ECE311F -2021
Lab3

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Output 1

• Appealing to the Internal Model Principle, show that in order to meet SPEC 1, C(s) must have a pole at zero.

According to the Internal Model Principle(IMP), to meet SPEC 1, which is to make the output of the closed-loop system asymptotically track reference signals, the product of the control signal C(s) and G(s) needs to have the poles of R(s). In this case, R(s) has a pole at 0, but G(s) only has a pole at -a, then C(s) must have the pole at 0 to meet SPEC 1.

• The PI controller (2) has a pole at zero. Give a clear explanation of the Internal Model Principle, and clearly show that it implies that if the gang of four transfer functions (see lecture notes) are BIBO stable, then SPEC 1 and SPEC 2 are met.

The Internal Model Principle(IMP) states that the controller needs to i) make the closed-loop system BIBO stable, ii) the product of the controller and the plant has the poles of R(s) and iii) controller has the poles of D(s). If the gang of four transfer functions are BIBO stable, then we get i) for free. In other words, if the gang of four transfer functions are BIBO stable, the closed-loop system is BIBO stable. As such, SPEC 2 is met. Based on the conclusion from the last question, and we are given that the PI controller has a pole at zero, SPEC 1 is met as well. At last, for iii, we do not need to worry it for now as SPEC 1 is not taking account of what D(s) is and SPEC 2 assumes D(s) is 0.

• A necessary and sufficient condition for the gang of four transfer functions to be BIBO stable is that
(a) all poles of the transfer function 1/(1 + CG) have negative real part, and
(b) the product C(s)G(s) has no pole-zero cancellations in the closed right-half plane.
Show that for any K, TI > 0 the PI controller above satisfies conditions (a) and (b). Present clear and concise arguments.

As given in the lab handout, the PI controller is designed as C(s) = K*((TI*s+1)/(TI*s)) for K, TI>0.

As G(s) = 1/(s+a), there is no pole-zero cancellation for C(s)*G(s), condition (b) is satisfied.

The value of 1/(1+C(s)*G(s)) is $\frac{TI*s^2+TI*a*s}{TI*s^2+TI*(a+K)*s+K}$. It is obvious that the poles have negative real parts because the denominator is a second-order polynomial and its coefficients are all positive for all TI, K, a>0. Condition (a) is satisfied.

Thus the gang of four transfer functions are BIBO stable.

• Now we need pick K, TI > 0 to meet SPECS 3,4 and 5. We begin with SPEC 3. Find the transfer function T(s) = Y(s)/R(s) (assuming D(s) = 0). In order for this transfer function to have negative real poles at − p1,−p2, its denominator must have the form s2 + (p1 + p2)s + p1p2. By equating coefficients, find the unique values of K, TI > 0 such that the poles of T(s) are − p1,−p2. Present a clear derivation of the relationship between (K, TI) and (p1, p2). You've now met SPEC 3, and you can still choose p1, p2 > 0 to meet SPECS,4 and 5. You will next work on SPEC 5. SPEC 4 will be addressed via tuning in simulation.

As this is a closed-loop system and D(s) = 0, the transfer function of the system is
T(s) = C(s)*G(s)/(1+C(s)*G(s))

By plug in the value of the C(s) = K*((TI*s+1)/(TI*s)) and G(s) = 1/(s+a),

T(s) = $\frac{K*TI*s+K}{TI*s^2+TI*(a+K)*s+K}$ and thus p1+p2 = a+K, p1*p2=K/TI

Thus, K = p1+p2-a, TI = (p1+p2-a)/(p1*p2), where a = 1/1000

• Now we investigate SPEC 5. The car is initialized at zero speed (because we are modelling it as a transfer function), and is asked to accelerate and reach a speed vdes. It is natural to assume that the largest acceleration will occur at t = 0. Assuming that this is the case (you'll verify this assumption later via simulation), we need to guarantee that |u(0)| < 30 m/s2.

Based on the derivation and setting R(s) = 14/s, we have the following value for U(s):

(C(s)/(1+C(s)G(s)))*R(s) = $\frac{K(TI*s+1)(s+a)*14}{((s+a)TI*s+K(TI*s+1))*s}$

We now apply the initial value theorem to this transfer function to find u(0) a s a function of K and TI: u(0) = 14K

because |u(0)| < 30, and u(0)=14K, we have K<30/14.

Recall from the previous question, K = p1+p2-a. Thus, p1+p2<30/14+a

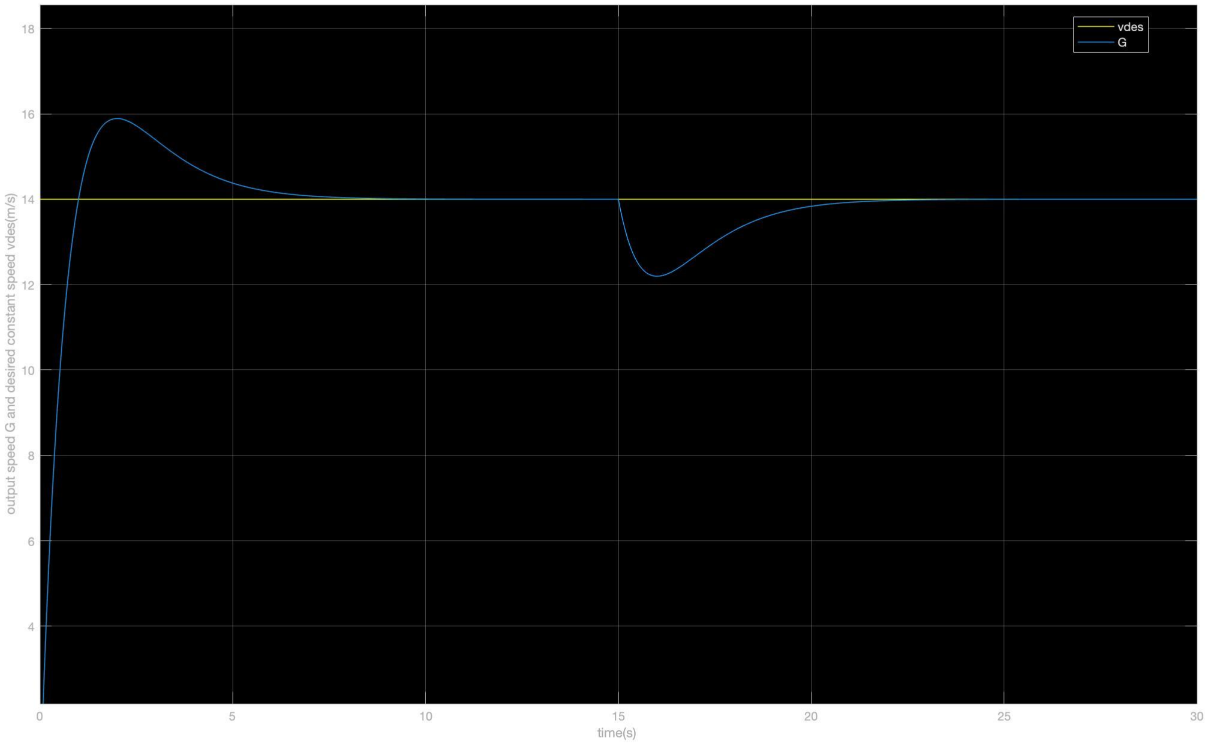
Output 2

• Print your initial choices of p1, p2, and the corresponding values of K and TI . Produce three plots corresponding to the three scopes in the Simulink diagram, and estimate the settling time based on the system response over the first 15 seconds. Then, estimate the settling time after the disturbance kicks in at t = 15 s. Your plots should have titles describing their content.

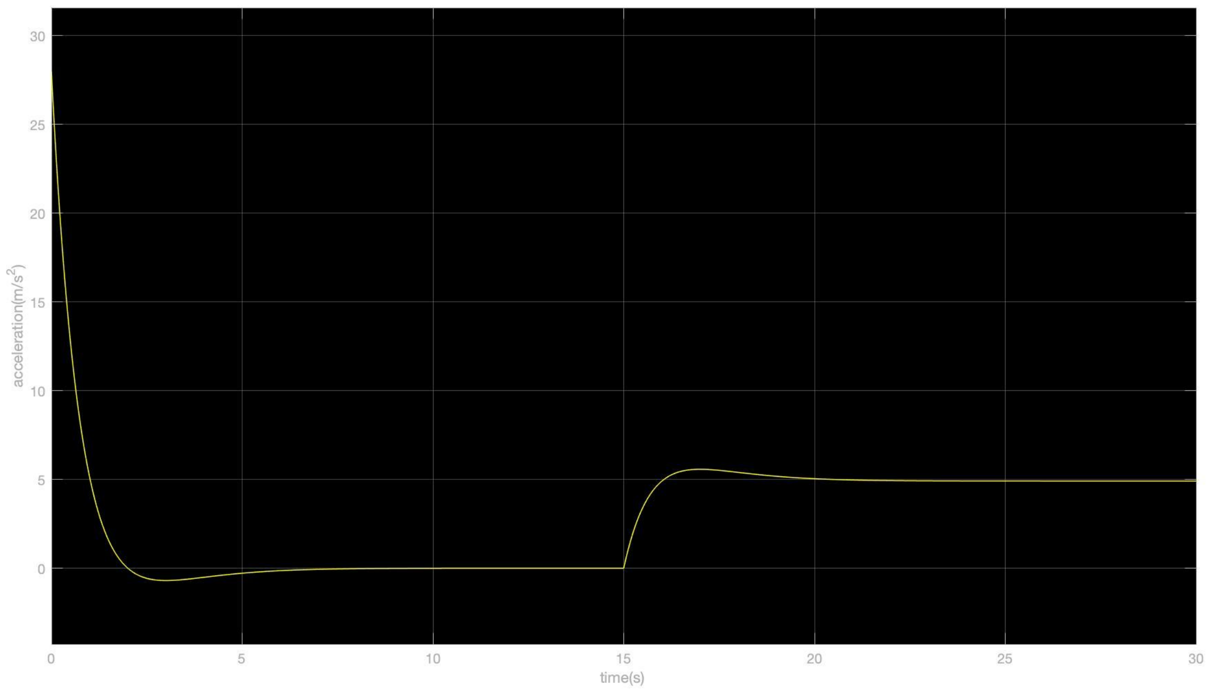
Initially, we pick p1 and p2 to be both 1

Based on the calculation of K = p1+p2-a, TI = (p1+p2-a)/(p1*p2) we derived in output 1, K =1.9990 and TI = 1.9990

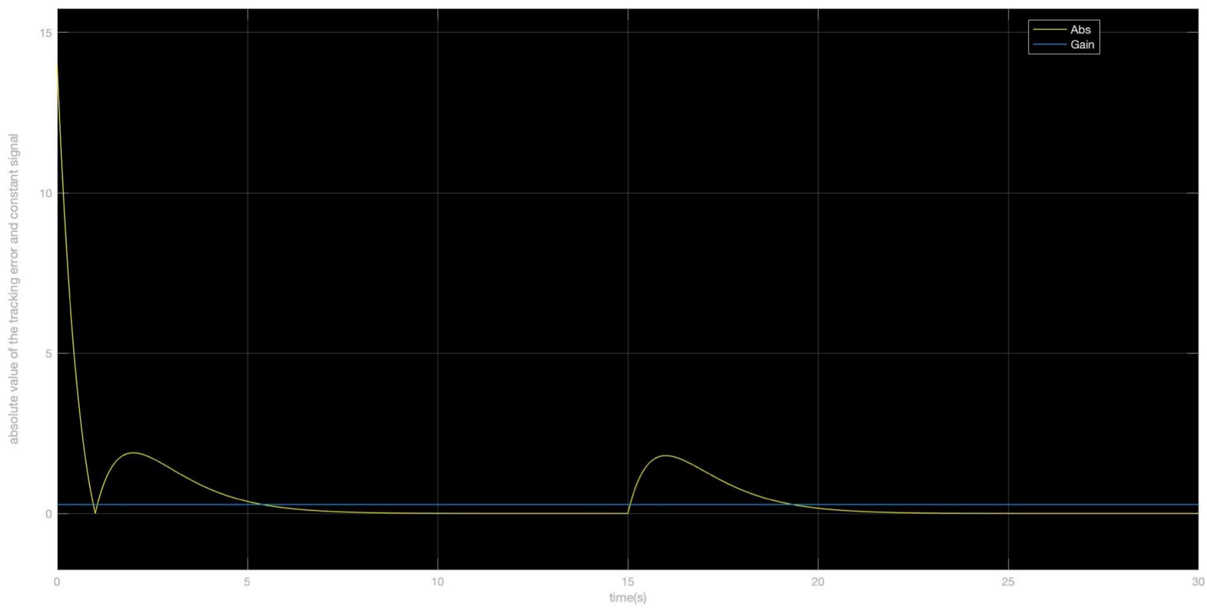
The three plots of the scope output and the reference signal, input u and tracking error is shown below:



scope: output and reference signal



scope: u(input)



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scope: tracking error

To estimate the settling time for the system, we use the cursor to find out the intersection between the absolute value of the tracking error and the constant signal. According to the cursor, the settling time in the first 15s is 5.43 seconds. The settling time after the disturbance kicks in at t = 15 s is 4.46 seconds.

• Comment on your initial findings before tuning. Are all the five specs met? Which ones aren't? Guide the reader of your report through the verification of which specs are met and which ones aren't. Explain the tuning you plan to do to improve the performance and show your reasoning behind it.

Based on the IMP and the justification we showed in output 1, since C(s) = K*(TI*s+1)/(TI*s) for K, TI>0 and G(s) = 1/(s+a), SPEC 1 and 2 are satisfied no matter what positive values K and TI are.

From Output 1, we have set s2 + (p1 + p2)s + p1p2 as the form of the denominator of the transfer function, and the poles are -p1 and -p2. Now in this case, we have poles both at -1, which is definitely on the real axis, so SPEC 3 is met.

The settling time we discovered is 5.43 seconds, which meets SPEC 4.

According to the graph from scope u(input), the initial acceleration u(0) = 28 m/s^2, which satisfies the condition that the control input signal u(t) should be less than 30 m^2/s, and so SPEC 5 is met.

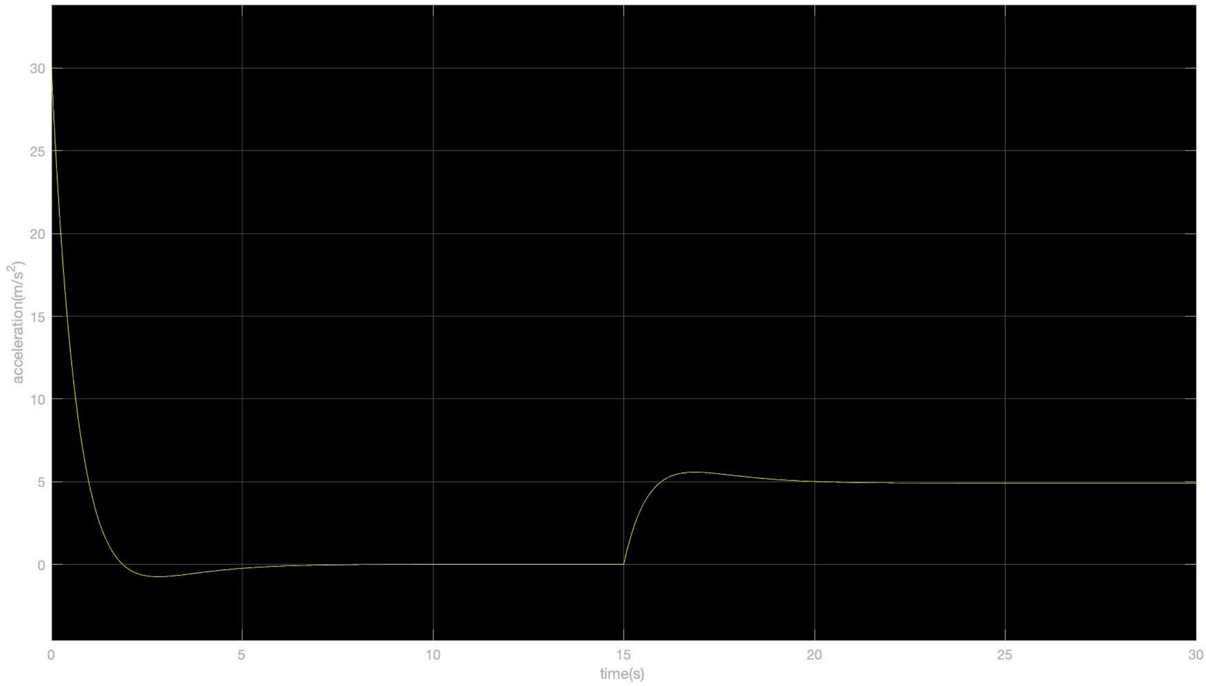
Since we are having a fair settling time, to improve the performance, first we will lower the value for both p1 and p2 to see if the settling time decreases or not. If the settling time is decreased, we will keep lowering the values for p1 and p2 until no better performance can be achieved. If the settling time is increased, we will try to increase the values for p1 and p2 until no better performance can be achieved.

Theoretically, we do not need to test different pairs of values for p1 and p2, and the best performance should be achieved when p1 and p2 are as large as possible. Recall from the lectures that the settling time is the time for y(t) to reach and stay within plus or minus 2% of the steady-state value, and we know poles at -p1 and -p2 in the s domain is e^(-p1*t) and e(-p2*t) in the time domain. As we assign positive values for p1 and p2, e^(-p1*t) and e^(-p2*t) will decay faster if both of them are larger. However, recall we have restrictions on p1 and p2 where 0<p1+p2<30/14, if one of them is larger, the other one must be smaller to satisfy the inequality, and if this is the case, then the smaller value of p1 or p2 will dominate the decay time. Therefore, to achieve the best performance, we should pick a value called alpha for both p1 and p2, and alpha needs to be as large as possible.

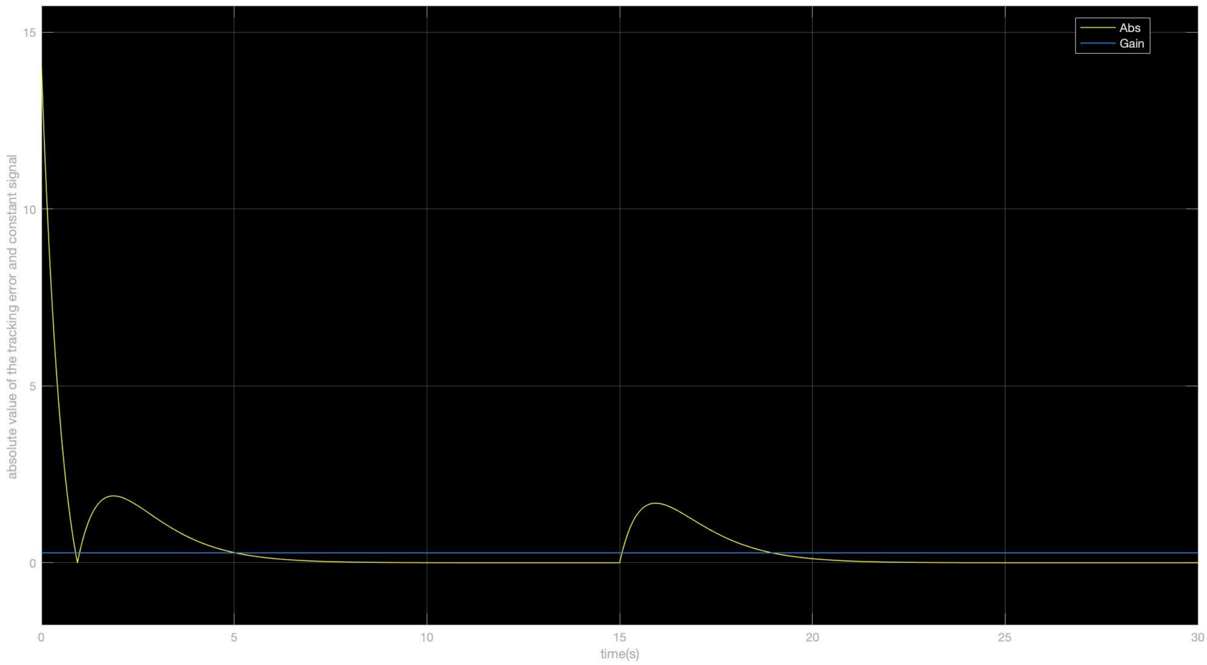
• After tuning, print the values of p1, p2 minimizing the settling time while respecting the bound on u(t). Repeat again your analysis verifying whether all the specs are met. What is the best settling time you could get over the time interval 0 to 15 seconds? What is the settling time over the time interval 15 to 30 seconds? Print the settling time you found using step info and verify that it's the same as the one over the time interval 0 to 15 seconds that you deduced from the plots.

After tuning, the values we select for p1 and p2 are both 15/14.

Below is the tuned scope of u:



Below is the tuned scope of tracking error:



As justified in the previous part the SPEC 1,2,3 are met as long as the p1 and p2 are not exceeding 30/14 + a. The settling time we discovered is 5.03 seconds, which meets SPEC 4.

According to the graph from scope u(input), the initial acceleration u(0) = 29.98 m/s^2, which satisfies the condition that the control input signal u(t) should be less than 30 m^2/s and so SPEC 5 is met.

According to the cursor, the best settling time in the first 15s is 5.03 seconds. The settling time after the disturbance kicks in at t = 15 s is 3.97 seconds.

Obtained from the step info function in Matlab, the settling time should be 5.0309 seconds, which is almost the same as the one we find in the first 15s from the plot.

• You will notice that, after the disturbance signal is enabled, the control signal u(t) will settle to a nonzero value. Find this value, and explain how it conforms with the physics of the problem (a mass on an inclined plane subject to gravity).
From the control signal graph, we can see that it converges to 4.9m/s^2 after the disturbance signal is enabled.
Recall that the disturbance is equal to g*sin(theta), and g=9.81 m/s^2 and theta = -pi/6, so the numerical value for the disturbance is -4.905m/s^2. Consider these in real life: a car is moving on a flat surface for the first 15 seconds, at 15 seconds, we apply a disturbance, that is, the road inclination now becomes uphill. The control input signal will then converge to 4.9m/s^2 so that it can be cancelled out with the disturbance caused by gravity. By making this cancellation, now our model becomes$v dot\ =\ -\frac{B}{M}v$and $y\ =\ v$. Because B/M=1/1000 and now our net acceleration is not dependent on u and disturbance, the change in acceleration is very small, and as such, our goal of converging the car speed to a desired constant speed is achieved.