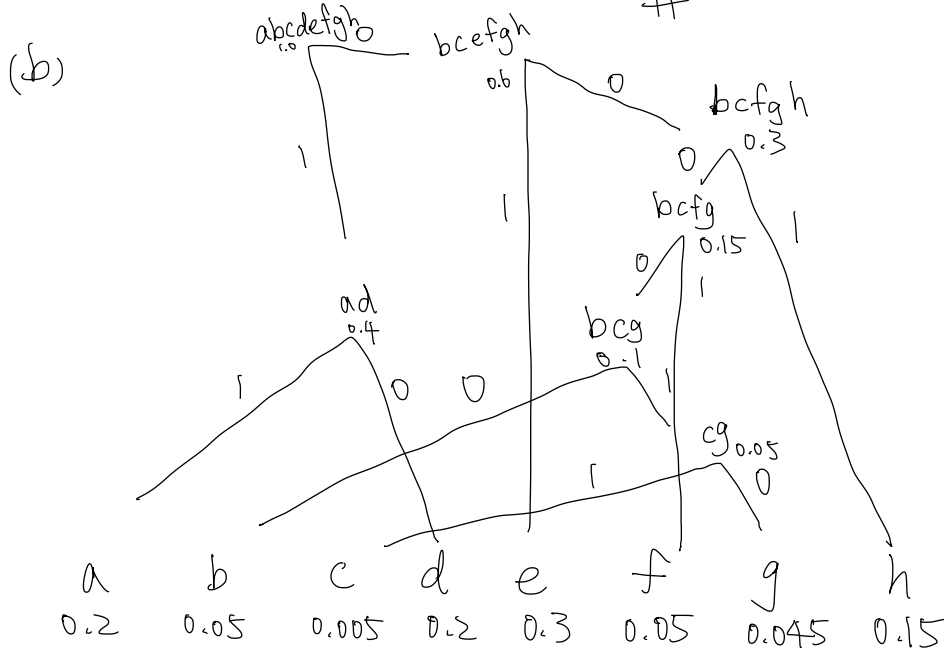


$$(a) - (0.2 \log 0.2 + 0.05 \log 0.05 + 0.005 \log 0.005 + 0.2 \log 0.2 + 0.3 \log 0.3 + 0.05 \log 0.05 + 0.045 \log 0.045 + 0.15 \log 0.15)$$

$$\approx 2.53214 \text{ bits (base 2)} \quad \#$$



	prob	left child	right child	bits
a	0.2			11
b	0.05			00000
c	0.005			000011
d	0.2			10
e	0.3			01
f	0.05			0001
g	0.045			000010
h	0.15			001
cg	0.05	g	c	(00001)
bfg	0.1	b	cg	(0000)
bctfg	0.15	bctfg	f	(000)
bctfgh	0.3	bctfgh	h	(00)
ad	0.4	d	a	(11)
bctfgh	0.6	bctfgh	e	(0)
abcdefgh	1.0	abcdefgh	ad	

(c)

$$2^{-2} + 2^{-5} + 2^{-6} + 2^{-2} + 2^{-2} + 2^{-4} + 2^{-6} + 2^{-3} = 1$$

Satisfy #

(d) #

$$0.2 \times 2 + 0.05 \times 5 + 0.005 \times 6 + 0.2 \times 2 + 0.3 \times 2 + 0.05 \times 4 + 0.045 \times 6 + 0.15 \times 3 = 2.6$$

by (a) $H[x] \approx 2.53 \leq 2.6 < H[x] + 1$ satisfy #

$\bar{L} = 2.6$

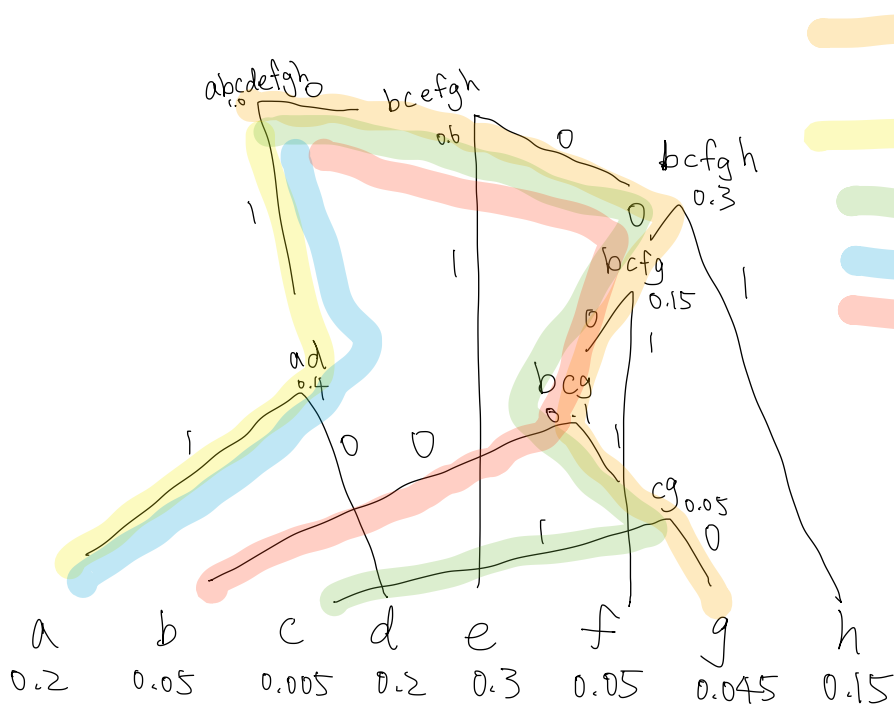
(e)

$$\{g, a, c, a, b\} = \{000010, 11, 000011, 11, 00000\}$$

$$\rightarrow 000010110000111100000$$

#

(f)



g 000010
a 11
c 000011
a 11
b 000000

→ gacab

#

(g)

find sequences s.t. $\left| \frac{-\log p(x^n)}{n} - H[x] \right| < 0.1$
ahahahahah, haahaaahah, aaaaaahhhhh

$$\left| \frac{-\log(0.2 \times 0.15)^5}{10} - 2.53214 \right| \approx 0.00269 < 0.1$$

hehhhhh, ehhehhhhh, hheehhhhhh

$$\left| \frac{-\log(0.15)^8 \times (0.3)}{10} - 2.53214 \right| \approx 0.00483 < 0.1$$

beeebeee, ebeeebeee, eebeeebeee, eeeeeeebb

$$\left| \frac{-\log(0.05)^3 \times (0.3)}{10} - 2.53214 \right| \approx 0.01969 < 0.1$$

#

2.

(a)

```
dict =
15x5 cell array

{'a'      } {[0.2000]} {0x0 double} {0x0 double} {'11'      }
{'b'      } {[0.0500]} {0x0 double} {0x0 double} {'00000'   }
{'c'      } {[0.0050]} {0x0 double} {0x0 double} {'000011'  }
{'d'      } {[0.2000]} {0x0 double} {0x0 double} {'10'      }
{'e'      } {[0.3000]} {0x0 double} {0x0 double} {'01'      }
{'f'      } {[0.0500]} {0x0 double} {0x0 double} {'0001'    }
{'g'      } {[0.0450]} {0x0 double} {0x0 double} {'000010'  }
{'h'      } {[0.1500]} {0x0 double} {0x0 double} {'001'     }
{'gc'     } {[0.0500]} {[      7]} {[      3]} {'00001'   }
{'bgc'    } {[0.1000]} {[      2]} {[      9]} {'0000'    }
{'bgcf'   } {[0.1500]} {[     10]} {[      6]} {'000'     }
{'bgcfh'  } {[0.3000]} {[     11]} {[      8]} {'00'      }
{'da'     } {[0.4000]} {[      4]} {[     11]} {'1'       }
{'bgcfhe' } {[0.6000]} {[     12]} {[      5]} {'0'       }
{'bgcfheda'} {[      1]} {[     14]} {[     13]} {0x0 double}
```

(b) same as Problem 1 (e).

```
>> bin_seq = huffman_enc(sym_seq, dict)

bin_seq =

'000010110000111100000'
```

(c) same as Problem 1 (f).

```
>> sym_seq = huffman_dec(bin_seq, dict)

sym_seq =

1x5 cell array

{'g'} {'a'} {'c'} {'a'} {'b'}
```

3.
(a)

```
>> huffman_sample(10, 1, false, false)

symbols =
    1×10 cell array

    Columns 1 through 9

        {'h'}    {'a'}    {'e'}    {'d'}    {'e'}    {'d'}    {'d'}    {'e'}    {'a'}

    Column 10

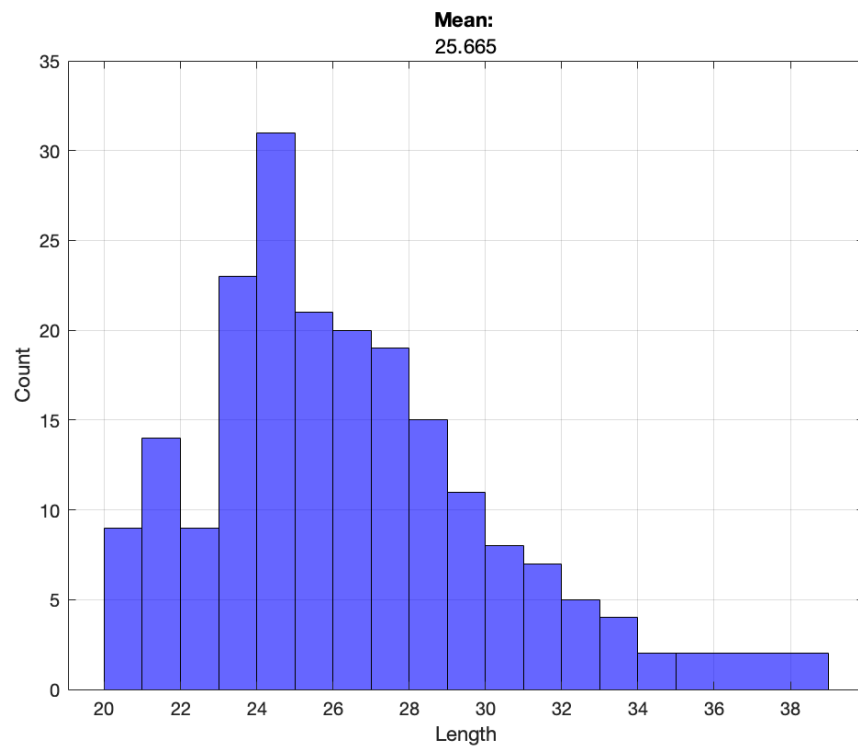
        {'h'}

bin_seq =

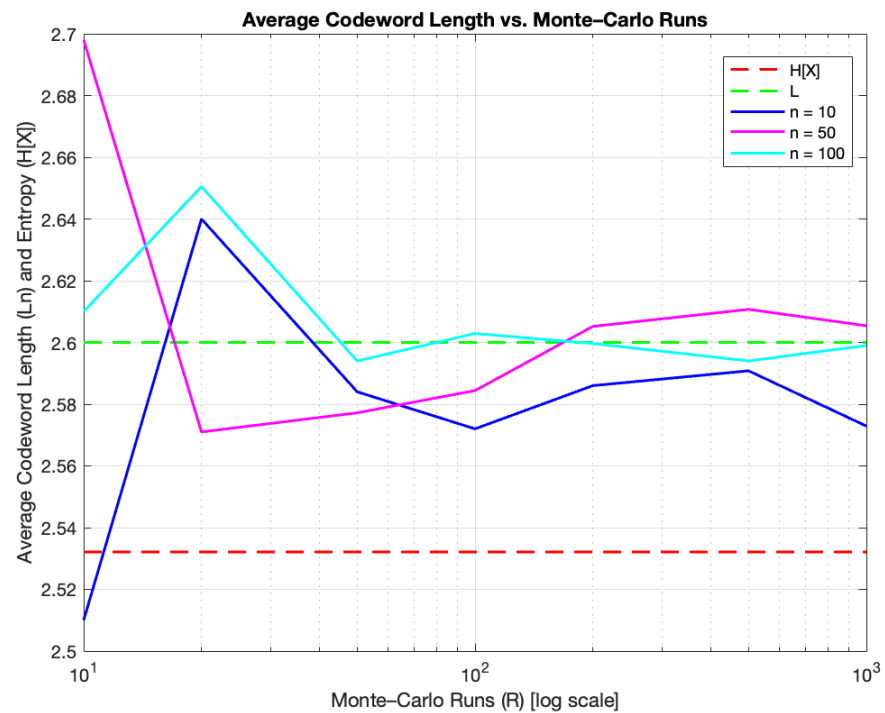
    '0011101100110100111001'

len =
    22
```

(b)



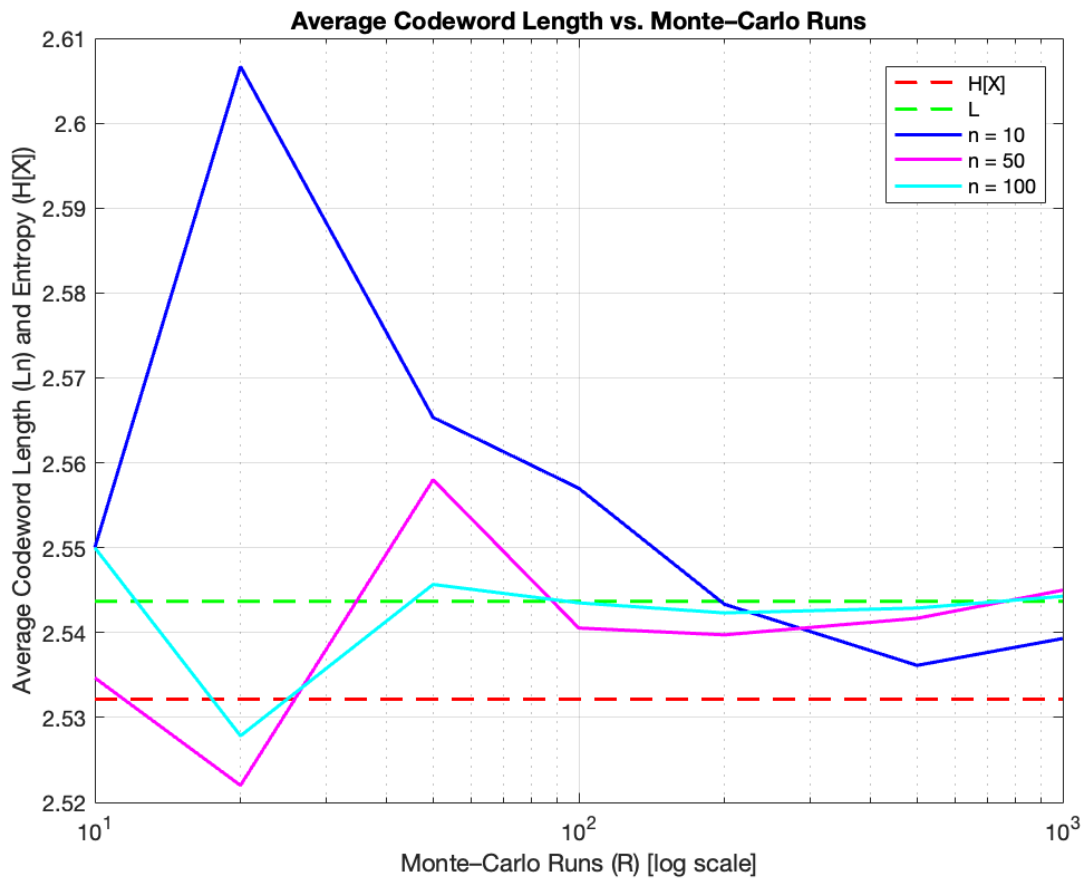
(c)



(d)

As the number of runs increases, the average of the average of length approximates the true mean. As the number of samples, the trends are also the same. This tells us that the length under the Monte-Carlo sample is valid when the number of sampling points and the number of runs approach infinity, which is good since when tend to transmit a lot of data.

4.



The difference between the entropy and the mean of the length of code decreases compared to only one system. This is the fact mentioned in class, the $1/N$ term that bounds the average L decreases as N increases.