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Homework 1

Problem # 1

a) $P_a' = [0,1,2]^T$, $P_b' = [-1,1,1]$, $R_x = 30^\circ$, Find P_a and P_b

$$R_x(30^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

Point

$P_{new} = J P_{old}$

$P_{old} = J^{-1} P_{new}$

$$P_a = R_x(30^\circ)^{-1} P_a' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \sin 30 \\ 0 & -\sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ \cos 30 + 2 * \sin 30 \\ -\sin 30 + 2 * \cos 30 \end{bmatrix}$$

$$P_a = \begin{bmatrix} 0 \\ 1.8660 \\ 1.2321 \end{bmatrix}$$

$$P_b = R_x(30)^{-1} P_b' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \sin 30 \\ 0 & -\sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ \cos 30 + \sin 30 \\ -\sin 30 + \cos 30 \end{bmatrix}$$

$$P_b = \begin{bmatrix} -1 \\ 1.366 \\ 0.366 \end{bmatrix}$$

Problem # 1

b) $P_a = [0,1,2]^T$, $P_b = [-1,1,1]^T$, $R_x = 30°$, Find P_a' and P_b'

$$R_x(30°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30' & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

$P_{New} = J P_{old}$

$$P_a' = R_x(30°) P_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ \cos 30 - 2\sin 30 \\ \sin 30 + 2\sin 30 \end{bmatrix}$$

$$P_a' = \begin{bmatrix} 0 \\ -0.1340 \\ 2.2321 \end{bmatrix}$$

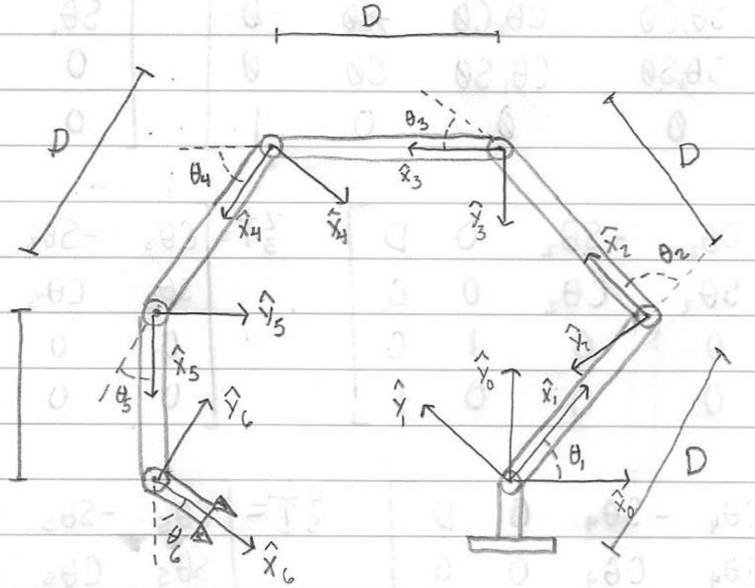
$$P_b' = R_x(30°) P_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ \cos 30 - \sin 30 \\ \sin 30 + \cos 30 \end{bmatrix}$$

$$P_b' = \begin{bmatrix} -1 \\ 0.3660 \\ 1.3660 \end{bmatrix}$$

Problem #2

Planar RRRRRR manipulator, frames, DH parameters, transforms

length = D



DH Parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	D	0	θ_2
3	0	D	0	θ_3
4	0	D	0	θ_4
5	0	D	0	θ_5
6	0	D	0	θ_6

* Manipulator is planar

* No movement along the z-axis

$$ {}^{i-1}_{i}T = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Problem # 2

$$ {}^{0}_{1}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 C0 & C\theta_1 C0 & -S0 & 0 \\ S\theta_1 S0 & C\theta_1 S0 & C0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & D \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & D \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & D \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{4}_{5}T = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & D \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}_{6}T = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & D \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Final frame with respect to the base

$$ {}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T $$

$$ \underbrace{\begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^{0}_{1}T} \underbrace{\begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & D \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^{1}_{2}T} = \begin{bmatrix} (C\theta_1 C\theta_2 - S\theta_1 S\theta_2) & -(C\theta_1 S\theta_2 + S\theta_1 C\theta_2) & 0 & DC\theta_1 \\ (S\theta_1 C\theta_2 + C\theta_1 S\theta_2) & (C\theta_1 C\theta_2 - S\theta_1 S\theta_2) & 0 & DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} C(\theta_1 + \theta_2) & -S(\theta_1 + \theta_2) & 0 & DC\theta_1 \\ S(\theta_1 + \theta_2) & C(\theta_1 + \theta_2) & 0 & DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Problem #2

$$\begin{array}{c} \\ \begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & DC\theta_1 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} {}^2_3T \\ \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & D \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} =$$

$$\begin{bmatrix} (C(\theta_1+\theta_2)C\theta_3 - S(\theta_1+\theta_2)S\theta_3) & -(C(\theta_1+\theta_2)S\theta_3 + S(\theta_1+\theta_2)C\theta_3) & 0 & DC(\theta_1+\theta_2) + DC\theta_1 \\ (S(\theta_1+\theta_2)C\theta_3 + C(\theta_1+\theta_2)S\theta_3) & (C(\theta_1+\theta_2)C\theta_3 - S(\theta_1+\theta_2)S\theta_3) & 0 & DS(\theta_1+\theta_2) + DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C(\theta_1+\theta_2+\theta_3) & -S(\theta_1+\theta_2+\theta_3) & 0 & DC(\theta_1+\theta_2) + DC\theta_1 \\ S(\theta_1+\theta_2+\theta_3) & C(\theta_1+\theta_2+\theta_3) & 0 & DS(\theta_1+\theta_2) + DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^3_4T$$

$$= \begin{bmatrix} C(\theta_1+\theta_2+\theta_3)C\theta_4 - S(\theta_1+\theta_2+\theta_3)S\theta_4 & -(C(\theta_1+\theta_2+\theta_3)S\theta_4 + S(\theta_1+\theta_2+\theta_3)C\theta_4) & 0 & DC(\theta_1+\theta_2+\theta_3) + DC(\theta_1+\theta_2) + DC\theta_1 \\ S(\theta_1+\theta_2+\theta_3)C\theta_4 + C(\theta_1+\theta_2+\theta_3)S\theta_4 & C(\theta_1+\theta_2+\theta_3)C\theta_4 - S(\theta_1+\theta_2+\theta_3)S\theta_4 & 0 & DS(\theta_1+\theta_2+\theta_3) + DS(\theta_1+\theta_2) + DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

${}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T =$

$$\begin{bmatrix} C(\theta_1+\theta_2+\theta_3+\theta_4) & -S(\theta_1+\theta_2+\theta_3+\theta_4) & 0 & DC(\theta_1+\theta_2+\theta_3) + DC(\theta_1+\theta_2) + DC\theta_1 \\ S(\theta_1+\theta_2+\theta_3+\theta_4) & C(\theta_1+\theta_2+\theta_3+\theta_4) & 0 & DS(\theta_1+\theta_2+\theta_3) + DS(\theta_1+\theta_2) + DS\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Repeating pattern
  - increment θ's

Problem #2

$${}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_6T=$$

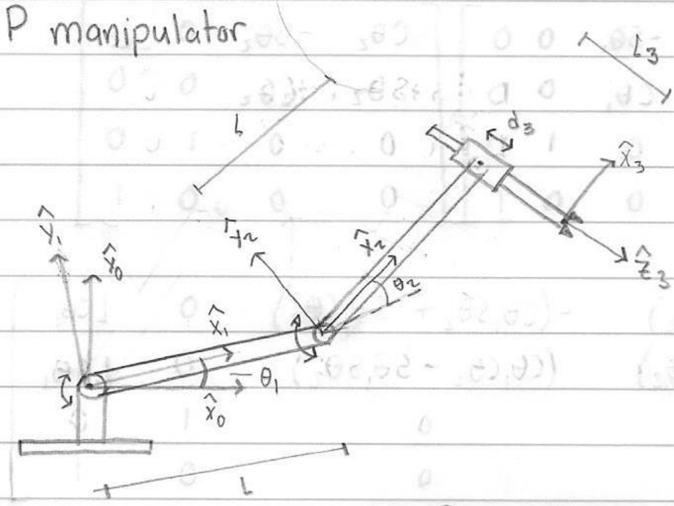
$$\begin{bmatrix} c(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5) & -S(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5) & 0 & Dc(\theta_1+\theta_2+\theta_3+\theta_4)+Dc(\theta_1+\theta_2+\theta_3)+Dc(\theta_1+\theta_2)+Dc\theta_1 \\ S(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5) & C(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5) & 0 & Ds(\theta_1+\theta_2+\theta_3+\theta_4)+Ds(\theta_1+\theta_2+\theta_3)+Ds(\theta_1+\theta_2)+Ds\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_6T={}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T\,{}^5_6T=$$

$$\begin{bmatrix} C(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5+\theta_6) & -S(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5+\theta_6) & 0 & D\big(c(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5)+c(\theta_1+\theta_2+\theta_3+\theta_4)+c(\theta_1+\theta_2+\theta_3)+c(\theta_1+\theta_2)+c\theta_1\big) \\ s(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5+\theta_6) & C(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5+\theta_6) & 0 & D\big(s(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5)+s(\theta_1+\theta_2+\theta_3+\theta_4)+s(\theta_1+\theta_2+\theta_3)+s(\theta_1+\theta_2)+s\theta_1\big) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem # 3

Planar RRP manipulator



i	α_{i-1}	a_{i-1}	d_i	θ
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	d_3	θ_3

$\rightarrow$

* DH Parameters :

i	α_{i-1}	a_{i-1}	d_i	θ
1	0	0	0	θ_1
2	0	L	0	θ_2
3	0	L	L_3	Ω_3

* Ω_3 is fixed

$$T_3^0 = {}^0_1T \, {}^1_2T \, {}^2_3T$$

$${}^0_1T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} C\Omega_3 & -S\Omega_3 & 0 & L \\ S\Omega_3 & C\Omega_3 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{0}_{1}T\,{}^{1}_{2}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{bmatrix} (C\theta_1 C\theta_2 - S\theta_1 S\theta_2) & -(C\theta_1 S\theta_2 + S\theta_1 C\theta_2) & 0 & LC\theta_1 \\ (S\theta_1 C\theta_2 + C\theta_1 S\theta_2) & (C\theta_1 C\theta_2 - S\theta_1 S\theta_2) & 0 & LS\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{2}T\,{}^{2}_{3}T =
$$

$$
\begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & LC\theta_1 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & LS\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\Omega_3 & -S\Omega_3 & 0 & L \\ S\Omega_3 & C\Omega_3 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =
$$

$$
\begin{bmatrix} (C(\theta_1+\theta_2)C\Omega_3 - S(\theta_1+\theta_2)S\Omega_3) & -(C(\theta_1+\theta_2)S\Omega_3 + S(\theta_1+\theta_2)C\Omega_3) & 0 \\ (S(\theta_1+\theta_2)C\Omega_3 + C(\theta_1+\theta_2)S\Omega_3) & (C(\theta_1+\theta_2)C\Omega_3 - S(\theta_1+\theta_2)S\Omega_3) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} LC(\theta_1+\theta_2) + LC\theta_1 \\ LS(\theta_1+\theta_2) + LS\theta_1 \\ L_3 \\ 1 \end{bmatrix}
$$

* Transform from the base to the last frame:

$$
{}^{0}_{3}T = \begin{bmatrix} C(\theta_1+\theta_2+\Omega_3) & -S(\theta_1+\theta_2+\Omega_3) & 0 & L(C(\theta_1+\theta_2)+C\theta_1) \\ S(\theta_1+\theta_2+\Omega_3) & C(\theta_1+\theta_2+\Omega_3) & 0 & L(S(\theta_1+\theta_2)+S\theta_1) \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
X = L[C(\theta_1+\theta_2) + C\theta_1] \qquad \phi = \theta_1+\theta_2+\Omega_3
$$

$$
Y = L[S(\theta_1+\theta_2) + S\theta_2]
$$

Problem #4

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \qquad R'_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & b \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = R_x(\alpha) R_y(\phi) =$$

$$\begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ \sin\alpha\sin\phi & \cos\alpha & -\cos\alpha\sin\phi & b \\ -\cos\alpha\sin\phi & \sin\alpha & \cos\alpha\cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T$$