

# EXAMINATION

## BLUE BOOK

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SUBJECT Algorithms

INSTRUCTOR Professor Chen

EXAM SEAT NO. \_\_\_\_\_ SECTION Sebasation (TA)

DATE 10/18/17 GRADE 45/100



University of Colorado®  
Boulder

### Honor Code Pledge

On my honor, as a University of Colorado® Boulder student, I have neither given nor received unauthorized assistance on this work.

Austin Griffith

Signature

10/18/17

Date

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DIVISION OF THE PHYSICAL SCIENCES  
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1a)  $f(n) = 10n \log n$   $g(n) = n^{4/3}$

$\frac{10}{20}$

1b)  $T(n) = 3T(n/3) + cn$   
 $a=3$   $b=3$   $f(n)=cn$

$\log_b a = c$   $3^? = 3$   
 $\log_3 3 = 1$   $3^1 = 3$   
 $c=1$

$1 = \log_3 3$   
 $f(n) = \Theta(n^1 \log n)$   $^2$  most, therm. case 2

1c)  $T(n) = 4T(n/2) + n^2$   $a=4$   $b=2$   $f(n)=n^2$

$\log_b a \approx \log_2 4 = 2$   $2^? = 4$   $2^2 = 4$   $c=2$

$2 = \log_2 4$

so case 2 Most therm

$c = \log_b a$

1d) False  $^2$

1e) The worst case performance would be if the elements that need to be sorted are all greater than the pivot picked which is the 1<sup>st</sup> element of the array.

1f)

element	freq	code
b	.12	1111
e	.13	1110
d	.15	110
c	.2	10
a	.4	1

$$\{k, l\} : k \neq l \text{ and } h(k) = h(l)$$

1g)

$$K = 2000 \quad L = 1000$$

$$\Pr(h(K) = h(L)) = \frac{1}{m}$$

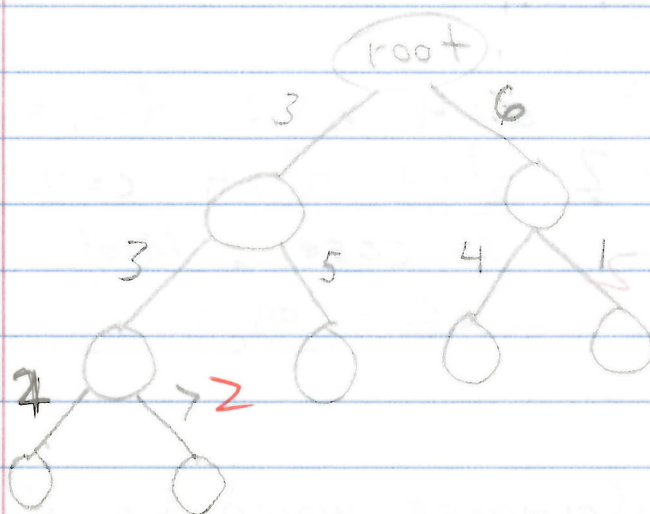
$$\sum_{\substack{k \neq l \\ i=1}}^n \frac{1}{m} = \frac{(n-1)}{m} = \frac{n^2 - n}{2m}$$

? 1h)

$$h(K) = K \bmod 11$$

$$\text{input } K : [11, 22, 11, 22, 11, 22, 11, 22]$$

1i)



1j)

start node A, want 4<sup>th</sup> edge

choice 1 A - E choice 2 E - F

vertex now E vertex now F

choice 3 F - C choice 4 F - B

vertex now C or F

0



$$\begin{matrix} 10 & 2 \\ n & m \end{matrix}$$

$$\begin{matrix} 2 \times 1 = 2 \\ n & m \end{matrix}$$

$$\begin{matrix} 2 \times 2 = 4 \\ n & m \end{matrix}$$

$$\begin{matrix} 2 \times 3 = 6 \\ 2+3 = 5 \\ m & n \end{matrix}$$

$$\begin{matrix} 2 \times 4 = 8 \\ 2+4 = 6 \end{matrix}$$

$$\begin{matrix} 2 \times 5 = 10 \\ 2+5 = 7 \end{matrix}$$

2a) ComputeProduct(m, n)

if (n > m)

{ x = n - m

return m + n + x

else { y = m - n

return m + n + y

$$\begin{matrix} 5 \\ 10 \end{matrix}$$

$$\begin{matrix} m & n \\ (2, 3) \end{matrix}$$

$$3 > 2 \checkmark$$

$$x = 3 - 2$$

$$\text{return } 2 + 3 + 1 = 6$$

$$2 \times 3 = 6 \checkmark$$

$$\begin{matrix} m & n \\ (3, 2) \end{matrix}$$

$$y = 3 - 2 = 1$$

$$m + n + 1 = 6 \checkmark$$

2b) The time complexity of this Algorithm is done in const time bc no loops or other function calls so  $\Theta(1)$

3)  $A = [1, 3, 5, 7, 9]$

$B = [2, 4, 6, 8, 10]$

$$5/5$$

4) a)  $T(n) = 2T(n/2) + cn$

$a = 2 \quad b = 2 \quad f(n) = cn$

$1 = \log_2 2$

so case 2 most, then

$$\frac{10}{10}$$

$\log_2 2 = ?$

$2^? = 2$

$2^1 = 2$

$c=1$

$c = \log_b a \Rightarrow f(n) = \Theta(n^c \log n)$

$\approx \Theta(n^1 \log n) \approx \Theta(n \log n)$

4b)  $i \neq ii$

$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ [2, & 6, & 4, & 1, & 5, & 3] \end{matrix}$

5a) ①  $X=4$   $i=-1$   $j=0$   $A[j] \leq X$   
 $A[0] \leq 4$  no

②  $X=4$   $i=-1$   $j=1$   $A[j] \leq 4$  yes  
 $i++ \Rightarrow$  now  $i=0$   $\text{swap}(A[i+1], A[j+1])$   
 $A[1], A[2]$

$[2, 4, 6, 1, 5, 3]$

③  $X=4$ ,  $i=0$ ,  $j=2$   $A[j] \leq 4$  yes  
 now  $i=1$   $\text{swap}(A[2], A[3])$

$[2, 4, 1, 6, 5, 3]$

④  $X=4$   $i=1$ ,  $j=3$   $A[j] \leq 4$  no

⑤  $X=4$   $i=1$   $j=4$   $A[j] \leq 4$  no

$X=4$   $i=1$   $j=5$   $A[j] \leq 4$  yes

now  $i=2$   $\text{swap } A[i+1], A[j+1]$

$\text{swap}(A[3], A[6])$  - out of bound

- The line of code that would need to change is the first call to swap. It should be:

$\text{swap}(A[i+1], A[j+1])$

You also need to account for the 2nd swap call which should be:

$\text{swap}(A[i-1], A[r])$

$$x=4 \quad i=1 \quad j=4$$

[2, 4, 1, 6, 5, 3]

swap( $A[i-1]$ ,  $A[j]$ )

swap( $A[0]$ ,  $A[5]$ )

[3, 4, 1, 6, 5, 2]

$$\frac{3}{10}$$

1 | 2 | 3 4 6 5  
 [1] 2 3 4 6 | 5  
 [1, 2] 3 4 | 5 | 6  
 [1, 2, 3, 4, 5, 6]

5b) The values used as pivots for the array are:

3, 2, 5

$$\frac{7}{10}$$

6a) Selection sort  $\Theta: \Theta(n^2)$

$O: O(n^2)$

$\Omega: \Omega(n^2)$

b) LI: for  $j=0$  and some array  $A$  the property hold true prior to the 1<sup>st</sup> iteration of the loop.

Initialization: the array  $A$  is unsorted prior to the 1<sup>st</sup> itt. and  $j=0$  is still true on the first itt.

Maintenance:  $j$  is increased upon each itt. and the  $A$  has sub arrays which are being sorted as the Algo runs.

Termination: upon term, the array is sorted and the loop exits.



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01

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$$A = [a_1, a_2, \dots, a_n]$$

$$B = [b_1, \dots, b_n]$$

7) Algo (A, B)  
 for (i=0 to length(A))  
     j=0  
     if (A[i] == negative of B[i] OR  
         negative of A[i] == B[i])  
         { sum = A[i] + B[i] }

the negative  
of the value  
of B[i]

explan: iterate through array A and B to check if any values are equal regardless of sign and take negate. If 2 exists in A check if negative of -2 is in B so  $2 = 2$  and will know one value is negative so added together will be zero

$$\frac{0}{10}$$



8) Base case:  $T(0) = 0$   $T(1) = 2$

a)  $T(n) = 2T(n/2) + n^2$   
 $a = 2$   $b = 2$   $f(n) = n^2$   
 $\log_b a = c$   
 $\log_2 2 = ? = 1$   
 $2^? = 2$   $2^1 = 2$

$$\frac{5}{15}$$

5

~~15~~

b)  $1 = \log_2 2$  so case 2 mast, then  
 $\Rightarrow \theta(n \log n)$

0



$$16 + 25 + 9 = 50$$

---

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

Source: <http://www.census.gov/hhes/education/data/tables/2000/2000.ed.education.attainment.attainment.html>

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