AEP 4500 PSET 10

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(1) Question 1:

(a) Recall that the density of states per volume for 3D electron gas is

$$D_{\text{3DGas}}(E \geq 0) = \frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E}$$

Which represents the conduction band with the exception of that the bottom of the band is E_c and have effective mass m_e. This implies DOS is

$$D_c(E \geq E_c) = \frac{(2m_e)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E - E_c}$$

Which implies at fixed μ,

$$n(T) = \int_{E_c}^{\infty} dE D_c(E) n_F(\beta(E-\mu)) = \int_{E_c}^{\infty} dE \frac{D_c(E)}{e^{\beta(E-\mu)}+1}$$

Now since $\beta(E-\mu) \gg 1$ implies

$$\frac{1}{e^{\beta(E-\mu)}+1} \approx e^{-\beta(E-\mu)} \implies$$

$$n(T) \approx \int_{E_c}^{\infty} dE D_c(E) e^{-\beta(E-\mu)} = \frac{(2m_e)^{\frac{3}{2}}}{2\pi^2\hbar^3}\int_{E_c}^{\infty} dE \sqrt{E-E_c} e^{-\beta(E-\mu)}$$

Evaluating this integral given $x = \sqrt{E - E_c}$ we get

$$\int_0^{\infty} dx \sqrt{x} e^{-\beta x} = -\frac{\partial}{\partial b}\sqrt{\pi}\beta = \frac{1}{2}\beta^{\frac{-3}{2}}\sqrt{\pi} \implies$$

$$n(T) = \frac{1}{4}(\frac{2m_e k_B T}{\pi\hbar^2})^{\frac{3}{2}} e^{-\beta(E_c-\mu)} = 2(\frac{m_e k_B T}{2\pi\hbar^2})^{\frac{3}{2}} e^{-\beta(E_c-\mu)}$$

(b) Similarly to the other case

$$D_c(E \leq E_v) = \frac{(2m_h)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E_v - E}$$

Which implies at fixed μ,

$$n(T) \int_0^{\infty} dE D_c(E) n_F(\beta())$$

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