Austin Bennett
Homework #3, Part 2

Transition Model

	A	B	C	D	E	F
A	.2	.8	0	0	0	0
B	0	.2	.8	0	0	0
C	0	0	.2	.8	0	0
D	0	0	0	.2	.8	0
E	0	0	0	0	.2	.8
F	0	0	0	0	0	1

Observation model

	hot	cold
A	1	0
B	0	1
C	0	1
D	1	0
E	0	1
F	0	1

#1: $P(X_3 \mid hot_1, cold_2, cold_3)$
$= \langle 0, .2, .8, 0, 0, 0 \rangle$

#2: $P(X_2 \mid hot_1, cold_2, cold_3)$
$= \langle 0, 1, 0, 0, 0, 0 \rangle$

#3: $P(hot_4 \mid hot_1, cold_2, cold_3)$
$= 0.64$

#4: $P(X_4 \mid hot_1, cold_2, cold_3)$
$= \langle 0, 0.2, 0.8, 0, 0, 0 \rangle$

#1 A-F

$P(X_3 \mid hot_1, cold_2, cold_3)$
$= \alpha P(cold_3 \mid X_3) \sum_{X_2} P(X_2 \mid hot_1, cold_2) P(X_3 \mid X_2)$

$P(X_2 \mid hot_1, cold_2)$
$= \alpha P(cold_2 \mid X_2) \sum_{X_1} P(X_1 \mid hot_1) P(X_2 \mid X_1)$

$P(X_1 \mid hot_1)$
$= \alpha P(hot_1 \mid X_1) P(X_1) =$
$P(hot_1 \mid X_1 = A) P(X_1 = A) = 1 \times 1 = 1$
$P(hot_1 \mid X_1 = B) P(X_1 = B) = 0 \times 0 = 0$
$X_1 = B \quad X_1 = C = 0 \times 0 = 0$
$X_1 = D \quad X_1 = D = 1 \times 0 = 0$
$X_1 = E \quad X_1 = E = 0 \times 0 = 0$
$X_1 = F \quad X_1 = F = 0 \times 0 = 0$

$P(X_1 \mid hot_1) = \langle 1, 0, 0, 0, 0, 0 \rangle$

$\sum_{X_1} P(X_1 \mid hot_1) P(X_2 \mid X_1)$

$P(X_1 = A \mid hot_1) P(X_2 = A \mid X_1 = A) \; 1 \times .2 = .2$
$P(X_1 = B \mid hot_1) P(X_2 = A \mid X_1 = B) \; 0 \times 0 = 0$
$P(X_1 = C \mid hot_1) P(X_2 = A \mid X_1 = C) \; 0 \times 0 = 0$
$P(X_1 = D \mid hot_1) P(X_2 = A \mid X_1 = D) \; 0 \times 0 = 0$
$P(X_1 = E \mid hot_1) P(X_2 = A \mid X_1 = E) \; 0 \times 0 = 0$
$P(X_1 = F \mid hot_1) P(X_2 = A \mid X_1 = F) \; 0 \times 0 = 0$
$P(X_1 = A \mid hot_1) P(X_2 = B \mid X_1 = A) \; 1 \times .8 = .8$
$P(X_1 = A \mid hot_1) P(X_2 = C \mid X_1 = A) \; 1 \times 0 = 0$

$\sum_{X_1} P(X_1 \mid hot_1) P(X_2 \mid X_1) = \langle .2, .8, 0, 0, 0, 0 \rangle$

$P(cold_2 \mid X_2) \sum_{x_1} P(\text{[illegible]} \cdot X_1 \mid hot_1) P(X_2 \mid x_1)$

$= P(cold_2 \mid X_2) \cdot \langle .2, .8, 0, 0, 0, 0 \rangle$

$= \langle 0, 1, 1, 0, 1, 1 \rangle \cdot \langle .2, .8, 0, 0, 0, 0 \rangle$

$= \alpha \langle 0, .8, 0, 0, 0, 0 \rangle$

$\alpha = \frac{1}{.8} = 1.25$

$= \langle 0, 1, 0, 0, 0, 0 \rangle = P(X_2 \mid hot_1, cold_2)$

$\sum_{x_2} P(X_2 \mid hot_1, cold_2) P(X_3 \mid x_2)$

$P(X_2 = A \mid hot_1, cold_2) P(X_3 = A \mid X_2 = A) = 0$

$P(X_2 = B \mid hot_1, cold_2) P(X_3 = A \mid X_2 = B) = 1 \times 0$

$P(X_2 = C \quad P(X_3 = A \mid X_2 = C) = 0$

$P(X_2 = D \quad P(X_3 = A \mid X_2 = D) = 0$

$P(X_2 = E \quad P(X_3 = A \mid X_2 = E) = 0$

$P(X_2 = F \quad P(X_3 = A \mid X_2 = F) = 0$

$P(X_2 = B) P(X_3 = B \mid X_2 = B) = 1 \times .2 = .2$

$P(X_2 = B) P(X_3 = C \mid X_2 = B) = 1 \times .8 = .8$

$\sum_{x_2} P(X_2 \mid hot_1, cold_2) P(X_3 \mid X_2) = \langle 0, .2, .8, 0, 0, 0 \rangle$

$P(cold_3 \mid X_3) = \langle 0, 1, 1, 0, 1, 1 \rangle$

$\alpha \langle 0, 1, 1, 0, 1, 1 \rangle \cdot \langle 0, .2, .8, 0, 0, 0 \rangle$

$= \alpha \langle 0, .2, .8, 0, 0, 0 \rangle$

$= \langle 0, .2, .8, 0, 0, 0 \rangle$

#2

$P(X_2 \mid hot_1, cold_2, cold_3)$
$= \alpha P(X_2 \mid hot_1, cold_2) P(cold_3 \mid X_2)$

$P(cold_3 \mid X_2)$
$= \sum_{X_3} P(cold_3 \mid X_3) P(X_3 \mid X_2)$

$P(cold_3 \mid X_3) = \langle 0, 1, 1, 0, 1, 1 \rangle$

$P(cold_3 \mid X_3 = A) P(X_3 = A \mid X_2 = A) = 0 \times .2 = 0$
$P(cold_3 \mid X_3 = A) P(X_3 = A \mid X_2 = B) = 0 \times 0$
$P(X_3 = A \mid X_2 = C) = 0 \times 0$
$P(X_3 = A \mid X_2 = D) = 0 \times 0$
$P(X_3 = A \mid X_2 = E) = 0 \times 0$
$P(X_3 = A \mid X_2 = F) = 0 \times 0$
$P(cold_3 \mid X_3 = B) P(X_3 = B \mid X_2 = A) = 1 \times .8 = .8$
$P(cold_3 \mid X_3 = B) P(X_3 = B \mid X_2 = B) = 1 \times .2 = .2$
$P(cold_3 \mid X_3 = C) P(X_3 = C \mid X_2 = B) = 1 \times .8 = .8$
$P(cold_3 \mid X_3 = C) P(X_3 = C \mid X_2 = C) = 1 \times .2 = .2$
$P(cold_3 \mid X_3 = E) P(X_3 = E \mid X_2 = D) = 1 \times .8 = .8$
$P(cold_3 \mid X_3 = E) P(X_3 = E \mid X_2 = E) = 1 \times .2 = .2$
$P(cold_3 \mid X_3 = F) P(X_3 = F \mid X_2 = E) = 1 \times .8 = .8$
$P(cold_3 \mid X_3 = F) P(X_3 = F \mid X_2 = F) = 1 \times 1 = 1$

$P(cold_3 \mid X_2) = \langle 0, 1, 1, 0, 1, 1.8 \rangle$
$= \langle 0.8, 1, .2, .8, 1, 1 \rangle$

$P(X_2 \mid hot_1, cold_2, cold_3)$
$= \langle 0, 1, 0, 0, 0, 0 \rangle \cdot \langle 0, 1, 1, 0, 1, 1.8 \rangle$
$= \langle 0, 1, 0, 0, 0, 0 \rangle$

$= \langle 0, 1, 0, 0, 0, 0 \rangle \cdot \langle 0.8, 1, 0.2, 0.8, 1, 1 \rangle$
$= \langle 0, 1, 0, 0, 0, 0 \rangle$

#3

$P(hot_4 \mid hot_1, cold_2, cold_3)$

~~$P(hot_4 \mid x_4) \sum_{x_3} P(x_3 \mid hot_1, cold_2, cold_3) P(x_4 \mid x_3)$~~

$\sum_{x_3} P(x_3 \mid hot_1, cold_2, cold_3) P(hot_4 \mid x_3)$

$P(hot_4 \mid x_3)$

$= \sum_{x_4} P(hot_4 \mid x_4) P(x_4 \mid x_3)$

$P(hot_4 \mid x_4 = A) P(x_4 = A \mid x_3 = A) = 1 \times .2 = 0.2$

$P(hot_4 \mid x_4 = B) P(x_4 = B \mid x_3 = A) = 0 \times .8 = 0$

$P(hot_4 \mid x_4 = D) P(x_4 = D \mid x_3 = C) = 1 \times .8 = .8$

$P(hot_4 \mid x_4 = D) P(x_4 = D \mid x_3 = D) = 1 \times .2 = .2$

~~$= \{0.2, 0, 0.8, 0.2\} = P(hot_4 \mid x_3)$~~

$P(hot_4 \mid x_3) = \{0.2, 0, 0.8, 0.2, 0, 0\}$

~~$\sum_{x_3} \{0.2, 0, 0.8, 0.2, 0, 0\} \cdot \{0.2, 0, 0.8, 0.2, 0, 0\}$~~

$\sum_{x_3} \{0, .2, .8, 0, 0, 0\} \cdot \{0.2, 0, .8, .2, 0, 0\}$

$= 0.64$

#4

$P(X_4 \mid hot_1, cold_2, cold_3)$

$\sum_{X3} P(X_3 \mid hot_1, cold_2, cold_3) P(X_4 \mid X_3)$

$P(X_3 = B \mid$) $P(X_4 = B \mid X_3 = B) = .2 \times .2 = .04$

$P(X_3 = B \mid$) $P(X_4 = C \mid X_3 = B) = .2 \times .8 = .16$

$P(X_3 = C \mid$) $P(X_4 = C \mid X_3 = C) = .8 \times .2 = .16$

$P(X_3 = C \mid$) $P(X_4 = D \mid X_3 = C) = .8 \times .8 = .64$

$= \langle 0, .2, .8, 0, 0, 0 \rangle$

$P(X_4 \mid hot_1, cold_2, cold_3) = \langle 0, .2, .8, 0, 0, 0 \rangle$

Problem 2

State	π(State)
A	~~1~~
B	1

	V_0^π(state)	V_1^π(state)	V_2^π(state)
A	0	0	0
B	0	5	10

$$V_1^\pi(A) = R(A,1) + \gamma(T(A,1,A)V_0^\pi(A) + T(A,1,B)V_0^\pi(A))$$
$$= 0 + 1(1 \cdot 0 + 0 \cdot 0)$$
$$= 0$$

$$V_1^\pi(B) = R(B,1) + \gamma(T(B,1,A)V_0^\pi(B) + T(B,1,B)V_0^\pi(B))$$
$$= 5 + 1(0 \cdot 0 + 1 \cdot 0)$$
$$= 5$$

$$V_2^\pi(A) = R(A,1) + \gamma(T(A,1,A)V_1^\pi(A) + T(A,1,B)V_1^\pi(A))$$
$$= 0 + 1(1 \cdot 0 + 0 \cdot 0)$$
$$= 0$$

$$V_2^\pi(B) = R(B,1) + \gamma \sim$$
$$= 5 + 1(0 \cdot 5 + 1 \cdot 5)$$
$$= 10$$

state	Action	Qvalue(S,A)
A	1	0
A	2	-1
B	1	15
B	2	10

$$Q(A,1) = R(A,1) + \gamma(T(A,1,A)V_2^{\pi}(A) + T(A,1,B)V_2^{\pi}(A))$$
$$= 0 + 1(1 \cdot 0 + 0 \cdot 0)$$
$$= 0$$

$$Q(A,2) = R(A,2) + \gamma(T(A,2,A)V_2^{\pi}(A) + T(A,2,B)V_2^{\pi}(A))$$
$$= -1 + 1(0.5 \cdot 0 + 0.5 \cdot 0)$$
$$= -1$$

$$Q(B,1) = R(B,1) + \gamma(T(B,1,A)V_2^{\pi}(B) + T(B,1,B)V_2^{\pi}(B))$$
$$= 5 + 1(0 \cdot 10 + 1 \cdot 10)$$
$$= 15$$

$$Q(B,2) = R(B,2) + 1(T(B,2,A)V_2^{\pi}(B) + T(B,2,B)V_2^{\pi}(B))$$
$$= 0 + 1(0 \cdot 10 + 1 \cdot 10)$$
$$= 10$$