CF1063C - Dwarves, Hats and Extrasensory Abilities

https://codeforces.com/contest/1063/problem/D

(In this outline, to document my full thought process, I first show my wildly incorrect initial approach before moving on to the correct solution. To skip the wrong ideas, jump to the red text)

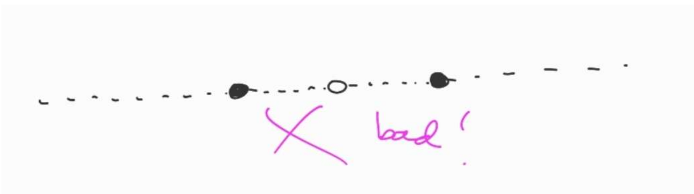
First, I summarize the gist of the problem to myself in plain language: I must place n integer points, each black or white, in $2D$ space such that when I'm done, I can draw a line with all blacks on one side and all whites on the other. I only know the color of each point *after* it is placed, and the judge is malicious and adaptive.

Now, to begin, I note that $n \leq 30$ is a suspiciously small constraint, and I conjecture that extremely brute force approaches might be possible for this problem; in particular, $n^5 \approx 2.4 \times 10^7$ (heavy brute force? DP?) and $2^{n/2} = 32768$ (meet in the middle?). I also note that $2^{30} \approx 10^9$, and the coordinates in the problem can be at most 10^9 in value (binary shenanigans?)

I don't know if any of this is relevant, but I keep it in the back of my mind.

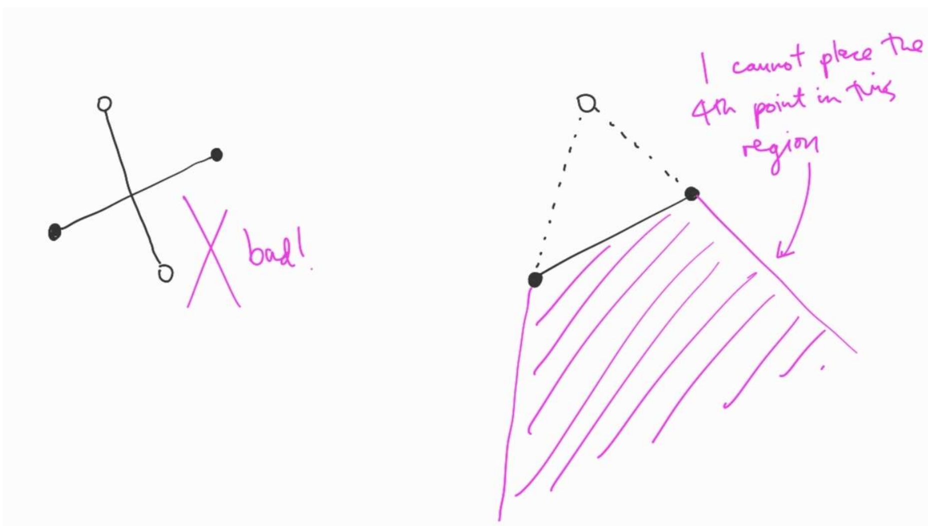
I begin by trying the problem-solving strategy of simulating interactions for small values of n using pen and paper, and seeing if I can get any insights by assuming the role of malicious judge myself.

For $n = 1$ and $n = 2$, it's impossible to fail. In $n = 3$, I note that if I have three collinear points with at least one black and one white among them, then the partition is impossible.



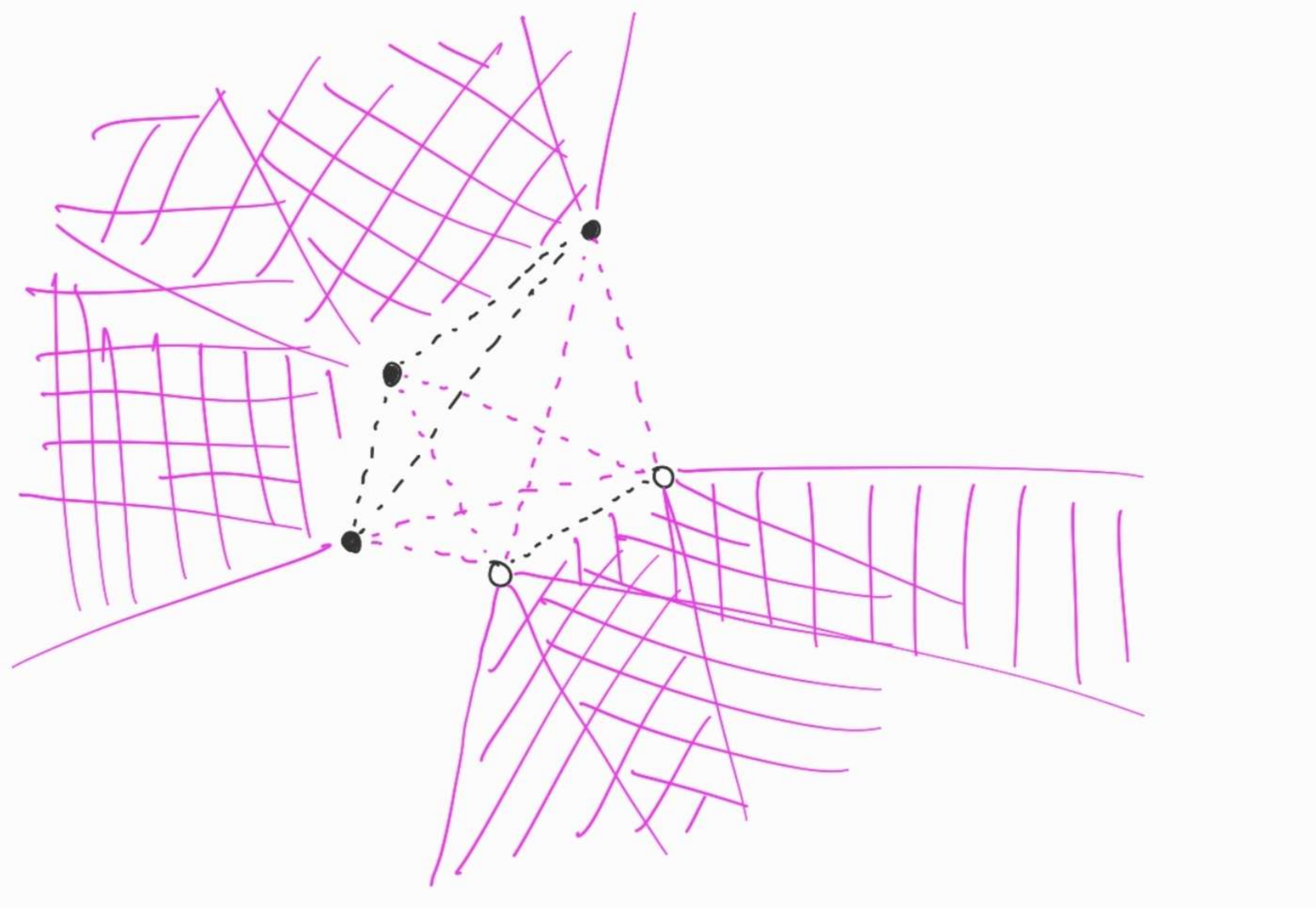
I thus resolve that any three points in my solution should *not* be collinear **(this is incorrect, and was a huge mistake; had I thought about this case more deeply at the time, I would have been able to solve this problem immediately, rather than fall down a dark path)**

For $n = 4$, I note that a necessary condition is that if I draw a segment between every pair of black points and every pair of white points, then none should intersect.



Thus, I begin to sketch out a comp-geom-heavy solution that involves considering all possible lines through pairs of points, partitioning space into different regions, then banning myself from placing new points into certain forbidden regions depending on the color conditions.

Emboldened by the very lenient $n \leq 30$ only, I reason that something like this should theoretically be able to work.



This is all completely the wrong way to go. The solution is messy, ugly, involves tons of floating point computations, and I can't even prove its correctness.

Embarrassingly, I waste about 20 minutes on this nonsense, drawing diagrams and going absolutely nowhere.

Eventually I (metaphorically) slap myself and pull away from this approach. I recognize that this is going nowhere, so I abandon this approach and reorient myself. I regroup and reflect.

What went wrong? **Answer:** Am I possibly overcomplicating this?

The point of this being an interactive problem is that I can place the points wherever I want. I don't need to consider the ugly general case; I can (and should) place the points in a "nice" arrangement.

I give up on the yucky geometry solution, but see if I can salvage any insights from it.

- If I arrange the points in a "circle", then the inside is safe...
- Also, the regions next to where the points change colors are **safe**!

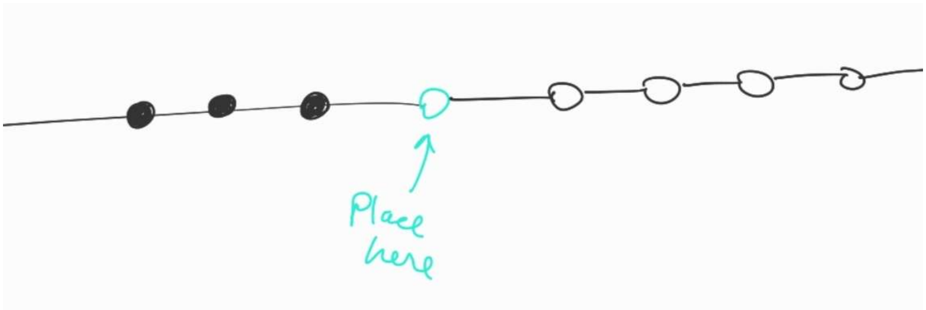
So, I can arrange the points in a circle, and then always insert the next point at a position in the circle where the color changes from black to white (or vice versa)!

Hm, but implementing this still feels somewhat complicated? Wait a minute...

Something clicks in my mind.

Can I just place all the points on a single line?

Suppose the points are arranged in a line, with all black points on one side and all white points on the other. If I am to insert a new point, I should place it between the last black and first white point. Then, whichever color this point is revealed to be, all the blacks and all the whites are still grouped together.

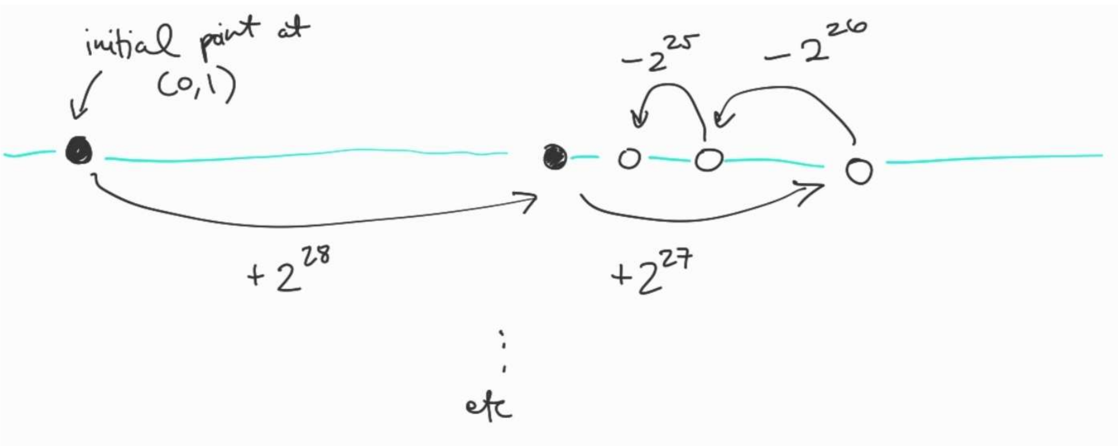


Formally, "all black points are on one side, and all white points are on the other" is an *invariant* that I maintain at each step.

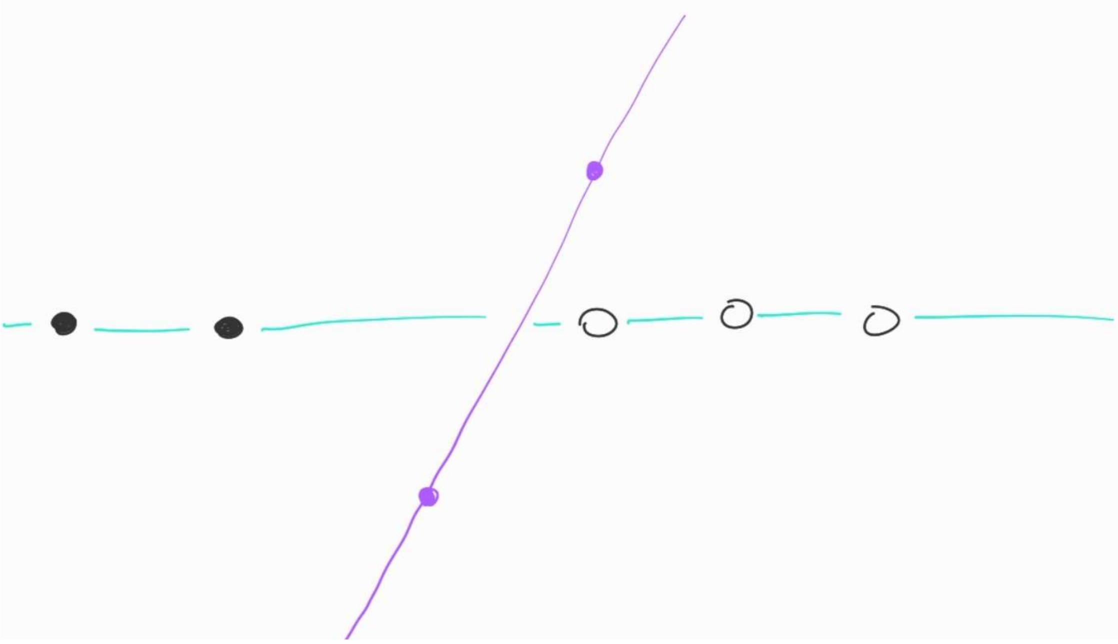
Totally arbitrarily, let's place all points on the horizontal line $y = 1$. Let the last black point be $(x_b, 1)$ and let the last white point be $(x_w, 1)$. Then, to insert the next point in between them, I can place it at $\left(\frac{x_b + x_w}{2}, 1\right)$.

Issue: The problem requires all placed points to be on integer coordinates, but $\frac{x_b + x_w}{2}$ might not be an integer anymore.

I then recall the fact that $n \leq 30$ and $2^{30} \approx 10^9$. Aha, it *was* relevant! I preempt the issue by sufficiently spacing out the initial points by a very large power of 2, so that even if this distance gets halved ≈ 30 times, all points will still have integer x coordinates.



Finally, I need to construct a line separating the black and white points. But the arrangement is so neat that this is barely a problem. Let $(x, 1)$ be the first point that is a different color from our initial point. Then, it should be clear (just by drawing it) that we can use the line passing through $(x - 1, 0)$ and $(x, 2)$.



This solves the problem.

Useful takeaways for this problem:

- Force simple/nice constructions for constructive problems
- Finding a nice invariant can be key to a good solution + easy proof of correctness
- Need to practice learning how to tell when an idea is leading nowhere and should be abandoned

Consider the following: If you are the problem setter, how do you write the judge program (which determines what color each point should be) which punishes the contestant whenever possible?