$u = \sin(3x) \quad u' = \cos(3x) \cdot 3$

$v = e^{\sqrt{x}} \quad v' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

4.2

a) $y = e^{\sqrt{x}} \cdot \sin 3x$

$e^{\sqrt{x}} = e^{\frac{1}{2}x}$

$f'(x) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot \sin 3x + e^{\sqrt{x}} \cdot \cos(3x) \cdot 3$

b) $\sqrt{x+1} + \sqrt{4-2x}$

$f'(x) = \frac{1}{2\sqrt{x+1}} \cdot 1 + \frac{1}{2\sqrt{4-2x}} \cdot (-2)$

$= \frac{1}{2\sqrt{x+1}} - \frac{1}{\sqrt{4-2x}}$

c)

$$\sqrt{x\sqrt{1-x}} = \sqrt{x\cdot(1-x)^{\frac{1}{2}}}$$

$$= \left(x\cdot(1-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= x^{\frac{1}{2}}\cdot\left((1-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= \sqrt{x}\cdot(1-x)^{\frac{1}{4}}$$

$u = \sqrt{x}$ $\qquad v = (1-x)^{\frac{1}{4}}$

$u' = \frac{1}{2\sqrt{x}}$ $\qquad v' = \frac{1}{4}(1-x)^{-\frac{3}{4}}\cdot(-1)$

$$f'(x) = \frac{1}{2\sqrt{x}}\cdot(1-x)^{\frac{1}{4}} + \sqrt{x}\cdot\frac{1}{4}\cdot(1-x)^{-\frac{3}{4}}\cdot(-1)$$

$$f'(x) = \frac{(1-x)^{\frac{1}{4}}}{2\sqrt{x}} + \frac{(-1)\cdot\sqrt{x}}{4\cdot(1-x)^{\frac{3}{4}}}$$

$$f'(x) = \frac{\sqrt[4]{1-x}}{2\sqrt{x}} - \frac{\sqrt{x}}{4\sqrt[4]{(1-x)^3}}$$

d)

$u = \cos x$

$u' = -\sin(x)$

$$f(x) = \ln\left(\tan\frac{x}{2}\right) + \frac{\cos x}{\sin^2 x}$$

$v = \sin^2(x)$

$v' = 2\cdot\sin(x)\cdot\cos x$

$$\frac{1}{\tan\frac{x}{2}}\cdot\left(\frac{1}{\cos^2\left(\frac{x}{2}\right)}\cdot\frac{1}{2}\right) + \frac{\sin^2 x\cdot(-\sin x) - \cos x\cdot 2\sin x\cdot\cos x}{\sin^4 x}$$

$$= \frac{1}{\tan\left(\frac{x}{2}\right)\cdot\cos^2\left(\frac{x}{2}\right)\cdot 2} + \frac{-\sin^3 x - 2\cdot\cos^2(x)\cdot\sin(x)}{\sin^4 x}$$

$$+ \frac{\cancel{\sin x}\left(-\sin^2 x - 2\cos^2 x\right)}{\cancel{\sin x}\left(\sin^3 x\right)}$$

$$f'(x) = \frac{1}{\tan\left(\frac{x}{2}\right)\cdot\cos^2\left(\frac{x}{2}\right)\cdot 2} + \frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x}$$

c)

$$f(x) = \arctan 2x + \arctan \frac{1}{x} \quad \Rightarrow \cancel{1}\ x^{-1} \rightarrow -x^{-2}$$

$$f'(x) = \frac{1}{(2x)^2+1} \cdot 2 + \frac{1}{\left(\left(\frac{1}{x}\right)^2+1\right)} \cdot \frac{-1}{x^2}$$

$$f'(x) = \frac{2}{4x^2+1} - \frac{1}{1+x^2}$$