REVIEW: MLA SAM PULSE } ANALOG | DIGITAL: ANALOG → SAMP → CH → MAP

01 '0

TO MANY IS NOT QUANTIZATION

3.9 ⇒ 4

EASIER TO TRANSMIT

WIDE BIT'S MORE BAND WIDTH

Analog and Digital Communication

CHAPTER 4: BASEBAND DATA TRANSMISSION AND DIGITAL BANDPASS MODULATION

ECE 4270

Communications Systems

Dr. Carl Baum

Department of Electrical and Computer Engineering
Clemson University

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PROJECT

NON-UNIFORM QUANTIZATION

Analog and Digital Communication

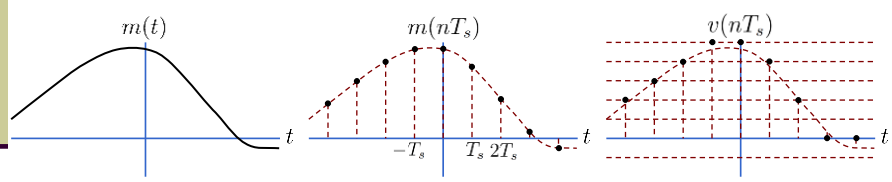
Section 4.1: Quantization and Digital Pulse Modulation

Overview of Section 4.1

- Quantization
- Quantization error
- Nonuniform quantization
- Pulse-code modulation
- Binary PCM mapping illustration
- Non-amplitude-based PCM

Quantization

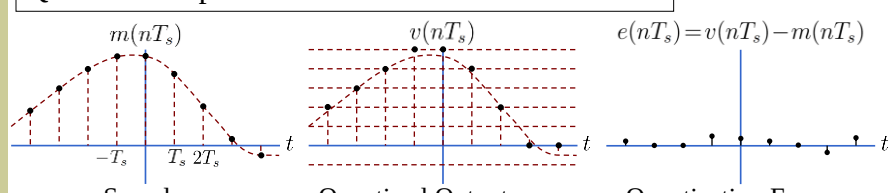
Quantization is the process of transforming the sample amplitudes $m(nT_s)$ of a message signal $m(t)$ into discrete amplitudes $v(nT_s)$ taken from a finite set of possible amplitudes.



Data Signal Sampling Quantization

Quantization Error

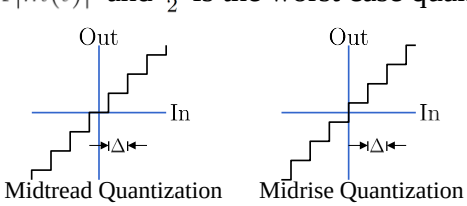
Quantization produces error that is unrecoverable.



Samples Quantized Outputs Quantization Error

For L equally spaced quantization levels with step size Δ, $\Delta = \frac{2}{L}\max_t |m(t)|$ and $\frac{\Delta}{2}$ is the worst case quantization error.

{FIND MID POINT



Midtread Quantization Midrise Quantization

. TO MINIMIZE POTENTIAL ERROR

Example: Quantization

A signal with bandwidth 10 kHz whose values range between +50 mV and -50 mV is to be quantized uniformly such that the maximum quantization error is no more than 1 mV. Assume Nyquist sampling is used and the number of quantization levels must be a power of two. Determine the minimum acceptable number of quantization levels and determine the resulting bit rate at the output of the quantizer.

MAX - MIN / L

MAX - MIN

$1\text{mV} \geq \frac{100}{2L} = 50 - (-50)$

In units of mV, $\frac{\Delta}{2} \leq 1 \Rightarrow \Delta \leq 2$ ERROR STEP SIZE

$$\Delta = \frac{2}{L}\max_t |m(t)| = \frac{2(50)}{L} = \frac{100}{L} \leq 2 \Rightarrow L \geq \frac{100}{2} = 50 \Rightarrow L = 64 = 2^6$$

POWER OF 2 : NEED BIT'S

$$f_s = 2(10,000) = 20,000 \text{ samples/s}$$

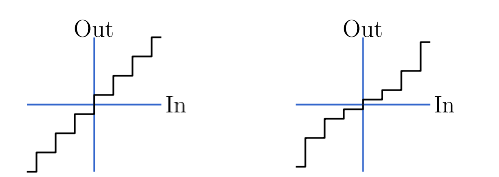
SEND n-BIT'S

$$\frac{20,000 \text{ samples}}{\text{s}} \times \frac{6 \text{ bits}}{\text{sample}} = 120 \text{ kbits/s}$$

NOT VOLTAGE VALUE

Nonuniform Quantization

Voice signals are predominantly transmitted at low amplitudes. Thus, quantization error is smaller if there are more quantization levels at low amplitudes and fewer at large amplitudes.



Uniform Quantization Nonuniform Quantization

Nonuniform Quantization

FILTER ON INPUT SIGNAL

A nonlinear device called a compressor can be used prior to uniform quantization to produce nonuniform quantization.

$$y = \frac{\log(1+\mu|x|)}{\log(1+\mu)}\operatorname{sgn}(x)$$

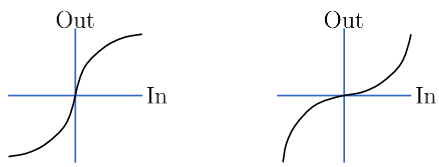
μ-law Compression

$$y = \begin{cases} \frac{Ax}{1+\log A} & \text{if } 0 \le |x| \le \frac{1}{A} \\ \frac{1+\log(A|x|)}{1+\log A}\operatorname{sgn}(x) & \text{if } \frac{1}{A} \le |x| \le 1 \end{cases}$$

A-law Compression

A matching expander is used to restore a compressed signal. The expander reverses the "in" and "out" of the compressor. The combination of the two devices is called a compander.

COMPRESSOR



Compressor Expander

In practice, piecewise linear approximations are used.

HOW TO CHOOSE MODULATION/LEVELS FOR QUANTIZATION? A→D

{ POWER ERROR BANDWIDTH }

↳ COMM. OVERHEAD

WHAT YOU DESIRE

ENCODE INFO. IN A PULSE

1.) AMP.

2.) POSITION

3.) WIDTH

Pulse-Code Modulation

Pulse-code modulation (PCM) sends a message with a sequence of coded pulses taken from a finite set; i.e.,

$$s(t) = \sum_{n=-\infty}^{\infty} p_{d_n}(t - nT_p)$$

where $f_p = \frac{1}{T_p}$ is the pulse rate and $d_n \in \{0, 1, \cdots, M-1\}$ is the message data. $M = 2^k$ is the PCM alphabet size.

Amplitude-based PCM is defined via

$$s(t) = \sum_{n=-\infty}^{\infty} A_{d_n} p(t - nT_p)$$

where the A_{d_n} are a set of M amplitude values and $p(t)$ is a pulse-shaping waveform.

COUPLE BOUND → NYQUIST RATE

1.) PULSE RATE

2.) SAMPLE RATE

3.) BIT RATE

↑ QUANT. LEVELS ↓ OVERHEAD # OF BITS POWER ↑

(L=128, K=7 ; $L=2^K$, $K=\log_2 L$)

NYQUIST RATE { HANDLE DISTORTION WHILE SAMPLING

Ex.) $L = 256$ $A_n = 110$ $2^6 + 2^5 + 2^3 + 2^2 + 2$ 256 (110) 01 00 10 11 → 00110110

BINARY

- FAST (ADD) ALGORITHM

↓

PACK OF 4 → M=4

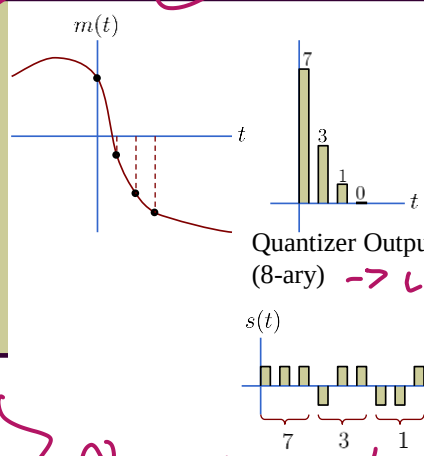
0011 0110

↓

RECEIVER KNOWS M

SO WE CAN STILL UNPACK

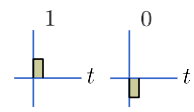
Binary PCM Mapping Illustration



Quantizer Output (8-ary) → LEVELS

$7 = 111$
$3 = 011$
$1 = 001$
$0 = 000$

3-BITS



PCM Pulse Set (binary)

ALPHABET SIZE = 2

QUANTIZAT N MOD. SCHE

Non-amplitude-based PCM

Position-based PCM (pulse position modulation) is defined via

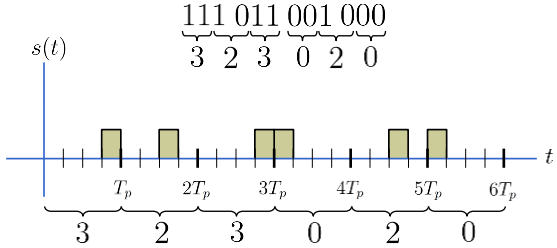
$$s(t) = \sum_{n=-\infty}^{\infty} p\left(t - nT_p - \frac{d_n T_p}{M}\right)$$

where $d_n \in \{0, 1, \cdots, M-1\}$ is the message data and $p(t)$ is a pulse-shaping waveform.

Illustration ($M = 4$):

7 3 1 0

$\underbrace{111}_{3}\ \underbrace{011}_{2}\ \underbrace{001}_{3}\ \underbrace{000}_{0\ 2\ 0}$



Non-amplitude-based PCM

Width-based PCM (pulse width modulation) is defined via

$$s(t)=\sum_{n=-\infty}^{\infty}\sum_{\ell=0}^{d_n} p(t-nT_p-\tfrac{\ell T_p}{M})$$

where $d_n \in \{0, 1, \cdots, M-1\}$ is the message data and $p(t)$ is a pulse-shaping waveform.

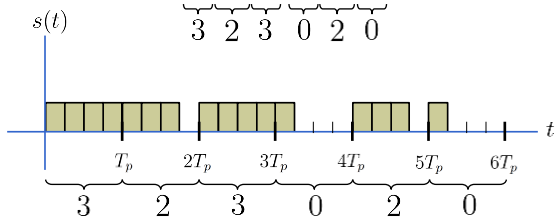
d_n is IN TIME DOMAIN

TYPE

CHANGE IN POSITION

Illustration ($M=4$):

$$\begin{array}{cccc} 7 & 3 & 1 & 0 \\ \underbrace{111}_{3} & \underbrace{011}_{2} & \underbrace{001}_{3} & \underbrace{000}_{0\ 2\ 0} \end{array}$$



$0 < \frac{d_n}{M} < 1$

Example: PCM Pulse Rates

Suppose $m(t)$ has maximum frequency 8 kHz, is sampled at the Nyquist rate, and is quantized using 128-level quantization. Determine the PCM pulse rate if (a) $M=128$, (b) $M=16$, (c) $M=2$.

$f_s = 2B = 16$ ksamples/s

128 levels $= 2^7 \Rightarrow 7$ quantizer output bits/sample

Quantizer output bit rate $= 16,000 \times 7 = 112,000$ bits/s

(a) $M=128=2^7 \Rightarrow 7$ bits/PCM pulse

$\Rightarrow \frac{112,000}{7} = 16$ kpulses/s

(b) $M=16=2^4 \Rightarrow 4$ bits/PCM pulse

$\Rightarrow \frac{112,000}{4} = 28$ kpulses/s

(c) $M=2 \Rightarrow 1$ bit/PCM pulse $\Rightarrow 112$ kpulses/s

NOT ALWAYS EQUAL!

128 ALPHABET CHOICES FOR PULSE

M = PULSE RATE

THINK: ERROR <-> BW
POWER

Analog and Digital Communication

Section 4.2: Enhancements to Digital Pulse Modulation

FINAL EXAM

GOOD MULTIPLEXING

TOO NOT MORE SIGNAL.

USE PULSE

DOES NOT CHANGE SPEED. DO NOT NEED HIGH SPEED.

Q/A

AD → BASEBAND → MOD → CH.

SAMPLING QUAN.

BASEBAND → MOVE TO HIGH SPEED.

COS(2π f_c(t) + θ)

CARRIER

Overview of Section 4.2

- Delta modulation
- Implementation of delta modulation
- Adaptive delta modulation
- Sigma-delta modulation
- Differential pulse code modulation → REDUCE OVERHEAD
- Adaptive differential pulse code modulation

OVER HEAD

PCM: LOOK @ VALUES INDEPENDENTLY.

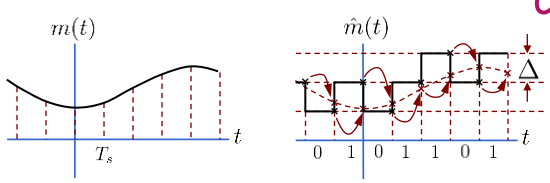
SEND THE DIFF. B/W SAMPLES

Delta Modulation

$\int \Delta y \to \Delta$
$\Delta x \to T_s$

Delta modulation is a "staircase" approximation to a message signal. The sampling rate f_s is oversampled sufficiently so that in time duration $T_s = \frac{1}{f_s}$ the signal does not change by more than Δ, where Δ is the height of a "step."

IF Δ IS SMALL WORK BEST WHEN OVERSAM. SIGNAL TO BEAT.



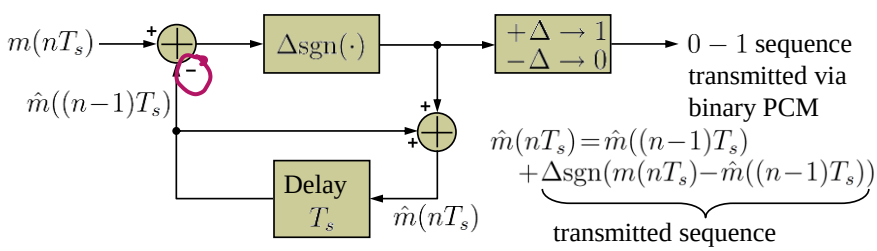
GOOD
• LOW POWER & BANDWIDTH

BAD
• LOSE MUCH INFORMATION
– DISTORTION

$$\hat{m}(nT_s) = \hat{m}((n-1)T_s) + \underbrace{\Delta \mathrm{sgn}(m(nT_s) - \hat{m}((n-1)T_s))}_{\text{transmitted sequence: } +\Delta \to 1,\ -\Delta \to 0}$$

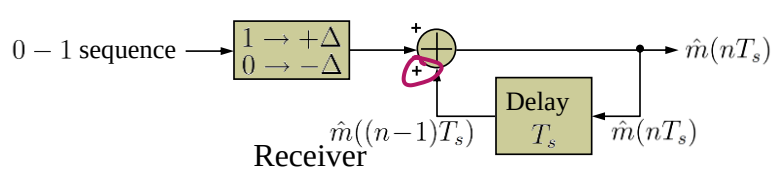
$$\boxed{\frac{\Delta}{T_s} \geq \max_t \left| \frac{dm(t)}{dt} \right|}$$

Implementation of Delta Modulation



$$\hat{m}(nT_s) = \hat{m}((n-1)T_s) + \underbrace{\Delta \mathrm{sgn}(m(nT_s) - \hat{m}((n-1)T_s))}_{\text{transmitted sequence } (+\Delta \to 1,\ -\Delta \to 0)}$$

Transmitter



Receiver

Example of Delta Modulation

Suppose $m(t) = a\text{sinc}(bt)$. Determine a formula for the minimum value of $\frac{\Delta}{a}$ that avoids slope-overload distortion. Assume f_s is M times the Nyquist rate.
Hint: $\max_t \left|\frac{d}{dt}\frac{\sin(t)}{t}\right| \simeq 0.342$

USE DERIVATIVE THAT YOU KNOW TO SOLVE

Let $w(t) = \frac{\sin(t)}{t}$. Then $m(t) = \frac{a\sin(\pi bt)}{\pi bt} = aw(\pi bt)$.

$$\frac{d}{dt}m(t) = aw'(\pi bt)\pi b$$

$$\Rightarrow \max_t \left|\frac{d}{dt}m(t)\right| = a\pi b \max_t |w'(\pi bt)| \simeq 0.342a\pi b \simeq 1.07ab$$

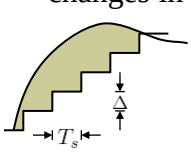
$$M(f) = \frac{a}{b}\text{rect}\left(\frac{f}{b}\right) \Rightarrow B = \frac{b}{2}$$

$$\Rightarrow \text{Nyquist bandwidth} = 2B = b \Rightarrow f_s = Mb$$

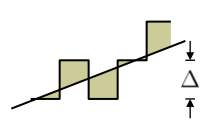
$$\Delta \geq T_s \max_t \left|\frac{dm(t)}{dt}\right| \simeq \frac{1}{Mb}(1.07ab) = \frac{1.07a}{M} \Rightarrow \frac{\Delta}{a} \geq \frac{1.07}{M}$$

Adaptive Delta Modulation

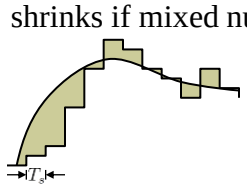
Δ too small: cannot track changes in $m(t)$



Δ too large: large quantization errors



Δ adaptive: grows with large numbers of zeroes or ones; shrinks if mixed numbers of zeroes and ones

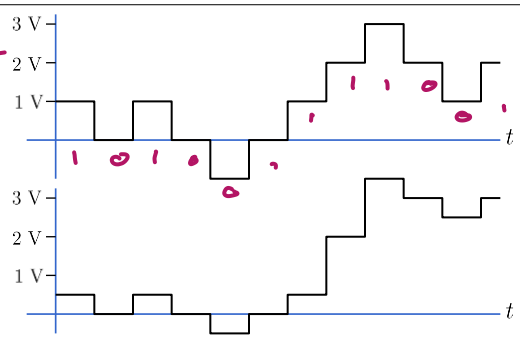


Example of Adaptive Delta Modulation

Plot the output that results from the input sequence 101001111001 for (a) delta modulation with $\Delta = 1$ V, and (b) adaptive delta modulation based on last 3 samples in which $\Delta = 1.5$ V if all 0s or 1s and $\Delta = 0.5$ V otherwise, assuming 0s occur before time zero. Start at 0 V.

$\Delta = 1$

+/- RISE



FROM A: (REMAP)

101
001
111
001 → +1.5V

22

$\boxed{?.?1}$ 00

ASSUMTION ON .5

Sigma-Delta Modulation

In delta modulation,

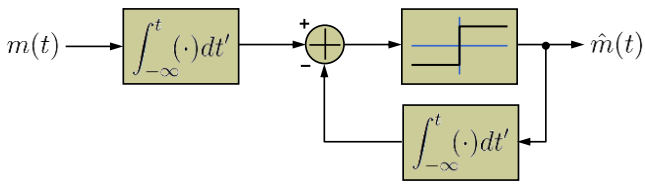
$$\hat{m}(nT_s) = \hat{m}((n-1)T_s) + \Delta \operatorname{sgn}(m(nT_s) - \hat{m}((n-1)T_s))$$

$$\Rightarrow \hat{m}(nT_s) = \sum_{i=1}^{n} \Delta \operatorname{sgn}(m(iT_s) - \hat{m}((i-1)T_s))$$

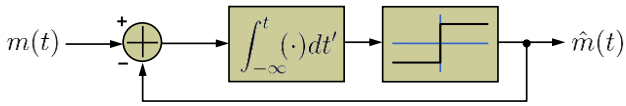
$\Rightarrow$ errors accumulate/propagate

Sigma-delta modulation is based on delta modulation but incorporates accumulation at the input so that no accumulation is needed at the output.

Sigma-Delta Modulation



Equivalently:



Sigma-delta modulation can also be made to be adaptive in step size.

NOT ON EXAMPLE TO SEE CORRELATION

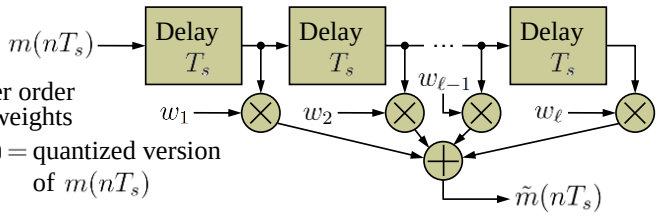
• REF. MEAN TO USE

Differential Pulse Code Modulation

If data in sample $m(nT_s)$ is highly correlated with its neighboring samples, there is redundant information embedded in the signal.

Differential pulse code modulation (DPCM) takes as the input to the quantizer the difference between $m(nT_s)$ and a prediction of it, $\tilde{m}(nT_s)$.

A tapped-delay line can be used as a predictor.



$\ell =$ filter order
w_ks $=$ weights
$\hat{m}(nT_s) =$ quantized version of $m(nT_s)$

LOOK @ PREV. SAMPLES TO FIND CORRELATION

Differential Pulse Code Modulation

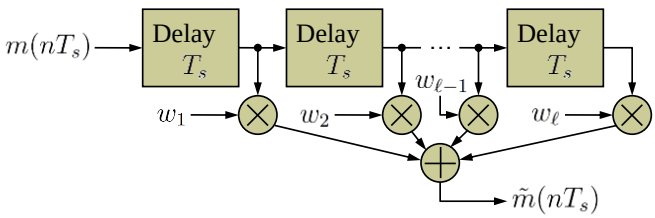
Examples of predictors:

$$\tilde{m}(nT_s) = \tfrac{1}{\ell} m((n-1)T_s) + \tfrac{1}{\ell} m((n-2)T_s) + \cdots + \tfrac{1}{\ell} m((n-\ell)T_s)$$

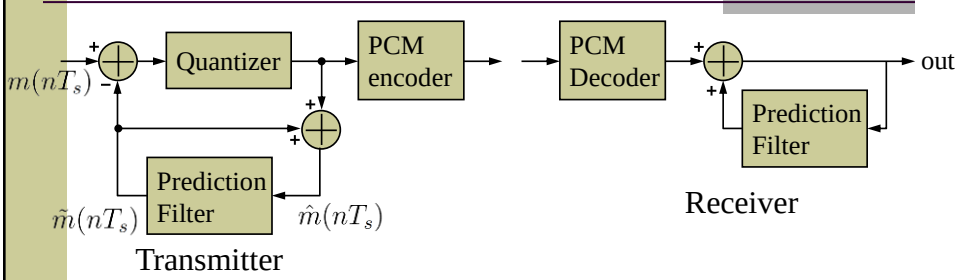
(equal averaging: past is equally weighted)

$$\tilde{m}(nT_s) = \tfrac{(1-p)}{1-p^{\ell}}[m((n-1)T_s) + pm((n-2)T_s) + \cdots + p^{\ell-1} m((n-\ell)T_s)] \qquad 0 < p < 1$$

(geometric averaging: recent past is overweighted)



Differential Pulse Code Modulation

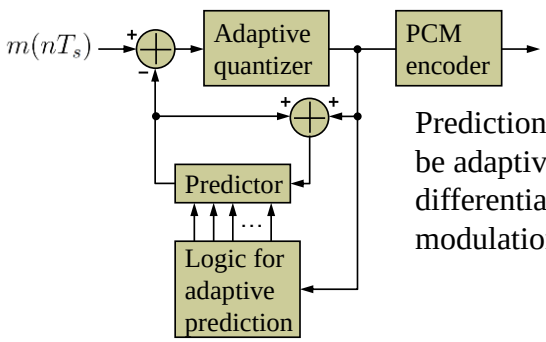


DPCM is a generalization of delta modulation; replace the prediction filter with a T_s delay and the quantizer with $\Delta \mathrm{sgn}(\cdot)$ and the result is delta modulation.

MULTIPLE BANDS OF PREDICTION

Adaptive Differential Pulse Code Modulation

If step size is too small, like delta modulation, DPCM may fail to track rapid changes. The step size can be made to be adaptive.



Prediction can also be made to be adaptive. Result: Adaptive differential pulse code modulation (ADPCM).

FROM LECTURE:

- GEN PCM V.S. Δ MOD.
- HOW TO EXTEND Δ MOD INTO DIFF & MULTIPLAY,

F. N AC EXAM

SAM → QUAN. → MOD. → CH.

→ OUTPUT 0,1,0,1,0

① BASEBAND: PULSE

② PASSBAND: HIGH FREQ.

Analog and Digital Communication

Section 4.3: Line Codes

"REAL" COMM. SYSTEM

Overview of Section 4.3

- Line codes
- Non-return-to-zero (NRZ) line codes
- Return-to-zero (RZ) line codes
- Biphase (Manchester) line codes
- Dicode line codes
- DC component
- Maximum run length
- Built-in clocking
- Bandwidth and minimum run length
- Polarity equivalence
- Modified AMI line codes
- Nonbinary line codes
- Line codes based on bit insertion

Line Codes

A line code is a rectangular pulse-based continuous-time digital signal waveform used to convey a sequence of binary data. Line codes are a special case of PCM.

Applications of line codes:

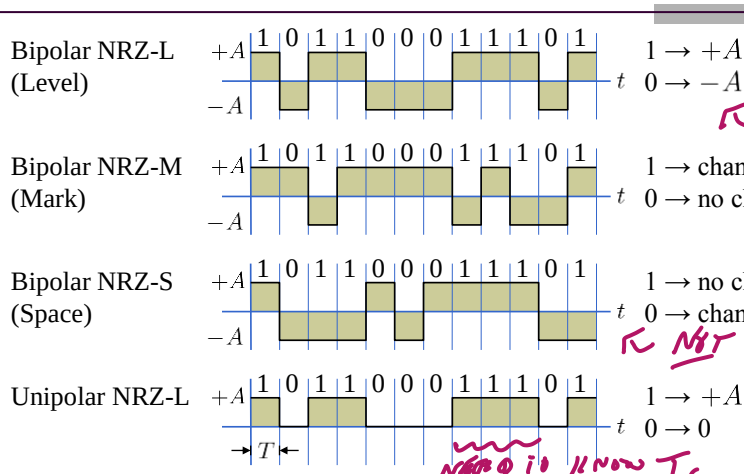
- Used directly on a transmission line via voltage or current.
- Pulse shaped, modulated into RF, and sent wirelessly.
- Used to turn on and off a laser/light for optical fiber/infrared remote.
- Printed on paper to create barcodes.
- Converted to magnetized spots/transistor charges/pits on hard drive/flash card/optical disc.

WAY: IMPULSE INFAS.

PULSE → HIGH BANDWIDTH

↳ USE COS(t)

Non-Return-to-Zero (NRZ) Line Codes



Unipolar NRZ-M and NRZ-S follow similarly.

• BETTER NOISE AGAINST. BIG, LARGE VOLTAGE

INFO IS INSIDE AMPLITUDE

AMP. SENSITIVE TO NOISE

NEED TO KNOW T

NOT

• GOOD POWER

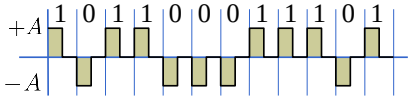
SELF. SYNC.

SYNC ≡ SCALING (TO KNOW WHEN TO READ DATA)

2-SIGNAL SIGNAL

Return-to-Zero (RZ) Line Codes

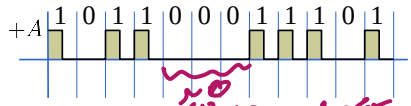
Bipolar RZ (L)



$1 \rightarrow +A, 0$
$0 \rightarrow -A, 0$

BW ↑ ALL LOS OF SWITCHING

Unipolar RZ (L)



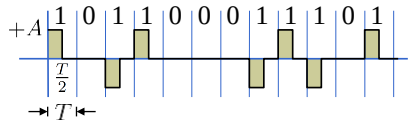
$1 \rightarrow +A, 0$
$0 \rightarrow 0$

NO SYNC. LOST T_S

Bipolar and unipolar RZ-M and RZ-S follow similarly.

GOOD

RZ-AMI (Alternate mark inversion)



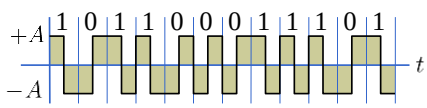
$1 \rightarrow$ alternating $+A, 0$ and $-A, 0$
$0 \rightarrow 0$

SAVE BW DATA(T_S) LOST IN DOD

* IF HAVE SYNC CLK: DONT CARE ABOUT T_S BUT... *

Biphase (Manchester) Line Codes

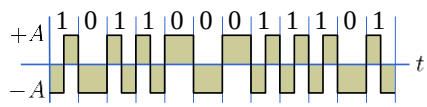
Biphase-L



$1 \rightarrow +A, -A$
$0 \rightarrow -A, +A$

SELF, GOOD SYNC.

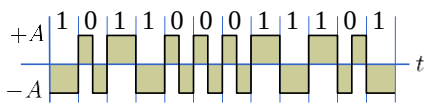
Biphase-M



Always transitions on symbol boundaries
$1 \rightarrow$ midsymbol transition
$0 \rightarrow$ no midsymbol transition

GOOD AGAINST NOISE (IGNORE MIDDLE)

Biphase-S



Always transitions on symbol boundaries
$1 \rightarrow$ no midsymbol transition
$0 \rightarrow$ midsymbol transition

Delay Modulation (Miller Coding)

Only transitions on symbol boundaries when 0 follows a 0
$1 \rightarrow$ midsymbol transition
$0 \rightarrow$ no midsymbol transition

* ACTUALLY USED

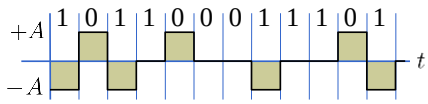
SELF-SYNC, NOISE RESISTANCE
HARD TO DECODE INFO NOT IN AMP

BW IS HIGH

FINAL EXAM: ENCODE DECODE & COMPARE (BW, POWER, TIME SYNC)

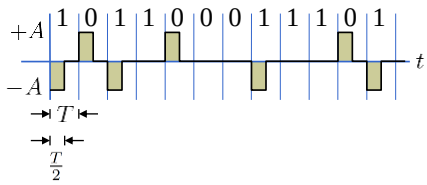
Dicode Line Codes

Dicode NRZ



change in data → polarity flip from past nonzero level
no change in data → 0

Dicode RZ



change in data → polarity flip from past nonzero level, 0
no change in data → 0

DC Component

For a block of M symbols, the <u>DC component</u> is given by

$$\frac{1}{MT}\int_0^{MT} s(t)dt$$

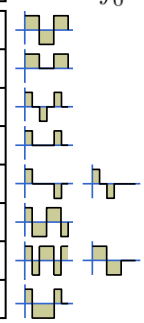
In general, the DC component depends on the particular data being transmitted.

If the limit of the DC component goes to 0 as $M \to \infty$, the line code is said to be <u>DC balanced</u>.

DC Component

Line Code	Range of DC Component
Bipolar NRZ	$[-A, A]$
Unipolar NRZ	$[0, A]$
Bipolar RZ	$[-\frac{A}{2}, \frac{A}{2}]$
Unipolar RZ	$[0, \frac{A}{2}]$
RZ-AMI, Dicode RZ	$[\frac{-A}{2M}, \frac{A}{2M}] \rightarrow 0$ as $M \rightarrow \infty$
Biphase-L	0
Biphase-M,S, Dicode NRZ	$[\frac{-A}{M}, \frac{A}{M}] \rightarrow 0$ as $M \rightarrow \infty$
Delay Modulation	$[\frac{-A}{3M}, \frac{A}{3M}] \rightarrow 0$ as $M \rightarrow \infty$

$$\frac{1}{MT}\int_0^{MT} s(t)dt$$



NEED CLK TO KNOW

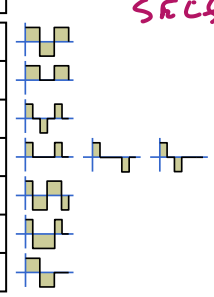
A DC component in a line code prevents its use with capacitive/transformer coupling.

Maximum Run Length

The <u>maximum run length</u> is the maximum time duration for which the signal does not change value.

Line Code	Maximum Run Length	
	$s(t)=\pm A$	$s(t)=0$
Bipolar NRZ	∞	–
Unipolar NRZ	∞	∞
Bipolar RZ	$\frac{T}{2}$	$\frac{T}{2}$
Unipolar RZ, RZ-AMI, Dicode RZ	$\frac{T}{2}$	∞
Biphase	T	–
Delay Modulation	$2T$	–
Dicode NRZ	T	∞

$\infty \rightarrow$ NO WAY TO DO SELF SYNC.



Long non-zero runs are difficult to generate; voltage sag is a common problem. Long runs also make it difficult to maintain symbol synchronization.

Built-in Clocking

Some line codes have features that are certain to occur every symbol duration. Such line codes are said to have <u>built-in clocking</u>.

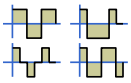
Bipolar RZ		guaranteed change from $\pm A$ to 0 at $t = T/2$, $3T/2$, etc.
Biphase-L		guaranteed change from $\pm A$ to $\mp A$ at $t = T/2$, $3T/2$, etc.
Biphase-M,S		guaranteed change from $\pm A$ to $\mp A$ at $t = T$, $2T$, etc.

If maximum run length (zero and nonzero) is small, clocking, although not built-in, is easier than if long runs occur. Example: delay modulation with maximum run length of $2T$.

Bandwidth and Minimum Run Length

Due to the time-scaling property of the Fourier transform, $x(at) \leftrightarrow \frac{1}{a}X(\frac{f}{a})$, the bandwidth of a line code is greatly influenced by the minimum run length; the larger this length, the smaller the bandwidth.

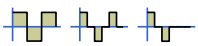
Line Code	Minimum Run Length
NRZ, Delay Modulation	T
RZ, Biphase	$\frac{T}{2}$



Polarity Equivalence

A line code is said to have <u>polarity equivalence</u> if the data can be recovered even if the received signal experiences amplitude inversions, i.e., $-s(t)$ is received.

Bipolar NRZ-M, S,
Bipolar RZ-M, S,
Dicode

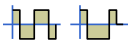


symbol transitions used to convey data

Unipolar (all kinds),
RZ-AMI

can use $|s(t)|$ for making decisions

Biphase-M, S,
Delay Modulation



midsymbol transitions used to convey data

ACTUAL | NOT TESTED

Modified AMI Line Codes

Modified AMI line codes purposefully insert bipolar violations (non-altering marks) so as to limit the maximum run length and help maintain synchronization.

Notation: V = bipolar violation mark, N = normal (non-violation) mark

B#ZS = binary #-bit zero substitution

B8ZS (used on 1.544 Mbit/s T1 lines)

Replaces each string of 8 zeroes with 000VN0VN.

B6ZS (used on 6.312 Mbit/s T2 lines)

Replaces each string of 6 zeroes with 0VN0VN.

B3ZS (used on 44.736 Mbit/s T3 lines)

Replaces each string of 3 zeroes with 00V or N0V.
The choice is made to ensure that consecutive violations are of opposite polarity.

Example: Modified AMI Line Codes

Encode 1100 0000 0000 1000 via AMI, B8ZS, B6ZS, and B3ZS using +,−,0 sequences. Assume the first mark is positive. For B3ZS, assume the first violation uses 00V.

```
        1100 0000 0000 1000
AMI:    +−00 0000 0000 +000

        +−00 0000 0000 +000
B8ZS:   +−00 0VN0 VN00 +000
   ⇒    +−00 0−+0 +−00 +000

        +−00 0000 0000 +000
B6ZS:   +−0V N0VN 0000 +000
   ⇒    +−0− +0+− 0000 +000

        +−00 0000 0000 +000
B3ZS:   +−X0 VX0V X0V0 +X0V     X = 0 or N
   ⇒    +−00 −+0+ −0−0 +00+     (00V or N0V)
```


Example: Modified AMI Line Codes

Decode the B3ZS sequences (a) 0+−+ 00+0 −00− +0+0, (b) 0+0− 00−+ 000− +−00, and (c) +0−+ −+−0 0−0+ 0+0+ into 0-1 sequences if possible, or state that there is an error.

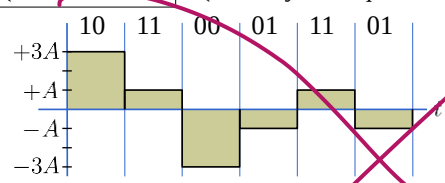
```
(a)  0+−+ 00+0 −00− +0+0
     0111 00V0 100V 10V0
     0111 0000 1000 0000

(b)  0+0− 00−+ 000− +−00
     0101 00V1 0001 1100
                ERROR: cannot have 3 consecutive 0s

(c)  +0−+ −+−0 0−0+ 0+0+
     1011 1110 0V01 0V0V
                    ERROR: not in form 00V or N0V
```

Nonbinary Line Codes

2B1Q (used in ISDN) (2 binary in, 1 quaternary out)



$10 \to +3A$
$11 \to +A$
$01 \to -A$
$00 \to -3A$

D = cumulative offset $(+\mathrm{s})-(-\mathrm{s})$

4B3T (used in ISDN) (4 binary in, 3 ternary out)

$0001 \to 0-+$
$0010 \to +-0$
$0100 \to -+0$
$0111 \to -0+$
$1011 \to +0-$
$1110 \to 0+-$

$0000 \to \begin{cases} +0+ & \text{if } D=-1 \\ 0-0 & \text{if } D\geq 0 \end{cases}$

$0011 \to \begin{cases} 00+ & \text{if } D\leq 1 \\ --0 & \text{if } D=2 \end{cases}$

$0101 \to \begin{cases} 0++ & \text{if } D=-1 \\ -00 & \text{if } D\geq 0 \end{cases}$

$1000 \to \begin{cases} +00 & \text{if } D\leq 1 \\ 0-- & \text{if } D=2 \end{cases}$

$1101 \to \begin{cases} 0+0 & \text{if } D=2 \\ -0- & \text{if } D\leq 1 \end{cases}$

$1111 \to \begin{cases} ++0 & \text{if } D=-1 \\ 00- & \text{if } D\geq 0 \end{cases}$

$-1 \leq D \leq 2$

$0110 \to \begin{cases} -++ & \text{if } D\leq 0 \\ --+ & \text{if } D\geq 1 \end{cases}$

$1001 \to \begin{cases} +-+ & \text{if } D\leq 1 \\ --- & \text{if } D=2 \end{cases}$

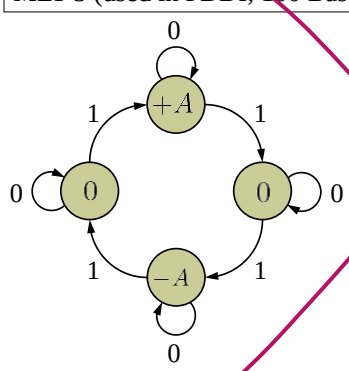
$1010 \to \begin{cases} ++- & \text{if } D\leq 0 \\ +-- & \text{if } D\geq 1 \end{cases}$

$1100 \to \begin{cases} +++ & \text{if } D=-1 \\ -+- & \text{if } D\geq 0 \end{cases}$

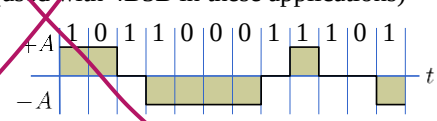
8B6T (used in 100 Base-T4 Ethernet) same idea

Nonbinary Line Codes

MLT-3 (used in FDDI, 100 Base-TX) (multi-level transmit 3)



(used with 4B5B in these applications)



4B1H (used in SHDSL, HDSL2) Maps 4 bits to 16-level amplitude-based PCM via powerful state-based encoder (trellis coded modulation)

Line Codes Based on Bit Insertion

Coding blocks of bits with extra redundancy bits can limit run length and reduce DC component.

4B5B (used in 100 Base-TX, FDDI) (4 binary in, 5 binary out)

0000	→	11110	1000	→	10010
0001	→	01001	1001	→	10011
0010	→	10100	1010	→	10110
0011	→	10101	1011	→	10111
0100	→	01010	1100	→	11010
0101	→	01011	1101	→	11011
0110	→	01110	1110	→	11100
0111	→	01111	1111	→	11101

Longest block of 0s: 3
Longest block of 1s: 8

Also several control symbols

8B10B (used in IEEE 13946, Gigabit Ethernet, DVI, HDMI, USB 3.0)

$\underbrace{\text{5 bits}}_{\text{6 bits}}$ $\underbrace{\text{3 bits}}_{\text{4 bits}}$

Two tables, depend on D
(similar to 4B3T)

Also several control symbols

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Line Codes Based on Bit Insertion

64B66B (used in 10 Gigabit Ethernet)

Preamble 01: next 64 bits are data
10: 8 bit "type" field plus 56 control/data bits

Bit transition (0 to 1 or 1 to 0) guaranteed every 66 bits

Data bits also scrambled using a self synchronous scramble function designed to make a relatively even distribution of 0s and 1s even if data is highly repetitive. Scrambling achieved using a linear feedback shift register.

Eight-to-Fourteen Modulation (EFM) (used on CDs)

Guarantees at least 2 zeroes between every 2 ones—reduces playback demands
Guarantees at most 10 zeroes between every 2 ones—reduces demands on clock
3 or more "merging" bits used for total of 17

EFM Plus (used on DVDs)

16 bits total instead of 17; same zero runlength properties as EFM

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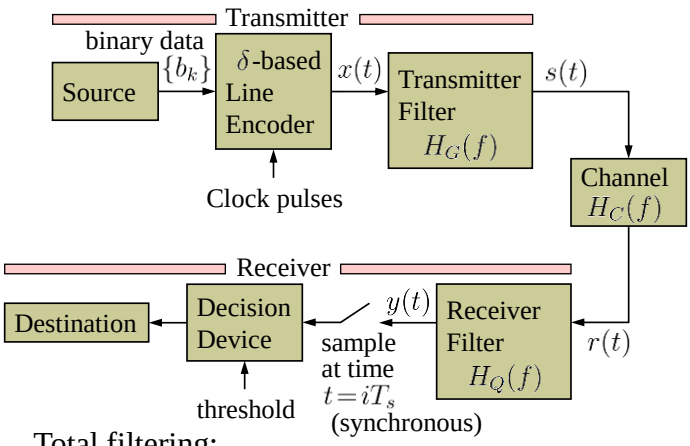
Analog and Digital Communication

Section 4.4: Baseband Transmission and Intersymbol Interference

Overview of Section 4.4

- Block representation of digital baseband communication system
- Total effects of filtering
- Intersymbol interference (ISI)
- Pulse shaping and the Nyquist solution
- Achieving minimum transmission bandwidth
- Channel filtering
- Nyquist binary level signaling
- Drawbacks of Nyquist solution
- Bandwidth-inefficient signaling

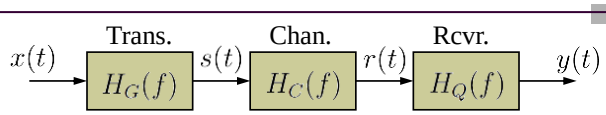
Block Representation of Digital Baseband Communication System



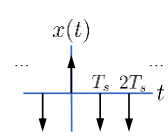
Total filtering:

$$P(f) = H_G(f)H_C(f)H_Q(f) \qquad p(t) = [h_G * h_C * h_Q](t)$$

Total Effects of Filtering



$$x(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s)$$



$$s(t) = [x * h_G](t) = \sum_{k=-\infty}^{\infty} a_k h_G(t - kT_s)$$

$$r(t) = [s * h_C](t) = \sum_{k=-\infty}^{\infty} a_k [h_G * h_C](t - kT_s)$$

$$y(t) = [r * h_Q](t) = \sum_{k=-\infty}^{\infty} a_k [h_G * h_C * h_Q](t - kT_s) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_s)$$

Intersymbol Interference (ISI)

Receive-filter output: $y(t)=\sum_{k=-\infty}^{\infty} a_k p(t-kT_s)$

where $p(t)=[h_G * h_C * h_Q](t)$

Sampler output: $y(iT_s)=\sum_{k=-\infty}^{\infty} a_k p((i-k)T_s)$

Discrete-time signal notation: $y[n]=y(nT_s)$, $p[n]=p(nT_s)$, $a[n]=a_n$

$$y[i]=\sum_{k=-\infty}^{\infty} a[k]p[i-k]=(a*p)[i]$$

Ideally, $y[i]$ should only depend on $a[i]$. This will only happen if $p[i-k]=0$ for all $k\neq i$; i.e., $p[n]=0$ for all $n\neq 0$.

In general,

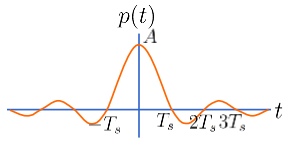
$$y[i]=\underbrace{\cdots+p[2]a[i-2]+p[1]a[i-1]}_{\text{intersymbol interference}}+p[0]a[i]+\underbrace{p[-1]a[i+1]+p[-2]a[i+2]+\cdots}_{\text{intersymbol interference}}$$

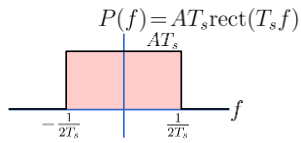
Pulse Shaping and the Nyquist Solution

Pulse shaping goal: Given the channel $H_C(f)$, determine the transmitter $H_G(f)$ and the receiver $H_Q(f)$ such that there is no ISI and transmission bandwidth is conserved.

The Nyquist solution: $\boxed{p(t)=[h_G * h_C * h_Q](t)=A\,\mathrm{sinc}(\frac{t}{T_s})}$

Because $\mathrm{sinc}(k)=0$ if $k\neq 0$, $p[n]=p(nT_s)=0$ if $n\neq 0 \Rightarrow$ no ISI.





The Nyquist solution uses the minimum possible system bandwidth, $\frac{1}{2T_s}$.

Achieving Minimum Transmission Bandwidth

Nyquist solution uses $P(f)=H_G(f)H_C(f)H_Q(f)=AT_s\text{rect}(T_sf)$.

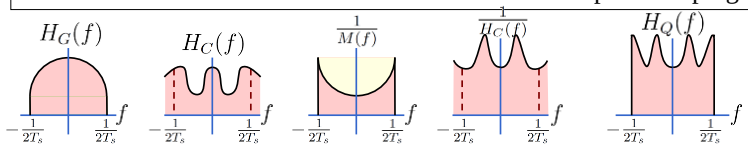
To achieve minimum <u>transmission</u> bandwidth, $H_G(f)$ must, in and of itself, be bandlimited to have maximum frequency $\frac{1}{2T_s}$.

If $H_G(f)=M(f)\text{rect}(T_sf)$, then for the Nyquist solution we must have

$$H_Q(f)=\frac{AT_s}{M(f)H_C(f)}\text{rect}(T_sf)=\frac{1}{M(f)}\frac{1}{H_C(f)}AT_s\text{rect}(T_sf)$$

inverse of channel filtering

inverse of pulse shaping filter



Example: Nyquist Channel

Suppose $H_G(f)=\text{rect}(T_sf)$ and $h_C(t)=\delta(t)+a\delta(t-b)$, where $|a|<1$. Determine the $H_Q(f)$ required for the Nyquist solution.

Using the fact that, if $|x|<1$, $\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^nx^n$, obtain an expression for $h_Q(t)$.

$$H_C(f)=1+ae^{-j2\pi fb} \qquad M(f)=1$$

$$H_Q(f)=\frac{1}{M(f)}\frac{1}{H_C(f)}AT_s\text{rect}(T_sf)=\frac{AT_s}{1+ae^{-j2\pi fb}}\text{rect}(T_sf)$$

$$H_Q(f)=AT_s\sum_{n=0}^{\infty}(-1)^n(ae^{-j2\pi fb})^n\text{rect}(T_sf)$$

$$=AT_s\sum_{n=0}^{\infty}(-1)^na^ne^{-j2\pi fbn}\text{rect}(T_sf)$$

$$\Rightarrow h_Q(t)=A\sum_{n=0}^{\infty}(-a)^n\text{sinc}\left(\frac{t-nb}{T_s}\right)$$

Channel Filtering

$$H_Q(f) = \frac{AT_s}{M(f)H_C(f)}\text{rect}(T_s f)$$

Generally, $H_C(f)$ is unknown prior to turning on the system.

$\Rightarrow$ $H_C(f)$ must be estimated by the receiver

$\Rightarrow$ The receiver must be able to adaptively implement $\frac{1}{H_C(f)}$.

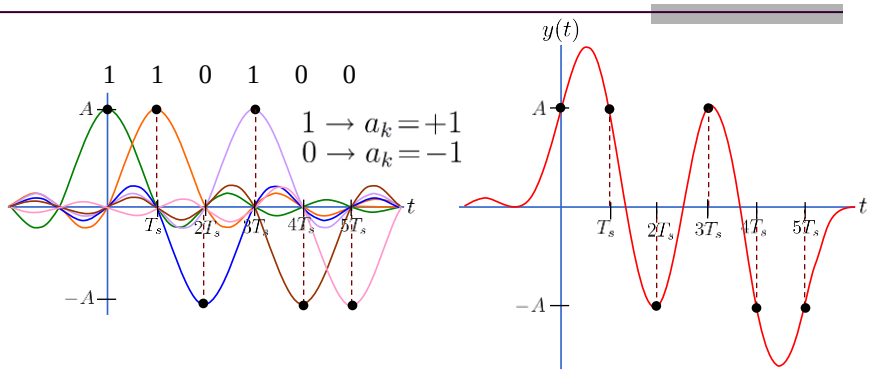
If there is mobility, $H_C(f)$ may be constantly changing.

$\Rightarrow$ The receiver must regularly adaptively update $\frac{1}{H_C(f)}$.

A <u>spectral null</u> is a frequency f' for which $H_C(f') = 0$.

$\Rightarrow$ If a spectral null lies in the frequencies between 0 and $\frac{1}{2T_s}$, then the Nyquist solution requires a value of infinity at the null frequency.

Nyquist Binary Level Signaling



$$y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - Tk_s) \text{ where } p(t) = [h_G * h_C * h_Q](t) = A\text{sinc}(\tfrac{t}{T_s})$$

Drawbacks of Nyquist Solution

The system requires that $P(f)$ be perfectly flat from $-\frac{1}{2T_s}$ to $\frac{1}{2T_s}$ and exactly zero elsewhere.

Noncausal filtering is required.

The system requires that $p(t) = A\text{sinc}(\frac{t}{T_s})$, a waveform with extremely slow decay (rate $\frac{1}{t}$) as $t \to \infty$.

There is no margin for error in sampling times.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_s) \qquad \sum_{k=1}^{\infty} \frac{1}{k} = \infty$$

$$= \sum_{k=-\infty}^{\infty} a_k \frac{\sin(\frac{\pi t}{T_s} - k)}{\frac{\pi t}{T_s} - k}$$

Slow decay requires large waiting times before signal can be truncated.

Bandwidth-Inefficient Signaling

Even with a bandwidth-inefficient signal like rectangular pulses, the channel $H_C(f)$ can cause severe ISI.

ISI can be avoided with Nyquist solution applied solely to the receiver.

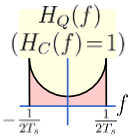
Example: Rectangular pulses

$$h_G(t) = \text{rect}(\tfrac{t}{T_s})$$

$$H_G(f) = T_s \text{sinc}(fT_s)$$

$$P(f) = AT_s \text{rect}(T_s f) = H_G(f) H_C(f) H_Q(f)$$

$$\Rightarrow H_Q(f) = \frac{AT_s \text{rect}(T_s f)}{H_G(f) H_C(f)} = \frac{A\text{rect}(T_s f)}{\text{sinc}(fT_s) H_C(f)}$$



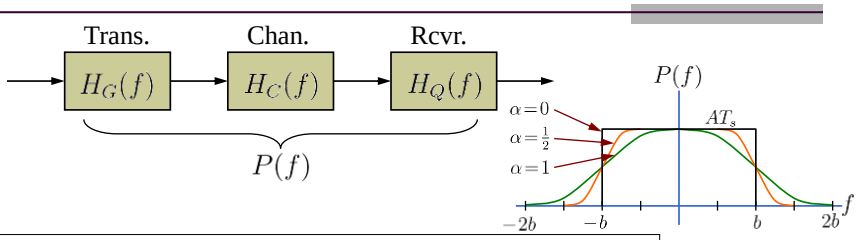
Analog and Digital Communication

Section 4.5: Raised Cosine Filtering and Eye Diagrams

Overview of Section 4.5

- Raised cosine pulse spectrum
- Time-domain representation of raised cosine pulse
- The raised cosine pulse and ISI
- Shared pulse shaping and root raised cosine filtering
- Eye diagram
- Observations from eye diagrams
- Binary eye patterns

Raised Cosine Pulse Spectrum



$$P(f)=\begin{cases} AT_s & \text{if } |f|<b-c \\ \frac{AT_s}{2}(1+\cos(\frac{\pi}{2}\frac{|f|-b+c}{c})) & \text{if } b-c<|f|<b+c \\ 0 & \text{otherwise} \end{cases}$$

$$b=\frac{1}{2T_s}$$

$$\text{Roll-off factor}=\alpha=\frac{c}{b}$$

$$0\le\alpha\le 1$$

$\alpha=0 \Rightarrow$ Nyquist solution

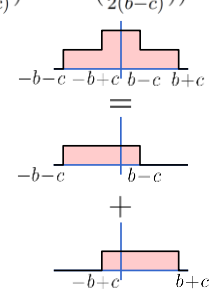
$$\text{Excess bandwidth}=c$$

Time-Domain Representation of Raised Cosine Pulse

$$P(f)=AT_s\text{rect}(\tfrac{f}{2(b-c)})+\tfrac{AT_s}{2}(1+\cos(\tfrac{\pi}{2}\tfrac{|f|-b+c}{c}))(\text{rect}(\tfrac{f}{2(b+c)})-\text{rect}(\tfrac{f}{2(b-c)}))$$

$$=\tfrac{AT_s}{2}\cos(\tfrac{\pi}{2}\tfrac{|f|-b+c}{c})(\text{rect}(\tfrac{f}{2(b+c)})-\text{rect}(\tfrac{f}{2(b-c)}))$$

$$+\tfrac{AT_s}{2}(\text{rect}(\tfrac{f}{2(b-c)})+\text{rect}(\tfrac{f}{2(b+c)}))$$



$$\frac{dP(f)}{df}=\frac{-AT_s}{2}(\tfrac{\pi}{2c})\sin(\tfrac{\pi}{2}\tfrac{|f|-b+c}{c})(\text{rect}(\tfrac{f}{2(b+c)})-\text{rect}(\tfrac{f}{2(b-c)}))$$

$$\frac{d^2P(f)}{df^2}=\frac{-AT_s}{2}(\tfrac{\pi}{2c})^2\cos(\tfrac{\pi}{2}\tfrac{|f|-b+c}{c})(\text{rect}(\tfrac{f}{2(b+c)})-\text{rect}(\tfrac{f}{2(b-c)}))$$

$$\Rightarrow P(f)=-(\tfrac{2c}{\pi})^2\frac{d^2P(f)}{df^2}+\tfrac{AT_s}{2}(\text{rect}(\tfrac{f}{2(b-c)})+\text{rect}(\tfrac{f}{2(b+c)}))$$

$$=-(\tfrac{2c}{\pi})^2\frac{d^2P(f)}{df^2}+\tfrac{AT_s}{2}(\text{rect}(\tfrac{f-c}{2b})+\text{rect}(\tfrac{f+c}{2b}))$$

$$\Rightarrow p(t)=-(\tfrac{2c}{\pi})^2(-j2\pi t)^2p(t)+\tfrac{AT_s}{2}(2b\ \text{sinc}(2bt)e^{j2\pi ct}+2b\ \text{sinc}(2bt)e^{-j2\pi ct})$$

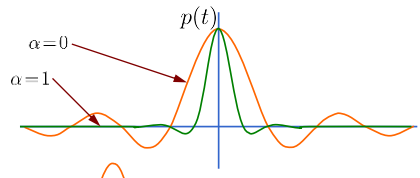
$$=16c^2t^2p(t)+2AbT_s\,\text{sinc}(2bt)\cos(2\pi ct)$$

$$b=\frac{1}{2T_s}$$

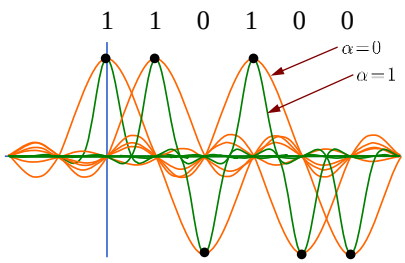
$$\boxed{p(t)=\frac{A\ \text{sinc}(\frac{t}{T_s})\cos(2\pi ct)}{1-16c^2t^2}}$$

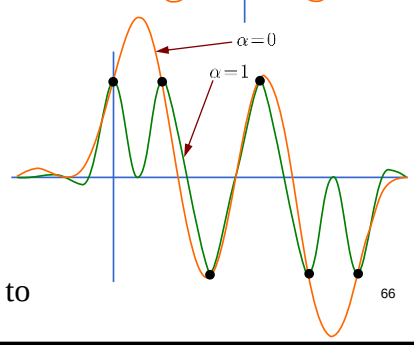
The Raised Cosine Pulse and ISI

$$p(t)=\frac{A\ \mathrm{sinc}(\frac{t}{T_s})\cos(2\pi ct)}{1-16c^2t^2}\Rightarrow p[n]=p(nT_s)=0 \text{ if } n\neq 0 \Rightarrow \text{ no ISI}$$



Binary level signaling:





Raised cosine is much less sensitive to timing errors than Nyquist solution.

Examples: Raised Cosine Channel

If a raised cosine channel has bandwidth $75\ \mathrm{kHz}$ and the symbol duration is $10\ \mu\mathrm{s}$, determine the roll-off factor and the excess bandwidth.

$$T_s=10\times 10^{-6}\ \mathrm{s}\Rightarrow b=\frac{1}{2T_s}=\frac{1}{2\times 10^{-5}}=5\times 10^4=50\ \mathrm{kHz}$$
$$\text{Excess bandwidth}=c=75-50=25\ \mathrm{kHz}$$
$$\text{Roll-off factor}=\alpha=\frac{c}{b}=\frac{25}{50}=\frac{1}{2}=50\%$$

If a raised cosine channel has bandwidth $75\ \mathrm{kHz}$ and the roll-off factor is $\frac{7}{8}$, determine the symbol duration.

$$\alpha=\frac{c}{b}=\frac{7}{8}\Rightarrow c=\frac{7}{8}b$$
$$b+c=75\Rightarrow \frac{15}{8}b=75\Rightarrow b=40\ \mathrm{kHz}$$
$$T_s=\frac{1}{2b}=\frac{1}{2\times 40\times 10^3}=12.5\ \mu\mathrm{s}$$

Shared Pulse Shaping and Root Raised Cosine Filtering

$$P(f) = \underbrace{H_G(f)}_{\text{Trans.}}\underbrace{H_C(f)}_{\text{Chan.}}\underbrace{H_Q(f)}_{\text{Rcvr.}}$$

$$= \begin{cases} AT_s & \text{if } |f| < b-c \\ \frac{AT_s}{2}\left(1+\cos\left(\frac{\pi}{2}\frac{|f|-b+c}{c}\right)\right) & \text{if } b-c < |f| < b+c \\ 0 & \text{otherwise} \end{cases}$$

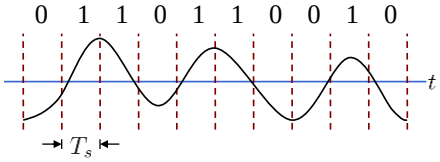
Transmitter shaping: $H_G(f) = P(f) \quad H_Q(f) = \frac{1}{H_C(f)}\text{rect}\left(\frac{f}{2(b+c)}\right)$

Shared pulse shaping: $H_G(f) = \sqrt{P(f)} \quad H_Q(f) = \frac{\sqrt{P(f)}}{H_C(f)}$
(Root-raised-cosine)

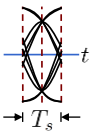
Shared pulse shaping has the best performance in noise.

Eye Diagram

The eye diagram is a synchronized superposition of many successive symbol intervals of an ISI distorted waveform appearing at the output of the receiver filter.

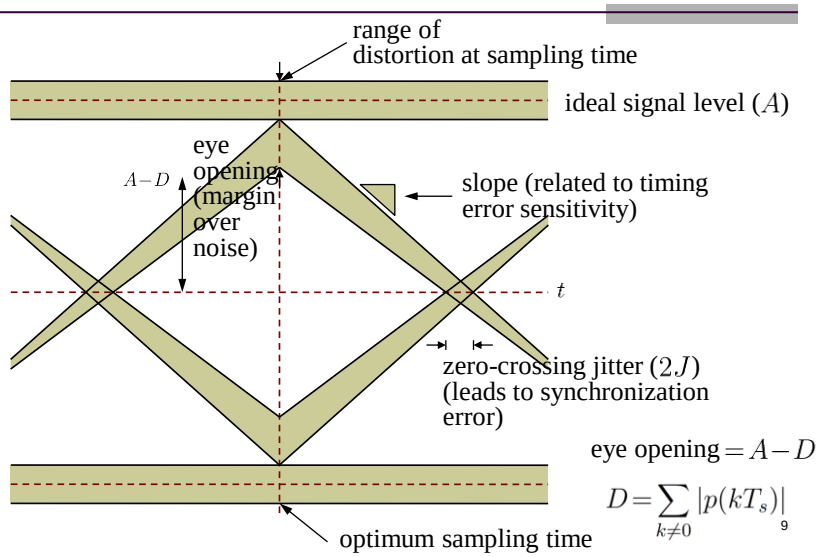


ISI-corrupted signal



Eye diagram
(binary)

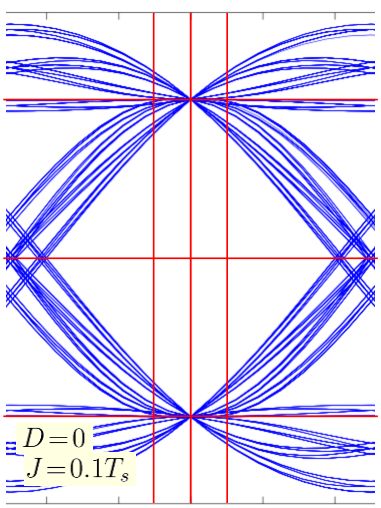
Observations from Eye Diagrams



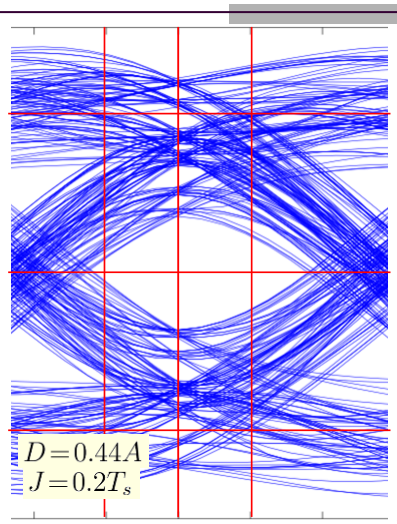
eye opening $= A - D$

$$D = \sum_{k \neq 0} |p(kT_s)|$$

Binary Eye Patterns



No ISI



Significant ISI

Analog and Digital Communication

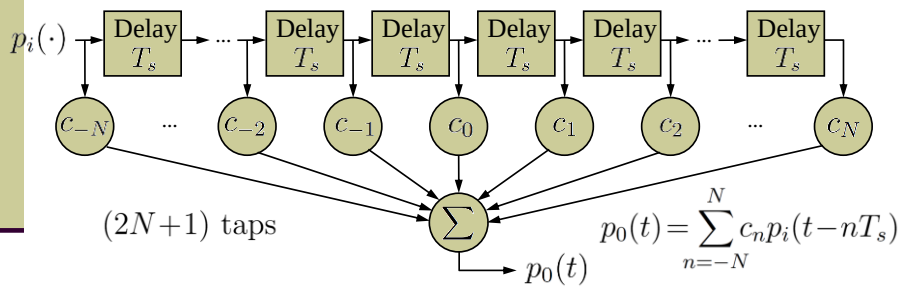
Section 4.6: Equalization and Partial Response Signaling

Overview of Section 4.6

- Equalization
- Zero-forcing equalizer
- Minimum mean square error (MMSE) equalizer
- Partial-response signaling
- Time analysis of partial-response signaling
- Frequency analysis of partial-response signaling
- Decoding of partial-response signaling
- Precoding and simplified decoding of partial-response signaling

Equalization

An <u>equalizer</u> is a filter designed to remove ISI at the decision-making instants $t = iT_s$. Often, a <u>transversal filter</u> is used.



Zero-Forcing Equalizer

$$p_0(kT_s) = \sum_{n=-N}^{N} c_n p_i((k-n)T_s) \Rightarrow \boxed{p_0[k] = \sum_{n=-N}^{N} c_n p_i[k-n]}$$

Zero-forcing solution: $\boxed{p_0[k] = \begin{cases} 1 \text{ if } k = 0 \\ 0 \text{ if } k = \pm 1, \pm 2, \ldots, \pm N \end{cases}}$

The zero-forcing solution does not remove <u>all</u> ISI; it only removes it for $-N \le k \le N$. Terms like $p_0[\pm(N+1)]$ and $p_0[\pm(N+2)]$ are not zero, although they are typically small.

Example: Zero-Forcing Equalizer

Determine the zero-forcing three-tap equalizer if $p_i[k]=(\frac{1}{2})^{|k|}$.

$$3 \text{ taps} \Rightarrow 2N+1=3 \Rightarrow N=1$$

$$p_0[k]=\sum_{n=-1}^{1} c_n p_i[k-n]=c_{-1}p_i[k+1]+c_0 p_i[k]+c_1 p_i[k-1]$$

$$p_0[-1]=0=c_{-1}p_i[0]+c_0 p_i[-1]+c_1 p_i[-2] \quad \Rightarrow c_{-1}+\tfrac{1}{2}c_0+\tfrac{1}{4}c_1=0$$

$$p_0[0]=1=c_{-1}p_i[1]+c_0 p_i[0]+c_1 p_i[-1] \quad \Rightarrow \tfrac{1}{2}c_{-1}+c_0+\tfrac{1}{2}c_1=1$$

$$p_0[1]=0=c_{-1}p_i[2]+c_0 p_i[1]+c_1 p_i[0] \quad \Rightarrow \tfrac{1}{4}c_{-1}+\tfrac{1}{2}c_0+c_1=0$$

$$\Rightarrow \tfrac{3}{4}c_{-1}-\tfrac{3}{4}c_1=0 \Rightarrow c_{-1}=c_1 \Rightarrow \tfrac{1}{2}c_0+\tfrac{5}{4}c_1=0,\ c_0+c_1=1$$

$$\Rightarrow \tfrac{1}{2}(1-c_1)+\tfrac{5}{4}c_1=0 \Rightarrow \tfrac{3}{4}c_1=-\tfrac{1}{2}$$

$$\Rightarrow c_{-1}=-\tfrac{2}{3}, c_0=\tfrac{5}{3}, c_1=-\tfrac{2}{3} \Rightarrow p_0[k]=-\tfrac{2}{3}p_i[k+1]+\tfrac{5}{3}p_i[k]-\tfrac{2}{3}p_i[k-1]$$

Minimum Mean Square Error (MMSE) Equalizer

$\text{MSE}=\sum_{k=-K}^{K}(p_0[k]-\delta[k])^2$ where $p_0[k]=\sum_{n=-N}^{N} c_n p_i[k-n]$ and $K \geq N$. $(2N+1)$ taps.

$$\frac{\partial}{\partial c_\ell}\text{MSE}=\sum_{k=-K}^{K} 2\left(\sum_{n=-N}^{N} c_n p_i[k-n]-\delta[k]\right)p_i[k-\ell]=0 \qquad \delta[k]=\begin{cases}1 & \text{if } k=0\\ 0 & \text{if } k\neq 0\end{cases}$$

$$\Rightarrow \sum_{n=-N}^{N} c_n \sum_{k=-K}^{K} p_i[k-n]p_i[k-\ell]=\sum_{k=-K}^{K}\delta[k]p_i[k-\ell]=p_i[-\ell]$$

$$\Rightarrow \boxed{\sum_{n=-N}^{N} v_{n,\ell}c_n=p_i[-\ell],\ -N\leq \ell \leq N, \text{ where } v_{n,\ell}=\sum_{k=-K}^{K} p_i[k-n]p_i[k-\ell]}$$

This minimizes MSE over $(2K+1)$ positions. The total MSE is $\sum_{k=-\infty}^{\infty}(p_0[k]-\delta[k])^2$.

MMSE equalization can also incorporate known statistical information about noise.

Example: MMSE Equalizer

Determine the MMSE 3-tap equalizer if $K=2$ and $p_i[k]=(\frac{1}{2})^{|k|}$. Compare with the zero-forcing equalizer.

$$v_{n,\ell}=\sum_{k=-2}^{2} p_i[k-n]p_i[k-\ell] \begin{array}{l} =p_i[-2-n]p_i[-2-\ell]+p_i[-1-n]p_i[-1-\ell] \\ +p_i[-n]p_i[-\ell]+p_i[1-n]p_i[1-\ell]+p_i[2-n]p_i[2-\ell] \end{array}$$

$$\Rightarrow v_{-1,-1}=\tfrac{1}{2}\cdot\tfrac{1}{2}+1\cdot 1+\tfrac{1}{2}\cdot\tfrac{1}{2}+\tfrac{1}{4}\cdot\tfrac{1}{4}+\tfrac{1}{8}\cdot\tfrac{1}{8}=\tfrac{101}{64}=v_{1,1}$$

$$v_{0,0}=\tfrac{1}{4}\cdot\tfrac{1}{4}+\tfrac{1}{2}\cdot\tfrac{1}{2}+1\cdot 1+\tfrac{1}{2}\cdot\tfrac{1}{2}+\tfrac{1}{4}\cdot\tfrac{1}{4}=\tfrac{13}{8}$$

$$v_{0,1}=\tfrac{1}{4}\cdot\tfrac{1}{8}+\tfrac{1}{2}\cdot\tfrac{1}{4}+1\cdot\tfrac{1}{2}+\tfrac{1}{2}\cdot 1+\tfrac{1}{4}\cdot\tfrac{1}{2}=\tfrac{41}{32}=v_{-1,0}=v_{1,0}=v_{0,-1}$$

$$v_{-1,1}=\tfrac{1}{2}\cdot\tfrac{1}{8}+1\cdot\tfrac{1}{4}+\tfrac{1}{2}\cdot\tfrac{1}{2}+\tfrac{1}{4}\cdot 1+\tfrac{1}{8}\cdot\tfrac{1}{2}=\tfrac{7}{8}=v_{1,-1}$$

$$\sum_{n=-1}^{1} v_{n,\ell}c_n=p_i[-\ell] \Rightarrow \tfrac{101}{64}c_{-1}+\tfrac{41}{32}c_0+\tfrac{7}{8}c_1=\tfrac{1}{2} \Rightarrow \tfrac{45}{64}c_{-1}-\tfrac{45}{64}c_1=0 \Rightarrow c_{-1}=c_1$$

$$-1\le\ell\le 1 \qquad \tfrac{41}{32}c_{-1}+\tfrac{13}{8}c_0+\tfrac{41}{32}c_1=1 \Rightarrow \tfrac{13}{8}c_0+\tfrac{41}{16}c_1=1,\ \tfrac{41}{32}c_0+\tfrac{157}{64}c_1=\tfrac{1}{2}$$

$$\tfrac{7}{8}c_{-1}+\tfrac{41}{32}c_0+\tfrac{101}{64}c_1=\tfrac{1}{2} \Rightarrow \tfrac{41}{32}(1-\tfrac{41}{16}c_1)\tfrac{8}{13}+\tfrac{157}{64}c_1=\tfrac{1}{2}$$

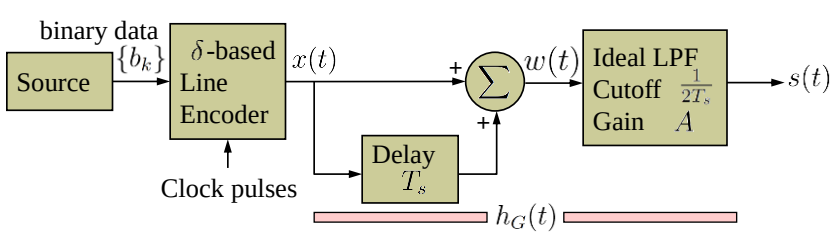
$$\Rightarrow c_{-1}=-\tfrac{2}{3},\ c_0=\tfrac{5}{3},\ c_1=-\tfrac{2}{3}$$

$$\Rightarrow p_0[k]=-\tfrac{2}{3}p_i[k+1]+\tfrac{5}{3}p_i[k]-\tfrac{2}{3}p_i[k-1]$$

(same as zero-forcing equalizer in this case)

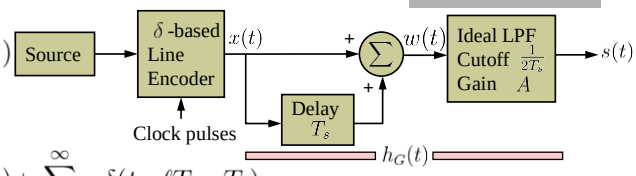
Partial-Response Signaling

Partial-response signaling (also called correlative coding and duobinary signaling) purposely inserts ISI and then removes its effects so as to use the Nyquist bandwidth without sharp cutoffs at the maximum frequency $\frac{1}{2T_s}$.



To achieve the same performance in noise as the Nyquist solution, partial-response signaling requires more power.

Time Analysis of Partial-Response Signaling



$$x(t)=\sum_{k=-\infty}^{\infty} a_k\delta(t-kT_s)$$

$$w(t)=x(t)+x(t-T_s)$$

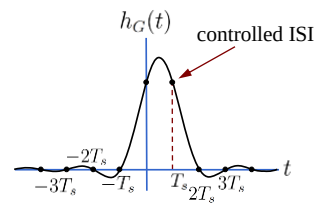
$$=\sum_{k=-\infty}^{\infty} a_k\delta(t-kT_s)+\sum_{\ell=-\infty}^{\infty} a_\ell\delta(t-\ell T_s-T_s)$$

$$k=\ell+1$$

$$=\sum_{k=-\infty}^{\infty} a_k\delta(t-kT_s)+\sum_{k=-\infty}^{\infty} a_{k-1}\delta(t-kT_s)$$

$$=\sum_{k=-\infty}^{\infty}(a_k+a_{k-1})\delta(t-kT_s)=\sum_{k=-\infty}^{\infty} d_k\delta(t-kT_s)$$

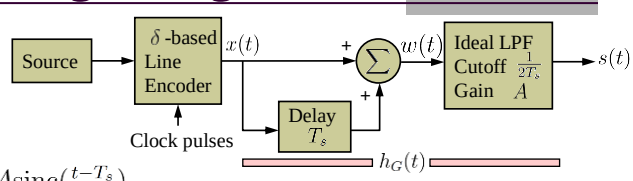
where $d_k=a_k+a_{k-1}$



Impulse response of LPF is $A\text{sinc}(\frac{t}{T_s})$.

$$\Rightarrow h_G(t)=A\text{sinc}(\tfrac{t}{T_s})+A\text{sinc}(\tfrac{t-T_s}{T_s})$$

Frequency Analysis of Partial-Response Signaling

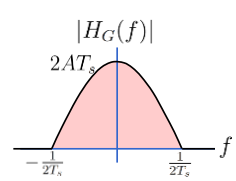


$$h_G(t)=A\text{sinc}(\tfrac{t}{T_s})+A\text{sinc}(\tfrac{t-T_s}{T_s})$$

$$\Rightarrow H_G(f)=AT_s\text{rect}(T_sf)+AT_se^{-j2\pi fT_s}\text{rect}(T_sf)$$

$$=AT_se^{-j\pi fT_s}(e^{j\pi fT_s}+e^{-j\pi fT_s})\text{rect}(T_sf)$$

$$=2AT_se^{-j\pi fT_s}\cos(\pi fT_s)\text{rect}(T_sf)$$



If $H_Q(f)=\frac{1}{H_C(f)}$,

$$P(f)=\underbrace{H_G(f)}_{\text{Trans.}}\underbrace{H_C(f)}_{\text{Chan.}}\underbrace{H_Q(f)}_{\text{Rcvr.}}=H_G(f)$$

Decoding of Partial-Response Signaling

If $a_k = +1$ or -1, $d_k = a_k + a_{k-1} = 2,\ 0,$ or -2.

Decoding rule:

If $\hat{d}_k = +2$, decide that $\hat{a}_k = 1$.
If $\hat{d}_k = -2$, decide that $\hat{a}_k = -1$.
If $\hat{d}_k = 0$, decide that $\hat{a}_k = -\hat{a}_{k-1}$.
(opposite of previous decision)

Note that errors tend to propagate, especially when there is a long sequence in which $\hat{d}_k$s equal zero.

Error propagation can be prevented with <u>precoding</u>.

Precoding and Simplified Decoding of Partial-Response Signaling

If bipolar level-based signaling is used, <u>precoding</u> consists of computing $b'_k = b_k \oplus b'_{k-1}$ on the binary 0-1 data prior to converting it to a_ks via $b'_k = 1 \Rightarrow a_k = -1$, $b'_k = 0 \Rightarrow a_k = +1$ and then computing $d_k = a_k + a_{k-1}$.

$$0 \oplus 0 = 0 \qquad 0 \oplus 1 = 1 \qquad 1 \oplus 0 = 1 \qquad 1 \oplus 1 = 0$$

Decoding rule for use with precoding:

If $\hat{d}_k = \pm 2$, decide that $\hat{b}_k = 0$.
If $\hat{d}_k = 0$, decide that $\hat{b}_k = 1$.

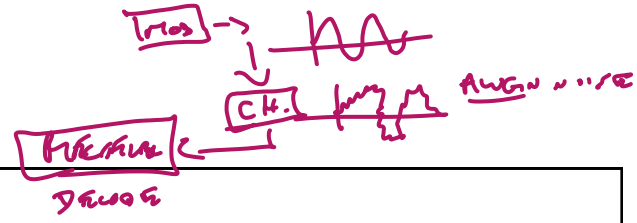
There is no error propagation with precoding and this decoding rule.

Example: Partial-Response Signaling

Precode (0)1011001110 and determine the a_ks and d_ks. Then decode using the decoding rule for use with precoding.

b_k		(0)	1	0	1	1	0	0	1	1	1	0
$b'_k = b_k \oplus b'_{k-1}$	(0)	(0)	1	1	0	1	1	1	0	1	0	0
a_k $\begin{matrix} 1 \to -1 \\ 0 \to +1 \end{matrix}$		(+1)	-1	-1	+1	-1	-1	-1	+1	-1	+1	+1
$d_k = a_k + a_{k-1} = \hat{d}_k$			0	-2	0	0	-2	-2	0	0	0	+2
$\hat{b}_k$ $\begin{matrix} \pm 2 \to 0 \\ 0 \to 1 \end{matrix}$			1	0	1	1	0	0	1	1	1	0

START



Analog and Digital Communication

Section 4.7 Signal Space Concepts and Bandpass Signaling

Overview of Section 4.7

- Orthogonal and orthonormal waveforms
- Signal space
- Energy and correlation
- Euclidean distance
- Signal space diagrams
- Orthonormal bandpass signals over phase
- Orthonormal bandpass signals over frequency
- Modulation of orthonormal baseband signals
- Orthonormal modulated signal sets

TEST 3; ANNOUNCEMENT

... → RCM - LINE CODE

Orthogonal and Orthonormal Waveforms

INNER PRODUCT

The signals $\phi_1(t)$ and $\phi_2(t)$ are said to be orthogonal over $(0,T)$ if $R_{\phi_1,\phi_2}=0$. The signals are said to be orthonormal if they are orthogonal and $E_{\phi_1}=1$ and $E_{\phi_2}=1$.

$$R_{\phi_1,\phi_2}=\int_0^T \phi_1(t)\phi_2^*(t)dt \quad E_{\phi_1}=\int_0^T |\phi_1(t)|^2 dt \quad E_{\phi_2}=\int_0^T |\phi_2(t)|^2 dt$$

ORTHOGONALITY ORTHONORMALITY

The signals $\phi_1(t)$, $\phi_2(t)$, …, $\phi_n(t)$ are said to be mutually orthogonal if each pair is orthogonal and mutually orthonormal if each pair is orthonormal. If mutually orthonormal,

$$\int_0^T \phi_i(t)\phi_j^*(t)dt=\begin{cases}1 & \text{if } i=j\\ 0 & \text{otherwise}\end{cases}$$

Signal Space

An orthonormally-generated signal space is the set of all functions $s_i(t)$ that can be written as linear combinations of the associated set of orthonormal functions. That is,

$$s_i(t)=\sum_{j=1}^{n} a_{i,j}\phi_j(t)$$

CONSTANT TERM

The $a_{i,j}$s can be obtained from the equation

$$a_{i,j}=\int_0^T s_i(t)\phi_j^*(t)dt$$

Proof:

$$\int_0^T s_i(t)\phi_j^*(t)dt=\int_0^T \sum_{\ell=1}^{n} a_{i,\ell}\phi_\ell(t)\phi_j^*(t)dt$$
$$=\sum_{\ell=1}^{n} a_{i,\ell}\int_0^T \phi_\ell(t)\phi_j^*(t)dt=a_{i,j}$$

Energy and Correlation

Energy in $s_i(t)$:

$$E_i = \int_0^T |s_i(t)|^2 dt = \int_0^T s_i(t) s_i^*(t) dt = \int_0^T \left(\sum_{\ell=1}^n a_{i,\ell}\phi_\ell(t)\right)\left(\sum_{k=1}^n a_{i,k}\phi_k(t)\right)^* dt$$

$$= \sum_{\ell=1}^n \sum_{k=1}^n a_{i,\ell} a_{i,k}^* \int_0^T \phi_\ell(t)\phi_k^*(t) dt = \sum_{\ell=1}^n a_{i,\ell} a_{i,\ell}^* \Rightarrow \boxed{E_i = \sum_{\ell=1}^n |a_{i,\ell}|^2}$$

Correlation between $s_i(t)$ and $s_j(t)$:

$$R_{i,j} = \int_0^T s_i(t) s_j^*(t) dt = \int_0^T \left(\sum_{\ell=1}^n a_{i,\ell}\phi_\ell(t)\right)\left(\sum_{k=1}^n a_{j,k}\phi_k(t)\right)^* dt$$

$$= \sum_{\ell=1}^n \sum_{k=1}^n a_{i,\ell} a_{j,k}^* \int_0^T \phi_\ell(t)\phi_k^*(t) dt = \sum_{\ell=1}^n a_{i,\ell} a_{j,\ell}^* \Rightarrow \boxed{R_{i,j} = \sum_{\ell=1}^n a_{i,\ell} a_{j,\ell}^*}$$

Euclidean Distance

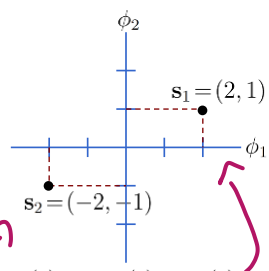
The <u>Euclidean distance</u> between $s_i(t)$ and $s_j(t)$ is defined as

$$d_{i,j} = \sqrt{\mathrm{MSE}_{i,j}} \quad \text{where} \quad \mathrm{MSE}_{i,j} = \int_0^T |s_i(t) - s_j(t)|^2 dt$$

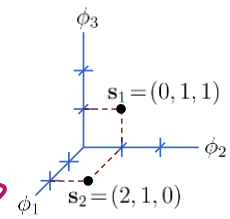
$$\begin{aligned}
\mathrm{MSE}_{i,j} &= \int_0^T (s_i(t) - s_j(t))(s_i(t) - s_j(t))^* dt \\
&= \int_0^T \left(\sum_{\ell=1}^n (a_{i,\ell} - a_{j,\ell})\phi_\ell(t)\right)\left(\sum_{k=1}^n (a_{i,k} - a_{j,k})\phi_k(t)\right)^* dt \\
&= \sum_{\ell=1}^n \sum_{k=1}^n (a_{i,\ell} - a_{j,\ell})(a_{i,k}^* - a_{j,k}^*) \int_0^T \phi_\ell(t)\phi_k^*(t) dt \\
&= \sum_{\ell=1}^n |a_{i,\ell} - a_{j,\ell}|^2 = \sum_{\ell=1}^n a_{i,\ell}a_{i,\ell}^* + a_{j,\ell}a_{j,\ell}^* - a_{i,\ell}a_{j,\ell}^* - a_{i,\ell}^* a_{j,\ell} \\
&= \sum_{\ell=1}^n |a_{i,\ell}|^2 + |a_{j,\ell}|^2 - 2\mathrm{Re}\{a_{i,\ell}a_{j,\ell}^*\} = E_i + E_j - 2\mathrm{Re}\{R_{i,j}\} \\
&\Rightarrow \boxed{\mathrm{MSE}_{i,j} = \sum_{\ell=1}^n |a_{i,\ell} - a_{j,\ell}|^2 = E_i + E_j - 2\mathrm{Re}\{R_{i,j}\}}
\end{aligned}$$

Signal Space Diagrams

The signal $s_i(t)$ can be viewed as the point $\mathbf{s}_i = (a_{i1}, a_{i2}, \ldots, a_{in})$ in an n-dimensional coordinate system with axes corresponding to $\phi_1(t),\ \phi_2(t),\ \ldots,\ \phi_n(t)$.



$s_1(t) = 2\phi_1(t) + \phi_2(t)$
$a_{11} = 2 \quad a_{12} = 1$
$s_2(t) = -2\phi_1(t) - \phi_2(t)$
$a_{21} = -2 \quad a_{22} = -1$



$s_1(t) = \phi_2(t) + \phi_3(t)$
$a_{11} = 0 \quad a_{12} = 1 \quad a_{13} = 1$
$s_2(t) = 2\phi_1(t) + \phi_2(t)$
$a_{21} = 2 \quad a_{22} = 1 \quad a_{23} = 0$

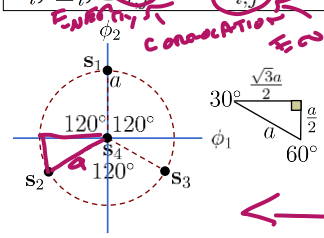
$$s_i(t) = \sum_{j=1}^{n} a_{i,j}\phi_j(t)$$

A signal space diagram is only useful if all quantities are real and n is no greater than 3.

Example: Signal Space Concepts

Suppose $s_1(t)$ through $s_4(t)$ are represented below. Determine $\mathbf{s}_i$, E_i, $R_{i,j}$ and $d_{i,j}$ for all i, j.

ENERGY CORRELATION ENG. DIST.



$\mathbf{s}_1 = (0, a)$ $\quad E_1 = 0^2 + a^2 = a^2$

$\mathbf{s}_2 = (-\frac{\sqrt{3}a}{2}, -\frac{a}{2})$ $\quad E_2 = (-\frac{\sqrt{3}a}{2})^2 + (-\frac{a}{2})^2 = a^2$

$\mathbf{s}_3 = (\frac{\sqrt{3}a}{2}, -\frac{a}{2})$ $\quad E_3 = (\frac{\sqrt{3}a}{2})^2 + (-\frac{a}{2})^2 = a^2$

$\mathbf{s}_4 = (0, 0)$ $\quad E_4 = 0^2 + 0^2 = 0$

PLUG-IN

NOTED
SIGNAL SPACE
EUCLIDEAN DIST.
E_i, $R_{j,i}$

$R_{1,2} = (0)(-\frac{\sqrt{3}a}{2}) + (a)(-\frac{a}{2}) = -\frac{a^2}{2}$ $\quad \mathrm{MSE}_{1,2} = a^2 + a^2 - 2(-\frac{a^2}{2}) = 3a^2$

$R_{1,3} = (0)(\frac{\sqrt{3}a}{2}) + (a)(-\frac{a}{2}) = -\frac{a^2}{2}$ $\quad \mathrm{MSE}_{1,3} = a^2 + a^2 - 2(-\frac{a^2}{2}) = 3a^2$

$R_{2,3} = (-\frac{\sqrt{3}a}{2})(\frac{\sqrt{3}a}{2}) + (-\frac{a}{2})(-\frac{a}{2}) = -\frac{a^2}{2}$ $\quad \mathrm{MSE}_{2,3} = a^2 + a^2 - 2(-\frac{a^2}{2}) = 3a^2$

$R_{1,4} = R_{2,4} = R_{3,4} = 0$ $\quad \mathrm{MSE}_{1,4} = \mathrm{MSE}_{2,4} = \mathrm{MSE}_{3,4} = a^2 + 0^2 - 2(0) = a^2$

$d_{1,2} = \sqrt{3}a \quad d_{1,3} = \sqrt{3}a \quad d_{2,3} = \sqrt{3}a \quad d_{1,4} = d_{2,4} = d_{3,4} = a$

B/C PERPENDICULAR → ORTHOGONALITY / ORTHONORMALITY : SPLIT SEND DATA OVER 2-SIGNAL'S W/ SAME BW, & FREQUENCY

STARTS @ ORIGIN

← BANDPASS AT HIGHER FREQ.

Orthonormal Bandpass Signals Over Phase

HAVE PHASE SHIFT: COS SIN

Over $(0,T)$, the signals $\phi_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t)=\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$ are approximately orthonormal if $f_c \gg \frac{1}{T}$.

$$\int_0^T |\phi_1(t)|^2 dt = \frac{2}{T}\int_0^T \cos^2(2\pi f_c t)dt = \frac{2}{T}\int_0^T (\tfrac{1}{2}+\tfrac{1}{2}\cos(4\pi f_c t))dt$$
$$= \frac{2}{T}(\tfrac{T}{2}+\tfrac{1}{8\pi f_c}\sin(4\pi f_c T)) = 1+\tfrac{1}{4\pi f_c T}\sin(4\pi f_c T) \simeq 1$$

$$\int_0^T |\phi_2(t)|^2 dt = \frac{2}{T}\int_0^T \sin^2(2\pi f_c t)dt = \frac{2}{T}\int_0^T (\tfrac{1}{2}-\tfrac{1}{2}\cos(4\pi f_c t))dt$$
$$= \frac{2}{T}(\tfrac{T}{2}-\tfrac{1}{8\pi f_c}\sin(4\pi f_c T)) = 1-\tfrac{1}{4\pi f_c T}\sin(4\pi f_c T) \simeq 1$$

$$\int_0^T \phi_1(t)\phi_2^*(t)dt = \frac{2}{T}\int_0^T \cos(2\pi f_c t)\sin(2\pi f_c t)dt = \frac{2}{T}\int_0^T \tfrac{1}{2}\sin(4\pi f_c t)dt$$
$$= \frac{2}{T}(\tfrac{1}{8\pi f_c})(\cos(0)-\cos(4\pi f_c T))$$
$$\Rightarrow 0 \le \int_0^T \phi_1(t)\phi_2^*(t)dt \le \tfrac{1}{2\pi f_c T} \ll 1 \Rightarrow \boxed{\int_0^T \phi_1(t)\phi_2^*(t)dt \simeq 0}$$

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Orthonormal Bandpass Signals over Frequency

Over $(0,T)$, the signals $\phi_i(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_i t)$ and $\phi_j(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_j t)$ are approximately orthonormal if $f_i \gg \frac{1}{T}$, $f_j \gg \frac{1}{T}$, and $f_i - f_j = \frac{k}{2T}$, where k is an integer.

Already shown that $\int_0^T |\phi_i(t)|^2 dt \simeq 1$ and $\int_0^T |\phi_j(t)|^2 dt \simeq 1$ if $f_i \gg \frac{1}{T}$ and $f_j \gg \frac{1}{T}$.

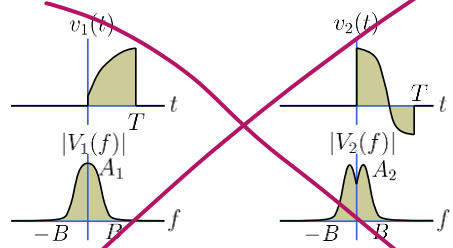
$$\int_0^T \phi_i(t)\phi_j^*(t)dt = \frac{2}{T}\int_0^T \cos(2\pi f_i t)\cos(2\pi f_j t)dt$$
$$= \frac{2}{T}\int_0^T \tfrac{1}{2}\cos(2\pi(f_i+f_j)t)+\tfrac{1}{2}\cos(2\pi(f_i-f_j)t)dt$$
$$= \frac{2}{T}\frac{1}{4\pi(f_i+f_j)}\sin(2\pi(f_i+f_j)T)+\frac{2}{T}\frac{1}{4\pi(f_i-f_j)}\sin(2\pi(f_i-f_j)T)$$
$$\simeq \frac{1}{2\pi T(f_i-f_j)}\sin(2\pi(f_i-f_j)T) \text{ because } f_i+f_j \gg \frac{1}{T}$$
$$= 0 \text{ if } 2\pi(f_i-f_j)T = k\pi \Rightarrow f_i - f_j = \frac{k}{2T}$$

Smallest acceptable spacing between frequencies: $\frac{1}{2T}$

95

Modulation of Orthonormal Baseband Signals

Suppose $v_1(t)$ and $v_2(t)$ are orthonormal baseband signals over $(0, T)$.



Modulated signals:

$$\phi_1(t) = \sqrt{2} v_1(t) \cos(2\pi f_c t)$$
$$\phi_2(t) = \sqrt{2} v_2(t) \cos(2\pi f_c t)$$

Assume $f_c \gg B$.

B is not the absolute bandwidth, but assume there is negligible energy beyond frequency B.

$$\int_0^T |v_1(t)|^2 dt = 1 \quad \int_0^T |v_2(t)|^2 dt = 1$$
$$\int_0^T v_1(t) v_2^*(t) dt = 0$$

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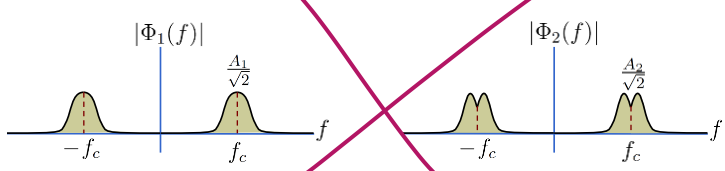
Modulation of Orthonormal Baseband Signals

$$\phi_1(t) = \sqrt{2} v_1(t) \cos(2\pi f_c t) = \frac{\sqrt{2}}{2} v_1(t)(e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$

$$\phi_2(t) = \sqrt{2} v_2(t) \cos(2\pi f_c t) = \frac{\sqrt{2}}{2} v_2(t)(e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$



$$\Phi_1(f) = \frac{\sqrt{2}}{2}(V_1(f+f_c) + V_1(f-f_c)) \quad \Phi_2(f) = \frac{\sqrt{2}}{2}(V_2(f+f_c) + V_2(f-f_c))$$

$$E_{\phi_1} \simeq 2(\tfrac{1}{\sqrt{2}})^2 E_{v_1} = 1 \quad E_{\phi_2} \simeq 2(\tfrac{1}{\sqrt{2}})^2 E_{v_2} = 1$$

A&D Communication: Section 4.7

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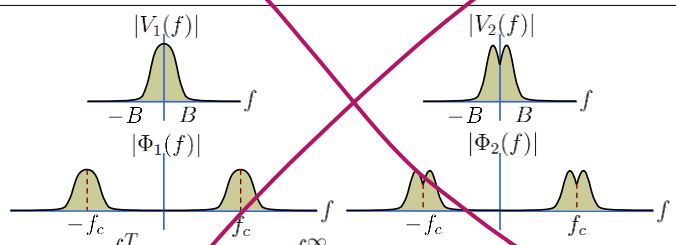
Modulation of Orthonormal Baseband Signals

Parseval's correlation identity: $\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$



$$R_{\phi_1,\phi_2} = \int_0^T \phi_1(t)\phi_2^*(t)dt = \int_{-\infty}^{\infty} \Phi_1(f)\Phi_2^*(f)df$$

$$\simeq 2\left(\tfrac{1}{\sqrt{2}}\right)\left(\tfrac{1}{\sqrt{2}}\right)\int_{-\infty}^{\infty} V_1(f)V_2^*(f)df = \int_0^T v_1(t)v_2^*(t)dt = 0$$

$\Rightarrow$ $\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{2}\,v_2(t)\cos(2\pi f_c t)$ are orthonormal if $v_1(t)$ and $v_2(t)$ are orthonormal and $f_c \gg B$.

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Orthonormal Modulated Signal Sets

Assumption: $v_1(t)$ and $v_2(t)$ are orthonormal with bandwidth B.

$$\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_c t + \theta_c)$$
$$\phi_2(t) = \sqrt{2}\,v_2(t)\cos(2\pi f_c t + \theta_c)$$

$$\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_c t + \theta_c)$$
$$\phi_2(t) = \sqrt{2}\,v_1(t)\sin(2\pi f_c t + \theta_c)$$

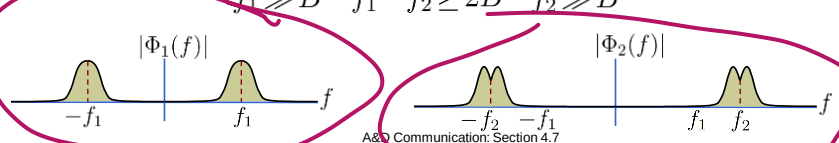
$$\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_c t + \theta_c)$$
$$\phi_2(t) = \sqrt{2}\,v_2(t)\sin(2\pi f_c t + \theta_c)$$

$$f_c \gg B$$

$$\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_1 t + \theta_1)$$
$$\phi_2(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_2 t + \theta_2)$$

$$\phi_1(t) = \sqrt{2}\,v_1(t)\cos(2\pi f_1 t + \theta_1)$$
$$\phi_2(t) = \sqrt{2}\,v_2(t)\cos(2\pi f_2 t + \theta_2)$$

$$f_1 \gg B \qquad f_1 - f_2 \geq 2B \qquad f_2 \gg B$$



A&D Communication: Section 4.7

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PASSBAND MODULATION

1

$s_1 = A\cos(\ )$ $E_{s_1} = s_0^{T_s} s_1^2$

$s_2 = B\cos(\ )$

$s_1^T s_2 = 0$

↳ ORTHOGONAL

CORRELATION

$\cos(\ )$ → ORTHOGONAL

$\cos 2\cos(\ )$ USING CORRELATER

→ 2

SPACE REP. "VECTOR"

ϕ_2

Analog and Digital Communication

ϕ_1

BASIS FUNCTION $= \sqrt{\frac{2}{T_s}}\cos(\ )$

Section 4.8 Binary Digital Carrier Modulation

$\{\phi_1 \perp \phi_2$

$\|\phi_1\| = 1$ → ORTHO NORMAL

UNIT NRG

3

ONE VECTOR SHOWS LOSE CORRELATION

MORE IS LESS → $\sqrt{E}$ $-\sqrt{E}$

USE ← SHOWS HOW BASS. SIG. IS.

Overview of Section 4.8

- Binary amplitude shift keying
- Demodulation of BASK
- Binary frequency shift keying
- Demodulation of BFSK
- Binary phase shift keying
- Demodulation of BPSK
- Differential phase shift keying
- Bandwidth of BASK/BPSK/DPSK
- Bandwidth of BFSK

SOURCE → [ ] – MOD → BASEBAND (RCM) OR

PASSBAND

SIN COS

HIGH (FREQ.)

Binary Amplitude Shift Keying

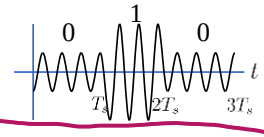
Binary amplitude shift keying (BASK) is defined as

$$s(t)=\sum_{k=-\infty}^{\infty} A_{d_k}\,\mathrm{rect}\left(\frac{t-\frac{1}{2}T_s-kT_s}{T_s}\right)\cos(2\pi f_c t)$$

where $d_k\in\{0,1\}$ and T_s is the symbol duration.

For $kT_s\le t<(k+1)T_s$,

$$s(t)=\begin{cases} A_0\cos(2\pi f_c t) & \text{if } d_k=0\\ A_1\cos(2\pi f_c t) & \text{if } d_k=1\end{cases}$$



UNIT NRG

$\int_0^{T_s} s(t)$

$= A^2\frac{T_s}{2}$

Letting $\phi_1(t)=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c(t-kT_s))=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t+\theta)$ gives

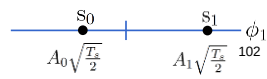
$$s_0(t)=A_0\sqrt{\frac{T_s}{2}}\phi_1(t) \text{ and } s_1(t)=A_1\sqrt{\frac{T_s}{2}}\phi_1(t)$$

ONE BASIS → DOES NOT MEAN ORTH.

IS ENOUGH

BUT UNIT NRG

BASK is a digital version of DSB AM.



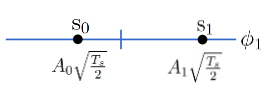
$\frac{s(t)}{\phi(t)} = A\sqrt{T_s/2}$

BINARY → "0" → "1"

$A\cos(2\pi f_c t+\theta)$ → P. BPSK → A. BASK → F. BFSK

$A\sqrt{T_s/2}$

Binary Amplitude Shift Keying



$$\mathbf{s}_0=(A_0\sqrt{\tfrac{T_s}{2}}) \qquad E_0=\frac{A_0^2T_s}{2}$$

$$\mathbf{s}_1=(A_1\sqrt{\tfrac{T_s}{2}}) \qquad E_1=\frac{A_1^2T_s}{2}$$

SHOW DIFF.

CORRELATION

$$R_{0,1}=(A_0\sqrt{\tfrac{T_s}{2}})(A_1\sqrt{\tfrac{T_s}{2}})=\frac{A_0A_1T_s}{2}$$

$$\mathrm{MSE}_{0,1}=E_0+E_1-2\mathrm{Re}\{R_{0,1}\}=\frac{A_0^2T_s}{2}+\frac{A_1^2T_s}{2}-2\left(\frac{A_0A_1T_s}{2}\right)=\frac{(A_0-A_1)^2T_s}{2}$$

$$d_{0,1}=\sqrt{\mathrm{MSE}_{0,1}}=|A_0-A_1|\sqrt{\frac{T_s}{2}}$$

ACSO

$\frac{A}{1/\sqrt{T}} = A\sqrt{T}$

On-off keying (OOK): $A_0=0$

Antipodal signaling: $A_1=-A_0$

FIND SIG UNIT NRG:

$E_{s_1} = \int_0^T A^2\,dt = A^2T$

$\frac{1}{\sqrt{T}}\Pi_T = 1$ $\xleftrightarrow{0\ A\sqrt{T}}$ $\phi_1 = \frac{1}{\sqrt{T}}$

$\int s_1\phi_1\,dt = A\sqrt{T}$

DIRECTION

Ex.) 0 1 0 1 0 (-A)

$s_1 = 0$

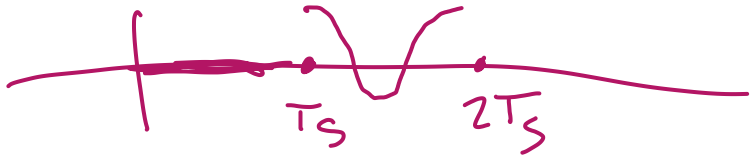
$s_2 = A\cos(2\pi f_c t)$

EX.) FIND BASIS FUNCTION

$s_1 \begin{cases} A \\ 0 \end{cases}$

$s_2 \begin{cases} -A \\ 0 \end{cases}$

ONLY NEED ONE



WHEN YOU SEND OVER CHANNEL USE BY 1 TIME SLOT A TIME

Binary Amplitude Shift Keying

AS BASIS FUNC. IS ALWAYS GIVEN. NEED TO BE ORTHOGONAL

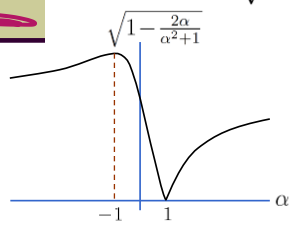
COS SIN

With no loss of generality, $A_0 = \alpha A_1 \Rightarrow d_{0,1} = |A_0 - A_1|\sqrt{\frac{T_s}{2}}$
$= |\alpha - 1||A_1|\sqrt{\frac{T_s}{2}}$

Suppose s_0 and s_1 are equally likely.

$\Rightarrow E_{\text{avg}} = \frac{1}{2}E_0 + \frac{1}{2}E_1 = \frac{T_s}{4}(A_0^2 + A_1^2) = \frac{T_s}{4}(1+\alpha^2)A_1^2$
$\Rightarrow |A_1| = 2\sqrt{\frac{E_{\text{avg}}}{T_s(1+\alpha^2)}}$

$\Rightarrow d_{0,1} = \frac{2|\alpha-1|\sqrt{E_{\text{avg}}}}{\sqrt{T_s(1+\alpha^2)}}\sqrt{\frac{T_s}{2}} = \sqrt{2E_{\text{avg}}}\sqrt{\frac{(\alpha-1)^2}{\alpha^2+1}} = \sqrt{2E_{\text{avg}}}\sqrt{\frac{\alpha^2+1-2\alpha}{\alpha^2+1}}$
$= \sqrt{2E_{\text{avg}}}\sqrt{1 - \frac{2\alpha}{\alpha^2+1}}$



$\Rightarrow$ maximum occurs at $\alpha = -1$ (antipodal)
$\Rightarrow (d_{0,1})_{\max} = 2\sqrt{E_{\text{avg}}}$

For $\alpha = 0$ (OOK), $d_{0,1} = \sqrt{2E_{\text{avg}}}$

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VECTOR DOMAIN

HARD IN TIME DOMAIN

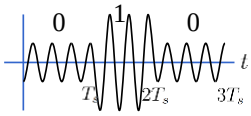
LOOK @ CALC.: d_{s_1} CD d_{s_2} → EXPECTED

MOD BIT INDEP.

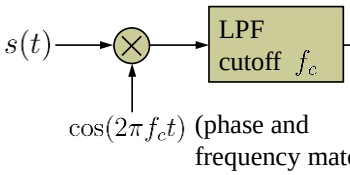
DIFF. TYPES

DOT PRODUCT → EASY ANALYSIS DISTRIB (TWO VARIANTS)

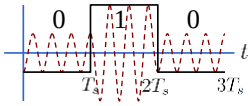
Demodulation of BASK



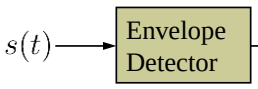
Coherent demodulation:



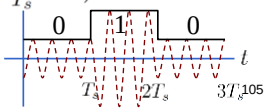
$\frac{1}{2}\sum_{k=-\infty}^{\infty} A_{d_k}\text{rect}\left(\frac{t-\frac{1}{2}T_s-kT_s}{T_s}\right)$



Noncoherent demodulation: (requires $|A_0| \neq |A_1|$)



$\sum_{k=-\infty}^{\infty} |A_{d_k}|\text{rect}\left(\frac{t-\frac{1}{2}T_s-kT_s}{T_s}\right)$



A&D Communication: Section 4.8

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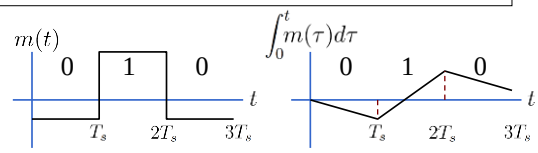
Binary Frequency Shift Keying

Binary frequency shift keying (BFSK) is defined as

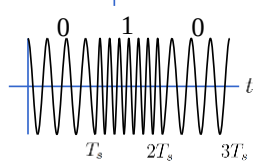
$$s(t) = A\cos(2\pi f_c t + 2\pi \int_0^t m(\tau)d\tau)$$

where $m(t) = \sum_{k=-\infty}^{\infty} f_{d_k} \mathrm{rect}(\frac{t-\frac{1}{2}T_s - kT_s}{T_s})$, $d_k \in \{0, 1\}$, and T_s is the symbol duration.



For $kT_s \le t < (k+1)T_s$,

$$s(t) = \begin{cases} A\cos(2\pi(f_c+f_0)t+\theta_{0,k}) & \text{if } d_k = 0 \\ A\cos(2\pi(f_c+f_1)t+\theta_{1,k}) & \text{if } d_k = 1 \end{cases}$$



BFSK is a digital version of FM.

Binary Frequency Shift Keying

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi(f_c+f_0)t+\theta_0)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi(f_c+f_1)t+\theta_1)$$

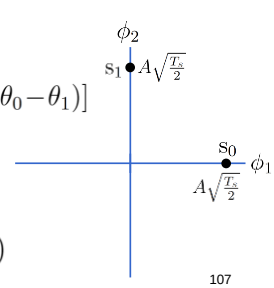
$$\int_0^{T_s} \phi_1(t)\phi_2^*(t)dt = \frac{2}{T_s}\int_0^{T_s} \cos(2\pi(f_c+f_0)t+\theta_0)\cos(2\pi(f_c+f_1)t+\theta_1)dt$$

$$= \frac{1}{T_s}\int_0^{T_s} \cos(2\pi(2f_c+f_0+f_1)t+\theta_0+\theta_1)+\cos(2\pi(f_0-f_1)t+\theta_0-\theta_1)dt$$

$$\simeq \frac{1}{T_s}\int_0^{T_s} \cos(2\pi(f_0-f_1)t+\theta_0-\theta_1)dt$$

$$= \frac{1}{2\pi(f_0-f_1)T_s}[\sin(2\pi(f_0-f_1)T_s+\theta_0-\theta_1)-\sin(\theta_0-\theta_1)]$$

$$= 0 \text{ if } 2\pi(f_0-f_1)T_s = 2\pi m, \; m \text{ an integer}$$

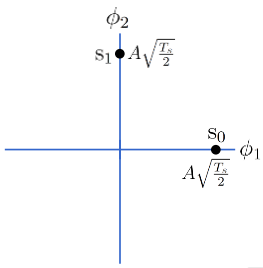


If $|f_1 - f_0| = \frac{m}{T_s}$, the signals are orthonormal

$\Rightarrow s_0(t) = A\sqrt{\frac{T_s}{2}}\phi_1(t)$ and $s_1(t) = A\sqrt{\frac{T_s}{2}}\phi_2(t)$

A&D Communication: Section 4.8

Binary Frequency Shift Keying



$$\mathbf{s}_0 = (A\sqrt{\tfrac{T_s}{2}}, 0) \qquad E_0 = \tfrac{A^2 T_s}{2}$$

$$\mathbf{s}_1 = (0, A\sqrt{\tfrac{T_s}{2}}) \qquad E_1 = \tfrac{A^2 T_s}{2}$$

$$R_{0,1} = (0)(A\sqrt{\tfrac{T_s}{2}}) + (A\sqrt{\tfrac{T_s}{2}})(0) = 0$$

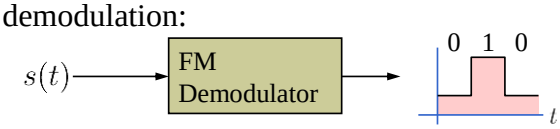
$$\mathrm{MSE}_{0,1} = E_0 + E_1 - 2\mathrm{Re}\{R_{0,1}\} = \tfrac{A^2 T_s}{2} + \tfrac{A^2 T_s}{2} - 2(0) = A^2 T_s$$

$$d_{0,1} = \sqrt{\mathrm{MSE}_{0,1}} = A\sqrt{T_s}$$

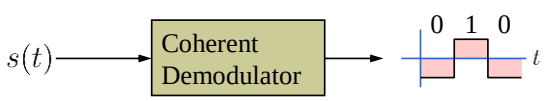
$$E_{\text{avg}} = E_0 = E_1 = \tfrac{A^2 T_s}{2} \Rightarrow d_{0,1} = \sqrt{2E_{\text{avg}}}$$ (same as OOK)

Demodulation of BFSK

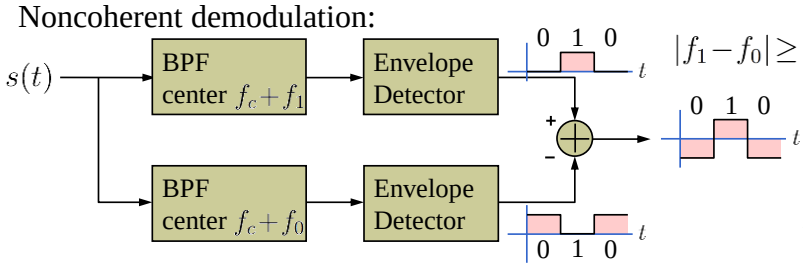
FM demodulation:



Coherent demodulation: discussed later $\quad |f_1 - f_0| = \frac{1}{T_s}$



Noncoherent demodulation: $\quad |f_1 - f_0| \geq \frac{2}{T_s}$



Binary Phase Shift Keying

Binary phase shift keying (BPSK) is defined as

$$s(t) = A\cos(2\pi f_c t + m(t))$$

where $m(t) = \sum_{k=-\infty}^{\infty} \theta_{d_k} \mathrm{rect}(\frac{t-\frac{1}{2}T_s - kT_s}{T_s})$, $d_k \in \{0, 1\}$, and T_s is the symbol duration. Normally $\theta_1 = \pi$ and $\theta_0 = 0$.

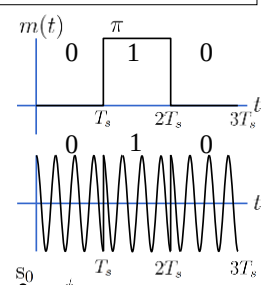
For $kT_s \le t < (k+1)T_s$,

$$s(t) = \begin{cases} A\cos(2\pi f_c t + \theta_0 + \beta_k) \text{ if } d_k = 0 \\ A\cos(2\pi f_c t + \theta_1 + \beta_k) \text{ if } d_k = 1 \end{cases}$$

If $\theta_1 = \pi$ and $\theta_0 = 0$,

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + \beta_k)$$

$$\Rightarrow s_{d_k}(t) = A\sqrt{\frac{T_s}{2}}(-1)^{d_k}\phi_1(t)$$



BPSK is a digital version of PM.

BPSK is antipodal BASK.

Binary Phase Shift Keying

$$\mathbf{s}_0 = (A\sqrt{\frac{T_s}{2}}) \qquad E_0 = \frac{A^2 T_s}{2}$$

$$\mathbf{s}_1 = (-A\sqrt{\frac{T_s}{2}}) \qquad E_1 = \frac{A^2 T_s}{2}$$

$$R_{0,1} = (A\sqrt{\frac{T_s}{2}})(-A\sqrt{\frac{T_s}{2}}) = \frac{-A^2 T_s}{2}$$

$$\mathrm{MSE}_{0,1} = E_0 + E_1 - 2\mathrm{Re}\{R_{0,1}\} = \frac{A^2 T_s}{2} + \frac{A^2 T_s}{2} - 2(\frac{-A^2 T_s}{2}) = 2A^2 T_s$$

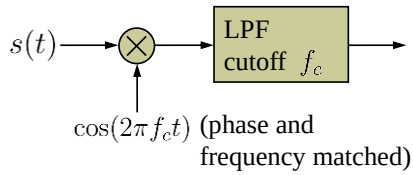
$$d_{0,1} = \sqrt{\mathrm{MSE}_{0,1}} = A\sqrt{2T_s}$$

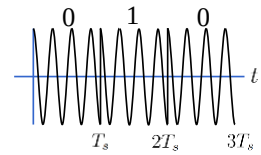
$$E_{\mathrm{avg}} = E_0 = E_1 = \frac{A^2 T_s}{2} \Rightarrow A^2 T_s = 2E_{\mathrm{avg}}$$

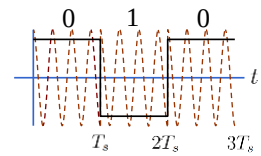
$$d_{0,1} = \sqrt{4E_{\mathrm{avg}}} = 2\sqrt{E_{\mathrm{avg}}}$$

Demodulation of BPSK

Coherent demodulation:







Noncoherent demodulation not possible.

Differential Phase Shift Keying

Differential phase shift keying (DPSK) is defined as BPSK with differential precoding. That is, BPSK is transmitted with the sequence d'_k where

$$d'_k = d'_{k-1} \oplus d_k = \begin{cases} d'_{k-1} & \text{if } d_k = 0 \\ 1 - d'_{k-1} & \text{if } d_k = 1 \end{cases}$$

d_k		0	1	1	0	0	1	0	1	1	1
d'_k	(0)	0	1	0	0	0	1	1	0	1	0

Although coherent demodulation can be used, a phase demodulator is lower in complexity. There will be an unknown phase offset, but the differential nature of the data permits demodulation by declaring a 0 if there is no phase transition and a 1 if a 180° phase transition; i.e., $d_k = d'_k \oplus d'_{k-1}$.

d'_k	(0)	0	1	0	0	0	1	1	0	1	0
d_k		0	1	1	0	0	1	0	1	1	1

Bandwidth of BASK/BPSK/DPSK

Viewing BASK/BPSK/DPSK as AM, note that the building block of the underlying baseband signal, the pulse $\mathrm{rect}(\frac{t}{T_s})$, has Fourier transform $T_s\mathrm{sinc}(fT_s)$ which has first-null bandwidth $\frac{1}{T_s}$. It follows that BASK/BPSK/DPSK has null-to-null bandwidth $\frac{2}{T_s}$.

Generally speaking, the great majority of the signal energy lies in the main spectral lobe.

Bandwidth of BFSK

Viewing BFSK as the juxtaposition of two OOK AM signals, the bandwidth is equal to the sum of $|f_1 - f_0|$, the LSB bandwidth of the lower frequency carrier, and the USB bandwidth of the upper frequency carrier. The total bandwidth is $\frac{2}{T_s} + |f_1 - f_0|$.

The minimum frequency separation for orthogonality is $|f_1 - f_0| = \frac{1}{T_s}$. In this case the bandwidth is $\frac{3}{T_s}$.

The minimum frequency separation for noncoherent demodulation is $|f_1 - f_0| = \frac{2}{T_s}$. In this case the bandwidth is $\frac{4}{T_s}$.

Example: Bandwidth

A BPSK system has bit rate 240 kbps. Suppose the modulation is changed to BFSK using the minimum frequency separation for orthogonality. If the bandwidth remains the same, determine the resulting bit rate.

The bandwidth of BPSK is $\frac{2}{T_s}$.
The bit rate is $\frac{1}{T_s}$ so the bandwidth is 480 kHz.
Switching to BFSK with the minimum frequency separation for orthogonality, the bandwidth is now $\frac{3}{T_s}$.
Setting this equal to 480 kHz gives $\frac{1}{T_s}$ equal to 160 kbps.

Analog and Digital Communication

Section 4.9 Nonbinary Digital Carrier Modulation

Overview of Section 4.9

- M-ary amplitude shift keying
- Demodulation of MASK
- M-ary frequency shift keying
- Demodulation of MFSK
- M-ary phase shift keying
- Demodulation of MPSK
- Differential M-ary phase shift keying
- Alternate view of QPSK
- Offset QPSK (OQPSK)
- Minimum shift keying (MSK)
- Rectangular quadrature amplitude modulation (QAM)
- Bandwidth of MASK
- Bandwidth of MFSK
- Bandwidth of MPSK and related signals

M-ary Amplitude Shift Keying

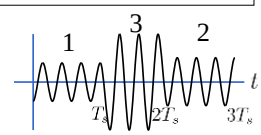
M-ary amplitude shift keying (MASK) is defined as

$$s(t)=\sum_{k=-\infty}^{\infty} A_{d_k}\,\mathrm{rect}\left(\frac{t-\frac{1}{2}T_s-kT_s}{T_s}\right)\cos(2\pi f_c t)$$

where $d_k \in \{0,1,\ldots,M-1\}$ and T_s is the symbol duration.

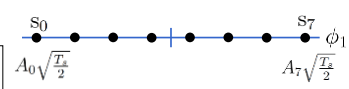
For $kT_s \le t < (k+1)T_s$,

$$s(t)=A_{d_k}\cos(2\pi f_c t)$$



Letting $\phi_1(t)=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c(t-kT_s))=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t+\theta)$ gives

$$s_k(t)=A_k\sqrt{\frac{T_s}{2}}\phi_1(t)$$



MASK is a digital version of DSB AM.

Example: MASK

Determine $\min_{i,j} d_{i,j}$ as a function of E_{avg} if MASK is used with $M=4$ and $A_0=-\frac{3}{2}A$, $A_1=-\frac{1}{2}A$, $A_2=\frac{1}{2}A$, and $A_3=\frac{3}{2}A$. Assume the four signals are equally likely.

$$\mathbf{s}_0=\left(-\frac{3A}{2}\sqrt{\frac{T_s}{2}}\right)\quad \mathbf{s}_1=\left(-\frac{A}{2}\sqrt{\frac{T_s}{2}}\right)\quad \mathbf{s}_2=\left(\frac{A}{2}\sqrt{\frac{T_s}{2}}\right)\quad \mathbf{s}_3=\left(\frac{3A}{2}\sqrt{\frac{T_s}{2}}\right)$$

$$E_0=E_3=\left(\frac{3A}{2}\sqrt{\frac{T_s}{2}}\right)^2=\frac{9A^2T_s}{8}\qquad E_1=E_2=\left(\frac{A}{2}\sqrt{\frac{T_s}{2}}\right)^2=\frac{A^2T_s}{8}$$

$$E_{\text{avg}}=\frac{1}{4}(E_0+E_1+E_2+E_3)=\frac{5A^2T_s}{8}$$

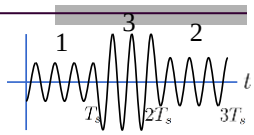
$$R_{0,1}=\left(-\frac{3A}{2}\sqrt{\frac{T_s}{2}}\right)\left(-\frac{A}{2}\sqrt{\frac{T_s}{2}}\right)=\frac{3A^2T_s}{8}$$

$$\mathrm{MSE}_{0,1}=E_0+E_1-2\mathrm{Re}\{R_{0,1}\}=\frac{A^2T_s}{2}$$

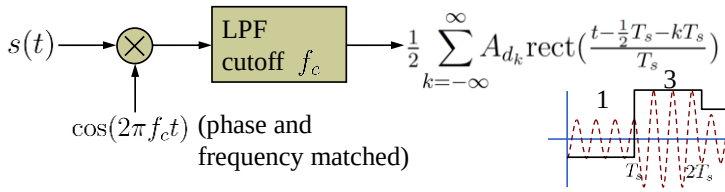
$$=\frac{1}{2}\frac{8E_{\text{avg}}}{5}=\frac{4E_{\text{avg}}}{5}$$

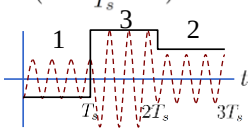
$$\min_{i,j} d_{i,j}=d_{0,1}=\sqrt{\frac{4}{5}E_{\text{avg}}}$$

Demodulation of MASK

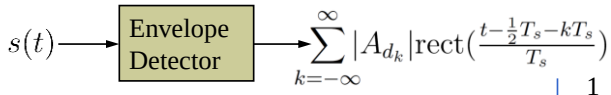


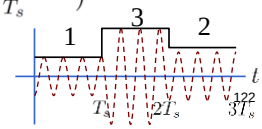
Coherent demodulation:





Noncoherent demodulation: (requires $|A_0| \neq |A_1| \neq \cdots \neq |A_{M-1}|$)





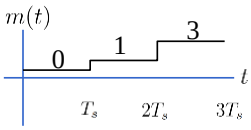
A&D Communication: Section 4.9

M-ary Frequency Shift Keying

M-ary frequency shift keying (MFSK) is defined as

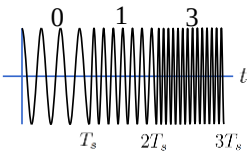
$$s(t) = A\cos(2\pi f_c t + 2\pi\int_0^t m(\tau)d\tau)$$

where $m(t) = \sum_{k=-\infty}^{\infty} f_{d_k}\mathrm{rect}(\frac{t-\frac{1}{2}T_s-kT_s}{T_s})$, $d_k \in \{0, 1, \ldots, M-1\}$, and T_s is the symbol duration.



For $kT_s \leq t < (k+1)T_s$,

$$s(t) = A\cos(2\pi(f_c + f_{d_k})t + \theta_{d_k,k})$$



MFSK is a digital version of FM.

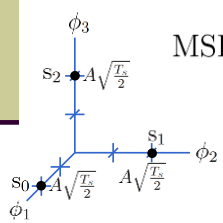
M-ary Frequency Shift Keying

If $|f_k - f_j| = \frac{m}{T_s}$, m an integer,

$$\phi_k(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi(f_c + f_k)t + \theta_k),\ k \in \{0, 1, \ldots, M-1\}$$

$$\Rightarrow s_k(t) = A\sqrt{\frac{T_s}{2}}\phi_{k+1}(t), \quad k \in \{0, 1, \ldots, M-1\}$$

$$E_k = \frac{A^2 T_s}{2} \qquad R_{j,k} = 0$$



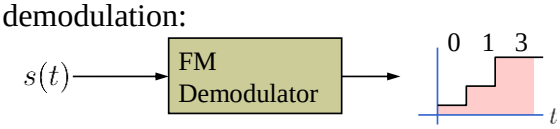
$$\mathrm{MSE}_{j,k} = E_j + E_k - 2\mathrm{Re}\{R_{j,k}\} = \frac{A^2 T_s}{2} + \frac{A^2 T_s}{2} - 2(0) = A^2 T_s$$

$$d_{j,k} = \sqrt{\mathrm{MSE}_{j,k}} = A\sqrt{T_s}$$

$$E_{\text{avg}} = E_k = \frac{A^2 T_s}{2} \Rightarrow d_{j,k} = \sqrt{2E_{\text{avg}}}$$

Demodulation of MFSK

FM demodulation:

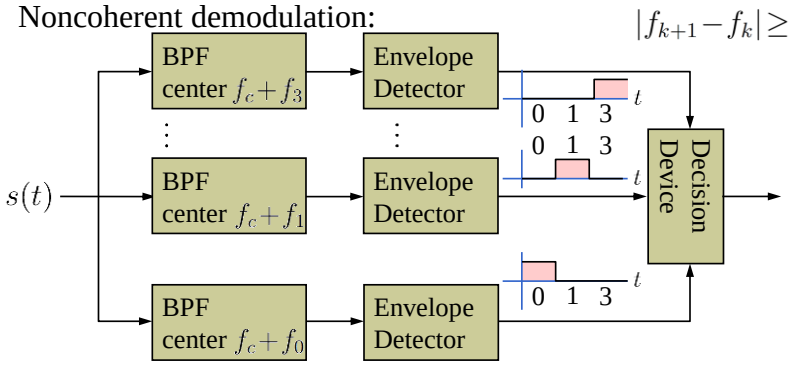


Coherent demodulation: discussed later

$|f_{k+1} - f_k| = \frac{1}{T_s}$

Noncoherent demodulation:

$|f_{k+1} - f_k| \geq \frac{2}{T_s}$



M-ary Phase Shift Keying

M-ary phase shift keying (MPSK) is defined as

$$s(t) = A\cos(2\pi f_c t + m(t))$$

where $m(t) = \sum_{k=-\infty}^{\infty} \theta_{d_k} \mathrm{rect}(\frac{t-\frac{1}{2}T_s - kT_s}{T_s})$, $d_k \in \{0, 1, \ldots, M-1\}$, and T_s is the symbol duration. Normally $\theta_k = \frac{2\pi k}{M}$.

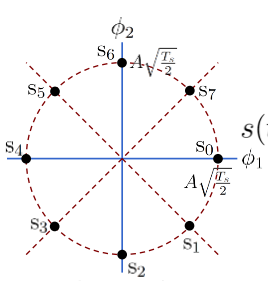
For $kT_s \le t < (k+1)T_s$,

$$s(t) = A\cos(2\pi f_c t + \theta_{d_k} + \beta_k)$$

MPSK is a digital version of PM.

For $M = 4$, M-ary phase shift keying is called quaternary phase shift keying (QPSK).

M-ary Phase Shift Keying



$(M = 8)$

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + \beta_k)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t + \beta_k)$$

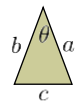
$$\begin{aligned} s(t) &= A\cos(2\pi f_c t + \theta_{d_k} + \beta_k) \\ &= A\cos(\theta_{d_k})\cos(2\pi f_c t + \beta_k) \\ &\quad - A\sin(\theta_{d_k})\sin(2\pi f_c t + \beta_k) \\ &= A\sqrt{\frac{T_s}{2}}\cos(\theta_{d_k})\phi_1(t) - A\sqrt{\frac{T_s}{2}}\sin(\theta_{d_k})\phi_2(t) \end{aligned}$$

MPSK is a digital form of QAM.

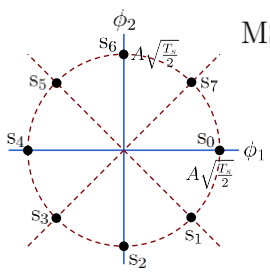
M-ary Phase Shift Keying

$\theta_k = \frac{2\pi k}{M} \qquad E_k = \frac{A^2 T_s}{2} = E_{\text{avg}}$

$\min_{j,k} d_{j,k} = d_{0,1}$



$c^2 = a^2 + b^2 - 2ab\cos(\theta)$



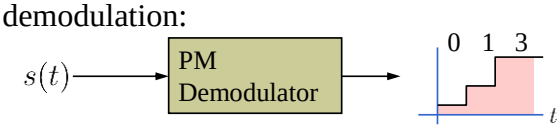
$(M=8)$

$$\begin{aligned}\text{MSE}_{0,1} &= (A\sqrt{\tfrac{T_s}{2}})^2 + (A\sqrt{\tfrac{T_s}{2}})^2 - 2(A\sqrt{\tfrac{T_s}{2}})^2\cos(\tfrac{2\pi}{M}) \\ &= A^2 T_s(1-\cos(\tfrac{2\pi}{M})) \\ &= A^2 T_s(1-(\cos^2(\tfrac{\pi}{M}) - \sin^2(\tfrac{\pi}{M}))) \\ &= 2A^2 T_s \sin^2(\tfrac{\pi}{M}) \\ &= 4E_{\text{avg}}\sin^2(\tfrac{\pi}{M})\end{aligned}$$

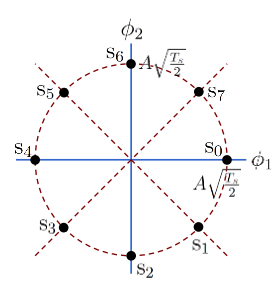
$$d_{0,1} = \sqrt{\text{MSE}_{0,1}} = 2\sqrt{E_{\text{avg}}}\sin(\tfrac{\pi}{M})$$

Demodulation of MPSK

PM demodulation:



Coherent demodulation: discussed later



$(M=8)$

Noncoherent demodulation not possible.

Differential M-ary Phase Shift Keying

Differential phase shift keying (DMPSK) is defined as MPSK with differential precoding. That is, MPSK is transmitted with the sequence d'_k where $d'_k = (d'_{k-1} + d_k) \bmod M$.

$$d'_k = \begin{cases} d'_{k-1} + d_k & \text{if } d'_{k-1} + d_k < M \\ d'_{k-1} + d_k - M & \text{otherwise} \end{cases}$$

$(M=4)$

d_k		2	3	1	1	2	0	0	1	2	3
d'_k	(0)	2	1	2	3	1	1	1	2	0	3

Although coherent demodulation can be used, a phase demodulator is lower in complexity. Recovery of the original data can be achieved via $d_k = (d'_k - d'_{k-1}) \bmod M$.

d'_k	(0)	2	1	2	3	1	1	1	2	0	3
d_k		2	3	1	1	2	0	0	1	2	3

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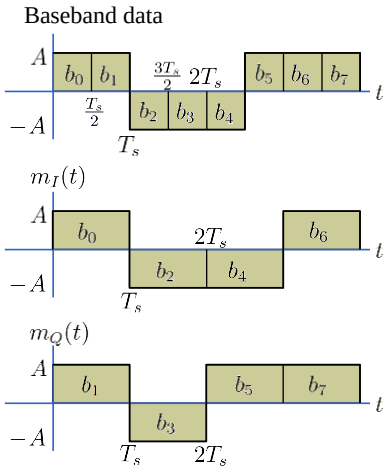
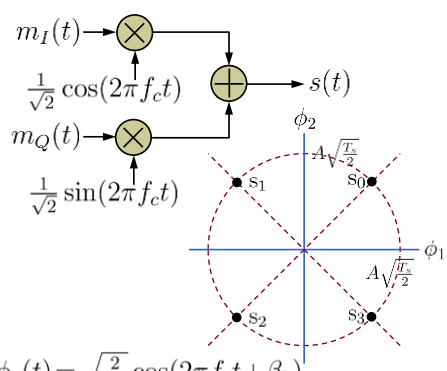
Alternate View of QPSK

Baseband data





$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + \beta_k)$$
$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t + \beta_k)$$

Transitions: every T_s seconds.
Phase shifts: $0,\ \pm\frac{\pi}{2},\ \pi$.

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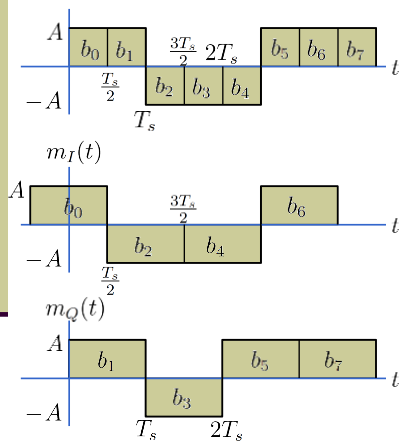
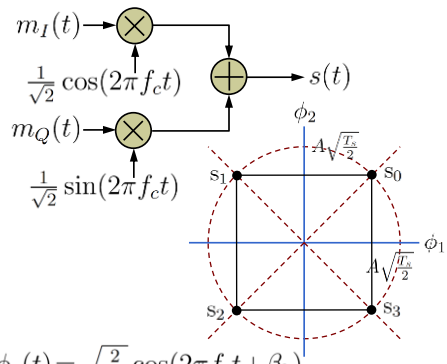
Offset QPSK (OQPSK)

Baseband data



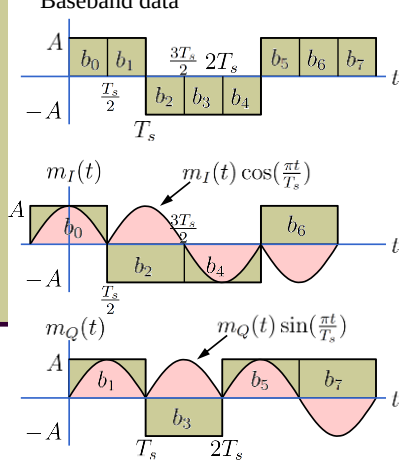


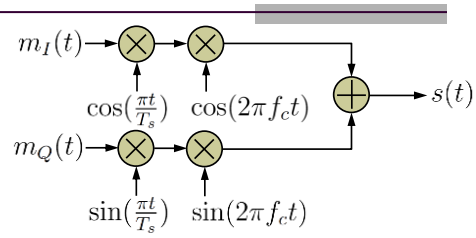
$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + \beta_k)$$
$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t + \beta_k)$$

Transitions: every $\frac{T_s}{2}$ seconds.
Phase shifts: $0,\ \pm\frac{\pi}{2}$ only.

Minimum Shift Keying (MSK)

Baseband data



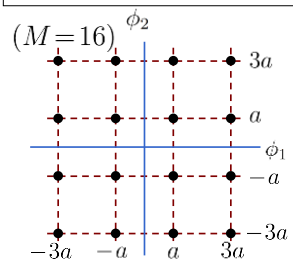


Phase shifts: None.
An example of <u>continuous phase modulation</u>.

Rectangular Quadrature Amplitude Modulation (QAM)

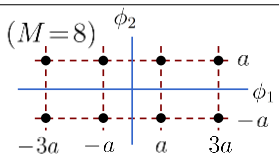
MPSK is a special case of digital quadrature amplitude modulation (QAM). In rectangular QAM, the signal points lie on a rectangular grid.

$(M=16)$



$$E_{\text{avg}}=\frac{a^2}{16}(4(\sqrt{2})^2+8(\sqrt{10})^2+4(\sqrt{18})^2)$$
$$=\frac{a^2}{16}(160)=10a^2 \Rightarrow a=\sqrt{\frac{E_{\text{avg}}}{10}}$$
$$\min_{j,k} d_{j,k}=2a=\sqrt{\frac{2E_{\text{avg}}}{5}}$$

$(M=8)$



$$E_{\text{avg}}=\frac{a^2}{8}(4(\sqrt{2})^2+4(\sqrt{10})^2)=6a^2$$
$$\Rightarrow a=\sqrt{\frac{E_{\text{avg}}}{6}}$$
$$\min_{j,k} d_{j,k}=2a=\sqrt{\frac{2E_{\text{avg}}}{3}}$$

Bandwidth of MASK

Viewing MASK as AM, note that the building block of the underlying baseband signal, the pulse $\text{rect}(\frac{t}{T_s})$, has Fourier transform $T_s\text{sinc}(fT_s)$ which has first-null bandwidth $\frac{1}{T_s}$. It follows that MASK has null-to-null bandwidth $\frac{2}{T_s}$.

Generally speaking, the great majority of the signal energy lies in the main spectral lobe.

For <u>all</u> M-ary signaling schemes, the following equations apply:

$$M=2^k \qquad T_s=kT_b \qquad R_s=f_s=\frac{1}{T_s}=\frac{1}{kT_b}=\frac{R_b}{k}$$

$R_s=$ symbol rate

$R_b=$ bit rate

Bandwidth of MFSK

Assume the frequencies are equally separated. Viewing MFSK as the juxtaposition of M OOK AM signals, the bandwidth is equal to the sum of $(M-1)|f_1-f_0|$, the LSB bandwidth of the lower frequency carrier, and the USB bandwidth of the upper frequency carrier. The total bandwidth is $\frac{2}{T_s}+(M-1)|f_1-f_0|$.

The minimum frequency separation for orthogonality is $|f_1-f_0|=\frac{1}{T_s}$. In this case the bandwidth is $\frac{M+1}{T_s}$.

The minimum frequency separation for noncoherent demodulation is $|f_1-f_0|=\frac{2}{T_s}$. In this case the bandwidth is $\frac{2M}{T_s}$.

Bandwidth of MPSK and Related Signals

MPSK (including QPSK), OQPSK, and digital QAM are all forms of QAM. Because the building block of the underlying baseband signal is $\mathrm{rect}(\frac{t}{T_s})$, which has first-null bandwidth $\frac{1}{T_s}$, it follows that MPSK/QPSK/OQPSK/QAM has null-to-null bandwidth $\frac{2}{T_s}$.

For MSK, the null-to-null bandwidth is $\frac{3}{T_s}$ which is 50% larger than that of QPSK/OQPSK. However, the 99%-of-power and 99.9%-of-power bandwidths are better (smaller) for MSK than for QPSK/OQPSK.

Example: Bandwidth

An 8-ary MFSK system using noncoherent demodulation has bandwidth 480 kHz. Determine the bit rate.

The bandwidth of MFSK with noncoherent demodulation is $\frac{2M}{T_s}$.
Here $M = 2^k$ is 8.
The symbol rate is $R_s = \frac{1}{T_s}$ so the symbol rate is 30 ksymbols/s.
Using $R_s = \frac{R_b}{k}$ gives a bit rate of 90 kbits/s.