ECE311F -2021
Lab2

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Output 1.

• Write in your report the equilibrium x¯ ∈ R3 and the control u¯ that you determined above. These quantities should contain ¯ y, and M, La, Ra, g, km as parameters.

$$\bar{x} = [\bar{y},\ 0,\ \sqrt{\frac{g*M}{k_m} * \bar{y}}]^T$$

$$\bar{u} = \sqrt{\frac{g*M}{k_m} * \bar{y}} * R_a$$

• Write the linearization of (3) at x¯ with control u¯, defining all error variables.

$$\dot{\tilde{x}} = \begin{matrix} 0 & 1 & 0 \\ \frac{2*g}{\bar{y}} & 0 & \frac{-2*k_m*\sqrt{\frac{g*M}{k_m}}}{M*\bar{y}} \\ 0 & 0 & \frac{-R_a}{L_a} \end{matrix} * \tilde{x} + \begin{matrix} 0 \\ 0 \\ \frac{1}{L_a} \end{matrix} * \tilde{u}$$

$$\tilde{y} = [\,1\,0\,0\,] * \tilde{x}$$

$$\tilde{x} = [x - \bar{y}, x\,,\ x - \sqrt{\frac{g*M}{k_m} * \bar{y}}]^T$$

$$\tilde{u} = u - \sqrt{\frac{g*M}{k_m} * \bar{y}} * R_a$$

$$\tilde{y} = y - \bar{y}$$

• Double-check carefully your work. Verify that f (x¯, u¯) = 0 and that the various partial derivatives were computed correctly.

Output 2.

• Print the numerically derived matrices A, B and their theoretically derived counterparts, A1, B1.

$$A = \begin{matrix} 0 & 1 & 0 \\ 196.2 & 0 & -62.6418 \\ 0 & 0 & -60 \end{matrix}$$

$$B = \begin{matrix} 0 \\ 0 \\ 20 \end{matrix}$$

$$A1 = \begin{matrix} 0 & 1 & 0 \\ 196.2 & 0 & -626418 \\ 0 & 0 & -60 \end{matrix}$$

$$B1 = \begin{matrix} 0 \\ 0 \\ 20 \end{matrix}$$

• Print the errors described above, and comment on the accuracy of the numerical approximation performed by Matlab/Simulink.

norm(A-A1) = 4.7481^{-6}

norm(B-B1) = 1.1898^{-11}

The difference in A and A1, B and B1 are very small compared to the value of A and B, which is on the scale of 10^-6 and 10^-11. This indicates that the accuracy of the numerical approximation is acceptable.

• Print the eigenvalues of A1 and the poles of G. Note that the poles of the transfer function coincide with the eigenvalues of the matrix A1. Is the linearized model internally stable? Is it BIBO stable? Comment on how your findings about stability conform with physical intuition

$\lambda_1 = 14.0071,\ \lambda_2 = -14.0071,\ \lambda_3 = -60$

poles of G = 14.01, -14.01, -60

The linearized model is neither internally stable nor BIBO stable since there is a positive value of 14.01, which is on the open right half-plane.

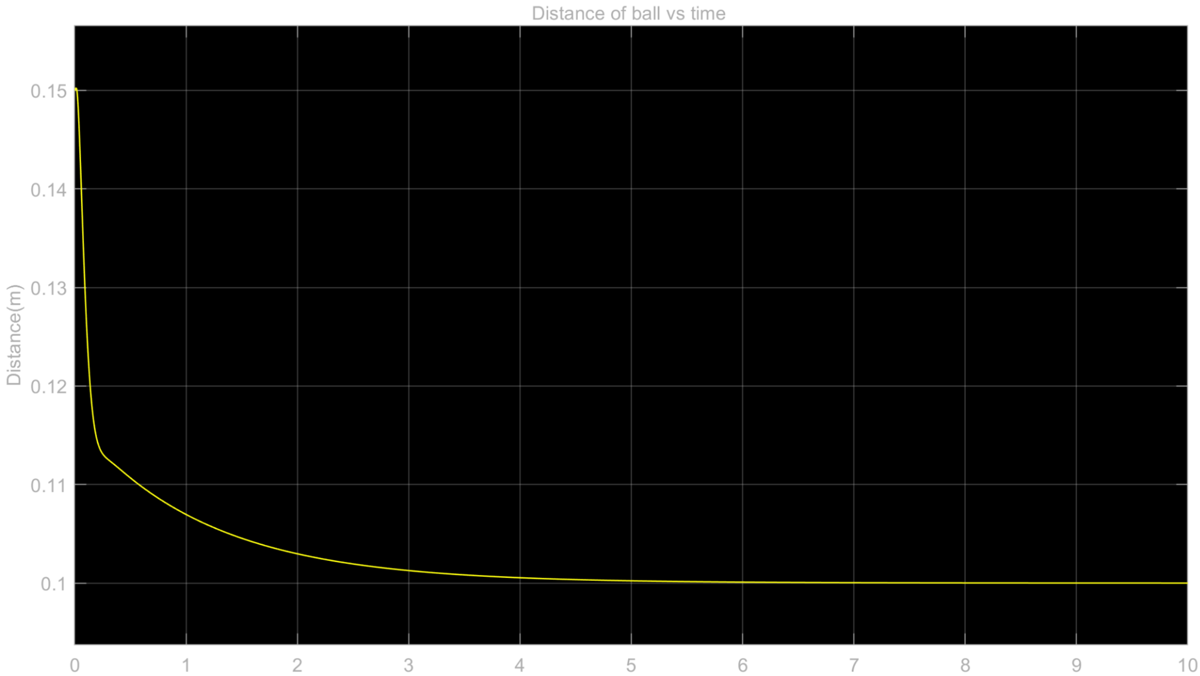
In the physical situation, if the distance of the ball from the bottom of the electromagnet becomes smaller and smaller, the force imparted by the electromagnet will become larger and larger. As a result, this system will not be able to levitate the ball at a fixed distance, when the initial distance of the ball from the bottom of the electromagnet is zero, as time goes by. i.e. this system is NOT stable.

Output 3.

• Print the value of K you found above. Produce a figure of the output y(t) when y(0) = 0.15 m.

The value we found is K = 0.15

The figure of y(t) we produced is as follows



• Describe the procedure you followed in finding K, and any observations you made along the way.

The procedure we followed in finding K is we maintained the initial value y(0) started with the suggested value of 70 and increased with the value 10 for every step.

When K = 70, we could observe the transfer function 1-CONTROLLER*G has two zeros on the real axis with one on the left(OLHP) and one on the right(ORHP) of the imaginary axis. With the increment in K and, we could observe both zeros on the real axis became in the right of the imaginary axis(OLHP).

For the output of y(t) in the time domain. with the increment in K from 70 to 100, the y(t) in the time domain started with blowing up to infinity gradually converges to 0.1.

• Print the range of initial conditions y(0) for which your controller succeeds in stabilizing the ball, and comment on why your controller does not work for initial conditions that are far from the equilibrium.
The range that we found is from 0.0525 to 18.805 meters.

The controller that could not work for the initial condition that far from the equilibrium is the distance that between the ball and the electromagnet is too far. The electromagnet force is too small and could not control it. The ball would not become stable in a fixed position.