Brain-Inspired Computing
Class 7
Bursting (multi-scale excitability) and CPGs

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Contents

The phase plane of excitability in the presence of slow positive feedback

A simple model with multi-scale excitability

Excitability is created by the co-existence (and cooperation) of fast positive and slow negative feedback. We can literally copy-paste the excitability feedback motif at multiple timescales.

Consider model

$$\begin{aligned} \tau \dot{x}(t) &= -x(t) + S\left(k \cdot x(t) + I(t) - (x_s(t) - x_{s_0})^2\right) \\ \dot{x}_s(t) &= \varepsilon\left(x(t) - x_s(t)\right) \end{aligned}$$

The feedback loop created by the slow variable x_s has gain

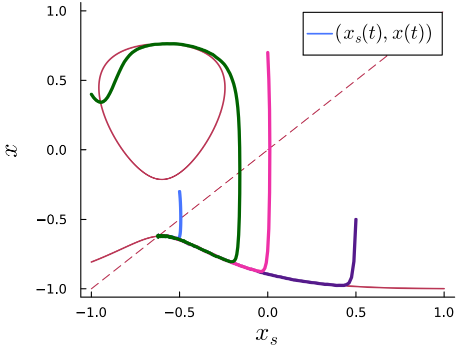
$$\frac{\partial \dot{x}}{\partial x_s}\frac{\partial \dot{x}_s}{\partial x} = 2\varepsilon\tau^{-1}S'(\star)\left(x_{s_0} - x_s\right)\begin{cases} < 0 & \text{if } x_s > x_{s_0} \\ > 0 & \text{if } x_s < x_{s_0} \end{cases}$$

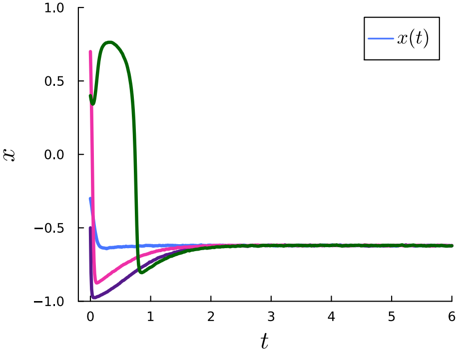
The slow feedback is positive at low ranges and negative at high ranges. Thus, at high ranges, the model should behave like a normal excitable model (it exhibits the fast positive, slow negative feedback structure of excitability), whereas at low ranges it should be bistable (its only localized feedback loops are positive, both in the fast and the slow timescales).

A simple model with multi-scale excitability

Parameter x_{s_0} determines the relative strength of slow positive and negative feedback: the larger x_{s_0} the more dominant is the slow positive feedback.

$$x_{s_0} = -0.6, \quad I(t) \equiv 0.05$$





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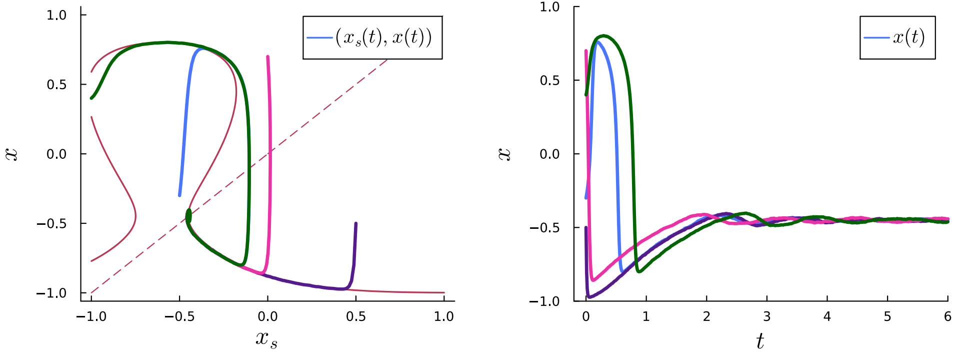

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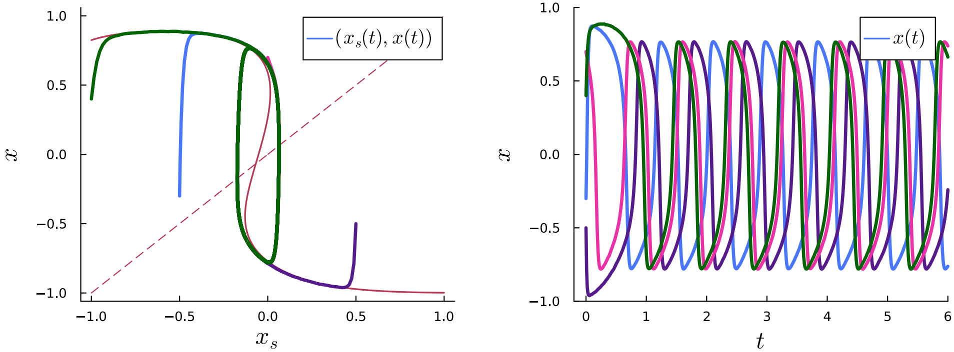
Parameter x_{s_0} determines the relative strength of slow positive and negative feedback: the larger x_{s_0} the more dominant is the slow positive feedback.

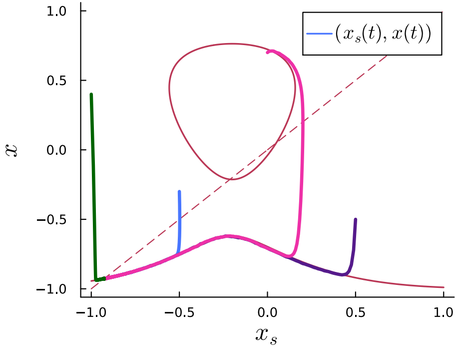
$$x_{s_0} = -0.6, \quad I(t) \equiv 0.3$$

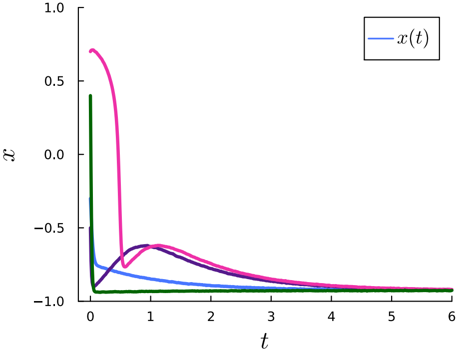


A simple model with multi-scale excitability

Parameter x_{s_0} determines the relative strength of slow positive and negative feedback: the larger x_{s_0} the more dominant is the slow positive feedback.

$$x_{s_0} = -0.2, \quad I(t) \equiv 0.05$$





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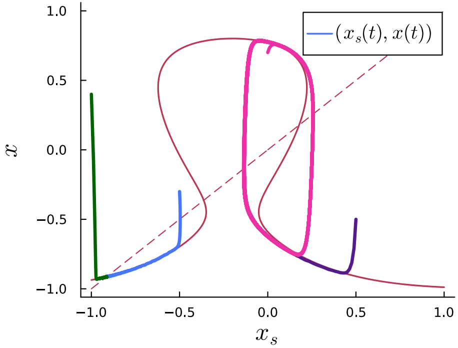

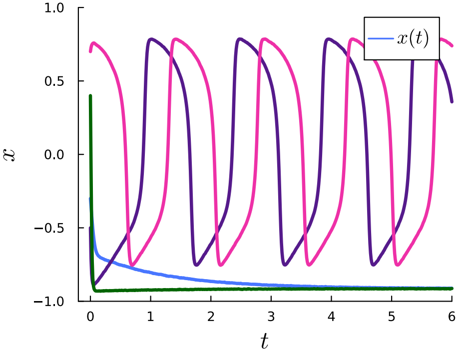

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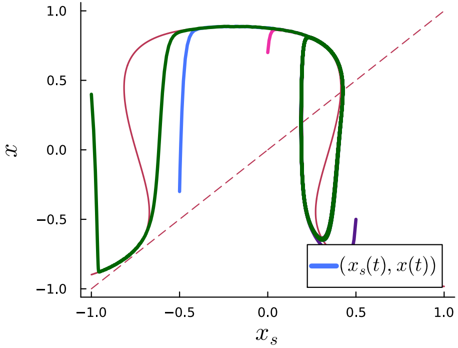
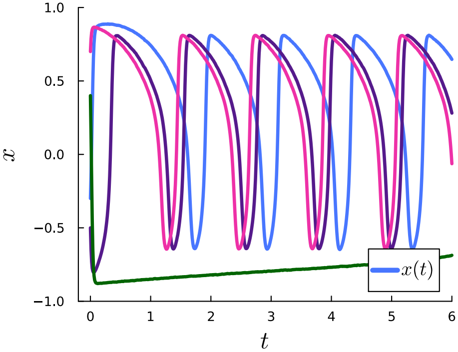




Rest-spike bistability

For sufficiently small x_{s_0}, there exist a range of applied currents for which a stable equilibrium and a spiking limit cycle co-exist. At the upper limit of this range, the stable equilibrium disappears in a saddle-node bifurcation. At the lower limit of this range, the limit cycle disappears in a **saddle-homoclinic bifurcation**. See [1] for more details. **Project:** apply the singularity theory of [1] to the model used here.

A simple model with multi-scale excitability

Adding **ultra-slow negative feedback** to the model above, we obtain a model of multi-scale excitability:

$$
\begin{aligned}
\tau \dot{x}(t) &= -x(t) + S\left(k \cdot x(t) + I(t) - \left(x_s(t) - x_{s_0}\right)^2 - k_u \cdot x_u(t)\right) \\
\dot{x}_s(t) &= \varepsilon \left(x(t) - x_s(t)\right) \\
\dot{x}_u(t) &= \varepsilon_u \left(x(t) - x_u(t)\right)
\end{aligned}
$$

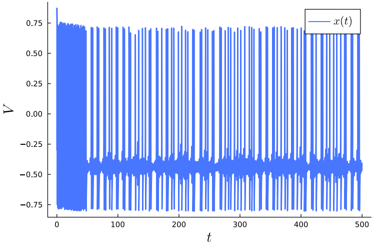
[1] Alessio Franci, Guillaume Drion, and Rodolphe Sepulchre. "Modeling the modulation of neuronal bursting: a singularity theory approach". In: *SIAM Journal on Applied Dynamical Systems* 13.2 (2014), pp. 798–829.

Bursting and its “phase plane”

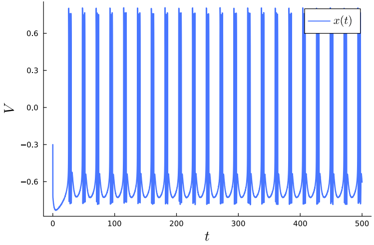
The spiking-bursting transition

As x_{s_0} is increased in the multi-scale excitability model, its behavior transition from tonic spiking to bursting

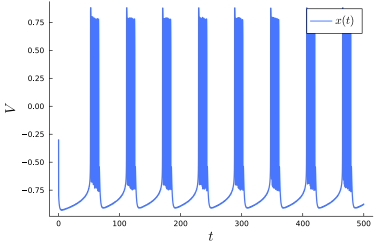
$x_{s_0} = -0.6$



$x_{s_0} = -0.4$



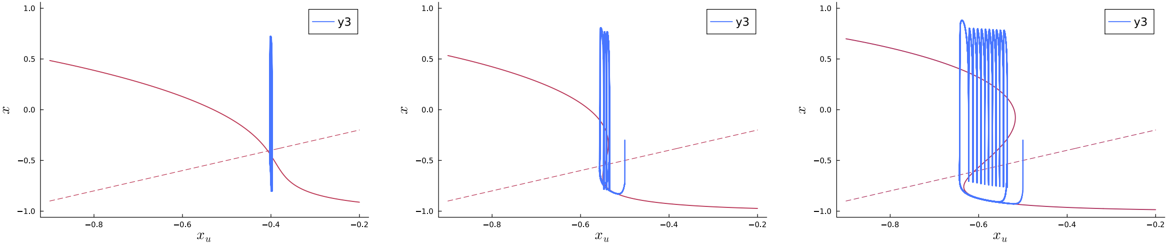
$x_{s_0} = -0.2$



The phase-plane of bursting

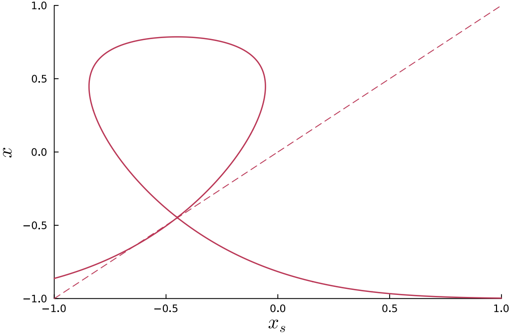
We can understand the tonic bursting transition in the (x_u, x) phase plane, similarly to how we understand spiking in the (x_s, x) phase plane.

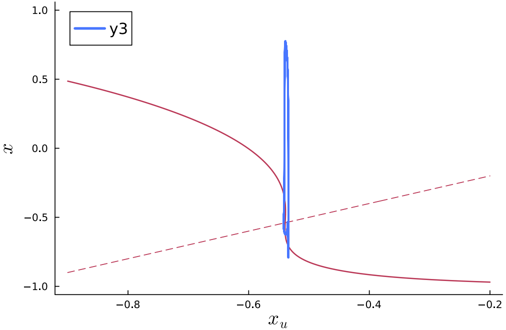
$x_{s_0} = -0.6$ $\qquad$ $x_{s_0} = -0.4$ $\qquad$ $x_{s_0} = -0.2$



The ultra-slow variable modulates the slow-fast subsystem across the rest-spike bistable range.

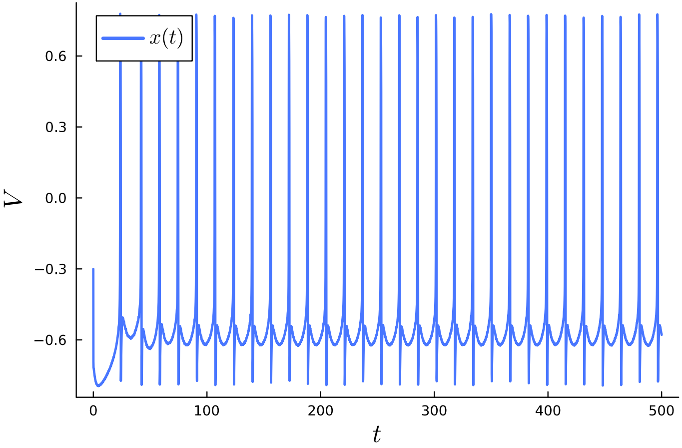
Slow tonic spiking at the transition between spiking and bursting





Slow tonic spiking corresponds to a saddle-node on limit cycle bifurcation of the slow-fast system. See [1] for more details.

Slow tonic spiking at the transition between spiking and bursting



Modulation of bursting wave

Exercise:

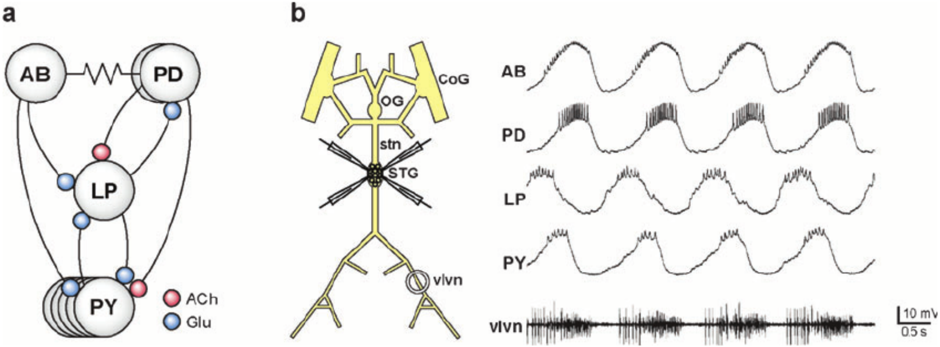
Play with the other model parameters to determine how the "shape" the bursting wave and create a parameter chart of the various wave forms (including tuning frequency and duty cycle of the bursting oscillation) you can generate.

[1] Alessio Franci, Guillaume Drion, and Rodolphe Sepulchre. "Modeling the modulation of neuronal bursting: a singularity theory approach". In: *SIAM Journal on Applied Dynamical Systems* 13.2 (2014), pp. 798–829.

CPGs

Central pattern generators

Central pattern generators are (relatively) small group of neurons that are densely interconnected in specific **network motifs** to generate **prototypical rhythmic patterns**.



The STG pyloric rhythm CPG. From [2]

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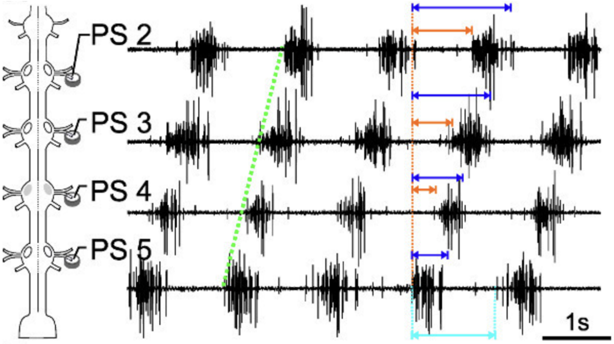

The crayfish swimmeret CPG. From [3].

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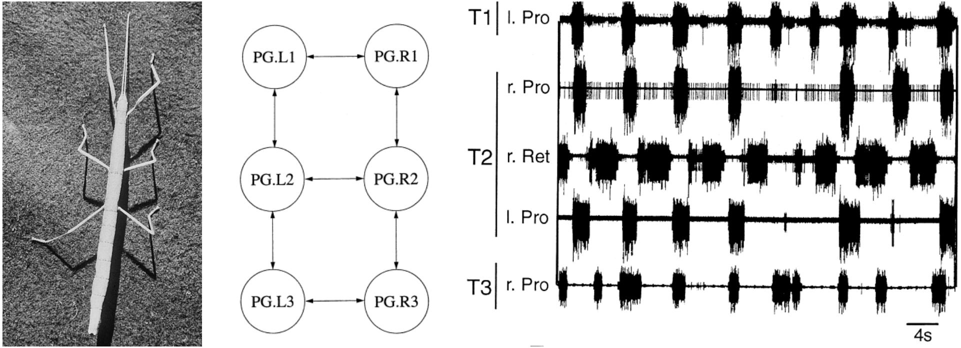
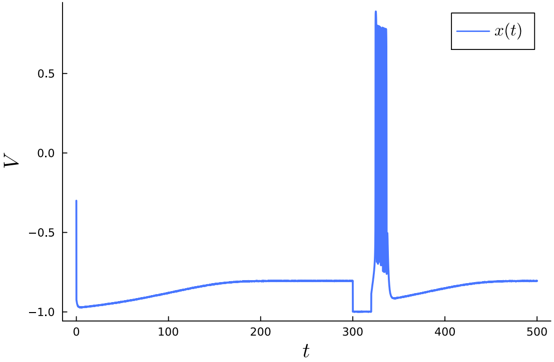


The stick insect locomotion CPG. From [1].

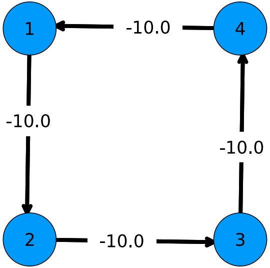
Rebound bursting: a basic mechanism for pattern generation

Response with a **rebound excitation** (a burst if in burst mode or a spike, if in tonic mode) in response to transient inhibition.



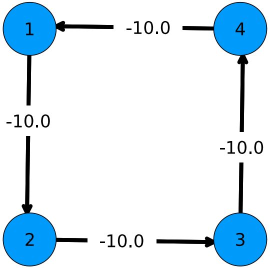
Inhibitory interconnections and bursting neurons underlie CPG through rebound mechanisms: each time a neuron bursts, its neighbors are silenced but rebound in response with a certain delay. The pattern of delays (or phase differences, in terms of the oscillation) is what determines the CPG rhythm.

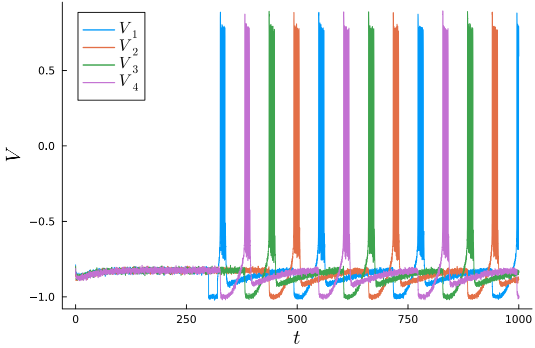
A 4-node inhibitory cycle CPG

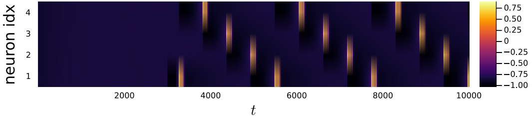


- Each neuron is in rebound mode.
- Neuron 1 receives a starting inhibitory pulse.

A 4-node inhibitory cycle CPG







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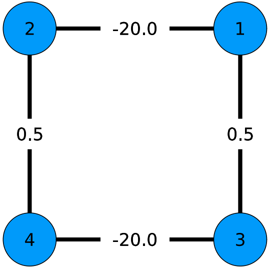
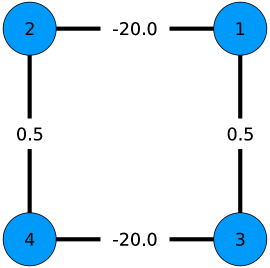
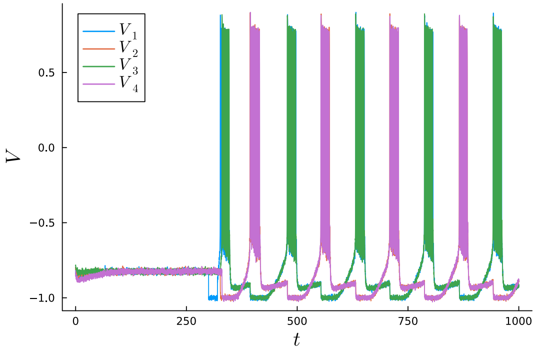
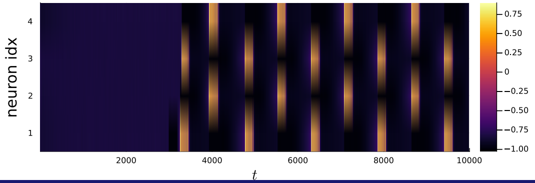


- Homolateral neurons mutually excite each other
- Contralateral neurons mutually inhibit each other

A 4-node inhibitory cycle CPG







[1] Ulrich Bässler and Ansgar Büschges. "Pattern generation for stick insect walking movements—multisensory control of a locomotor program". In: *Brain research reviews* 27.1 (1998), pp. 65–88.

[2] Eve Marder and Dirk Bucher. "Understanding circuit dynamics using the stomatogastric nervous system of lobsters and crabs". In: *Annu. Rev. Physiol.* 69 (2007), pp. 291–316.

[3] Henriette A Seichter, Felix Blumenthal, and Carmen R Smarandache-Wellmann. "The swimmeret system of crayfish: A practical guide for the dissection of the nerve cord and extracellular recordings of the motor pattern". In: *Journal of Visualized Experiments: JoVE* 93 (2014).

CPGs in the real world

CPGs in the real world

PowerPoint/Keynotes slides.