Experimental Physics Lab
Skin Depth in Conductors

CNNAUA001

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Introduction and Theory

Skin depth is defined as the depth at which the field's amplitude is reduced by a factor of $1/e$ and is given by the equation:

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \tag{1}$$

When alternating current goes through a conductor, most of the current flows close to the surface of the conductor. This phenomenon is called the skin effect. The current does this as a changing electric field produces a magnetic field, which in turn generates eddy currents on the surface of the conductor, and they cause an addition of current along the surface and a cancellation of current at the centre of the conductor. This is a result of Maxwell's equations as we have changing electric and magnetic fields, and we have mini currents flowing in a circle generating magnetic fields as well. All predicted by Maxwell's equations. This is seen in the image below:

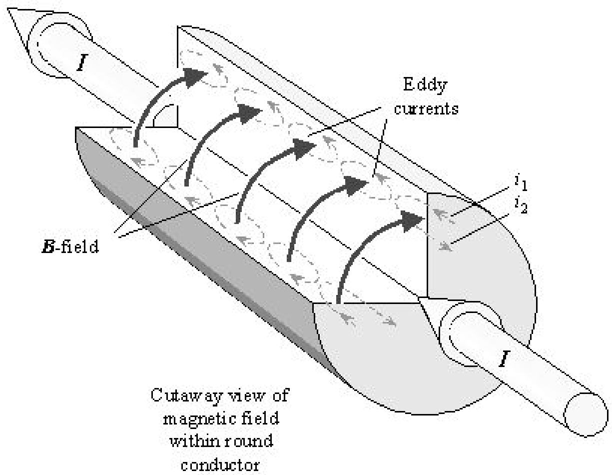

Figure 1: This image shows the Eddy currents generated from the magnetic field in the conductor generated from the changing current[2]

In this practical the skin effect will be simulated by using a copper tube. This replicates the skin effect found in solid conductors by means of Eddy currents being created and surface charge moving around. When the magnetic field is generated, and flows through the conductor, Eddy currents are set up, as seen in the image below:

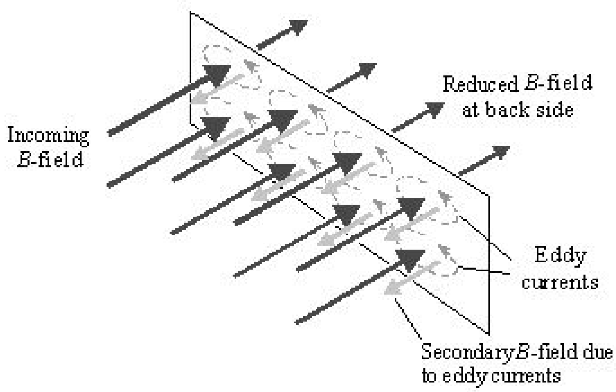

Figure 2: This image shows how Eddy currents are set up from magnetic fields[2]

These Eddy currents themselves creates magnetic fields as seen in Figure (2), which counteract the ones flowing through the conductor. In this experiment, we have a copper tube placed over an axial cable and a magnetic field created by two coils placed in the Helmholtz arrangement. The Eddy currents in the tube generates a magnetic field flowing in the opposite direction to the magnetic field generated by the main coils. As the axial cable is induced by the magnetic field, if there is a magnetic field that opposes the one generated by the main coils, the axial coil will feel less magnetic field than if the tube was not there and will therefore have a lower induced voltage. The higher the frequency, the stronger the magnetic field, which means the stronger the Eddy currents in the conductor are which means that it produces a stronger magnetic field which opposes the one from the coils which means that the induced voltage in the axial cable is lowered. This means that skin depth decreases in the conductor and as it decreases, the current flowing through the centre decreases. This effect of lowering the induced voltage is called screening. This practical will look at how skin depth is inversely proportional to the frequency of the signal as well as how screening takes place.

Aim

To investigate the effect of damping of electric fields or induced voltage due to a conductor in an alternating electromagnetic field, as well as to observe the skin depth in conductors.

Apparatus

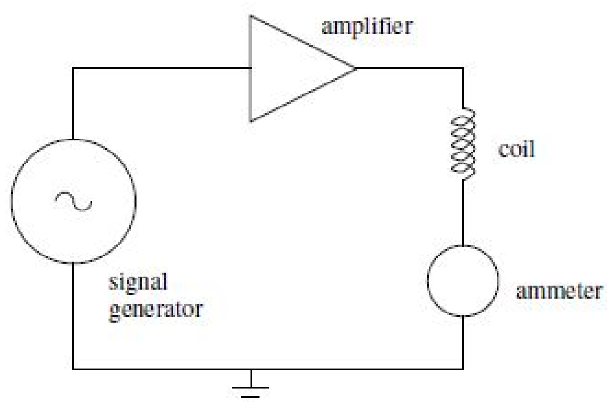

Figure 3: This image shows the setup of the practical

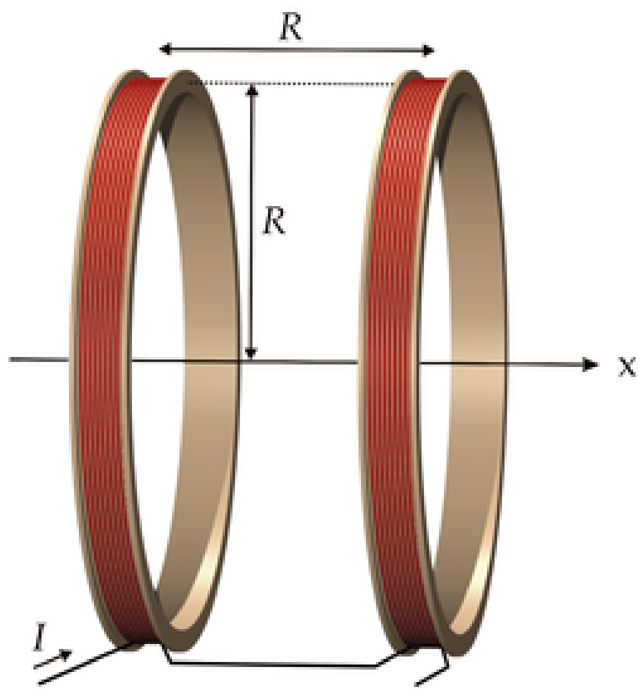

Figure 4: This figure shows the Helmholtz arrangement[3]

In the setup two large coils were set up in the Helmholtz arrangement as seen in Figure (4) with each coil having 80 turns. These are connected for a maximum magnetic field at the centre where the small or axial coil is placed, this will pick up the magnetic field and will be measured as the emf induced in the axial coil. The small coil has 200 turns. The coils are connected to an ammeter, which measures the rms current, to measure the current in the circuit which will be kept under 1A as if it exceeds this value the coil will heat up and the coil will be damaged. The signal generator is connected in series to the amplifier which is in series with the coils as seen in Figure (3). The signal generator will be kept fixed at 2 V_{pp}, as if it is not, a clipping of the sine wave will be seen. The frequency range is chosen between 100Hz and 5kHz. The amplifier is used to vary the current in the circuit. An oscilloscope is also set up to record the emf induced in the axial coil. A copper tube is used to put over the axial coil to shield the coil. This is the conductor in the practical.

Method

Getting Data

The circuit was set up as seen in Figure (3). A frequency was chosen from the signal generator in the range of 100Hz to 5kHz, then the current is adjusted with the amplifier in order to get a nice sinusoidal wave on the oscilloscope. The induced voltage was taken down when the copper tube was not over the axial coil and the induced voltage was taken when the copper tube was over the axial cable. This was done for a few frequencies and then these were put in a text file with each induced voltage for with the copper tube and without the copper tube corresponding to the frequency that they were measured at.

Curve_fit Program

This data was taken from the text file and using the Bootstrap Method and curve_fit program, the value for σ and its uncertainty was calculated. The uncertainty put into the uncertainty section in the curve_fit program was given by equation (12) which multiplied the values of the ratio of induced voltages with the shield to without the shield by $\sqrt{2}/50$ in order to get the uncertainties into an array which was passed through the curve_fit function. When the value for sigma was calculated the graph in Figure (6) was attained by plotting the values of the angular frequency against the skin depth equation given by equation (1).

Data

Conductivity values for copper and other metals

These values were obtained from a table on the website in reference[2].
Copper = 5.80×10^{7}S/m
Aluminium = 3.55×10^{7}S/m
Stainless Steel = 5.8×10^{6}S/m

Graphs

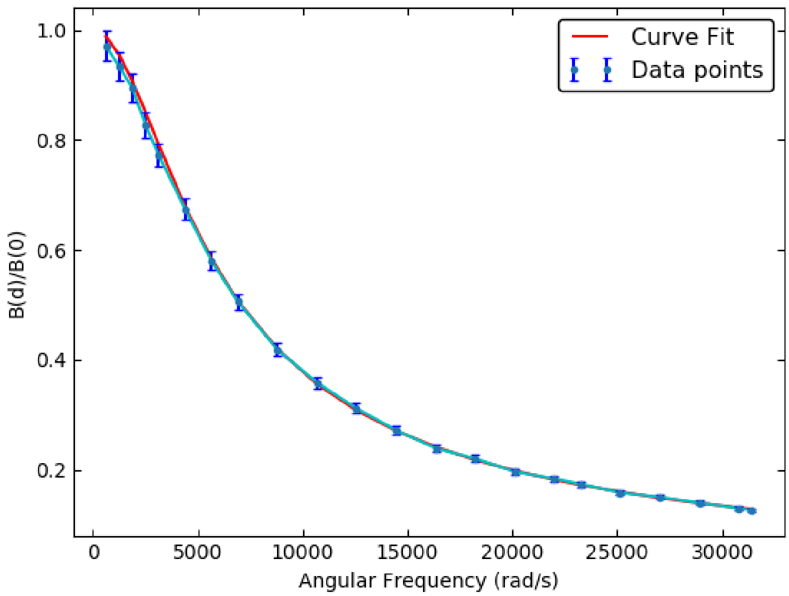

Figure 5: This graph shows the ratio of B(d) to B(0) vs the angular frequency

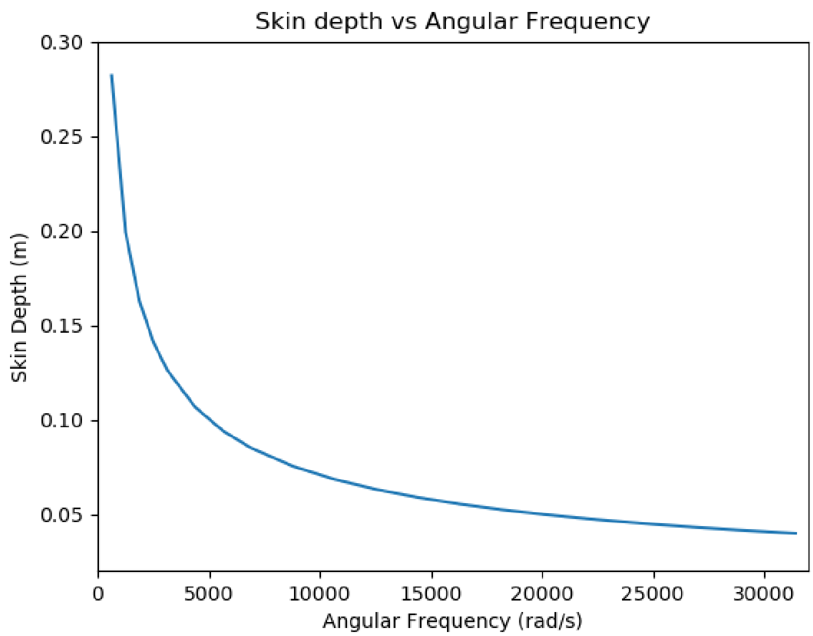

Figure 6: This graph shows the skin depth of the copper tube vs the angular frequency

Calculations

Showing that the ratio of the magnetic fields is equal to the ratio of the induced voltages by using Faraday's Law

Starting with Faraday's Law we have:

$$\epsilon = -N_a \frac{d}{dt} \int \mathbf{B} \bullet d\mathbf{A} \approx -N_a A \frac{dB}{dt} \tag{2}$$

where N_a is the number of turns in the coil experiencing the magnetic field, A is the cross-sectional area of the coil and B is the component of the magnetic field that is perpendicular to the area.
As we are using a coil, the magnetic field is given by:

$$B = \frac{\mu_0 N I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}} \tag{3}$$

Where z is the distance to the centre of the coil, N is the number of turns in the coil generating the magnetic field, μ_0 is the permeability of free space, a is the radius of the coil and I is given by $I = I_0 \cos \omega t$.

Incorporating this into equation (2):

$$\epsilon \approx -N_a A \frac{d}{dt}\Big(\frac{\mu_0 N(I_0 \cos \omega t)}{2} \frac{a^2}{(a^2+z^2)^{3/2}}\Big)$$
$$\therefore \epsilon \approx -N_a A \frac{\mu_0 N I_0}{2} \frac{a^2}{(a^2+z^2)^{3/2}} \frac{d}{dt}\Big(\cos \omega t\Big)$$
$$\therefore \epsilon \approx N_a A\omega \frac{\mu_0 N I_0}{2} \frac{a^2}{(a^2+z^2)^{3/2}} \sin \omega t$$

As we are only considering the amplitude of the induced voltage, we can 'get rid of' the sine function as anything in front of it, is the amplitude:

$$\epsilon = N_a A \omega B \tag{4}$$

As we are considering two different magnetic fields, we rewrite these as:
For the magnetic field experienced by the axial coil without the copper tube:

$$B(0) = \frac{\epsilon_0}{N_a A \omega} \tag{5}$$

and for the magnetic field experienced in the coil with the copper tube present will be of the form:

$$B(d) = \frac{\epsilon_d}{N_a A \omega} \tag{6}$$

Skin Depth

When a conducting body is put into a time varying magnetic field, induced currents are set up inside the conductor to reduce the magnetic field. This can be characterized by the following equation, which is used if there is a sheet of conducting material:

$$B(z) = B(0)e^{-z/\delta} \tag{7}$$

where z is the distance from the conductor surface, $B(0)$ is the magnetic field when the conductor is not present, and δ is called the skin depth and is defined by equation (1). As there is circular geometry given by the fact that it is a copper cylinder, the magnetic field can be rewritten as:

$$B(d) = B(0)\frac{1}{\sqrt{1+(Rd/\delta^2)^2}} \tag{8}$$

where R is the inner radius of the tube and $R+d$ is the outer radius, where $R = (20 \pm 0.1)$mm and $d = (1 \pm 0.1$mm. As we want the ratio of $B(d)$ to $B(0)$ we can rewrite equation (8):

$$\frac{B(d)}{B(0)} = \frac{1}{\sqrt{1+(Rd/\delta^2)^2}} \tag{9}$$

Now substituting equations (6) and (5) into this gives:

$$\frac{B(d)}{B(0)} = \frac{\frac{\epsilon_d}{N_a A\omega}}{\frac{\epsilon_0}{N_a A\omega}} = \frac{\epsilon_d}{\epsilon_0}$$

This equates the ratio of the magnetic fields to the ratio of the induced voltages. Putting this into equation (9):

$$\therefore \frac{\epsilon_d}{\epsilon_0} = \frac{1}{\sqrt{1+(Rd/\delta^2)^2}} \tag{10}$$

This will be used to find the conductivity σ. As we have the values for ϵ_d, the induced voltage when the conductor is present and we have ϵ_0 the induced voltage when the conductor is not present.

Uncertainty Analysis

Uncertainty calculation for curve_fit program

As we need to find the ratio $B(d)/B(0)$, they come with their own uncertainties as they were measured in volts from the oscilloscope which has an uncertainty of 2%, then the uncertainty equation used when multiplying is:

$$u(R) = R\sqrt{\Big(a\frac{u(A)}{A}\Big)^2 + \Big(b\frac{u(B)}{B}\Big)^2} \tag{11}$$

$$\therefore u\Big(\frac{B(d)}{B(0)}\Big) = \Big(\frac{B(d)}{B(0)}\Big)\sqrt{\Big(\frac{u(B(d))}{B(d)}\Big)^2 + \Big(\frac{u(B(0))}{B(0)}\Big)^2}$$

$$\therefore u\Big(\frac{B(d)}{B(0)}\Big) = \Big(\frac{B(d)}{B(0)}\Big)\sqrt{\Big(\frac{0.02\times B(d)}{B(d)}\Big)^2 + \Big(\frac{0.02\times B(0)}{B(0)}\Big)^2}$$

$$\therefore u\Big(\frac{B(d)}{B(0)}\Big) = \Big(\frac{B(d)}{B(0)}\Big)\sqrt{(0.02)^2 + (0.02)^2}$$

$$\therefore u\Big(\frac{B(d)}{B(0)}\Big) = \Big(\frac{B(d)}{B(0)}\Big)\Big(\frac{\sqrt{2}}{50}\Big) \tag{12}$$

Uncertainty of σ

There are two uncertainties that will be calculated, the one is given by the curve_fit function, by taking in the uncertainties of the ratio $B(d)/B(0)$ as shown in equation (12) and the other is given from rearranging equation (9) to get σ as the subject of the formula as there is uncertainty in the measurements of the radii.

Curve_fit program

The uncertainty generated by the curve_fit code is a Type A uncertainty as it is the standard deviation of all of the values of sigma calculated in the Bootstrap method. This value is:

$$u(\sigma_{code}) = 34833.45$$

Other uncertainties from rearranging equation (9)

Equation (9) is:

$$\begin{aligned}
\frac{B(d)}{B(0)} &= \frac{1}{\sqrt{1+(Rd/\delta^2)^2}} \\
\therefore \left(\frac{B(d)}{B(0)}\right)^2 &= \frac{1}{1+(Rd/\delta^2)^2} \\
\therefore \left(\frac{B(0)}{B(d)}\right)^2 &= 1+(Rd/\delta^2)^2 \\
\therefore \frac{Rd}{\delta^2} &= \left(\frac{B(0)}{B(d)}\right)^2 - 1 \\
\therefore \frac{1}{\delta^2} &= \frac{\left(\frac{B(0)}{B(d)}\right)^2 - 1}{Rd}
\end{aligned}$$

Substituting in δ:

$$\begin{aligned}
\left(\sqrt{\frac{\sigma\omega\mu}{2}}\right)^2 &= \frac{\left(\frac{B(0)}{B(d)}\right)^2 - 1}{Rd} \\
\therefore \frac{\sigma\omega\mu}{2} &= \frac{\left(\frac{B(0)}{B(d)}\right)^2 - 1}{Rd} \\
\therefore \sigma &= \frac{2\left(\left(\frac{B(0)}{B(d)}\right)^2 - 1\right)}{Rd\omega\mu} \\
\therefore \sigma &= \frac{2\left(\left(\frac{B(0)}{B(d)}\right)^2\right)}{Rd\omega\mu} - \frac{2}{Rd\omega\mu}
\end{aligned}$$

This shows that we need to get the uncertainty for the value Rd and then add this to the uncertainty given by the code. For the uncertainty of the radii:

$$u(R) = 0.1 \times 10^{-3} m$$

and

$$u(d) = 0.1 \times 10^{-3} m$$

When multiplying uncertainties the formula in equation (11). The value Rd is:

$$Rd = (20)(1) \times 10^{-6}$$
$$\therefore Rd = 20 \times 10^{-6} m^2$$

Substituting the values into the uncertainty equation gives:

$$u(Rd) = (Rd)\sqrt{\left(\frac{u(d)}{d}\right)^2 + \left(\frac{u(R)}{R}\right)^2}$$
$$\therefore u(Rd) = (20 \times 10^{-6})\sqrt{\left(\frac{0.1 \times 10^{-3}}{1 \times 10^{-3}}\right)^2 + \left(\frac{0.1 \times 10^{-3}}{20 \times 10^{-3}}\right)^2}$$
$$\therefore u(Rd) = 2.00 \times 10^{-6} m^2$$

When propagating these uncertainties, the equation below is used:

$$u(C) = \sqrt{(u(A))^2 + (u(B))^2} \tag{13}$$

Substituting the uncertainties in:

$$u(\sigma) = \sqrt{(u(Rd))^2 + (u(\sigma_{code}))^2}$$
$$\therefore u(\sigma) = \sqrt{(2.00 \times 10^{-6})^2 + (34833.45)^2}$$
$$\therefore u(\sigma) = 34833.45 S/m$$

It is seen that the uncertainty of the measuring of the radii do not play a large role in the uncertainty for sigma. This means that most of the uncertainty comes from the code.

Interpretation and Discussion

The value of σ

After running the code, and propagating the uncertainties, the value of the conductivity is:

$$\sigma = 1957.01 \times 10^4 \pm 3.48 \times 10^4 S/m$$

It is seen from the Data section that copper has a very high conductivity compared to the other metals, the actual value is: 5.80×10^7S/m. It is seen that the value calculated is close as it is of the same order of magnitude but is not the same number. The uncertainty is also very high, this is because, even though the bootstrap method was used, there was not enough data points for the curve to accurately navigate the path through the points.

Skin Depth

As seen in Figure (6) as well as in equation (1) Skin depth is inversely proportional to the frequency of the current, this means that the faster the changing current in the conductor, or from our experiment, the more the magnetic field changed or alternated the current flowing in the conductor, the less current went through the centre of the conductor, and more flowed along the edges, causing less induced voltage in the axial cable as a higher magnetic field was set up to counteract the one from the main coils. This means in conductors that it can be stated that if current alternates more, it sets up a stronger magnetic field on the inside of the conductor, leading to a lower skin depth and less current flowing through the centre of the conductor.

The Screening effect

It can be seen in Figure (5) that as frequency increases, the induced voltage decreases. This shows that in fact the conductor does shield the axial cable, and the reason for this is given in the introduction.

Why is this useful?

When there is a high frequency of the signal, there is a lower skin depth, this means that more current is flowing through less area, which means that there is a higher resistance, but as copper has a high conductivity, which is how easily it allows current to flow, it allows more current to pass through, these two almost counteract each other meaning that at high frequencies, more current will flow through copper than another metal with a lower conductivity. Which is useful when dealing with high frequencies of power stations.

Conclusion

In conclusion, it is seen that there is a shielding of the axial cable to the magnetic field as stated above. It is seen that as the frequency of the signal is increased the skin depth decreases, showing the inverse proportionality.

Exercise 1

If a DC current is supplied instead of an AC current, there will still be a magnetic field produced, however as this is not changing, we do not get the effect of Eddy currents in the conductor, as there are no Eddy currents present, there will be no current on the surface of the conductor, meaning that there will be no other magnetic field produced which can counteract the one generated from the coils.

References

[1] Marc Grenier. *Back to Basics-Skin Depth.* URL: `https://www.eddyfi.com/en/blog/back-to-basics--skin-depth`. (accessed:18.10.2020).

[2] Howard Johnson and Martin Graham. *Skin Effect.* URL: https://flylib.com/books/en/1.389.1/skin_effect.html#:~:text=To%5C%20understand%5C%20the%5C%20skin%5C%20effect,operate%5C%20within%5C%20a%5C%20solid%5C%20conductor.&text=In%5C%20a%5C%20good%5C%20conductor%5C%20the,material%5C%2C%5C%20shown%5C%20as%5C%20dotted%5C%20lines.. (accessed:18.10.2020).

[3] Wikipedia. *Helmholtz coil.* URL: https://en.wikipedia.org/wiki/Helmholtz_coil. (accessed:18.10.2020).