Note 0

Review of Sets and Notation

Set: well-defined collection of objects (**elements** or **members**)

- usually denoted by capital letters
- defined by listing its elements and surrounding the list by curly braces

$$A = \{2, 3, 5, 7, 11\}$$

If x is an element of A, then we write $x \in A$.
If y is not an element of A, then we write $y \notin A$.

Two sets A and B are said to be **equal**, written as $A = B$, if they have the same elements.

- order and repetition of elements do not matter
  - $\{\text{red}, \text{white}, \text{blue}\} = \{\text{blue}, \text{white}, \text{red}\} = \{\text{red}, \text{white}, \text{white}, \text{blue}\}$

More complicated sets might be defined using a different notation. The set of all rational numbers, denoted by $\mathbb{Q}$, can be written as:

$$\mathbb{Q} = \{\frac{a}{b} \mid a, b \text{ are integers}, b \neq 0\}$$

In English, this is read as "$\mathbb{Q}$ is the set of all fractions such that the numerator is an integer and the denominator is a non-zero integer."

Cardinality

Cardinality: the size of a set

If $A = \{1, 2, 3, 4\}$, then the cardinality of A, denoted by $|A|$, is 4. It is possible for the cardinality of a set to be 0. The **empty set**, denoted by the symbol $\emptyset$, is a unique such set.

A set can also have an infinite number of elements, such as the set of all integers, prime numbers, or odd numbers.

Subsets and Proper Subsets

If every element of a set A is also in set B, then A is a **subset** of B, written as $A \subseteq B$. Equivalently we can write $B \supseteq A$, or B is a **superset** of A.

A **proper subset** is a set A that is strictly contained in set B, written as $A \subset B$, meaning that A excludes at lest one element of B.

Consider the set $B = \{1, 2, 3, 4, 5\}$. Then $A = \{1, 2, 3\}$ is both a subset and a proper subset of B, while $C = \{1, 2, 3, 4, 5\}$ is a subset but not a proper subset of B.

Basic properties regarding subsets:

- The empty set, denoted by $\{\}$ or $\emptyset$, is a proper subset of any nonempty set A: $\emptyset \subset A$
- The empty set is a subset of every set B: $\emptyset \subseteq B$
- Every set A is a subset of itself: $A \subseteq A$

Intersections and Unions

The **intersection** of a set A with a set B, written as $A \cap B$, is the set containing all elements that are in both A and B.

Two sets are said to be **disjoint** if $A \cap B = \emptyset$.

The **union** of the set A with a set B, written as $A \cup B$, is the set of all elements which are in either A or B or both.

If A is the set of all positive even numbers, and B is the set of all positive odd integers, then $A \cap B = \emptyset$, and $A \cup B = \mathbb{Z}^+$, or the set of all positive integers.

Properties of intersections and unions:

- $A \cup B = B \cup A$
- $A \cup \emptyset = A$
- $A \cap B = B \cap A$
- $A \cap \emptyset = \emptyset$

Complements

If A and B are two sets, then the **relative complement** of A in B, or the **set difference** between B and A, written as $B - A$ or $B \setminus A$, is the set of elements in B, but not in A: $B \setminus A = \{x \in B \mid x \notin A\}$.

If $B = \{1, 2, 3\}$ and $A = \{3, 4, 5\}$, then $B \setminus A = \{1, 2\}$.