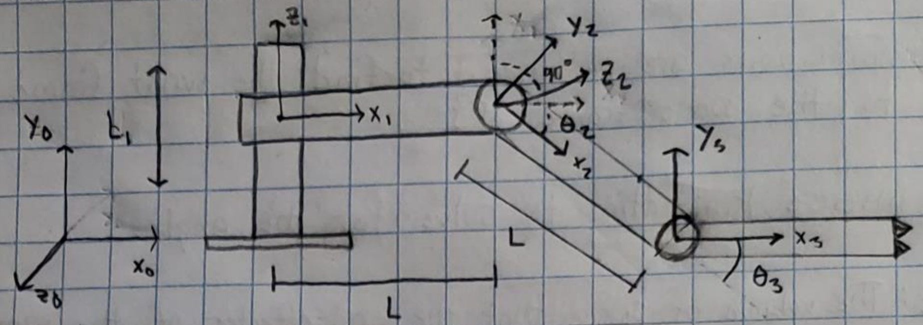
Extra Cedit:

PRR Manipulator; find L_1, θ_2, and θ_3 ; $L_2 = L_3 \Rightarrow L$, Ω_1

Use inverse kinematics and compute Jacobian.



DH-Parameters

i	α_{i-1}	a_{i-1}	d_i	θ
1	$\omega_1+90°$	0	L_1	0
2	$90°$	L	0	θ_2
3	0	L	0	θ_3

$$ {}^{i-1}_{i}T = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = {}^0_1T \, {}^1_2T \, {}^2_3T $$

$$ {}^0_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_1 & -S_1 & -S_1L_1 \\ 0 & S_1 & C_1 & C_1L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & L \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & -S_2 & 0 & L \\ -S_1S_2 & -S_1C_2 & S_1 & -S_1L_1 \\ C_1S_2 & C_1C_2 & C_1 & C_1L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} C_2 & -S_2 & 0 & L \\ -S_1S_2 & -S_1C_2 & S_1 & -S_1L_1 \\ C_1S_2 & C_1C_2 & -C_1 & C_1L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & L \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{23} & -S_{23} & 0 & LC_2+L \\ -S_1S_{23} & -S_1C_{23} & S_1 & -LS_1S_2-S_1L_1 \\ C_1S_{23} & C_1C_{23} & C_1 & LC_1S_2+C_1L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} C_{23} & -S_{23} & 0 & LC_2+L \\ -S_1S_{23} & -S_1C_{23} & S_1 & -LS_1S_2-S_1L_1 \\ C_1S_{23} & C_1C_{23} & C_1 & LC_1S_2+C_1L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$\omega_1 \neq 0$

$X = LC_2 + L$

$Y = -LS_1S_2 - S_1L_1$

$Z = LC_1S_2 + C_1L_1$

$\omega_1 = 0$

$X = LC_2 + L$

$Y = LS_2 + L$

$Z = 0$

$$ C_2 = \frac{X - L}{L} \qquad \theta_2 = \operatorname{Atan2}(s_2, c_2) $$

$$ S_2 = \frac{Y - L}{L} \qquad \phi = \theta_2 + \theta_3 $$

$$ \theta_3 = \phi - \theta_2 = \phi - \operatorname{Atan2}(s_2, c_2) $$