ECE311F -2021
Lab4

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Course: ECE311F 2021
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Output 1

• Produce a list of the parameters you found, and explain in your own words how you determined these parameters. You need to show an understanding of the procedure that you followed to get them, and of why various steps were taken in the design of the lead controller.

The following are parameters we found:

- K = 72.0044.
  The following identity is given, K|G(i(initial_crossover))|=1, for finding K, as such, we have K=1/(|G(i(initial_crossover))|). Then we use the command evalfr to find the final value of K.
- $\omega_{MAX} = \overline{\omega}$ = 35.5663.
  As stated in the question, we guarantee that the frequency ω_{MAX} coincides with the crossover frequency of the product $C_1(s)G(s)$, so that the peak phase contribution of $C_1(s)$ is placed at the frequency where it is most needed. Later, we use the command margin to find the final value of ω_{MAX} and $\overline{\omega}$
- T = 0.0889.
  As we impose $\omega_{MAX} = \overline{\omega}$, the basic property of the lead controller can be written as $\overline{\omega} = \frac{1}{T\sqrt{\alpha}}$, which implies that $T = \frac{1}{\overline{\omega}\sqrt{\alpha}} = 0.0889$
- α = 0.1. This parameter is given.

Now, all the parameters have been found for the controller C_1

• Display in one figure the Bode plots of G(s), KG(s), and C1(s)G(s). Explain the effect that the parameter K has on the frequency response of KG(s) when compared to the frequency response of G(s). Then explain the further effect that the lead controller has on the frequency response of C1(s)G(s) when compared to that of KG(s).

The following (figure 1) is the Bode plots of G(s), KG(s), and $C_1(s)G(s)$

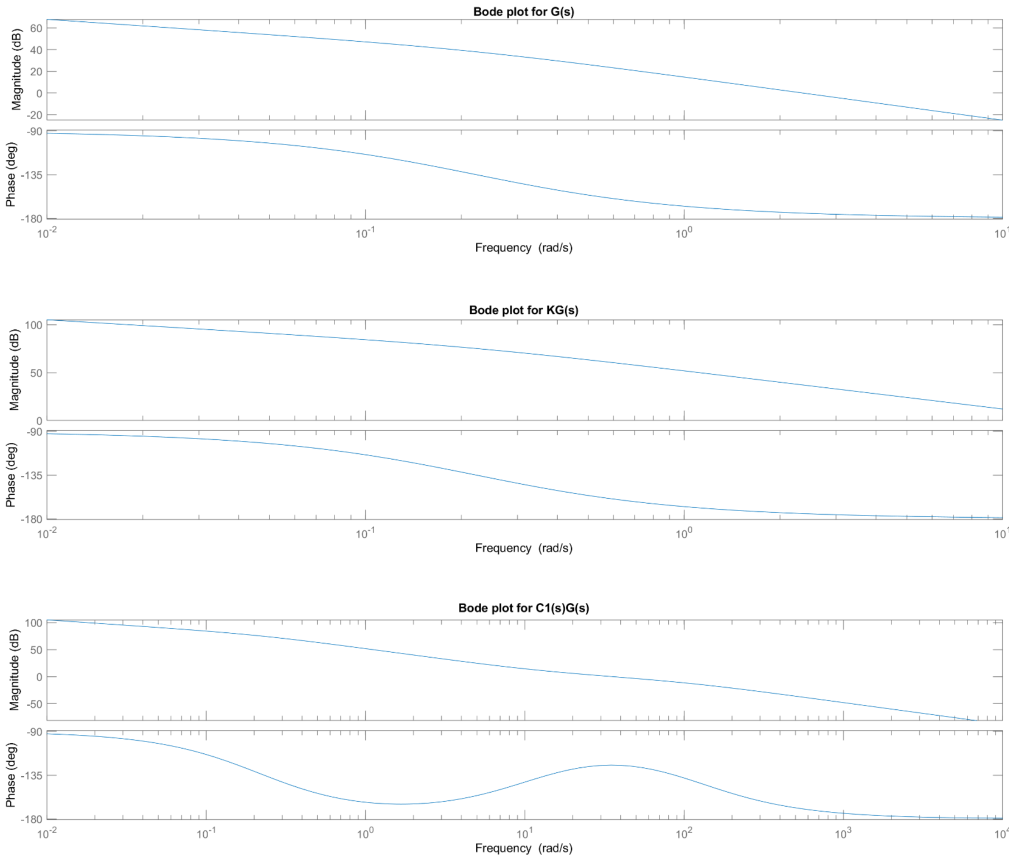

Figure 1: Bode Plots of G(s), KG(s), and C1(s)G(s)

Based on the comparison between G(S) and KG(s), the extra parameter K will cause a new offset to the original G(s) bode plots. In other words, the original G(s) will shift upwards or downwards, depending on the value of K. In this case, since K = 72, the magnitude bode plot of G(s) will shift upwards.

Based on the comparison between KG(s) and $C_1(s)G(s)$, the controller $C_1(s)$ will slightly affect the magnitude of the original system, shift the crossover frequency and increase the phase at frequency ω_{MAX} as well. As a result, we will have a larger phase margin in $C_1(s)G(s)$, compared to the one in $KG(s)$

• Explain how your lead controller meets SPECS 2, 3 but does not yet meet SPEC 1.

As illustrated in previous question, as we found $\omega_{MAX} = \overline{\omega}$ = 35.5663, and the bandwidth needs to be between $\overline{\omega}$ (35.5663) and $\overline{2\omega}$ (71.1326). Thus, the requirements of crossover frequency to be between 20 rad/s and 50 rad/s and the bandwidth to be between 20 rad/s and 100 rad/s, are satisfied. As such, SPEC 2 is met.

The goal of using a lead controller is to increase the phase margin while only slightly shifting the crossover frequency, and in this case, adding this specific lead controller leads to a PM of 55.2612 degrees. This is larger than 45 degrees as required. As such, SPEC 3 is met.

To meet SPEC 1, the tracking error needs to converge to zero asymptotically. By Internal Model Principle, the controller needs to have a pole at zero, however, the lead controller does not have a pole at 0, thus, SPEC 1 is not met at this moment.

Output 2.

• Display the value of the parameter TI that you've chosen, and indicate specifically its relationship to the crossover frequency of C1G. Comment on the extent to which the PI controller changes the gain crossover frequency and the phase margin of C1(s)G(s).

Initially, we choose TI to be 10 times smaller than the gain crossover frequency of $C_1(s)G(s)$, and their relation is given as $0.01\omega_{c1} < 1/TI < 0.1\omega_{c1}$, where ω_{c1} stands for the crossover frequency of $C_1(s)G(s)$. In this case, our selection on TI is TI = 10/(ω_{c1}) ≈ 0.2812. In general, the PI controller will not affect the gain crossover frequency above 1/TI and the phase margin above 10/TI.

• Explain why, in order to check whether SPEC 4 is met, we need to verify whether the magnitude plot of G/(1 + CG) is below the − 34 dB line. Your explanation should rely on the meaning of frequency response, and it should be as clear and precise as possible. Comment: does your controller meet SPEC 4? If it doesn't, there is an error in your design and you need to fix it.

The SPEC 4 says that if the disturbance d(t) is a sinusoid, in steady-state the tracking error is no greater than 2% of the maximum value of |d(t)|.

If the disturbance d(t) is a sinusoid, and assume all poles of $\frac{G}{1+CG}$ are in OLHP (we design it in this way), then we know that in steady state, $e_D(t)$ will be sinusoidal with certain frequency and amplitude $A * |\frac{G(i\omega)}{1+C(i\omega)G(i\omega)}|$, where A is the same coefficient as in d(t). To make sure in steady state $e_D(t)$ is no greater than 2% of the maximum value of |d(t)|, it is the same as saying that we need to make sure $|\frac{G(i\omega)}{1+C(i\omega)G(i\omega)}|$ is less than or equal to 0.02, which is 20log(0.02) = -34dB.By plotting the bode plots of $\frac{G}{1+CG}$, we verify that its magnitude is below the -34dB line, and as such, SPEC 4 is met.

• Produce one figure with the two step responses described earlier as subplots. Comment on these step responses and explain whether or not they confirm that SPEC 1 (asymptotic tracking and disturbance rejection) has been met.

The following (figure 2) is the figure with two step responses of thetades*T_R and d_bar*T_D.

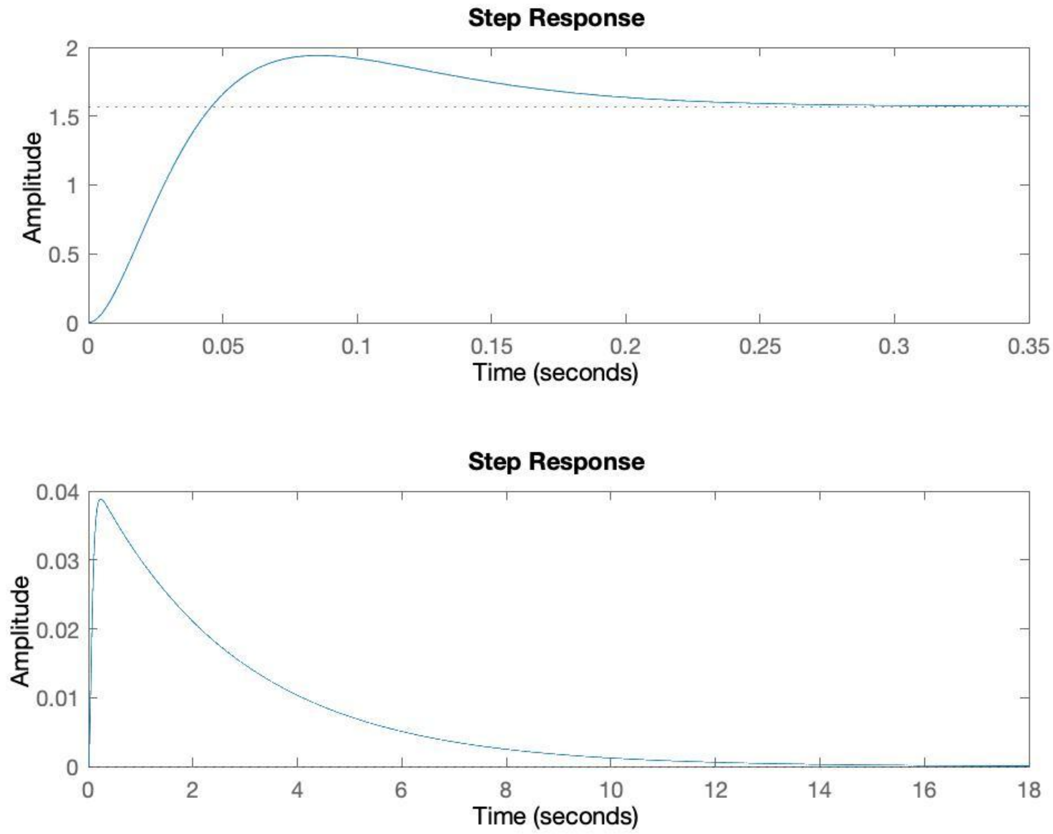

Figure 2: step responses of thetades*T_R and d_bar*T_D

The upper graph in this figure is for $T_R(s)R(s)$, and the lower graph in this figure is for $T_D(s)D(s)$. From upper the graph, the amplitude for $T_R(s)R(s)$ converges to pi/2, this can only be achieved when $e_R(t) = 0$, in other words, asymptotic tracking is achieved. From lower graph, the amplitude for $T_D(s)D(s)$ converges to 0, this can only be achieved when $e_D(t) = 0$, in other words, disturbance rejection is achieved. Therefore, SPEC 1 is met.

• Display the settling time and percent overshoot of y(t) (more precisely, the contribution to it due to r(t)) for your controller.

In this case, we are setting $\frac{1}{TI} = 0.01\omega_{cp1}$.

Due to the contribution of r(t), the settling time is 0.2337s and the percent overshoot is 23.542%.

***Output 3*.**

• Describe your tuning of the parameter TI and its effects on settling time and percent overshoot. Is there a clear trend emerging?

In this case, we would vary 1/TI from 10 to 100 times smaller than the gain crossover frequency of $C_1(s)G(s)$, and keeping the initial_crossover to 20. Some results are summarized in the following table.

TI Value	Settling Time	Percent Overshoot
2.8116	0.2337	23.5420
1.4058	0.2313	24.4437
0.9372	0.2279	25.3325
0.7029	0.2240	26.2066
0.5623	0.2197	27.0655

Table 1. Table for the settling time and percent overshoot when varying the TI value

From the table above, we see the trend that as we decrease the value of TI, the settling time decreases, but the percent overshoot increases.

• To understand the trend you've just observed, we need to develop some intuition about the operation of PI controllers. We will resort to an electrical analogy. Recall that, in the time domain, a PI controller takes the input e(t) and produces the output (assuming K = 1). The parameter 1/TI is the weight of the integral term. Think of the integral term as a capacitor, and of the tracking error as the capacitor current. The tracking error charges the capacitor; the greater the tracking error, the faster the capacitor will accumulate charge; if e(t) is positive (or negative) for a long time, the capacitor will have more time to charge. The integrator term, therefore, will make u(t) increase until e(t) changes sign or tends to zero. If e(t) changes sign from positive to negative, then the integrator will begin discharging, but the time it will take for it to discharge will be longer if the capacitor has a lot of charge to begin with. The parameter 1/TI controls how fast the capacitor charges and discharges in response to a tracking error. If a constant disturbance or constant reference signal causes e(t) to be nonzero, the integrator term will accumulate "charge" until u(t) is large enough to drive e(t) ! 0. This is obviously a beneficial effect. With it, however, there comes a price. Using the intuition just described, explain why more weight on the integral term affects overshoot and settling time in the way you've observed.

First of all, we know that $\frac{1}{TI}$ controls the speed for the capacitor to charge and discharge. When we have a relatively small TI, the weight $\frac{1}{TI}$ will be relatively large, this definitely will make the capacitor discharge faster, and therefore the settling time decreases.

But at the same time, when the value of TI is relatively small, the capacitor charges relatively fast, and the total amount of u(t) will be relatively large. This corresponds to the trend in table 1, as the value of TI gets smaller, the percent overshoot gets larger.

• Describe your tuning of the parameter initial_crossover and its effects on settling time and percent overshoot, and disturbance attenuation. Is there a clear trend emerging?
In this case, we would increase the initial_crossover from 10 to 50 and keep TI to be 10 times smaller to seek for a clear tread, and the results are summarized in the following table.

initial_crossover	Settling Time	Percent Overshoot	Approximate Range of Magnitude of C	Approximated Maximum Magnitude of $\frac{G}{1+CG}$	Phase Margin
10	0.4006	30.6035	28 - 70 dB	-28 dB	49.9289
20	0.1986	31.1692	40 - 66 dB	-39 dB	49.5725
30	0.1321	31.3579	47.5 - 78 dB	-45 dB	49.4537
40	0.0989	31.4523	52 - 86 dB	-50 dB	49.3943
50	0.0791	31.5090	55 - 92 dB	-55 dB	49.3586

Table 2. Table for the settling time, percent overshoot and approximated maximum magnitude of $\frac{G}{1+CG}$ when varying the initial_crossover

From the table above, we see the trend that as we increase the initial_crossover, the settling time decreases, the percent overshoot increases slightly and the disturbance attenuation is getting better. (Recall from the class, good control design will make the magnitude of $\frac{G}{1+CG}$ small over a range of frequencies. Thus, smaller the magnitude of $\frac{G}{1+CG}$, the CLS attenuates the disturbance better.)

• Clearly, increasing the parameter initial_crossover will increase the gain crossover frequency of C(s)G(s), and hence the bandwidth of the closed-loop transfer function CG/(1 + CG). Using this fact and the relationships seen in class between phase margin and closed-loop system bandwidth on one hand, and overshoot and settling time on the other, explain the trend you observed for settling time and overshoot.
Recall the following relations:

- $BW = \frac{L}{1+L} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$.
- PM = $atan(\frac{2\zeta}{-2\zeta^2+\sqrt{1+4\zeta^4}})$

- %OS = $exp(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}})$
- For a fixed PM, (therefore a fixed ζ), the settling time is proportional to $\frac{1}{\omega_b}$

Also recall from class, we conclude that smaller phase margin will lead to higher overshoot and larger crossover frequency will lead to smaller settling time.
Therefore, as the phase margin is slightly decreasing, the ζ is slightly increasing, and we will have a slightly increasing percent overshoot.
Note, when we increase the initial_crossover, the bandwidth of the closed-loop system gets increased, and since the change in phase margin is negligible, the settling time is decreasing.

• Explaining the trend you observed for disturbance attenuation requires some more thought. The transfer function responsible for disturbance attenuation is G/(1+CG). The smaller its magnitude, the greater the attenuation. When you increase initial_crossover, what is the primary effect that causes the magnitude of G/(1 + CG) to change in a way consistent with the trend you observed? You might want to investigate this question by investigating the magnitude plot of C.
Based on our observation(4th column in table 2), when we increase the initial_crossover, the magnitude of C will increase. Thus, the magnitude of $\frac{G}{1+CG}$ will decrease, and we could conclude that this will have greater attenuation.

Output 4.

 • Using the baseline control parameters, produce three figures by saving the scopes (do not take screenshots): the output versus the reference signal, the tracking error, and the control input.

In this case, we are setting $\frac{1}{TI} = 0.1\omega_c$ and initial_crossover = 20 rad/s.

The three figures below are the outputs versus the reference signal, the tracking error, and the control input.

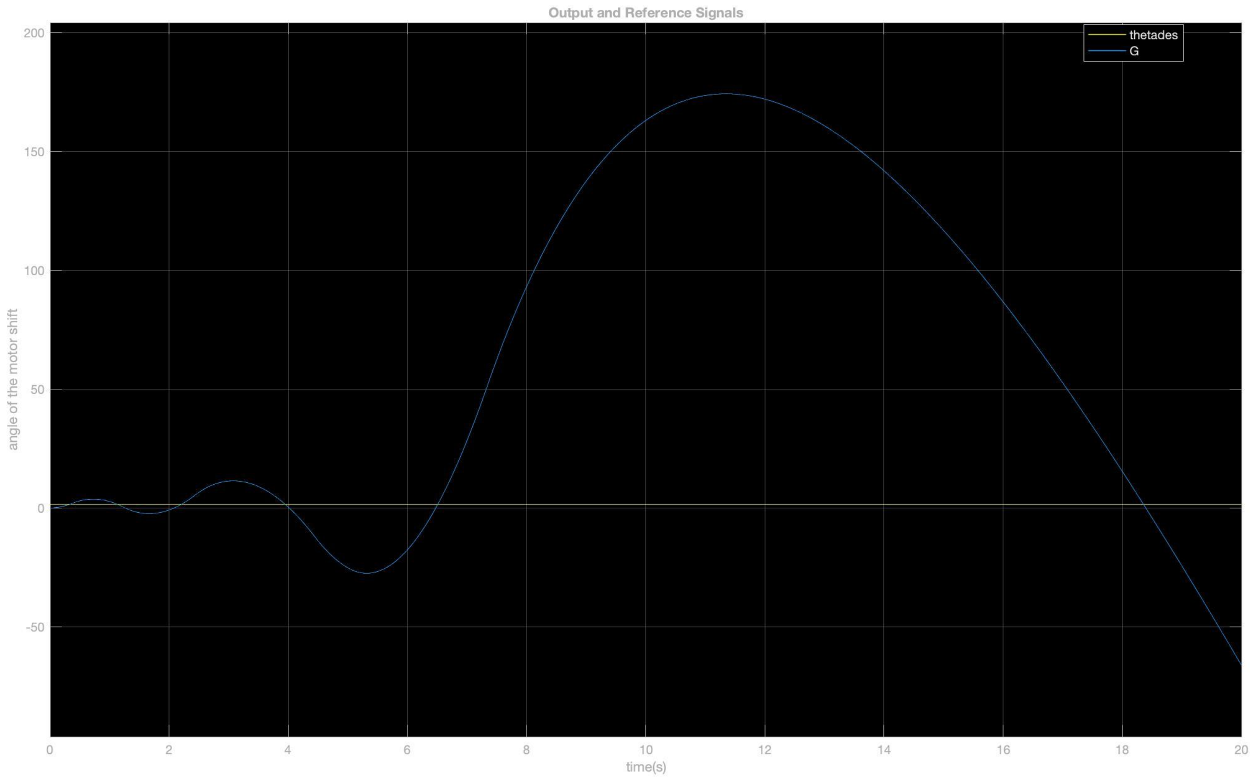

Figure 3: Output and Reference Signals with $\frac{1}{TI} = 0.1\omega_c$

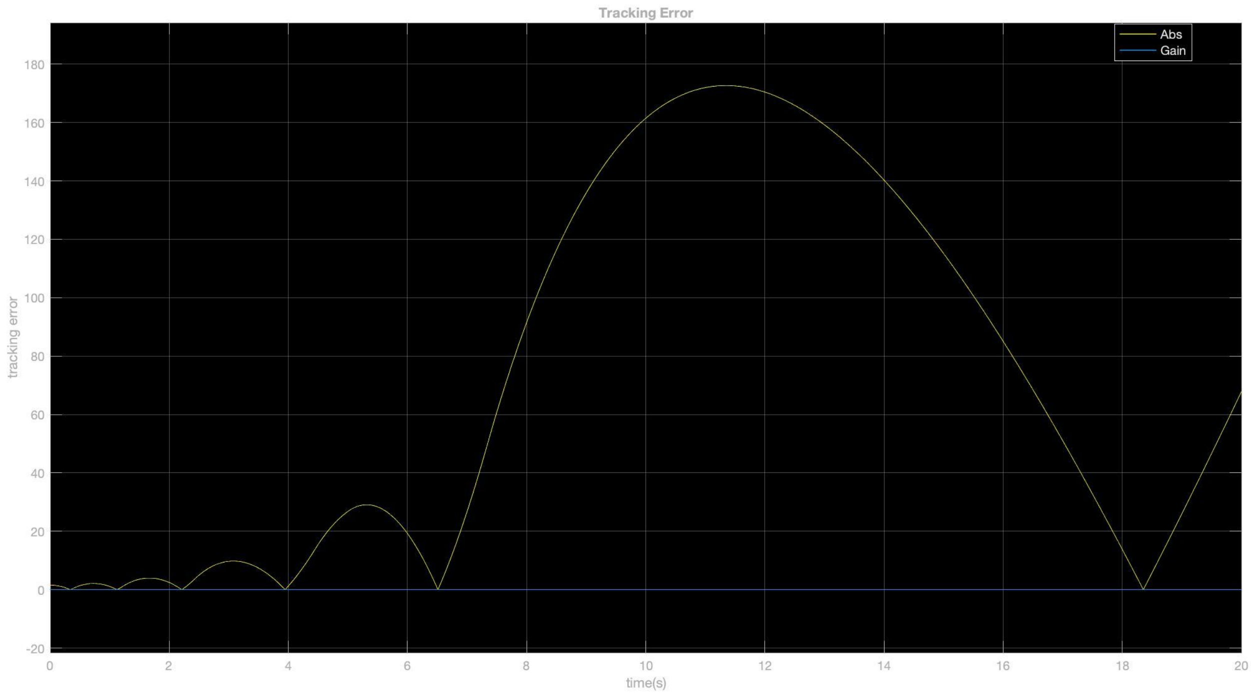

Figure 4: Tracking Error with $\frac{1}{TI} = 0.1\omega_c$

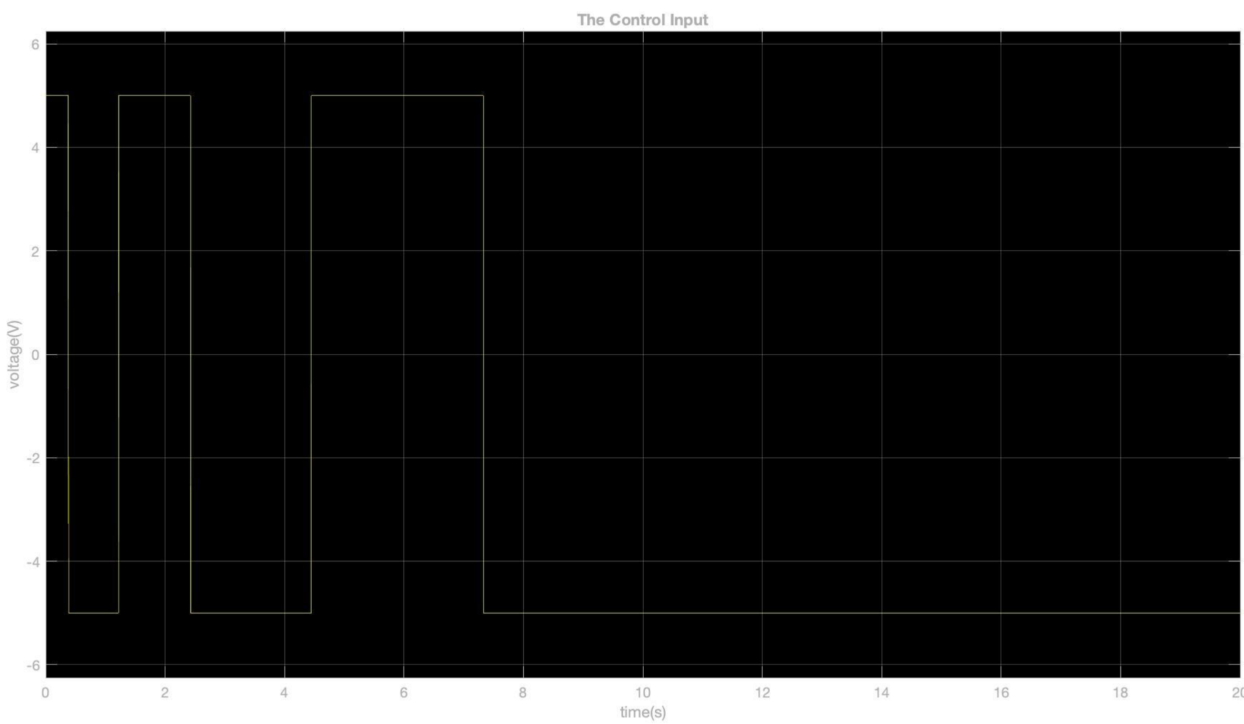

Figure 5: The Control Input with $\frac{1}{TI} = 0.1\omega_c$

Since we used $\frac{1}{TI} = 0.01\omega_c$ in output 2 and this cause the crossover frequency and phase margin of $C_1(s)C_2(s)G(s)$ much closer to those of $C_1(s)G(s)$. Following are figures produced by setting $\frac{1}{TI} = 0.01\omega_c$ and initial_crossover = 20 rad/s.

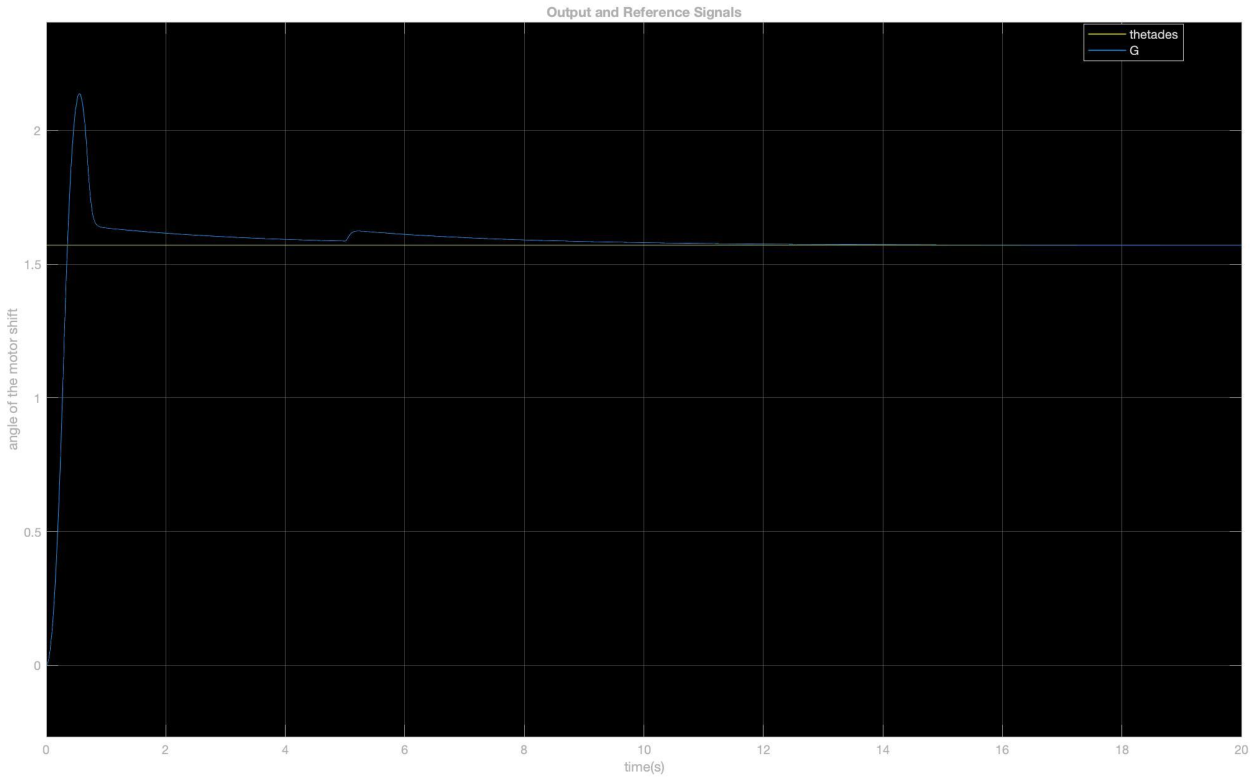

Figure 6: Output and Reference Signals with $\frac{1}{TI} = 0.01\omega_c$

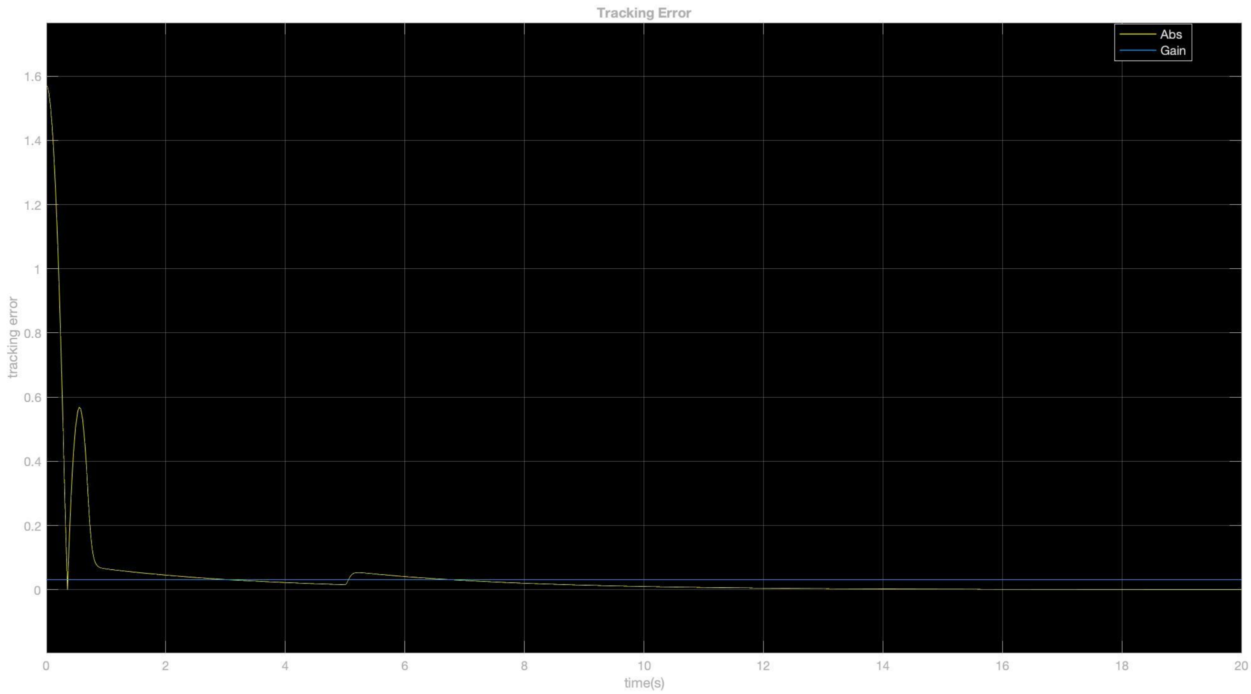

Figure 7:Tracking Error with $\frac{1}{TI} = 0.01\omega_c$

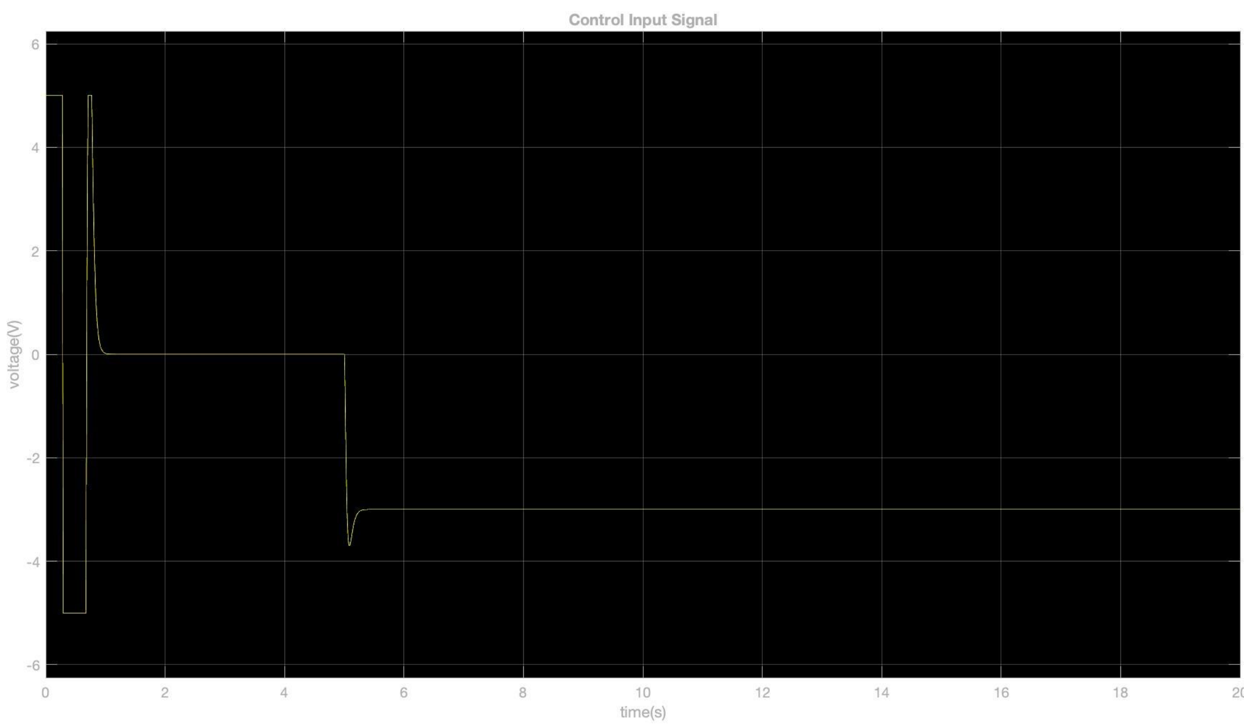

Figure 8: The Control Input with $\frac{1}{TI} = 0.01\omega_c$

• **Using the three figures above, comment on how the presence of actuator saturation degrades the settling time of the closed-loop system as compared to your results in**

Section 3. Observe the control signal u. Why does the saturation have such drastic effects on performance?
The presence of actuator saturation degrades the settling time of the closed-loop system, as it causes the output signal and the tracking error to infinity, e.g. having instability.
The reason for such a drastic effect is due to the nature of the actuator saturation as would work as a threshold and it often takes time for the loop to perform the error tracking and maintain the stability. Thus, this could result in significantly large tracking error and instability.

• Explain what tuning of the parameter initial_crossover you performed, and the effects you noticed on the settling time. What is the limiting value of this parameter before instability ensues? Why does instability occur for large values of initial_crossover? You may want to look at the control signal to gain intuition.
In this case, we are setting $\frac{1}{TI} = 0.01\omega_c$. The limiting value of the initial_crossover is 53.5, in other words, the system is stable for initial_crossover between [20, 53.5]
Here is our tuning procedure: We continuously increase the initial_crossover by 10 rad/s until we reach an instability, which is 60 rad/s in this case. Then, we use this relation $new\ bound = \frac{lower\ bound + upper\ bound}{2}$, where the lower bound is 50 rad/s and the upper bound is 60 rad/s. Then we will get a new bound value of 55 rad/s, and then to check if this value makes the system insatiable. If not, this new bound value will be the lower/upper bound, and then repeat the above procedure until instability occurs.
As the initial_crossover increases, the OS% would increase and the settling time would decrease. However, we would require a controller to input and comply with a stronger control signal. Due to the existence of the saturation, the input for the controller is limited and thus, it could not comply with the system when the initial_crossover is too big.

• Explain what tuning of the parameter TI you performed, and the effects you noticed on the settling time. What is the limiting value of this parameter before instability ensues? You should notice that the effects of TI on settling time (and instability) are pronounced. Using the intuition about PI controllers that we developed in Output 3, explain why integral action is problematic in the presence of actuator saturation.
In this case, we are setting initial_crossover = 20. The limiting value of TI is 0.4373, in other words, the system is stable for TI between [0.4373, 2.8116], or TI needs to be $0.01\omega_c < 1/TI < 0.0643\omega_c$.
The tuning procedure is the same as tuning the initial_crossover, except for the relation is this equality: $0.01\omega_c < \frac{1}{TI} < 0.1\omega_c$, and we can further rewrite as $TI = \frac{1}{x * \omega_c}$ where x is between 0.01 and 0.1.
The integral action is problematic, because when the control is saturated, the PI controller still gets charged by the tracking error. When the tracking error starts to become small, the integral still has a large amount of charge, and this will cause the control input to stay high and saturated.

Output 5.

• Setting Kaw = 1/TI, produce a figure displaying the tracking error and comment on the settling time your controller achieves over the first five seconds of operation. Compare the performance of the antiwindup controller to that of the baseline controller in Section 4. Has the settling time improved, by how much? What about the percent overshoot? How does the response to the onset of a disturbance at t = 5 seconds compare with and without antiwindup mechanism?

In this case, we are setting $\frac{1}{TI} = 0.01\omega_c$ and initial_crossover = 20.

Below is the generated output figure of tracking error versus time with the antiwindup controller.

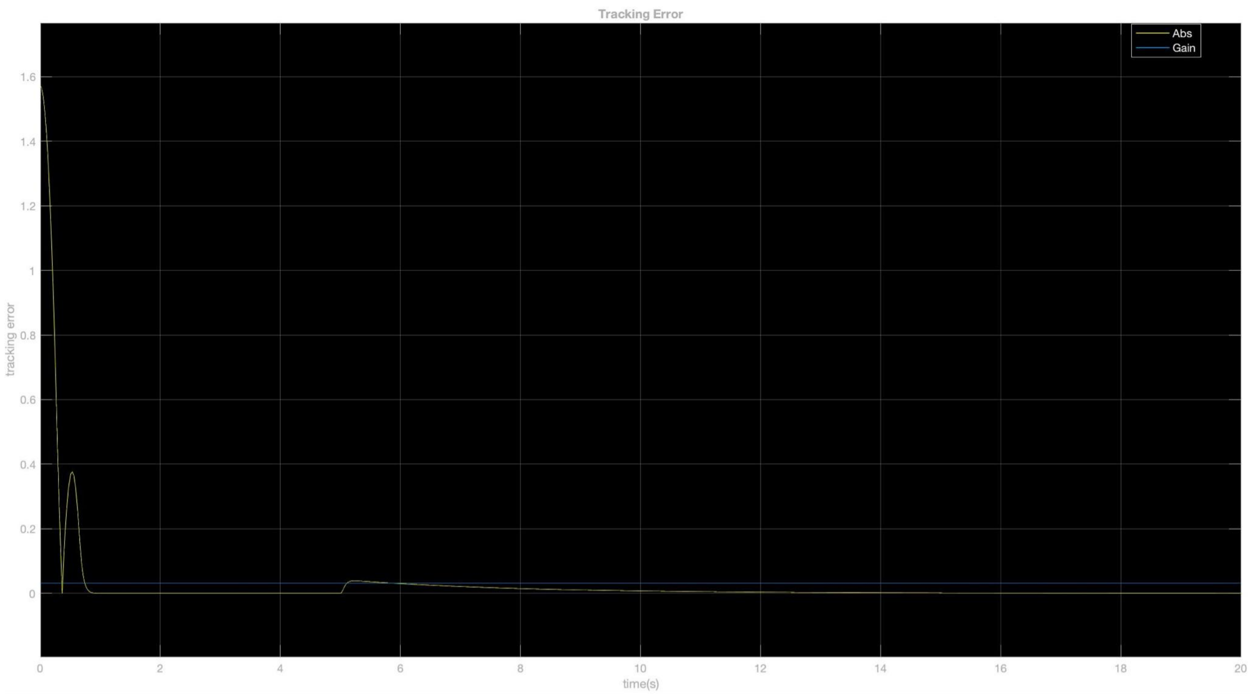

Figure 9: Tracking Error

Based on the figure, the settling time of the tracking error is 0.741s, which means our settling time has improved. The percent overshoot is 23.9% calculating from the output of the system.
If we compare to the tracking error figure we produced in figure 7(settling time is 3.028s and percent overshoot is 36.1%), the settling time is reduced by 2.287 seconds and the percent overshoot is reduced by 12.2%. With the antiwindup mechanism(figure 9), the CLS attenuates the disturbance better compared to the one without the antiwindup(figure7).

• Describe the tuning you performed of the parameter Kaw and display the optimal value you found. Produce a figure with the tracking error and compare it to the result you got in the item above. Has the performance improved by much? Comment on the effects of different values of Kaw on the effectiveness of the antiwindup mechanism.

We tune the Kaw using the following relationship: $Kaw = \frac{x}{TI}$, where x ranges from 0.9 to 1.5, results are summarized in the following table

Kaw	Settling Time
$\frac{0.9}{TI}$	0.760s
$\frac{1}{TI}$	0.741s
$\frac{1.1}{TI}$	0.727s
$\frac{1.2}{TI}$	0.713s
$\frac{1.3}{TI}$	0.7s
$\frac{1.4}{TI}$	0.689s
$\frac{1.5}{TI}$	1.255s

Table 3. Table of different settling times when we vary the value of Kaw

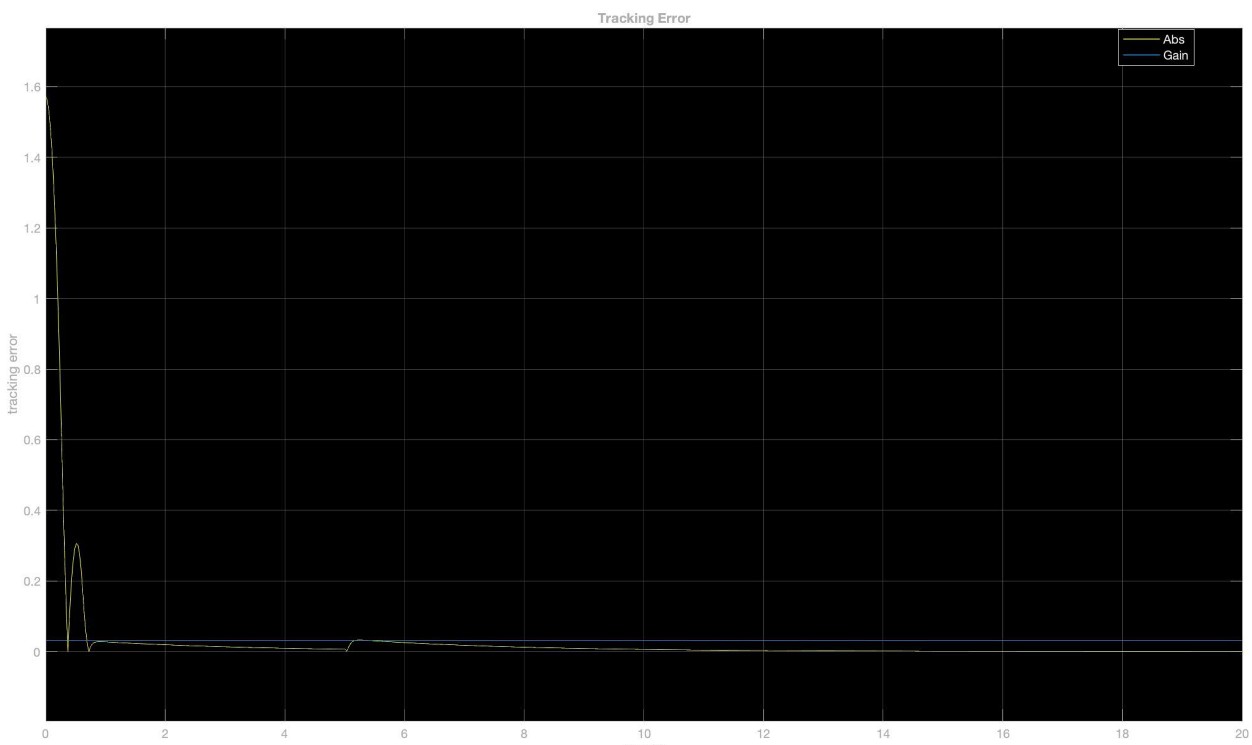

Figure 9: Tracking Error with Kaw = $\frac{1.4}{TI}$

From the table above we could clearly observe the decrease in the settling time which proves that the Kaw is as a threshold at Kaw = 0.4979. For values below Kaw = 0.4979, the settling time decreases, while above Kaw = 0.4979 the settling time increases again. Simultaneously, we take a look at the output, the percentage overshoot has a subtle increase throughout the entire tuning compared to the system without the antiwindup controller.
This is an effective mechanism in improving the performance of the system as it sacrifices a few percent overshoot in the output exchanged for a shorter settling time.