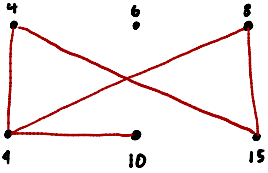
1. The greatest common divisor (GCD) of two positive integers is the largest positive integer that divides exactly into both. For example, $GCD(5, 20) = 5$, $GCD(6, 15) = 3$, and $GCD(7, 25) = 1$.

Austin Barrett

(a) **Construct** the graph with vertices $V = \{4, 6, 8, 9, 10, 15\}$ such that there is an edge between vertices v_1 and v_2 iff $GCD(v_1, v_2) = 1$. On your graph, **identify** any vertices that are isolated or pendant (if there are any).



isolated vertices
6
pendant vertices
10

(b) Give the **degree** of each vertex.

deg(4) = 2 deg(6) = 0 deg(8) = 2

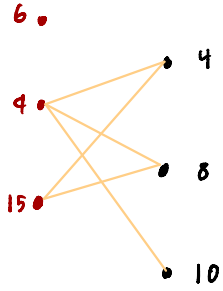
deg(9) = 3 deg(10) = 1 deg(15) = 2

(c) **Determine** if the graph is bipartite, and if it is complete bipartite. Justify your answers.

With our vertex's lets make two partitions red and black

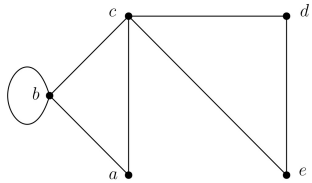
partition red : {6, 9, 15}
partition black : {4, 8, 10}

Making our graph from these seperated partitions we get



∴ All the vertices in this graph connect to vertices from different partitions but not to every vertice from the opposite partition. making it only bipartite but not complete.

2. Consider the graph



(a) Give the **adjacency list** of this graph.

a: [b, c]
b: [b, c, a]
c: [b, d, e, a]
d: [c, e]
e: [c, d]

(b) Give the **adjacency matrix** of the graph using alphabetical order for the vertices.

	a	b	c	d	e
a	0	1	1	0	0
b	1	1	1	0	0
c	1	1	0	1	1
d	0	0	1	0	1
e	0	0	1	1	0

(c) In general, how can you determine the number of edges in a pseudograph from its adjacency matrix? Explain your answer.

To do This we must avoid counting the non-zero entrys on the central diagnol downward from 0 more then once. once we count those (represent as n) we count all other non zero entrys (represent as m). Then to get the total # of edges we just do the following equation.

$\left(\frac{m}{2}\right) + n$ = number of edges

example

	a	b	c	d	e
a	0	1	1	0	0
b	1	1	1	0	0
c	1	1	0	1	1
d	0	0	1	0	1
e	0	0	1	1	0

$n = 1$

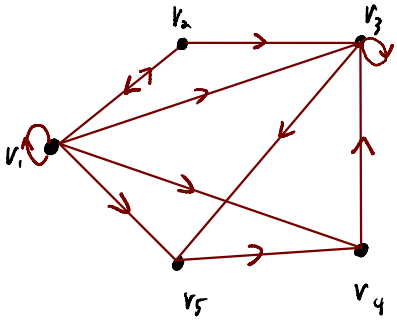
$m = 12$

$\left(\frac{12}{2}\right) + 1 = 7$

$\therefore$ There are 7 edges

3.(a) Assuming that you label/order your vertices as v_1, v_2, etc., draw a **directed graph** having adjacency matrix

$$A = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}$$

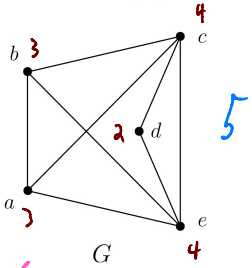


(b) Is it possible to travel along arcs (possibly more than one, laid end to end) of the above digraph from any vertex to any other vertex? Explain your answer.

IT is not! For example if you start at v_4 and wished to travel to v_1 you could not. this is because the only vertexes a route from v_4 connects to are v_3 and v_5.

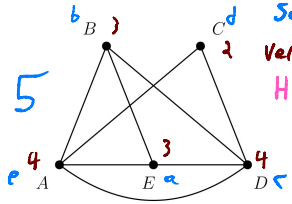
wm = degree

4. Determine if the two graphs below are isomorphic. **Justify your answer.**



G

5



H

5

Same # of verticies ✓

Verticies are same degrees ✓

Have the same degree structure (cycle structure) ✓

G

Checking the degree of the vertices with $D = d_0, d_1, d_2 \dots d_i$ where d = # of verticies of Degree i

$G = 0,0,1,2,2$

or

$H = 0,0,1,2,2$

$G = \{\overset{a}{3}, \overset{b}{3}, \overset{c}{4}, \overset{d}{2}, \overset{e}{4}\}$ $H = \{\overset{A}{4}, \overset{B}{3}, \overset{C}{2}, \overset{D}{4}, \overset{E}{3}\}$

Since G and H have the same # of vertices and same # of vertices of the same degree, we can try matching up the vertices like...

$f(a) = E$ $f(b) = B$

$f(c) = D$ $f(d) = C$

$f(e) = A$

because the degrees and verticies match up, we can test this with adjacency matrix

	a	b	c	d	e
a	0	1	1	0	1
b	1	0	1	0	1
c	1	1	0	1	1
d	0	0	1	0	1
e	1	1	1	1	0

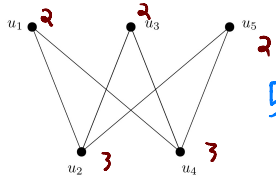
	E	B	D	C	A
E	0	1	1	0	1
B	1	0	1	0	1
D	1	1	0	1	1
C	0	0	1	0	1
A	1	1	1	1	0

Match up!

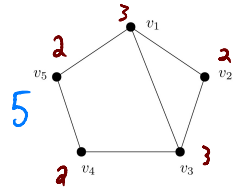
$\therefore$ is isomorphic!

$\# = \text{degree}$

5. Determine if the two graphs below are isomorphic. **Justify your answer.**



G



H

5

Same # or verticies ✓

Have same degree sequence structure ✓

Have same cycle structure? ✗

$D = d_0, d_1, d_2 \ldots$ where $d_i = \#$ of vertices of degree i

$G = 0, 0, 3, 2$

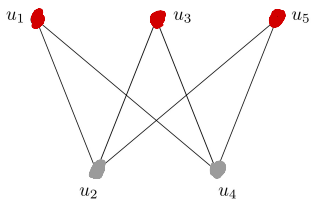
$H = 0, 0, 3, 2$

or

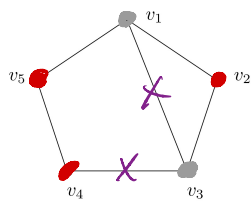
$G = \{\overset{v_1}{2}, \overset{v_2}{3}, \overset{v_3}{2}, \overset{v_4}{3}, \overset{v_5}{2}\}$

$H = \{\overset{v_1}{3}, \overset{v_2}{2}, \overset{v_3}{3}, \overset{v_4}{2}, \overset{v_5}{2}\}$

These two graphs are not isomorphic. This is because the cycle structure for both G and H are different. We can tell because structure G is bipartite where as structure H is not!



G

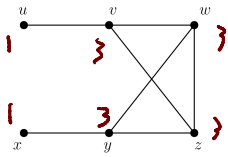


H

Indicating the structure is fundementaly different

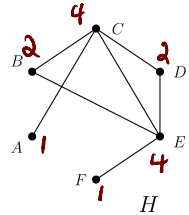
$\therefore$ Not isomorphic

6. Determine if the two graphs below are isomorphic. **Justify your answer.**



G

6 vertices



H

6 vertices

Same # of vertices? ✓

Same degree sequence structure? ✗

Since G and H do not have the same degree structure, they cannot be isomorphic

$G = \{\overset{u}{1}, \overset{v}{3}, \overset{w}{3}, \overset{x}{1}, \overset{y}{3}, \overset{z}{3}\}$ $H = \{\overset{A}{1}, \overset{B}{2}, \overset{C}{4}, \overset{D}{2}, \overset{E}{4}, \overset{F}{1}\}$ or

$D = d_0, d_1, d_2 \cdots, d_i$ where d = # of vertices at degree i

$G = 0, 2, 0, 4$

$H = 0, 2, 2, 0, 2$

$\therefore$ Not isomorphic