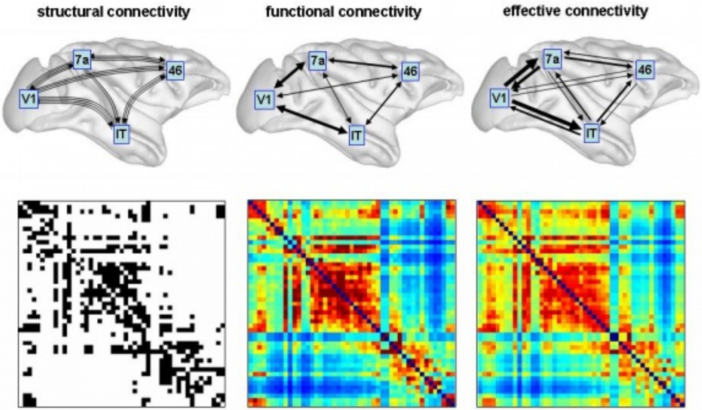
Causal Modeling

NEUR 608

October 19^{th} 2018

Connectivity



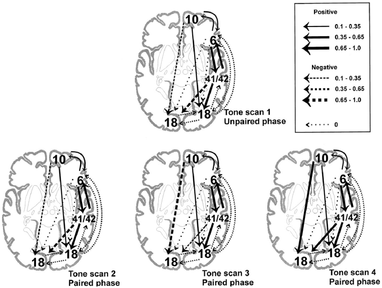
Sporns (2007) *Scholarpedia*

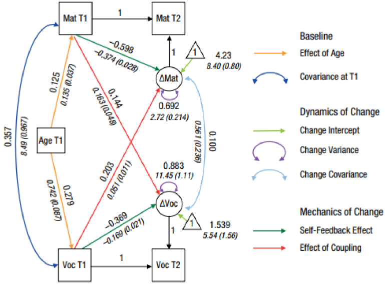
Inferring effective connectivity

1. physical influence
2. temporal precedence

- structural equation modeling (SEM)
- dynamic causal modeling (DCM)
- “Granger causal” methods

Structural equation modeling

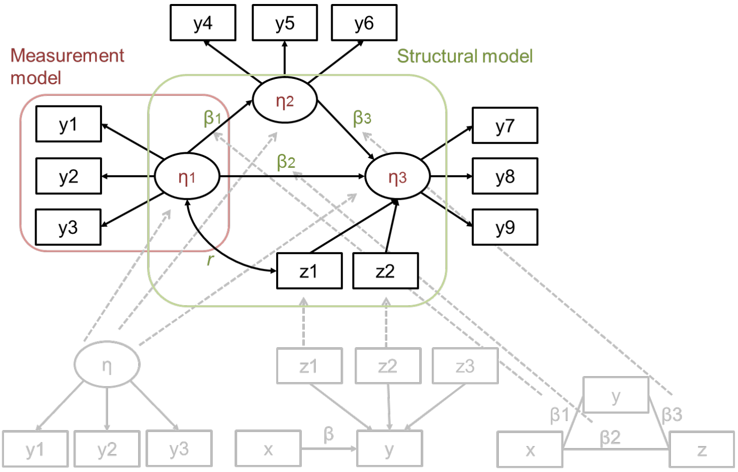




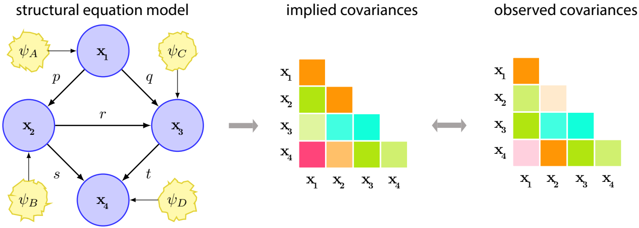
McIntosh et al. (1998) *J Neurophysiol*

Kievit et al. (2017) *Psych Sci*

Structural equation modeling



Structural equation modeling



structural equations:

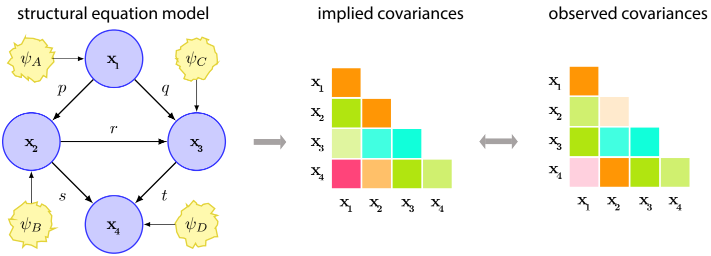
$$x_1 = \psi_{x_1}$$
$$x_2 = px_1 + \psi_{x_2}$$
$$x_3 = qx_1 + rx_2 + \psi_{x_3}$$
$$x_4 = sx_2 + tx_3 + \psi_{x_4}.$$

pairwise correlations:

$$R_{x_1,x_2} = p$$
$$R_{x_1,x_3} = q + pr$$
$$R_{x_1,x_3} = ps + prt + qt$$
$$R_{x_2,x_3} = r + pq$$
$$R_{x_2,x_3} = s + rt + pqt$$
$$R_{x_3,x_4} = t + sr + qps$$

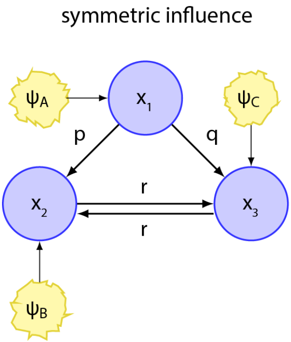
McArdle & McDonald (1984) *Br J Math Stat Psychol*

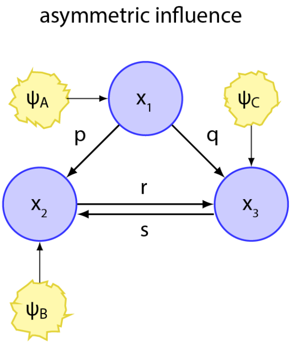
Fitting structural models



- compare implied and observed covariance matrices
- fewer parameters than variances/covariances
- fit indices: chi-sq, BIC/AIC, CFI, RMS, etc.
- non-significant chi-sq implies not-poor fit

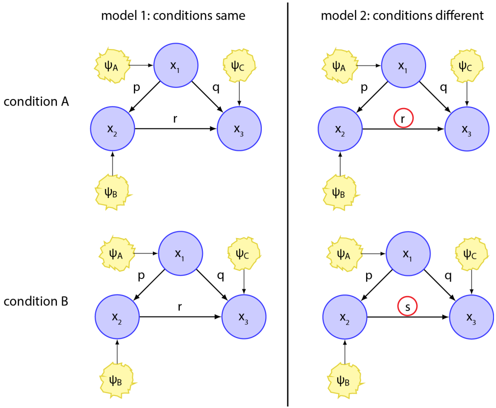
Example 1: compare models





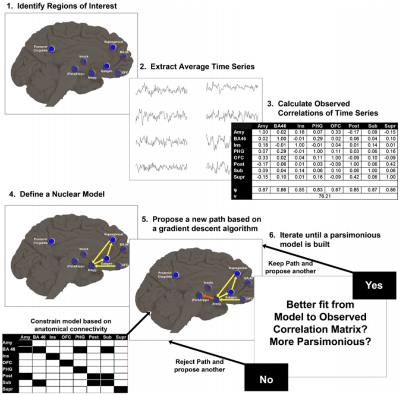
- is influence from x_1 to x_2 symmetric or asymmetric?
- difference test $= \chi^2_{sym}$ - χ^2_{asym}, df $=$ df(sym) - df(asym)
- null hypothesis: no significant difference in fit

Example 2: compare groups/conditions



- are parameters different across groups/conditions?
- evaluate using difference test
- bootstrap to get confidence intervals on path coefficients

Example 3: relativistic models



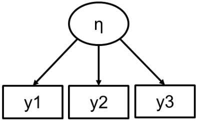
- modification index: improvement in fit if the path coefficient were unconstrained

Bullmore et al. (2000) *NeuroImage*

Stein et al. (2007) *NeuroImage*

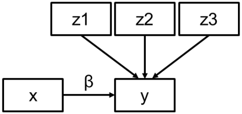
SEM as a general modeling framework

Factor Analysis



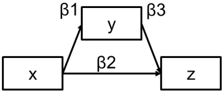
Estimate a construct (η) underlying values on various related indicator variables (y1 – y3)

Multiple Regression



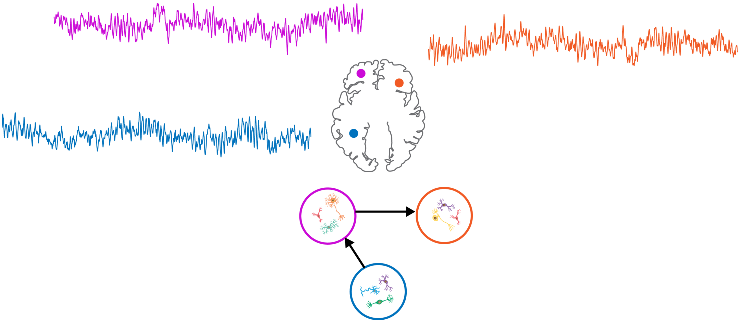
Estimate x -> y (parameter β) while controlling for confounders z1, z2, z3 that are related to x and to y

Path Analysis



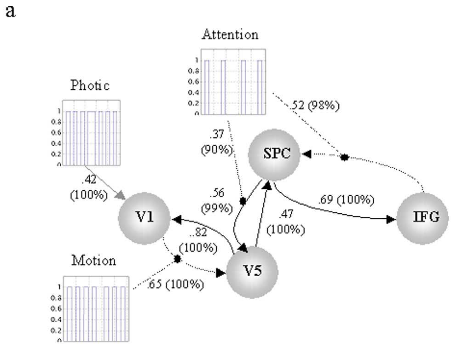
Estimate relations (parameters β1, β2, β3) between various construcs (x, y, z) at the same time

Dynamic Causal Models



- SEM estimates causality at level of observations
- DCM estimates causality at level of neural interactions
- observed: BOLD, EEG, MEG
- unobserved: neural states and interactions

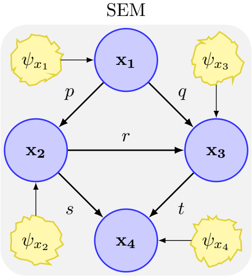
Dynamic Causal Models



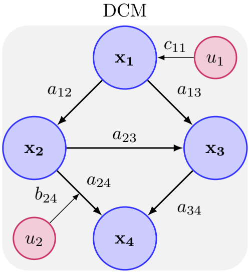
- neuronal model
- forward model
- Bayesian model inversion

Friston et al. (2003) *NeuroImage*

Neuronal model



$$\mathbf{X} = \beta\mathbf{X} + \psi$$

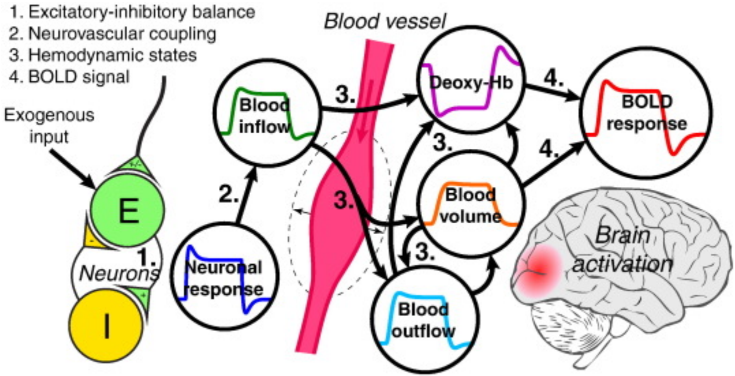


$$\frac{\partial\mathbf{X}}{\partial t} = \mathbf{AX} + \mathbf{uBX} + \mathbf{Cu}$$

$$\text{SEM: } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ q & r & 0 & 0 \\ 0 & s & t & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} \psi_{x_1} \\ \psi_{x_2} \\ \psi_{x_3} \\ \psi_{x_4} \end{pmatrix}$$

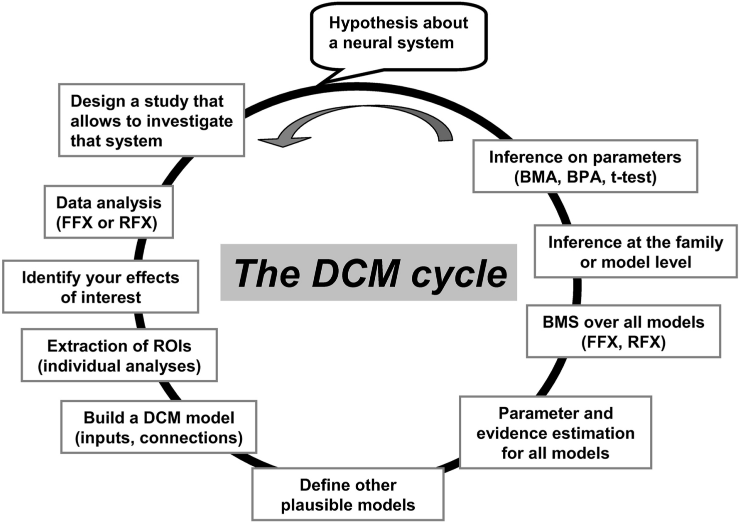
$$\text{DCM: } \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \left(\begin{pmatrix} 0 & a_{12} & a_{13} & 0 \\ 0 & 0 & a_{23} & 0 \\ 0 & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix} + u_2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{24} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} c_{11} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Forward model



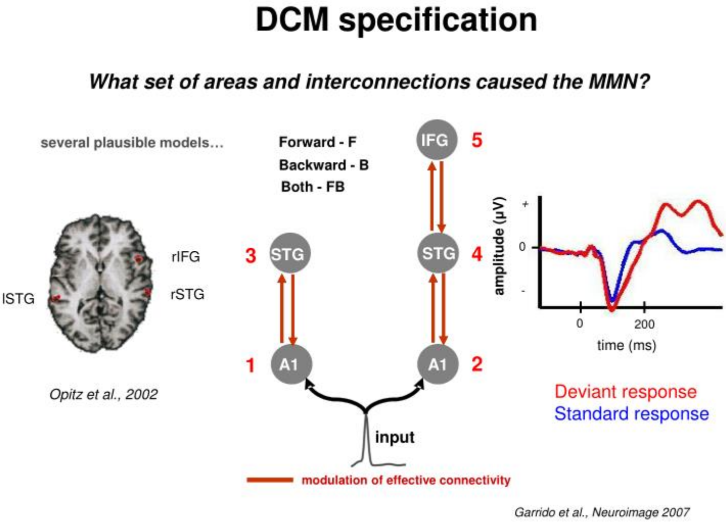
Havlicek et al. (2015) *NeuroImage*

Model inversion



Seghier et al. (2010) *Front Syst Neurosci*

Example: mismatch negativity



Granger causal models

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \gamma_p \end{bmatrix} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \gamma_{p-1} & \gamma_{p-2} & \gamma_{p-3} & \cdots \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \varphi_p \end{bmatrix}$$

- γ_i "causes" γ_j if past of $_i$ predicts future of γ_j
- if effects are linear, problem can be formulated as a multivariate vector autoregressive model (MVAR)
- current value of γ_i is a linear combination of the previous *m* values of all other γ_j

Granger (1969) *Econometrica*

Goebel et al. (2003) *Magn Reson Imag*

Extensions

- frequency domain; proportion of total power in x_j that can be attributed to x_i ("spectral Granger causality" / "directed transfer function")
- mutual information; proportion of uncertainty in x_j that is reduced by knowledge of x_i ("transfer entropy")
- nonparametric inference via surrogate models

Kaminski et al. (2001) *Biol Cybern*

Schreiber (2000) *Phys Rev E*

Strengths and limitations

- confirmatory
- mechanisms
- novel questions
- robust against some violations of assumptions

- hypotheses $\approx$ assumptions
- causal inference vs. model comparison
- size limit (SEM, DCM)
- parameter estimation

Constructive criticism