Conservation Laws and SHM F=ma Exam Practice

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This problem set will focus on solving problems in topics such as momentum, work, energy, and simple harmonic motion that have appeared on past F=ma Exams, the first qualifying test in the US Physics Olympiad. Do them to the best of your ability and collaborate with others when needed. Have fun!

1. **EASY** (2012 $F = ma$ #4) A particle at rest explodes into three particles of equal mass in the absence of external forces. Two particles emerge at a right angle to each other with equal speed v. What is the speed of the third particle?

   (A) v (B) $\sqrt{2}v$ (C) $2v$ (D) $2\sqrt{2}v$ (E) The third particle can have a range of different speeds

2. **EASY** (2012 $F = ma$ #5) A 12 kg block moving east at 4 m/s collides head on with a 6 kg block that is moving west at 2 m/s. The two blocks move together after the collision. What is the loss in kinetic energy in this collision?

   (A) 36 J (B) 48 J (C) 60 J (D) 72 J (E) 96 J

3. **EASY** (2015 $F = ma$ #6) Three trolley carts are free to move on a one dimensional frictionless horizontal track. Cart A has a mass of 1.9 kg and an initial speed of 1.7 m/s to the right; Cart B has a mass of 1.1 kg and an initial speed of 2.5 m/s to the left; cart C has a mass of 1.3 kg and is originally at rest. Collisions between carts A and B are perfectly elastic; collisions between carts B and C are perfectly inelastic. What is the velocity of the center of mass of they system after both collisions happen?

   (A) 0.11 m/s (B) 0.16 m/s (C) 1.4 m/s (D) 2.0 m/s (E) 3.23 m/s

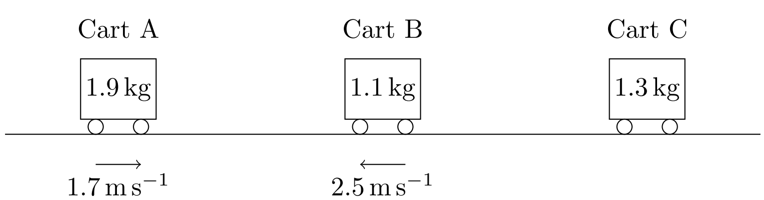

Figure 1

4. **EASY** (2015 $F = ma$ #7,8) Carts A, B, and C are on a long horizontal frictionless track. The masses of the carts are m, $3m$, and $9m$. Originally cart B is at rest at the 1.0 meter mark and cart C is at rest on the 2.0 meter mark. Cart A is originally at the zero meter mark moving toward the cart B at a speed of v_0.

   (a) Assuming all collisions are completely *inelastic*, what is the final speed of cart C?
   (A) $v_0/13$ (B) $v_0/10$ (C) $v_0/9$ (D) $v_0/3$ (E) $2v_0/5$

   (b) Assuming all collisions are *elastic*, what is the final speed of cart C?
   (A) $v_0/8$ (B) $v_0/4$ (C) $v_0/2$ (D) v_0 (E) $2v_0$

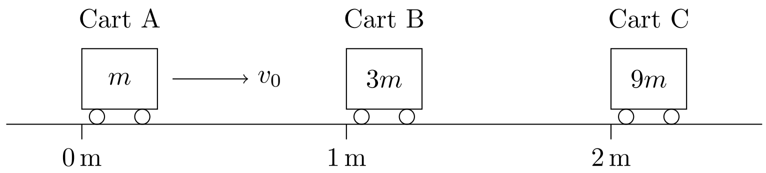

Figure 2

5. **EASY** (2012 $F = ma$ #8) A block of mass $m = 3.0\ kg$ is moving on a horizontal surface towards a massless spring with spring constant $k = 80.0\ N/m$. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.50$. The block has a speed of 2.0 m/s when it first comes in contact with the spring. How far will the spring be compressed?

   (A) 0.19 m (B) 0.24 m (C) 0.39 m (D) 0.40 m (E) 0.61 m

6. **MEDIUM** (2010 $F = ma$ # 15,16) A small block of mass m is moving on a horizontal table surface at initial speed v_0. It then moves smoothly onto a sloped big block of mass M. The big block can also move on the table surface. Assume that everything moves without friction.

   (a) After the small block reaches the height h on the slope, it slides down. Find the height h.
   (A) $h = \frac{v_0^2}{2g}$ (B) $h = \frac{1}{g}\frac{Mv_0^2}{m+M}$ (C) $h = \frac{1}{2g}\frac{Mv_0^2}{m+M}$ (D) $h = \frac{1}{2g}\frac{mv_0^2}{m+M}$ (E) $h = \frac{v_0^2}{g}$

   (b) Following the previous set up, find the speed v of the small block after it leaves the slope.
   (A) $v = v_0$ (B) $v = \frac{m}{m+M}v_0$ (C) $v = \frac{M}{m+M}v_0$ (D) $v = \frac{M-m}{m}v_0$ (E) $v = \frac{M-m}{m+M}v_0$

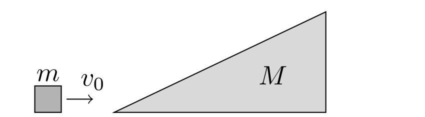

Figure 3

7. **MEDIUM** (2013 $F = ma$ # 19,20) A simple pendulum experiment is constructed from a point mass m attached to a pivot by a massless rod of length L in a constant gravitational field. The rod is released from an angle $\theta_0 < \pi/2$ at rest and the period of motion is found to be T_0. Ignore air resistance and friction.

   (a) At what angle θ_g during the swing is the tension in the rod the greatest?
   (A) $\theta_g = \theta_0$ (B) $\theta_g = 0$ (C) $0 < \theta_g < \theta_0$ (D) The tension is constant (E) None of the above

   (b) Following the previous set up, find the speed v of the small block after it leaves the slope.
   (A) mg (B) $2mg$ (C) $mL\theta_0/T_0^2$ (D) $mgsin\theta_0$ (E) $mg(3 - 2cos\theta_0)$

8. **MEDIUM** (2008 $F = ma$ # 19) A car has an engine which delivers a constant power. It accelerates from rest at time $t = 0$, and at $t = t_0$ its acceleration is a_0. What is its acceleration at $t = 2t_0$? Ignore energy loss due to friction.

   (A) $\frac{1}{2}a_0$ (B) $\frac{1}{\sqrt{2}}a_0$ (C) a_0 (D) $\sqrt{2}a_0$ (E) $2a_0$

9. **HARD** (2014 $F = ma$ # 6) A cubical box of mass 10 kg with edge length 5 m is free to move on a frictionless horizontal surface. Inside is a small block of mass 2 kg, which moves without friction inside the box. At time $t = 0$, the block is moving with velocity 5 m/s directly towards one of the faces of the box, while the box is initially at rest. The coefficient of restitution for any collision between the block and box is 90%, meaning that the relative speed between the box and block immediately after a collision is 90% of the relative speed between the box and block immediately before the collision.

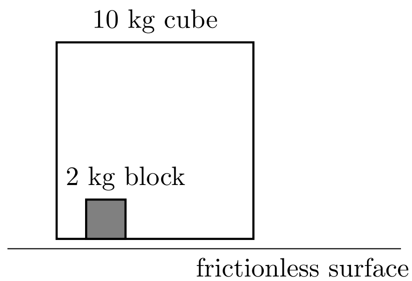

Figure 4

After 1 minute, the block is a displacement x from the original position. Which of the following is closest to x?

(A) 0 m (B) 50 m (C) 100 m (D) 200 m (E) 300 m

10. **HARD** (2012 $F = ma$ # 16) Inside a cart that is accelerating horizontally at acceleration a, there is a block of mass M connected to two light springs of force constants k_1 and k_2. The block can move without friction horizontally. Find the vibration frequency of the block.

(A) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}+a}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}+a}$ (D) $\frac{1}{2\pi}\sqrt{\frac{|k_1-k_2|}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}}$

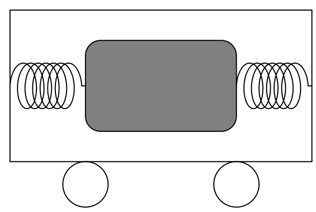

Figure 5

11. **HARD** (2012 $F = ma$ # 21) A spring system is set up as follows: a platform with a weight of 10 N is on top of two springs, each with spring constant $75N/m$. On top of the platform is a third spring with spring constant $75N/m$. If a ball with a weight of 5.0 N is then fastened to the top of the third spring and then slowly lowered, by how much does the height of the spring system change?

(A) 0.033 m (B) 0.067 m (C) 0.100 m (D) 0.133 m (E) 0.600 m

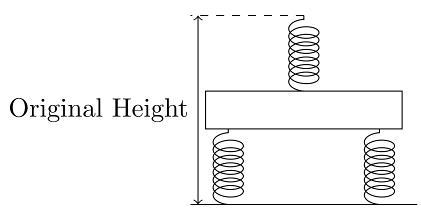

Figure 6