ECE 106 - Physics of Electrical Engineering 2

Andy Zhang

Winter 2014

Contents

Chapter 1

Electric Fields

1.1 Properties of an Electric Charge

- Two types of charge: **Positive** and **Negative**
- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Electric charge is always conserved.

1.2 Charging Objects by Induction and Conduction

1.2.1 Insulators and Conductors

- Conductors are a material where electrons not bound to atoms and are free to move through the material.
- Insulators are materials in which all electrons are bound to atoms and cannot move freely through the material.

1.2.2 Charging by Induction

Using a charged object, charges in a conductor can be repulsed out of an object through ground. The objects do not have to touch for this to happen.

1.2.3 Charging by Conduction

Rub two objects so that charges on one rub off to the other.

1.3 Coulomb's Law

To generalize the properties of the electric force, we find that the magnitude of the electric force between two point charges is given by Coulomb's Law:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

where k_e is known as Coulomb's constant and has the value:

$$k_e = 8.9876 \times 10^9 N \cdot m^2/C^2 = \frac{1}{4\pi\epsilon_0}$$

Note that the force is a vector and with multiple charges, the force is equal to the sum of the vector forces. As mentioned previously, the direction depends on the two charges.

1.4 Particle in an Electric Field

An electric field is said to exist in the region of space around a charged object, the source charge. We define the electric field vector $\vec{E}$ at a point to be:

$$\vec{E} \equiv \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

Like the force, an electric field at a certain point is the sum of all electric fields due to multiple point charges.

1.5 Electric Field of a Continuous Charge Distribution

When calculating the electric field at a point due to a continuous distribution of charge such as a surface, we can divide the charge distribution into many small charge Δq.
We then take the sum of all of these charges:

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

Popular examples would include a uniform ring of charge where many of the forces would cancel out due to symmetry and you'd calculate the formula with a $\cos\theta$ to only get 1 component. This approach would be the same for a disk of uniform charge.

1.6 Electric Field Lines

The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The denser the number of electric field lines, the stronger the field and vice versa.
For **positive charges** the field lines are directed radially outward.
For **negative charges** the field lines are directed radially inward.
The number of lines drawn leaving a charge is proportional to the **magnitude** of the charge.

1.7 Motion of a Charged Particle in a Uniform Field

Since the force is denoted as $\vec{F}_e = q\vec{E} = m\vec{a}$, then we can isolate $\vec{a}$ and determine the motion of the particle based on the acceleration. If the electric field is uniform, then we know the particle is under constant acceleration.

Chapter 2

Gauss's Law

2.1 Electric Flux

The electric flux is defined as the number of total electric field lines multiplied by the area of the surface penetrated. We generalize the electric flux as:

$$\Phi_E = EA\cos\theta$$

This formula can be generalized as

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

The direction of the area vector is chosen so that the vector points outward from the surface. If the electric field line points the same side as the area vector, then the electric flux through that area is positive, otherwise, it's negative.

2.2 Gauss's Law

The net flux through *any* closed surface surrounding a point charge q is given by q/ϵ_0 and is independent of the shape of that surface. We define the net flux as:

$$\Phi_E = \frac{q_{in}}{\epsilon_0}$$

This implies that the net electric flux is the same through all closed surfaces. If the charge was outside the surface, the net flux due to that charge would be 0.

2.3 Application of Gauss's Law to Various Charge Distribution

Example: When calculating the electric field inside an **insulating** sphere, we define the internal charge by its density times its charge. We then replace it in $E = \frac{q_{in}}{\epsilon_0}$ to obtain

$$E = k_e \frac{Q}{a^3} r \text{ for } r < a$$

Example: When calculating the electric field from an infinite line of uniform charge, we define $q_{in} = \lambda l$ where λ is the linear charge density. By then measuring the flux of a closed cylinder around the line, we define its area as $2\pi rl$ and we can then isolate the electric field as:

$$E = 2k_e \frac{\lambda}{r}$$

This would not be the case if the line was not infinitely long.
Example: When finding the electric field due to an infinite plane of charge, we define $q_{in} = \sigma A$. By then using Gauss's Law, we find that

$$E = \frac{\sigma}{2\epsilon_0}$$

Using Gauss's only works for charge distributions with sufficient symmetry.

2.4 Conductors in Electrostatic Equilibrium

When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium**. A conductor in electristatic equilibrium has the following properties:

1. $E = 0$ everywhere inside conductor

2. If conductor is isolated and carries a charge, the charge resides on the surface.
3. The electric field at a point outside the conductor is perpendicular to the surface of the conductor and has magnitude σ/ϵ_0 where σ is the surface charge density at that point.
4. On irregularly shaped conductors, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest

Chapter 3

Electric Potential

3.1 Electric Potential and Potential Difference

Like in Mechanics, the difference in potential energy is the force times the displacement or:

$$\Delta U = -q \int_B^A \vec{E} \cdot d\vec{s}$$

A charge in an electric field has a potential U. Dividing the potential energy by q gives a physical quantity defined as the **electric potential**:

$$V = \frac{U}{q}$$

We can then define the potential difference ΔV as:

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{s}$$

It's worth nothing that potential energy only exists in a system of two or more charges. If we have the potential difference and want to check the work done with a given charge, we define work as:

$$W = q\Delta V$$

We can then define electric field as this: The **electric field** is a measure of the rate of change of the electric potential with respect to position.

3.2 Potential Difference in a Uniform Electric Field

Since the electric field is uniform, the potential difference can be defined as:

$$\Delta V = -Ed\cos\theta$$

where θ is the angle between the electric field line and the direction of the displacement.
Also, all points in a plane perpendicular to a uniform electric field are at the same potential.

3.3 Electric Potential and Potential Energy due to Point Charges

The equations speak for themselves:

$$\text{Potential Difference: } V_B - V_A = k_e q[\frac{1}{r_B} - \frac{1}{r_A}]$$
$$\text{Total Energy: } U = ke(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}})$$

3.4 Obtaining the Value of the Electric Field from the Electric Potential

If we know the value of the electric potential, then assuming the electric field has only one component E_x,

$$E_x = -\frac{dV}{dx}$$

This adds onto the previously mentioned fact that the potential difference across an equipotential surface is 0.

3.5 Electric Potential Due to Continuous Charge Distributions

We can express the electric potential at a point due to a continuous charge distribution as the sum of the potential due to each point:

$$V = k_e \int \frac{dq}{r}$$

This is similar to calculating the electric. However, since electric potential is not a vector, the components do not cancel out.

3.6 Electric Potential Due to a Charged Conductor

The electric field is always going to be perpendicular to the equipotential surfaces surrounding a charged conductor. The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential. Furthermore, since the electric potential field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.