

# Unitary circuits

- Consider time evolution generated by two-site **unitary circuits**

$$U = \text{blue square with four legs and a corner symbol}$$

$$U^\dagger = \text{red square with four legs and a corner symbol}$$

E.g.  $U_{n,n+1} = \exp[-ih_{n,n+1}\Delta t]$

- Unitarity can be graphically represented as

$$\begin{array}{c} a \quad b \\ \diagdown \quad \diagup \\ \text{blue square} \\ \diagup \quad \diagdown \\ e \quad f \\ \text{red square} \\ \diagdown \quad \diagup \\ c \quad d \end{array} = \begin{array}{c} a \quad b \\ \diagdown \quad \diagup \\ \text{red square} \\ \diagup \quad \diagdown \\ e \quad f \\ \text{blue square} \\ \diagdown \quad \diagup \\ c \quad d \end{array} = \begin{array}{cc} a & b \\ | & | \\ c & d \end{array}$$

$$\sum_{ef} U_{ab,ef} U_{ef,cd}^\dagger = \sum_{ef} U_{ab,ef}^\dagger U_{ef,cd} = \delta_{ac} \delta_{bd}$$