

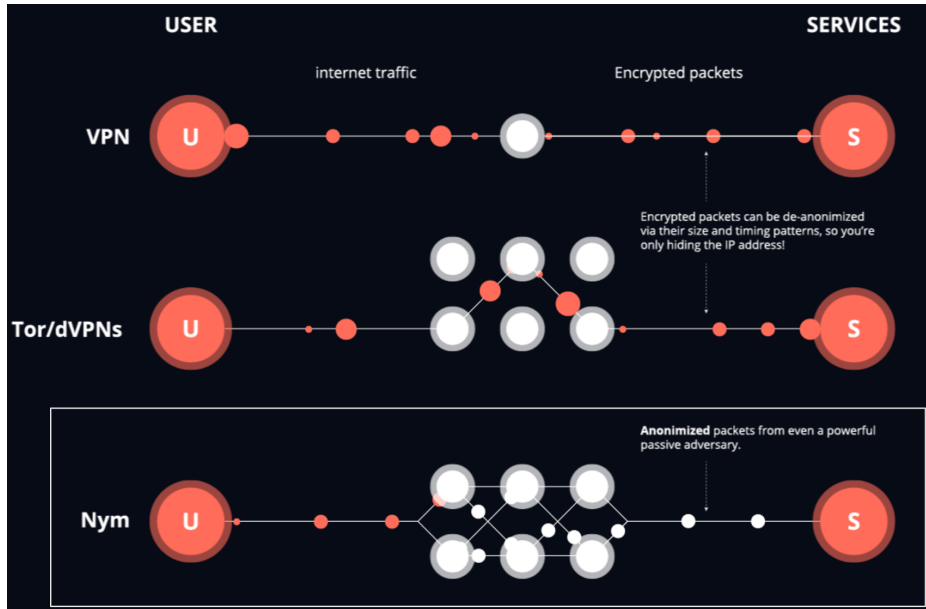
Sphinx Packets

Decentralized Header Construction

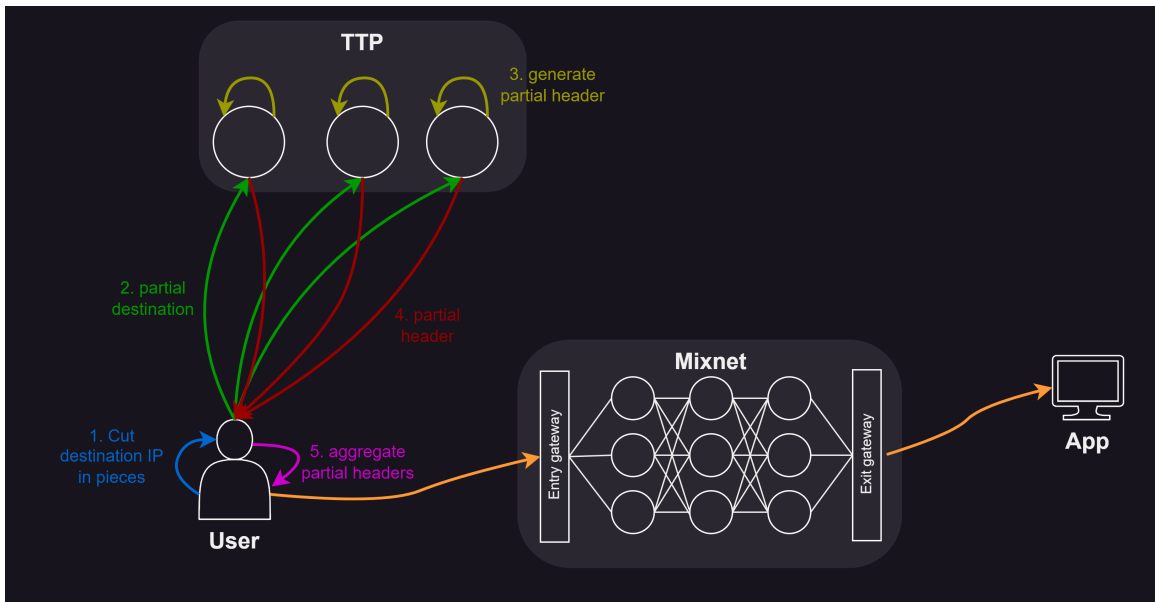
Aurélien Chassagne

February 18, 2025

Mixnet



Schema overview



Desired properties

- Generic properties
 - **Correctness:** schema works without adversary
 - **Compactness:** Minimal overhead
 - **Efficiency:** Easy and fast to compute (e.g. XOR, hash, exponentiation,...)

Desired properties

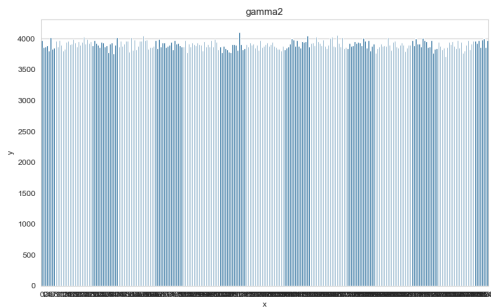
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 - **Replay attack resistant**: Cannot reused previous packet

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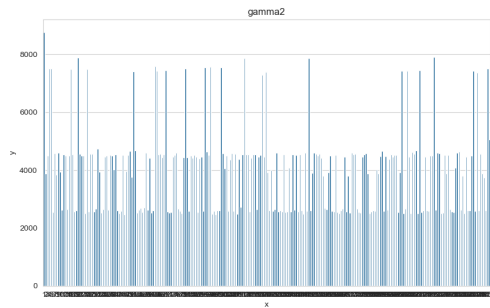
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 - **Replay attack resistant**: Cannot reused previous packet
- Depends on the header
 - **Integrity**: Maximum size path
 - **Wrap-resistance**: Unable to increase the initial path
 - **Unlinkability**: Cannot link incoming and outgoing packet from a mixnode

Unlinkability compromised

Original schema

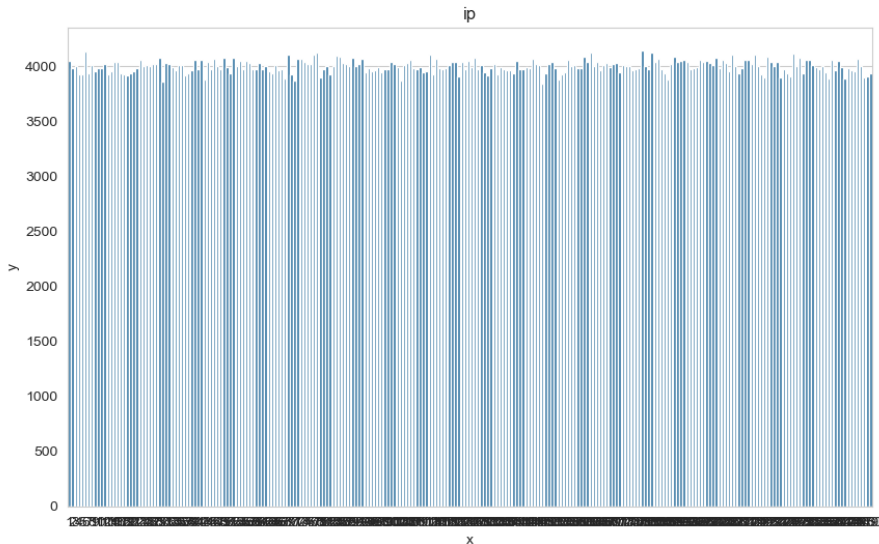


My schema



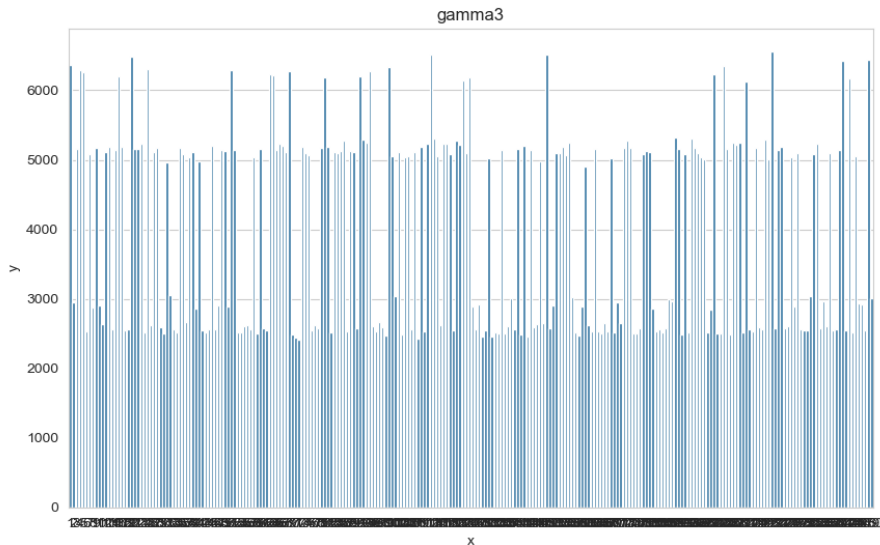
Unlinkability compromised

n_1	γ_1	n_2	γ_2	n_3	γ_3	<i>IP</i>
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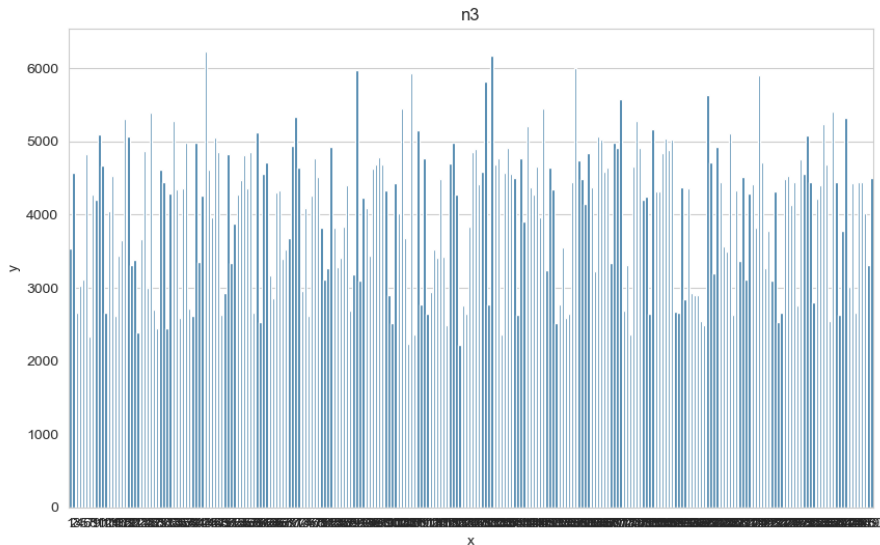
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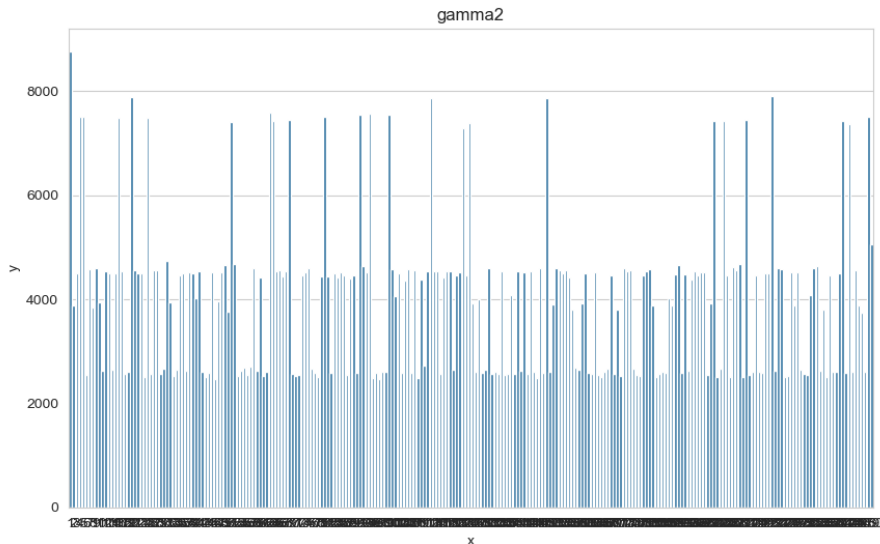
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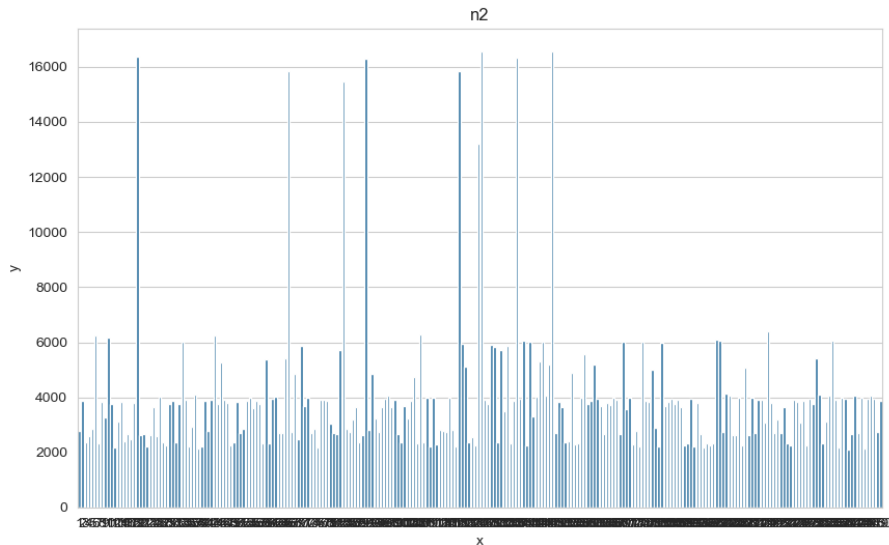
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n_1	γ_1	n_2	γ_2	n_3	γ_3	IP
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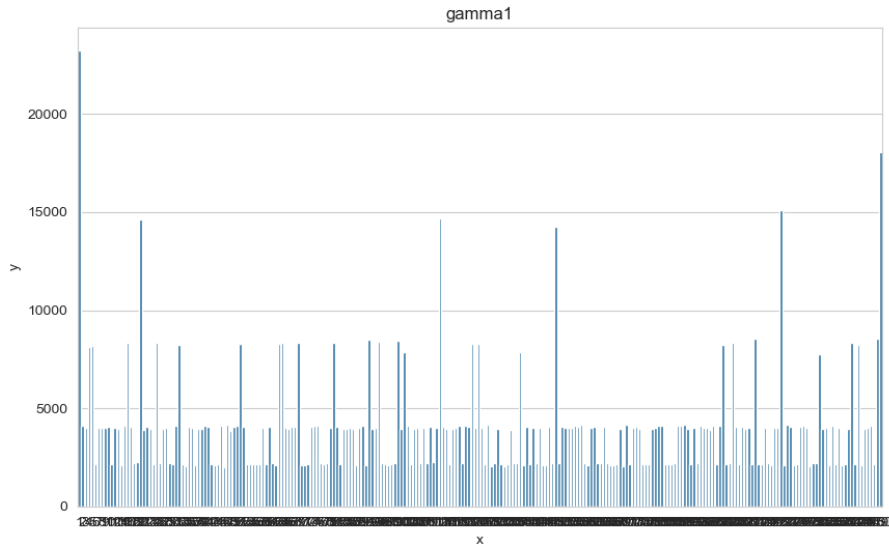
Unlinkability compromised

n_1	γ_1	n_2	γ_2	n_3	γ_3	IP
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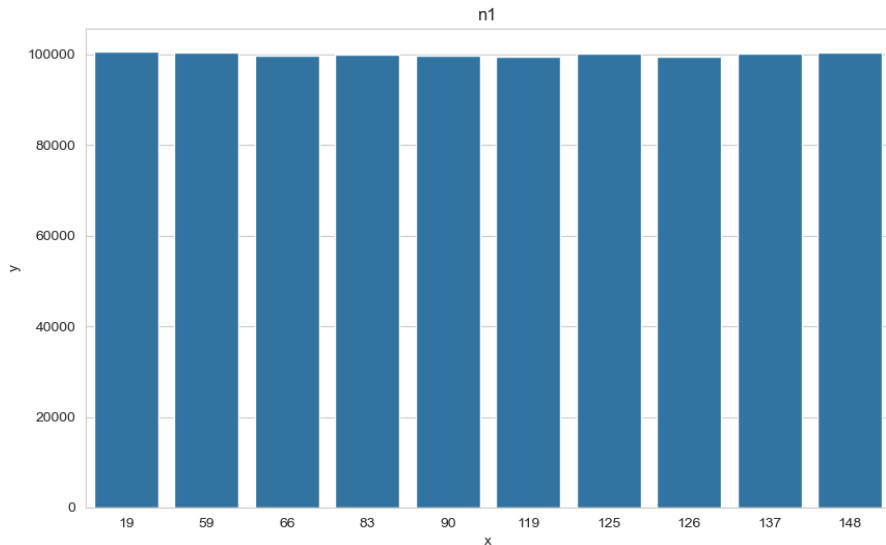
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n_1	γ_1	n_2	γ_2	n_3	γ_3	IP
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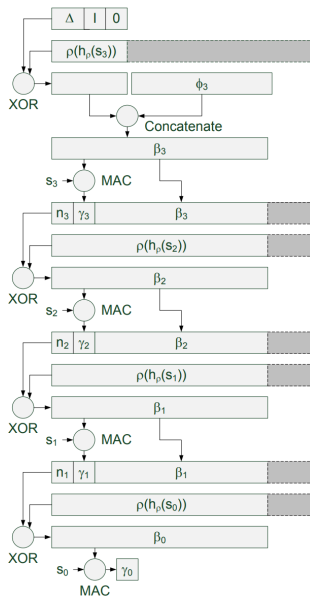


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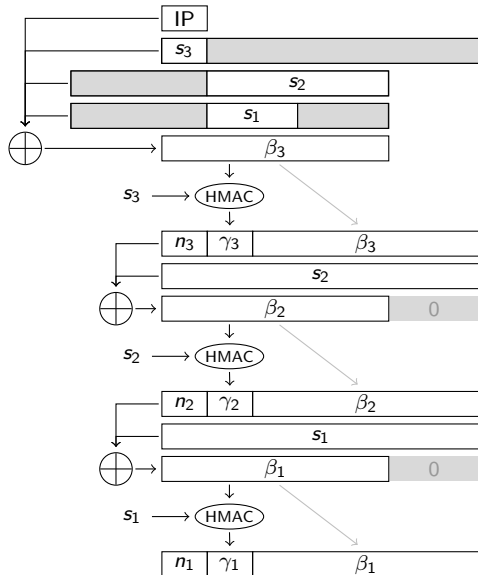
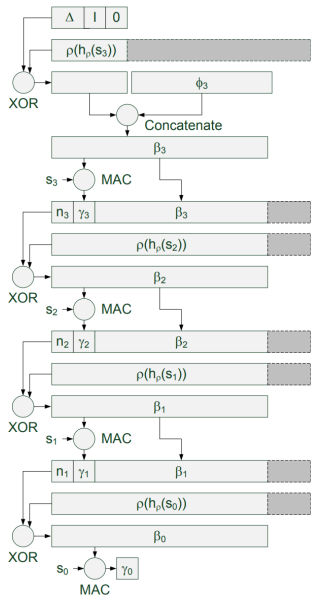
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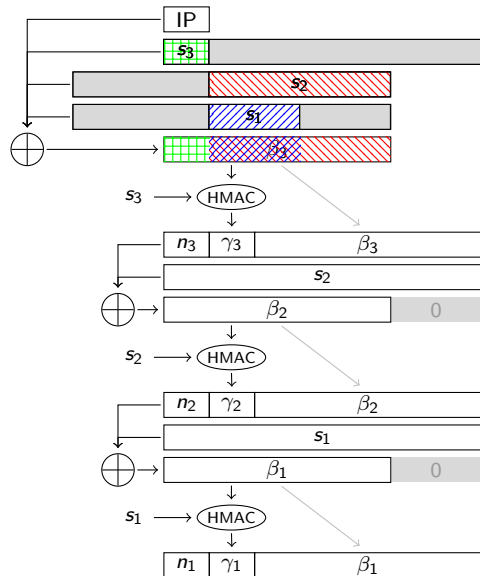
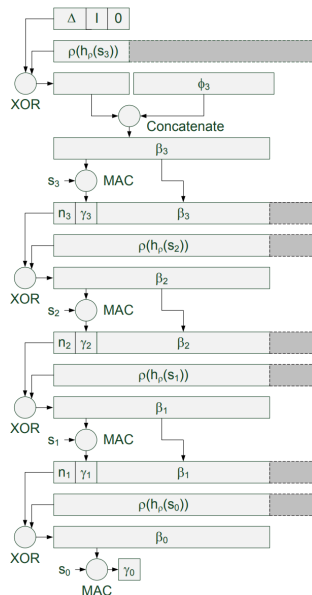
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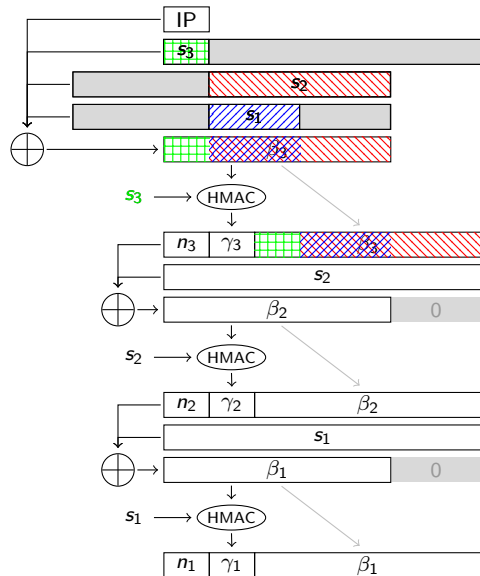
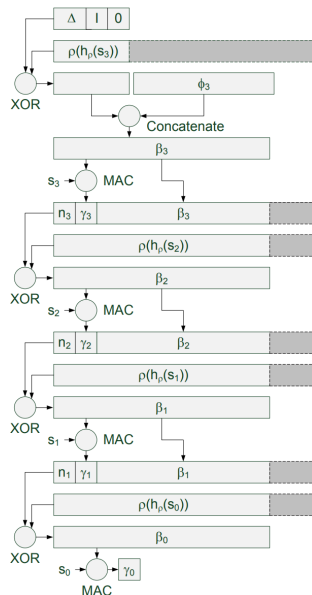
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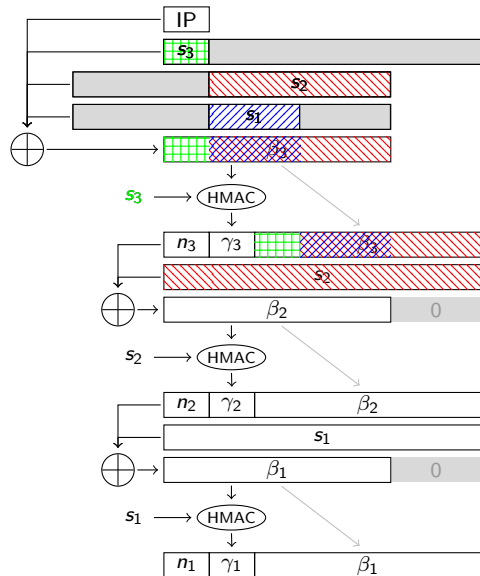
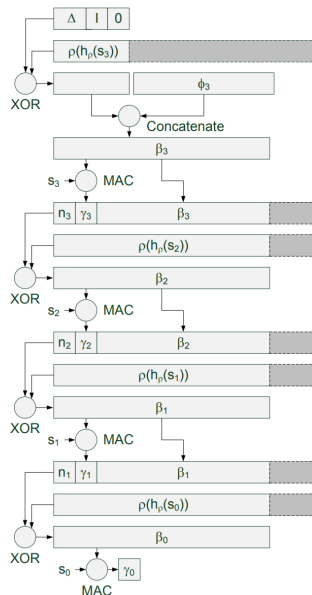
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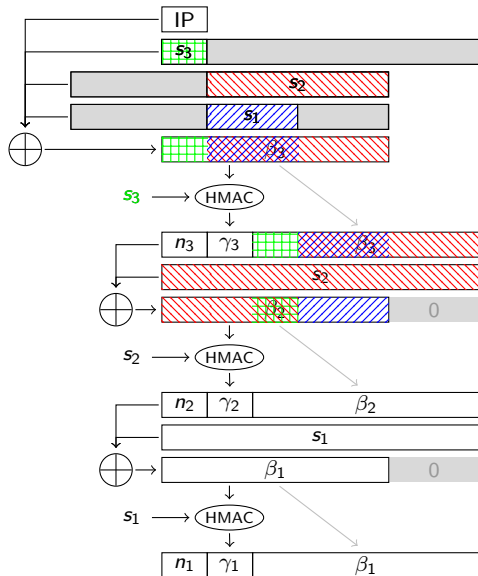
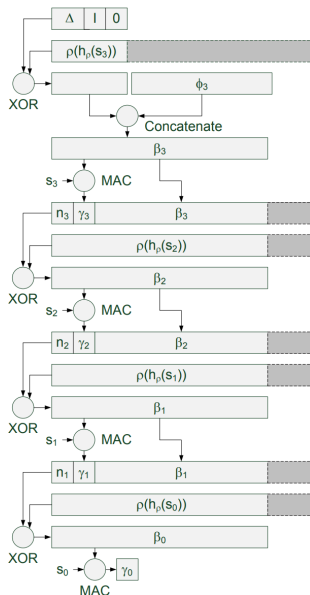
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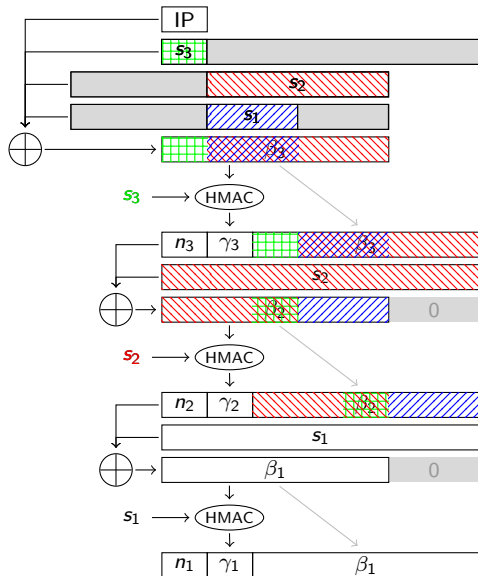
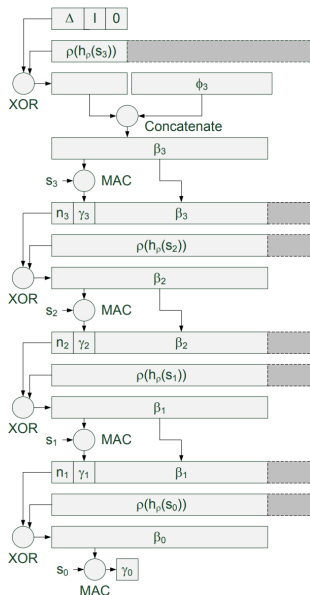
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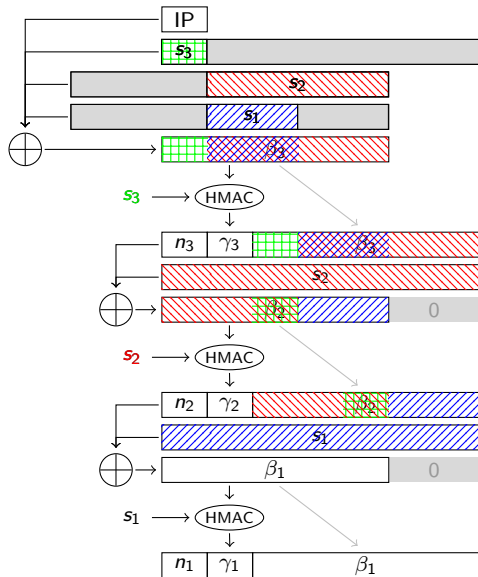
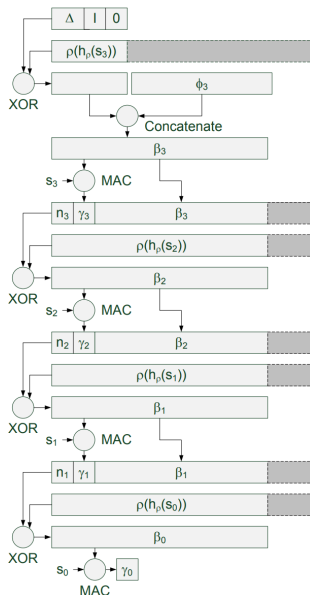
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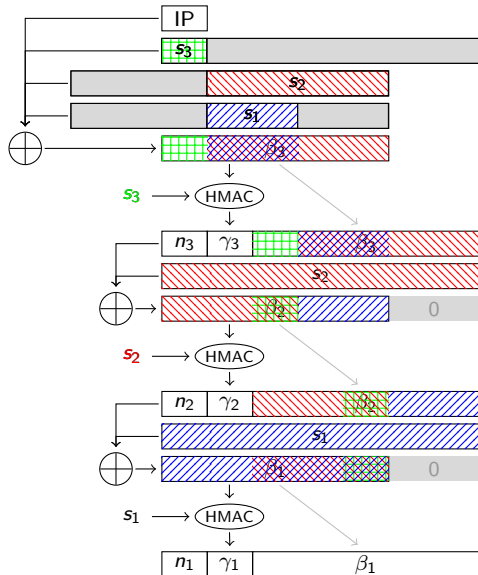
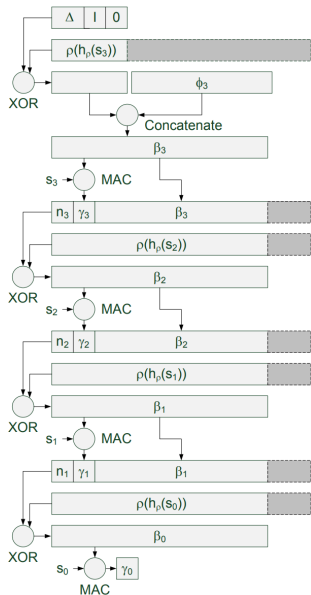
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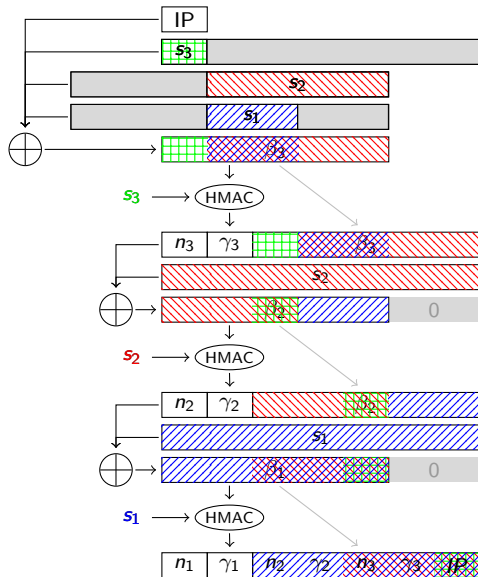
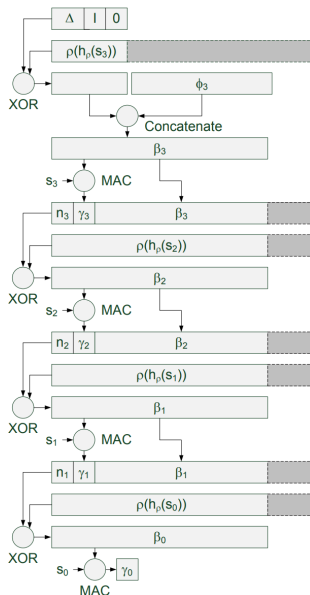
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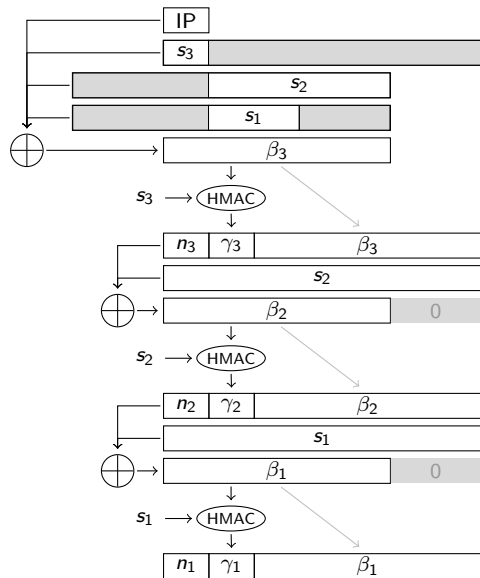
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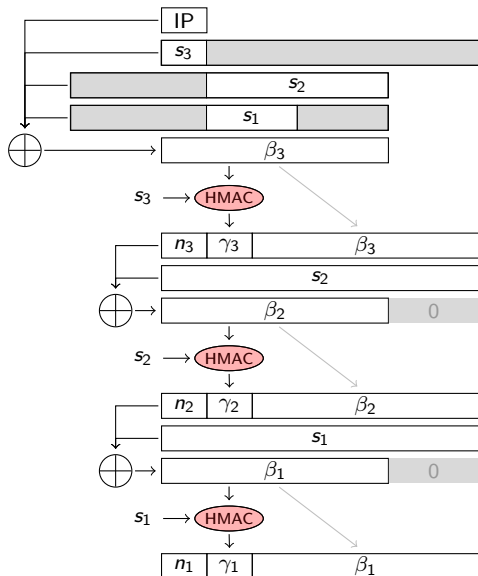


Adapting HMAC



Adapting HMAC

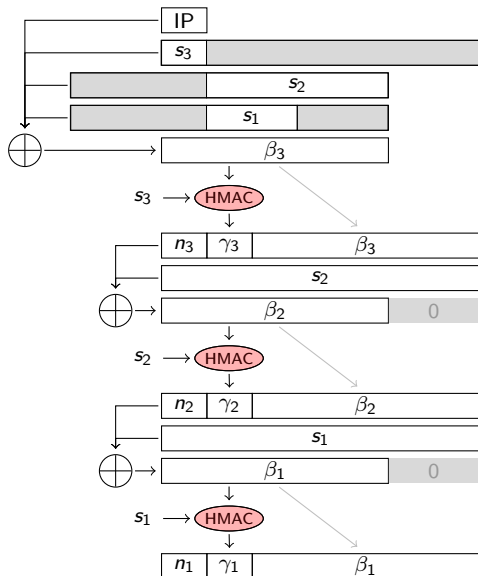
Main problem: Decentralizing a **Hash** ?



Adapting HMAC

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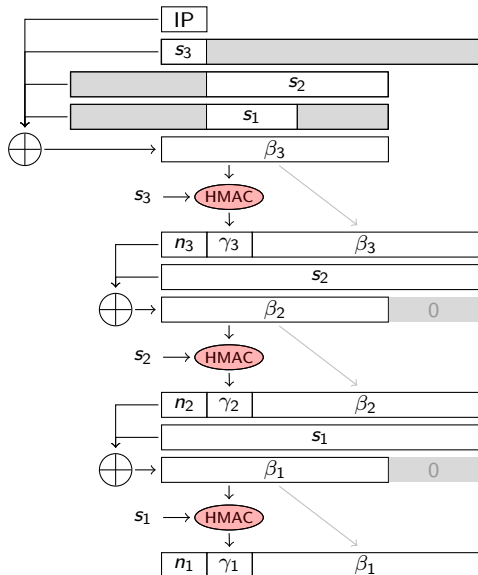
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Adapting HMAC

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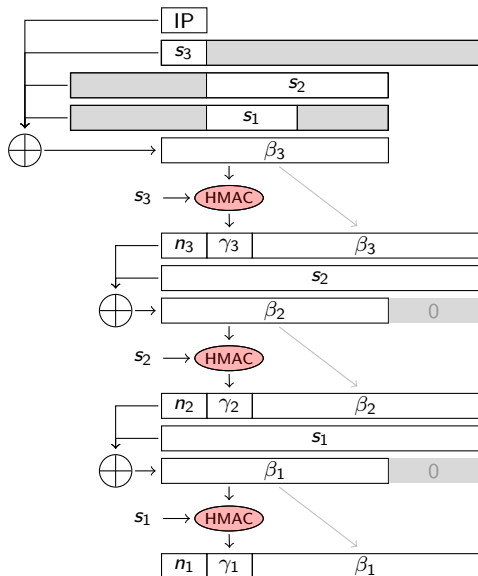
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Main problem: Decentralizing a **Hash** ?

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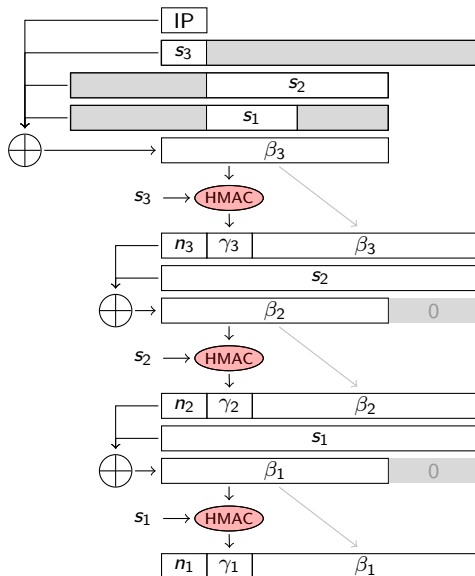


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- 1 RSA
- 2 ElGamal
- 3 Paillier



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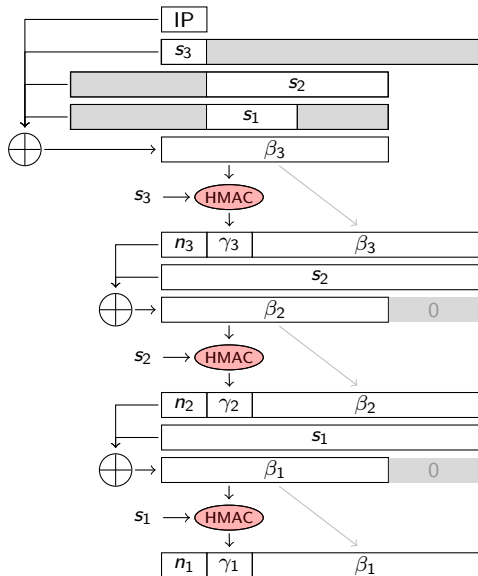
3 Paillier

$$\mathcal{E}(m_1) \cdot \mathcal{E}(m_2) = (g^{m_1} r_1^n)(g^{m_2} r_2^n) \bmod n^2$$

$$= g^{m_1+m_2} (r_1 r_2)^n \bmod n^2$$

$$= \mathcal{E}(m_1 + m_2).$$

Problem: Mix of different operations... order matters!



Adapting HMAC

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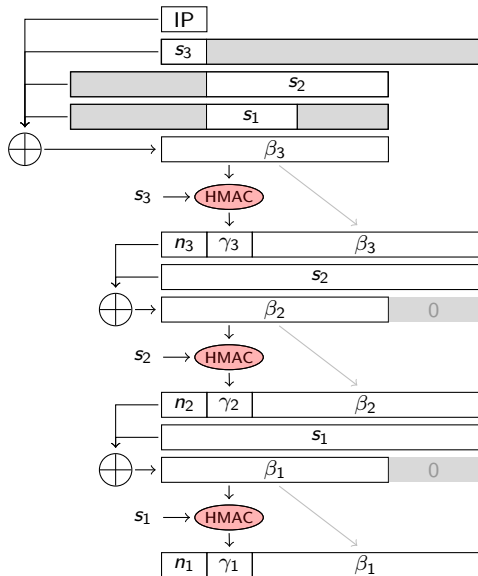
$$\mathcal{E}(m_1) \cdot \mathcal{E}(m_2) = (g^{r_1}, m_1 \cdot h^{r_1})(g^{r_2}, m_2 \cdot h^{r_2})$$

$$= (g^{r_1+r_2}, (m_1 \cdot m_2)h^{r_1+r_2})$$

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3 Paillier

Limitation: Increase ciphertext size...



Adapting HMAC

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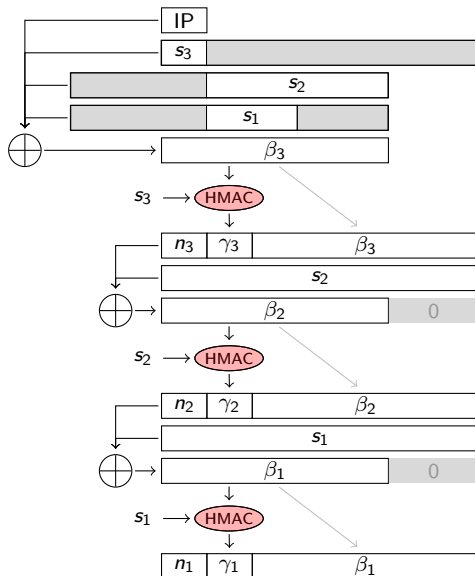
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① RSA

$$\begin{aligned}\mathcal{E}(m_1) \cdot \mathcal{E}(m_2) &= m_1^e m_2^e \bmod n \\ &= (m_1 m_2)^e \bmod n \\ &= \mathcal{E}(m_1 \cdot m_2)\end{aligned}$$

② ElGamal

③ Paillier



Adapting HMAC

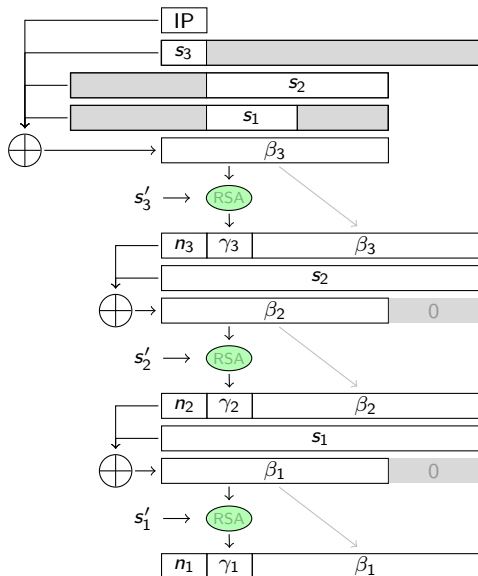
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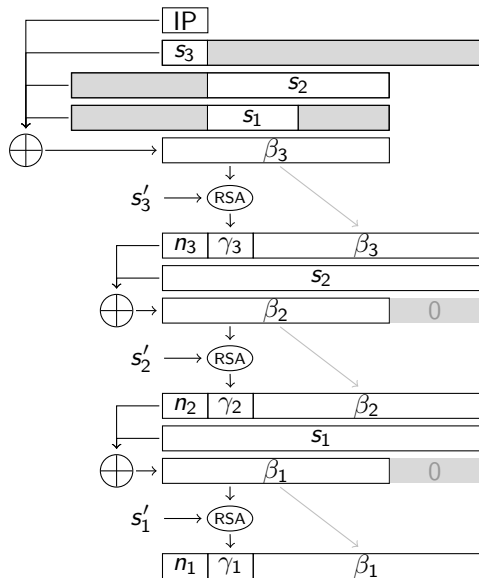
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Selected solution: **RSA** for integrity tag

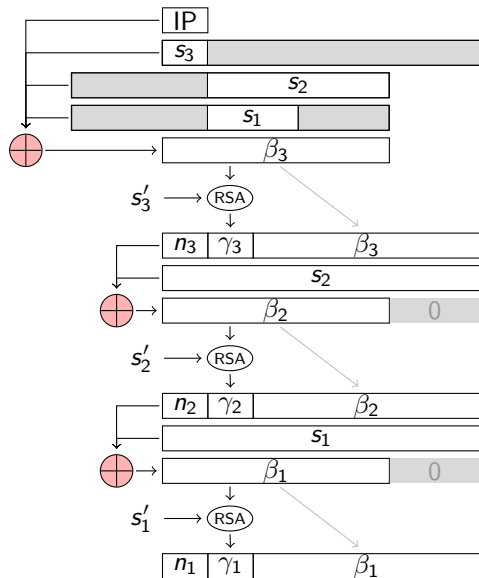
NB: s_i is different for each TTP but RSA required the same e...
Thus, create a new shared secret s'_i common to all TTP



Adapting XOR

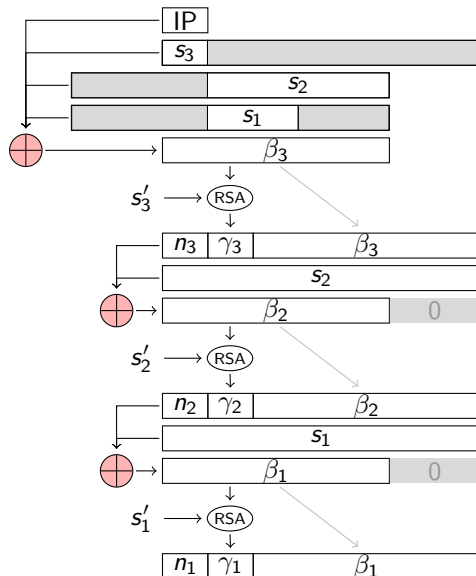


Adapting XOR



Adapting XOR

Since we use **RSA** for integrity tag γ_i



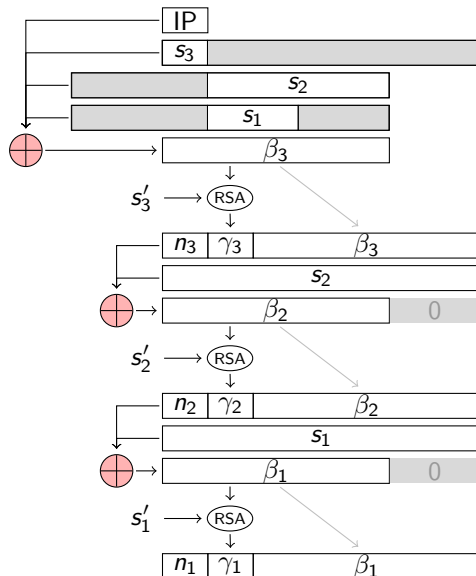
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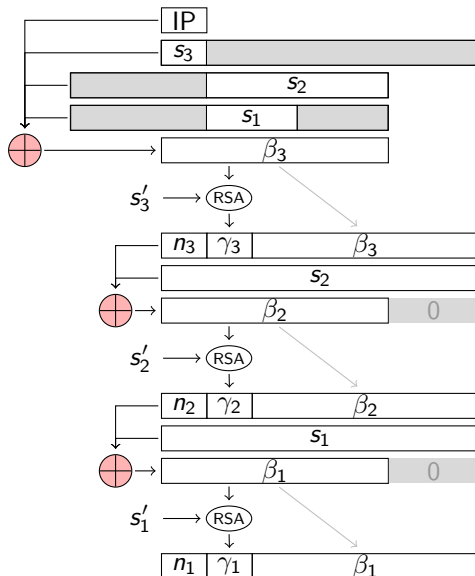
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Modular multiplication of integrity tags gives integrity tag of headers' modular product.



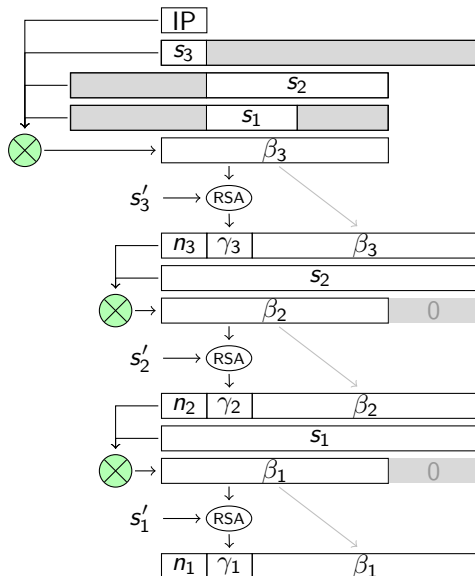
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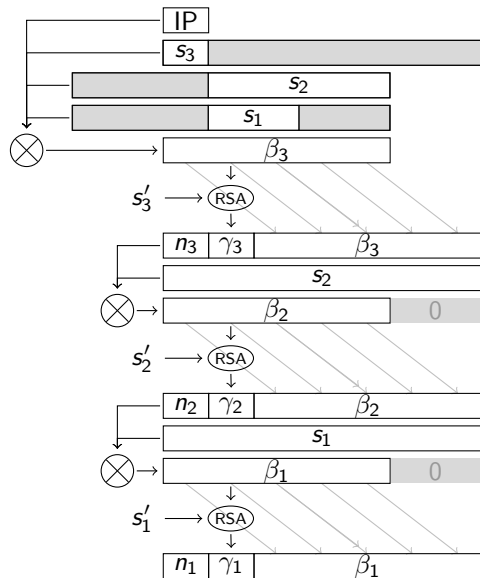
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Modular multiplication of integrity tags gives integrity tag of headers' modular product.

Thus, header elements must be combined via **modular multiplication** rather than **XOR**.

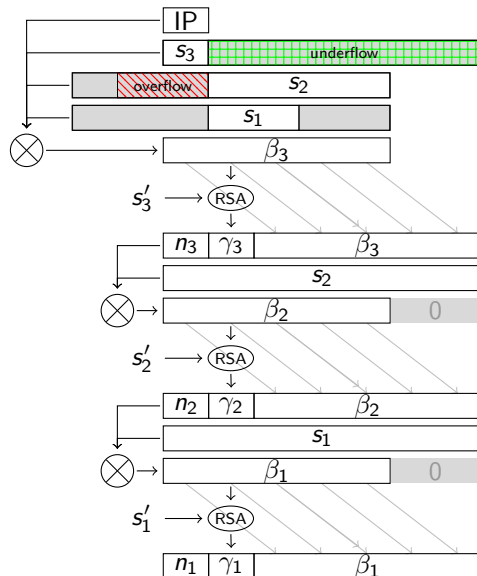


Handle overflows



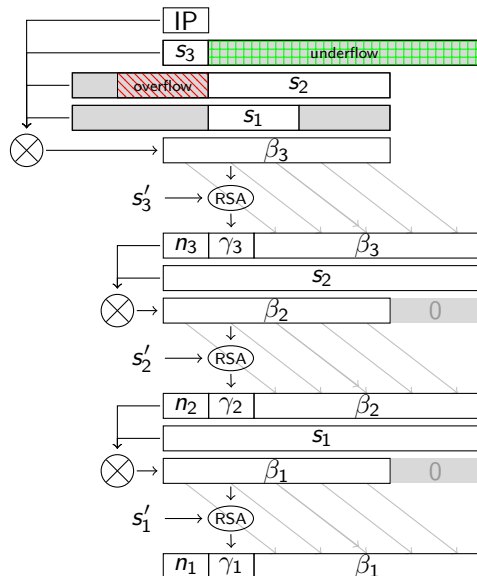
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- Handling these issues is challenging.
It may lead to **information loss** (further research is required).



Handle overflows

- **Current Challenge:** **Overflow** issues
- Handling these issues is challenging.
It may lead to **information loss** (further research is required).
- **Proposed solution:** Simplify by **dividing data into small chunks** and processing each chunk modulo its size.

