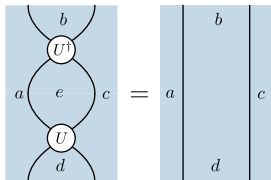


# Dual-unitarity from biunitarity

- A fully shaded biunitary corresponds to **dual-unitary clockwork**

**Vertical unitarity:**



$$\sum_e (U_{a,c})_{b,e}^\dagger (U_{a,c})_{e,d} = \delta_{bd}, \quad \forall a, c$$

- Can express fully shaded biunitary as **2-controlled 1-site unitary**

$$(U_{a,c})_{b,d} = \begin{array}{c} \text{Diagram of } U_{a,c} \end{array} = \begin{array}{c} \text{Diagram of 2-controlled 1-site unitary} \end{array}$$

The diagram on the left shows a vertical rectangle with input lines 'b' and 'c' from above and output lines 'a' and 'd' from below. The diagram on the right shows a horizontal line with a blue square in the middle. The square has input lines 'a' and 'd' from the left and output lines 'b' and 'c' from the right.

- **Horizontal** unitarity enforces unitarity of  $\tilde{U}_{a,c}$  with  $(\tilde{U}_{a,c})_{b,d} = (U_{b,d})_{a,c}$