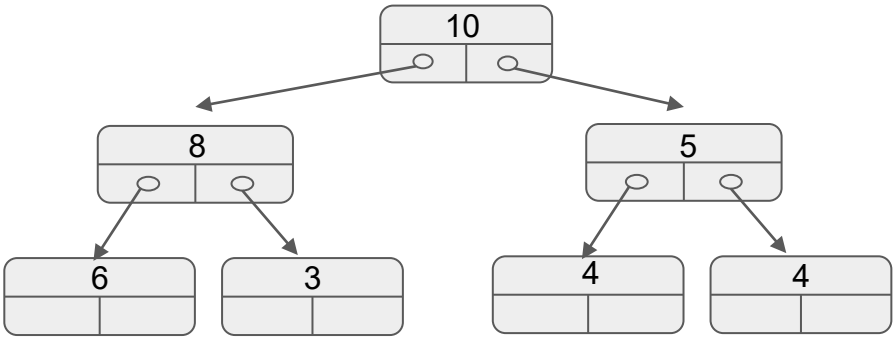
Definition: A tree is a **complete tree** if every level but the last level is completely full, and the last level has its nodes all the way to the **left**.

Property: A complete tree's size and height are related by: *height ~ log(size)*

Definition: A tree is in **max** (min) **heap order** if every node's key is **greater** (less) than or equal to all of its childrens' keys.

Definition: A **max** (min) **heap** is a **complete tree** that is in **max** (min) **heap order**.



10	8	5	6	3	4	4

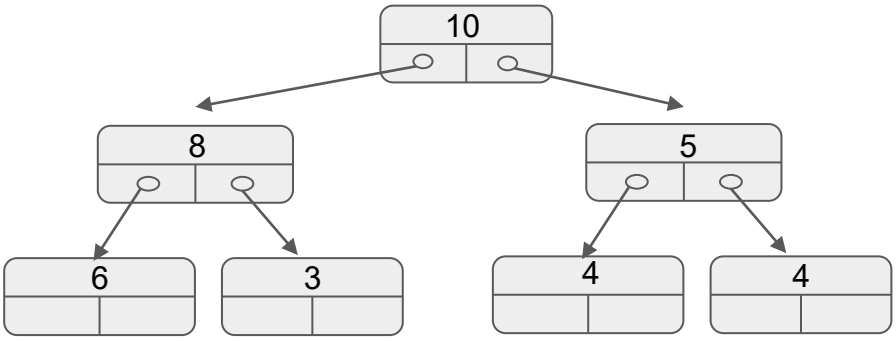
Key	index	Parent	Left	Right
10	0	N/A	1	2
8	1	0	3	4
5	2			
6	3	1	N/A	N/A
3	4			
4	5	2	N/A	N/A
4	6			

```
class BT<K,V> {

Node<K,V> root;

...


V get(Node<K,V> node, K key) {
    if(node == null) { return null; }

    if(node.key.equals(key)) { return node.value; }

    V leftResult = get(node.left, key);
    V rightResult = get(node.right, key);
    if(leftResult != null) { return leftResult; }
    if(rightResult != null) { return rightResult; }
    return null;
}

}
```

```
class Heap<K,V> {

List<Entry<K,V>> entries;

int left(int index) {



}

int right(int index) {



}

V get(int index, K key) {
    if(index >= this.entries.size()) { return null; }
    Entry<K,V> entry = entries.get(index);

    if(entry.key.equals(key)) { return entry.value; }

    V leftResult = get(          left(index)       , key);
    V rightResult = get(          right(index)       , key);
    if(leftResult != null) { return leftResult; }
    if(rightResult != null) { return rightResult; }
    return null;
}

}
```

K poll()

// remove and return largest element

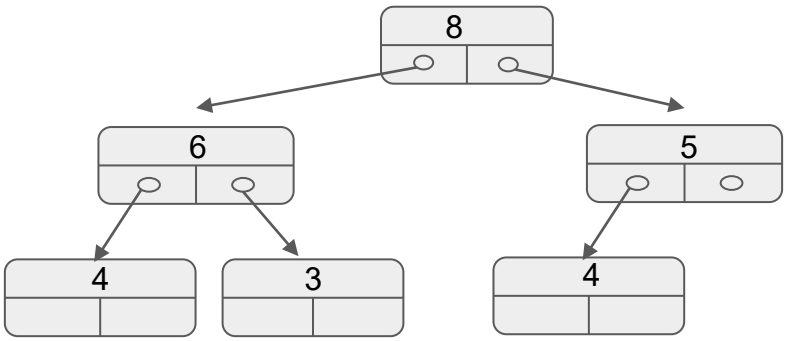


10	8	5	6	3	4	4

void add(K k)
// add the element, ensuring heap - ness

add(2)

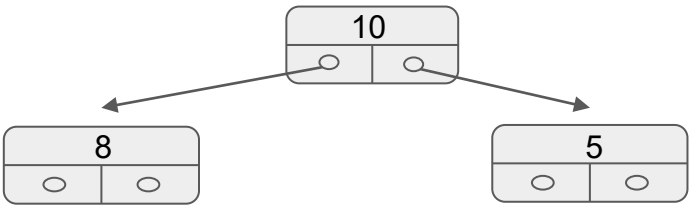
add(9)

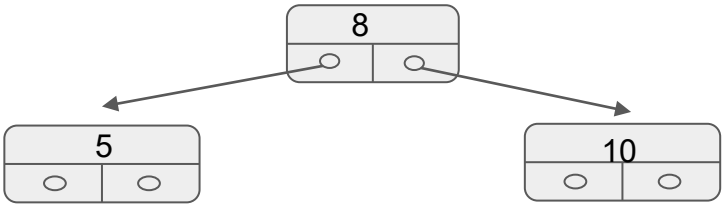


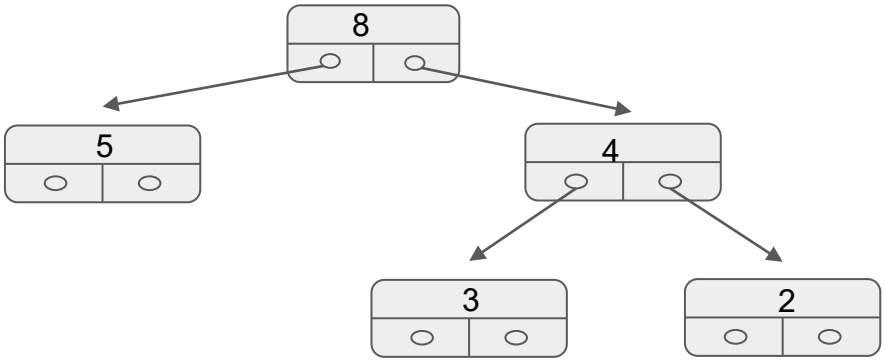
8	6	5	4	3	4	

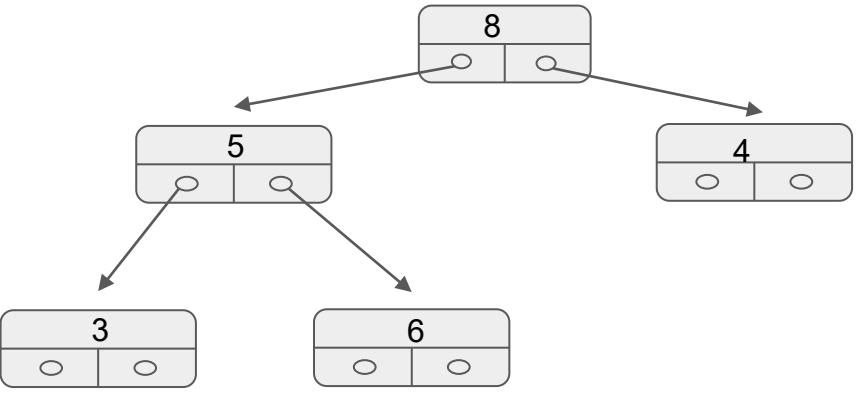
```
void bubbleDown(int index) {
    if(index >= this.entries.size()) { return; }
    int leftIndex = left(index);
    if(leftIndex >= this.entries.size()) { return; }
    int largerChildIndex = leftIndex;
    int rightIndex = right(index);
    if(existsAndGreater(rightIndex, leftIndex)) {
        largerChildIndex = rightIndex;
    }
    if(existsAndGreater(largerChildIndex, index)) {
        swap(index, largerChildIndex);
        bubbleDown(largerChildIndex);
    }
}
```

```
void bubbleUp(int index) {
    if(index <= 0) { return; }
    Entry<K,V> e = this.entries.get(index);
    Entry<K,V> parent = this.entries.get(parent(index));
    int comp = this.comparator.compare(e.key, parent.key);
    if(comp > 0) {
        swap(index, parent(index));
        bubbleUp(parent(index));
    }
    else {
        return;
    }
}
```


10
8
5


8
5
10


8
5
4
3
2


8
5
4
3
6