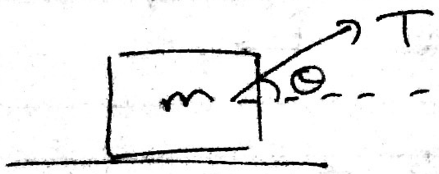
10/17/14

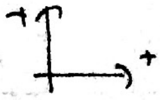
AP Forces

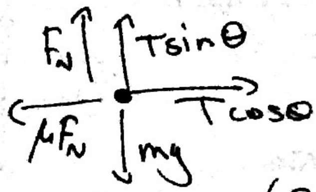
Problem Set 2: Solutions

Arun Kannan

1.







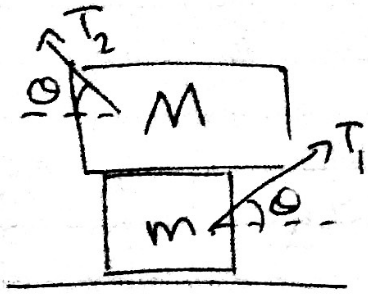
y: $F_N + T\sin\theta - mg = 0.$ (Block isn't moving)

$F_N = mg - T\sin\theta$

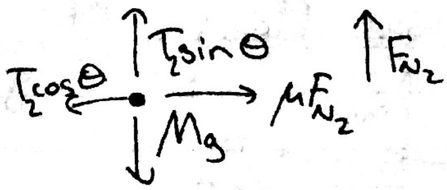
x: $T\cos\theta - \mu F_N = 0$

$T\cos\theta - \mu[mg - T\sin\theta] = 0 \Rightarrow \boxed{T = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}}$

2.



Top Block:



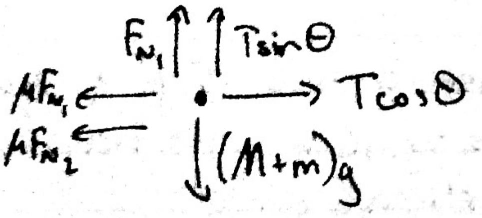
y: $F_{N_2} + T_2\sin\theta - Mg = 0.$

$F_{N_2} = Mg - T_2\sin\theta$ (1)

x: $\mu F_{N_2} - T_2\cos\theta = 0.$ (2)

$\mu F_{N_2} = T_2\cos\theta$

Bottom Block:



y: $F_{N_1} + T_1\sin\theta - (M+m)g = 0$

$F_{N_1} = (M+m)g - T_1\sin\theta$ (3)

x: $T_1\cos\theta - \mu F_{N_1} - \mu F_{N_2} = 0$

$T_1\cos\theta = \mu F_{N_1} + \mu F_{N_2}$ (4)

Now we solve for tension's ratio:

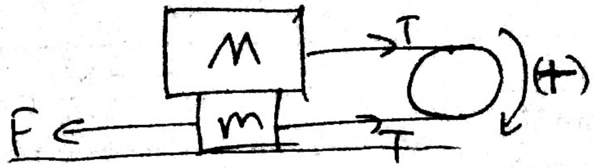
From (4) and (2) and (3):

$T_1\cos\theta = \mu[(M+m)g - T_1\sin\theta] + T_2\cos\theta$

$T_1(\cos\theta + \mu\sin\theta) = \mu(M+m)g + T_2\cos\theta$

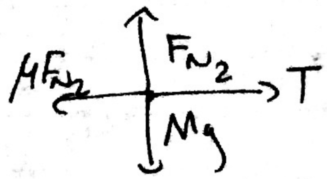
$$\boxed{T_1 = \frac{T_2\cos\theta}{\cos\theta + \mu\sin\theta} + \frac{\mu(M+m)g}{\cos\theta + \mu\sin\theta}}$$

3.



(we treat the clockwise motion of the string of the pulley as positive so that both blocks have the same acceleration, even though in opposite directions).

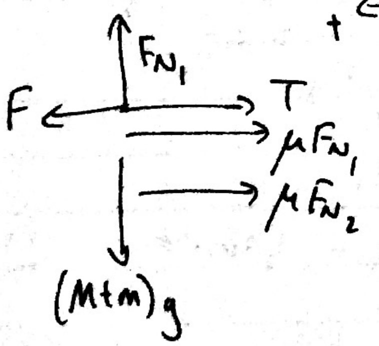
Top Mass



$$y: F_{N_2} - Mg = 0 \Rightarrow F_{N_2} = Mg$$

$$x: T - \mu F_{N_2} = Ma$$
$$T - \mu Mg = Ma$$

Bottom Mass



$$y: F_{N_1} - (M+m)g = 0$$
$$F_{N_1} = (M+m)g$$

$$x: F - T - \mu F_{N_1} - \mu F_{N_2} = ma$$

$$F - T - \mu(M+m)g - \mu Mg = ma \quad a = \frac{T - \mu Mg}{M}$$

$$F - T - \mu g(2M+m) = \frac{m}{M}(T - \mu Mg)$$

Solving for T, we have:

$$\boxed{T = \frac{F - 2\mu Mg}{1 + \frac{m}{M}}}$$

(Observe that if $F \leq 2\mu Mg$, the system will have no acceleration)

Plugging back in, we have a:

$$\frac{F - 2\mu Mg}{1 + \frac{m}{M}} - \mu Mg = Ma \Longrightarrow \boxed{a = \frac{F - \mu g(3M+m)}{M+m}}$$