Politecnico di Milano

Prova finale: Introduzione all'analisi di missioni spaziali
AA 2023-2024

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Data di consegna: 29/11/2023

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1. Introduction

Orbital impulsive maneuvers transfer a spacecraft from an orbit to another. According to the initial and final orbit shapes and planes, these actions differ in energy and time required. The aim of this report is to inspect the range of possible alternatives that allows to get from the initial to the final orbits (both given) and minimize the costs in terms of fuel and time.

After studying the orbits given our team hypothesized some combinations of orbital transfers using the notions acquired in this course and some information found in publications cited at the end of this document. Then we discussed the convenience of these strategies and examined other different options that seemed to be interesting in terms of time or fuel cost. We included all the results in the tables found in the appendix, however we described in detail only the most suitable maneuvers.

Transfers were chosen and elaborated with the full collaboration of each member of the team. All the calculations made have been carried out by each member of the team with a personal MATLAB code. In this way we had a triple check on the results obtained and, to gain even more certainty on the results, we decided to write a testing code which checks the Δv calculated by finding out the new velocity vector and applying it in the maneuver point, then the new orbital parameters. This process was repeated for every impulse, the final orbit thus found is confronted with the assigned one.

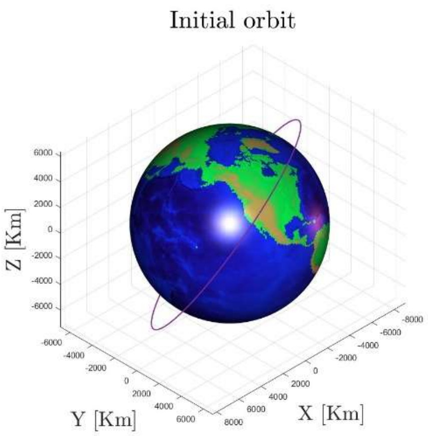
The tables in the appendix contain the calculated values reported with a chosen number of decimal places: time intervals are approximated with integer numbers, while all the other results are expressed to four decimal places.

2. Initial and final orbit characterization

2.1. Initial orbit characterization

The initial orbit is described starting from the following vectors of position and velocity:

```
position = [-7203.4669, -4889.1562, -503.0398]
velocity = [1.8390, -3.1230, -5.4180]
```



these parameters characterize the position taken from the satellite at a specific true anomaly. The function *car2kep* calculates the orbital parameters using the appropriate formulas. c*ar2kep* returns the following values for the initial orbit (a, e, i, Ω, ω, θ):

kep_in = [8146.8018, 0.1091, 0.9814, 0.5577, 0.8578, 2.3532]

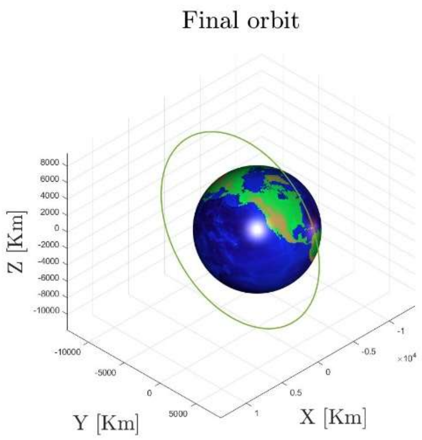
2.2. Final orbit characterization

The final orbit is identified by the given orbital parameters (a, e, i, Ω, ω, θ):

```
kep_fin = [10810.0000, 0.2329, 1.5080, 1.5980, 0.4748, 1.0100]
```

Using function *kep2car*, based on rotation matrices, it is possible to determine the final position and velocity of the satellite:

```
Position_fin = [-589.8035, 765.5691, 9045.3336]
Velocity_fin = [0.0725, -6.8864, 1.8261]
```



As the initial orbit, the final one is elliptical. It is more eccentric than the first and it has a bigger semimajor axis. This means that the final orbit and the initial one have a different shape.
They also have a different inclination:

$$\Delta i = 0.5266\, rad = 30.173\,°$$

and a different right ascension of the ascending node:

$$\Delta\Omega = 1.0403\, rad = 59.6074\,°$$

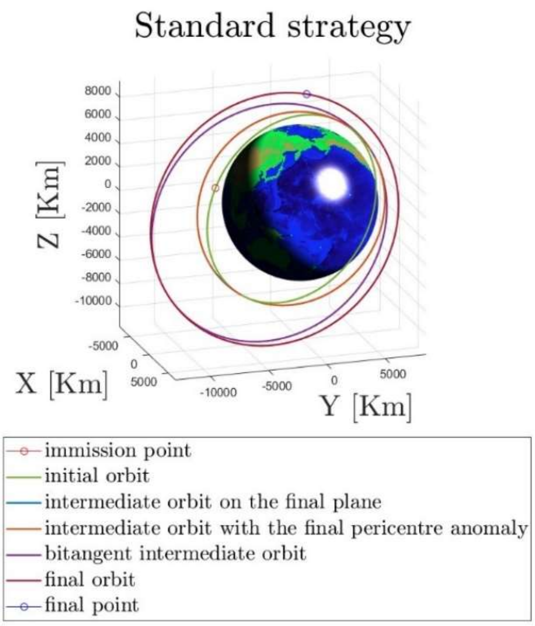
this means that also the pericenter anomaly will be measured from a different axis.
These parameters will affect the orbital transfer choice.

3. Transfer trajectory definition and analysis

The orbits characterization points out the possible transfer strategies. Initial and final orbit don't intersect, are not coplanar and differ for their shape, therefore all the strategies analyzed will contemplate a variation of each orbital parameter previously calculated.

3.1. Standard Strategy [transfer 1]

The standard strategy involves three different maneuvers: change of orbital plane, change of pericenter anomaly and change of shape.
First, the satellite must move on the initial orbit in order to reach the right point for the first transfer; this happens at a true anomaly, reported in table "transfer 1". There, a single impulse is given to the satellite, that starts moving on a new orbit characterized by the same shape but coplanar to the final orbit.
Now, after a waiting time, it is possible to change the pericenter anomaly, entering the satellite on a second transfer orbit.
The last maneuver is a bitangent transfer starting from the pericenter of the second transfer orbit and ending in the apocentre of the final one.
This strategy allows to achieve the final position and velocity of the satellite in a total time of:



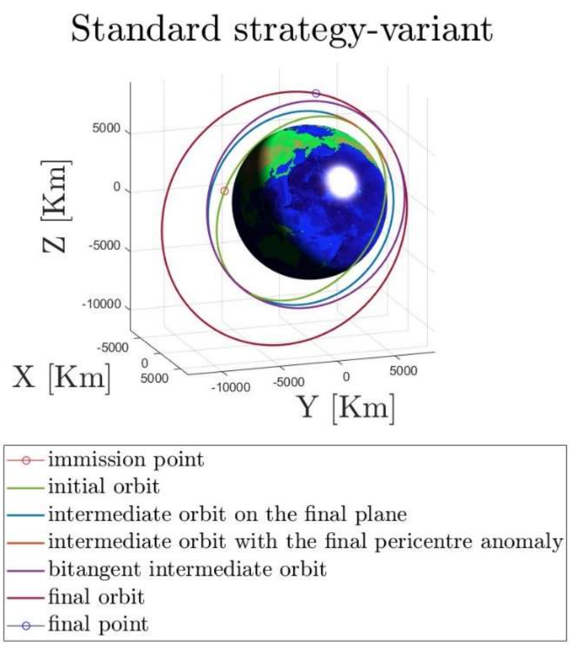
$$\Delta t_{tot} = 24028 \text{ s}$$

with a total variation of velocity:

$$\Delta v_{standard} = 7.4600\ km/s$$

3.2. Standard Strategy – variant [transfer 2]

In the standard strategy the shape of the orbit is changed through a bitangent transfer from the pericenter of an intermediate orbit to the apocentre of the final orbit. Since the initial true anomaly is included between the pericenter and apocenter, a strategy which uses a bitangent transfer from the apocentre to the pericenter results to be more efficient in terms of time spent.
This option is similar to the previous one, performs the same plane change and the same periapsis change, resulting, in part, with the same parameters.
The first advantage is in the time saved to get to the apocentre, from the injection point.



This drop influences the results of the succeeding time intervals calculated. The total time spent to accomplish this strategy is:

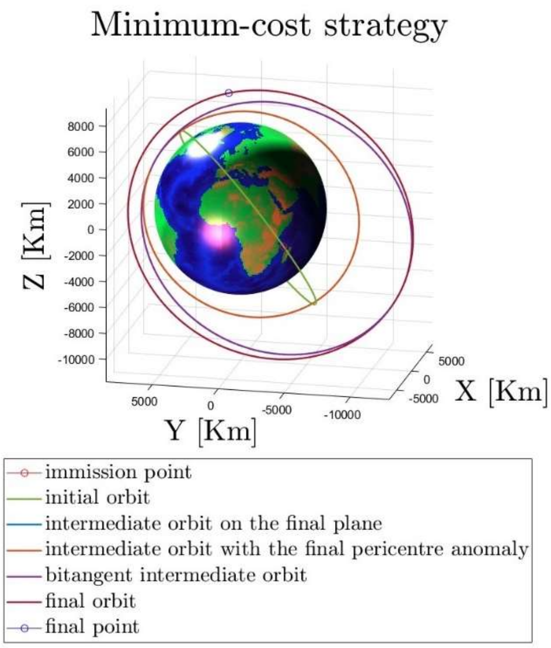
$$\Delta t_{tot} = 13593s$$

Shifting the positions of application of the impulses relative to the bitangent transfer, the variation of velocity changes in comparison to the standard transfer:

$$\Delta v_{tot} = 7.4811\text{km/s}$$

The current strategy, compared to the standard one, provides a 43.43% decrease in time and a 0.28% increase in Δv. We find the added cost as negligible compared to the time saved so we consider this transfer as a solid alternative.

3.3. Minimum cost strategy [transfer 3]



In this alternative strategy the final orbit is reached after four minimized Δv. This means that each of these maneuvers is chosen to get the minimum possible variation of velocity.
The first and second impulses produce a shape change obtained with a bitangent maneuver (from the pericenter to the apocentre). Thanks to a Δv of 0.6283 km/s the satellite first enters the transfer orbit, characterized by the same pericenter radius of the initial orbit and the same apocentre radius of the final orbit. Reached this position another Δv of 0.1974 km/s moves the satellite on an intermediate orbit that has the same shape as the final orbit but lies on the initial plane.
The satellite must now get to a specific point of intersection, between the current orbit and the desired orbit on the new plane. This point is chosen between the two available options, to minimize the necessary velocity for the maneuver.
The third impulse applied in the position just described, returns a new orbit that is on the plane of the last orbit and has the same shape of the last orbit (thanks to the previous maneuver).
The last variation of velocity ($\Delta v = 0.0108$ km/s) produces a rotation on the orbital plane which changes the pericenter anomaly. The last impulse is small because of the lower value of the variation of anomaly at the pericenter between initial and final orbit and does not affect much the total Δv.
The resultant values of the whole strategy are:

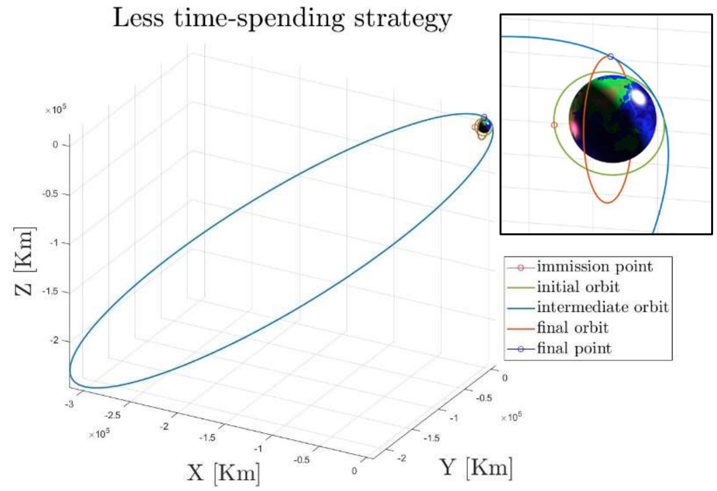
$$\Delta \text{v}_{\text{tot}} = 5.9925\ km/s$$

$$\Delta t_{tot} = 16709s$$

3.4. Less time-spending strategy [transfer 8]

In this strategy the satellite reaches the ascending node of the initial orbit, there enters the transfer orbit that intersects the final one exactly at the ending point given.
The right ascension of the ascending node was set equal to the one of the initial orbit, while the pericenter anomaly of the intermediate orbit was chosen with a time-optimizing code. This influences the value of the true anomaly of the point of intersection with the initial orbit. Using spherical

geometry, it is possible to calculate the transfer orbit inclination, semimajor axis and eccentricity and have a complete characterization of the intermediate orbit.



The first impulse is given in the ascending node of the initial orbit, the second one exactly in the final point (that belongs both to the final and the transfer orbit) to correct inclination, pericenter anomaly and right ascension of the ascending node.

The total cost is:

$$\Delta v = 13.4279\ km/s$$

while the total time needed is:

$$\Delta t = 5145\ s$$

These values underline this strategy short time and the increasing cost in Δv compared to the others.

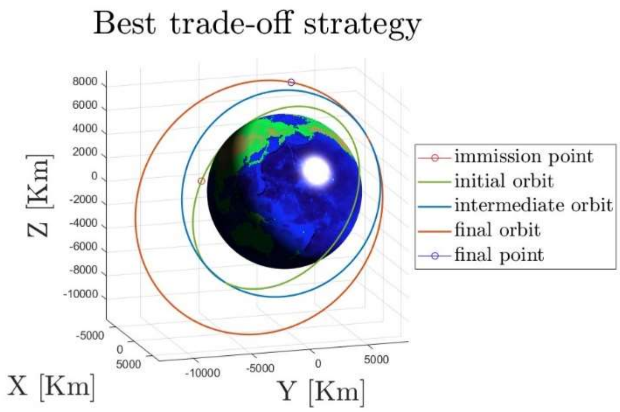
In the optimization code some corrections were necessary: we verified the eccentricity value was included between 0 and 1, so we excluded parabolic and hyperbolic orbits, and that the minimum radius of the orbit was a sufficient distance from the Earth's surface.

3.5. Best trade-off strategy [transfer 7]

This orbital transfer was initially thought with a saving time aim. It is based on two single impulses, the first given in order to change the orbital plane and the pericenter anomaly, that means changing parameters i, Ω, ω, the second in order to change the orbit shape, so the parameter a.

The satellite moves on the initial orbit from the starting position to the nearest point useful to the plane change, where it enters the transfer orbit. To characterize it, it is necessary to set the pericenter radius, the inclination and the right ascension of the ascending node equal to the ones of the final orbit. The true anomaly of the transfer orbit corresponding to the point of plane change is obtained using the spherical triangle. It is also fundamental to equalize the radius formula of the initial orbit and the one of the transfer orbit. In this way all the orbital parameters can be obtained, and the transfer orbit is completely defined.

The satellite travels up to the pericenter of the transfer orbit, that coincides with the one of the final orbit. There the second impulse is given, and the orbit shape becomes the same as the final one.



The total time is:

$$\Delta t_{tot} = 6275\ s$$

This is the second most convenient maneuver in terms of time.

The Delta-v becomes:

$$\Delta v_{tot} = 7.3808 km/s$$

This strategy, born to save time, appears to be also the second most convenient maneuver in terms of Δv.

4. Discussion and conclusions

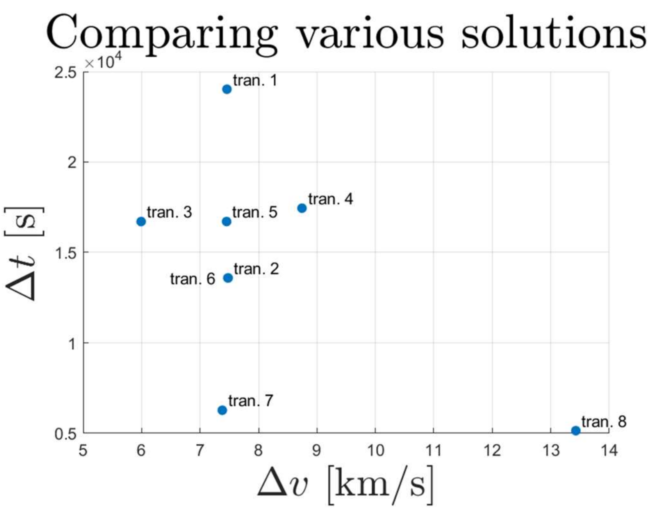
This report aimed to analyze different maneuvers to minimize Δv and Δt necessary to get from the injection point to the final one. We first considered the standard strategy as required an then we tried to find better strategies in terms of performance.

Considering the same standard strategy maneuvers there are two ways to reduce the total mission time: keeping the same order but modifying the initial point of the bitangent transfer or switching the maneuvers order. Both these alternatives cause a decrease in terms of time; among these strategies the most convenient is the second one (Table 2).
Afterwards we unified the change of shape and change of pericenter anomaly maneuvers in a one-tangent transfer (Table 5 and Table 6). In transfer 5 and 6 we first changed plane and then used this combined transfer, starting from the pericenter in the former and then from the apocentre in the latter. The quickest transfer is number 6; its spending time is similar to transfer 2.
We also hypothesize two strategies that need a single intermediate orbit to reach the final one. In strategy 7 the satellite reaches the point of change plane calculated in transfer 2, then enters a transfer orbit that reaches the pericenter of the final one, while in strategy 8 the transfer orbit passes through the ascending node of the initial orbit and the final point. These strategies change all the orbital parameters at the same time and the total variation of velocity is determined by only two single impulses.
Considering all these strategies, transfer 8 appears to be the most convenient in terms of time.

Aiming to minimize the Δv, we used the combination of the maneuvers described before and asked our MATLAB functions to return the results minimized in terms of velocity. As expected, transfer 8 (the quickest) resulted to be the most expensive. Transfers 1, 2, 5, 6 and 7 have similar Δv, therefore their convenience is based on time. The cheapest strategy is transfer 3.

In our study we evaluated the possible bielliptic transfers (change of plane or change of shape) too but did not develop the strategy since it was not convenient for the two orbits given. As a matter of facts, the difference of inclination ($\Delta i = 30.1730°$) resulted to be out the suitability range ($38.94° < i < 60°$) for the plane change transfer and, considering the ratio of semimajor axes ($a2/a1 = 1.3269$), the inequality $(a2/a1) > 11.94$ was not satisfied so neither the shape change is convenient.

In conclusion, as shown in the graph on the right, depending on the mission target, the strategy choice would be different. Among the different strategies studied, transfer 7 is characterized by a favorable Δt and does not need a high variation of velocity, while transfer 3 is the most suitable in terms of Δv. Our choice would fall on these two transfers.



5. Appendix

SYMBOL	MANOEUVRES		
S	Change of shape (tangent)	C	Change of shape (Secant
P	Change of plane	**O**	Change of anomaly at pericenter

Transfer 1 (Standard)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1757	*P*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*6.6286*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*205.9848*	
4765	*O*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*359.7878*	*0.0057*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.2040*	*0.2122*	
12079	*S*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.2040*	*0*	*0.6283*
		10292.9006	*0.2948*	*86.4020*	*91.5587*	*27.2040*	*0*	
17276	*S*	*10292.9006*	*0.2948*	*86.4020*	*91.5587*	*27.2040*	*180*	*0.1974*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*180*	
24028	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*7.4600*

Transfer 2 (Standard-variant)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1757	*P*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*6.6286*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*205.9848*	
4765	*O*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*359.7878*	*0.0057*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.2040*	*0.2122*	
8420	*S*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.2040*	*180*	*0.2287*
		8663.9012	*0.0429*	*86.4020*	*91.5587*	*27.2040*	*180*	
12433	*S*	*8663.9012*	*0.0429*	*86.4020*	*91.5587*	*27.2040*	*0*	*0.6180*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*0*	
13593	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*7.4811*

Transfer 3 (Minimum cost strategy)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
4768	*S*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*0*	*0.6283*
		10292.9006	*0.2948*	*56.2291*	*31.9512*	*49.1496*	*0*	
9964	*S*	*10292.9006*	*0.2948*	*56.2291*	*31.9512*	*49.1496*	*180*	*0.1974*
		10810.0000	*0.2329*	*56.2291*	*31.9512*	*49.1496*	*180*	
11201	*P*	*10810.0000*	*0.2329*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*5.1560*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.6284*	*205.9848*	
15553	*O*	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.6284*	*359.7878*	*0.0108*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*0.2122*	
16709	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*5.9925*

Transfer 4 (Shape-plane from apoapsis)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1109	*S*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*180*	*0.2287*
		8663.9012	*0.0429*	*56.2291*	*31.9512*	*49.1496*	*180*	
5122	*S*	*8663.9012*	*0.0429*	*56.2291*	*31.9512*	*49.1496*	*0*	*0.6180*
		10810.0000	*0.2329*	*56.2291*	*31.9512*	*49.1496*	*0*	
5617	*P*	*10810.0000*	*0.2329*	*56.2291*	*31.9512*	*49.1496*	*25.9848*	*7.8866*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.6284*	*25.9848*	
10704	*O*	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.6284*	*179.7878*	*0.0108*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*180.2122*	
17447	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*8,7441*

Transfer 5 (Change of plane-direct maneuver to the final apoapsis)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1757	*P*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*6.6286*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*205.9848*	
4761	*OC*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*359.5756*	*0.6283*
		10292,9104	*0.2948*	*86.4020*	*91.5587*	*5*	*0*	
9958	*S*	*10292,9104*	*0.2948*	*86.4020*	*91.5587*	*27.2040*	*180*	*0.1974*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*180*	
16710	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*7.4544*

Transfer 6 (Change of plane- direct maneuver to the final periapsis)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1757	*P*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*6.6286*
		8146.8018	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*205.9848*	
8416	*OC*	*8146.8018*	*0.1091*	*86.4020*	*91.5587*	*27.6284*	*179.5756*	*0. 2288*
		8663.8860	*0.0429*	*86.4020*	*91.5587*	*27.2040*	*180*	
12429	*S*	*8663.8860*	*0.0429*	*86.4020*	*91.5587*	*27.2040*	*0*	*0.6180*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*0*	
13590	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*7.4755*

Transfer 7 (Best trade-off strategy)

t (s)	type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
1757	*POC*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*205.9848*	*6.7490*
		8627.4910	*0.0388*	*86.4020*	*91.5587*	*27.6284*	*206.4092*	
5115	*S*	*8627.4910*	*0.0388*	*86.4020*	*91.5587*	*27.2040*	*0*	*0.6318*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*0*	
6275	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*7.3808*

Transfer 8 (Less time spending)

t (s)	Type	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	-	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*134.8296*	
3951	*POC*	*8146.8018*	*0.1091*	*56.2291*	*31.9512*	*49.1496*	*310.8504*	*5.2015*
		229150.0303	0.9697	*83.9311*	*31.9512*	32.0000	328.0000	
5145	*POC*	229150.0303	0.9697	*83.9311*	*31.9512*	*32.0000*	*58.6003*	*8.2264*
		10810.0000	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	
5145	-	*10810.0000*	*0.2329*	*86.4020*	*91.5587*	*27.2040*	*57.8687*	*13.4279*

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