Electic Potential

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February 27, 2015

1 Introduction

We continue our study of systems involving electric charge. We have spent some time describing these systems through interactions, forces, which arise from the electric field. Just as we did with mechanics earlier, we are now going to look at the same systems through the use of energy.

2 Electric Potential Energy

Recall from our study of gravitation the definition of the gravitational force and potential energy

$$F_g = \frac{GMm}{r^2}, U_g = -\frac{GMm}{r}$$

It turns out that the electric force behaves in the same way

$$F_e = \frac{kQq}{r^2}, U_e = \frac{kQq}{r}$$

Notice that we dropped a negative sign in the potential energy definition. When we talked about gravitation, two objects with mass were acted upon by an attraction force. In our study of charges, we have found that two particles with the same sign (positive or negative) of charge *repel* instead. This accounts for the change of sign. We now have yet another form of energy which we can include in a conversation of energy statement for a particular system.

3 Electric Potential

When we talked about forces involving electric charge, we found that it was convenient to develop a *force per unit charge*, or electric field, since this would be constant for a particular system. We will develop a similar notion for energy. We define the **electric potential**, often denoted by V to be the *energy per unit charge* due to a particular electric field. For a point particular this would be defined (per the above definition of potential energy) as

$$V = \frac{U}{q} = \frac{kQ}{r}$$

and so now to find the potential energy of a particular particle, we just multiple the potential at its location by its charge. Rather than absolute values, we are most often interested in the change in electic potential, also called the *potential difference*, in applications including a conservation of energy statement.

4 Capacitors

We are now ready to discuss our first electrical circuit component, the capacitor. A capacitor is simply a device that *stores charge*. The most basic form is comprised of two oppositely charged parallel plates. The spread of charge induces an electric field across the plates, and therefore a potential difference ΔV is generated. By convention, the negatively charged plate is considered to have a potential of 0 and the postiviely charges one a potential of V. This potential difference depends on both the electric field present as well as the separation of the plates d as

$$\Delta V = E\Delta d$$

We can also calculate the electric potential at any point P between the plates. If we know the distance d_P between P and the plate with potential 0 and the electric field E present, then the potential at the point is calculated by

$$V_p = Ed_p$$

A useful way to qualitatively describe a capacitor is through its *capacitance*, the amount of charge stored per unit potential or

$$C = \frac{Q}{\Delta V}$$

For a parallel-plate capacitor, the capitance depends on some physical parameters including the separation between the plates d, the surface area of each plate A, and the permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N * m^2$

$$C = \frac{\epsilon_0 A}{d}$$

The amount of energy stored by the capacitor is

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

5 Problems

1. Two charged plates have a charge of 3.0 coulombs, are separated by a distance of 10 cm, and have a potential difference of 12 volts. Determine (a) the magnitude of the electric field between the plates, (b) the capacitance of the charged plates, and (c) the energy stored by the charged plates.

2. A conducting sphere with a mass of 2.0 kg and a charge of 1.0 coulomb is initially at rest. Determine its speed after being accelerated through a 16-volt potential difference.

3. Four charges of equal magnitudes are arranged at the vertices of a square, with two charges of the same sign at opposite corners. Which is true at P, the point at the exact center of the square?

   A. $E \neq 0$ and $V < 0$

   B. $E \neq 0$ and $V = 0$

   C. $E \neq 0$ and $V > 0$

   D. $E = 0$ and $V = 0$

   E. $E = 0$ and $V > 0$