CS 205 Final Question 1: Grab Bag

16:198:205

1) Prove that the function $f(x) = (8x + 5) \bmod 17$ is injective on the set $\{0, 1, 2, 3, \ldots, 15, 16\}$ **(10 points)**.

2) Is $f(x) = (8x + 5) \bmod 17$ invertible on this set? If so, give its inverse function **(5 points)**.

3) Is $f(x) = (8x + 5) \bmod 18$ invertible on this set? If so, give its inverse function **(5 points)**.

4) True or false - give a mathematical justification **(10 points)**:

   a) $n = O(n^2)$.

   b) $n(n+1)(n+2) - n^3 = O(n^3)$.

   c) $n(n+1)(n+2) - n^3 = O(n^2)$.

   d) $n \ln n = O(n^2)$.

   e) $n^2 = O(n \ln n)$.

   f) $1/n = O(1)$.

   g) $1000000n = O(n)$.

   h) $2^n = O(3^n)$.

   i) $3^n = O(2^n)$.

   j) $\sum_{i=1}^{n} i(i+1)(i+2) = O(n^4)$.

5) The 'double factorial' is a variant on the factorial function where every other number is multiplied together: for instance, $9!! = 9 * 7 * 5 * 3 * 1 = 945$, $10!! = 10 * 8 * 6 * 4 * 2 = 3840$. In general, $n!!$ is much smaller than $n!$ **(20 points)**.

   - Argue that $n!!/n!$ goes to 0 as $n \to \infty$, i.e., $n!$ dominates $n!!$ severely. *Hint: Be wary of cases.*
   - Argue, however, that in the limit for even n, $\ln(n!!)/(n \ln n)$ is bound from above and below by some constants.
   - Argue that for odd n, $\ln(n!!)/(n \ln n)$ is bound from above and below by some constants as well. *Hint: Show that* $n! = n!!(n-1)!!$.
   - Conclude therefore that while $n!$ is a lot larger than $n!!$, we have that $\ln n!! = \Theta(\ln n!)$.