KIIRA COLLEGE BUTIKI MATHEMATICS CLUB

MATHLETICS CONTEST 2019

Senior Mathletes Category

SECTION A (5 marks each)

Qn. 1. If $3\cos x + 4\cos y = 5$, what is the greatest value of $3\sin x + 4\sin y$?

Qn. 2. For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2019^{th} term of the sequence?

Qn. 3. A function f is defined for all real numbers and $f(x) = 2019x - x^2$ for all x.
Find the value of $f(1) \times f(2) \times f(3) \times \ldots\ldots\ldots \times f(2017) \times f(2018) \times f(2019)$.

Qn. 4. You are provided with a 10×10 draughts board, how many squares of any size are on the board?

Qn. 5. The addition shown below is in a certain base and each of the letters a, b and c represents a different digit. Find the values of a, b and c as wells as this base.

$$a\ b + a\ b = a\ c\ a$$

Qn. 6. Without using a calculator, find the value of:

$$\sqrt[3]{2\sqrt{13}+5} - \sqrt[3]{2\sqrt{13}-5}$$

Qn. 7. If two real non-positive numbers p and q satisfy

$$\frac{q}{1+pq} = 2019$$

and $k = \left(\frac{1}{q} + p\right) - \frac{1}{2019}$, what is $\left(\frac{q}{p}\right)^k$?

Qn. 8. A unique line ℓ has its slope equal to its y-intercept. Find that particular point through which ℓ is guaranteed to pass.

Qn. 9. Suppose two real positive numbers a and x satisfy $\log_x a + \log_a x = 3$

What is $(\log_x a)^2 - (\log_a x)^2$?

Qn. 10. For real a and b

$$a \,@\, b = \sqrt[3]{(a-b)^{\frac{1}{5}}}$$

What is $(m-n)^2 \,@\, (n-m)^2$?

SECTION B (10 marks each)

Qn. 11. Determine all pairs of real numbers b and c that satisfy the simultaneous equations below:

$$5c^3 + bc^2 + 8 = 0$$
$$5c^3 + 8c^2 + b = 0$$

Qn. 12. An exponential function f is such that $f(x) = \frac{1}{a^{\log_a x}}$. What is the value of $f(2) - f(3) + f(4) - f(9) + f(8) - f(27) + \cdots$?

Qn. 13. The letters in MATHLETICS and CONTEST are cycled up separately as shown. The next correct spelling of both words will appear in row number n. find the value of n.

1 MATHLETICS	CONTEST
2 ATHLETICSM	ONTESTC
3 THLETICSMA	NTESTCO
.	
.	
.	
n MATHLETICS	CONTEST

Qn. 14. A parking lot has 16 spaces in a row. Yesterday, twelve cars arrived each of which required one parking space, and their drivers chose the spaces at random from among the available spaces. Uncle Ismail then arrived in his Katrina Toyota, which required two adjacent spaces. What is the probability that he was able to park?

Qn. 15. A census taker approached Mrs. Em in the morning and asked her about her children. Mrs. Em said "I have three children, each aged above 1 year and the product of their ages is 90. The sum of their ages is the number on the door." The census taker did some calculations and found the given information insufficient. However, Mrs. Em hurried off into the house saying she had to see her youngest child with measles in bed. At this, the census taker departed satisfied. Find the ages of Mrs. Em's children.