Work/Energy F=ma PSet Solutions

12/3/15

1. Momentum conserved. $x]\ mv = mv_x$ $\quad v_f = \sqrt{v_x^2+v_y^2} = \sqrt{v^2+v^2} = \boxed{v\sqrt{2}}$
$y]\ mv = mv_y$

2. $\boxed{12\text{ kg}} \xrightarrow{4\text{ m/s}}$ $\quad \xleftarrow{2\text{ m/s}} \boxed{6\text{ kg}}$ $\quad KE_i = \frac{1}{2}(12)(4)^2 + \frac{1}{2}(6)(2)^2 = 108\text{ J}$
momentum conserved. $\quad KE_f = \frac{1}{2}(18)(2)^2 = 36\text{ J}$
$12(4)+6(-2) = 18v_n,\ v_n = 2\text{ m/s}$

3. No external forces, V_{com} unchanged after collision.
$$V_{com} = \frac{m_1v_1+m_2v_2+m_3v_3}{m_1+m_2+m_3} = \frac{1.9(1.7)+1.1(-2.5)+1.3(0)}{1.9+1.1+1.3} = \boxed{0.11\text{ m/s}}$$

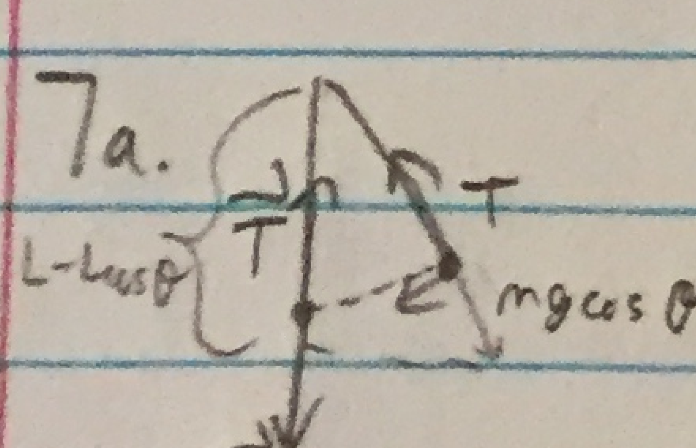
4a. Momentum conserved.
$$mv_0 = (13m)v_c,\ v_c = \boxed{\frac{v_0}{13}}$$

4b. $$u_{Bx} = \underbrace{\frac{2m_A}{m_A+m_B}v_{Ax}}_{} + \frac{m_B-m_A}{m_A+m_B}\cancelto{0}{v_{Bx}}$$
$$u_{Bx} = \frac{2m}{m+3m}v_0 = v_0/2 \qquad u_{Bx} = \frac{2(3m)}{3m+9m}\left(\frac{v_0}{2}\right) = \boxed{\frac{v_0}{4}}$$

5. $E_{mec,i} = E_{mec,f} + W_{friction}$
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu_k mgx$$
$$40x^2+15x-6=0,\ x = \boxed{0.24\text{ m}}$$

6a. Momentum conserved. $mv_0 = (m+M)v_n,\ v_n = \dfrac{mv_0}{m+M}$
$$K_i + \cancelto{0}{U_i} = K_f + U_f$$
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(M+m)\left(\frac{mv_0}{m+M}\right)^2 + mgh. \text{ Solve for } h:\ h = \boxed{\frac{Mv_0^2}{2(m+M)g}}$$

6b. $$u_{Ax} = \frac{m_A-m_B}{m_A+m_B}v_{Ax} + \frac{2m_B}{m_A+m_B}\cancelto{0}{v_{Bx}}$$ (Elastic b/c p & KE are conserved)
M initially at rest.
$$u_A = -\boxed{\frac{m-M}{m+M}v_0} = \boxed{\frac{M-m}{m+M}v_0}$$

7a.


7b. $$K_i + U_i = \cancelto{0}{K_f} + \cancelto{0}{U_f}$$
$$mgL(1-\cos\theta) = \frac{1}{2}mv^2$$
$$v^2 = 2gL(1-\cos\theta)$$

$$T - mg = m\frac{v^2}{r},\ T = m\left(g+\frac{v^2}{L}\right)$$
$$= m(g + 2g(1-\cos\theta))$$
$$= mg(1+2(1-\cos\theta))$$
$$= \boxed{mg(3-2\cos\theta)}$$

8. 5m w/ 5 m/s ≈ 1 collision per second.
$0.9^{60} \approx 0.0017$. Box 3 block will eventually be same speed.
Momentum Conserved.

$2(5) = 12 v_n$, $v_n \approx 0.8$ m/s.

$60s\,(0.8\ m/s) \approx 48m \approx \boxed{50m}$

9. $m\vec{a} \leftrightarrow (k_1+k_2)x$

$(k_1+k_2)x = ma$

$T = 2\pi\sqrt{\frac{M}{k_1+k_2}}$. $f = \frac{1}{T} = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}}}$

10. Spring circuits: series: $\frac{1}{k_n} = \frac{1}{k_1} + \frac{1}{k_2}$
parallel: $k_n = k_1 + k_2$

$\frac{1}{k_n} = \frac{1}{k_1} + \frac{1}{k_2+k_3}$, $k_n = \frac{1}{\frac{1}{75} + \frac{1}{75+75}} = 50\ N/m$

$\uparrow k_n x$

$\downarrow mg$

$k_n x = mg$

$x = \frac{mg}{k_n} = \frac{5}{50} = \boxed{0.1\ m}$

11. $KE_i = 0$, so $P = \frac{dW}{dt} = \frac{d(KE)}{dt}$ is constant.

$t=0$ $\quad t = t_0$ $\quad t = 2t_0$

$KE_i = 0$ $\quad KE = \frac{1}{2}mv_0^2$ $\quad KE = mv_0^2 = \frac{1}{2}mv_1^2$

$v_0 = \sqrt{\frac{v_1^2}{2}} = \frac{v_1}{\sqrt{2}}$

$\cancel{m}a_0 v_0 = \cancel{m}a_1 v_1$

$a_1 = \frac{a_0 v_0}{v_1} = \frac{a_0\left(\frac{v_1}{\sqrt{2}}\right)}{v_1} = \boxed{\frac{a_0}{\sqrt{2}}}$

All solutions were written by Ray Liu.
If you have any questions, please feel free to email him at 2016rliu@tjhsst.edu