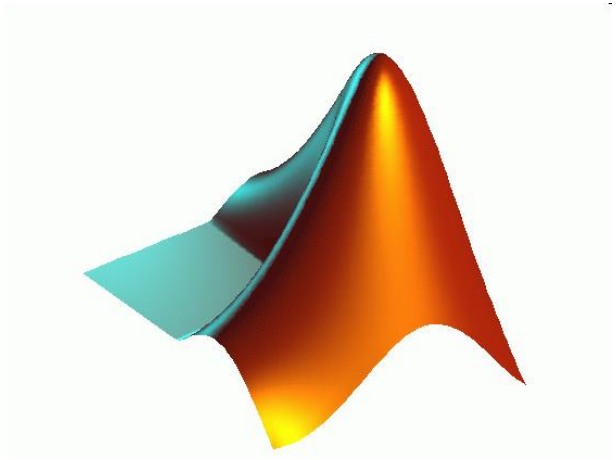
ME 203
Introduction to MATLAB

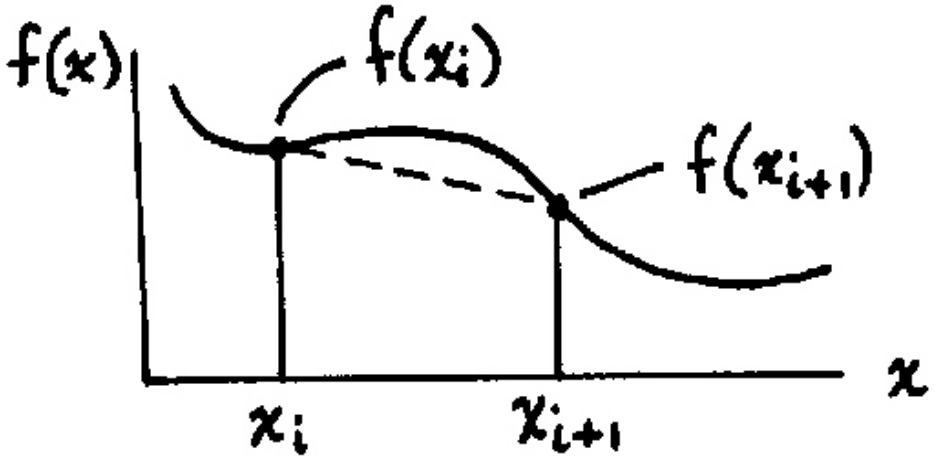


Integration

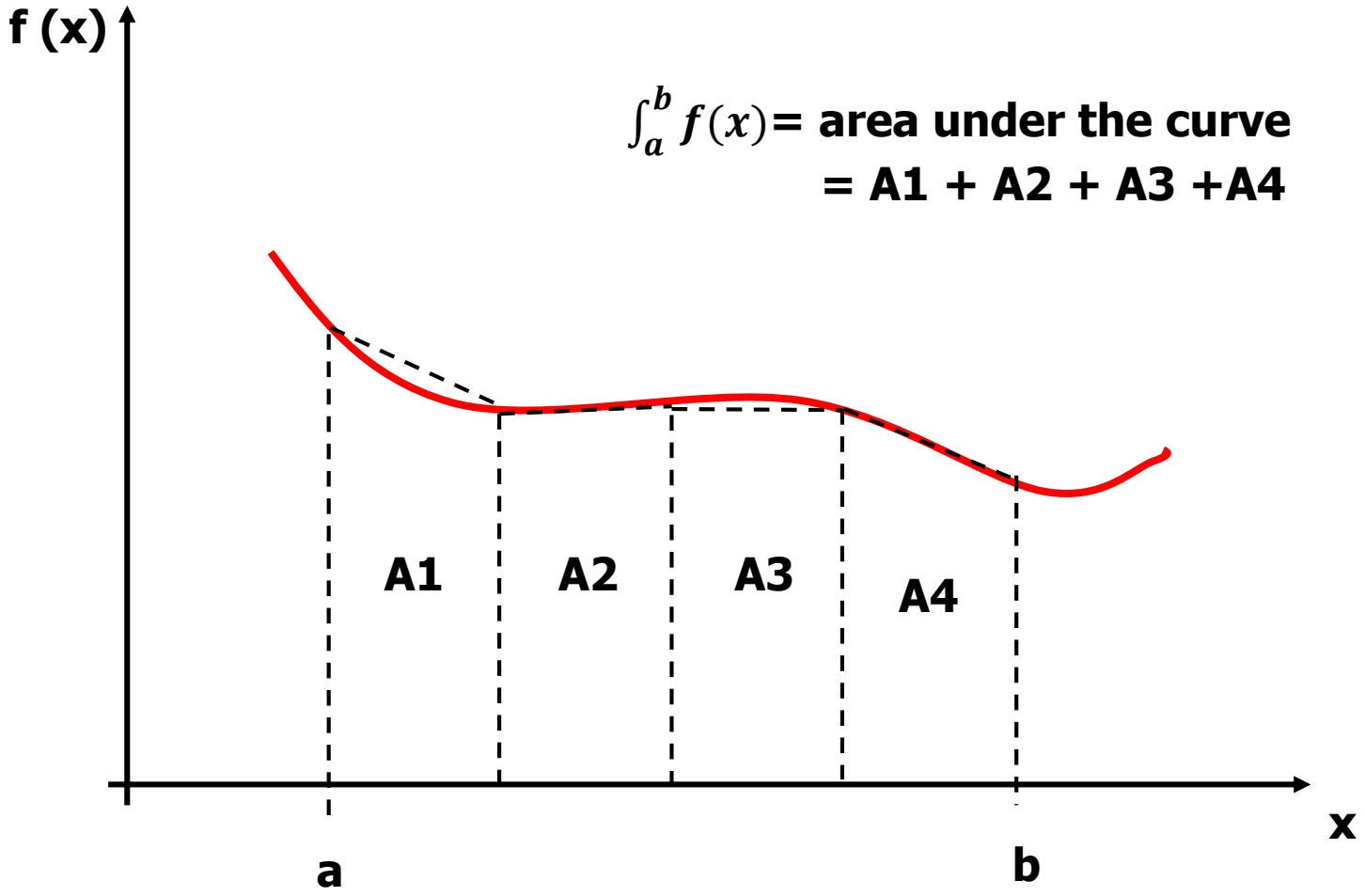
There are many integration methods in MATLAB. We are only going to look at a few of them. The rest of the functions in Matlab work similarly.

Single Integrals

Trapezoidal Rule – calculates the area under the curve by dividing the range into a series of points and connecting each point using a straight line



Trapezoidal Rule



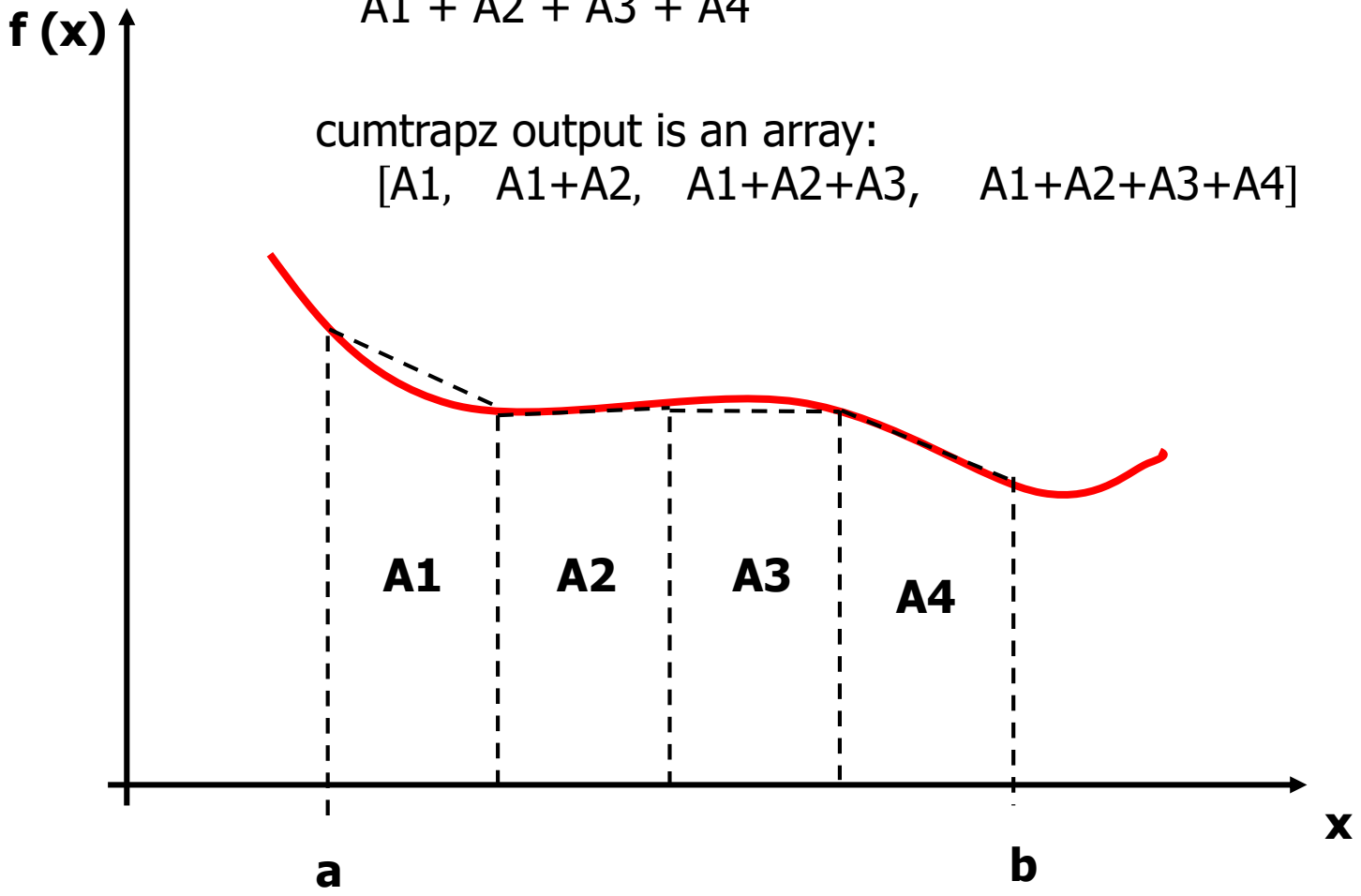
Trapezoidal Numerical Integration

z = trapz(x,y)	Integral of y with respect to x by the trapezoidal rule. x and y must be vectors of the same length, or x must be a column vector and y an array whose first non-singleton dimension is length(x). trapz operates along this dimension.
z = cumtrapz(x,y)	Computes the cumulative integral of y with respect to x using trapezoidal integration. x and y must be vectors of the same length, or x must be a column vector and y an array whose first non-singleton dimension is length(x). cumtrapz operates across this dimension.

Trapezoidal Rule

trapz output is a scalar:
A1 + A2 + A3 + A4

cumtrapz output is an array:
[A1, A1+A2, A1+A2+A3, A1+A2+A3+A4]



Example 1

Calculate the integral of sin(x) in the interval $[0,\pi]$ (exact solution is 2)

```
% Integral example 1
clc; clear all; format compact

%x range
x = linspace(0,pi,100);

%function
y=sin(x);

%calculate integral
z=trapz(x,y)
```

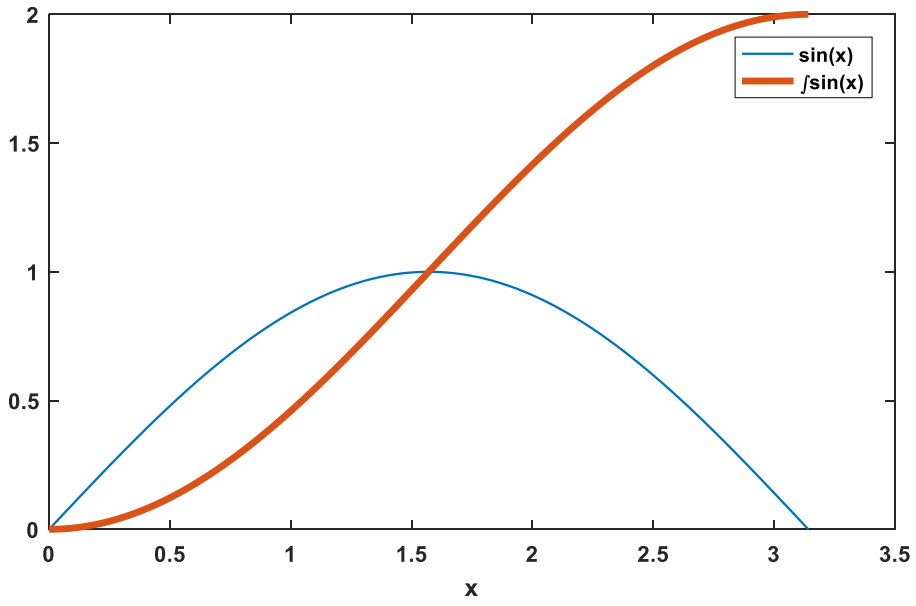
Result

```
z =
   1.9998
```

Example 2

Calculate the integral of sin(x) in the interval [0,π]. Plot the function and its integral over the range of x.

```
% Integral example 2
clc; clear all; format compact

%x range
x = linspace(0,pi,100);

%function
y=sin(x);

%calculate integral
z=cumtrapz(x,y);

%plot results
plot(x,y,x,z)
xlabel('x')
legend('sin(x)','\intsin(x)')
```



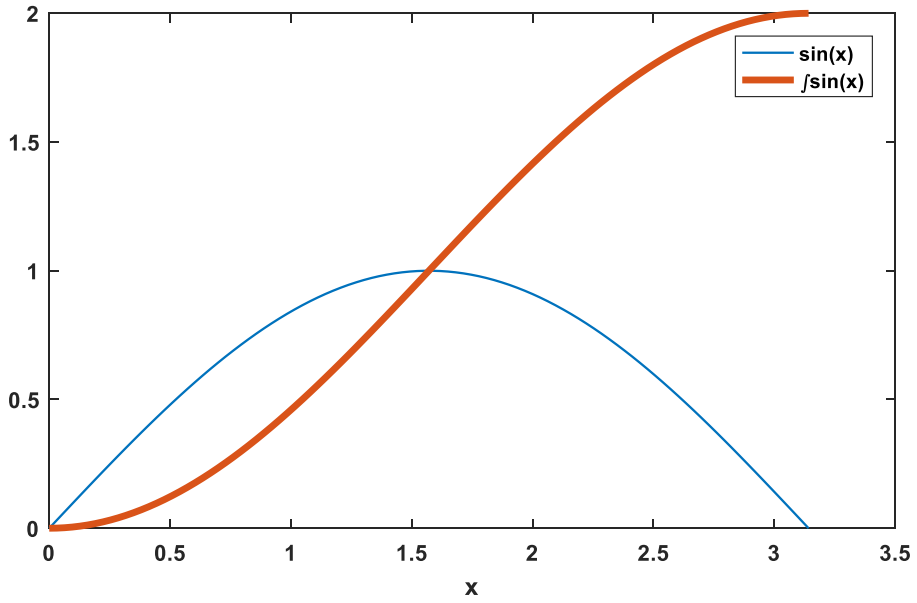
Example 3

Calculate the integral of sin(x) in the interval [0,π]. Plot the function and its integral over the range of x. **You cannot use cumtrapz.**

```
% Integral example 3
clc; clear all; format compact

%x range
x = linspace(0,pi,100);

%function
y=sin(x);
```

Example 3 (continued)

```
z=zeros(1,100);

%calculate integral
for i=2:100
    z(i) = trapz(x(1:i),y(1:i));
end

%plot results
plot(x,y,x,z)
xlabel('x')
legend('sin(x)','\intsin(x)')
```



Single Integrals – Alternative method

q = integral (function,xmin,xmax)

Example: Calculate the integral of [sin(x) + cos(x)] in the interval [0,π] (exact solution = 2)

```
% Integral example 1
clc; clear all; format compact

%given
xmin = 0; xmax = pi;

%function handle
F = @(x) sin(x)+cos(x);

%calculate integral
q = integral (F, xmin, xmax)
```

Multiple Integrals

$$\int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} f(x,y)\,dx\,dy$$

Matlab Function for Double Integrals

q = integral2 (function, xmin, xmax, ymin, ymax)

$$\int_{z_{\min}}^{z_{\max}} \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} f(x,y)\,dx\,dy\,dz$$

Matlab Function for Triple Integrals

q = integral3(function, xmin, xmax, ymin, ymax, zmin, zmax)

Double Integral Example 1

Calculate the following integral: $\int_2^4 \int_1^2 6xy^2 \, dydx$

```
% Double integral example 1
clc; clear all; format compact

%given
xmin = 2; xmax = 4;
ymin = 1; ymax = 2;

%function handle
F = @(x,y) (6*x.*y.^2);

%compute integral
q = integral2 (F, xmin, xmax, ymin, ymax)
```

Double Integral Example 2

Calculate the following integral: $\int_0^1 \int_0^{1-x} \frac{1}{\sqrt{x+y}(1+x+y)}\, dydx$

```
% Double integral example 2
clc; clear all; format compact

%given
xmin = 0; xmax = 1;
ymin = 0;

%function handles
F = @(x,y) 1./(sqrt(x+y).*(1+x+y));
ymax = @(x) (1-x);

%compute integral
q = integral2 (F, xmin, xmax, ymin, ymax)
```

Result

```
q =
   0.4292
```

Triple Integral Example 1

Calculate the following integral: $\int_0^1 \int_0^{\pi} \int_{\pi}^{2\pi} [z\, y\, sin(x) + x\, cos(y)]\, dxdydz$

```
% Triple integral example 1
clc; clear all; format compact

%given
xmin = pi; xmax = 2*pi;
ymin = 0; ymax = pi;
zmin = 0; zmax = 1;

%function handle
F = @(x,y,z) z.*y.*sin(x)+x.*cos(y);

%compute integral
q = integral3 (F, xmin, xmax, ymin, ymax, zmin, zmax)
```

Result

```
q =
   -4.9348
```

Triple Integral Example 1

Calculate the following integral: $\int_0^{x+y} \int_0^{\pi} \int_{\pi}^{2\pi} [z\ (\ sin(x) + x\ cos(y))]\ dxdydz$

```
% Triple integral example 1
clc; clear all; format compact

%given
xmin = pi; xmax = 2*pi;
ymin = 0; ymax = pi;
zmin = 0;

%function handle
F = @(x,y,z) z.*y.*sin(x)+x.*cos(y);
zmax = @(x,y) x+y;

%compute integral
q = integral3 (F, xmin, xmax, ymin, ymax, zmin, zmax)
```

Result

```
q =
  -319.2827
```