USAPHO Relativity

February 2016

1. (2014 A3) When studying problems in special relativity it is often the invariant distance Δs between two events that is most important, where Δs is defined by

$$(\Delta s)^2 = (c\Delta t)^2 - [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2].$$

where $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$ is the speed of light.

   (a) Consider the motion of a projectile launched with initial speed v_0 at angle of θ_0 above the horizontal. Assume that g, the acceleration of free fall, is constant for the motion of the projectile.

      i. Derive an expression for the invariant distance of the projectile as a function of time t as measured from the launch, assuming that it is launched at $t = 0$. Express your answer as a function of any or all of θ_0, v_0, c, g, and t.

      ii. The radius of curvature of an object's trajectory can be estimated by assuming that the trajectory is part of a circle, determining the distance between the end points, and measuring the maximum height above the straight line that connects the endpoints. Assuming that we mean "invariant distance" as defined above, find the radius of curvature of the projectile's trajectory as a function of any or all of θ_0, v_0, c, and g. Assume that the projectile lands at the same level from which it was launched, and assume that the motion is not relativistic, so $v_0 \ll c$, and you can neglect terms with v/c compared to terms without.

   (b) A rocket ship far from any gravitational mass is accelerating in the positive x direction at a constant rate g, as measured by someone inside the ship. Spaceman Fred at the right end of the rocket aims a laser pointer toward an alien at the left end of the rocket. The two are separated by a distance d such that $dg \ll c^2$; you can safely ignore terms of the form $(dg/c^2)^2$.

      i. Sketch a graph of the motion of both Fred and the alien on the space-time diagram provided in the answer sheet. The graph is not meant to be drawn to scale. Note that t and x are reversed from a traditional graph. Assume that the rocket has velocity $v = 0$ at time $t = 0$ and is located at position $x = 0$. Clearly indicate any asymptotes, and the slopes of these asymptotes.

      ii. If the frequency of the laser pointer as measured by Fred is f_1, determine the frequency of the laser pointer as observed by the alien. It is reasonable to assume that $f_1 \gg c/d$.

2. (2013 A3) A beam of muons is maintained in a circular orbit by a uniform magnetic field. Neglect energy loss due to electromagnetic radiation. The mass of the muon is 1.88×10^{-28} kg, its charge is -1.602×10^{-19} C, and its halflife is $1.523\,\mu$s.

   (a) The speed of the muons is much less than the speed of light. It is found that half of the muons decay during each full orbit. What is the magnitude of the magnetic field?

   (b) The experiment is repeated with the same magnetic field, but the speed of the muons is increased; it is no longer much less than the speed of light. Does the fraction of muons which decay during each full orbit increase, decrease, or stay the same?

3. (2012 A1) A newly discovered subatomic particle, the *S meson*, has a mass M. When at rest, it lives for exactly $\tau = 3 \times 10^{-8}\,\mathrm{s}$ before decaying into two identical particles called *P mesons* (peons?) that each have a mass of αM.

   (a) In a reference frame where the S meson is at rest, determine

      i. the kinetic energy,

      ii. the momemntum, and

      iii. the velocity

   of each P meson particle in terms of M, α, the speed of light c, and any numerical constants.

   (b) In a reference frame where the S meson travels 9 meters between creation and decay, determine

      i. the velocity and

      ii. the kinetic energy of the S meson.

   Write the answers in terms of M the speed of light c, and any numerical constants.