EXAMINATION

BLUE BOOK

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SUBJECT Algorithms

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EXAM SEAT NO. SECTION Sebasation (TA)

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Honor Code Pledge

On my honor, as a University of Colorado® Boulder student, I have neither given nor received unauthorized assistance on this work.

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Signature Date

10/20

1a) $f(n) = 10n \log n$ $g(n) = n^{4/3}$ 0

1b) $T(n) = 3T(n/3) + cn$ $\log_b a = c$ $3^? = 3$

$a = 3$ $b = 3$ $f(n) = cn$ $\log_3 3 = 1$ $3^1 = 3$

$c = 1$

$1 = \log_3 3$

$f(n) = \Theta(n^1 \log(n))$ 2 most. therm. case 2

1c) $T(n) = 4T(n/2) + n^2$ $a = 4$ $b = 2$ $f(n) = n^2$

$\log_b a = \log_2 4 = 2$ $2^? = 4$ $2^2 = 4$ $c = 2$

$2 = \log_2 4$ 2 so case 2 Most therm

$c = \log_b a$

1d) False 2

1e) The worst case performance would be if the elements that need to be sorted are all greater than the pivot picked which is the 1st element of the array. 0

1f)

element	freq	code
b	.12	1111
e	.13	1110
d	.15	110
c	.2	10
a	.4	1

$\{k, \ell\} : k \neq \ell$ and $h(k) = h(j)$

1g) $k = 2000 \quad \ell = 1000$

$\Pr(h(k) = h(j)) = \frac{1}{m}$

$\sum_{\substack{k \neq \ell \\ i=1}}^{n} \frac{1}{m} = \frac{(n-i)}{m} = \frac{n^2 - n}{2m}$

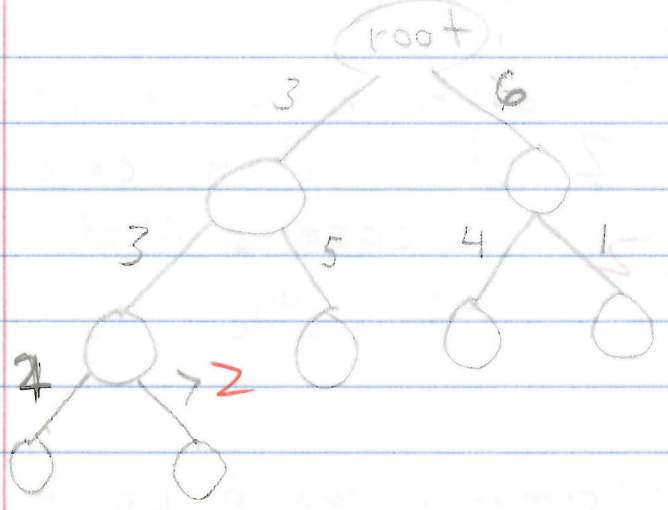
0

?. 1h) $h(k) = k \bmod 11$

input k : [11, 22, 11, 22, 11, 22, 11, 22]

≥

1i)



1j) start node A, want 4th edge

choice 1 A - E choice 2 E - F

vertex now E vertex now F

choice 3 F - C choke 4 F - B

vertex now C or F

0

10 2
n m

2 × 1 = 2
n m

2 × 2 = 4
n m

2 × 3 = 6
2 + 3 = 5
m n

2 × 4 = 8
2 + 4 = 6

2 × 5 = 6
2 + 5 = 7

5
10

2a) ComputeProduct(m, n)

```
if (n > m)
{  x = n - m
return m + n + x
else { y = m - n }
return m + n + y
```

0

m n
(2, 3)
3 > 2 ✓
x = 3 - 2
return 2 + 3 + 1 = 6

2 × 3 = 6 ✓

m n
(3, 2)
y = 3 - 2 = 1
m + n + 1 = 6 ✓

2b) The time complexity of this Algorithm is done in const time bc no loops or other function calls so $\Theta(1)$

3) A = [1, 3, 5, 7, 9]
B = [2, 4, 6, 8, 10]

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4) a) $T(n) = 2T(n/2) + cn$

$a = 2 \quad b = 2 \quad f(n) = cn$

$1 = \log_2 2$ so case 2 most, therm,

$\log_2 2 = ?$
$2^? = 2$
$2^1 = 2$ $(c = 1)$

10
10

$c = \log_b a \Rightarrow f(n) = \Theta(n^c \log n)$
$\approx \Theta(n^1 \log n) = \Theta(n \log n)$

4b) i ✗ ii

0 1 2 3 4 5

[2, 6, 4, 1, 5, 3]

j=0

5a) ⑥ x=4 i=-1 A[j] ≤ x

A[0] ≤ 4 no

⑦ x=4 i=-1 j=1 A[1] ≤ 4 yes

i++ ⇒ now i=0 swap(A[i+1], A[j+1])

A[1], A[2]

[2, 4, 6, 1, 5, 3]

③ x=4, i=0, j=2 A[2] ≤ 4 yes

now i=1 swap(A[2], A[3])

[2, 4, 1, 6, 5, 3]

④

x=4 i=1, j=3 A[3] ≤ 4 no

③ x=4 i=1 j=4 A[4] ≤ 4 no

x=4 i=1 j=5 A[5] ≤ 4 yes

now i=2 swap A[i+1], A[j+1]

swap [A[3], A[6] - out of bound

- The line of code that would need to change is the first call to swap. It should be:

swap(A[i+1], A[j+1])

You also need to account for the 2nd swap call which should be:

swap(A[i-1], A[r])

x = 4 i = 1 j = 4

[2, 4, 1, 6, 5, 3]

swap(A[i-1], A[r])

swap(A[0], A[5])

[3, 4, 1, 6, 5, 2]

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1 | 2 | 3 4 6 5

[1] 2 3 4 6 | 5 |

[1, 2,] 3 4 | 5 | 6

[1, 2, 3, 4, 5, 6]

5b) The values used as pivots for the array are:

3, 2, 5

2

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6a) Selection sort

Θ : $\Theta(n^2)$

O : $O(n^2)$

Ω : $\Omega(n^2)$

5

b) LI: for j = 0 and some array A the property hold true prior to the 1st iteration of the loop.

0

Initialization: the array A is unsorted prior to the 1st itt. and j = 0 is still true on the first itt.

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Maintence: j is increased upon each itt. and the A has sub arrays which are being sorted as the Algo runs.

Termination: upon term, the array is sorted and the loop exits.

$A = [a_1, a_2 \ldots a_n]$

$B = [b_1 \ldots b_n]$

7)

```
Algo( A, B )
for( i=0 to length(A))
    j=0,
    if ( A[i] == negative of B[i] OR
         negative of A[i] == B[i] )
    { sum = A[i] + B[i] }
```

the negative of the value of B[i]

explain: iterate through array A and B to check if any values are equal regardless of sign and take negate. If 2 exsits in A check if negative of -2 is in B so 2 = 2 and will know one value is negative so added together will be zero

0

/

10

8) Base Case: $T(0) = 0$ $T(1) = 2$

a) $T(n) = 2T(n/2) + n^2$

$a = 2$ $b = 2$ $f(n) = n^2$

$\log_b a = c$

$\log_2 2 = ?^{?} = 1$

$2^? = 2$ $2^1 = 2$

b) $1 = \log_2 2$ so case 2 mast. therm

$\Rightarrow \Theta(n \log n)$

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