ECE 106 - Physics of Electrical Engineering 2

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Chapter 1

Electric Fields

1.1 Properties of an Electric Charge

- Two types of charge: **Positive** and **Negative**
- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Electric charge is always conserved.

1.2 Charging Objects by Induction and Conduction

1.2.1 Insulators and Conductors

- Conductors are a material where electrons not bound to atoms and are free to move through the material.
- Insulators are materials in which all electrons are bound to atoms and cannot move freely through the material.

1.2.2 Charging by Induction

Using a charged object, charges in a conductor can be repulsed out of an object through ground. The objects do not have to touch for this to happen.

1.2.3 Charging by Conduction

Rub two objects so that charges on one rub off to the other.

1.3 Coulomb's Law

To generalize the properties of the electric force, we find that the magnitude of the electric force between two point charges is given by Coulomb's Law:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

where k_e is known as Coulomb's constant and has the value:

$$k_e = 8.9876 \times 10^9 N \cdot m^2/C^2 = \frac{1}{4\pi\epsilon_0}$$

Note that the force is a vector and with multiple charges, the force is equal to the sum of the vector forces. As mentioned previously, the direction depends on the two charges.

1.4 Particle in an Electric Field

An electric field is said to exist in the region of space around a charged object, the source charge. We define the electric field vector $\vec{E}$ at a point to be:

$$\vec{E} \equiv \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

Like the force, an electric field at a certain point is the sum of all electric fields due to multiple point charges.

1.5 Electric Field of a Continuous Charge Distribution

When calculating the electric field at a point due to a continuous distribution of charge such as a surface, we can divide the charge distribution into many small charge Δq.
We then take the sum of all of these charges:

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

Popular examples would include a uniform ring of charge where many of the forces would cancel out due to symmetry and you'd calculate the formula with a $\cos\theta$ to only get 1 component. This approach would be the same for a disk of uniform charge.

1.6 Electric Field Lines

The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The denser the number of electric field lines, the stronger the field and vice versa.
For **positive charges** the field lines are directed radially outward.
For **negative charges** the field lines are directed radially inward.
The number of lines drawn leaving a charge is proportional to the **magnitude** of the charge.

1.7 Motion of a Charged Particle in a Uniform Field

Since the force is denoted as $\vec{F}_e = q\vec{E} = m\vec{a}$, then we can isolate $\vec{a}$ and determine the motion of the particle based on the acceleration. If the electric field is uniform, then we know the particle is under constant acceleration.

Chapter 2

Gauss's Law

2.1 Electric Flux

The electric flux is defined as the number of total electric field lines multiplied by the area of the surface penetrated. We generalize the electric flux as:

$$\Phi_E = EA\cos\theta$$

This formula can be generalized as

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

The direction of the area vector is chosen so that the vector points outward from the surface. If the electric field line points the same side as the area vector, then the electric flux through that area is positive, otherwise, it's negative.

2.2 Gauss's Law

The net flux through *any* closed surface surrounding a point charge q is given by q/ϵ_0 and is independent of the shape of that surface. We define the net flux as:

$$\Phi_E = \frac{q_{in}}{\epsilon_0}$$

This implies that the net electric flux is the same through all closed surfaces. If the charge was outside the surface, the net flux due to that charge would be 0.

2.3 Application of Gauss's Law to Various Charge Distribution

Example: When calculating the electric field inside an **insulating** sphere, we define the internal charge by its density times its charge. We then replace it in $E = \frac{q_{in}}{\epsilon_0}$ to obtain

$$E = k_e \frac{Q}{a^3} r \text{ for } r < a$$

Example: When calculating the electric field from an infinite line of uniform charge, we define $q_{in} = \lambda l$ where λ is the linear charge density. By then measuring the flux of a closed cylinder around the line, we define its area as $2\pi rl$ and we can then isolate the electric field as:

$$E = 2k_e \frac{\lambda}{r}$$

This would not be the case if the line was not infinitely long.

Example: When finding the electric field due to an infinite plane of charge, we define $q_{in} = \sigma A$. By then using Gauss's Law, we find that

$$E = \frac{\sigma}{2\epsilon_0}$$

Using Gauss's only works for charge distributions with sufficient symmetry.

2.4 Conductors in Electrostatic Equilibrium

When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium**. A conductor in electristatic equilibrium has the following properties:

1. $E = 0$ everywhere inside conductor

2. If conductor is isolated and carries a charge, the charge resides on the surface.
3. The electric field at a point outside the conductor is perpendicular to the surface of the conductor and has magnitude σ/ϵ_0 where σ is the surface charge density at that point.
4. On irregularly shaped conductors, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest

Chapter 3

Electric Potential

3.1 Electric Potential and Potential Difference

Like in Mechanics, the difference in potential energy is the force times the displacement or:

$$\Delta U = -q\int_B^A \vec{E}\cdot d\vec{s}$$

A charge in an electric field has a potential U. Dividing the potential energy by q gives a physical quantity defined as the **electric potential**:

$$V = \frac{U}{q}$$

We can then define the potential difference ΔV as:

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \vec{E}\cdot d\vec{s}$$

It's worth nothing that potential energy only exists in a system of two or more charges. If we have the potential difference and want to check the work done with a given charge, we define work as:

$$W = q\Delta V$$

We can then define electric field as this: The **electric field** is a measure of the rate of change of the electric potential with respect to position.

3.2 Potential Difference in a Uniform Electric Field

Since the electric field is uniform, the potential difference can be defined as:

$$\Delta V = -Ed\cos\theta$$

where θ is the angle between the electric field line and the direction of the displacement.
Also, all points in a plane perpendicular to a uniform electric field are at the same potential.

3.3 Electric Potential and Potential Energy due to Point Charges

The equations speak for themselves:

$$\text{Potential Difference: } V_B - V_A = k_e q[\frac{1}{r_B} - \frac{1}{r_A}]$$
$$\text{Total Energy: } U = ke(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}})$$

3.4 Obtaining the Value of the Electric Field from the Electric Potential

If we know the value of the electric potential, then assuming the electric field has only one component E_x,

$$E_x = -\frac{dV}{dx}$$

This adds onto the previously mentioned fact that the potential difference across an equipotential surface is 0.

3.5 Electric Potential Due to Continuous Charge Distributions

We can express the electric potential at a point due to a continuous charge distribution as the sum of the potential due to each point:

$$V = k_e \int \frac{dq}{r}$$

This is similar to calculating the electric. However, since electric potential is not a vector, the components do not cancel out.

3.6 Electric Potential Due to a Charged Conductor

3.6.1 Electric Potential due to a Charged Conductor

The electric field is always going to be perpendicular to the equipotential surfaces surrounding a charged conductor. The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential. Furthermore, since the electric potential field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

3.6.2 A Cavity within a Conductor

Due to the equipotential property of a conductor, a cavity surrounded by a conductor is a field free region as long as there are no charges inside the cavity such as a insulating charged object.

3.7 Applications of Electrostatics

Wtf is a Van de Graaff Generator?? When a charged conductor touches the inside of a hollow conductor, all of the charge goes to the hollow conductor. Van de Graaff decided to build an electrostatic generator that uses an insulating belt to transfer charge to a metal dome. This metal dome can have extremely high potential. A 1m radius sphere can have a maxmimum potential of $3 \times 10^6 V$. This is useful to create large potential differences.

Wtf is a Electrostatic Precipitator???! It's a device that removes particulate matter from combustion gases, thereby reducing air pollution. The device has a wire with a negative electric potential relative to the walls to create an electric field towards the wire. The electric field becomes high enough to cause a corona discharge, causing electrons and negative ions to charge dirt during collisions which then become attracted by the outer walls that are positively charged. Das cool..

Chapter 4

Current and Resistance

4.1 Electric Current

Let's say charges are moving through a cross section of surface area A. Current is defined as how many charge flows through this surface in terms of time. We define instantaneous current I as:

$$I \equiv \frac{dQ}{dt} \text{ where } 1A = 1C/s$$

4.2 Resistance

We define the current density as

$$J \equiv \frac{I}{A} = \sigma E$$

where σ is defined as conductivity.
As previously defined, $\Delta V = El$. We can redefine the current density as

$$J = \sigma \frac{\Delta V}{l}$$

Using this and $\Delta V = El$, we find that

$$J = \sigma \frac{\Delta V}{l}$$

We can rearrange this to obtain

$$\Delta V = \frac{l}{\sigma} J = (\frac{l}{\sigma A}) I = RI$$

which we define as Ohm's law($1\Omega \equiv 1V/A$). We also define the resistivity as $\rho = \frac{1}{\sigma}$, meaning the resistivity is inversely proportional to the conductivity. We can sum this up as:

$$R = \rho \frac{l}{A}$$

where ρ also depends on temperature:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

4.3 Electrical Power

The equations speak for themselves:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V = P$$

Chapter 5

Magnetic Fields

5.1 Analysis Model: Particle in a Field(Magnetic)

- The magnetic force is proportional to the charge q of the particle
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction
- The magnetic force is proportional to the magnitude of the magnetic filed vector $\vec{B}$
- The magnetic force is proportional to the speed v of the particle
- If the velocity vector makes an angle θ with the magnetic filed, the magnitude of the magnetic force is proportional to $\sin\theta$
- When a charged particle moves *parallel* to the magnetic field vector, the magnetic force on the charge is zero
- When a charged particle moves in a direction not parallel to the magnetic field vector, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$(use right hand rule)

We define the magnetic force as:

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

Note: It's important to know how to use the right hand rule. Seriously. We can calculate a numerical value of the force with:

$$F_B = |q|vB\sin\theta$$

Differences to electric field:

- The electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field
- The electric force acts on a chraged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application

5.2 Motion of a Charged Particle in a Uniform Magnetic Field

We denote $\cdot$ as a magnetic field coming out of the paper and $\times$ when the magnetic field is going into the paper. If a charge has a velocity perpendicular in a uniform magnetic field, it will move in a circular path. We can then define several properties of its path:

$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

5.3 Magnetic Force Acting on a Current Carrying Conductor

Rather than applying a magnetic force on a charge, what if we wanted to apply it on a current? We'd need to redefine our formula:

$$\vec{F_B} = (q\vec{v}_d \times \vec{B})nAL$$

where n is defined as the number of charge per unit volume. Hence, we define the magnetic for a current as:

$$\vec{F_B} = I\vec{L} \times \vec{B}$$

where $\vec{L}$ is a vector that points in the direction of the current I. As the path of the current may vary, we can generalize this formula as:

$$\vec{F_B} = I \int_a^b d\vec{s} \times \vec{B}$$

5.4 Torque on a Current Loop in a Uniform Magnetic Field

Imagine a square loop with current flowing through it and a magnetic field going across it, parallel to 2 of its sides. The 2 sides which are perpendicular would then have a force but since the current is flowing in the opposite direction, the square loop would feel a torque:

$$F_2 = F_4 = IaB \text{ where } a \text{ is the length of the side}$$

$$\tau_{max} = F_2\frac{b}{2} + F_4\frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB \text{ where b is the length of the other side}$$

$$\tau_{max} = IAB$$

With incline, we would define the torque as:

$$\tau = IAB\sin\theta$$

We redefine $I\vec{A} = \vec{\mu}$ as the magnetic dipole moment so that:

$$\tau = \vec{\mu} \times \theta$$

In the previous section, we defined the potential energy to be $U_E = -\vec{p} \cdot \vec{E}$. We can define the magnetic potential energy as:

$$U_B = -\vec{\mu} \cdot \vec{B}$$

This is the basis of how a motor works.

5.5 The Hall Effect

When a current carrying conductor is placed in a magnetic field, a force pushes the current towards a direction, causing there to be a potential difference in a direction perpendicular to the magnetic field.
We can measure this potential difference through:

$$E_H = v_dB$$

$$\Delta V_H = E_Hd = v_dBd$$

Chapter 6

Sources of the Magnetic Field

6.1 Biot-Savart Law

The equation speaks for itself:

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{s}\times\hat{r}}{r^2}$$

where μ_0 is the permeability(not permitivity) of free space:

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$

Common Conductors:

- Thin Straight Conductor: $B = \frac{\mu_0 I}{2\pi a}$
- Circular Current Loop: $B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$

6.2 The Magnetic Force Between Two Parallel Conductors

Just draw it, draw the forces and you'll see that if there are 2 conductors with current flowing in the same direction, then they'll be attracted with a force of :

$$F_1 = I_1 l B_2 = I_1 l(\frac{\mu_0 I_2}{2\pi a}) = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

and if the current flows in opposite directions, then the force would repel both conductors.

6.3 Amphere's Law

Ampere's Law is extremely similar to Gauss's Law in terms of approaching the problem. We define Ampere's Law as follows:
The line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

Ampere's Law describes the creation of magnetic fields by all continuous current configurations and is mainly useful for calculating the magnetic field with configurations that have symmetry.
Ampere's Law makes it simpler to solve for the magnetic field of conductors like an infinitely long conductor with current
Note: If there are several loops such as a solenoid, note that Ampere's Law becomes:

$$\oint B \cdot d\vec{s} = \mu_0 N I$$

where N is the amount of loops.

6.4 The Magnetic Filed of a Solenoid

We find that the magnetic field of a solenoid is

$$B = \mu_0 \frac{N}{l} I = \mu_0 n I$$

where n is the number of loops per unit length

6.5 Gauss's Law in Magnetism

The flux associated with magnetic field is similar to that of the electric flux. We define the magnetic flux as:

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

Like the electric flux, if the magnetic field is parallel with the surface, the magnetic field is zero and if it's not perpendicular, it's $BA\cos\theta$. Additionally, for all magnetic flux going through a closed surface:

$$\int \vec{B} \cdot d\vec{A} = 0$$

6.6 Magnetism in Matter

Chapter 7

Faraday's Law

7.1 Faraday's Law of Induction

Based on experiments done by Michael Faraday, an emf is induced in a loop when the magnetic flux trough the loop changes with time. This statement can be written as:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

where N is the amount of loops and ε has V as units. This implies that an induced emf is created to resist the change in magnetic flux.

7.2 Motional emf

When a moving bar is a part of a closed conducting path with a resistor, free charges in the bar move due to the field, causing an induced current. If we define the position of the bar as x, we can define the magnetic flux as:

$$\Phi_B = Blx$$

Using Faraday's Law, we find that the induced motional emf is:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -Bl\frac{dx}{dt} = -Blv$$

$$\text{Induced Current:} I = \frac{Blv}{R}$$

Since the bar is undergoing constant velocity, we can define the magnetic force as:

$$P = F_{app}v = (IlB)v = \frac{B^2l^2v^2}{R} = \frac{\varepsilon^2}{R}$$

7.3 Lenz's Law

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.
This can further be explained with the example mentioned in the previous section where a bar was moving in a closed conductor loop. As the area increases, an induced current tries to decrease the magnetic flux and vice versa.

7.4 Induced emf and Electric Fields

Even without the presence of a conducting loop, a changing magnetic field generates an electric field in empty space, a field that is nonconservative. To illustrate this, consider a conducting loop of radius r situated in a uniform magnetic field that is perpendicular to the plane $\varepsilon = -\frac{d\Phi_B}{dt}$. An induction of a current in the loop implies the presence of an induced electric field which must be tangent to the loop. The work done by the electric field in moving a charge is equal to $q\varepsilon$ and the electric force is $q\vec{E}$. This implies:

$$q\varepsilon = qE(2\pi r)$$

$$E = \frac{\varepsilon}{2\pi r}$$

Using this result and that $\Phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as:

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

We can generalize Faraday's Law for all shapes as:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

7.5 Generators and Motors

Imagine a closed loop between 2 magnets, north and south. Due to the magnetic flux through the loop, an induced force causes a torque on the loop, leading to a rotational oscillation of the loop which leads to an oscillating induced emf:

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$
$$\varepsilon_{max} = NBA\omega$$

Depending on how the ends of the loop are received, the generator could be AC or DC. If the ends loop are constantly connected to the same end of the external circuit, then it's AC. If it depends on the angle of the end of the loop, then it is DC. Both emf graphs are similar except DC remains positive.

7.6 Eddy Currents

Due to a change of magnetic flux on a conducting surface, circulating currents called eddy currents are induced to oppose the change in magnetic flux as stated by Lenz's Law. This often happens when a conducting surface is moved into a magnetic field. Because the induced eddy current always produces a magnetic retarding force when the plate enters or leaves the field.

Chapter 8

Inductance

8.1 Self-Inductance and Inductance

Note:We use the adjective induced when a current is caused by a change of magnetic field.

Imagine a regular circuit, a resistor, a voltage source and a switch. As the switch is closed, the current is not maximized immediately. Due to the voltage difference, the current increases, causing a magnetic field which goes through the loop. As a result, an induced current opposes the change and this current prevents the current to change immediately.

To quantify this:

$$\varepsilon_L = -L\frac{di}{dt}$$

Combining the idea of magnetic flux, we obtain:

$$L = \frac{N\Phi_B}{i}$$

We can then define the inductance as the ratio:

$$L = -\frac{\varepsilon_L}{di/dt}$$

8.2 RL Circuits

We define a circuit with a large inductance as an inductor. Given a circuit with a resistor, battery and an inductor, we define KVL as:

$$\varepsilon - iR - L\frac{di}{dt} = 0$$

By applying some differential equation methods, we can isolate the current as:

$$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$$

By graphing this, it can be seen that the current slowly increases and converges to a value over time. Assuming this current still exists, if the circuit element is changed to an inductor and a resistor, it can be seen that the KVL is:

$$iR + L\frac{di}{dt}$$

By applying some differential equation methods again, we obtain:

$$i = \frac{\varepsilon}{R}e^{-t/\tau} = I_i e^{-t/\tau}$$

In this case, the equation shows that as time passes by, the current slowly converges to 0.

8.3 Energy in a Magnetic Field

By multiplying the KVL equation previously used by i, we obtain

$$i\varepsilon = i^2R + Li\frac{di}{dt}$$

We know that $i\varepsilon$ is the rate at which energy is delivered to the resistor. We can then redefine it and obtain

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

Solving this differential equation, we obtain

$$U_B = \frac{1}{2}Li^2$$

8.4 Mutual Inductance

Two closely wound coils have N_1 and N_2 number of coils.The magnetic flux induced by the first coil passes through coil 2 is defined as Φ_{12}. We can identity the mutual inductance M_{12} of coil 2 with respect to coil 1 as:

$$M_{12} = \frac{N_2\Phi_{12}}{i_1}$$

We can then define the induced emf from coil 2 as:

$$\varepsilon_2 = -N_2\frac{d\Phi_{12}}{dt} = -N_2\frac{d}{dt}(\frac{(M_{12}i_1)}{N_2}) = -M_{12}\frac{di_1}{dt}$$