Part A

Question A1

Single bubble sonoluminescence occurs when sound waves cause a bubble suspended in a fluid to collapse so that the gas trapped inside increases in temperature enough to emit light. The bubble actually undergoes a series of expansions and collapses caused by the sound wave pressure variations.

We now consider a simplified model of a bubble undergoing sonoluminescence. Assume the bubble is originally at atmospheric pressure $P_0 = 101$ kPa. When the pressure in the fluid surrounding the bubble is decreased, the bubble expands isothermally to a radius of 36.0 μm. When the pressure increases again, the bubble collapses to a radius of 4.50 μm so quickly that no heat can escape. Between the collapse and subsequent expansion, the bubble undergoes isochoric (constant volume) cooling back to its original pressure and temperature. For a bubble containing a monatomic gas, suspended in water of $T = 293$ K, find

a. the number of moles of gas in the bubble,

b. the pressure after the expansion,

c. the pressure after collapse,

d. the temperature after the collapse, and

e. the total work done on the bubble during the whole process.

You may find the following useful: the specific heat capacity at constant volume is $C_V = 3R/2$ and the ratio of specific heat at constant pressure to constant volume is $\gamma = 5/3$ for a monatomic gas.

Question A2

A thin, uniform rod of length L and mass $M = 0.258$ kg is suspended from a point a distance R away from its center of mass. When the end of the rod is displaced slightly and released it executes simple harmonic oscillation. The period, T, of the oscillation is timed using an electronic timer. The following data is recorded for the period as a function of R. What is the local value of g? Do not assume it is the canonical value of 9.8 m/s^2. What is the length, L, of the rod? No estimation of error in either value is required. The moment of inertia of a rod about its center of mass is $(1/12)ML^2$.

R (m)	T (s)
0.050	3.842
0.075	3.164
0.102	2.747
0.156	2.301
0.198	2.115

R (m)	T (s)
0.211	2.074
0.302	1.905
0.387	1.855
0.451	1.853
0.588	1.900

You *must* show your work to obtain full credit. If you use graphical techniques then you must plot the graph; if you use linear regression techniques then you must show all of the formulae and associated workings used to obtain your result.

Part A

Question A1

A newly discovered subatomic particle, the *S meson*, has a mass M. When at rest, it lives for exactly $\tau = 3 \times 10^{-8}$ seconds before decaying into two identical particles called *P mesons* (peons?) that each have a mass of αM.

a. In a reference frame where the S meson is at rest, determine

   i. the kinetic energy,
   ii. the momentum, and
   iii. the velocity

   of each P meson particle in terms of M, α, the speed of light c, and any numerical constants.

b. In a reference frame where the S meson travels 9 meters between creation and decay, determine

   i. the velocity and
   ii. kinetic energy of the S meson.

   Write the answers in terms of M, the speed of light c, and any numerical constants.

Question A2

An ideal (but not necessarily perfect monatomic) gas undergoes the following cycle.

- The gas starts at pressure P_0, volume V_0 and temperature T_0.
- The gas is heated at constant volume to a pressure αP_0, where $\alpha > 1$.
- The gas is then allowed to expand adiabatically (no heat is transferred to or from the gas) to pressure P_0
- The gas is cooled at constant pressure back to the original state.

The adiabatic constant γ is defined in terms of the specific heat at constant pressure C_p and the specific heat at constant volume C_v by the ratio $\gamma = C_p/C_v$.

a. Determine the efficiency of this cycle in terms of α and the adiabatic constant γ. As a reminder, efficiency is defined as the ratio of work out divided by heat in.

b. A lab worker makes measurements of the temperature and pressure of the gas during the adiabatic process. The results, in terms of T_0 and P_0 are

Pressure	units of P_0	1.21	1.41	1.59	1.73	2.14
Temperature	units of T_0	2.11	2.21	2.28	2.34	2.49

   Plot an appropriate graph from this data that can be used to determine the adiabatic constant.

c. What is γ for this gas?

Part A

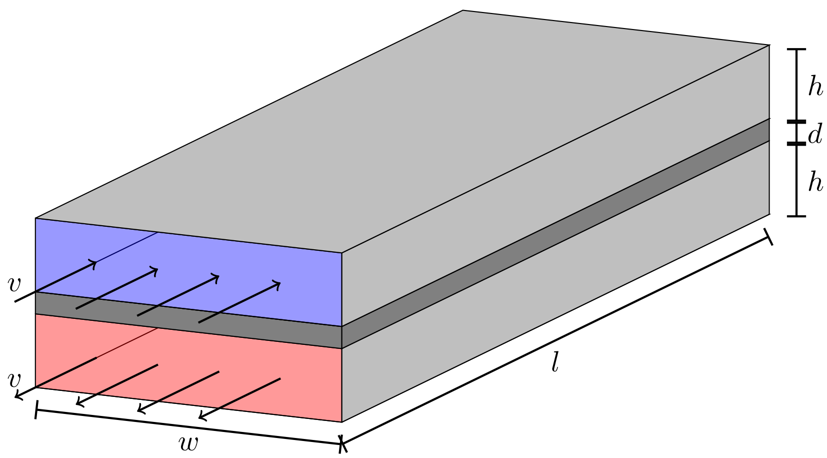
Question A1

The flow of heat through a material can be described via the *thermal conductivity* κ. If the two faces of a slab of material with thermal conductivity κ, area A, and thickness d are held at temperatures differing by ΔT, the thermal power P transferred through the slab is

$$P = \frac{\kappa A \Delta T}{d}$$

A *heat exchanger* is a device which transfers heat between a hot fluid and a cold fluid; they are common in industrial applications such as power plants and heating systems. The heat exchanger shown below consists of two rectangular tubes of length l, width w, and height h. The tubes are separated by a metal wall of thickness d and thermal conductivity κ. Originally hot fluid flows through the lower tube at a speed v from right to left, and originally cold fluid flows through the upper tube in the opposite direction (left to right) at the same speed. The heat capacity per unit volume of both fluids is c.

The hot fluid enters the heat exchanger at a higher temperature than the cold fluid; the difference between the temperatures of the entering fluids is ΔT_i. When the fluids exit the heat exchanger the difference has been reduced to ΔT_f. (It is possible for the exiting originally cold fluid to have a *higher* temperature than the exiting originally hot fluid, in which case $\Delta T_f < 0$.)



Assume that the temperature in each pipe depends only on the lengthwise position, and consider transfer of heat only due to conduction in the metal and due to the bulk movement of fluid. Under the assumptions in this problem, while the temperature of each fluid varies along the length of the exchanger, the temperature *difference* across the wall is the same everywhere. You need not prove this.

Find ΔT_f in terms of the other given parameters.

Part A

Question A1

Inspired by: http://www.wired.com/wiredscience/2012/04/a-leaning-motorcycle-on-a-vertical-wall/

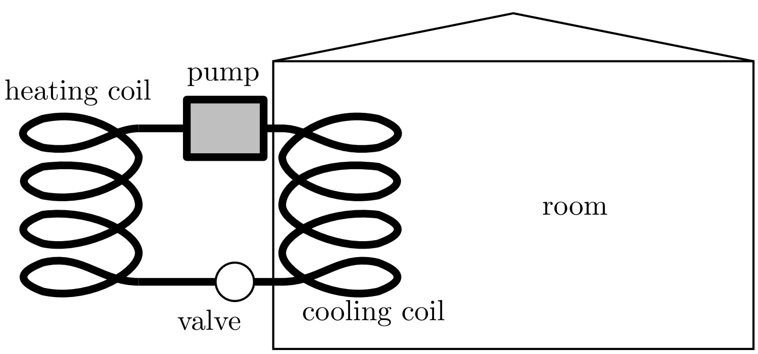
A unicyclist of total height h goes around a circular track of radius R while leaning inward at an angle θ to the vertical. The acceleration due to gravity is g.

a. Suppose $h \ll R$. What angular velocity ω must the unicyclist sustain?

b. Now model the unicyclist as a uniform rod of length h, where h is less than R but not negligible. This refined model introduces a correction to the previous result. What is the new expression for the angular velocity ω? Assume that the rod remains in the plane formed by the vertical and radial directions, and that R is measured from the center of the circle to the point of contact at the ground.

Question A2

A room air conditioner is modeled as a heat engine run in reverse: an amount of heat Q_L is absorbed from the room at a temperature T_L into cooling coils containing a working gas; this gas is compressed adiabatically to a temperature T_H; the gas is compressed isothermally in a coil *outside* the house, giving off an amount of heat Q_H; the gas expands adiabatically back to a temperature T_L; and the cycle repeats. An amount of energy W is input into the system every cycle through an electric pump. This model describes the air conditioner with the best possible efficiency.



Assume that the outside air temperature is T_H and the inside air temperature is T_L. The air-conditioner unit consumes electric power P. Assume that the air is sufficiently dry so that no condensation of water occurs in the cooling coils of the air conditioner. Water boils at 373 K and freezes at 273 K at normal atmospheric pressure.

a. Derive an expression for the maximum rate at which heat is removed from the room in terms of the air temperatures T_H, T_L, and the power consumed by the air conditioner P. Your derivation must refer to the entropy changes that occur in a Carnot cycle in order to receive full marks for this part.

b. The room is insulated, but heat still passes into the room at a rate $R = k\Delta T$, where ΔT is the temperature difference between the inside and the outside of the room and k is a constant. Find the coldest possible temperature of the room in terms of T_H, k, and P.

c. A typical room has a value of $k = 173$ W/°C. If the outside temperature is 40°C, what minimum power should the air conditioner have to get the inside temperature down to 25°C?

Question A4

A heat engine consists of a moveable piston in a vertical cylinder. The piston is held in place by a removable weight placed on top of the piston, but piston stops prevent the piston from sinking below a certain point. The mass of the piston is $m = 40.0$ kg, the cross sectional area of the piston is $A = 100\ \text{cm}^2$, and the weight placed on the piston has a mass of $m = 120.0$ kg.

Assume that the region around the cylinder and piston is a vacuum, so you don't need to worry about external atmospheric pressure.

- At point **A** the cylinder volume V_0 is completely filled with liquid water at a temperature $T_0 = 320$ K and a pressure P_{min} that would be just sufficient to lift the piston alone, except the piston has the additional weight placed on top.
- Heat energy is added to the water by placing the entire cylinder in a hot bath.
- At point **B** the piston and weight begins to rise.
- At point **C** the volume of the cylinder reaches V_{max} and the temperature reaches T_{max}. The heat source is removed; the piston stops rising and is locked in place.
- Heat energy is now removed from the water by placing the entire cylinder in a cold bath.
- At point **D** the pressure in the cylinder returns to P_{min}. The added weight is removed; the piston is unlocked and begins to move down.
- The cylinder volume returns to V_0. The cylinder is removed from the cold bath, the weight is placed back on top of the piston, and the cycle repeats.

Because the liquid water can change to gas, there are several important events that take place

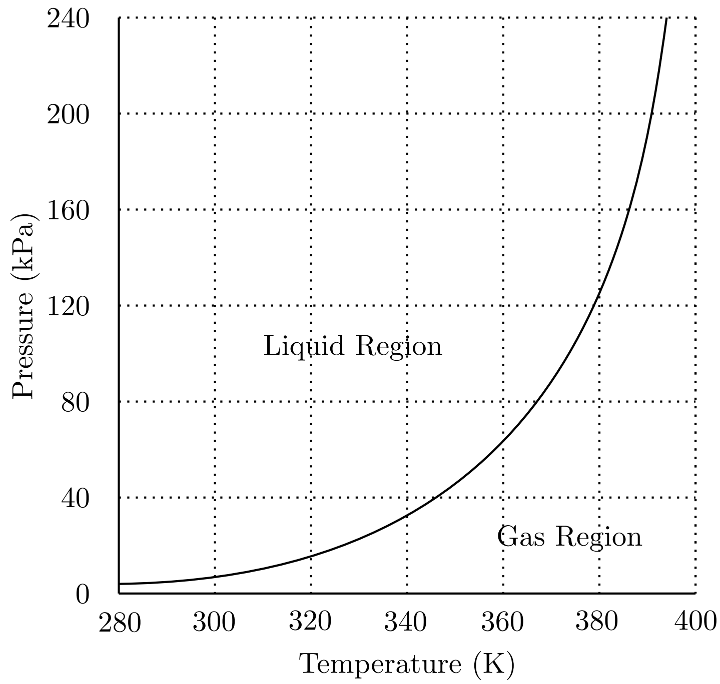
- At point **W** the liquid begins changing to gas.
- At point **X** all of the liquid has changed to gas. This occurs at the same point as point **C** described above.
- At point **Y** the gas begins to change back into liquid.
- At point **Z** all of the gas has changed back into liquid.

When in the liquid state you need to know that for water kept at constant volume, a change in temperature ΔT is related to a change in pressure ΔP according to

$$\Delta P \approx (10^6\ \text{Pa/K})\Delta T$$

When in the gas state you should assume that water behaves like an ideal gas.

Of relevance to this question is the pressure/temperature phase plot for water, showing the regions where water exists in liquid form or gaseous form. The curve shows the coexistence condition, where water can exist simultaneously as gas or liquid.



The following graphs should be drawn on the answer sheet provided.

a. Sketch a PT diagram for this cycle on the answer sheet. The coexistence curve for the liquid/gas state is shown. Clearly and accurately label the locations of points **B** through **D** and **W** through **Z** on this cycle.

b. Sketch a PV diagram for this cycle on the answer sheet. You should estimate a reasonable value for V_{max}, note the scale is logarithmic. Clearly and accurately label the locations of points **B** through **D** on this cycle. Provide reasonable approximate locations for points **W** through **Z** on this cycle.