Experimental Physics Lab
The Magnetic Field of a Circular Coil: Induction and Inductance

CNNAUA001

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Introduction and Theory

In this report we are looking at how an alternation current produces a changing magnetic field, which in turn creates an emf. This can be seen when looking at Faraday's Law:

$$\epsilon = -N_a \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \tag{1}$$

The way we are going to experiment the relationship between the magnetic field and emf and the frequency of the signal, is by using two coils. The magnetic field in the main coil is given by:

$$B(z,t) = \frac{\mu_0 N I(t)}{2} \frac{a^2}{(a^2 + z^2)^{3/2}} \tag{2}$$

where $\mu_0 = 4\pi \times 10^{-7}$, N is the number of turns in the coil, a is the radius of the coil, z is the distance away from the coil and $I(t) = I_0 \cos \omega t$. From these equations, and from this experiment we can better understand how induction works.

Aim

To investigate the magnetic field of a coil due to an alternating current by measuring the induced voltage produced in a search coil. To investigate the relationship between emf and magnetic field and to determine the proportionality constant between them. To investigate the relationship between induced voltage and frequency of the signal. To determine the internal resistance and inductance of the main coil.

Apparatus

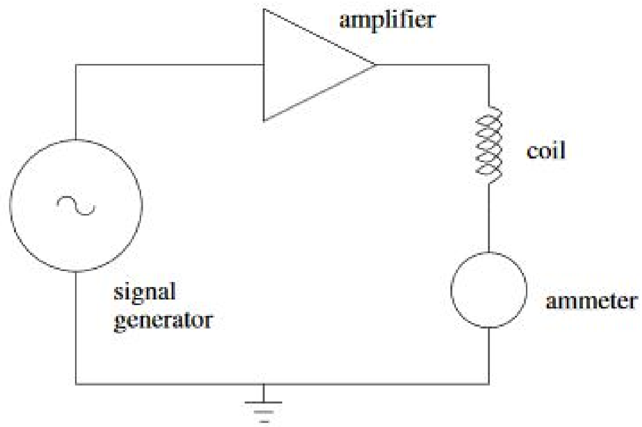

Figure 1: Setup of the Inductance Practical[1]

- 1 120 turn main coil
- 1 175 turn search/axial coil
- 1 Signal Generator
- 1 Digital Ammeter
- 1 Digital Oscilloscope
- 1 Ruler
- 1 Laptop
- 1 Audio Amplifier
- 1 Vernier Caliper

Setup of the circuit

The signal generator generates the current with a sinusoidal wave, this current then goes into the amplifier which increases the amount of current, this then goes to the coil and then to the digital ammeter. The oscilloscope measures the voltage across the main coil and the axial coil.

Method

Field on the axis of a circular coil

The peak-to-peak voltage of the current from the signal generator was set to $2V_pp$ with a frequency of $1kHz$, and the current generated from the amplifier was $500mA$ or $353mA$ RMS. The axial coil was then moved to different positions along the z-axis of the coil, in and out of the main coil, and the voltage across the search coil and its respective distance was measured and saved into a text file.
The data from this text file was then extracted and used to plot Figure (2). The data was then used in code in order to plot the experimental magnetic field and the theoretical magnetic field. This was done by sending the relevant data points through equation (4) in order to determine the experimental magnetic field and then the theoretical graph was plotted by sending the relevant data points from the text file through the amplitude version of equation (2). A scaling constant had to be introduced in order to compare the graphs. From this Figure (3) was obtained. The data was then passed through equation (5) and the voltage data was used in order to generate Figure (4).

Induction

The axial coil was moved to the centre of the main coil and the frequency was changed to a range between $100Hz$ and $2kHz$. The amplitude of the voltage across the main coil and the axial coil were then measured with the frequency associated and this data was put into a text file. The data from the file was then used to generate Figure (5) and the line of best fit came from using the frequency and voltage across the axial coil points in the file. The voltage across the main coil was then used to determine the inductance and resistance of the large coil by linearising $V = IZ$ as done in equation (10).

Collecting Data

All of the data for this practical was collected before hand in the lab and uploaded to Vula. This data was then downloaded and extracted from the zip file.

Graphs

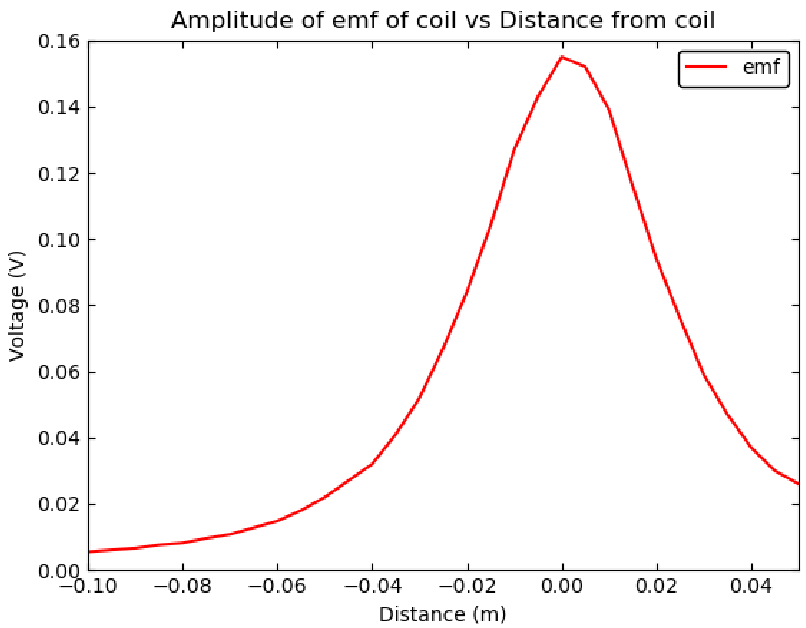

Figure 2: This graph shows the amplitude of the emf of the axial coil vs the distance from the main coil

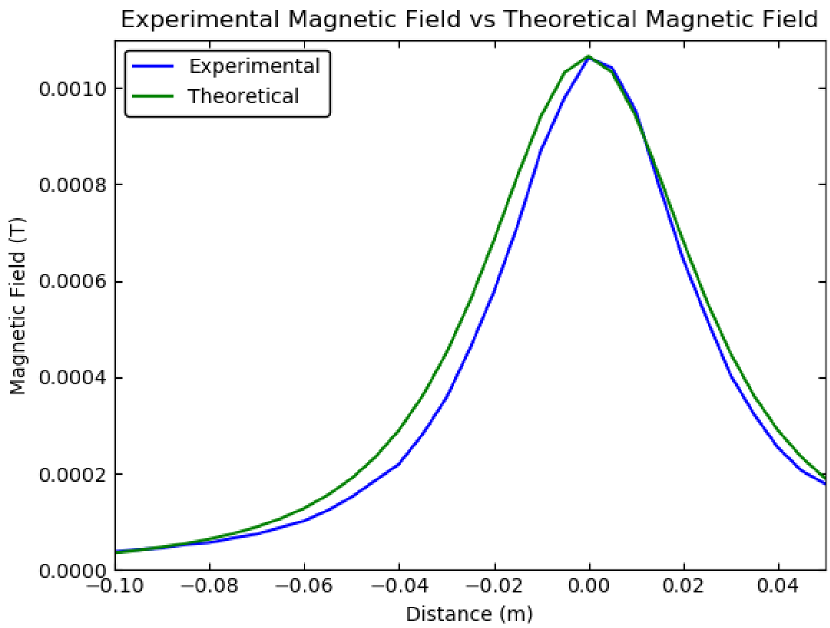

Figure 3: This graph shows the experimental magnetic field vs the theoretical magnetic field of the main coil

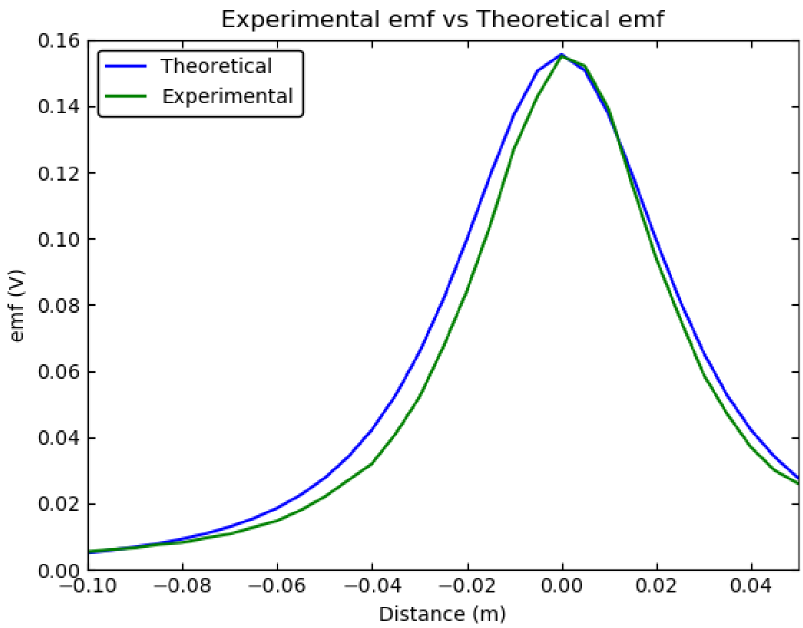

Figure 4: This graph shows the experimental emf of the axial coil and the theoretical emf of the axial coil due to the magnetic field of the main coil

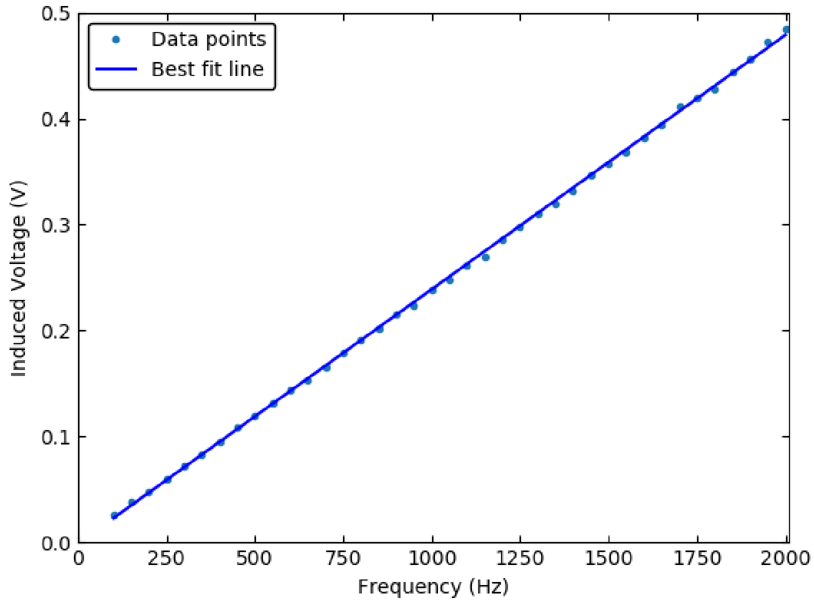

Figure 5: This graph shows the relationship between induced voltage and frequency

Calculations

Emf and Magnetic field relation

Starting with Faraday's Law:

$$\epsilon = -N_a \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \approx -N_a A \frac{dB(z,t)}{dt} \tag{3}$$

where $B(z,t) = B(z) \cos \omega t$. Plugging this in gives:

$$\epsilon \approx -N_a A \frac{d}{dt}\Big(B(z) \cos \omega t\Big)$$
$$\therefore \epsilon \approx N_a A B(z) \omega \sin \omega t$$

where N_a is the number of turns in the coil, A is the cross-sectional area and B is the component of the magnetic field that is perpendicular to the area. As we are only interested in the amplitude for the calculations, $\sin \omega t$ can be ignored as the coefficients of $\sin \omega t$ is the amplitude of the sine wave. Therefore with just the amplitude:

$$\epsilon = N_a A \omega B(z) \tag{4}$$

This means that the emf is directly proportional to the magnetic field with the proportionality constant $N_a A\omega$.
In order to determine the theoretical emf, the magnetic field from the main coil has to be included in the code to generate Figure(4). Starting with equation (4):

$$\epsilon = N_a A\omega B(z)$$

And now the magnetic field from the main coil is given by equation (2), however we need the amplitude and therefore leave out the $\cos\omega t$ in the current. This gives:

$$B(z) = \frac{\mu_0 N I_0}{2}\frac{a^2}{(a^2+z^2)^{3/2}}$$

and putting this into equation (4):

$$\epsilon = N_{axial}A_{axial}\frac{\mu_0 N_{main} I_0}{2}\frac{a^2}{(a^2+z^2)^{3/2}} \tag{5}$$

Verifying that induced voltage is proportional to the frequency

In order to compare that the induced voltage is proportional to the frequency, we will need the gradient of both the line of best fit from the experimental data as well as an equation using the theoretical data. From the code that plotted Figure (5), the gradient was outputted for the ϵ vs f graph. The gradient was: $2399.47 \times 10^{-7} \pm 6.55 \times 10^{-7}$. In order to find the theoretical version, using the amplitude equation (4):

$$\epsilon = N_a AB\omega$$

and $\omega = 2\pi f$

$$\therefore \epsilon = (BN_a A2\pi)f \tag{6}$$

therefore in this equation the gradient is given by: $BN_a A2\pi$. In order to find this we need to find the magnetic field generated by the main coil using equation (2) as well as finding the cross-sectional area of the search coil. As we are considering the magnetic field right at the centre of the coil, $z = 0$, and we are only considering the amplitude of the magnetic field, therefore the $\cos\omega t$ falls away.

$$B = \frac{\mu_0 N I_0}{2}\frac{1}{a} \tag{7}$$

where the N value here is the number of coils in the main coil generating the magnetic field, a is the radius of the main coil which is half of the diameter given and I_0 is the amplitude of the current running through the main coil. The amplitude of the current is given by multiplying the RMS current value by $\sqrt{2}$.

The RMS value is $570mA$, therefore $I_0 = \sqrt{2}570 \times 10^{-3} = 0.806A$.
The magnetic field is therefore:

$$B = \frac{(4\pi \times 10^{-7})(120)(0.806)}{2(1/2)(6.8 \times 10^{-2})}$$
$$B = 1.787 \times 10^{-3}T$$

Finding the area of the search coil:

$$A = \pi r^2$$

where r is the radius of the small coil, which is half of the diameter given.

$$\therefore A = \pi(1/2(1.3 \times 10^{-2}))^2$$
$$\therefore A = 1.327 \times 10^{-4}m^2$$

Now substituting this into the gradient equation:

$$\begin{aligned} m_{Theoretical} &= BN_aA2\pi \\ \therefore m_{Theoretical} &= (1.787 \times 10^{-3})(175)(1.327 \times 10^{-4})(2\pi) \\ \therefore m_{Theoretical} &= 2607.43 \times 10^{-7} \end{aligned}$$

And including the uncertainty:

$$m_{Theoretical} = 2607.43 \times 10^{-7} \pm 386.61 \times 10^{-7}$$

Linearising $V = IZ$ to find the Resistance and Inductance of the main coil

Starting with the equation:

$$V = IZ \tag{8}$$

where V is the voltage, I is the current through the coil and Z is the magnitude of the impedance given by:

$$Z = \sqrt{R^2 + (2\pi fL)^2} \tag{9}$$

Now substituting $Z = V/I$ into the impedance equation:

$$\frac{V}{I} = \sqrt{R^2 + (2\pi fL)^2}$$
$$\therefore \frac{V^2}{I^2} = R^2 + 4\pi^2 f^2 L^2$$
$$\therefore V^2 = (I^2 4\pi^2 L^2)f^2 + I^2 R^2 \tag{10}$$

Now we can use the linear fit function to determine the gradient of this graph running the induced voltage of the large coil and frequency through the above equation to determine the gradient and constant. Then by dividing the gradient by the current through the main coil squared (I^2) and $4\pi^2$ and then square rooting the value gives the Inductance of the main coil. For the resistance the constant is divided by the current through the coil squared (I^2) and then square rooted to get the resistance. This looks like:

$$m = I^2 4\pi^2 L^2$$
$$\therefore L^2 = \frac{m}{I^2 4\pi^2}$$
$$\therefore L = \sqrt{\frac{m}{I^2 4\pi^2}}$$

and for the constant $+c$:

$$c = I^2 R^2$$
$$\therefore R^2 = \frac{c}{I^2}$$
$$\therefore R = \sqrt{\frac{c}{I^2}}$$

where the current used is the RMS value of the current multiplied by square root of two in order to get the amplitude of the current through the coil. I included this calculation in my code, the inductance and resistance produced from the code is:

$$L = 1502.00 \times 10^{-6} H$$
$$R = 3.17\Omega$$

And with uncertainties:

$$L = 15.02 \times 10^{-4} \pm 1.90 \times 10^{-4} H$$
$$R = 3.17 \pm 0.41\Omega$$

Uncertainty Analysis

Uncertainty of the theoretical gradient for induced voltage vs frequency

The uncertainties in this calculation comes from the radius of the search coil in order to get the area, from the current amplitude, and the radius of the main coil when determining the magnetic field. As all of these values are being multiplied together I will use the uncertainty equation given below:
For an equation of the form:

$$R = cA^a B^b \tag{11}$$

The uncertainty is given by:

$$u(R) = R\sqrt{\left(a\frac{u(A)}{A}\right)^2 + \left(b\frac{u(B)}{B}\right)^2} \tag{12}$$

Rewriting this in the form of the gradient:

$$u(m_{Theoretical}) = m_{Theoretical}\sqrt{\Big(\frac{u(B)}{B}\Big)^2 + \Big(\frac{u(A)}{A}\Big)^2}$$

Finding $u(B)$:

$$u(B) = B\sqrt{\Big(\frac{u(I_0)}{I_0}\Big)^2 + \Big(-1\frac{u(a_{maincoil})}{a_{maincoil}}\Big)^2}$$

For I_0: As the reading is off of a multimeter, it is triangular pdf and the uncertainty is:

$$u(I_0) = \frac{0.5}{2\sqrt{6}}$$
$$\therefore u(I_0) = 0.102$$

The uncertainty in the diameter of the main coil is given as 0.1cm, now we multiply it by 10^{-2} to get it into meters and we divide it by two to get the radius.

$$u(a_{maincoil}) = 1/2(0.1 \times 10^{-2})$$
$$\therefore u(a_{maincoil}) = 5 \times 10^{-4}$$

Now finding the uncertainty of the magnetic field:

$$u(B) = B\sqrt{\Big(\frac{u(I_0)}{I_0}\Big)^2 + \Big(-1\frac{u(a_{maincoil})}{a_{maincoil}}\Big)^2}$$
$$\therefore u(B) = (1.787 \times 10^{-3})\sqrt{\Big(\frac{(0.102)}{(0.806)}\Big)^2 + \Big(\frac{(5 \times 10^{-4})}{(6.8 \times 10^{-2})}\Big)^2}$$
$$\therefore u(B) = 2.265 \times 10^{-4}$$

Now finding uncertainty of the area of the search coil:
As the uncertainty of the radius of the search coil is the same as the uncertainty of the uncertainty of the radius of the main coil, as they are both given, $u(a_{maincoil}) = u(a_{searchcoil})$

$$u(A) = A\sqrt{\Big(2\frac{u(a_{searchcoil})}{a_{searchcoil}}\Big)^2}$$
$$\therefore u(A) = A\Big(2\frac{u(a_{searchcoil})}{a_{searchcoil}}\Big)$$
$$\therefore u(A) = \Big(1.327 \times 10^{-4}\Big)\Big(2\frac{(5 \times 10^{-4})}{(1.3 \times 10^{-2})}\Big)$$
$$\therefore u(A) = 1.021 \times 10^{-5}$$

Now putting this into the uncertainty of the theoretical gradient uncertainty calculation:

$$u(m_{Theoretical}) = m_{Theoretical}\sqrt{\Big(\frac{u(B)}{B}\Big)^2 + \Big(\frac{u(A)}{A}\Big)^2}$$

$$\therefore u(m_{Theoretical}) = (2607.43 \times 10^{-7})\sqrt{\Big(\frac{(2.265 \times 10^{-4})}{1.787 \times 10^{-3}}\Big)^2 + \Big(\frac{(1.021 \times 10^{-5})}{(1.327 \times 10^{-4})}\Big)^2}$$

$$\therefore u(m_{Theoretical}) = 386.61 \times 10^{-7}$$

Uncertainty for calculating the inductance and resistance of the main coil

As the gradient is being used, the 'equation' of inductance is:

$$L = \sqrt{\frac{m}{I^2 4\pi^2}}$$

$$\therefore L = \sqrt{mI^{-2}(4\pi^2)^{-1}}$$

$$\therefore L = m^{\frac{1}{2}} I^{-1} (4\pi^2)^{-\frac{1}{2}}$$

Therefore the uncertainty is given by:

$$u(L) = L\sqrt{\Big(\frac{1}{2}\frac{u(m)}{m}\Big)^2 + \Big(-1\frac{u(I)}{I}\Big)^2}$$

and the gradient and uncertainty of the gradient outputted by the linear fit code is: $m = 578.73 \times 10^{-7} \pm 1.37 \times 10^{-7}$.

$$\therefore u(L) = (1502 \times 10^{-6})\sqrt{\Big(\frac{1}{2}\frac{(1.37 \times 10^{-7})}{(578.73 \times 10^{-7})}\Big)^2 + \Big(\frac{(0.102)}{(0.806)}\Big)^2}$$

$$\therefore u(L) = 1.90 \times 10^{-4}$$

For resistance:

$$R = \sqrt{\frac{c}{I^2}}$$

$$\therefore R = c^{\frac{1}{2}} I^{-1}$$

Therefore the uncertainty is given by:

$$u(R) = R\sqrt{\Big(\frac{1}{2}\frac{u(c)}{c}\Big)^2 + \Big(\frac{u(I)}{I}\Big)^2}$$

the constant and the uncertainty of the constant is given by the code, where $c = 6.55 \pm 0.26$

$$\therefore u(R) = (3.17)\sqrt{\Big(\frac{1}{2}\frac{0.26}{6.55}\Big)^2 + \Big(\frac{0.102}{0.806}\Big)^2}$$

$$\therefore u(R) = 0.41$$

Interpretation and Discussion

Amplitude of the emf in the search produced from the main coil

As seen in Figure (2) the closer the axial coil is to the centre of the main coil, where the magnetic field is strongest, it produces the highest emf, or induced voltage in the axial coil, and as the axial coil is moved further away from the centre of the main coil the emf decreases. This is true in both directions, away from in the $-z$-direction as well as away in the positive z-direction.

The relationship between emf and magnetic field

As discussed above it is seen that there is a directly proportional relationship between distance from the centre of the coil and emf in the axial coil, this can be interpreted further as the further away from the centre of the main coil one is, the weaker the magnetic field is, this means that the further away the axial coil is from the main coil, the weaker the emf experienced by the coil as the magnetic field is also weaker. This means that there is a directly proportional relationship between emf and magnetic field strength. This is not only shown in Figure (2) but is also shown mathematically by equation (4), where it is seen that the emf is equal to some constants being multiplied by the magnetic field, this can be shown as:

$$\begin{aligned} \epsilon &= N_a A \omega B(z) \\ \therefore \epsilon &\propto B(z) \end{aligned}$$

The proportionality constant is thus $N_a A\omega$. This shows that using Faraday's Law, one can also see that the emf is directly proportional to the magnetic field, as experiment and theory agree it can be conclusively see that the emf produced in the axial coil is directly proportional to the magnetic field produced by the main coil.

Accuracy of the Theoretical Data

It can be seen in Figure (3) and Figure (4) The theoretical graphs almost match the experimental data exactly, although there was a scaling constant that had to be multiplied in order to compare the two graphs, it can be seen from shape that the experimental and theoretical graphs for both Figure (3) and Figure (4) match. It can be seen that the gradient from Figure (5) and the theoretical gradient calculated although not exactly the same, the experimental gradient lies within the uncertainty of the theoretical gradient as seen in the "Calculations" section under "Verifying that induced voltage is proportional to the frequency". This means that there is experimental evidence to show that the theory is correct.

The relationship between induced voltage and frequency of the signal

Looking at Figure (5), the line of best fit shows that there is a directly proportional relationship between induced voltage and frequency of the signal. This is also shown in equation (6), where it can also be shown that induced voltage is directly proportional to the frequency of the signal:

$$\epsilon = (BN_a A 2\pi)f$$
$$\therefore \epsilon \propto f$$

As there is experimental and theoretical evidence, it can be said that the induced voltage is directly proportional to the frequency of the signal.

The internal resistance and inductance of the main coil

The internal resistance and inductance of the main coil were calculated to be:

$$L = (15.02 \pm 1.90) \times 10^{-4} H$$
$$R = 3.17 \pm 0.41\Omega$$

As the main coil has a decent amount of turns, however not high current, the Inductance value being to the power of -4 makes sense. The internal resistance of the wire is not a large value as the voltage and current through the main coil was not large, according to Ohm's Law, the small value of resistance also makes sense.

Conclusion

In conclusion, there is a directly proportional relationship between emf or induced voltage and the magnetic field from the main coil. There is a directly proportional relationship between induced voltage and the frequency of the signal. It was shown that there is a proportionality constant which can be used to go between magnetic field and emf. The internal resistance and the inductance of the main coil was determined and can be found in the "Interpretation and Discussion" section under "The internal resistance and inductance of the main coil".

Exercises

Exercise 1 - If we operate the coil below this frequency range (100Hz-2kHz) this will lead to the coil heating up. Think of why this would be the case.

Using an adaption of Ohms Law, if we decrease the frequency, we decrease the reactance of the coil. This leads to the impedance becoming the resistance. As

V is constant and R decreases current must increases which means that the coil heats up.

Exercise 2 - Write down reasons as to why the current needs to be adjusted as the frequency is changed

The reason that the current needs to be adjusted is so that the magnetic field can remain constant, as the magnetic field is directly proportional to current in a loop, if the current increases the magnetic field increases and we no longer can investigate induced voltage and frequency as the magnetic field is changed.

References

[1] Assoc. Professor Mark Blumenthal. *PHY2004W: PHYLAB 2, Experimental Physics Lab Session, The Magnetic FIeld of a Circular Coil: Induction and Inductance*. UCT, 2020. URL: `https://vula.uct.ac.za/access/content/group/21e34359-1eaa-44b3-a8d8-20da306e1181/PHYLAB%202%20-%20Online%20Manuals/Induct.pdf`.