Midpoint and Trapezoidal Project

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1 Table of Values

$f(x)$	$[a,b]$	N	M	T	E	$\frac{E-T}{E-M}$
$f(x)=2x^3-5x^2+6$	$[0,1]$	20	4.83375	4.8325	$4.8\overline{3}$	-1.999976
$f(x)=3x^4-x^3+7x$	$[0,1]$	20	3.849063047	3.851874375	3.85	-2.0000500559
$sin(\frac{\pi x^2}{9})$	$[0,1]$	20	0.1152781773	0.115383	0.1153465199	-1.996998944
e^{2x}	$[0,1]$	20	3.193197384	3.197189713	3.194528049	-2.000251002
$\sqrt{x+1}$	$[0,1]$	20	1.218966669	1.21892091	1.218951416	-2.0

2 Sides on the Integral

Based on your table, are the midpoint and trapezoid approximations generally on the same side of the exact value of the integral? Justify your answer.

Midpoint and Exact	Trapezoid and Exact
$4.83375 > 4.8\overline{3}$ $(M > E)$	$4.8325 < 4.8\overline{3}$ $(T < E)$
$3.849063047 < 3.85$ $(M < E)$	$3.851874375 > 3.85$ $(T > E)$
$0.1152781773 < 0.1153465199$ $(M < E)$	$0.115383 > 0.1153465199$ $(T > E)$
$3.193197384 < 3.194528049$ $(M < E)$	$3.197189713 > 3.194528049$ $(T > E)$
$1.218966669 > 1.218951416$ $(M > E)$	$1.21892091 < 1.218951416$ $(T < E)$

Based on the information presented, the midpoint and trapezoidal approximations are never on the same side.

3 Approximation Accuracy

Which of the approximations, midpoint or trapezoidal, is generally closer to the exact value of the integral?

$$\delta\,(\%) = \frac{x_{estimate} - x_{actual}}{x_{actual}}$$

Percent Error Formula

Midpoint and Exact	Trapezoid and Exact
$\delta = 0.008621\%$	$\delta = 0.017241\%$
$\delta = 0.024336\%$	$\delta = 0.048685\%$
$\delta = 0.059250\%$	$\delta = 0.031627\%$
$\delta = 0.041655\%$	$\delta = 0.083319\%$
$\delta = 0.001251\%$	$\delta = 0.002503\%$
$\%\text{ avg} = 0.0270226\%$	$\%\text{ avg} = 0.036675\%$

The midpoint approximation is, in most cases, closer to the exact value of the integral than the trapezoidal approximation. The average for the midpoint approximation is closer.