Finding the Rydberg Constant:
An exploration using the Balmer Series
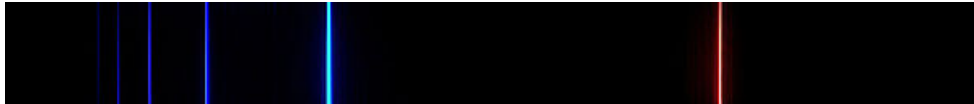
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Figure 1: This Figure shows the Balmer Series[4]

Introduction and Theory

In 1885 Johann Balmer, came up with an equation to describe the wavelengths of light released when exciting the hydrogen atom. This equation is[5]:

$$\lambda = h \frac{m^2}{m^2 - n^2} \qquad (1)$$

where $n = 2$, $m = 3, 4, 5, 6, ...$ and $h = 3.6456 \times 10^{-7} m$ which he called the "fundamental number of hydrogen"[5]. Johann Balmer was a mathematician, and so he modelled this from wavelengths found by Anders Angstrom[5], and his model did very well[3] at producing the wavelengths for the different values of m. At this time not much was known about the atom, the electron wasn't even discovered yet, and so an explanation was needed. In 1888, Johannes Rydberg[5] came up with his formula, the Rydberg formula[5]:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \qquad (2)$$

where R_H is the Rydberg constant for Hydrogen, $n = 2$ and $m = 3, 4, 5....$ Balmer died in 1898, so he never found out the reason for these spectral lines. In his honour the visible spectrum released from the Hydrogen atom was called the Balmer Series[4]. Near the beginning of the quantum mechanical era, Neils Bohr postulated a new model of the atom, one where electrons can only orbit with certain energies and none in between, and each of these energies were quantized using Max Planck's idea of the quantum. This model predicted that for an electron to move between energy levels, it either has to gain or release energy, and this energy was related to a frequency of light by the photoelectric effect: $E = hf$, where E is the energy, h is Planck's constant and f is the frequency of the light. From this model and the new quantum mechanics, he was able to determine the wavelengths of the light that is released when the electrons jumped down energy levels. However, as this is an experimental investigation, we need to talk about the equipment. A HeNe Laser is used to excite hydrogen gas and then this is fed to a spectrometer where the Balmer series is attained. However calibration will need to be done, and a brief understanding of the laser will be needed. The Helium Neon laser works by exciting Neon atoms which then release light of frequency $\approx$6238A[2], this is then used for calibration of the data.

Aim

To experimentally determine the Rydberg Constant, and compare it to the known value.
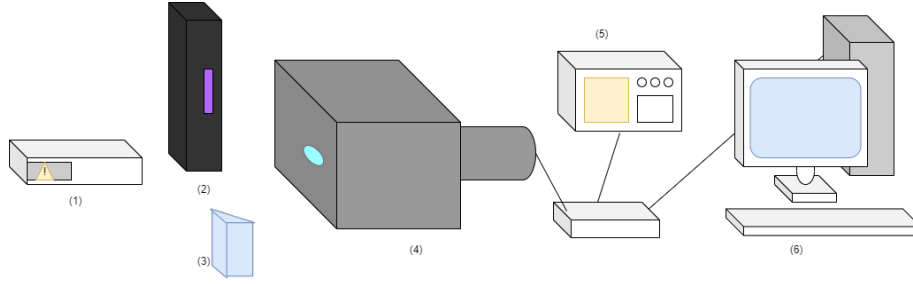
Apparatus

Figure 2: This image shows a diagram of the setup used

The HeNe laser (1) is used to excite a tube of hydrogen gas (2), by hitting a mirror and entering the tube and then this goes into the monochrometer (3). This measures the amount of gammas hitting a detector at a certain wavelength. This is then fed to an oscilloscope (5) where one can see the activity inside the monochrometer. From the monochrometer is also connected to a computer (6) which using the OpSpec GUI, reads in the data from the monochrometer and stores it in a CSV file.

Method

Calibration Data

Once the HeNe laser was turned on it was aligned with the detector by means of setting the monochromator to detect the wavelength of approximately 6350A. Then using the oscilloscope the laser was maneuvered so that it could align with the detector. Once it was aligned the monochrometer dial was set to 6271A. The going onto the computer and opening the OpSpec GUI, the relevant inputs were filled in and for the first run, it was set to a slit width of 25μm and a dwell time of 100ms and it recorded the counts of gammas at every 1A until it reached 6425A. The monochrometer dial was then set to 6334A and once more the same information was inputted except the end wavelength was 6375A and the slit width was decreased to 20μm and it recorded counts of gammas at every 0.5A. Then for the last run the monochrometer dial was set to 6334A and the slit width was set to 15μm and it recorded counts of gammas at every 0.2A and it ended at 6375A.

Hydrogen Data

For getting the Hydrogen emissions, the same method as above was used, however the diffence is that instead of the laser going straight into the monochrometer, using a mirror is was reflected into the hydrogen gas tube and the light emissions from the tube were fed into the monochrometer. The settings for the monochrometer were as follows:

Start Wavelength (A)	End Wavelength (A)	Interval (A)	Slit Width μm	Dwell time (ms)
6496	6650	1.0	100	300
6562	6600	0.2	100	300
4858	4900	1.0	20	300
4861	4900	0.2	20	300
4300	4450	1.0	100	300
4338	4380	0.2	100	300
4051	4200	1.0	100	300
4099	4150	0.2	100	300

Table 1: This table shows the wavelengths being measured for the Hydrogen gas emission

Code for Analysing the Data

When coding, the CSV files for the calibration, as attained by the Calibration data section, were first read in using the python pandas library, then this data was then used for the curvefit program where a Gaussian was fit to the data so that the μ-value, the peak wavelength, and it's standard uncertainty given by σ. These three values were then averaged, as well as the uncertainty. The value of the wavelength of light that neon released was attained for the NIST website[2]. This calibration constant was then calculated by subtracting the value attained by the code from the value attained from the NIST website. When working with the Hydrogen data, the data files were read in and the same process as for the calibration data was followed. Once these wavelengths were attained from the data, the wavelengths from the same energy level jump and their uncertainties were averaged. Then the calibration constant was subtracted from these four values. This was then fed to the weighted least squares fit code, where the $'y'$ values were given by $1/\lambda$ and the $'x'$ values were given by $(1/4 - 1/m^2)$, from equation (2). Then the gradient from this code was attained.

Data

Wavelength from Data (m)	Uncertainty (m)	Wavelength from NIST and Uncertainty
65631.1×10^{-11}	9.7×10^{-11}	$65685.175(7) \times 10^{-11}$
48600.7×10^{-11}	6.3×10^{-11}	$48612.83363(24) \times 10^{-11}$
43389.3×10^{-11}	10.7×10^{-11}	$43404.72(6) \times 10^{-11}$
41003.2×10^{-11}	9.1×10^{-11}	$41017.07462(21) \times 10^{-11}$

Table 2: This Table shows the wavelengths acquired from the hydrogen gas emissions as well as their respective values attained from the NIST website[1]
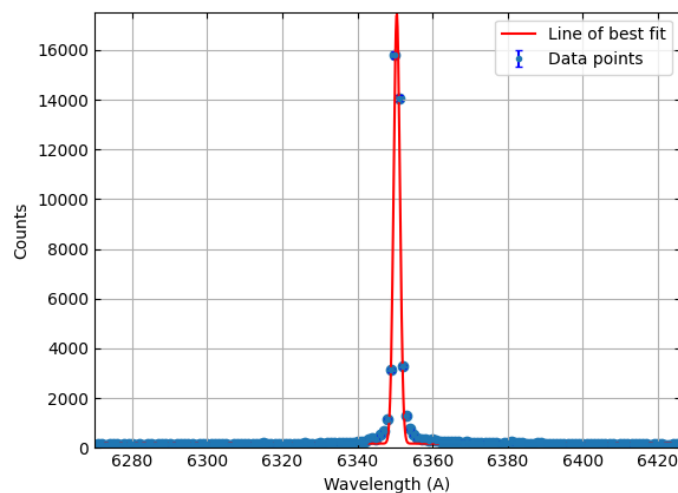
Graphs

Calibration

Figure 3: This graph shows one of the Gaussians produced in order to find the wavelength of Neon detected by the Monochrometer in order to get a calibration constant

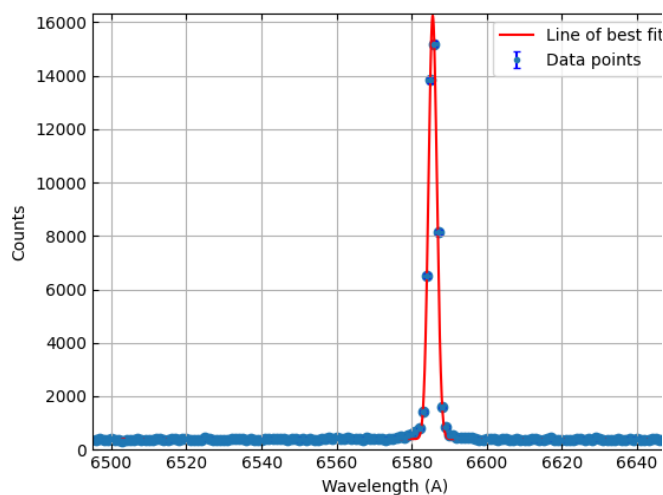## Light emissions from the excited states of the hydrogen gas

Figure 4: This graph shows the wavelengths of light released from hydrogen's 2nd excited state to its 1st
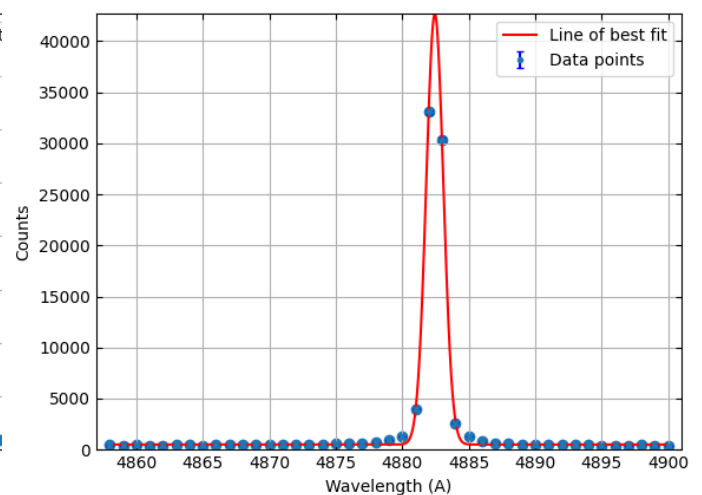

Figure 5: This graph shows the wavelengths of light released from hydrogen's 3rd excited state to its 1st
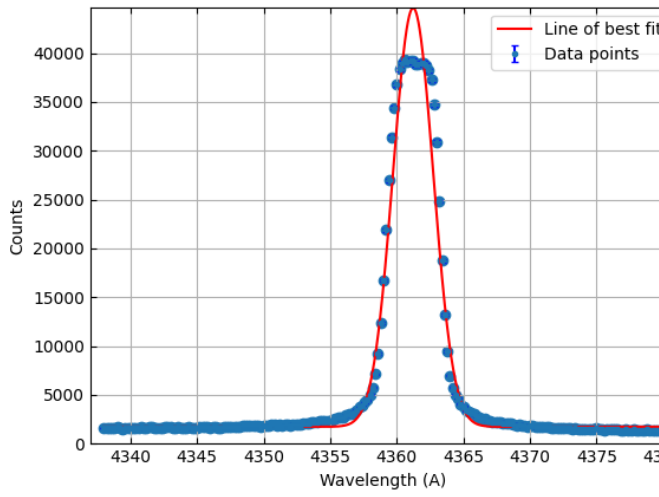
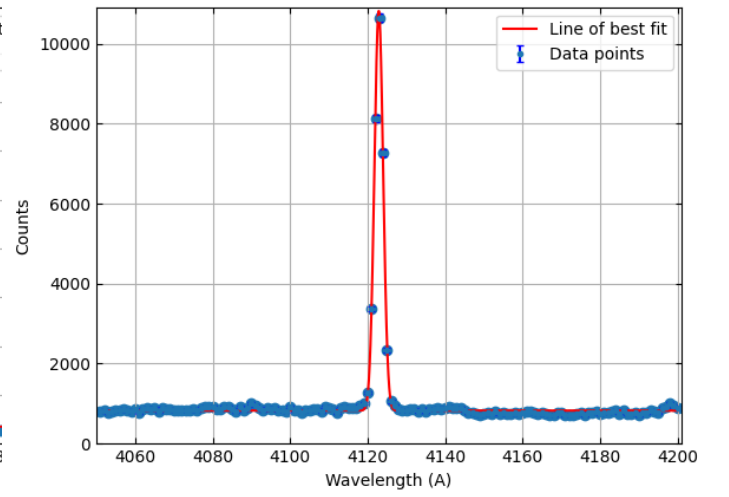Figure 6: This graph shows the wavelengths of light released from hydrogen's 4th excited state to its 1st

Figure 7: This graph shows the wavelengths of light released from hydrogen's 5th excited state to its 1st
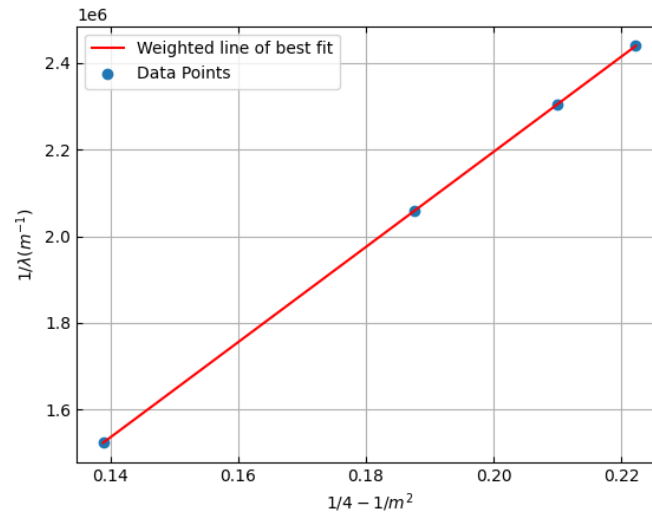
The Least Squares fit graph

Figure 8: This graph shows the Least Squares fit of the data in Table (2) to determine the Rydberg constant for Hydrogen

Calculations

Calibration constant

The calibration constant was attained from the calibration data as explained in the Method section. The peak wavelength found is: $6350.52 \pm 0.43A$
The wavelength from NIST[2]: $6328.16460 \pm 0.00040A$.
Therefore the calibration value is: $22.36 \pm 0.43A$

The Rydberg constant from the code

Using equation (2) and the data from table (1), the Rydberg constant for Hydrogen was found to be:

$$R_{H(experimental)} = 10982826.27 \pm 5142.88 m^{-1} \tag{3}$$

Calculating the Rydberg constant for Hydrogen

As the NIST website only gives the Rydberg constant and not the constant for hydrogen, we need to calculate the Rydberg constant for hydrogen. To calculate the Rydberg constant for Hydrogen we use the equation[6]:

$$R_H = R_\infty \frac{m_p}{m_e + m_p} \tag{4}$$

where R_∞ is the Rydberg constant given by $R_\infty = 10973731.568160$, m_e is the mass of an electron and m_p is the mass of a proton. Substituting it all into equation (4) gives:

$$R_H = (10973731.568160)\frac{(1.672622 \times 10^{-27})}{(9.109384 \times 10^{-31}) + (1.672622 \times 10^{-27})}$$

$$\therefore R_H = 10967758.340357 m^{-1} \tag{5}$$

Uncertainty Analysis

Rydberg Constant for Hydrogen

In order to calculate the uncertainty equation (4) needs to be rewritten:

$$R_H = R_\infty (m_p)(m_e + m_p)^{-1}$$

Now we can find the uncertainty of individual parts.

For $m_e + m_p$

$$u(m_e + m_p) = \sqrt{u(m_e)^2 + u(m_p)^2}$$

where $u(m_e) = 2.80 \times 10^{-40}$ and $u(m_p) = 5.10 \times 10^{-37}$.

$$\therefore u(m_e + m_p) = \sqrt{(2.80 \times 10^{-40})^2 + (5.10 \times 10^{-37})^2} = 5.10 \times 10^{-37} kg \tag{6}$$

For $R_\infty(m_p)$

$$u((R_\infty)(m_p)) = (R_\infty)(m_p)\sqrt{\left(\frac{u(R_\infty)}{R\infty}\right)^2 + \left(\frac{u(m_p)}{m_p}\right)^2} \tag{7}$$

where $u(R_\infty) = 2.10 \times 10^{-5}$ and $u(m_p) = 5.10 \times 10^{-37}$, then:

$$u((R_\infty)(m_p)) = (10973731.568160)(1.672622 \times 10^{-27})\sqrt{\left(\frac{2.10 \times 10^{-5}}{10973731.568160}\right)^2 + \left(\frac{5.10 \times 10^{-37}}{1.672622 \times 10^{-27}}\right)^2}$$

$$\therefore u((R_\infty)(m_p)) = 5.5937 \times 10^{-30} \tag{8}$$

Now multiplying it all together:

$$u(R_H) = R_H\sqrt{\left(\frac{u((R_\infty)(m_p))}{(R_\infty)(m_p)}\right)^2 + \left(\frac{u(m_e + m_p)}{m_e + m_p}\right)}$$

$$\therefore u(R_H) = 0.0047 m^{-1}$$

Uncertainty in coding

When coding and averaging the values, for the calibration data, the uncertainty was divided by three as three data points were averaged:

$$u(av) = \sqrt{data1^2 + data2^2 + data3^2}/3$$

This same idea was used when averaging between the data that came from the code to find the Rydberg constant.

Uncertainty in reading off of the monochrometer

The starting points of the wavelengths was done using a dial, therefore the uncertainty is Type B and we use the rectangular pdf:

$$u(dial) = \frac{0.5}{2\sqrt{3}}$$

$$\therefore u(dial) = 0.144 A$$

where 0.5A is how much a reading could have been off by.

Total uncertainty for experimental Rydberg constant for hydrogen

$$u(R_{H(Experimental_T)}) = \sqrt{u(R_{H(experimental)})^2 + (u(dial) \times 10^{-7})^2}$$

$$\therefore u(R_{H(Experimental_T)}) = 5142.88m^{-1}$$

Interpretation and Discussion

When taking the readings it was seen that a larger slit width as well as a short dwell time led to sufficient data, otherwise the peak would become inverted and the data could not be used. The experimental Rydberg constant was found to be $R_{H(experimental)} = 10982826.27 \pm 5142.88m^{-1}$. This differs from the actual value calculated being $R_H = 10967758.3403 \pm 0.0047m^{-1}$. It is seen that the results obtained were very close however they do not agree within the standard uncertainty. This is due to the fact that if one looks at table (1) it is seen that the values are off. Meaning the value obtained for the Rydberg constant for Hydrogen would be off as well.

Conclusion

In conclusion the Rydberg constant found by experiment is $R_{H(experimental)} = 10982826.27 \pm 5142.88m^{-1}$, and it agrees with the actual value but is off by $\approx 20000m^{-1}$.

References

[1] NIST Atomic Spectra Database Lines Data. *H (all spectra)*. URL: https://physics.nist.gov/cgi-bin/ASD/lines1.pl?spectra=Hydrogen&limits_type=0&low_w=4000&upp_w=7000&unit=0&submit=Retrieve+Data&de=0&format=0&line_out=0&en_unit=0&output=0&bibrefs=1&page_size=15&show_obs_wl=1&show_calc_wl=1&unc_out=1&order_out=0&max_low_enrg=&show_av=2&max_upp_enrg=&tsb_value=0&min_str=&A_out=0&intens_out=on&max_str=&allowed_out=1&forbid_out=1&min_accur=&min_intens=&conf_out=on&term_out=on&enrg_out=on&J_out=on. (accessed:22.08.2021).

[2] NIST Atomic Spectra Database Lines Data. *Ne (all spectra)*. URL: https://physics.nist.gov/cgi-bin/ASD/lines1.pl?spectra=Neon&limits_type=0&low_w=6300&upp_w=6400&unit=0&submit=Retrieve+Data&de=0&format=0&line_out=0&en_unit=0&output=0&bibrefs=1&page_size=15&show_obs_wl=1&show_calc_wl=1&unc_out=1&order_out=0&max_low_enrg=&show_av=2&max_upp_enrg=&tsb_value=0&min_str=&A_out=0&intens_out=on&max_str=&allowed_out=1&forbid_out=1&min_accur=&min_intens=&conf_out=on&term_out=on&enrg_out=on&J_out=on. (accessed:20.08.2021).

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[6] Wikipedia. *Rydberg constant*. URL: https://en.wikipedia.org/wiki/Rydberg_constant. (accessed:20.08.2021).