Contents

Modeling Discourse Resolution: On Transforming SL into (Segmented) DRT and Developing Its Revised Model Theory $\mathcal{M}_{(S)DRT}$

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SDRT classifies a discourse as coherent just in case it forms a connected discourse structure with all underspecifications resolved (Asher & Lascarides 2003:165).

The computational semantics [presented] here, together with the semantics of rhetorical relations, [...] form the foundations of the account (2003:50-1).

We begin by reviewing the foundations of our theory. Following Asher and Lascarides (2003:50-1), we define these in two parts: a computational semantics (1) and a set of well-defined rhetorical relations (2). In this essay, however, we simple discuss the properties of components (1), examining how it is exactly we get a $\mathcal{M}_{(S)DRT}$, and why.

Due to the complex composition and theoretical dependencies of SDRT, the only of way cutting the Gordian knot of theory-specific explication, so to speak, is to provide ourselves with a systematic basis for understanding SDRT, complete with discussion of its borrowed and rudimentary concepts. The goal then is to develop a "big-pictured"conception of what SDRT is all about by means of explaining its ancestor theory DRT and the FS notions implicit therein. Additionally, this essay is designed to meet existing expository demands, typically corollary to research in formal semantics (**FS**), as well as to impress upon the reader the overall structure and elegance of basic components of SDRT research. Hopefully, taking time to work through these initial sections will contribute to the promise we deem SDRT research to possess and, just as importantly, clue us in to its potential weaknesses.

1 Establishing Constraints on Resolution

In FS, the notion of *simple semantic import* is not treated as a given. And yet, no attempt is made to provide a general, essentialist definition of what a "delimitable", linguistic semantic instance is (Dowty 1991:14-5). Rather, as the name indicates, FS broaches the question formally, maintaining there to be no such *tertium quid* beyond promising formal analysis, where all formal analysis in FS is understood to proceed compositionally.

On the contrary, the ideas of a *world*, and *extension* into some one world w_i, are assumed to be given. No attempt is made to work out the ontological purchase these notions may or may not possess. FS, like mathematics proper, leaves this concern to metaphysics. In part, this expository absence flows from a belief that such details form an unnecessary component to the integrity of proposed SDRT.[1]

[1] This excision of interest represents more an operating mind-set in SDRT research than it does the concerns I personally have regarding the "integrity"of SDRT. My opinion is that, just as the question of the ontological purchase of mathematics is and should be a lively question in the philosophy of mathematics, so also, I think, the question of the ontological purchase of SDR-theoretic entities *should* be a live, and serious, question in the philosophy of language and metaphysics. What perhaps distinguishes me from others writing on the present subject is that I believe conclusions drawn in these disciplines should bear an impact on our commitments to theory, since afterall, presumably, SDRT is designed to address *actual* natural language

Notions such as the two above, we call the *primitives* of our theory, taking after David Hilbert [3] in his assessment of Euclidean geometry and the notions of point, line, plane, "between", and congruent. We agree with Barker that "...a system is more elegant the simpler is its list of primitive terms and simpler is its list of axioms" (1964:25). To be sure, having only a select number of undefined terms, SDRT is much the better for it. When we encounter SDRT's set of axioms in the final section of this paper, we may however be inclined to consider the theory less elegant than once hoped.

1.1 Introductory FS Notions

J. van Eijck and H. Kamp[2] [13] write, "The difference between first order logic and basic DRT has nothing to do with expressive power but resides entirely in the different way in which DRT handles context"(1997:194). Eijck and Kamp continue on to say "...meaning in the narrow sense of truth conditions does not exhaust the concept of meaning for DRSs"(194-5). To be sure, the context change potential (**CCP**) is what distinguishes Discourse Representation Theory (or *dynamic semantics*) from sentential logic (**SL**); but, it is the traditional and philosophical concern with *truth-conditions* which has formed the backbone of much FS research, starting with Montague Grammar and carrying into the present. With this in mind, we'll examine relevant components of this research, opening the way for investigation into DRT and ultimately to the foundations of SDRT, our concern in the final part of this essay.

1.1.1 The Role of Compositionality

One of the nice properties of SL is that it abides by the principle of bivalence, and so is discretely composed. Given a statement φ, we have for any statement in SL[3] $\varphi = 1$ or $\varphi = 0$. That is, φ can only be true (1) or false (0). This is a nice property of SL statements in that it makes for a clean space of *provable* values, given certain SL rules.

The comparison of SL and DRT is made with respect to the structure each possess. It should be said however that the function of these two theories is quite different. To state the obvious, DRT is about discourse semantics, or, equivalently, the semantics of discourse update. This calls for an account of CCP phenomena, which Eijck and Kamp mentioned. By contrast, SL is concerned with modeling valid deduction (See Bergmann, Moor, Nelson [5]) and, as such, is topically concerned with a product rather than process. DRT and SDRT alike serve to model the semantic processes of discourse update, a process quite different from mere logical cogitation. Important to note here though is again another point of similarity, which actually ties into the similarity of structure mentioned above; this special point of similarity is that both SL and DRT abide by the *principle of compositionality*, which may be defined as follows:

The Principal of Compositionality The meaning of a complex expression is a function of the meanings of its [discrete] parts and of the syntactic rules by which they are [concatenated].[4]

semantics, and not some fiction. I agree with Devitt an Sterelny [12] when they say, "An explanation of [linguistic] meaning must somehow relate language to the external world"(1997:33). For more on worlds and a theory of reference, see Devitt and Sterelny's discussion in *Language and Reality*. To quote, "The objection to all these explanations is that they leave us with a notion in our semantic theory *that badly needs explaining*: the notion of possible worlds"(27; emphasis added). I'm inclined to agree with this assessment; but, for the purposes of concision, I check this concern at the door, maintaining a more customary model-theoretic approach to a potentially suspect set of "entities"such as this. Also see Kamp and Reyle [17] for how they address their concerns about the ontological issue of worlds (§1.2; Cf. also §8.5 in [19]). One might reasonably wonder if the notion of a world meets Tarski's criterion for a primitive, which maitains that a primitive is only that concept which fit into "...a small group of expressions of [a given] discipline *that seem to us immediately understandable*"(1941:118). To be sure classifying a world as a primitive, "[W]e employ [it] without explaining [its] meaning";but, usually this status as a primitive is reserved for those ideas not "in need of"such explanation, being agreeably "immediately understandable".

[2] "One of the most important uses linguists and especially semanticists make of formal languages is to *represent meaning* of natural languages. Characterizing meaning is the main goal of the semantic component of a grammar, whether it be a grammar of a formal language or a grammar of a natural language. Like any scientific enterprise, semantics chooses particular aspects and parts of meaning as objects of study and employs formal languages as analytic tools"(Partee, Meulen, Wall 1990:92-3; emphasis in text).

[3] We may consider SL as a well-defined set such that, for some φ, $\varphi \in$ SL.

[4] Partee, Meulen, Wall (1990:318); for a more rigorous definition see Janssen (1997:417-73).

For any structure or theory (like SL and DRT) which operates under compositional constraints, we find these to manifest what is termed an "*algebra* of meanings"(Dowty 1991:14; emphasis in text). The simplest examples found in SL are truth tables.

ϕ	ψ	θ	β	$\phi \wedge \psi$	$\beta \equiv \phi$	$\theta \vee (\phi \wedge \psi)$	$\theta \vee (\phi \wedge \psi) \longrightarrow \beta \equiv \phi$
1	1	1	1	1	1	1	1
1	1	1	0	1	0	1	0
1	1	0	1	1	1	1	1
1	1	0	0	1	0	1	0
1	0	1	1	0	1	1	1
1	0	1	0	0	0	1	0
1	0	0	1	0	1	0	1
1	0	0	0	0	0	0	1
0	1	1	1	0	0	1	0
0	1	1	0	0	1	1	1
0	1	0	1	0	0	0	1
0	1	0	0	0	1	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	1	1	1
0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	1

Table 1: Truth Table Assignment for $\varphi := \theta \vee (\phi \wedge \psi) \longrightarrow \beta \equiv \phi$

This is a very nice and discrete example, demonstrating as it does the compositional interface of syntax and semantics. To explain, we have the following two recursive schemata, one for the syntax of SL and one for its semantics (The compositional interface of these two emerges when we take both schemata together):

The Set of all Well-formed Formulae (wffs) of SL [5] is defined recursively as follows:

1. **Basic Clause:** Every atomic statement is a wff.
2. **Recursive Clauses:**
   (i) If ϕ is a wff, then the result of prefixing a $\neg$ to ϕ is a wff.

   (ii) If ϕ and ψ are wffs, then the result of concatenating (, ϕ, $\wedge$, ψ, and) in that order is a wff.

   (iii) If ϕ and ψ are wffs, then the result of concatenating (, ϕ, $\vee$, ψ, and) in that order is a wff.

   (iv) If ϕ and ψ are wffs, then the result of concatenating (, ϕ, $\longrightarrow$, ψ, and) in that order is a wff.

   (v) If ϕ and ψ are wffs, then the result of concatenating (, ϕ, $\equiv$, ψ, and) in that order is a wff.

The recursive schema for the semantics of these formal rules is stated similarly, by *assigning semantic values to complex wffs as functions of the semantic values of their constituent wffs and of the rules by which they are constructed* (Partee, Meulen, Wall 1990:320).

[5] Partee, Meulen, Wall (1990:320); So we see that φ is simply a variable over wffs in SL.

Assignment of Semantic Values to Atomic Statements: Let f be a function which assigns to each atomic statement of SL one of the two truth values 1 and 0.[6]

The Semantics of Interpretation of the set of all well-formed formulae of SL, given an initial valuation f for the atomic statements, is defined recursively as follows:

1. **Basic Clause:** If ϕ is an atomic statement, then the semantic value of ϕ is $f(\phi)$.
2. **Recursive Clauses:**
   (i) The semantic value of the result of prefixing $\neg$ to ϕ is 1 iff the semantic value of ϕ is 0.

   (ii) The semantic value of $(\phi \wedge \psi)$ is 1 iff the semantic values of ϕ and ψ are both 1.

   (iii) The semantic value of $(\phi \vee \psi)$ is 1 iff the semantic value of ϕ is 1 or the semantic value of ψ is 1.

   (iv) The semantic value of $(\phi \longrightarrow \psi)$ is 1 iff the semantic value of ϕ is 0 or the semantic value of ψ is 1.

   (v) The semantic value of $(\phi \equiv \psi)$ is 1 iff the semantic value of ϕ is identical to the semantic value of ψ.

Utilizing the two recursive definitions above, it's not difficult to see how the compositional semantics of SL proceeds. To use the language of the basic clause of the recursive interpretation schema, where $\varphi := \theta \vee (\phi \wedge \psi) \longrightarrow \beta \equiv \phi$, $f(\varphi)$ is a function of its constituent wffs and the rules by which each is combined to the other.

This is no less the case regarding the structure of Discourse Representation Structures (**DRSs**). Putting it loosely, to begin we have a set of discourse referents, the *Universe* (or U), and a set of relations over members of U, the *Conditions* (or K). The template is then as follows:

(1)

$Universe$
$Condition_1, \ldots, Condition_n$

To explain the compositional semantics of DRSs we need to provide definitions defining the DRS syntax-semantics interface. Again, for now, we restrict ourselves to simple examples, particularly those embodying principles present in early DRT development; so, as a matter of course, we'll abide by a strictly truth conditional interpretation of DRSs, which will hopefully make clearer DRT's connections to SL. Hans Kamp and Uwe Reyle [17] write in their seminal work on DRT, "...the truth conditional account should define not only what the *actual* truth values are of the sentences or discourses to which it applies, but also what their truth values are in each of the *possible* (but non-actual) worlds"(1993:92; emphasis in text). This is just as in the case of SL, where "...we have to consider all possible assignments to the variable in a formula in arriving at the truth conditions for the whole formula"(Partee, Meulen, Wall 1990:326). Kamp and Reyle continue on to insist that our objectives aimed at building such a truth conditional account, complete with definitions of actual and possible truth values, may be attained by the use of *model-theoretic semantics*. They maintain:

[6] Partee, Meulen, Wall (1990:321).

In model-theoretic semantics the place of possible worlds is taken by models. *A model is a certain information structure*, relative to which it is possible to evaluate the expressions of some given language [natural or otherwise].... The truth or falsity of ... DRSs depend[s] on what *individuals* [(or discourse referents)] there are, on what *properties* these individuals [possess]... and in what *relations* they stand to each other, and finally on which individuals are denoted by which *names*. (1993:93; initial italics mine)

To give an example of what Kamp and Reyle are intending, we have the following two relevant definitions:

Definition 1.1 (Kamp & Reyle DRT Syntax) [7]

(i) A *DRS* K *confined to* V [(Vocabulary)] *and* R [(Discourse Referents)] is a pair, consisting of a subset of U_K (possibly empty) of R and a set $\texttt{Con}_K$ of indexed DRS-conditions confined to V and R;

(ii) An indexed DRS-condition confined to V and R is a pair $\langle\gamma, \mathrm{I}\rangle$, where γ is a DRS-condition confined to V and R and I is an Index.

(iii) An index is a pair $\langle\delta, m\rangle$, where δ is a DRS-condition and m is a natural number.

(iv) A *DRS-condition confined to* V *and* R is an expression of one of the following forms:

- (a) $x = y$, where x, y belong to R
- (b) $\pi(x)$, where x belongs to R and π is a name from V
- (c) $\eta(x)$, where x belongs to R and η is a unary predicate (corresponding to a common noun) from V
- (d) $x\zeta$, where x belongs to R and ζ is a unary predicate (corresponding to an intransitive verb) from V
- (e) $x\,\xi\,y$, where x, y belong to R and ξ is a binary predicate from V
- (f) $\neg$K, where K is a DRS confined to V and R
- (g) $\mathrm{K}_1 \Rightarrow \mathrm{K}_2$, where K_1 and K_2 are DRSs confined to V and R
- (h) $\mathrm{K}_1 \vee \ldots \vee \mathrm{K}_n$, where for some $n \geq 2$, $\mathrm{K}_1, \ldots, \mathrm{K}_n$ are (completed) DRSs confined to V and R.

Definition 1.2 (Kamp & Reyle DRT Model Theory)

Let K be a DRS confined to V and R, $\langle\gamma, \mathrm{I}\rangle$ be an indexed DRS-condition, and let f be an *embedding from* R *into* `M`, i.e. a function whose Domain is included in R and whose Range is included in $\mathrm{U}_{\texttt{M}}$.

(i) f *verifies* the DRS K in `M` iff f verifies each of the indexed conditions belonging to $\texttt{Con}_K$ in `M`

(ii) f *verifies* the indexed condition $\langle\gamma, \mathrm{I}\rangle$ in `M` iff

- (a) γ is of the form $x = y$ and f maps x and y onto the same element of $\mathrm{U}_{\texttt{M}}$
- (b) γ is of the form $\pi(x)$ and f maps x onto the element a of $\mathrm{U}_{\texttt{M}}$ such that $\langle\pi, a\rangle$ belongs to $\texttt{Name}_M$
- (c) γ is of the form $\eta(x)$ and f maps x onto an element a of $\mathrm{U}_{\texttt{M}}$ such that a belongs to $\texttt{Pred}_M(\eta)$
- (d) γ is of the form $x\zeta$ and f maps x onto an element a of $\mathrm{U}_{\texttt{M}}$ such that a belongs to $\texttt{Pred}_M(\zeta)$

[7] Kamp and Reyle (1993:229-30), definitions 2.4.1 and 2.4.2

(e) γ is of the form $x\,\xi\,y$ and f maps x and y onto elements a and b of $\mathrm{U}_{\mathtt{M}}$ such that $\langle a,\, b\rangle$ belongs to $\mathtt{Pred}_M(\xi)$

(f) γ is of the form $\neg\mathrm{K}'$ and there is no embedding g from R into $\mathtt{M}$ which extends f, such that $\mathrm{Dom}(g) = \mathrm{Dom}(f) \cup \mathrm{U}_{K'}$ and g verifies K′ in $\mathtt{M}$

(g) γ is of the form $\mathrm{K}_1 \Rightarrow \mathrm{K}_2$ and for every extension g of f such that $\mathrm{Dom}(g) = \mathrm{Dom}(f) \cup \mathrm{U}_{K_1}$ which verifies K_1 in $\mathtt{M}$ there is an extension h of g such that $\mathrm{Dom}(h) = \mathrm{Dom}(g) \cup \mathrm{U}_{K_2}$ and h verifies K_2 in $\mathtt{M}$.

(h) γ is of the form $\mathrm{K}_1 \vee \mathrm{K}_2 \vee \ldots \vee \mathrm{K}_n$ and for some i ($i = 1, \ldots, n$) there is an extension g_i of f such that $\mathrm{Dom}(g_i) = \mathrm{Dom}(f) \cup \mathrm{U}_{K_i}$ and g_i verifies K_i in $\mathtt{M}$.

Several of these notions Kamp and Reyle mention are familiar enough—like the notion of individuals, mentioned moments ago, and the notion of relations, also mentioned moments ago, as the set of all conditions. Less familiar are the notions of a world and of a model. Also there is the notion of property "possession", the notion of names, and the technical notion of *verification* within a model $\mathcal{M}$ (or $\mathtt{M}$ in definition 1.2). How might all these relate? To see how, we'll return to our objective of developing a syntax-semantics interface for DRSs, and, from this construction, we'll advance our discussion of worlds, names, individuals, relations, properties, compositionality and the like.

Definition 1.3 (DRSs—a preliminary definition)

terms $t := u \,|\, c.$

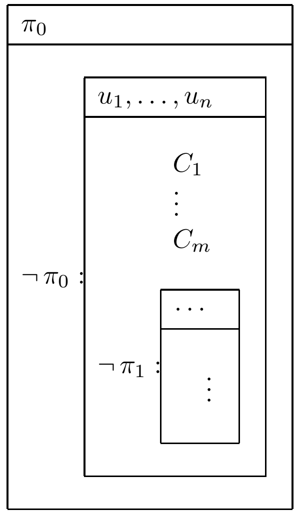
conditions $K := \top \,|\, Rt_1 \ldots t_k \,|\, u \doteq t \,|\, u \neq t \,|\, \neg D.$

DRSs $D := (\{u_1, \ldots, u_n\},\, \{C_1, \ldots, C_m\}).$

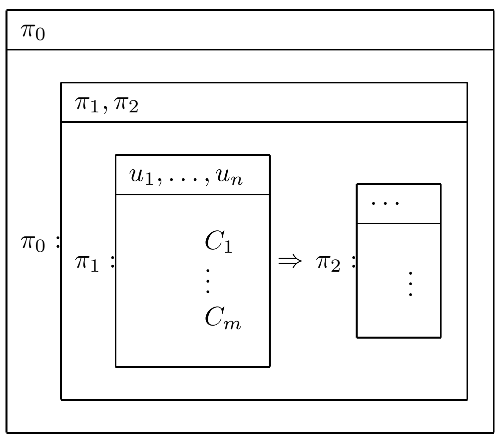
Now, if we let A represent the set of all constants, and, as agreed earlier, if we let U represent the set of all variables over constants (or, as Eijck and Kamp call them in *Representing discourse in context* [13], "reference markers"), then we may observe that c ranges over A and u ranges over U. In a similar fashion, letting P represent the set of all predicates with k arity, for $k \geq 1$, we see that R_k ranges over P, just as c and u range over their respective sets. Taken together, we have the loose outlines of a syntax for atomic DRSs, out of which complex DRSs may be formed.[8] To demonstrate what we've accomplished so for, we'll discuss a template DRS useful for later purposes. By convention, we say that $D_1 \Rightarrow D_2$ is shorthand for $\neg(\{u_1, \ldots, u_n\},\, \{C_1, \ldots, C_m, \neg D_2\})$, where $D_1 = (\{u_1, \ldots, u_n\},\, \{C_1, \ldots, C_m\})$. The consequence of this convention yeilds the following equivalence among DRSs depicted in 2 and 3:

[8] Eijck and Kamp observe "... [I]t turns out to be possible to view DRSs as built from atomic building blocks which are also DRSs"(1997:194). If this observation is true of DRS composition, then this compositional property suffices to abide by desired compositionality constraints already mentioned.

(2)



(3)



About the π_is appended to the DRSs above, for our purposes, we won't bother discussing the role and properties of these DRS-markers. For now suffice it to say that this designation denotes DRS dependency, a hierarchical concept related to the notion of *accessibility* relations, about which I'll have something brief to say later as an aside.To continue our discussion of DRS compositionality, we need to explore in more depth the notion of a model. Having further discussed the properties of a model, we'll provide this rudimentary syntax with a semantic counterpart—a model theory, in fact.

Recall what Kamp and Reyle say of a model: that "... [a] model is a certain *information structure*"(1993:93). They go on to say that there are "... many different ways in which this kind of information could be represented". Elsewhere, in Partee, Meulen, and Wall [19] we find that the relationship between a *theory* and a model in mathematics and logic is no less applicable in natural language studies. Much of what we would consider a theory of some one ℓ-particular language (from the set of all natural languages) will depend on which ℓ-particular language we're talking about, and perhaps which grammar handbook we consult. However, if we're intending our notion of theory to be a more expansive concept in the context of our linguistic research, we may consider defining "theory"by all and only those derivations which are possible under Minimalist constraints—or some other L-universal theory of Syntax.[9]

[9] See Faulkner [14] in which the notions of a theory, Θ_α and its defining derivations (or productions) are discussed in the context of the Minimalist Program; see also Partee, Meulen, Wall (1990:94-5).

1.1.2 A Brief Treatment of Model Theory Proper

Developing an intuition for what the notion of a model is requires us to distinguish it from its complementary notion of a theory. For the sake of space we'll forego a more rigorous definition of a theory, opting for definitions suitable enough to our purposes.[10]

Defining a Theory Θ: A set of axioms together with all the theorems formally derivable from this set constitutes a theory. In general, we say a theory is defined in terms of a proper subset of independent statements axiomatic to the theory under consideration. A theory is said to be closed under logical consequence—that is, if φ is a logical consequence of some one $\varphi\prime \in \Gamma$ where $\Gamma \in \Theta$, then φ is in Θ as well. In symbols: If $\Gamma \models \varphi$, where $\Gamma \in \Theta$, $\longrightarrow$ $\varphi \in \Gamma$. Taken together, we say a theory defines a formal system.

Definition 1.4 (Constituents of a Formal System Θ_α)

Any given formal system Θ_α is said to be comprised of the following constituent sets:

(i) A non-empty set of primitives

(ii) A set of statements about the primitives, the *axioms*

(iii) A means of deriving further statements from the axioms, either:

  (a) An explicit set of recursive rules of derivation; or

  (b) Appeal to a background logic [(as in the case of mathematics)] for the language in which in the axioms are stated, usually predicate logic [(**PL**)][11] ; or

  (c) No explicit means of derivation; one is to derive "whatever logically follows" from the axioms, [(as in the case of much mathematical research)].[12]

A model then is an interpretation of a given formal system. In the case of mathematics and logic, a model $\mathcal{M}$ serves to establish the conditions under which all expressions derivable in Θ_α are true, secured upon the precise interpretation $\mathcal{M}$ gives to Θ_α's primitive expressions. A standard example of a model is Plane geometry for the Euclidean axioms.[13] Plane geometry, it's said, constitutes that (infinite) body of *true* statements that follow logically from members of Euclid's axiom set. Now to define a given $\mathcal{M}$:

Defining a Model $\mathcal{M}$: Let $\mathcal{M} = \langle M, I\rangle$ be an appropriate model for DRS D. An assignment s for $\mathcal{M} = \langle M, I\rangle$ is a mapping of the set of reference markers U to elements of M. The term valuation determined by $\mathcal{M}$ and s is the function $V_{\mathcal{M},s}$ defined by $V_{\mathcal{M},s}(t) := I(t)$ if $t \in A$ and $V_{\mathcal{M},s}(t) := s(t)$ if $t \in U$ (recall definition 1.3)[14] .

Regarding our notion of a model $\mathcal{M}$ for DRSs, we have the following three relevant definitions. We use $s[X]s'$ to mean s' agrees with s except possibly on the values of the members of X.

Definition 1.5 (Assignments Verifying a DRS)

An assignment s verifies $D = (\{u_1, \ldots, u_n\}, \{C_1, \ldots, C_m\})$ in $\mathcal{M}$ if there is an assignment s' with $s[\{u_1, \ldots, u_n\}]s'$ which satisfies every member of $\{C_1, \ldots, C_m\}$ in $\mathcal{M}$.

[10] See Faulkner [14] and Partee, Meulen, Wall [19] more details on the subject.
[11] Tarski discusses the interesting notion of *disciplines preceding the given discipline* and goes on to say "...logic itself does not presuppose any preceding discipline"(1941:119).
[12] Partee, Meulen, Wall (1990:92)
[13] See Barker (1964:15-55; Partee, Meulen, Wall [19] §8.5; Penrose (2004:25-50).
[14] Eijck and Kamp (1997:192-3)

Definition 1.6 (Assignments Satisfying a Condition K)

(i) s always satisfies $\top$ in $\mathcal{M}$

(ii) s satisfies $P(t_1, \ldots, t_n)$ in $\mathcal{M}$ iff $\langle V_{\mathcal{M},s}(t_1), \ldots, V_{\mathcal{M},s}(t_n)\rangle \in I(P)$.

(iii) s satisfies $u \doteq t$ in $\mathcal{M}$ iff $s(u) = V_{\mathcal{M},s}(t)$

(iv) s satisfies $u \neq t$ in $\mathcal{M}$ iff $s(u) \neq V_{\mathcal{M},s}(t)$

(v) s satisfies $\neg D$ in $\mathcal{M}$ iff s does not verify D in $\mathcal{M}$

Definition 1.7 (DRS Truth in $\mathcal{M}$)

Structure D is true in $\mathcal{M}$ if there is an assignment which verifies D in $\mathcal{M}$.

A few remarks about models are in order here before we proceed to give definition 1.3 a simplified model theory, fit for its simple syntax. We want to call attention to several important properties of models. One is that if a theory Θ_α "...has an axiomatic characterization," something is a model $\mathcal{M}$ for this theory if and only if $\mathcal{M}$ is a model for the axioms (Partee, Meulen, Wall 1993:200). This is an important point since model theory is principally concerned with the interpretation of members of the axiom set α and the resulting semantic consequences $I(\alpha)$. Equally worthy of note is that models in model theory "...are always models of axioms or other expressions *in some language*, never of concrete objects"(1993:201; emphasis added).[15]

More generally, we have these properties for any given model $\mathcal{M}$: Recalling the notion of a theory as a formal system, we must recognize that it is in point of fact by means of a given theory Θ_α that we make sense of a certain $\mathcal{M}$ relative to Θ_α. Tarski maintains that a model $\mathcal{M}$ satisfies *not the statements of* Θ_α *themselves*; but rather, it is the *sentential functions obtained by replacing the primitive terms by variables* which a given $\mathcal{M}$ relative to some Θ_α satisfies (1941:123). So, we see that a model emerges from the *form* of a given theory, rather than its content. As such, upon finding a model for some Θ_α, we may consider the model to be *a model of the theory itself* (123). The form sets up a set of sentential functions, which taken together, secure the validity or affirmative truth value of a certain set of possibly different interpretations. We may consider an example:

Example 1 (Geometric Congruence)

Axiom I. For any element x of the set S (Segments), $x \cong x$ (i.e., $\forall s \in S$, where $x = s$, s is congruent to itself).

Axiom II. For any elements x, y and z of the set S, if $x \cong z$ and $y \cong z$, then $x \cong y$ (i.e., for any two (distinct) $s \in S$, if both are congruent to some (distinct) third $s \in S$, then they're congruent to each other as well).

Theorem I (Symmetry of Congruence). For any elements x and y of the set S, if $x \cong y$, then $y \cong x$.

Theorem II (Transitivity of Congruence). For any elements x, y and z of the set S, if $x \cong y$ and $y \cong z$, then $x \cong z$.

[15] It follows then that model theory is quite different from what you would find in a course in mathematical modeling or in a course in mathematical physics.

Proof of Theorem I. For any elements x and y of the set S, if $x \cong x$ and $y \cong x$, then $x \cong y$.

The proof of Theorem II proceeds similarly and so is omitted. Important about this simple example is that a "model of the theory itself"may be formed. Observe that replacing "S"with "K"and "$\cong$"with "R", the axioms in I and II become considerably more general, as do their set of theorems:

Axiom I′. For any x in K, $x\mathrm{R}x$.

Axiom II′. For and x, y and z in K, if $x\mathrm{R}z$ and $y\mathrm{R}z$, then $x\mathrm{R}y$.

And from these more general renderings of Axioms I and II, we have:

Theorem I′. Every relation R which is reflexive in K and has the property P in K is also symmetrical in K.

Theorem II′. Every relation R which is reflexive in K and has the property P in K is also transitive in K.

From example 1, we see that, if a relation R is reflexive and has some property P in K, both K and R together form a model or realization of the axioms (Tarski 1941:123). More importantly, we may observe that, after translation into variables over specific sets and relations, the statements of our theory are then no longer sentences with obvious content, but have become sentential functions which contain free variables (or "placeholders", if you will) ranging over specific interpretations. In mathematics alone, there are many different interpretations which *satisfy* these axioms alone, and thereby their derivable theory $\Theta_{example\,1}$. To name just one obvious example, equality is satisfied by this model:

Proof by Interpretation with $\mathcal{M}_{equality}$ For any x in N (natural numbers), if $x = x$ and $y = x$, then $x = y$ (symmetry satisfied); furthermore, for any x, y and z in N, if $x = y$ and $y = z$, then $x = z$ (transitivity satisfied).

So, we see we have two phenomena here: both (1) a model of the deductive theory itself—i.e., the more general form—and (2) a model predicated on interpretations of the model of the theory itself, which in turn *prove* the model of the deductive theory (as in the case of $\mathcal{M}_{equality}$). Writes Tarski, the type of models characterized by classification (1) take "...no distinguished place among the other models"(1941:126). He reasons,

> [W]hen deducing this or that theorem from the axioms, we do not think of the specific properties of this model, and we make use only of those properties which are explicitly stated in the axioms and, therefore, belong to every model of the axiom system. Consequently, every proof of a particular theorem of our theory can be extended to every model of the axiom system and can be thus transformed into a much more general argument no longer belonging to our theory [alone] but to logic; and as a result of this generalization we obtain a *general logical statement*, which establishes the fact that the theorem in question is satisfied by every model of our axiom system.

A general result follows from this, the formal details of which we may be allowed to skip over; and that is the *Law of Deduction*:

Definition 1.8 (The Law of Deduction)

> Every theorem of a given deductive theory Θ_α is satisfied by any model $\mathcal{M}_\alpha$ of the axiom system of Θ_α; and moreover, to every theorem there corresponds a general statement which can be formulated and proved within the framework of logic and which establishes the fact that the theorem in question is satisfied by any such model $\mathcal{M}_\alpha$.[16]

[16] Tarski (1941:127) considers the low of deduction to be a general methodological postulate of the deductive sciences.

1.1.3 Static versus Dynamic Models

We return now to our discussion earlier of SL, and to its compositional properties. We will now define a model for SL, call it $\mathcal{M}_{SL}$:

Definition 1.9 ($\mathcal{M}_{SL}$—A Compositional Definition)

The denotation of an expression φ relative to a model $\mathcal{M}$ and an assignment g, written $[\![\varphi]\!]^{\mathcal{M},\,g}$, is defined recursively, in concert with its syntactic rules (See page 4):

(0) (a) If φ is a non-logical constant in $\texttt{CON}^L_A$, then $[\![\varphi]\!]^{\mathcal{M},\,g} = F(\varphi)$

(b) If φ is a variable in $\texttt{VAR}_a$, then $[\![\varphi]\!]^{\mathcal{M},\,g} = g(\varphi)$.

(1) If P is an n-ary predicate and t_1, t_n are all terms, then $[\![P(t_1, \ldots, t_n)]\!]^{\mathcal{M},\,g} = 1$ iff $\langle [\![t_1]\!]^{\mathcal{M},\,g}, \ldots, [\![t_n]\!]^{\mathcal{M},\,g} \rangle \in [\![P]\!]^{\mathcal{M},\,g}$ (i.e. $[\![P(t_1, \ldots, t_n)]\!]^{\mathcal{M},\,g}$ is true iff $t_1, \ldots, t_n$ are in the extension of P).

If φ, $\psi \in$ SL, then:

(2) $[\![\neg\varphi]\!]^{\mathcal{M},\,g} = 1$ iff $[\![\varphi]\!]^{\mathcal{M},\,g} = 0$

(3) $[\![\varphi \,\&\, \psi]\!]^{\mathcal{M},\,g} = 1$ iff $[\![\varphi]\!]^{\mathcal{M},\,g} = 1$ and $[\![\psi]\!]^{\mathcal{M},\,g} = 1$

(4) $[\![\varphi \vee \psi]\!]^{\mathcal{M},\,g} = 1$ iff $[\![\varphi]\!]^{\mathcal{M},\,g} = 1$ or $[\![\psi]\!]^{\mathcal{M},\,g} = 1$

(5) $[\![\varphi \rightarrow \psi]\!]^{\mathcal{M},\,g} = 1$ iff $[\![\varphi]\!]^{\mathcal{M},\,g} = 0$ or $[\![\psi]\!]^{\mathcal{M},\,g} = 1$ (Note that (5) is reducible to (2) and (4))

(6) $[\![\varphi \leftrightarrow \psi]\!]^{\mathcal{M},\,g} = 1$ iff $[\![\varphi]\!]^{\mathcal{M},\,g} = [\![\psi]\!]^{\mathcal{M},\,g}$ (Note that (6) is reducible to (5))

And then we have *logical truth* and *logical falsity*:

(LT) For any formula φ, $[\![\varphi]\!]^{\mathcal{M}}$ (i.e. φ is true *simpliciter* with respect to $\mathcal{M}_{SL}$) iff, for all assignments g, $[\![\varphi]\!]^{\mathcal{M},\,g} = 1$

(LF) For any formula φ, $[\![\varphi]\!]^{\mathcal{M}} = 0$ iff, for all assignments g, $[\![\varphi]\!]^{\mathcal{M},\,g} = 0$[17]

$\mathcal{M}_{SL}$ represents a static model theory. For SL, a static model theory is perfectly alright in that SL deals in only static productions, and, as mentioned earlier, is not process oriented. Having seen $\mathcal{M}_{SL}$ and a number of other examples of a model theory, we may now provide definition 1.3 with a model, call it $\mathcal{M}_{prelim}$. We begin with a static model and then revise our model to account for CCP phenomena.

Definition 1.10 ($\mathcal{M}_{DRSs}$)

$$[\![(\{u_1, \ldots, u_n\}, \{C_1, \ldots, C_m\})]\!]_{\mathcal{M}} := (\{u_1, \ldots, u_n\}, [\![C_1]\!]_{\mathcal{M}} \cap \ldots \cap [\![C_m]\!]).^{18}$$

Definition 1.11 (Model $\mathcal{M}_K$ of conditions K for definition 1.3)

(i) $[\![P(t_1, \ldots, t_n)]\!]_{\mathcal{M}} := \{s \in M^U \mid \langle V_{\mathcal{M},\,s}(t_1), \ldots, V_{\mathcal{M},\,s}(t_n)\rangle \in I(P)\}$

(ii) $[\![u \doteq t]\!]_{\mathcal{M}} := \{s \in M^U \mid s(u) = V_{\mathcal{M},\,s}(t)\}$

[17] Partee, Meulen, Wall (1990:325-7); note that $g\colon \texttt{VAR} \mapsto D$, where D is understood to be equivalent to our A, which we've defined as our set of constants.

[18] See Eijck and Kamp (1997:193-5) for more details on the definitions and propositions that follow.

(iii) $[\![u \neq t]\!]_{\mathcal{M}} := \{s \in M^U \mid s(u) \neq V_{\mathcal{M},\,s}(t)\}$

(iv) $[\![\neg D]\!]_{\mathcal{M}} := \{s \in M^U \mid$ for no $s' \in M^U : s[X]s'$ and $s' \in F\}$, where $(X, F) = [\![D]\!]_{\mathcal{M}}$

Connecting $\mathcal{M}_{SL}$ and $\mathcal{M}_{prelim}$ have the following meaning preserving translation procedure:[19]

Definition 1.12 (Translation. $^\circ$: $DRT \mapsto PL$**)**

(i) For DRSs: if $D = (\{u_1, \ldots, u_n\}, \{C_1, \ldots, C_m\})$ then $D^\circ := \exists u_1 \cdots \exists u_n(C_1^\circ \wedge \cdots \wedge C_m^\circ)$.

(ii) For atomic conditions K_i (i.e. atoms or links): $C^\circ := C$.

(iii) For negations: $(\neg D)^\circ : \neg D^\circ$.

From these, have the following translation instruction for implications:

First, assume $D_1 = (\{u_1, \ldots, u_n\}, \{C_1, \ldots, C_m\})$.

(iv) $(D_1 \Rightarrow D_2)^\circ := \forall u_1, \cdots, \forall u_n((C_1^\circ, \cdots, C_m^\circ) \rightarrow D_2^\circ)$.

Proposition 1.1 (Tarskian Satisfaction$^\circ$)

s verifies D in $\mathcal{M}$ iff $\mathcal{M}, s \models D^\circ$, where $\models$ is Tarski's definition of satisfaction for first-order PL.

We may also give a meaning preserving translation procedure from PL to basic DRT, as at least specified in definitions 1.3, 1.10 and 1.11: Let $\varphi^\bullet$ be the DRS corresponding to the PL formula φ. Using this allowance, we say $\varphi_i^\bullet$ and $\varphi_j^\bullet$ are two *consecutive* components $i, j \in \mathrm{N}$, where $i \neq j$. From this we have:

Definition 1.13 (Translation. $^\bullet$: $PL \mapsto DRT$**)**

(i) For atomic formulas: $C^\bullet := (\emptyset, C)$.

(ii) For conjunctions: $(\varphi \wedge \psi)^\bullet := (\emptyset, \{\varphi^\bullet, \psi^\bullet\})$.

(iii) For negations: $(\neg\varphi)^\bullet := (\emptyset, \varphi^\bullet)$.

(iv) For quantifications: $(\exists u\varphi)^\bullet := (\varphi_i^\bullet \cup \{u\}, \varphi_j^\bullet)$.

As earlier, from this meaning preserving translation procedure, we have the corresponding proposition, regarding Tarskian satisfaction:

Proposition 1.2 (Tarskian Satisfaction$^\bullet$)

$\mathcal{M}, s \models \varphi$ iff s verifies $\varphi^\bullet$ in $\mathcal{M}$, where $\models$ is Tarski's definition of satisfaction for first-order PL.

[19] I realize we haven't discussed predicate logic (**PL**. Suffice it to say that predicate logic is merely the extension of SL by adding the two additional operators $\exists$ and $\forall$, which respectively mean *there exists an* and *for all.*)

1.1.4 Dynamic Distinctions. Beyond Truth-Conditions and the Extension of DRT for CCP

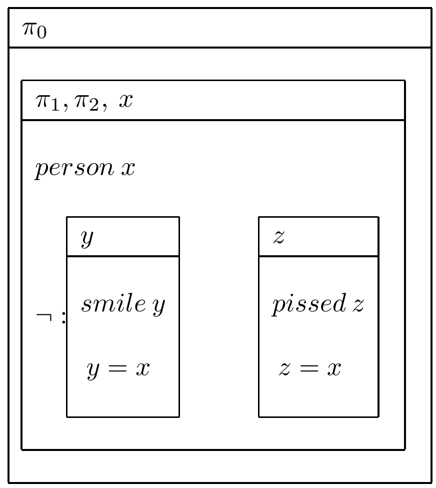
At this point we wish to distinguish SDRT from PL, or for that matter SL, by calling attention to CCP phenomena and how SDRT handles these. Eijck and Kamp (1997:194) provide of us with the following two sentences to elucidate the distinction in functionality between PL and DRT. It'll work just as well for SDRT.

(A) Someone didn't smile. He was pissed.

(B) Not everyone smiled. *He was pissed.

From (A) we find that DRS 4 represents a suitable depiction of its semantics relationships (of course, ignoring for tense, which isn't a necessary condition on DRS capabilities (See Kamp and Reyle (1993:483-689))).[20]

(4)



[20] There are details to DRSs 4 and 5 which we will not discuss in the course of this essay; nevertheless, for the interested reader, I provide the constraints on anaphoric dependencies in the form of the two definitions seen below:

Definition 1.14 (DRS Subordination)

(i) A DRS K_j is *immediately subordinate* to K_h if the DRS conditions C_{K_h} of K_h contains K_j (or $\neg K_j$) — that is, if $C_{K_h} \subseteq K_j$.

(ii) *Transitive Closure*: K_j is subordinate to K_i (e.g., $K_j \leq K_i$) if (*i*) $K_j = K_i$; or (*ii*) K_j is immediately subordinate to K_h; or (*iii*) there is a DRS K_i such that K_j is immediately subordinate to K_i and $K_i \leq K_h$.

Definition 1.15 (DRS Accessibility)

A discourse referent u is *accessible* to an anaphoric DRS condition in K_i iff u is introduced in U_{K_h} where:

(i) $K_i \leq K_h$; or

(ii) $K_h \Rightarrow K_j$; or, K_h quantifies/scopes K_j *and* $K_i \leq K_j$.

Given the subordination of $\neg : \begin{array}{|l|}\hline y \\ \hline \textit{smile}\ x \\ y = x \\ \hline \end{array}$, variable x has access to the variable in $\begin{array}{|l|}\hline x \\ \hline \textit{person}\ x \\ \hline \end{array}$, and thus is resolved. Updating, we see the same may be said for $\begin{array}{|l|}\hline z \\ \hline \textit{pissed}\ z \\ z = x \\ \hline \end{array}$, that z is resolved in $\begin{array}{|l|}\hline x \\ \hline \textit{person}\ x \\ \hline \end{array}$. This matches our intuitions in that it produces the meaning (A) $\Longleftrightarrow$ (A′):

(A) Someone didn't smile. He was pissed.

(A′) There is at least one person who didn't smile and the person who is pissed is that same person.

Note that this stands in contrast to the form given in DRS 5 below, which lacks several forms of resolution:

$$(5)\qquad \begin{array}{|l|}\hline \pi_0 \\ \hline \begin{array}{|l|}\hline \pi_1, \pi_2 \\ \hline \neg : \begin{array}{|l|}\hline \\ \hline \begin{array}{|l|}\hline x \\ \hline \textit{person}\ x \\ \hline \end{array} \Rightarrow \begin{array}{|l|}\hline y \\ \hline \textit{smile}\ y \\ y =? \\ \hline \end{array} \\ \hline \end{array} \quad \begin{array}{|l|}\hline z \\ \hline \textit{pissed}\ z \\ z =? \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array}$$

In (B), *He* in *He was angry* has no clear anaphoric relationship with any one member from set *not everyone* in the prior processed *Not everyone smiled*. This ambiguity is modeled above by the unresolved structures $\mathtt{v} = ?$, where $\mathtt{v} \in \mathtt{VAR}$. These features of anaphoric resolution in discourse update represent just a few of the difficulties which arise in attempts to model context change potential. PL, let alone SL, are ineffectual to model such behavior and, as such, are in need of considerable revision. Both DRT and SDRT represent projects underway, which are trying to do just that: to account for CCP phenomena, and more. Kamp and Reyle [17] write,

> The incremental nature of [discourse] interpretation is closely connected with a ubiquitous feature of discourse, its *semantic cohesiveness.* Typically the sentences that make up a coherent piece of discourse are connected by various kinds of cross-reference. As a consequence it is often impossible to analyze the meaning of cohesive discourse as a simple conjunction of the separate meanings of the individual sentences that make it up [(An affront to the principle of compositionality)]. The meaning of the whole is more, one might say, than the conjunction of its parts. The connection between cohesiveness and incremental discourse processing is, in rough outline, this: to understand what information is added by the next sentence of a discourse to what he has learned already from the sentences preceding it, the interpreter must *relate* that sentence to the information structure he has already obtained from those preceding sentences. Thus the interpretation of the new sentence must rely on two kinds of structures, *the syntactic structure of the sentence itself* and the *structure representing the context of the earlier sentences.* (1993:59; emphasis added)

The updating properties of CCP phenomena coupled with properties of other unmentioned updating constraints together call for a dynamic semantics. What we've seen so far are models which constitute the basis for a static semantics. And although, as Eijck and Kamp write, "they are quite different from a conceptual point of view, the dynamic and static semantics for formalisms like those of DRT are nonetheless closely connected"(1997:197). The distinction arises with respect to a dynamic semantic's ability to model sequentiability, a notion of processing update information in a given order (198). Below we'll display a dynamic model for the DRT theory we've put forward thus far, and then we'll extend this model a bit making room for the computational semantics SDRT requires.

Definition 1.16 (A Dynamic Model for DRT $\mathcal{M}_{DRT}^{dynamic}$)

(i) ${}_s[\![(\{u\},\,\emptyset)]\!]_{s'}^{\mathcal{M}}$ iff $s[u]s'$.

(ii) ${}_s[\![(\emptyset,\,\{\top\})]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$.

(iii) ${}_s[\![(\emptyset,\,\{Pt_1,\,\ldots\,,t_n\})]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $\langle V_{\mathcal{M},\,,s}(t_1),\,\ldots\,,V_{\mathcal{M},\,s}(t_n)\rangle \in I(P)$.

(iv) ${}_s[\![(\emptyset,\,\{u \doteq t\})]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $s(u) = V_{\mathcal{M},\,s}(t)$.

(v) ${}_s[\![(\emptyset,\,\{u \neq t\})]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $s(u) \neq V_{\mathcal{M},\,s}(t)$.

(vi) ${}_s[\![\neg D]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $\neg\exists\, s''$ such that ${}_s[\![D]\!]_{s''}^{\mathcal{M}}$.

(vii) ${}_s[\![D \oslash D']\!]_{s'}^{\mathcal{M}}$ iff ${}_s[\![D]\!]_{s'}^{\mathcal{M}}$ and ${}_{s'}[\![D']\!]_{s'}^{\mathcal{M}}$.[21]

Unpacking some of this notation we find the symbol $\oslash$ which is an asymmetric merge operation defined as follows:

If $D = (X,\,C)$ and $D' = (Y,\,C')$ then $D \oslash D' := (X, C \cup C')$ is a DRS.

Notice though that $\oslash$ drops Y in the merge process. This is insufficient to our causes, and so will have to be revised, so as to preserve access to discourse referents in Y. "To give a true account of the context change potential of DRSs,"write Eijck and Kamp,"one has to be able to answer the question how the context change potential of DRS D_1 and that of D_2 which follows it determine the context change potential of their composition"(1997:198).

Further, we find ${}_s[\![D]\!]_{s'}^{\mathcal{M}}$ which serves to denote the dynamic value of DRS D in $\mathcal{M}$, where D is understood to determine the relationship between assignments s and s' of $\mathcal{M}$. Defining ${}_s[\![D]\!]_{s'}^{\mathcal{M}}$, we have:

If $D = (X,\,C)$ then: ${}_s[\![D]\!]_{s'}^{\mathcal{M}}$ iff $s[X]s'$ and s' verifies D in $\mathcal{M}$.

[21] Eijck and Kamp (1997:197-8)

Having given ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ a proper definition we may proceed to define the computational semantic basis of Segmented DRT, an extension over DRT. In words, we say ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ constitutes a function in which s and s' are an input/output state pair determined by D in model $\mathcal{M}$. We say $s[u]s'$ represents the fact that s and s' differ at most in value for u. We'll refrain from going into the details of modal logic, but it is important to note at this juncture that much more could be said about the notion of an input/output structure and about how these relate to worlds, mentioned earlier in our discussion of primitives.

1.1.5 Defining the Segmented DRT's Computational Semantics

We close by defining the whole of SDRT's computational semantics to be the following:[22]

Definition 1.17 (An Embellished Syntax for the DRS Language)

Vocabulary:

(i) *Discourse-Referents* is a set of variables x, y, z, ..., with or without subscripts.

(ii) *Predicates* is a set of predicate symbols associated with the various natural language nouns, verbs, adjectives, and adverbials.

Discourse Representation Structures (DRSs)

Suppose $U \subseteq$ *Discourse-Referents*. Then the well-formed DRSs K and the DRS conditions γ are defined recursively:

$$K \quad := \quad \langle U, \emptyset \rangle \,|\, K^{\frown}\gamma$$

where K is understood to be a DRS condition, indicative of one or more conditions in K, and γ is some one new update DRS condition appended to these prior existing conditions K. In effect, $K^{\frown}\gamma$, which may be defined as $K^{\frown}\gamma =_{def} \langle U_K, append(C_K, \gamma) \rangle$.

Let $R \in Predicates$ be an n-ary predicate, x_1, ..., x_n be discourse referents from U, and $K_{i,j}$ be DRSs. Then, we find the following structural definitions:

$$\gamma \quad := \quad R(x_1, \ldots, x_n) \,|\, \neg K \,|\, K_i \Rightarrow K_j \,|\, K_i \vee K_j \,|\, K_i > K_j \,|\, \Box K \,|\, \Diamond K.$$

Notice that many of the above operators are those familiar to us from first-order logic. In addition to these, there are the operators $\Box$, $\Diamond$, and $>$. To be sure, over time the modal operators have taken on numerous interpretations (See Garson 2006), many of which are clearly inapplicable to the purposes of SDRT; so, not unreasonably, SDRT restricts its use of these operators to a strictly alethic interpretation—that is, to an interpretation concerned solely with $\Box$ and $\Diamond$'s functions as truth-functional predicates of necessity and possibility. It is from these modal operators—and the assignment pair notion ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ mentioned above—that we find ourselves working with the notion of *worlds* (Asher and Lascarides (2003:47); Eijck and Kamp (1997:197); Garson (2006:93-116); Kripke (1980)), an intuitively useful concept. The other operator, $>$, might be termed the *normality* operator and works roughly in the following way: $K_i \Rightarrow_{normally} K_j$. To use an example, in words we have something like the following: "If it's raining outside (K_i), (then) one will (normally) take an umbrella (K_j)."[23] Note the similarity between this operator and the material conditional familiar from PL. Notably, however, the truth-values predicated by $>$ are markedly different.

Having developed a proper base-syntax, we may now define the semantic counterpart to the structures outlined above.

[22] Adaptations of Asher and Lascarides (2003:46-8); you'll find that much of what is presented here is pretty much the same as what we've already established, with of course the exception of the revised elements.

[23] Some may recognize this as one of the oft used anecdotes of early behaviorism (Searle 2003:51-2).

Definition 1.18 (A Model Theory for Segmented DRT—The Clause Axioms)

(i) $(w,f)[\![\langle U,\emptyset\rangle]\!]_M(w',g)$ iff $w=w' \;\wedge\; f\subseteq g \;\wedge\; dom(g)=dom(f)\cup U$

(ii) $(w,f)[\![K^{\frown}\gamma]\!]_M(w',g)$ iff $\exists w''\exists h(w,f)[\![K]\!]_M(w'',h) \;\wedge\; (w'',h)[\![\gamma]\!]_M(w',g)$

(iii) $(w,f)[\![R(x_1,\ldots,x_n)]\!]_M(w',g)$ iff $(w,f)=(w',g) \;\wedge\; \langle f(x_1),\,\ldots,\,f(x_n)\rangle\in I_M(R)(w)$

(iv) $(w,f)[\![\neg K]\!]_M(w',g)$ iff $(w,f)=(w',g) \;\wedge\; \neg\exists w''\exists h$ such that $(w,f)[\![K]\!]_M(w'',h)$

(v) $(w,f)[\![K_i\Rightarrow K_j]\!]_M(w',g)$ iff $(w,g)=(w',g) \;\wedge$

$$\forall h\forall w''(w,f)[\![K_i]\!]_M(w'',h) \;\longrightarrow\; \exists k\exists w'''(w'',h)[\![K_j]\!]_M(w''',k)$$

(vi) $(w,f)[\![K_i\vee K_j]\!]_M(w',g)$ iff $(w,f)=(w',g) \;\wedge\; (\exists h(w,f)[\![K_i]\!]_M(w',h) \;\vee\; \exists k(w,f)[\![K_j]\!]_M(w',k))$

(vii) $(w,f)[\![K_i>K_j]\!]_M(w',g)$ iff $(w,f)=(w',g) \;\wedge$

$$\forall w''\forall h((w,f)[*(w,[\![K_i]\!])_M](w'',h) \;\longrightarrow\; \exists w''\exists k(w'',h)[\![K_j]\!]_M(w''',k)$$

(viii) $(w,f)[\![\Box K]\!]_M(w',g)$ iff $(w,f)=(w',g) \;\wedge$

$$\forall w''(wR_{\Box}w'' \;\longrightarrow\; \exists h\exists w'' \text{ such that } (w',g)[\![K]\!]_M(w''',h))$$

As always, the computational semantics employed in SDRT proceeds relative to a fixed model $\mathcal{M}_{SDRT}$, where $\mathcal{M}_{SDRT}$ is understood to be a quintuple $\mathcal{M}_{SDRT}=\langle A_M,\, W_M,\, *_M,\, R_{\Box},\, I_M\rangle$. We have that $A_M=U$ from discussions earlier and so constitutes a set of individuals. Regarding W_M, we say W_M is a set of possible worlds. I_M is a function which assigns n-ary predicates P_n at a world w_i a set of n-tuples of A_M, to be written as $I_M(P_n)(w)$. $R_{\Box}$ is a binary relation on W_M which defines relative to a world w_i all the possibilities relative to w_i. To define the DRS condition $K_i>K_j$, we have the added structure $*M$, a dynamic conditional selection function, which maps a world and a dynamic proposition to another dynamic proposition. To explain, the notion of a dynamic proposition, recall ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ (See Asher and Lascarides (2003:47) for more details on $*M$ and the mechanics of how it defines normality in some one world w_i. We observe that ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ equivalent to the template function $(w,f)[\![K]\!]_M(w',g)$, where it can easily be seen that $K=D$, $f=s$, $g=s'$ and $M=\mathcal{M}$, the variable over model designations. The only additional notational component we find are the ws. These serve to make more explicit the notion of truth-in-a-model, where truth-in-a-model is thought of holding true relative to a select number of worlds $w_1,\,\ldots\,,\,w_n\in W_M$ (See Kamp and Reyle (1993); Partee, Meulen, Wall (1990)).

2 Toward Rhetorical Relations: Concluding Remarks

To be sure we're still quite a ways a away from justifying a structure like the ones below:

(6)

$$\begin{array}{|l|}\hline \pi_0 \\ \hline \pi_0 : \begin{array}{|l|}\hline \pi_1, \pi'' \\ \hline \pi_1 : K_{\pi_1}, \mathit{Elaboration}(\pi_1, \pi'') \\ \pi'' : \begin{array}{|l|}\hline \pi_2, \pi_5, \pi' \\ \hline \pi_2 : K^+_{\pi_2}, \pi_5 : K^+_{\pi_5}, \\ \mathit{Narration}(\pi_2, \pi_5 \\ \mathit{Elaboration}(\pi_2, \pi') \\ \pi' : \begin{array}{|l|}\hline \pi_3, \pi_4 \\ \hline \pi_3 : K^+_{\pi_3}, \pi_4 : K^+_{\pi_4} \\ \mathit{Narration}(\pi_3, \pi_4) \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array}$$

(7)

$$\begin{array}{|l|}\hline \pi'' \\ \hline \pi'' : \begin{array}{|l|}\hline \pi_d \\ \hline \pi_d : \partial \left(\begin{array}{|l|}\hline x, e_3 \\ \hline \mathit{dog}(x) \\ \mathit{own}(e_3, j, x) \\ \hline \end{array} \right) \\ R(u, v) \\ R = ?\ u = ?\ v = ? \\ i - \mathit{scopes}(\pi'', \pi_d) \\ \hline \end{array} \\ \hline \end{array}$$

Nevertheless, we have come rather far in that we've developed a deeper intuition about the constraints necessary for the monotonic basis of our theory. Cognitive processing of these structures in 6 and 7 would requires us to do a lot more in the way of theory building and development of our intuitions. For instance, it would require us to grapple with details concerning a nonmonotonic logic possibly used in the event of discourse processing. A glue logic would have to be investigated (See Asher and Lascarides (2003:179-248;375-451)). Additionally, as Kamp and Reyle mention above (1993:59), more research would have to be done concerning the set of structural properties, forming a single sentence φ, which, together or individually, positively inform semantic content at the level of discourse. All that aside, considering the foundations we've laid, we may proceed in a better informed manner to discuss those structures—rhetorical relations—which constitute the second major component of Segmented DRT.

Pointing the way to further discussion of the model-theoretic elements presented here, as well as their relationship to rhetorical relations, you'll find in the bibliographic contents that there are texts specially marked with an asterisk beside the authors name; these texts extend the scope and depth of the present discussion and are presented here for the interested reader. According to interest, the interested reader is encouraged to consult these marked texts so as to pick up where the present essay leaves off.

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