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Aerospace Engineering Department
Aircraft systems

Laboratory N. 10-flight instruments

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A.A. 2022-23

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1 Altimeter: an introduction

All the calculations proposed in this laboratory are made in order to enhance the characteristics of some specific flight tools based on pressure measurement.
First the barometric altimeter is an instrument that withdraws outside pressure from the static port and from the variation of this pressure senses the alteration of the aircraft's altitude, which is the value returned to the cabin.
Pressure is conveyed into a chamber where it is captured by a corrugated metal capsule transducer (disk shaped). The aneroid capsule (as defined), in which the void is made, is able to deform when pressurized and record the variations, which kinematics convert into alterations of altitude (displayed on the altimeter).
The usual configuration of the static ports provides two intakes, situated at the sides of the airplane and connected one with each other. This is accomplished in order to minimize the errors that might arise due to a yaw angle given to the vehicle. Connecting the two ports it is possible to have an average pressure (between the pressures measured at the two different sides) at the middle of the pipeline, that is going to be the pressure used by the instruments. Usually this couple of intakes is actually doubled to divide the values given to the captain, from the ones given to the other pilot, so to have another reduction of the error (the two lines can also be connected if necessary).
The pressure measured from the static port is the ambient atmospheric pressure in which the aircraft is flying, and since the variation of pressure with altitude is known (equation 1), this rate can be used from the altimeter to determine aircraft's altitude.
Consider an air particle of a certain density ρ and height dz, subjected to the gravity g and pressure p (on the lower surface) and $p + dp$ (on the upper surface). Writing the equation of equilibrium along the vertical directions, we obtain the following result: $dp = -\rho \cdot g \cdot dz$. Being air an ideal gas, the pressure is calculated as: $p = \rho RT$[1]. The rate of degrowth of Temperature (considering the troposphere, with the highest altitude of 11.000 m) with altitude is a=6.5 °C/Km.

$$\begin{cases} dp = -\rho \cdot g \cdot dz \\ p = \rho RT \\ \frac{dT}{dz} = -a \end{cases}$$

incorporating the results in the system of equations above, we can explicit the connection between pressure and temperature as following:

$$\frac{P}{P_0} = \left(1 - \frac{az}{T_0}\right)^{\frac{g}{Ra}} \tag{1}$$

P_0 and T_0 are the ground values of pressure and temperature respectively.
The altitude indicated from the altimeter is calculated using this equation (1), where the value of P_0 varies depending on the altimeter's regulation chosen (analyzed forward in this essay), while a and T_0 cannot be altered. The use of the standard temperature $T_0 = 288K$ in the altimeter induces a permanent error.

[1] R is the thermodynamic constant of air (an ideal gas) of $\frac{8{,}314}{29} \frac{J}{mol \cdot K}$

2 Description of the problem and given values

In this laboratory I am going to study the flight mission of an airplane flying from Lima, Perù to La Paz, Bolivia, which corresponding air characteristics are described in the following table:

	Place	Altitude [m]	Local pressure [hPa]	Local temperature[°C]
Take off airport	Lima	35	1020	20
Landing airport	La paz	4060	625	5

As requested, I am going to:

1. esteem the theoretical conditions of sea level pressure and temperature corresponding to each airport;
2. calculate QFE, QNH, QNE altitudes for both the airports (for a maximum height of 11.000 m)
3. draw the profiles of aircraft and cabin's altitude during all the flight mission's time. (assuming the vehicle flying at FL320, with a maximum altitude of 38.000ft[2], stationary weather condition during landing and take off, and a linear variation of cruising altitude between the two airports);
4. draw TAS(z), EAS(z) graphics during the climb to the given flight level (assuming to have a constant CAS of 280kts[3]).

3 Problem resolution

3.1 Sea level estimations

To calculate the theoretical sea level values of pressure P_{lm} and temperature T_{lm} in correspondence of the two airports I am using this equations:

$$T_{lm} = T + a \cdot z$$

$$p_{lm} = p \cdot \left(1 - \frac{a \cdot z}{T_{lm}}\right)^{-\frac{g}{R \cdot a}}$$

As T, p and z I am using the given local values for the two airports. Pressure and temperature at sea level obtained are an accurate estimation of the real standard conditions, which differ from the standard ones (1.013 hPa [4], 288K). Here are the results of the calculations:

airport	sea level temperature [K]	sea level pressure [hPa]
Lima	293.4	1024
La paz	304.5	1007

[2]ft=1 foot=0,3048 m

[3]kts=1 knots=0,514444 m/s

[4]hPa=$10^{-2}Pa$

3.2 Airports' tables

Altimeters can be calibrated according to three different regulations possible of the P_0 value:

- QNE regulation uses $P_0 = P_{STD}$
- QFE regulation uses $P_0 = P_A$ (where P_A is the airport's pressure)
- QNH regulation, instead of P_0 uses a specific pressure, calculated with the formula:

$$p_{QNH} = p_A \cdot \left(1 - \frac{a \cdot z_A}{T_{STD}}\right)^{\frac{-g}{Ra}}$$

Being P_A and z_A airports' values and $T_{STD} = 288K$ the standard value of temperature.

In the following tables (one for each airport) every estimation is made at values of altitude which vary from the airport's altitude to 11.000 meters (the limit altitude for troposphere linear approximation of pressure). The pressure in the second column is the real pressure at the given altitude, calculated with the standard equation (using sea level values we are expecting a realistic value of pressure):

$$p = p_{LM} \cdot \left(1 - \frac{a \cdot z}{T_{LM}}\right)^{\frac{g}{Ra}}$$

$z_{Qfe}[m]$, $z_{Qne}[m]$ and $z_{Qnh}[m]$ are the altitudes indicated by the altimeter in the corresponding conditions of regulation. When the altimeter captures the profile of real pressure (calculated in the second column) a Q regulation of the instrument is based on the resolution of this formula:

$$p = p_Q \cdot \left(1 - \frac{a \cdot z_Q}{Tstd}\right)^{\frac{g}{Ra}}$$

ε_{Qfe}, ε_{Qne} and ε_{Qnh} are the errors recorded on the altitude due to the use of the standard temperature in the altimeter's formula, obtained simply from a difference of values(real altitude-Q altitude).
Here are the values estimated as just described above.[5]

El Alto(LPB) estimations

$z[m]$	$p[hPa]$	$z_{Qfe}[m]$	ε_{Qfe}	$z_{Qne}[m]$	ε_{Qne}	$z_{Qnh}[m]$	ε_{Qnh}
4060	625	0	0	3888	172	4060	0
5000	556	974	-34	4776	224	4945	55
6000	490	2010	-70	5721	279	5886	114
7000	430	3046	-106	6666	334	6827	173
8000	376	4082	-142	7611	389	7768	232
9000	328	5118	-178	8556	444	8709	291
10000	285	6154	-213	9501	498	9650	350
11000	246	7190	-249	10447	553	10591	409

[5] All the approximations are done on the final results, in order to obtain the most accurate estimations without losing any important information.

Lima(LIM) estimations

$z[m]$	$p[hPa]$	$z_{Qfe}[m]$	ε_{Qfe}	$z_{Qne}[m]$	ε_{Qne}	$z_{Qnh}[m]$	ε_{Qnh}
35	1020	0	0	-56	91	35	0
1000	910	949	16	894	106	983	17
2000	807	1931	33	1878	122	1965	35
3000	713	2914	51	2862	138	2947	53
4000	628	3897	68	3846	154	3929	71
5000	552	4880	85	4830	169	4911	88
6000	483	5863	102	5815	185	5894	106
7000	422	6846	119	6799	201	6876	124
8000	367	7829	136	7783	217	7858	142
9000	318	8812	153	8767	233	8840	160
10000	274	9795	170	9751	249	9822	178
11000	235	10778	187	10736	264	10804	195

Drawing the trend of the different regulations' altitudes (Figure 1) compared with the real altitude course I can state that errors are not so relevant for lower altitudes, while become more noticeable at higher altitudes. In QNE calibration (used at cruise flight) temperature and pressure errors are not relevant because it is fundamental to have the correct difference of relative arcrafts altitudes in order to have the correct separation between the vehicles, not to return the real altitude. QFE regulation is mainly used during landing operations and produces an exacts indication at the airport's altitude, so no relevant errors are made. For QNH calibration it might be necessary to induce a correction at a low level flight calculated according to the temperature.

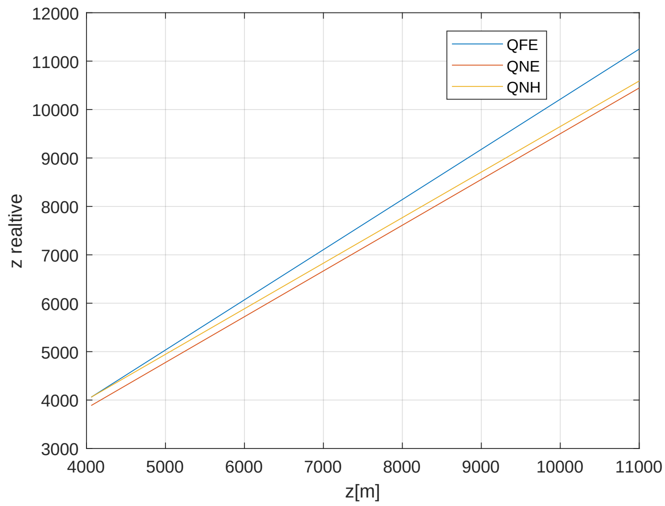


Figure 1: Altimeter's regulations La Paz(LPB)

3.3 Altitude-time profiles

To draw the altitude-time graphics of aircraft and cabin is necessary to find the effective height of flight. Being the flight level given, the isobaric level corresponding to FL320 (32.000 ft) is calculated as following:

$$p_{FL} = p_{STD} \cdot \left(1 - \frac{a \cdot z_{FL}}{T_{STD}}\right)^{\frac{-g}{Ra}}$$

FL320 isn't the real altitude at which the aircraft is going to fly, it only indicates the relative pressure. To find the effective altitude z_{EFF}:

$$p_{FL} = p_{lm} \cdot \left(1 - \frac{az_{EFF}}{T_{lm}}\right)^{\frac{g}{Ra}}$$

Resulting 10.002 m at Lima and 10.267m at La Paz with the corresponding value of pressure of 273,91 hPa.
Assuming these estimations and a normal cabin pressurization, with the use of MATLAB, the profiles of cabin's pressure altitude and aircraft's altitude during the flight mission are drawn considering a cabin rate of ascent of 400 ft/min (=2m/s) and the descent rate of −300 ft/min (=1.524m/s).

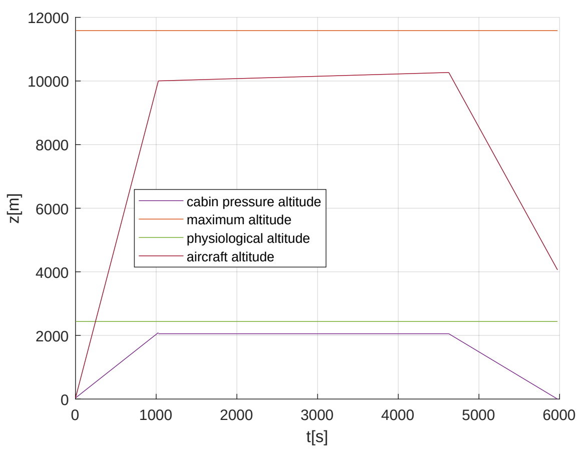


Figure 2: Cabin-aircraft altitudes

Aircraft's altitude during cruise flight is not simply linear horizontal as an effect of the real flight level evaluated. The cabin pressure altitude represented in the graphic is not accurate, since it would allow a landing cabin pressure different from the ambient pressure of the airport (located at 4.060 meters of altitude) with all the leading consequences. El Alto, La Paz's airport, needs a different pressurization of cabin (as discussed in the last section of this laboratory).

3.4 CAS, TAS, EAS

In order to complete the last request of the problem the notions required concern the airspeed indicator, another pressure based flying instrument. Similarly to the altimeter a capsule (receiving both the static and total pressure from the intakes), subjected to the differential pressures is able to return an indication of speed thanks to a specific law. The equation that rules the total pressure of a fluid P_T, with density ρ, in a stagnation point in non-compressible conditions, is given by:

$$p_T = p + \frac{1}{2}\rho v^2$$

In compressible conditions this law changes, and becomes more complex.
Another fundamental instrument is the mach meter, necessary in order to consider additional loads caused by high velocity. The mach meter combines the two values of air speed and altitude read separately from two capsules, and returns a more complete indication.
To draw the graphic (Figure 3) I considered the troposphere, the relative profile of pressure, the total pressure calculated using the given value of CAS (Calibrated Air Speed) and the standard air density ρ_{STD}:

$$P_{TOT} = P + \frac{1}{2} \cdot \rho_{STD} CAS^2$$

I evaluated Mach numbers (considering an isentropic equation) in order to find the TAS (True Air Speed). K is the isentropic coefficient (=1.4 for a bi-atomic gas as air) and c the speed of sound (calculated using as $T = T_{lm} - az$, varying with altitude).

$$P_{TOT} = P \cdot \left(1 - \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$$

$$M = \frac{TAS}{c}, \qquad c = \sqrt{KRT}$$

Last I calculated the EAS (Equivalent Air Speed), considering ρ as variable with altitude:

$$\frac{1}{2} \cdot \rho_{STD} EAS^2 = \frac{1}{2}\rho TAS^2$$

The resulting graphic plots CAS, TAS, EAS velocities and shows their variation with altitude modification (Figure 3).

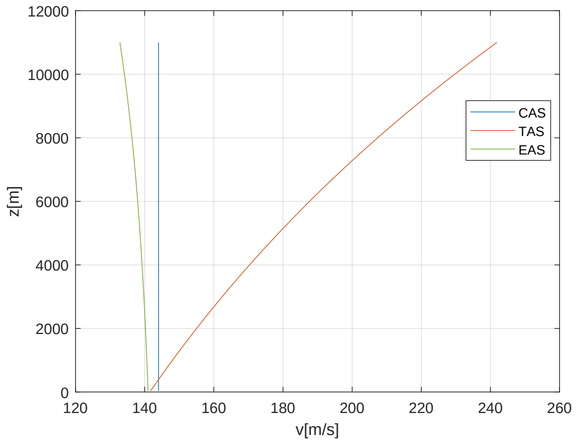
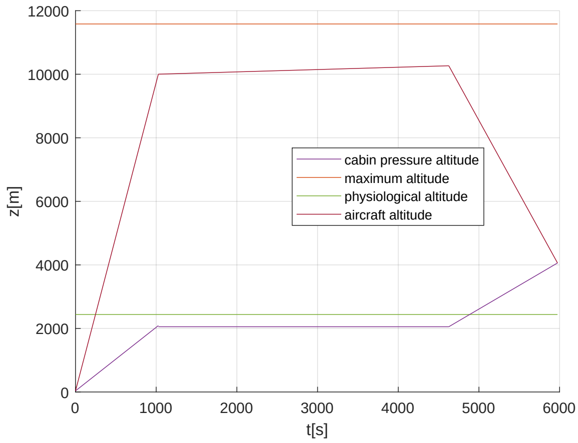


Figure 3: Mach meter velocities

4 El Alto airport-La Paz

La Paz is the highest capital of the world (around 3.625 meters of altitude), so the high altitude airport of El Alto (4.060 m) needs to be approached with some specific procedures, that differ from the usual mainly for the sequence of cabin's pressurization.
While an airplane is on the ground, the interior cabin pressure must be equal to the outside ambient air pressure to allow for easy opening of the exit doors in case of an emergency evacuation. So flying to/from an high elevation airport an aircraft can't be pressurized at 8.000 feet (the physiological altitude that ensures passenger's welfare, the maximum cabin pressure altitude allowable).
As stated from Federal Aviation Administration (FAA) (extracted from a rule proposed on the 04/05/2019) when an airplane takes off from an airport with an elevation greater than 8.000 feet, the cabin pressure altitude must begin at the higher equivalent altitude and slowly decrease until it is less than 8.000 feet.
To land at a high elevation airport, such as El Alto (LPB), the interior cabin pressure altitude will start near 8.000 feet and slowly rise as the airplane descends into the airport, until the interior cabin pressure altitude is the same as the equivalent pressure altitude at the airport when the airplane lands. The maximum cabin pressure altitude of 8.000 feet, is then exceeded when operating in these conditions. Here is a new graphic of aircraft and cabin's altitude considering the given information.



12000
10000
8000
6000
4000
2000
0
z[m]
0
1000
2000
3000
4000
5000
6000
t[s]
cabin pressure altitude
maximum altitude
physiological altitude
aircraft altitude