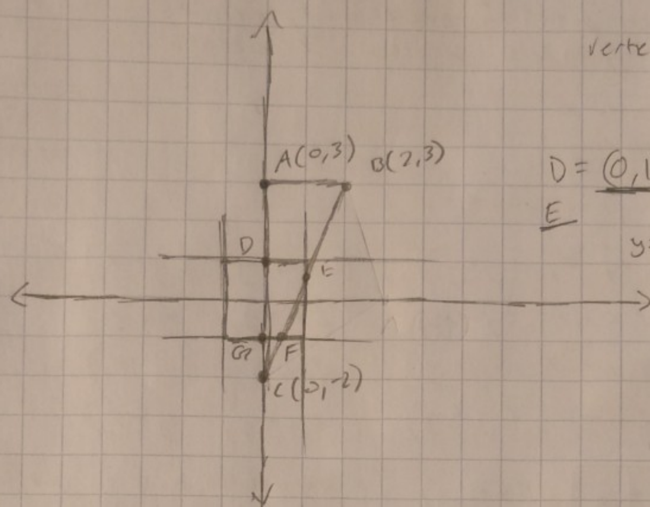
HW3

1)



vertex outcodes?

← found by inspection

$D = (0,1)$

E

$y = mx - 2$ ← eqn of line $\overline{BC}$

$m = \frac{-2-3}{0-2} = \frac{-5}{-2} = \frac{5}{2}$

$y = \frac{5}{2}x - 2$

E when $x = 1$

$y = \frac{5}{2} - \frac{4}{2}$

$y = \frac{1}{2}$

$E = \left(1, \frac{1}{2}\right)$

F = when $\overline{BC}$ $y = -1$

$-1 = \frac{5}{2}x - 2$

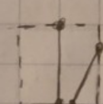
$1 = \frac{5}{2}x$

$x = \frac{2}{5}$

$F = \left(\frac{2}{5}, -1\right)$

$G = (0,-1)$ by inspection

Final clip:



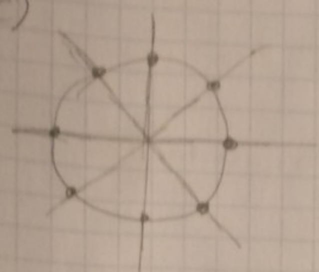
2) a) $(x - x_0)^2 + (y - y_0)^2 = r^2$

$F(x,y) = x^2 + y^2 - R^2 = 0$

b) $y = \pm\sqrt{R^2 - x^2}$

```
for ( x from x1 to x2 )
begin
    setPixel ( x, floor(y));
    y = sqrt(R^2 - x^2)
end
```

c)



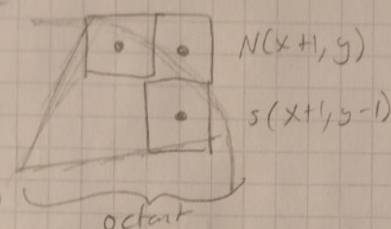
For each pixel in one octant, draw corresponding pixels in other 7 octants

Cx = center x
Cy = center y

The 8 points are

$(cx+x,\ cy+y)$ $(cx+y,\ cy+x)$
$(cx+x,\ cy-y)$ $(cx-y,\ cy+x)$
$(cx-x,\ cy+y)$ $(cx+y,\ cy-x)$
$(cx-x,\ cy-y)$ $(cx-y,\ cy-x)$

Using Bresenham, choose either $N(x+1,y)$ or $S(x+1,y-1)$. Make choice based on the one with less distance from desired point (Error)



We use a decision variable d, which is the sum of the two errors $E(N)$ and $E(S)$. If d is $<= 0$, choose N as the pixel to display. Otherwise, choose S.

Error from N is $E(N) = (x+1)^2 + y^2 - r^2$
" " S is $E(S) = (x+1)^2 + (y-1)^2 - r^2$

$$\therefore d_{initial} = 2(x_i+1)^2 + (y_i)^2 + (y_i-1)^2 - 2r^2$$

To find the next d_{i+1}, $d_{i+1} = 2(x_i+2)^2 + (y_{i+1})^2 + (y_{i+1}-1)^2 - 2r^2$

$$d_{i+1} - d_i = 2((x_i+2)^2 - (x_i+1)^2) + (y_{i+1}^2 - y_i^2) + ((y_{i+1}-1)^2 + (y_i-1)^2)$$

$$d_{i+1} = d_i + 2((x_i+2+x_i+1)(x_i+2-x_i-1)) + ((y_{i+1}+y_i)(y_{i+1}-y_i)) + ((y_{i+1}-1+y_i-1)(y_{i+1}-1-y_i+1))$$

$$d_{i+1} = d_i + 2(2x_i+3) + ((y_{i+1}+y_i)(y_{i+1}-y_i)) + ((y_{i+1}-1+y_i-1)(y_{i+1}-1-y_i+1))$$

if $d_i \leq 0$,

$x_{i+1} = x_i + 1$ $y_{i+1} = y_i$

$$\therefore d_{i+1} = d_i + 2(2x_i+3) + ((y_{i+1}+y_i)(0)) + ((y_i-1+y_i-1)(0))$$

$$d_{i+1} = d_i + 4x_i + 6$$

if $d_i > 0$

$$d_{i+1} = d_i + 4x_i + 6 + ((2y_i-1)(-1)) + ((2y_i-3)(-1))$$

$$d_{i+1} = d_i + 4x_i + 6 - 2y_i - 2y_i + 4$$

$$d_{i+1} = d_i + 4(x_i - y_i) + 10$$

d_0 is found if $x=0$, $y=r$

$d_0 = 2 + r^2 + (r-1)^2 - 2r^2$

$d_0 = 2 + r^2 + r^2 + 1 - 2r - 2r^2$

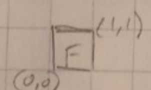
$d_0 = 3 - 2r$

<u>Algorithm</u> R = radius

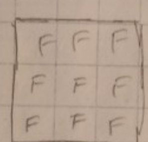
```
x = 0
Y = R
D = 3 - 2R
while (x < Y)
    DrawCircle (X_center, Y_center, x, y)
    x = x + 1
    if (D < 0) {
        D = D + 4x + 6
    else
        y = Y - 1
        D = D + 4 (x - Y) + 10
    }
    DrawCircle (Xc, Yc, X, Y)
    X++;
```

DrawCircle() draws the 8 points found by symmetry

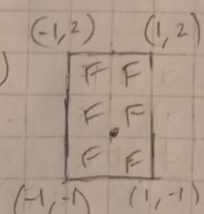
3)



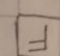
a)



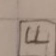
b)



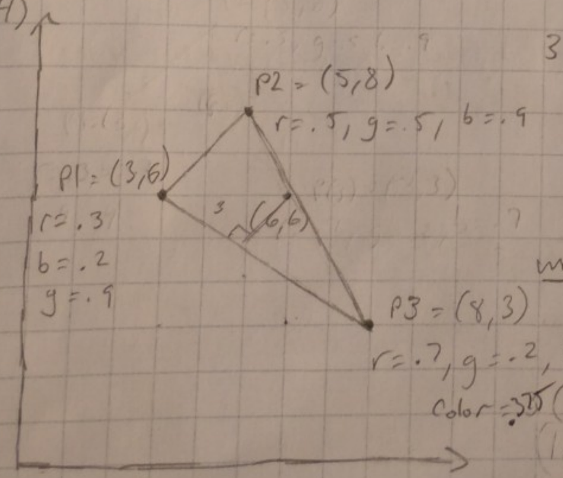
c)



d)



4)



3 triangles: $A_{13P} = 4.5u^2$

$A_{23P} = .5u^2$

$A_{12P} = 3u^2$

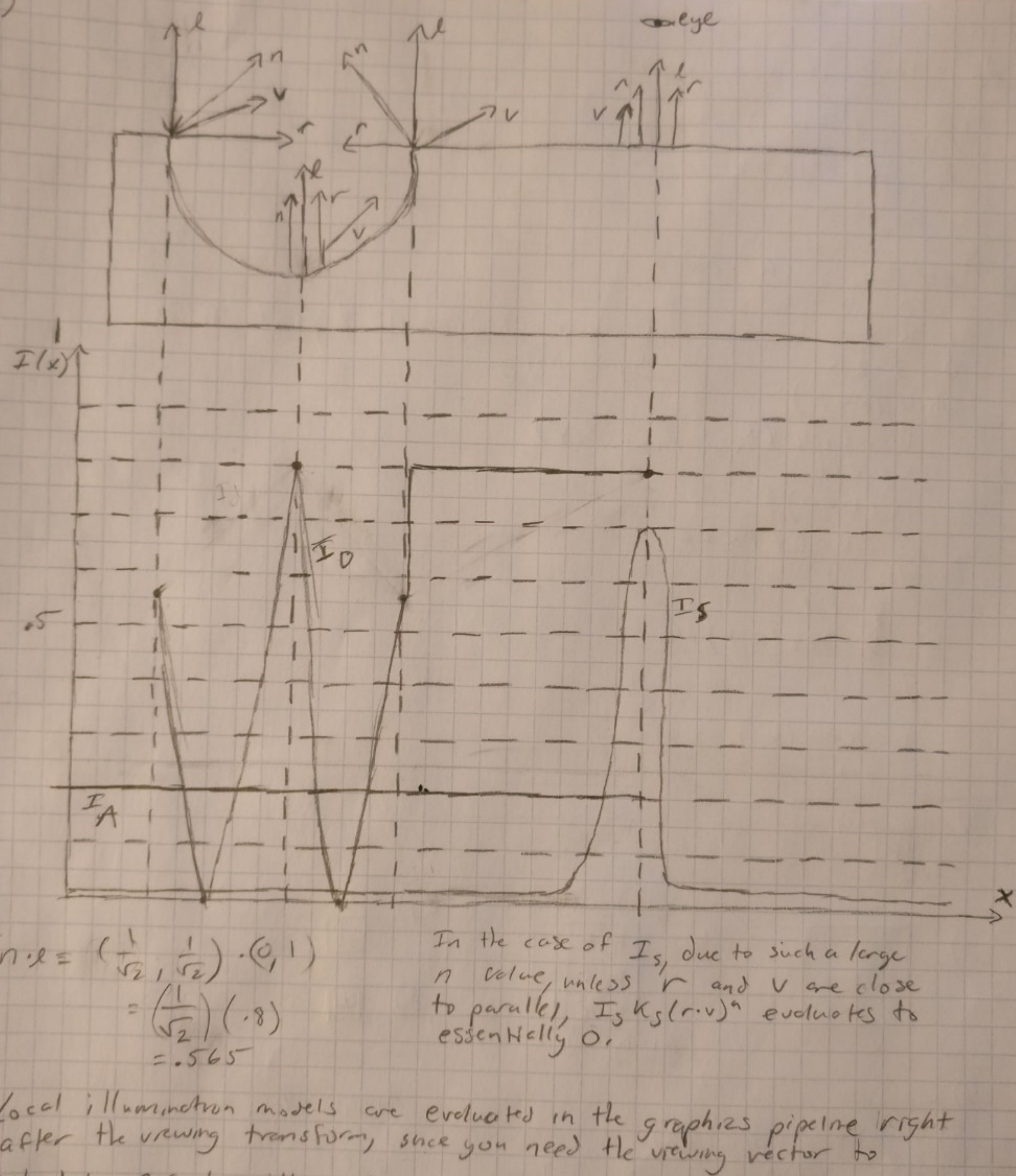
Total area = 8

<u>weights</u> $\boxed{\begin{array}{l} u = .5625 \\ v = .0625 \\ w = .375 \end{array}}$

Color = .375(.5, .5, .9) + .0625(.7, .2, .7) + .5625(.3, .9, .2)

(.4, .70625, .49375)

5)



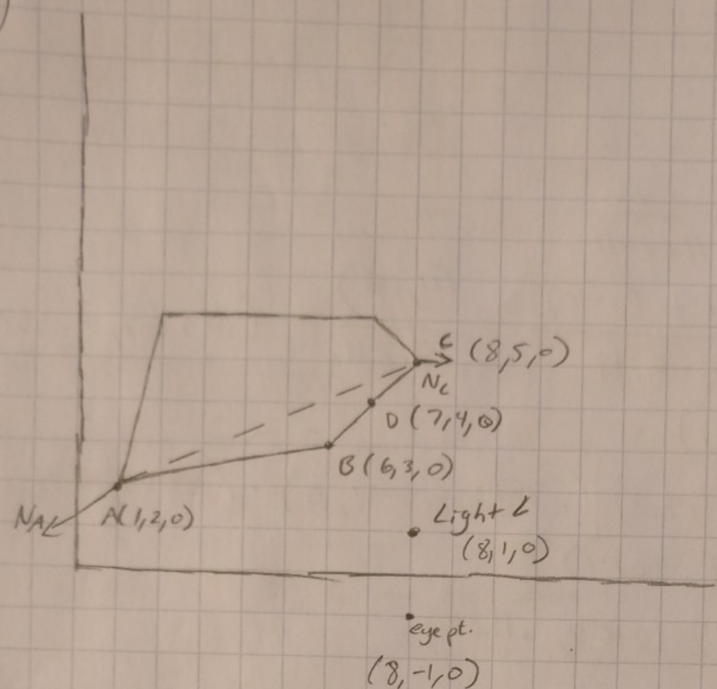
$I_d = n \cdot l = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \cdot (0, 1)$
$= \left(\frac{1}{\sqrt{2}}\right)(.8)$
$= .565$

In the case of I_s, due to such a large n value, unless r and v are close to parallel, $I_s k_s (r \cdot v)^n$ evaluates to essentially 0.

b) Local illumination models are evaluated in the graphics pipeline right after the viewing transform, since you need the viewing vector to calculate specular illumination.

(C) cont.

6) a)



$\overline{AC} = \frac{5-2}{8-1} = \frac{3}{7}$

perpendicular slope $= -\frac{7}{3}$

$2 = \frac{3}{7}(1) + b$

$\frac{11}{7} = b$

$y = \frac{3}{7}x + \frac{11}{7}$

$3 = -\frac{7}{3}(6) + b$

$17 = b$

$y = -\frac{7}{3}x + 17$

$\frac{3}{7}x + \frac{11}{7} = -\frac{7}{3}x + 17$

$\frac{58}{21}x = \frac{108}{7}$

$x = \frac{162}{29}$

$y = -\frac{7}{3}\left(\frac{162}{29}\right) + 17$

$y = \frac{115}{29}$

$B' = \left(\frac{162}{29}, \frac{115}{29}\right)$

$N_A = \frac{(-1,-1)}{\sqrt{2}}$

$= \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$N_B = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\left(\frac{2.6261}{7.6158}\right) + (1,0)\left(\frac{4.9896}{7.6158}\right)$

$= (-.2434, -.2434) + (.6557, 0)$

$= (.4118, -.2434)$

Normalize: $\left(\frac{.4118}{.4781}, \frac{-.2434}{.4781}\right)$

$N_B = (.86131, -.5091)$

(C)ont

b) $B(6,3,0)$

$C(8,5,0)$

$D(7,4,0)$

(B) Halfway vector $H = \frac{L+V}{\|L+V\|}$

$$H = \frac{(2,-2)+(2,-4)}{\|(4,-6)\|} = \frac{(4,-6)}{7.211} = (.5547, -.8321)$$

$N = (.8613, -.5091)$

$$I_S = (1,1,.9)(1,1,1)\left[(.8613,-.5091,0)\cdot(.5547,-.8321)\right]^{20}$$

$$=(1,1,.9)(.9014)^{20}$$

$$I_{S@B} = (.1254, .1254, .1128)$$

$$I_A = (.1,.2,.1)(.1,.1,.1)$$

$$=(.01,.02,.01)$$

$$I_D = I_L k_d (n \cdot L)$$

$$=(1,1,.9)(.3,.8,.9)\left[(.8613,-.5091)\cdot(.7071,-.7071)\right]$$

$$(.3,.8,.81)[.96902]$$

$$=(.2907,.7752,.78491)$$

Total $I = I_S + I_A + I_D = (.4261, .9206, .90771)$ ← cap at 1

(C) $L = (0,-4)$
$V = (0,-6)$

$$H = \frac{(0,-4)+(0,-6)}{\|0,-10\|} = \frac{(0,-10)}{10} = (0,-1)$$

~~$N = (.8613, -.5091)$~~

$$I_S = (1,1,.9)(1,1,1)\left[(0,-1,0)\cdot(.1,0,0)\right]^{20}$$

$$=(0,0,0)$$

(C) Cont.

$I_A = (.01, .02, .01)$

$I_D = I_L k_d (n \cdot L)$

$= (1, 1, .9)(.3, .8, .9)[(1, 0, 0) \cdot (0, -1, 0)]$

$= (0, 0, 0)$

Total Illumination $= (.01, .02, .01)$

(D) Illumination for D matches B or C, depending on implementation.

c) Illumination at B, C are the same as in part (B). However, in Gouraud shading, we average the illuminations at B and C, the corners, to interpolate. We get

$\left.\begin{array}{l} I_{S@B} = (.1254, .1254, .1128) \\ I_{S@C} = (0, 0, 0) \end{array}\right\} I_{S@D} = (.0627, .0627, .0564)$

$\left.\begin{array}{l} I_{A@B} = (.01, .02, .01) \\ I_{A@C} = (.01, .02, .01) \end{array}\right\} I_{A@D} = (.01, .02, .01)$

$\left.\begin{array}{l} I_{D@B} = (.2907, .7752, .78491) \\ I_{D@C} = (0, 0, 0) \end{array}\right\} I_{D@D} = (.1454, .3876, .39246)$

Total illumination @ D in Gouraud shading is

$(.2081, .4503, .47886)$

d) In Phong, the illumination at points B and C are still the same, but at point D we interpolate corner normals.

$N_B = (.8613, -.5091, 0)$

$N_C = (1, 0, 0)$

$N_D = (.93065, -.25455, 0)$

Normalized $N_D = (.96456, -.26383, 0)$

Phong #6d cont

$I_{s@D} =$ $L = (1, -3, 0)$

$V = (1, -5, 0)$

$$H = \frac{(2, -8, 0)}{8.246} = (.24254, -.97014, 0)$$

$N = (.96456, -.26383, 0)$

$$I_S = (1, 1, .9)\left[(.24254, -.97014, 0) \cdot (.96456, -.26383\ \ )\right]^{20}$$

$$(6.3399 \times 10^{-7}, 6.3399 \times 10^{-7}, 5.7059 \times 10^{-7})$$

$I_A = (.01, .02, .01)$

$$I_D = (.3, .8, .81)\left[(.96456, -.26383, 0) \cdot \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}, 0\right)\right]$$

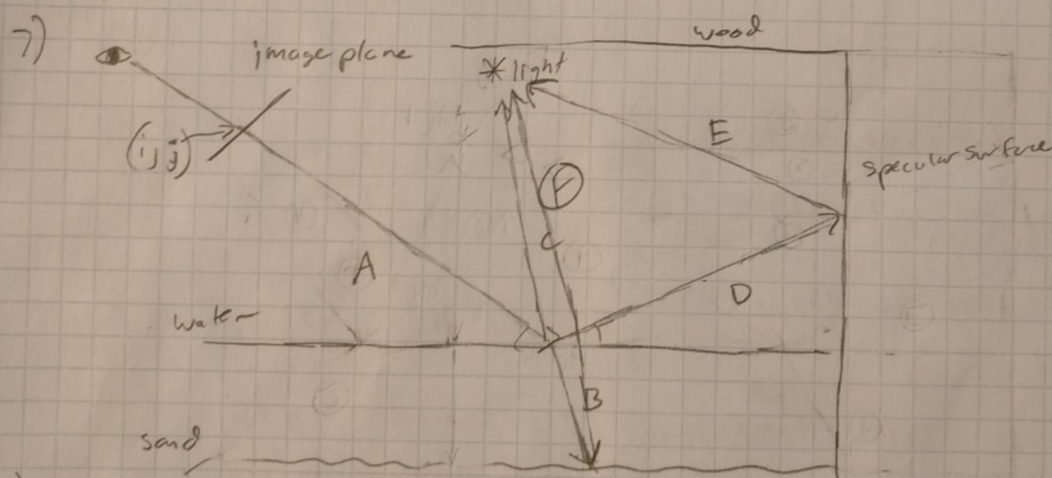
$$(.3, .8, .81)[.5553]$$

$$= (.1666, .4442, .4498)$$

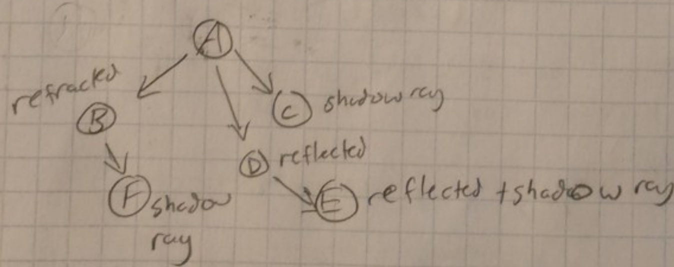
Total illumination at D

$$= (.1766, .4642, .4598)$$

7)



b)



#8

a) $\rho(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$\rho_0 = a_0$

b) $\rho'(t) = a_1 + 2a_2 t + 3a_3 t^2$

$\rho''(t) = 2a_2 + 6a_3 t$

$T_0 = a_1$

$A_0 = 2a_2$

$\rho_1 = a_0 + a_1 + a_2 + a_3$

c) $$\begin{bmatrix} \rho_0 \\ \rho_1 \\ T_0 \\ A_0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\text{Basis} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}^{-1}$$