UNIT - III
Artificial Intelligence

III / II CSE, R 16 - JNTUK

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Contents

- **Logic concepts:**
  - Propositional calculus
  - Proportional logic
  - Natural deduction system
  - Axiomatic system
  - Semantic tableau system in proportional logic
  - Resolution refutation in proportional logic
  - Predicate logic

Propositional Calculus

- A set of rules are used to combine simple propositions to form compound propositions
- Few logical operators are used, which are known as **connectives**
- E.g. not(~), and(Ʌ), or(Ѵ), implies(->), etc.
- A well formed formula is defined as a symbol or a string of symbols generated by the formal grammar of a formal language

Truth table

- A Truth Table is used to provide operational definitions of important logical operators
- The logical constants in PC are true and false, these are represented as T and F
- The truth values of well formed formulae are calculated by using the truth table approach

Table 4.1 Truth Table for Logical Operators

A	B	~A	A ∧ B	A ∨ B	A → B	A ↔ B
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Equivalence laws

- Equivalence laws are used to reduce or simplify given well formed formula
- Or to derive a new formula from the existing formula
- These laws can be verified using the truth table approach

Table 4.3 Equivalence Laws

Name of Relation	Equivalence Relations
Commutative Law	$A \vee B \equiv B \vee A$ $A \wedge B \equiv B \wedge A$
Associative Law	$A \vee (B \vee C) \equiv (A \vee B) \vee C$ $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$
Double Negation	$\sim(\sim A) \equiv A$
Distributive Laws	$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
De Morgan's Laws	$\sim(A \vee B) \equiv \sim A \wedge \sim B$ $\sim(A \wedge B) \equiv \sim A \vee \sim B$

Table 4.3 (Contd.)

Name of Relation	Equivalence Relations
Absorption Laws	$A \vee (A \wedge B) \equiv A$ $A \wedge (A \vee B) \equiv A$ $A \vee (\sim A \wedge B) \equiv A \vee B$ $A \wedge (\sim A \vee B) \equiv A \wedge B$
Idempotence	$A \vee A \equiv A$ $A \wedge A \equiv A$
Excluded Middle Law	$A \vee \sim A \equiv$ T (True)
Contradiction Law	$A \wedge \sim A \equiv F$ (False)
Commonly used equivalence relations	$A \vee F \equiv A$ $A \vee T \equiv T$ $A \wedge T \equiv A$ $A \wedge F \equiv F$ $A \rightarrow B \equiv \sim A \vee B$ $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$ $\equiv (A \wedge B) \vee (\sim A \wedge \sim B)$

Proportional logic

- It deals with the validity, satisfiability (also known as consistency), and unsatisfiability of a formula and the derivation of a new formula using equivalence laws
- A formula α is said to be tautology iff the value of α is true for all its interpretations
- The validity, satisfiability, and unsatisfiabilityof a formula may be determined as follows:

- A formula α is said to be valid iff it is tautology
- A formula α is said to be satisfiable if there exists at least one interpretation for which α is true
- A formula α is said to be unsatisfiable if the value of α is false under all interpretations
- E.g. if it is humid then it will rain and since it is humid today it will rain
- Solution:
  - A: It is humid
  - B: It will rain

- The formula α corresponding to the given sentence is α: [(A->B)^A]->B

Table 4.5 Truth Table for $[(A \rightarrow B) \wedge A] \rightarrow B$

A	B	$A \rightarrow B = (X)$	$X \wedge A = (Y)$	$Y \rightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

- Truth table approach is easy for evaluating consistency, validity, etc. of a formula
- But the size of truth table grows exponentially
- If a formula contains n atoms, then the truth table will contain 2^n entries
- Some other methods are concerned are:
  - Natural deduction system
  - Axiomatic system
  - Semantic tagleu method
  - Resolution refutation method

Natural deduction system

- It mimics the pattern of natural reasoning
- The system is based on a set of deductive inference rules
- Assuming that A_1, A_2,…A_k, where 1<=k<=n, are set of atoms and α_j, where 1<=j<=m, and β are well formed formula
- The inference rules may be stated as in table

Table 4.6 NDS Rules Table

Rule Name	Symbol	Rule	Description
Introducing ∧	$(I{:}\wedge)$	If $A_1, \ldots, A_n$ then $A_1 \wedge \ldots \wedge A_n$	If $A_1, \ldots, A_n$ are true, then their conjunction $A_1 \wedge \ldots \wedge A_n$ is also *true*.
Eliminating ∧	$(E{:}\wedge)$	If $A_1 \wedge \ldots \wedge A_n$ then $A_i\ (1 \le i \le n)$	If $A_1 \wedge \ldots \wedge A_n$ is *true*, then any A_i is also *true*.
Introducing ∨	$(I{:}\vee)$	If any $A_i\ (1 \le i \le n)$ then $A_1 \vee \ldots \vee A_n$	If any $A_i\ (1 \le i \le n)$ is *true*, then $A_1 \vee \ldots \vee A_n$ is also *true*
Eliminating ∨	$(E{:}\vee)$	If $A_1 \vee \ldots \vee A_n, A_1 \to A, \ldots, A_n \to A$ then A	If $A_1 \vee \ldots \vee A_n, A_1 \to A, A_2 \to A, \ldots,$ and $A_n \to A$ are *true*, then A is *true*.
Introducing →	$(I : \to)$	If from $\alpha_1, \ldots, \alpha_n$ infer β is proved then $\alpha_1 \wedge \ldots \wedge \alpha_n \to \beta$ is proved	If given that $\alpha_1, \alpha_2, \ldots,$ and α_n are *true* and from these we deduce β then $\alpha_1 \wedge \ldots \wedge \alpha_n \to \beta$ is also *true*.

Table 4.6 (Contd.)

Rule Name	Symbol	Rule	Description
Eliminating $\rightarrow$	$(E: \rightarrow)$	If $A_1 \rightarrow A, A_1$, then A	If $A_1 \rightarrow A$ and A_1 are *true* then A is also *true*. This is called *Modus Ponen* rule.
Introducing $\leftrightarrow$	$(I: \leftrightarrow)$	If $A_1 \rightarrow A_2$, $A_2 \rightarrow A_1$ then $A_1 \leftrightarrow A_2$	If $A_1 \rightarrow A_2$ and $A_2 \rightarrow A_1$ are *true* then $A_1 \leftrightarrow A_2$ is also *true*.
Elimination $\leftrightarrow$	$(E: \leftrightarrow)$	If $A_1 \leftrightarrow A_2$ then $A_1 \rightarrow A_2, A_2 \rightarrow A_1$	If $A_1 \leftrightarrow A_2$ is *true* then $A_1 \rightarrow A_2$ and $A_2 \rightarrow A_1$ are *true*
Introducing ~	$(I: \sim)$	*If from A infer* $A_1 \wedge \sim A_1$ is proved then $\sim A$ is proved	If from A (which is *true*), a contradiction is proved then truth of $\sim A$ is also proved
Eliminating ~	$(E: \sim)$	*If from* $\sim A$ *infer* $A_1 \wedge \sim A_1$ is proved then A is proved	If from $\sim A$, a contradiction is proved then truth of A is also proved

- A theorem in NDS written as, from α1,… α2 infer β leads to the interpretation that β is deduced from a set of hypotheses {α1,… α2} are assumed to be true in a given context
- Therefore the theorem β is also true in the same context
- Thus we can conclude that β is consistent
- A theorem that is written as infer β implies that there are no hypotheses and β is true under all interpretations, i.e. β is a tautology

- E.g. prove that A ∧ (B∨C) is deduced from A∧B
- Solution: The theorem in NDS can be written as from A∧B infer A∧(B∨C) in NDS

Table 4.7 Proof of the Theorem for Example 4.3

Description	Formula	Comments
Theorem	*from* $A \wedge B$ *infer* $A \wedge (B \vee C)$	*To be proved*
Hypothesis (given)	$A \wedge B$	1
E: ∧ (1)	A	2
E: ∧ (1)	B	3
I: ∨ (3)	$B \vee C$	4
I: ∧ (2, 4)	$A \wedge (B \vee C)$	Proved

Axiomatic system

- It is based on a set of three axioms and one rule of deduction
- It is as powerful as Truth table & NDS approach
- In this, a guess is required in selection of appropriate axiom(s)
- In this, only two logical operators, not(~) and implication(->) are allowed to forma a formula
- Any formula can be written using the ~ and ->

- E.g.
  - A∧B≡ ~(~A∨~B) ≡ ~(A->~B)
  - A∨B ≡ ~A->B
  - A<->B ≡ (A->B)∧(B->A) ≡ ~[(A->B)->~(B->A)]
- In axiomatic system, there are three axioms, which are always true(or valid), one rule is called Modus Ponen (MP) as follows:
  - Axiom1: A->(B->A)
  - Axiom2: [A->(B->C)]-> [(A->B)->(A->C)]
  - Axiom3: (~A->~B)->(B->A)