise 0.

Where r% is RAN annual coupon rate and n is number of assessment periods in year

3yr Capped Bonus Certificate Plus

Description

Enables full participation in the underlying index from a “bonus” level up to a certain maximum and with downside protection at the bonus level though contingent on a knock in barrier level not being breached during the life of the note

Construction

3 Year ZCB

+

Long Call at Product Strike (115%)

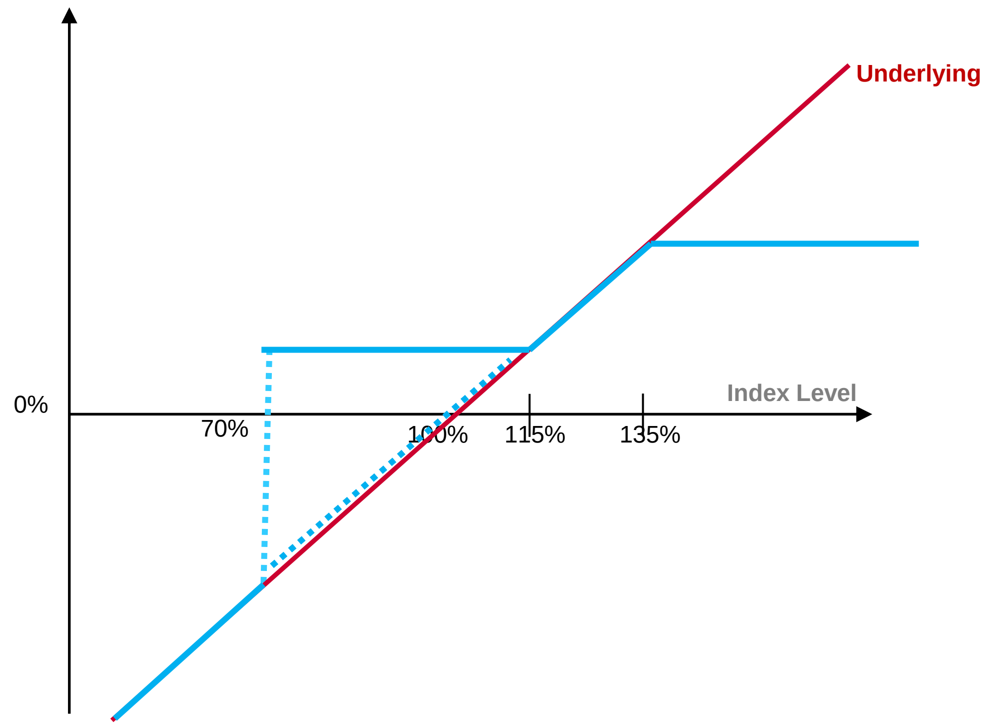
−

Short Call at Product Cap (135%)

−

Short RKI put, Barrier=70%

Payoff

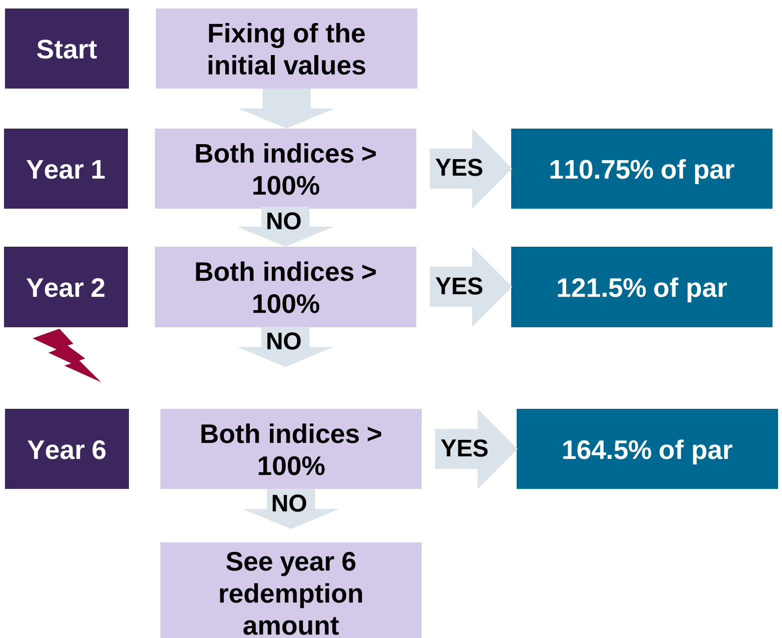


The 3yr zero coupon investment consists of product notional plus net premium ensuring at maturity proceeds pay off the 115% call option strike

Structured Notes Example - 6 Year 10.75% FTSE100/S&P500 Autocall

Description

- Opportunity to receive a return by **selling volatility** and accepting potential downside risk at maturity should the market have fallen by 50% or more on either index.
- Each anniversary has potential for early redemption
- Designed for investors looking for a fixed capital return when the note is redeemed.



Redemption Amount at Year 6

- If both indices are above 100% of it's initial level, the note is redeemable at 164.5% of Par.
- If either index is below 100% of its initial level, the note matures as follows.
  - If neither index has fallen to 50% of its initial value, then investors receive 100% of par.
  - If either index has at maturity fallen to or below 50% of its initial value, then investors receive a 1 for 1 capital loss in line with the worst performing index.

Autocalls

Autocallables offer investors the potential early return (redemption) of their capital plus a return

Investor takes additional risk that coupons may not be paid in exchange for higher potential yield

Structure may be suitable for investors that:

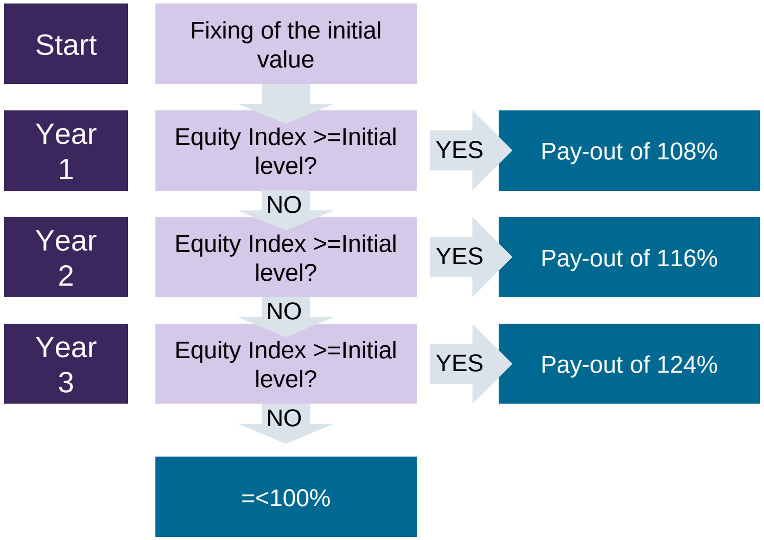
- Seek enhanced yield
- Are willing to take downside risk and the risk of receiving fewer or no coupons
- Are comfortable with potential early redemption

Many investors like liquidity and consider structured products with maturities > 5yrs as undesirable

Example: A three-year product offering an 8% cumulative return linked to the performance of an Index.

If the Index is higher than inception at each determination date, early redemption plus payment of the cumulative coupon. Otherwise, it continues into the next year.

If a knock in (KI) barrier to the downside is breached during the life of the product, this might result in a final redemption amount of less than originally invested.



4

Case Study

FitchLearning

Case study: Structured Product Term Sheets

- There are 5 term sheets
- For each term sheet the describe the risk and return profile for the investor
- For each term sheet explain the products that have been used to create the profile
- Be prepared to share in class and explain your solutions

FitchLearning

BNP PARIBAS

Risk and Regulation

Agenda

1. Case Study
   – Case Study 1:GARP Risk Model
2. Risk and Regulation
3. Case Studies
   – Case Study 2: Risk Analysis and Mitigation
   – Case Study 3: Managing Bank Risks

1

Case Study

GARP Risk Model

FitchLearning

What is Risk?

Identifying Risk

Risk exists when the outcome of taking a particular decision or course of action is uncertain and could potentially impact whether, or how well, we deliver on our objectives.

Citi faces risks throughout its business, every day, in everything it does

- We choose to take some risks after appropriate consideration
  - *Lending money to a customer*
- Other risks arise from unintended consequences of internal actions
  - *An IT system failure or poor sales practices*

Some risks are the result of events outside the Group but have an impact on our business

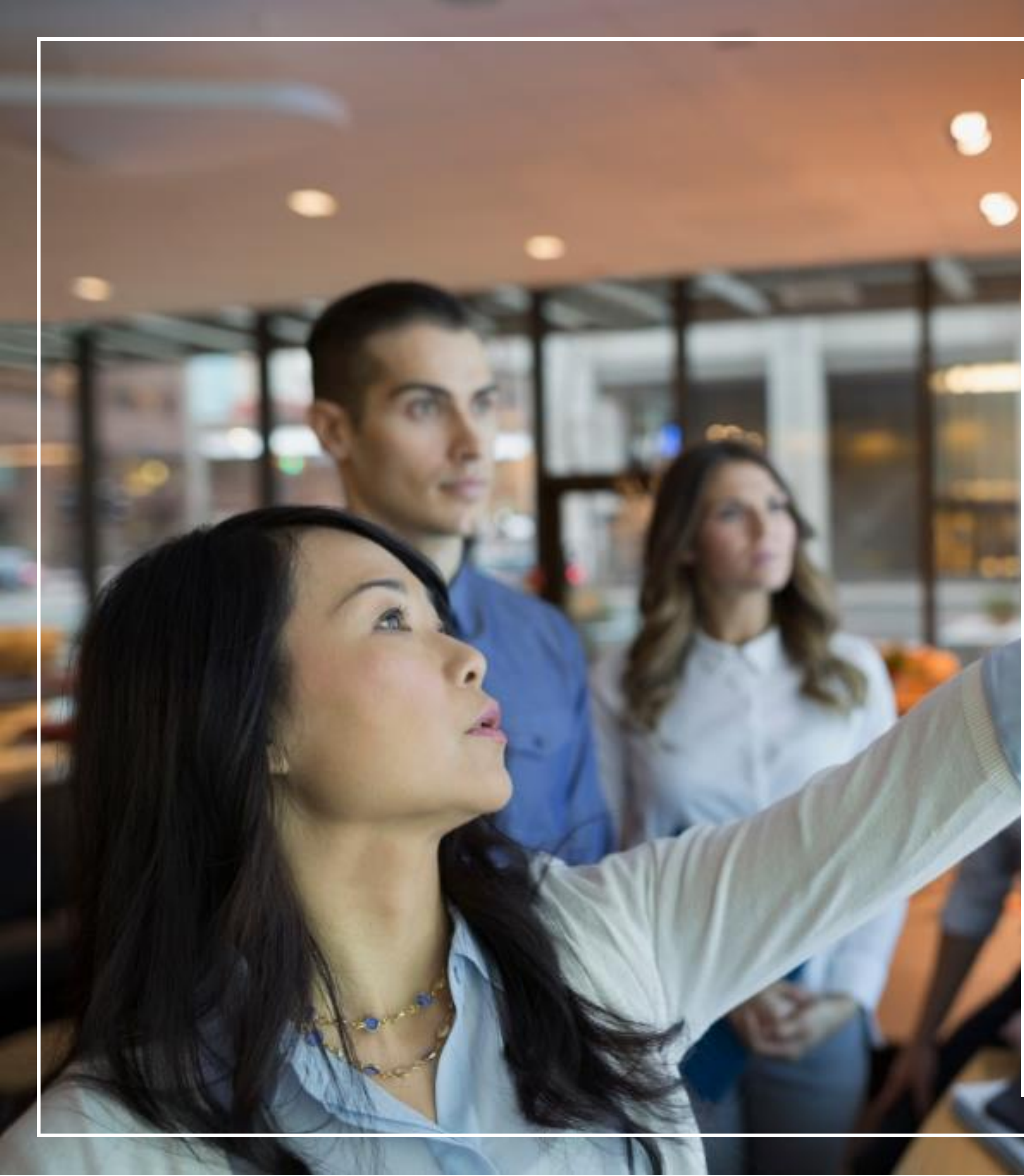
– *Natural disasters, a cyber attack or political or legal actions*

We attempt to mitigate risks by establishing controls around our processes

Case Study 1

Case Study 1: GARP Risk Model

- Discuss the risks listed
- Sort the risks into buckets of related risks
- Name the buckets
- Be prepared to explain your decisions in class



2

Risk and Regulation

FitchLearning

Principal Risk Definitions

Credit risk - The risk of loss to the firm from the failure of clients, customers or counterparties, including sovereigns, to fully honour their obligations to the firm, including the whole and timely payment of principal, interest, collateral and other receivables.

Market risk - The risk of loss arising from potential adverse changes in the value of the firm's assets and liabilities from fluctuation in market variables including, but not limited to, interest rates, foreign exchange, equity prices, commodity prices, credit spreads, implied volatilities and asset correlations.

Treasury and capital risk – This comprises:

- **Liquidity risk**: The risk that the firm is unable to meet its contractual or contingent obligations or that it does not have the appropriate amount, tenor and composition of funding and liquidity to support its assets.
- **Capital risk**: The risk that the firm has an insufficient level or composition of capital to support its normal business activities and to meet its regulatory capital requirements under normal operating environments or stressed conditions (both actual and as defined for internal planning or regulatory testing purposes). This includes the risk from the firm's pension plans.
- **Interest rate risk in the banking book**: The risk that the firm is exposed to capital or income volatility because of a mismatch between the interest rate exposures of its (non-traded) assets and liabilities.

Principal Risk Definitions -2

Operational risk – Risk of loss from inadequate process, systems, human factors or external events where the root cause is not credit or market risks

Model risk – Risk of adverse consequences from financial assessments or decisions based on incorrect or misused model outputs/reports

Reputation risk - Risk that an action/transaction/investment /event reduces stakeholder trust in the firm’s integrity and competence

Conduct risk – risk of detriment to customers/clients/market integrity/competition/Barclays from inappropriate supply of financial services, includes willful or negligent misconduct

Legal risk – Risk of loss/penalties/damages/fines from the failure to meet legal obligations including regulatory or contractual requirements

Types of Retail Credit Risk

Retail credit risk occurs across the banking book and is managed in alignment with the following product offerings:

Home loans
- Arise from mortgage lending and are repaid on either a fixed or floating basis where the customer is paying interest and/or principal regularly

Credit cards
- Arise from transactions where borrowers are utilising unsecured credit limits via their credit card.

Business lending
- Arises from lending to Small and Medium Enterprise but within the definition of retail lending.

Overdrafts
- Arise from revolving credit facilities that do not require contractual monthly payments. Overdrafts are generally expected to be used for short term financial requirements. May be secured or unsecured

Other secured lending
- Arises from lending where non-mortgage security is held with an agreed schedule of repayment on a fixed or floating interest rate basis

Other unsecured lending
- Arises from lending where no security is held with an agreed schedule of repayment on a fixed or floating interest rate basis

Types of Wholesale Credit Risk

Wholesale credit risk occurs across the banking and trading book. Five types of wholesale credit risk are defined:

Primary risk

- Arises from a default or in anticipation of a default by an obligor on direct lending (e.g. loans or bonds held in the banking book) or contingent obligations (e.g. letters of credit)

Counterparty risk

- Arises from the failure of an obligor to meet payment obligations under a derivative, securities or margin lending agreement. The Bank's loss is the cost of replacing or closing out the contract and is recognized as a trading loss. Mitigated by central clearing

Issuer risk

- Arises from the default or in anticipation of a default of a financial instrument held in the trading book or the Available for Sale account

Country risk

- Arises where an obligor is unable to meet obligations denominated in foreign currency because of actions taken by the relevant authority in the obligor's country of operations, rather than through their own financial distress. Typically these actions restrict or prevent remittance of foreign currency reserves outside of the country

Settlement risk

- The risk that an obligor fails to deliver cash or securities after cash/securities have been irrevocably released by the Bank in settlement of a transaction or remitted through a payment system

Credit Risk: Measurement

Three key parameters are assessed to determine the level of credit risk an obligor represents. Changes in any of these parameters can have a detrimental impact on the Bank's P&L and capital position in advance of an obligor's default:

- **Probability of Default (PD)** – the likelihood that the individual obligor will default within a defined outcome period[*]
- **Exposure at Default (EAD)** – generally assigned at the account level (retail) or facility level/or across total exposure where netting applies (wholesale), is an estimate of the credit risk should an obligor default occur during the defined outcome period[*] and takes into account the potential for changes in exposure prior to default, e.g. increase or decrease in lines.
- **Loss Given Default (LGD)** –assigned at an account level (retail), facility or product level (wholesale) it represents an estimate of the percentage of EAD that will be lost should an obligor default occur during the defined outcome period[*]

** For wholesale credit risk = rolling 12 month period*

Probability of Default

Credit Risk is a Function of Default Probability

This probability can be estimated in several ways.

Real world probability of defaults

- Default probabilities can be contributed by rating agencies or calculated internally based on historical data

Risk-neutral default probabilities

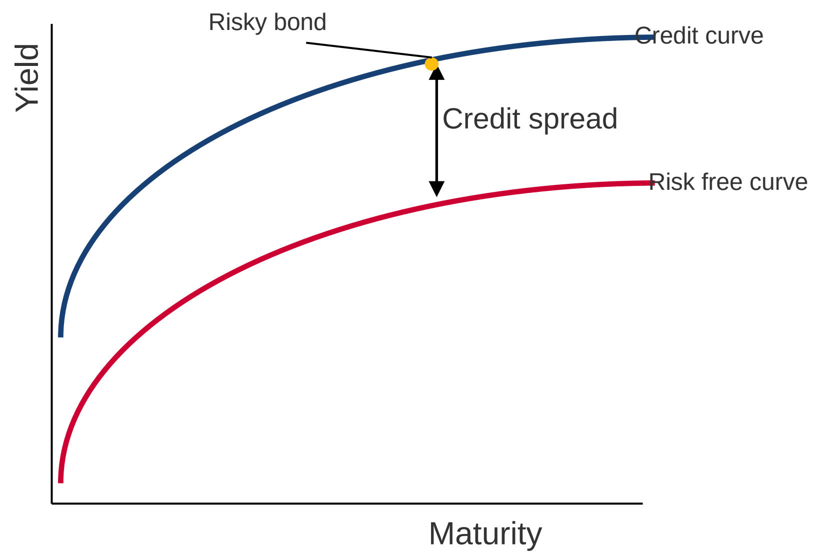
- Obtained from credit spreads on notes or bonds issued by the debtor

$$PD \approx \frac{s(T)}{1-RR}$$

S(T) is the credit spread for maturity T

RR is the recovery rate

Yield Curve Representation



Loss Given Default

Central to creditor default is the actual loss suffered by the lender

In most default events, lenders can recover at least some of the debt owed to them.

This is called the **recovery rate**

Recovery rate = Value recovered/Value of debt

The recovery rate depends on several factors

- Seniority of the debt
- Economic conditions
- Industry
- Value of any collateral

Recovery rates are available from rating agencies or internal estimates

- Recovery rates vary significantly but historically average around 70% for secured bank loans and 40% for unsecured debt

The recovery rate is used to estimate the **loss given default (LGD)**

Loss Given Default = 1- Recovery rate

Credit Risk Exposure (Exposure at Default)

For loans and principal lending, the credit risk exposure is equivalent to the outstanding loan principal value plus of any accrued interest.

For securities and derivatives, the credit risk exposure is more complex to quantify:

- Can be measured as **current exposure (CE)** - a measure of the replacement cost of the instrument based on today's prices
- Or as a **potential future exposure (PFE)** - a measure of the replacement cost of the instrument based on forecast future prices

Credit risk can be estimated as the product of the credit exposure (EAD), the probability of default (PD) and the loss given default (LGD)

Credit risk = EAD x PD x LGD

If a bank is owed $10m by a borrower and the probability of default is estimated to be 2%, the average recovery rate for loans to this type of borrower is 40%. What is the credit risk?

Concentration Risk

In Addition to Assessing Credit Risk Exposure, We Must Assess Concentration Risks

Concentration risk arises if:

- An individual exposure/group of similar exposures (e.g. mortgages with similar LTV), is sufficiently large
- Or there is a high degree of correlation between one or more of the credit risk metrics across the following parameters:

Single name: the default of a single obligor or group of aggregated obligors

Collateral: deterioration in value or liquidity of similar collateral held against one or many obligors

Industry: deterioration in the economic or external factors affecting obligors in a specific industry

Geography: deterioration in the economic environment or other external factors affecting obligors in a specific geographic location

Product: change in the risk profile of a specific product or set of products resulting from either internal factors (e.g. modelled risk) or external events (eg legal change impacting enforceability)

Foreign currency risk: lending where the amount borrowed, or the basis in which the amount borrowed is calculated, is in a currency different to the income or collateral of the customer eg foreign currency mortgages versus local currency collateral for local borrowers.

Types of Market Risk

Individual risk factors can be bucketed in to risk sub-types to understand the mix and weighting of similar risk factors. For example:

- **Interest rates**: Changes can impact values of rates sensitive assets such as bonds and derivatives (IRS)
- **Traded credit**: Uncertainty of credit quality impacts asset prices such as corporate bonds securitized products and credit derivatives (CDS)
- **Spread**: Difference between bond yields and swap rates where we have positions in both . The assets may trade at different levels but exposed to similar risk
- **Foreign exchange**: Impacts of changes in FX rates and volatilities
- **Equity**: Risk due to changes in equity prices, volatilities and dividend yields as part of market making activity, syndication or underwriting IPOs
- **Commodity**: Arises from providing hedging solutions and access to a range of derivative and physical products and movements in spot/forward curves
- **Inflation**: Impact of changes in inflation rates and volatilities on cash and derivative instruments as part of market making activity or syndications
- **Basis**: Impact of changes in interest rate tenor basis and cross currency basis from market making activity but occurs from discounting collateralized derivatives

Market Risk Measurement

Traded Market Risk framework has four key measurement approaches that must be calculated for any Traded Market Risk exposure:

- **Value at Risk (VaR)** – an estimate of the potential loss arising from unfavourable market movements if the current positions were held unchanged for one business day
- **Primary stress tests** – an estimate of the potential losses that might arise from extreme market moves or scenarios to key liquid risk factors
- **Secondary stress tests** – modelled losses to unfavourable market movements to illiquid market risk exposures
- **Aggregate scenario stresses** – multi-asset scenario analysis of extreme, but realistic events that may simultaneously impact market risk exposures across all primary and secondary stresses

Which of These Has More Market Risk?

A	B	
EUR1m long in a 1-year bond	EUR1m long in a 30-year bond	
EUR1m 3-month position in EUR/CHF	EUR1m 3-month position in EUR/TRL	
EUR1m sold DAX 5000 call option	EUR1m bought DAX 5000 call option	
EUR1m long position in Deutsche Telecom + EUR1m short position in Telefonica	EUR1m long position in Deutsche Telecom	
EUR3m long in a 1-year bond	EUR1m long in a 3-year bond	
EUR1m long position in 3-month copper	EUR1m long position in Vodafone stock	

Value at Risk (VaR)

VaR is an estimate of the potential loss arising from unfavourable market movements if the current positions were to be held unchanged for one business day.

- For internal market risk management purposes, a historical simulation methodology with a two-year equally weighted historical period, at the 95% confidence level for 1 day could be used for the trading books
- For regulatory purposes, the VaR measure is based on the same historical simulation methodology but set at a 99% confidence level for 10 days

Treasury and Funding Risk

The Risk that the Group is Unable to Maintain Appropriate Capital and Liquidity Ratios

Funding risk results in the Group being unable to achieve its business plans due to:

Capital risk: the risk that the Group is unable to maintain appropriate capital ratios which could lead to: an inability to support business activity; a failure to meet regulatory requirements; and/or changes to credit ratings, which could also result in increased costs or reduced capacity to raise funding;

- CET1 Ratio, Leverage Ratio

Structural risk: impact on the Group's balance sheet of changes in primarily interest rates on income or foreign exchange rates on capital ratios

Liquidity risk: the risk that the Group is unable to meet its obligations as they fall due resulting in: an inability to support normal business activity, a failure to meet liquidity regulatory requirements; and/or changes to credit ratings.

- Liquidity Coverage Ratio (LCR) (the Bear Stearns rule)
- Net Stable Funding Ratio (NSFR) (the Northern Rock rule)

Bank Regulation - Capital

Main objectives of regulation are:

- Investor and consumer protection
- Market efficiency and competition
- Prevention of financial crime
- **Financial stability**

National Regulators

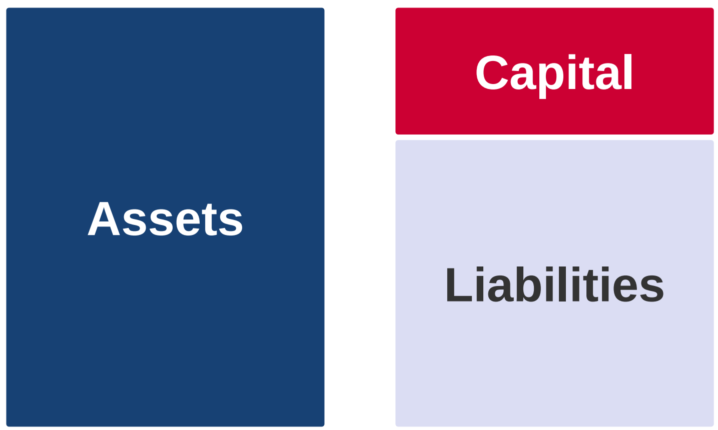
Federal Reserve Board, Bank of England, ECB

International Regulators

BIS (Basel Committee), IMF, World Bank

Basel III

- Minimum Capital requirements for banks to ensure they have enough reserves to protect against losses.
- Concerns how much capital that must be held against a bank's assets



- CET1 Ratio, Leverage Ratio, Liquidity Ratio

Capital Ratio Requirements

- Common Equity Tier1 (CET1) is a component of Tier 1 Capital, and it encompasses ordinary shares and retained earnings
- **The Tier 1 Capital Ratio** is a measure of a bank's ability to withstand financial distress. This must be greater than or equal to 4.5%

$$\textbf{Common Equity Tier 1 Capital Ratio} = \frac{\textbf{CET 1 Capital}}{\textbf{Risk Weighted Assets}} \times 100$$

- Total Capital Ratio (Capital Adequacy Ratio) which must be at least 12.9%
- Tier 1 is a bank's core capital, equity, and disclosed reserves
- Tier 2 is a bank's supplementary capital

$$\textbf{Total Capital Ratio} = \frac{\textbf{Tier 1 Capital} + \textbf{Tier 2 Capital}}{\textbf{Risk Weighted Assets}} \times 100$$

Tier 1 Leverage

- The Tier 1 leverage ratio compares a bank's Tier 1 capital to its total assets to evaluate how leveraged a bank is.
- This ratio must be at least 3%

$$\textbf{Tier 1 Leverage Ratio} = \frac{\textbf{Tier 1 Capital}}{\textbf{Consolidated Assets}} \times 100$$

Liquidity

- Liquidity Coverage Ratio

- The liquid assets that a bank has to hold on hand to cover a period of stress in the market

- Liquid assets include those that can be converted easily and quickly into cash

$$\textbf{Liquidity Coverage Ratio} = \frac{\textbf{Liquid Assets}}{\textbf{Net Cash flow amount}}$$

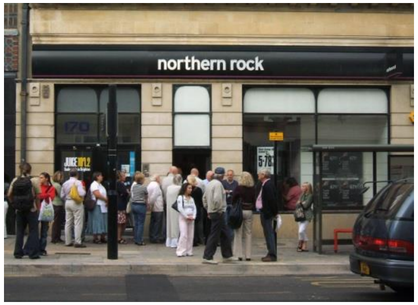
- This ratio must be at least 100%

Liquidity Crisis: The Demise of Bear Stearns

Date		Share price
Dec 2006	Bear Stearns posts record earnings, touting huge profit gains from then-booming businesses advising on mergers and arranging credit derivative, distressed debt and leveraged finance deals	$159
May 2007	Goldman Sachs slashed its quarterly earnings target for the rival investment bank, citing concern about Bear's heavy exposure to the mortgage securitization business	$147
July 2007	As losses from subprime mortgages begin to threaten credit markets around the world, Bear Stearns informs investors in its two struggling hedge funds that the funds have "very little value" remaining.	$139
October 2007	Prosecutors launch a criminal probe into the collapse of the two Bear Stearns hedge funds.	$131
Dec 2007	Bear reports its first-ever quarterly loss, driven by $1.9 billion of bad debt write-downs.	$91
Jan 2008	CEO replaced	$71
March 12 2008	Responding to market rumours of a cash crunch at the bank, new Bear CEO Alan Schwartz goes on CNBC television and assures viewers that the firm has ample liquidity	$61
March 14 2008	JPMorgan, backed by the Federal Reserve, provides an undisclosed amount of emergency financing to Bear Stearns. Bear says its liquidity position had deteriorated dramatically in the previous 24 hours.	$30
March 16 2008	JPMorgan agrees to buy Bear for $2 a share	$4 (optimism of another buyer coming forward)

Funding Crisis: Northern Rock

Northern Rock, a British bank, was founded in 1997. In 2007, it was one of the top five mortgage lenders in the UK. It had **76 branches** (**Barclays had over 1700**) and offered deposit accounts, savings accounts, loans and insurance.



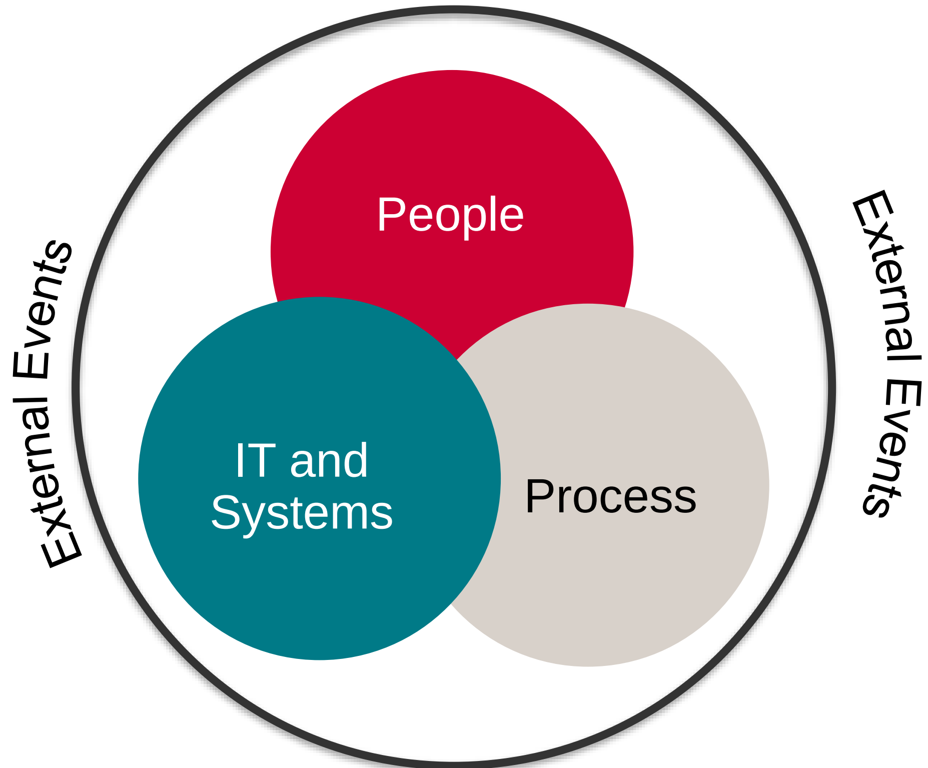
Northern Rock relied on **selling short-term debt instruments for much of its funding**. Following the subprime crisis of August 2007, the bank found it **very difficult to replace maturing instruments**. This is because institutional investors became very nervous about lending to Banks that were heavily involved in the mortgage market. **The Banks assets were sufficient to cover its liabilities so it was not insolvent.**

Date	
Sept 2007	The FSA stated in September 2007 “The FSA judges that Northern Rock is solvent ,exceeds its regulatory capital requirement and has a good quality loan book”.
September 12th 2007	Northern Rock borrowed £3 billion from the Bank of England. The story was broken by the BBC on September 13th and led to the first run on a British bank in 150 years as investors queued to withdraw their funds.
September 17th 2007	The British Chancellor of the Exchequer , Alistair Darling, announced that the British government would guarantee all deposits held at Northern Rock
February 2008	Northern Rock’s emergency borrowing had reached £25 billion and the bank was nationalised with the management of the bank being changed

Definition of Operational Risk

Basel/Bank of International Settlements (BIS) definition:

The risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events.



The Causes of Operational Risk

Process	Technology	People	Environment
• Lack of effective procedures • Lack of capacity • Volume sensitivity • Lack of effective controls • Failure to review controls when process changes	• Lack of systems availability due to operational failure • Lack of systems integrity • Inadequate control functionality • Poor systems development and Beta testing • Lack of strategic approach to systems design • Lack of understanding of (complex) systems	• Human error • Unauthorised activities • Lack of accountability for operational risk management • Lack of integrity/ honesty • Lack of customer care • Skills/training deficit • Poor communication • Concentration of expertise • Lack of appropriate supervision	**External** • War, terrorism and natural disasters • Regulatory and tax changes • Lack of experienced staff, high costs and use of contractors • The management of change **Internal** • Increased reliance on technology • Increased volumes as a result of increased marketing activity and increased internet trading • Continuing industry rationalisation –need to integrate processes, systems and corporate cultures

Top 10 Operational Risks for 2021

Risk.net top 10 operational risks of 2021, as chosen by risk practitioners

1. IT Disruption
2. Data compromise
3. Resilience risk
4. Theft and fraud
5. Third-party risk
6. Conduct risk
7. Regulatory risk
8. Organizational change
9. Geopolitical risk
10. Employee wellbeing

Source: Risk.net

Top 10 Operational Risks for 2022

Risk.net top 10 operational risks of 2022, as chosen by risk practitioners

1. IT disruption
2. Theft and fraud
3. Talent risk
4. Geopolitical risk
5. Information security
6. Resilience risk
7. Third-party risk
8. Conduct risk
9. Climate risk
10. Regulatory risk

Source: Risk.net

3

Case Studies

FitchLearning

Case Study 2: Risk Analysis and Mitigation

- Answer questions and be prepared to share in class

Case Study 3: Managing Bank Risks

- Source your Bank Annual Report
- Which risks are included?
- How does the Bank manage risk?
- Answer questions and be prepared to share in class