Dis 00

1 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x, y)$ that would make the implication false).

(a) $\forall x \forall y P(x, y) \Rightarrow \forall y \forall x P(x, y)$

The implication is true because quantifiers of the same type can be switched around (i.e. they are commutative):

$$\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y) \text{ and } \exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$$

(b) $\forall x \exists y P(x, y) \Rightarrow \exists y \forall x P(x, y)$

The implication is false. Let $P(x, y)$ be $x = y$; then it is easy how the antecedent is true. For any x chosen, there exists a y that equals x, which is true. However, there does not exist one value of y that can equal all values of x.

(c) $\exists x \forall y P(x, y) \Rightarrow \forall y \exists x P(x, y)$

The implication is true. The first statement says that there exists an x, where x_1 for every y, is true. Setting $x = x_1$ (the second x in the implication) means that for every y, there is an x that makes the second statement true.

2 Equivalences with Quantifiers

3 XOR