Note 1

Propositional Logic

To become fluent in working with mathematical statements, you need to understand the basic framework of the language of mathematics.

Proposition: a statement which is either true or false

These statements are propositions:

1. $\sqrt{3}$ is irrational.
2. $1 + 1 = 5$.

These statements are not propositions:

1. $2 + 2$.
2. $x^2 + 3x = 5$. (since x is unknown)

Propositions should not include fuzzy terms, so these statements aren't propositions either (although some sources may say they are):

1. Arnold Schwarzenegger often eats broccoli. (”often” is fuzzy)
2. Henry VIII was unpopular. (”unpopular” is fuzzy)

Propositions may be joined together to form more complex statements. Let P, Q, and R be variables representing propositions. The simplest way of joining these propositions together is to use the connectives ”and,” ”or,” and ”not.”

1. **Conjunction**: $P \wedge Q$ (”P and Q”). True only when both P and Q are true.
2. **Disjunction**: $P \vee Q$ (”P or Q”). True when at least one of P and Q is true.
3. **Negation**: $\neg P$ (”not P”). True when P is false.

Statements with variables (like these) are called **propositional forms**.

A fundamental principle known as the **law of the excluded middle** says that, for any proposition P, either P is true or $\neg P$ is true (but not both). Thus $P \vee \neg P$ is always true, regardless of the truth value of P. A propositional form is always true regardless of its truth values is called a **tautology**. Conversely, a statement such as $P \wedge \neg P$, which is always false, is called a **contradiction**.

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