Perfect Writeup: Assignment 31 Exercise 2

Henry Yu

Exercise 2

Find the area of the region inside the hypocycloid parametrized by $\vec{r}(\theta) = \langle \cos^3\theta, \sin^3\theta \rangle$ where $0 \leq \theta \leq 2\pi$. (Note from its graph in figure 7.5 that the area is slightly less than $\sqrt{2} \cdot \sqrt{2} = 2$, the area of the smallest circumscribing square.)

We can use $\overrightarrow{F}(x,y) = (0, x)$ because the curl equals 1.

$$\int_0^{2\pi} \langle 0, \cos^3\theta \rangle \cdot \langle -3\cos\theta\sin\theta, 3\sin^2\theta\cos\theta \rangle \, d\theta$$
$$= 3\int_0^{2\pi} \cos^4\theta \sin^2\theta \, d\theta$$
$$= \frac{3\pi}{8}, \text{ which is less than 2.}$$