

BN learning

Advances in Learning with Bayesian **Networks**

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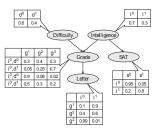
DUKe (Data User Knowledge) research group, LINA UMR 6241, Nantes, France

Nantes Machine Learning Meetup, July 6, 2015, Nantes, France



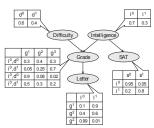


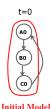
- Bayesian networks (BNs) are a powerful tool for graphical representation of the underlying knowledge in the data and reasoning with incomplete or imprecise observations.

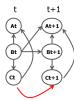




- Bayesian networks (BNs) are a powerful tool for graphical representation of the underlying knowledge in the data and reasoning with incomplete or imprecise observations.
- BNs have been extended (or generalized) in several ways, as for instance, causal BNs, dynamic BNs, relational BNs, ...





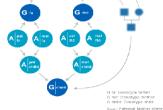


Transition Model



Victim identification system

BN learning



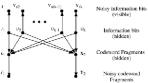
After-sale services

O in succes

Steve

G mn: Gosofype mother A par Meternel Methor Alicle

Turbo-codes (GSM, ...)



Noisy codeword (visible)

Anti Spam

Mail	Junk E-mail		
Pavorite Poblera	Amenged Dirt Date		
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Street Street (200)	Home Security		
Total Supposed	Gradientini		

MS Office assistant



Assistant iPhone SIRI







We would like to learn a BN from data... but which kind of data?

complete

- ABCD 2 3 5 6



BN learning

We would like to learn a BN from data... but which kind of data?

- complete /incomplete [François et al. 06]



We would like to learn a BN from data... but which kind of data?

- high n,

Α	В	C	D	 	 x
					100000
0	1	2	3	 	 7
4	6	1	0	 	 5
2	3	5	6	 	 4
1	3	2	6	 	 7
3	8	9	0	 	 1
1	2	4	5	 	 3
1	4	3	7	 	 2
8	5	4	3	 	 4



We would like to learn a BN from data... but which kind of data?

- high n, n >> p [Ammar & Leray, 11]

A	В	C	D	 	 X ₁₀₀₀₀₀
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We would like to learn a BN from data... but which kind of data?

- stream [Yasin and Leray, 13]

```
X<sub>100000</sub>
```

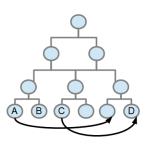


We would like to learn a BN from data... but which kind of data?

BN learning

• + prior knowledge / ontology [Ben Messaoud et al., 13]

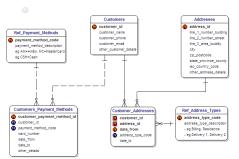






We would like to learn a BN from data... but which kind of data?

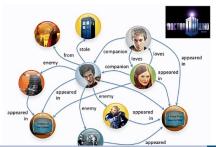
- structured data [Ben Ishak, Coutant, Chulyadyo et al.]





We would like to learn a BN from data... but which kind of data?

- not so structured data [Elabri et al.]





Even the learning task can differ: generative

• modeling P(X, Y)

- no target variable
- more general model
- better behavior with incomplete data



Even the learning task can differ: generative vs. discriminative

• modeling P(X, Y)

- no target variable
- more general model
- hetter behavior with incomplete data

- modeling P(Y|X)
- one target variable Y
- dedicated model



Even the learning task can differ: generative vs. discriminative

• modeling P(X, Y)

BN learning

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Objectives of this talk

- how to learn BNs in such various contexts?
- state of the art : founding algorithms and recent ones
- pointing out our contributions in this field



Outline ...



- **BN** learning
 - Definition
 - Parameter learning
 - Structure learning
 - - Definition
 - Learning
- - Definition
 - Learning
 - Graph DB?
- - Last words
 - References



Bayesian network

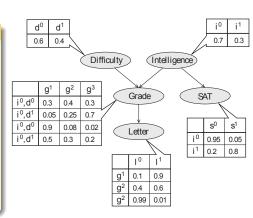
[Pearl, 1985]

Definition

G qualitative description of conditional dependences / independences between variables directed acyclic graph (DAG)

•00000000000

 quantitative description of these dependences conditional probability distributions (CPDs)





Bayesian network

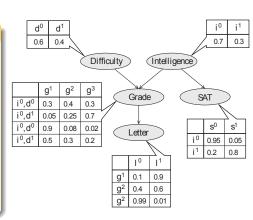
[Pearl, 1985]

Definition

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Main property

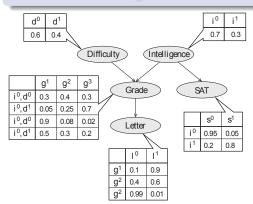
 the global model is decomposed into a set of local conditional models



One model... but two learning tasks

BN = graph G and set of CPDs Θ

BN learning



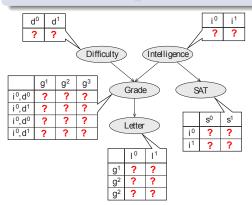


One model... but two learning tasks

BN = graph G and set of CPDs Θ

- parameter learning / G given

BN learning





One model... but two learning tasks

BN = graph G and set of CPDs Θ

- parameter learning / G given
- structure learning





Complete data \mathcal{D}

BN learning

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- max. of likelihood (ML) : $\hat{\theta}^{MV}$ = argmax $P(\mathcal{D}|\theta)$
- closed-form solution :

$$\hat{P}(X_i = x_k | Pa(X_i) = x_j) = \hat{\theta}_{i,j,k}^{MV} = \frac{N_{i,j,k}}{\sum_k N_{i,j,k}}$$

 $N_{i,i,k}$ = nb of occurrences of $\{X_i = x_k \text{ and } Pa(X_i) = x_i\}$



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Other approaches

$P(\theta) \sim \mathsf{Dirichlet}(\alpha)$

- max. a posteriori (MAP) : $\hat{\theta}^{MAP}$ = argmax $P(\theta|\mathcal{D})$
- expectation a posteriori (EAP) : $\hat{\theta}^{EAP} = E(P(\theta|D))$ $\hat{\theta}_{i,j,k}^{MAP} = \frac{N_{i,j,k} + \alpha_{i,j,k} - 1}{\sum_{k} (N_{i,j,k} + \alpha_{i,j,k} - 1)}$ $\hat{\theta}_{i,i,k}^{EAP} = \frac{N_{i,j,k} + \alpha_{i,j,k}}{\sum_{i,j,k} (N_{i,j,k} + \alpha_{i,j,k})}$



no closed-form solution

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 EM (iterative) algorithm [Dempster, 77], convergence to a local optimum

$$\theta_{i,j,k} = \frac{N^{old} \theta_{i,j,k}^{old} + N_{i,j,k}}{N^{old} + N}$$



Incomplete data

- no closed-form solution
- EM (iterative) algorithm [Dempster, 77], convergence to a local optimum

Incremental data

advantages of sufficient statistics

$$\theta_{i,j,k} = \frac{N^{old}\theta_{i,j,k}^{old} + N_{i,j,k}}{N^{old} + N}$$

• this Bayesian updating can include a forgetting factor



Parameter learning (discriminative)

Complete data

no closed-form

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iterative algorithms such as gradient descent

Parameter learning (discriminative)

Complete data

no closed-form

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iterative algorithms such as gradient descent

Incomplete data

- no closed-form
- iterative algorithms + EM :-(

Relational BN learning



BN structure learning is a complex task

Size of the "solution" space

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- the number of possible DAGs with n variables is super-exponential w.r.t n [Robinson, 77] NS(5) = 29281 $NS(10) = 4.2 \times 10^{18}$
- an exhaustive search is impossible for realistic n!

One thousand millenniums = 3.2×10^{13} seconds



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• an exhaustive search is impossible for realistic n!

One thousand millenniums = 3.2×10^{13} seconds

Identifiability

- data can only help finding (conditional) dependences / independences
- Markov Equivalence : several graphs describe the same dependence statements
- causal Sufficiency: do we know all the explaining variables?



Structure learning (generative / complete)

Constraint-based methods

- BN = independence model
 - ⇒ find CI in data in order to build the DAG
 - ex: IC [Pearl & Verma, 91], PC [Spirtes et al., 93]
- problem : reliability of CI statistical tests (ok for n < 100)



Structure learning (generative / complete)

Constraint-based methods

000000000000

- BN = independence model
- problem : reliability of CI statistical tests (ok for n < 100)

Score-based methods

- BN = probabilistic model that must fit data as well as possible ⇒ search the DAG space in order to maximize a scoring function
 - ex: Maximum Weighted Spanning Tree [Chow & Liu, 68], Greedy Search [Chickering, 95], evolutionary approaches [Larranaga et al., 96] [Wang & Yang, 10]
- problem : size of search space (ok for n < 1000)



Structure learning (generative / complete)

Constraint-based methods

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- BN = independence model
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Score-based methods

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- problem : size of search space (ok for n < 1000)

Hybrid/ local search methods

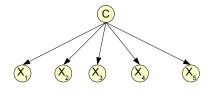
- local search / neighbor identification (statistical tests)
- global (score) optimization
- usually for scalability reasons (ok for high n)
- ex : MMHC algorithm [Tsamardinos et al., 06]

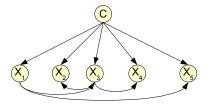


Structure learning (discriminative)

Specific structures

- naive Bayes, augmented naive Bayes
- multi-nets
- ...





- usually, the structure is learned in a generative way
- the parameters are then tuned in a discriminative way

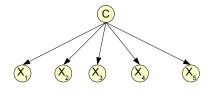


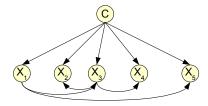
Structure learning (discriminative)

Specific structures

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Structure learning

- usually, the structure is learned in a generative way
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Structure learning

Incomplete data

- hybridization of previous structure learning methods and EM
- ex: Structural EM
 [Friedman, 97]
 ≃ Greedy Search + EM
- problem : convergence







Structure learning

 robustness and complexity issues

- application of Perturb & Combine principle
- ex : mixture of randomly perturbed trees [Ammar & Leray, 11]

A	В	С	D	 	 X ₁₀₀₀₀₀
0	1	2	3	 	 7
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Structure learning

Dynamic BN learning

Incremental learning and data streams

- Bayesian updating is easy for parameters
- Bayesian updating is complex for structure learning
- and other constraints related to data streams (limited storage, ...)
- ex : incremental MMHC [Yasin and Leray, 13]

A	В	С	D	 	 X ₁₀₀₀₀₀
0	1	2	3	 	 7
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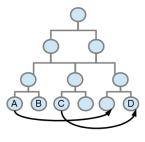


Structure learning

Integration of prior knowledge

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- in order to reduce search space: white list, black list, node ordering [Campos & Castellano, 07
- interaction with ontologies [Ben Messaoud et al., 13]





Outline ...



- Definition
- Parameter learning
- Structure learning
- **Dynamic BN learning**
 - Definition
 - Learning
- - Definition
 - Learning
 - Graph DB?
- - Last words
 - References



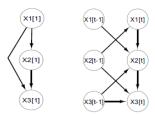
Dynamic Bayesian networks (DBNs)

k slices temporal BN (k-TBN) [Murphy, 02]

 \bullet k-1 Markov order

BN learning

- prior graph G_0 + transition graph G_{\rightarrow}
- for example : 2-TBNs model [Dean & Kanazawa, 89]



(a) Prior network (b) Transition network



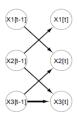
Dynamic Bayesian networks (DBNs)

k slices temporal BN (k-TBN) [Murphy, 02]

- k-1 Markov order
- ullet prior graph G_0 + transition graph $G_{
 ightarrow}$
- for example: 2-TBNs model
 [Dean & Kanazawa, 89]

Simplified k-TBN

 k-TBN with only temporal edges [Dojer, 06][Vinh et al, 12]



(c) Transition network with only inter time-slice arcs



DBN structure learning (generative)

Score-based methods

- dynamic Greedy Search [Friedman et al., 98], genetic algorithm [Gao et al., 07], dynamic Simulated Annealing [Hartemink, 05], ...
- for k-TBN (G_0 and G_{\rightarrow} learning)
- but not scalable (high n)



DBN structure learning (generative)

Score-based methods

BN learning

- dynamic Greedy Search [Friedman et al., 98], genetic algorithm [Gao et al., 07], dynamic Simulated Annealing [Hartemink, 05], ...
- for k-TBN (G₀ and G_→ learning)
- but not scalable (high n)

Hybrid methods

- [Dojer, 06] [Vinh et al., 12] for simplified k-TBN, but often limited to k = 2 for scalability
- dynamic MMHC for "unsimplified" 2-TBNs with high n [Trabelsi et al., 13]

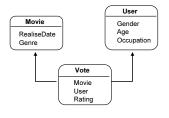


Outline ...



- Definition
- Parameter learning
- Structure learning
- - Definition
- **Relational BN learning**
 - Definition
 - Learning
 - Graph DB ?
 - - Last words
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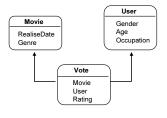


BN learning

A relational schema $\mathcal R$

- classes + relational variables



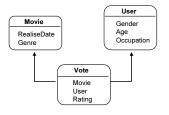


BN learning

A relational schema $\mathcal R$

- classes + relational variables
- reference slots (e.g., Vote. Movie, Vote. User)



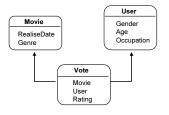


BN learning

A relational schema \mathcal{R}

- classes + relational variables
- reference slots (e.g., Vote. Movie, Vote. User)
- slot chain = a sequence of reference slots
 - allow to walk in the relational schema to create new variables





BN learning

A relational schema \mathcal{R}

- classes + relational variables
- reference slots (e.g., Vote. Movie, Vote. User)
- slot chain = a sequence of reference slots
 - allow to walk in the relational schema to create new variables
 - ex: Vote. User. User⁻¹. Movie: all the movies voted by a particular user



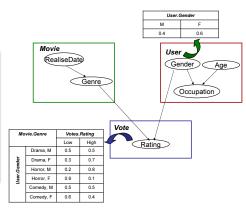
Probabilistic Relational Models

[Koller & Pfeffer, 98]

Definition

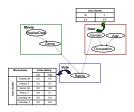
A PRM Π associated to \mathcal{R} :

- a qualitative dependency structure S (with possible long slot chains and aggregation functions)
- a set of parameters $\theta_{\mathcal{S}}$



Probabilistic Relational Models

Definition



Aggregators

- Vote. User. $User^{-1}$. Movie. $genre \rightarrow Vote. rating$
- movie rating from one user can be dependent with the genre of all the movies voted by this user
 - how to describe the dependency with an unknown number of parents?
 - ullet solution : using an aggregated value, e.g. $\gamma = \textit{MODE}$



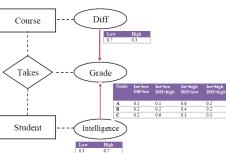
DAPER

Another probabilistic relational model

[Heckerman & Meek, 04]

Definition

Probabilistic model associated to an Entity-Relationship model





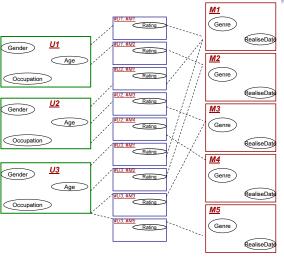
Learning from a relational datatase

GBN

PRM/DAPER learning = finding the probabilistic dependencies and the probability tables from an instantiated database

BN learning

the relational schema/ER model is given





Relational variables

- finding new variables by exploring the relational schema
- ex: student.reg.grade, registration.course.reg.grade, registration.student reg.course.reg.grade, ...
- ⇒ adding another dimension in the search space
- ⇒ limitation to a given maximal slot chain length



Relational variables

BN learning

- ⇒ adding another dimension in the search space
- ⇒ limitation to a given maximal slot chain length

Constraint-based methods

- relational PC [Maier et al., 10] relational CD [Maier et al., 13]
- don't deal with aggregation functions



Relational variables

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Score-based methods

• Greedy search [Getoor et al., 07]

Hybrid methods

• relational MMHC [Ben Ishak et al., 15]



Relational variables

BN learning

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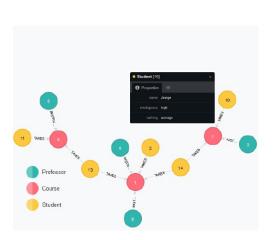
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Hybrid methods

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Graph database



Definition

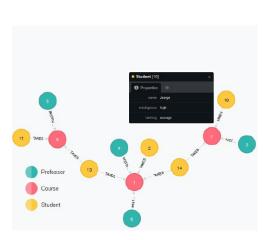
- Data is organized as a graph, with "labelled" nodes and relationships
- Attributes can be associated to both
- Seems nice for ER model but

Schema-free





Graph database



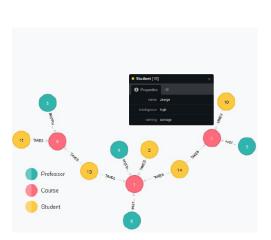
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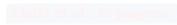


Graph database



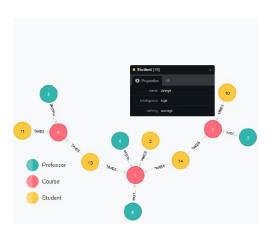
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Graph database



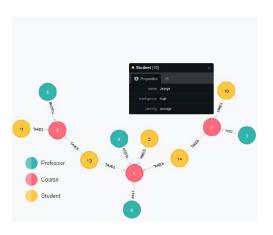
Definition

Schema-free

- Only data, no "relational schema"



Graph database



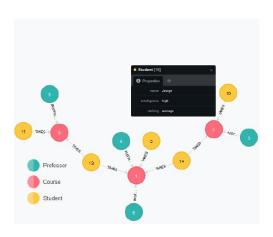
Definition

Schema-free

- Only data, no "relational schema"
- No warranty that the data has been "stored" by following some meta/ER model.



Graph database



BN learning

Definition

Schema-free

Elabri et al., in progress

- Learning a probabilistic relational model from a graph DB
- Extension to Markov Logic Networks



Outline ...



- - Definition
 - Parameter learning
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 - - Definition
 - Learning
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 - Graph DB?
- **Conclusion**
 - Last words
 - References



Visible face of this talk

- BNs = powerful tool for knowledge representation and reasoning
- ⇒ interest in using structure learning algorithms for knowledge discovery



Visible face of this talk

- BNs = powerful tool for knowledge representation and reasoning
- ⇒ interest in using structure learning algorithms for knowledge discovery
 - BN structure learning is NP-hard, even for "usual" BN/data
 - but we want to learn more and more complex models with more and more complex data
- ⇒ many works in progress in order to develop such learning algorithms



Visible face of this talk

BN learning

Hidden face of this talk

- BN learning and causality: causal discovery



Visible face of this talk

BN learning

Hidden face of this talk

- BN learning and causality: causal discovery
- BN learning tools: no unified programming tools, often limited to simple BN models / simple data
- \Rightarrow coming soon : PILGRIM our GPL platform in C++, dealing with BN, DBN, RBN, incremental data, ...



Visible face of this talk

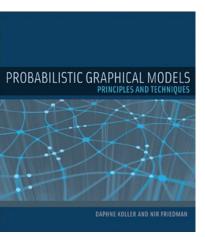
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- \Rightarrow coming soon : PILGRIM our GPL platform in C++, dealing with BN, DBN, RBN, incremental data, ...
 - BN versus other probabilistic graphical models: Qualitative probabilistic models, Markov random fields, Conditional random fields, Deep belief networks, ...



References



BN learning

One starting point

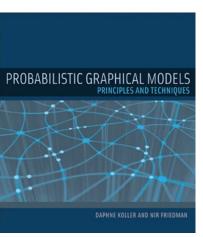
[Koller & Friedman, 09] Probabilistic Graphical Models: Principles and Techniques. MIT Press.

Our publications

http://tinyurl.com/PhLeray



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Thank you for your attention