CF1504D - 3-Coloring

https://codeforces.com/contest/1504/problem/D

First, I summarize the gist of the problem to myself in plain language: I must successfully 3-color an $n \times n$ grid, while Alice chooses one color to *ban* each turn.

Now, to begin, I try the problem-solving strategy of simplifying the problem. What if there is no malicious Alice that is meddling with my moves, i.e. I can do whatever I want with no restrictions? Then the problem is trivially easy—there are lots of ways to 3-color a grid. Actually, I recall the fact that **I only need 2 colors**, since it is a classical result that can 2-color a grid using a chessboard coloring.

Can I use this chessboard coloring idea in my solution? If I just tried to force a chessboard coloring, can Alice stop me? I think about it and sketch out the following partial solution idea.

Suppose I want to try to chessboard-color the grid using 1 or 2 only (ignoring 3 for now); let's label each empty cell with "a 1 should go here" and "a 2 should go here" (which I'll call 1 cells and 2 cells, respectively). Alice can't stop me from coloring in this pattern because she can only ban one color at a time!

- If she bans 1, I place a 2 in a 2-cell.

- If she bans 2, I place a 1 in a 1-cell.

- If she bans 3, I can do either.
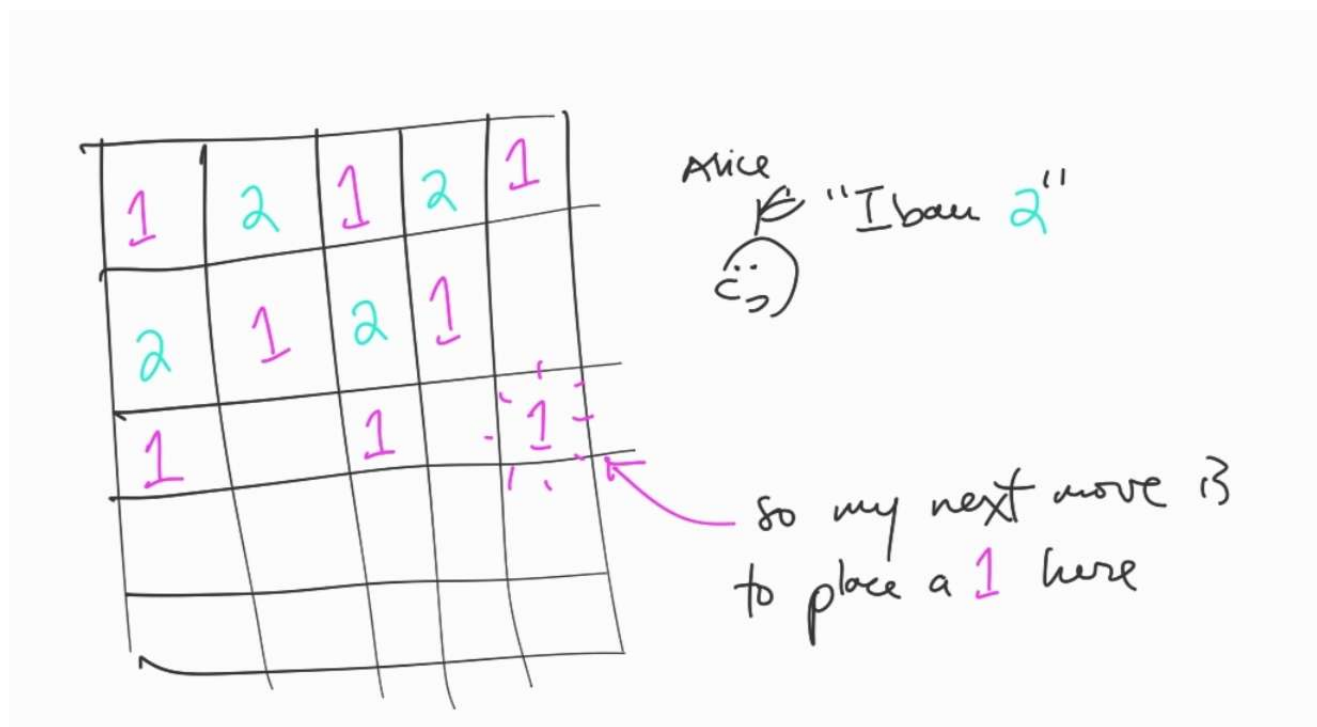
Is this a full solution? Hmm, I'm not sure. Where can this go wrong...?

Well, suppose that I have filled in all the 1 cells of the grid already, and then Alice bans 2. My partial solution dictates that I should now place a 1 somewhere, but I can't, since I've already filled up all the places where a 1 should go.

I draw a picture of this scenario:

Highlighted in block are the empty cells
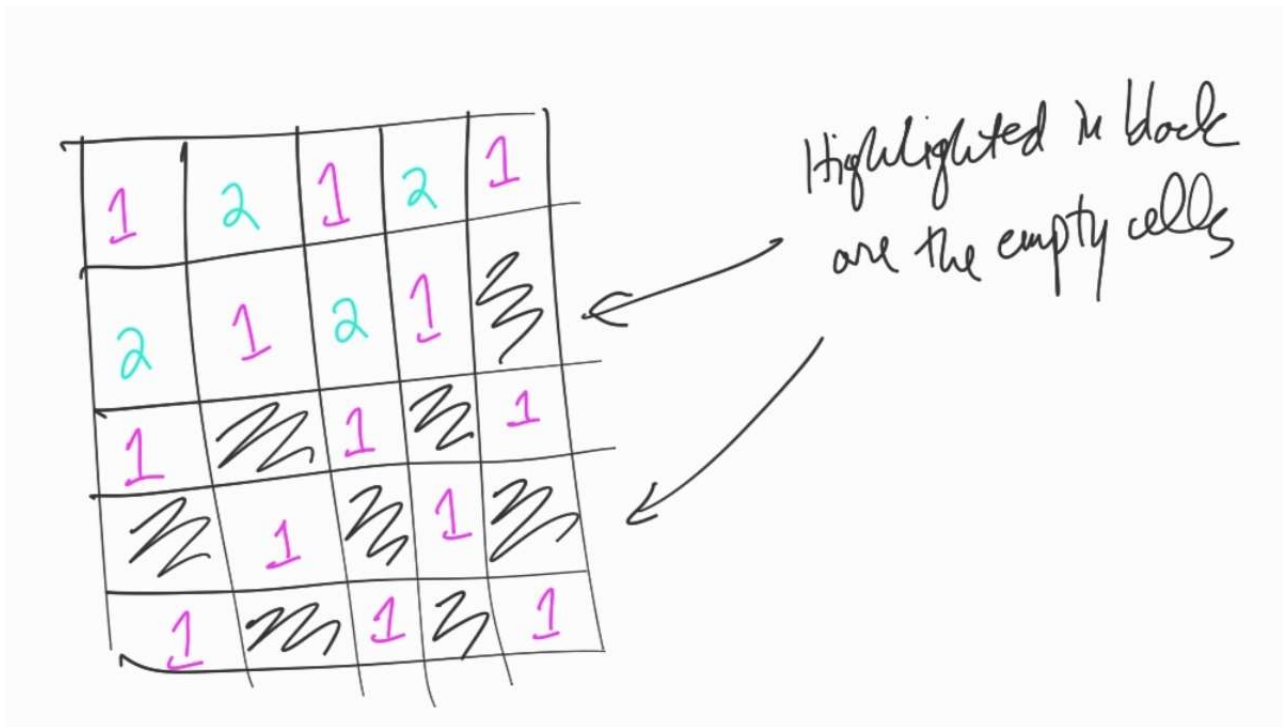
Well, looking at this picture, it's pretty clear to me what I can do. When we reach this point, we switch strategies.

If all the 1 cells have been filled in already, then that means all remaining empty cells are adjacent to only 1s, and no two empty cells are adjacent to each other. So, I can independently place a 2 or 3 (whichever one Alice didn't ban for that turn; again, Alice can only ban up to one color at a time) in each remaining empty cell.

This turns our partial solution into a full solution. Yay!

Useful takeaways for this problem:

- Simplifying-the-problem strategy

- Drawing inspiration from solutions to past related problems

- Starting with a partial solution and then later figuring out how to patch up the holes

- Drawing figures helped with insights

Consider the following: If you are the problem setter, how do you write the Alice Program which punishes Bob for any misplays?